

# Term Structure, Inflation and Real Activity

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## Abstract

This paper estimates an internally consistent structural model that imposes cross-sectional restrictions on the dynamics of the term structure of interest rates, inflation, and output growth. Distinct from previous term structure settings, this model introduces both time varying central tendencies and a stochastic conditional mean of output growth. The estimation of the model, which is based on U.S. data over a 1960 to 2005 sample period, provides reliable estimates for the implicit term structures of real interest rates, expected inflation rates, and inflation risk premia, as well as for expectations of macroeconomic variables. The model has better out-of-sample forecasting properties than a number of alternative models, and contradicts the puzzling evidence that during the ‘Great Moderation’ in inflation subsequent to the mid-1980s, the forecasting ability of structural models deteriorated with respect to atheoretic statistical models.

## I. Introduction

The relationship between the term structure of interest rates, inflation, and real activity has long been recognised as central to both macroeconomic and finance theory, and as critical in formulating economic policy and in investment decisions. Inflation expectations and predictions of economic growth also play a key role in pricing financial instruments, as well as in determining the actions of governments and central banks in their attempts to smooth the business cycle. Conversely, changes in asset prices and economic policies influence current levels of inflation and real growth, and therefore, expectations about their future direction.

Extensive theoretical and empirical work has been devoted to the relationship between real activity, inflation, and the term structure of interest rates. Thus far,

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however, there is neither a commonly accepted theoretical framework, nor agreement on empirical regularities.

According to Harvey (1988 and 1989) and subsequent analyses by, among others, Estrella and Hardouvelis (1991), Plosser and Rouwenhorst (1994), Chapman (1997), Kamara (1997), and Roma and Torous (1997), the shape of the term structure reflects information about future economic growth. In particular, a positive slope in the yield curve predicts an increase in the level of real activity, whereas a flattening or a negative slope in the yield curve is associated with a future recession. Conversely, Evans and Marshall (2001), Ang and Piazzesi (2003), and Dewachter and Lyrio (2006) show that macroeconomic factors have a significant impact on the term structure of interest rates.

Empirical studies on the information content of the term structure for future inflation have been undertaken by, among others, Fama (1990), and Mishkin (1990). These studies suggest that only the long end of the term structure of interest rates contains information about future inflation, whereas the short end of the term structure provides almost no such information.

All the above cited studies relate to interest rate theory, but do not satisfy the problem of specifying a complete model of the term structure of interest rates which fully accounts for the links between inflation, economic growth, and bond prices, and which might be usable for forecasting purposes.

The inclusion of macroeconomic relationships in a general equilibrium term structure framework has its origins in the continuous time model of Cox, Ingersoll and Ross (hereafter CIR) (1985a and 1985b), which describes a complete economy with production and a stochastic investment opportunity set and endogenously determined optimal consumption, portfolio choice, and equilibrium asset prices.

Among the CIR-type term structure models which explicitly consider the role of macroeconomic variables, we make note of Breeden (1986), Pennacchi (1991), Sun (1992), Bakshi and Chen (1996) and, more recently, Wu (2001), Ang and Piazzesi (2003), Buraschi and Jiltsov (2005), and Dewachter and Lyrio (2006).

Considering data on macroeconomic variables such as inflation and output, along with data on bonds, highlights the macroeconomic underpinnings of the yield curve and makes a term structure model suitable for both investment and economic policy purposes.

This paper adopts such a macroeconomic approach and presents a structural model which allows (i) investigation of the cross-sectional restrictions which link the dynamics of the term structure of interest rates to output growth and inflation, (ii) reliable estimation of the unobservable term structures of real interest rates, expected inflation rates, and inflation risk premia, and (iii) generation of endogenous, accurate forecasts of future inflation and gross domestic product (GDP) growth rates.

This paper relates to its field as follows. First, it builds an internally consistent structural model, which provides a bridge between fully specified equilibrium models (see, for example, Marshall (1992) or, more recently, Wu (2001)) and non-structural term structure models with macroeconomic factors, such as the vector autoregression (VAR)-based models of Evans and Marshall (2001), Ang and Piazzesi (2003), and Ang, Piazzesi and Wei (2006). In particular, the model complements non-structural models in documenting how macroeconomic factors explain the bulk of term structure movements. However, unlike Ang and Piazzesi (2003), the model does not incorporate any latent factors lacking clear economic interpretation. Rather, all factors are clearly identified as components of those processes pertaining to inflation, short-term real interest rates, and returns to the production process. In this respect, our approach is close in spirit to that of Dewachter and Lyrio (2006).

Second, this paper obtains the term structures of real interest rates and expected inflation rates from observations on nominal bond prices using a more structured approach than Pennacchi (1991), and Sun (1992), who focus only on inflation data, and Evans (2003), Goto and Torous (2003), and Ang and Bekaert (2005), who adopt regime-switching models.

Third, we derive endogenous estimates of time varying bond risk premia using a framework which extends the yield-only approach of Duffee (2002), and Duarte (2004), and which differs substantially from those recently proposed by Buraschi and Jiltsov (2005), and Cochrane and Piazzesi (2005). Finally, we exploit the equilibrium cross-sectional restrictions imposed by our model to obtain accurate forecasts of future inflation and GDP growth rates by making use of term structure data. The model has better out-of-sample forecasting properties than a number of alternative models, including atheoretic statistical models, and seems to contradict the puzzling evidence described by Stock and Watson (2007); that is, the fact that during the so-called ‘Great Moderation’ in inflation subsequent to the mid-1980s, the forecasting ability of structural models deteriorated with respect to naive univariate models.

This paper proceeds as follows. Section II illustrates the structural term structure model and derives the main equations describing the relationship between interest rates and macroeconomic variables. In Section III, using U.S. data over a 1960–2005 sample period, we present the maximum likelihood–Kalman filter estimates of the model for the term structures of real interest rates, expected inflation rates, and inflation risk premia. Moreover, we provide statistical evidence of the fact that the model produces more accurate forecasts of future inflation and GDP growth than some alternative models. Section IV presents our conclusions.

## II. The Model

In this section, we outline the structure of the macroeconomic term structure model.<sup>1</sup>

### A. Production, Inflation, and State Variables

The underlying basic assumptions of the model are consistent with the CIR (1985a) general equilibrium framework,<sup>2</sup> in which the state variables driving the dynamics of the economy are assumed to be the real interest rate and the expected inflation rate. In common with most term structure models including inflation (such as CIR (1985b), Pennacchi (1991), Sun (1992), Ang and Bekaert (2005), and Buraschi and Jiltsov (2005)), we do not explicitly model money and monetary policy and assume an

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<sup>1</sup> A full derivation of the model is available upon request.

<sup>2</sup> These assumptions are: (i) a continuous time competitive economy composed of a fixed number of identical individuals with rational expectations and time-additive logarithmic utility functions; (ii) a single physical good is produced, which may be allocated to consumption or investment; (iii) a single endogenous production process exists, which is affected by stochastic technological changes; (iv) there are markets for instantaneous borrowing and lending at the same riskless interest rate as well as markets for default-free nominal and real bonds. The solution of the individual’s intertemporal problem determines the total consumption and production plan, the price level, the optimal allocation of wealth among activities, and the equilibrium value of the riskless interest rate and nominal and real bond prices. As shown by CIR (1985a), in equilibrium all wealth should be invested in the physical production process. Therefore, equilibrium is characterised by a zero supply of bonds and no borrowing and lending.

exogenously given process for the price level. In other words, we assume that there exists an underlying equilibrium in the money market that supports the observed price level. However, we depart from the above-mentioned models in allowing a time varying central tendency for both the real interest rate and the expected inflation rate variables.<sup>3</sup> This allows us to indirectly account for monetary policy in our framework in that these two variables can be interpreted as proxies for Fed interest rate targets, as in Balduzzi, Das and Foresi (1995), and Jegadeesh and Pennacchi (1996) for example.

As regards the real economy, we assume a single technology producing a single physical good and that production output follows a stochastic process. In line with a CIR-type general equilibrium framework, the logarithmic utility hypothesis underlying the model implies that the expected rate of return on the production process should be equal to the sum of the real interest rate and the variance of the production process (see Breeden (1986)). However, in practice, the relationship between the real interest rate and expected output growth can change significantly over time. Chapman (1997) shows that this relationship has been quite unstable in the U.S. over the last 50 years and has also exhibited a negative sign for some periods. In fact, recursively regressing quarterly GDP growth rates against a naive estimator of the real interest rate, we observe that the slope coefficient varies significantly over time.<sup>4</sup>

For this reason, in order to better describe the dynamics of output, we relax the CIR general equilibrium restriction by allowing the conditional mean of output growth to be modelled as an exogenous stochastic variable, which is assumed to be related to the real interest rate, both in the drift and in the innovation term.

We assume that, under the physical probability measure, the state variables, i.e., the instantaneous real interest rate  $r$ , and expected inflation rate  $\pi$ , and their stochastic long-term means,  $\theta$  and  $\xi$ , respectively, along with the conditional mean of output growth  $\alpha$ , follow a joint elastic random walk process,

$$(1) \quad ds = (\phi + \Gamma s)dt + \Sigma dz,$$

$$\text{where } s \equiv \begin{pmatrix} r \\ \pi \\ \theta \\ \xi \\ \alpha \end{pmatrix}, \quad \phi \equiv \begin{pmatrix} 0 \\ 0 \\ \phi_\theta \\ \phi_\xi \\ \phi_\alpha \end{pmatrix}, \quad \Gamma \equiv \begin{pmatrix} \gamma_{rr} & \gamma_{r\pi} & -\gamma_{rr} & -\gamma_{r\pi} & \gamma_{r\alpha} \\ \gamma_{\pi r} & \gamma_{\pi\pi} & -\gamma_{\pi r} & -\gamma_{\pi\pi} & 0 \\ 0 & 0 & \gamma_{\theta\theta} & 0 & 0 \\ 0 & 0 & 0 & \gamma_{\xi\xi} & 0 \\ \gamma_{\alpha r} & 0 & 0 & 0 & \gamma_{\alpha\alpha} \end{pmatrix}.$$

$\Sigma$  is a diagonal matrix and  $z$  is a vector of standard Brownian motion.

