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The role of fund size in the performance of mutual funds assessed with DEA models

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Abstract. This contribution studies the role of the size of mutual funds in the evaluation of the fund performance with a data envelopment analysis (DEA) approach, with the aim of studying the issue from different angles and with different statistical tools and of testing the presence of economies or diseconomies of scale in the mutual funds market. Firstly, we discuss the role of fund size in the performance evaluation and wonder whether it is appropriate to include size information among the variables of DEA models. Secondly, we analyse the presence of a relationship between the performance scores and the size of mutual funds using different statistical tests and carry out an empirical investigation on a set of European equity mutual funds.

Keywords: Fund size, Mutual fund performance evaluation, Data envelopment analysis (DEA).

JEL Classification Numbers: C65, G1, G23.

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1. Introduction

This contribution studies in detail an issue often overlooked in the analysis of the performance of mutual funds: the role of the size of mutual funds in the evaluation of the fund performance.

This issue is particularly relevant when the performance assessment is carried out with a data envelopment analysis (DEA) approach. DEA is a nonparametric technique that can be used to perform a relative efficiency analysis of a set of decision making units of various kinds (just to mention a couple of examples from different fields: hospitals and banks). In the last years, this methodology has been increasingly applied in finance to evaluate the performance of mutual funds.

An initial analysis of the effects of the size of mutual funds on the fund performance is outlined in Basso and Funari (2014b). The present contribution deepens the investigation, still within the framework of a DEA approach, with the aim of studying the issue from different angles and with different statistical tools and of testing the presence of economies or diseconomies of scale in the market of mutual funds.

First, we discuss the role of fund size in the performance evaluation of mutual funds and wonder whether it is appropriate to include size information among the input or output variables of DEA models.

The advisability of including size in the performance evaluation of mutual funds arises especially when the analysis is focused on the point of view of financial investors, since investors would primarily like to maximize the financial results without being exposed to high risk levels, and may not care much about fund size.

Furthermore, it is interesting to analyse the presence of a dependence of the performance scores on the size of mutual funds.


Alternatively, the presence of a linear relationship between the fund size
and the performance measure can be investigated also by computing the correlation coefficient between the DEA performance measure and the fund size and by testing the significance of the correlation coefficient (see Murthi et al. (1997) and Choi and Murthi (2001)).

In order to analyse if there is any effect of fund size on performance, we carry out an empirical investigation on a set of European equity mutual funds in the period June 2006 to June 2009. In this investigation the measure of relative efficiency of mutual funds is computed by using a suitable DEA model that has been recently proposed in the literature to assess fund performance (Basso and Funari (2014a)); this model allows to take into consideration the main elements of an investment in mutual funds and allows for variable returns to scale.

In particular, in the empirical analysis we first test if the linear correlation coefficient is significantly different from 0, as should be in case economies or diseconomies of scale are present.

Secondly, it is interesting to study not only the value of the performance measure but also the resulting performance ranking. Therefore, we also analyse the rank correlation between the DEA performance measure and the fund size.

Thirdly, in order to see if size causes any significant effects on fund performance, we also compare the performance values obtained by small and large mutual funds by means of appropriate statistical tests.

The structure of the paper can be outlined as follows. In Section 2 we briefly present the DEA-V model used to compute the performance measure of mutual funds. In Section 3 we discuss the role of fund size in the evaluation of the performance of mutual funds with the DEA methodology. Sections 4 and 5 present the results of the empirical analysis carried out on European mutual funds. Finally, Section 6 synthesises the conclusions.

2. The DEA model

In order to study the role of size in the performance of mutual funds, we apply a methodology, namely data envelopment analysis (DEA), which is used to evaluate the relative efficiency of decision making units of both non-profit institutions, such as hospitals, libraries and universities, and profit-oriented organisations, such as banks and mutual funds.

This methodology computes an efficiency score which is comprised between 0 and 1, where 1 is the score assigned to the efficient decision making
units of the set of units under consideration and a lower score characterises the inefficient units. The DEA efficiency score is obtained as the ratio of a weighed sum of the outputs produced by the decision making unit over a weighed sum of the inputs required. In the DEA approach the weights assigned to the input and output values are computed by solving a suitable optimization problem which can be written as a linear program. For an introduction to the DEA methodology see for example Cooper et al (2000) and Cooper et al (2011).

