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ARE WE BETTER-OFF FOR WORKING HARD?

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Abstract

When traders are uncertain on being informed and make effort to reduce their uncertainty, we would expect an improvement in both the welfare and price efficiency. By considering the disutility of the effort, we characterize the non-cooperative information game on traders' decision of making effort through a Nash equilibrium and asset price through a noisy rational expectation equilibrium. We show that making effort to be informed is harmful for social welfare. Also improving market efficiency is always at the cost of welfare reduction. Therefore, with the disutility of making effort to reduce the uncertainty on being informed, social welfare can be improved when traders make less effort, and more importantly, social welfare and price efficiency cannot be improved simultaneously.

Key words: Uncertainty and effort, Nash equilibrium, endogenous information, asset pricing, efficiency, and social welfare.

JEL Classification: G02, G12, G14

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1. Introduction

With increasing uncertainty and multidimensional information in financial markets, traders become more uncertain about certain information and need to make some effort in order to reduce their uncertainty. Traders can benefit from being more informative about the information when they make more effort. This however also increases the disutility of their effort. Such a trade-off plays a central role in traders' decision making and can have important implications to market efficiency, and social welfare. In this paper we consider traders' uncertainty on being informed about certain information and disutility of their effort to become informed in an otherwise standard Grossman and Stiglitz (1980) model (GS model) of noisy rational expectation equilibrium (NREE). We demonstrate that making effort to be informed is harmful for social welfare and improving market efficiency is always at the cost of reducing the welfare. Therefore traders cannot be better off by making more effort and social welfare cannot be improved in more efficient markets.

In a competitive economy, Grossman and Stiglitz (1980) show that information efficiency of a price system depends on the number of individual who are informed. When information is imperfect and costly, the more individuals choose to be informed, the more efficient the price becomes, the less valuable the information is, and the less incentive individuals choose to be informed. Therefore in equilibrium, *"the number of individuals who are informed is itself an endogenous variable"*. In the GS model, traders can decide to a pay fixed cost to becoming informed for sure. In this paper traders are uncertain on being informed. To reduce the uncertainty, traders need to make some effort. The more effort a trader makes, the more likely he becomes informed. We examine how traders' uncertain on being informed and their effort affect their information acquisition decision, price efficiency, and more importantly the social welfare. In particular, we investigate how traders' risk aversion and market information structure determine the optimal effort for traders to becoming informed and what are the implications to social welfare and its relation to market efficiency.

We first consider a baseline GS model when the information acquisition decision is exogenous; that is the fraction of informed is given as a parameter. Consistently with the GS model, the price becomes more informative when there are more informed traders. However uninformed traders are less willing to trade when the price becomes more informative. In NREE, the aggregate risk faced by the uninformed traders increases in the informed trading. Intuitively, more informed trading reduces dividend risk but increases information risk for uninformed traders, which dominates the dividend risk. This effect becomes more significant when traders are less risk averse. The risk premium decreases in informed trading when traders are highly risk averse; but has a hump-shaped relation to the informed trading when traders are less risk averse. Concerning the welfare, informed trading always improves (marginal) welfare in the sense that individual is always better-off for being informed rather uninformed, however more informed trading always reduces the welfare for both informed and uninformed traders. In aggregation, the social welfare is always higher when traders are all uninformed than when they are all informed. Put differently, we detect a sort of *Prisoner's dilemma* situation. The social welfare would be better off if nobody is informed. However, individual is rationally driven to being informed. Therefore, at the equilibrium, the market ends up into a sub-optimal equilibrium (from a welfare viewpoint) typical of a coordination-failure situation. In addition, informed trading improves the social welfare only when price is less informative, the supply is less noisy, and traders are less risk averse; otherwise, the social welfare is typically reduced with informed trading. Therefore for the first time (to our knowledge) we show that high price efficiency corresponds to low social welfare in general.

The baseline model suggests that both price efficiency and social welfare depend on the fraction of informed traders. When traders face the uncertainty on becoming informed, we endogenize traders' decision on their optimal effort to become informed. We model a continuum of agents playing an *information game* inspired by global games related to public and private information (see Morris and Shin (2002)). Differently from classical global games, the strategy of the players is expressed in terms of the *probability of being informed*. With this respect, our model resembles some recent literature on *probabilistic choice models* (Mattsson and Weibull (2002)) and classical results in information theory (Hobson (1969)). In Mattsson and Weibull

(2002), an individual optimally makes an effort to deviate from the status-quo (a reference probability) and *change the likelihood* of a finite set of possible scenarios in order to get closer to implementing a more desired outcome. Given that the reward is always higher for informed than uninformed, traders choose their optimal information acquisition strategy to maximize the trade-off between a higher expected reward of being more informed and a higher disutility of the effort. When individual makes an optimal trade-off between the expected reward and the cost of deviating from the status-quo, Mattsson and Weibull (2002) show that the disutility of the optimal effort is related to the information entropy. The resulting choice probabilities are a distortion of the logit model, in which the degree of distortion is governed by the default distribution.

We incorporate this probabilistic-based trade-off mechanism into our two-stage optimization scheme based, firstly, on a strategic information game and, secondly, on a classical mean and variance investment decision problem. We characterize a unique Nash equilibrium in the vector of probabilities of traders being informed and a NREE in asset pricing. With the disutility being characterized by information entropy, we show that traders' optimal effort depends on their risk aversion and the information structure. The resulting endogenous information equilibrium leads to outcomes that are significantly different from the GS model.

Firstly, on the equilibrium fraction of informed, informed trading does not monotonically increase in the noise supply. In fact, when traders are less risk averse, they make more effort to being informed as the noise supply increases. However when traders are more risk averse, there is a hump-shaped relation between the endogenous informed trading and the noise supply. Traders make more effort when the supply noise is neither too small nor too large. This effect becomes more pronounced when traders are very risk averse. Also, an increase in informed trading cannot perfectly offset the noises in supply and dividend payoff. Furthermore, with an increase in the dividend noise and the informativeness of the information, traders are making more effort to be informed. Overall, when the noise in the information is neither too small nor too large relative to the noise supply and the unobserved noise, the information becomes more valuable and traders make more effort to becoming informed.

Secondly, on the equilibrium implications for price efficiency, informed trading improves price efficiency in general, but this is not necessarily always the case. It depends on traders' risk aversion and the information structure characterized by the noise in supply, the informativeness of the information, and the dividend noise. When traders are less risk averse, as the supply noise increases, traders make more effort to become informed but the price becomes less efficient. When traders are more risk averse, an increase in the supply noise at low levels leads to more effort for traders to become informed, which however reduces the price efficiency. More generally, an increase in the supply noise at high levels always leads traders to make less effort to becoming informed, making price less efficient. Furthermore, with an increase in the dividend noise and the informativeness of the information, traders are making more effort to be informed, which however has different impact on price efficiency. High dividend noise makes the price less efficient, while high informativeness of the information improves the price efficiency. Therefore, when traders make more effort to becoming informed, the price efficiency is not necessarily improved.

Thirdly, on the social welfare, it is always low when price is more efficient. This implies that improving market efficiency is always at the cost of reducing the social welfare. Also, when traders make effort to reduce their uncertainty on being informed, it is always harmful to the social welfare comparing to making no effort, which is underlined by the Prisoner's dilemma situation in the exogenous NREE. Therefore traders are better off by making less effort and social welfare cannot be improved in more efficient markets.

The structure of the paper is as follows. We introduce the model and define the equilibrium in Section 2. In Section 3, we consider a baseline GS model with exogenous fraction of informed traders and examine the impact of informed trading on aggregate risk, risk premia, and welfare. Section 4 extend the analysis to an endogenous information equilibrium and explore the equilibrium informed trading, price efficiency and welfare analysis. In Section 5 we explicitly model trading motives as a possible route to endogenous supply. Section 6 concludes and all the proofs are collected in the Appendix.

2. The Model

We consider a static one-period model. There is a measure-one continuum of traders, indexed by $i \in (0,1)$ who can invest in two assets: a risk-free asset with a rate of return $R > 1$ and a risky asset with payoff $\tilde{D}$ at the end of the period. Traders are risk averse with a CARA utility function, i.e., $u(\tilde{W}_i) = -e^{-\alpha \tilde{W}_i}$, where α is the absolute risk aversion and $\tilde{W}_i$ is trader i's terminal portfolio wealth. As in Grossman and Stiglitz (1980) (GS henceforth), the payoff of the risky asset is given by

$$\tilde{D} = d + \tilde{\theta} + \tilde{\epsilon}, \tag{2.1}$$

where $\tilde{\theta} \sim \mathcal{N}(0, v_\theta)$ represents the fundamental information component of the risky payoff, which is observable to the informed traders, $\tilde{\epsilon} \sim \mathcal{N}(0, v_\epsilon)$ represents the noise component, which is unobservable to all traders, and $d = \mathbb{E}[D](> R)$ is a constant. We assume that the risk-free asset is in zero net supply and the risky asset has a noisy net supply of $\tilde{z} \sim \mathcal{N}(0, v_z)$. Note that the supply shock $\tilde{z}$ can be due to liquidity demand or noise trading. In Section 5, we model the behaviour of liquidity/noise traders explicitly using endowment shocks. For now we simply take the noisy supply as given.

In the GS model, trader can pay a fixed cost to be informed about the information $\tilde{\theta}$ for sure. Therefore, the information acquisition decision depends on the information cost and the difference in the expected utilities between informed and uninformed. If the cost is less than the difference, some individuals switch from being uninformed to being informed.

In our model there is an uncertainty about how likely the trader can be informed about the fundamental information and he needs to make some effort in order to reduce this uncertainty. In other words, trader i faces the possibility to be informed. The more effort the trader makes, the more likely he becomes informed, and the more disutility he has to bear. More explicitly, with a *probability* $p_i \in [p_0, 1]$, trader

i can be informed to observe the fundamental information $\tilde{\theta}$.[1] To increase his probability to be informed, the trader needs to make an effort. On the information acquisition decision, the trader decides the effort he wants to put in place in order to be informed. This effort is signalled by the probability p_i, the more the effort, the higher the probability. However there is a disutility associated with the effort, $c(p_i)$, which is an increasing function of the effort and hence probability p_i. Therefore, trader's information acquisition decision depends on the trade-off between the expected utility to be informed and the disutility of his effort. Intuitively, in equilibrium the probability of becoming informed is same for all traders, $p_i^* = p^*$, which also corresponds to the fraction of traders to be informed $\lambda = p^*$. For convenience, we denote type I as informed and type U as uninformed.

