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Gender gap, Labor market, Self-confidence, Affirmative action

JEL codes:
D03, D83, J16, J24
The wrong man for the job: biased beliefs and job mismatching

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In this paper we build a theoretical model to show the role of self-confidence in leading to inefficient job matching equilibria: underconfident highly-qualified workers do not apply for highly-skilled jobs, because mistakenly perceive themselves as having relatively lower abilities with respect to other candidates, and firms are no longer selecting their workers from a pool containing the best fitted ones. Policies to foster underconfident workers to apply for highly-skilled jobs cannot easily be implemented, because under-confidence is not an observable characteristic, and any attempt to elicit this information from workers can be easily manipulated. However, if gender is correlated with this psychological bias, and there more underconfident female workers than male workers, a second best policy based on gender affirmative action may enhance the efficiency of matching in the job market. We show that increasing the gender diversity of the qualified applicants by imposing an affirmative action may positively affect the selection of candidates because it increases the average quality of the pool of candidates for high-qualified jobs.

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What a man thinks of himself, that is which determines, or rather indicates, his fate.

- Henry David Thoreau -

1 Introduction

There is strong evidence that women, while globally facing higher unemployment rates than men, seem also to be segregated in some segments of the labor market: they are underrepresented in managerial and legislative occupations and over-represented in mid-skill occupations\(^1\) (Bourmpoula et al., 2012). In this paper, we provide a theoretical foundation to explain the emergence of gender gap and segregation in the job market, as a consequence of different levels of self-confidence of men and women, when abilities are equally distributed among them. We then show that well-calibrated gender quotas can be beneficial for the employers themselves because they improve the average quality of their workers.

Different reasons have been proposed to explain the existence of the gender gap in the workplace. First, women may have innate lower (higher) abilities than men in some sectors and are thus less (more) likely to be selected when applying. However, even if discussion about this topic is still open, recent research suggests that men and women do not differ much in their cognitive abilities and it is rather social and cultural factors that influence perceived or actual performance differences (Hyde, 2005, Spelke, 2005).

Second, women and men face a different trade-off when formulating their career and family plans, and a link between relative wages and fertility has been showed (Erosa et al., 2002, Galor and Weil, 1996). In particular, Dessy and Djebbary (2010) show that the shorter reproductive capability of women with respect to men causes them to be more constrained in their career-family choices, so that failure in coordination of women’s marriage-timing decisions lead to persisting gender differences in career choices.

Third, some studies have suggested the existence of a glass ceiling (Cotter et al., 2001)\(^2\), which leads organization to discriminate women’s promotion and thus prevent

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\(^1\)The global female labour force was estimated to be 1.3 billion in 2012, - about 39.9 per cent of the total labour force of 3.3 billion.

\(^2\)Carol Hymowitz and Timothy D. Schellhardt were the first to use the term “glass ceiling” in their March 24, 1986 article in the Wall Street Journal, “The Glass Ceiling: Why Women Can’t Seem to Break the Invisible Barrier That Blocks Them from the Top Job.”
them to achieve the highest rank in the firm, when having equal abilities than men (Bassanini and Saint-Martin, 2008). However, a recent research by the Institute of Leadership & Management (2011), claims that women managers are rather impeded in their careers by lower ambitions and expectations, which lead them to a cautious approach to career opportunities, than by a glass ceiling.

A fourth possible explanation of gender segregation has thus been developed, which relies on different preferences of men and women regarding the job environments where they would like to work, ultimately affecting their job entry decisions. Laboratory (Gneezy et al., 2003, Masclet et al., 2015) and natural field experiments (Flory et al., 2015) provide evidence of women being less likely to apply to competitive work-settings (for a review of these studies, see Niederle and Vesterlund (2011)). In particular, this phenomenon is associated with women i) having different distributional preferences (Balafoutas et al., 2012), ii) being more risk averse (Charness and Gneezy, 2012) and iii) less (over)confident than men about their relative performance in a (mathematical) task (Niederle and Vesterlund, 2007) 3.

Self-confidence in the ability to successfully win a contest and the gender of the competitors seems to play an important role in these studies. Wieland and Sarin (2012) and Kamas and Preston (2012a) indeed show that there is no difference in the choice of the payment scheme (i.e. the decision to compete in tournament) with respect to gender when considering gender neutral tasks. In particular, Kamas and Preston (2012a), in a recent experiment investigate the extent to which differences in a taste to compete or differences in ability, confidence, risk aversion, or personality characteristics explain gender differences in willingness to compete and conclude that gender differences in confidence, and to a lesser extent risk attitudes, explain this pattern. Moreover, Günther et al. (2010) observe that "women tend not to compete with men in areas where they (rightly or wrongly) think that they will lose anyway". Whether women have lower self-confidence that men is a long lasting question (Lenney, 1977), which is sustained by studies in social psychology (Furnham, 2001, Haynes and Heilman, 2013, Bleidorn et al., 2016) showing females as less likely to perceive themselves as qualified to run for political office (Lawless and Fox, 2005), or expressing lower career-entry and career-peak pay expectations (Bylsma and Major, 1992, Schweitzer et al., 2014). In particular, when considering the job market, Barbulescu and Bidwell. (2012) showed that, among MBA

3For a more general discussion of the contributions of laboratory and field experiments in explaining gender differences on labor market outcomes see Azmat and Petrongolo (2014).
students, women’s lower expectations of job demand’s success is one of the causes of their lower number of applications to finance and consulting jobs with respect to men. Even if such expectations had not had an empirical foundation (i.e. the authors found no evidence that women were less likely to receive job offers in any of these fields), they lead women to accept lower salary offers than the ones accepted by their male counterparts (Bowles et al., 2005). In line with these results, in a recent experiment Mobius et al. (2014) found that women are significantly more conservative than men in updating their beliefs about their own ability. One implication is that high-ability women who receive the same mix of signals as high-ability men will tend to end up less confident.4

Our main contribution is to provide a first theoretical foundation to explain the crucial role of self-confidence in explaining the observed gender gap in the workplace, as put into evidence by previous experimental results (Kamas and Preston, 2012a, Buser et al., 2014). If women and men are different in how they perceive themselves as having the abilities to fulfill the job responsibilities with respect to other candidates, this will affect in turn their job application decision, with women being less likely to apply to the skilled segment of the market. Gender segregation occurs due to a different psychological attitude among men and women. Women, more frequently than men, self-select into lower ranked job positions due to their (mis)perceptions about their opportunity to be successfully recruited. The unequal distribution of males and females between the skilled and unskilled sector is thus endogenous on workers’ application choices.

In our model we consider a stylized labor market segmented in two levels, represented by two firms: firm G offers a small number of high-skilled jobs while firm B offers a large number of low-skilled jobs. Each worker receives a (not fully) informative binary signal of her own ability, which can be high (H) or low (L), and can apply at most to one firm: a worker who applies to firm B is hired for sure, while, if she applies to firm G, she is hired only if receiving an offer. Firm G, when hiring, gives priority to high ability workers when being able to screen candidates’ ability with some positive probability p. Differently, with complementary probability 1 − p firm G hires workers at random among the pool of candidates, being unable to rank them according to their abilities. Workers face a trade-off between getting a sure but low paying job in firm B and a risky application to firm G for a high salary. We assume that salaries are fixed and such that

if $p = 0$ all workers apply to $G$.

Without any behavioral bias, results are very intuitive. When $p$ is small all workers apply to $G$; when it increases workers who received signal $L$ apply with probability lower than one, while workers who received signal $H$ apply with probability one and, finally, when $p$ is larger than a certain threshold $\bar{p}$ only workers who received signal $H$ apply to firm $G$. It is easy to see that the larger is $p$ the higher is the average quality of workers hired by firm $G$, because of two reasons: first, firm $G$ increases the probability to hire the right candidates and, second, the average quality of the pool of candidates increases too. As a consequence, firm $G$ hires, on average, better workers also when it is not able to screen them. However, an increase in the ability of screening candidates may have a negative effect on the average quality of the pool of applicants to firm $G$, when there exist underconfident individuals who assign themselves a high probability of being low ability workers even if having received signal $H$. In presence of underconfidence, for intermediate values of $p$ all workers except those who are underconfident and have received signal $L$ apply to firm $G$. When $p$ increases only selfconfident workers apply, independently on the signal received. Underconfidence may lead to an inefficient job matching equilibrium because firms are no longer selecting their workers from a pool containing the best fitted ones.