In the last 15 years the DEA methodology has been applied to the evaluation of the performance of mutual funds, and by now the number of papers on applications of DEA to mutual funds adds up to a good hundred (for a comprehensive review see Basso and Funari (forthcoming)).

In order to study the role of size on the performance of mutual funds, we apply the DEA-V model, which is a recent model proposed in Basso and Funari (2014a) that considers the most significant variables for an investor with a well diversified portfolio and can be used even in the presence of negative mean returns.

In this model we first consider the capital $K_j$:

$$K_j = \frac{1}{1 - c_{Ij}},$$

which is the payout required to an investor by fund $j$ (with $j \in \{1, 2, \ldots, n\}$) in order to start with an initial investment, net of the initial fee $c_{Ij}$, of value equal to 1. In addition, we consider the $\beta$-coefficient of fund $j$, $\beta_j$, as a risk measure that is able to take into account the risk of an investment in fund $j$ for an investor with a well diversified portfolio.

Besides the variables $K_j$ and $\beta_j$, which are included in the DEA model as input variables, we consider as output variable the final value of the investment after a holding period of $T$ years, net of the exit fee $c_{Ej}$:

$$M_j = (1 + R_j)^T(1 - c_{Ej}),$$

where $R_j$ denotes the annual rate of return and we consider an initial value of the investment (net of the initial fee) equal to 1.

Let us observe that this model allows us to deduct the initial and exit fees from the final value, taking into account the net profit of the investment.

In order to grasp the idea underlying the DEA model used, let us begin with the DEA-C model (Basso and Funari (2014a)), which is a DEA model
with constant returns to scale that can be written in its dual version as follows:

\[
\begin{align*}
\max & \quad z_0 + \varepsilon s_1^+ + \varepsilon s_1^- + \varepsilon s_2^- \\
\text{s.t.} & \quad M_0 z_0 - \sum_{j=1}^{n} M_j \lambda_j + s_1^+ = 0 \\
& \quad \sum_{j=1}^{n} K_j \lambda_j + s_1^- = K_o \\
& \quad \sum_{j=1}^{n} \beta_j \lambda_j + s_2^- = \beta_o \\
& \quad \lambda_j \geq 0 \quad j = 1, 2, \ldots, n \\
& \quad s_1^+, s_1^-, s_2^+ \geq 0
\end{align*}
\]

where \(z_0, \lambda_j(j = 1, 2, \ldots, n), s_1^+, s_1^-, s_2^-\) are the dual variables associated to the constraints of the linear programming problem which is the primal of problem (3)–(8), \(\varepsilon\) is a non-Archimedean constant (see e.g. Cooper et al (2000)) and \(o\) (with \(o \in \{1, 2, \ldots, n\}\)) denotes the fund which is being evaluated.

By solving problem (3)–(8) we obtain a DEA performance measure for fund \(o, I_{o,C}\), computing the reciprocal of the optimal value of \(z_0\):

\[
I_{o,C} = \frac{1}{z_0^*} = \frac{u^* M_o}{v_1^* K_o + v_2^* \beta_o},
\]

where the asterisk denotes the optimal values of \(z_0\), of the input weights \(v_1, v_2\) and of the output weight \(u\).

Let us note that the value of the optimal weights used in ratio (9) to compute the DEA score \(I_{o,C}\) comes from the solution of the optimization problem (3)–(8), so that for each mutual fund \(o \in \{1, 2, \ldots, n\}\) we have to solve a different optimization problem. This also means that the DEA performance score is computed using the most favorable values of the input and output weights for each fund.

On the other hand, it is possible to generalise a DEA model so that the returns to scale of the mutual funds are no longer constrained to be constant (see for example Cooper et al (2000)). This can be done by adding to the model the following constraint that characterises the DEA models
with variable returns to scale:

\[ \sum_{j=1}^{n} \lambda_j = 1. \]  \hspace{1cm} (10)

By adding constraint (10) to the DEA-C model (3)–(8) we obtain the output oriented DEA-V model with variable returns to scale; for further details see Basso and Funari (2014a).

