Modeling intrinsic and extrinsic tax compliance

Fernando Garcia Alvarado
Paolo Pellizzari

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GARCIA ALVARADO Fernando∗
PELLIZZARI Paolo†

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Abstract

This paper attempts to fill the gap between economic and behavioral literature on tax evasion. We propose a micro-founded model whose results are consistent with the ‘Slippery Slope’ Framework and with the Expected Utility Theory. Individual taxpayers face the problem of deciding the fraction of income they wish to disclose to the government, based on their trust and perceived power of authorities. Trust is modeled as the voluntary compliance originated by the umbrella-term tax morale and power is shaped by the perceived enforcement, mainly motivated by higher audit probabilities and penalty fees. Tax rates, however, play a negative effect on tax compliance, as agents are less willing to pay taxes whenever facing larger obligations. Additionally, we study taxpayer behavior when the audit rate is zero and when agents are inclined to make charitable donations. Above all, we derive the conditions under which individuals would fully-evade, fully-comply or even over-comply as charitable giving in the absence of audits.

JEL: H26, H31, C63, A12

Keywords: Tax Evasion, Power, Trust, Simulations.

1 Introduction

In this paper we attempt to bridge together the two main theoretical approaches to tax evasion: economic and behavioral. Economic literature considers the rational decisions of individuals as in Becker (1968), Allingham and Sandmo (1972) and Yitzhaki (1974); behavioral literature, in turn, considers power and trust as the main drivers of tax compliance as in Kirchler et al. (2008), Prinz et al. (2014) and Kogler et al. (2015). We present a parsimonious model for individual tax compliance on self-reported income, distinguishing between intrinsic and extrinsic motivations for tax compliance. As in Kirchler et al. (2008) we seek to determine tax compliance as a combination of two factors: trust as an intrinsic measure of voluntary disclosures, and power as an extrinsic predictor of enforced compliance.

On the one hand, the intrinsic motives are understood as the voluntary compliance stimulated by trust in the government or desire to conform with morality and peer-behavior, which is independent of the audit probabilities, effective tax rates and applicable penalty fees. In this paper trust is modeled as tax morale, which may be interpreted as the main predictor of voluntary compliance and driven by a degree of perceived procedural and distributive justice. On the other hand, extrinsic influences encompass the enforcement of tax compliance by the government’s power, which can be described in terms of audit schemes, penalties, retaliation and tax rate fixation. The perceived power of the authorities, which consists in the government’s potential to detect tax evasion and to punish illegal behavior, turns out to be the main predictor of enforced tax compliance. In our approach, risk-aversion is the channel or link through which power affects the behavior of agents. Thus, perceived power of authorities is shaped

∗Ca Foscari University of Venice and Paris 1 Pantheon-Sorbonne. e-mail: fernando.garcia@unive.it
†Ca Foscari University of Venice. e-mail: paolop@unive.it
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by individual risk-aversion, along with other key parameters such as audit probabilities, tax rates and penalty fees.

Prinz, Muehlbacher and Kirchler (2014) adequately developed a mathematical model of tax compliance at a society level with heterogeneous agents. The probability of individual tax compliance was derived considering the shares of evasion-minded and compliance-minded taxpayers. Although this model successfully replicated the well-known ‘slippery slope’ framework, it did not include a micro-founded explanatory model for rational agents attempting to optimize an expected utility in function of audit probabilities, tax rates, penalty fees, and other parameters. A key difference between our paper and previous literature relies on the fact that we mathematically model individual taxpayer behavior and do not restrict the analysis to aggregate values of tax compliance.

The structure of this paper is as follows. Section 2 surveys the extensive literature on economics of crime and tax evasion. Section 3 defines the mathematical model underlying the taxpayer’s decision to evade or comply with its tax duties. Section 4 offers an econometric analysis performed on the Agent-Based simulations of individual taxpayers and their decisions. Section 5 summarizes the findings of this paper and discusses the concluding remarks.

2 Background

Becker (1968) wrote a seminal economic approach to crime and punishment. A first notion of optimality conditions to combat illegal behavior was derived by entitling individuals with utility functions, costs, behavior shifts and punitive fines. Allingham and Sandmo (1972) and Srinivasan (1973) developed in parallel a formal analysis of tax evasion on self-reported income. In particular, Allingham and Sandmo (1972) adopted this framework and employed it to model tax evasion in a setting where rational agents seek to maximize their expected utility based on tax rates, audit probabilities and penalty fees which are applied to evaded taxes. Myles and Naylor (1996) extended the individual tax evasion models to account for group conformity and social norms. Andreoni, Erard and Feinstein (1998) argued in favor of implementing morale and social dynamics in economic modeling, as well as a psychological cost for cheating. The authors argued that it is the perceived audit rate and not the objective audit rate which should be employed to model taxpayer behavior. Additionally, their research extended tax modeling to include for impure altruism and donations to public goods, introducing the ‘warm glow’ effect of giving.

Although the expected utility models have shown how rational agents might increase tax compliance proportionately to tax rates, many experiments have disproved this to occur in real life as in Slemrod (1985) and Alm, Jackson & McKee (1992). Alm, Jackson & McKee (1992) thoroughly explored why people pay taxes and found a positive relationship between tax rates and evasion, while larger audit probabilities and fine rates increased the taxes paid by individuals. Moreover, Alm, McClelland & Schulze (1992) found how a share of individuals fully complied with their tax duties even when the probability of audit was zero. Such phenomenon occurred in the presence of a public investment game, where society benefited from tax collections. An alluring result from this laboratory experiment was the finding that tax disclosures follow an almost dichotomous distribution, where most agents either fully comply or fully evade, while only a rather small share of individuals appears to stay in the middle, strategically selecting the optimal fraction of income to declare.

Although persuasive power predicts trust and voluntary compliance, it is very complicated to device policies targeted at enhancing persuasiveness. Eriksen and Fallan (1996) found that whenever individuals acquired additional tax knowledge they tended to increase their perceived tax fairness, and thus take their taxpaying decisions more seriously and frown on tax evasion. Cullis and Lewis (1997) and Adams and Webley (2001) remark the importance of having an efficient, favorable interaction between taxpayers and tax authorities. Different studies have found how the presence of intrusive, unreasonable, or unfair audits may lead to negative effects on tax compliance. Bergman (2003) found how Chile managed to enhance tax compliance by enforcing legitimate power through tax reforms, whereas Argentina was unable to do so by coercive means. Moreover, Frey (2003) found that constant monitoring could be perceived as a lack of trust or resemble a ‘cops and robbers’ situation. In such cases, when authorities are overly inquisitive or extremely punitive, an increment of power would provoke a decrement in trust. As mentioned by Turner (2005) power can arise from either legitimate or coercive mechanisms. Consequently, the interactions between power and trust can follow different paths depending on the enforcement mechanisms that the government may decide to follow.
Perceived fairness may be divided in three justice categories as in Wenzel (2003): distributive, procedural and retributive. Distributive justice encompasses the equal treatment and similar tax burden for individuals; procedural justice relates to the proper use of money and resources; retributive justice refers to the application of proper sanctions to tax evaders. Wenzel (2005) found evidence of misconceptions whenever individuals leaned to converge at the social norm. In some cases, people would undermine their tax compliance as a consequence of wrongly assuming that evasive behavior is the common practice. Another aspect of taxpayers’ perceptions about the tax system is the limited liability assumption, where agents can be fined, at most, the entirety of their income. Ueng and Yang (2006) concluded that in the presence of limited liability, excessively increasing tax rates would lead to a worse-off state of the economy in a Pareto-sense. Germane to this idea, Andreoni et al. (1998) had previously suggested how unreasonably high fine rates would anyway not be attainable due to bankruptcy or finite maximal losses. Consequently, recent literature has endorsed limited liability as a common assumption, as in Piolatto and Rablen (2017).

