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Four Essays on social Interaction Models

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Ciclo: 30

Titolo della tesi[1] : Four essays on social interaction models.

Abstract:

This PhD dissertation explores the effects of social interactions on individual's behaviour (at the micro level) and the collective equilibrium emerging (at the macro level). The first chapter empirically explores the role that *social interactions* have on natives' attitudes towards migrants in Latin American countries. The second chapter studies the effects that multiple equilibria, due to social interactions, have in the context of duopoly competition. The theoretical model formalizes a two-stage Bertrand economy where firms simultaneously choose optimal prices, and then potential adopters decide upon their preferred technology.

The last part of this thesis consist of two chapters containing two independent extensions of the model developed in Chapter 2. In Chapter 3 firms are allowed to decide their level of differentiation (location). Therefore, on the first stage firm's decide upon their differentiation, then their price, and finally consumers decide between the two products. Finally, Chapter 4 focuses on the demand side of the market where an outside option (not buying) is offered to the consumers.

Firma dello studente

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[1] Il titolo deve essere quello definitivo, uguale a quello che risulta stampato sulla copertina dell'elaborato consegnato.

Four Essays on Social Interaction Models

Jorge Yépez

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Abstract

This PhD dissertation explores the effects of *social interactions* on individual's behaviour (at the micro level) and the collective equilibrium emerging (at the macro level). *Social interactions* refer to situations where the payoff or utility an individual receives from a given action depends directly on the choices of others.

The first chapter explores the role that *social interactions* have on natives' attitudes towards migrants in Latin American countries. In addition, the effects of other economic motives are also explored. The results found evidence that conformity behaviour is a strong determinant of natives views towards migrants.

The second chapter studies the effects that multiple equilibria, due to social interactions, have in the context of duopoly competition. We model agents with heterogeneous preferences who decide to buy from one of the two firms on the market. The firms are differentiated by the strength of their *social recognition*. In this sense, we speak about *firm-specific network effect*: the firm with the higher parameter is perceived by consumers to have a higher social recognition (exert higher social pressure). The model formalizes a two-stage Bertrand economy where firms simultaneously choose optimal prices, and then potential adopters decide upon their preferred technology. The market equilibrium confirms intuition: the firm with the higher network effect exerts a higher social pressure on potential adopters and obtains a higher market share. Surprisingly, this outcome is not always observed: an excessive network effect may be detrimental for the stronger firm, which, eventually, can be thrown out of the market with a positive probability. This is due to the reinforcing effect of consumers' behaviour that could upset the competition in favour of the apparently weaker firm.

The last part of this thesis consist of two chapters containing two independent extensions of the model developed in Chapter 2. In Chapter 3 firms are allowed to decide their level of differentiation (location). Therefore, on the first stage firm's decide upon their differentiation, then their price, and finally consumers decide between the two products. For low levels of social interactions, we show that it is optimal for the firms to be highly differentiated. On the opposite, for high levels of social interactions, the other situation applies: firms offer the same standard product, although in this case, only one firm survives and monopolizes the market.

Finally, Chapter 4 focuses on the demand side of the market where an outside

option (not buying) is offered to the consumers. Moreover, we benefit from this generalization to show an application of our model to the smartphone industry. Here we associate the concept of brand awareness to the definition of *firm-specific network effects*. As a case study, the model is calibrated with real data from the smartphone industry obtaining an estimate of the value of the brand awareness of two dominant brands.

Contents

Introduction

Social norms, tipping points, multiple equilibria and social traps are some features related to complex systems that make *social interaction* models very appealing for economic and social research, especially because they are useful to explain phenomena that are difficult to capture with standard economic models. These features characterize the essence of this PhD dissertation. Indeed, the main point that has motivated this thesis is to propose a framework where *social interactions* affect the emergent equilibria in social and economic domains.

An interest in the role of *social interactions* in economic decision making has been growing over the years. Pioneering studies are Schelling (1971) segregation model, Granovetter (1978) threshold model of collective behaviour, or Bass (1969) model on diffusion on innovation that incorporate *social interactions* on individuals decision making. In general, discrete choice models with *social interactions* refer to scenarios where the payoff function of a given agent takes into account the choices of other agents. These models assume that social decisions are not simple choices based primarily on individual considerations, but also take into account the collective behaviour, such as social norms, peer effects, or network effects. Quoting Akerlof (1997), *"There is a significant difference between these social decisions and the conventional economic decision-making epitomized in intermediate microeconomic theory as choices among alternative fruits available at the supermarket"*.

The objective of *social interaction* models is to analyze how group (macro) behaviour emerges from the interdependence across individuals' (micro) behaviour. When externalities are in place, the decision of each single agent is influenced by others' decisions; in turn, at the aggregate level, each single decision impacts on the global outcomes of the economy. For instance, conformity effects[1] have been studied for different situations such as students achievement (Alcalde, 2014), voting behaviour (Li and Lee, 2009), smoking (Soetevent and Kooreman, 2007), criminal activity (Glaeser et al., 1996) and several others.

A second feature of *social interaction* models is the probability of generating cascade effects (or tipping points) and sticky behaviour. At one extreme we have cascade effects that could dramatically change the emerging equi-

[1] These externalities can also influence agents' choices when people try to distance themselves from the society, a behaviour often called status seeking or vanity behaviour. See Grilo et al. (2001) for an example of conformity behaviour and vanity behaviour on a duopoly model.

librium. The reason is that when an individual changes his behaviour, this could induce, via the social effect, more individuals to reassess their choices and change their behaviour, which affects others' behaviour too and so on. This cascading effect suggests that a moderate perturbation of the model's parameters may result in a large change in the aggregate behaviour (Granovetter, 1978). In contrast, when the social effect is dominant, it would require a substantial shock to create a non-trivial change in the collective behaviour. These implications are of great interest for economic outcomes, because they make it difficult to predict the effect of a shock in the parameters of the model. An economic policy could generate a change on the collective behaviour via cascade effects or simply have no effect at all because of the sticky behaviour of individuals. More importantly, this stickiness could generate *social traps*, which is the attitude of individuals to conform to the social norms that could generate a resilient behaviour yielding suboptimal outcomes, such as unwillingness to work (Lindbeck et al., 1997), harmful political opinions (Brock, 2003), or low growth levels (Marsiglio and Tolotti, 2016).

Mathematically speaking, the cascade effects and the presence of multiple equilibria are often related to the presence of a "bifurcation" in the parameters diagram. This means that there exist thresholds in the level of the parameters of the model where the behaviour of the system abruptly changes[2]. A typical example is the case where a system exhibits a unique or multiple equilibria depending on the value of (some) parameters. This transition from a unique to multiple equilibria makes *social interaction* models very appealing in economic research. Intuitively, this characteristic implies that it is possible to observe groups of people, which are otherwise identical, behaving in different ways.

Characteristics such as conformity, multiple equilibria and social traps, are some of the typical traits related to *social interactions* that make this topic interesting. Indeed, the core of this PhD dissertation is the analysis of social and economic environments in which the individual choice is socially influenced. More precisely, the interest of this thesis is twofold: to empirically determine the influence of conformity on a social context such as the attitudes of natives towards migrants; and to theoretically analyse the impact of social effects on a market context such as a duopoly competition model.

To meet this challenge, one of the main analytical frameworks used in this thesis was developed by Brock and Durlauf (2001b, 2003) and Blume and

[2]See for example Phan and Semeshenko (2008) that summarized on a phase diagram the regions of qualitatively different collective behaviour as a function of the model parameters.

Durlauf (2002). The main feature of this framework is that it extends a random utility model (RUM)[3], which is well known in the economic literature, to explicitly account for *social interactions* in the context of a discrete choice model. The model outlines a payoff function with three main features. Firstly, by using a RUM the authors are able to describe a society composed by individuals with different preferences, and these tastes are modelled by a stochastic term in the payoff function. Secondly, each action has a defined characteristic, which is well known by all agents, a so called public utility. Thus, the payoff of each individual also includes a term homogeneous across agents (for example in the context of a discrete choice among different goods on the market, each product poses a different price which is well known by consumers). Finally, the agent's payoff function explicitly incorporates the choices of others and a parameter measuring the importance of this social component on the payoff. The combination of these characteristics provides the decisional model with a description of individual's behaviour under social influence, as well as the interdependence across individuals' actions.

Several authors have used and extended Brock and Durlauf (2001b) model to empirically estimate the role of *social interactions* in everyday life scenarios, such as teenagers behaviour (Soetevent and Kooreman, 2007, Lin, 2014), recycling attitudes (Kipperberg, 2005) or movies consumption (Masood, 2015). The first chapter of this thesis relies on this type of analysis; more precisely, the chapter uses a binary choice model to study the role that *social interactions* have on natives' attitudes towards migrants in a set of Latin American countries. As a benchmark, the analysis includes standard economic variables. For example, the ratio of skilled to unskilled labour in the native to the immigrant labour force that is well studied by several authors such as Mayda (2006), Facchini and Mayda (2009, 2012), O'Rourke and Sinnott (2004). They conclude that the more skilled natives to migrants are, the higher the support of skilled natives to migrants. The results of this chapter reveals that conformist behaviour is a strong determinant of natives' views on migrants. Moreover, this social norm is more important than the economic motives.

Conformity is crucial in order to understand the actions taken by individuals and the emerging collective outcome. Moreover, several authors[4] show that under positive social externalities (i.e. conformist behaviour), multiple equilibria emerge. The presence of extremely different equilibria contributes

[3]See Anderson et al. (1992) for an extended description of random utility models and some of its applications on economic theory.

[4]See for example Brock and Durlauf (2001b), Kalai (2004), Phan and Semeshenko (2008).

to exploring why some social systems may get stuck in social traps. In other words, the presence of *social interactions* could generate an equilibrium that is not socially optimal. The appearance of multiple equilibria, in the context of duopoly competition, is studied in the second chapter. This chapter presents a Bertrand economy, where two firms compete on prices, while a large population of potential adopters decide which one of the two goods to buy. The model describes this heterogeneous population of consumers relying on social interaction, similar to Anderson et al. (1992) and Grilo et al. (2001). This chapter also relies on the literature devoted to the study of *large populations* of heterogeneous agents, by assuming that the size of the population increases to infinity, such as Kalai (2004). In this situation, it is easier to obtain closed form solutions and characterize the equilibria of the economy, which makes easier to analyse the results obtained. More precisely, the model of the second chapter of this thesis formalizes a two-stage game in which, firstly, firms simultaneously choose optimal prices and, secondly, a large population of potential adopters decide upon the preferred technology. In addition, the role of the strength of the *social interactions* is extended to be firm-specific. For example, suppose that the two firms are characterized by a different level of *network strength*; then, *ceteris paribus*, the firm with higher *firm-specific network effect* experiences a higher social recognition/pressure across consumers, and therefore consumers have a higher incentive to conform and buy from this firm. Under low levels of social interaction, the firm with the higher *network effect* exerts a higher social pressure on potential adopters. Surprisingly, this outcome is not the only possibility: an excessive *network effect* may be detrimental for the strongest firm, which, eventually, can be thrown out of the market with a positive probability. Indeed, the aforementioned bad equilibrium resembles the social traps discussed before. This is due to the appearance multiple equilibria related to the reinforcing behaviour effect.

Finally, Chapters 3 and 4 are extensions of the model presented on Chapter 2. In Chapter 3, goods characteristics and consumers preferences are represented by points in an interval. In addition, firms are able to optimize their location on the interval; in other words, they are able to decide their level of product differentiation. This is known in the literature as a *Location model*, introduced by Hotelling (1929), and extended by d'Aspremont et al. (1979). The introduction of this differentiation approach was motivated by the fact that similar products are desired by their attributes that form the basis for consumer preferences (Anderson et al., 1992). The novelty of this chapter is that it introduces *firm-specific network effects*. The model is formalized by a game which incorporates one stage more than the previous chapter. On the first stage firms decide upon their level of differentiation, then their price, and finally consumers decide between the two products. The results show that under low *network effects* levels, firms differentiate as much as possible from each other, attending different niches of the market. Therefore, under low *network effects*, this model is equivalent to the one described on Chapter 2. However, when *network effects* are strong, firms converge to offer a single standard product and only one firm monopolizes the market. Although this result may appear counter-intuitive, it is in line with the literature of political competition, in which two different ideologies converge to serve the median voter (Downs, 1957).

Finally, Chapter 4 extends the model presented in Chapter 2 by modelling the demand side of the market where an outside (not-buying) option is offered to the consumers. In addition, the concept of brand awareness is introduced and associated to the definition of *firm-specific network effects*. This association is an interpretation given by the authors[5], based on management literature in which a strong brand awareness and positive brand image result in customers bringing others' attention to the brand; therefore, attracting new customers via social influences (Chung et al., 2008, McColl and Moore, 2011, O'Cass and Siahtiri, 2013, Virvilaite et al., 2015). As a case study, the model is calibrated with real data from the smartphone industry obtaining an estimate of the value of the brand awareness of two leading brands.

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1 Discrete model for social choices: an application to attitudes toward migration in Latin America

Jorge Yépez

Abstract

This article explores the effects that the relative skills composition of natives to migrants, and the diversity of migration, have on natives attitude towards migrants. We also study the role that *social interactions* have on individuals' attitudes. To our knowledge, this conformity effect has not been yet analysed on the international migration literature. We used a barely explored dataset from 13 Latin American countries. We find evidence both supporting economic motives and social conformity effects. Precisely, the more skilled natives relative to migrants are, the higher the support of skilled natives to migrants. Also the higher the diversity of migrants, the more pro-migration skilled natives are. We also find a strong significance for the social norms effect: conformist behaviour is a stronger determinant than economic motives when natives form their views on migrants.

1.1 Introduction

Which factors contribute in shaping attitudes towards migrants? Economic literature has extensively described rational economic reasons as the main motives that influence individuals attitudes towards migration. According to the *Labour Market Hypothesis* people take into account the impact of migration on their relative wage, therefore, a highly skilled migrant labour force would hurt highly skilled natives wages (see O'Rourke and Sinnott (2004) and Facchini and Mayda (2012)). On the other hand, the *Diversity Complementarity Hypothesis* has found empirical evidence that a diverse labour force might increase productivity of skilled workers in areas related to creativity and research (see Alesina et al. (2016)). Thus, skilled natives will favour diverse migration. Finally, fear of taxation has also being explored on the economic literature. It has been shown that skilled natives would oppose low skilled migration, in a taxation system where high income individuals tax burn increases with low income migrants (see Facchini and Mayda (2008)).

We also analyse the potential role of social interactions and conformity on individual's attitudes. To this aim we use Brock and Durlauf (2001b) approach to explicitly take into account social interactions in the context of discrete choices. In line with the literature in this field, we assume that individuals are influenced by the (expected) average country attitude towards migrants. To our knowledge, although, several authors have studied the effect of social interaction on different social phenomena, this is the first attempt to explore this effect on attitudes towards migration.

This chapter addresses these different arguments relying on Latin American census data from 13 countries. This is important because most of the empirical literature on this field relies on developed economies data, such as OECDs or the U.S.. To our knowledge, this is the first attempt to explore available but barely used Latin American data.

We find that the more skilled natives to migrants are, the higher the support of skilled natives to migrants. We also find that the higher the diversity of migrants, the more pro-migration skilled natives are. Our results show a statistical significance for the social interaction effect: conformity behaviour may be a stronger determinant than classical economic motives when natives form their attitudes towards migrants.

The rest of the chapter is organized as follows: Section 1.2 discusses the determinants of migration attitudes and social interaction effects. Section 1.3 describes the dataset (1.3.1) and the empirical analysis in place (1.3.2). Section 1.4 shows the econometric results, section 1.4.2 addresses endogeneity while section 1.4.3 describes the elasticities and non linearities of the model. Finally, section 1.5 concludes. Tables and figures are displayed in the Appendix.

1.2 Determinants on attitudes towards migration

Individual attitudes towards migrants are affected by different economic and non-economic factors. Economists assume that individuals are rational utility maximizers. Thus, people takes into account the impact of migration on their utility function in order to take a part, against or in favour, of the arrival of immigrants on their homeland. According to the *Labour Market Hypothesis*, the main economic factors that shape individual's preferences is the impact of foreigners on wages. Standard economic trade models, such as the Heckscher-Ohlin model (HOM) or the factor proportions model (FPM) are often used to analyse the impact of immigration on wages. The main intuition behind both aforementioned models is that immigration affects relative wages of skilled and unskilled workers through changes in the relative supply of the labour force[1]. The FPM describes two economies distinguished only by their endowment of primary factors of production which can be classified into skilled and unskilled labour force. These primary factors of production are combined to produce a single good, according to constant returns to scale. Other things being equal, the relative wage of skilled workers will be lower in the country where skilled workers are abundant, compared to a country where skilled workers are scarce. We can say that

$$(w_s/w_u)^R < (w_s/w_u)^P,$$

where w_s and w_u denote the wages of skilled and unskilled workers respectively, while R and P denote the country with skilled abundant labour (*Rich*) and unskilled abundant labour (*Poor*). The prediction of the model about who are the winners and losers of the migration process is straight forward. If immigrants are on average more skilled than natives, a positive migration flow will decrease the wages of skilled natives, while benefit unskilled natives wages. On the other hand, if immigrants are on average less skilled than natives, their arrival will impact negatively on the wages of unskilled workers. Therefore, if migrants are on average less (more) skilled than natives in the *Rich* (*Poor*) country, the skilled workers should be pro (against) migration, while unskilled workers should favour (be against) migration on the *Poor* (*Rich*).

Prediction 1 *According to the Labour Market Hypothesis, the impact of skills attitudes towards migrants is related to the relative skilled composition*

[1]A HOM assumes a diversified production, where 2 goods are traded. Each good is produced by a mix of the 2 primary production factors, but one is intensive on on skilled work, while the other is intensive on unskilled work. Each country's specialization depends on the initial endowments of production factors, the national technology and the world output prices. Therefore the difference on production factors endowments determine the wages for skilled and unskilled workers. If the country produces both goods and the immigration shock is small, changes in factor supplies are absorbed through the reallocation of factors across sectors (Rybczynski effects), and wages remain unchanged. If (i) the country only produces one good, (ii) the immigration shock is big enough, or (iii) the country is small enough, so that the magnitude of the immigration shock is sufficiently large, the wages of the labour force change due to a shock on the labour force produced by a positive flow of migrant workers. For a more detailed discussion see Mayda (2006).

of natives to migrants.

Mayda (2006) used the following index:

$$RSC = Log\left(1 + \frac{\frac{n_s}{n_u}}{\frac{m_s}{m_u}}\right). \tag{1}$$

RSC is a logarithmic transformation of the ratio of skilled (s) to unskilled (u) labour in the native (n) to the immigrant (m) labour force. Therefore, skilled natives are prone to be in favour (against) of migration the higher (lower) the RSC index is. An interaction term between individual skills and RSC should enter with a positive sigh in a regression explaining pro-migration sentiment.

Facchini and Mayda (2009) and Mayda (2006), showed a positive relation between positive attitudes towards migration on skilled natives and the RSC index using two data sources, the International Social Survey Programme (ISSP) and the World Value Survey (WVS). Facchini and Mayda (2012) showed also a positive relation between skills and positive attitudes towards high skilled migrants using the European Social Survey (ESS). O'Rourke and Sinnott (2004), tested the *Labour market hypothesis* assuming that high skilled workers migrate from rich to poor countries and low skilled workers migrate from poor to rich countries. They predicted that migration hurts skilled wages and benefit unskilled on poor countries. Their study shows a negative relation between anti-migrants attitudes and the interaction term between GDP per capita and a skill individual indicator measured by education level attended by the individual. The authors used a cross-section analysis with individual data from ISSP.

A strong assumption about the *Labour Market Hypothesis* is that natives and immigrants can be seen as perfect substitutes on the labour market. This assumption has received some criticisms from recent empirical works. Ottaviano and Peri (2012), using a general equilibrium model, find that natives and immigrants are imperfect substitutes. Moreover, skilled natives experience a positive increase on their wage according to U.S. data. Similarly, Card (2009) finds lower degrees of substitution between skilled natives and skilled immigrants using regional U.S data. Likewise, recent contributions show that a diverse labour force has a positive effect on productivity. The *Diversity Complementarity Hypothesis* claims that, although migrants and natives are in some degree substitutes, there are also positive spillovers on productivity that can be translated into an increase on wages for those natives who work closely with immigrants. This positive spillover is linked to complementarity among workers from different origins. People from different cultures approach to problems differently. When a group of people with different backgrounds work together, they may come up with different and more creative solutions; see Page (2007). In addition, firm's expansion may be constrained by local native scarcity of specialised labour; in this case, recruitment from abroad may be essential; see Beaverstock and Hall (2012). Of course, diverse group of people may also come with a cost, such as communication problems. So the benefits and costs of diversity can generate either a positive or a negative impact, depending on the composition of the group. Recent empirical research has shown a positive impact of diversity on sectors with highest share of skilled natives, notably on innovation-intensive industries. Alesina et al. (2016) showed a positive relation between birthplace diversity of skilled immigrants and economic development at a macro level. Trax et al. (2012) also found positive spillovers effects from the regional diversification of the workforce, especially for plants in technology intensive industries; the authors used data for German establishments. Using longitudinal data on Dutch firms, Ozgen et al. (2013) found that diversity among firm's foreign workers is positively associated with innovation activity especially in knowledge intensive sectors; although, firms that employ fewer foreign workers are generally more innovative.

Prediction 2 *According to the Diversity Complementarity Hypothesis, the highest the diversity of migrant workers the highest the impact on productivity, specially on high skilled labour force.*

Therefore, skilled natives will be in favour of migration when the diversity migrants is high. Similarly to Alesina et al. (2016), Trax et al. (2012), we use the Herfindahl Index (HHI) to messure the diversity among migrants.

$$HHI_g = 1 - \sum_{k=1}^{k} sm_{k,g}^2 \tag{2}$$

where $sm_{k,g}$ is the share of workers from nation k among all foreign workers in the hosting nation g.

An interaction term between individual skills and migrants diversity should have a positive sign in a regression explaining pro-migration sentiment.

If we consider that foreign workers both contribute to and benefit from the welfare state, immigration may also effect public finances through the redistributive policies. This effect should impact on people's attitudes towards migration. The aggregate net effect on the welfare state could be positive or negative, depending on the socio-economic status of immigrants relative to natives. Facchini and Mayda (2008) consider a simple redistributive system[2], in which government relocates resources from high-income to low-income individuals. A country can set its tax system in two ways. On the one hand, under a system called tax adjustment model (TAM), migration can produce changes in the tax rate, while per capita benefits remain constant. In this scenario, rich natives will pay higher taxes if immigration increases the number welfare recipients while poor people remain unaffected. On the other hand, the taxation system may variate the per capita benefits transferred, while tax rates are unchanged, this may happens under the so-called benefit adjustment model (BAM). If taxes remain the same, rich people will not be affected if the migrants increase the net number of welfare recipients, but poor people may need to split the fixed amount of transfers among more recipients. Facchini and Mayda (2008) and Facchini and Mayda (2012) find some evidence for the tax-adjustment model using a cross country dataset on OECD countries.

Prediction 3 *Under a tax adjustment model (TAM) individuals with high pre-tax income should be less supportive of immigration. While under the benefit adjustment model (BAM) individuals with low pre-tax income may be less supportive to migrants.*

Therefore, an interaction variable between income and RSC must show a negative coefficient under the TAM, and a positive or non significant coefficient under the BAM. Unfortunately, we don't have an income variable in our data; however, we will use socio-economic status as a proxy of income.

Literature on social interactions shows that when facing a decision that would have social consequences, herding behaviour, conformity and homophily generally impact individuals decisions. For example, early applications of social interactions include analyses of patterns of residential segregation (see Schelling (1971)), household income (see Becker (1974)), riots (see Granovetter (1978)), social space (see Akerlof (1997)) and crime (see Glaeser et al. (1996)). Social behaviour could generate both social and poverty traps, where the attitude of individuals to conform to the social norm could deviate collective behaviour to an inferior sub optimal equilibrium.

On their seminal paper Brock and Durlauf (2001b) extend a standard Random Utility Model to explicitly account for social interactions in the context of discrete choice models. A binary choice model á la Brock and Durlauf

[2] All income sources are taxed at the same rate and all individuals in the economy are entitled to an equal lump sum per capita benefit.

considers a large population of N individuals in which the social interaction specification is global, so individuals are seen as being influenced by the (expected) average group behaviour. Individuals, indexed by i, choose an alternative w, where $w \in \{-1, 1\}$. The utility $V_i(w)$ is assumed to be additively separable in the three components: (i) a vector of observables X_i and unobservable ϵ_i individual specific characteristics; (ii) a vector of group specific characteristics y_g; (iii) and the subjective expectation of the individual about the average choice $\bar{m}^e_{i,g}$ of the social group to which the decision maker belongs. Given these assumptions the individual choice is the result of the comparison between the payoffs the two choices $V_i(1)$ and $V_i(-1)$, representing being pro-migration and against migration respectively. Following Blume et al. (2010) the difference in the payoffs is given by:

$$V_i(1) - V_i(-1) = k + cX_i + dy_g + J\bar{m}^e_{i,g} - \epsilon_i, \tag{3}$$

where k, c, d, and J are suitable regression parameters to be estimated. It is worth noting that if the social interaction parameter, J, is zero, social interaction is not significant and the decision making process does not depend on the behaviour of others; hence, the model becomes a standard Random Utility Model without social interaction. On the other hand, a positive social interaction parameter ($J > 0$) implies preferences for conformity[3].

The decision problem is given by individual i choosing $w = 1$ if and only if $V_i(1) - V_i(-1) > 0$. Assuming that $(\epsilon_{ij})_{i,j}$ is an i.i.d sequence of random variables[4], the probability of the event $\{w = 1\}$ is given by:

$$\begin{aligned} P(V_i(1) - V_i(-1) \geq 0) &= P(\epsilon_i \leq k + cX_i + dy_g + J\bar{m}^e_{i,g}) \\ &= F(k + cX_i + dy_g + J\bar{m}^e_{i,g}), \end{aligned} \tag{4}$$

[3]On the other hand, $J < 0$ would mean that the higher is the fraction of the population in favour of immigration the higher is the probability that individual i is against immigration. However, if this applies to all individuals, as it can be interpreted if we empirically find a negative coefficient, thus the majority would be against migration. Therefore, preference for non-conformity would imply that individual i would be against being against immigration. In order to avoid this problem, conformity to the social norm ($J > 0$) is assumed and tested, as this is the standard case of interest in the social interaction literature (Brock and Durlauf, 2001b). Therefore, in this framework a negative coefficient, $J < 0$, cannot be interpreted as preference for not-conformity, but as not significant social interaction.

[4]The model make two assumptions on noise components: (i) the expected value of ϵ_i is independent of observable features of the individual and any features of his group and (ii) any pair i and j of the errors are conditionally independent within and across groups.

where $F(\cdot)$ is the cumulative distribution of ϵ. The model is closed by imposing an equilibrium condition on individual beliefs. Each person is assumed to know y_g, F, and $F_{X|g}$ the empirical within-group distribution of X_i.[5] This specification is equivalent to assume *rational expectation*, in particular, by assuming common knowledge of (i) the choice-rule (including the preference weights) and (ii) the distribution of factors that affect private utility among group members. Given this information, when the population size is large, equilibrium requires that the subjective individual expectation $m^e_{i,g}$ coincides with the mathematical expectation of the average choice of the group $\bar{m}_g$.

$$m^e_{i,g} = \bar{m}_g = 2 \int F(k + cX_i + dy_g + J\bar{m}_g) dF_{X|g} - 1. \tag{5}$$

Generally, there is no closed form solution for m_g given that it m_g appears on both sides of the non-linear equation (5). However, as seen in Brock and Durlauf (2001b), using Brower's fixed point theorem, at least one solution exists. Multiple equilibria arise when the social interaction parameter is positive and sufficiently large.

Several authors have extended Brock and Durlauf (2001b) model to estimate discrete choice models with social interactions to study social dynamics in everyday life. For example, Soetevent and Kooreman (2007) investigate several types of high school teenagers behaviour such as smoking. Lin (2014) studied the peer influences in adolescents' deviant behaviours, including drinking alcohol, skipping school and physical fighting. Li and Lee (2009) studied voting behaviour from the 1996 US presidential election. Kipperberg (2005) provides an econometric analysis of the determinants of consumer recycling. Masood (2015) investigates the influence of social interactions on the taste of local versus foreign films on the French population. Although widely used in different subjects, social interactions have not been estimated for describing individual attitudes towards migration. We rely on a binary choice social interaction model in our study.

