

# KNOWING HEAVEN AND EARTH THROUGH MUSIC<sup>1</sup>

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## 1. Introduction: Musical theory and practice intertwined

In 1581 Vincenzo Galilei compared the collapse of a boy's angelic soprano voice due to its inevitable mutation with the equally inevitable, declining beauty of a woman advancing in age. In his view, either loss is apparent only, as it is a certain kind of secret knowledge that constitutes the true repository of beauty. 'Neither hoarseness nor a voice's mutation can divest someone of knowledge', so Galilei observes. Similarly, 'as long as her face maintains the desirable proportion of lines and colors that converge to form its beauty, all the world admires her ... as beautiful thanks to the agreement of those traits'.<sup>2</sup> In this well-considered view of a late sixteenth-century highly original music theorist who was also an accomplished lute-player and a composer, then, beauty is not so much in the eye of the beholder as, rather, the expression of certain universal rules in the object of knowledge. All over the *Dialogo della musica antica, et della moderna* that has this enticing comparison for its final clause, Galilei maintains a centuries-old Pythagorean tradition in regarding such universally valid mathematical rules as residing in historical conceptions of *world harmony* and of a concomitant *numerical model* that jointly underlie all possible music theory and practice. Galilei asked himself why the music of the ancient Greeks had been so effective, and why the music of his own time – just as all aesthetic judgement –, in his mind at least, was so superficial and frivolous. It became one of the ruling quests of Galilei's life to understand this problem and to rediscover the lost harmonic knowledge of the universe. However, in his *Dialogo* Galilei was also out to reconcile this hoary model with a new, *rhetorical-expressive model* that was inspired by ancient Greek ideas about the ethical power of music to affect the human soul.

In the sixteenth century innovation was sought in two directions: in the reordering and synthesis of past knowledge of world harmony and the ethical power of music, and in first attempts to set music theory and practice free from what now came to look like the trammels of traditional conceptions. The story to be told in the present chapter, then, concerns centrally how the doctrine of world harmony and the numerical model that went with it came into a situation of profound tension with a new ideal of musical expression advocated and tried out by Galilei and many other sixteenth-century music scholars about the nature and ultimate end of music. In describing and analysing the two models and this rising tension between them, we shall be concerned to show how closely universal theorizing and musicians' everyday practical concerns were intertwined in the making of, and the thinking about, music over the length and breadth of the period here treated. This is so because Galilei's (and many another's) quest had vast practical implications for the significance of certain intervals, for the question of what musical notes actually to use, and for how to tune one's instruments. The intimate

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<sup>2</sup> Vincenzo Galilei, *Dialogue*, p. 374.

connection between our various topics will become readily apparent even as we discuss them as much as possible one by one – musical humanism in the sense of competing doctrines of world harmony and musical *ethos*, and tuning systems and temperaments as practice-imbued consequences of how number and sound serve as the indispensable foundation for musical theory. Treatment of these various topics is followed by two case studies of how all this worked out with string and with keyboard instruments. With a view to placing all this in a wider frame we shall now sketch out a thought-provoking vision of the dynamics of musical history in the Western tradition that the German sociologist and cultural historian Max Weber outlined in 1913.<sup>3</sup>

At the heart of Weber's uncompleted treatise on decisive turning points in musical history is a conception of the extraordinary development of Western music, from the early Middle Ages onwards, in the world-historically unique direction of an ongoing subjection to rational rules of a particularly strict character. In his view, the development of our diatonic tonal system, like so many other developments in the West, is due to an ongoing historical process of *rationalization*, with tonal material and the corresponding musical theory being stripped step by step of extra-musical elements, notably of idioms and specific qualities, which are to the maximum extent possible reduced to functional elements. The diatonic principles of a functional harmonic system overrule more and more the vast variety it displayed in earlier times – a development that, *in spite of all the subtle losses incurred thereby*, would in the end culminate in equal temperament as a rational foundation of modern Western music practices. As Weber insisted, ever-increasing rationalization is not a one-dimensional, unidirectional historical process – to the contrary, it is precisely in the sixteenth century that, by way of a fresh countermovement, an allegedly very old (but actually quite new) rhetorical model arose that allowed the creation of novel, unheard-of outlets for musical expression. In Weber's view, it was specifically 'the great musical experimenters of the Renaissance period [who] created [this novel form of expression] in their tempestuously-rational striving for discovery'.<sup>4</sup>

According to Weber, during the sixteenth century music was subject to a relentless process of disenchantment, i.e., the cultic melodies that had once enchanted the world were being modernized and turned into an efficient means of harmonic production. Whereas Weber only summarily explained in normative terms that at the end of the sixteenth century new ideas and practices of tuning and temperament began to desensitize the ears of musicians and their audience with a 'dulling effect' and shackled music in 'dragging chains', from a more neutral historiographical point of view we demonstrate in this chapter how ideas about cosmic harmony were transformed in order to suit new

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<sup>3</sup> Weber, *Rational and Social Foundations of Music*.

<sup>4</sup> Weber, 'Sinn der Wertfreiheit', p. 484.

musical ideas.<sup>5</sup>

Several more steps of major significance induced the process of the rationalization of musical materials. In each step, so Weber insisted, *technology* was the decisive element. He lists the following large-scale musical innovations made from the early Middle Ages onwards: (1) music notation; (2) mensuration according to varied, non-metronomic rhythmical patterns; (3) polyphony governed throughout by harmonic considerations; (4) chromatic alteration applied by way of a refinement of these harmony-based musical structures; (5) musical instruments, notably string instruments and keyboard instruments, serving in effect as the principal carriers of the entire, unparalleled development.

Central to Weber's conception of the world history of music is the paradox that precisely the art form that more directly than any other may affect us to the core of our emotional being finds itself circumscribed by rigorous rules, imposed by elementary arithmetic, and on that very basis allows (yet does not require) the most thorough-going rationalization. The entire rationalization process, that quite inadvertently began with early medieval monks and then received boost after boost in later times, finally culminated in equal temperament. This seemingly natural yet really odd and artificial temperament is the ultimate, yet not at all inevitable outcome of the highly unusual method for selecting notes from the infinite multitude available undertaken for the first time in the early Middle Ages.

Not, as elsewhere, by means of a division of the *fourth* in view of their mutual *distances*, but of the *fifth* in terms of *ratios* is how Western harmony was in the end to come about. In ancient Greece as in other 'non-Western' civilizations, the space of the fourth was customarily filled with notes chosen in view of their respective distances from (for example) C on one side and F on the other, with melodic requirements serving as the supreme arbiter. Quite unlike this universal pattern, by means of Pythagorean ratios but greatly extending them in the process so as now to include the consonances with the number 5 in their numerical definition as well, music masters in medieval Europe uniquely opted for dividing the fifth 2:3, thus producing the major third 4:5 and the minor third 5:6 (in the next section we explain how musical intervals and numerical ratios were held at the time to be connected). Undertaken for its own sake to be sure, this world-historically unique, arithmetical division of the fifth nonetheless proved in due time to be the decisive step toward a rationalized music governed in the first place by harmonic considerations and rooted in the triad – the chord harmony all of us in the West have become so thoroughly familiar with in our own musical upbringing.

Not that Weber was a one-sided supporter, or even a neutral chronicler, of the relentless rationalization process that he recognized in musical history. All kinds of considerations that

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<sup>5</sup> Here we build forth on Chua's argument in his *Absolute Music*, p. 12-28, and in his 'Vincenzo Galilei', p. 17-29.

governed and still govern ‘non-Western’ tonal systems came in every now and then to counteract, upset or at least disturb the drive, built into the music of the West, in the direction of ever-enhanced rationalization. They did so from the inside, in that the arithmetically determined, mutual incompatibility of the consonances makes for all kinds of inherent asymmetries (e.g., between the major scales and their minor counterparts) and irregularities (e.g., intervals that are marked by numerical ratios with 7). They did so, too, from the outside. The flow of the melody, primary in other musical traditions, is in Western harmony out all the time to regain a place of its own. Dissonances, elsewhere unproblematic from the outset, regain more and more territory in the guise of special effects, starting in the late sixteenth century with the dominant-seventh chord. The utter *irrationality* that centrally marks what music means in the life of a human being thus kept intruding time and again, at all kinds of spots in all kinds of ways and doing so (as we shall have ample occasion to observe) with unprecedented insistence in the course of the sixteenth century. Even so, the drive toward musical rationalization has over time proved relentless indeed, as notably in the development of equal temperament. Not that the flexibility that (unlike any other tuning system) equal temperament possesses in allowing unlimited transposition and unrestrained modulation inside the Western harmonic tonal structure made its final predominance easy or even inevitable. To the contrary, equal temperament comes with grave drawbacks of its own, which up to the early nineteenth century stood in the way of its final acceptance. In sum, then, the history of Western music is uniquely characterized by a drive towards ever enhanced rationalization that nonetheless is neither complete nor invariably positive in its unintended consequences – what you gain in one respect, you often stand to lose in another.

In this chapter, Weber’s paradox will be investigated in further detail. First, a transformation of the concept of *musica mundana* (the presence of harmony in the cosmos) that took place during the sixteenth century will be analysed.<sup>6</sup> Central in this part of the chapter are reconsiderations about the nature of number and sound, which had big implications for the numerical model of the cosmos. Second, new ideas about music’s power to affect man’s soul will be discussed. This part of the chapter is focused on the question of how sixteenth-century scholars attempted to reconcile a new rhetorical model centred round the concept of musical expression with the old numerical model. Finally, in two case studies in the last part of the chapter, the interaction between emerging new music-theoretical

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<sup>6</sup> On this transformation, see Stefano Mengozzi, *The Renaissance Reform of Medieval Music Theory* (Cambridge University Press, 2010), Klaus Wolfgang Niemöller, “Zum Paradigmenwechsel in der Musik der Renaissance”, Jessie Ann Owens, “Music Historiography and the Definition of “Renaissance” and Philippe Vendrix, “L’impossible Renaissance musicale: Les débats sur l’histoire de la musique de la Renaissance au XXe siècle” and “Music and Model in the Renaissance”.

ideas and musical practice will be discussed. This part of the essay focuses on musical instruments as a source for the historical study of music theory to illustrate how “the great musical experimenters of the Renaissance period” used musical instruments as tools in the context of scientific experimentation.

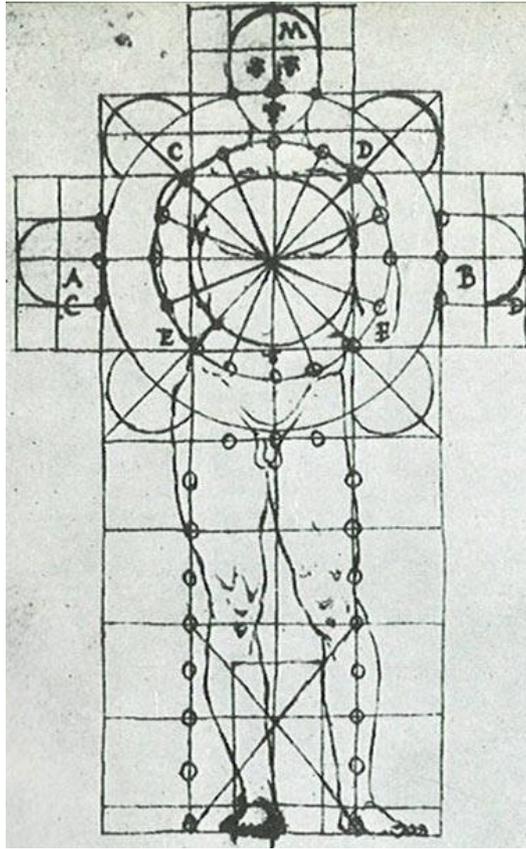
## **2. Musical Humanism: competing models of world harmony and musical *ethos***

### **2.a. World harmony: the numerical model**

Central to all Western musical thought until far into the seventeenth century was a conception of cosmic harmony. Consonant intervals are represented by ratios of integral numbers, as the octave by 1:2, or the fifth by 2:3. These ratios, originally derived from the lengths of the strings that produce the consonances, were in the Pythagorean tradition regarded as nothing less than the fundamental constituents of the world. According to the Pythagorean model, there is a cosmic harmony, not really audible but still universally present in that we can grasp its presence by means of our intellect. In the definitive guise that was given to it by the late Roman mathematical theorist Boethius, it takes three closely connected forms. There is what in the sixteenth century was still being called *musica mundana* – cosmic harmony in the widest sense of the ‘music of the spheres’, that is, of the harmonic relations held to obtain between, notably, the orbits of the planets around the stationary Earth. There is *musica humana*, the manner in which cosmic harmony stands reflected in each of us individually – in our bodies no less than in our souls. And there is *musica instrumentalis*, the audible music that (whether it be sung or played on a musical instrument) we actually make and listen to.<sup>7</sup> Consequently, all art and all music making should be an expression of the harmony of the world, and in experiencing any art music one is not just enjoying a well-written and well-executed piece, but one also partakes in what holds the cosmos together, that is, in the arithmetical ratios constituting the ‘tuning of the world’ [Figure 1].

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<sup>7</sup> Boethius, *Fundamentals of Music*. I, ii. p.9-10.