A paramount contribution by Kirchler et al. (2008) bridged behavioral and economic motives for taxpayers in a single framework, commonly known as the ‘Slippery Slope’. The ‘Slippery Slope’ relates two main drivers of tax compliance: trust as a predictor for voluntary disclosures and power as a factor of enforced compliance. The authors influentially argue how tax rates may play different roles depending on the level of trust. Under low trust, high tax rates strike taxpayers as being an unfair treatment; whereas for high trust, higher taxes may be perceived as an additional contribution to the common good. Trust in government may be linked by two different channels: as an individual’s perceived quality of authorities or as the agent’s tax morale. On government quality, Torgler and Schneider (2009) look at tax evasion from a macroeconomic perspective, and argue that the shadow economy may be tackled by improving the institutional quality of authorities and in such way earning taxpayer’s confidence. On the other hand, defining tax morale is a complex endeavor. Luttmer and Singhal (2014) provided an umbrella definition for tax morale, which encompasses all the mechanisms that may entice compliance above the level expected by a rational agent: intrinsic motivation to comply, reciprocity, peer pressure, social customs, information misconceptions or overestimation of the audit rate.

Mittone and Patelli (2000) proposed a novel way to look at tax compliance from a computational point of view by developing Agent-Based Models (ABM) of tax compliance. An extensive list of computational approaches to tax evasion followed. Davis et al. (2003) studied the evolution of compliant and non-compliant groups involving social norms and enforcement dynamics. Mittone (2006) adapted a model with psychological costs under repetitive dynamic choices made by individuals to study the complex behavior of taxpayers. Bloomquist (2006) highlighted the importance of adopting computational social science techniques to validate models that represent real-world problems which require attention from policy makers. Korobow, Johnson & Axtell (2007) linked agents in a Networked Agent Based-Model which permitted to measure the individual heterogeneity of the agents’ perceptions of audit probabilities and fine rates. A network approach to tax evasion allowed the authors to computationally examine how peer effects affected the aggregate level of tax compliance in a world with limited knowledge. Hokamp and Pickhardt (2010) improved the ABM framework by endowing agents with an exponential utility function and lapse time effects to study the evolution of alternative government policies to deter tax evasion. Hokamp (2014) incorporated back auditing and aging to study the dynamics of social norm updating for heterogeneous agents inside a population. Calimani and Pellizzari (2014) opted for a power utility function and investigated the optimal audit schemes from a social planner’s point of view aiming to curb tax evasion. Hashimzade, Myles, Page and Rablen (2014) grouped akin agents by occupation and argued in favor of endogenous attitude formations to account for individual perceptions. Not surprisingly, Andrei, Coner & Koehler (2014) found that the underlying network structure of agents in an ABM has a direct effect on the aggregate level of tax compliance, and networks which account for higher connectedness allow for a faster and more efficient spread of information and peer effects, leading to higher levels of aggregate tax compliance.

Field experiments have also had their saying in tax compliance and evasion. Kleven et al. (2011) performed a tax experiment in Denmark and found that augmenting marginal tax rates had an increasing effect on tax evasion. Additionally, the authors extended the seminal expected utility model to combine prior audits and threat of audits as incentives to further comply for individuals with self-reported income. Kastlunger et al. (2013) found how excessive enforcement may aggravate tax evasion, experimentally revealing a complex and multifaceted relation between enforcement and compliance. Dwenget et al.
(2016) performed a field experiment to study local church tax compliance in Germany, which are legally binding but no audit schemes are enforced to monitor payments. The experiment showed how around 20% of the individuals fully complied even in the absence of audits, while most of the remaining subjects fully evaded. The authors found substantial intrinsic motivation to comply and found no crowding out effect between extrinsic and intrinsic motivations to comply. Notably, this field experiment confirmed the binary behavior of taxpayer compliance obtained ‘in vitro’ by Alm, McClelland & Schulze (1992), where the majority of individuals ‘bunch’ at kink points, namely either full evasion or full compliance.

Tax evasion is an ongoing research topic and extensive literature continues to investigate this practice. For the case of self-reported income Kogler, Muehlbacher and Kirchler (2015) presented both procedural and distributive justice as predictors for voluntary compliance and trust, while retributive justice and sanctions predicted enforced compliance and power. The authors empirically proved that trust and power significantly predict tax compliance regardless of their underlying motives: trust induces voluntary compliance, power stimulates enforced disclosures, trust and power have a negative yet not statistically significant relation, social norms are important but do not predict trust nor power, and women comply more than men but no other demographic characteristic seems to have an effect on compliance. Similarly, Hofmann et al. (2017) analyzed data from 111 countries and concluded that demographic characteristics had little to no effect on tax compliance. Gobena and Dijke (2016) established that procedural justice encourages voluntary tax compliance in developing countries where legitimate power is considered to be low. Non-linearity effects are also present in taxpayer behavior. Fochmann and Kroll (2016) showed that voluntary disclosures increase with respect to public good provisions up to a certain extent, after which they start to marginally decrease. Mendoza et al. (2017) showed how auditing could backfire as exercising additional audits decreases tax evasion up to some extent, after which it commences to acquire a U-shaped effect. Moreover, another complexity level was added by Alm, Bloomquist and McKee (2017), who showed how disclosure behavior of individuals is directly affected by their neighbors. In this sense, agents behave proportionately as they perceive their neighbors are behaving.

3 A taxpayer's decision to evade

The current section scrutinizes the individual optimal actions for a single agent. Tax compliance decision-making is ordinarily modeled as a gamble or an investment opportunity involving one risky asset (undisclosed income) and a risk-free asset (disclosed income). The expected utility to be optimized with respect to the fraction of income declared \( d \) by each individual \( i \) at time \( t \), yet discarding the subindex for simplicity, may well be defined as:

\[
EU[d] = p \cdot U(X) + (1 - p) \cdot U(Y),
\]

where \( X = I - \tau(d \cdot I) - \theta \tau(I - d \cdot I) \) is the net income after taxes and penalties in case an audit takes place and \( Y = I - \tau(d \cdot I) \) is the net income after taxes in case no audit takes place\(^1\). The wealth levels \( X \) and \( Y \) can be defined in terms of the gross earned income \( I \), the penalty rate \( \theta \) applied to the undisclosed fraction of income in case an audit occurs, and the tax rate applicable as a function of the income declared \( \tau(\cdot) \). Following, \( X \) and \( Y \) can be expressed as functions of \( d \). Thus, we can reformulate Equation 1 similar to Allingham and Sandmo (1972) and Yitzhaki (1974):

\[
EU[d] = p \cdot U[I - \tau(d \cdot I) - \theta \tau(I - d \cdot I)] + (1 - p) \cdot U[I - \tau(d \cdot I)].
\]

Akin to Hokamp and Pickhardt (2010), a power utility function is imputed into the model outlined in Equation 2. Following, the utility function of every single agent is:

\[
U(d) = (1 + d)^\rho W^{(1-\rho)},
\]

where \( d \in [0,1] \) is the fraction of declared income, \( W = \{X,Y\} \) is the period-wealth (which is, in fact, a function of \( d \)), \( \rho \in (0,1) \) is the risk-aversion , and \( \kappa \in [0,0.5] \) is the tax-morale\(^2\). In this sense,

\(^1\)A remark for the current tax decision model is the non-matchable income assumption, meaning that an auditor from the Tax Agency does not know beforehand the individual’s income. If a society would happen to account for a non-negligible matching system for its labor market, the assumption may be relaxed to take into consideration only the non-matchable portion of the agents’ stipends without sacrificing any of the models’ intuitions and results.

