Title: Dynamic Tax Evasion with Habit Formation in Consumption

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Abstract:
We model the optimal intertemporal decision of an agent who chooses tax evasion and consumption, over an infinite lifetime horizon, where consumption is driven by habits. We find that: (i) tax evaders reduce consumption in the early stages of habit accumulation and increase it over time; (ii) habit formation has a dampening effect on tax evasion; (iii) neglecting tax evasion may lead to habit overestimation; (iv) the effect of the tax rate on tax evasion is ambiguous; and (v) heavy fines are more efficient than frequent controls in reducing tax evasion.

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I Introduction

Tax evasion is a severe plague in many countries. Estimates show that intentional under-reporting of income in the US is about 18-19 percent of reported income in the US, leading to a tax gap of about $500 billion per year (Feige and Cebula, 2012). In Europe the level of tax evasion is about 20 percent of GDP, which accounts for a potential loss of about €1 trillion per year (Murphy, 2014; Buehn and Schneider, 2016). The shadow economy, the size of which is closely related to tax evasion, and tax evasion both reflect weak enforcement of fiscal rules (Schneider, 2005; Alm and Embaye, 2013).

The literature has explored several avenues to identify motivations for tax evasion by enriching traditional portfolio models with behavioural considerations and social norms (see Alm, 2012 for an extensive review), but little attention has been paid to the age-dependent structure of tax evasion. To show this, in Figure 1 we plot the age-dependent paths of both the shadow economy and the aggregate private consumption in several high and upper-middle-income countries around the world (Alm and Embaye, 2013).¹

The size of the shadow economy decreases with the population’s median age, more rapidly at until the median age reaches about 35 and then slowing as it approaches a steady state. Such a connection between the dynamic path of household consumption and the relative weight of the shadow economy cannot be explained by the recent literature on dynamic tax evasion (Wen-Zhung and Yang, 2001; Niepelt, 2005; Dzhumashev and Gahramanov, 2011; Levaggi and Menoncin, 2012, 2013; Bernasconi et al., 2014). In this paper we argue that the decision to evade taxes is dynamically embedded with consumption decisions, which we assume are driven by consumption habits. Those habits make more strict the usual trade-off between evasion and the risk of being caught. In fact, habit implies a form of time non-separability of preferences which is highly intuitive: present utility depends positively on current consumption and negatively on a time-varying sub-

¹We use the subset of high and upper-middle-income countries because data on consumption for low income countries, also considered in Alm and Embaye (2013), are less reliable; the overall dynamics of evasion is even more pronounced in the whole set of countries than in the subset we use.
Figure 1: Shadow economy and consumption versus population age for 49 high-income and upper-middle-income countries. The data source are Alm and Embaye (2013) for the shadow economy and the UN database for household consumption and median age.

![Graphs showing shadow economy and consumption versus population age](image)

...sistence level which is a positive function of the consumer’s past consumption (the so-called “intrinsic habit” as in Constantinides, 1990; Chapman, 1998; Rozen, 2010; Chetty and Szeidl, 2016). The economic literature first introduced habit formation in the early 1970s to analyse several issues in economics and finance.²

We present a closed form solution for the optimal consumption and evasion paths in the presence of habit using a dynamic framework with an infinite horizon and whose state variable is the endogenous dynamic capital accumulation. A representative consumer gets utility in the form of a Hyperbolic Absolute Risk Aversion (HARA) from consumption...

²See Havranek et al. (2017); Thimme (2017) for a review.
that exceeds a time dependent threshold given by “intrinsinc” habit. Capital is accumulated through savings (i.e., income that is not consumed) and the evaded taxes. The evasion may be audited with a probability driven by a Poisson jump process; in case of detection, the consumer must pay a fine that is an affine transformation of the evaded income.

Our paper makes several contributions to the existing literature. Because of the presence of habits, evasion and consumption follow specific age-dependent paths, while the level of capital accumulation varies over time, either with or without tax evasion. If tax evasion is not taken into account, the effect of habits in consumption is overestimated. In our setting the attitude toward risk changes over time according to the speed with which the consumer’s habit accumulates. If the process of habit formation has a sufficiently long memory, the consumer becomes increasingly less willing to take the risk associated with tax evasion while habit accumulates. We show that these paths are consistent with actual data (Figure 1) and with the empirical literature on the demographic factors that affect tax evasion (Jackson and Milliron, 1986; Andreoni et al., 1998; Richardson and Sawyer, 2001). The reaction to a change in tax rate depends on the ratio of habit to income; this finding provides a possible explanation for Yitzhaki (1974)’s paradox. In particular, the presence of consumption habits allows our model to be consistent with both signs in the derivative of optimal evasion with respect to the tax rate. Thus, we reconcile theory with empirical evidence.

Finally, when the fine is proportional either to the evaded income or to the evaded tax, the elasticity of evasion with respect to a change in the fine itself is higher than the elasticity computed with respect to the frequency of audits. This suggests that governments could control evasion more effectively by increasing fines rather than by making more frequent audits.

Some studies have introduced persistent memory in tax evasion through reference dependent preferences (Yaniv, 1999; Bernasconi and Zanardi, 2004; Dhami and Al-Nowaihi, 2007), but they all rely on a static framework, an approach that has been questioned re-
cently (e.g., Piolatto and Rablen, 2017). Here, we argue that such a problem may be more conveniently cast in a dynamic framework with consumption habits.

The paper is organised as follows. Section II presents the model with tax evasion, capital accumulation, and preferences with habit formation. Section III derives optimal consumption and evasion both in closed form. Section IV presents simulations and Section V concludes. Some technical proofs are gathered in appendices.

II The model

Over an infinite horizon, we model the behaviour of a representative consumer whose income is produced by capital through a production function. In each period, the consumer decides the proportion of income to consume and to evade in order to maximise his expected inter-temporal HARA utility with habit.

Accumulation of capital

Over the period \([t_0, \infty)\), total income \(y_t\) is produced by using capital \(k_t\) through the deterministic production function \(y_t = A k_t\), where \(A\) measures total factor productivity. It is reasonable to assume \(0 < A < 1\) since a given amount of capital cannot produce an income greater than the capital itself. Although \(A\) is deterministic and constant, the process of capital accumulation becomes endogenous through the individual’s consumption choices \((c_t)\).