*Figure 1: Architectural drawing of a church based on ‘musica humana’ – measures of the divinely created human body (Francesco di Giorgio Martini; Codex Magliabechiano, fol. 42v)*

The traditional manner in which Boethius and his medieval successors conceived of world harmony confined musical ratios to just three consonances: the octave (2:1), the fifth (2:3), and the fourth (3:4). With the twelfth-century codification of polyphony in terms of notation and with the early-fourteenth-century admission of pure thirds and sixths in the practice of art- (not just folk-) music, scholars given to speculation about the nature of music began to face a need to catch up in theory with what was actually happening in musical practice. Among the resources that late-fifteenth and sixteenth-century thinkers began to call upon to that end were the following two.

First, numerous source materials in musical theory became newly available to European humanists following the fall of Byzantium in 1453. These materials were foreign to the traditional philosophy of music derived from Boethius, their neglect being most often due to sheer lack of awareness of their very existence. Works by ancient scholars like Aristoxenus, Plutarch, Ptolemy, Plotinus, Proclus and Aristides Quintilianus enabled musical humanists to raise new questions, but also, even more importantly, to seek new answers to old questions. Not only did theoretical knowledge of the Greek tonal system and of the ancient doctrine of the modes and their specific effects on the human mind become newly available. A few surviving fragments in ancient Greek musical notation were also

recovered, along with the Alypius table that might make it possible to decipher them.

Secondly, the recovery of these sources went together with a new, more empiricist approach toward those realities of everyday musical life that now gave occasion to reviewing quite thoroughly what the very idea of ‘music’ really is. This urge to take the phenomena of musical experience themselves, not their customary, *a priori* intellectualization for point of departure is reinforced by similar tendencies in other domains (e.g., painting, or the pursuit of knowledge of nature, both exemplified in stupendous fashion by Leonardo da Vinci). This was likewise reinforced by the ever more frequent observation that musical practice had meanwhile outpaced musical theory by large stretches.

These two developments led to re-evaluation of the standing conception of world harmony from a variety of novel perspectives that, in their ever more intricate interplay, gave rise eventually to an allegedly very ancient (but actually quite novel), *rhetorical* model of musical *expression*. This model forms at one and the same time the polar opposite of, and a significant complement to, the still standard, *numerical* model of musical *harmony*. The many attempts that many musical theorists undertook (at first almost exclusively in Italy) to transform these two distinct models into one all-encompassing, coherent whole are central to the present section, and provide the story of what happened in music practice during the period with much of its underlying dynamics. It should not be thought, to be sure, that sixteenth century musical theorists were all that clearly aware that they were in the business of defining a new model for music. Rather, most of them were convinced that their ideas about world harmony and the effect of music upon the human mind were not new at all, but rather a faithful reconstruction of notions first entertained by the ancient Greeks.

The process of gradual rethinking along the lines here sketched out begins with the Italian philosopher, Marsilio Ficino, who in 1489 published his *De vita triplici* (‘Three books on life’), and in 1496 the final version of his *Compendium in Timaeum*. The two treatises may well be regarded as bridges between medieval and early-modern conceptions of harmony and music. After the (to his mind) ‘dark’ Middle Ages in which true musical knowledge went into oblivion, Ficino seeks to revive the age-old Pythagorean-Platonic doctrine of world harmony, which he envisions as a ‘cosmic mystery’ hidden in nature behind the ‘mathematical images’ described in sources such as Plato’s *Timaeus*.<sup>8</sup> By linking the ancient Greek doctrine of world harmony up with biblical passages such as Solomon’s ‘Thou hast ordered all things in measure and number and weight’ he feels able to find the key to the secret harmonic knowledge of the world, because unlike Pythagoras and Plato he is a Christian, so God can speak directly to him by way of revelation.<sup>9</sup> As against polyphonic musical

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<sup>8</sup> Marsilio Ficino, *Opera Omnia* I, Epist. Lib. XI to Paulus Middelburgensis, p. 944.

<sup>9</sup> Wisdom 11:20e; Ficino, *Compendium in Timaeum*, Cap. XVIII, p. 64<sup>f</sup>.

culture and the normative ideal of Pythagorean tuning he pleads for monody and for new ways of tuning and temperament. In his *Timaeus* commentary, for example, Ficino still praises the fourth as one of the three Pythagorean consonances, but admits that it is ‘not indeed approved by the hearing in itself.’ Contrary to the Pythagorean tradition, but in line with contemporary musical practice, he admits the third and the sixth as beautiful consonances in his music theory ‘for the third and the sixth are similar in sweetness just as the second ... and the seventh ... are in harshness’.<sup>10</sup>

Ficino thoroughly studied all accounts of the ethical and healing power of music and asked himself what specific means enabled the ancient Greeks and Jews to produce music to so spectacular effect as the glowing descriptions thereof suggested, and to work so wonderful and admirable effects upon body and mind alike. He reports the famous story of the Puglian who after being stung by a venomous spider was cured by music, because ‘he dances along with the sound, works up a sweat, and gets well’.<sup>11</sup> Moreover, he is convinced that ‘that singing through which the young David used to relieve Saul’s insanity ... one might attribute to nature’.<sup>12</sup> Nature, however, in the mind of Ficino is an integral part of the very metaphysical harmonic structure of the universe. Ficino’s philosophy of music represents an attempt to bridge the chasm he observes between theory and practice by using the doctrine of cosmic harmony in the context of magical and astrological musical practices.

Initially, the same is true of Franchino Gaffurio, who likewise exemplifies the novel, humanist orientation in thinking about music. His *Theorica musice* (1492) and his *Practica musice* (1496) still maintain the customary distinction between theory and practice. But his *De harmonia musicorum instrumentorum opus* (1518) testifies to a new view of music in which the two go hand in hand. This development is reflected in his ideas about world harmony. In a discussion about the musical modes in relation with *musica mundana* Gaffurio concludes that ‘there are those who believe that the modes participate in celestial harmony. They say the sun rules the Dorian, and ascribe the Phrygian to Mars, the Lydian to Jupiter, and the Mixolydian to Saturn. Although they are ascribed to the eight lower strings in [chapter] I.2 of [my] *Theorica*, a later description will show them more clearly.’<sup>13</sup> As promised here, in *De harmonia* he develops the relation between musical modes in earthly music and the heavenly music of the planets further, with a view to aligning theory and practice. Just like Ficino, by projecting contemporary ideas about tuning and temperament on the music of the heavens, Gaffurio transforms it into something new [*Figure 2*].

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<sup>10</sup> Ficino, *Compendium in Timaeum*, Cap. XXXIII, p. 73<sup>r</sup>.

<sup>11</sup> Ficino, *De vita* III.xxi, p. 362-363.

<sup>12</sup> Ficino, *De vita* III.xxi, p. 354-355.

<sup>13</sup> Gaffurio, *De harmonia* IV, 5 (p. 184 in the Miller 1977 edition).

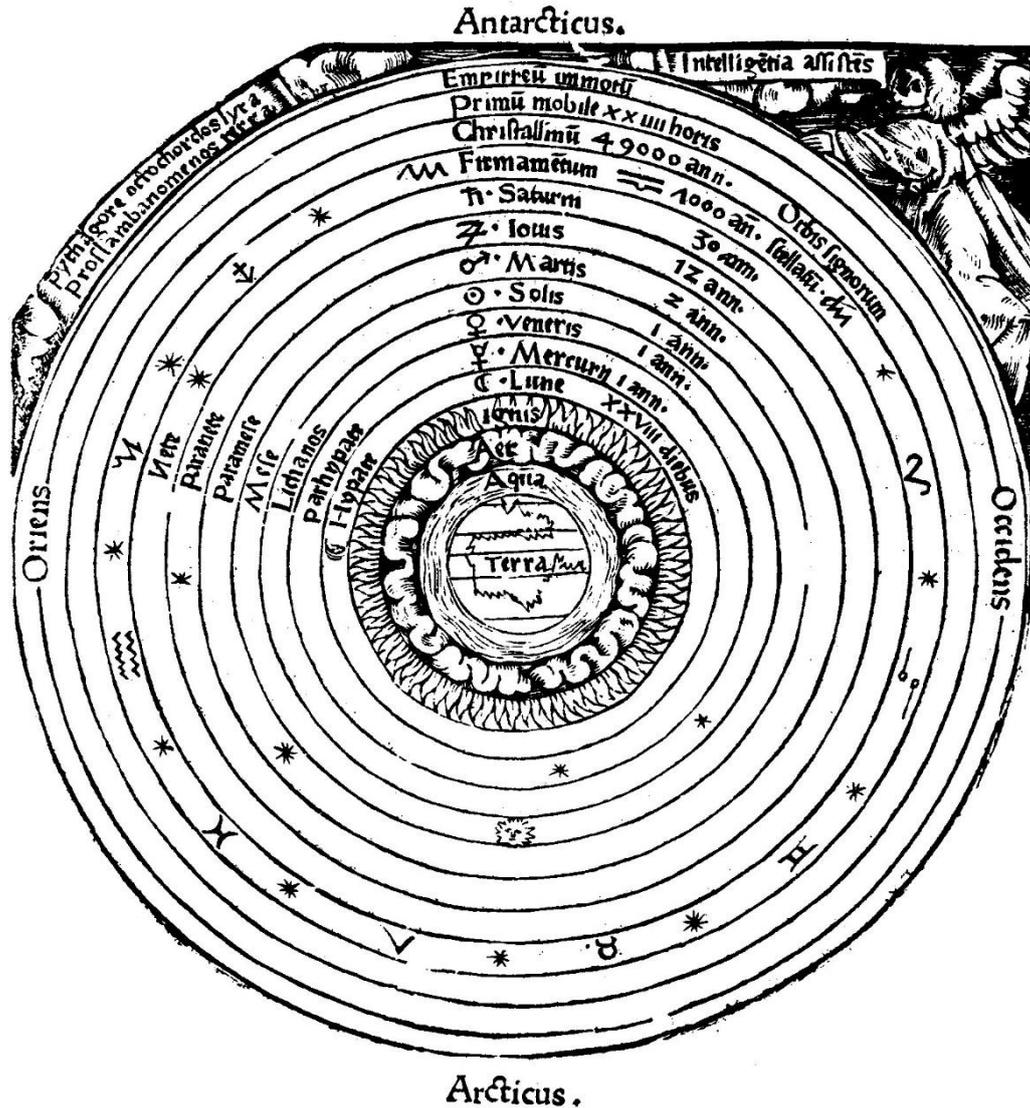


Figure 2: Harmony in heaven and on earth (from Aristotle's *Libri de Caelo* in Johann Eck's edition; Augsburg, 1519)

By mid-sixteenth century, Gioseffo Zarlino (choir master at St Mark's in Venice) is the philosopher of music who aims most comprehensively for a grand synthesis. He takes as his point of departure Glareanus, who in his *Dodecachordon* (1547) designed a system of twelve musical modes. Zarlino seeks to attain an acceptable balance between the metaphysical-arithmetical model of world harmony and the expressive urges of an emerging new musical style, in which each finds a harmonious place of its own, all the while giving both speculative philosophy and contemporary musical practice their due while binding them together in the tightest possible way. Consequently, in his authoritative *Istitutioni harmoniche* (1558) as well as in his later *Dimostrationi harmoniche* (1571) and *Sopplimenti musicali* (1588) he remains, on the one hand, true to traditional world harmony, which offers a

foundation for the standard conviction of a natural connection between musical intervals and numerical ratios – ultimately, the connection that holds the universe together. Here speaks a Platonist with a vision of the ideal world lurking behind the everyday world of the senses. On the other hand, however, Zarlino is thoroughly aware that practical obstacles and objections prevent this ideal of harmony from being always and everywhere realized in musical practice. Even so he believes that contemporary vocal practice is solidly rooted in a perfect, eternal, rational tonal system. He acknowledges that with musical instruments this system is particularly hard to realize, yet this circumstance does not give him occasion to doubt the correctness of the Pythagorean doctrine of the arithmetical-musical model underlying the very structure of the universe. Just like scholars within the tradition of the harmony of the spheres up to and including Ficino and Gaffurio, Zarlino still respects Platonic doctrine as ancient and venerable, fully in line with the biblical Creation story. He argues that, if the very idea of the harmony of the spheres ‘would seem incredible to anyone, then I would refer him to the testimony of Holy Scripture to the harmony of heaven, as when the Lord speaks to Job, saying: “Who will tell of your ordinances, O voices of the heavens? And who will make their music sleep?”’<sup>14</sup>

In order to link *musica mundana* to modern music practice, and well aware that Pythagorean tuning has meanwhile become obsolete, Zarlino appeals to Ptolemy’s *syntonon diatonic*. In this tuning system not only the fifths and fourths but also all thirds and sixths appear as pure (at least at first sight), which is why it is customarily called ‘just intonation’. Even though in his time it was already well known that just intonation was not really appropriate as a tuning system for keyboards, he tried to save the belief that there is a natural connection between the numerical ratios of the intervals in just intonation and the sounds they exemplify. Temperament (a slight alteration of these just tunings that involves non-simple ratios) he saw as an earthly practice belonging to imperfect human beings, who are unable to fathom perfect harmony. Hence, Zarlino maintains a broadly Pythagorean conception of things musical while extending it at the same time so as now to include pure thirds and sixths as well. In this resolute reorganization of all tonal material lies surely Zarlino’s most lasting achievement as a philosopher and theorist of music.