\(^2\)An analogous case would occur if the tax morale could take values from zero to one and then the value of the parameter would be divided by half in Equation 3.
a higher tax morale yields a larger utility of complying; while a higher risk-aversion would yield a lower utility on wealth. Considering Equation 3, it might be advantageous to remark that the only parameter over which agents optimize is the fraction of declared income \( d \), given that wealth \( W \) is dependent on income, fine rates, tax rates and \( d \). Furthermore, the utility function is concave with respect to both \((1 + d)\) and \( W \). The parsimonious representation of power and trust in our setup relies in the empirical findings of Kogler et al. (2015) where both parameters are main predictors of compliance, while there is no established relationship between the two of them.

Furthermore, in order for agents not to evade systematically, the fine rate must such that \( \theta > 1 \) and, under the power utility function specified in Equation 3, this condition is necessary and sufficient for agents to fully comply whenever audits happen with certainty (see Appendix). Referring to the tax rate function \( \tau(\cdot) \) the most common examples of such tax functions may be either a flat (constant) tax rate or a stepped-tax regime, where agents are taxed progressively according to their income. For analytical convenience, we will use a flat tax rate \( \tau \) in what follows.

**Optimality under certain audits and null audits**

Considering the expected utility optimization problem specified in Equation 2, where the utility function is the one expressed in Equation 3, there are closed-form solutions for two scenarios: when the audit probability is null \((p = 0)\) and whenever the audits occur with certainty \((p = 1)\). It turns out that compliance depends on a function of parameters that can be named ‘willingness-to-pay’, as in the following proposition.

**Proposition 1** Whenever the probability of being audited at time \( t \) is zero, individual \( i \) might fully evade, partially evade or fully comply depending on its own willingness-to-pay taxes \((\gamma)\).

\[
[d^*]_{p=0} = \begin{cases} 
0 & \text{if } \gamma \leq \tau, \\
(0, 1) & \text{if } \tau < \gamma < \frac{2\tau}{1-\tau}, \\
1 & \text{if } \gamma \geq \frac{2\tau}{1-\tau},
\end{cases}
\]

where \( \gamma = \frac{\kappa}{1-\rho} \) is the willingness-to-pay taxes.

**Proposition 2** Whenever the probability of being audited at time \( t \) is one, individual \( i \) will fully comply.

\[
[d^*]_{p=1} = 1 \quad \forall \{i, t\}.
\]

**Proposition 1** evidences how taxpayers might comply (and even fully comply) under certain conditions, even if the probability of being audited is zero. This proposition may offer a mathematical explanation for the startling results found by the experimental approach of Alm et al. (1992) and by the field experiment of Dwenger et al. (2016). Furthermore, the intuition behind **Proposition 2** is quite straightforward. The proofs for **Proposition 1** and **Proposition 2** are available in the Appendix.

A crucial question arises whenever talking about the effects of parameters on evasion models: How is compliance affected by the tax rate \( \tau \)? In other words, is the derivative of the expected utility function with respect to the tax parameter positive, negative or zero?

**Proposition 3** Assume the minimal net income an individual may receive after audits is zero, then in fact, the expected utility of agents is worse-off after paying taxes. Moreover, the expected utility function is reduced whenever the effective tax rate \( \tau \) increases.

\[
\frac{\partial EU(d)}{\partial \tau} \leq 0.
\]

**Proposition 3** establishes that in our mathematical model individuals are worse-off whenever tax rates are increased, as suggested by Ueng and Yang (2006). Albeit this result may seem trivial, many tax evasion models which do not account for a ‘limited liability’ propose an ambiguous effect of tax rates on expected utility. In fact, some models asserve that, under certain conditions, the expected utility
of agents increases whenever they face a larger tax rate. The ‘limited liability’ assumption postulates that the minimal net income after audits is zero. Analogously, it establishes the notion that the maximal amount of money the government or tax authority may collect in taxes and fines is the entirety of an agent’s income. Nonetheless, limited liability is a common assumption in tax evasion literature (See Piolatto and Rablen (2017)). More precisely, a necessary condition for the income in case of an audit to become negative, is that \( \theta \) must be strictly larger than \( \frac{1}{\tau} \). In real life, limited liability holds almost surely given that tax rates seldom go over 50% and penalty rates frequently oscillate between 1.2 and 1.75 (See Hindriks and Myles (2006)). In several expected utility models there is a somewhat counter intuitive property of high tax rates promoting tax compliance, however this is a non-linear effect that arises given the fact that applied penalties are set by \( \theta \tau \); thus increasing the tax rate is in some sense enhancing the effect of fine rates. The proof for Proposition 3 is provided in the Appendix.

**Remark:** Among the working assumptions of the model, we have that income is strictly positive, otherwise there would be no evasion problem to begin with. The tax rate \( \tau \) is neither 0% nor 100%. In the first case, everyone would be fully compliant by definition; in the second case (given the lack of a public game or a re-distributive mechanism) all agents would leave the labor market as their wage would be nominally zero after taxes. Further, the results found in this paper hold for any non-regressive tax function \( \tau(\cdot) \). Moreover, there are no retrospective audits, and we require limited liability assumptions. Namely, there is no possibility to send tax evaders to jail and the maximum penalty equals the entire income of the individual. Lastly, given the power utility functions are, in fact, iso-elastic, individuals have a Constant Relative Risk Aversion (CRRA). The latter means that the fraction of wages or profits that they decide to disclose each fiscal period is independent of their income level.

**The ‘Slippery Slope’ Framework**

Kirchler et al. (2008) assume that agents in a ‘cop-robber’ scenario may be tempted to behave rationally and to weigh audit and penalty rates, whereas in a ‘service-client’ scenario they would be more comfortable following the social norm and contribute their fair share. Consequently, the authors introduced the ‘Slippery Slope Framework’ in which a ‘service-client’ approach to deter tax evasion is preferred over the classical ‘cop-robbers’ system where individuals are targeted as criminals and not as customers. The model we introduced in Section 3 is also reproducible as a ‘slippery slope’ in a three-dimensional fashion.

Figure 1: Representation of the ‘Slippery Slope’ from the front (left image) and from the side (right image). These images map individual tax compliance \( (d^*) \) in function of voluntary \( (\kappa) \) and enforced \( (1 - \rho) \) motivations.

Figure 1 represents the individual disclosures for varying levels of tax morale and risk aversion under a setting with a flat tax rate of \( \tau = 30\% \), an audit probability of \( p = 5\% \) and fine rates of \( \theta = 2 \). Figure 1 shows the ‘slippery slope’ as seen from the front (left image) and from the side (right image). For larger values of enforced and voluntary motivations to comply, tax evasion may be fully eradicated. However,
such stable conditions are not attainable for an entire society in real life. As suggested by Kirchler et al. (2008) the main area of action consists on the section where both enforced and voluntary compliance are low. Lisi (2012) confirmed the effects of both trust and power on income disclosures and concluded that trust was a more important driver of tax compliance than power. Gobena and Dijke (2016) studied tax evasion in developing and industrialized countries, finding a remarkable difference between both situations. The authors suggested that for developing countries, where power and trust is perceived to be low, it would be optimal to focus on enhancing trust with respect to power.