To characterize the equilibrium information acquisition, for trader i, we denote by $\omega_i \in \{0, 1\}$, where $\omega_i = 1$ corresponds to type I and $\omega_i = 0$ corresponds to type U.[2] Therefore, for all $i \in (0, 1)$,

$$\mathbb{P}(\omega_i = 1) = p_i; \quad \mathbb{P}(\omega_i = 0) = 1 - p_i.$$

Each trader makes a decision about his desired effort signalled by p_i. The trader also chooses his optimal demand x_i, the number of shares invested in the risky security, after knowing which type (informed or uninformed) he belongs to. Therefore the objective of trader i is to maximize

$$\mathcal{U}(p_i; \lambda) = p_i V_I(\lambda) + (1 - p_i) V_U(\lambda) - \mu\, c(p_i), \tag{2.2}$$

with respect to his probability p_i, where

$$V_I(\lambda) = \max_{x_i} \mathbb{E}\Big[u\big(W(x_i)\big)\Big|\omega_i = 1,\, \lambda\Big], \;\; V_U(\lambda) = \max_{x_i} \mathbb{E}\Big[u\big(W(x_i)\big)\Big|\omega_i = 0,\, \lambda\Big], \tag{2.3}$$

c is an increasing and convex cost function such that $c(p_0) = 0$ and $\mu \geq 0$ a constant parameter.

[1] Here $p_0 \geq 0$ is a reference probability, depending on the specification of the model, p_0 may be 0 or a strictly positive value.

[2] We assume everyone starts off being uninformed.

The optimization scheme of trader i is separated into two stages and solved *backward*. First, trader i decides his portfolio choice x_i^* *given his type*, that is the realization of ω_i. Second, by averaging on the likelihood of being informed and forming an expectation about other traders' actions, trader i strategically chooses p_i^*, the *optimal* probability to become informed; we refer to this stage as the information game. The vector of optimal strategies, $p^* = (p_i^*)_{i\in(0,1)}$, is then determined by a Nash equilibrium, while the equilibrium price P^* of the risky asset is determined by the market clearing condition as in the standard noisy rational expectation equilibrium (NREE)(Admati (1985), Admati and Pfleiderer (1987)). Note that the equilibrium price for the risky asset depends on the *fraction of informed traders*, λ, where

$$\lambda = \int_0^1 \mathbb{I}_{\{\omega_i=+1\}} di.$$

Therefore, when needed, we will denote the price as P_λ.[3]

Before introducing a formal definition of equilibrium, we briefly discuss the structure of the two-stage optimization problem. Both V_I and V_U in (2.3) represent, respectively, the expected utilities of the wealth for traders of type I and U, conditional on the result of the information game.[4] Concerning $\mathcal{U}$ defined in (2.2), it provides the payoff for trader i related to the information game played by the traders. Since each trader optimally chooses the probability of being informed, $p_i \in [p_0, 1]$, our approach is strictly related to a genreal class of *probabilistic choice models* (see Mattsson and Weibull (2002)). In this context, a very natural choice for the payoff of the game is the difference between the expected reward and the disutility from effort due to the change in the status-quo (p_0 in our model). We will specify in more details the shape of the cost function c is Section 4 when solving the information game. We now introduce the following definition of *market equilibrium*.

Definition 2.1. We say that probability $p^* = (p_i^*)_{i\in(0,1)}$, fraction of informed λ^*, and price P^* of the risky asset are in equilibrium if

[3]More precisely, we see in Section 3 that $P_\lambda = h_\lambda(\theta, z)$ is a random variable, where h_λ is a deterministic function depending on λ.

[4]At this stage, we assume that individual traders do know the realization of the information game; the vector p of probabilities and the fraction of informed λ. Knowledge of λ is crucial since the price of the risky asset depends on λ.

(i) $p^* = (p_i^*)_{i\in(0,1)}$ is a Nash equilibrium, meaning that for every $i \in (0,1)$,

$$\mathcal{U}(p_i^*;\lambda) \geq \mathcal{U}(p_i;\lambda);$$

(ii) the following *consistency equation* is satisfied[5]

$$\lambda = \mathbb{E}\left[\int_0^1 \mathbb{I}_{\{\omega_i^*=1\}}\, di\right] = \int_0^1 p_i^*\, di \equiv \lambda^*,$$

here ω_i^* is the random variable associated to the optimal probability p_i^*;

(iii) the price $P^* = P_{\lambda^*}$ satisfies market clearing condition

$$\int_0^1 x_i^*\, di = \tilde{z}, \tag{2.4}$$

where $x^* = (x_i^*)_{i\in(0,1)}$ is the optimal investment strategy profile.

In the following section, we first turn our attention to the second stage of the problem, solving for the optimal demand x_i, conditioning on the outcome of the information game. In particular, we examine the NREE when the fraction of informed λ is given exogenously. We then explore the implication of endogenous information equilibrium on price efficiency and social welfare in Section 4.

3. A Welfare Analysis with Asymmetric Information

To better understand the implication of the information uncertainty and the effort for market equilibrium and social welfare, in this section we first consider a baseline GS model of NREE with information asymmetry, assuming that the fraction of the informed λ is exogenously given.

3.1. **The Securities.** Concerning the investment decision, each trader maximizes his expected utility conditional on his information set, i.e, $\mathbb{E}[u(\tilde{W}_i)|\mathcal{F}_i]$. Let x_i be the number of shares trader i holds and $\tilde{P}$ be the price of the risky asset, then trader i's terminal wealth becomes

$$\tilde{W}_i = x_i(\tilde{D} - R\tilde{P}) + W_{i,0}R, \tag{3.1}$$

[5]At the equilibrium, expectations realize so that the fraction of informed, λ, exactly matches the value expected by the traders when using the revealed vector of probabilities p^*. Finally, the equilibrium price P^* will be consistent with the outcome of the game.

where $W_{i,0}$ is trader i's initial wealth (assumed to be zero for simplicity). Since dividend payoff is normally distributed, the standard solution for trader i's optimal holding of the risky asset is given by

$$x_i^* = \frac{\mathbb{E}[\tilde{D} - R\tilde{P}|\mathcal{F}_i]}{\alpha \mathbb{V}ar[\tilde{D} - R\tilde{P}|\mathcal{F}_i]}. \tag{3.2}$$

Informed traders observe both the signal and the price, hence $\mathcal{F}_i = \{\tilde{\theta}, \tilde{P}; \lambda\}$ for $i = I$ ($\omega_i = +1$). Their optimal demand becomes[6]

$$x_I^*(\theta, P) = \frac{\mathbb{E}[\tilde{D} - R\tilde{P}|\tilde{\theta} = \theta, \tilde{P} = P]}{\alpha \mathbb{V}ar[\tilde{D} - R\tilde{P}|\tilde{\theta} = \theta, \tilde{P} = P]} = \frac{d + \theta - RP}{\alpha v_\epsilon}. \tag{3.3}$$

For the uninformed traders, they observe only the price, i.e, $\mathcal{F}_i = \{\tilde{P}; \lambda\}$ for $i = U$ or $\omega_i = 0$. Their optimal demand is given by

$$x_U^*(P) = \frac{\mathbb{E}[\tilde{D} - R\tilde{P}|\tilde{P} = P]}{\alpha \mathbb{V}ar[\tilde{D} - R\tilde{P}|\tilde{P} = P]} = \frac{d + \mathbb{E}[\tilde{\theta}|\tilde{P} = P] - RP}{\alpha(v_\epsilon + \mathbb{V}ar[\tilde{\theta}|\tilde{P} = P])}, \tag{3.4}$$

where, since $\tilde{\theta}$ and $\tilde{P}$ are both normally distributed,

$$\mathbb{E}[\tilde{\theta}|\tilde{P} = P] = \frac{\mathbb{C}ov[\tilde{\theta}, \tilde{P}]}{\mathbb{V}ar[\tilde{P}]}(P - \mathbb{E}[\tilde{P}]), \;\; \mathbb{V}ar[\tilde{\theta}|\tilde{P} = P] = (1 - \mathbb{C}orr[\tilde{\theta}, \tilde{P}]^2)v_\theta.$$

Evaluating the above requires the exact form of the price $\tilde{P}$. Following the NREE literature (Admati (1985), Admati and Pfleiderer (1987)), we postulate a linear price

$$\tilde{P} = \frac{1}{R}(d + b_\theta \tilde{\theta} - b_z \tilde{z}), \tag{3.5}$$

where $b_\theta(\geq 0)$ and $b_z(\geq 0)$ are determined in equilibrium. By inserting (3.5) into (3.4) we obtain that

$$x_U^*(P) = \frac{d - RP}{\alpha\, v_U}, \tag{3.6}$$

where

$$v_U = \underbrace{\mathbb{V}ar[\tilde{D}|\tilde{P} = P]}_{\text{dividend risk}} + \underbrace{\frac{\beta}{1-\beta}\mathbb{V}ar[\tilde{D}|\tilde{P} = P]}_{\text{information risk}}, \qquad \beta = \frac{b_\theta v_\theta}{b_\theta^2 v_\theta + b_z^2 v_z}.$$

One important observation from (3.6) is that the uninformed trader trades as if his expected payoff is $d - RP$ and the risk of his payoff is v_U, which is made up

[6]Note that the optimal demand for both informed and uninformed do not explicitly depend on $i \in (0, 1)$, rather on the type of the agent.

of two components: *dividend risk* and *information risk*. Intuitively, the uninformed trader reduces his demand when price becomes more sensitive to the signal, which is measured by b_θ. In other words, the uninformed trader becomes unwilling (or requires a larger risk premium) to trade when the price contains more information. When $b_\theta = 0$ and thus $\mathbb{C}ov[\tilde{\theta}, \tilde{P}] = 0$, the aggregate risk faced by the uninformed trader is the same as the unconditional variance of the dividend payoff, i.e., $v_U = \mathbb{V}ar[\tilde{D}] = v_\theta + v_\epsilon$ since information risk disappears in this case. Therefore, price's sensitivity towards the signal, or *price efficiency*, has two offsetting effects; it reduces dividend risk (conditional variance of dividend), but increases information risk. In order to determine which effect dominates, we need to solve for the coefficients b_θ and b_z in equilibrium, which we do next.

3.2. **Equilibrium.** Based on (3.3) and (3.4), the market clearing condition requires that

$$\int_0^1 x_i^* \, di = \lambda \, x_I^*(\theta, P) + (1 - \lambda) \, x_U^*(P) = \tilde{z}, \tag{3.7}$$

where λ is the fraction of traders who are informed. The equilibrium price can be characterized as follows.