In an attempt to reconceptualize the nature of music, Vincenzo Galilei in his *Dialogo della musica antica, et della moderna* (1581) critically investigates Greek harmonic theory as a rational foundation for musical practice. The treatise does not contain a section devoted as such to the theme of world harmony, but no effort is made to combat it, either. In a passage on Orpheus’s lyre, the idea of a harmony of the spheres linking planets with notes in a musical scale is referred to as an ancient

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<sup>14</sup> Zarlino, *Istitutioni Harmoniche* I.6, referring to Job 38:137 (translation adopted with some alterations from Godwin’s *Harmony of the Spheres*, p. 207).

doctrine, and so is the way in which Ptolemy compared the aspects of the planets with the musical intervals.<sup>15</sup>

The revival of the harmony of the spheres which started in Ficino's work was by the end of the sixteenth century taken up in France, Germany, England and the Netherlands. Praetorius, for example, handed down Ficino's description of the universe as a cosmic lyre, which also became a favourite subject for scholars such as Marin Mersenne and Robert Fludd.<sup>16</sup>

On the whole, however, the numerical model of world harmony is overruled by a new, rhetorical model of musical expression, in which there is no direct relationship anymore between number and sound, and also by accompanying innovations in tuning and temperament. In order to understand this secular shift in the rational foundations of music, we take a closer look at this rival model.

## **2.b. Musical *ethos*: a rhetorical model**

Sixteenth-century philosophers and humanists writing about music from the point of view of earlier doctrines found themselves in a rather uncomfortable position in reconciling conceptions of world harmony with ideas about musical *ethos* and expressiveness. During the century influential Italian musical theorists were in search of a universal musical language based on a frame of cosmic regularities (the metaphysical-arithmetical model) inside which it now became possible to communicate equally universal, human affects (the rhetorical model). The domain of practical music (*musica instrumentalis*), that until far into the fifteenth century occupied a place subordinate to speculative musical theory (*musica mundana* and *humana*), became an integral part of sixteenth-century humanist musical culture.<sup>17</sup> This process, which started with Ficino, actually involves a great deal of idealization of ancient Greek culture, now glorified as one in which simplicity, clarity, and rationality were the normative standards. Plato's dialogues, for example, provide a major source of inspiration for humanist scholars in search of the miraculous power of ancient music. In his *Republic*, Plato argued that 'even before a child is old enough to reason ... rhythm and harmony sink deep into the recesses of the soul and take the strongest hold there, bringing that grace of body and mind which is only to be found in one who is brought up in the right way'.<sup>18</sup> The theorists' fascination with the moral and healing power of music is a sixteenth century echo of Plato's, thus giving us an insight into how they conceive of the relation between cosmos, man and music.

The Italian physician, encyclopaedist, mathematician, and music theorist Girolamo Cardano places

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<sup>15</sup> Vincenzo Galilei, *Dialogue*, p. 286 and p. 33-36, respectively.

<sup>16</sup> Praetorius, *Syntagma musicum I*, p. 401 (referring to Ficino's *Compendium in Timaeum* XXX, 69°).

<sup>17</sup> This story is presented in chapters 2-5 of *The Routledge Companion to Music, Mind, and Well-being*.

<sup>18</sup> Plato, *Republic* 401e (p. 90 in the Cornford translation).

himself in sharp contrast to Ficino, who thought that the reintroduction of early Greek musical practice was only a matter of time and effort. Inspired by the humanist ideal of an entirely natural and (in his case) also magical vocal practice, Cardano proves himself aware that even if it were possible fully to resolve the tuning problem and the difficulty of making out the words in much vocal music of his time, it would *still* be out of the question to perform musical wonders like those of Orpheus. He undertakes an experiment of a kind by forming the ‘recesses of the soul’ of his two sons using the right kind of music, and he concludes that

if we do it [i.e. to make music] at home the singers will ... corrupt the characters of our young boys and adolescents, for most of them are drunkards and gluttons, also wanton, fickle, impatient, coarse, indolent, and tainted with every kind of unlawful desire. The best of them are fools.<sup>19</sup>

That is, the emotional underbrush of Renaissance man, and the ‘recesses of the soul’ of the singer more especially, have meanwhile been mucked up by their involvement in present-day culture to such an extent as to prevent for good a return of the lost musical paradise.

This kind of sceptical voice aside, the ancient Greek musician remains an ideal-type in this period, in that he finds attributed to himself meanwhile lost knowledge of how music may help shape and elevate our souls. The closer a sixteenth-century musician approximates this ideal, by working directly on the mood of the audience by rhetorical means, the higher his status. The circumstance that in reality far too little ancient Greek music has been preserved to enable a faithful reconstruction creates plenty of room to fill the lost musical paradise with the most varied contents of one’s own making.

Consequently, Ficino’s and Gaffurio’s call to revive the musical practice of the ancient Greeks is taken up by many music philosophers, theorists and composers who turn monody and melodic simplicity and a direct understanding of the text, that is, its rhetorical force of expression, into the core of their musical style (*seconda prattica*). Also, ancient Greek myths about the miraculous power of music, notably the story of Orpheus, are made the subject of much vocal music. The Florentine *Camerate* bring together one such group of music theorists in search of a Greek musical paradise lost. Here the idea comes up that polyphonic structure is responsible for what they perceive as a regrettable state of contemporary music making.

To some extent, to be sure, the men of the *Camerate* were quite right in this. Problems of tuning and temperament of considerable import in their own time arise because Pythagorean tuning, conceived long ago in a monodic context, presents major problems for polyphonic music, even if (as

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<sup>19</sup> Cardano, *Opera Omnia* II, p. 116-117 (‘De utilitate ex adversis capienda’, III.2).

with Zarlino) it is altered and expanded so as to turn into ‘just intonation’, with fifths *and* thirds allegedly pure. The monodic nature of ancient Greek music confined its harmonic character to a succession of notes rather than to the simultaneity of a plurality of relatively autonomous voices (as practised first in the Notre Dame school). Musical humanists took Greek music to have been sung by one voice, accompanied by one instrument, preferably a string instrument such as a lyre or a lute. Free of complicated rules of counterpoint, this music was believed to move spontaneously inside some fitting tuning system. The members of the *Camerate* further believed that polyphony undermines the expressive power of the text. Due to the difficulty (if not impossibility) of understanding texts distributed over many voices, polyphonic music may, so they argued, from a superficial point of view sound beautiful, yet it leaves the human heart cold in that it cannot give expression to the musical substance that resides in the very combination of ‘word’ (representative of the rhetorical model) and ‘tone’ (representative of the numerical model).

The varied efforts at reconceptualization just discussed come clearly to the fore in ongoing, fierce debates raging over the period (notably between Zarlino and his former pupil Vincenzo Galilei) about the true nature of music and what it entails for what the world and we ourselves at bottom are like. The medieval philosophy of music in the Boethian tradition was abstract and intellectualist to the core, founded as it was upon metaphysical and arithmetical principles extraneous to the sphere of practical music itself. This preponderant intellectualism of musical thought, already under siege for some time, now receives one blow after another in various efforts to bring about a new consensus between *intellectus*, of which there should be less, and *sensus*, of which there should be much more. A new ideal of music as expressive of clearly articulated words that can be grasped and understood at once; a new vision of music as moving the human heart in accordance with ancient as well as current doctrines of the affections, rob the standard, Pythagorean-Boethian conception of world harmony of its monopoly. The philosophical-arithmetical model of music is enjoined to yield at least in part to a new, rhetorical model, in terms of which the expressive force of vocal music becomes more important than the proportional value of musical intervals.

Not only do these debates serve modern musicologists to document the course of musical history – in their own time they shaped and conditioned the very course of this history itself. Far more is changing here than the discovery of (to use a modern phrase) ‘tonal harmony’ by philosophers of music and its ever-increasing usage in musical practice. How music is being performed, listened to, and understood, alters in the process as well. All this goes together with profound changes in the social and cultural roles of musical theorist, composer, performer, and audience alike.<sup>20</sup> Slowly but surely art music loses its near-exclusive connection with church ritual and invades the secular sphere.

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<sup>20</sup> Weber, *Rational and Social Foundations*, p. 104.

Musical genres such as the madrigal quickly gain popularity during the sixteenth century; instruments begin more and more often to accompany or even replace vocal music; in performance acts, the roles of composer, performer, and audience develop not only distinct identities but also separate. Up to then, composers wrote their music most often in the service of church and court, but in the new climate they began to write music increasingly with a view to entertaining audiences not actively involved in music making. The repercussions for the style of composing are considerable, in that composers begin to use a simpler, more accessible musical language, which enables them to communicate more effectively with their audience. These new musical realities require a less ambiguous, more straightforward semantics, and music becomes more rational in the specific sense of more functional.

None of this contradicts the customary view of sixteenth-century musical culture as marked centrally by an effort to invoke, and to play upon, the senses, so as to move the affections or to touch as it were the strings of the human heart. The audience becomes more differentiated than before, it attends a performance with the express aim of undergoing a musical experience, so composer and performer alike seek to meet this novel requirement. Ongoing secularisation of music now alters what music is taken to be meant for. The final objective of music, determined in a more accurate manner than before, is that it must provide pleasure and entertainment and in doing so stir the listener to joy or sorrow or any other affect available in the human emotional spectrum. Hence, to bring about successful musical communication, composer and performer must think up and maintain some rhetorical strategy. The philosophical-arithmetical and the rhetorical models for music appear to complement each other nicely when it comes to realizing the new requirements involved in the new musical practice. Emerging ideas about tuning and temperament, associated with the development of tonal harmony, are used to formulate a functional musical grammar, on the basis of which a clear, flexible rhetorical musical structure can be built of music capable of affecting the senses and of providing entertainment.

One effort to reconcile the two models with each other is a discussion of hearing in Girolamo Cardano's *De subtilitate* (1552). In this bestselling work on subtle phenomena which are difficult to fathom the author defines both 'sound' and 'hearing' in the standard Pythagorean terms of proportionality. He then goes on to argue that since musical sound is of the very same proportional substance as our hearing, listening to music gives pleasure – musical sounds, or consonances, 'arise in a proportion, for as such they are known and thus are pleasing'.<sup>21</sup> However, when he seeks to define the pleasure of listening to music in a stricter way, this quickly turns out not to derive from its objective numerical structure but from a quite subjective rhetorical play with those rules. From the

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<sup>21</sup> Cardano, *De subtilitate* (p. 572 in *Opera Omnia* III).

psychological circumstance that ‘better things are always pleasing after worse ones’ Cardano argues that consonant intervals are not pleasing in themselves – as was thought in the Pythagorean tradition – but derive their sweetness from their following upon dissonances. Furthermore, music has a powerful effect on the affections because

each sense is subject to change, and the change is to the opposite, as from good to evil, which then results in sadness. Thus pleasure will come from a change of evil into good [or from dissonance to consonance], yet it is necessary that evil had been present before.

This functional explanation of how certain chord progressions work on our minds has no longer much in common with the ontological power of certain consonances and their ratios, but rather fits in with an Aristotelian-epistemological conception of musical effects. The passage illustrates that in the musical aesthetics of the period the sense of hearing acquires an important place of its own beside the intellect as a criterion for judging certain musical phenomena. After Cardano, many music theorists and composers further explored the ways and means available to a musician to express certain affects. To that end, affects were contrasted in opposing pairs such as vigour and weakness, which could be associated with size of interval (the larger, the more vigorous) and with the movement rising or falling (C-B generally weaker than C-D). The traditional Pythagorean idea of number as a constituent of a so-called ‘passion of the mind’ (i.e. an emotion) disappears in this new kind of music theory.

So as to move the listener’s ‘affections’ with ever greater effectiveness, theorists such as Zarlino and Galilei held it to be desirable to make music and text correspond very closely. When the exact meaning of a word is supported by a musical equivalent, so the theory went, this will enhance communication with the audience. By way of an endorsement of this view, Zarlino appeals to Plato, who ‘suggested that speech should have priority and that the other two elements [harmony and rhythm] should be subservient to it’. In order to be successful, then, a composer should ‘use joyful harmonies and rapid rhythms in joyful matters, and in mournful ones mournful harmonies and heavy rhythms ...’.<sup>22</sup>

Most often the resulting recommendations concern melody, yet Galilei, for instance, extended the issue to include music in parts, as a demonstration of why he found polyphony unfit for affective expression in that it may so easily set a word to a rising phrase while at the same moment another voice, with the same text, falls. It is not too clear to what extent such theorizing on the musical expression of the affects matches what composers did in practice, even though Cipriano de Rore was praised by many musical humanists (Galilei included) for how he managed to bring the texts of his

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<sup>22</sup> Zarlino, *Istitutioni harmoniche*, 4, p. 32 (translation adopted from Strunk, *Source readings*, p. 256-257).

madrigals to musical expression<sup>23</sup>.

In his *Dialogo* of 1581 Galilei also makes a sustained effort to defend the claim that nature invented an effective, monodic kind of diatonic singing, which should once again become the foundation of modern singing practice so as to make it more efficacious.<sup>24</sup> He clearly envisions a new idealized form of expressive music in which the ideas expressed by the words are imitated in monodic music in such a way that they can move the soul of the listener directly.<sup>25</sup> His view of nature, however, is highly idealized in itself. Hence, his belief that to replace number-oriented Pythagorean tuning principles with the more sound-oriented ones instigated by Aristoxenus can bring back the ethical and healing power of ancient music, is founded on a fruitful misunderstanding (as will become apparent below).