We assume that both the macroeconomic variables, the production output  $q$ , and the price level  $p$ , evolve according to a Gaussian process,

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<sup>3</sup> Dewachter and Lyrio (2006) also assume a stochastic central tendency for the inflation rate and the real interest rate. They adopt a rather simple specification: the central tendency of the real interest rate is assumed to be a linear function of the central tendency of the inflation rate and the long-term means of these variables are fixed at their initial values.

<sup>4</sup> The regressions use a fixed 15-year window of quarterly data beginning in 1960:Q1. Proceeding recursively, we obtain parameter estimates for the period from 1975:Q1 to 2005:Q4. The estimator of the real interest rate is constructed by estimating the expected inflation rate from the one-step forecast of an ARIMA (1,0,1) model applied to the logarithm of the GDP deflator and then subtracting this series from the observed 3-month zero coupon yield. Consistent with Chapman (1997), we find that the estimated slope coefficient is, on average, around zero and ranges between 0.68 and -0.36.

$$(2) \quad \frac{dm}{m} = \Gamma_m s dt + \Sigma_m dz_m,$$

$$\text{where} \quad \frac{dm}{m} \equiv \begin{pmatrix} dq/q \\ dp/p \end{pmatrix}, \quad \Gamma_m \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}.$$

$\Sigma_m$  is a diagonal matrix and  $z_m$  a vector of standard Brownian motion.

Innovations in the state variables, output and price level could be mutually correlated and the correlation matrices  $\Psi \equiv E_t\{dz, dz'\}$ ,  $\Psi_m \equiv E_t\{dz_m, dz'_m\}$ , and  $\Xi \equiv E_t\{dz, dz'_m\}$  full.

From equation (2), expressions for logarithmic changes in production and price level over time interval  $[t, t+\tau]$  can be obtained. For both these expressions and the stochastic differential equations (1), a closed form solution can be derived and the dynamics of the economy summarised by the following set of equations:

$$(3) \quad E_t\{s(t+\tau)\} = a(\tau) + B(\tau)s(t),$$

$$(4) \quad \text{Cov}_t\{s(t+\tau), s(t+\tau)'\} = C(\tau),$$

$$(5) \quad E_t\{\ln[m(t+\tau)/m(t)]\} = h(\tau) + J(\tau)s(t),$$

$$(6) \quad \text{Cov}_t\{\ln[m(t+\tau)/m(t)], \ln[m(t+\tau)/m(t)]'\} = M(\tau),$$

$$(7) \quad \text{Cov}_t\{s(t+\tau), \ln[m(t+\tau)/m(t)]'\} = Q(\tau),$$

where  $\tau > 0$ , and where  $a(\tau)$ ,  $B(\tau)$ ,  $C(\tau)$ ,  $h(\tau)$ ,  $J(\tau)$ ,  $M(\tau)$  and  $Q(\tau)$  are non-linear transformations of the original set of coefficients in equations (1-2).

Equations (3-7) completely describe the dynamics of the economy. This structure implies that cross-equation restrictions link the dynamics of all the variables in the economy, that is, the state variables, output, and the price level.

## B. Risk Premia and Term Structure of Interest Rates

In the CIR (1985a) framework, the volatility and correlation structures endogenously determine the market prices of risk,

$$(8) \quad \Lambda_0 \equiv \Sigma^{-1} \left( \text{Cov}_t \left\{ ds, \frac{dq}{q} \right\} + \text{Cov}_t \left\{ ds, \frac{dp}{p} \right\} \right) = \Xi \Sigma_m \mathbf{1},$$

with  $\mathbf{1}' \equiv (1 \ 1)$ .

The homoskedasticity assumption underlying the Gaussian structure of the model implies that risk premia are constant. However, following Duffee (2002) (see also Duarte (2004)), we adopt the ‘essentially affine’ specification, which allows the market

price of risk to vary independently of interest rate volatility. In this case, market prices of risk are assumed to be affine in the state variables,

$$(9) \quad \Lambda(t) \equiv \Lambda_0 + \Lambda_s s(t),$$

where  $\Lambda_0$  is specified as above and  $\Lambda_s$  is a matrix of free parameters. This specification implies that the matrix of mean reversion coefficients under the risk-adjusted probability measure is given by  $\tilde{\Gamma} \equiv \Gamma - \Sigma \Lambda_s$ . However, some constraints on the form of matrix  $\Lambda_s$  must be imposed in order to preserve a structure consistent with  $\theta$  and  $\xi$  serving as the stochastic central tendencies of  $r$  and  $\pi$  under the risk-adjusted probability measure. Therefore, we have

$$\Lambda_s \equiv \begin{pmatrix} \lambda_{rr} & \lambda_{r\pi} & -\lambda_{rr} & -\lambda_{r\pi} & \lambda_{r\alpha} \\ \lambda_{\pi r} & \lambda_{\pi\pi} & -\lambda_{\pi r} & -\lambda_{\pi\pi} & 0 \\ 0 & 0 & \lambda_{00} & 0 & 0 \\ 0 & 0 & 0 & \lambda_{\xi\xi} & 0 \\ \lambda_{\alpha r} & 0 & 0 & 0 & \lambda_{\alpha\alpha} \end{pmatrix}.$$

This model admits a closed form solution for the equilibrium price of a nominal unit discount bond with maturity  $\tau$ . The nominal term structure takes the form

$$(10) \quad Y(t; t + \tau) = \kappa_0(\tau) + \kappa(\tau)s(t),$$

where the coefficients  $\kappa_0(\tau)$  and  $\kappa(\tau)$  depend on the underlying model parameters.

Instantaneous expected excess returns on nominal bonds are time varying. In fact, by defining with  $F(t; t + \tau)$  the price at current time  $t$  of a nominal unit discount bond with maturity  $\tau$  and with  $R(t)$  the instantaneous nominal interest rate at time  $t$ ,  $R(t) \equiv Y(t; t + 0)$ , we have

$$(11) \quad E_t \left\{ \frac{dF(t; t + \tau)}{F(t; t + \tau)} \right\} - R(t)dt = -\tau \kappa(\tau) \Sigma \Lambda(t).$$

Instantaneous expected excess returns can be decomposed into their components such that the contribution of each state variable can be identified.

We can also determine the closed form solution for the equilibrium price of a real unit discount bond with maturity  $\tau$  and, therefore, an expression for the real term structure,

$$(12) \quad y(t; t + \tau) = \omega_0(\tau) + \omega(\tau)s(t),$$

where the coefficients  $\omega_0(\tau)$  and  $\omega(\tau)$  are non-linear functions of the original set of parameters.

The actual<sup>5</sup> expected inflation rate for the maturity  $\tau$  can be obtained from expression (5) above and is affine in the state variables. The difference between the  $\tau$ -maturity nominal yield and the sum of the  $\tau$ -maturity real interest rate and actual expected inflation rate is a variable that comprises an inflation risk premium and the Jensen's inequality term. The inflation risk premium included in the nominal term structure depends on the covariance between inflation and output growth rates, as this premium is linked to the bond's ability to hedge against a decrease in consumption.<sup>6</sup> The essentially affine specification implies that the inflation risk premium at time  $t$  on a bond with maturity  $\tau$  is time varying and can be calculated as

$$(13) \quad W(t; t + \tau) = Y(t; t + \tau) - y(t; t + \tau) - \frac{1}{\tau} E_t \left\{ \ln \frac{p(t; t + \tau)}{p(t)} \right\} + \frac{1}{2\tau} \text{Var}_t \left\{ \ln \frac{p(t; t + \tau)}{p(t)} \right\}.$$

This expression shows that the greater is the variability in the price level, the higher is the inflation risk premium required on nominal bonds.

### III. Empirical Results

In this section, we present the results of the estimation of the model for U.S. data over the 1960 to 2005 sample period. First, we show the model estimates of the unobservable term structures of real interest rates, expected inflation rates, and inflation risk premia. Then, we evaluate the forecasting ability of the model for future inflation and GDP growth and compare its performance with that of alternative models.

#### A. Data

The empirical investigation is based on U.S. quarterly data from the first quarter of 1960 through the fourth quarter of 2005. Bond yields are end-of-quarter annualised zero coupon yields with maturities of 3 months, 1, 3, 5, 10 and 20 years. For the period between the first quarter of 1960 and the first quarter of 1991, the data are taken from McCulloch and Kwon (1993). From the second quarter of 1991 through the fourth quarter of 2005, zero coupon yields calculated from U.S. Treasury STRIP prices are used.<sup>7</sup>

As a measure of economic activity, we use seasonally adjusted data on real GDP, expressed in constant dollars, from the U.S. Department of Commerce's Bureau of Economic Analysis. The seasonally adjusted GDP deflator is used as a proxy for the price level.

#### B. Estimation Method

The model is estimated using data available only at discrete observation intervals. The discrete time transformation of the continuous time model can be obtained by

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<sup>5</sup> The term 'actual' indicates that these are the inflation expectations calculated under the physical, rather than the risk-adjusted, probability measure.

<sup>6</sup> See, among others, Benninga and Protopapadakis (1983), Breeden (1986), and Abken (1993).

<sup>7</sup> These data were kindly provided by Stephen Schaefer, who computed bond yields from U.S. Treasury STRIP prices supplied by Street Software.

exploiting the solution to the stochastic differential equations that describe the dynamics of the variables. The resulting discrete time state space form is composed of state transition equations related to unobserved state variables and measurement equations related to bond yields, output growth and inflation.

The parameters of the state space model are estimated by the maximum likelihood method with the Kalman filter algorithm used to calculate the values of the unobserved state variables.<sup>8</sup>

## C. Estimation Results

### Specification tests

The model imposes several moment restrictions and encompasses more parsimonious models such as, for example, the Pennacchi (1991) model. In order to test the adequacy of the model, we perform some preliminary specification tests following Duffee (2002), and Ang and Bekaert (2005).