The government levies a proportional tax \(0 \leq \tau \leq 1\) on income. Without evasion, the net change in capital becomes:

\[
d k_t = (1 - \tau) y_t - c_t \ dt.
\]  (1)

Capital accumulation may be improved if the agent hides a fraction \(\varepsilon_t \in [0, 1]\) of his
income \( y_t \). If evasion is detected, a fine \( \eta (\tau) \) must be paid on the evaded income (where \( \eta (\tau) \) is a non-decreasing function of \( \tau \)). Thus, the total fine is:

\[
\eta (\tau) e_t y_t.
\] (2)

The specification of \( \eta (\tau) \) allows several fine regimes to be considered: (i) \( \eta (\tau) = \eta_0 \), where the fine is applied on evaded income as in Allingham and Sandmo (1972); (ii) \( \eta (\tau) = \eta_1 \tau \), where the fine is applied on evaded tax as in Yitzhaki (1974); and (iii) \( \eta (\tau) = \eta_0 + \eta_1 \tau \), where the fine is a linear combination of the two previous cases.

When the fraction \( e_t \) of income is evaded, the tax bill that is not paid \( (\tau e_t y_t) \) is added to the capital accumulation process. Therefore, the capital accumulation is an endogenous process since it depends on the optimal choice of two control variables. The expected change in capital becomes:

\[
dk_t = ((1 - \tau + \tau e_t) y_t - c_t) dt.
\]

As in Wen-Zhung and Yang (2001) and Dzhumashev and Gahramanov (2011), evasion introduces risk in Equation (1) because a fine must be paid, but only if evasion is discovered. Therefore, a stochastic term must be added to the differential equation that describes capital accumulation.

We model auditing as a Poisson jump process \( d\Pi_t \) that can be thought of as a limit of a binomial model whose value is either 1 (i.e. there is a fiscal audit) with probability \( \lambda dt \) or 0 (i.e. there is no fiscal audit) with probability \( 1 - \lambda dt \) (Levaggi and Menoncin, 2012, 2013). Thus, when there is an audit, the Poisson jump process creates a sudden decrease in the capital level (because of the payment made for fines and evaded taxes). The first
two moments of the distribution are:

\[ \mathbb{E}_t [d\Pi_t] = \lambda dt, \]  \hspace{1cm} (3)

\[ \mathbb{V}_t [d\Pi_t] = \lambda dt, \]  \hspace{1cm} (4)

where \( \mathbb{E}_t [\cdot] \) and \( \mathbb{V}_t [\cdot] \) are the expected value and the variance operators, respectively, conditional on the information set at time \( t \), and \( \lambda \in [0, \infty) \) is the “intensity” of the process which determines the frequency of audits within a time interval. The use of a continuous time Poisson jump process whose jumps occur in discrete time implies that, during a finite time period, audit may occur only a finite number of times.

The parameter \( \lambda \) determines the probability of the audit itself. If \( N_t \) is the number of audits that have taken place until time \( t \), the probability of having \( n \) audits is \( \mathbb{P} (N_t = n) = e^{-\lambda t} (\lambda t)^n / n! \). Accordingly, the probability of having at least one audit is:

\[ \mathbb{P} (N_t \geq 1) = 1 - \mathbb{P} (N_t = 0) = 1 - e^{-\lambda t}. \]  \hspace{1cm} (5)

Finally, with tax evasion, the stochastic process of capital accumulation can be written as:

\[ dk_t = ((1 - \tau + \tau e_t) y_t - c_t) dt - \eta (\tau) e_t y_t d\Pi_t, \]  \hspace{1cm} (6)

whose expected value is:

\[ \mathbb{E}_t [dk_t] = ((1 - \tau + \tau e_t) y_t - c_t) dt - \mathbb{E}_t [\eta (\tau) e_t y_t d\Pi_t] \]

\[ = ((1 - \tau + (\tau - \eta (\tau) \lambda) e_t) y_t - c_t) dt, \]  \hspace{1cm} (7)
and whose variance is:

$$\mathbb{V}t [dk_t] = \eta (\tau)^2 e_t^2 y_t^2 \lambda dt.$$

(8)

Comparing Equation (1) with Equation (7) shows that evasion is, on average, profitable if $\tau - \eta (\tau) \lambda > 0$, so from now on we will assume that $\frac{\eta (\tau) \lambda}{\tau} < 1$, that is we consider only interior solutions.

In this framework, we take into account neither the cost of evadings differing from the fine nor any cost for the government to perform audits.

**Consumer preferences**

Consumer preferences are represented by a HARA utility function with a minimum level of consumption ($h_t$) given by a weighted mean of the past optimal consumption (i.e. an intrinsic habit). Formally, the instantaneous utility function is:

$$U (c_t) = \frac{(c_t - h_t)^{1-\delta}}{1-\delta},$$

(9)

in which $\delta > 1$ measures the consumer’s risk aversion and $h_t$ is the solution to the differential equation:

$$dh_t = (\alpha c_t - \beta h_t) \, dt,$$

(10)

with an exogenous initial value $h_0$, sometimes called the subsistence consumption level, which is assumed to be lower than the initial income ($h_0 < y_0$).

Equation (9) entails addiction in consumption, since consumption over time must always remain higher than the habits. Given $h_0$, the (unique) solution to (10) is:

$$h_t = h_0 e^{-\beta t} + \alpha \int_0^t c_se^{-\beta (t-s)} ds.$$

(11)
The model in Equations (9) and (10) is sometimes called “addictive” habit formation (see e.g. Chapman 1998, p. 1224) with intrinsic habit, since habit depends on past consumption levels (Thimme, 2017).

The utility function (Equation 9) is well defined only for \(c_t > h_t\). This function works like a reference dependent utility where an infinite penalty is associated with downward deviation from the habit. Thus, it is optimal to choose evasion at a level that may imply a total fine that does not prevent the optimal consumption from reaching at least the habit level. In fact, in our model, the agent decides the total amount of risk through the level of evasion, so we can argue that the risk is fully endogenous.\(^3\)

The parameter \(b\) measures habit persistence (or memory) such that the higher the \(b\) the lower the weight of the past consumption on the present habit. In particular, for \(b = 0\), past consumption levels are all taken into account with the same weight, independent of the time they were available. On the other hand, if \(b \to \infty\), there is no habit formation and the utility function is traced back to the Constant Relative Risk Aversion case (with a subsistence consumption level equal to zero).

The parameter \(a\) measures how the weighted sum of past consumption contributes to create the minimum consumption level \(h_t\), and could be interpreted as the intensity at which past consumption becomes a habit.

The habit \(h_t\) is a positive function of \(a\) and a negative function of \(b\), but the two parameters should be interpreted together and we should expect \(\beta \geq \alpha \geq 0\) (e.g. Constantinides, 1990).