The very construction of the rhetorical model, up against but also (prior to Galilei) complementary with the earlier metaphysical-arithmetical model, was very much an Italian affair in the sixteenth century – the rest of Europe would catch up later.

### 3. Conceptions of musical consonance: Sound and number reconsidered

The humanists' search for the ways in which Greek music achieved its effects was guided by many sources containing technical music details. In Boethius's *De Institutione Musica* they read, for example, that

it is common knowledge that song has many times calmed rages and that it has often worked wonders on affections of bodies and minds. Who does not know that Pythagoras by performing a spondee [in the Dorian mode], restored a drunk adolescent of Taormina incited by the sound of the Phrygian mode to a calmer and more composed state?<sup>26</sup>

In studying this kind of source, sixteenth-century scholars wondered how Greek musical modes referred to in this passage, such as the Phrygian associated with excitement and the Dorian associated with a calming orderly rhythm, were constructed, and what they sounded like. To answer their questions, they studied Pythagorean and Platonic technical sources in which the musical intervals were associated with numerical ratios. But even though the association made in the tradition of the

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<sup>23</sup> On p. 79 of *Studies* D.P. Walker gives as a rare example 'Rore, *Works*: IV, 80; cf. Introduction to Vol. II, p. 5 on the use of major and minor chords to "imitar le parole".'

<sup>24</sup> Galilei, *Dialogue*, p. 221-222.

<sup>25</sup> Galilei, *Dialogue*, p. 171-172 and p. 200-201.

<sup>26</sup> Boethius, *Fundamentals of music* (ed. Bower, p. 5).

harmony of the spheres between music and the planetary orbits provided a base for mathematical investigations of the entire universe, increasingly musical humanists such as Girolamo Mei or Giovanni Bardi perceived a gap between music theory and practice. In their ambition to found modern musical practice on the rules of ancient Greek music, they started to look for alternative ancient sources such as Aristoxenus, who dealt with music primarily in terms of sound, not of number. In order to understand the difference between these approaches, let us look at those two influential traditions.

### 3.a. Numerical models in music theory

The arithmetic that underlies all music making within the Pythagorean tradition, in theory as in practice, is relatively easy to understand and can be reduced to four basic facts:

- (1) Every musical interval may be rendered as a numerical ratio.
- (2) In musical harmony as understood throughout the period here treated, the consonant intervals derive from very simple ratios, composed of no more than three numbers (beyond 1) and their multiples, to wit, 2, 3, and 5. The number 2 comes in to produce the octave (1:2); the number 3 to produce the fifth (2:3) and the fourth (3:4), and the number 5 to produce the thirds (4:5 and 5:6) and the sixths (3:5 and 5:8).
- (3) When one interval is ‘added’ to another, as when for instance an octave C-c is formed by stacking the fourth G-c upon the fifth C-G, then the corresponding arithmetic is not *addition* but *multiplication*: in the present case  $(2:3) \times (3:4) = 1:2$  (the short answer to why this is so being that our hearing happens to work logarithmically).
- (4) Powers of figures not divisible by each other can never be equal. Thus,  $1/2$  to the power of 7, which corresponds to seven octaves stacked upon each other, can be told in advance not to equal  $2/3$  to the power of 12, i.e., twelve fifths stacked upon each other. So the c reached by going up seven octaves from C is not the same note as the b sharp attained by departing from the same C and going up twelve fifths. This is what modern textbooks on the history of music theory mean when they say that the circle of fifths is not closed. The ‘difference’, a very complex fraction and, hence, a harshly dissonant mini-interval, is known as the *Pythagorean comma*.

Once these four rules are understood, we are able to survey those principal issues of consonance, tuning, and temperament that plagued sixteenth century musical theorists and practising musicians alike.

Some of the basics were discovered in the sixth century BC by Pythagoras (or at least by the sect that named itself after this legendary hero). These were notably the intriguing correspondence between, on the one hand, our sense-given experience of how in some rare cases two musical notes

do not jar when sounded together but almost blend into each other, with, on the other hand, the ratios of precisely the simplest integral numbers. With Pythagoras, this concerned the consonances built with 2 and 3, i.e., none but the octave, fifth, and fourth. After all, with the octave and also the fifth and therefore the fourth, too, as pure, the whole tone, which of course is the ‘difference’ between a fifth and a fourth, becomes  $(2:3) : (3:4) = 8:9$ , which yields for major third 8:9 squared, i.e., 64:81. This in its turn ‘differs’ from a pure major third by 80:81, a fraction known as the *syntonic comma*. Hence, in Pythagorean harmony thirds and sixths cannot stand on their own but have to be resolved on truly consonant intervals.

As has been discussed above, Pythagoras’ original discovery gave rise to a conception of cosmic harmony. So pervasive did that conception remain throughout the European Middle Ages that the incisive harmonic innovation arising by the early fifteenth century, the introduction of thirds and sixths into art music, did not just threaten to upset the standard theoretical account of the scale and of the range of consonant intervals, but of the entire order (or tuning) of the cosmos. The accepted conception of how life on Earth and in the Heavens hangs together now came to depend crucially on the tough problem of how in a plausible manner to eliminate the potentially disastrous theoretical consequences of that seemingly tiny disturber of harmonic peace, the syntonic comma.

To pull off this feat of accounting for musical consonance in such a manner that the thirds and sixths were satisfactorily included as well in the list of theoretically and, therefore, cosmically acceptable musical consonances, Zarlino arrived at the solution of the *senario*, that is, the range of integers 1 through 6. All intervals whose ratios contain numbers inside the *senario* are consonant, those outside yield none but dissonances. An obvious problem with this ingenious way to rationalize the thirds and sixths into the harmony of the world is that it seems to fail with the minor sixth, which after all corresponds to 5:8. This is explained, or rather explained away, by a distinction typical of the Aristotelian philosophy prominently involved in the vast thought-construction in which Zarlino enveloped his account of consonance. He argued that the 8 in the ratio for the minor sixth is only potentially 8, that is, it should really be regarded as twice 4, so that, luckily, it falls within the *senario* after all.

Now what makes the number six so special as to constitute the ‘sonorous’, i.e., the consonance-producing number? To answer the question, Zarlino reasoned that six is the first perfect number:  $1 + 2 + 3 = 1 \times 2 \times 3$  (the next figure which does this is 28); further that God created the world in six days, that there are six planets (Moon, Mercury, Venus, Mars, Jupiter, Saturn), and so on and so forth.

In this manner the integrity of cosmic harmony, dangerously threatened by the rise of the triad in art music, was triumphantly restored. And indeed, if the manner in which Zarlino managed to reason his way to this (so he hoped) final outcome had not fitted so neatly into the larger structure of thinking about cosmic harmony that we have examined above, the shortcomings in his argument might have

become manifest at an earlier time and on more massive scale. Still, the ineluctable, built-in imperfections of current thinking about cosmic harmony could hardly remain hidden forever. No more than a fairly slight change in perspective sufficed to reveal a few such imperfections, which then quickly gathered into the veritable avalanche that was to upset cosmic harmony for good in the course of the seventeenth century. The first cracks in the ice on which Zarlino had been skating as if it were a solid floor, came to light in the work of two Italian thinkers who were also composers. One, primarily a philosopher, was Giambattista Benedetti (whom we discuss in the next section), the other was Vincenzo Galilei.

In the late 1580s, some thirty years after Zarlino started his campaign for the *senario*, Galilei opened up a new line of argument regarding the nature of consonance. Sustained by practical experimentation of a kind that his eldest son Galileo was soon to seize upon and famously direct to mostly quite different objectives, Vincenzo upset the customary conception of consonance at a profound level never before considered. Against Zarlino (and in fact against Pythagoras, too) he argued that there is another way than the traditional one to produce consonant intervals. Pythagoras' derivation of the consonant intervals had been by way of the division of a vibrating string. If you sound first the whole string and then its half, or two strings simultaneously with lengths in the same ratio 1:2, this yields the octave; similarly so with the fifth for string lengths in the ratio 2:3, etc. Galilei now argued that you can *also* attain a fifth by different means, to wit, by suspending from two strings of equal length weights in *another* ratio, namely, 4:9, that is,  $(2:3)^2$ . In other words, consonances may indeed be represented by just the familiar ratios, but also by those same ratios squared – the former stand for string lengths, the latter for string tensions. What, then, about the *senario*, the numbers 1 through 6, as the truly sonorous, the truly harmonic number? The integrity of cosmic harmony, lost earlier by the introduction of the triad and then restored seemingly for good by Zarlino's *senario*, might well appear for a second time, but now on quite other grounds, to have been blown to pieces; this time, not by the introduction of a new consonance, but by an unruly disciple.

What made Galilei so unruly is, above all, his readiness to adopt Aristoxenus' style of reasoning about music to the full, and to draw the consequences thereof to a greater extent than had been considered so far. As a true disciple of Aristotle, Aristoxenus deviated quite consciously from the Pythagorean way of thinking in terms of number and harmony. His primary entity is the free flow of the melody, and the quantities he works with are not those of the consonances as defined by Pythagoras but rather those involved in the filling up of his basic musical unit, the *tetrachord*. His scale is organized in two tetrachords, with the respective outer notes removed from each other by an interval of a fourth. When the two disjunct tetrachords, subdivided each as TTS (tone – tone – semitone) are made heard consecutively, this yields a *diatonic* scale with imperfect thirds and sixths. But (as we saw at the end of our introduction when dealing with Praetorius) the tetrachord can also

be subdivided in a different manner, as for instance in a double whole tone and two quarter tones. So the place of the two notes at the inside of the tetrachord can be shifted more or less at will, which yields a very large spectrum of possible scales.

Although less consistent in this than he thinks (or claims) to be, Galilei rejects the customary idea, maintained from Pythagoras up to and including Zarlino, that, in a profound sense, music *is* arithmetic. In that tradition one could prove a musical point by means of calculation, even if musical practice might fail to conform to it. To a fuller extent than before, in how Galilei employs Aristoxenus the empiricist leanings of the period now come together with the recovery of all those so far unknown ancient texts. To Zarlino, following Ptolemy, music was bound up numerically with the motions of the heavenly spheres, hence, with heavenly harmony. The whole notes in the scale favoured by Zarlino in just intonation must correspond with the intervals of 8:9 and 9:10, not because that is how they sound but because how calculation shows them to fit into heavenly harmony. In stark contrast with such *normative-theoretical* procedures, Aristoxenus used some basic arithmetic to *describe pragmatically* what musicians in his time actually did and what scales they actually used. In Aristoxenus' own words:

Some of our [Platonic]<sup>27</sup> predecessors introduced extraneous reasoning, and rejecting the senses as inaccurate, fabricated rational principles, asserting that height and depth of pitch consist in certain numerical ratios and relative rates of vibration – a theory utterly extraneous to the subject and quite at variance with the phenomena ... Our subject-matter then being all melody, whether vocal or instrumental, our method rests in the last resort on an appeal to the two faculties of hearing and intellect. By the former we judge the magnitudes of the intervals, by the latter we contemplate the functions of the notes.<sup>28</sup>

This renders neatly how Galilei, too, uses mathematics in the discipline of music. He seeks to describe, order, and where possible explain the principal musical phenomena that he knows from his own experience as a lute-player, with what calculation is needed to that end being used by way of a handy tool only. Just like Aristoxenus, Galilei allows for both irrational numbers and a certain amount of indeterminacy in his measurements. The ear is a fine measuring instrument for musical phenomena, only, just like other tools it is subject to certain limitations of its own. In the same vein, even though he never denies the existence of world harmony, Galilei has no time for the innate moral meaning attributed to Pythagorean arithmetic in its close involvement in heavenly harmony. Moreover, during his whole life he continued to believe that he was reviving ancient Greek music practices and their accompanying tuning systems, whereas in fact he was developing a whole new conception of tuning

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<sup>27</sup> Presumably, Aristoxenus is referring here to Plato's *Republic* 531a-b.

<sup>28</sup> Aristoxenus, *Harmonics*, p. 188-89.

and temperament, thus instituting a movement that was due to end in our Western (meanwhile near-universal) equal temperament.

Not that, in practice as distinct from his ‘official’ views on the matter, Galilei goes all the way. On grounds of sheer symmetry he extends without more ado the experimental argument against the *senario* he derived from weights suspended from the vibrating string to the quite mistaken idea that with pipes the consonant intervals are represented by cubed ratios (e.g., the fifth by 8:27). So his departure from Zarlino-style reasoning about the consonances is far from complete.

In another unpublished treatise of his, written close to his death in 1591, Galilei undertook a range of experiments which allowed him to conclude that not just the length or just the tension of a vibrating string determines the pitch it produces. Other determinants of pitch, so he asserted, are the thickness of a vibrating string and the material (e.g., gut, or steel) it is made of.