First, we apply a generalised method of moments (GMM)-type test to assess the closeness of the estimated unconditional moments to the sample moments. This test is based on the point statistic  $H = (h - \bar{h})' \Sigma_h^{-1} (h - \bar{h})$ , where  $\bar{h}$  represents the estimates of the unconditional moments obtained from the sample,  $h$  represents the unconditional moments estimated by the model,  $\Sigma_h$  represents the covariance matrix of the sample estimates of the unconditional moments, which is estimated using GMM with the Newey–West correction for heteroskedasticity and autocorrelation. Under the null hypothesis, this statistic is distributed as  $\chi^2(n)$ , where  $n$  is the number of overidentifying restrictions. We test for first and second moments of bond yields and yield changes, for maturities of 3 months, 1, 3, 5, 10 and 20 years, and GDP growth and inflation rates with time horizons of 3, 6, 9 and 12 months.

Second, we use a Lagrange multiplier (LM) statistic to test for the presence of autoregressive conditional heteroskedasticity (ARCH) effects in the estimation residuals, which might arise from the fact that we are using homoskedastic Gaussian-type processes in modelling financial and macroeconomic variables.

Third, we apply a GMM-based test for first order serial correlation in the estimated scaled residuals of the measurement equation. We test the following null hypothesis:  $E(e_t e_{t-1}) = 0$ .

Finally, using a Wald statistic, we test a null hypothesis involving restrictions on the model parameter vector. The restricted models are represented by: (i) a two-factor model, in which central tendencies are constant and the conditional mean of output growth is fixed at a constant value; (ii) a three-factor model in which central tendencies are constant; (iii) a four-factor model in which the conditional mean of output growth is non-stochastic; (iv) a five-factor completely affine specification in which risk premia are constant (the coefficients of matrix  $\Lambda_s$  in equation (9) are all equal to 0).

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<sup>8</sup> For the covariance matrices relating error terms in the state variables, output growth, and inflation we impose all the cross-equation restrictions implied by the theoretical model, which further helps to identify the model. The deviation of actual yields from their theoretical values is assumed to be due to ‘observation error’. In contrast, the errors in the equations for output growth and inflation are ‘forecasting errors’, which arise from the difference between actual and expected values for output growth and inflation. In the state space form, both these error types are referred to as ‘measurement errors’. We assume that observation errors in bond yields are mutually uncorrelated and are not correlated with forecasting errors in output growth and inflation. Moreover, in order to limit the number of parameters which must be estimated, we assume that the errors on bond yields are cross-sectionally homoskedastic.



The results in Table 1 show that the model performs relatively well. In Panel A we observe that the unconditional moments implied by the model estimates are within 1 standard error of data moments, with the exception of the variance of the GDP growth rate. The LM statistic shows that the model captures the heteroskedasticity within the bond yield data, whereas the presence of ARCH-type effects in the estimated residuals does not look negligible in the case of GDP growth and inflation rates. The goodness of fit of the model is reflected in Panel B, where the  $\chi^2$  test does not allow us to reject the null hypothesis that the first and second moments of yields, yield changes, and inflation rates over various time horizons estimated by the model match the sample moments. Consistent with the previous tests, we find that the model tends to underestimate the volatility of GDP growth rates and that the null hypothesis must be rejected.

Panel C shows that, as predicted by the model, the estimated residuals for bond yields and quarterly GDP growth and inflation rates are not serially correlated. Moreover, we observe that the various restrictions on the model's parameters are rejected, indicating that the time varying central tendencies, the stochastic conditional mean of output growth, and the essentially affine specification for the risk premia all play a significant role in fitting the relationship between the term structure and macroeconomic variables.

As regards goodness of fit of the model, the estimated bond yield errors are relatively low, with no evidence of systematic under/over pricing. In fact, the errors have both positive and negative mean values at the various maturities and the root mean squared errors are below 20 basis points. It is worth noting that the model also fits well the yields not included in the estimation sample (2-, 4-, and 6-to-9-year maturities).

### **Estimated parameters and state variables**

Table 2 reports the estimated parameter values under the physical probability measure. We note that the real interest rate  $r$  exhibits significant mean reversion toward its time varying central tendency  $\theta$ , which is more persistent. In contrast, both the expected inflation rate  $\pi$  and its central tendency  $\xi$  are close to a random walk. The estimated long-term means of the central tendencies  $\theta$  and  $\xi$  are equal, respectively, to 1.7% and 3.8%.

Figure 1 shows the Kalman filter estimated series of the instantaneous real interest rate and expected inflation rate, their time varying central tendencies and the conditional mean of output growth. Consistent with previous estimates (see, for example, Pennacchi (1991), Sun (1992), and Ang and Bekaert (2005)), the real interest rate becomes negative in the 1970s and exhibits a sharp increase in level and volatility in the 1979–82 period, when the Federal Reserve changed its monetary policy procedures. Thereafter, especially in the second half of the sample, the real interest rate seems to revert quickly toward its central tendency. The expected inflation rate declines considerably after the so-called ‘Fed experiment’. The series estimated for the stochastic central tendencies appears reasonable.<sup>9</sup>

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<sup>9</sup> Following Jegadeesh and Pennacchi (1996), we might interpret the sum of the central tendencies as a proxy for the unobservable nominal target rate of Fed monetary policy. Long-run targets reflect individuals' expectations about medium- to long-term inflation and economic growth and, implicitly, individuals' beliefs regarding future monetary policy. Our estimates imply that the half-life (the time it takes for the variable to revert half way to its initial value) of the short-term nominal rate is about 2 years, whereas its central tendency is very persistent. This result appears to be in contrast with the evidence presented in Jegadeesh and Pennacchi (1996), where the central tendency displays higher mean reversion than the short rate.

The estimated series of the conditional mean of output growth exhibits a relatively high mean reversion toward a 3.5% long-term mean, and a much higher volatility than the other state variables. As shown in Figure 1, this variable seems to be highly related to the cyclical component of output growth, which is estimated by applying the Hodrick–Prescott filter to the annual rate of growth of real GDP. We note that the volatility of the conditional mean of output growth sharply decreases during the post-1984 Great Moderation in U.S. inflation. In this period, its correlation with both the real interest rate and the expected inflation rate declines considerably. This evidence seems to be consistent both with Chapman (1997), who claims that the relationship between the real interest rate and expected output growth has been quite unstable in the last few decades, and with Stock and Watson (2007), who show that some of the macroeconomic relationships implicit in models that use the Phillips curve have weakened in the post-1984 period.

#### D. Real and Expected Inflation Term Structures

The model allows extrapolation from nominal bond prices of the unobservable term structures of real interest rates and inflation expectations. A natural benchmark to evaluate model estimates would be represented by market values extracted from inflation-indexed bonds. However, a liquid market with a relatively long series of observations exists only in the U.K.;<sup>10</sup> the U.S. Treasury has only recently begun to issue inflation-indexed bonds. Estimates of the U.S. real term structure calculated from such bonds begin only in 1997. A different benchmark must be used for analysis of the properties of the real and expected inflation term structures estimated by the model for the sample period 1960–2005. In particular, we use ex post realised real yields and inflation rates. The  $\tau$ -maturity real interest rate realised at time  $t$  is calculated as the difference between the time  $t$  observed  $\tau$ -maturity nominal yield and the annualised inflation rate realised between time  $t$  and  $t+\tau$ . The latter term is used as a benchmark to test the predictive ability of the model term structure estimates for future inflation rates.

Panel A of Table 3 contains summary statistics on the estimated real and expected inflation term structures for maturities of up to 5 years. On average, both these term structures are upward sloping, with volatility tending to decrease with the time horizon. As in Ang and Bekaert (2005), we find that real rates are procyclical and expected inflation is countercyclical. In fact, estimated real rates are generally higher during expansions than recessions, whereas expected inflation rates behave in the opposite way, with the volatilities of both variables increasing during recessions.<sup>11</sup>

We compare the moments estimated by the model with the sample moments of ex post realised real yields and inflation rates with maturities from 1 to 5 years using the  $\chi^2$  test based on the point statistic  $H$  described above.

We observe that the unconditional moments implied by the model predictions for real interest rates and inflation rates are within 1 standard error of realised data moments, with the exception of the variance of real yields. Indeed, the volatility of

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<sup>10</sup> Empirical evidence on this market has been provided by, among others, Brown and Schaefer (1994), Barr and Campbell (1997), and Evans (1998).

<sup>11</sup> For example, we estimate that the 2-year real rate is, on average, 2.46% (with Newey–West heteroskedasticity and autocorrelation consistent standard error of 0.30%) during expansions, and 2.19% (s.e. 0.78%) during recessions. The corresponding 2-year expected inflation rate is on average 3.51% (s.e. 0.34%) during expansions and 4.92% (s.e. 0.92%) during recessions. We consider expansion and recession periods as defined by the NBER.

estimated real yields is relatively low with respect to that observed ex post. To a certain extent, this might be due to the fact that the ex post real yields include an implicit inflation risk premium. In general, however, the model seems to provide relatively accurate predictions on the dynamics of the levels of both the real and the inflation term structures. In fact, the correlation between the series estimated by the model and the corresponding ex post realised values is very high.

Turning to the time series of the estimated term structures of real interest rates and expected inflation rates, we observe a change in the sign of their correlation following the 1979–82 ‘Volcker experiment’. Indeed, consistent with the evidence presented by Goto and Torous (2003), we show in Panel B of Table 3 that the correlation between real interest rates and expected inflation rates is significantly negative before 1979 and becomes positive after 1982. The average shape of the term structures also changes. The term structure of real rates is upward sloping before 1979, and humped, at much higher levels, after 1982. The term structure of expected inflation rates moves from downward sloping to upward sloping, with a sharp decline in level and volatility.