Proposition 3.1. *For given* $\lambda \in (0, 1)$, *there exists a unique linear equilibrium price of the risky asset,*

$$\tilde{P} = \frac{1}{R}(d + b_\theta \tilde{\theta} - b_z \tilde{z}), \tag{3.8}$$

where

$$b_\theta = \frac{\lambda \bar{v}}{v_\epsilon}, \qquad\qquad b_z = \alpha \bar{v}, \tag{3.9}$$

and

$$\frac{1}{\bar{v}} = \frac{\lambda}{v_\epsilon} + \frac{1-\lambda}{v_U}, \qquad v_U = v_\epsilon + v_\theta + \frac{1}{\alpha}\left(\frac{v_\theta}{v_z}\right)\left(\frac{\lambda}{\alpha v_\epsilon}\right). \tag{3.10}$$

Proposition 3.1 shows that the aggregate market risk, measured by $\bar{v}$, is the harmonic mean of the risk of informed v_ϵ and uninformed v_U weighted by their market fractions λ and $1 - \lambda$, respectively. Also, the aggregate risk faced by the uninformed traders, v_U, is larger than the unconditional variance of the dividend. Intuitively, an increase in the fraction of informed traders λ makes the price more sensitive to the signal θ, which reduces dividend risk while increases information risk as per discussion previousely. The expression of v_U in (3.10) shows the former is dominated by the latter in equilibrium, thus the net effect is positive. Also, the effect is stronger when the informed traders are less risk averse (and trader more aggressively), i.e., $\frac{1}{\alpha}$ increases, and when the information to noise ratio $\frac{\lambda}{\alpha v_\epsilon} = \frac{b_\theta}{b_z}$ and the informativeness of the information $\frac{v_\theta}{v_z}$ are higher; both indicate a more informative price.

The fact that the aggregate risk faced by the uninformed traders can be larger than the unconditional variance of the the dividend payoff, i.e., $v_U \geq \mathbb{V}ar[\tilde{D}]$, has important implication on aggregate market risk $\bar{v}$ and the risk premium $\alpha\bar{v}$. First, we investigate how an increase in informed trading λ can affect the aggregate risk and risk premium. From (3.10), we can see that an increase in λ reduces the aggregate risk $\bar{v}$ more towards v_ϵ, which is the risk faced by the informed traders, however, it also increases information risk for the uninformed traders. The net effect of λ on the aggregate risk is characterized as follows.

Corollary 3.2. *On the aggregate risk $\bar{v}$ (per unit of supply, that is $\frac{1}{z}\frac{\partial \mathbb{E}[\tilde{D}-R\tilde{P}|\tilde{z}=z]}{\partial \lambda}$)*

(i) *when $\alpha \geq \bar{\alpha}$, it decreases in λ;*
(ii) *when $\alpha < \bar{\alpha}$, it increases in λ if and only if $\lambda < \alpha^2 v_\epsilon^2 v_z^2 Q$,*

where $\bar{\alpha} = \frac{1}{\sqrt{v_z(v_\epsilon + v_\theta)}}$ and

$$Q = \sqrt{(1/v_\theta + 1/v_\epsilon)^2 + v_\theta v_z v_\epsilon^2(1/\alpha^2 - 1/\bar{\alpha}^2)} - (1/v_\epsilon + 1/v_\theta) > 0.$$

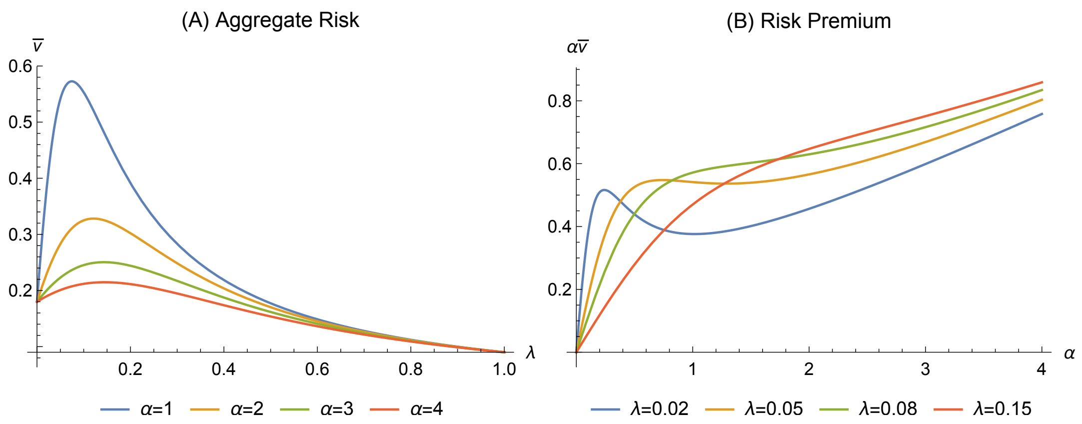

FIGURE 3.1. Relationship between aggregate risk $\bar{v}$ and fraction of informed traders λ in panel (A), and between risk premium $\alpha\bar{v}$ and risk aversion α in panel (B).

Corollary 3.2 shows that whether informed trading increases the aggregate risk (per unit of supply) depends on traders' risk aversion. Note that the aggregate risk $\bar{v}$ is larger in a market with all uninformed ($\lambda = 0$) than the one with all informed ($\lambda = 1$). When traders are highly risk averse ($\alpha > \bar{\alpha}$), informed trading always reduces the risk premium. Intuitively, when traders are more risk averse, they trade less aggressively. However more informed trading makes the price more informative, which then reduces the aggregate risk. Note that $\bar{\alpha}$ decreases in the dividend noise ($v_\epsilon + v_\theta$) and supply noise (v_z), this becomes more pronounced when the dividend and supply noises are large (so that $\alpha \geq \bar{\alpha}$).

When traders are less risk averse ($\alpha < \bar{\alpha}$), there is a hump-shaped relation between the aggregate risk ($\bar{v}$) and the informed trading (λ). This means that, when there are less informed traders in the market, the aggregate risk increases in the informed trading; however the aggregate risk decreases when there are more informed traders. This, somewhat unexpected, non-monotonic behavior is due to the information risk component of uninformed risk. When α is small, v_U is highly susceptible on shifts in λ. Moreover, if λ is also small, uninformed are predominant on the market. Therefore the impact of v_U on the harmonic mean is higher compared to v_ϵ. When λ increases, this effect vanishes due to the shift of the weight towards the informed fraction of the traders. The results are illustrated in Figure 3.1 (A) showing that aggregate risk is significantly higher in market with a small fraction of informed traders than in a market with either no informed trading or all informed trading. For instance, for $\alpha = 1$, the aggregate risk is more than triples when $\lambda \approx 0.075$ ($\bar{v} = 0.573$) than in the case with $\lambda = 0$ ($\bar{v} = 0.18$). This becomes more pronounced when the dividend and supply noises are small (so that $\alpha < \bar{\alpha}$).

Moreover, quite remarkably, panel (B) shows that the risk premium $\alpha\bar{v}$ can increase in the risk aversion when the risk aversion is either low or high. This interesting phenomenon occurs because a reduction in α increases information risk faced by uninformed traders, which dominates the net effect on the risk premium when risk aversion is low and fraction of informed traders λ is small.[7]

[7]This interesting phenomenon occurs because, as seen before, when λ and α are both small the information risk component takes comparatively large values which bump up the aggregate risk $\bar{v}$. Therefore, the natural monotonically increasing behavior of risk premium as a function of α suffers

3.3. **Welfare.** We now analyze trader i's welfare and start by some preliminary results related to expected utilities conditioned on the type. Using notations as in (2.3) for $V_I(\lambda)$ and $V_U(\lambda)$, expressions for x_I^* and x_U^* as in (3.3) and (3.6) and the linear equilibrium in Proposition 3.1, we obtain the following result.

Proposition 3.3. *For given $\lambda \in [0,1]$, the welfare of each trader, conditional on his type $k \in \{I, U\}$, is given by*

$$V_k(\lambda) = -\frac{1}{\sqrt{1+\xi_k(\lambda)}}, \qquad \xi_k(\lambda) = \frac{\mathbb{V}ar\left[\mathbb{E}\left[\tilde{D} - R\tilde{P}|\mathcal{F}_k\right]\right]}{\mathbb{V}ar\left[\tilde{D} - R\tilde{P}|\mathcal{F}_k\right]}, \tag{3.11}$$

where

$$\xi_I(\lambda) = \frac{(1-b_\theta)^2 v_\theta + b_z^2 v_z}{v_\epsilon}, \qquad \xi_U(\lambda) = \frac{(1-\beta)^2(b_\theta^2 v_\theta + b_z^2 v_z)}{v_\epsilon + (1-\beta b_\theta)v_\theta}.$$

Also, $V_I(\lambda) > V_U(\lambda)$.

Proposition 3.3 shows that traders' welfare increases with the variance of the conditional expectation of excess return of the risky asset, i.e., $\tilde{D} - R\tilde{P}$, and decreases with the conditional variance of excess return. Intuitively, the ratio between the two aforementioned quantities determines the aggressiveness of the traders' portfolios and thus, their expected utilities. Proposition 3.3 also shows that, if information is costless, every trader would choose to become informed, i.e., $\lambda = 1$. However, as we show in the next corollary, having all traders to be informed does not collectively maximize their overall welfare.

Corollary 3.4. *Each trader i's expected utility in a market with only informed traders ($\lambda = 1$) and in a market with only uninformed traders ($\lambda = 0$) are given by respectively,*

$$V_I(1) = -\frac{1}{\sqrt{1+\alpha^2 v_\epsilon v_z}} \tag{3.12}$$

and

$$V_U(0) = -\frac{1}{\sqrt{1+\alpha^2(v_\epsilon + v_\theta)v_z}}, \tag{3.13}$$

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a sort of *transient bull in the risk premium*, observable only for small values of λ and in the region where α is positive but not too large.

Corollary 3.4 implies that the overall welfare of all traders is actually higher when they are all uninformed ($\lambda = 0$) than when they are all informed ($\lambda = 1$), which may seem counter-intuitive since traders are always better off to be informed than uninformed. As a textbook example, consider the case where we just have two players and, for simplicity, information is costless (players may decide their state). Fixing all the relevant parameters equal to one; $v_\epsilon = v_\theta = v_z = \alpha = 1$, we obtain $V_U(0.5) < V_I(1) < V_U(0) < V_I(0.5)$, where $\lambda = 0.5$ accounts for the situation where the two players choose a different action. Eventually, the normal-form game has a payoff matrix as in Table 1, which is evidently a Prisoner's dilemma situation.

	I	U
I	-0.70; -0.70	-0.56; -0.76
U	-0.76; -0.56	-0.57; -0.57

Table 1. Two-player payoff matrix.

The resulting Prisoner's dilemma illustrates a situation in which traders fail to coordinate towards the best outcome (represented by $V_U(0)$) and came up with a socially less preferable Nash equilibrium.