Prediction 4 *Given herding behaviour or preferences for conformity, we expect a significant positive social interaction parameter J. Therefore, the more pro-migration a country is on average, the higher the probability an individual will express a pro-migration view.*

[5] The mathematical expectation of equation (5) is taken over a distribution function $F_{X|g}$, which means that each agent is assumed to condition the probabilities of the individual characteristics in a group on the aggregates which determine his payoffs.

1.3 Data and empirical model

1.3.1 Data

The analysis is based on the *Latinobarometro* conducted by the Coorporation Latinobarometro[6]. This yearly survey has being taken since 1995 and include 17 Latin American countries[7]. From this survey, we draw the following question to construct the dependent variable: *"What impact, if any, do the citizens of other countries who come to live here have on your country? They come to compete for our jobs"* 1 "Strongly Agree", 2 "Agree", 3 "Neither Agree or Disagree", 4 "Disagree" and 5 "Strongly Disagree". Besides the five ordered answers, the survey allows for "Don't Know / Not Answer". According to van Dijk (2015), a phrase like "migrants come to compete for our jobs" is a clear separation between us (natives) from them (migrants), which appeals essentially to various types of threat, such as income/money, jobs or welfare [8]. Moreover, this phrase is adequate to test the LMH, because, natives will be against migration the higher is the competition with migrants at the job market. We also used the following question, which is coded as the one described above, as a robustness check, *"There ought to be laws to prevent immigrants from entering into (country)"*. Both questions were asked only on the Latinobarometro waves 2002, 2009 and 2015. We also took from the survey questions about age, gender, educational level attained, socio-economic status, political orientation, level of trust, national pride, occupational status, urban/rural residence, marital status and crime victimization.

As a second source, we rely on Latin American census data to extract bilateral data of migration by skill category and birthplace country. We used

[6]With the financial support of the European Union, IDB (Inter Americal Development Bank), UNDP (United Nations Development Programme), AECI (Agencia Espaola de Cooperacin Internacional), SIDA (Swedish International Development Cooperation Agency), CIDA (Canadian International Development Agency), CAF (Development Bank of Latin America), OEA (Organization of American States), United States Office of Research, IDEA Internacional, UK Data Archive.

[7]Argentina, Bolivia, Brazil, Chile, Colombia, Costa Rica, Ecuador, El Salvador, Guatemala, Honduras, Mexico, Nicaragua, Panama, Paraguay, Peru, Uruguay and Venezuela

[8]According to van Dijk (2015), the semantics of discourse is not limited to the meanings of isolated words and sentences, but also on the ways such meanings are combined with each other. "Thus, immigration may be translated as a perceived threat to "our" territory, with arguments such as: these immigrants may be presented as taking away our jobs, and cheap immigrant labour may be denounced as unfair competition for the wages of ordinary native workers" (Van Dijk. 1991)

the "Data Recovery for Small Areas by Microcomputer" (REDATAM by its acronym in Spanish), which is an on-line system that facilitates processing, analysing and the dissemination of information of Latin American census. The system was created by the Latin American and Caribbean Demographic Center (CELADE), the Economic Commission for Latin America and the Caribbean (ECLAC) and the United Nations (UN). We extract data from the following census, Argentina (2001, 2010), Bolivia (2001, 2012), Chile (2002), Colombia (2005), Dominican Republic (2012), Ecuador (2001, 2010), Honduras (2001, 2013), Nicaragua (2005), Panama (2001, 2010), Paraguay (2002), El Salvador (2007), Uruguay (2011), Venezuela (2001, 2011). To extract the number of skilled and unskilled workers, we identify the labour population with people aged 24 or more. We harmonize the data in order to define the population of skilled an unskilled. We identify a skilled worker a person with at least some tertiary education, while unskilled workers are individuals that have reached inferior levels of education[9].

1.3.2 Empirical specification

We aim to investigate the predictions stated on Section 1.2. We explore the Labour Market Hypothesis, in which the relative skilled composition of natives to migrants plays an important role on the attitudes natives take towards migration. We also test the role of diversity and socio-economic condition, as well as the social interaction effect. According to the previous discussion, the choice of an individual attitude towards migration can be approximated by the following function:

$$\begin{aligned} Y_i^* &= V_i(1) - V_i(0) \\ &= k + \beta_1 RSC_g \cdot Skills_i + \beta_2 HHI_g \cdot Skills_i + J\bar{m}_g + C \cdot X_i + D \cdot y_g + \theta t - \epsilon_i \end{aligned} \tag{6}$$

[9]The questions about the maximum level of education attended is not harmonized among Latin American census, which includes different terminologies for each country. Therefore in order to differentiate the tertiary level of education from the secondary level (high-school), we had to revise the different categories included on the census forms. We define skilled worker every individual with tertiary level of education, which includes the following categories: tertiary, superior, technical (formation center, institutional or superior), military or police degree, specialization, school-teaching (or normal), bachelor, university, master, doctorate and post-university level. The rest of the labour force is classified into unskilled labour force, and includes the population that has not attended any formal education or have reached any education level not described above, such as literary center, preschool, primary or secondary.

where i indicates the individual agent, g the country. Y_i^* is the underlying continuous (or unobservable) variable with a zero threshold.

$$w_i = \begin{cases} 0, & \text{if } Y_i^* \leq 0 \\ 1, & \text{if } Y_i^* > 0 \end{cases} \tag{7}$$

The *Pro.Migration* variable w takes the value 0 if individuals agree or strongly agree with the statement that citizens of other countries compete with their jobs (or there ought to be laws to prevent immigrants from entering into their country) and 1 if individuals disagree or strongly disagree. We can interpret w_i as a dummy being 1 if the individual has chosen to express a positive attitude towards migration, and 0 if the person has chosen a negative attitude. Therefore, we have performed a change of variables ($k = (w_i + 1)/2$) in order to shift the support of the individual decisions from $\{-1, 1\}$ to $\{0, 1\}$ (see Brock and Durlauf (2001b)). This allows us to interpret $\bar{m}_g$ as the proportion of individuals with *Pro.migration* attitudes. Assuming that the error term ϵ in equation (6) follows a normal distribution (logistic distribution), the choice of the agent can be modelled using a binomial choice model such as probit (or logit) model:

$$\begin{aligned} P(Y_i = 1) &= P(Pro.Migration = 1) \\ &= P(V_i(1) - V_i(0) > 0) \\ &= P(X\beta > \epsilon_i) \\ &= F(X\beta), \end{aligned} \tag{8}$$

where F is the standard cumulative normal distribution (cumulative logistic distribution).

RSC_g and HHI_g are calculated as described on section 1.2 in equations (1) and (2), respectively. Both variables were taken from the national census data. The individual skills variable, $Skills_i$, is coded 1 if the respondent has reached some higher education, and 0 of the individual has lower education levels. Latinobarometro also provides individual-level measures of demographic, socio-economic and political variables described on the equation above by the vector X_i. Although the survey does not provide the wage of the respondent, there is a question answered by the interviewer containing a score from 1 (very high) to 5 (very low), to the following statement. *"Assessment of the interviewee's socio-economic level. Take as reference the quality of dwelling, quality of furniture and the interviewee general appearance"*. We use this question, reverting the scores, as a measure of socio-economic level. We also construct a measure of household assets, summing up a group of

dummies that report if the household posses the following items: own home, computer, washing machine, car, hot running water and sewage system. This property variable goes from 0 to 6, being 0 if the household does not have any mentioned item, and 6 if the household possesses all items. Of course, the two variables are positively correlated but far from being perfect correlated; the correlation between socio-economic status and property items, is 0.43. Among the socio-economic variables we include dummies of occupational status, such as: employed, unemployed, retired, housewife and student.
Among the demographic variables we include age, gender, urban residence (city > 50.000 habitants) and marital status, including dummies for married (or living with partner), separated (or divorced or widow/er) and single. We also include a variable related to life satisfaction, in which individuals were asked to describe how satisfied they feel with their life, being 1 "Not at all" to 4 "Very satisfied". We also include non-economic determinants of individual attitudes such as pride, crime victimization, trust and political orientation. We include victimization variable (dummy coded 1 if the respondent or some relative has been a victim of a crime on the past 12 months) to explore what Mayda (2006) indicates as a security concern related to the perception that immigrants are more likely than natives to be involved in criminal activity[10]. We also include a dummy variable of political orientation to the right-wing, which has being associated with negative views towards foreigners (see Facchini and Mayda (2009), Facchini and Mayda (2008), Facchini and Mayda (2012)). Likewise, national pride is well explored by Mayda (2006) and O'Rourke and Sinnott (2004), thus, we use a variable from national pride (0 if "Not at all or a little proud " and 1 if "Proud or very proud"). We explore the possibility that anti-immigrant attitudes may be related to nationalistic and patriotic feelings that include a sense of national superiority or the belief that the host country's culture may be endangered by the arrival of foreigners. Immigration may feed cultural and national-identity worries, and/or may just be the effect of the antipathy towards anything different from the host country identity such as cultural and racial intolerance. We also include a dummy of trust in people to reflect the level of social trust. Table 1.B.1 shows some summary statistics from the variables taken from Latinobarometro. The following indicators are limited to the set of individuals in our sample who are not migrants, for the countries and years for which we have all available data. The first row shows the dependent variable, $Pro.Migration$, coded from 1 to 5. It shows that on average public opinion does not support migration. The second row recodes $Pro.Migration$ into a

[10]To disentangle whether this perception is due to an objective situation or just due to racist or intolerant feelings is out of the scope of our study.

dummy variable, as it was described above. In the third row $Pro.Migration$ is a dummy variable, but this time we include people without an opinion ("neither agree nor disagree") into the supporting groups which is coded 1. On average, our sample group consists of not skilled, mid-socio-economic status, employed, married and resident of an urban area. On average, the levels of life satisfaction is 2.8 over 4, the level of trust is low (only 22% respond that "one can trust most people"), 40% of individuals were victims of crime or know that some relative was victim of crime during the past 12 months, and 67% are proud of their nationality.

Following the social interaction literature, see Section 1.2, we assume that the individual attitudes is in part driven by herding behaviour, by aspiration of being part of a group or social recognition. Therefore, individuals preferences are influenced by the preferences of the collectivity, which justifies the inclusion of other's choices in equation (6) with the term $\bar{m}_g$.

$$\bar{m}_g = \frac{1}{n_g} \sum_j w_j \tag{9}$$

Where n_g is the size of the group, and w_j the preference choice of individual j in group g. Thus, $\bar{m}_g$ is defined as the proportion of individuals in country g that has a positive attitude towards migrants.

As originally recognized by Manski (1993) and further analysed in Brock and Durlauf (2001a), a difficulty in the estimation of social effects is the reflection problem, which refers to the impossibility of disentangling the endogenous effect ($\bar{m}_g$) to the contextual effect (y_g). The first effect is when an individual's outcome (e.g. attitudes toward migration) is affected by his or her peers' outcomes (e.g. attitudes toward migration), while the so-called contextual effect refers to the influence of peers characteristics (e.g. social background). The endogenous effect, which is what we are trying to estimate here, gives rise to the possibility of social multipliers, while contextual effect does not. The contextual effect arises when individuals display similar behaviour simply due to their shared contextual environment. As a result, these two types of social effects have different policy implications. Manski (1993) showed that for linear models, the collinearity between contextual and endogenous effects, which can be due to self-consistent beliefs, can induce non identification problems. However, in contrast to the linear case, identification problems do not hold for our model. Brock and Durlauf (2001a) showed that this problem is not present in the binary choice case[11], because the binary choice models is inherently non-linear in the relationship among control variables such as $\bar{m}_g$ and y_g. Since choice probabilities are bounded, contextual effects and endogenous effects (which are non linear) cannot be linearly dependent. According to Brock and Durlauf (2001a), we require three things: (i) it is necessary that the data contain sufficient intra-group variation within at least one group to ensure that C can be identified; (ii) there must be enough inter-group variation in y_g to ensure that D and J are identified; (iii) there cannot be collinearity between the regressors contained in X_i and y_g , so that individual and contextual effects may be distinguished.

Firstly, the Latinobarometro survey provides enough observations (around 1000) for every country, which ensures enough intra-group variation. Secondly, we include as covariates the average of every variable in X_i for every group, which gives us enough inter-group variation. Finally, the last point may arise through endogenous group formation, causing group members to behave similarly even in the absence of social effect. Failing to control for confounding effect will lead to spurious estimate of peer effects. The above specification does not completely solve this problem; however, by including in the model variables that are both individual and region specific, we should lower the impact of omitted variable problem. We also believe that group formation might not be a serious problem, because we limit our sample to non-migrant respondents[12]; so we can assume that natives won't migrate because of the national average attitudes towards migration [13]. Table 1.B.2 shows summary statistics[14] taken from the census data and the national average $Pro.Migration$ computed from the Latinobarometro. Uruguayans have on average more positive attitudes towards migration, while Chileans have on

[11] See Appendix.

[12] We used the TRUST question, "How proud are you to be (nationality)? Are you very proud, fairly proud, a little proud, or not proud at all?" where the answer choose was "I'm not (nationality)". Also, on the 2009 survey, individuals were asked "Are you a citizen of (country)?.". With this two questions we were able to differentiate locals from migrants.

[13] Groups (in this case countries) should not be formed because other members (in this case other natives) like or dislike migrants. If natives migrate for this reason, then countries will be against migrants because the locals who like migrants abandon their country and migrate to a pro migrant nation. Hence, countries that have a high fraction of natives against migrants can be explained because the other natives, the ones who like migrants, already migrate. This could create reverse causality problems. Therefore, we assume that natives don't emigrate because of the average attitudes toward migrants. They can migrate for other reasons (unemployment, poverty, crime, etc.) of course, but not because the other natives are against immigrants.

[14] For the countries that we have two years (*), the information presented on the table is the bi-annual average, although the regression used separate.

average less positive preferences. Argentina shows higher Relative Skilled Coefficient of natives to migrants, while Honduras and Colombia have the lowest index. Colombia also shows the most diverse migrants labour force, although the share of migrants is the lowest within our sample, while Venezuela and Argentina host more migrants than the other countries on our sample. The summary statistics also provide information on the per capita GDP which comes from the World Bank International Comparison Program data, and the GINI index taken from CEPALstats (the statistical information collected, systematized and published by Economic Commission for Latin America and the Caribbean).

1.4 Results

1.4.1 Main results

Using the dummy *Pro.Migration* as dependent variable, we estimate equation (6) using a probit model with robust standard errors clustered by country, to account for correlation of individual observations within a country[15]. Table 1 shows the main interesting covariates, the set of regressors is completed with the demographic variables on Table 1.B.4 and the contextual variables on Table 1.B.5. All tables are on the Appendix.

In Column (1), we include our covariates of the *Labour Market* and *Diversity Complementarity Hypothesis*, which are the interaction between the dummy variable of Skilled worker and RSC, as well as the interaction between Skilled worker and HHI. Both coefficients are positive as it was expected. Although skilled workers show a negative coefficient, this negative effect is reduced when RSC is high. Similarly, consistent with the *Diversity Complementarity Hypothesis*, educated individuals show more pro-immigration attitudes if the diversity of migrants is high. The hypothesis claims that migrants and natives are, in some degree, complements on the labour force, so skilled natives may benefit from the positive spillovers on productivity that can be translated into an increase of wages. We have also included the total share of migrant workers[16] as control, and the interaction between skilled and the total share of skilled migrant workers. We also included socio-demographic variables,

[15]Using cluster command in STATA, the standard errors are allowed for intra-group correlation. That is, the observations are independent across groups (clusters) but not necessarily within groups, relaxing the usual requirement that the observations must be independent. This affects the standard errors and variance-covariance matrix of the estimators but not the estimated coefficients.

[16]Individuals over 24 years old.

displayed on Table 1.B.4, although none of these variables are significant. Column (2) includes non-economic variables as covariates, such as right-wing political orientation, trust, life satisfaction and crime victimization. However, none of these variables were significant, neither their collective contribution was significant (according a F-Test performed).

In column (3) we include socio-economic status and the interaction of socio-economic status and RSC. Perhaps, due to our lack of a precise income variable or in support to the BAM system, we don't find evidence supporting an income effect of the TAM through our socio-economic variable. However, the coefficient of the interaction between Skilled and RSC increased its level of significance when the socio-economic variable is included. Facchini and Mayda (2012) highlight the importance of incorporate variables of education and income at the same time on the set of regressors in order to avoid omitted variable bias. On the one hand, individual's education can be related to our dependent variable through the *Labour market* channel. The higher the education the more pro-migrant the individual will be if the RSC is high, due to the positive effect on relative wages, as we discuss in section 1.2. On the other hand, income will have the opposite effect under a TAM system, because rich natives will be paying more taxes for poor migrants welfare. Since, the correlation between income and education should be positive, the coefficient of the interaction variable between Skilled and RSC will be downward biased due to the omission of income as a covariate. This result is consistent with the *Labour Market Hypothesis:* the higher the relative skill composition of natives, the smaller the relative supply of skilled to unskilled labour in the destination economy, the higher the skilled wage. The coefficient of skills is negative but the interaction term with RSC is positive. Educated individuals are more likely to be pro-immigration if the latter variable is above a given threshold. The relationship between the individual skill level and pro-immigration attitudes indeed depends on the RSC.

In column (4) we investigate the impact of the social interaction. As expected the coefficient is positive and significant. The contextual effects, on Table 1.B.5, of age, gender, employment, crime-victimization and political orientation are also significant, although the coefficients of the latter two are counter-intuitive. It is worth noting that the inclusion of this set of contextual variables and the social effect variable improved the model considerable by their level of significance and the R^2 level of the regression. Column (5) confirm our results, with the same set of covariates using a Logit model. Finally, in column (6) the dependent variable include people who *"neither agree or disagree"* about migrants taking their jobs. Their answers were coded into $Y = 1$. It is worth noting that our main variables, such as skilled, the interaction between skilled and RSC, the interaction between skilled and HHI and the social interaction coefficient J, remain stable over the estimations presented.

Column (1) in Table 1.B.6 shows the same model that was presented in column (4) of Table 1. The demographic and contextual covariates are not shown; although, they were also employed. Column (2) and (3) limit the observations to the set of individuals that are economical active and not active respectively. Column (2) which takes into account only employed and unemployed individuals, shows similar coefficients to column (1), although the threshold of RSC is shift. This is due to the fact that the coefficient of skilled is more strongly negative, while the interaction between skilled and RSC is slightly higher. Similarly, the effect of diversity is stronger for skilled workers. Column (3) which only includes students, housewives (and house-husbands) and retired people, shows no significance on our interest variables. Our results confirm the *Labour Market Hypothesis* and the *Diversity Complementarity Hypothesis*, therefore individuals out of the labour force are not affected by the diversity or the relative skills composition of workers and the total share of migration plays a more important role by the magnitude of its coefficient. Column (4) includes country fixed effects. As expected, the significance of many contextual variables were considerable lowered. What is important to notice is that the coefficients of the interest variables remain unchanged. Finally, column (5) includes the household asset variable (property) instead of the socio-economic variable. Although the *Labour Market* and *Diversity Complementarity* variables remain stable, the coefficient of household assets is not significant.

Table 1.B.7 reports the same set of covariates as shown in table 1.B.6, but the dependent variables change to a dummy equal to 1 if the individual disagrees with the statement *"There ought to be laws to prevent immigrants from entering into (country)."*. Although this variable represents an attitude towards migration, it is not as explicit on the labour market as the one we used before. The social interaction coefficient remains significant and positive as it was expected. The interaction variable between RSC and Skilled is positive and significant only on column (2) where the sample is limited to individuals on the labour force, and on column (5) where the household asset replaces the socio-economic variable. Our diversity variable interacting with the level of skills is also positive, although the level of significance is lower.

1.4.2 Instrumental variable

If from one side, we can assume that natives do not set their level of education according to their attitudes to migration, assuming that individual's attitudes do not affect the skills composition of immigration might be too strong. Migration and attitudes could be jointly determined by unobserved country level variables. Moreover, migrants can choose or avoid a host country because of the attitudes of natives towards migrants of that country. In other words, migrants might prefer to go to migrant friendly countries. This will create a reverse causation problem. Thus, we address this problem instrumenting the relative skills composition indicator (RSC).

On a HOM and the FPM model, labour migration flows may be driven by differences on the rate of return of skill labour. Therefore, in a country with a higher (lower) per capita GDP, skilled workers may experience lower (higher) returns for their labour (lower (higher) relative wages) and unskilled workers may experience higher (lower) returns, than in countries with lower per-capita GDP. Therefore, migration of skilled workers will flow from high per-capita GDP countries to low per capita GDP countries and migration of unskilled workers will go from low per-capita GDP countries to high per-capita GDP countries. We assume that taking into account our set of individuals and contextual covariates, which includes variables of individual and aggregate socio-economic level, the per-capita GDP will explain the country RSC level but not the individual's attitudes towards migration. Therefore, we use the interaction between our dummy of skills and the country per capita GDP as an instrument variable of the interaction variable of skills and RSC. The per-capita GDP (constant 2010 US$) was taken by the World Development Indicators provided by the World Bank International Comparison Program[17].

Similarly, Borjas (1987) explained the different migration flows through the level of income inequality of the receiving country. On highly unequal countries, skilled workers enjoy a higher wage than in equal countries. Therefore, skilled workers migrate to highly unequal countries (positive self-selection), while unskilled migrants migrate to highly equal countries (negative self-selection). We assume that the country level of inequality explains the RSC but not the individual attitudes to migration. Thus, we use the interaction between skilled and country Gini index as a second instrument of the interaction term of skilled and RSC. The Gini index taken from CEPALstats (the statistical information collected, systematized and published by Economic

[17]http://data.worldbank.org

Commission for Latin America and the Caribbean). It's important to mention that not all the countries have the GINI index it the national level, but only at urban level (Argentina and Uruguay). In addition, some countries don't have the index for the required years, so we average the index if the missing year is between two reported years, or extrapolate the closest year if the missing year is followed by another missing year (Chile), to fill the missing years.

We used a structural approach that explicitly models both the non-linearity of our probit model and the endogeneity[18] (see p. 265 on Cameron and Trivedi (2010)). The endogenous regressors $E_{i,g}$ ($Skilled \cdot RSC$) is modelled as linear to our exogenous variables and the instruments $Z_{i,g}$ ($Skilled{\cdot}GDPpc$ and $Skilled. \cdot Gini$).

$$Y_i^* = k + \beta_1 E_{i,g} + J\bar{m}_g + C \cdot X_i + D \cdot y_g + \theta t - \epsilon_i \tag{10}$$

$$E_i = k + \pi_1 Z_{i,g} + \pi_2 \bar{m}_g + \pi_3 \cdot X_i + \pi_4 \cdot y_g + \pi_5 t + u_i \tag{11}$$

Table 1.B.8 shows the coefficients of our main interest variables. Column (1) uses a Probit model which will be used as a benchmark from our Instrumented Variable (IV) Probit structural model. Column (2) uses $Skilled \cdot GDPpc$ as an instrument, while column (3) uses $Skilled \cdot Gini$. Column (4) uses both variables as instruments. The coefficient of $Skilled{\cdot}GDPpc$ on the first stage is positive and significant as it was expected, although the size is considerably small. In column (3), counter-intuitively the first stage shows a negative coefficient of $Skilled \cdot Gini$. This could be due to the inconsistency that the Gini index present, that we mentioned before, which limits the usefulness of this index. When we use both instruments, none of the two are significant on the first stage. The significance of our main interest variable $Skilled \cdot RSC$ is affected on the model presented on column (3), but the size and level of significance is practically invariant in the structural model and the standard Probit model presented on columns (1), (2) and (4). Moreover, according to the Wald test on the null hypothesis of exogeneity of $Skilled \cdot RSC$, we don't find evidence to reject exogeneity of our main interest variable. Therefore, those unmeasured factors that make pro-migration attitudes and RSC jointly determined are not supported by our model. Thus, we don't find enough evidence to justify the use an instrument variable for correcting possible biased coefficient for the *Labour Market Hypothesis*.

[18] The structural model assumes that ϵ_i and u_i are jointly normally distributed.

1.4.3 Marginal effects

Table 1.B.9 shows the coefficients of a Probit model similar to the one of Table 1 column (4), on the first column, and the respective elasticities on the second column, computed at the average. Yet, it is interesting to see how the effects of these variables may vary according to the value of other characteristics. Figure 1.1 shows the predicted probability of being pro-migrant for skilled workers (at means values), for different levels of RSC and $\bar{m}_g$ (left panel), and the predicted probability of being pro-migrant for skilled workers (at means values), for different levels of HHI (right panel).

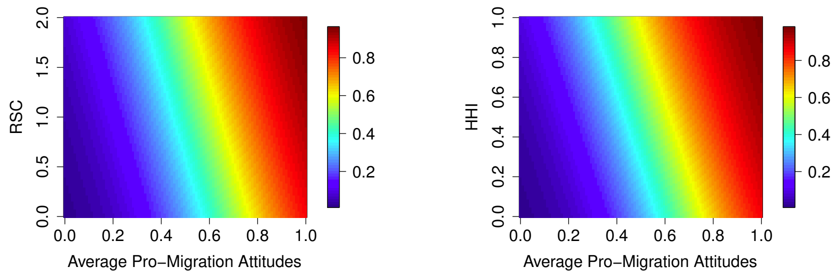


Figure 1.1: Predicted model for skilled workers at means values, at different levels of $\bar{m}_g$ and RSC(left panel) and HHI (right panel)

We can infer by the graph, that social interaction greatly impacts on attitudes, compared to the rational economic variables such as RSC or HHI. When fixing social interaction at a low or high level, the other variables barely affect the probability of the individual to choose a positive attitude toward migrants. Meanwhile, fixing social interaction at medium levels, the effect of the economic variables becomes more important. Figure 1.2 describes this relationship by showing the marginal effects of RSC at different levels of $\bar{m}_g$. These effects are present because of the strong social interaction behaviour and the non-linearities present in our model.

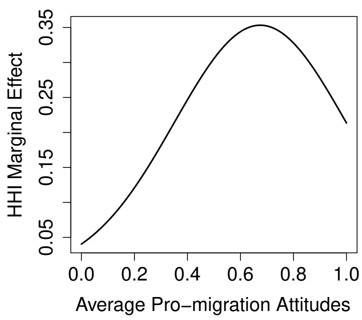


Figure 1.2: Marginal Effect of RSC for skilled workers, at different levels of $\bar{m}_g$

Finally, Figure 1.3 shows the predicted model at different levels of $Pro.MigrationAverage$, at the average values for the other set of covariates. The 45^o line crosses our cumulative distribution at different points, which implies the possibility of multiple equilibria. These points will exert attraction on natives point of view, leading to social traps, in favour or against migration at extreme levels. Due to our low model explanatory level ($R^2 = 0.05$)[19], the graph could only be taken as an illustrative example of the reference equilibria points at $P(\bar{w}) = \bar{w} = \{0.11, 0.43, 0.92\}$.

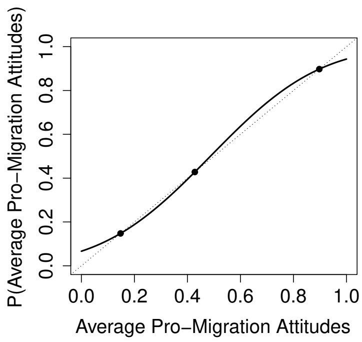


Figure 1.3: Predicted model at means values, for different levels of $\bar{m}_g$

[19]Low levels of R^2 are common across this literature. For example, O'Rourke ad Sinnot O'Rourke and Sinnott (2004) report a R^2 around 0.08, Facchini and Mayda Facchini and Mayda (2012) report on average 0.01, Mayda Mayda (2006) using the WVS show an R^2 around 0.07.

1.5 Conclusion

The economic literature has extensively recognized rational economic reasons as the main motives that influence individuals attitudes towards migrants. We analysed the effect of the skill and the variety of migration, following the *Labour Market* and the *Diversity Complementarity Hypothesis*. Supporting the former hypothesis, we find that the higher the relative skills of natives to migrants, the higher the support of skilled natives to migration. We also find that the higher the diversity of migrants, the more pro-migration skilled natives are, which supports the latter hypothesis. We didn't find evidence that attitudes towards migration can be affected by the welfare system transfers to migrants through the TAM model, although our lack of a direct income variable might bias our results. We used Latin American data from 13 countries, integrating census data at a macro level, with the Latinobarometro survey at the individual level. To our knowledge, this is the first attempt to explore data different from OECD or U.S. sources for constructing macro indicators related to this stream of literature.

We also explored non-economic factors, especially the potential role of social interactions in decision-making. We demonstrated that conformity or herding behaviour influence individuals attitudes to migration. We used a model à la Brock and Durlauf to explicitly take into account social interactions in the context of discrete choices. Individuals are seen as being influenced by the (expected) average country attitude towards migrants. Although, several authors have extended Brock and Durlauf's (2001) model to estimate social interactions in everyday life, these effects had not yet applied to explore attitudes to migration. Our results find a strong support for this effect. Moreover, conformity behaviour may be a stronger determinant of attitudes to migrants than classical economic motives.