The critique on policy measures introduced by Gobena and Dijke (2016), who claim optimal policy should be country dependent, is taken into account by an asymmetric ‘slippery slope’ represented in Figure 1. Our simulations do not give a symmetric importance to trust and power as in the seminal work of Kirchler et al. (2008), but instead convey different parametric impact depending on the initial conditions of the population. That is, policy implementation varies according whether a society is in the lower or upper part of the slope. The steepness of our simulated ‘slippery slope’ is more pronounced than the one pictured in Kirchler et al. (2008). Notwithstanding, this steepness is the result of accounting for two large fractions of the population being either fully evading or fully complying with their tax duties, producing a bimodal behavior of income disclosures.

Tax Compliance Area

Alm et al. (1992) and Dwenger et al. (2016) found that a fraction of people are willing to fully-comply with their tax obligations even under the absence of audits. This section derives the mathematical conditions which help explain why some individuals opt to disclose their full income even when the audit rate is zero. From the third equation of Proposition 1 one can reformulate the following result.

If $\gamma \geq \frac{2\tau}{1-\tau} \Rightarrow [d^*|p=0] = 1$. Substituting the willingness-to-pay taxes in terms of $\kappa$ and $\rho$, and by applying some basic algebra one can derive a linear relation as in Equation 4.

If $\rho \geq 1 - \kappa \left(\frac{1-\tau}{2\tau}\right) \Rightarrow [d^*|p=0] = 1. \quad (4)$

Equation 4 represents a linear function in the $(\kappa, \rho)$ plane, where the intercept is 1 and the term $\left(\frac{1-\tau}{2\tau}\right)$ is the absolute value of the slope. For increasing tax rates, the slope steepness decreases, making the linear function flatter. Consequently, the conditions under which an individual would fully comply with its tax duties may be expressed in terms of the taxpayer’s tax morale, risk aversion and tax rate. Figure 2 shows three ‘slippery slopes’ where the audit rate is zero and varying flat tax rates were applied to individuals. From left to right the effective tax rates are 15%, 20% and 30%, respectively. The $(\kappa, \rho)$ coordinates where individuals fully comply, partially comply and fully evade are colored in red, yellow and blue, respectively (we use the same color-code as in Figure 1). Moving from left to right we see how the area of full-compliance, colored in red, shrinks with increasing tax rates. For low tax rates (left), full compliance can be achieved by high voluntary motivations even without enforcement. Notwithstanding, as the tax rate increases, we see that this effect fades away. For scenarios with relatively high trust in authorities, as noted by Kirchler et al. (2008), risk-aversion would become irrelevant as agents would stop optimizing rationally and would opt for following the common norm.

There exists a complex relationship between voluntary and enforced tax compliance, which is itself affected by the tax rate faced by individuals. Bordignon (1993) claimed that agents would be prone to evade taxes unless they faced a non-zero probability of being detected. Nonetheless, such is not necessarily true for real life scenarios as shown in Dwenger et al. (2016). Although Prinz et al. (2014) based their model on the assumption that three fractions of people (full-evaders, optimizers and full-payers) always existed, the paper did not tested for settings where the audit probability would be set to zero. Figure 2 shows the presence and survival of all three sets of taxpayers even in the extreme case scenario where audits never occur.
Donations

Proposition 1 suggests that \( d^* \) may be larger than 1 in some cases. Whenever the expected utility of declaring \( d = 1 + \delta, \delta > 0 \) would be larger than the one attained by fully complying, agents would be willing to further contribute through donations. If we assume that \( d = 1 + \delta \), with \( \delta > 0 \), can be interpreted as the fraction of wealth that would optimally be declared even though full compliance has already been achieved, Proposition 4 shows the conditions where an agent’s expected utility would be better-off by overstating its income. Alternatively, one may interpret this behavior as a donation of \( \tau \delta \).

**Proposition 4** An individual would be willing to over-comply its tax duties by a fraction \( \tau \delta \), with \( \delta > 0 \), whenever the following condition is met:

\[
d^* = 1 + \delta \iff \gamma > \frac{-\ln\left(1 - \frac{\tau \delta}{1 - \tau}\right)}{\ln\left(1 + \frac{\delta}{2}\right)},
\]

where the right-hand term is proportional to the effective tax rate \( \tau \) and to the charity giving \( \delta \); while it is independent of the audit rate \( p \).

Moreover,

\[
\lim_{\delta \to 0^+} \frac{-\ln\left(1 - \frac{\tau \delta}{1 - \tau}\right)}{\ln\left(1 + \frac{\delta}{2}\right)} = \frac{2\tau}{1 - \tau}.
\]

For infinitesimally small values of \( \delta \), the condition stated in Proposition 4 reduces to the result previously derived in Proposition 1. Following, there exists an inverse relationship between charitable giving and tax rates \( \tau \), donation parameters \( \delta \) and their interactions \( \tau \delta \). Figure 2 illustrates this inverse relationship, where all individuals inside the red-colored area are potentially willing to donate. An implication from Proposition 4 is that the conditions under which an agent would donate a share of its income are independent of the audit rate \( p \). A philanthropist would do charitable actions independently of the true audit probability that is faced and only depending on its \( \gamma \) and other parameters. Their reason to do this, understood as a high value of tax morale or germane intrinsic reasons, may be motivated by social norms, reputation, status, ‘warm glow’ or other factors. Furthermore, if the donations would be such that \( \delta = 1/\tau \) the agent’s utility would be zero. Given that the utility function is concave, \( \delta \) is bounded by \( \delta < 1/\tau \).
4 An Agent-Based Model of Tax Compliance

The previous section provides some analytical treatment of the model in some important, but special, cases described by \( p = 0 \) or \( p = 1 \). We now turn to an agent-based model to investigate other less tractable and more general situations, as well as to extend the analysis to a population of heterogeneous agents who may have different individual levels of tax morale \( \kappa \) and risk-aversion \( \rho \) levels. In order to better understand the effect of each parameter on tax decisions, we simulated individual taxpayer behavior. A total of 10,000 heterogeneous tax payers were generated. Each agent had a unique set of parameters. Respectively, each individual had a perceived audit probability between 0% and 10%, faced an effective tax rate from 10% to 50%, was subjected to a penalty fee between 1 to 5 times the evaded taxes, with a tax morale sampled from a uniform distribution from 0.10 to 0.40 and a risk-aversion level uniformly sampled from 0.10 to 0.90. Additionally, individual income oscillated from 15000 to 30000 units yet it had no impact on the fraction of income declared given that agents have constant relative risk aversion.