Positive interventions to push talented but underconfident workers to apply for highly skilled positions cannot easily be implemented, simply because underconfidence is not an observable characteristic and any attempt to elicit this information from workers can be easily manipulated. Nonetheless, when an observable characteristic, such as gender, is positively correlated with the unobservable one, a second best policy based on gender affirmative action may enhance the efficiency of matching in the job market. Our second contribution is to show that, in the presence of a positive correlation between underconfidence and gender, an affirmative action imposing a gender quota on the labour force composition may not only increase the job market diversity, but also increases market efficiency.

1.1 Literature Review

A number of studies in psychology and economics have analyzed the role of self-confidence both from a practical and theoretical point of view. In particular, when considering the role played by self-confidence in the labor market, most of the literature has been pri-
marily focused on the agency model. In Sautmann (2013) and Santos Pinto (2008, 2010) studies, the principal, who is aware of the agent’s overconfidence\(^5\), takes advantage of it by paying the worker a lower wage. Bénabou and Tyrole (2002) analyze the role of self-confidence in influencing how people process information and make decisions in order to explain some “irrational” behaviors such as self-handicapping or self-deception. In the studies by Falk et al. (2006a,b) and Andolfatto et al. (2009) self-confidence is analyzed in a job searching framework. While Andolfatto et al. (2009) apply the ideas of Bénabou and Tyrole (2002) in a model of labor market search, Falk et al. (2006a,b) show that wrong beliefs about relative ability affects unemployment duration, in turn determining worker’s potential starting wages.

Our results are consistent with the recent study by Flory et al. (2015). They show, in a natural field experiment where almost 9000 job-seekers are randomized into different compensation regimes, that women are less likely than men to apply to competitive work settings as much as they shy away from jobs characterized by uncertainty over the payment, where is the worker’s ability that influence the labour outcome. Thus, even if the worker is not performing against anyone else, so that competition does not play any role, females are less confident than men about their ability to successfully fulfil the job request.

When considering sorting decisions in the labor market and in educational attainments, Filippin and Paccagnella (2012) provide evidence that the level of confidence of young agents may consistently affect their future lives: they show that even small differences in initial confidence of people about their ability may lead to diverging patterns of human capital accumulation between otherwise identical individuals. Larkin and Leider (2012) and Dohmen and Falk (2011) experimentally demonstrate that different incentive schemes invite different employees to join the organization, depending on their behavioral biases. In a study closely relate to our topic, Santos Pinto (2012) analyze the emergence of the gender pay gap as a result of males and females different levels of self-confidence in the classic labor market signaling model by Spence (1973): overconfident men are more likely to invest in education than underconfident women, which in turn lead to a higher productivity of men with respect to women, thus generating a gender pay gap.

Differently than in our study, Santos Pinto (2012) aims to explain the gender gap

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\(^5\)Santos Pinto (2008) refers to the term positive (negative) self-image as the agent’s over(under) estimation of the productivity of effort.
in wages while we focus our attention at the previous level: we want to investigate the gender gap in the job application process which results in gender segregation, due to high qualified but underconfident women renouncing to apply to the high skilled job market. Moreover, the author explains the existence of a gender pay gap according to different (observable) educational investments made by biased males and females, while in our model the role of the bias does affect their career choice, when holding the same level of ability and education. This job mismatching causes an efficiency loss in the job market, which can be canceled out by calibrating a suitable affirmative action that guarantees the participation of high skilled women in the job market.

Previous researchers have analyzed models of directed search where workers do not randomly search among all possible jobs, but apply for jobs that are more likely to be appropriate for their skills and interests. In particular, Galenianos and Kircher (2009) develop a model where both the success probability of unemployed workers and the wages posted by firms are equilibrium outcomes. Wage dispersion is the result of allowing every worker to apply for multiple jobs. Differently that in their paper, in our model we allow for heterogeneity of both workers and firms, rather than assuming that all workers and all firms are identical. Moreover, as in Chade et al. (2006) and Nagypal (2004) the wages are exogenously given but, a fundamental modification of our paper is played by the role of biased beliefs of (female) workers regarding their relative abilities.

During the past years, several policies have been proposed to establish gender (and minorities) equality in the job market (see Anderson (2004), for an historical view of the affirmative action agenda). Affirmative action, first instituted in US in the 1960s and 1970s by employers and educational institutions, is a policy designed to increase the employment and educational opportunities available for disadvantage groups. Often, it is addressed to qualified women and other minorities and gives them preference in hiring, promotion, and admission. Coate and Loury (1993) provide mixed results regarding whether positive discrimination policies eliminate negative stereotypes in the employer’s beliefs. Affirmative action such as exogenous imposed quotas on the labour force composition have been often criticized for being unfair and inefficient. Opponents to such a policy, claim that it is unfair to hire an individual for a job on anything other than his qualifications and skills.

In a recent laboratory experiment, Balafoutas and Sutter (2012) provide evidence that affirmative actions encourage women to enter competition more often, without negatively affecting efficiency. Niederle et al. (2013) obtained a similar result when
experimentally testing the effect of quotas in favor of women in competitive tournaments. Moreover, in contrast with a common critique to the implementation of quotas, they did not find a decrease in the minimum performance threshold when achieving a more diverse set of winners. Finally, in a recent field experiment investigating affirmative actions in Colombia (Ibaezy et al., 2015), the authors find that the gains of attracting female applicants far outweigh the losses in male applicants. In the present study we thus provide a theoretical foundation of the positive effect of quotas in favor of women in the labor market. However, we are not claiming that positive discrimination policies, such as gender quotas, are the only solution to the gender segregation problem. Our model suggests that the gender gap may be a result of a structural bias in the society (Gneezy and Leonard, 2009), so that an effective desegregation policy should intervene early in life in order to provide educational programmes designed to positively encourage the correct development of self-image in both men and women and to promote new role models.

In the following section, we compute the equilibrium of the model when workers have unbiased beliefs about their abilities and when female workers exhibit underconfidence. In Section 3 we analyze the role of affirmative actions. Finally, Section 4 discusses the results and conclusions. Proofs of all results are in the Appendix.

2 The model

Consider a job market with the following matching process. There is a mass equal to one of workers, balanced in terms of gender. Workers may either decide to apply for a job offered by firm $G$ or by firm $B$. Firm $B$ has a number of vacancies equal to one and, therefore, all workers who apply for this job are hired. Firm $G$ only offers $z < 1$ posts. Wages are fixed and equal to $w_G$ and $w_B$, for firm $G$ and $B$ respectively, with $w_G > w_B > 0$.

Each worker is of type (ability) $k = H$, with $k \in \{H, L\}$, with probability $\sigma$ and of type $k = L$ with complementary probability $1 - \sigma$. Each worker receives an informative signal $s_i \in \{H, L\}$ about her/his type. The signals are independent from each other and informative: for both $k \in \{H, L\}$ let $\Pr(i = k|s_i = k) = \pi^k$ denote the probability that worker $i$ is of type $k$ conditioned on having received signal $s_i = k$. We assume that $\pi^H > \frac{1}{2}$ and $\pi^L > \frac{1}{2}$. Both the quality of the signals and the proportion of $H$ and
L types in the job market is common knowledge so we define $\gamma$ as the proportion of candidates who received the signal $s_i = H$ and $1 - \gamma$ as the proportion of candidates who received the signal $s_i = L$. After having observed the signal, each worker decides whether to apply to firm $G$ or $B$ of the labor market. Each worker can apply at most to one firm.\footnote{Alternatively, we could assume that the worker has to pay a specific cost $C_k$ with $C_G > 0$ and $C_B$ normalized to zero (investment in education, for instance) to apply to sector $k$. Results are similar in a model with sunk costs.} The utility of an employed worker is equal to her wage while the payoffs of unmatched workers are normalized to zero. Since there is no unemployment benefit, participation constraints are satisfied.