Appendix

1.A Theorem

Blume et al. (2010) pg.67, emphasizes in the following theorem the sufficient conditions, not necessary, that allows for the identification of the parameters on a discrete choice model.

Theorem 1 *Under the following assumptions:*

1. *conditional on* (X_i, y_g) *the random payoff terms* ϵ_i *are i.i.d according to* $F(\cdot)$ *and* $F(0) = 0.5$;
2. $F(\cdot)$ *is absolutely continuous with associated density* $dF(\cdot)$, $dF(\cdot)$ *is positive almost everywhere on its support, the interval (L,U), which may be* $(-\infty, \infty)$*;*
3. *for at least one group g, conditional on* y_g*, each element of the vector* X_i *varies continuously over all R and supp(*X_i*) is not contained in a proper linear subspace of* R^R*;*
4. y_g *does not include a constant; each element of* y_g *varies continuously over all R; at least one element of D is non-zero; and supp(*y_g*) is not contained in a proper linear subspace of* R^S*;*

Then k, D, C*, and* J *are identified up to scale.*

Within a given group, $Dy_g + Jm_g$ are constant for all agents. Assumptions 1) - 3) are sufficient to ensure that within that group, C and $F(\cdot)$ are identified. Assumption 4) ensures that k, D, and J are identified up to scale. The reason why D and J are identified is that the unbounded support on the y_g element with a non-zero coefficient ensures that m_g and y_g cannot be linearly dependent. This follows from the fact that m_g is bounded.

1.B Tables

Table 1.B.1: Descriptive Statistics: Average individual variables

Variable	Mean	StdDev	Min	Max	Obs
Pro-mig	2.74	1.18	1	5	18400
Pro-mig (Dummy 1)	0.40	0.49	0	1	15016
Pro-mig (Dummy 2)	0.51	0.50	0	1	18400
Skilled	0.16	0.37	0	1	19435
Socio-economic status	3.29	0.92	1.00	5	19435
Employed	0.53	0.50	0	1	19435
Unemployed	0.08	0.28	0	1	19435
Retired	0.07	0.26	0	1	19435
Househusband/wife	0.24	0.43	0	1	19435
Student	0.08	0.26	0	1	19435
Urban	0.63	0.48	0	1	19435
Male	0.48	0.50	0	1	19435
Age	39.65	16.44	16	98	19435
Married or Cohabit	0.56	0.50	0	1	19268
Single	0.32	0.47	0	1	19268
Divorced/Separated/Widow	0.12	0.32	0	1	19268
Life Satisfaction	2.88	0.88	1	4	19326
Crime Victimization	0.40	0.49	0	1	19146
Trust	0.22	0.41	0	1	18806
Right-wing Pol. Orientation	0.42	0.49	0	1	14714
National Proud	0.67	0.47	0	1	19324

Table 1.B.2: Descriptive Statistics: Country level variables. Countries(*) show the average values from the two periods (2002, 2009).

Country	Pro-Mig	RSC	HHI	Share Migrants	Share Skilled Migrants	GDP per capita	GINI
Argentina*	0.37	1.01	0.86	0.065	0.042	8723	0.54
Bolivia*	0.39	0.21	0.90	0.014	0.038	1624	0.61
Chile	0.27	0.22	0.89	0.014	0.034	10133	0.55
Colombia	0.55	0.15	0.91	0.003	0.008	4826	0.57
Dominican Republic	0.28	0.72	0.43	0.037	0.035	5092	0.57
Ecuador*	0.33	0.33	0.70	0.014	0.027	4210	0.52
El Salvador	0.42	0.27	0.84	0.008	0.020	3509	0.48
Honduras*	0.40	0.15	0.85	0.007	0.035	1899	0.57
Nicaragua	0.59	0.15	0.86	0.006	0.024	1326	0.58
Panama*	0.31	0.39	0.89	0.045	0.075	6457	0.55
Paraguay	0.50	0.45	0.65	0.052	0.081	2570	0.56
Uruguay	0.68	0.45	0.82	0.030	0.044	11112	0.43
Venezuela*	0.36	0.94	0.58	0.073	0.054	12351	0.46

Table 1.B.3: Regression table.

Dependent Variable:
Pro.migration (Disagree that migrants come to compete for our jobs)

	(1) Probit	(2) Probit	(3) Probit	(4) Probit	(5) Logit	(6) Probit
Skilled	-0.782*	-0.729*	-0.772*	-0.693*	-1.117*	-0.567*
	(-2.46)	(-2.00)	(-2.29)	(-2.37)	(-2.35)	(-2.54)
(Skilled)*(RSC)	0.498**	0.485**	0.427***	0.371***	0.592***	0.322***
	(3.12)	(2.79)	(3.62)	(4.35)	(4.31)	(5.71)
Share Mig	-3.404*	-2.376	-5.510	2.158*	5.026**	-1.762
	(-1.98)	(-1.44)	(-1.28)	(2.16)	(3.18)	(-1.09)
(Skilled)*(% Skilled Mig)	-2.870	-2.721	-1.692	-1.734	-2.765	-0.919
	(-0.68)	(-0.60)	(-0.59)	(-0.84)	(-0.81)	(-0.45)
(Skilled)*(HHI)	0.981**	0.906*	0.932*	0.885**	1.431**	0.715**
	(2.87)	(2.37)	(2.56)	(2.85)	(2.85)	(3.09)
(Socio-ec)*(RSC)			0.0556	0.0502	0.0835	0.0648
			(0.56)	(0.54)	(0.55)	(0.59)
Socio-economic			0.00216	-0.00352	-0.00513	-0.0222
			(0.04)	(-0.07)	(-0.07)	(-0.41)
J				3.086***	5.081***	3.012***
				(9.94)	(10.10)	(5.86)
Constant	-0.0956	-0.143	-0.132	-4.474	-7.152	-5.812
	(-0.47)	(-0.63)	(-0.66)	(-1.46)	(-1.44)	(-1.24)
PseudoR2	0.009	0.008	0.010	0.054	0.054	0.031
Observations	14903	10963	14903	14903	14903	18253

t statistics in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 1.B.4: Regression table
(Continuation).

Dependent Variable:
Pro.migration (Disagree that migrants come to compete for our jobs)

	(1)	(2)	(3)	(4)	(5)	(6)
Employed	-0.0627	-0.0627	-0.0561	-0.0527	-0.0833	-0.0374
	(-1.37)	(-1.11)	(-1.17)	(-1.49)	(-1.45)	(-1.39)
Unemployed	0.107	0.0840	0.121	0.108	0.174	0.0904
	(1.47)	(1.09)	(1.54)	(1.58)	(1.58)	(1.76)
Retired	-0.000149	0.00920	0.00691	-0.0866	-0.139	-0.0509
	(-0.00)	(0.10)	(0.09)	(-1.56)	(-1.53)	(-0.99)
Housewife/husband	-0.0837	-0.0697	-0.0730	-0.0403	-0.0629	-0.0222
	(-1.05)	(-0.74)	(-0.90)	(-0.68)	(-0.65)	(-0.50)
Student	0	0	0	0	0	0
	(.)	(.)	(.)	(.)	(.)	(.)
Urban	-0.117	-0.112	-0.123	-0.0535	-0.0860	-0.0357
	(-1.81)	(-1.65)	(-1.93)	(-1.08)	(-1.06)	(-0.84)
Male	0.00389	0.0144	0.00615	0.0268	0.0426	0.0260
	(0.13)	(0.41)	(0.21)	(1.00)	(0.99)	(1.22)
Age	-0.000485	-0.000521	-0.000585	-0.000595	-0.000982	-0.000538
	(-0.41)	(-0.44)	(-0.48)	(-0.75)	(-0.76)	(-0.87)
Married or Cohabit	0.0117	-0.0126	0.00854	0.0146	0.0229	0.0109
	(0.28)	(-0.24)	(0.21)	(0.37)	(0.35)	(0.30)
Single	0.0244	-0.0136	0.0220	0.0205	0.0325	0.0326
	(0.58)	(-0.30)	(0.53)	(0.61)	(0.60)	(1.23)
Divorced	0	0	0	0	0	0
	(.)	(.)	(.)	(.)	(.)	(.)
Life satisfaction		0.0187				
		(0.80)				
Right-wing		0.00988				
		(0.22)				
Nationalist		0.0298				
		(0.62)				
Trust		-0.0393				
		(-0.36)				
Crime victim		-0.0714				
		(-1.14)				
PseudoR2	0.009	0.008	0.010	0.054	0.054	0.031
Observations	14903	10963	14903	14903	14903	18253

t statistics in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 1.B.5: Regression table
(Continuation).

Dependent Variable:
Pro.migration (Disagree that migrants come to compete for our jobs)

	(1)	(2)	(3)	(4)	(5)	(6)
J				3.086***	5.081***	3.012***
				(9.94)	(10.10)	(5.86)
Urban				0.444	0.803	0.377
				(0.90)	(1.00)	(0.61)
Male				-2.505***	-5.169***	-0.155
				(-4.06)	(-5.02)	(-0.26)
Age				0.0497	0.0946*	0.0270
				(1.77)	(2.06)	(0.85)
Married or Cohabit				5.867	10.42	4.896
				(1.35)	(1.48)	(0.84)
Single				1.007	1.365	2.425
				(0.86)	(0.71)	(1.00)
Divorced				0	0	0
				(.)	(.)	(.)
Life satisfaction				0.0117	-0.0150	0.0676
				(0.11)	(-0.09)	(0.67)
Socio-economic				-1.679	-3.134	-0.877
				(-1.50)	(-1.73)	(-0.73)
Trust				2.382	4.422	1.469
				(1.39)	(1.59)	(0.72)
Right-wing				2.268	4.341*	0.844
				(1.73)	(2.03)	(0.66)
Nationalist				0.395	0.613	0.629
				(0.92)	(0.88)	(1.01)
Crime victim				2.154	4.079*	0.947
				(1.68)	(1.96)	(0.71)
Household Asset				0.185	0.307	0.174
				(1.63)	(1.71)	(1.21)
Employed				0.415*	0.734**	0.166*
				(2.38)	(2.66)	(2.02)
Unemployed				4.731	8.744	2.819
				(1.53)	(1.74)	(0.79)
Retired				-0.267	-0.495	0.00107
				(-0.71)	(-0.77)	(0.00)
Housewife/husband				0	0	0
				(.)	(.)	(.)
Student				0	0	0
				(.)	(.)	(.)
PseudoR2	0.009	0.008	0.010	0.054	0.054	0.031
Observations	14903	10963	14903	14903	14903	18253

t statistics in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 1.B.6: Regression table
Labour force, country fix effects and property.

Dependent Variable:
Pro.migration (Disagree that migrants come to compete for our jobs)

	(1) Model 4	(2) Lab. For.	(3) Out Lab. For.	(4) Country FE	(5) Property
J	3.086^{***}	3.094^{***}	3.146^{***}	3.375^{***}	2.935^{***}
	(9.94)	(11.86)	(7.74)	(5.45)	(16.01)
Skilled	-0.693^{*}	-0.979^{**}	-0.179	-0.693^{*}	-0.770^{*}
	(-2.37)	(-3.17)	(-0.33)	(-2.37)	(-2.32)
(Skilled)*(RSC)	0.371^{***}	0.397^{***}	0.337	0.371^{***}	0.372^{***}
	(4.35)	(8.42)	(1.41)	(4.35)	(3.93)
(Skilled)*(HHI)	0.885^{**}	1.263^{***}	0.206	0.885^{**}	0.937^{**}
	(2.85)	(3.90)	(0.36)	(2.85)	(2.68)
Share Mig	2.158^{*}	-2.326^{*}	12.22^{***}	-4.000	3.311^{***}
	(2.16)	(-2.17)	(7.03)	(-0.98)	(3.31)
(Skilled)*(% Skilled Mig.)	-1.734	-2.149	-0.919	-1.734	-1.184
	(-0.84)	(-1.12)	(-0.26)	(-0.84)	(-0.52)
Socio-economic	-0.00352	-0.0170	0.0253	-0.00352	
	(-0.07)	(-0.36)	(0.49)	(-0.07)	
(Socio-ec)*(RSC)	0.0502	0.0423	0.0496	0.0502	
	(0.54)	(0.51)	(0.43)	(0.54)	
Househ. Asset					0.0162
					(0.59)
(Househ. Asset)*(RSC)					-0.000207
					(-0.00)
PseudoR2	0.054	0.053	0.062	0.054	0.055
Observations	14903	9279	5624	14903	14226

t statistics in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 1.B.7: Regression table
Labour force, country fix effects and property.

Dependent Variable:
Pro.migration (Disagree that should be a law preventing migrants from entering).

	(1) Model 4	(2) Lab. For.	(3) Out Lab. For.	(4) Country FE	(5) Property
J	1.968***	1.947***	1.967***	2.000***	1.842***
	(10.88)	(12.56)	(8.32)	(6.70)	(16.47)
Skilled	-0.526	-0.806**	0.110	-0.558	-0.681*
	(-1.88)	(-3.10)	(0.26)	(-1.89)	(-2.23)
(Skilled)*(RSC)	0.220	0.298*	0.0529	0.236	0.293*
	(1.53)	(2.36)	(0.23)	(1.55)	(1.98)
(Skilled)*(HHI)	0.911**	1.212***	0.236	0.931**	1.039**
	(3.18)	(4.37)	(0.56)	(3.13)	(3.28)
Share Mig	-20.28***	-12.66***	-30.83***	2.233	-23.05***
	(-12.90)	(-8.51)	(-12.50)	(0.39)	(-25.38)
(Skilled)*(% Skilled Mig.)	-1.613	-1.013	-2.938	-1.382	-1.249
	(-0.46)	(-0.34)	(-0.58)	(-0.38)	(-0.34)
Socio-economic	0.0256	0.0274	0.0262	0.0386	
	(0.68)	(0.86)	(0.49)	(1.24)	
(Socio-ec)*(RSC)	0.0527	0.0168	0.101	0.0232	
	(0.65)	(0.25)	(0.91)	(0.36)	
Househ. Asset					0.0222
					(1.00)
(Househ. Asset)*(RSC)					-0.0141
					(-0.34)
PseudoR2	0.050	0.049	0.055	0.050	0.050
Observations	18191	11340	6851	18191	17383

t statistics in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 1.B.8: Regression table.
Instrumental Variables

Dependent Variable:
Pro.migration (Disagree that migrants come to compete for our jobs)

	(1) Probit	(2) IVProbit	(3) IVProbit	(4) IVProbit
J	3.086^{***}	3.088^{***}	3.018^{***}	3.102^{***}
	(9.94)	(10.05)	(7.96)	(10.65)
Skilled	-0.693^{*}	-0.719	0.0519	-0.898^{**}
	(-2.37)	(-1.91)	(0.08)	(-2.63)
(Skilled)*(RSC)	0.371^{***}	0.392^{*}	-0.242	0.537^{**}
	(4.35)	(2.00)	(-0.54)	(2.97)
(Skilled)*(HHI)	0.885^{**}	0.908^{*}	0.242	1.062^{**}
	(2.85)	(2.42)	(0.42)	(2.99)
Share Mig	2.158^{*}	2.108^{*}	3.693^{**}	1.761
	(2.16)	(1.97)	(2.67)	(1.61)
(Skilled)*(% Skilled Mig)	-1.734	-1.784	-0.0631	-2.130
	(-0.84)	(-0.85)	(-0.02)	(-1.03)
(Skilled)*(RSC) (First Step)				
(Skilled)*(gdp)		0.0000506^{**}		0.0000686
		(3.24)		(1.93)
(skilled)*(gini)			-1.989^{*}	1.787
			(-2.39)	(0.85)
Wald Test (P-value)		0.908	0.246	0.213
Observations	14903	14903	14903	14903

t statistics in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 1.B.9: Regression table
Marginal Effects.

	(1) Probit	(2) Elasticities
Disagree Mig. compete jobs		
J	3.086***	1.188***
	(9.94)	(9.94)
Skilled (d)	-0.693*	-0.240**
	(-2.37)	(-2.77)
(Skilled)*(RSC)	0.371***	0.143***
	(4.35)	(4.35)
(Skilled)*(HHI)	0.885**	0.341**
	(2.85)	(2.85)
Share Mig	2.158*	0.830*
	(2.16)	(2.16)
(Skilled)*(% Skilled Mig)	-1.734	-0.667
	(-0.84)	(-0.84)
Socio-economic	-0.00352	-0.00135
	(-0.07)	(-0.07)
(Socio-ec)*(RSC)	0.0502	0.0193
	(0.54)	(0.54)
PseudoR2	0.054	0.054
Observations	14903	14903

Marginal effects; t statistics in parentheses
(d) for discrete change of dummy variable from 0 to 1
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

2 Duopoly competition under firm-specific network effects

Marco Tolotti and Jorge Yépez

Abstract

We consider a duopoly where competing firms are differentiated solely by the intensity of the network externality that their products produce on consumers. We formalize a two-stage game in which, firstly, firms simultaneously choose optimal prices and, secondly, potential adopters decide upon their preferred technology by playing a variant of a consumer choice game. We fully characterize the subgame perfect Nash equilibria w.r.t. prices and market share. The firm with the highest network effect exerts a higher social pressure on potential adopters and, eventually, becomes a monopolist. Surprisingly, this outcome is not persistent: an excessive network effect may be detrimental for the strongest firm, that, can eventually be thrown out of the market with positive probability. This is due to the existence of suboptimal equilibria causing a negative reinforcing effect on consumers' behavior.

2.1 Introduction

Consider a Bertrand economy where two firms, say A and B, compete on prices, while a large population of consumers choose among the two offered technologies. Suppose, that the two firms are characterized by a different level of *social recognition* they are able to produce in customers mind. Put differently, when consumers face the decision about which product to buy, they will consider differently product A and product B, not only because of product specific properties (such as price) but also because of this firm specific social strength, say J_A and J_B. Let A be the *leader*, in that its social strength is higher. *Ceteris paribus*, one expects profits and market shares for the leader, A, to increase in J_A. This is exactly what happens under the standard *weak network effects* scenario. Now, what happens if $J_A \gg J_B$? Do the market share for A and its revenues increase accordingly? We show the emergence of a second regime, called *strong network effects* scenario, where monotonicity is lost. Indeed, high network externalities may cause an abrupt instability: consumers coordinate *in the wrong way* from the point of view of the leader, who can eventually result to be out of the market with positive probability. The main goal of this paper is to analyse the impact of firm specific network externalities on the Nash equilibria expressed in terms of prices and market shares. In particular, we show that the aforementioned result, which could appear somewhat paradoxical at a first glance, can be explained in terms of social interactions.

The paramount role of network effects in social sciences has been widely investigated during the last decades. Among the others, Schelling (1971) explores patterns of residential segregation, Becker (1974) households' income distribution, Katz and Shapiro (1985) competition and compatibility among technologies, Akerlof (1997) studies how social distance impacts on decisions. Glaeser et al. (1996) empirically studies the impact of social interactions on crime rates in different regions. One of the main consequences of network effects is the appearance of what are commonly called *social traps*. Quoting Akerlof (1997), *"These externalities [...] will create long-run low-level equilibrium traps that are far from socially optimal."* To the best of our knowledge, the first attempt to incorporate network effects in the context of a duopoly competition is Grilo et al. (2001). In this paper, the authors show how the level of conformity (i.e., the strength of the network effects) in consumer behavior impacts on the pricing policies of the two competitors. The stronger the network effects, the higher is the competition and the lower are prices; eventually, assuming very high network effects, a *winner-takes-all* effect prevails and one of the companies may become a monopolist.

A second strand of literature in the field of social interactions devotes its attention to the study of *large populations* of heterogeneous agents linked by social ties. The pioneering contribution is Granovetter (1978) where riots' formation is analyzed in relation to the characteristics of the underlying population. In the context of discrete choices, Brock and Durlauf (2001b) describe the aggregate outcomes of the economy when the size of the population increases to infinity. In this case, it is easier to obtain closed-form solutions and characterize the equilibria of the economy. A similar asymptotic perspective is fruitful when players act strategically as in Kalai (2004) who identifies a general class of large games for which existence and robustness of pure-strategy Bayesian Nash equilibria is guaranteed.[1]

Our aim is to unite these two branches of literature in modeling a duopoly

[1] This literature falls within the context of network externalities of the *mean-field type*: the economy is not endowed with a proper geometric structure and agents are, indistinguishable to the modeler. As we make clear below, our work fits exactly this setting.

competition where network effects are firm-specific and where demand arises as in a consumer game with many players. More precisely, a large population of heterogeneous consumers are asked to choose between two technologies. In taking decisions, agents weight her personal taste (signaled by a type) and social norms (being the majority is relevant). With this respect, our approach resembles the *computer-choice game* in Kalai (2004), where prices for the two technologies matter and the levels of network externality are firm-specific. We assume that prices, p_A and p_B, are determined by the two competitors by playing a preliminary Bertrand duopoly game. Denote by q the proportion of agents choosing A; we analyze the subgame perfect Nash equilibria (p^*_A, p^*_B, q^*) emerging in the economy. We show that the leader takes advantage of his dominant position to increase its market share and, eventually, *force* the competitor out of the market. However, in case of very a large network strength,multiple equilibria emerge and, for some of them, the leader could be worse off and, eventually, even end up out of the market. As far as we know, this non-linearity in market outcomes due to social interactions has never been documented in the context of market competition.

The rest of the paper is organized as follows. Section 2.2 describes the demand side of the economy: we solve the consumer choice game. In section 2.3 we model the Bertrand duopoly competition, discussing the emerging equilibria. Section 2.4 is devoted to the analysis of the equilibria in terms of profits, market shares and prices. In section 2.5 we draw some conclusions, whereas Appendix A contains all the technical proofs.

2.2 The consumer choice game

Given a duopoly where two competitors are in charge to set prices and a large population of potential adopters reacts to prices, we study two subgames separatly. In the first stage, firms optimally choose prices in a non-cooperative way. Secondly, consumers choose between the two technologies, given the pair of prices selected previously by the firms and depending on the other characteristics of the economy. This follows standard literature in the context of duopoly competition on prices; see Grilo et al. (2001) and Anderson et al. (1992).

We start from the second stage, assuming that prices are fixed. Consider two firms $\{A, B\}$ selling a homogeneous product to a large population of n potential buyers. Each agent, $i = 1, \ldots, n$ has to decide between two mutually exclusive actions $w_i = \{A, B\}$: buying product A or buying product B.[2] Each player i is privately endowed with a type t_i, describing her personal taste for technology B with respect to A. We assume that $(t_i)_{i=1,\ldots,n}$ are independent and identically distributed according to a continuous and zero-mean distribution, so that there is no a-priori bias towards one of the two technologies. As in the computer-choice game by Kalai (2004), we assume that consumers have separable and linear preferences for public, private and social effects.

The (indirect) utility V of each agent i is the weighted sum of three components: public effects (prices), individual effects (personal taste signalled by the type) and network effects. Concerning the former, it positively depends on the reservation value Y of the technology and negatively on its price.[3] The individual component accounts for the (personal) propensity for product B w.r.t. A. To avoid technicalities, we simply assume a negative (positive) contribution for technology A (B, respectively), signaled by the type. Indeed, we add $-0.5\, t_i$ to the indirect utility for technology A and $+0.5\, t_i$ for B, the factor 0.5 makes computations convenient.

Concerning the network effects, the larger is the (expected) share for one technology, the higher are its social recognition and the personal advantage for the agent in adopting it. Therefore, we make the payoff depend upon the *expected market share* of each product. We denote by q^i, the expectation of agent i about the market share of product A. Indeed,

$$q^i = \frac{1}{n}\, \mathbb{E}^i \left[\sum_{j=1}^{n} \mathbb{I}_{\{w_j = A\}} \right],$$

where $\mathbb{E}^i[\cdot]$ denotes the expectation taken with respect to the joint distribution of the vector of types $(t_j)_{j \neq i}$.[4] Finally, we introduce the parameters related to the firm-specific network strength, (J_A, J_B), respectively: an higher J reflects an higher social recognition/pressure assigned by consumers to the

[2] To simplify the analysis, no exit option is allowed. As it will become clear shortly, the *reservation value* for the technology is large enough that all the agents enter the market.

[3] We assume that Y is constant across technologies and large enough so that each consumer enters the market. These assumptions are standard in the context of horizontal product differentiation models; see, Belleflamme and Peitz (2015). An interpretation can be found in Anderson et al. (1992) where the authors assume Y to be a constant income across all individuals; therefore, the payoff function is interpreted as an indirect utility function.

[4] We could equivalently use $\frac{1}{n-1} \sum_{j \neq i} \mathbb{I}_{\{w_j = A\}}$, in place of q^i. Indeed, when n is large, the marginal contribution of the choice of agent i becomes negligible and the two aggregate statistics are indistinguishable.

brand. In turn, the social component of the utility is $J_A\, q^i$ (respectively, $J_B\,(1-q^i)$) for technology A (B).

As previously discussed, each of these three components is treated as a distinct argument and summed to form the final indirect utility function $V_i(w_i, t_i)$.[5] Summing up,

$$\begin{array}{l} V_i(A, t_i) = Y - p_A - 0.5\, t_i + J_A\, q^i; \\ V_i(B, t_i) = Y - p_B + 0.5\, t_i + J_B\,(1 - q^i). \end{array} \tag{1}$$

The decision faced by each consumer depends on the action of other agents through the participation shares shaping the social component term. This eventually results in setting a non-cooperative game, in which each agent makes her choice given an expectation of the population outcome. It is assumed that the agents know the characteristics of the economy (prices and parameters) as well as the probability distribution for the types. Moreover, they know the structure of the individual choice problems. Evidently,

$$V_i(A, t_i) > V_i(B, t_i) \iff -p_A + p_B - t_i + J_A\, q^i - J_B\,(1 - q^i) > 0. \tag{2}$$

By defining agent-specific threshold levels $t_i^{th} = -p_A + p_B + J_A\, q^i - J_B\,(1 - q^i)$, it turns out that, for each $i = 1, \ldots, n$,

$$w_i = A \iff t_i < t_i^{th}. \tag{3}$$

Note that $\mathbb{P}(t_i = t_i^{th}) = 0$, because the distribution of types is continuous; therefore, decision under equality is immaterial. Finally, as stated in the literature on large games, it is convenient to explore the behavior of the system when the number of agents is getting larger and larger. In this case, the effect of the single agent on the market outcomes (such as the expectations q^i) becomes negligible and aggregate statistics are easier to determine in closed form. The next proposition characterizes the Nash equilibrium of the consumer choice game. Proofs are postponed to the Appendix.

Proposition 1 *Consider the n-player game with payoff structure expressed by thresholds as in* (3) *and where the types $(t_i)_{i=1,\ldots,n}$ are independent and identically distributed according to some continuous distribution function F, where $F(s) = \mathbb{P}(t_i \le s)$. There exists at least one Nash equilibrium in pure strategies.*

Moreover, when $n \to \infty$, let the marginal agent, by her type t_m, be indifferent between A or B. Indeed,

$$t_m(q) = p_B - p_A + J_A\, q - J_B\,(1 - q), \tag{4}$$

[5]This is also rather common in the literature on network competition, e.g., see, Katz and Shapiro (1985), Ambrus and Argenziano (2009), Hagiu and Hałaburda (2014).

where q solves the consistency (fixed point) equation

$$F(t_m(q)) = q. \tag{5}$$

We benefit from rational expectations: each agent shares the same expectation about other players' actions and this expectation, when $n \to \infty$, matches the limiting value q. Eventually, the decision for an action is based upon a comparison between the individual type t_i and the threshold t_m: the population splits so that all agents with $t_i < t_m$ choose technology A, and all agents with $t_j \geq t_m$ will choose B.[6] In the next section we analyze the market share emerging as the fixed point of (5).

2.2.1 Demand under weak network effect

We hence forth consider the situation where the types $(t_i)_{i=1,\dots,n}$ are independent and uniformly distributed on the interval $[-a, a]$.[7] In this case,

$$F(t_m) = \begin{cases} 0 & \text{if} \quad t_m \leq -a \\ \frac{t_m+a}{2a} & \text{if} \quad -a < t_m \leq a \\ 1 & \text{if} \quad a < t_m \end{cases} \tag{6}$$

It is convenient to separate our analysis in two cases. The first case, referred to as *weak network effects*[8], describes the situation where the sum of the network effect parameters is smaller than the range of the support of the distribution of types:

$$J_A + J_B < 2a. \tag{7}$$

This, corresponds to a diverse population, with respect to the network effects. Under (7), an explicit expression for q is

$$q = \begin{cases} 0 & \text{if} \quad p_B - p_A \leq J_B - a \\ \frac{p_B - p_A - J_B + a}{2a - J_B - J_A} & \text{if } J_B - a < p_B - p_A \leq a - J_A \\ 1 & \text{if} \quad p_B - p_A > a - J_A \end{cases} \tag{8}$$

[6] As said, being the distribution of t_i continuous, it is not relevant where we put the equality sign.