As pointed out by Goto and Torous (2003), this could have resulted from an aggressive anti-inflationary monetary policy implemented by the Federal Reserve beginning in 1982, which implies that nominal interest rates move more than one-for-one with inflation expectations, inducing a positive relationship between real interest rates and expected inflation. Indeed, by regressing  $\tau$ -maturity nominal yields against  $\tau$ -maturity real interest rates and expected inflation rates for this period, we obtain coefficients significantly higher than 1. This implies that the Fisher relationship, that is, the independence between real interest rates and inflation expectations, is violated. Moreover, this evidence shows that the so-called Mundell (1963)–Tobin (1965) effect, which has been documented extensively in the empirical literature (see, among others, Mishkin (1992) and Boudoukh (1993)), does not hold in the face of the correlation between real interest rates and expected inflation rates in the post-1982 period. We show below that this result has a significant impact on the inflation risk premium.

## E. Risk Premia

Figure 2 shows that the estimated inflation risk premia vary significantly over time and across maturities. We note a clear change in the sign of inflation risk premia at the beginning of the 1980s, i.e., during the Volcker experiment. This result is strikingly similar to that obtained by Goto and Torous (2003) using a switching regime approach, and implies that a positive premium is required by investors on nominal bonds when monetary policy strictly controls inflation. As in Buraschi and Jiltsov (2005), we find that the average term structure of inflation risk premia has a positive slope, with values ranging between 10 (1-year bond) and 90 (10-year bond) basis points.

Figure 2 also shows the time series behaviour of the average (across maturities) instantaneous expected excess bond returns. We observe that these values are, on average, around zero before 1982 and that they become significantly positive thereafter. Panel A of Table 4 presents summary statistics on the estimated instantaneous expected excess bond returns for various maturities and for each state variable. We observe that the instantaneous risk premium on the real interest rate remains almost constant with maturity (around 25 basis points), whereas that for expected inflation increases with bond maturity and is around 35 basis points on average. The total instantaneous risk premium has an increasing structure and ranges between 42 (1-year bond) and 286 (10-year bond) basis points.

As in Cochrane and Piazzesi (2005), we find that expected excess bond returns are linked to the business cycle and tend to be more volatile during recessions than during expansions. A similar result applies to estimated inflation risk premia.

In Panel B of Table 4 we compare the predictive power of model estimates for 1-year excess bond returns with that of the Cochrane and Piazzesi (2005) single-factor model. The single factor in the Cochrane–Piazzesi model is a linear function of forward rates which is obtained by running the following regression

$$(14) \quad \overline{ER}(t+1) = \beta_0 + \sum_{i=1}^5 \beta_i f(t;0,i) + \bar{u}(t+1) = (\beta'f(t)) + \bar{u}(t+1),$$

where  $ER(t+1;T) \equiv [F(t+1;T-1)/F(t;T)] - [1/F(t;1)]$  is the 1-year excess return on a  $T$ -maturity bond and  $\overline{ER}(t+1)$  is the average of excess bond returns for  $T = 2, 3, 4, 5$ .  $F(t;T)$  represents the price at time  $t$  of a  $T$ -maturity unit zero coupon bond and  $f(t;T-1,T)$  is the forward rate for loans between time  $t+T-1$  and  $t+T$ .

In a second step, the following forecasting regressions are estimated

$$(15) \quad ER(t+1;T) = b_T (\beta'f(t)) + u(t+1;T), \quad T = 2, 3, 4, 5.$$

We run the same two regressions by using as a predictor the annualised estimated  $T$ -maturity model risk premia,  $T = 2, 3, 4, 5$ , instead of the forward rates.

We observe, consistent with Cochrane and Piazzesi (2005), that the normalised coefficients  $b_T$  tend to increase with the maturity of the bonds. The results show that the model's predictive power, measured in terms of  $\bar{R}^2$ , is comparable to that produced by the Cochrane–Piazzesi framework and explains about one-third of the time variation in 1-year excess bond returns. As pointed out by Cochrane and Piazzesi (2005), this is considerably higher than the value obtained in the benchmark studies of Fama and Bliss (1987), and Campbell and Shiller (1991).<sup>12</sup> According to this evidence, it can be argued that including macroeconomic variables in a term structure model does indeed provide relevant information for the prediction of bond returns.

## F. GDP Growth and Inflation Forecasts

In this section, we compare both the in-sample and out-of-sample accuracy of the macro term structure model (MTS) in predicting GDP growth and inflation rates with respect to other models. In particular, as competing models, we consider an essentially affine version of the Pennacchi (1991) model (PEN), and the Ang, Piazzesi and Wei (2006) model (APW).

The PEN two-factor model with constant long-term means and no adjustment for the conditional mean of output growth is nested within our term structure framework.<sup>13</sup>

<sup>12</sup> We estimate the Fama–Bliss regressions for our data set by regressing the 1-year excess return on a  $T$ -maturity bond,  $T = 2, 3, 4, 5$ , against the forward spread of the corresponding maturity. The estimated  $\bar{R}^2$  ranges between 10% and 16%. For brevity, we do not report these regressions.

<sup>13</sup> In fact, the Pennacchi (1991) model assumes that the state variables driving the dynamics of the economy are the instantaneous real interest rate and expected inflation rate following a joint elastic random walk process with constant long-term means. The dynamics of the price level and the production output are described by Gaussian-type processes, with no stochastic conditional mean of output growth. The model is estimated using the maximum likelihood–Kalman filter method.

A comparison with the results obtained by this model allows us to evaluate the role of time varying central tendencies and the conditional mean of output growth in the empirical properties of the model.

Ang, Piazzesi and Wei (2006) estimate the relationship between the term structure and macroeconomic variables in the context of a VAR model, in which no-arbitrage restrictions are imposed on the dynamics of bond yields. The APW model can be considered a non-structural term structure model with macro factors (see, for example, Evans and Marshall (2001), and Ang and Piazzesi (2003)) and, in this respect, it represents an interesting alternative approach to our own.<sup>14</sup>

Moreover, as a benchmark for the evaluation of GDP growth and inflation predictions, we estimate the Harvey (1989) term spread regression model (HAR), and the random walk model (RW).<sup>15</sup>

### **In-sample predictions**

Table 5 shows the results of the in-sample regression of realised GDP growth (Panel A) and inflation rates (Panel B) against the corresponding expectations generated by the models.

The output growth expectations implicit in the MTS model estimates represent relatively accurate predictions of actual GDP growth rates, although the accuracy of the fit tends to decrease as the time horizon lengthens. In fact, the  $\bar{R}^2$  of the regression is 85% at the 3-month horizon, and 57% at the 1-year horizon. The competing models, in contrast, are poor in their ability to predict short-term GDP growth rates and to improve performance as time horizon lengthens. The HAR and APW estimators appear to be unbiased, whereas in the case of the PEN model we observe that the sign of the regression coefficient is wrong. This is an effect of the restriction imposed by the Pennacchi (1991) framework (and, in general, by general equilibrium term structure models of the CIR type) on the relationship between expected output growth and real yields. In particular, it implies that output growth expectations are determined in direct relationship with the real interest rate. However, as illustrated above, the sign of this relationship can change over time because of the effect of exogenous factors, such as productivity shocks. This evidence underlines the relevance of the variable  $\alpha$ , which captures the impact of such exogenous shocks on the dynamics of output.

The expectations produced by the MTS model represent accurate predictions of future inflation at all time horizons up to 2 years. Moreover, the estimator is unbiased. The inflation predictions produced by the model are more precise than those provided by alternative models. Again, the comparison with the PEN two-factor model allows us to appreciate the role of both the time varying central tendencies, and the conditional

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<sup>14</sup> In the original APW specification, the vector of state variables includes the short rate (3-month yield), the term spread (20-year minus 3-month yield) and lagged annualised quarterly GDP growth rate. With respect to this model, we add two more factors: the ‘curvature’ factor (3-month plus 20-year yield minus twice the 5-year yield), and lagged annualised quarterly inflation rate. The five factors are assumed to follow a Gaussian VAR with one lag. Cross-equation restrictions stemming from the no-arbitrage condition are imposed and allow the model to endogenously produce expectations of future inflation and GDP growth rates over any time horizon. In estimating the parameters of the model, we adopt the consistent two-step procedure used by Ang, Piazzesi and Wei (2006). The analytical derivation of the PEN and APW models, and the estimated coefficients are available upon request.

<sup>15</sup> The HAR model uses as a predictor for the annualised real GDP rate of growth between time  $t$  and  $t+\tau$  the slope of the term structure observed at time  $t$ . We estimate the model using the difference between the 5-year and the 1-year nominal yield as a measure of the slope of the term structure. Consistent with Atkeson and Ohanian (2001) and Stock and Watson (2007), the RW model uses the average of the current quarter, and the preceding three quarters of inflation as its forecast for future periods.

mean of output growth in improving the fit of the relationship between bond yields and macroeconomic variables.

The model's predictive properties are robust with respect to multiple time periods. According to the empirical evidence presented by Atkeson and Ohanian (2001), Fisher, Liu and Zhou (2002), and Stock and Watson (2007), since the mid-1980s most modern macroeconometric models (for example, Stock and Watson (1999) and related models) fail to produce inflation forecasts that are more accurate than those generated by a naive RW model. Stock and Watson (2007) point out that the Great Moderation in U.S. inflation after 1984 has led to a marked decrease in inflation volatility. They observe that on the one hand, this has lowered the size of forecasting errors; but on the other hand, it has induced a deterioration in the forecasting accuracy of structural macro models with respect to univariate naive models.

In Panel C of Table 5, we test this hypothesis by comparing the in-sample forecast root mean square errors (RMSE) of the MTS and RW models in the pre-1984 and post-1984 periods. Consistent with Stock and Watson (2007), we observe that the MTS model yields more accurate predictions in the pre-1984 sample and that RMSE sharply decreases in the post-1984 sample. However, in contrast with the empirical findings cited above, we find that the MTS model significantly outperforms the RW model in the post-1984 period as well. This result is reinforced by the following analysis, which illustrates the out-of-sample forecasting properties of the models.