Some special cases require highlighting: (i) when \(a = 0\), \(h_t\) coincides with \(h_0\) reducing over time at rate \(\beta\), so that there is no true habit formation; (ii) when \(\beta = \alpha = 0\), \(h_0\) can be interpreted as the classical subsistence consumption level of the linear expenditure models

\(^3\)Standard models of habit formation are not equivalent to models of reference-dependent preferences. A major difference is that, whereas the restriction \(c_t \geq h_t\) is common in the habit literature, in reference-dependent models the case \(c_t \leq h_t\) corresponds to a situation in which the consumer experiences a psychological loss and it falls in a range where utility is convex. Such a case may occur, for example, when taxpayers are myopic, in which case habit formation may require an increasing level of tax evasion (Gagnonossi degl’Innocenti and Rablen, 2018).
(see Levaggi and Menoncin, 2013); (iii) when $\alpha = 0$ and $\beta < 0$, there is no habit formation ($\alpha = 0$), but there is a subsistence consumption level that is increasing over time. If, for instance, we assume that $\beta$ is a (constant) interest or inflation rate, the consumer will want to increase his minimum consumption over time to take into account a kind of opportunity cost of consumption.

Finally, the HARA specification entails decreasing absolute risk aversion in the difference between instantaneous consumption and habit. In particular, with $z_t \equiv c_t - h_t$, the absolute risk aversion is $-\frac{u''}{u'} = \frac{\delta}{z_t}$, which is decreasing with respect to $z_t$. This, of course, implies that the closer the consumption is to the habit, the higher the consumer’s risk aversion.

## III Optimal consumption and evasion

The consumer optimises his inter-temporal utility by solving the following problem:

$$\max_{\{c_s,e_s\}\in [0,\infty]} E_0 \left[ \int_0^\infty \frac{(c_s - h_s)^{1-\delta}}{1-\delta} e^{-\rho s} ds \right],$$

(12)

where $\rho > 0$ is a subjective discount rate and the state variables $k_t$ and $h_t$ solve Equations (6) and (10), respectively. The solution to Problem (12) is shown in the Proposition 1.

**Proposition 1.** The optimal consumption and evasion that solve Problem (12), given the capital accumulation in Equation (6) and the habit formation in Equation (10), are:

$$e^{*}_t = \frac{1}{\eta A} \left( 1 - \frac{h_t}{k_t} \right) \left( \frac{1}{1 - \alpha} - \frac{1}{A + \beta} \right) \left( 1 - \frac{\lambda \eta}{\tau} \right)^{\frac{1}{\delta}},$$

(13)

$$c^{*}_t = h_t + \frac{k_t ((1 - \tau) A + \beta - \alpha)}{(1 - \tau) A + \beta} \left( \frac{\rho + \lambda}{\delta} + \frac{\delta - 1}{\delta} \left( \frac{\tau}{\eta} + (1 - \tau) A \right) - \frac{\tau}{\eta} \left( \frac{\lambda \eta}{\tau} \right)^{\frac{1}{\delta}} \right).$$

(14)

**Proof.** See Appendix B.

□

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Equation (13) measures the fraction of income that is concealed from the tax authority. An interior solution $e_t^*$ exists when specific restrictions on the parameters are satisfied. Without habit (i.e. with $h_t = 0$ for any $t$), the condition $\frac{\eta(t)\lambda}{\tau} < 1$ is sufficient for having an interior solution to the optimal evasion. For the model with habits, we add

$$1 - \frac{\eta A}{1 - \left(\frac{\lambda \eta}{\tau}\right)^\frac{1}{2}} < \frac{h_t}{k_t (1 - \tau) A + \beta - \alpha} < 1,$$

which, when $(1 - \tau) A + \beta - \alpha > 0$, simplifies to

$$((1 - \tau) A + \beta - \alpha) \left(1 - \frac{\eta A}{1 - \left(\frac{\lambda \eta}{\tau}\right)^\frac{1}{2}}\right) < \frac{h_t}{k_t} < (1 - \tau) A + \beta - \alpha. \quad (16)$$

Evasion is not expedient when $\lambda = \frac{\tau}{\eta}$; in all other cases the relationship between the parameters is highly convoluted and simple conditions cannot be stated. However, when an interior solution exists, several conclusions can be drawn from the results presented in Table 1. First, habit affects the decision to evade because it creates a genuinely dynamic pattern: without habit, evasion would be a constant percentage of wealth (last column of Table 1). The evolution of tax evasion over a lifetime depends on the ratio $k_t / h_t$. The dynamics of this ratio is computed through Itô’s lemma and has the following form:

$$\frac{1}{dt} \mathbb{E}_t \left[ d \left( \frac{k_t}{h_t} \right) \right] = \gamma_0 + \gamma_1 \frac{k_t}{h_t} + \gamma_2 \left( \frac{k_t}{h_t} \right)^2, \quad (17)$$

in which

$$\gamma_0 = -\frac{\tau}{\eta} \left(1 - \frac{\lambda \eta}{\tau}\right) \left(1 - \left(\frac{\lambda \eta}{\tau}\right)^\frac{1}{2}\right) - 1 + \frac{\rho + \lambda}{\theta} + \frac{\xi - 1}{\theta} \left(\frac{\tau}{\eta} + (1 - \tau) A\right) - \frac{\tau}{\eta} \left(\frac{\lambda \eta}{\tau}\right)^\frac{1}{2}, \quad (18)$$
\[ \gamma_1 = \frac{2\alpha - (1 - \tau) A - \beta}{(1 - \tau) A + \beta} \left( \frac{\rho + \lambda}{\delta} + \frac{\delta - 1}{\delta} \left( \frac{\tau}{\eta} + (1 - \tau) A \right) - \frac{\tau}{\eta} \left( \frac{\lambda \eta}{\tau} \right)^{\frac{1}{2}} \right) \]

\[ + (1 - \tau) A - \alpha + \beta + \frac{\tau}{\eta} \left( 1 - \frac{\lambda \eta}{\tau} \right) \left( 1 - \left( \frac{\lambda \eta}{\tau} \right)^{\frac{1}{2}} \right), \]

Equation (17) is a Riccati second-order differential equation with constant coefficients. We can see that when \( \alpha = 0 \) – that is, when there are no intrinsic habits since \( h_t \) does not depend on previous consumption – Equation (17) becomes linear, as happens in most of the macroeconomic models.

The asymptotic equilibrium of the expected value in Equation (17) is the positive solution of the quadratic form:

\[ \lim_{t \to \infty} \frac{1}{d} \mathbb{E}_t \left[ d \left( \frac{k_t}{h_t} \right) \right] = \frac{-\gamma_1 + \sqrt{\gamma_1^2 - 4\gamma_0 \gamma_2}}{2\gamma_2}, \]

which is a highly non-linear transformation of all the parameters. It is difficult to study algebraically. Furthermore, although the optimal path of \( k_t/h_t \) can be computed algebraically, its interpretation is not unequivocal. For this reason, Section IV presents a simulation exercise that shows that decreasing evasion over time is optimal for sufficiently strong habit formation (i.e. a sufficiently high value of \( \alpha \)), combined with a relatively high level of capital productivity (\( A \)) and risk aversion (\( \delta \)).