[7] The choice of the uniform distribution is made for tractability. In this case, in fact, all the equilibria in prices and market share can be derived explicitly. Much of the arguments we derive in the next sections generalize to several continuous and unimodal distribution (normal, logistic, etc.).

[8] This terminology and classification is rather standard in the literature, see Grilo et al. (2001) or Belleflamme and Peitz (2015) for more details.

Equation (8) implies that, once prices have been fixed, the equilibrium level q for the market share is unique. Figure 2.1 describes the demand of technology A, according to equation (8). Demand q decreases with p_A and increases with p_B, yielding negative own-price elasticity and positive cross-price elasticity.

For an intuition about the dependence of the equilibrium demand on the parameters of the model, Figure 2.2 shows the phase diagram for q, when varying J_A and the difference in prices $p_B - p_A$; in particular, we set $a = 4$ and $J_B = 2$. We fix types distribution and the network level of firm B and let J_A and the difference in prices $p_B - p_A$ vary. For each combination of prices (on the vertical axis) and level of J_A (on the horizontal axis) we have a corresponding (unique) demand q. There are three different regions. Region [1] is characterized by a non-trivial market share $q \in (0, 1)$. It is bounded by two lines: the red horizontal line corresponding to equation $p_B - p_A = J_B - a$ and the black line given by equation $p_B - p_A = a - J_A$. Note that the former corresponds to a combination mix for which $q = 0$, whereas the latter to $q = 1$. Therefore, in region [2], (above the black line $p_B - p_A = a - J_A$), $q = 1$; conversely, below the red line, in region [3], $q = 0$.

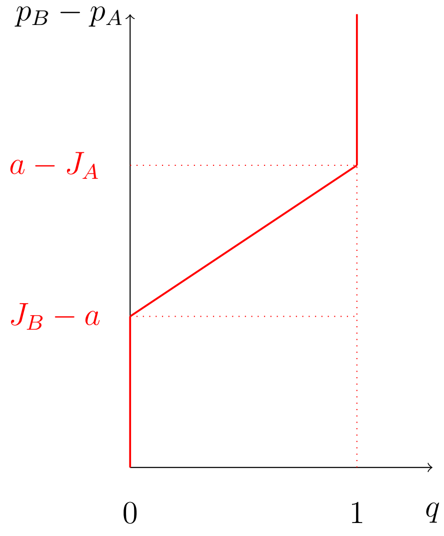


Figure 2.1: Demand q under weak network effects.

Finally, the rightmost vertical dashed line represents equation (7) which holds for the values of J_A smaller that $2a - J_B$. It is worth noting that when J_A increases, the region in which $q = 1$ expands.

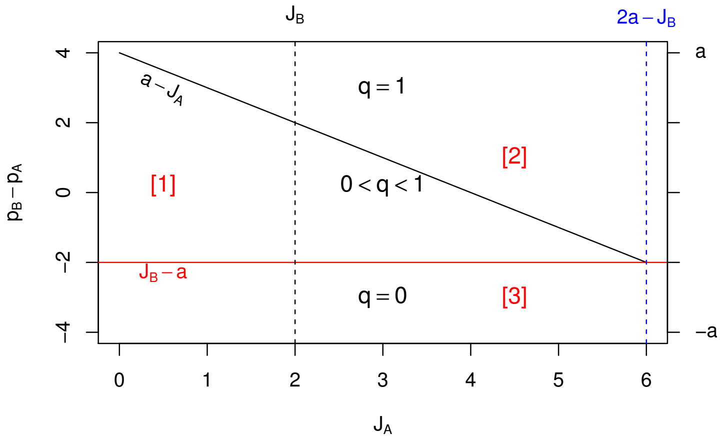


Figure 2.2: Phase Diagram for q under weak network effects, when $a = 4$ and $J_B = 2$.

2.2.2 Demand under strong network effect

The second situation, when

$$J_A + J_B > 2a, \tag{9}$$

deals with *strong network effects*[9]. Under condition (9), the values of q are

$$q = \begin{cases} 0 & \text{if} \quad p_B - p_A \le J_B - a \\ \{0;\theta;1\} & \text{if } a - J_A < p_B - p_A \le J_B - a \\ 1 & \text{if} \quad p_B - p_A > a - J_A, \end{cases} \tag{10}$$

where

$$\theta = \frac{p_B - p_A - J_B + a}{2a - J_B - J_A}. \tag{11}$$

This situation is very different compared to the case of weak network effects. When (9) holds, $a - J_A < J_B - a$; therefore, as depicted on Figure 2.3, for intermediate prices ($a - J_A < p_B - p_A \le J_B - a$), three self-consistent equilibria coexist. In particular, for a given combination of prices $\hat{p} = p_B - p_A$, there are two extreme equilibria ($q = 0$ and $q = 1$) and an intermediate equilibrium $\theta \in (0, 1)$. As an example, fixed the parameters and p_B. Suppose

[9]The case in which $J_A + J_B = 2a$, usually called the critical case, is basically unfeasible due to the zero-measure of the parameters set; we briefly describe it in section 2.4.

$p_A < p_B - J_B + a$; in this case, the only possible equilibrium demand is $q = 1$. Suppose now that p_A increases so that $p_A > p_B - J_B + a$. We enter in the region where firm A may lose its market power and even go out of the market (in case the equilibrium $q = 0$ prevails). Note that, differently from the weak network effect regime, this transition is non-smooth (see Figure 2.1).[10] We

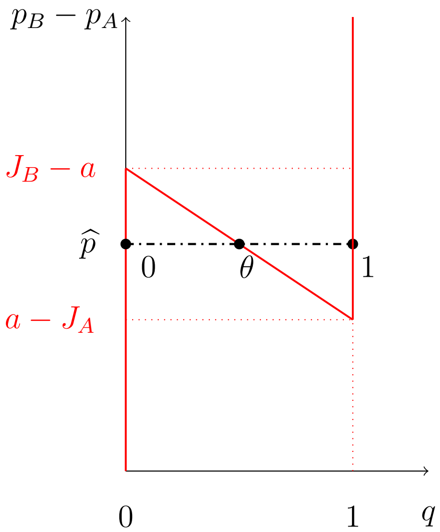


Figure 2.3: Demand q under strong network effects.

summarize the outcome of the consumer choice game in the following

Proposition 2 *Assume that the type distribution is uniform with support* $[-a, a]$.

The Nash equilibria as follows:

- *under Weak Network Effects,*

  if $p_B - p_A \leq J_B - a, \quad q = 0;$

  if $J_B - a < p_B - p_A \leq a - J_A, \quad q = \frac{p_B - p_A - J_B + a}{2a - J_B - J_A};$

  if $p_B - p_A > a - J_A, \quad q = 1;$

- *under Strong Network Effects,*

  if $p_B - p_A \leq J_B - a, \quad q = 0;$

  if $a - J_A < p_B - p_A \leq J_B - a$, $q \in \{0; \theta; 1\}$, *where* $\theta = \frac{p_B - p_A - J_B + a}{2a - J_B - J_A}$.

  if $p_B - p_A > a - J_A, \quad q = 1.$

[10] We discuss how the two firms take this *uncertainty* into account in the next section .

2.3 The two-player Bertrand competition

Up to now, the prices of the two technologies have been given. We now model the first stage of the game where firms choose prices prices. As before, it is convenient to distinguish two cases based on the strength of the network effects.

2.3.1 Supply under weak network effects

Each firm selects a price, $p_k \geq 0$, where $K \in \{A, B\}$. The firms' profits $\{\pi_A, \pi_B\}$ are the normalized per-capita profit of firm A and B, respectively. These profits depend on the firms prices $\{p_A, p_B\}$ and the market shares[11] $\{q, (1-q)\}$:

$$\begin{aligned} \pi_A =& p_A \cdot q\,; \\ \pi_B =& p_B \cdot (1-q). \end{aligned} \tag{12}$$

We search for a Nash equilibrium, $\{p_A^*; p_B^*\}$, in pure strategies, such that each firm K maximizes its own profit with respect to p_K conditioned upon the (optimal) price of the competitor:

$$\begin{aligned} \pi_A(p_A^*, p_B^*) &\geq \pi_A(p_A, p_B^*), \quad \text{for all } p_A \geq 0; \\ \pi_B(p_B^*, p_A^*) &\geq \pi_B(p_B, p_A^*), \quad \text{for all } p_B \geq 0. \end{aligned}$$

Replacing equation (8) in equation (12), we get quadratic and concave functions (see Figure 2.1). Assuming that the competitor's price is fixed, with a slight abuse of notation, we write, respectively,

$$\pi_A(p_A) = p_A \left(\frac{p_B - J_B + a}{2a - J_A - J_B} \right) - \frac{p_A^2}{2a - J_A - J_B}; \tag{13}$$

$$\pi_B(p_B) = p_B \left(\frac{p_A - J_A + a}{2a - J_A - J_B} \right) - \frac{p_B^2}{2a - J_A - J_B}. \tag{14}$$

Therefore, there are two values for which π_A (π_B resp.) are zero:

$$\begin{aligned} \pi_A(p_A) = 0 \Leftrightarrow p_A = 0 \text{ or } p_A^0 = p_B - J_B + a; \\ \pi_B(p_B) = 0 \Leftrightarrow p_B = 0 \text{ or } p_B^0 = p_A - J_A + a. \end{aligned} \tag{15}$$

[11] To focus on the effects of network externalities, we set the marginal cost for each firm equal to zero, and we assume no fixed costs.

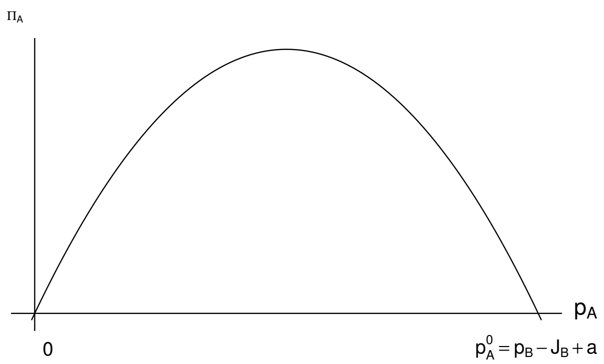


Figure 2.1: $\pi_A(p_A)$ for p_B fixed.

From (8), we infer that, when $p_A^0 = p_B - J_B + a$, then $q = 0$. Therefore, in this case, firm B becomes a monopolist. Conversely, when $p_B^0 = p_A - J_A + a$, the market share is $q = 1$, letting firm A be the monopolist. The possible outcomes of the market are summarized in the following

Proposition 3 *[Subgame perfect equilibria under weak network effects] Consider the model where q is described by equation (8) and two firms, A and B, simultaneously optimize their profits as in (12). Assume, moreover, that equation (7) holds, then the unique subgame perfect Nash equilibrium (p_A^*, p_B^*, q^*) can be described as follows*

1. *If $3a \geq J_A + 2J_B$ and $3a \geq 2J_A + J_B$, then*

$$p_A^* = \frac{3a - J_A - 2J_B}{3};\ p_B^* = \frac{3a - 2J_A - J_B}{3};\ q^* = \frac{3a - J_A - 2J_B}{3(2a - J_A - J_B)}.$$

   And the market is a proper duopoly. Moreover,

   – *if $J_A \leq J_B$, $p_B^* > p_A^*$ then $0 < q^* \leq \frac{1}{2}$;*

   – *if $J_A \geq J_B$, $p_A^* > p_B^*$ then $\frac{1}{2} \leq q^* < 1$.*

2. *If $3a < 2J_A + J_B$, then $p_A^* = p_A^M = J_A - a$, $p_B^* = 0$ and $q^* = 1$. Therefore, firm A monopolizes the market.*

3. *If $3a < J_A + 2J_B$, then $p_B^* = p_B^M = J_B - a$; $p_A^* = 0$ and $q^* = 0$. Therefore, firm B monopolizes the market.*

Figure 2.2 resembles the phase diagram depicted in Figure 2.2 where $a = 4$ and $J_B = 2$. The green line represents the *critical difference in prices*, i.e., $p_B^* - p_A^*$ as resulting from Proposition 3. When $3a < 2J_A + J_B$, we are into the region called M, where firm A monopolizes the market. In this region, the optimal price for firm A is the *monopolistic price* p_A^M. The green line shows a kink at this level because of the change in the optimal pricing strategy. We also include a dashed line corresponding to the level of prices under which $q = \frac{1}{2}$. The equation of this line is $p_B^h - p_A^h = \frac{J_B - J_A}{2}$. We can see that when $J_A < J_B = 2$, $p_B^* > p_A^*$ and $q^* < \frac{1}{2}$. This means that firm B takes advantage of the higher network parameter and charges a higher price because it is still able to obtain a higher market share compared to its competitor. When $J_A = J_B = 2$, the model is perfectly symmetric and prices and market shares are equal: $p_A^* = p_B^*$ and $q^* = \frac{1}{2}$. When $J_A > J_B$, then $p_A^* > p_B^*$ and $q^* > \frac{1}{2}$. Finally, when $J_A \geq \frac{3a - J_B}{2} = 5$, (i.e., after the second dashed red vertical line in Figure 2.2), firm A monopolizes the market ($q^* = 1$), and charges the monopoly price $p_A^M = a - J_A$.

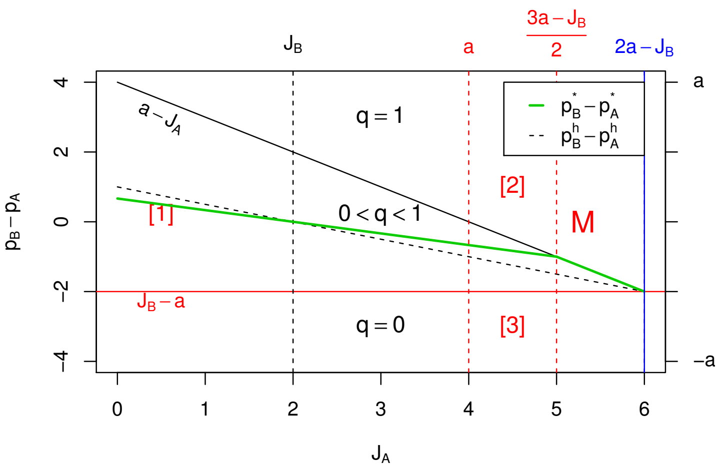


Figure 2.2: Phase Diagram for q under weak network effects, when $a = 4$ and $J_B = 2$.

Finally, in Figure 2.3, we depict the optimal market share by letting J_A and J_B vary ($a = 4$ is fixed). The admissible values of J_A and J_B, under weak network effects are represented by the area below the blue line corresponding to equation $J_A + J_B = 2a$. We recognize the three regions corresponding to situations [1]-[3] of Proposition 3.

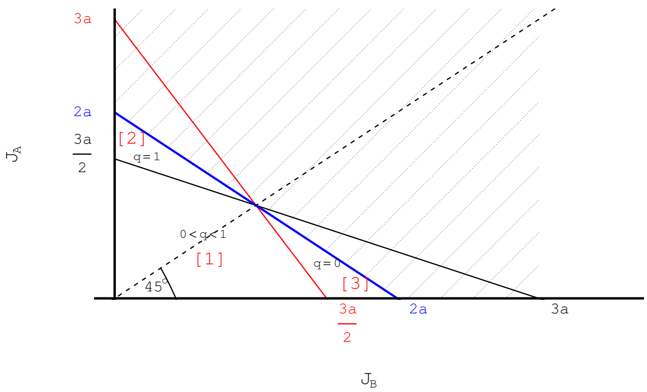


Figure 2.3: Phase diagram in J_A and J_B for $a = 4$, under Weak Network Effects.

2.3.2 Supply under strong network effects

As discussed in Section 2.2.2, when equation (9) holds, *strong network effects* are in place and multiple equilibria coexist for intermediate prices, $a - J_A < p_B - p_A < J_B - a$. In this situation, as seen in Proposition 2, the prevailing market share is not a-priori determined because the actual equilibrium is selected in the consumer choice sub game. Therefore, in order to form an expectation, we assume that the two firms consider the emerging equilibrium in a *probabilistic sense*, by assigning a certain probability to the three Nash equilibria. In particular, we assume that the probability of a Nash equilibrium is proportional to the size of its *basin of attraction*.[12] In a static setting, this amounts to saying that the two firms consider as plausible only the equilibria that can be reached by the consumers as fixed points of the best response map. This argument is made mathematically precise in the following lemma.

Lemma 1 *Under strong network effects, suppose $a - J_A < p_B - p_A \leq J_B - a$. In this case, the three solutions to* (5)*, found in Proposition 2, can be interpreted as the long-run attractors of the map $q \mapsto F(t_m(q))$. In this respect,*

[12]We say that q_0 belongs to the domain of attraction of the equilibrium $\bar{q}$, if, when starting at q_0 and iterating the map $q(t) = F(t_m(q_{t-1}))$, for $t \geq 1$, the system converges to $\bar{q}$.

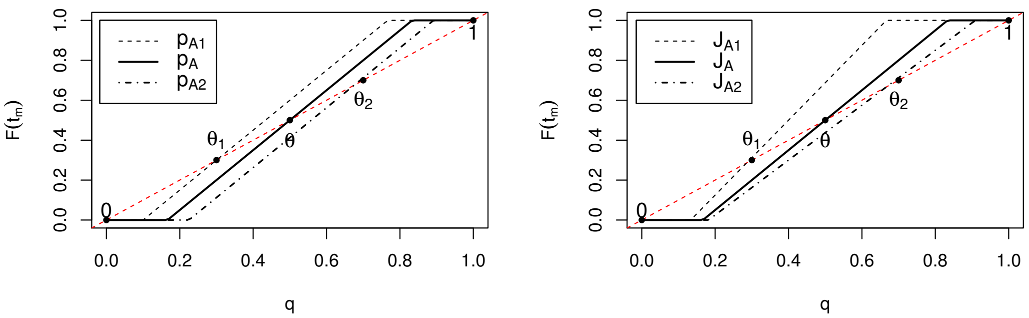
- $q = \frac{p_B - p_A - J_B + a}{2a - J_B - J_A}$ *is a linearly unstable equilibrium;*
- $q = 0$ *and* $q = 1$ *are locally stable and their domains of attraction are, respectively, of size* θ *and* $1 - \theta$*, where* $\theta = \frac{p_B - p_A - J_B + a}{2a - J_B - J_A}$.

Lemma 1 says that the intermediate equilibrium is not *reachable* by agents best-responding to the actions of other players. Therefore, if the two firms consider only reachable equilibria as possible outcomes of the economy necessarily $q \in \{0; 1\}$. Moreover, under this assumption, the size of the domain of attraction of $q = 0$ is the probability of reaching it.

We assume that the two firms evaluate the market share as the realization of a Bernoullian random variable as follows:[13]

$$Q = \begin{cases} 0 & \text{with probability} \quad \theta \\ 1 & \text{with probability} \quad 1 - \theta. \end{cases} \tag{16}$$

The two locally stable equilibria correspond to the *winner-takes-all* situation where the entire population purchases one of the two technologies.[14]



(a) $p_{A1} < p_A < p_{A2}$ (b) $J_{A1} > J_A > J_{A2}$.

Figure 2.4: $F(t_m)$ as a function of q for different values of prices p_A (left panel) and network levels J_A (right panel).

[13]In this case, we denote the equilibrium demand by a capital letter to emphasize its random nature. Note that the choice we made about the probability θ presumes that the iteration procedure of Lemma 1 starts by a uniformly randomly chosen $q_0 \in [0, 1]$.

[14]The fact that $q \in \{0; 1\}$ strongly depends on the choice we made about the uniform distribution of types. By considering any other continuous and unimodal probability distribution, we would still see three equilibira where the intermediate one is unstable, but the two external ones would belong to the open set $(0, 1)$.

Concerning θ, Figure 2.4a depicts this relationship for three different values of p_A: $p_{A1} < p_A < p_{A2}$ and their corresponding $\{\theta_1, \theta, \theta_2\}$. It is easy to notice that θ_1 is closest to 0. Thus, by iterating the map $F(t_m(q))$ from a random initial level of q, it is less likely to end up at $q = 0$ that at $q = 1$. A similar relationship can be found for different levels of J_A. The lower (higher) J_A, the higher (lower) is θ. Figure 2.4b shows the fixed points corresponding to three different values of J_A, where $J_{A1} > J_A > J_{A2}$.

In this sense, under *strong network effects* and intermediate prices, the social influence is so strong that the population ends up buying one of the goods unanimously, but we cannot predict with certainty which one of the two prevails. This *paradoxical result* is related to social interactions: when coordination is huge, the direction of the coordination is unclear but crucial. Both firms, especially the leader, are aware of this intrinsic uncertainty. As a consequence, firms A and B maximize their *expected profits*, which turn out to be, respectively[15],

$$\begin{aligned} \mathbb{E}(\pi_A) &= p_A \cdot \mathbb{E}[Q] \\ &= p_A \cdot (\theta \cdot 0 + (1-\theta) \cdot 1) \\ &= p_A \cdot (1-\theta) \\ &= p_A \cdot \left(\tfrac{p_A - p_B - J_A + a}{2a - J_A - J_B}\right); \end{aligned} \tag{17}$$

$$\begin{aligned} \mathbb{E}(\pi_B) &= p_B \cdot (1 - \mathbb{E}[Q]) \\ &= p_B \cdot (1 - (\theta \cdot 0 + (1-\theta) \cdot 1)) \\ &= p_B \cdot \theta \\ &= p_B \cdot \left(\tfrac{p_B - p_A - J_B + a}{2a - J_A - J_B}\right). \end{aligned} \tag{18}$$

The following proposition summarizes the outcome of the model under strong network effects.

Proposition 4 *[Subgame perfect equilibria under Strong Network Effects] Consider the model where q is described by equation* (10) *and two firms, A and B, simultaneously optimize their expected profits as in* (17)-(18).
Assume, moreover, that equation (9) *holds, then the following subgame perfect Nash equilibria (p_A^*, p_B^*, q^*) emerge:*

1. *If $3a \le J_A + 2J_B$ and $3a \le 2J_A + J_B$, then $E^*[Q] = 1 - \theta^*$. Optimal prices are*

$$p_A^* = \frac{2J_A + J_B - 3a}{3}; \; p_B^* = \frac{J_A + 2J_B - 3a}{3}.$$

[15] For simplicity we assume that firms are risk neutral.

Moreover, with probability θ^*, firm B monopolizes the market and, with probability $1-\theta^*$, firm A monopolizes the market. Finally,

– *if $J_A \leq J_B \Longrightarrow p_B^* \geq p_A^*$ then $\theta^* \geq \frac{1}{2}$;*

– *if $J_A \geq J_B \Longrightarrow p_A^* \geq p_B^*$ then $\theta^* \leq \frac{1}{2}$.*

2. *If $3a \geq J_A + 2J_B$, then $p_A^* = p_A^M = J_A - a$, $p_B^* = 0$ and $q^* = 1$. Therefore, firm A monopolize the market.*

3. *If $3a \geq 2J_A + J_B$, then $p_B^* = p_B^M = J_B - a$, $p_A^* = 0$ and $q^* = 0$. Therefore, firm B monopolizes the market.*

In Figure (2.5), we show the diagram of q^* as a function of J_A and J_B for $a = 4$. The admissible values of parameters under strong network effects (the non-shaded area) are above the blue line, $2a = J_A + J_B$. In region [3], $E^*[Q] = 0$ and $q^* = 0$. Similarly, in region [2] where $3a > J_A + 2J_B$, $E^*[Q] = 1$ and $q^* = 1$. Finally, in region [1], the two extreme equilibria coexist and $E^*[Q] = 1 - \theta^* \in (0, 1)$.

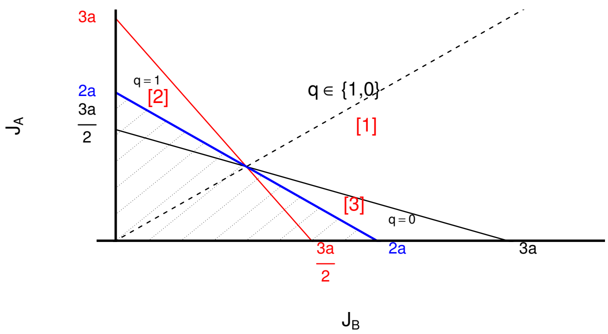


Figure 2.5: Phase diagram for q^* with $a = 4$ and under strong network effects.

2.4 Discussion of market equilibria

In this section we discuss some market implications of equilibria, in terms of market shares, prices and profits. Figure 2.1 (left panel) depicts the market shares of the two firms at the equilibrium for different values of J_A, assuming that $a = 4$ and $J_B = 2$. First of all, notice that for these parameters, scenario 3. of Proposition 3 and Proposition 4 are not attainable. As a consequence, firm B cannot be the monopolist (unless multiple equilibria are present). The blue dashed vertical line divides the graph into *weak network effects* (on the left) and *strong network effects* (on the right). As can be inferred by q^* [16] that under weak network effects, firm A market share increases with J_A up to a point where the firm monopolizes the market. On the other hand, as seen in section 2.2.2, when $J_A > 3a - 2J_B$, $q^* \in \{0; 1\}$ and the two extreme values are locally stable; for this values of the parameters, we plot the expected market shares as described in Proposition 4. Recall the interpretation: with probability θ^*, firm B monopolizes the market, otherwise A does. Moreover, by θ^*,[17], when $J_A > J_B$, $\lim_{J_A \to \infty} \mathbb{E}^*(1 - Q) = \theta^* = \frac{1}{3}$. Therefore, the probability for firm A to be out of the market is high even when J_A becomes huge.

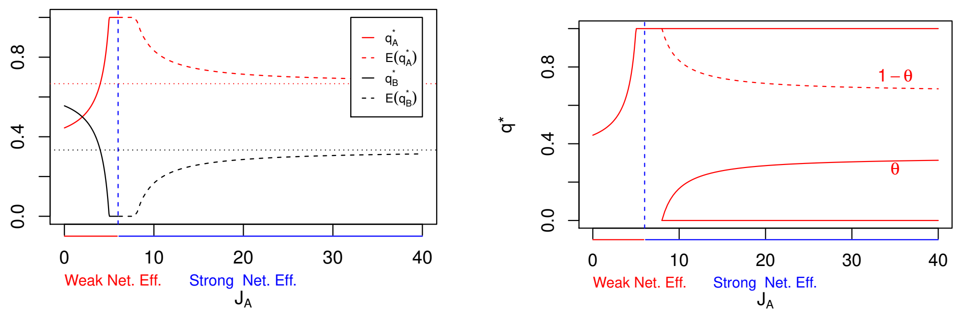


Figure 2.1: Optimal market shares (left panel) and bifurcation diagram for q^* (right panel) varying J_A, for $a = 4$ and $J_B = 2$.

It might seen counter-intuitive that the market share of firm A decreases with J_A. A closer look to the microstructure of the decision process for the consumer population shows that this phenomenon is explained by the network externalities. Let us take equation (2) and replace $p_B^* - p_A^* = \frac{J_B - J_A}{3}$ as derived in (25). We obtain

$$V_i(A, t_i) > V_i(B, t_i) \iff \underbrace{\frac{J_B - J_A}{3}}_{\text{Price Impact (-)}} + \underbrace{J_A q^* - J_B(1 - q^*)}_{\text{Social Impact (+)}} > t_i. \qquad (19)$$

We can distinguish two occurrences in J_A, with opposite sign: a "positive contribution" (positive externality) on the social component of the utility

[16] See equation (26) in the appendix

[17] See equation (34) on the appendix

and a "negative contribution" (negative externality) in terms of the price (due to the fact that p_A increases with J_A under the strong network effects). Therefore, it is no longer obvious that the market share has to be monotonically increasing in J_A. To reinforce the intuition, in Figure 2.1 (right panel) we plot the bifurcation diagram for q^*. Under weak network effects, the level of q^* is unique whereas, under strong network effects, there are two locally stable equilibria and an intermediate unstable equilibrium (called θ). Recall that the unstable equilibria is interpreted as the probability to end up in the scenario where $q^* = 0$; therefore, the probability that firm A monopolizes the market is described by the dashed line, $(1 - \theta)$.