Table 1 surveys the characteristics of all the relevant parameters employed in our simulations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Setting</th>
<th>Distribution</th>
<th>Support</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau ): tax rate</td>
<td>Societal</td>
<td>Uniform</td>
<td>( \tau \in [0.10, 0.50] )</td>
</tr>
<tr>
<td>( p ): audit rate</td>
<td>Societal</td>
<td>Uniform</td>
<td>( p \in [0.00, 0.10] )</td>
</tr>
<tr>
<td>( \theta ): fines</td>
<td>Societal</td>
<td>Uniform</td>
<td>( \theta \in [1.00, 5.00] )</td>
</tr>
<tr>
<td>( \rho ): risk aversion</td>
<td>Individual</td>
<td>Uniform</td>
<td>( \rho \in [0.10, 0.90] )</td>
</tr>
<tr>
<td>( \kappa ): tax morale</td>
<td>Individual</td>
<td>Uniform</td>
<td>( \kappa \in [0.10, 0.40] )</td>
</tr>
</tbody>
</table>

Number of simulated tax payers: 10,000

The income disclosures resulting from the simulated environments may be statistically analyzed to assess the effect that each parameter has on tax compliance. Following, the dependent variable \( d^* \) was regressed on the five variables specified in Table 1 in a Tobit model censored at \( [0, 1] \), which includes linear and quadratic terms to test for non-linearity. Equation 5 shows the regression model employed to analyze the simulated taxpayer decisions.

\[
\begin{align*}
d^* = \alpha + \beta_1 \tau + \beta_2 \tau^2 + \beta_3 p + \beta_4 p^2 + \beta_5 \theta + \beta_6 \theta^2 + \beta_7 \rho + \beta_8 \rho^2 + \beta_9 \kappa + \beta_{10} \kappa^2
\end{align*}
\]

Equation 5 shows the regression model employed to analyze the simulated taxpayer decisions.

The results obtained from this mathematical procedure are presented in Table 2. Column 1 of Table 2 shows the estimates and significance levels for all parameters under a limited liability assumption. The applied tax rate \( \tau \) has a negative and significant effect on tax compliance, with a positive estimate for its quadratic term. This result suggests that as the tax rate increases, people would tend to evade more yet with decreasing marginal effects. Audit probabilities \( p \) have a positive and significant deterrence impact on tax evasion. The probability of being audited has a direct and linear effect on tax compliance. Fine rates \( \theta \) have a positive and significant effect to enhance income disclosure, yet the estimate is rather small compared to the effect attained by other parameters. Moreover, fine rates seem to have a decreasing marginal effect. These results confirm previous studies, as in Alm et al. (1992), where it is argued that fine rates do improve tax compliance but are not the most effective mechanism to do it. The level of risk aversion \( \rho \) has a positive and significant effect on tax compliance, while its quadratic term suggests increasing marginal effects. Consequently, if risk aversion is low an increment of it would have a small effect; however, if risk aversion is high then an increment of it would considerably enhance income disclosures. Tax morale \( \kappa \) has a positive, significant and very large effect on tax compliance. The impact of tax morale, however, has a decreasing effect for larger values of tax morale. Similar responses were found by Fochmann and Kroll (2016) for public good provisions. Our results are in line with the findings of Kircher et al. (2008) who suggested how, whenever a society faces low trust and low power, the government should prioritize to motivate voluntary compliance as it would produce larger benefits for tax compliance.

A similar model specification is detailed in Column 2, where an interaction effect between the risk aversion level and tax morale was included in the regression. Once we consider the reciprocal actions
Table 2: Tobit regression for individual declarations \( d^* \in [0, 1] \)

<table>
<thead>
<tr>
<th>Variables</th>
<th>Limited Liability (1)</th>
<th>Limited Liability (2)</th>
<th>No Limited Liability (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>2.2730***</td>
<td>-0.3041***</td>
<td>1.5340***</td>
</tr>
<tr>
<td></td>
<td>(0.0525)</td>
<td>(0.0054)</td>
<td>(0.1027)</td>
</tr>
<tr>
<td>Intercept</td>
<td>-2.2020***</td>
<td>-2.1940***</td>
<td>-1.2610***</td>
</tr>
<tr>
<td></td>
<td>(0.0104)</td>
<td>(0.0011)</td>
<td>(0.0094)</td>
</tr>
<tr>
<td>( \tau ): tax rate</td>
<td>-8.4080***</td>
<td>-9.1140***</td>
<td>-6.9130***</td>
</tr>
<tr>
<td></td>
<td>(0.0988)</td>
<td>(0.1082)</td>
<td>(0.1769)</td>
</tr>
<tr>
<td>( \tau^2 ): tax rate(^2)</td>
<td>6.7400***</td>
<td>7.6800***</td>
<td>8.5260***</td>
</tr>
<tr>
<td></td>
<td>(0.151)</td>
<td>(0.1615)</td>
<td>(0.2845)</td>
</tr>
<tr>
<td>( p ): audit rate</td>
<td>3.4050***</td>
<td>3.3970***</td>
<td>1.9650***</td>
</tr>
<tr>
<td></td>
<td>(0.2215)</td>
<td>(0.2204)</td>
<td>(0.4536)</td>
</tr>
<tr>
<td>( p^2 ): audit rate(^2)</td>
<td>3.2343</td>
<td>2.1420***</td>
<td>2.5850</td>
</tr>
<tr>
<td></td>
<td>(2.152)</td>
<td>(2.1130)</td>
<td>(4.4270)</td>
</tr>
<tr>
<td>( \theta ): fines</td>
<td>0.1121***</td>
<td>0.1259***</td>
<td>0.1674***</td>
</tr>
<tr>
<td></td>
<td>(0.0081)</td>
<td>(0.0081)</td>
<td>(0.0168)</td>
</tr>
<tr>
<td>( \theta^2 ): fines(^2)</td>
<td>-0.0092***</td>
<td>-0.0012***</td>
<td>-0.0023</td>
</tr>
<tr>
<td></td>
<td>(0.0013)</td>
<td>(0.0013)</td>
<td>(0.0028)</td>
</tr>
<tr>
<td>( \rho ): risk aversion</td>
<td>0.3343***</td>
<td>1.1200***</td>
<td>-0.1271</td>
</tr>
<tr>
<td></td>
<td>(0.0363)</td>
<td>(0.0582)</td>
<td>(0.0739)</td>
</tr>
<tr>
<td>( \rho^2 ): risk aversion(^2)</td>
<td>2.0340***</td>
<td>1.7060***</td>
<td>1.4550***</td>
</tr>
<tr>
<td></td>
<td>(0.0388)</td>
<td>(0.0429)</td>
<td>(0.0746)</td>
</tr>
<tr>
<td>( \kappa ): tax morale</td>
<td>8.0450***</td>
<td>9.8150***</td>
<td>5.5080***</td>
</tr>
<tr>
<td></td>
<td>(0.1250)</td>
<td>(0.1591)</td>
<td>(0.1276)</td>
</tr>
<tr>
<td>( \kappa^2 ): tax morale(^2)</td>
<td>-7.8960***</td>
<td>-9.5020***</td>
<td>-5.7040***</td>
</tr>
<tr>
<td></td>
<td>(0.2411)</td>
<td>(0.2560)</td>
<td>(0.3009)</td>
</tr>
<tr>
<td>( \rho \kappa ): Interaction</td>
<td>-2.0220***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.1155)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Signif. codes: ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05

Notes: Individual tax compliance predicted by applied tax rates \( \tau \), audit probability \( p \), fine rates \( \theta \), risk aversion \( \rho \), tax morale \( \kappa \) and their respective quadratic terms. A total of 10,000 simulations were generated, where \( \tau \in [10\%, 50\%] \), \( p \in [0\%, 10\%] \), \( \theta \in [1, 5] \), \( \rho \in [0.10, 0.90] \), \( \kappa \in [0.10, 0.40] \). It is worth mentioning that the highly statistically significant results found are partially inflated given that this is a simulation without any external effects nor policy shocks, therefore this is an explanatory regression and not a forecasting model.