If the habit intensity is lower than net capital productivity increased by the persistence of habit – that is, if \( \alpha < (1 - \tau) A + \beta \) – intrinsic habit reduces tax evasion. The size of the difference depends on \( h_t/k_t \): all else being equal, the higher the ratio – that is the closer the level of habit to income – the lower the tax evasion. We have already shown that the lower \( c_t - h_t \), the higher the risk aversion.

Table 1 shows the optimal consumption and evasion in four frameworks (with and
Table 1: Optimal tax evasion under varying assumptions on preferences and habit formation.

<table>
<thead>
<tr>
<th>Without habit</th>
<th>With tax evasion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evasion $e^*_t$</td>
<td>$E$</td>
</tr>
<tr>
<td>Cons. $c^*_t$</td>
<td>$k_t F$</td>
</tr>
</tbody>
</table>

Without habit, with and without evasion. It indicates that, in the presence of habit, the optimal consumption without tax evasion is lower than it is with tax evasion. In fact, if evasion is expedient, $\lambda$ must be greater than $\frac{T}{\eta}$.

Therefore, a tax-evader may have the same consumption level as an honest consumer with a lower habit and higher capital. This finding has implications for the debate on the strength and persistence of habit formation. For example, the variability of the effect of habit on consumption that is often observed in international comparisons could be due in part to different rates of evasion across countries.

**Comparative statics**

Because of the presence of habit, preferences are not time-separable, that is decisions are made not only on the basis of present values of the state variables, but also on their whole history. Thus, the optimal behaviour is no longer a simple feedback strategy, and the optimal solution at time $t$ is a function of the whole optimal path starting from $t_0$. Despite this difficulty, a closed-form solution for the relationship between tax rate and evasion can be computed.

**Proposition 2.** The elasticity of optimal tax evasion to a change in the tax rate may be either
Table 2: Sign of the elasticity of optimal tax evasion with respect to tax under various functional forms of the fine. The results are obtained under the hypotheses $\frac{\lambda \eta (\tau)}{\tau} < 1$, and 

\[(1 - \tau) A + \beta - \alpha > 0.\]

<table>
<thead>
<tr>
<th>Fine $\eta (\tau)$</th>
<th>Sign of elasticity $\frac{\partial e^<em>_t}{\partial \tau} \frac{\tau}{e^</em>_t}$</th>
<th>Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$= \eta_0$</td>
<td>$&gt; 0$</td>
<td>$\frac{h_t}{k_t} &lt; \frac{(1 - \tau) A + \beta - \alpha}{1 + \frac{A \tau}{(1 - \tau) A + \beta - \alpha} \left( \frac{\lambda \eta (\tau)}{\tau} \right)^{\frac{1}{\delta}} - 1}$</td>
</tr>
<tr>
<td>$= \eta_1 \tau$</td>
<td>$&lt; 0$</td>
<td></td>
</tr>
<tr>
<td>$= \eta_0 + \eta_1 \tau$</td>
<td>$&gt; 0$</td>
<td>$\frac{h_t}{k_t} &lt; \frac{T_H}{1 + T_H} \left( (1 - \tau) A + \beta - \alpha \right)$</td>
</tr>
</tbody>
</table>

Positive or negative according to the sign of the following inequality:

\[
\frac{\partial e^*_t}{\partial \tau} \frac{\tau}{e^*_t} \begin{cases} 
> 0 \iff \frac{h_t}{k_t} < \frac{1}{1 - \frac{h_t}{k_t} \frac{1}{(1 - \tau) A + \beta - \alpha}} \\
< 0 \iff \frac{h_t}{k_t} > \frac{1}{1 - \frac{h_t}{k_t} \frac{1}{(1 - \tau) A + \beta - \alpha}}
\end{cases}
\]

\[
\left( \frac{(1 - \tau) A + \beta - \alpha}{A \tau} \right)^2 \begin{cases} 
1 - \left( 1 - \frac{\partial \eta (\tau)}{\partial \tau} \frac{\tau}{\eta (\tau)} \right) \left( 1 + \frac{1}{\delta} \frac{\left( \frac{\lambda \eta (\tau)}{\tau} \right)^{\frac{1}{\delta}}}{1 - \left( \frac{\lambda \eta (\tau)}{\tau} \right)^{\frac{1}{\delta}}} \right)
\end{cases}
\]

**Proof.** See Appendix C

While most of the current literature on tax evasion has either $\eta_0 = 0$ with $\eta_1 > 0$, or $\eta_1 = 0$ with $\eta_0 > 0$, our model is more general since either $\eta_0$ and $\eta_1$ may be greater than zero, which facilitates interpretation of fines in the real world. Our results are summarised in Table 2.

As long as $(1 - \tau) A + \beta - \alpha > 0$, if the fine is proportional to the evaded income ($\eta_1 = 0$), the model shows a positive effect only if the ratio of the subsistence consumption level to capital ($h_t / k_t$) is smaller than a threshold.

The ambiguity, which is consistent with the classical Allingham and Sandmo (1972) model, arises because HARA preferences with positive subsistence level $h_t$ presents de-
creasing relative risk aversion (Levaggi and Menoncin, 2013). When the fine is proportional to evaded taxes ($\eta_0 = 0$), Yitzhaki (1974)'s paradox of a negative impact is confirmed, but if the fine is an affine transformation of the tax rate, the elasticity is positive only if the ratio of habit to capital ($h_t / k_t$) is lower than a constant threshold.

The presence of a threshold in the ratio $h_t / k_t$ around which the elasticity of evasion with respect to the tax rate changes in sign contributes to explaining why the empirical evidence on Yitzhaki’s paradox is often not clear-cut. Since the ratio $h_t / k_t$ is not constant over time and it has a dynamic pattern, the agent’s reactions to changes in the tax rate over the habit-accumulation path may differ.