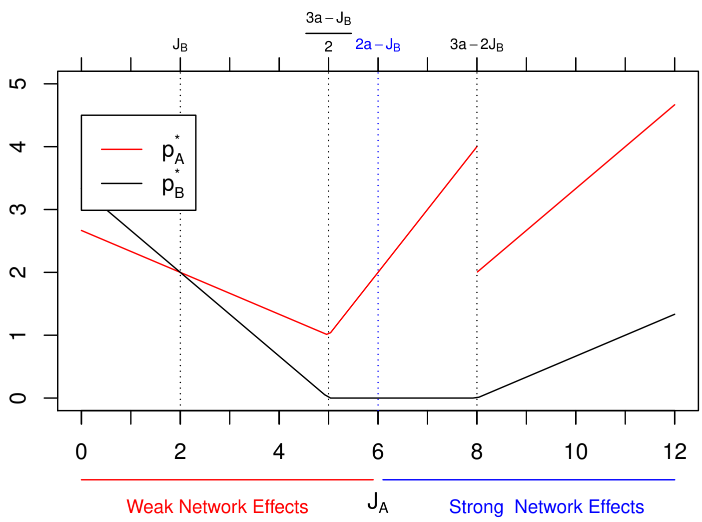


Figure 2.2: Optimal prices for firm A (red) and firm B (black), varying J_A, for $a = 4$ and $J_B = 2$.

Concerning optimal pricing policies, they are depicted in Figure 2.2. We can identify three regions corresponding to three different scenarios. The first one corresponds to the region where $J_A < \frac{3a-J_B}{2}$, here prices decrease with J_A. This is in line with Grilo et al. (2001) where a similar monotonicity of prices in network externalities is found. One remark may be useful. In that paper, authors motivate this evidence as follows: an increase in network effects signals a fiercer market competition and, finally, a reduction in prices. In our model, where network power is firm-specific, the interpretation of network effects as *market competition* is still plausible but the effect of a fiercer competition on the two firms is disentangled. Firm A, thanks to a higher network power, has a relative advantage due to the relative increase of J_A compared to J_B: its price decreases sensibly less compared to the competitor. For $\frac{3a-J_B}{2} < J_A < 3a - 2J_B$, firm A acts as a monopolist and its price increases in J_A. Finally, for $J_A > 3a - 2J_B$, we enter a region of multiple equilibria. As discussed before, there is a positive probability for firm A to lose its market power. As a consequence of this fact, it is no longer optimal for the leader (firm A) to use the monopolistic price $p_A^M = J_A - a$. Strictly speaking, the risk of ending up in a *bad scenario* makes the competition even more severe and the leading firm is forced to adjust its price[18] consequently. Conversely, under this latter scenario, firm B takes advantage of this uncertainty and, back to the market, charges a non-zero price accordingly.

The discussion about profits is in line with the previous analysis. In Figure 2.3 we see how profits of both firms decrease under the first scenario (when $J < \frac{3a-J_B}{2}$).Under weak network effects, an increase in J_A signals an increase in competition, hence a decrease in prices and, eventually, a decrease in profits. On the other hand, as noticed before, the fiercer competition has a non-symmetric effect on the two firms: the leading firm (A in this case) can maintain almost constant profits whereas firm B loses large part of its. Therefore, even if slightly loosing profits, firm A increases its market power. When entering in the second scenario, where A acts as a monopolist, the picture changes completely: here profits π_A^* increase in J_A, since now $p_A^* = p_A^M = J_A - a$ (and $q^* = 1$ is constant). Finally, under the third scenario, there is a discontinuity in (expected) profits for firm A, due to the appearance of multiple equilibria. Nevertheless, profits still grow in J_A due to the increase in the optimal price p_A^*, which offsets the decrease in (expected) market share.

[18]If we interpret an option value as the value that is placed on firm A for preserving a chance to monopolize the market in the future; then, the difference between the monopolistic price p^M and the optimal price p_A^* after the (downward) jump, could express an option value, a situation already found in Wirl (2008), yet in a different framework.

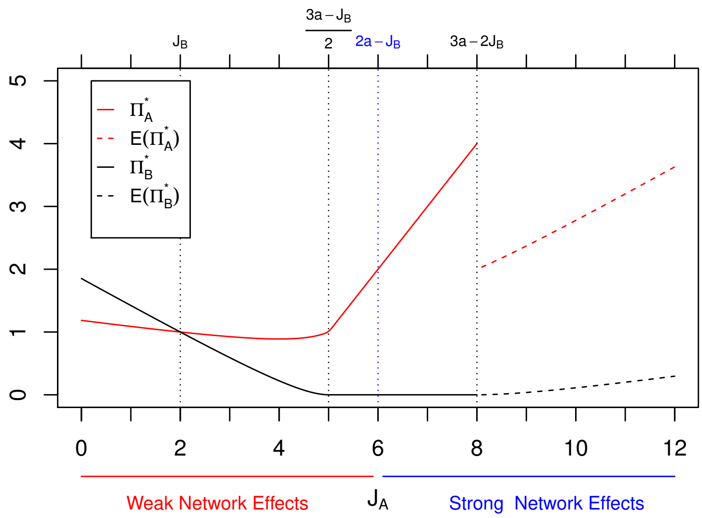


Figure 2.3: Profits under the optimal pricing policy, varying J_A, for $a = 4$ and $J_B = 2$.

2.5 Conclusions

We have studied a duopoly model where firms compete on prices and where a large population of heterogeneous potential adopters strategically choose one of the two available technologies in a consumer choice game. The peculiarity of our model is the presence of firm-specific network effects. This generalization opens up a fiercer market competition, signaled by an increase in network externality, trumping the relative market advantage/disadvantage of the competitors on the market.

By assuming that random types of population's taste are uniformly distributed, we are able to fully characterize the emergent equilibria in prices and market share in closed-form. Our analysis confirms previous studies in showing that, under weak network effects, competition increases when network externality increases, thus, abating prices. Indeed, when network externalities increase, the leading firm (the one that sees its market power getting relatively larger) is able to knock down the competitor and become a monopolist.

A novel result is that when network effects are very high, the leading firm partially loses its market power due to the uncertainty related to the presence of multiple equilibria. In particular, *bad equilibrium*, where the leader has a zero-market share, gets positive probability. Apart from the interesting discontinuity in the profit function which, the appearance of multiple equilibria relates our duopoly model to the literature of social choices. Indeed, the bad equilibrium resembles the *low-level equilibria* (or social traps) widely discussed in the literature in social dynamics. However, in our case, it is not correct to speak about *socially suboptimal equilibria*: for intermediate values of J_A, the potential adopters (the demand side) take advantage of lower prices on the market. When J_A exceeds a certain level, p_A (and even p_B) turns out to be even higher than in the monopoly region, thus causing worse market conditions for consumers.

Appendix

2.A Proofs

Proof of Proposition 1

To prove existence of a Nash equilibrium in the n-player consumer choice game, it is more convenient to consider the *optimal thresholds*, $(t_i^{th})_{i=1,\dots,n} \in \overline{\mathbb{R}}^n$, rather than the vector of binary actions. Indeed, equation (3) suggests a one-to-one relationship between ω_i and t_i^{th}. The idea is to show that the *best response map* is continuous over a convex and compact domain and thus it admits at least one fixed point. Let consider t^*_{-i} be the $(n-1)$-dimensional vector formed by the optimal thresholds excluding agent i. Note that t_i^{th} can be written as

$$t_i^{th} = -p_A + p_B - J_B + \frac{J_A + J_B}{N}\,\mathbb{I}_{\{t_i < t_i^{th}\}} + \frac{J_A + J_B}{N} \sum_{j \neq i} \mathbb{I}_{\{t_j < t_j^{th}\}}. \tag{20}$$

Consider two different vectors $t^{*,1}_{-i}$ and $t^{*,2}_{-i}$. As said, we want to show that t_i^{th} is continuous in t^*_{-i}. Equation (20) suggests that

$$\left|t_i^{th}(t^{*,1}_{-i}) - t_i^{th}(t^{*,2}_{-i})\right| \leq c\,\mathbb{P}\Big(\bigcup_{j\neq i} \min\{t^{*,1}_{-i}(j), t^{*,2}_{-i}(j)\} \leq t_j \leq \max\{t^{*,1}_{-i}(j), t^{*,2}_{-i}(j)\}\Big).$$

By continuity of the distribution of the types, the r.h.s. of the previous inequality goes to zero as $t^{*,1}_{-i} \to t^{*,2}_{-i}$. This ensures continuity of the best response map w.r.t the players' thresholds. Given that the best response map continuous on $\overline{\mathbb{R}}^n$, it admits at least one fixed point and so at least one Nash equilibrium in pure strategies exists.

Concerning the limit as $n \to \infty$ we need to introduce an aggregate variable. Consider equation (3), aggregate on the number of agents and divide by n to we obtain

$$\tilde{q}^n := \frac{1}{n} \sum_{i=1}^{n} \mathbb{I}_{\{w_i = A\}} = \frac{1}{n} \sum_{i=1}^{n} \mathbb{I}_{\{t_i < t_i^{th}\}}. \tag{21}$$

The sequence $(\tilde{q}^n)_n$, taking values on $[0,1]$, is tight. Therefore, all subsequences almost surely converge to a limit $q \in [0,1]$. It remains to show that the limits of such sequences are the solutions to the fixed point equation $F(t_m(q)) = q$. This at least at an heuristic level, follows directly from (21). As said, the l.h.s. converges to q. Concerning the r.h.s. it basically represents the *empirical distribution* of the n-dimensional system; it can be formally proved that, almost surely,

$$\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \mathbb{I}_{\{t_i < t_i^{th}\}} = F(t_m).$$

Therefore, for $n \to \infty$, we obtain $q = F(t_m(q))$.[19]

Proof of Proposition 3

Since the demands are linear and decreasing in their own prices, a Nash equilibrium in pure strategies exists. Moreover, because the profit functions concave and quadratic, the global maximum can be interior or at the boundary; in this latter case the firm is indifferent between selling nothing ($q = 0$) or setting the price equal to zero.

The *FOCs* are

$$\begin{cases} \dfrac{d\pi_A}{dp_A} = q + p_A \dfrac{dq}{dp_A} = 0 \\ \dfrac{d\pi_B}{dp_B} = 1 - q - p_B \dfrac{dq}{dp_B} = 0 \end{cases} \tag{22}$$

Replacing equation (8) on (22) and solving for p_K, $K = \{A; B\}$, the reaction functions of each firm is

$$\begin{cases} p_A = \dfrac{p_B - J_B + a}{2} \\ p_B = \dfrac{p_A - J_A + a}{2} \end{cases} \tag{23}$$

Solving the system of equations we obtain a unique pair of critical point

$$\begin{cases} p_A^* = \dfrac{3a - J_A - 2J_B}{3} \\ p_B^* = \dfrac{3a - 2J_A - J_B}{3} \end{cases} \tag{24}$$

Since

$$p_B^* - p_A^* = \frac{J_B - J_A}{3}, \tag{25}$$

[19] A formal proof of this limit is out of the scope of this paper. Details can be found in Dai Pra et al. (2013).

the critical market share q^* related to (p_A^*, p_B^*), resulting from equation (8), reads

$$q^* = \frac{3a - J_A - 2J_B}{3(2a - J_A - J_B)}. \tag{26}$$

According to equation (26), we are able to define the regions where the solution is feasible, i.e., $0 \leq q^* \leq 1$, depending on the parameters (J_A, J_B, a). Given that equation (7) holds, we have:

$$q^* < 0 \Leftrightarrow 3a < J_A + 2J_B; \tag{27}$$

$$0 \leq q^* \leq 1 \Leftrightarrow 3a \geq J_A + 2J_B \text{ and } 3a \geq 2J_A + J_B; \tag{28}$$

$$1 < q^* \Leftrightarrow 3a < 2J_A + J_B. \tag{29}$$

Indeed,

$$p_B^* = \frac{3a - 2J_A - J_B}{3} < 0 \Leftrightarrow 3a < 2J_A + J_B.$$

Moreover, because the demand of a good is decreasing in its price, region M is characterized by $p_A^* < p_A^M$, where p^M is the price such that $q = 1$. Following equation (7), $p_A^M = p_B + J_A - a$. Therefore, the critical price suggested by the *FOC*, p_A^*, is not admissible on region M because $q^* > 1$ is not feasible. Figure 2.A.1 represents π_A under the scenario depicted, respectively, by equation (28) (left panel) and (29) (right panel).[20] The price levels p_A^*, p_A^M and p_A^0 are represented on the horizontal axis with their corresponding levels of q.

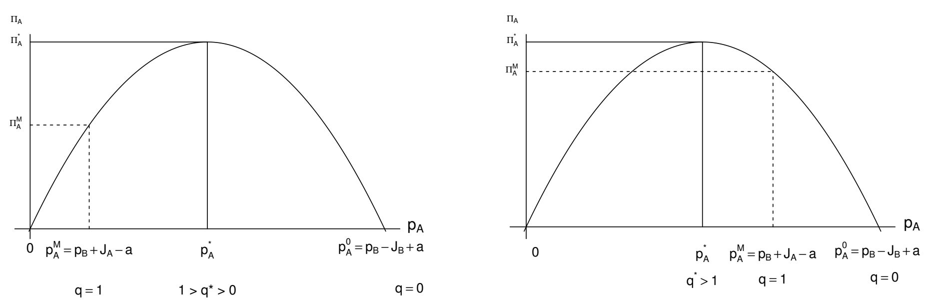


Figure 2.A.1: $\pi_A(p_A)$ in two different situations: when equation (28) holds (left panel) or when (29) holds (right panel).

[20] With the values of the parameters we are using, the scenario as in equation (27), where firm B becomes the monopolist, is not admissible.

Furthermore, region M is the only region where $p_A^* < p_A^M$ and so, the only region where $p_B^* < 0$. Indeed, all the following inequalities are equivalent:

$$\begin{aligned} p_A^* &< p_A^M \\ p_A^* &< p_B^* - a + J_A \\ p_B^* - (p_B^* - p_A^*) &< p_B^* - a + J_A \\ -\frac{J_B - J_A}{3} &< -a + J_A \\ 3a &< J_B + 2J_A. \end{aligned}$$

Finally, consider that firm's B negative price is not a feasible strategy; moreover, when $q^* > 1$, firm B has a negative profit for all its admissible prices. Therefore, the best strategy for firm B is the *boundary solution*, i.e., to choose $p_B = 0$ (or, equivalently, to be out of the market); conversely, firm A will choose its most profitable feasible price. In this case, because $p_A^* < p_A^M$ is not feasible and $p_A > p_A^M$ is not optimal, the most profitable choice for A is the monopolist price p_A^M. Figure 2.A.2 depicts the feasible prices (the non-shaded areas), in the two situations corresponding to parameters as for Figure 2.A.1.

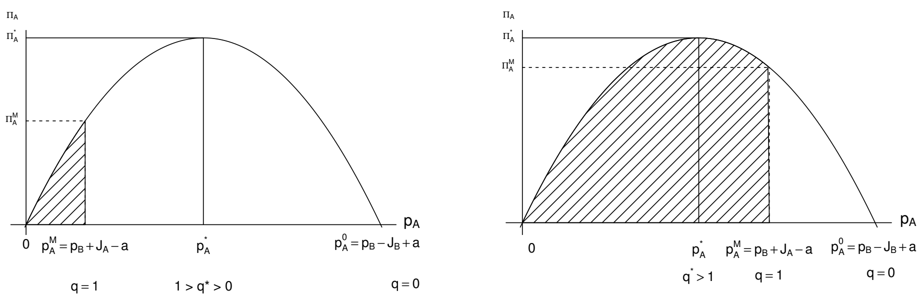


Figure 2.A.2: $\pi_A(p_A)$ in two different situations: when equation (28) holds (left panel) or when (29) holds (right panel). Unfeasible regions are shaded.

Therefore, considering that prices and quantities cannot be negative, firm B (A) will step out of market when $3a < J_B + 2J_A$ ($3a < J_A + 2J_B$), while, the competitor will take the most profitable feasible price p_A^M (p_B^M):

$$\begin{array}{llll} p_A^* = p_A^M = J_A - a & \text{and} & p_B^* = 0 & \text{when} \quad 3a \leq J_B + 2J_A; \\ p_B^* = p_B^M = J_B - a & \text{and} & p_A^* = 0 & \text{when} \quad 3a \leq J_A + 2J_B. \end{array} \tag{30}$$

Consequently, when equation (30) holds, either $q = 1$ or $q = 0$ and the market becomes a monopoly.

Proof of Lemma 1

It can be shown that the intermediate equilibrium θ is unstable. Indeed, assuming a monotonically increasing cumulative distribution function $F(\cdot)$, an equilibrium point is locally stable if the slope of the 45-degree line in Figure 2.A.3 exceeds that of the $F(\cdot)$ at the intersection of the diagram. Standard arguments[21] show that an equilibrium point is locally stable if and only if

$$\frac{dF(t_m(q))}{dq} < 1.$$

In our model,the stable equilibria are $q = \{0, 1\}$ because $dF(t_m(q))/dq = 0$, as soon as $q = \{0, 1\}$.

Conversely, condition (9) implies that any intermediate equilibrium θ, as in equation (11), is unstable. Indeed,

$$\frac{dF(\epsilon_m(q))}{dq} = \frac{J_A + J_B}{2a} > 1 \quad \text{when} \quad 0 < q < 1.$$

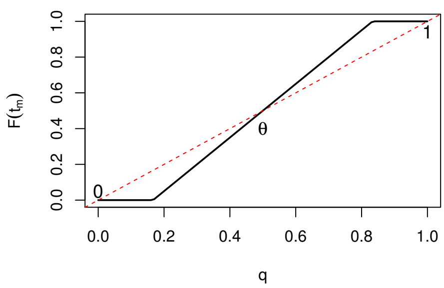


Figure 2.A.3: $F(t_m)$ as a function of q.

[21]See, for instance, Granovetter and Soong (1983) for an application in the context of threshold models.

Proof of Proposition 4

Although conceptually different, the algebraic form of the maximization problem looks very similar to the one of Proposition 3. The FOC read

$$\begin{cases} (1-\theta) - p_A \dfrac{d\theta}{dp_A} = 0 \\ \theta + p_B \dfrac{d\theta}{dp_B} = 0 \end{cases} \tag{31}$$

Therefore,

$$\begin{cases} p_A = \dfrac{p_B + J_A - a}{2} \\ p_B = \dfrac{p_A + J_B - a}{2} \end{cases} \tag{32}$$

Hence,

$$\begin{cases} p_A^* = \dfrac{2J_A + J_B - 3a}{3} \\ p_B^* = \dfrac{J_A + 2J_B - 3a}{3} \end{cases} \tag{33}$$

Finally, by replacing p_B^* - p_A^* = $\frac{J_B - J_A}{3}$ in the intermediate equilibrium of equation (10), we obtain

$$\theta^* = \frac{3a - J_A - 2J_B}{3(2a - J_A - J_B)}. \tag{34}$$

Thanks to equation (34), we identify two scenarios: in the first one, q^* can be either 1 or 0 with positive probability (corresponding to the case where $0 < \theta^* < 1$); in the second scenario, just one of the two border solutions is admissible. In this latter case, either firm A monopolizes the market, which happens when $E^*[Q] = 1 \Leftrightarrow 1 - \theta^* \geq 1 \Leftrightarrow \theta^* \leq 0$, or firm B monopolizes the market, when $1 - E^*[Q] = 1 \Leftrightarrow \theta^* \geq 1$. Specifically,

$$q^* = 1 \Leftrightarrow 3a \geq J_A + 2J_B \tag{35}$$

$$q^* = Q \Leftrightarrow 3a < J_A + 2J_B \text{ and } 3a < 2J_A + J_B \tag{36}$$

$$q^* = 0 \Leftrightarrow 3a \geq 2J_A + J_B \tag{37}$$

where Q is a Bernoullian random variable with parameter $(1 - \theta^*)$.

Similarly as before, for firm B (A) is optimal to be out of market when $3a \geq 2J_A + J_B$ ($3a \geq J_A + 2J_B$); conversely, the competitor takes the most profitable feasible price p_A^M (p_B^M), respectively. Therefore,

$$\begin{cases} p_A^* = p_A^M = J_A - a \quad \text{and} \quad p_B = 0 \quad \text{when} \quad 3a > J_B + 2J_A \\ p_B^* = p_B^M = J_B - a \quad \text{and} \quad p_A = 0 \quad \text{when} \quad 3a > J_A + 2J_B \end{cases} \tag{38}$$

Consequently, when $3a \geq 2J_A + J_B$ ($3a \geq J_A + 2J_B$), $q = 1$ ($q = 0$), firm A (B) becomes the monopolist.

3 Location model under firm-specific network effects

Marco Tolotti and Jorge Yépez

Abstract

In the present section we extend the model shown in Chapter 2 by letting firms decide on their location on the consumer's preference spectrum. On the first stage firms decide their location, then their price, and finally consumers decide between the two products. On the one hand, our analysis partially confirms what we see on the previous chapter, where under weak network effects, competition increases when network externalities increase, thus abating prices. Under this scenario, each firm differentiates as much as possible; therefore, each firm locates at an extreme of the interval and each firm serves a niche of the market. On the other hand, under *strong network effects*, both firms end up meeting at the center of the interval and only one succeeds in monopolizing the market. Under this regime, although the firm with the lowest network effect has a chance to monopolize the market, the strongest firm has a higher probability to monopolize the market, set higher prices, and expect higher profits.

3.1 Introduction

In Chapter 2 we modelled a duopoly where competing firms, differentiated by their levels of network effect, play a two-stage game in which, firstly, they simultaneously choose optimal prices and, secondly, potential adopters decide upon their preferred good. We fully characterized the equilibria expressed in prices and market shares. In the present section we extend this model by letting firms decide on their location on the consumer's preference spectrum. On the first stage firms decide their location, then their price, and finally consumers decide between the two products. Similarly to the previous model, it is natural to distinguish two regimes: under *weak network effects*, the two firms differentiate as much as possible; in other words, they locate at the extremes of the interval. Under *strong network effects* firms converge to the center of the interval, satisfying the interest of the median consumer.

This result is in line with the *median voter*[1] evidence where two political parties with different ideologies, right-wing and left-wing, converge to a central ideology, satisfying the median voter preference. In our model, under *strong network effects* only one firm survives, thus monopolizing the market; locating at the center of the interval of consumer preferences is therefore optimal for firms in order to increase their expected profits. Under this regime, and in Chapter 2, the firm with strong network effects has higher chances to monopolize the market, set higher prices, and expects higher profits.

We are inspired by the location model[2] of Hotelling (1929) where a duopoly model of horizontal differentiation, without social interactions, is presented. In this paper, goods characteristics and consumers preferences are represented by particular points in a unit interval. On the consumer side, preferences are uniformly distributed over an interval, while on the supply side, the two goods are represented by two points on the same interval. Consumer faces a binary choice problem in which they have to decide between the two mutually exclusive products and demand only one unit from the chosen good. The consumers' choice depends negatively on prices and the distance between the consumer's own preference and the position of the good. Therefore, assuming that there is no price competition on the market and the prices are fixed, each agent prefers to choose the product closer to its individual location. Hotelling showed that a unique Nash Equilibrium exists in which firms will locate at the same mid point on the interval and where both firms get the same market share. Therefore, the duopoly competition leads to minimal product differentiation[3]. d'Aspremont et al. (1979) extended Hotelling (1929) into a two stage game where firms first decide their location and then their prices. They found two effects: on the one hand, as one firm chooses to locate closer to its rival, it benefits in terms of the market share; on the other hand, when getting closer, the intensity in price competition increases, hence prices and profits deteriorate. This makes the firm to change its position and move away from its rival. As a matter of facts, product differentiation lowers price competition, enabling firms to make higher profits. In other words, firms gain an advantage by segmenting the market and locating themselves

[1] See Downs (1957) and Enelow and Hinich (1984) for a deeper description of the median model model.

[2] The location problem is sometimes refered as *adress model* or *characteristics model* (see Anderson et al. (1992))

[3] This model was also applied to the concept of electoral competition on political economy models. The interval can be interpreted as political preferences towards a party platform or a public policy. Thus, defining consumers as voters and firms as political parties, the model concludes on the convergence of political platforms to the median voter's preferred outcome.

in opposite locations, thus performing a degree of monopoly on their niche. Firms market power and prices increase with the distance between firms and an equilibrium where firms locate at the extremes of the interval emerges.

The aim of this chapter is to incorporate network effects in the setting of d'Aspremont et al. (1979). Firms first decide their location, then their prices, and finally consumers choose a product. We assume that prices are determined by the two competitors, say A and B, by playing a Betrand duopoly game. However, we include the idea of location, in which firms chooses their position in advance. Denote by α and β the location of firms A and B respectively, and by q the proportion of agents choosing A; we analyse the subgame perfect Nash equilibria $(p_A^*,\, p_B^*,\, q^*, \alpha^*, \beta^*)$ emerging in the economy. Because the model consists on three stages, we proceed backwards, $q^* \Rightarrow (p_A^*, p_B^*) \Rightarrow (\alpha^*, \beta^*)$. On the next section we review the consumer choice game, which is the third stage of the game. In section 3.3 we go over the optimal price strategy of the firms, taken location as given; whereas in section 3.4 we analyse the first stage in which firms optimize over their location. In section 3.5 we discuss the market equilibria obtained. Finally, in section 3.6 we conclude.

3.2 The consumer choice game

In order to match the aforementioned location literature, we slightly review the formation of our model. More precisely, consumer preferences are uniformly distributed over an interval , $t \sim U\,[-\sigma, \sigma]$, and firms A and B launch their products over this space. Firms products are placed at two points, α and $\sigma - \beta$[4], respectively for firm A and B. We assume that each firm represents a segment of the population preferences, therefore, firm A can locate its product at any place on the left side of the interval, $\alpha \in [-\sigma, 0]$, while firm B can choose any point on the right side of the interval, $\beta \in [0, \sigma]$.

[4]Therefore, α (β) represent the distance of the firm to the left (right) extreme.

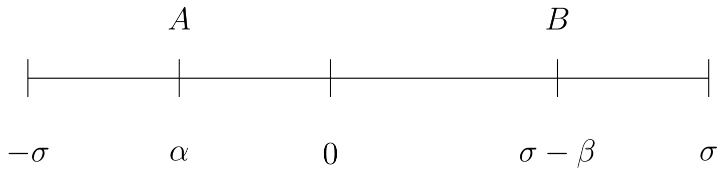


Figure 3.1: Address Interval in which firms A and firm B locate their products on the positions α and $\sigma - \beta$ respectively; and consumers preferences are uniformly distributed over $t \sim U\,[-\sigma, \sigma]$.

It is worth noting that two extreme scenarios could emerge. Firms can choose *maximal differentiation* and be located at each extreme of the consumers preferences, as we can see in the left panel of Figure 3.2, where $\alpha = -\sigma$ and $\beta = 0$; and *no differentiation*, where firms converge to the center of the interval, as in the right panel of Figure 3.2 where $\alpha = 0$ and $\beta = \sigma$.

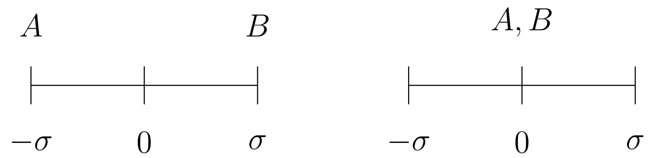


Figure 3.2: Maximal differentiation (left panel) and No differentiation (right panel)

As before, we proceed by solving the game backward, assuming that prices and locations are fixed. Consider two firms $\{A, B\}$ selling a homogeneous product to a large population of n potential buyers. Each agent, indexed by $i = 1, \ldots, n$, has to decide between two mutually exclusive actions $w_i = \{A,\, B\}$: buying product A or buying product B, and each action yields an indirect utility $V_i(w_i, t_i)$:

$$\begin{aligned} V_i(A, t_i) &= Y - p_A + J_A q^i - \tau(t_i - \alpha)^2 \\ V_i(B, t_i) &= Y - p_B + J_B(1 - q^i) - \tau(\sigma - \beta - t_i)^2. \end{aligned} \tag{1}$$

The terms $\{p_A, p_B, J_A, J_B, q^i\}$ preserve the same meaning as described in Chapter 2 (see section 2.2 for details on the utility structure of the agents.). However, we slightly modify the interpretation of the random type t_i describing the personal taste/preference of agent i. It represents the position of the individual *ideal* preference on the interval where firms A and B are located. The last term of the utility represents now a quadratic cost: the euclidean distance between the consumer's ideal preference point and the location of each good. The parameter $\tau > 0$ measures the consumer sensibility to distance in this linear space, which can be interpreted as the psychological cost or the cognitive dissonance that a consumer faces when buying a product far from her own individual taste, t_i. This quadratic cost is in line with d'Aspremont et al. (1979) and Grilo et al. (2001).