The optimal solution of the DEA-V model enables to compute the DEA performance score in presence of variable returns to scale, \( I_{o,V} \), that is the performance measure which will be used in the subsequent analysis.

3. The role of fund size in DEA models for mutual funds

The analysis carried out in this contribution is focused on the point of view of a financial investor that utilises the performance scores in order to choose the best mutual fund to invest on or to evaluate an investment he made in a mutual fund.

Let us observe that, from the point of view of a financial investor, two funds with identical features in terms of mean return, risk level, initial and exit fees, can be considered as equivalent: if the values of these variables are identical, the two funds are clearly perceived as equivalent by investors. This evaluation does not depend on the fund size in terms of total market value. Of course, if the aim were to evaluate the skill of the fund managers, the changes in the size of the funds might well be relevant.

On the other hand, if we include size among the input variables, two funds with identical values of mean return, risk level and initial and exit fees but different size could have different performance scores.

In this regard, let us consider the instance illustrated in table 1, in which funds F11 to F20 have the same values of mean return, risk level and initial and exit fees as funds F1 to F10, but they have four times their size. Hence, from the investors’ point of view, fund F1 is judged equivalent to fund F11, fund F2 is judged equivalent to fund F12, and so on.

On the other hand, let us consider the fund size \( S_j \) and let us define as input variable the initial capital paid in fund \( j \) by all investors (including the initial fee) as follows:

\[ K_{Sj} = \frac{S_j}{1 - c_{ij}} = S_j K_j. \]  \hspace{1cm} (11)
Table 1: Data of the instance on the effect of size on the fund performance.