between the two main drivers of compliance, the signs and significance levels of the original parameters remain stable. Slight differences arise, as the coefficients for tax rate \( \tau \) and tax morale \( \kappa \) attain higher absolute values, whereas the estimate for risk-aversion \( \rho \) diminished. In our simulations there is a negative interaction effect between voluntary and enforced motives, analogous to previous empirical studies, Kastlunger et al. (2013), and theoretical models, Prinz et al. (2014). Another interesting robustness exercise is to analyze the same scenarios without assuming limited liability. These results are shown in Column 3 of Table 2. Although the influence of tax rates keeps being negative and significant, its estimate is lower than in the case with limited liability. Moreover, the quadratic term is larger, meaning that the convexity is more pronounced. Audit rates \( p \) directly enhance tax compliance. A second difference applies to the fine rate \( \theta \), which slightly increased its estimate value and possesses now a linear and direct effect on tax compliance. Since there is no limited liability, the impact of fine rates does not dissipates as fast as in the baseline scenario in Column 1. The last variation between both columns is the negative and insignificant effect of risk-aversion on tax compliance. This is counter intuitive and may be associated with the fact that its predictive power was absorbed by the audit and fine rate parameters. Another explanation is given in Kastlunger et al. (2013), who found this situation in an experiment with Italian
self-reporting taxpayers. The authors detected a negative yet small impact of enforcement on compliance, and attributed the effect to an heterogeneity of intimidated and defiant taxpayers. Following, it may be that such ‘intimidated’ individuals followed a limited liability mental-accounting, and were willing to lose a defined maximal amount as penalties. Analogously, the ‘defiant’ subjects would have been those whose mental-accounting process did not consider a limited liability assumption, and thus were considering the possibility of losing unlimited quantities of money. Consequently, an increase on tax enforcement provoked different reactions among heterogeneous groups of taxpayers. At last, the impact of tax morale on compliance is consistent with the one found under limited liability, but the effects of risk aversion are harder to interpret. The limited liability stays along the ‘slippery slope’ framework more accurately than the case where no limited liability exists.

Marginal effects are useful to assess how the outcome variable changes with respect to a specific predictor ceteris paribus. Table 3 shows the marginal effects attained by the linear and quadratic terms of the parameters estimated in Equation 5 and computed from the results in Column (1) of Table 2. Moreover, the marginal effects evaluated at the minimum and maximum values of the support are provided for each explanatory variable. The benchmark fraction of income declared by individuals is of 68%, computed as the average value of each parameter weighted by its coefficient on the model specified in Equation 5.

Table 3: Marginal effects of parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Linear Marginal effect</th>
<th>Quadratic Marginal effect</th>
<th>Marginal effect at Support Minimum</th>
<th>Marginal effect at Support Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau$: tax_rate</td>
<td>-1.2720</td>
<td>1.0422</td>
<td>-0.1147</td>
<td>-0.3733</td>
</tr>
<tr>
<td>$p$: audit_rate</td>
<td>0.4538</td>
<td>0.5843</td>
<td>0.0000</td>
<td>0.0530</td>
</tr>
<tr>
<td>$\theta$: fines</td>
<td>0.0194</td>
<td>-0.0018</td>
<td>0.0149</td>
<td>0.0483</td>
</tr>
<tr>
<td>$\rho$: risk_aversion</td>
<td>0.0454</td>
<td>0.2993</td>
<td>0.0078</td>
<td>0.2819</td>
</tr>
<tr>
<td>$\kappa$: tax_morale</td>
<td>1.1939</td>
<td>-1.1848</td>
<td>0.1061</td>
<td>0.2864</td>
</tr>
</tbody>
</table>

Benchmark disclosed income: 68%

A graphic representation of the marginal effects listed in Table 3 is illustrated in Figure 3. Tax morale has a strong concave effect on tax compliance, while the impact of risk aversion appears in a convex manner. An inverse growth relationship between voluntary and enforced compliance has been assessed both empirically by Kastlunger et al. (2013) and theoretically by Prinz et al. (2014). In this sense, an increment in the level of enforcement would decrease the marginal effect of an increment in the voluntary compliance. Therefore, in scenarios with low trust and low power, governments should opt to foster tax morale and voluntary compliance, as the marginal effect in this area of the curve is larger for tax morale than for risk-aversion. However, for settings where both trust and power are high, the government might be better-off by increasing the enforcement as the marginal gain for risk-aversion is larger than the one for tax morale at the end side of the curve. Nonetheless, as mentioned by Kirchler et al. (2008) such case would be of little interest as taxpayers would already be in a compliant stable area of the curve. Figure 4 shows how audit probabilities and fine rates enhance tax compliance, but their influence is comparatively modest.
Increasing tax rates foster evasion, which has been suggested by Alm et al. (1992) and Kleven et al. (2011). Figure 5 shows the different impact of tax rates on tax compliance, both under limited liability and without. Whenever the maximum possible penalty an agent may pay is its entire income, tax rates play a negative effect on tax compliance (black line). However once we remove this assumption, the relation becomes more complicated as the marginal effects acquire a curvature for large tax rate values (red line). Without limited liabilities and for lower values of tax rates, increasing taxes enhances evasion; yet for higher tax rate values, a raise in taxes commences to augment individual tax compliance. Such outcome conforms to the scenario explored by Allingham and Sandmo (1972) in their seminal model.

Simulated environments allow to analyze different facets of taxpayer behavior. Figure 6 shows the density distribution of the willingness-to-pay taxes $\gamma$ for different societies, where the tax morale of agents is bound from above at three different levels: 0.25 (low), 0.50 (mid) and 0.75 (high), and the risk-aversion levels remain as before $\rho \in [0.1, 0.9]$. The black line shows how the willingness-to-pay follows a humped
distribution whenever tax morale is relatively low. In this case, a large fraction of the population would be unwilling to comply, while a relatively small fraction would be law abiding. The blue line shows a society where mid-level tax morale can attain any of the possible values previously defined in Section 3. The distribution becomes much more flatter and includes a heavy tail of highly compliant individuals. Moreover, the red line depicts very high values of tax morale which are beyond the ones defined in our model. This extreme case scenario is merely illustrative to show how the willingness-to-pay becomes flatter and with heavier tails as the individual values of tax morale increase.

Figure 7 shows the distribution of individual optimal income disclosures. The simulated data matches extremely well the experimental results in Alm et al. (1992) and the field experiment by Dwenger et al. (2016). The black line evidences that for a society with low levels of tax morale, the share of full-evaders is slightly larger than the one of full-compliant taxpayers. For a collective of agents with a medium tax morale, the blue line shows how a fair share of agents fully-comply while the share of full-evaders is substantially reduced. At last, an extreme scenario is framed with a red line. Allowing for unattainable high values of tax morale, the graph shows a large group of agents converging to full compliance. However, this scenario is merely exploratory as the parameter values are unattainable inside the support defined for individual tax morale.