Another issue that policy-makers face in designing a fiscal system is the relative effectiveness of the two audit instruments, that is, the level of fines and the number of controls. The effects of a change in the audit parameters can be found by comparing three elasticities computed with respect to $\lambda$, $\eta_0$, and $\eta_1$:

\[
\frac{\partial e_t^*}{\partial \lambda} \frac{\lambda}{e_t^*} = -\frac{1}{\delta} \left( \frac{\lambda \eta}{\tau} \right)^{\frac{1}{3}} < 0, \quad (23)
\]
\[
\frac{\partial e_t^*}{\partial \eta_0} \frac{\eta_0}{e_t^*} = - \left( 1 + \left| \frac{\partial e_t^*}{\partial \lambda} \frac{\lambda}{e_t^*} \right| \right) \frac{\eta_0}{\eta} < 0, \quad (24)
\]
\[
\frac{\partial e_t^*}{\partial \eta_1} \frac{\eta_1}{e_t^*} = - \left( 1 + \left| \frac{\partial e_t^*}{\partial \lambda} \frac{\lambda}{e_t^*} \right| \right) \frac{\eta_1 \tau}{\eta} < 0. \quad (25)
\]

We can conclude that: (i) the elasticity of optimal evasion with regard to $\eta_0$ is higher (in absolute value) than that with regard to $\eta_1$ if and only if $\eta_0 > \eta_1 \tau$, (ii) the elasticity of optimal evasion with regard to $\eta_0$ is higher (in absolute value) than that with regard to $\lambda$ if and only if $\frac{\eta_0}{\eta_1 \tau} > \frac{1}{\delta} \left( \frac{\lambda \eta}{\tau} \right)^{\frac{1}{3}}$, and (iii) the elasticity of optimal evasion with regard to $\eta_1$ is higher (in absolute value) than that with regard to $\lambda$ if and only if $\frac{\eta_1 \tau}{\eta_0} > \frac{1}{\delta} \left( \frac{\lambda \eta}{\tau} \right)^{\frac{1}{3}}$.

These results suggest that, in extreme cases, when either $\eta = \eta_0$ or $\eta = \eta_1 \tau$, the elasti-
city with regard to the fine is always greater than the elasticity with regard to the frequency of controls. Accordingly, evasion can be fought more effectively by increasing the fine, than it can by increasing the number of controls. This result is in line with empirical evidence (Chiarini et al., 2009; Feige and Cebula, 2012) showing that tax evasion is decreasing in the audit rate, but it may also explain why the number of audits is decreasing over time (Slemrod, 2007). If fines are more effective in reducing tax evasion and are less costly than audits, it may make sense to reduce audits. The result is also consistent with Chen (2003), who showed, in a model with tax evasion and endogenous growth, that an increase in fines reduces tax evasion, whereas an increase in tax auditing reduces tax evasion only if the cost of tax enforcement is not too high. On the other hand, fines should be credible: if they are very high, their social cost may be too high to be enforced.

IV Simulations

In this section we propose a simulation exercise to study the optimal dynamic behaviour of the optimal tax evasion and consumption decisions as obtained in Proposition 1. As many authors have stressed (e.g. Rebelo, 2005), such exercises can be used to illustrate the long-run effects of public policies when they are calibrated with parameters drawn from the literature to generate artificial data that mimic long-term properties of actual economies. In the present case, simulations may also help to resolve some of the ambiguities from the theoretical analysis.

To this purpose, we conducted several simulations using a large set of parameters. We start from a benchmark scenario based on the most common values of the parameters found in the literature, as summarised in Table 3. Then we conduct sensitivity analyses to illustrate how the results are affected by changes in the parameters within various ranges, as also shown in Table 3.
Table 3: Parameters for simulations (the values used in the benchmark scenario are in bold).

<table>
<thead>
<tr>
<th>Definition</th>
<th>Parameters</th>
<th>Values</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Habit formation</td>
<td>((a, \beta))</td>
<td>((0, +\infty))</td>
<td>No habit</td>
</tr>
<tr>
<td></td>
<td></td>
<td>((.093, .1) - (.25, .3) - (.493, .6))</td>
<td>Constantinides (1990)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>((.1, .174))</td>
<td>Kraft et al. (2017)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>((.47, 1))</td>
<td>Havranek et al. (2017)</td>
</tr>
<tr>
<td>Utility curvature</td>
<td>(\delta)</td>
<td>(2.2 - 4 - 6.11)</td>
<td>Constantinides (1990) – Kraft et al. (2017) – Fuhrer (2000)</td>
</tr>
<tr>
<td>Rate of time preference</td>
<td>(\rho)</td>
<td>(.037 - .1)</td>
<td>Constantinides (1990) – Kraft et al. (2017)</td>
</tr>
<tr>
<td>Habit at (t = 0)</td>
<td>(h_0)</td>
<td>(0)</td>
<td>Assumption: starting with no habit for simulating the whole path of habit accumulation</td>
</tr>
<tr>
<td>Total factor productivity</td>
<td>(A)</td>
<td>(.107 - .457 - .613)</td>
<td>Assumption: chosen so that the economic growth rate with neither habits nor evasion is in the range 1.5% – 5%</td>
</tr>
<tr>
<td>Capital at (t = 0)</td>
<td>(k_0)</td>
<td>(100)</td>
<td>Normalisation assumption</td>
</tr>
<tr>
<td>Tax rate</td>
<td>(\tau)</td>
<td>(.172 - .343 - .459)</td>
<td>Minimum – average – maximum tax rate in OECD countries (OECD, 2017)</td>
</tr>
<tr>
<td>Fine</td>
<td>((\eta_0, \eta_1))</td>
<td>((.3, 1.2) - (.3, 2) - (.3, 3))</td>
<td>(\eta_1) is chosen to be consistent with values reported in the literature (Alm, 2019, and references therein), and (\eta_0) is by assumption</td>
</tr>
<tr>
<td>Audit intensity</td>
<td>(\lambda)</td>
<td>(.01715 - .0429 - .09431 - .223)</td>
<td>Consistent with yearly audit rates (equal to (p = 1 - e^\lambda)) discussed in the literature (Alm, 2019, and references therein)</td>
</tr>
</tbody>
</table>
Benchmark simulation

A large empirical literature has measured the intensity of habit. Estimates confirm that habit matters to describe consumption behaviour, but the magnitude varies widely, ranging from low effects (Dynan, 2000), to more sizeable effects (Ferson and Constantinides, 1991; Braun et al., 1993; Browning and Collado, 2007; Carroll et al., 2011), to very strong effects (Fuhrer, 2000; Kueng and Yakovlev, 2014). Several factors may contribute to explaining this variability (Havranek et al., 2017 provide a meta-analysis). In the benchmark scenario we initialised the parameters of habit preference, starting with the values in Constantinides (1990). For the benchmark simulation we used the intermediate values $a = .3$ and $b = .250$ (Table 3). The rate of time preference and the parameter of the curvature of the utility function are set to $\rho = .1$ and $\delta = 4$, consistent with values in this literature.

The total factor productivity $A$ is set at a level that indicates a desired real growth rate in an economy that has neither tax evasion nor habit equal to $\left(\frac{1}{\delta} (A (1 - \tau) - \rho)\right)$. In the benchmark framework, this growth rate is set to 5 per cent, which can be interpreted as a steady state in our model.