Widely read as Vincenzo’s published treatises on the subject were in his own time and beyond, they appear to have opened the doors for a thorough undermining of the belief that the universe is ordered by the same numerical proportions that produce harmonies in earthly music. Within thirty years of his publications, not only the belief itself but also, even more importantly, the style of reasoning behind it changed drastically. This happened in two different directions, one *geometric*, one *physical*. Pioneers of the former, geometric approach were, c. 1605-8, Simon Stevin, further young René Descartes in 1618-9 and then a few months later Johannes Kepler. Pioneers of the latter, physical approach were Benedetti in the 1560s and Vincenzo’s eldest son, Galileo, in the 1610s. We start with the three geometers.

Stevin replaced Zarlino’s *senario* with his *a priori* conviction that what we now call equal temperament is really the tuning system given by nature. In his view, the circle of fifths is *naturally* closed. For instance, the pure fifth is not given in nature by 2:3, but by the twelfth root of 2 to the seventh power  $\sqrt[12]{2^7}$ .

Kepler replaced Zarlino’s *senario* with an alternative criterion to distinguish consonance from dissonance. It involved the reiterated division of a circle by means of successive regular polygons.

Young Descartes likewise replaced Zarlino’s *senario* with an alternative criterion, likewise geometric but attained in his case by means of three successive bisections of a right line representing the vibrating string.

What all three alternatives had in common was a wholesale rejection of Zarlino’s arithmetical practices, which in the eyes of their respective authors were just a matter of unscientific number play. Clearly, the way of reasoning adopted by Zarlino and his contemporaries had by the early seventeenth century been rejected for good, at least by such individuals as were at the same time pioneering science of a radically novel kind – the science, at bottom, that we are still familiar with nowadays.

### 3.b. Acoustic models in music theory

In the decades around 1600, then, an ancient debate about the status of numbers in music was revived in the context of the philosophy of nature, with far-reaching consequences for arithmetic-driven accounts of consonance and dissonance. For instance, the Italian philosopher Francesco Patrizi argued in 1591 that the Pythagorean belief that numbers are the ultimate constituents of reality, which is the very foundation of the numerical model of music, is nothing but superstition:

The Ancients based themselves on divination rather than knowing the cause.... Continuous quantity [i.e. lines] exists by nature, while [the discrete quantity of] number is the work of the human mind.<sup>29</sup>

Hence, a philosophy of nature (which must include a discourse on the nature of sound) cannot be based on numbers, because these are merely conventional constructs. As a consequence, Patrizi abandoned the Pythagorean belief that the universe is ordered by numerical proportions that produce harmonies in earthly music. In his discussion of the science of music he even goes a step further: instead of a mathematical science, he envisions music as a sub-discipline of acoustics, which is all about the quality, not the quantity, of sound. Sound should be dealt with as a phenomenon brought about by wave-like motions which manifest themselves in the air.<sup>30</sup> Patrizi might have found inspiration for this view in Vincenzo Galilei's *Dialogo*, at the point where the latter argued that

Aristoxenus knew very well that the quality of sound was what had to be distributed in equal parts, and not the quantity of the line, string, or space. He was operating as a musician on a sonorous body, not as a pure mathematician on a continuous quantity.<sup>31</sup>

Before Patrizi and Galilei, a first, tentative effort to link an idea of the wave-like propagation of sound up with the nature of consonance had emerged in 1563, when Benedetti, a mathematician / philosopher but also a composer, wrote a letter to Cipriano de Rore. The letter ends with forty terse lines about the proper derivation of the consonances. It appeared in 1585 in a large book on mathematical and philosophical problems, that was read by but very few. In these forty lines Benedetti made an unheard-of connection between the nature of consonance and the nature of sound.

Antiquity had yielded two almost opposite accounts of how sound is produced, propagated, and

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<sup>29</sup> Patrizi, *Nova de Universis Philosophia* Book 4, II, 68<sup>r</sup>.

<sup>30</sup> Patrizi, *Nova de Universis Philosophia*, Book 4, II, 68<sup>v</sup>.

<sup>31</sup> Vincenzo Galilei, *Dialogue*, p. 127.

perceived. One, an emission account, ran in terms of the atomist philosophy of nature, the other of Stoic natural philosophy, hence, in terms of wave-like motion. Atomists thought that a vibrating string, or a pipe, emits invisibly tiny particles which insensibly fly through the air and in touching our sense of hearing make themselves perceived by us as sound. Stoics rather employed a comparison with a quiet pond in which you throw a stone. Sound, in their view, starts from a disturbance of the air which is subsequently transmitted to that air. Just as with those water ripples brought about by the stone, the air now begins to ripple, and the ripples propagate outwards until they reach our sense of hearing, where we perceive them as sound. Note that in this account it is not the air itself that moves, just as the water in the pond is not in motion itself, it only ripples. The water or air affected by the disturbance goes up and down without being displaced horizontally – it is only the disturbance that keeps being displaced until, weakening all the time, it comes to a stop.

The new thing about Benedetti's argument is that, in associating himself with the latter, wave-like account, he linked it up with the hoary, Pythagorean correspondence between the consonances and the ratios of the first few integral numbers. In his summarily stated view, a vibrating string regularly 'strikes' the air around it, and the regularity with which it vibrates, that is, the number of 'strokes' per second determines pitch. If a note of given pitch is yielded by a certain amount of strokes and another note by twice that amount, then they together sound the octave. So it is with strokes in the ratio 2:3, which yield a pure fifth. We are still dealing with the same consonance-generating numerical ratios as before, only, they now stand for something quite different. Surely Pythagoras' original observation stems from strings vibrating, yet in pursuing this newly discovered, numerical relationship between their respective lengths he had at once taken leave of the original vibrating, never to return to it. With Benedetti for the first time, the actual, physical vibration is now being brought to bear on the nature of the phenomenon of consonance. His quite novel conclusion (a novelty almost hidden behind the laconic style in which he expresses it) is that consonance emerges from how often the vibrations, or strokes, or wavelets, concur, or coincide: with the octave every  $2 \times 1 =$  second time, with the fifth every  $2 \times 3 =$  sixth time, and so on, up to and including the minor sixth  $5 \times 8 =$  the fortieth time. Not that Benedetti regards the list from 1 through 40 that thus ensues as indicative of *degrees* of consonance; he is satisfied with pointing out that the numbers in the list provide neat ratios in their own right. In his final line he summarily points at what makes us perceive those regularly coinciding strokes as consonant, unlike those that coincide but rarely or not at all: 'The pleasure that the consonances give to hearing comes from their softening the senses, while, to the contrary, the pain that originates from the dissonances is born from sharpness, as you can easily see when organ pipes are tuned.'<sup>32</sup>

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<sup>32</sup> Final clause of Giovan Battista Benedetti, 'De intervallis musicis'.

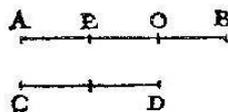
At the beginning of the seventeenth century, Vincenzo Galilei's eldest son Galileo and the Dutch theologian / candle maker Isaac Beeckman independently hit upon the same train of thought, albeit attained from opposite directions, and extended it so as to come up with a likewise physical yet fuller account of the nature of consonance and of what makes it principally distinct from dissonance. These two men really stand at the origin of a 'coincidence' conception of consonance – one indispensable ingredient of Rameau's much later, triadic account of harmony.

In the most revolutionary of all his books, the *Discorsi* of 1638, Galileo devoted some ten pages to the problem of consonance. He adopted the arguments that his father had put forward against Zarlino and extended these (in all likelihood independently) in the direction of points made so maddeningly briefly by Benedetti. Yes, so Galileo agreed with his father, 'there are three Ways by which we may sharpen the Tone of a String, viz. by shortening it, by stretching it, or by making it thinner.'<sup>33</sup> Still, it does not follow from this that musical intervals may be represented by ratios in either straightforward or squared proportion – the fifth as either 2:3 or 4:9. What really counts is the *frequency* with which a given string vibrates, as this is what determines pitch: the greater the frequency, the higher the pitch. Consequently, the original ratios for musical intervals as Pythagoras had conceived them (1:2 for the octave, etc.) are now being reinstated, albeit with a radically altered meaning – they no longer stand for numerical ratios but for the ratios of vibrational frequencies.

Upon this authoritatively presented, novel basis Galileo now erects in a delightfully playful (also delightfully translated) passage his explanation of the phenomena of consonance and dissonance:

The Offence [the Dissonances] give, proceeds, I believe, from the discordant and jarring Pulsations of two different Tones, which without any Proportion, strike the Drum of the Ear. And the Dissonances will be extreme harsh, in case the Times of the Vibrations are incommensurable ... Those Pairs of Sounds shall be Consonances, and will be heard with Pleasure, which strike the *Timpanum* in some Order; which order requires, in the first Place, that the Percussions made in the same Time be commensurable in Number, that the Cartilage of the *Timpanum* or Drum may not be subject to a perpetual Torment of bending itself two different Ways, in submission to the ever disagreeing Percussion.<sup>34</sup>

This is followed by a neat illustration, meant to explain the consonance of the fifth (*Figure 3*):



*Figure 3: From Galileo's Discorsi (end of First Day)*

<sup>33</sup> Galileo Galilei, *Opere* 8, p. 143 (Weston's translation: p. 146).

<sup>34</sup> Galileo Galilei, *Opere* 8, p. 146-7 (Weston's translation: p. 151).

A vibration is supposed to ‘strike’ (‘percuss’) at its beginning and its end. AB represents the vibration of the string that emits the lower note, and CD the higher. AB is divided into three equal segments, CD into two. The time needed for passing from A to E counts for one moment. After two moments the higher string strikes in D, but the lower one, having arrived in O, does not yet strike. When it does, at the third moment, in B, the other one is half-way back, at DC, so again there is only one stroke that reaches the ear; also, the directions of the vibrations are now opposite. At the fourth moment again there is only one stroke, in C, and not until six moments have passed do the strokes finally coincide; the procedure is then repeated for as long as the vibrations continue. This, so Galileo asserts, not only explains why the octave is more consonant than any other interval, but it also accounts for the peculiar nature of the fifth, which ‘produces such a Titillation upon the Cartilage of the *Timpanum*, that, allaying the Sweetness by a Mixture of Tartness, it seems at one and the same Time to kiss and bite.’<sup>35</sup>

Not quite so implicitly as with Benedetti, yet hardly worked out to the full yet, we have here the beginnings of a momentous transition from consonance and dissonance conceived as absolute opposites to a scale of greater or lesser consonance. With the exception of solely the passage here quoted, Galileo just leaped over that scale, and over its dire consequences for any viable distinction between consonance and dissonance at all. In the same vein, he had nothing more to say about the anatomy and physiology of the ear where these regularly coinciding strokes somehow make themselves heard as pleasantly sweet rather than so jarringly harsh as with those that never (or at least not so often) coincide.

It is precisely these two tough topics circumvented by Galileo that at about the same time began to be addressed with considerably greater sophistication, at first by the Dutch natural philosopher Isaac Beeckman, and in subsequent decades by Mersenne, Descartes, and many a later music theorist as well. Also in the first decades of the seventeenth century, the prime methodologist of empiricism of his time, Francis Bacon, took an interestingly different approach. He rejected the Greek penchant for intellectualist top-down constructions in favour of bottom-up experiments meant to describe, list, and compare natural phenomena. Consistent with his denial of any connection between sound and number, he regarded the nature of the consonances as still a great secret. Bacon’s chief interest in the domain of sound was in keeping with his general concern for the improvement of human destiny by harnessing natural effects to human ends. The end in this particular case was the artificial ‘majoration’ (increase) of sound, by means of devices to make sounds louder or carry farther. Speaking trumpets, ‘ear spectacles’, echoes, and whispering galleries all found a place in Bacon’s program for a natural

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<sup>35</sup> Galileo Galilei, *Opere* 8, p. 149 (Weston’s translation: p. 155).

history of sound and how to make proper use of it. He further suspected that makers of musical instruments had amassed treasures of empirical knowledge on how sound behaves under artificial conditions. From their instruction one might infer how pitch, loudness, and timbre vary with such factors as the material and the shape of viols and clarions and bells and a whole plethora of other instruments with strings and pipes and resonating bodies.

With Bacon and like-minded experimentalists of the early seventeenth century we are meanwhile far removed indeed from any theorizing in terms of world harmony. This is only fitting, as we are left to conclude that, by way of a model for understanding the universe, world harmony was definitely on its way out, even though it was to emerge all over again in many another sphere of human life.

#### **4. Coda**

In spite of what world harmony may look like at the surface, we have seen throughout the present chapter how it remained a quite elusive doctrine in several ways. During the sixteenth century it stimulated a variety of interpretations of how the harmonic ratios of the universe are reflected in the harmonies used in earthly music. Time and again the harmonies employed in musical practice, be it in Pythagorean tuning or in one or another variety of just intonation, were not so much reflections of heavenly harmonies as, rather, projections of earthly music-theoretical conceptions. During the long sixteenth century, the Pythagorean notion of world harmony and the very idea that heaven and earth are knowable through music continued to determine new theories and experiments to a large extent. Even if musical scholars wanted to read God's book of Nature, they continued to do so through the lens of the books of the Ancients.