5 Conclusions

There are two broad research lines dealing with tax evasion and the decision making process of taxpayers. On the one hand, economic literature has attempted to explain this problem through expected utility models, where rational agents choose the optimal amount or fraction of income to disclose to the government. On the other hand, the behavioral approaches have discovered many interesting findings on how people differently perceive enforcement parameters and display heterogeneous levels of trust in authorities. The ‘Slippery Slope’ Framework introduced by Kirchler et al. (2008) has triggered a wide debate among economist and psychologists, offering new insights to policy makers who are advised to drop the ‘cop robber’ model to embrace a ‘service-client’ approach. Our paper attempts to bridge both aforementioned strands of research introducing a micro-founded model which explains many of the stylized facts of individual and collective tax evasion, while staying faithful to the ‘slippery slope’ paradigm.

We model trust in authorities as the voluntary compliance of individuals which is motivated by their tax morale. The perceived power of authorities is introduced as the enforced compliance, driven by the risk aversion of individuals, the probability of being audited, the penalty fees, and the tax rate faced by taxpayers. Some properties of the model can be analytically derived and we show conditions under which agents fully comply even in the (known) absence of audits by the tax authority. Such behavior is mainly motivated by tax morale under low tax rates, but it is primarily induced by risk aversion for high tax rates, adding a level of complexity to the ‘slippery slope’. Additionally, conditions were studied to
explain the rationale behind altruistic charitable giving. The main results of this paper can be expressed in terms of the willingness-to-pay taxes, which may be interpreted as a tax morale corrected for risk aversion. Following, individual tax compliance is predominantly driven by this metric.

Using Agent-Based simulations we further study the two main incentives to comply, tax morale (trust) and risk aversion (power). Under a limited liability assumption, we find that higher tax rates significant augment tax evasion. Meanwhile, audit probabilities and penalties have a significant, yet secondary, proportional effect on tax compliance. In addition to the analysis on parameter sensibility, many other stylized facts where found. Namely, the individual responses seem to follow a dichotomous distribution, where the majority of agents either fully evade or fully disclose their income, while only a relatively small fraction of individuals choose an interior solution. Furthermore, there is a complex relationship between tax morale, risk aversion and tax compliance. The outcome of tax simulations hinted at a concave impact of tax morale on compliance, while risk aversion played a convex effect. Consequently, for societies where trust in government is low and enforcement power is also low, then the government should give high priority to find ways to boost voluntary compliance. However, for scenarios where both trust and power are high, then the government might be inclined to increment its perceived power to conduct audits and apply penalties. Nonetheless, as mentioned by Kirchler et al. (2008), this second setting is of little interest as in such case individual risk-aversion would be to a big extent irrelevant given that people would follow the social norm of complying with authorities.

References


Appendix

Proposition 1: Whenever the audit probability is zero

Assume that the audit probability is set to zero such that the expected utility form can be expressed as $EU(d) = U(Y)$. Then, for each agent $i$ at time $t$, but removing the subindex for simplicity:

$$EU(d) = (1 + d)\kappa[I(1 - \tau d)]^{1-\rho}$$

and

$$\frac{\partial EU(d)}{\partial d} = (1 + d)\kappa(1 - \rho)[I(1 - \tau d)]^{-\rho} (\tau I) + \kappa(1 + d)\kappa^{-1}[I(1 - \tau d)]^{1-\rho}.$$

Equalizing the partial derivative to zero and dividing both sides by $(1 + d)\kappa^{-1}[I(1 - \tau d)]^{1-\rho}$ we get:

$$(1 + d)(1 - \rho)(\tau I) + \kappa[I(1 - \tau d)] = 0.$$

Solving for $d$ we can derive the optimal fraction of declared income whenever the probability of being audited is assumed to be null as a function of $\tau, \rho, \kappa$ as follows:

$$[d^*|p=0] = 1 - \frac{(1-\rho)\tau}{\kappa(1-\tau d)},$$

(6)

where $d^* \in [0,1]$.

Given that the denominator is strictly positive, the boundary conditions of full-evasion and full-compliance may be computed from the numerator term. An agent will be a full-evader whenever the numerator is smaller or equal than zero. That is, whenever:

$$\frac{1 - \rho_{i,t}}{\kappa_{i,t}} \geq \frac{1}{\tau},$$

or analogously, whenever:

$$\frac{\kappa_{i,t}}{1 - \rho_{i,t}} \leq \tau.$$

Following, a new parameter $\gamma$ is defined as the willingness-to-pay taxes, and may be understood as an agent’s measure of tax morale corrected by its own risk aversion: $\gamma \equiv \frac{\kappa}{1 - \rho}$. The willingness-to-pay taxes is increasing with respect to both tax morale and risk aversion. Consequently, whenever an agents’ willingness-to-pay taxes is smaller or equal to the tax rate, and the audit probability is zero, full-evasion will take place.

∴ If $\gamma \leq \tau \Rightarrow [d^*|p=0] = 0.$

(7)

Following, agents may be fully-compliant even in scenarios where the audit probability is zero. Substituting for $\gamma$ in Equation (6), agents fully comply even in the absence of audits whenever:

$$\frac{1 - \frac{\tau}{\gamma_{i,t}}}{\frac{\tau}{\gamma_{i,t}}} \geq 1.$$

(8)

Applying basic algebra, it is straightforward to derive from Equation (8) the following condition:

∴ If $\gamma \geq \frac{2\tau}{1 - \tau} \Rightarrow [d^*|p=0] = 1.$

(9)

Consequently, if the willingness to pay taxes is high enough with respect to the tax rate, then agents may be full-compliers even in the absence of audits. The term on the right is increasing with respect to $\tau$. This suggests that if taxes increase, less individuals would be fully-compliant whenever the audit
probability is zero.

From Equation (6) and Results (7) and (9) it is immediate to summarize these conditions as the following expression:

\[
[d^*]_{p=0} = \begin{cases} 
0 & \text{if } \gamma \leq \tau, \\
(0,1) & \text{if } \tau < \gamma < \frac{2\tau}{1-\tau}, \\
1 & \text{if } \gamma \geq \frac{2\tau}{1-\tau}.
\end{cases}
\]

\[\square\]

\section*{Proof of Proposition 2: Whenever the audit probability is one}

Assume audits are certain, thus making \( p = 1 \) and allowing to reformulate the expected utility as \( EU(d) = U(X) \). Then, for each agent \( i \) at time \( t \), but removing the subindex for simplicity:

\[
\frac{\partial EU(d)}{\partial d} = (1 + d)^\kappa (1 - \rho) [I(1 - \tau d - \theta \tau (1 - d))]^{-\rho}[I(-\tau + \theta \tau)] + \kappa(1 + d_n)^{\kappa-1}[I(1 - \tau d - \theta \tau (1 - d))]^{1-\rho} = 0
\]

Equalizing the partial derivative to zero and dividing both sides by \( (1 + d_n)^{\kappa-1}[I(1 - \tau d - \theta \tau (1 - d))]^{-\rho} \) one can simplify to:

\[
(1 + d)(1 - \rho)(-\tau + \theta \tau) + \kappa(1 - \tau d - \theta \tau (1 - d)) = 0.
\]

\[
\Rightarrow (1 + d)(1 - \rho)(\tau - 1) = -\kappa(1 - \tau d - \theta \tau (1 - d)).
\]

\[
\Rightarrow (1 + d) \frac{(1 - \rho)}{\kappa}(\tau - 1) = d(1 - \theta) + \frac{1}{\tau}.
\]

Applying basic algebra and substituting for \( \gamma \) it is straightforward to derive \( d^* \) as:

\[
[d^*]_{p=1} = \frac{\theta \tau - 1}{\theta - 1} - \frac{\tau}{\gamma},
\]

which is similar to Equation (6), where \( p = 0 \). Solving for \( \gamma \), the optimal fraction of income declared is equal or larger than one whenever:

\[
\gamma \geq \frac{2\tau}{\theta \tau - 1 - \tau}.
\]

\[\square\]
Proof of Proposition 3: Are people worse-off after paying taxes?