Therefore, similarly as in the Chapter 2,

$$\begin{aligned} &V_i(A, t_i) > V_i(B, t_i) \iff \\ &\quad - p_A + p_B + \tau \left[(\sigma - \beta - t_i)^2 - (t_i - \alpha)^2\right] + J_A\, q^i - J_B\, (1 - q^i) > 0 \end{aligned}$$

By defining agent-specific threshold levels $t_i^{th} = \frac{p_B - p_A + \tau\left[(\sigma-\beta)^2 - \alpha^2\right] + J_A\, q^i - J_B\,(1-q^i)}{2\tau(\sigma-\beta-\alpha)}$, it turns out that, for each $i = 1, \ldots, n$,

$$w_i = A \iff t_i < t_i^{th}. \tag{2}$$

The vector of binary actions translates, therefore, into a vector of real thresholds $(t_i^{th})_{i \in n} \in \bar{\mathbb{R}}^n$. Being the *best response map* continuous on the compact and convex domain $\bar{\mathbb{R}}^n$, it admits at least one fixed point, so that at least one Nash equilibrium in pure strategies exists. As corollary of Proposition 1, in Chapter 2, we obtained the following:

Corollary 1 *Consider the n-player game with payoff structure expressed by thresholds as in (2). At least one Nash equilibrium in pure strategies exists. Moreover, when $n \to \infty$, it is possible to define a marginal agent, signalled by her type t_m, indifferent between A or B. Indeed,*

$$t_m(q) = \frac{p_B - p_A + \tau\left[(\sigma - \beta)^2 - \alpha^2\right] + J_A\, q - J_B\,(1 - q)}{2\tau(\sigma - \beta - \alpha)}, \tag{3}$$

where $q = lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \mathbb{I}_{\{w_i = A\}} = lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \mathbb{I}_{\{t_i < t_i^{th}\}}$.

Finally, when $n \to \infty$, self-consistency requires $\mathbb{P}(t_i \leq t_m) = q$. We thus obtain

Corollary 2 *Any Nash equilibrium of the infinite-player game satisfies the consistency (fixed point) equation*

$$F(t_m(q)) = q, \tag{4}$$

where F is the distribution function of the types of agents.

Finally, we analyse the situation where the types $(t_i)_{i=1,\dots,n}$ are independent and uniformly distributed on the interval $[-\sigma, \sigma]$.

$$F(t_m(q)) = \begin{cases} 0 & \text{if} \quad t_m \leq -\sigma \\ \frac{t_m+a}{2a} & \text{if} \quad -\sigma \geq t_m \geq \sigma \\ 1 & \text{if} \quad \sigma \geq t_m \end{cases} \tag{5}$$

The decision for an action is based upon the comparison between the individual random type t_i and the threshold t_m. It follows from equation (2) that the population splits in a way that all agents with $t_i \leq t_m$ will choose technology A, and all agents with $t_j > t_m$ will choose B.

3.2.1 Demand under weak network effects

The first case, referred to as *weak network effects*, describes the situation where the sum of the network effect parameters does not overpass the size of the support of the distribution of types:

$$J_A + J_B < 4\sigma\tau(\sigma - \beta - \alpha). \tag{6}$$

Note that, in this setting, the upper bound for the network strength, $4\sigma\tau(\sigma - \beta - \alpha)$, depends on the decisional variables, thus making the analysis more involved in some sense: the firms may "decide" if they want to stay in the *weak* or the *strong* regime.

Under assumption (6), it is easy to derive an explicit expression for q, the solution of (4) as a function of the locations, prices and the other parameters of the model. Eventually,

$$q = \begin{cases} 0 & \text{if} \quad p_B - p_A \leq J_B - \tau\left[(\sigma-\beta)^2 - \alpha^2\right] - 2\tau\sigma(\sigma-\beta-\alpha) \\ 1 & \text{if} \quad p_B - p_A \geq -J_A - \tau\left[(\sigma-\beta)^2 - \alpha^2\right] + 2\tau\sigma(\sigma-\beta-\alpha) \\ \theta & \text{otherwise} \end{cases} \tag{7}$$

Where

$$\theta = \frac{p_B - p_A - J_B + \tau\left[(\sigma-\beta)^2 - \alpha^2\right] + 2\tau\sigma(\sigma-\beta-\alpha)}{4\tau\sigma(\sigma-\beta-\alpha) - J_B - J_A}. \tag{8}$$

It is worth noting that equation (6) ensures that $-\tau\left[(\sigma-\beta)^2-\alpha^2\right]+2\sigma\tau(\sigma-\beta-\alpha)-J_A > -\tau\left[(\sigma-\beta)^2-\alpha^2\right]-2\sigma\tau(\sigma-\beta-\alpha)+J_B$. Therefore, once prices have been fixed, the equilibrium level q for the market share is unique. Figure 3.3 describes the demand of technology A, according to equation (7). Demand q decreases with p_A and increases with p_B, yielding traditional negative own-price elasticity and positive cross-price elasticity.

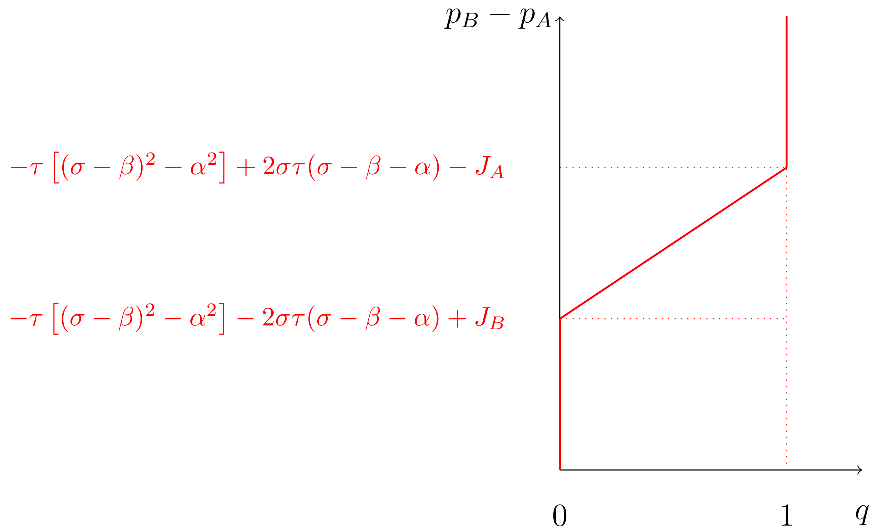


Figure 3.3: Self-consistent expectations demand for good A under *Weak Network Effects*.

3.2.2 Demand under strong network effect

The second situation deals with *strong network effects*, which are in place when

$$J_A + J_B > 4\sigma\tau(\sigma-\beta-\alpha). \tag{9}$$

Under condition (9), the values of q solving equation (4) are

$$q = \begin{cases} 0 & \text{if} \quad p_B - p_A \leq J_B - \tau\left[(\sigma-\beta)^2-\alpha^2\right] - 2\tau\sigma(\sigma-\beta-\alpha) \\ 1 & \text{if} \quad p_B - p_A \geq -J_A - \tau\left[(\sigma-\beta)^2-\alpha^2\right] + 2\tau\sigma(\sigma-\beta-\alpha) \\ \{0;\theta;1\} & \text{otherwise,} \end{cases} \tag{10}$$

where θ as in 8. Similarly to what we see in Chapter 2, this situation offers a very different picture compared to the case of weak network effects. When equation (9) holds, $-\tau\left[(\sigma-\beta)^2-\alpha^2\right]+2\sigma\tau(\sigma-\beta-\alpha)-J_A < -\tau\left[(\sigma-\beta)^2-\alpha^2\right]-2\sigma\tau(\sigma-\beta-\alpha)+J_B$; therefore, as depicted on Figure 3.4, for intermediate prices ($J_B-\tau\left[(\sigma-\beta)^2-\alpha^2\right]-2\tau\sigma(\sigma-\beta-\alpha) < p_B-p_A < -J_A-\tau\left[(\sigma-\beta)^2-\alpha^2\right]+2\tau\sigma(\sigma-\beta-\alpha)$), three self-consistent equilibria coexist. In particular, for a given combination of prices $\hat{p}=p_B-p_A$, the two extreme equilibria ($q=0$ and $q=1$) are possible plus an intermediate equilibrium $\theta\in(0,1)$.

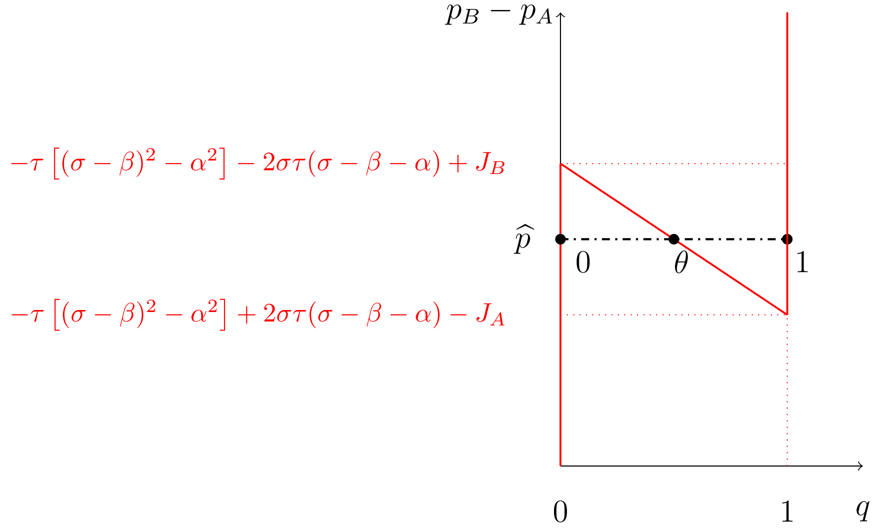


Figure 3.4: Self-consistent expectations demand q under strong network effects.

As before, it can be shown that the intermediate equilibrium θ is unstable in the sense of q being the fixed point (long-run attraction) of the best response map $q\to F(t_m(q))$.

Therefore, we can better specify the emerging equilibrium in a probabilistic sense as follows:

$$Q=\begin{cases} 0 & \text{with probability} \quad \theta \\ 1 & \text{with probability} \quad 1-\theta \end{cases} \tag{11}$$

where θ is as defined in (8).

Summarizing, under strong network effects and intermediate prices, the social influence is so strong that the population ends up buying one of the goods unanimously, although we cannot predict with certainty which one of the two will prevail. For a given value of p_B, the larger is p_A, the higher is the probability that the population ends up buying the technology B. Similarly, for a given value of J_B, the lower is J_A, the higher is the probability that the lower stable equilibrium, $q = 0$, realizes.

We summarize the outcome of the consumer choice game in the following

Proposition 5 *Assume that the type distribution is uniform with support* $[-\sigma, \sigma]$. *Define*

$$\theta = \frac{p_B - p_A - J_B + \tau\left[(\sigma-\beta)^2 - \alpha^2\right] + 2\tau\sigma(\sigma-\beta-\alpha)}{4\tau\sigma(\sigma-\beta-\alpha) - J_B - J_A}.$$

The stable self-consistent Nash equilibria of the infinite-players choice game are as follows.

- *Weak Network Effects. If* $J_A + J_B < 4\sigma\tau(\sigma-\beta-\alpha)$, *q is such that*

$$q = \begin{cases} 0 & \text{if} \quad p_B - p_A \leq J_B - \tau\left[(\sigma-\beta)^2 - \alpha^2\right] - 2\tau\sigma(\sigma-\beta-\alpha) \\ 1 & \text{if} \quad p_B - p_A \geq -J_A - \tau\left[(\sigma-\beta)^2 - \alpha^2\right] + 2\tau\sigma(\sigma-\beta-\alpha) \\ \theta & \text{otherwise} \end{cases} \tag{12}$$

- *Strong Network Effects. If* $J_A + J_B > 4\sigma\tau(\sigma-\beta-\alpha)$, *q is such that*

$$q = \begin{cases} 0 & \text{if} \quad p_B - p_A \leq J_B - \tau\left[(\sigma-\beta)^2 - \alpha^2\right] - 2\tau\sigma(\sigma-\beta-\alpha) \\ 1 & \text{if} \quad p_B - p_A \geq -J_A - \tau\left[(\sigma-\beta)^2 - \alpha^2\right] + 2\tau\sigma(\sigma-\beta-\alpha) \\ Q & \text{otherwise} \end{cases} \tag{13}$$

  where Q is a Bernoulli random variable with parameter $(1-\theta)$ *as described in equation* (11).

3.3 The two-player Bertrand competition

Up to now, the prices of the two technologies have been treated as fixed parameters. We now model the second stage of the game where firms are in charge to choose their prices in a two-player non-cooperative game. As before, it is convenient to separate the two cases related to the strength of the network effects. At this stage, we take the location of the firms as given.

3.3.1 Supply under weak network effects

Firms select their price on a strategy space of admissible prices $p_K \subseteq [0, \infty)$, where $K = \{A, B\}$. We define the firms' profits $\{\pi_A, \pi_B\}$ as the normalized per-capita profit of firm A and B respectively. These profits depend on the firms prices $\{p_A, p_B\}$ and the market shares $\{q, (1-q)\}$:

$$\begin{aligned} \pi_A =& p_A \cdot q\,; \\ \pi_B =& p_B \cdot (1-q). \end{aligned} \tag{14}$$

We search for a Nash equilibrium in pure strategies, meaning that a price equilibrium is a pair $\{p_A^*;\, p_B^*\}$, such that each firm K maximises its own profit with respect to p_K conditioned upon the (optimal) price of the competitor. p_A^* and p_B^* are such that

$$\begin{aligned} \pi_A(p_A^*, p_B^*) \geq \pi_A(p_A, p_B^*), \quad \text{for all } p_A \subseteq [0, \infty)\,; \\ \pi_B(p_B^*, p_A^*) \geq \pi_B(p_B, p_A^*), \quad \text{for all } p_B \subseteq [0, \infty)\,. \end{aligned}$$

Replacing equation (7) in equation (14), under weak network effects as in equation (6), we get continuous quadratic and concave functions in prices. Assuming that the competitor's price is fixed, with a slight abuse of notations, we write, respectively,

$$\pi_A(p_A) = p_A \left(\frac{p_B - J_B + \tau\left[(\sigma-\beta)^2 - \alpha^2\right] + 2\tau\sigma(\sigma-\beta-\alpha)}{4\tau\sigma(\sigma-\beta-\alpha) - J_B - J_A} \right) - \frac{p_A^2}{4\tau\sigma(\sigma-\beta-\alpha) - J_B - J_A}; \tag{15}$$

$$\pi_B(p_B) = p_B \left(\frac{p_A - J_A - \tau\left[(\sigma-\beta)^2 - \alpha^2\right] + 2\tau\sigma(\sigma-\beta-\alpha)}{4\tau\sigma(\sigma-\beta-\alpha) - J_B - J_A} \right) - \frac{p_B^2}{4\tau\sigma(\sigma-\beta-\alpha) - J_B - J_A}. \tag{16}$$

Therefore, there must be two values for which π_A (π_B resp.) are zero:

$$\begin{aligned}
\pi_A(p_A) = 0 \Leftrightarrow p_A = 0 \text{ or } p_A^0 = p_B - J_B + \tau\left[(\sigma-\beta)^2 - \alpha^2\right] + 2\tau\sigma(\sigma-\beta-\alpha);\\
\pi_B(p_B) = 0 \Leftrightarrow p_B = 0 \text{ or } p_B^0 = p_A - J_A - \tau\left[(\sigma-\beta)^2 - \alpha^2\right] + 2\tau\sigma(\sigma-\beta-\alpha).
\end{aligned} \tag{17}$$

From equation (7), we infer that, when $p_A^0 = p_B - J_B + \tau\left[(\sigma-\beta)^2 - \alpha^2\right] + 2\tau\sigma(\sigma-\beta-\alpha)$, then $q = 0$. Therefore, in this case, firm B becomes a monopolist. Conversely, when $p_B^0 = p_A - J_A - \tau\left[(\sigma-\beta)^2 - \alpha^2\right] + 2\tau\sigma(\sigma-\beta-\alpha)$, the market share is $q = 1$, letting firm A be the monopolist.

Since the demands are linear and decreasing in their own prices, a Nash equilibrium in pure strategies exists. Moreover, being the profit functions concave and quadratic, the global maximum can be either at the critical point (where the *first order conditions (FOC)* are met) or where equation (17) is satisfied (*boundary solution (BS)*); in this latter case, firm is indifferent between selling nothing ($q = 0$) or setting the price equal to zero.

The *FOC*, applied to equation (14), read

$$\begin{cases}
\dfrac{\partial \pi_A}{\partial p_A} = q + p_A \dfrac{\partial q}{\partial p_A} = 0\\
\dfrac{\partial \pi_B}{\partial p_B} = 1 - q - p_B \dfrac{\partial q}{\partial p_B} = 0
\end{cases} \tag{18}$$

Replacing equation (7) on (18) and solving for p_K, $K = \{A; B\}$, we get the reaction functions of each firm

$$\pi_A(p_A) = \frac{p_B - J_B + \tau\left[(\sigma-\beta)^2 - \alpha^2\right] + 2\tau\sigma(\sigma-\beta-\alpha)}{2}; \tag{19}$$

$$\pi_B(p_B) = \frac{p_A - J_A - \tau\left[(\sigma-\beta)^2 - \alpha^2\right] + 2\tau\sigma(\sigma-\beta-\alpha)}{2}. \tag{20}$$

Solving the system of equations we obtain the unique critical point

$$\begin{aligned}
p_A^* &= \frac{6\sigma\tau(\sigma-\beta-\alpha) + \tau\left[(\sigma-\beta)^2 - \alpha^2\right] - J_A - 2J_B}{3}\\
p_B^* &= \frac{6\sigma\tau(\sigma-\beta-\alpha) - \tau\left[(\sigma-\beta)^2 - \alpha^2\right] - 2J_A - J_B}{3},
\end{aligned} \tag{21}$$

Since
$$p_B^* - p_A^* = \frac{J_B - J_A - 2\tau\left[(\sigma-\beta)^2 - \alpha^2\right]}{3}, \tag{22}$$
the critical market share q^* related to (p_A^*, p_B^*), resulting from equation (7), reads

$$q^* = \frac{6\sigma\tau(\sigma-\beta-\alpha) + \tau\left[(\sigma-\beta)^2 - \alpha^2\right] - J_A - 2J_B}{3(4\sigma\tau(\sigma-\beta-\alpha) - J_A - J_B)} \tag{23}$$

According to equation (23), we are able to define the regions where the solution is feasible, i.e., $0 \leq q^* \leq 1$, depending on the parameters $(J_A, J_B, \alpha, \beta, \tau, \sigma)$. Given that equation (6) holds, we have:

$$q^* < 0 \Leftrightarrow 6\sigma\tau(\sigma-\beta-\alpha) + \tau\left[(\sigma-\beta)^2 - \alpha^2\right] < J_A + 2J_B; \tag{24}$$
$$1 < q^* \Leftrightarrow 6\sigma\tau(\sigma-\beta-\alpha) - \tau\left[(\sigma-\beta)^2 - \alpha^2\right] < 2J_A + J_B. \tag{25}$$
$$0 \leq q^* \leq 1 \Leftrightarrow \quad \text{otherwise;} \tag{26}$$

As in the Chapter 2, it can be shown that considering that prices and quantities cannot be negative, firm B will "decide" to step out of market when $6\sigma\tau(\sigma-\beta-\alpha) - \tau\left[(\sigma-\beta)^2 - \alpha^2\right] < 2J_A + J_B$, while, on the meantime, the competitor will take the most profitable feasible price p_A^M. Similarly, firm A will do the same when $6\sigma\tau(\sigma-\beta-\alpha) + \tau\left[(\sigma-\beta)^2 - \alpha^2\right] < J_A + 2J_B$ while, on the meantime, the competitor will take the most profitable feasible price p_B^M. Therefore,

- when $6\sigma\tau(\sigma-\beta-\alpha) - \tau\left[(\sigma-\beta)^2 - \alpha^2\right] < 2J_A + J_B$
  $p_B^* = 0$ and $p_A^* = p_A^M = J_A + \tau\left[(\sigma-\beta)^2 - \alpha^2\right] - 2\tau\sigma(\sigma-\beta-\alpha)$
- when $6\sigma\tau(\sigma-\beta-\alpha) + \tau\left[(\sigma-\beta)^2 - \alpha^2\right] < J_A + 2J_B$
  $p_A^* = 0$ and $p_B^* = p_B^M = J_B - \tau\left[(\sigma-\beta)^2 - \alpha^2\right] - 2\tau\sigma(\sigma-\beta-\alpha)$

Consequently, either $q = 1$ or $q = 0$ and the market becomes a monopoly. We summarize the results related to the supply under weak externality effects in the following

Proposition 6 *Consider the model where q is described by equation* (7) *and two firms, A and B, simultaneously optimize their profits as in* (14)*. Assume, moreover, that equation* (6) *holds, and that α and β are fixed, then the optimal prices p_A^* and p_B^* are*

1. *If* $6\sigma\tau(\sigma-\beta-\alpha)+\tau\left[(\sigma-\beta)^2-\alpha^2\right]\geq J_A+2J_B$ *and* $6\sigma\tau(\sigma-\beta-\alpha)-\tau\left[(\sigma-\beta)^2-\alpha^2\right]\geq 2J_A+J_B$, *then*

   $p_A^*=\frac{6\sigma\tau(\sigma-\beta-\alpha)+\tau\left[(\sigma-\beta)^2-\alpha^2\right]-J_A-2J_B}{3}$;
   $p_B^*=\frac{6\sigma\tau(\sigma-\beta-\alpha)-\tau\left[(\sigma-\beta)^2-\alpha^2\right]-2J_A-J_B}{3}$.

   In this case,

   $q^*=\frac{6\sigma\tau(\sigma-\beta-\alpha)+\tau\left[(\sigma-\beta)^2-\alpha^2\right]-J_A-2J_B}{3(4\sigma\tau(\sigma-\beta-\alpha)-J_A-J_B)}$.

2. *If* $6\sigma\tau(\sigma-\beta-\alpha)-\tau\left[(\sigma-\beta)^2-\alpha^2\right]\leq 2J_A+J_B$, *then* $p_A^*=p_A^M=J_A-2\sigma\tau(\sigma-\beta-\alpha)+\tau\left[(\sigma-\beta)^2-a^2\right]$, $p_B^*=0$ *and* $q^*=1$. *Therefore, firm A monopolizes the market.*

3. *If* $6\sigma\tau(\sigma-\beta-\alpha)+\tau\left[(\sigma-\beta)^2-\alpha^2\right]\leq J_A+2J_B$, *then* $p_B^*=p_B^M=J_B-2\sigma\tau(\sigma-\beta-\alpha)-\tau\left[(\sigma-\beta)^2-a^2\right]$; $p_A^*=0$ *and* $q^*=0$. *Therefore, firm B monopolizes the market;*

3.3.2 Supply under strong network effects

As already discussed in section 3.2.2, when equation (9) holds, *strong network effects* are in place and multiple equilibria coexist for intermediate prices. In this situation, as seen in proposition 5, the prevailing market share Q is a proper random variable taking value $q=0$ with probability θ and value $q=1$ with probability $1-\theta$.
Therefore, it seems natural to assume that, in this case, firms A and B maximize their *expected profits*, which turn out to be, repsectively,

$$\begin{aligned}\mathbb{E}(\pi_A) &= p_A\cdot\mathbb{E}\left[Q\right]\\ &= p_A\cdot(\theta\cdot 0+(1-\theta)\cdot 1)\\ &= p_A\cdot(1-\theta)\\ &= p_A\cdot\left(\frac{p_A-p_B-J_A+a}{2a-J_A-J_B}\right);\end{aligned}\tag{27}$$

$$\begin{aligned}\mathbb{E}(\pi_B) &= p_B\cdot(1-\mathbb{E}[Q])\\ &= p_B\cdot(1-(\theta\cdot 0+(1-\theta)\cdot 1))\\ &= p_B\cdot\theta\\ &= p_B\cdot\left(\frac{p_B-p_A-J_B+a}{2a-J_A-J_B}\right).\end{aligned}\tag{28}$$

Similarly as before we obtain the following prices:

$$p_A^* = \frac{2J_A + J_B + \tau\left[(\sigma-\beta)^2 - \alpha^2\right] - 6\sigma\tau(\sigma-\beta-\alpha)}{3}$$
$$p_B^* = \frac{J_A + 2J_B - \tau\left[(\sigma-\beta)^2 - \alpha^2\right] - 6\sigma\tau(\sigma-\beta-\alpha)}{3} \quad (29)$$

And the following value for θ^*:

$$\theta^* = \frac{6\sigma\tau(\sigma-\beta-\alpha) + \tau\left[(\sigma-\beta)^2 - \alpha^2\right] - J_A - 2J_B}{3(4\sigma\tau(\sigma-\beta-\alpha) - J_A - J_B)}. \quad (30)$$

Thanks to equation (30), we identify two scenarios: in the first one, q^* can be either 1 or 0 with positive probability (corresponding to the case where $0 < \theta^* < 1$); in the second scenario, just one of the two border solution is admissible. In this latter case, either firm A monopolizes the market, which happens when $E^*[Q] = 1 \Leftrightarrow 1 - \theta^* \geq 1 \Leftrightarrow \theta^* \leq 0$, or firm B monopolizes the market, when $1 - E^*[Q] = 1 \Leftrightarrow \theta^* \geq 1$. More in details,

$$q^* = 0 \Leftrightarrow 6\sigma\tau(\sigma-\beta-\alpha) - \tau\left[(\sigma-\beta)^2 - \alpha^2\right] > 2J_A + J_B; \quad (31)$$

$$q^* = 1 \Leftrightarrow 6\sigma\tau(\sigma-\beta-\alpha) + \tau\left[(\sigma-\beta)^2 - \alpha^2\right] > J_A + 2J_B. \quad (32)$$

$$q^* = Q \Leftrightarrow otherwise \quad (33)$$

As in Chapter 2, it can be shown that, firm B will "decide" to step out of market when $6\sigma\tau(\sigma-\beta-\alpha) + \tau\left[(\sigma-\beta)^2 - \alpha^2\right] > J_A + 2J_B$, while, on the meantime, the competitor will take the most profitable feasible price p_A^M. Similarly, firm A will do the same when $6\sigma\tau(\sigma-\beta-\alpha) - \tau\left[(\sigma-\beta)^2 - \alpha^2\right] > 2J_A + J_B$ while, on the meantime, the competitor will take the most profitable feasible price p_B^M. Therefore,

- when $6\sigma\tau(\sigma-\beta-\alpha) + \tau\left[(\sigma-\beta)^2 - \alpha^2\right] > J_A + 2J_B$
  $p_B^* = 0$ and $p_A^* = p_A^M = J_A + \tau\left[(\sigma-\beta)^2 - \alpha^2\right] - 2\tau\sigma(\sigma-\beta-\alpha)$
- when $6\sigma\tau(\sigma-\beta-\alpha) - \tau\left[(\sigma-\beta)^2 - \alpha^2\right] > 2J_A + J_B$
  $p_A^* = 0$ and $p_B^* = p_B^M = J_B - \tau\left[(\sigma-\beta)^2 - \alpha^2\right] - 2\tau\sigma(\sigma-\beta-\alpha)$

The following proposition summarizes the outcome of the model under strong network effects.

Proposition 7 *Consider the model where q is described by equation* (10) *and two firms, A and B, simultaneously optimize their expected profits as in* (27)-(28).
Assume, moreover, that equation (9) *holds, and that α and β are given, then the following optimal prices p_A^* and p_B^* emerge:*

1. *If* $6\sigma\tau(\sigma-\beta-\alpha)+\tau\left[(\sigma-\beta)^2-\alpha^2\right] \leq J_A+2J_B$ *and* $6\sigma\tau(\sigma-\beta-\alpha)-\tau\left[(\sigma-\beta)^2-\alpha^2\right] \leq 2J_A+J_B$, *then*

   $$p_A^* = \frac{2J_A+J_B+\tau\left[(\sigma-\beta)^2-\alpha^2\right]-6\sigma\tau(\sigma-\beta-\alpha)}{3};$$
   $$p_B^* = \frac{J_A+2J_B-\tau\left[(\sigma-\beta)^2-\alpha^2\right]-6\sigma\tau(\sigma-\beta-\alpha)}{3}.$$

   Moreover, with probability θ^*, firm B monopolizes the market and, with probability $1-\theta^*$, firm A monopolizes the market.