To see why $V_I(1) < V_U(0)$, note that as λ increases, price becomes more sensitive to the signal, i.e, b_θ increases. When $\lambda = 1$, we obtain that $d + \tilde{\theta} - R\tilde{P} = \alpha v_\epsilon \tilde{z}$. Therefore, the informed traders are only compensated by the risk premium since the information they receive have been fully reflected by the equilibrium price, i.e., $b_\theta = 1$. On the other hand, when $\lambda = 0$, which would be the outcome when information is extremely costly, the price is uninformative since $b_\theta = 0$. Therefore, traders are compensated by the risk premium, i.e, $d - R\tilde{P} = \alpha(v_\epsilon + v_z)\tilde{z}$, which is however larger than in the case of $\lambda = 1$, because traders perceive a larger dividend risk, thus a larger price discount is required. Furthermore, we can show that $\xi_i'(\lambda) < 0$ for $i \in \{I, U\}$ and $\xi_I'(\lambda) - \xi_U'(\lambda) < 0$, from which we have the following result on the welfare change.

Corollary 3.5. *For $\lambda \in (0, 1)$, the welfare of both informed and uninformed traders is decreasing in λ. In addition, the difference between their welfare is also decreasing* *in λ, i.e.,*

$$\frac{d}{d\lambda}V_k(\lambda) < 0, \quad k \in \{I, U\} \qquad and \qquad \frac{d}{d\lambda}\left(V_I(\lambda) - V_U(\lambda)\right) < 0. \tag{3.14}$$

Corollary 3.5 provides further insight into the result in Corollary 3.4. It implies that the welfare of both informed and uninformed traders is decreasing in the informed trading. Therefore more informed trading is harmful for traders' welfare. More interestingly, with informed trading, the welfare is reducing faster for the informed than for the uninformed, meaning that the informed trading is more harmful for the informed than for the uninformed traders. Let social welfare be defined as the weighted average of traders' welfare.[8]

$$\mathcal{U}(\lambda) = \lambda\, V_I(\lambda) + (1-\lambda)\, V_U(\lambda), \tag{3.15}$$

then we have

$$\mathcal{U}'(\lambda) = \underbrace{\left(V_I(\lambda) - V_U(\lambda)\right)}_{>0} + \underbrace{\frac{d}{d\lambda}V_U(\lambda)}_{<0} + \underbrace{\lambda\frac{d}{d\lambda}\left(V_I(\lambda) - V_U(\lambda)\right)}_{<0}. \tag{3.16}$$

Therefore, informed trading improves the average welfare only when the welfare difference between the informed and uninformed traders is large enough, which is more likely to occur when λ is small and the price is less informative. Furthermore, we can obtain the necessary and sufficient condition for the social welfare to be increasing when the informed trading level is very low ($\lambda \approx 0$).

Corollary 3.6. *In market populated by uninformed traders, i.e., $\lambda \approx 0$, the social welfare is increasing in λ, i.e., $\lim_{\lambda\to 0}\mathcal{U}'(\lambda) > 0$ if and only if*

$$1 + \frac{\alpha^2 v_z}{v_\epsilon}(v_\epsilon^2 - v_\theta^2) > \frac{1}{1+\frac{v_\theta}{v_\epsilon}}\sqrt{\left(1+\alpha^2 v_z(v_\epsilon + v_\theta)\right)\left(1 + \frac{v_\theta}{v_\epsilon} + \frac{v_z}{v_\epsilon}\alpha^2(v_\epsilon + v_\theta)^2\right)} \tag{3.17}$$

Corollary 3.6 shows that, in the case where $v_\theta/v_\epsilon = 1$, the average welfare $\mathcal{U}(\lambda)$ is more likely to increase in λ when the noise supply v_z or risk aversion α is small. In fact, in the limits of either $v_z \to 0$ or $\alpha \to 0$, condition (3.17) reduces to $1 + \frac{v_\theta}{v_\epsilon} > 1$, which always holds.

[8]In this section, we set $\mu = 0$ since the fraction of informed is exogensously given and the information game is not explicitly modeled. In Section 4 we provide a general version of the social welfare where the cost of effort $\mu\, c(p)$ is taken into account.

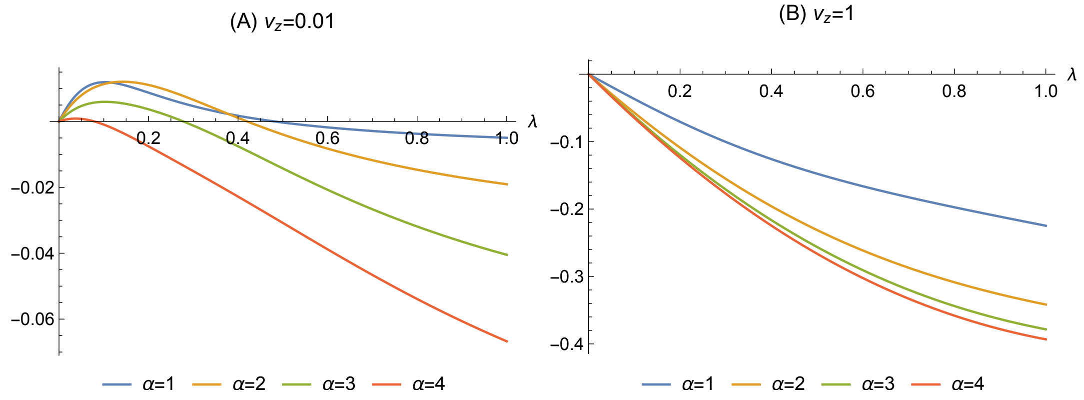

FIGURE 3.2. Percentage change in welfare $1 - \frac{\mathcal{U}(\lambda)}{\mathcal{U}(0)}$ as a function of λ. Parameter values are given by $\{v_\epsilon = 1, v_\theta = 1, v_z = 0.1^2\}$ in panel (A) and $\{v_\epsilon = 1, v_\theta = 1, v_z = 1\}$ in panel (B).

In Figure 3.2, we plot the percentage change in the welfare or the relative social welfare, define by $1 - \frac{\mathcal{U}(\lambda)}{\mathcal{U}(0)}$ (since $\mathcal{U}(\lambda) < 0$). We have the following observations. First, Panel (A) shows that when v_z is relatively small, there is initially a welfare improvement when λ is small, which gradually diminishes as λ increases and becomes negative when λ becomes large. However, the hump shape weakens when traders become more risk averse, consistent with Corollary 3.6. Second, Panel (B) shows that, when v_z is relatively large, informed trading always leads to a welfare loss, which monotonically decreases with the fraction of informed traders λ.

In summary, when λ is given exogenously, a small to moderate amount of informed trading (depending on traders' risk aversion and information structure with respect to v_θ, v_ϵ and v_z) could be welfare improving when the informed trading results in a large difference in the welfare between informed and uninformed traders, which is shown to be the case as long as the noise in supply is small enough. Otherwise, informed trading always reduces the welfare. Therefore, how informed trading affects the welfare depending on market fraction of informed traders. With the information uncertainty and disutility of the effort, the market fraction of informed traders is determined endogenously by the trade off between the utility of being informed and the disutility of the effort. This leads to the endogenous information equilibrium to be explored in the following section.

4. Endogenous Informed Trading and Social Welfare

In the previous section, we have taken λ as an exogenous parameter, i.e., a certain proportion of the traders are born as informed and the rest are born as uninformed. In this section, we allow traders to play the strategic information game, whose outcome is a Nash equilibrium p^*, corresponding to λ^*, hence the equilibrium price P^*.

4.1. **Endogenous Information Equilbrium.** As already discussed in Section 2, each trader optimally chooses a probability, $p_i \in [p_0, 1]$, to be informed (about $\tilde{\theta}$). To model the information game, we rely on the payoff structure as in (2.2), inspired by probabilistic choice models. It remains to specify the cost function c and to analyze in details the equilibrium information acquisition of the game. To this aim, we set

$$c(p) = \int_{p_0}^{p} g(t)\, dt, \tag{4.1}$$

where the marginal cost of effort, $g(t) = c'(t)$, is positive and increasing.[9] Therefore, the first order condition related to (2.2) suggests that p_i^* solves

$$g(p_i^*) = h(\lambda) \equiv \frac{1}{\mu} \left(V_I(\lambda) - V_U(\lambda) \right). \tag{4.2}$$

First of all, since $h(\lambda)$ is independent from i, at the equilibrium, we have $p_i^* = p^*$ for all $i \in (0,1)$. Therefore, the Nash equilibrium is symmetric. Moreover, at the information acquisition equilibrium,

$$\lambda^* = \int_0^1 p_i^*\, di = p^*, \tag{4.3}$$

which shows that the fraction of informed emerging at the equilibrium coincides with the probability of the (representative) agent to become informed. Therefore, we can rewrite the first order condition (4.2) in terms of the unique variable λ (from now on, we write λ in place of λ^*):

$$g(\lambda) = h(\lambda).$$

[9]Note that, $c(p_0) = 0$ meaning that, in general, even if traders make no effort, they can still become informed purely by chance with a probability of p_o.

Furthermore, since $g'(\lambda) > 0$ and $h'(\lambda) < 0$, we have a unique equilibrium λ for $p_0 < \lambda < 1$ when $g(0) < h(0)$ and $g(1) > h(1)$. In summary, the following proposition characterizes the endogenous information and noise rational expectation equilibrium.

Proposition 4.1. *With the disutility function* (4.1), *if* $g(\lambda)$ *is positive and increasing and* $h(\lambda)$ *is defined by* (4.2) *for* $\lambda \in [p_0, 1]$ *satisfying* $g(p_0) < h(p_0)$ *and* $g(1) > h(1)$, *then there exists a unique equilibrium price* $\tilde{P}$ *and equilibrium market fraction of informed traders* λ *satisfying* (3.8) *and* $g(\lambda) = h(\lambda)$, *respectively, where the parameters* b_θ *and* b_z *defined in* (3.9) *and* (3.10) *are evaluated at the equilibrium* λ.

When considering the information uncertain and disutility of the effort in a more general setup, Mattsson and Weibull (2002) show that the only rational choice for the disutility function is given by the entropic cost function,

$$c(p) = p \ln \frac{p}{p_0} + (1-p) \ln \frac{1-p}{1-p_0}, \tag{4.4}$$

which implies that the marginal cost g reads

$$g(p) = \ln \frac{p}{1-p} - \ln \frac{p_0}{1-p_0}.$$

Since $g(p_0) = 0 < h(p_0)$ and $\lim_{p \to 1} g(p) = +\infty$, all assumptions of Proposition 4.1 are met. The unique equilibrium $\lambda \in (p_0, 1)$ is determined by

$$\lambda = g^{-1}(h(\lambda)) = \frac{p_0}{p_0 + (1-p_0)e^{-h(\lambda)}},$$

which is equivalent to the discrete choice model with a choice intensity of μ (see, for instance, Anderson, de Palma and Thisse (1992)). This provides the microfoundation for the effort function (4.4) in the following analysis on the price efficiency first and then the social welfare.