<table>
<thead>
<tr>
<th>Fund</th>
<th>S</th>
<th>c_I</th>
<th>c_E</th>
<th>R</th>
<th>β</th>
<th>K_S</th>
<th>M_S</th>
<th>DEA score</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>7</td>
<td>0.02</td>
<td>0.02</td>
<td>0.05</td>
<td>1.3</td>
<td>7.143</td>
<td>7.941</td>
<td>1.000</td>
<td>1</td>
</tr>
<tr>
<td>F2</td>
<td>11</td>
<td>0.04</td>
<td>0</td>
<td>0.07</td>
<td>1.7</td>
<td>11.458</td>
<td>13.475</td>
<td>1.000</td>
<td>1</td>
</tr>
<tr>
<td>F3</td>
<td>24</td>
<td>0.03</td>
<td>0.01</td>
<td>0.04</td>
<td>0.8</td>
<td>24.742</td>
<td>27.114</td>
<td>1.000</td>
<td>1</td>
</tr>
<tr>
<td>F4</td>
<td>18</td>
<td>0.04</td>
<td>0</td>
<td>-0.02</td>
<td>1.2</td>
<td>18.750</td>
<td>16.941</td>
<td>0.800</td>
<td>15</td>
</tr>
<tr>
<td>F5</td>
<td>20</td>
<td>0.025</td>
<td>0</td>
<td>0.025</td>
<td>0.8</td>
<td>20.513</td>
<td>21.538</td>
<td>1.000</td>
<td>1</td>
</tr>
<tr>
<td>F6</td>
<td>37</td>
<td>0.03</td>
<td>0</td>
<td>-0.07</td>
<td>0.9</td>
<td>38.144</td>
<td>29.761</td>
<td>0.705</td>
<td>19</td>
</tr>
<tr>
<td>F7</td>
<td>23</td>
<td>0</td>
<td>0.025</td>
<td>-0.04</td>
<td>1.1</td>
<td>23.000</td>
<td>19.840</td>
<td>0.764</td>
<td>17</td>
</tr>
<tr>
<td>F8</td>
<td>13</td>
<td>0.02</td>
<td>0.01</td>
<td>0.010</td>
<td>0.95</td>
<td>13.265</td>
<td>13.260</td>
<td>0.998</td>
<td>9</td>
</tr>
<tr>
<td>F9</td>
<td>5</td>
<td>0.025</td>
<td>0</td>
<td>-0.15</td>
<td>1.0</td>
<td>5.128</td>
<td>3.071</td>
<td>1.000</td>
<td>1</td>
</tr>
<tr>
<td>F10</td>
<td>9</td>
<td>0.035</td>
<td>0</td>
<td>-0.05</td>
<td>1.1</td>
<td>9.326</td>
<td>7.716</td>
<td>0.831</td>
<td>13</td>
</tr>
<tr>
<td>F11</td>
<td>28</td>
<td>0.02</td>
<td>0.02</td>
<td>0.05</td>
<td>1.3</td>
<td>28.571</td>
<td>31.765</td>
<td>0.967</td>
<td>10</td>
</tr>
<tr>
<td>F12</td>
<td>44</td>
<td>0.04</td>
<td>0</td>
<td>0.07</td>
<td>1.7</td>
<td>45.833</td>
<td>53.902</td>
<td>1.000</td>
<td>1</td>
</tr>
<tr>
<td>F13</td>
<td>96</td>
<td>0.03</td>
<td>0.01</td>
<td>0.04</td>
<td>0.8</td>
<td>98.969</td>
<td>108.456</td>
<td>1.000</td>
<td>1</td>
</tr>
<tr>
<td>F14</td>
<td>72</td>
<td>0.04</td>
<td>0</td>
<td>-0.02</td>
<td>1.2</td>
<td>75.000</td>
<td>67.766</td>
<td>0.808</td>
<td>14</td>
</tr>
<tr>
<td>F15</td>
<td>80</td>
<td>0.025</td>
<td>0</td>
<td>0.025</td>
<td>0.8</td>
<td>82.051</td>
<td>86.151</td>
<td>0.958</td>
<td>11</td>
</tr>
<tr>
<td>F16</td>
<td>148</td>
<td>0.03</td>
<td>0</td>
<td>-0.07</td>
<td>0.9</td>
<td>152.577</td>
<td>119.045</td>
<td>1.000</td>
<td>1</td>
</tr>
<tr>
<td>F17</td>
<td>92</td>
<td>0</td>
<td>0.025</td>
<td>-0.04</td>
<td>1.1</td>
<td>92.000</td>
<td>79.361</td>
<td>0.783</td>
<td>16</td>
</tr>
<tr>
<td>F18</td>
<td>52</td>
<td>0.02</td>
<td>0.01</td>
<td>0.010</td>
<td>0.95</td>
<td>53.061</td>
<td>53.040</td>
<td>0.903</td>
<td>12</td>
</tr>
<tr>
<td>F19</td>
<td>20</td>
<td>0.025</td>
<td>0</td>
<td>-0.15</td>
<td>1.0</td>
<td>20.513</td>
<td>12.283</td>
<td>0.541</td>
<td>20</td>
</tr>
<tr>
<td>F20</td>
<td>36</td>
<td>0.035</td>
<td>0</td>
<td>-0.05</td>
<td>1.1</td>
<td>37.306</td>
<td>30.866</td>
<td>0.733</td>
<td>18</td>
</tr>
</tbody>
</table>
in place of the initial capital paid in by a single investor, $K_j$. Of course, we have to modify the final value accordingly, so that the output variable is defined as follows:

$$M_{Sj} = K_{Sj} (1 + R_j)^T (1 - c_{Ej}) = S_j M_j.$$ \hfill (12)

The last but one column of table 1 shows the DEA score obtained with a DEA model analogous to model DEA-V in which the input variable $K_j$ is substituted by $K_{Sj}$ and the output variable $M_j$ is substituted by $M_{Sj}$, while the last column displays the relative ranking based on this score.

As can be seen, the DEA score of fund $j$ is by no means always the same as that of fund $j + 10$. For example, the score of fund F16 is much higher than the score of fund F6, while the score of fund F19 is much lower than that of Fund F9, and their relative rank is very different, too.

This shows that the performance scores computed in this way exhibit a bias. For this reason, even if in the DEA literature we can find a few contributions that include the fund size among the variables of the model, we deem correct not to consider it in the model and not insert fund size among the inputs of the DEA model.