For the benchmark simulation, the tax rate is set at $\tau = .343$, which matches the average tax rate in the OECD countries (OECD, 2017). The instantaneous audit intensity is $\lambda = .09431$. This corresponds to a per-period probability of being selected for an audit $(1 - e^{-\lambda})$ equals to .09, which is consistent with an average of taxpayers’ subjective assessments that have been reported in many surveys like Harris et al., 1987. The parameters of the fine function are $\eta_0 = .03$ for the part on evaded income as in Allingham and Sandmo (1972), and $\eta_1 = 2$ for the part on evaded taxes as in Yitzhaki (1974). The value of $\eta_1$ entails the payment of both evaded taxes and a fine of the same amount; whereas the value of

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The value is higher than the actual frequency of controls in several countries, which is between 2 percent and 5 percent. Since not all incomes can be evaded, audits are concentrated on a smaller fraction of taxpayers (Kleven et al., 2011). In addition, as many scholars have suggested, “what matters for the taxpayer’s decision is not the objective frequency of audit, but his subjective perception” (Sandmo, 2012, p. 10). In fact, there is evidence that a typical taxpayer tends to overestimate and/or overweight the probability of controls (Alm et al., 1992; Andreoni et al., 1998; Alm, 2019).
\( \eta_0 \) may reflect additional costs, including administrative, legal, or even reputational costs, that an evader incurs when he or she is discovered (Sandmo, 2012).

We perform \( N = 100 \) simulations of monthly data (\( dt = \frac{1}{12} \)), the parameters of which are all in annual terms. We adopt the normalization \( k_0 = 100 \) and set \( h_0 = 0 \) to show how a habit is created from the beginning.

The results of the benchmark simulation are presented in Figure 2. The upper-left and upper-right panels show the optimal paths of the control variables, respectively evasion and consumption, both as a percentage of income; the bottom-left and bottom-right panels show the paths for the ratios \( h_t/k_t \) and \( h_t/c_t \), respectively. In each panel, each grey line is a simulated path, the solid line is the average of the 100 simulated paths, and the dotted line shows the behaviour of the optimal solution in the case without evasion (i.e. in a fully deterministic framework).

Over time, the optimal consumption tends toward a constant percentage of the habit that can be interpreted as a steady state in our model. The presence of habit reduces tax evasion over time, while, without habit (\( \alpha = \beta = h_0 = 0 \): CRRA preferences), evasion would be constant over time as stated in Proposition 1. With habit, the evolution of tax evasion depends on the ratio \( h_t/k_t \), as Equation (13) suggests. The simulation in the bottom-left panel shows that this ratio increases over time. Therefore, as predicted by Equation (13), the fraction \( c^*_i \) of evaded income decreases because the property of the HARA utility function causes the consumer to become less willing to take risk over time.

The upper-right panel shows that, as in standard models of habit formation, in the first periods the consumption-to-income ratio is increasing until it reaches its long run equilibrium. The higher speed with respect to the case without evasion is due to the greater amount of capital that is saved (and invested) through evasion. The results of this simulation are remarkably close to the paths presented in Figure 1, which are based on real data.

Since habit accumulates over time, a consumer with a higher accumulated habit tends
Figure 2: Benchmark simulation with the parameters in bold in Table 3. Each grey line is the path of a single simulation. The continuous line is the average of 100 simulated paths. The dotted line is the path without evasion.
to evade less than does a consumer at an earlier stage of habit accumulation. This result is in line with the literature on risk taking (Holt and Laury, 2002). However, the policy implications of our model differ from those in the standard risk-taking literature, where people become increasingly risk averse as they age. Instead, in our model the parameter of the curvature of the utility function ($\delta$) does not change over time, and the consumer becomes more conservative because his/her consumption is closer to his/her habit. Many empirical studies have reported evidence that is consistent with such a behaviour, usually with no reference to a theoretical model that can explain the forces that drive this effect.

The evolution of habit as a percentage of consumption (bottom-right panel) is also interesting. If we compare the dotted line (without tax evasion) with the continuous one (with tax evasion) we see that habits are stronger for honest individuals. This clearly indicates that if on the one hand tax evasion increases the opportunity for consumption, on the other hand it also moderates habit. This result also confirms the need for empirical estimates of consumption habits to take evasion into account.

The simulations’ dispersion around the mean shows that the edges of the individual simulations are above the mean in the simulations of optimal evasion, while the edges are below the mean in simulations of the other variables, especially consumption. This result is consistent with intuition: since consumption must always be greater than habit (i.e., the argument of the utility function cannot be negative), there is an upper limit to evasion that cannot be so big as to leave the consumer with an income that is below the consumption habit if evasion is discovered and the consumer is fined.

When an agent is caught evading, the fine results in sudden drop in his income (which can be seen in the grey lines of Figure 2). Income shrinks because of payments for fines. After each decrease in income, the optimal evasion increases again until the next drop. On

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5For example, econometric studies on tax evasion that find a significant positive impact of age on compliance include Spicer and Lundstedt (1976); Slemrod (1985); Dubin and Wilde (1988); Feinstein (1991). A weaker effect is in Witte and Woodbury (1985). Clotfelter (1983) found a greater tendency toward compliance in the oldest and youngest segments of the population than was present among those of middle age. Experiments and/or surveys that have documented a positive effect of age have been reported by Vogel (1974); Tittle (1980); Kirchler (1999); Wenzel (2002, 2004).
average, optimal tax evasion is decreasing over time (upper-right panel), but the payment of a fine causes a jump in consumption as a percentage of income (upper-left panel), which also causes a positive jump in the ratio $h_t/k_t$. Since habit $h_t$ is always increasing over time, any jump that is due to paying a fine suddenly reduces $k_t$. In addition, the ratio $h_t/k_t$ has positive jumps because of the drop in the optimal consumption that is due to paying the fine. After this payment, the ratio starts decreasing, but its dynamics is increasing on average (lower-right panel).

**Sensitivity analysis and policy implications**

We ran several additional simulations to check the robustness of the results shown in Figure 2 and to see how the results change depending on the model’s parameters. One robust result is that the trends of the benchmark simulation are confirmed with any combination of parameters in Table 3. In fact, the driving path of the habit-to-capital ratio, $h_t/k_t$, is always increasing, which leaves unaffected the drifts in the other variables’ dynamics. In particular, evasion is decreasing towards the steady state, whereas the path of consumption-to-income and that of habit-to-consumption are both increasing.

Changing the parameters affects the speed of adjustments and the levels of the variables. As discussed in the theoretical section, an increase in $\alpha$ produces higher weight for the past consumption in the habit, so it increases the difference between the values at the earlier stage of the simulations and those at the steady state. Instead, an increase in $\beta$ reduces the persistence of past consumption in habit, so it shortens the periods before obtaining the steady state. Both effects are illustrated in the simulation in Figure 3, where we set $\alpha = .47$ consistent with the average values of the estimates Havranek et al. (2017) reviewed. The simulation shows that a model with such a simple structure is likely to have short transitional dynamics. In general, the empirical literature is based on simpler specifications of habit, where the present consumption depends only on the consumption in the previous period (Muellbauer, 1988; Dynan, 2000; Havranek et al., 2017). Our sim-
Figure 3: Simulation with short persistence ($\alpha = 0.47, \beta = 1$). All other parameters are the same as in the basic model.
ulation shows that these models may not capture the essential features of consumption habits, but a slower convergence toward the steady state.