Firm $G$ gives priority to $H$ workers when hiring. With probability $p$ firm $G$ observes the applicants' type, while with complementary probability firm $G$ is not able to observe their type. If more than $z$ workers apply to $G$ then a rationing occurs. When firm $G$ is able to observe the applicants' type, it hires $H$ workers first and, if some jobs are still vacant, workers of type $L$ are enrolled too.\footnote{In our model, we assume that employers do not have prior beliefs regarding gender ability, i.e. negative stereotypes. In particular, firms select workers only considering the applicants' abilities.} Differently, when applicants' type is not observed by firm $G$, it randomly selects $z$ workers among their applicants. Workers, hence, decide whether to compete for a job position in firm $G$, which provides a higher payoff conditional on success, or to apply to firm $B$, that offers a sure job but for a low pay.

Along the rest of the paper we assume the following:

**Assumptions A1:** $\gamma \pi^H \geq z$ and $zw_G > w_B$.

The assumption that $\gamma \pi^H \geq z$ implies that at least $z$ workers who are $H$-type receive the signal $s_i = H$. Consequently, if firm $G$ perfectly observes the candidates' type, so that $p = 1$, and all workers who receive a signal $s_i = H$ apply to firm $G$, then $L$-type workers are not hired in firm $G$. According to our second assumption, if firm $G$ is unable to observe candidates' type and hire at random, so that $p = 0$, then all workers apply to firm $G$.

In our model, search is directed, firms publicly post vacancies and commit to a wage and each worker chooses to which job to apply for. We assume that agents can only apply to one firm, not to both. This is a reasonable assumption since job applications is a time consuming process: workers need to adjust their resume in order to present themselves as the ones perfectly fitting the job, according to the firm's specific requirements. As
a consequence, candidates do not apply to all the job offers in the market, but just select the ones that they think to be the more achievable from their own point of view. Finally, to define whether a position is achievable or not a candidate has to take into consideration many factors: the expected salary, the working environment, but also his (perceived) relative ability with respect to others potential competitors in the application process. Our model resembles the academic market: universities are vertically differentiated (in our model the market is segmented in two levels), entry salaries are fixed at large extent and there are a limited number of jobs available; as in our model, candidates may apply to a limited number of jobs (due to time and effort constraints).

2.1 The job matching equilibrium

We first derive the job matching equilibria in the benchmark case in which workers do not have biased beliefs regarding their ability. In both subsections 2.1.1 and 2.1.2, we examine the job matching equilibria when considering different levels of firm $G$’s ability to screen its applicants’ types.

2.1.1 The benchmark case

In this section we examine the matching equilibria that will be used as a benchmark when we introduce the assumption that some workers have biased beliefs about their own ability.

**Proposition 1.** If $p \geq \bar{p}$, with $\bar{p} \equiv \frac{\pi_H (\pi_B - \gamma \pi_B \gamma)}{\pi_G \gamma (\pi_B - \pi_B)}$ and $\pi_G \leq \gamma \pi_B \frac{\pi_H}{1 - \pi_H}$, then in equilibrium workers who receive a signal $s_i = H$ apply to firm $G$ and those who receive a signal $s_i = L$ apply to firm $B$ (separating equilibrium). If $p \leq \frac{(\pi_G - \pi_B)}{\pi_G \gamma (\pi_B - \pi_B - 1)} \equiv \bar{p}$, both types of workers apply to firm $G$ (pooling equilibrium). If $p \in [\bar{p}, \bar{p}]$, there is a semiseparating equilibrium such that workers who receive a signal $s_i = H$ apply to firm $G$ and those who receive a signal $s_i = L$ are indifferent: a fraction $\xi(p) \in (0, \frac{1}{2})$ of them apply to firm $B$ and a fraction $(1 - \xi(p))$ apply to firm $G$, with $\xi(p)$ satisfying the following equality $p \left[ (1 - \pi^L) \frac{\pi_G}{\xi(1-\pi^L)(1-\gamma)} + (1 - p) \frac{\pi_B}{\gamma \xi(1-\gamma)} \right] = \pi_G = \pi_B$.

**Corollary 1.** An increase in $p$ increases the average quality of the pool of applicants.

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8In the American job market each candidate can send a limited number of expressions of interests to the departments they applied (currently only five), which are considered as credible signal of a sincere interest in the position.
Proof of proposition 1 is in the Appendix. Previous proposition states that, when firm $G$’s ability to screen candidates is high enough ($p \geq \bar{p}$), only workers receiving a signal $s_i = H$ apply to $G$ and, among them, $\gamma \pi^H$ are truly $H$ types and $\gamma (1 - \pi^H)$ are $L$ types who received the wrong signal. Since we assume that $z \leq \gamma \pi^H$, the probability to hire a high ability worker is equal to $p + (1 - p)\pi^H$. When the probability $p$ to screen candidates decreases, the efficiency of the matching decreases as well. Namely, if $p < \bar{p}$, the probability of hiring a $H$ worker is $p + \frac{(1-p)}{2}$. Thus, since $\pi^H \geq \frac{1}{2}$, an increased ability of hiring based on merit by firm $G$ induces a more efficient matching in the job market.

2.1.2 Underconfidence and job matching

In this section we introduce the following modification with respect to the benchmark case: a fraction of workers $\delta$ has now biased beliefs regarding their ability in the job market. In particular, when an underconfident worker receives a signal $s_i = H$, (s)he underestimates the quality of the signal and believes that (s)he is of type $L$ with a larger probability: parameter $\rho$ measures the worker’s bias, that we define as underconfidence, with $0 \leq \rho \leq +\infty$.

Formally, an underconfident individual $i$ who observes $s_i = H$ at time $t = 0$ assigns probability $\Pr(i = H \mid s_i = H) = \frac{1}{1+\rho} \pi^H$ to be a $H$-type worker. Therefore, $\Pr(i = L \mid s_i = H) = \frac{1+\rho-\pi^H}{1+\rho}$ is the probability that an underconfident worker $i$ assigns to be a $L$ type worker when she receives a signal $s_i = H$. Similarly, $\Pr(i = L \mid s_i = L) = \min(1, \frac{1+\rho}{1+\pi^L})$. \footnote{Think, for example, of a student who receives a good mark on an exam (i.e. a signal about her/his ability): s/he may think it was not due to her/his ability but the result of a mistake of the professor, or that it depends on luck; s/he does not correctly infer the information of the signal, because of her/his prior beliefs about her/his own ability.}

Assumption A2: $\rho > \frac{\pi^H}{1-\pi^L} - 1$.

We focus on the more interesting case for our analysis, that is when underconfidence is sufficiently severe so that $\frac{1}{1+\rho} \pi^H < 1 - \pi^L$: an underconfident individual who gets a signal $s_i = H$ assigns a lower probability to be a $H$-type with respect to the probability that an unbiased individual who observes $s_i = L$ assigns to his own type. Now workers differ not only with respect to the signal they receive of their ability, but also with respect to their level of self-confidence (which, as we put into evidence in the introduction, might be correlated with their gender). We have thus to analyse the decision of four different
types of workers: self and underconfident workers who received signal $H$, and those who received signal $L$.

We focus on the case $p \geq \bar{p}$ such that in equilibrium without biased confidence, only workers who receive the signal $s_i = H$ apply to firm $G$. When the fraction $\delta$ of underconfident workers in the population is sufficiently high, we observe the following equilibria depending on how $p$ varies:

- if $p$ is sufficiently high and there are "enough" selfconfident workers with $s_i = H$, then only these workers (who received $s_i = H$ and are selfconfident) apply to $G$ and all the other workers apply to $B$.

- if $p$ is smaller and included in a certain interval, then selfconfident workers with $s_i = H$ will still apply with probability one, while underconfident workers (irrespectively of their signal) do not apply. Selfconfident workers who got $s_i = L$ now apply with positive probability.