2. *If* $6\sigma\tau(\sigma-\beta-\alpha)+\tau\left[(\sigma-\beta)^2-\alpha^2\right] > J_A+2J_B$, *then* $p_A^* = p_A^M = J_A+\tau\left[(\sigma-\beta)^2-\alpha^2\right]-2\tau\sigma(\sigma-\beta-\alpha)$, $p_B^*=0$ *and* $q^*=1$. *Therefore, firm A monopolizes the market.*

3. *If* $6\sigma\tau(\sigma-\beta-\alpha)-\tau\left[(\sigma-\beta)^2-\alpha^2\right] > 2J_A+J_B$, *then* $p_B^* = p_B^M = J_B-\tau\left[(\sigma-\beta)^2-\alpha^2\right]-2\tau\sigma(\sigma-\beta-\alpha)$; $p_A^*=0$ *and* $q^*=0$. *Therefore, firm B monopolizes the market;*

3.4 Hotelling-Bertrand location model

Up to now we have considered (α,β) as given; however, these are decisional variables that firms choose in order to maximize their profits. As we described before, firms decide on their product location. In particular, in the first stage firm A chooses α and at the same time firm B chooses β. On the second stage, after observing their product type chosen in the first stage, firms choose prices simultaneously. In this section we solve the first stage of the game in which firms decide the their optimal locations (α^*,β^*).

3.4.1 Location under weak network effects

On the first stage, firms select their location strategy on the space of admissible locations. Firm's A chooses $\alpha \in [-\sigma, 0]$, and firm B, $\beta \in [0,\sigma]$[5]. Both firms search for a Nash equilibrium in pure strategies, meaning that a location equilibrium is a pair $(\alpha^*,\ \beta^*)$, such that each firm maximises its own

[5] $1-\beta \in [0,\sigma] \Leftrightarrow \beta \in [0,\sigma]$

profit with respect it's location conditioned upon the (optimal) location of the competitor.

$$\pi_A(\alpha^*, \beta^*) \geq \pi_A(\alpha, \beta^*), \quad \text{for all } \alpha \in [-\sigma, 0]\,;$$
$$\pi_B(\beta^*, \alpha^*) \geq \pi_B(\beta, \alpha^*), \quad \text{for all } \beta \in [0, \sigma]\,.$$

Let's assume that $0 < q^* < 1$, so the optimal prices and quantities are given by:

$p_A^*(\alpha,\beta) = \frac{6\sigma\tau(\sigma-\beta-\alpha)+\tau\left[(\sigma-\beta)^2-\alpha^2\right]-J_A-2J_B}{3};$
$p_B^*(\alpha,\beta) = \frac{6\sigma\tau(\sigma-\beta-\alpha)-\tau\left[(\sigma-\beta)^2-\alpha^2\right]-2J_A-J_B}{3};$
$q^*(\alpha,\beta) = \frac{6\sigma\tau(\sigma-\beta-\alpha)+\tau\left[(\sigma-\beta)^2-\alpha^2\right]-J_A-2J_B}{3(4\sigma\tau(\sigma-\beta-\alpha)-J_A-J_B)}.$

By substituting the previous expressions in the profit function ($\pi = p \cdot q$), we obtain the profits defined in terms of (α, β):

$$\begin{aligned}
\pi_A^*(\alpha,\beta) &= \frac{\left(6\sigma\tau(\sigma-\beta-\alpha)+\tau\left[(\sigma-\beta)^2-\alpha^2\right]-J_A-2J_B\right)^2}{9(4\sigma\tau(\sigma-\beta-\alpha)-J_A-J_B)};\\
\pi_B^*(\beta,\alpha) &= \frac{\left(6\sigma\tau(\sigma-\beta-\alpha)-\tau\left[(\sigma-\beta)^2-\alpha^2\right]-2J_A-J_B\right)^2}{9(4\sigma\tau(\sigma-\beta-\alpha)-J_A-J_B)},
\end{aligned} \tag{34}$$

The FOCs read:

$$\frac{\partial \pi_A^*}{\partial \alpha} = -4\tau q^*\left(\sigma(1-q^*)+\frac{\alpha^*}{3}\right) = 0 \Leftrightarrow \alpha^* = -3\sigma(1-q^*) \tag{35}$$

$$\frac{\partial \pi_B^*}{\partial \beta} = -4\tau(1-q^*)\left(\sigma q^* - \frac{\sigma-\beta}{3}\right) = 0 \Leftrightarrow \beta^* = 3\sigma(1-3q^*) \tag{36}$$

where the later equivalence follows since, by assumption, $q^* \in (0,1)$. Replacing (35) in equation (36), we find a necessary condition on (α^*, β^*):

$$\beta^* = -(2\sigma - \alpha^*). \tag{37}$$

By rearranging the previous expressions, we obtain

$$\alpha^* = -\sigma\left(1 + \frac{3\tau\sigma^2 - J_A}{6\tau\sigma^2 - J_A - J_B}\right); \quad (38)$$

$$\beta^* = -\sigma\left(1 + \frac{J_A - 3\tau\sigma^2}{6\tau\sigma^2 - J_A - J_B}\right). \quad (39)$$

We now analyse the algebraic form of (38) and (39) in order to describe the optimal locations. Finally, it can be shown that the optimal location of the firms (α^*, β^*) cannot belong to the interior $(-\sigma, \sigma)$ for both firms at the same time. Indeed, if both α^* and β^* belong to the interval $(-\sigma, \sigma)$, then need $\alpha^* > -\sigma$ and $\beta^* > 0$. Hence, according to equations (38) and (39), two possibilities apply:

- Assuming $6\tau\sigma^2 > J_A + J_B$, if (α^*, β^*) are interior points then $3\tau\sigma^2 < J_A$ and $3\tau\sigma^2 < J_B$. This contradicts the assumption that $6\tau\sigma^2 > J_A + J_B$.
- Similarly, assuming $6\tau\sigma^2 < J_A + J_B$, if (α^*, β^*) are interior points then $3\tau\sigma^2 > J_A$ and $3\tau\sigma^2 > J_B$. This contradicts the assumption that $6\tau\sigma^2 < J_A + J_B$.

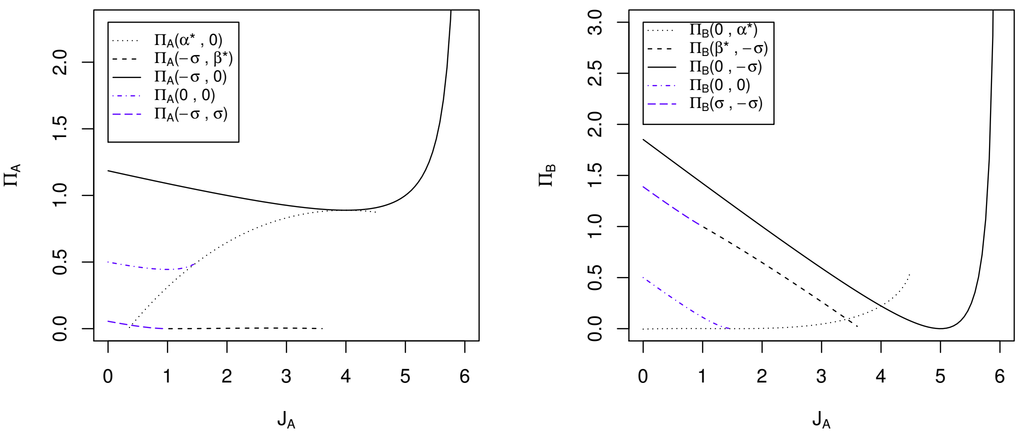
Because both firms cannot be at the interior at the same time, we study the situation where one firm is located at the extreme while the other optimizes its location given the boundary position of its rival. More precisely, we solve the F.O.C for α^* in the cases where $\pi_A(\alpha, 0)$ and $\pi_A(\alpha, \sigma)$; and β^* for the case where $\pi_B(\beta, -\sigma)$, $\pi_B(\beta, 0)$. We compare these profits and the profits obtained if both firms decide to stay at the boundaries.

The following table describes the possible strategies played by the firm. The strategy on the shadowed cell at the center of the table was already discarded before when we showed that an interior solution is not optimal. Finally, the other shadowed cell at the bottom right violates the *weak network effects* assumption, and will be analysed on the next section[6].

		Firm B		
		0	β^*	σ
Firm	$-\sigma$	$\pi_A(-\sigma, 0), \pi_B(0, -\sigma)$	$\pi_A(-\sigma, \beta^*), \pi_B(\beta^*, -\sigma)$	$\pi_A(-\sigma, \sigma), \pi_B(\sigma, -\sigma)$
A	α^*	$\pi_A(\alpha^*, 0), \pi_B(0, \alpha^*)$	$\pi_A(\alpha^*, \beta^*), \pi_B(\beta^*, \alpha^*)$	$\pi_A(\alpha^*, \sigma), \pi_B(\sigma, \alpha^*)$
	0	$\pi_A(0, 0), \pi_B(0, 0)$	$\pi_A(0, \beta^*), \pi_B(\beta^*, 0)$	$\pi_A(0, \sigma), \pi_B(\sigma, 0)$

[6] If we let $\alpha = 0$ and $\beta = \sigma$ then $4\sigma\tau(\sigma - \beta - \alpha) = 4\sigma\tau(0) = 0 \leq J_A + J_B$, which violates the *weak network effects* assumption 6.

The following figure shows the payoffs of firm A for each strategy. We take as an example the case where $J_B = 2$, $\tau = \sigma = 1$. As we can see, the best payoff is the one received when both firms stay at the extremes $\pi_A(-\sigma, 0)$.



(a) $p_{A1} < p_A < p_{A2}$ (b) $J_{A1} > J_A > J_{A2}$.

Figure 3.1: Firm's A (left panel) and firm's B (right panel) profit levels for different levels of J_A, when $\tau = \sigma = 1$ and $J_B = 2$.

The same can be observed for firm B, for which its best payoff is the one received when both firms stay at the extremes $\pi_B(0, -\sigma)$. (Although, there is a small region when $J_A > 4$ where $\pi_B(0, \alpha^*) > \pi_B(0, -\sigma)$, this location is not feasible since for this region $\alpha^* < -\sigma$.)

Thus, the unique equilibrium is such that firms locate at the extremes of the linear interval; $\alpha^* = -\sigma$ and $\beta^* = 0$. In line with the previous literature, firms choose a position yielding to a market with maximal product differentiation. Similarly to d'Aspremont et al. (1979) the closer the firms, the higher is the price competition; therefore they prefer to position away from each other, lowering price competition, segmenting the market and positioning their products on the extreme niches.

Figure 3.2: Location Interval in equilibrium where firm A sets $\alpha = -\sigma$ and firm B sets $\beta = 0$, locating their products at the extremes of the interval.

At equilibrium, firms set prices:

$$\begin{aligned} p_A^* &= \frac{12\sigma^2\tau - J_A - 2J_B}{3} \\ p_B^* &= \frac{12\sigma^2\tau - 2J_A - J_B}{3}, \end{aligned} \tag{40}$$

and the emerging market share is given by:

$$q^* = \frac{12\sigma^2\tau - J_A - 2J_B}{3(8\sigma^2\tau - J_A - J_B)}. \tag{41}$$

It is worth noting that firms increase their prices when the cognitive dissonance, τ, that consumers experience when buying a product away from their individual preferences is high, and also when the length of the interval of consumers' heterogeneity, σ, is high.

In Figure 3.3 we show the equilibrium prices (left panel) and firm's A market share (right panel), for different levels of J_A, when $\tau = \sigma = 1$ and $J_B = 2$. We can see that when $J_A < J_B = 2$, $p_B^* > p_A^*$ and $q^* < \frac{1}{2}$. This means that firm B takes advantage of the higher network strength and charges a higher price being still able to obtain a higher market share compared to its competitor. When $J_A = J_B = 2$, the model is perfectly symmetric so that prices and market shares are equal: $p_A^* = p_B^*$ and $q^* = \frac{1}{2}$. When $J_A > J_B$, $p_A^* > p_B^*$ and $q^* > \frac{1}{2}$.

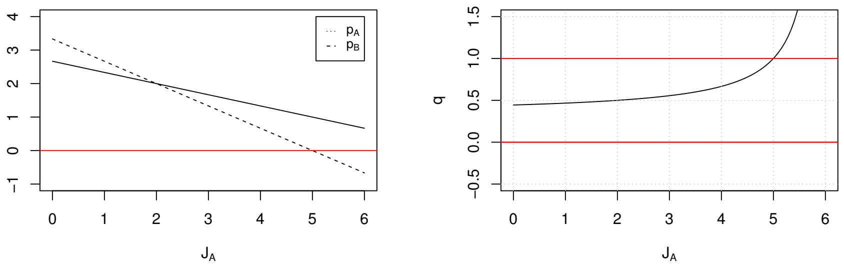


Figure 3.3: *Left Panel:* Firm's optimal prices for different levels of J_A, when $\tau = \sigma = 1$ and $J_B = 2$. *Right Panel:* Optimal firm's A market share, q^*, for different levels of J_A, when $\tau = \sigma = 1$ and $J_B = 2$.

Finally, when $J_A > 5$, we enter to the second case in Proposition 7, where firm A monopolizes the market and charges the monopoly price $p_A^M = J_A - 2\sigma\tau(\sigma - \beta - \alpha) + \tau\left[(\sigma - \beta)^2 - a^2\right]$. If the firms remain in the same *maximal differentiation* position, the monopoly price set by firm A is $p_A^M = J_A - 4\tau\sigma$.

However, the firm can optimize its position, by optimizing its monopolist profit, $\pi_A^M = p_A^M \cdot q^* = p_A^M$, with respect to its location. Computing this F.O.C, we have:

$$\frac{\partial \pi_A^M}{\partial \alpha} = -2\tau\alpha + 2\tau\sigma = 0 \Leftrightarrow \alpha^* = \sigma. \tag{42}$$

However, this position is not feasible since we assume that A can locate it's product on the left side of the interval, $\alpha \in [-\sigma, 0]$. Therefore, firm's A at best can be located at the center of the interval, $\alpha^* = 0$. However, firm B can react to this strategy by moving also to the center of the interval, $\beta = \sigma$. In this case, firms will switch to the *strong network effects* regime[7]. As we will see in the next section, by doing this firm B will ensure an expected market share to be at least $\frac{1}{3}$. The following section describes the location under *strong network effects.*

[7]As we described before, when $\alpha = 0$ and $\beta = \sigma$ then $4\sigma\tau\left(\sigma - \beta - \alpha\right) \leq J_A + J_B$, which violates the *weak network effects* assumption 6.

3.4.2 Location under strong network effects

When equation (9) holds, *strong network effects* are in place and multiple equilibria coexist for intermediate prices. As discussed before, the market share Q is a proper random variable taking value $q = 0$ with probability θ and value $q = 1$ with probability $1 - \theta$. As in the previous section, on the first stage of the game firms select their location strategy. Therefore, both firms search for a Nash equilibrium in pure strategies, meaning that a location equilibrium is a pair (α^*, β^*), such that each firm maximises its own expected profit with respect to it's location conditioned upon the (optimal) location of the competitor.

$$\mathbb{E}(\pi_A(\alpha^*, \beta^*)) \geq \mathbb{E}(\pi_A(\alpha, \beta^*)), \quad \text{for all } \alpha \in [-\sigma, 0]\,;$$
$$\mathbb{E}(\pi_B(\beta^*, \alpha^*)) \geq \mathbb{E}(\pi_B(\beta, \alpha^*)), \quad \text{for all } \beta \in [0, \sigma]\,.$$

In section 3.3.2 we solved the second stage subgame Nash equilibrium for any given pair of locations, $(\alpha, \sigma - \beta)$, as described in Proposition 7. Let's assume that $\mathbb{E}(Q) \in (0, 1)$ (i.e. $0 < \theta^* < 1$), so the optimal prices and θ on the second stage are given by:

$$p_A^* = \tfrac{2J_A + J_B + \tau\left[(\sigma-\beta)^2 - \alpha^2\right] - 6\sigma\tau(\sigma-\beta-\alpha)}{3};$$
$$p_B^* = \tfrac{J_A + 2J_B - \tau\left[(\sigma-\beta)^2 - \alpha^2\right] - 6\sigma\tau(\sigma-\beta-\alpha)}{3};$$
$$\theta^* = \tfrac{6\sigma\tau(\sigma-\beta-\alpha) + \tau\left[(\sigma-\beta)^2 - \alpha^2\right] - J_A - 2J_B}{3(4\sigma\tau(\sigma-\beta-\alpha) - J_A - J_B)}.$$

Arguing similarly as before, we obtain

$$\mathbb{E}(\pi_A^*(\alpha, \beta)) = -\frac{\left(6\sigma\tau(\sigma - \beta - \alpha) - \tau\left[(\sigma-\beta)^2 - \alpha^2\right] - 2J_A - J_B\right)^2}{9(4\sigma\tau(\sigma - \beta - \alpha) - J_A - J_B)};$$
$$\mathbb{E}(\pi_B^*(\beta, \alpha)) = -\frac{\left(6\sigma\tau(\sigma - \beta - \alpha) + \tau\left[(\sigma-\beta)^2 - \alpha^2\right] - J_A - 2J_B\right)^2}{9(4\sigma\tau(\sigma - \beta - \alpha) - J_A - J_B)}. \quad (43)$$

Then F.O.C. read:

$$\frac{\partial \mathbb{E}(\pi_A^*)}{\partial \alpha} = 4\tau(1 - \theta^*)\left(\theta\sigma - \frac{\alpha^*}{3}\right) = 0 \Leftrightarrow \alpha^* = 3\sigma\theta^* \quad (44)$$

$$\frac{\partial \mathbb{E}(\pi_B^*)}{\partial \beta} = 4\tau\theta^* \left(\sigma(1-\theta^*) + \frac{\sigma - \beta^*}{3} \right) = 0 \Leftrightarrow \beta^* = \sigma(4 - 3\theta^*) \tag{45}$$

As before, we replace (44) in equation (45),

$$\beta^* = 4\sigma - \alpha^*, \tag{46}$$

and using this relationship in equation (44), we obtain

$$\alpha^* = \sigma \left(1 + \frac{3\tau\sigma^2 + J_B}{6\tau\sigma^2 + J_A + J_B} \right); \tag{47}$$

$$\beta^* = \sigma \left(2 + \frac{3\tau\sigma^2 + J_A}{6\tau\sigma^2 + J_A + J_B} \right). \tag{48}$$

As we discussed previously, the optimal location of the firms (α^*, β^*) cannot belong to the interior for both firms at the same time. Indeed, if both α^* and β^* belong to the interval $(-\sigma, \sigma)$, then $\alpha^* < \sigma$ and $\beta^* < 2\sigma$. Hence, according to equations (47) and (48), the two possibilities apply:

- Assuming $6\tau\sigma^2 + J_A + J_B > 0$, if (α^*, β^*) are interior points then $J_A < -3\tau\sigma^2$ and $J_B < -3\tau\sigma^2$. This contradicts the assumption that J_A and J_B are greater or equal to zero.
- Similarly, assuming $6\tau\sigma^2 + J_A + J_B < 0$, if (α^*, β^*) are interior points then $J_A > -3\tau\sigma^2$ and $J_B > -3\tau\sigma^2$. Then summing both equations, $J_A + J_B > -6\tau\sigma^2 \Leftrightarrow J_A + J_B + 6\tau\sigma^2 > 0$, this contradicts the assumption that $6\tau\sigma^2 + J_A + J_B < 0$.

As before, we study the situation where firm A is on the left side and firm B is on the right side. We look for boundary solutions where one firm is located at its extreme or at the center, while the other optimizes its location given the boundary position of its rival. More precisely, we solve the F.O.C for α^* in the cases where $\mathbb{E}(\pi_A(\alpha, 0))$ and $\mathbb{E}(\pi_A(\alpha, \sigma))$; and β^* for the case where $\mathbb{E}(\pi_B(\beta, -\sigma))$, $\mathbb{E}(\pi_B(\beta, 0))$. We compare these profits and the profits obtained if both firms decide to stay at the boundaries.

As before, to illustrate this, we use our example where we let J_A variates and fix $\tau = \sigma = 1$ and $J_B = 2$. Figure 3.4 shows (left panel) the optimal α^*, when $\beta = 0$. Firm's A optimal location is above the boundary ($\alpha^* > 0$) in both cases. Because, this a is non-feasible region, at the boundary firm A at best can set $\alpha = 0$.

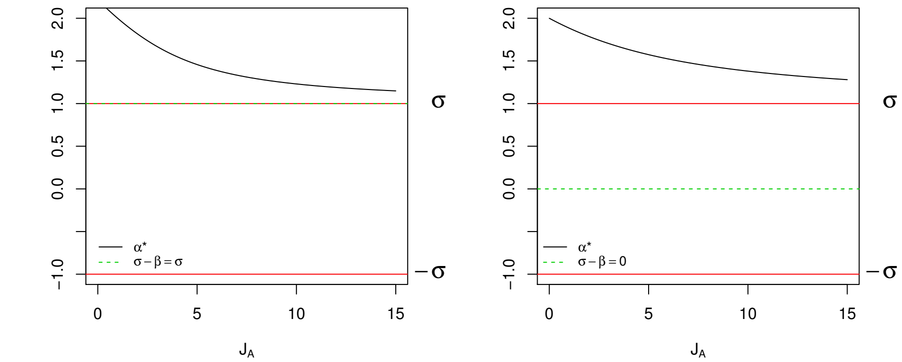


Figure 3.4: Firm's A optimal location, α^*, for different levels of J_A, when $\tau = \sigma = 1$ and $J_B = 2$, when $\beta = 0$ (left panel) and when $\beta = \sigma$ (right panel).

Similarly, the following figure describes the optimal strategy $\sigma - \beta^*$, when $\alpha = -\sigma$ on the left panel, and when $\alpha = 0$ on the right panel. We can notice that firm's B optimal location is below the boundary ($\beta^* > \sigma$) in both cases. As before, this is not feasible region, so firm B at the boundary set $\beta = \sigma$.

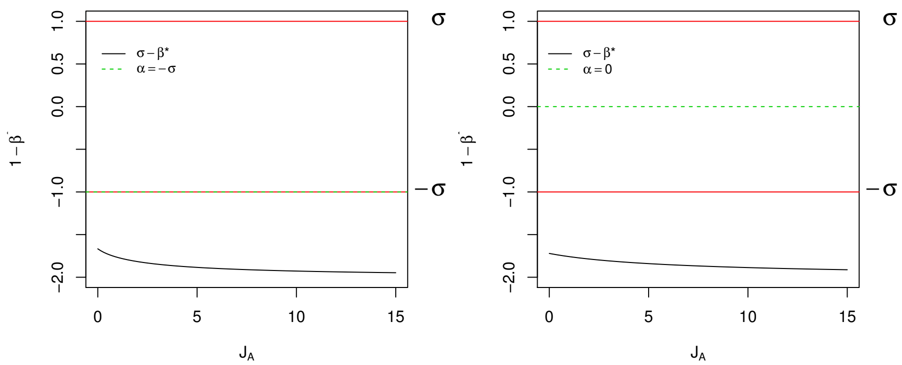


Figure 3.5: Firm's B optimal location, $\sigma - \beta^*$, for different levels of J_A, when $\tau = \sigma = 1$ and $J_B = 2$, when $\alpha = -\sigma$ (left panel) and when when $\alpha = 0$ (right panel).

The following table describes the possible strategies played by the firms.

		Firm B	
		0	σ
Firm	$-\sigma$	$\pi_A(-\sigma, 0)$, $\pi_B(0, -\sigma)$	$\pi_A(-\sigma, \sigma)$, $\pi_B(\sigma, -\sigma)$
	0	$\pi_A(0, 0)$, $\pi_B(0, 0)$	$\pi_A(0, \sigma)$, $\pi_B(\sigma, 0)$

Figure 3.6 shows the corresponding positions of each strategy, following the same order as in the table.

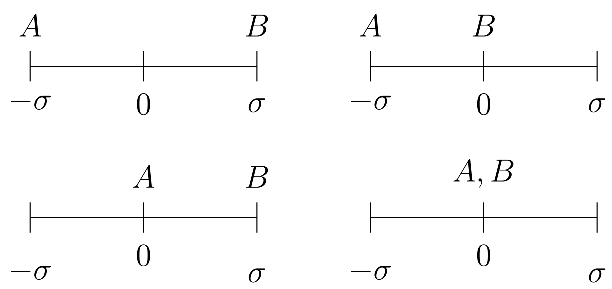


Figure 3.6: Possible location strategies.

The following figure show the payoffs of firm A. As we can see, the best payoff is the one received when both firms stay at the center of the interval $\pi_A(0, \sigma)$.

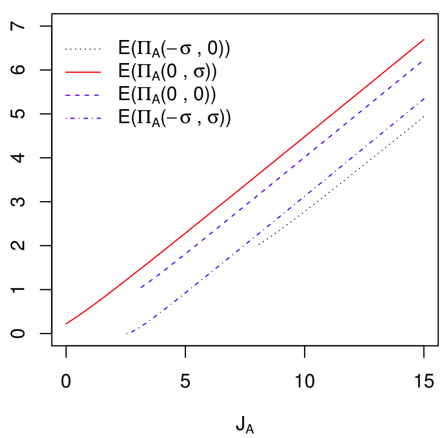


Figure 3.7: Firm's A optimal expected profits for different levels of J_A, when $\tau = \sigma = 1$ and $J_B = 2$.

The same can be observed for firm B, in which its best payoff is the one received when both firms converge in the middle[8], $\pi_B(\sigma, 0)$.

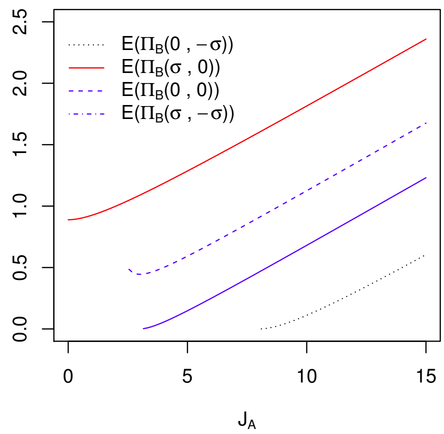


Figure 3.8: Firm's B optimal expected profits for different levels of J_A, when $\tau = \sigma = 1$ and $J_B = 2$.

Thus, in equilibrium, under *Strong Network Effects*, the firms converge at the center of the interval, therefore the market maximal differentiation, under *Weak Network Effects*, is lost and firms offer the same good. The only parameter that differentiates the firms is the network strength (J_A, J_B). This is reflected on the optimal prices that depend only on the network effects, and the firm with the strongest network effect can benefit from a higher price:

$$p_A^* = \frac{2J_A + J_B}{3}$$
$$p_B^* = \frac{J_A + 2J_B}{3} \tag{49}$$

The following figure shows the prices at equilibrium.

[8]If we relax the assumption that firm A can locate its product at any place on the left side of the interval, $\alpha \in [-\sigma, 0]$, while firm B can choose any point on the right side of the interval, $\beta \in [0, \sigma]$; and we let firms to be at any point of the interval but restricting that A cannot be at the right side of B (so assuming $\alpha \leq \sigma - \beta$), one more equilibrium appear, where A and B end up in the same extreme, right or left. The profits obtained are the same as if firms end up in the center. We show this in the appendix.

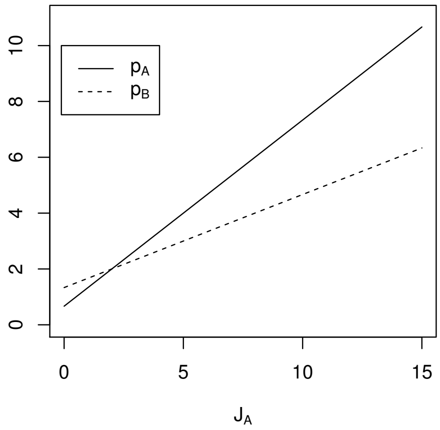


Figure 3.9: Firm's optimal prices for different levels of J_A, when $\tau = \sigma = 1$ and $J_B = 2$.

The expected market shares also depend uniquely on the strength of the network effect.

$$\mathbb{E}(q_A) = 1 - \theta^* = \frac{1}{3} + \frac{1}{3}\left(\frac{J_A}{J_A + J_B}\right)$$
$$\mathbb{E}(q_B) = \theta^* = \frac{1}{3} + \frac{1}{3}\left(\frac{J_B}{J_A + J_B}\right) \tag{50}$$

Figure 3.10 shows the expected value of q, $1 - \theta^*$. Thanks to equation (50) it can be shown that for a given value of $J_B > 0$, firm A will have an expected market share of one-third, $\mathbb{E}(q_A) = \frac{1}{3}$, in the worst case scenario where its network strength is zero, $J_A = 0$. In contrast, for a given value of $J_B < \infty$, if J_A goes to infinite, $q \to \frac{2}{3}$. Thus, the expected market share lies between $\left[\frac{1}{3}, \frac{2}{3}\right)$.