Using a finite horizon setting, Kraft et al. (2017) estimated a model similar to ours with US data. We used their habit preferences ($\alpha = .1; \beta = .174$) in the simulations shown in Figure 4. Although the consumption paths are close to those of the two previous simulations and they reach similar steady states (around 55% of income), evasion remains much higher in Figure 4 (almost 40% in the steady state) than it is in Figure 2 (about 27%), while habit is lower (40% in steady state in Figure 4 versus more than 60% in Figure 2). This result supports the intuition of our theoretical analysis that the same level of consumption may result from very different combinations of habit and evasion, so it once more confirms the potential effect of taking evasion into account when estimating consumption habits.

The responses to the other parameters indicate that, as expected, making the curvature of the utility function more pronounced (namely increasing $\delta$) reduces tax evasion. From the base scenario, if we set $\delta = 2.2$, evasion raises by about 15 percentage points, but increasing $\delta$ to 6.11 (as in Fuhrer, 2000) decreases evasion by 20 percent. The model also allows us to draw some conclusions regarding the effects of tax evasion on economic growth. A lower value of $A$ reduces the economic growth rate and increases tax evasion. On the other hand, since evasion declines over time, the losses in income that are due to tax evasion when tax evasion is high along a path are partially offset by a higher growth rate.

Finally, we conducted simulations for the fiscal variables. Reducing the tax rate to $\tau = .172$, which is the lowest level among OECD countries (see Table 3), increases tax evasion to 35 percent in the steady state. On the other hand, setting $\tau$ to the maximum rate ($\tau = .459$) reduces tax evasion to 25 percent in the steady state. Therefore, each percent point increment in the tax rate in the benchmark simulation reduces tax evasion as a percent of income by about a third of a point. Consistent with the theoretical discussion,
Figure 4: Simulation based on Kraft et al. (2017) parameters ($\alpha = 0.1, \beta = 0.174$). All other parameters are as in the basic scenario.
the effect here depends on the form the fine takes. In particular, if more weight is given to evaded income (large \( \eta_0 \)) and less weight to evaded tax (low \( \eta_1 \)), the direction of the effect becomes more ambiguous and may even overturn the Yitzhaki (1974)’s paradox.

Regardless of their interactions with the tax rate, fines and intensity of audits have the expected effects. Moreover, the simulations confirm that evasion responds proportionately more to fines (with an average elasticity in the simulations of about .88 in the steady state) than it does to controls (average elasticity of about .5 in the steady state).

V Conclusion

Decision to evade tax have an intertemporal dimension that the traditional literature ignored until recently. Using a fully dynamic framework of capital accumulation and intertemporal utility maximisation, we introduce habit formation in consumption and we are able to compute closed-form solutions for the optimal dynamic tax evasion and the optimal dynamic consumption.

Our model shows that risk aversion is not the only determinant of consumer’s propensity to evade tax: as consumption habits play an important role that the literature has not considered. Even for a constant level of risk aversion, the consumer tends to reduce his level of tax evasion over time because of habit formation. In the long term, the consumer wants to keep his standard of living but is less willing to bear the risk that payment of a fine will reduce his/her income and prevent him/her from consuming at least his/her habit.

The comparative statics shows that the response of tax evasion to an increase in tax rate is ambiguous. In particular, the response can be positive when the habit-to-capital ratio is high, while it can be negative when the ratio is low. Optimal tax evasion is more responsive to changes in the fine level than it is in the frequency of audits. The intuitive explanation for this result is that higher fines produce a more pronounced income loss for consumers who are found cheating, increasing the difficulty of recovering from these
losses. In fact, recovery is possible only at the price of a longer period of evasion, and correlated high risk.

The results of our research signal that models of consumption with habits that neglect tax evasion may produce biased estimations of the effects on consumption of the persistence and intensity of habits. Therefore, future empirical research on consumption with habits should take tax evasion into account to obtain more reliable estimates for the impact of habit on consumption behaviour. In the model that we propose, tax evasion is a choice variable in a kind of portfolio optimisation problem, so it does not take personal and subjective variables like the tax morale and social stigma into account. In addition, we model the behaviour of a representative consumer and ignore the social interactions in a community of agents, a line of research that could be pursued in the future.

References


### A Data for Figure 1

The data on the shadow economy come from recent estimates reported by Alm and Em-baye (2013). The whole data set refer to 111 countries, including both developed and underdeveloped countries, for the years 1984–2006. In Table 4 we show the values for 49 high and upper-middle income countries of the last year of the dataset (2006).

Table 4: The data on household consumption and on the population median age both come from the United Nation database: household consumption data are based on the United Nation’s System of National Accounts (SNA) and refer to year 2006 while data on the population median age refer to the closest available year which is 2012. Both data set have been downloaded from the United Nation database (http://data.un.org) on June 2018.

<table>
<thead>
<tr>
<th>Country</th>
<th>Population median age</th>
<th>Shadow economy as % of GDP</th>
<th>Household consumption as % of GDP</th>
<th>Income group</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>30.8</td>
<td>33.4%</td>
<td>63.3%</td>
<td>Upper middle inc.</td>
</tr>
<tr>
<td>Australia</td>
<td>37.1</td>
<td>15.8%</td>
<td>56.5%</td>
<td>OECD</td>
</tr>
<tr>
<td>Austria</td>
<td>42.4</td>
<td>13.8%</td>
<td>53.2%</td>
<td>OECD</td>
</tr>
<tr>
<td>Bahrain</td>
<td>30.1</td>
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<td>34.7%</td>
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</tr>
<tr>
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<td>33.2%</td>
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<td>Country</td>
<td>GDP per Capita</td>
<td>High Income (%)</td>
<td>Upper Middle (%)</td>
<td>Category</td>
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<td>---------------------</td>
<td>---------------</td>
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<td>Trinidad &amp; Tobago</td>
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<td>42.5%</td>
<td>29.2%</td>
<td>Upper middle inc.</td>
</tr>
</tbody>
</table>
B Proof of Proposition 1