Previous intuitions are formalized in the following proposition. Sorting is defined as inefficient when (some) workers who have received signal $s_i = H$ apply to firm $B$ and (some) workers who received a signal $s = L$ apply to firm $G$. We will first derive the equilibria when considering a low proportion of underconfident workers in the population and, secondly, when such a fraction is higher than a certain threshold.

**Proposition 2.** If $z \leq \gamma \pi^H (1 - \delta)$ and $p \geq \frac{zw_G - zw_B(1-\delta)\pi^H}{zw_G (\pi^H + \pi^L - 1)}$, only selfconfident workers who observe $s_i = H$ apply to firm $G$. If $z \leq \gamma \pi^H + (1 - \pi^L)(1-\gamma)(1 - \delta)$, $p \in \left[ \frac{zw_G - zw_B(1-\delta)[\gamma \pi^H + (1 - \pi^L)(1-\gamma)]}{zw_G [\gamma \pi^H + (1 - \pi^L)(1-\gamma)][(1+\rho)(1+\delta) - \pi^H]} \right]$, and $\delta > \frac{zw_G \gamma}{w_B - 1}$ all selfconfident workers, irrespective of their signal, apply to firm $G$ while all underconfident workers, irrespective of their signal, apply to firm $B$. If $z \leq \pi^H \gamma + (1 - \delta)(1 - \gamma)(1 - \pi^L)$ and $p \leq \frac{[zw_G - zw_B\gamma(1-\delta)(1-\gamma)]^{\gamma \pi^H + (1 - \delta)(1-\gamma)}[\gamma \pi^H + (1 - \pi^L)(1-\gamma)]}{zw_G [\gamma \pi^H + (1 - \pi^L)(1-\gamma)]^{\gamma \pi^H + (1 - \delta)(1-\gamma)}}$, all selfconfident workers and underconfident workers who receive $s_i = H$ apply to firm $G$, while underconfident workers who receive signal $s_i = L$ apply to firm $B$. If $z \leq \gamma \pi^H + (1 - \pi^L)(1 - \gamma)$ and $p \leq \frac{zw_G - zw_B[\gamma \pi^H + (1-\gamma)(1-\pi^L)]}{zw_G [\gamma \pi^H + (1-\gamma)(1-\pi^L)]}$, then all workers apply to firm $G$.

**Corollary 2.** When $z \leq [\gamma \pi^H + (1 - \pi^L)(1-\gamma)](1 - \delta)$ there exist two thresholds $p'$ and $p''$ with $p'' > p'$ such that the average quality of the pool of applicants when $p < p'$ is higher than in case $p \in [p', p'']$.
We observe a similar characterization of equilibria when considering a higher proportion of underconfident workers in the population. If \( \gamma \pi^H (1 - \delta) \leq z \leq \gamma (1 - \delta) \) and \( p \geq \frac{[z w_G - y w_B (1 - \delta)](1 - \pi^H)}{w_G [\pi^H + \pi^I - 1] (1 - \delta) - z} \), only selfconfident candidates who observe \( s_i = H \) apply to firm \( G \). If \( p \geq \frac{[\gamma \pi^H + (1 - \pi^L)(1 - \gamma)](1 - \delta)}{[\gamma (1 - \delta) + \pi^I (1 - \gamma)](1 - \delta) - z} \), and \( \rho \geq \frac{z [\pi^H + (1 - \gamma) \pi^I] + (1 - \gamma) [1 - \delta]}{[1 - \delta] \gamma \pi^H + (1 - \gamma) [1 - \delta] + z [1 - \delta] \gamma \pi^I + \pi^H - 1} \), only selfconfident candidates apply to firm \( G \). If \( z \geq \gamma \pi^H + (1 - \gamma)(1 - \pi^L) \) and \( p \leq \frac{[z w_G - y w_B (1 - \delta)](1 - \pi^H) + (1 - \gamma) \pi^I}{w_G [1 - z] [\pi^H + \gamma (1 - \pi^H) - 1]} \), all workers apply to firm \( G \).

According to the previous proposition, when \( p \) decreases and the fraction of underconfident workers is sufficiently large, we move from an equilibrium where only selfconfident workers who received a signal \( s_i = H \) apply to firm \( G \) to an equilibrium where also selfconfident workers who observed \( s_i = L \) apply to firm \( G \), while underconfident workers with \( s_i = H \) are still enrolled in firm \( B \). Differently, an equilibrium where workers who observe \( s_i = H \) apply to firm \( G \), irrespective of their level of self-confidence, while other workers apply to firm \( B \) (no gender segregation and efficient sorting) cannot exist because of our assumption A2.\(^{10}\)

When the fraction of underconfident workers is sufficiently large we observe a negative consequence on matching: if \( \delta = 0 \) an increase in \( p \) increases the probability of an efficient matching (i.e. workers who receive a signal \( s_i = H \) apply to firm \( G \) while those who receive a signal \( s_i = L \) apply to firm \( G \)). If \( \delta > 0 \) an increase in \( p \) may have a detrimental effect because it increases the probability to exclude the underconfident workers who received a signal \( s_i = H \). Previous evidence (Barbulescu and Bidwell., 2012, Kamas and Preston, 2012b, Buser et al., 2014) supports the hypothesis that underconfidence is correlated with gender; as a consequence, we observe a gender gap in the labor market, even in absence of any discrimination or negative stereotype against women.

2.1.2.1 A test for the emergence of the gender gap Our model allows to test whether the emergence of inefficient sorting and, consequently, of a gender gap, depends on gender differences in self-confidence or in other characteristics, like aversion to competition or risk aversion. In our model if \( p \leq \frac{[1 + \rho] z w_G - y w_B (1 - \delta) (1 - \gamma)](1 - \pi^H)}{w_G (1 + \rho) [\pi^H + (1 - \gamma) (1 - \delta)]} \), and

\(^{10}\)Proof of Proposition 2 and of the following non-existence of additional equilibria is provided in the Appendix.
all workers who received a signal \( s_i = H \) and self-confident candidates who received signal \( s_i = L \) apply to firm G. When \( p \) increases and is included in the interval 
\[
p \in \left\{ \frac{[zw_G - w_B (1-\delta)][(1+\rho)(\gamma \pi^H + (1-\pi^L)(1-\gamma))]}{zw_G \gamma (\pi^H + \pi^L - 1)}, \frac{[zw_G - w_B (1-\delta)][\gamma \pi^H + (1-\pi^L)(1-\gamma)]}{zw_G \gamma (\pi^H + \pi^L - 1)} \right\},
\]
however, the number of workers who got \( s_i = H \) and decides to apply to G decreases, because underconfident workers do not apply anymore. This prediction cannot occur if female and male workers differ in their aversion to risk or competition. In particular:

1. Aversion to competition: since firm B is characterized by a non competitive environment, competition averse workers should prefer to apply to it even when receiving a signal \( s_i = H \), because they assign a non monetary benefit in working in a non competitive environment. However, since in our model the expected benefit of applying to firm G for a worker who received a signal \( s_i = H \) is increasing in \( p \) then, in presence of competition aversion, the number of workers with \( s_i = H \) who applies in G should increase and not decrease when \( p \) increases.

2. Risk-aversion: Consider first a low value of \( p \). When \( p = 0 \), every worker applies to firm G so that the probability of being hired is \( z \). On the contrary, when \( p = 1 \), conditional on having received \( s_i = H \), the probability of being hired is higher. As a consequence, when increasing \( p \), the risk of competing for a job in firm G decreases. In our model, irrespective of workers' risk attitude, we therefore should observe an increasing number of candidates receiving a signal \( s_i = H \) applying to G, as the level of \( p \) increases (and there are not any biases in self-confidence).

It follows that if an increase in firm G's ability to screen their candidates is associated with a decrease in the number of workers receiving a signal \( s_i = H \) applying to G, then only the presence of underconfidence can rationalize this evidence. Policies aim to reduce underconfidence might be hard to design (underconfidence has been showed to be related to educational models, cultural bias, etc). However, if underconfidence is correlated with an observable characteristic such as gender, affirmative actions may represent a second best policy to increase market efficiency.