#### **Case study I: Meantone tuning for keyboard instruments**

Better perhaps than any other musical instrument, the church organ<sup>36</sup> exemplifies the close intertwinement of the subjects of world harmony, of the rhetorical power of music, of tuning and temperament, that we discuss in the accompanying chapter. The organ was held of old to possess near-magical powers, witness for instance what Girolamo Cardano wrote about it:

But those who take extreme pleasure in sound and pursue it without restraint become excessively enamoured of music, as in the case of Nero, who was captivated by the wonderfully

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<sup>36</sup> We want to thank organist Peter van Dijk most cordially for his helpful comments on this part of the text.

pleasant sound of hydraulic organs. Even in the midst of danger to his life and empire ... he did not neglect them.<sup>37</sup>

Cardano attributed the magical power of the organ to the perfect proportions of its pipes, yet in the second half of the sixteenth century music theorists and organ players gradually stepped out of this magical way of thinking and concentrated on functional problems of the instruments caused by emerging new musical ideals. In Max Weber's words, 'when any sizable church obtained an organ, ... organ-building and with it to a considerable degree the practical leadership in development of the tone system lay in the hands of professional secular organ-builders'.<sup>38</sup> They were the ones who brought about changes far surpassing the problems of temperament in general importance.

An eyewitness report of 'the great musical experiments of the Renaissance period' discussed by Weber is given in Michael Praetorius' treatise 'De organographia' (volume II of his *Syntagma Musicum* of 1619). In chapter 40 of part II, entitled 'The universal, or perfect harpsichord', Praetorius discusses a most remarkable instrument, built in the 1580s in Vienna, that he has seen with his own eyes in Prague. He still believed that ancient Greek music has been very effective because it was built on the harmonic rules of the universe. He was interested in the music theory and musical instruments of his sixteenth-century predecessors because, just like them, he strove to reconstruct a lost idealized performance practice by examining historical accounts of the association of the musical intervals with arithmetical ratios – Pythagoras' legendary discovery. He aimed at inventing musical instruments which were capable of imitating heavenly harmonies, while also meeting the demands of contemporary earthly musical practice. Unlike in Praetorius' time, when a 'black' key represented *either C sharp or D flat*, etc., and also unlike in equal temperament, where one and the same 'black' key is made to represent *both C sharp and D flat*, etc., this particular harpsichord was fitted out with *separate* keys for both C sharp and D flat, etc., as shown by Praetorius in *Figure 4*. Note here that even between E and F, where ordinarily no black key appears at all, a special key has been inserted for E sharp as distinct from F!

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<sup>37</sup> Cardano, *De subtilitate* (*Opera Omnia* III, p. 572).

<sup>38</sup> Weber, *Rational and social foundations*, p. 115.

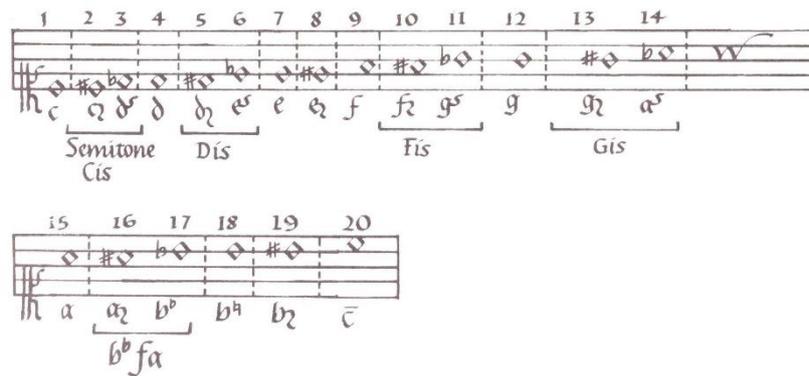


Figure 4: The keyboard in Praetorius' perfect harpsichord

What is the point of all this seemingly vain exertion? Praetorius explains the matter thus:

All three kinds of *genera* [Greek scales of four notes], to wit, diatonic, chromatic, and enharmonic, can thus be observed here. So this may fairly be called a perfect or even *the* most perfect instrument, since such variation through all super- and semitones cannot be found on other instruments.

For although on the viol [*viola da gamba*], but most of all on the lute a motet or madrigal can be played through all semitones, so that the chromatic *genus* can be played by a well-trained and experienced master lutenist, nonetheless this is not so pure and just as can be attained on a harpsichord like this one. Here is why: it is because on viols and lutes all frets are equally far (...) removed from each other, so that the semitones can and should be called neither major nor minor but rather intermediate. After all, in my estimation every fret ... contains in itself  $4\frac{1}{2}$  comma's, as otherwise the major semitone would comprise five, but the minor semitone only four comma's ... so that half a comma is missing at both sides ...<sup>39</sup>

In contrast to viols and lutes, so Praetorius continues, keyboard instruments are generally unsuitable for playing in any other than the diatonic genus, since the strings or pipes must by necessity be intoned ahead of their being played. However, the great virtue of that perfect harpsichord in Prague is that on it one can play in all three *genera* and also that one can accompany any instrument, regardless of how it is intoned. But the biggest advantage of all is that all its semitones are pure, not equal and therefore half a comma less than pure as with the viol and the lute.

In this connection Praetorius refers in passing to several madrigals that Luca Marenzio wrote expressly in the chromatic genus. Now what does the term 'chromatic' (as also 'enharmonic') stand for in this connection? In ways to be discussed in greater detail in section 3.a., this refers to how the ancient Greeks used to fill up the *tetrachord*, not just with tones ('diatonic') but also with semitones ('chromatic') and/or quarter tones ('enharmonic'). In Italy, too, attempts were made (e.g., by Nicola Vicentino) to construct keyboard instruments capable of producing the corresponding micro-

<sup>39</sup> Praetorius, *Syntagma Musicum* II ('De organographia'), p. 65-6.

intervals; Praetorius himself mentions in passing that counterparts of that perfect Prague harpsichord have been built in Italy.<sup>40</sup>

‘Chromatic’ and ‘enharmonic’ are indicative in their turn of the ancients-inspired, rhetorical model of music making that has come up in the course of the sixteenth century to replace but also to supplement the standing numerical model of world harmony. On the whole, Praetorius’ description of the ‘universal, or perfect harpsichord’ bears witness to a failed reconstruction experiment. Its very failure goes to show that the ambition to found modern musical practices on ancient Greek ideas about perfect harmony was impossible. The underlying misunderstanding was nevertheless a most fruitful one, in that it led to the new musical conceptions of world harmony and to the new ideas about tuning and temperament that we discuss in the present chapter.

In order to understand how the general advance and technical elaboration of the organ coincides with the great innovations in polyphonic singing, we now cast a glance at a famous period organ. Suppose, for instance, that you are given a chance to play the 1596 organ ‘in cornu Evangelii’ in San Petronio cathedral in Bologna and you begin to play the ‘Recercar dopo l’elevazione’ in Frescobaldi’s *Messa delli Apostoli* (1583).<sup>41</sup> You happily play along until bar 49, where you find that the A flat prescribed there sounds extremely odd— a quite apparent misfit in its harmonic environment. Along the way you have hit many a G sharp too wonderfully harmonious to be attainable in equal temperament (as has already begun to dawn upon you). So the A flat that you at first thought you were sounding was really one more G sharp. But how on earth, then, could in bar 49 so delicate and accomplished a composer as Frescobaldi have prescribed A flat? Just possibly a passage by Girolamo Diruta flashes through your mind, where he summons the organist ‘to imitate with his playing at the elevation of the Most Holy Body and Blood of Our Lord Jesus Christ’ the hard and bitter torments of ‘the Most Holy Passion with the harmony of the fourth or second tones’.<sup>42</sup> May this perhaps account for the A flat? In any case you now look more closely at the key in question and find to your amazement that it is split crosswise. The organ builder, Baldassarre Malamini, has foreseen the difficulty, and he has resolved it by splitting the black key between F sharp and B flat. That is, he has built pipes for *both* G sharp *and* A flat and connected one to the foreside and the other to the rear of keys neatly split to that very purpose (similarly so for D sharp / E flat; for what a split key looks like, see Figure 5). From here on you play the piece with the utmost care, intent all the time on whether you come across a G sharp (keep finger up front) or rather another A flat (move finger to the rear).

Key splitting, then, provided one viable way out of the quandary that faced a keyboard player in

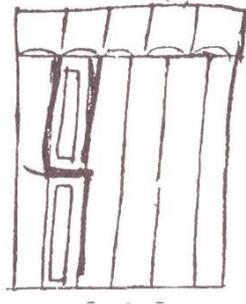
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<sup>40</sup> See “5. The Archicembalo” in Rehding, “Instruments of Music Theory”.

<sup>41</sup> On p. 44/5 of the Bärenreiter-Ausgabe no. 2205, Band V: *Fiori Musicali 1635*.

<sup>42</sup> Diruta, *The Transylvanian* (vol. 2, book 3, p. 116).

the age prior to equal temperament:



*Figure 5: Drawing of a split key by Christiaan Huygens (c. 1660)*

At the heart of the quandary are two basic facts of the history of Western music. One is the reliance on consonant intervals in the philosophic-arithmetical sense given (if not always in practice then at least in theory) to the notion of consonance from Pythagoras onwards – a reliance powerfully reinforced once the rise of polyphony turned harmony into the keystone of art music. The other is the familiar circumstance that (to use a later expression) the circle of fifths is not closed. As a consequence, diatonic and chromatic semitones are not equal, unless (and that is what equal temperament in effect comes down to) you *make* them equal by strictly artificial means. And musicians had very good reasons not to take that way out.

The basic problem (elucidated in the main text of the chapter) is that you cannot have all octaves, fifths, and thirds as pure at the same time. Whether or not this mathematically given, ineluctable fact of musical life can be avoided or at least circumvented in practice by singers, was an issue fiercely debated between Gioseffo Zarlino and Vincenzo Galilei. But whatever practical way out might have been open to singers was definitely closed to keyboard players, who cannot of course alter pitch while playing. You can have a pure major third C – E and a pure major third E – G sharp, but then G sharp – c is no longer pure or even a major third at all, but rather a harshly dissonant diminished fourth. On keyboard instruments, the more chromatic alteration you allow, the further this complicates the matter. Since on your manual or pedal you have only five black keys available, in order to maintain pure consonant intervals you are obliged to define them as C sharp *or* D flat, D sharp *or* E flat, F sharp *or* G flat, G sharp *or* A flat, A sharp *or* B flat. Hence, transposition and modulation are possible only within fairly narrow bounds. But harmonic experimentation beyond traditionally prescribed limits is precisely what the more daring among late-sixteenth and early-seventeenth century composers were after. On this very point split keys were meant to help them out — by enabling you to choose at will between playing G sharp or A flat, you may enhance the harmonic range of the piece

you are composing.

This principle of key splitting might of course be extended indefinitely, and indeed, a slightly later age was to see a large variety of multiple-key solutions to the tuning problem. The lasting solution, however, was to be found in quite another direction, also broached in the sixteenth century. This was (and is) the practice of *temperament* — a deliberate, slight mistuning of such a kind as not to offend our ear in too indelicate a manner. An early recipe was given by Arnolt Schlick, who in 1511 wrote in wholly qualitative terms of ‘the fifth ascending from gamut F in the manual to tenor C: do not make it high enough, or completely pure, but hovering [‘schwebend’, i.e., beating] somewhat lower, as much as the ear can stand, yet in such a way that one does not easily notice the above mentioned deficiency ...’<sup>43</sup>

Not for nothing did you, while playing that Recercar by Frescobaldi, enjoy the purity of the organ’s triadic and other chords so much, until you found that you had hit the ‘wrong’ A flat – the organ is tuned in so-called mean-tone temperament, with eight (or on this particular organ nine) out of twelve major thirds as pure and the fifths flattened just a little. Not for nothing did it take equal temperament no less than over a century and a half finally to replace a large variety of temperaments proposed and practiced in the meantime. Indeed, all these ‘well-tempered’ systems were meant to serve as viable compromises between, on the one hand, enhanced transposition and modulation and, on the other, both tonal *purity* and subtle tonal *differences* — with any *unequal* temperament, a piece in, say, C sounds really, not just nominally, different than one in D, as the sizes of the tones and semitones are subtly different in each key.

To grasp why this is so, consider the *order* of the tones and semitones. Each church mode used to be characterized by a specific tone-semitone pattern (TS-pattern) of its own, for instance, Dorian TSTTTST, or Lydian TTTSTTS. Due chiefly to an ever increasing use of accidentals, two modes developed in such a way as eventually to turn into the only ones still in use in our tonal system, major and minor, each of course fitted out with its characteristic TS-pattern (TTSTTTS for major, TSTTSTT for minor). However, in so-called just intonation (with all consonant intervals purportedly pure) a mode is not sufficiently characterized by a TS-pattern of its own, as both the T and the S may stand for different magnitudes at different times. In just intonation the T may be 8:9 or 9:10, and the S any of no less than three different semitones. So the number of TS-patterns available, as given by the need to have all consonant intervals as pure, comes pretty close to being infinite.

Now did Baldassarre Malamini’s organ provision of an additional pure major third A flat – C leave the organ with none but pure major thirds? Far from it; in reality the number of pure major thirds on this organ has increased only from the usual eight to just nine. This is so because the organ has been

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<sup>43</sup> Schlick, *Spiegel der Orgelmacher und Organisten*, ch. 8 (translation: p. 79).

tuned in what by the end of the sixteenth century came to be known as *meantone temperament*. Its defining feature is to split the ‘difference’ between the two whole tones in just intonation. The major third, say C-E, is kept pure, hence, 4:5 exactly, so the whole tone is given by the square root of 4:5. In consequence, the fifths suffer somewhat – each fifth is one fourth part of a syntonic comma too small, which is a deviation even sensitive ears tend quite quickly to accept without difficulty. Phrased more generally, this apparent willingness of our sense of hearing to put up with slight deviations from tonal purity is what makes temperament as such possible in the first place.