### **Out-of-sample forecasts**

Out-of-sample forecasts are obtained by using the preceding 15 years of quarterly data to estimate the parameters needed to predict subsequent GDP growth and inflation rates over various time horizons. Proceeding recursively, we compute forecasts for all the competing models from 1975:Q1 to 2005:Q4 and compare these to realised GDP growth and inflation rates over the same period. As above, the models we consider are the MTS, the PEN, the APW, the HAR (for GDP growth), and the RW (for inflation). Moreover, we include the forecasts provided by the Survey of Professional Forecasters published by the Federal Reserve Bank of Philadelphia (SUR).<sup>16</sup>

Table 6 summarises the error properties of the estimated models over various time horizons and reports the values of the Diebold and Mariano (1995) statistic (DM), which tests the null hypothesis of no difference in the accuracy of the forecasts generated by the alternative models with respect to those produced by the MTS model.<sup>17</sup>

Panel A of the Table 6 shows that the GDP growth forecasts produced by the MTS model are more accurate than those generated by the alternative approaches for

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<sup>16</sup> This survey relies on macroeconomic forecasts supplied by several major economic research centres and financial institutions. Before 1990, this was known as the ASA/NBER survey. As in Ang, Bekaert and Wei (2007), we consider median forecasts. However, using mean forecasts causes the results to not differ significantly.

<sup>17</sup> The Diebold–Mariano statistic is calculated as the  $t$ -statistic obtained by regressing the difference in squared errors between the competing forecasts against a constant and correcting this equation for serial correlation. The statistical properties of the test can be summarised as follows. Let  $x_t$  denote the loss differential series (in our case, the difference in squared errors between the competing forecasts at time  $t$ ). Under the null hypothesis that the population mean of the  $\{x_t\}$  series is zero, the statistic  $\bar{x}/\sqrt{2\pi\hat{g}_x(0)/T}$  is asymptotically standard normal distributed, where  $\bar{x}$  is the sample mean loss differential and  $\hat{g}_x(0)$  is a consistent estimate of the spectral density of the loss differential at frequency 0. A consistent estimate of  $2\pi g_x(0)$  is obtained by using the Newey–West correction for serial correlation. The test is valid for a wide class of loss functions and allows the forecast errors to be non-Gaussian, nonzero mean, serially correlated, and contemporaneously correlated.

time horizons of up to 1 year. We observe that the MTS model does not systematically over/underestimate future GDP growth rates and that the mean of the forecast errors is very close to zero. The reduction in forecasting errors induced by the inclusion in the model of the stochastic central tendencies and the conditional mean of output growth is significant, as we can infer by comparing these errors with those of the PEN model. For longer time horizons, the Harvey's term spread appears to be the best predictor for future GDP growth.

The striking result that emerges from Panel B of Table 6 is the accuracy of the MTS model in forecasting inflation rates at time horizons of up to 2 years. The forecasting errors, measured in terms of RMSE, are around half those of the RW model. The DM test rejects the null hypothesis of no difference in the accuracy of MTS model forecasts with respect to competing models at all time horizons.

Consistent with Ang, Bekaert, and Wei (2007), we find that the SUR predictor outperforms the PEN and APW term structure models. However, according to the DM test, the SUR forecasts are not statistically more accurate than those generated by the RW process.

In order to test whether the forecasting properties of the MTS model are influenced by the change in the U.S. inflation regime beginning around the mid-1980s, in Panel C of Table 6 we compare the out-of-sample forecasts of the MTS and the RW models on a decade-by-decade basis. In particular, we report the ratio between the RMSE of the MTS and RW forecasts for three sub-periods: 1975–1983, 1984–1994, and 1995–2005. We find that this ratio does not change significantly across the three periods and is always below 1, meaning that the MTS model outperforms the RW model at all forecasting horizons.

Figure 3 shows the dynamic behavior of the ratio between the annual average of absolute inflation forecasting errors generated by the MTS and RW models, respectively, for various time horizons. We observe that this ratio is above 1 only in the case of the 2-year forecasting horizon for the 1992–95 sample period. Furthermore, in the last decade, the ratio has become relatively low (below 50%) at all forecasting horizons.

The fact that the MTS model performs better than the RW model at forecasting horizons of 1 and 2 years contradicts the evidence presented by Atkeson and Ohanian (2001), and Stock and Watson (2007), which show that during the post-1984 Great Moderation in inflation, the forecasting ability of structural models has deteriorated with respect to naive univariate models.

In our view, the superior performance of the MTS model in forecasting inflation in recent years can be explained by the fact that, in the post-1984 sample period, (i) the size of the correlation between actual inflation and other macroeconomic variables, such as GDP growth and the unemployment rate, has declined, whereas (ii) the tightness of the correlation between actual inflation rates and the level and slope of both the nominal and real term structures of interest rates has increased, especially in the last decade.

On the one hand, this evidence may explain the fact that the predictive power of backward-looking Phillips curve models that rely only on macroeconomic variables in forecasting future inflation has diminished. On the other hand, this may explain why the MTS model, which accounts for the forward-looking cross-sectional relationships between inflation, output growth, and nominal and real interest rates, generates relatively accurate forecasts. In this respect, our results seem to support the multi-country empirical analysis presented in Stock and Watson (2003), where a positive role in forecasting output and inflation is assigned to asset prices because of their forward-looking nature.

## IV. Conclusion

In this paper, we build and estimate an internally consistent structural model that analyses the relationship between the term structure of interest rates, inflation, and output growth. The equilibrium aspects of the model impose nontrivial restrictions on the joint dynamics of bond yields and macroeconomic variables and imply that real and monetary variables of the economy are interrelated and influence the shape of the yield curve, while bond yields convey information that is useful for forecasting economic fundamentals.

The results indicate that by considering data on inflation and GDP, along with data on yields in an affine term structure setting allows us to obtain reliable estimates of the implicit term structures of real interest rates, expected inflation rates, and inflation risk premia, and to derive sensible predictions for bond returns.

Moreover, the empirical evidence shows that the model generates forecasts of future inflation and GDP growth rates which are considerably more accurate than those produced by several well known alternative approaches, and it contradicts the notion that the Great Moderation in inflation subsequent to the mid-1980s has induced a deterioration in the forecasting ability of structural models with respect to naive univariate models.

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TABLE 1  
Specification Tests

Panel A. Model-estimated Sample Moments

Variable	Mean			Variance			ARCH test
	Model	Sample	S. E.	Model	Sample	S.E.	<i>p</i> -value
3-month yield	571	567	64	8.3	8.1	2.8	0.34
1-year yield	605	611	65	7.9	8.2	2.8	0.68
3-year yield	655	653	63	7.3	7.3	2.4	0.91
5-year yield	676	674	61	7.0	6.8	2.3	0.66
10-year yield	696	697	59	6.3	6.2	2.0	0.42
20-year yield	719	719	57	5.6	5.7	1.7	0.40
GDP growth rate	322	327	33	6.2	11.5	2.2	0.01
inflation rate	370	370	54	5.1	5.8	1.6	0.06

Panel B.  $\chi^2$  Test on Unconditional Moments

<i>p</i> -value	Yields		Changes in Yields		GDP Growth Rates		Inflation Rates	
	<i>n</i> = 6		<i>n</i> = 6		<i>n</i> = 4		<i>n</i> = 4	
	Mean	Variance	Mean	Variance	Mean	Variance	Mean	Variance
	1.00	1.00	1.00	0.93	0.13	0.04	1.00	0.99

Panel C.  $\chi^2$  Test on Residual Correlation and Walt Test on Parameter Restrictions

<i>p</i> -value	Residuals Correlation	Constant Means No $\alpha$ Variable	Constant Means	No $\alpha$ Variable	Constant Risk Premia
	<i>n</i> = 8	<i>n</i> = 30	<i>n</i> = 19	<i>n</i> = 13	<i>n</i> = 9
		1.00	0.00	0.00	0.00

Table 1 shows summary statistics on specification tests based on the maximum likelihood–Kalman filter estimates of the macro term structure model over the sample period 1960:Q1–2005:Q4. Panel A compares the moments estimated by the model with the sample moments of bond yields and annualised quarterly GDP growth and inflation rates. Data are expressed in basis points. The standard errors (S.E.) are calculated using GMM with the Newey–West correction. The last column reports the *p*-values of LM statistic testing for the presence of ARCH effects in the residuals. Panel B reports the *p*-values of a  $\chi^2$  test based on the point statistic  $H = (h - \bar{h})' \Sigma_h^{-1} (h - \bar{h})$ , where  $\bar{h}$  and  $h$  are the estimates of the unconditional moments provided by the sample and the model, respectively, and  $\Sigma_h$  is the covariance matrix of the sample estimates of the unconditional moments, estimated using GMM with the Newey–West correction. We test for first and second moments of bond yields and yield changes, with maturity of 3 months, 1, 3, 5, 10 and 20 years, and GDP growth and inflation rates, with time horizon 3, 6, 9 and 12 months. Panel C contains the *p*-value of a GMM-based test for the null hypothesis of no serial correlation in the model scaled residuals and the *p*-values of a Wald statistic testing the null hypothesis that *n* restrictions can be placed on the model parameter vector: (i) constant central tendencies and no stochastic conditional mean of output growth (the  $\alpha$  variable is a constant); (ii) constant central tendencies; (iii) no stochastic conditional mean of output growth; (iv) completely affine specification implying constant risk premia.