3 Affirmative Action

In our model firms' ability to screen workers is not sufficient to restore an efficient matching, because underconfidence prevents some good workers to apply for the skilled
segment of the market and therefore there is an inefficient candidates’ self-selection. As a consequence, a job market in which firm \( G \) is perfectly able to identify who are the best candidates, may be less efficient and induce more segregation than a market in which firm \( G \) has a lower screening ability.

In this section we want to study which policies firm \( G \) can implement to increase the quality of its hirings in presence of underconfidence of some of the (good) candidates.

An affirmative action is usually designed to improve the employment or educational opportunities of individuals in disadvantaged group. These policies sought to eliminate the injustices so frequently associated with discrimination, but there is disagreement about how to design them, and the introduction of exogenous quotas are particularly ostracised. Nevertheless, in our study we are presenting a theoretical explanation in favor of the introduction of a gender quota. If ability is equally distributed among males and females workers, but women are more underconfident on average, introducing an exogenously imposed quota would lead qualified but underconfident women to apply, thus increasing the diversity and efficiency of the labor force, without discriminating men. Recent studies have indeed supported the introduction of affirmative actions to increase women’s willingness to compete (Villeval, 2012). Laboratory (Balafoutas and Sutter, 2012, Niederle et al., 2013) and field (Ibaezy et al., 2015) experiments demonstrated that gender quotas do not result in less able women overtaking most able men. In the present study, besides providing a theoretical evidence which explains the emergence of gender segregation in the job market as a result of women’s underconfidence, we now demonstrate the efficacy of a calibrated quota in closing the gender gap and restoring efficiency.

Suppose that underconfidence and gender are correlated. In this case we aim to show that a quota policy might not only increase the average quality of the pool of applicants but also the average quality of workers hired by firm \( G \). Note that we do not provide an argument to support a quota policy based on equitable considerations, but we want to show that a quota policy can increase not only the pool of candidates applying to firm \( G \), but also the quality of the hired workers. To make the argument as simple as possible, we consider the (extreme) case where all male workers are selfconfident while all female workers are underconfident. The reasoning we provide can be easily generalized to the case in which the fraction of underconfident workers is larger among female than male candidates.

Let’s assume that \( \frac{1}{2} > \sigma > z > \frac{\sigma}{2} \), where \( \sigma \) is the number of \( H \)-type workers in
the pool of candidates; notice that we have assumed that the number of male $H$-type workers is lower than $z$, but the number of $H$-type workers is larger than $z$. Assume also for sake of simplicity that $p = 1$ and $\pi^H = 1$, so that, if no worker is underconfident firm $G$ only hires $H$ workers.

Consider the equilibrium where all men workers apply to $G$ while no female workers apply. This situation occurs only if underconfident (female) workers are sufficiently pessimistic, that is $\frac{1}{1+\rho} < 1 - \pi^L$, and the following conditions hold:

$$
\left(1 - \pi^L + \pi^L \frac{2z - \sigma}{1 - \sigma}\right) w_G \geq w_B,
$$

and

$$
\left(\frac{1}{1+\rho} + \frac{\rho}{1+\rho} \frac{2z - \sigma}{1 - \sigma}\right) w_G < w_B,
$$

Since by assumption $z > \frac{\sigma}{2}$, firm $G$ is forced to assume $L$-type workers simply because there are not enough $H$-type workers applying. It is straightforward to conclude that if this situation occurs, it is profitable for firm $G$ to push female workers to apply. We show that a policy in which firm $G$ commits to hire (at least) $z - \frac{\sigma}{2}$ female workers increases the average quality of its workers. It is important to notice that which equilibria arise when a gender quota is fixed, will depend on workers’ beliefs about the distribution of underconfidence among other workers. We consider underconfidence as a psychological trait of rational agents who only underestimate their own ability, but have correct beliefs about what occurs in equilibrium. Namely, underconfident workers know the correct distribution of ability not only in the pool of candidates, but also in the set of workers hired by firm $G$. We also restrict our attention to symmetric equilibria in which workers with the same ability and level of confidence play the same strategy.

Proposition 3. Suppose that firm $G$, with $p = 1$, only hires male workers because female workers are underconfident and do not apply to it. Then, there exists a quota policy that strictly increases the average quality of workers hired by firm $G$.

Notice that even a small quota can push female workers to apply to firm $G$, thus destroying the possibility of an equilibrium in which less than $z$ workers of high ability apply to firm $G$. Any policy that provides incentives to $H$ female workers to apply without undermining male workers’ incentives is clearly beneficial for firm $G$. However, finding the optimal policy for firm $G$ is not a trivial task because multiple equilibria
arise. Moreover, equitable considerations should also play a role in determining which policy is desirable for the society.

4 Discussion

In this paper we have built a stylized model which present the positive correlation between gender and underconfidence as an explanation of gender segregation in the job market. Our results are consistent with recent experimental evidence showing that females are more likely to be underconfident with respect to their abilities (Kamas and Preston, 2012a,b, Flory et al., 2015).

Our contribution is twofold. First, we show that talented female workers, when being underconfident, do not apply to the skilled segment of the market, and this result is exacerbated when the ability of the employers to screen their candidates is high. This generates an inefficient sorting because less talented, but selfconfident workers may be induced to apply to high-skilled jobs, due to the reduction in competition they face. Our results depend on the particular behavioural bias assumed since workers characterized by competition or risk aversion will behave as in our benchmark model. We show that the workers’ underconfidence has an effect on the pool of candidates applying to firms. As a consequence, employers should not only focus on the procedures used to select the best workers, but also on how to attract the best pool of candidates: a selection based on merit from the wrong pool can be worse than a biased selection from a better pool.

Second, in our model affirmative actions may restore the efficiency of the job matching. When in presence of a correlation between underconfidence and gender, a gender quota allows firms to enhance the quality of the pool of applicants, without negatively affecting the selection process. We show that incentivizing women to participate in the high skilled segment of the job market is not (only) a question of fairness. However, more effective solutions may be implemented to recover the efficient matching between firms and workers in the long term. In particular, the gender gap in self-confidence seems to develop early in life (Orenstein, 1994, Hoffman, 1972) and depending on factors such as socioeconomic environments and parental attitudes (Filippin and Paccagnella, 2012, Chowdry et al., 2011). A policy which intervenes to equally encourage the development of self-image in young women and men would be beneficial in improving the gender equality and the efficiency of the job market. It has indeed recently been showed that
educational and role model interventions in children can mitigate the gender gap, mainly acting through self-confidence and the response to performance feedback (Alan and Ertac, 2016).

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A Appendix

Proof of Proposition 1

First of all we prove the existence of the equilibrium such that only workers who receive a signal $s_i = H$ apply to firms in firm $G$ if $p \geq \bar{p}$. A worker who receives a signal $s_i = H$ has no incentive to deviate if and only if:

$$
\left\{ p \left[ \frac{\pi^H}{\gamma \pi^H} \left( \frac{z}{\gamma \pi^H} \right) + (1 - \pi^H) \left( \max \left( 0, \frac{z - \gamma \pi^H}{\gamma (1 - \pi^H)} \right) \right) \right] \right. + \left. (1 - p) \frac{z}{\gamma} \right\} w_G \geq w_B \tag{1}
$$

with probability $p$ firm $G$ is able to observe the applicants’ type. Since with probability $\Pr(i = H|s_i = H) = \pi^H$ the signal is correct, the worker will be enrolled in $G$ depending on the size of the segment $z$ and on the proportion of true $H$ types applicants. In particular, the probability of a (true) $H$ type worker getting a job in $G$ is equal to $\frac{z}{\gamma \pi^H}$.

With probability $\Pr(i = L|s_i = H) = 1 - \pi^H$ the worker receiving a signal $s_i = H$ is a $L$ type applicant and thus, if $z$ is small enough, she will be not enrolled and will get zero utility. However, if there are still vacant jobs in $G$ after that all true $H$ types applicants are hired, that is $z \geq \gamma \pi^H$, she will get a job competing with other $L$ type applicants $\gamma (1 - \pi^H)$. Finally, with probability $1 - p$, firm $G$ is not able to observe the applicants’ type and thus will select workers at random. In such a situation, all applicants will be hired with probability $\frac{z}{\gamma}$.