Still, meantone temperament does not quite allow you to have your cake and eat it, too. Its drawback is that (but for the ingenious yet rare and somewhat unwieldy artifice of split keys) it leaves you with unambiguously defined accidentals — *either C sharp or D flat*, etc. For as long as composers were happy or at least willing to stay within these limits, this was not a problem. However, in line with the entire direction in which *seconda prattica* experimentation was leading them, adventurous composers in Frescobaldi’s time began to overstep these limits, which already were a hindrance for easy transposition. From about mid-seventeenth century onward, the drive toward enhanced modulation set in motion a development away from meantone temperament. After the quite long-lived intermediary of an array of ‘well-tempered’ systems, the secular development towards wholesale rationalization of all tonal material was to find its logical end with equal temperament, by now so common to all of us that we tend to overhear its gross deficiencies.<sup>44</sup> Indeed, what with equal temperament you gain in terms of modulation and transposition you lose in terms of purity of the triad and subtlety of tonal variety. In music as elsewhere, modernity has definitely come with a price.

### **Case study II: Lutes, perfect harmony and temperaments**

During the sixteenth century string instruments were often considered through the lens of the predominant conception of world harmony. Many scholars believed that instruments owed their magical power to influence the human mind to numerical aspects, such as a perfect geometric shape or the balanced proportions of their parts, as for instance their string lengths. Thus, Marsilio Ficino firmly believed that a lute, in view of the similarity of its oval shape to the shape of the ear and the mouth but also of the human soul, could remedy human passions and emotions, and even restore the perfect harmony of the soul.<sup>45</sup> Next to the oval shape of its sound box, Ficino used the strings of the lute in his explanation of world harmony. While sketching how every part of the cosmos is harmoniously connected to all other parts, he invites his readers to imagine that

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<sup>44</sup> Konoval, “Pythagorean Pipe Dreams, Vincenzo Galilei, Marin Mersenne, and the Pneumatic Mysteries of the Pipe Organ”.

<sup>45</sup> Ficino, *Compendium in Timaeum* Cap. XXXI, 71<sup>r</sup>.

from one sounding lyre a note is suddenly communicated to another lyre tuned in the same way, then immediately from this vibrating string a similar vibration is passed on to the [other] string which is tuned equally.<sup>46</sup>

Behind the visible world hides the natural mystery of ‘cosmic sympathetic vibration’, which is modelled after a special type of resonance in which sound, travelling in air as waves of a kind, passes on its inner nature from one string to another.

As the sixteenth century advanced, however, this belief in the magical qualities of numerical aspects of instruments increasingly made room for a more pragmatic, functional approach. As Max Weber argued ‘The ever-present desire for expressive sonorous beauty, for a singing tone, and elegance of the instrument itself were the driving forces in [sixteenth-century] Italy of the orchestras and the instrument-makers.’<sup>47</sup> Weber explains the disenchantment of musical instruments by isolating equal temperament as the most modern mode of musical rationalization. Equal temperament, the near-universal tuning system nowadays, was held in the sixteenth century to be required only for fretted instruments such as the lute and viol. The driving force behind the requirement was new aesthetic ideals leading to important changes in tuning and temperament.

The starting point in this development is the ‘just intonation’ adopted by Gioseffo Zarlino, a prominent ‘maestro di capella’, musical theorist, and composer (see p. 8 above). Just intonation is meant to produce scales with none but pure consonances, which in the then current model of world harmony implies an octave with for numerical ratio 1:2, a fifth 2:3, a fourth 3:4, a major third 4:5, a minor third 5:6, a major sixth 3:5, and a minor sixth 5:8. Now take a very simple sequence such as the following: rise from C to G, then descend to D, then rise to A, then descend to E, then back to the original C. The *original* C indeed? For the case that the singer has intoned all these intervals as pure, it was demonstrated by Benedetti that the C sung at the end of the clause is no longer the original C but a C sharpened by a syntonic comma (see p. 14 above). In the course of one piece of music such sharpening need not occur more than nine times for pitch to have risen by a whole tone – the ostensible C has really become D. This is so because the syntonic comma (80:81) has the size of about one ninth whole tone. Not aware of the calculation, Zarlino and Vincenzo Galilei quarrelled for years about what singers do in practice.<sup>48</sup> Do they sing in just intonation and maintain pitch, as the former kept insisting, or, in order to maintain pitch, unconsciously adapt their intervals a little as they move on?

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<sup>46</sup> Ficino, *Compendium in Timaeum*, XXXI, 71<sup>r</sup>.

<sup>47</sup> Weber, *Rational and Social Foundations*, p. 109.

<sup>48</sup> Vincenzo Galilei, *Dialogue*, p. 10-13.

Instability of pitch is not the only practical difficulty that comes with just intonation. Another difficulty has to do with the tones and semitones, both in and of themselves and in regard of their order. In the Pythagorean division of the octave you have one whole tone, which is of course the ‘difference’ between the fifth and the fourth, hence, 8:9. In just intonation you have two. If the whole tone C-D is 8:9, and if the major third C-E is pure, hence, 4:5, then the whole tone D-E is necessarily  $(4:5) : (8:9) = 9:10$ , that is, a well-audible bit smaller than the other whole tone. With the semitones things are even more complicated. In Pythagorean intonation there are the diatonic semitones E-F and B-C, both calculated as the ‘difference’ between the pure fourth C-F and the Pythagorean major third, i.e.,  $(3:4) : (64:81) = 243:256$ . For the chromatic semitone this leaves 2048:2187, which actually makes it somewhat larger than its diatonic counterpart. In just intonation you rather end up, not only (as just shown) with two whole tones 8:9 and 9:10, but also with no less than three different chromatic semitones, one diatonic (15:16) and two chromatic (128:135 and 24:25).

In short, then, just intonation came with numerous, quite weighty problems for musical practice. How could one possibly make music with tonal material of such unwieldy complexity, innocently called into being by those who in the early fourteenth century insisted on introducing pure thirds in their music making? The problems are most acute for fretted and for keyboard instruments.

Singers, as Vincenzo Galilei rightly intuited, are flexible in their intonation, but the notes employed by lute and keyboard players must by necessity be determined before they start playing. For them there are solutions of two kinds. One is to increase the number of keys on the keyboard beyond the standard twelve per octave. The other is to adapt a little (or, in slightly more technical language, to *temper*) the purity of the consonant intervals employed. Specific solutions of both kinds began to be tried out in the course of the sixteenth century.

Equal temperament is the theoretical possibility of getting rid of all commas (the Pythagorean as well as the syntonic) by making all semitones, the diatonic and the chromatic, equal. Theoretically it was known by mid-sixteenth century at the latest. But how was it employed in musical practice? As an experienced lutenist, Galilei was well aware of the advantages equal temperament offered his own instrument. On a lute all whole tones are best given equal size in view of its neck and fingerboard being tied with frets. Galilei uses Aristoxenus in solving the problem of tuning lutes.<sup>49</sup> To arrive at the size of a musical interval of a third, Aristoxenus did not start from numerical ratios, but from the sound of a perfect third as it is constituted in the sense of hearing. The sound of a third can then easily be split into two halves, each of which forms an equally large, whole tone. According to Galilei, Aristoxenus’ approach fits in nicely with what lutenists habitually do in practice. He lets Bardi, one of the two interlocutors of the dialogue, describe the usage of practitioners on the lute: “the tones are

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<sup>49</sup> Vincenzo Galilei, *Dialogue*, p. 105.

equal (as I told you and proved) and divided into two equal parts, and the major third is consonant.”<sup>50</sup> This is the case, because they tune their instruments by ear, in ways which come very close to equal temperament. Indeed, Galilei gave a recipe for accomplishing this in practice to a very good approximation. The fifths in his lute are a little smaller than is required by their ideal proportions, and, consequently, the fourths are just a little larger to the same extent.<sup>51</sup> The frets on a lute, then, are located in different positions as would be the case in Pythagorean tunings such as just intonation.

Right upon establishing that lutes cannot be tuned in the same way as harpsichords, Galilei asked himself whether the harpsichord can perhaps be tuned in the same way as the lute. If his personal approximation of equal temperament were to become the standard way for tuning keyboard instruments, too, the whole notion of harmony in earthly music being based on the harmony of the universe would become obsolete. After a great deal of thought Galilei rejected this solution, arguing that “if we wanted to temper the keyboard instrument according to the usage of the lute, we cannot escape having the sense be offended in certain particular places”.<sup>52</sup> Hence, the harpsichord would sound too much out of tune for late sixteenth-century ears, which were accustomed to listening to perfect, i.e. not tempered, consonances

Seeming unaware of this development in Italy, at the beginning of the seventeenth century, the German Abraham Bartolus continued to advocate the idea that the tuning of instruments must be based on the harmonic ratios of the universe. He searched for a temperament which, while imitating the harmony of the spheres, could also be employed in modern performance practice. Just how familiar Bartolus was with fretted instruments appears from his diagrams employing lute tablature, according to which *a* (in French tablature) and *O* (in Italian tablature) refer to frets 1, 4 and 6; *e* and *4* refer to fret 3; and so on (*Figure 6*).<sup>53</sup>

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<sup>50</sup> Vincenzo Galilei, *Dialogue*, p. 107.

<sup>51</sup> Vincenzo Galilei, *Dialogue*, p. 113.

<sup>52</sup> Vincenzo Galilei, *Dialogue*, p. 114.

<sup>53</sup> Lindley, *Lutes, Viols and Temperament*, p. 70.

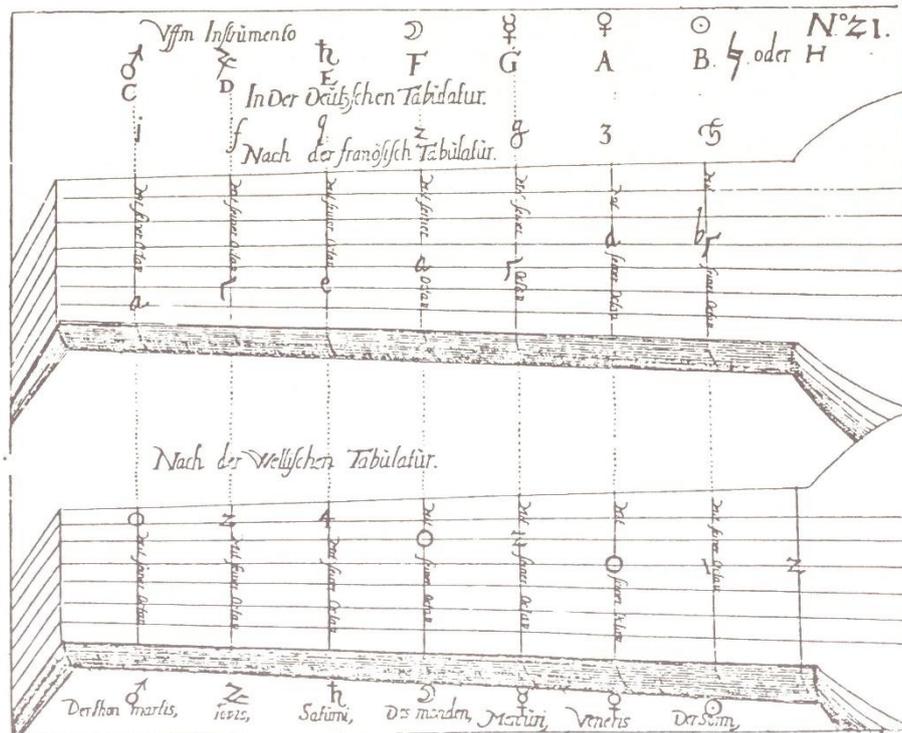


Figure 6: Abraham Bartolus' design for French and Italian tablature, in which the frets on the lute represent the music of the spheres (*Musica*, 1641, p. 172)

In addition, the frets on the lute represent a cosmic scale, in which the C on the first fret is linked to the planet Mars, the second to Jupiter, and so on.

The cosmic implications of Bartolus' just-intonation lute are a clear indication that, despite all difficulties it raised for tuning and temperament, the Pythagorean doctrine of world harmony continued to be a source of inspiration for the practice of music until the beginning of the seventeenth century.

## 5. Short bibliographic essay

### Source anthologies

- Godwin, Joscelyn (ed.), *Harmony of the Spheres: A Sourcebook of the Pythagorean Tradition in Music*. Rochester: Inner Traditions International, 1993.
- Strunk, Oliver (ed.), *Source Readings in Music History: from Classical Antiquity through the Romantic Era*. New York: Norton, 1950.