4.2. **Equilibrium Informed Trading and Price Efficiency.** With the closed form endogenous information equilibrium, we can numerically analyze the implications for the equilibrium informed trading and price informativeness. To be consistent with the GS model, we examine the impact from three aspects: the noise in supply v_z; the informativeness of the signal v_θ/v_ϵ (keeping $v_\epsilon + v_\theta$ constant); and the dividend risk $v_\epsilon + v_\theta$ (keeping v_θ/v_ϵ constant). Unless specified otherwise, the analysis is based on the reference probability of becoming informed $p_0 = 0.1$ and variance parameters $v_z = 0.1^2$ and $v_\theta = v_\epsilon = 1$. Figure 4.1 plots the impact on the equilibrium fraction of informed traders, λ, and the price informativeness, $\rho \equiv \mathbb{C}orr(\tilde{P}, \tilde{\theta})$.

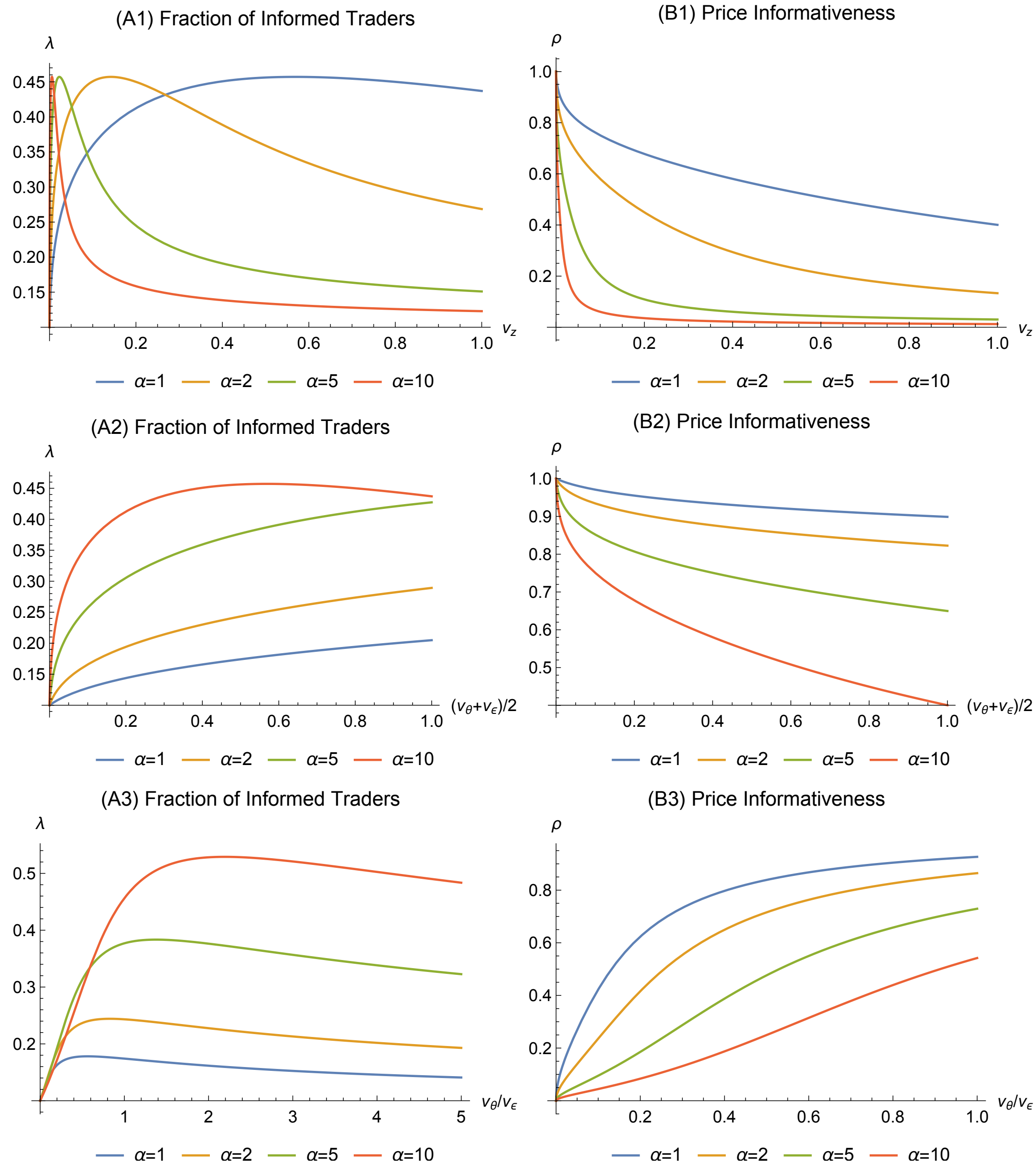

FIGURE 4.1. Impact of noise in supply v_z, dividend risk $v_\theta + v_\epsilon$ and signal informativeness v_θ/v_ϵ on fraction of informed traders λ in Panel A and price informativeness $\rho \equiv \mathbb{C}orr(\tilde{P}, \tilde{\theta})$ in Panel B.

On the effect of the supply noise, Figure 4.1 (A1) and (B1) lead to the following observations.

(i) The price informativeness $\rho = \mathbb{C}orr(\tilde{P}, \tilde{\theta})$ decreases monotonically in the supply noise v_z, illustrated in plot (B1). This implies that high noise in the supply always reduces the price efficiency.

(ii) On the equilibrium fraction of informed traders λ, in contrary to the GS model (Conjecture 5), it does not monotonically increase with the noise supply (v_z), illustrated in plot (A1), unless traders are very risk averse ($\alpha \leq 1$).[10]

(iii) When traders are less risk averse ($\alpha \leq 1$), they trade more aggressively, which increases the benefit of being informed, so traders make more effort to becoming informed. This is consistent with the GS model (Conjecture 5). Interestingly, in this case, more effort is actually reducing price efficiency, which is however inconsistent with the GS model (Conjecture 1). Therefore, when traders are less risk averse, more informed trading is harmful for price efficiency.

(iv) When the supply noise is large enough and traders are more risk averse ($\alpha > 1$), the equilibrium fraction of informed trading is decreasing in the noise supply. This result goes opposite to the GS model (Conjecture 5). Intuitively, with large supply noise, price becomes less informative. When traders are more risk averse ($\alpha > 1$), they trade less aggressively, which limits the benefit of being informed. Therefore traders make less effort to becoming informed.

(v) When the supply noise is small and traders are more risk averse, we observe a hump-shaped relation between the fraction of informed and the supply noise. This means that the information becomes more valuable (so that traders are making more effort to becoming informed) when the noise in the supply is neither too small nor too large. Intuitively, when the supply noise

[10]This is due to the fact that, in the GS model, λ is determined by the ratio $\frac{\mathbb{E}[u^*(\tilde{W}_I)|\lambda]}{\mathbb{E}[u^*(\tilde{W}_U)|\lambda]}$ rather than the difference $\mathbb{E}\left[u^*\left(\tilde{W}_I\right)|\lambda\right] - \mathbb{E}\left[u^*\left(\tilde{W}_U\right)|\lambda\right]$. As it turns out, the ratio is increasing in v_z whereas the difference follows a hump shape when traders are not too risk averse.

is too small, the price becomes more informative, reducing the informative advantage to becoming informed. On the other hand, when the supply noise is too large, the price becomes less informative, reducing trading of uninformed traders and hence the informative advantage to becoming informed. The hump shaped relationship between λ and v_z, and the reduction of the price informativeness in v_z are more pronounced when traders are more risk averse and the supply noise is very low, leading price to be less efficient even when traders make more effort. In the limit when there is no noise, traders make no effort to become informed (except with the default probability of p_0). This is consistent with the GS model (Conjecture 6)

In summary, whether traders make more effort to become informed and hence improves price efficiency depends on the size of noisy supply and traders' risk aversion. When traders are less risk averse, as the supply noise increases, traders make more effort to become informed but the price becomes less efficient. When traders are more risk averse, an increase in the supply noise at low levels leads to more effort for traders to become informed, which however does not improve the price efficiency; more generally, an increase in the supply noise at high levels always leads traders to make less effort to becoming informed, making price less efficient. Therefore informed trading improves price efficiency in general, but this is nor necessarily always the case.

On the effect of the dividend risk $v_\epsilon + v_\theta$, Figure 4.1 (A2) and (B2) lead to the following observation.

(vi) With an increase in the dividend risk, traders are making more effort to be informed but the price becomes less efficient; in contrary to the GS model (Conjecture 1). Intuitively, when dividend becomes more noisy, traders are making more effort to becoming informed, which improves the price efficiency. However, such improvement is dominated by the increasing dividend noise, resulting price to be less efficient. The effect becomes more significant when traders become more risk averse and the dividend is less noisy.

On the effect of the informativeness of the signal v_θ/v_ϵ, Figure 4.1 (A3) and (B3) illustrate the following result.

(vii) With an increase in the informativeness, traders are making more effort to be informed so that price becomes more informative. However, when the information becomes too informative for traders, their marginal effort to becoming informed is reducing, thought price efficiency still improves marginally. Intuitively, improving price efficiency and the informativeness of the information reduce the information advantage and hence traders' incentive top make effort. The effect become more significant when traders become more risk averse. More interestingly, the increase in the informed trading cannot perfectly offset the increase in the noise (in supply and in dividend), which is different from the GS model.

Overall, when the noise in the information is neither too small nor too large relative to the noise supply and the unobserved noise, the information becomes more valuable and traders make more effort to becoming informed. However more effort does not necessarily make the price to be more efficient.

4.3. **Equilibrium Social Welfare.** We now examine the impact on the social welfare $\mathcal{U}(\lambda)$ and the relative social welfare $1-\mathcal{U}(\lambda)/\mathcal{U}(p_0)$. The results are reported in Fig. 4.2 with respect to the noise in supply v_z (Panels (A1) and (B1)), the informativeness of the signal v_θ/v_ϵ (Panels (A2) and (B2)), and the dividend risk $v_\epsilon+v_\theta$ (Panels (A3) and (B3)), as in Fig. 4.1. They lead to the following results.

(viii) The social welfare increases in the supply noise and dividend noise, but decreases in the informativeness of the signal, as illustrated in the left panels of Fig. 4.2. Comparing to the right panels of Fig. 4.1, we observe a negative relation between the price efficiency and social welfare. This implies that the social welfare is always low when price becomes more efficient.