By assumption $z < \gamma \pi^H$ so that segment $G$ is smaller than the proportion of true $H$ type workers receiving a signal $s_i = H$. Therefore, equation 1 can be written as:

$$
2z w_G \geq w_B \gamma \tag{2}
$$

which holds by Assumption A1.
Workers who receive a signal $s_i = L$ have not incentive to deviate from the above equilibrium if and only if:

$$\left\{ p \left[ \pi^L \left( \max \left( 0, \frac{z - \gamma \pi^H}{\gamma (1 - \pi^H)} \right) \right) + (1 - \pi^L) \frac{z}{\gamma \pi^H} \right] + (1 - p) \frac{z}{\gamma} \right\} w_G \leq w_B$$

(3)

that is with probability $\pi^L$ the signal is correct and the worker is a true $L$ type so when deviating and applying to a position in firm $G$, with probability $p$ she will be enrolled only if there is a sufficient number of vacancies with probability $\frac{z w_G \gamma^H}{\gamma (1 - \pi^H)}$. With probability $1 - \pi^L$ the signal $s_i = L$ is not correct and thus the worker will compete with others $H$ type workers for a job in $G$ and will be enrolled with probability $\frac{z}{\gamma \pi^H}$. Finally, with complementary probability $1 - p$, firm $G$ is not able to observe the applicants’ type and thus will select workers at random.

By assumption $z w_G \geq w_B$ and $\pi^H + \pi^L > 1$, thus deviating from equilibrium is not profitable when:

$$p \geq \frac{\pi^H (z w_G - \gamma w_B)}{z w_G (\pi^H + \pi^L - 1)} = \tilde{p}$$

(4)

Consider now a strategy profile such that all workers apply to firm $G$.

Workers who observed $s_i = H$ apply to $G$ if and only if:

$$\left\{ p \left[ \pi^H \left( \max \left( \frac{z}{\sigma}; 1 \right) \right) + (1 - \pi^H) \left( \max \left( 0; \frac{z - \sigma}{1 - \sigma} \right) \right) \right] + (1 - p) z \right\} w_G \geq w_B$$

(5)

with $\sigma = \gamma \pi^H + (1 - \pi^H)(1 - \gamma)$. With probability $p$ firm $G$ observe applicants’ ability. If $z$ is sufficiently large the candidate is getting a job for sure. In the alternative case, the probability to be hired depends on the size of the segment $z$ and on the proportion of $H$ workers who applies to firm $G$, that is $\frac{z}{\gamma \pi^H \pi^H} \frac{z}{\gamma \pi^H}$. Since $z < \gamma \pi^H$, when being a true $L$ type, a worker is not hired by firm $G$. By rearranging equation 5 it follows that:

$$p \geq \frac{(w_H - z w_G) [\gamma \pi^H + (1 - \pi^H)(1 - \gamma)]}{z w_G (\pi^H + \pi^L - 1)(1 - \gamma)}$$

(6)

which always holds since the numerator of equation 6 is negative by assumption A1 while its denominator is positive.

Workers who receive signal $s_i = L$ apply to $G$ if and only if:

$$\left\{ p (1 - \pi^L) \left( \min \left( \frac{z}{G}; 1 \right) \right) + (1 - p) z \right\} w_G \geq w_B$$

(7)

if the signal is correct, (s)he will not be enrolled in $G$ when it is able to screen candidates: High ability types will be hired and no more vacant jobs will be available since, by
assumption A1, \( z \leq \gamma \pi^H \). With probability \( (1 - \pi^L) \) the worker is a truly \( H \) type, thus (s)he will get a position in \( G \) competing with other \( H \) type candidates. By rearranging equation 7 we get:

\[
p \leq \frac{(zw_G - w_B)[\gamma \pi^H + (1 - \pi^L)(1 - \gamma)]}{z\gamma w_G (\pi^H + \pi^L - 1)} = \tilde{p}
\]

Finally, let \( \xi \in (0, \frac{1}{2}) \) be the fraction of workers who observe \( s_i = L \) and apply to firm \( B \). These workers are indifferent whether to apply to firm \( G \) or \( B \) if and only if:

\[
\pi \left( \frac{z}{\xi} \left( (1 - \pi^L)(1 - \gamma) + \gamma \pi^H \right) + (1 - \pi) \frac{z}{\gamma + \xi (1 - \gamma)} \right) w_G = w_B, \tag{9}
\]

equality 9 can be written as \( p = \frac{(zw_G - w_B)[(1 - \pi^L)(1 - \gamma) + \gamma \pi^H]}{z\gamma w_G (\pi^H + \pi^L - 1)(1 - \gamma)} \) and this expression is decreasing in \( \xi \); therefore this equilibrium exists if \( p \in \left[ \frac{(zw_G - w_B)[\gamma \pi^H + (1 - \pi^L)(1 - \gamma)]}{z\gamma w_G (\pi^H + \pi^L - 1)(1 - \gamma)} \right] \).

A strategy profile such that no worker applies to firm \( G \) cannot exist, because any worker could deviate and successfully apply to firm \( G \).

**Proof of Proposition 2**

(i) *Inefficient Sorting (and gender segregation)* Consider first an equilibrium where all selfconfident workers, independently on whether they received a signal \( s_i = L \) or \( s_i = H \) apply to firm \( G \) and all underconfident workers apply to firm \( B \).

Selfconfident workers who receive a signal \( s_i = H \) will apply to \( G \) if the following condition is satisfied:

\[
\left\{ p \left[ \pi^H \min \left( 1; \frac{z}{\gamma w_G (\pi^H + \pi^L - 1)} \right) \right] + (1 - \pi^H) \left( \max \left( 0; \frac{z}{\gamma w_G (\pi^H + \pi^L - 1)} \right) \right) \right\} w_G \geq w_B \tag{10}
\]

depending on the fraction of selfconfident candidates in the population, we may encounter two possible situations. If the proportion of truly \( H \)-type selfconfident candidates is lower than the positions available in firm \( G \), that is if \( z \geq [\gamma \pi^H + (1 - \pi^L)(1 - \gamma)](1 - \delta) \), they will be enrolled with probability one. In the opposite situation, they will compete with other selfconfident \( H \)-type workers for a position. In the latter case previous equation is equal to:

\[
p \geq \frac{[w_B (1 - \delta) - zw_G][\gamma \pi^H + (1 - \pi^L)(1 - \gamma)]}{zw_G (\pi^H + \pi^L - 1)(1 - \gamma)} \tag{11}
\]

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while in the first case it is equal to:

\[
P \geq \frac{[w_B(1 - \delta) - zw_G][(1 - \pi^H)\gamma + \pi^H(1 - \gamma)]}{w_G \left[z(1 - [(1 - \pi^H)\gamma + \pi^L(1 - \gamma)] + (\pi^H + \pi^L - 1)(1 - \gamma)(1 - \delta)) \right]}
\]  

(12)

Both conditions 11 and 12 are always satisfied since, by assumption A.1, the numerator is negative and the denominator is positive.