Actually, in Haslem and Scheraga (2006) and Premachandra et al (2012) the fund size is considered as an output variable since the focus of their studies is on the managerial ability. On the other hand, Pendaraki (2012) includes the size among the outputs though it focuses on the fund performance, while other contributions include the fund size among the input variables of DEA models (Haslem and Scheraga (2003) and Alexakis and Tsolas (2011)).

4. The effect of size on the performance of European mutual funds

Although, as argued in the previous section, the fund size has not to be explicitly included among the input and output variables of the DEA model used to evaluate the performance of mutual funds from the point of view of an investor, it is interesting to wonder whether it affects the fund performance in any way. In the next two sections we investigate if we can find indication of the presence of either scale economies or scale diseconomies which affect the performance of mutual funds.

For example, Grinblatt and Titman (1989) in a study of US mutual funds finds that the mutual funds with the smallest net asset values also realized the largest performance, thus suggesting the presence of scale diseconomies,
and we may wonder if a similar result holds also for the present market of mutual funds.

Actually, if we look at the DEA contributions that study the dependence of the DEA scores on the fund size, the results presented in the literature are well diversified. Indeed, some results do not indicate the presence of a statistically significant relationship at all (Murthi et al (1997), Choi and Murthi (2001), Daraio and Simar (2006); also Darling et al (2004) for the performance of CTAs). On the other hand, other contributions find indication of a statistically significant dependence, but the sign of such a dependence is positive for some studies (Margaritis et al (2007), Hu et al (2012)) and negative for others (Babalos et al (2012), Majid and Maulana (2012), Tavakoli Baghdadabad and Noori Houshyar (2014)). Therefore, the results stated in the literature are by no means conclusive about the presence of either scale economies or scale diseconomies in the mutual funds market.

In order to verify if there is an effect of fund size on the performance of mutual funds, we have carried out an empirical analysis on European data in the period June 2006–June 2009. The data set comprises 260 equity mutual funds, randomly chosen from the western European countries (source: Morningstar database). In this analysis the fund size $S_j$ is measured by the total market value (expressed in millions of euros).

The analysis has been carried out by using the DEA-V model described in Section 2. The results obtained are summarised in table 3, while table 2 summarises the main statistics of the input and output variables, as well as the main information relevant for the analysis carried out.

From table 2 we may observe that in the period considered the average of the mean returns is negative, pointing out that we are considering a period of financial crisis. From table 3, which reports the main statistics on the DEA performance scores obtained by the mutual funds analysed, we note that 6 funds out of 260 are efficient (2.3%), while the lowest performance score is equal to 0.385 and the mean score is 0.613.

Moreover, let us consider the scatter plot of the European mutual funds analysed with respect to size and the performance score $I_{o,V}$, represented in figure 1, which might suggest an eventual dependence of the performance score on size. We have to report the presence in the data of an outlier (omitted from figure 1), represented by a mutual fund much bigger than the other funds in the set, with a size of 4869.79 millions of euros which is three times the size of the second biggest fund. Actually, it is difficult to visually identify a dependence from the scatter plot in figure 1.
Table 2: Some statistics on the relevant information for the analysis of the 260 European equity mutual funds analysed.

<table>
<thead>
<tr>
<th>Statistics</th>
<th>$S$</th>
<th>$c_I$</th>
<th>$c_E$</th>
<th>$R$</th>
<th>$\beta$</th>
<th>$K_S$</th>
<th>$M_S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min</td>
<td>2.09</td>
<td>0.00</td>
<td>0.00</td>
<td>-20.04</td>
<td>0.45</td>
<td>1.000</td>
<td>0.511</td>
</tr>
<tr>
<td>Max</td>
<td>4869.79</td>
<td>7.00</td>
<td>5.00</td>
<td>16.32</td>
<td>1.40</td>
<td>1.075</td>
<td>1.574</td>
</tr>
<tr>
<td>Mean</td>
<td>146.14</td>
<td>2.95</td>
<td>0.14</td>
<td>-5.36</td>
<td>0.97</td>
<td>1.031</td>
<td>0.853</td>
</tr>
<tr>
<td>Median</td>
<td>54.60</td>
<td>3.50</td>
<td>0.00</td>
<td>-6.24</td>
<td>0.98</td>
<td>1.036</td>
<td>0.824</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>364.28</td>
<td>2.16</td>
<td>0.61</td>
<td>4.78</td>
<td>0.13</td>
<td>0.023</td>
<td>0.136</td>
</tr>
</tbody>
</table>