(ix) Quite remarkably, regardless the change in the social welfare $\mathcal{U}(\lambda)$, the relative social welfare is always negative with respect to the supply noise, dividend risk, and informativeness of the signal, as illustrated in the right panels in Fig. 4.2. This implies that making effort always reduces the social welfare comparing to making no effort. This is underlined by the Prisoner's dilemma discussed in the previous section, also in contrast to Figure 3.2, where the

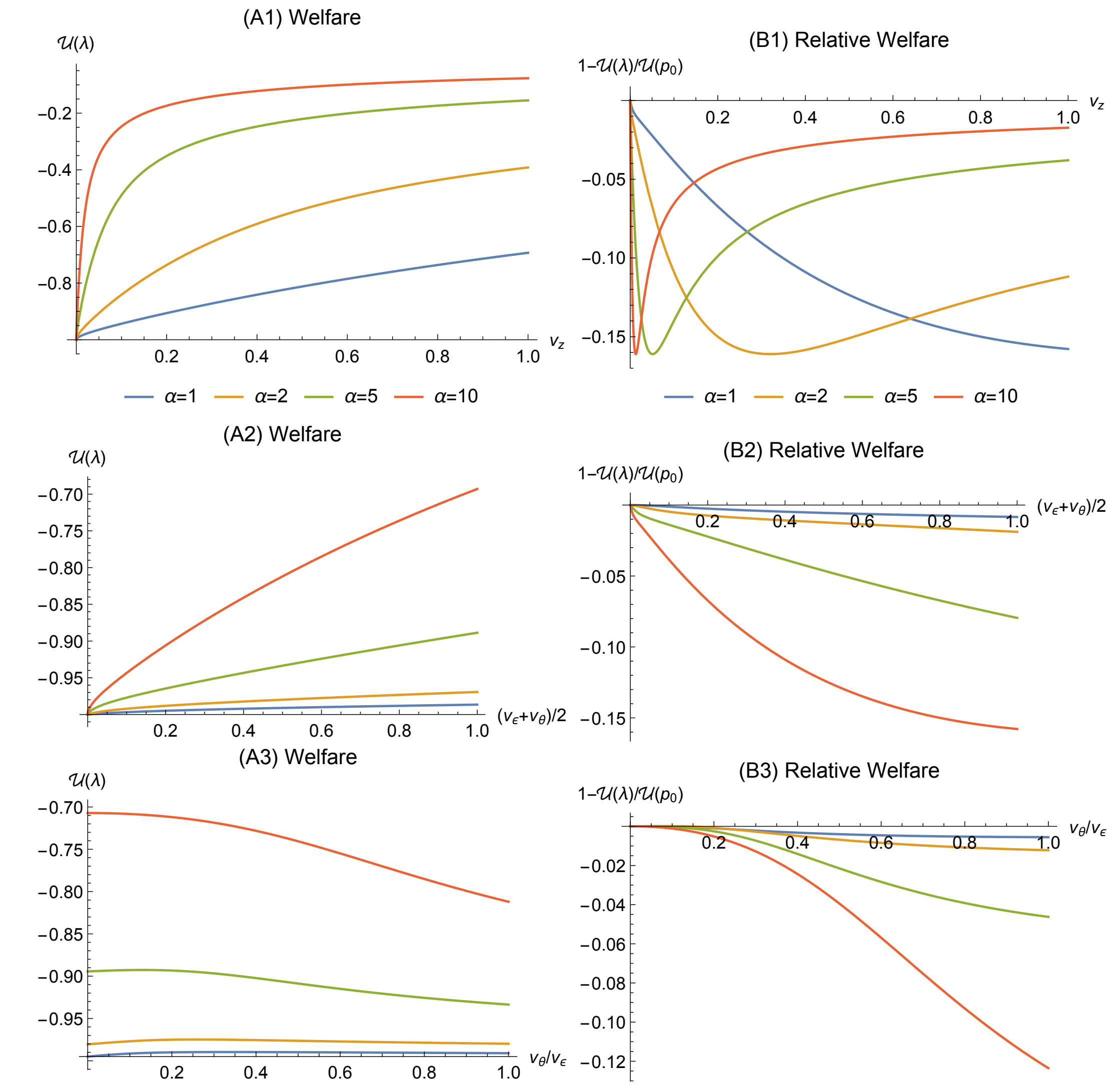

FIGURE 4.2. Impact of noise in supply v_z, dividend risk $v_\theta + v_\epsilon$ and signal informativeness v_θ/v_ϵ on traders' welfare $\mathcal{U}(\lambda)$ in Panel A and relative welfare $1 - \mathcal{U}(\lambda)/\mathcal{U}(p_0)$ in Panel B.

relative social welfare can be positive and increasing in λ when λ is given exogenously.

(x) The relative welfare decreases in the dividend noise and the informativeness of the signal, but the effect is ambiguous about the change in the supply noise, as illustrated in the right panels of Fig. 4.2. This means that the relative social welfare of making effort comparing to making no effort becomes even worse with high dividend noise and informativeness of the signal. When traders are less risk averse ($\alpha \le 1$), the relative welfare decreases in the noisy supply. However, when traders are more risk averse ($\alpha > 1$), it decreases quickly and then increases in the noisy supply. This effect becomes more significant when traders are very risk averse. Therefore, when traders are more risk averse, high noisy supply helps to improve the relative welfare of making effort, though still below the welfare of making no effort. More interestingly, comparing the right panels of Fig. 4.2 to the left panels of Fig. 4.1, we also observe a negative relation between the equilibrium fraction of informed traders and the relative social welfare. This implies that the relative social welfare is always low when traders make more effort.

Therefore, our findings suggest that improving market efficiency is always at the cost of reducing the social welfare. Also, when traders make effort to reduce their uncertainty on being informed, this is always harmful to the social welfare relative to the benchmark in which traders make no effort. Intuitively, when λ is given exogenously, an initial increase in λ from zero can improve traders' welfare if $V_I(\lambda) - V_U(\lambda)$ is large enough. However, when the disutility of the effort is considered, in equilibrium this gain in the social welfare is perfectly offset by the disutility $\mu\, c(\lambda)$. In fact, from the FOC

$$V_I(\lambda) - V_U(\lambda) = \mu c'(\lambda). \tag{4.5}$$

Thus it is no longer possible to improve welfare from $\mathcal{U}(p_0)$, which is the limiting case when $\mu \to \infty$. If we take the disutility into account and redefine the modified social welfare $\mathcal{V}(\lambda)$ as

$$\mathcal{V}(\lambda) = \mathcal{U}(\lambda) - \mu c(\lambda),$$

we can observe from (3.16) and (4.5) the following result.

Corollary 4.2. *In equilibrium, the modified social utility $\mathcal{V}(\lambda)$ is always decreasing in λ.*

Furthermore, we show in Figure 4.3 the relationship between the disutility of the effort, $\mu c(\lambda)$, for each trader, in (A), the relative welfare $1 - \mathcal{U}(\lambda)/\mathcal{U}(p_0)$ in (B) and the cost coefficient μ.[11] Based on Figure 4.3 (A), the disutility is a decreasing function of μ except when μ is very low ($\mu \approx 0$). Also more risk averse traders

[11]Being now λ endogenous, in both $c(\lambda)$ and $\mathcal{U}(\lambda)$ we are implicitly assuming that $\lambda = \lambda(\mu)$.

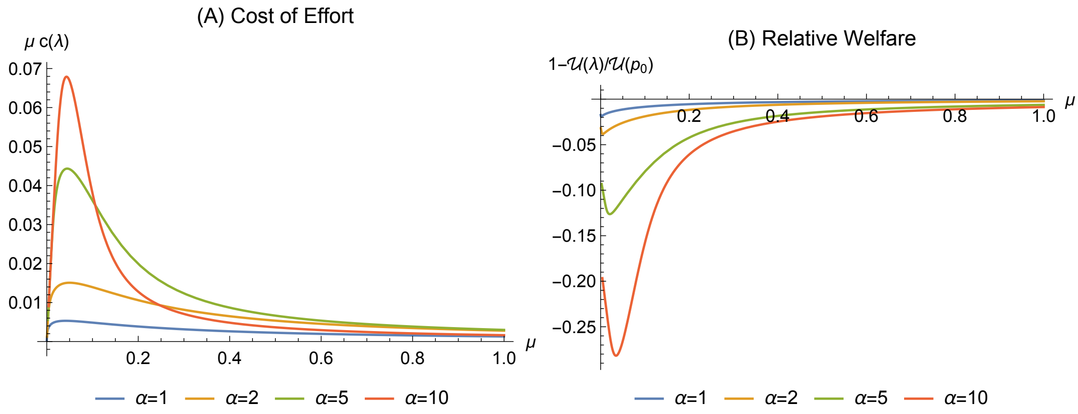

FIGURE 4.3. Impact of cost sensitivity μ on total utility cost of effort $\mu\, c(\lambda)$ in Panel A and relative welfare $1 - \mathcal{U}(\lambda)/\mathcal{U}(p_0)$ in Panel B.

tend to exert more effort to obtain information. Intuitively, traders need to make more effort when μ is relatively high. However, a higher μ also leads to a lower λ since traders optimally choose a lower p_i^*. Our finding shows that the latter effect dominates in determining $\mu\, c(\lambda)$ except when μ is close to zero. Based on Figure 4.3 (B), the relative welfare is mostly negative when traders exert most effort, i.e., when $\mu\, c(\lambda)$ is the highest. Therefore, when traders spent more effort to becoming informed, this actually makes them worse off in terms of their overall welfare.

5. Modelling Trading Motives Explicitly

In this section, rather assuming noise in supply, we follow Bond and García (2017) explicitly to motivate trading using *endowment shocks*.

Each trader i, $i \in \{I, U\}$, receives an endowment, $e_i\tilde{D}$, at the end of the trading period. Thus, trader i's terminal wealth is given by[12]

$$\tilde{W}_i = (x_i + e_i)(\tilde{D} - \tilde{P}) + e_i\tilde{P}. \tag{5.1}$$

We assume that e_i is known to trader i, but not known to other traders. Moreover $e_i = \tilde{z} + \tilde{u}_i$, where $\tilde{z} \sim \mathcal{N}(0, v_z)$ is an aggregate endowment shock and $\tilde{u}_i \sim \mathcal{N}(0, v_u)$

[12]For simplicity, we assume the the payoff of the risk-free asset, $R = 1$.

is an idiosyncratic shock. Therefore, trader i's optimal portfolio is given by

$$x_i^* = \frac{\mathbb{E}\left[\tilde{D} - \tilde{P}|\mathcal{F}_i\right]}{\alpha \mathbb{V}ar\left[\tilde{D} - \tilde{P}|\mathcal{F}_i\right]} - e_i, \tag{5.2}$$

where the information set for the informed and uninformed traders are given by $\mathcal{F}_{i\in I} = \{\theta, P, e_i\}$ and $\mathcal{F}_{i\in U} = \{P, e_i\}$ respectively. We conjecture that equilibrium price has the following form,

$$\tilde{P} = d + b_\theta \tilde{\theta} - b_z \tilde{z}. \tag{5.3}$$

If trader i is uninformed, his own endowment shock e_i provides additional information about the dividend payoff $\tilde{D}$ since e_i is positively correlated with the aggregate endowment shock $\tilde{z}$. Thus, the optimal portfolio for an uninformed trader is given by

$$x_{i\in U}^* = \frac{\left(1 - \frac{\beta_{P,D}}{1-\rho_{e_i,P}^2}\right)(d - P) - \frac{\beta_{e_i,P}\beta_{P,D}}{1-\rho_{e_i,P}^2} e_i}{\alpha\left(1 - \frac{\rho_{P,D}^2}{1-\rho_{e_i,P}^2}\right)(v_\epsilon + v_\theta)} - e_i, \tag{5.4}$$

where $\beta_{X,Y} \equiv \mathbb{C}ov[X,Y]/\mathbb{V}ar[X]$ and $\rho_{X,Y} \equiv \mathbb{C}orr[X,Y]$, and the optimal portfolio for an informed trader is given by

$$x_{i\in I}^* = \frac{d + \theta - P}{\alpha v_\epsilon} - e_i. \tag{5.5}$$

Since we assume the risky asset is in zero net supply, market clearing requires

$$\int_0^1 \lambda x_{i\in I}^* + (1-\lambda) x_{i\in U}^* di = 0,$$

where λ is the fraction of informed traders.