Underconfident workers who observe \(s_i = H\) prefer not to apply to \(G\) when all selfconfident workers apply if:

\[
\left\{ \begin{array}{l}
p \left[ \frac{\min (1; \frac{z w}{\pi}, \frac{z w}{\pi})}{\pi} \right] + \frac{\max (0; \frac{z w}{\pi}, \frac{z w}{\pi})}{\pi} \right) + (1 - p) \frac{w_L}{w_H} \leq 1
\end{array} \right.
\]  

(13)

while selfconfident workers receiving \(s_i = L\) apply to firm \(G\) only if

\[
\left\{ \begin{array}{l}
p \left[ (1 - \pi^L) \left( \frac{\min (1; \frac{z w}{\pi}, \frac{z w}{\pi})}{\pi} \right) \right] + \pi^L \left( \frac{\max (0; \frac{z w}{\pi}, \frac{z w}{\pi})}{\pi} \right) \right) + (1 - p) \frac{w_L}{w_H} \geq 1
\end{array} \right.
\]  

(14)

when \(z \leq [\gamma \pi^H + (1 - \pi^L)(1 - \gamma)](1 - \delta)\) previous equations are satisfied if \(\delta > \frac{zw_G}{w_B} - 1\) and \(\rho > \frac{(1 - \gamma)(\pi^H + \pi^L - 1)}{(1 - \gamma)(1 + \pi^L + \gamma \pi^L)}\), which is always true because of assumption A.2. As a consequence, this equilibrium exists when \(p \in \left[ \frac{zw_G - w_B (1 - \delta)}{zw_G - w_B (1 - \delta) + \rho \gamma \pi^H + (1 - \pi^L)(1 - \gamma)}, \frac{zw_G - w_B (1 - \delta) + \rho \gamma \pi^H + (1 - \pi^L)(1 - \gamma)}{zw_G - w_B (1 - \delta) + \rho \gamma \pi^H + (1 - \pi^L)(1 - \gamma) - \rho \gamma \pi^L - \rho} \right]\); the larger is \(\rho\) the larger is the set of parameter values for which this equilibrium exists.

Note that when \(z \geq \frac{w_B (1 - \delta)}{w_G (1 - \pi^L)}\), with \(w_G \geq \frac{w_B}{1 - \pi^L}\), equation 14 is satisfied irrespective of \(p\) so that only the constraint on equation 13 becomes binding: this equilibrium exists when \(p \geq \frac{zw_G - w_B (1 - \delta) - \rho \gamma \pi^H + (1 - \pi^L)(1 - \gamma)}{zw_G - w_B (1 - \delta) + \rho \gamma \pi^H + (1 - \pi^L)(1 - \gamma) - \rho \gamma \pi^L - \rho} \) and \(\rho > \frac{(1 - \gamma)(\pi^H + \pi^L - 1)}{(1 - \gamma)(1 + \pi^L + \gamma \pi^L)}\).

Let’s now analyse the case when \(z \geq [\gamma \pi^H + (1 - \pi^L)(1 - \gamma)](1 - \delta)\). If \(z \geq 1 - \delta\) then equation 14 is satisfied irrespective of \(p\). However, in order for equation 13 to be satisfied, it must be that \(z \leq \frac{[\gamma \pi^H + (1 - \pi^L)(1 - \gamma)](1 - \delta)}{1 - [\gamma \pi^H + (1 - \pi^L)(1 - \gamma)]} \leq 1 - \delta\). Therefore, this equilibrium exists when \(p \in \left[ \frac{zw_G - w_B (1 - \delta)}{zw_G - w_B (1 - \delta) + \rho \gamma \pi^H + (1 - \pi^L)(1 - \gamma)}, \frac{zw_G - w_B (1 - \delta) + \rho \gamma \pi^H + (1 - \pi^L)(1 - \gamma)}{zw_G - w_B (1 - \delta) + \rho \gamma \pi^H + (1 - \pi^L)(1 - \gamma) - \rho \gamma \pi^L - \rho} \right]\) with \(\rho > \frac{zw_G}{w_B} - 1\), and underconfidence is sufficiently severe \(\rho \geq \frac{z(\pi^H + (1 - \gamma)(1 - \pi^L + \pi^L(1 - \gamma)] + (\pi^H + \pi^L - 1)(1 - \gamma)(1 - \delta)}{(1 - \delta)\gamma \pi^H + (1 - \pi^L)(1 - \gamma) + z(\gamma \pi^H + \pi^L(1 - \gamma) - 1)}\).

(ii) Efficient Sorting (and Gender Segregation): Consider now an equilibrium where, among the selfconfident workers, only those who received a signal \(s_i = H\) apply to firm \(G\), while all underconfident workers apply to firm \(B\).
Underconfident workers who received a signal $s_i = H$ prefer to not apply to firm $G$ if:

$$p \left( \Gamma_{wG} \left( \min \left( 1; \frac{z}{\gamma \pi^H (1-\delta)} \right) \right) + \left( 1 - p \right) \Gamma_{wH} \left( \max \left( 0; \frac{z}{\gamma (1-\delta)} \right) \right) \right) + \left( 1 - p \right) \frac{z}{\gamma (1-\delta)} \leq w_B \quad (15)$$

when the market is able to screen candidates, $p > 0$, with probability $\frac{\pi^H}{1+\rho}$ they are truly $H$ type workers thus they are enrolled for sure if there are enough vacancies, otherwise they will compete with High ability selfconfident candidates for a job. If we assume that $z \leq \gamma \pi^H (1-\delta)$ the latter case applies.

Similarly, selfconfident workers who observe $s_i = L$ do not apply to firm $G$ if:

$$p \left( 1 - \pi^L \right) \left( \min \left( 1; \frac{z}{\gamma \pi^L (1-\delta)} \right) \right) + \pi^L \left( \max \left( 0; \frac{z}{\gamma (1-\delta)} \right) \right) + \left( 1 - p \right) \frac{z}{\gamma (1-\delta)} \leq w_B \quad (16)$$

The characterization of this equilibrium depends on the fraction of selfconfident workers in the population. Let’s first analyse the case such that $z \leq \gamma \pi^H (1-\delta)$: rearranging equations 15 and 16 we have that such equilibrium exists if $p \geq \frac{\pi^H + \pi^L - 1}{1 - \pi^H}$, which is always true because of assumption A.2, and $p \geq \frac{\left[ w_G - w_B \gamma \pi^H (1-\delta) \right] \pi^H}{w_G \gamma (1-\delta) \pi^H + \pi^L - 1}$. Similarly, when $z \geq \gamma \pi^H (1-\delta)$, it must be that $p \geq \frac{\left[ w_G - w_B \gamma (1-\delta) \right] \pi^H}{w_G \gamma \pi^H + \pi^L - 1} + \frac{z}{\gamma (1-\delta)}$, with $z \leq \gamma (1-\delta)$. In both cases, if $p$ decreases, selfconfident workers who received a signal $s_i = L$ are the first who deviate and apply to $G$, while underconfident candidates still apply to $B$.

(iii) Partial Gender segregation: All selfconfident workers, irrespective of their signals, and underconfident workers who observe $s_i = H$ apply to $G$ if the following conditions hold.

Selfconfident workers who observe $s = L$ apply if:

$$p \left( \pi^L \left( \min \left( 1; \frac{z}{\gamma \pi^L (1-\delta)} \right) \right) + \left( 1 - p \right) \left( \min \left( 1; \frac{z}{\gamma \pi^L (1-\delta)} \right) \right) \right) + \left( 1 - p \right) \frac{z}{\gamma (1-\delta)} \gamma \pi^H + (1-\delta) \leq w_G \quad (17)$$

that is if $p \leq \frac{\left[ w_G - w_B \gamma (1-\delta) \right] \gamma \pi^H + (1-\delta) \pi^L \left( 1-\gamma \right)}{w_G \gamma \pi^H + \pi^L - 1}$ when $z \leq \gamma \pi^H (1-\delta)$.

When and $\pi^H \gamma + (1-\delta) \left( 1-\pi^L \right) \left( 1-\gamma \right) \leq \gamma + (1-\delta) \left( 1-\gamma \right)$ previous equation is satisfied if $p \leq \frac{\left[ w_G - w_B \gamma (1-\delta) \right] \pi^H + (1-\delta) \pi^L \left( 1-\gamma \right)}{w_G \gamma \pi^H + \pi^L - 1} \left( 1-\gamma \right)$

Underconfident workers who observe $s_i = L$ do not apply if:

$$p \left( 1 - \pi^L \right) \left( \min \left( 1; \frac{z}{\gamma \pi^L (1-\delta)} \right) \right) + \left( 1 - p \right) \left( \min \left( 1; \frac{z}{\gamma \pi^L (1-\delta)} \right) \right) \gamma \pi^H + (1-\delta) \leq w_G \quad (18)$$

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If \( z \leq \pi^H \gamma + (1 - \delta) (1 - \pi^L) (1 - \gamma) \), this condition is satisfied when \( \rho > 0 \) and \( p \geq \frac{\sum_G - w_B (\gamma + (1 - \delta) (1 - \gamma)) [\gamma \pi^H + (1 - \delta) (1 - \pi^L) (1 - \gamma)]}{\sum_G [\gamma \pi^H (\pi^H + \pi^L - 1) + \pi^H \rho (\gamma + (1 - \delta) (1 - \gamma))]}. 