Table 3: Some statistics on the DEA performance scores $I_{o,V}$ obtained for the European equity mutual funds analysed.

<table>
<thead>
<tr>
<th>Statistics</th>
<th>$I_{o,V}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min</td>
<td>0.385</td>
</tr>
<tr>
<td>Max</td>
<td>1.000</td>
</tr>
<tr>
<td>Mean</td>
<td>0.613</td>
</tr>
<tr>
<td>Median</td>
<td>0.576</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.136</td>
</tr>
<tr>
<td>No. of efficient funds</td>
<td>6</td>
</tr>
<tr>
<td>% of efficient funds</td>
<td>2.3 %</td>
</tr>
<tr>
<td>Mean for small funds</td>
<td>0.596</td>
</tr>
<tr>
<td>Mean for large funds</td>
<td>0.629</td>
</tr>
</tbody>
</table>
Figure 1: Scatter plot of the European equity mutual funds analysed with respect to size and the performance score $I_{O,V}$ (without the outlier).
In order to see if there is a significant influence of the fund size on the DEA performance score for the set of European funds analysed or not, we firstly use a method similar to the one proposed by Murthi et al. (1997) and Choi and Murthi (2001), taking the fund size into consideration as an external variable.

We thus compute the correlation coefficient between the fund size and the DEA performance scores and evaluate its statistical significance at a confidence level of $\alpha = 0.05$. As discussed in Section 3, this choice seems natural, since the direct inclusion of size among the variables of the model may lead to distorted results when we adopt the investors’ point of view.

The results are presented in Table 4; from this table we can see that the correlation coefficient between the fund size and the DEA performance score is low and it is not statistically different from 0 for all usual significance levels. Note that the computations have also been repeated by omitting the outlier from the data set, but the conclusion does not change.

Hence, from this test there seems to be no significant linear relationship between size and performance, a conclusion that is similar to that reached by Murthi et al. (1997) and Choi and Murthi (2001) (but also by Daraio and Simar (2006) and Darling et al. (2004)).

### Table 4: Correlation coefficients between the fund size and the performance scores $I_{o,V}$ for the European equity mutual funds analysed.

<table>
<thead>
<tr>
<th>Set of funds</th>
<th>Correlation coefficient</th>
<th>$p$-value (two-tailed test)</th>
<th>Significance (for $\alpha = 5%$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>All 260 funds</td>
<td>-0.0256</td>
<td>0.6809</td>
<td>Not significant</td>
</tr>
<tr>
<td>259 funds (without outlier)</td>
<td>0.0339</td>
<td>0.5873</td>
<td>Not significant</td>
</tr>
</tbody>
</table>

5. Any effect of size?

We have seen in the previous section that there seems to be no linear relationship between the performance score and the fund size, at least for the data set considered. In this section we carry out two additional investigations, with different statistical methodologies, in order to see if we may find a size effect of any kind which is empirically verifiable.

To begin with, it is interesting to study not only the value of the performance measure but also the resulting ranking. We have therefore studied the rank correlation between the DEA-V performance score $I_{o,V}$ and the fund
Table 5: Rank correlation coefficients between the fund size and the performance scores $I_{o,V}$ for the European equity mutual funds analysed.

<table>
<thead>
<tr>
<th>Test</th>
<th>Correlation coefficient</th>
<th>$p$-value (two-tailed test)</th>
<th>Significance (for $\alpha = 5%$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spearman rank correlation</td>
<td>0.1312</td>
<td>0.0344</td>
<td>Significant</td>
</tr>
<tr>
<td>Kendall rank correlation</td>
<td>0.0868</td>
<td>0.0373</td>
<td>Significant</td>
</tr>
</tbody>
</table>

More in detail, to measure the degree of similarity between the two rankings and assess the significance of the relationship between them, we have computed the Spearman correlation coefficient and the Kendall correlation coefficient between the size and the performance score.