As before, we are interested in the impact of informed on the overall welfare of the traders. First, we consider two special cases where $\lambda = 0$ and $\lambda = 1$. We define

$$\xi_i(\lambda) \equiv \frac{\mathbb{V}ar\left[\mathbb{E}\left[\tilde{D} - \tilde{P}|\mathcal{F}_i\right]\right]}{\mathbb{V}ar\left[\tilde{D} - \tilde{P}|\mathcal{F}_i\right]}$$

as a profitability measure of trader i's portfolio, which depends on the fraction of informed traders λ. Note that for $\lambda = 0$, the equilibrium price becomes $\tilde{P} = d - \alpha(v_\theta + v_\epsilon)\tilde{z}$ and trader i's optimal portfolio is $x_{i\in U}^* = \frac{d-P}{\alpha(v_\theta+v_\epsilon)} - e_i$. On the other hand, when $\lambda = 1$, the equilibrium price and trader i's optimal portfolio are given by $\tilde{P} = d + \theta - \alpha v_\epsilon$ and $x^*_{i\in I} = \frac{d+\theta-P}{\alpha v_\epsilon} - e_i$. The following proposition characterizes traders' overall welfare.

Proposition 5.1. *The welfare of trader i is given by*

$$\mathbb{E}\left[u(\tilde{W}_{i\in U})|e_i\right] = -\exp\left\{-\alpha e_i\, d + \frac{1}{2}\frac{e_i^2}{v_z}\xi_{i\in U}(0)\right\}\left(1 + \frac{v_u}{v_z}\xi_{i\in U}(0)\right)^{-1/2} \tag{5.6}$$

when $\lambda = 0$*, where* $\xi_{i\in U}(0) = \alpha^2(v_\theta + v_\epsilon)v_z$*, and*

$$\mathbb{E}\left[u(\tilde{W}_{i\in I})|e_i\right] = -\exp\left\{-\alpha e_i\, d + \frac{1}{2}\frac{e_i^2}{v_z}\xi_{i\in U}(0)\right\}\left(1 + \frac{v_u}{v_z}\xi_{i\in I}(1)\right)^{-1/2} \tag{5.7}$$

when $\lambda = 1$*, where* $\xi_{i\in I}(1) = \alpha^2 v_\epsilon v_z$*.*

Proposition 5.1 shows that, given the same endowment shock e_i, informed trading incurs a welfare cost due to the reduction in the profitability measure ξ. Note that, when $e_i = 0$, we recover the results in Corollary 3.4.

6. Conclusion

With growing population, economy and technological innovations, we have witnessed increasing uncertainty and complexity in financial markets. Traders become more uncertain on multidimensional information and need to make some effort to reduce their uncertainty on certain information. We expect that such effort would improve traders' welfare as well as price efficiency. However such effort is associated with disutility; the more the effort, the high the disutility. In this paper we consider traders' effort to being informed and the disutility of their effort in an otherwise standard Grossman and Stiglitz (1980) model. In a baseline model with no uncertainty (and hence no effort), we show that informed trading improves prices efficiency, but reduces the social welfare in general, resulting a Prisoner's dilemma situation; the social welfare would be better off if nobody is informed but individual is rationally driven to being informed. When the disutility of the effort is taken into account, we are able to characterize the endogenous information equilibrium of the non-cooperative information game on traders' decision of making effort by a Nash equilibrium and asset price by a noisy rational expectation equilibrium. We show that making effort to be informed is harmful for social welfare and improving market efficiency is always at the cost of reducing the welfare. Therefore, with the disutility of making effort, social welfare can be improved when traders make less effort and social welfare and price efficiency cannot be improved simultaneously.

Appendix A. Proofs

Proof of Proposition 3.1: We substitute the linear equilibrium price in (3.5) into traders' optimal demand functions in (3.3) and (3.4) respectively, from which we obtain

$$x_I^*(\theta, P) = \frac{d + \theta - RP}{\alpha v_\epsilon} \tag{A.1}$$

and

$$x_U^*(P) = \frac{\left(1 - \frac{b_\theta v_\theta}{b_\theta^2 v_\theta + b_z^2 v_z}\right)(d - RP)}{\alpha \left(v_\epsilon + \frac{b_z^2 v_z}{b_\theta^2 v_\theta + b_z^2 v_z} v_\theta\right)} \tag{A.2}$$

Then, by applying the market clearing condition,

$$\lambda x_I^*(\theta, P) + (1 - \lambda) x_U^*(P) = \tilde{z}$$

we obtain the following equilibrium price,

$$\begin{aligned}\tilde{P} &= \frac{1}{R}\left[\frac{\frac{\lambda}{v_\epsilon}(d + \tilde{\theta}) + \frac{(1-\lambda)\left(1 - \frac{b_\theta v_\theta}{b_\theta^2 v_\theta + b_z^2 v_z}\right)}{v_\epsilon + \frac{b_z^2 v_z}{b_\theta^2 v_\theta + b_z^2 v_z} v_\theta} d - \alpha \tilde{z}}{\frac{\lambda}{v_\epsilon} + \frac{(1-\lambda)\left(1 - \frac{b_\theta v_\theta}{b_\theta^2 v_\theta + b_z^2 v_z}\right)}{v_\epsilon + \frac{b_z^2 v_z}{b_\theta^2 v_\theta + b_z^2 v_z} v_\theta}}\right] \\ &= \frac{1}{R}\left(d + \frac{\lambda \bar{v}}{v_\epsilon}\tilde{\theta} - \alpha \bar{v} \tilde{z}\right),\end{aligned} \tag{A.3}$$

where

$$\frac{1}{\bar{v}} = \frac{\lambda}{v_\epsilon} + \frac{(1 - \lambda)\left(1 - \frac{b_\theta v_\theta}{b_\theta^2 v_\theta + b_z^2 v_z}\right)}{v_\epsilon + \frac{b_z^2 v_z}{b_\theta^2 v_\theta + b_z^2 v_z} v_\theta}.$$

Thus, by matching coefficient to the conjectured equilibrium price in (3.5), we obtain

$$b_\theta = \frac{\lambda \bar{v}}{v_\epsilon} \qquad \text{and} \qquad b_z = \alpha \bar{v}.$$

Therefore, since $b_\theta = \lambda b_z / (\alpha v_\epsilon)$, we obtain an explicit solution for $\bar{v}$ by solving

$$\frac{\lambda}{v_\epsilon} + \frac{(1 - \lambda)\left(1 - \frac{(\lambda b_z/\alpha)\, v_\theta / v_\epsilon}{(\lambda b_z/\alpha)^2 v_\theta / v_\epsilon^2 + b_z^2 v_z}\right)}{v_\epsilon + \frac{b_z^2 v_z}{(\lambda b_z/\alpha)^2 v_\theta / v_\epsilon^2 + b_z^2 v_z} v_\theta} = \frac{b_z}{\alpha}$$

for b_z and substituting the solution back into the expression for $1/\bar{v}$.

Proof of Corollary 3.2: The risk premium per unit of supply is given by $\frac{1}{z}\mathbb{E}[\tilde{D} - R\tilde{P}|\tilde{z} = z] = \alpha\bar{v}$. For convenience, we let $\alpha_\theta = 1/v_\theta, \alpha_\epsilon = 1/v_\epsilon$ and $\alpha_z = 1/v_z$, hence (3.10) can be rewritten as

$$\frac{1}{\bar{v}} = \lambda\alpha_\epsilon + (1-\lambda)\frac{\alpha_\epsilon\alpha_\theta}{\alpha_\theta + \alpha_\epsilon + \lambda A}$$

where $A = \alpha_z\alpha_\epsilon^2/\alpha^2$. Then

$$\frac{\partial(1/\bar{v})}{\partial\lambda} = \frac{\alpha_\epsilon}{[\alpha_\theta + \alpha_\epsilon + \lambda A]^2}\{[\lambda A + \alpha_\theta + \alpha_\epsilon]^2 - [\alpha_\theta^2 + \alpha_\theta\alpha_\epsilon + \alpha_\theta A]\}. \tag{A.4}$$

Therefore $\frac{\partial(1/\bar{v})}{\partial\lambda} \geq 0$ if and only if

$$\lambda A \geq \sqrt{\alpha^2\theta + \alpha_\theta\alpha_\epsilon + \alpha_\theta A} - [\alpha_\theta + \alpha_\epsilon] := Q$$

Note that $Q \geq 0$ if and only if $1/\alpha^2 \geq (1/\alpha_z)[1/\alpha_\epsilon + 1/\alpha_\theta]$. Therefore, when $1/\alpha^2 < (1/\alpha_z)[1/\alpha_\epsilon + 1/\alpha_\theta]$, $\frac{\partial(1/\bar{v})}{\partial\lambda} > 0$, meaning that the risk premium decreases in λ. Otherwise, when $1/\alpha^2 \geq (1/\alpha_z)[1/\alpha_\epsilon + \alpha_\theta]$, the risk premium increases in λ if and only if $\lambda < Q/A$.