If \( z \geq \pi^H \gamma + (1 - \delta) (1 - \pi^L) (1 - \gamma) \) and \( z \geq \gamma + (1 - \delta) (1 - \gamma) \) it must be that \( p \leq \frac{\sum_G - w_B (\gamma + (1 - \delta) (1 - \gamma)) [\gamma \pi^H + (1 - \delta) (1 - \gamma)]}{\sum_G [\gamma \pi^H (\pi^H + \pi^L - 1) + \pi^H \rho (\gamma + (1 - \delta) (1 - \gamma)) (1 - z)]}. 

Finally, underconfident workers who received a signal \( s_i = H \) apply to \( G \) if:

\[
\left\{ p \left( \frac{\gamma \pi^H}{1 + \rho} \left( \min \left( 1; \frac{\pi^H}{\gamma \pi^H} \right) \right) + (1 + \rho) \frac{\pi^L}{1 + \rho} \left( \max \left( 0; \frac{\pi^H}{\gamma \pi^H} \right) \right) \right) \right\} w_G \geq w_B \quad (20)
\]

that is if \( p \leq \frac{\sum_G - w_B \gamma \pi^H (1 - z)}{\sum_G \pi^H (\pi^H + \pi^L - 1)} \) or \( p \leq \frac{\sum_G - w_B \gamma \pi^H (1 - z) \pi^H}{\sum_G \pi^H (\pi^H + \pi^L - 1)} \), respectively when \( z \leq \gamma \pi^H \) and \( z \geq \gamma \pi^H \).

Selfconfident workers who observe \( s_i = L \) do not apply to firm \( G \) only if:

\[
\left\{ p \left( (1 - \pi^L) \left( \min \left( 1; \frac{\pi^L}{\gamma \pi^L} \right) \right) \right) + \pi^L \left( \max \left( 0; \frac{\pi^H}{\gamma \pi^H} \right) \right) \right\} w_G \leq w_B \quad (21)
\]

that is if \( p \geq \frac{\sum_G - w_B \gamma \pi^H \pi^L}{\sum_G (\pi^H + \pi^L - 1)} \) or, when \( z \geq \gamma \pi^H, p \geq \frac{\sum_G - w_B \gamma \pi^H \pi^L}{w_G (\gamma - z) \pi^H (\pi^H + \pi^L - 1)} \). Notice that \( \frac{\sum_G - w_B \gamma \pi^H (1 - z)}{\sum_G \pi^H (\pi^H + \pi^L - 1)} \) implies \( \frac{\pi^H + \pi^L - 1}{\pi^H} > \rho \). The same reasoning applies when \( z \geq \gamma \pi^H \).

Therefore this equilibrium cannot exist by Assumption A2.

(iv) Efficient Sorting (No Gender Segregation): Consider the equilibrium where workers who observe \( s_i = H \) apply to \( G \) and while other workers apply to \( B \), irrespective of their level of confidence.

Underconfident workers who observe a signal \( s_i = H \) prefer to apply when all workers who have received \( s_i = H \) apply if:

\[
\left\{ p \left[ \gamma \pi^H \left( \min \left( 1; \frac{\pi^H}{\gamma \pi^H} \right) \right) + \pi^L \left( \max \left( 0; \frac{\pi^H}{\gamma \pi^H} \right) \right) \right] \right\} w_G \geq w_B \quad (20)
\]

that is if \( p \leq \frac{\sum_G - w_B \gamma \pi^H (1 - z)}{\sum_G \pi^H (\pi^H + \pi^L - 1)} \) or \( p \leq \frac{\sum_G - w_B \gamma \pi^H (1 - z) \pi^H}{\sum_G \pi^H (\pi^H + \pi^L - 1)} \), respectively when \( z \leq \gamma \pi^H \) and \( z \geq \gamma \pi^H \).

Poolconfident workers who observe \( s_i = L \) do not apply to firm \( G \) only if:

\[
\left\{ p \left( (1 - \pi^L) \left( \min \left( 1; \frac{\pi^L}{\gamma \pi^L} \right) \right) \right) + \pi^L \left( \max \left( 0; \frac{\pi^H}{\gamma \pi^H} \right) \right) \right\} w_G \leq w_B \quad (21)
\]

that is if \( p \geq \frac{\sum_G - w_B \gamma \pi^H \pi^L}{\sum_G (\pi^H + \pi^L - 1)} \) or, when \( z \geq \gamma \pi^H, p \geq \frac{\sum_G - w_B \gamma \pi^H \pi^L}{w_G (\gamma - z) \pi^H (\pi^H + \pi^L - 1)} \). Notice that \( \frac{\sum_G - w_B \gamma \pi^H (1 - z)}{\sum_G \pi^H (\pi^H + \pi^L - 1)} \) implies \( \frac{\pi^H + \pi^L - 1}{\pi^H} > \rho \). The same reasoning applies when \( z \geq \gamma \pi^H \).

(v) Pooling (No Gender Segregation): All workers apply to firm \( G \) if underconfident workers who observe \( s_i = L \) apply to \( G \), that is:

\[
\left\{ p \left[ (1 - \min(1, (1 + \rho) \pi^L)) \right] \right\} w_G \geq w_B \quad (22)
\]
The above condition is satisfied if \( p \leq \frac{(zw_G - w_B)[\gamma \pi^H + (1 - \gamma)(1 - \pi^L)]}{zw_G[\rho \pi^H + \gamma(\pi^H + \pi^L - 1)]} \) when \( z \leq \gamma \pi^H + (1 - \gamma)(1 - \pi^L) \). In the opposite situation, it must be that \( p \leq \frac{(zw_G - w_B)[\gamma (1 - \pi^H) + (1 - \gamma)\pi^L]}{w_G(1 - z)\rho \pi^H + \gamma(\pi^H + \pi^L - 1)} \).

**Proof of Proposition 3**

If only male workers apply to \( G \) then, by assumption, at least \( 2z - \sigma \) workers are \( L \text{-type} \). Consider a quota policy that reserves exactly \( 2z - \sigma \) job openings to female workers. First, notice that it does not exist an equilibrium in which no female worker applies to \( G \). Suppose, by contradiction, that no female worker applies to firm \( G \): then any female worker, irrespectively of her confidence, can deviate, apply to \( G \) and be hired with probability one, which is a profitable strategy since \( w_G > w_B \). Suppose that only \( L \text{-type} \) female workers apply with positive probability to firm \( G \). Then, the decision to apply to firm \( G \) is a best response for a (female) worker who assesses that her own probability of being \( H \text{-type} \) is \( \frac{1 - \pi^L}{1 + \rho} \). Consider any \( H \text{-type} \) female worker: if she deviates and apply to firm \( G \) her estimated payoff, given that her beliefs of being \( H \text{-type} \) are equal to \( \frac{\pi^H}{1 + \rho} > \frac{1 - \pi^L}{1 + \rho} \), is strictly larger than the the estimated payoff of an \( L \text{-type} \). Then, there cannot be an equilibrium in which only \( L \text{-type} \) workers apply to firm \( G \). It follows that in any symmetric equilibrium \( H \text{-type} \) female workers apply with larger probability than \( L \text{-type} \) female workers and therefore firm \( G \) hires \( H \text{-type} \) female workers with positive probability. Finally, we need to show that the average quality of male workers hired by firm \( G \) cannot be lower than their average quality without any quota policy. Since \( p = 1 \), a male worker who receives a signal \( s_i = H \) knows that he is hired with probability less than one only if firm \( G \) hires only \( H \text{-type} \) workers. So, in equilibrium, either firm \( G \) only hires \( H \text{-type} \) workers or \( H \text{-type} \) workers are hired with probability one and therefore all \( H \text{-type} \) male workers apply to firm \( G \).