The results are reported in Table 5 and indicate the presence of a statistically significant rank dependence. The positive sign of the rank correlation coefficients denotes that, on average, the larger funds exhibit a slightly higher performance score, thus showing the presence of scale economies.

In addition, in order to study the presence of scale economies more thoroughly, we have divided the set of mutual funds analysed into two groups according to their size and we have compared the mean performance scores of the two groups.

More precisely, let us define the set of small funds as the set of mutual funds whose size is lower than the median size, and the set of large funds as the set of funds whose size is higher than the median size.

We compare the mean performance scores of the two groups of small and large funds and we test whether their differences are statistically significant.

To this aim, we apply some statistical tests specially designed to compare the DEA inefficiencies between two groups of decision making units (see Banker and Natarajan (2011) and Banker et al (2010)). In this context, the DEA inefficiencies are defined as the reciprocal of the DEA efficiency scores, that is $1/I_{o,V}$. These tests are based on order statistics; they do not require specific assumptions on the probability distribution of the inefficiencies and can be used in the presence of a noise term independent of the inefficiencies.

The tests we adopt are the following:

1. a median test designed to evaluate the equality of the median of the inefficiencies between two groups;
2. a Mann–Whitney test which compares the DEA efficiency scores of two groups;
3. a Kolmogorov–Smirnov test for equality of the distributions of the inefficiencies of two groups.

The results of the tests are reported in table 6 and show that the differences in the performance score are statistically significant. On the other hand, the mean value of the performance score of the large funds is slightly higher than that of the small funds (see table 3). Therefore, again, we can conclude that the performance scores of large funds tend to be somewhat higher than those of small funds.

Table 6: Results of the statistical tests for the performance comparison of the inefficiencies of the sets of small and large mutual funds.

<table>
<thead>
<tr>
<th>Test</th>
<th>Test statistic</th>
<th>asymptotic p-value (two-tailed test)</th>
<th>Significance for $\alpha = 5%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median test</td>
<td>-1.9846</td>
<td>0.0472</td>
<td>Significant</td>
</tr>
<tr>
<td>Mann-Whitney test</td>
<td>-2.3009</td>
<td>0.0214</td>
<td>Significant</td>
</tr>
<tr>
<td>Kolmogorov-Smirnov test</td>
<td>0.1769</td>
<td>0.0298</td>
<td>Significant</td>
</tr>
</tbody>
</table>

6. Summary

The focus of this contribution is the analysis of the role of the fund size on the performance of mutual funds, evaluated from the point of view of investors using a DEA approach.

In the first place, this issue is studied by investigating if the fund size has a role to play in the DEA model. With the aid of an appropriate instance, we argue that it is better to exclude size from the variables of the DEA model.

On the contrary, the analysis of the effects of the fund size on the DEA performance measure is undertaken by taking size into consideration as an external variable.

We carry out an empirical investigation on a set of 260 European equity mutual funds. In this investigation we apply various statistical tests in order to:

1. verify the presence of a linear relationship between the DEA performance score and the fund size, by testing the significance of the correlation coefficient;
2. verify the presence of a rank correlation, by testing the significance of the Spearman and Kendall rank correlation coefficients;
3. compare the DEA inefficiencies of small and large mutual funds (with a size lower than or higher than the median, respectively), by using three appropriate statistical tests for the comparison of the DEA inefficiencies of two groups of decision making units.

The results of the analysis carried out indicate that:

1. there is no significant linear correlation between the performance score and the fund size;
2. the rank correlation coefficients are statistically significant;
3. the differences in the performance scores between small and large mutual funds are statistically significant.

Therefore, at least for the set of European equity mutual funds analysed, there does not exist a linear relationship between size and performance, but the large funds tend, on average, to exhibit a slightly higher performance score than the smaller ones, thus indicating the presence of scale economies.

References