Given Problem (12), the value function \( J_t(h_t, k_t) \) which solves it can be defined as

\[
J_t(h_t, k_t) e^{-\rho t} = \max_{c_t, e_t} \mathbb{E}_t \left[ \int_t^\infty \frac{(c_s - h_s)^{1-\delta}}{1-\delta} e^{-\rho s} ds \right],
\]

and \( J_t(h_t, k_t) \) must solve the following Hamilton-Jacobi-Bellman equation:

\[
0 = \frac{\partial J_t}{\partial t} - \rho J_t + \max_{c_t} \left[ \frac{(c_t - h_t)^{1-\delta}}{1-\delta} + \frac{\partial J_t}{\partial h_t} (\alpha c_t - \beta h_t) - \frac{\partial J_t}{\partial k_t} c_t \right] + \max_{e_t} \left[ (J_t(h_t, k_t - \eta e_Ak_t) - J_t) \lambda + \frac{\partial J_t}{\partial k_t} (1 - \tau + \tau e_t) Ak_t \right].
\]

The first order conditions on \( c_t \) and \( e_t \) are:

\[
\dot{c}_t^* = h_t + \left( \frac{\partial J_t}{\partial k_t} - \frac{\partial J_t}{\partial h_t} \alpha \right)^{-\frac{1}{\delta}},
\]

\[
\frac{\partial J_t}{\partial e_t^*} (h_t, k_t - \eta e_Ak_t) \lambda + \frac{\partial J_t}{\partial k_t} \tau Ak_t = 0.
\]

In this case the so-called “guess function” is

\[
J_t = F_t^s (k_t - h_t H_t)^{1-\delta},
\]

where the functions \( F_t \) and \( H_t \) must be found in order to solve the HJB equation. The two transversality conditions are

\[
\lim_{t \to \infty} F_t = 0, \quad \lim_{t \to \infty} H_t = 0.
\]
Given the guess function, the optimal consumption and evasion are

\[ e_t^* = \frac{k_t - h_t H_t}{\eta A k_t} \left( 1 - \left( \frac{\lambda \eta}{\tau} \right)^{\frac{1}{\delta}} \right), \]

\[ c_t^* = h_t + \frac{k_t - h_t H_t}{F_t} (1 + \alpha H_t)^{-\frac{1}{\delta}}. \]

After plugging these values into the HJB, it becomes

\[ 0 = \delta F_t^{\delta-1} \frac{(k_t - h_t H_t)^{1-\delta}}{1-\delta} \frac{\partial F_t}{\partial t} - h_t F_t^{\delta} (k_t - h_t H_t)^{-\delta} \frac{\partial H_t}{\partial t} - \rho F_t^{\delta} (k_t - h_t H_t)^{1-\delta} \]

\[ + \frac{1}{1-\delta} \frac{(k_t - h_t H_t)^{1-\delta}}{F_t^{1-\delta}} (1 + H_t \alpha) \frac{1-\delta}{\delta} - H_t F_t^{\delta} (k_t - h_t H_t)^{-\delta} (\alpha h_t - \beta h_t) \]

\[-\alpha H_t F_t^{\delta-1} (k_t - h_t H_t)^{1-\delta} (1 + \alpha H_t)^{-\frac{1}{\delta}} - F_t^{\delta-1} (k_t - h_t H_t)^{1-\delta} (1 + \alpha H_t)^{-\frac{1}{\delta}} + F_t^{\delta} (k_t - h_t H_t)^{-\delta} (1 - \tau) A k_t \]

\[ + F_t^{\delta} (k_t - h_t H_t)^{-\delta} \left( \left( \frac{\lambda \eta}{\tau} \right)^{\frac{1}{\delta}} - 1 \right) \lambda + F_t^{\delta} (k_t - h_t H_t)^{-\delta} \frac{\tau}{\eta} \left( 1 - \left( \frac{\lambda \eta}{\tau} \right)^{\frac{1}{\delta}} \right), \]

which can be simplified as two differential equations (one in \( F_t \) and one in \( H_t \)) as follows

\[ 0 = \frac{\partial F_t}{\partial t} - F_t \left( \frac{\rho + \lambda}{\delta} - \frac{1-\delta}{\delta} \left( \frac{\tau}{\eta} + (1-\tau) A \right) - \frac{\tau}{\eta} \left( \frac{\lambda \eta}{\tau} \right)^{\frac{1}{\delta}} \right) + (1 + \alpha H_t)^{1-\frac{1}{\delta}}, \]

\[ 0 = \frac{\partial H_t}{\partial t} - H_t ((1 - \tau) A + \beta - \alpha) + 1. \]

Given the transversality conditions, the solutions of these two differential equations are:

\[ H_t = \int_t^{\infty} e^{-\int_t^s ((1-\tau) A + \beta - \alpha) du} ds = \frac{1}{(1-\tau) A + \beta - \alpha}, \]

\[ F_t = \int_t^{\infty} (1 + \alpha H_s)^{1-\frac{1}{\delta}} e^{-\int_t^s \left( \frac{\rho + \lambda}{\delta} - \frac{1-\delta}{\delta} \left( \frac{\tau}{\eta} + (1-\tau) A \right) - \frac{\tau}{\eta} \left( \frac{\lambda \eta}{\tau} \right)^{\frac{1}{\delta}} \right) du} ds \]

\[ = \frac{(1-\tau) A + \beta}{(1-\tau) A + \beta - \alpha}^{1-\frac{1}{\delta}} \]

\[ = \frac{\rho + \lambda}{\delta} - \frac{1-\delta}{\delta} \left( \frac{\tau}{\eta} + (1 - \tau) A \right) - \frac{\tau}{\eta} \left( \frac{\lambda \eta}{\tau} \right)^{\frac{1}{\delta}}. \]

If \( F_t \) and \( H_t \) are substituted into the first order conditions, the result of Proposition 1 is obtained.
C Proof of Proposition 2

The elasticity of (13) with respect to $\tau$ is

$$\frac{\partial e^*_i}{\partial \tau} e^*_i = \frac{\partial}{\partial \tau} \left( \frac{1}{\eta(\tau) A} \right) \tau + \frac{\partial}{\partial \tau} \left( \frac{1 - \frac{h_l}{k_l} \frac{1}{(1-\tau)A+\beta-a}}{1 - \frac{h_l}{k_l} \frac{1}{(1-\tau)A+\beta-a}} \right) \tau + \frac{\partial}{\partial \tau} \left( \frac{1 - \left( \frac{\lambda(\tau)}{\tau} \right)^\frac{1}{\delta} \right) \tau,$$

and if the derivatives at the numerators are computed, we have

$$\frac{\partial e^*_i}{\partial \tau} e^*_i = -\frac{\partial \eta(\tau)}{\partial \tau} \eta(\tau) \tau + \frac{h_l}{k_l} \frac{A \tau}{(1-\tau)A+\beta-a} + \frac{1}{\delta} \left( \frac{\lambda(\tau)}{\tau} \right)^\frac{1}{\delta} \left( 1 - \frac{\partial \eta(\tau)}{\partial \tau} \eta(\tau) \right),$$

from which the result of the proposition is obtained.