### Sources cited in the main text

- Aristoxenus, *Harmonics* (fourth century BCE; transl. Henry S. Macran. Oxford: Clarendon Press, 1902).
- Abraham Bartolus, 'Musica mathematica' (in H. Zeisung (ed.) *Theatri machinarum*, vi. Leipzig, 1614).
- Giovanbattista Benedetti, 'De intervallis musicis' (1585; in: idem, *Diversarum speculationum mathematicorum et physicarum liber*, p. 277-283).
- Boethius, *De institutione musica* (early sixth century AD; translated in: Calvin M. Bower and Claude V. Palisca (eds.), *Anicius Manlius Severinus Boethius: Fundamentals of Music*. New Haven & London: Yale University Press, 1989).
- Girolamo Cardano, *Opera omnia* (facs. reprint of the 1663 edition, introduced by August Buck; Stuttgart, 1966).
- Girolamo Diruta, *Il Transilvano* (1593, 1609; facs. reprint with introd. by Edward J. Soehnlén and Murray C. Bradshaw; Buren: Knuf, 1983; translated by Murray C. Bradshaw and Edward J. Soehnlén as *The Transylvanian*, Henryville: Institute of Mediaeval Music, 1984).
- Marsilio Ficino, *Opera Omnia* (2 vols.), Basle, 1576 (reprint: Turin: Bottega d'Erasmus, 1962).
- *Compendium in Timaeum* (1496).
- *De vita triplici* (1489; Carol V. Kaske and John R. Clark (ed. and transl.), *Three Books On Life*, Binghamton, N.Y.: Medieval & Renaissance Texts & Studies, in conjunction with the Renaissance Society of America, 1989).
- Franchino Gaffurio, *Theorica musicae* (1492).
- *Practica musicae* (1496).
- *De harmonia musicorum instrumentorum opus* (1518; introd. and transl. by Clement A. Miller; Rome: American Institute of Musicology, 1977).
- Galileo Galilei, *Discorsi* (1638; in: idem, *Opere*, Edizione Nazionale, vol. 8, p. 143; here quoted from Thomas Weston's translation (1730: *Mathematical Discourses Concerning Two New Sciences*))
- Vincenzo Galilei, *Dialogo della musica antica, et della moderna* (1581; facsimile ed. by Fabio Fano; Rome, 1934; translated with introd. and notes by Claude V. Palisca as *Dialogue On Ancient and Modern Music*; New Haven: Yale University Press, 2003).
- Francesco Patrizi, *Nova de universis philosophia*. Ferrara, 1591 (reprinted with variants in Venice, retrodated to 1593, and in London in 1611).
- Michael Praetorius, *Syntagma Musicum*. Wittenberg and Wolfenbüttel, 1615-1620; 3 vols. (facs. reprint edited by Willibald Gurlitt (Kassel: Bärenreiter, 1974-1978); parts 1 and 2 of vol. 2 'De

Organographia' have been translated into English by David Z. Crookes; Oxford: Clarendon Press, 1991).

Arnolt Schlick, *Spiegel der Orgelmacher und Organisten* (1511; facsimile reprint, with transcription and translation by Elizabeth Berry Barber: Buren (Netherlands): Frits Knuf, 1980).

Gioseffo Zarlino, *Istitutioni harmoniche* (1558; portions of the book have been translated into English by Guy A. Marco and Claude V. Palisca, and into German by Michael Fend).

---- *Dimostrationsi harmoniche* (1571).

---- *Sopplimenti musicali* (1588).

## Further Reading

### *World history of music*

Our principal guiding thread in the present chapter is Max Weber, *The Rational and Social Foundations of Music* (South Illinois University Press, 1958). This is an unfinished treatise written in 1913 under the title *Die rationalen und soziologischen Grundlagen der Musik* (briefly summarized, with some additional insights, in his essay 'Der Sinn der "Wertfreiheit" der soziologischen und ökonomischen Wissenschaften'. In: *Gesammelte Aufsätze zur Wissenschaftslehre* (Tübingen: Mohr, 1922; p. 451-502). The idea of rationality in music that is developed in this most inspiring treatise does not at all involve a unilinear account of relentless progress. Rather, Weber provides an illuminating (albeit in several ways meanwhile obsolete) outline of how, starting in the early Middle Ages, religious and art music in Western Europe uniquely turned into our present-day functional harmony. Two excellent studies on the same subject are Christoph Braun, 'Grenzen der Ratio, Grenzen der Soziologie. Anmerkungen zum "Musiksoziologen" Max Weber'. *Archiv für Musikwissenschaft* 51, 1, 1994, p. 1-25, and Daniel K.L. Chua, *Absolute Music and the Construction of Meaning*. Cambridge University Press, 1999. A new view on the history of music and emotions is presented in *The Routledge Companion to Music, Mind, and Wellbeing*, Penelope Gouk, James Kennaway, Jacomien Prins, and Wiebke Thormahlen (eds.) (Abingdon: Routledge, 2018).

### *Renaissance humanism*

A very good introduction to the humanist context of much Renaissance thinking on music is a range of volumes (once path-breaking, and still very important) by Paul O. Kristeller jointly entitled *Renaissance Thought and the Arts: Collected Essays* (Princeton, NJ: Princeton University Press, 1951).

*Music philosophy and musical theory in the sixteenth century*

The great pioneers were D.P. Walker and Claude V. Palisca. Many of Walker's splendid studies have been collected in *Studies in Musical Science in the Late Renaissance* (Leiden: Brill, 1978) and in *Music, Spirit and Language in the Renaissance* (Penelope Gouk, ed. (London: Variorum Reprints, 1985)). Claude V. Palisca's numerous publications tend to stress in the many sources that he consulted the more 'modern' looking elements, with consequences at times for his interpretations and also for his translations: *Studies in the History of Italian Music and Music Theory* (Oxford: Clarendon, 1994); *Music and Ideas in the Sixteenth and Seventeenth Centuries* (University of Illinois Press, 2006), and *Humanism in Italian Renaissance Musical Thought* (New Haven: Yale University Press, 1985). In his tradition stands also Nancy Kovaleff Baker and Barbara Russano Hanning (eds.), *Musical Humanism and Its Legacy: Essays in Honor of Claude V. Palisca* (Stuyvesant, NY: Pendragon Press, 1992).

Other significant works on music philosophy and musical theory in the sixteenth century include (in chronological order):

- Willi Apel, *Renaissance and Baroque Music: Composers, Musicology and Music Theory* (Stuttgart : Steiner, 1989).
- Alberto Gallo *et al.*, 'Italienische Musiktheorie im 16. und 17. Jahrhundert. Antikrezeption und Satzlehre'. In: *Geschichte der Musiktheorie 7* (Darmstadt: Wissenschaftliche Buchgesellschaft, 1989).
- Anne Moyer, *Musica Scientia: Musical Scholarship in the Italian Renaissance* (Ithaca: Cornell University Press, 1992).
- James Haar (ed. Paul Corneilson), *The Science and Art of Renaissance Music* (Princeton University Press, 1997).
- Cristle Collins Judd, *Reading Renaissance Music Theory: Hearing with the Eyes* (Cambridge University Press, 2000).
- Philippe Vendrix (ed.), *Music and Mathematics* (Turnhout: Brepols, 2008).
- Cristle Collins Judd ed.), *Musical Theory in the Renaissance* (Ashgate, 2013).
- Stefano Mengozzi, *The Renaissance Reform of Medieval Music Theory* (Cambridge University Press, 2010).

There are further several useful studies about individual protagonists:

- on Ficino and Patrizi: Jacomien Prins, *Echoes of an Invisible World: Marsilio Ficino and Francesco Patrizi On Cosmic Order and Music Theory* (Leiden: Brill, 2014).
- on Vincenzo Galilei: ch. 2 'Vincenzo Galilei and Zarlino' in D.P. Walker's *Studies in Musical Science ...*; further Claude V. Palisca, 'Was Galileo's Father an Experimental Scientist?' In: V.

Coelho (ed.), *Music and Science in the Age of Galileo* (Dordrecht: Reidel, 1992); p. 143-151; p. 143-51; Daniel K.L. Chua, 'Vincenzo Galilei, Modernity and the Division of Nature'. In: Suzannah Clark and Alexander Rehding (eds.), *Music Theory and Natural Order from the Renaissance to the Early Twentieth Century* (Cambridge University Press, 2001); p. 17-29, and two articles by Carla Bromberg and Ana Maria Alfonso-Goldfarb entitled 'Vincenzo Galilei and Music: Some Socio-Cultural and Acoustical Discussions'. *Circumscribere* 6, 2009, p. 1-11, and 'A Preliminary Study of the Origin of Music in Cinquecento Musical Treatises'. In: *IRASM* 41, 2, 2010, p.161-183.

- on Benedetti: H. Floris Cohen, 'Benedetti's Views on Musical Science and Their Background in Contemporary Venetian Culture'. In: *Cultura, Scienze e Tecniche nella Venezia del Cinquecento. Atti del Convegno Internazionale di Studio 'Giovanni Battista Benedetti e il suo tempo'* (Venezia: Istituto Veneto di Scienze, Lettere ed Arti, 1987); p. 301-310.
- on Stevin: H. Floris Cohen, 'Simon Stevin's Equal Division of the Octave', in: *Annals of Science* 44, 1987, p. 471-488.

*Music philosophy and musical theory over longer stretches of time*

World harmony as such is treated in a very enlightening manner by James Haar in an unpublished dissertation (Harvard University, 1961) entitled *Musica Mundana: Variations on a Pythagorean Theme*.

Three large books full of pertinent contributions (all in the same series as the present volume) are A.M.B. Berger and J. Rodin (eds.), *The Cambridge History of Fifteenth Century Music*, Cambridge: Cambridge University Press, 2015); Thomas S. Christensen (ed.), *The Cambridge History of Western Music Theory* (Cambridge University Press, 2002), and Tim Carter and John Butt (eds.), *The Cambridge History of Seventeenth-century Music* (Cambridge University Press, 2005).

Four consecutive chapters in Charles Burnett, Michael Fend and Penelope Gouk (eds.), *The Second Sense: Studies in Hearing and Musical Judgement from Antiquity to the Seventeenth Century* (The Warburg Institute, University of London, 1991) brilliantly pursue the shifting relationship between 'intellectus' and 'sensus' from Aristoxenus and Ptolemy up to c. 1600.

Seventeenth century themes with more than a little overlap with the preceding period include (in chronological order):

- Jamie Kassler, 'Music as a Model in Early Science'. *History of Science* 20, 1982; p. 103-39.
- H. Floris Cohen, *Quantifying Music: The Science of Music at the First Stage of the Scientific Revolution, 1580-1650* (Dordrecht: Reidel, 1984).
- Penelope Gouk, *Music, Science and Natural Magic in Seventeenth-Century England* (New Haven/London: Yale University Press, 1999).

- Paolo Gozza (ed.), *Number to Sound: The Musical Way to the Scientific Revolution* (Dordrecht: Kluwer Academic Publishers, 2000).
- Wolfgang Fuhrmann, 'Antike Musiktheorie und die naturwissenschaftliche Revolution der frühen Neuzeit', in: *Musiktheorie* 22, 2007; p. 317–327.
- H. Floris Cohen, 'Music as Science and As Art – The 16<sup>th</sup>/17<sup>th</sup> Century Destruction of Cosmic Harmony'. In: R. Bod, J. Maat & T. Weststeijn (eds.), *The Making of the Humanities. Vol. 1: Early Modern Europe* (Amsterdam University Press, 2010); p. 59-71.

An eighteenth-century theme with implications for earlier periods is taken up in Downing A. Thomas, *Music and the Origins of Language: Theories from the French Enlightenment* (Cambridge University Press, 1995).

### *Tuning and temperament*

A near-exhaustive survey, in which every single tuning system is measured in terms of how close it came to equal temperament as if this were the final destination of the entire development, is James M. Barbour, *Tuning and Temperament: A Historical Survey* (New York: Da Capo Press, 1951). Other broad but not nearly so blatantly 'progressivist' works on the subject are Carl Dahlhaus *et al.*, *Hören, Messen und Rechnen in der frühen Neuzeit* (Darmstadt: Wissenschaftliche Buchgesellschaft, 1987); D. Devie, *Le tempérament musical: Philosophie, histoire, théorie et pratique* (Béziers: Société de Musicologie de Languedoc, 1990), and three works by Mark Lindley: *Lutes, Viols and Temperament* (Cambridge University Press, 1984); 'Stimmung und Temperatur'. In: *GMt* 6 (1987); p. 109–331, and a book written together with R. Turner-Smith: *Mathematical Models of Musical Scales: A New Approach* (Bonn: Verlag für Systematische Musikwissenschaft, 1993).

Please note that issues of tuning and temperament also come to the fore in most works mentioned above under 'music philosophy and musical theory'.

### *Musical instruments as a source for the historical study of music theory*

The role of the lute in sixteenth-century musical theory comes up for investigation in Julia Kursell, 'Falsche Strecken, leise Töne: die Laute in der Musiktheorie'. In F. A. Kittler and A. Ofak (eds.), *Medien vor den Medien* (Paderborn: Fink, 2007); p. 199-221.

A wonderful introduction to musical instruments as a source for the historical study of music theory is given in: Alexander Rehding, "Instruments of Music Theory", *Music Theory Online: A Journal of the Society for Music Theory*, vol. 22.4 (2016), pp. 1-22 and in Brandon Konoval, "Pythagorean Pipe Dreams, Vincenzo Galilei, Marin Mersenne, and the Pneumatic Mysteries of the Pipe Organ", *Perspectives on Science* vol. 26.1 (2018), pp. 1-51..