Proof of Proposition 3.3: First, compute trader i's expected utility given their information set $\mathcal{F}_i$, which yields (since wealth $\tilde{W}_i$ is normally distributed)

$$\begin{aligned}
\mathbb{E}\left[u(\tilde{W}_i)|\mathcal{F}_i\right] &= -\exp\left\{-\alpha\left(\mathbb{E}\left[\tilde{W}_i|\mathcal{F}_i\right] - \frac{1}{2}\alpha\mathbb{V}ar\left[\tilde{W}_i|\mathcal{F}_i\right]\right)\right\} \\
&= -\exp\left\{-\alpha\left(x_i^*\mathbb{E}\left[\tilde{D} - R\tilde{P}|\mathcal{F}_i\right] - \frac{1}{2}\alpha(x_i^*)^2\mathbb{V}ar\left[\tilde{D} - R\tilde{P}|\mathcal{F}_i\right]\right)\right\} \\
&= -\exp\left\{-\frac{1}{2}\frac{\left(\mathbb{E}\left[\tilde{D} - R\tilde{P}|\mathcal{F}_i\right]\right)^2}{\mathbb{V}ar\left[\tilde{D} - R\tilde{P}|\mathcal{F}_i\right]}\right\}.
\end{aligned} \tag{A.5}$$

For the informed,

$$\mathbb{V}ar\big[\tilde{D} - R\tilde{P}|\,\mathcal{F}_I\big] = \mathbb{V}ar\big[\tilde{D} - R\tilde{P}|\tilde{P} = P,\ \tilde{\theta} = \theta\big] = v_\epsilon, \tag{A.6}$$

and

$$\mathbb{E}[\tilde{D} - R\tilde{P}|\mathcal{F}_I] = d + \theta - RP = (1 - b_\theta)\theta - b_z z.$$

Therefore,

$$\mathbb{V}ar\big[\mathbb{E}[\tilde{D} - R\tilde{P}|\tilde{P} = P,\, \tilde{\theta} = \theta]\big] = \mathbb{V}ar\left[(1 - b_\theta)\theta - b_z z\right] = (1 - b_\theta)^2 v_\theta + b_z^2 v_z. \tag{A.7}$$

For the uninformed

$$\mathbb{V}ar\left[\tilde{D}-R\tilde{P}|\,\mathcal{F}_U\right] = \mathbb{V}ar\left[\tilde{D}-R\tilde{P}|\tilde{P}=P\right] = v_\epsilon+(1-\mathbb{C}orr[\tilde{\theta},\tilde{P}]^2)v_\theta == v_\epsilon+(1-\beta b_\theta)v_\theta,$$

and

$$\mathbb{E}[\tilde{D}-R\tilde{P}|\tilde{P}=P] = (1-\beta)(d-RP) = (1-\beta)(-b_\theta\theta - b_z z).$$

where we used the fact that $d - RP = d - (d + b_\theta\theta + b_z z)$. Therefore,

$$\mathbb{V}ar\left[\mathbb{E}[\tilde{D}-R\tilde{P}|\tilde{P}=P]\right] = \mathbb{V}ar\left[(1-\beta)(-b_\theta\theta - b_z z)\right] = (1-\beta)^2(b_\theta^2 v_\theta + b_z^2 v_z). \quad \text{(A.8)}$$

Based on the above results, we obtain the expressions for $\xi_I(\lambda)$ and $\xi_U(\lambda)$.

Next, since the conditional expectation $\mathbb{E}\left[\tilde{D}-R\tilde{P}|\mathcal{F}_i\right]$ itself is a normally distributed random variable for both informed and uninformed traders, we can use following standard result to compute trader i's unconditional expected utility.

Lemma A.1. *Let $X \in \mathbb{R}^n$ be a normally distributed random vector with mean μ and variance-covariance matrix Σ. Let $b \in \mathbb{R}^n$ be a given vector, and $A \in \mathbb{R}^{n\times n}$ a symmetric matrix. If $I - 2\Sigma A$ is positive definite, then $\mathbb{E}\left[\exp\{b^\top X + X^\top AX\}\right]$ is well defined, and given by*

$$\begin{aligned}\mathbb{E}\left[\exp\{b^\top X + X^\top AX\}\right] =& |I-2\Sigma A|^{-1/2}\exp\{b^\top\mu + \mu^\top\Sigma\mu \\ &+ \frac{1}{2}(b+2A\mu)^\top(I-2\Sigma A)^{-1}\Sigma(b+2A\mu)\}.\end{aligned}$$

Applying Lemma A.1 to the conditional expected utility in (A.5) with $X = \mathbb{E}\left[\tilde{D}-R\tilde{P}|\mathcal{F}_i\right]$, $A = -\frac{1}{2}(\mathbb{V}ar\left[\tilde{D}-R\tilde{P}|\mathcal{F}_i\right])^{-1}$, $\Sigma = \mathbb{V}ar\left[\mathbb{E}\left[\tilde{D}-R\tilde{P}|\mathcal{F}_i\right]\right]$, $b = 0$, $\mu = 0$ leads to the desired result.

It is straightforward to show that welfare is higher for the informed than for the uninformed traders, i.e., $\mathbb{E}[u^*(\tilde{W}_I)] \geq \mathbb{E}[u^*(\tilde{W}_U)]$, since by the law of total variance,

$$\mathbb{V}ar\left[\mathbb{E}\left[\tilde{D}-R\tilde{P}|\tilde{\theta},\tilde{P}\right]\right] = \mathbb{V}ar\left[\tilde{D}-R\tilde{P}\right] - \underbrace{\mathbb{E}\left[\mathbb{V}ar\left[\tilde{D}-R\tilde{P}|\tilde{\theta},\tilde{P}\right]\right]}_{v_\epsilon},$$

$$\mathbb{V}ar\left[\mathbb{E}\left[\tilde{D}-R\tilde{P}|\tilde{P}\right]\right] = \mathbb{V}ar\left[\tilde{D}-R\tilde{P}\right] - \underbrace{\mathbb{E}\left[\mathbb{V}ar\left[\tilde{D}-R\tilde{P}|\tilde{P}\right]\right]}_{v_\epsilon+\left(1-\mathbb{C}orr\left[\tilde{\theta},\tilde{P}\right]^2\right)v_\theta},$$

$$\mathbb{V}ar\left[\mathbb{E}\left[\tilde{D}-R\tilde{P}|\tilde{\theta},\tilde{P}\right]\right] - \mathbb{V}ar\left[\mathbb{E}\left[\tilde{D}-R\tilde{P}|\tilde{P}\right]\right] = \left(1-\mathbb{C}orr\left[\tilde{\theta},\tilde{P}\right]^2\right)v_\theta \geq 0.$$

Proof of Corollary 3.4: When all traders are informed, $\lambda = 1$ and the equilibrium price becomes $\tilde{P} = 1/R(d + \tilde{\theta} - \alpha v_\epsilon \tilde{z})$. Moreover, traders' conditional expectation and variance of the excess return are given by

$$\mathbb{E}\left[\tilde{D} - R\tilde{P}|\tilde{\theta}, \tilde{P}\right] = d + \theta - RP = \alpha v_\epsilon \tilde{z} \qquad \text{and} \qquad \mathbb{V}ar\left[\tilde{D} - R\tilde{P}|\tilde{\theta}, \tilde{P}\right] = v_\epsilon.$$

Substituting the above into (3.11) leads to (3.12).

On the other hand, when all traders are uninformed, i.e., $\lambda = 0$, the equilibrium price becomes $\tilde{P} = 1/R(d - \alpha(v_\theta + v_\epsilon)\tilde{z})$, traders' conditional expectation and variance of the excess return are given by

$$\mathbb{E}\left[\tilde{D} - R\tilde{P}|\tilde{P}\right] = d - RP = \alpha(v_\theta + v_\epsilon)\tilde{z} \qquad \text{and} \qquad \mathbb{V}ar\left[\tilde{D} - R\tilde{P}|\tilde{P}\right] = v_\theta + v_\epsilon.$$

Substituting the above into (3.11) leads to (3.13).

Proof of Corollary 3.6: Using the definition of the weighted average of traders' welfare in (3.15), we can obtain

$$\mathcal{U}'(0) = \frac{v_\epsilon(v_\epsilon + v_\theta) + v_z(v_\epsilon - v_\theta)(v_\epsilon + v_\theta)^2\alpha^2 - v_\epsilon^2\sqrt{\frac{(1+v_z(v_\epsilon+v_\theta)\alpha^2)(v_\epsilon+v_\theta+v_z(v_\epsilon+v_\theta)\alpha^2)}{v_\epsilon}}}{v_\epsilon(v_\epsilon + v_\theta)\left(1 + v_z(v_\epsilon + v_\theta)\alpha^2\right)^{3/2}},$$

which simplifies to the condition in (3.17).

Proof of Proposition 5.1: When $\lambda = 0$, the equilibrium price is given by $\tilde{P} = d - \alpha(v_\epsilon + v_\theta)\tilde{z}$ and trader i's optimal demand becomes

$$x^*_{i\in U} = \frac{d - P}{\alpha(v_\epsilon + v_\theta)} - e_i. \tag{A.9}$$

Thus, from (5.1), the conditional expectation and variance of trader i's terminal wealth are given by

$$\mathbb{E}\left[\tilde{W}_{i\in U}|\tilde{P}, e_i\right] = \frac{(d - P)^2}{\alpha(v_\epsilon + v_\theta)} + e_i P \qquad \text{and} \qquad \mathbb{V}ar\left[\tilde{W}_{i\in U}|\tilde{P}, e_i\right] = \frac{(d - P)^2}{\alpha^2(v_\epsilon + v_\theta)}, \tag{A.10}$$

respectively. Therefore, trader i's conditional expected utility is given by

$$\mathbb{E}\left[u\left(\tilde{W}_{i\in U}\right)|\tilde{P}, e_i\right] = -\exp\left\{-\frac{1}{2}\frac{(d - P)^2}{(v_\epsilon + v_\theta)} - \alpha\, e_i P\right\}. \tag{A.11}$$

Since $\tilde{P}$ is normally distributed, we can use Lemma A.1 to compute $\mathbb{E}\left[u\left(\tilde{W}_{i\in U}\right)|e_i\right]$, which leads to (5.6).

On the other hand, when $\lambda = 1$, price becomes $\tilde{P} = d + \tilde{\theta} - \alpha v_\epsilon \tilde{z}$ and $x^*_{i\in I} = (d+\theta-P)/(\alpha v_\epsilon) - e_i$, thus we have

$$\mathbb{E}\left[\tilde{W}_{i\in I}|\tilde{P},\tilde{\theta},e_i\right] = \frac{(d+\theta-P)^2}{\alpha v_\epsilon} + e_i P \qquad \text{and} \qquad \mathbb{V}ar\left[\tilde{W}_{i\in I}|\tilde{P},\theta,e_i\right] = \frac{(d+\theta-P)^2}{\alpha^2 v_\epsilon}, \tag{A.12}$$

and

$$\mathbb{E}\left[u\left(\tilde{W}_{i\in I}\right)|\tilde{P},\tilde{\theta},e_i\right] = -\exp\left\{-\frac{1}{2}\frac{(d+\theta-P)^2}{v_\epsilon} - \alpha\, e_i P\right\}. \tag{A.13}$$

Since $\tilde{P}$ and $\tilde{\theta}$ are both normally distributed, Lemma A.1 can help us to compute $\mathbb{E}\left[u\left(\tilde{W}_{i\in I}\right)|e_i\right]$.

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