TABLE 2  
Estimated Parameter Values

Variable	$\phi$	$\Gamma$				
		$r$	$\pi$	$\theta$	$\xi$	$\alpha$
$r$		-0.6110 (0.0125)	-0.1301 (0.0125)	0.6110	0.1301	-0.0487 (0.0186)
$\pi$		-0.0582 (0.0072)	-0.0721 (0.0071)	0.0582	0.0721	
$\theta$	0.0037 (0.0020)			-0.2187 (0.0059)		
$\xi$	0.0020 (0.0017)				-0.0515 (0.0304)	
$\alpha$	0.0191 (0.0013)	-0.0391 (0.0065)				-0.5419 (0.0021)

Variable	$\Sigma$ (diag.)	$\Lambda_s$				
		$r$	$\pi$	$\theta$	$\xi$	$\alpha$
$r$	0.0243 (0.0057)	4.7529 (0.1796)	5.9385 (0.1296)	-4.7529	-5.9385	--
$\pi$	0.0141 (0.0052)	2.6685 (0.3861)	12.6708 (0.1767)	-2.6685	-12.6708	
$\theta$	0.0222 (0.0060)			-2.2262 (0.0848)		
$\xi$	0.0091 (0.0067)				-4.6782 (0.2331)	
$\alpha$	0.0654 (0.0084)	--				1.9994 (0.4601)

Variable	$\Sigma_m$ (diag.)	$\Psi_m$ $P$	$\Psi$				$\Xi$	
			$\pi$	$\theta$	$\xi$	$\alpha$		
$r$			-0.2842 (0.0129)	0.4172 (0.0217)	--	-0.1926 (0.0026)	0.2571 (0.0097)	--
$\pi$					0.3353 (0.0113)	-0.1959 (0.0051)	--	-0.0431 (0.0209)
$\theta$					-0.3925 (0.0080)	--	0.0793 (0.0154)	--
$\xi$						--	-0.2241 (0.0155)	0.2775 (0.0098)
$\alpha$							0.5009 (0.0099)	--
$q$	0.0036 (0.0018)	0.2339 (0.0022)						
$p$	0.0032 (0.0010)							

Table 2 shows the parameter values of the macro term structure model estimated by the maximum likelihood–Kalman filter method over the sample period 1960:Q1–2005:Q4. Asymptotic standard errors are in parentheses.

TABLE 3  
Real and Expected Inflation Term Structures

Panel A. Estimated vs. Realised Sample Moments

	Mean			Variance			Correlation
	Model	Sample	S. E.	Model	Sample	S.E.	
<b>Real Interest Rates</b>							
1 year	222	240	54	4.4	6.1	1.7	0.97
2 years	245	268	62	3.9	7.5	2.1	0.94
3 years	261	288	64	3.3	8.1	2.3	0.92
4 years	272	305	67	2.8	8.5	2.4	0.90
5 years	276	318	70	2.4	8.8	2.4	0.87
<i>p</i> -value <i>H</i> statistic			0.96			0.00	
<b>Exp. Inflation Rates</b>							
1 year	374	371	54	5.0	5.2	1.5	0.97
2 years	379	375	54	5.0	4.9	1.4	0.91
3 years	384	379	53	4.9	4.6	1.2	0.86
4 years	389	383	53	4.9	4.3	1.1	0.81
5 years	395	387	52	4.8	4.1	1.0	0.77
<i>p</i> -value <i>H</i> statistic			1.00			0.97	

Panel B. Correlation Between Real Rates and Expected Inflation Rates

Maturity (years)	Pre-1979 (1960:Q1–1978:Q4)					Post-1979 (1982:Q1–2005:Q4)				
	Real Rates		Exp. Inflation			Real Rates		Exp. Inflation		
	Mean	St.Dev.	Mean	St.Dev.	Corr.	Mean	St.Dev.	Mean	St.Dev.	Corr.
1	105	118	436	246	−0.76	295	214	268	90	0.37
3	157	107	434	242	−0.77	314	194	284	93	0.44
5	178	96	429	237	−0.77	309	172	296	95	0.49
7	186	86	423	233	−0.76	297	152	307	96	0.51
10	187	73	415	225	−0.76	276	127	320	98	0.54

Table 3 shows summary statistics on the implicit unobservable real and expected inflation term structures estimated by the macro term structure model for the sample period 1960:Q1–2005:Q4. In panel A, we compare the moments estimated by the model with the sample moments of ex post realised real yields and inflation rates with maturity from 1 to 5 years. The standard errors (S.E.) are calculated using GMM with the Newey–West correction. The table also reports the *p*-values of a  $\chi^2$  test based on the point statistic  $H = (h - \bar{h})' \Sigma_h^{-1} (h - \bar{h})$ , where  $\bar{h}$  and  $h$  are the estimates of the unconditional moments provided by the sample and the model, respectively, and  $\Sigma_h$  is the covariance matrix of the sample estimates of the unconditional moments, estimated using GMM with the Newey–West correction. The last column of the table contains the correlation between the series estimated by the model and the corresponding ex post realised values. Panel B illustrates the change in the correlation between real interest rates and expected inflation rates after the so-called ‘Volcker experiment’. Data expressed in basis points.

TABLE 4  
Instantaneous Expected Bond Excess Returns

Panel A. Estimated vs. Realised Sample Moments

Maturity (years)	$r$	$\pi$	$\theta$	$\xi$	$\alpha$	Total	
	Mean	Mean	Mean	Mean	Mean	Mean	St.Dev.
1	17	15	4	4	1	42	54
3	27	32	24	26	3	111	113
5	28	40	42	55	3	169	147
7	27	46	55	87	4	219	174
10	26	50	69	137	4	286	207

Panel B. Excess Bond Returns Forecasting Regressions

Maturity (years)	CP		MTS	
	$b_T$	$\bar{R}^2$	$b_T$	$\bar{R}^2$
2	0.49 (0.09)	0.30	0.49 (0.09)	0.30
3	0.84 (0.16)	0.29	0.83 (0.18)	0.29
4	1.18 (0.23)	0.31	1.17 (0.30)	0.30
5	1.49 (0.30)	0.30	1.51 (0.33)	0.31

Table 4 examines the instantaneous expected bond excess returns on  $\tau$ -period nominal zero coupon bonds implicit in the estimates of the macro term structure model (MTS) for the sample period 1960:Q1–2005:Q4. Panel A contains summary statistics on estimated instantaneous expected risk premia on each state variable. Panel B compares the predictive power of the model estimates for 1-year excess bond returns with those provided by the Cochrane and Piazzesi (2005) model (CP). The following regressions are estimated in the CP case:

$$\overline{ER}(t+1) = \beta_0 + \sum_{i=1}^5 \beta_i f(t;0,i) + \bar{u}(t+1) = (\beta'f(t)) + \bar{u}(t+1)$$

$$ER(t+1;T) = b_T(\beta'f(t)) + u(t+1;T), \quad T = 2, 3, 4, 5,$$

where  $ER(t+1;T) \equiv [F(t+1;T-1)/F(t;T)] - [1/F(t;1)]$  is the 1-year excess return on a  $T$ -maturity bond and  $\overline{ER}(t+1)$  is the average of excess bond returns for  $T = 2, 3, 4, 5$ .  $F(t;T)$  represents the price at time  $t$  of a  $T$ -maturity unit zero coupon bond and  $f(t;T-1,T)$  is the forward rate for loans between time  $t+T-1$  and  $t+T$ .

In the MTS case, the same two regressions are estimated by using as regressors the annualised estimated  $T$ -maturity model risk premia,  $T = 2, 3, 4, 5$ , instead of the forward rates. In parentheses, Newey–West heteroskedasticity and autocorrelation consistent standard errors correcting for overlapping observations.

TABLE 5  
In-Sample Prediction of GDP Growth and Inflation Rates

Panel A. In-Sample Predictions of GDP Growth Rates

Horizon	MTS		PEN		APW		HAR	
	Slope Coeff.	$\bar{R}^2$	Slope Coeff.	$\bar{R}^2$	Slope Coeff.	$\bar{R}^2$	Slope Coeff.	$\bar{R}^2$
3 months	1.26 (0.04)	0.85	-0.71 (0.25)	0.05	1.00 (0.21)	0.13	0.92 (0.36)	0.04
6 months	1.07 (0.02)	0.84	-0.89 (0.26)	0.13	1.23 (0.22)	0.22	1.09 (0.35)	0.10
1 year	0.80 (0.06)	0.57	-0.95 (0.25)	0.18	1.25 (0.24)	0.26	0.99 (0.31)	0.13
2 years	0.47 (0.09)	0.22	-0.91 (0.24)	0.20	1.08 (0.23)	0.23	0.86 (0.22)	0.18

Panel B. In-Sample Predictions of Inflation Rates

Horizon	MTS		PEN		APW		RW	
	Slope Coeff.	$\bar{R}^2$	Slope Coeff.	$\bar{R}^2$	Slope Coeff.	$\bar{R}^2$	Slope Coeff.	$\bar{R}^2$
3 months	1.03 (0.02)	0.94	0.67 (0.15)	0.28	1.00 (0.04)	0.78	0.92 (0.08)	0.77
6 months	1.01 (0.02)	0.96	0.64 (0.15)	0.27	1.03 (0.05)	0.79	0.90 (0.09)	0.79
1 year	0.98 (0.04)	0.94	0.59 (0.16)	0.22	1.11 (0.08)	0.79	0.86 (0.10)	0.76
2 years	0.90 (0.08)	0.83	0.46 (0.16)	0.13	1.19 (0.14)	0.67	0.77 (0.11)	0.66

Panel C. Pre-1984 vs. Post-1984 RMSE of Inflation Predictions

Horizon	Pre-1984		Post-1984	
	MTS	RW	MTS	RW
3 months	67	145	48	73
6 months	51	141	36	61
1 year	70	152	36	57
2 years	116	181	53	67

Table 5 compares the in-sample predictive accuracy for GDP growth and inflation rates of the macro term structure model (MTS) and several competing models, i.e., the Pennacchi (1991) two-factor model (PEN), the Ang, Piazzesi and Wei (2006) VAR model (APW), a random walk model (RW) and the Harvey (1989) term spread regression model (HAR). Panels A and B show the estimation results of the regression of actual annualised logarithmic changes in the real GDP level and the GDP deflator, respectively, against the corresponding expectations generated by the model estimates for the sample period 1960:Q1–2005:Q4. In parentheses, Newey–West heteroskedasticity and autocorrelation consistent standard errors. Panel C contains the root mean squared errors (RMSE), expressed in basis points, of the inflation predictions produced by the MTS and RW models for the period from 1960:Q1 to 1983:Q4 and the period from 1984:Q1 to 2005:Q4.