This proof seeks to know how the expected utility of individuals is affected by a change in the tax rate. Under a limited liability assumption as in Piolatto and Rablen (2017), we consider non-negative net incomes after audits and penalties. The partial derivative is:

\[
\frac{\partial EU(d)}{\partial \tau} = (1 + d)^\kappa I^{1-\rho}(1 - \rho)X^{-\rho}[-d - \theta d] + (1 + d)^\kappa I^{1-\rho}(1 - \rho)Y^{-\rho}[-d].
\]  

(12)

In the right-hand-side of Equation (12), both terms are non-positive. For both terms \((1 + d)^\kappa I^{1-\rho}(1 - \rho)X^{-\rho}\) is strictly positive by definition of the model. Moreover, \(Y^{-\rho}\) is strictly positive and \(X^{-\rho}\) is non-negative by the limited liability assumption. Finally, \([-d - \theta d\) is strictly positive as \(\theta + d > \theta d\) and the term \([-d\) is non-positive by definition. Therefore, both adding terms are non-positive, and thus the partial derivative as well, giving:

\[
\Rightarrow \frac{\partial EU(d)}{\partial \tau} \leq 0,
\]

where the equality holds only for both \(d = 0\) and \(X = 0\) simultaneously. Such would be the scenario in which an agent fully-evaded its tax duties \((d = 0)\) and then was caught by the Tax Agency, losing its entire income \((X = 0)\) in penalty fees. In which case the tax rate itself would become irrelevant, once the individual lost all of its income.

\[\square\]

Proof of Proposition 4: Conditions for Donations

Consider the expected utility whenever agent \(i\) chooses to disclose its full income, i.e. \(EU[d = 1]\). Then, from Equation 2 one has:

\[
EU[d = 1] = p \cdot (1 + d)^\kappa [I - \tau(d \cdot I) - \theta \tau(I - d \cdot I)]^{(1-\rho)} + (1 - p) \cdot (1 + d)^\kappa [I - \tau(d \cdot I)]^{(1-\rho)}.
\]

In this scenario there is no uncertainty. Given that the taxes have been paid in full, the net income after audits and penalties is deterministic. Thus, substituting the value of \(d\) and applying some algebra:

\[
EU[d = 1] = p \cdot 2^\kappa[I(1 - \tau)]^{(1-\rho)} + 2^\kappa[I(1 - \tau)]^{(1-\rho)} - p \cdot 2^\kappa[I(1 - \tau)]^{(1-\rho)},
\]

giving the following result:

\[
EU[d = 1] = 2^\kappa[I(1 - \tau)]^{(1-\rho)}.
\]  

(13)

Consider now the expected utility whenever agent \(i\) chooses to disclose its full income plus an additional charitable giving \(\delta > 0\), i.e. \(EU[d = 1 + \delta]\). Then, from Equation 2 one has:

\[
EU[d = 1 + \delta] = p \cdot (1 + d)^\kappa [I - \tau(d \cdot I) - \theta \tau(I - d \cdot I)]^{(1-\rho)} + (1 - p) \cdot (1 + d)^\kappa [I - \tau(d \cdot I)]^{(1-\rho)}.
\]

Given that the agent has fully disclosed its income (surpassed it, in fact) there is no uncertainty about the audit or no-audit state. Even if audits take place, the agent will not be penalized and no fees will apply, as there was no tax evasion involved. The equation can be simplified to:

\[
EU[d = 1 + \delta] = p(2 + \delta)^\kappa [I - \tau(1 + \delta)I]^{(1-\rho)} + (1 - p)(2 + \delta)^\kappa [I - \tau(1 + \delta)I]^{(1-\rho)}
\]

\[
= (2 + \delta)^\kappa[p[I(1 - \tau - \tau \delta)]^{(1-\rho)} + [I(1 - \tau - \tau \delta)]^{(1-\rho)} - p[I(1 - \tau - \tau \delta)]^{(1-\rho)}.
\]

Giving the following result:

\[
EU[d^* = 1 + \delta] = (2 + \delta)^\kappa[I(1 - \tau - \tau \delta)]^{(1-\rho)},
\]  

(14)

The next step is to compare the results from Equation 13 and Equation 14 and to see when it is optimal for an agent to over-comply or endorse charity giving.

\[
EU[d = 1] < EU[d = 1 + \delta] \iff 2^\kappa[I(1 - \tau)]^{(1-\rho)} < (2 + \delta)^\kappa[I(1 - \tau - \tau \delta)]^{(1-\rho)},
\]
\[ \iff 2^\kappa (1 - \tau)^{(1 - \rho)} < (2 + \delta)^\kappa (1 - \tau - \tau \delta)^{(1 - \rho)}, \]
\[ \iff 1 < \left( \frac{2 + \delta}{2} \right)^\kappa \left( \frac{1 - \tau - \tau \delta}{1 - \tau} \right)^{(1 - \rho)}, \]
\[ \iff 0 < \kappa \ln \left( \frac{1 + \delta}{2} \right) + (1 - \rho) \ln \left( \frac{1 - \tau \delta}{1 - \tau} \right). \]

Which can be expressed in terms of \( \gamma \equiv \frac{\kappa}{1 - \rho} \) as:

\[ EU[d = 1 + \delta] > EU[d = 1] \iff \gamma > -\frac{\ln \left( \frac{1 - \tau \delta}{1 - \tau} \right)}{\ln \left( \frac{1 + \delta}{2} \right)}. \tag{15} \]

Moreover, for infinitesimally small values of \( \delta \), one can apply L’Hopital’s Rule and obtain:

\[ \lim_{\delta \to 0^+} -\ln \left( \frac{1 - \tau \delta}{1 - \tau} \right) \cdot -\frac{1}{\ln \left( \frac{1 + \delta}{2} \right)} \cdot \frac{1}{1 - \tau} = \frac{1}{1 - \tau} \cdot \frac{1}{1 - \tau} = 2 - \tau. \]

\[ \blacksquare \]

**Absolute Risk-Aversion and Relative Risk-Aversion**

Recalling the power utility function specified in Equation 3, it is easy to derive the first and second derivatives with respect to \( W \).

\[ U(d) = (1 + d)^\kappa W^{(1 - \rho)}. \]
\[ U_W'(d) = (1 + d)^\kappa (1 - \rho) W^{(-\rho)}. \]
\[ U_W''(d) = (1 + d)^\kappa (1 - \rho)(-\rho) W^{(-\rho - 1)}. \]

And so the coefficient of absolute risk-aversion \( A(W) \) is given by:

\[ A(W) = \frac{-u''}{u'} = \frac{-(1 + d)^\kappa (1 - \rho)(-\rho) W^{(-\rho - 1)}}{(1 + d)^\kappa (1 - \rho)W^{(-\rho)}}, \]

which simplifies to:

\[ A(W) = \frac{-u''}{u'} = \frac{\rho}{W}. \]

Following, the coefficient of relative risk-aversion \( R(W) \) is given by:

\[ R(W) = W \cdot A(W) = \rho. \]

\[ \blacksquare \]