TABLE 6  
Out-of-Sample Forecasts of GDP Growth and Inflation Rates

Panel A. RMSE of GDP Growth Forecasts

Horizon	MTS	PEN		APW		HAR		SUR	
3 months	137	375	(0.00)	329	(0.00)	296	(0.00)	300	(0.00)
6 months	107	329	(0.00)	260	(0.00)	231	(0.00)	230	(0.00)
1 year	154	280	(0.00)	224	(0.00)	197	(0.02)	181	(0.30)
2 years	187	230	(0.07)	186	(0.96)	141	(0.10)		

Panel B. RMSE of Inflation Forecasts

Horizon	MTS	PEN		APW		RW		SUR	
3 months	57	210	(0.00)	123	(0.00)	111	(0.00)	93	(0.00)
6 months	43	208	(0.00)	124	(0.00)	103	(0.00)	82	(0.00)
1 year	52	210	(0.00)	141	(0.00)	110	(0.01)	83	(0.00)
2 years	82	224	(0.00)	192	(0.00)	130	(0.02)		

Panel C. Ratio Between RMSE of MTS and RW Inflation Forecasts

Horizon	1975–1983		1984–1994		1995–2005	
3 months	0.44	(0.00)	0.64	(0.00)	0.68	(0.00)
6 months	0.34	(0.00)	0.61	(0.00)	0.57	(0.00)
1 year	0.43	(0.01)	0.65	(0.02)	0.60	(0.00)
2 years	0.59	(0.02)	0.85	(0.16)	0.65	(0.00)

Table 6 summarises the error properties in forecasting GDP growth and inflation rates over various time horizons of the macro term structure model (MTS) and several alternative models: the Pennacchi (1991) two-factor model (PEN); the Ang, Piazzesi and Wei (2006) VAR model (APW); a random walk model (RW); the Harvey (1989) term spread regression model (HAR); the Survey of Professional Forecasters published by the Federal Reserve Bank of Philadelphia (SUR). The forecasts in panel A and panel B are obtained by using 15-year windows of quarterly data to estimate the parameters needed to forecast subsequent GDP growth and inflation rates over the different horizons. Proceeding recursively, we compute forecasts for all the competing models for the period from 1975:Q1 to 2005:Q4. Root mean squared errors (RMSE) are expressed in basis points. In parentheses are the asymptotic  $p$ -values of the Diebold–Mariano statistic, which tests the null hypothesis of no difference in the accuracy of the forecasts provided by the competing models with respect to those of the MTS model. Panel C shows the ratio between the RMSEs of the forecasts estimated by the MTS model and the RW model in different subperiods. In parentheses are reported the asymptotic  $p$ -values of the Diebold–Mariano statistic testing the null hypothesis of no difference in the accuracy of the forecasts produced by the two models.



FIGURE 1  
Kalman Filter Estimates of the Unobservable State Variables

Panel A. Real Interest Rate

**Insert here: FIGURE 1 - Graph A**

Panel B. Expected Inflation Rate

**Insert here: FIGURE 1 - Graph B**

Panel C. Conditional Mean of Output Growth

**Insert here: FIGURE 1 - Graph C**

Figure 1 shows the Kalman filter estimated series of the unobservable state variables over the sample period 1960:Q1–2005:Q4. Panel A shows the instantaneous real interest rate and its time-varying central tendency, Panel B the instantaneous expected inflation rate and its time-varying central tendency, Panel C the conditional mean of output growth and, as a comparison, the cyclical component of output growth, which is estimated by applying the Hodrick–Prescott filter to the annual rate of growth of real GDP.

FIGURE 2  
Inflation Risk Premia and Expected Bond Excess Returns

**Insert here: FIGURE 2**

Figure 2 shows the estimated series of the inflation risk premia (IRP), for maturities of 1, 5 and 10 years, and of the average (across maturities) instantaneous expected excess bond returns (EER) over the sample period 1960:Q1–2005:Q4. Data expressed in basis points.

FIGURE 3  
Ratio Between MTS and RW Absolute Inflation Forecasting Errors

**Insert here: FIGURE 3**

Figure 3 shows the dynamic behaviour of the ratio between the annual average of absolute inflation forecasting errors generated by the macro term structure (MTS) and random walk (RW) models, respectively, for various time horizons. Out-of-sample forecasts are obtained by using the preceding 15 years of quarterly data to estimate the parameters needed to predict subsequent inflation rates. Proceeding recursively, we compute forecasts for the two models from 1975:Q1 to 2005:Q4.

FIGURE 1 - Graph A

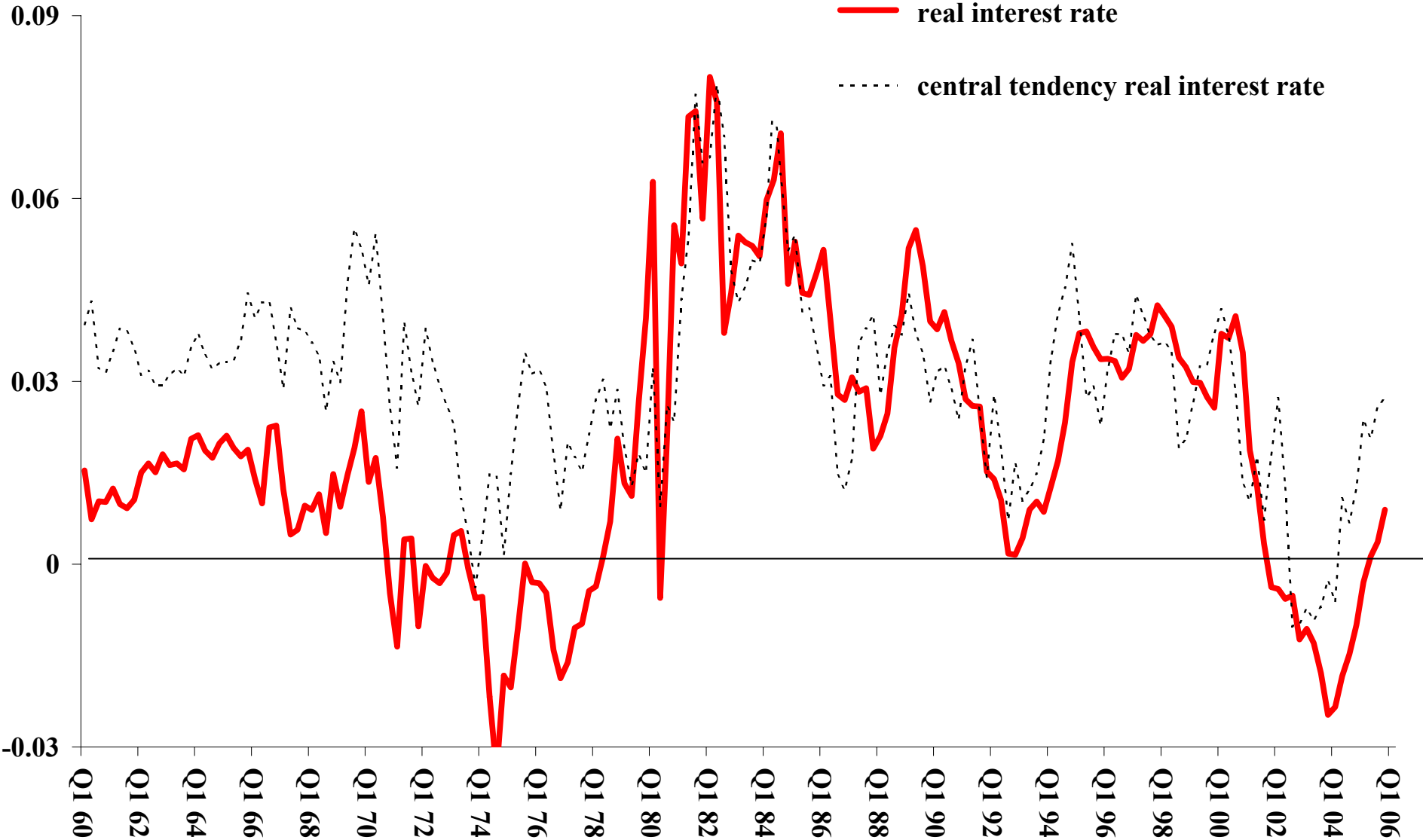


FIGURE 1 - Graph B

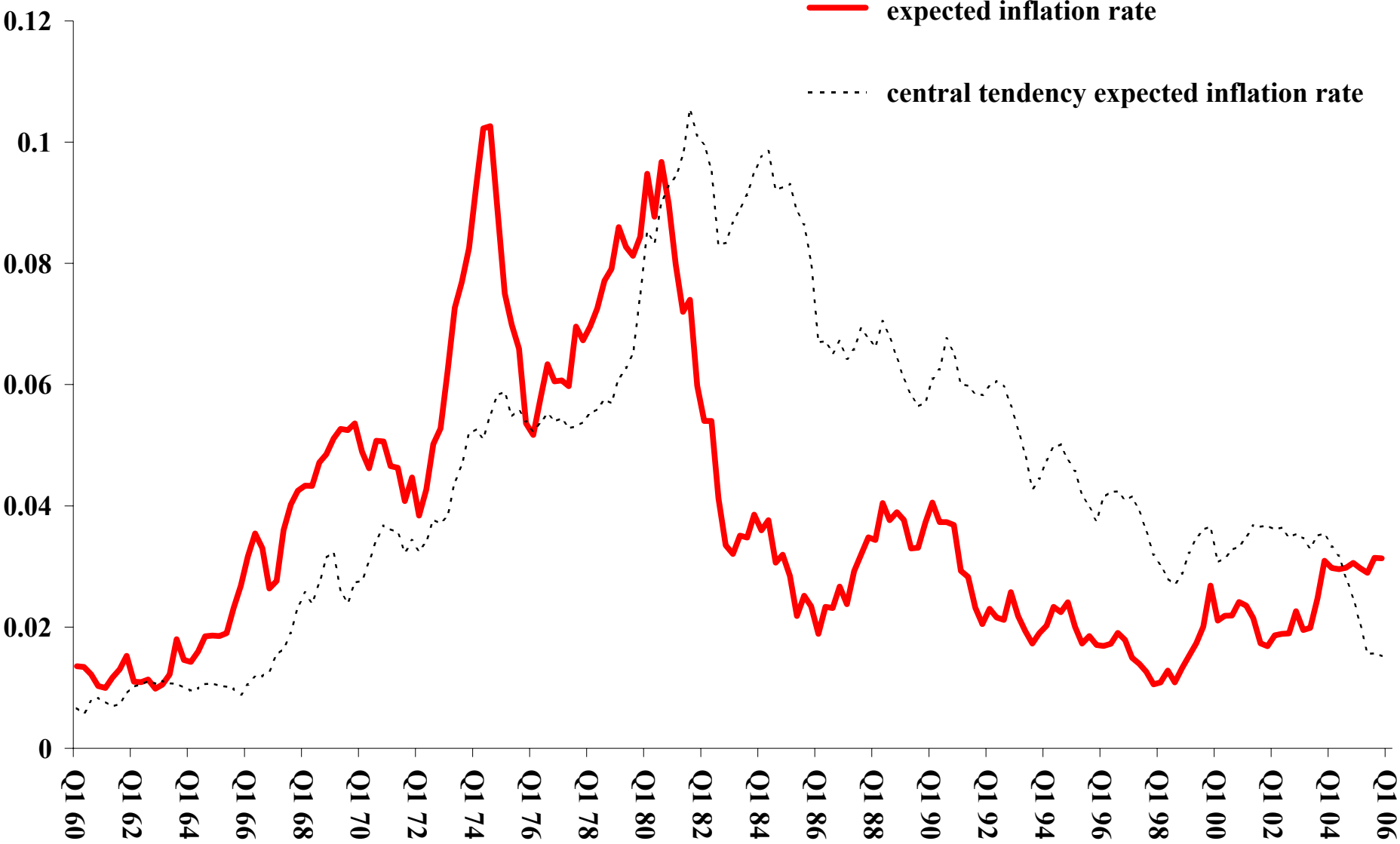


FIGURE 1 - Graph C

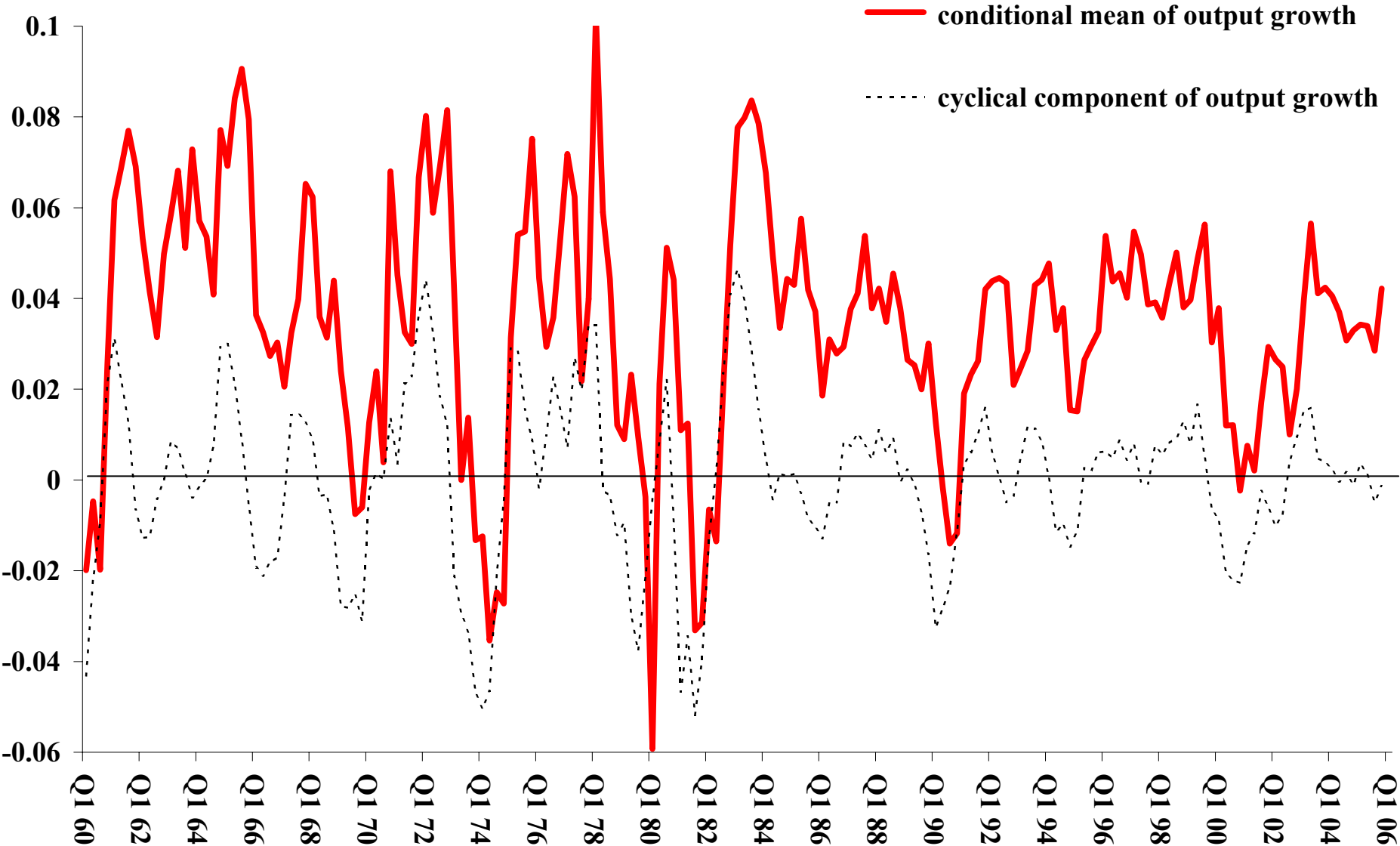


FIGURE 2

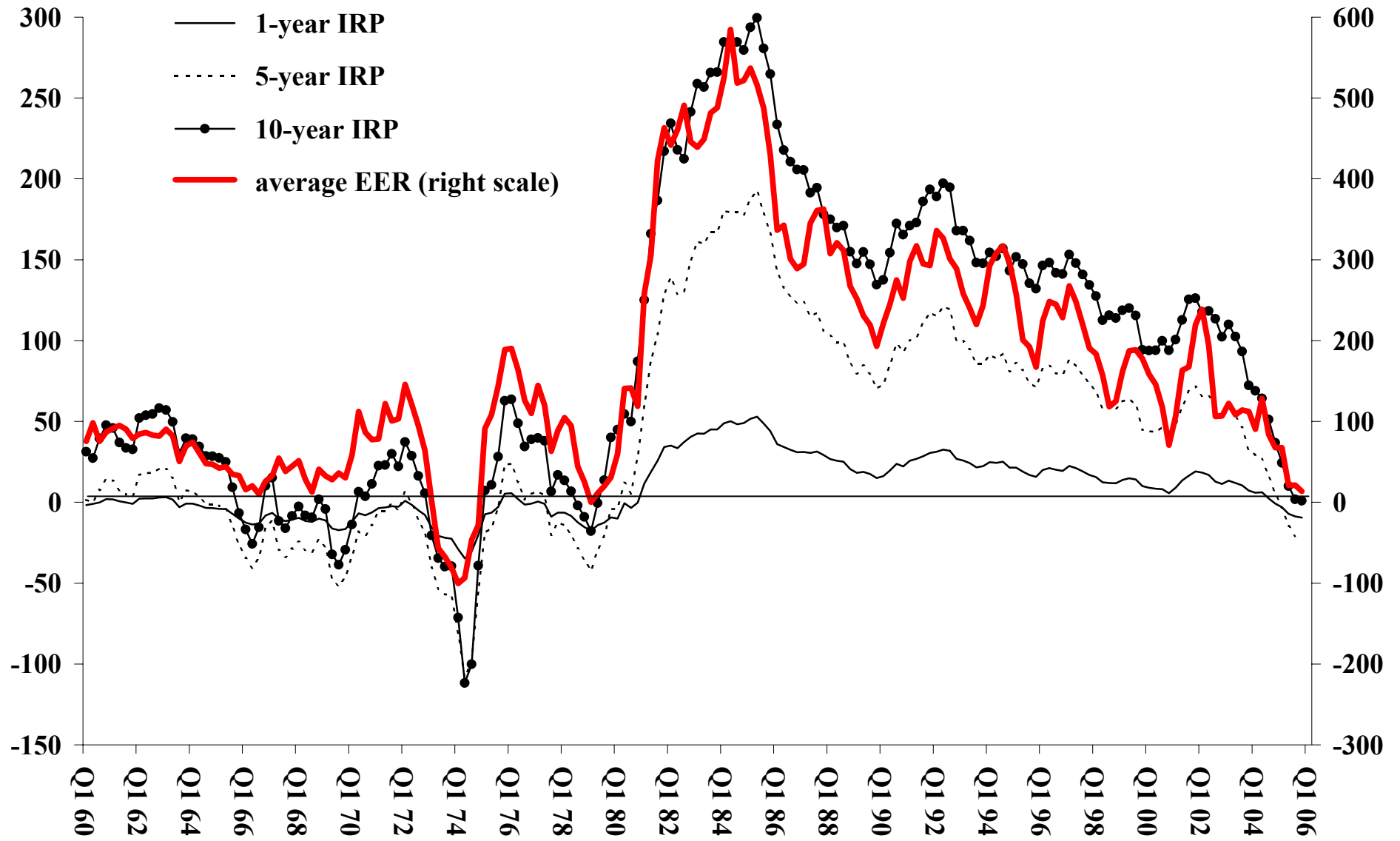


FIGURE 3