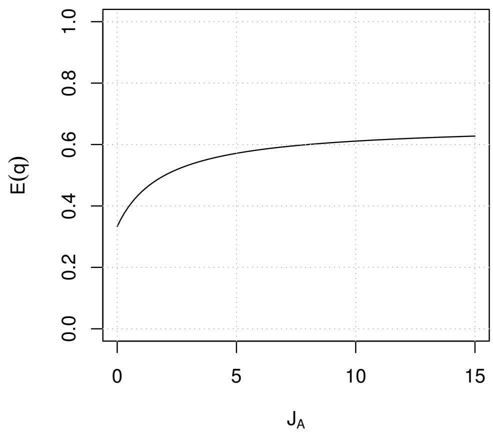


Figure 3.10: Firm's optimal expected market share for different levels of J_A, when $\tau = \sigma = 1$ and $J_B = 2$.

3.5 Discussion of market equilibria

In this section we discuss some market features at equilibria, in terms of market shares, prices and profits. Figure 3.1 compares the optimal profits under *weak network effects* when firms locate at different extremes (dashed line) and the optimal expected profits under *strong network effects* when firms converge at the center (blue line). The left panel shows this comparison for firm A and the right panel for firm B. We can see that it's optimal for the firms to be *maximally differentiated* when $J_A \leq 2$. In this region firms compete based on product differentiation and each firm serves a niche on the market. Thereafter, when $J_A \geq 2$ it's optimal for the firms to converge to the center switching to *strong network effects* and *no differentiation.* Under this regime, only one firm survives on the market (J_A with probability $1-\theta$ and J_B with probability θ), so differentiation is lost and firms prefer to serve to the median consumer in order to increase their survival chances. In other words, when the survival of the firm is probabilistic, firms locate at the center in order to increase their chances to monopolize the market. This is in line with the Hotelling (1929) model and with standard median voter election model in which two political parties converge to the median in order to increase their chances to win the election. Note finally that *no differentiation*, with the risk of being left out of the market, yields a higher expected profit than the monopolistic profit under *weak network effects* to firm A.

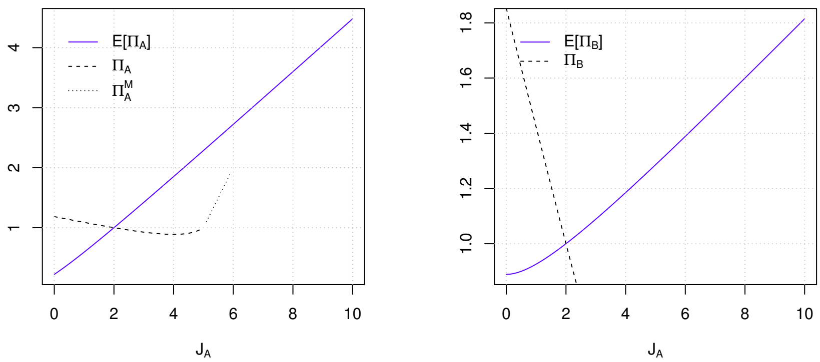


Figure 3.1: Optimal profits and expected profits for different levels of J_A, when $\tau = \sigma = 1$ and $J_B = 2$, for Firm A (left panel) and Firm B. (right panel)

Figure 3.2 shows firm's A profits, on the vertical axis, for different values of α, on the horizontal axis, when $J_A = 1$ (left panel), $J_A = 2$ (right panel) and $J_A = 3$ (bottom panel). We use our example where $J_B = 2$ and $\tau = \sigma = 1$. The figure shows the market profits under *weak network effects*, when $\beta = 0$, with the straight lines; and the expected profits under *strong network effects*, when $\beta = \sigma = 1$, with dashed lines. We can notice, on the left panel, that when $J_B > J_A = 1$ firm's A profits are higher under *weak network effects* when $\alpha = -\sigma = -1$ and $\beta = 0$. On the right panel, when $J_B = J_A = 2$, firm A is indifferent between locate at the left extreme, when $\alpha = -\sigma = -1$ under *weak network effects*, and locate at the center, when $\alpha = 0$ under *strong network effects*. On the same panel, we also denote with a red line, the monopoly profit of firm A under *weak network effects* when $\beta = 0$. It's noteworthy that any monopolistic profit cannot be an equilibrium because firm B can always relocate in a way to switch the regime to *strong network effects* with a positive probability to monopolize the market. Moreover, being in one or the other regime depend on the values of the parameters[9], J_A, J_B, σ and τ, but also on the variables β and α. Consequently, for our example, when $J_A = 3$ (bottom panel) and $\beta = 0$, firm A can switch the regime from *weak network effects* to *strong network effects* by locating closer to the center. The bottom panel shows with a red line the monopoly profits of firm A under *weak network effects* and with a red dashed line the monopoly profits under

[9]We are under *strong network effects* when $J_A + J_B > 4\tau\sigma(\sigma - \beta - \alpha)$; and under *weak network effects* when $J_A + J_B < 4\tau\sigma(\sigma - \beta - \alpha)$.

strong network effects, when $\beta = 0$. We can see that firm A can obtain higher profits when $\alpha = 0$ and $\beta = 0$. However, as we mentioned before, this could never be an equilibrium because firm B can always relocate at $\beta = 1$ and have a positive probability to monopolize the market. Therefore, it's optimal for firm A to stay at the center, $\alpha = 0$ and $\beta = 1$.

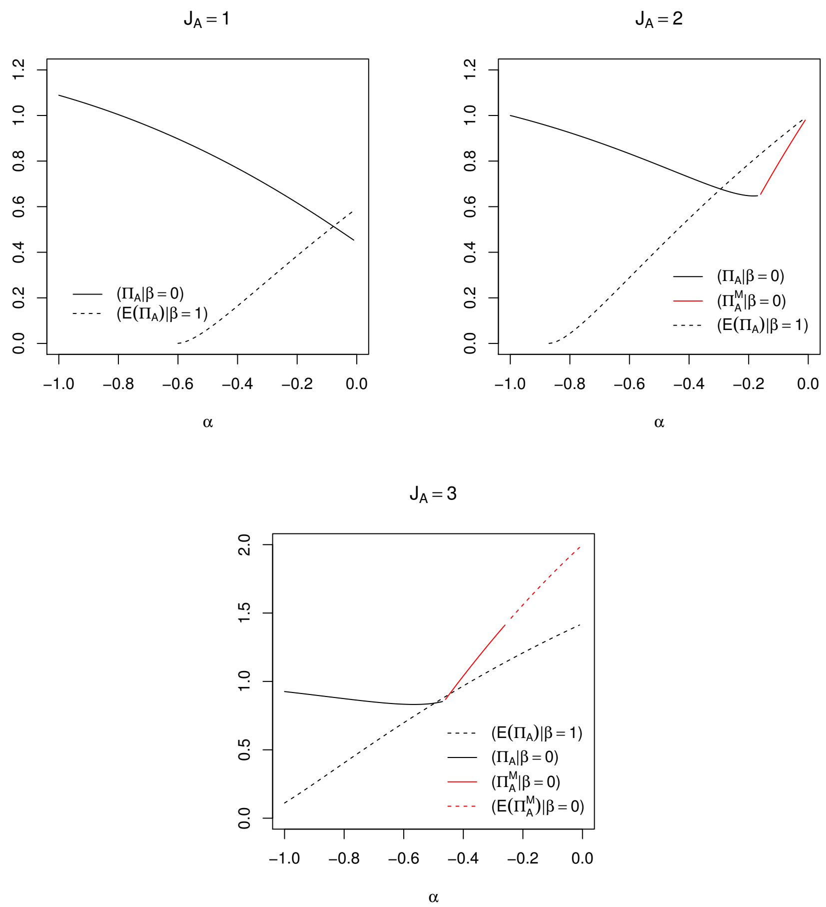


Figure 3.2: Firm's A market profits and expected profits for values of α, when $J_B = 2$ when $\tau = \sigma = 1$, $J_A = 1$ (left panel), $J_A = 2$ (right panel) and $J_A = 3$ (bottom panel).

Figure 3.3 (left panel) shows the market shares of the two firms at the equilibrium, for different values of J_A, assuming that $a = 4$ and $J_B = 2$. First of all, notice that for these parameters, firm B cannot result to be the monopolist (unless multiple equilibria are present). The blue dashed vertical line divides the graph into *weak network effects* (on the left) and *strong network* *effects* (on the right).

As we noticed in Chapter 2, under *weak network effects*, firm A market share increases with J_A; however, it never reaches to be a monopoly given that firms switch to *strong network effects* and *no differentiation* when $J_A \geq 2$. Recall that under this last regime the interpretation is as follows: with probability θ^*, firm B monopolizes the market, whereas with probability $(1-\theta)$, A is the monopolist. Moreover, as in the previous chapter, it is evident that $\lim_{J_A \to \infty} \mathbb{E}^*(1-Q) = \theta^* = \frac{1}{3}$, no matter of the values of the parameters. Therefore, the probability for firm A to be out of the market remains sensibly high even when J_A becomes huge.

Figure 3.3 (right panel) shows the bifurcation diagram for q^*. Under weak network effects, the level of q^* is unique, whereas, under strong network effects, there are two locally stable equilibria and an intermediate unstable equilibrium. The probability that firm A monopolizes the market is described by the dashed line, $(1-\theta)$. Note that the bifurcation happens when firms meet at the center of the interval. This happens when $J_A = J_B = 2$. Recall, that now firms are able to chose a point over the interval, which allow them to benefit from *no differentiation*, limiting the competition, and taking advantage of *strong network effects* earlier than on the previous chapter.

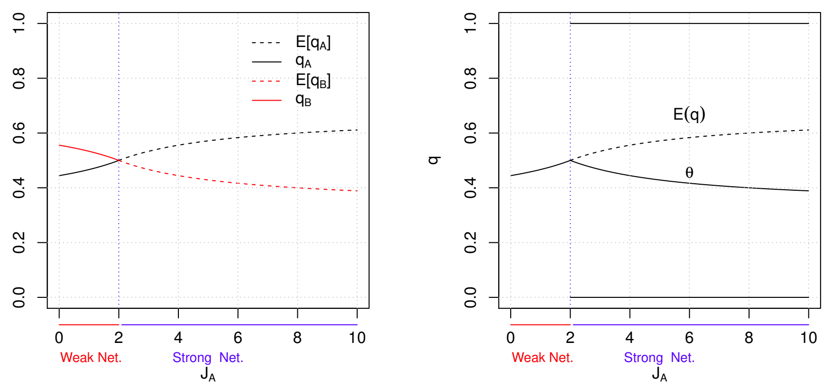


Figure 3.3: Optimal market shares (left panel) and Bifurcation Graph for q^* (right panel) varying J_A when $\tau = \sigma = 1$ and $J_B = 2$.

Finally, figure 3.4 shows the optimal prices. As we saw on the previous chapter, under *weak network effects* and *maximal differentiation*, prices decrease with J_A, an increase in network effects signals a fiercer market competition.

When, $J_A \geq 2$, firms switch to *strong network effects* and *no differentiation*, such that this price competition doesn't hold any more. Under this condition, there is a positive probability for each firm to monopolize the market, and product differentiation dissipates, so firms profit only on the strength of their social effect, as we can see by looking at equation (49). Although both firms experiment a price increase, firm A is able to set a higher price due to its higher network strength.

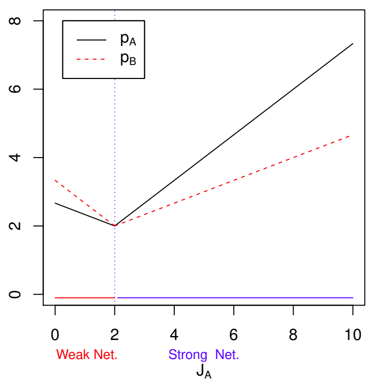


Figure 3.4: Firm's optimal prices for different levels of J_A, when $\tau = \sigma = 1$ and $J_B = 2$.

3.6 Conclusions

We have extended the duopoly model of Chapter 2 by relaxing the assumption that firms are horizontally differentiated: we let firms compete on location. This makes the analysis more involved because the nature of the regime, *strong* or *weak network effects*, becomes endogenous as it depends on the election of the location variables of the firms. Our analysis partially confirms what we saw in the previous chapter, and what is shown on the literature: under weak network effects, competition increases when network externalities increase, thus, abating prices. Under this scenario *maximal differentiation* emerges and each firm serves a niche of the market. However, a novel result emerges; under *strong network effects* firms converge to serve the median consumer. This result is in line with the median voter political competition model, where two parties with different ideologies, *right-wing* and *left-wing*, converge to satisfy to the median voter; thus, in equilibrium ideologies converge to the center. In our duopoly model, under *strong network effects*, and analogous to the political competition model, both firms end up meeting at the center of the interval, although only one succeeds to monopolies the market, while the other is out of the market. Under this regime, although the firm with the lowest network effect has chance to monopolize the market, the strongest firm has a higher probability to monopolize the market and can benefit from higher optimal prices.

Appendix

3.A Relaxing the segment location assumption

We assumed that each firm represent a segment of the population preferences, firm A locate its product at any place on the left side of the interval, $\alpha \in [-\sigma, 0]$, while firm B is located on the right side of the interval, $\beta \in [0, \sigma]$. Under this assumption we showed that the optimal strategy for the firms, under *strong network effects*, is to be located at the center of the interval, $\alpha = 0$ and $\beta = \sigma$. This strategy lets the profits be:

$$\begin{aligned}
\mathbb{E}(\pi_A^*(\alpha = 0, \beta = \sigma)) &= -\frac{(-2J_A - J_B)^2}{9(-J_A - J_B)};\\
\mathbb{E}(\pi_B^*(\beta = \sigma, \alpha = 0)) &= -\frac{(-J_A - 2J_B)^2}{9(-J_A - J_B)}.
\end{aligned}$$

Now lets relax this assumption and assume, as in d'Aspremont et al. (1979), that firm A is located to the left of firm B, therefore, we only assume that $\alpha \leq \sigma - \beta$. We will show that under this assumption, when the two firms converge into the same point in the interval, the profits are the same. To make the analysis simpler, let's define $\hat{\beta} = \sigma - \beta$. Thus, $\hat{\beta}$ and α represent the distance of the firms to the left extreme. Finally, let's assume that the two firms are in the same point, so $\alpha = \hat{\beta}$. Then, the profits are:

$$\begin{aligned}
\mathbb{E}(\pi_A^*(\alpha = \hat{\beta})) &= -\frac{\left(6\sigma\tau(\hat{\beta} - \alpha) - \tau\left(\hat{\beta}^2 - \alpha^2\right) - 2J_A - J_B\right)^2}{9(4\sigma\tau(\hat{\beta} - \alpha) - J_A - J_B)};\\
&= -\frac{(-2J_A - J_B)^2}{9(-J_A - J_B)};\\
\mathbb{E}(\pi_B^*(\alpha = \hat{\beta})) &= -\frac{\left(6\sigma\tau(\hat{\beta} - \alpha) + \tau\left(\hat{\beta}^2 - \alpha^2\right) - J_A - 2J_B\right)^2}{9(4\sigma\tau(\hat{\beta} - \alpha) - J_A - J_B)};\\
&= -\frac{(-J_A - 2J_B)^2}{9(-J_A - J_B)}.
\end{aligned}$$

Therefore, since the two firms are in a winner-takes-all monopoly, they are indifferent where to be located. What seems to be relevant is the fact that the two firms are overlapped.

4 Measuring brand awareness in a random utility model

Perfrancesco Dotta[1], Marco Tolotti and Jorge Yépez

Abstract

Brand Awareness is recognized to be an important determinant in shaping the success of durables, yet it is very difficult to be quantified. This is exactly the main goal of this chapter: propose a suitable model where brand awareness of two competing firms is modelled and, eventually, estimated. To this aim, we build a random utility model for a duopoly where each competitor is characterized by different pricing strategies and brand awareness. As a result, different levels of market shares will emerge at the equilibrium. As a case study, we calibrate the model with real data from the smartphone industry obtaining an estimate of the value of the brand awareness of two leading brands[2].

4.1 Introduction

Brand awareness plays an important role in consumer buying decision-making and is central on determining the success of companies. The objective of branding decisions in modern organizations is to generate a *brand image* for their products or services that is in line with the firm's target market and positioning decisions. The brand image can be thought of as the result of all the subjective perceptions and mental images that consumers' minds associate with a particular brand. From the consumer perspective, *brand awareness* is the extent to which a particular brand and its qualities are recognized, thus evoking its image (see Drummond and Ensor (2005) for more details). A strong brand awareness is essential in order to form brand image, because when a brand is well established in the consumer's memory it

[1]P. Dotta is a former Master student of M. Tolotti. This research originated by his master thesis.

[2]The material of this chapter has led to the publication: Dotta, P., Tolotti, M., and Yepez, J. (2017). Measuring brand awareness in a random utility model, *Advances in Complex Systems*, 20(1) 1750004 (11 pages). DOI: 10.1142/S0219525917500047.

is easier for the mental images and perceptions to be associated with a brand (see Esch et al. (2006)). Joseph (2010) and Keller (2013) illustrate how a brand's image can be present in the mind of consumers and how it impacts their buying decisions. O'Cass and Siahtiri (2013) argue that consumption behaviors characterize the desire to possess certain brands as a mean to achieve a particular status and self-fulfillment; therefore, branding plays an important role in order to project a certain image on potential customers and generate a positive brand awareness. For example, consumers buy certain clothes or cars because they want to be associated with the prestige of the brand itself (see McColl and Moore (2011)).

A strong brand awareness and positive brand image also result in customers spreading brand loyalty and devotion: devoted consumers act to bring others' attention to the brand and attract new customers. In addition, potential adopters are attracted towards the group of consumers with the highest recognition (see Chung et al. (2008)).[3] Positive correlation was found between word of mouth, brand loyalty and brand awareness on luxury goods (see Virvilaite et al. (2015)). Summarizing, it is evident that social interactions play an important role in consumers buying decision process: brand awareness produces a positive externality, which, in turn, contributes to the company position on the market.

Our goal is to model brand awareness of two competitive firms in a society made of heterogeneous individuals with heterogeneous preferences. More specifically, brand awareness plays a crucial role in shaping customers' decision process, by adding a positive externality on the perceived utility for the product . We micro found agents with heterogeneous preferences using a random utility model endowed with a social component, in line with the traditional discrete social choice literature. In their seminal paper, Brock and Durlauf (2001b) propose a binary decision model where the action of single agents is influenced by an aggregate signal represented by the (estimated) percentage of actors adopting the product. In particular, actors are prone to imitate the behavior of the majority, thus, proving to be influenced by social effects in their decision-making. In Pellizzari et al. (2015), the Brock and Durlauf paradigm is transferred into a game-theoretical language, thus emphasizing the strategic behavior of a large population of players subject to social interactions.

[3] As argued by Asch (1951), *"the primary mechanism in social influence is the change in the definition and meaning of an object"*. Wood and Hayes (2012) tell us how the consumers perform a *"social (re)construction of reality"*, reinterpreting the information about objects in relation to their reference groups, wondering whether this would be in line or not with their being, and thus leading to potential social reward or punishment.

In the present chapter, we extend the model in Pellizzari et al. (2015) to the case of a duopoly: two competitors, characterized by different levels of brand awareness and prices, offer a new technology on the market. These quantities enter as parameters into a well-posed random utility model and, eventually, shape the level of market shares for the two technologies at the equilibrium. As an illustrative example, we calibrate the model using real data of the market share and the prices of the two major players in the smartphone industry: Apple and Samsung.

4.2 A duopoly and a large population of possible buyers

Each agent in a large population of N potential buyers has to decide among three mutually exclusive options: buying product A (produced by firm A), buying product B (produced by firm B) or stay out of the market. Each action yields a utility U_A, U_B and U_0, respectively. The decision-making process can be visually represented by the decision tree depicted in Figure 4.1.[4]

Starting from the left, the agent faces two subsequent decisions: (i) to buy or not to buy a product, (ii) whether to buy product A or B. If $U_0 > max(U_A, U_B)$, we end up in *Event* 0. Without loss of generality, we set $U_0 = 0$. On the other hand, as soon as the utility of buying product A or B is positive, the agent follows the higher branch and chooses the preferable outcome according to utilities U_A and U_B.

[4]Square nodes are decisional nodes and indicate the moment at which the agent is required to take a decision; triangle nodes are terminal nodes endowed with a utility value that the agent receives if the node is selected. The share of the population that decides to enter the market is s, while x is the share of consumers in the market that purchase good A.

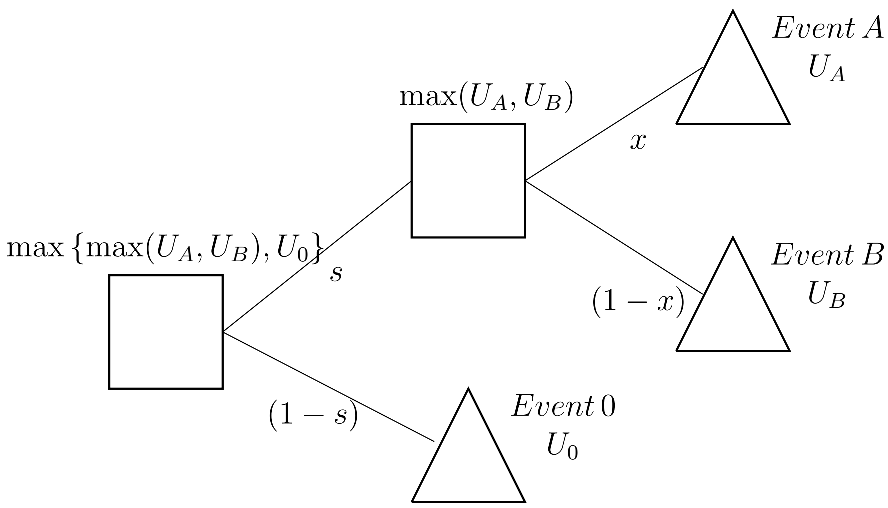


Figure 4.1: Decision Tree representing the decision process of any potential adopter.

Let us define $P_A = s \cdot x$ as the unconditional probability of buying A and $P_B = s \cdot (1-x)$ the unconditional probability of buying B. Consequently, s is the total probability of entering the market and x is the probability of *Event A* conditioned on the fact that the agent enters the market. Formally,

$$s = P_A + P_B = P(\max(U_A, U_B) > 0), \tag{1}$$

$$x = \frac{P_A}{P_A + P_B} = P(U_A > U_B | \max(U_A, U_B) > 0). \tag{2}$$

In order to compute the values of s and x at the equilibrium, we rely on a random utility model inspired by Brock and Durlauf (2001b) and Pellizzari et al. (2015). In detail, for each single actor $i = 1, ., N$, we set

$$U_A(i) = -p_A + J_A xs + t_A(i), \tag{3}$$

$$U_B(i) = -p_B + J_B(1-x)s + t_B(i), \tag{4}$$

$$U_0(i) = 0. \tag{5}$$

(3) and (4) resemble the standard shape of random utilities à la Brock and Durlauf (see Brock and Durlauf (2001b)) and are composed of three terms[5].

[5] We can see from equations (1) and (2) that x and s depend on U_A and U_B; therefore, equations (3) and (4) are indeed fix points. Later on in this chapter we explore this to find the market equilibria.

p_A and p_B are the market prices of technology A and B respectively; each of the second components introduces an externality due to social interactions. Indeed, xs and $(1-x)s$ denote the respective market shares[6] prevailing at the equilibrium whereas J_A and J_B measure the level of brand awareness;[7] finally, $t_A(i)$ and $t_B(i)$ are independent and identically distributed (i.i.d.) random variables introducing heterogeneity in the population of buyers.[8] Therefore, they can be seen as independent drawings from the same random variable t. Following standard literature in random utility models, we assume that $t_A(i)$ and $t_B(i)$ have a logistic probability distribution η with mean zero and variance $\sigma^2 = \frac{\pi^2}{3\beta^2}$.

$$\eta(z) = P(t \leq z) = \frac{1}{1 - \exp(-\beta z)}, \quad \beta > 0. \tag{6}$$

The bigger σ^2 (the smaller β) the more disperse the taste of the population of buyers.

Each agent compares among his utilities of adopting product A, product B, and not entering the market, $\{U_A(i), U_B(i), U_0\}$, as was described in Figure 1. The agent's utilities when entering the market, $\{U_A(i), U_B(i)\}$, depend on the action of other agents through the participation shares shaping the social component term of the utilities. This eventually results in setting a non-cooperative game, in which each agent makes his choice given an expectation of the population outcome. Similarly to Pellizzari et al. (2015), agents do not communicate or coordinate, rather each individual knows the common distribution of the heterogeneous shocks t_j, for $j \neq i$. In other words, we impose *rational expectations*: each agent has a correct belief about others' preferences; moreover, we assume that each agent shares the same expectation about other player's actions.[9] For a fixed population of N agents, where $N \to \infty$, at least one Nash equilibrium in pure strategies exists (see

[6] The number of agents, N, can be partitioned into $N = N^0 + N^A + N^B$. When $N \to \infty$, the market shares are thus defined as $\lim_{N\to\infty} \frac{N^0}{N} = 1 - s$, $\lim_{N\to\infty} \frac{N^A}{N} = sx$ and $\lim_{N\to\infty} \frac{N^A}{N} = (1-x)s$.

[7] The coefficient multiplying the social component of the utility is interpreted as the force of externality or as the imitation driver (see for example Bass (1969), where a similar interpretation applies to the context of diffusion of innovation). In the same spirit, we interpret it here as the brand awareness of the issuing firm.

[8] Products A and B are considered to have the same level of technology and quality. Consequently, the stochastic variables, t_A and t_B, reflect the individual preferences for each good, hence the products are not vertically but horizontally differentiated.

[9] We are aware that our model is over simplifying the micro structure behind the decision process of the agents. Information asymmetries, heterogeneous preferences, local interactions or network effects could be introduced; on the other hand, this would make the model much more complicated, causing a loss of tractability.

Dai Pra et al. (2013)). The result of this game theoretical setting -when we let $N \to \infty$- is therefore the emergence of a Nash equilibrium characterized by levels x^* and s^*. Because the logistic distribution is unimodal and S-shaped, we can have one or more equilibria depending on the set of prices $\{p_A, p_B\}$ and brand awareness $\{J_A, J_B\}$ (see Brock and Durlauf (2001b)).

Under these assumptions, when the number of buyers tends to infinity, it is possible to derive an explicit expression for the probabilities P_A and P_B.[10] All these results are summarized in the next proposition:

Proposition 8 *Assume a population of N agents as described by equations (3)-(5) where $\{\beta, p_A, p_B, J_A, J_B\}$ are fixed and where η_A and η_B have the form (6). Then, at least one Nash equilibrium (x_N^*, s_N^*) exists. Moreover, when $N \to \infty$,*

$$(x_N^*, s_N^*) \to (x^*, s^*)$$

where (x^*, s^*) solves the fixed point problem

$$\begin{cases} f(x,s) = 0 \\ g(x,s) = 0 \end{cases} \tag{7}$$

with $f(x,s) := P_A + P_B - s$ and $g(x,s) = \frac{P_A}{P_A+P_B} - x$. Moreover,

$$\begin{aligned} P_A = {} & \frac{\exp(\beta X_0)}{(\exp(\beta X_0)-1)^2} \cdot \log\left(\frac{\exp(-\beta X_0)+\exp(\beta X_A)}{\exp(\beta X_A)+1}\right) \\ & + \frac{\exp(\beta X_0)}{(\exp(\beta X_0)-1)(\exp(\beta X_A)+1)} \end{aligned} \tag{8}$$

and

$$\begin{aligned} P_B = {} & \frac{-\exp(\beta X_0)}{(\exp(\beta X_0)-1)^2} \cdot \log\left(\frac{\exp(\beta X_B)+1}{\exp(\beta X_0)+\exp(\beta X_B)}\right) \\ & - \frac{1}{(\exp(\beta X_0)-1)(\exp(\beta X_B)+1)} \end{aligned} \tag{9}$$

where $X_A = p_A - J_A x s$, $X_B = p_B - J_B(1-x)s$ and $X_0 = p_B - p_A - J_B s + (J_A + J_B) s x$.

Note that the problem is intrinsically bi-dimensional in that x and s have to be determined at the equilibrium as the solutions to (7). Depending on the values of the parameters $\{\beta, J_A, J_B, p_A, p_B\}$, different equilibria emerge. Indeed, all the equilibrium solutions (x^*, s^*) can be found by solving

[10] For a derivation, see the Appendix

$$(x^*, s^*) = \underset{(x,s)\in[0,1]\times[0,1]}{\text{argmin}} \left\{ \phi(x,s) \right\}, \tag{10}$$

where

$$\phi(x,s) = f(x,s)^2 + g(x,s)^2. \tag{11}$$

As an example, we run a simulation where we consider a fixed population (represented by $\beta = 2$). In Figure 4.2 we plot the contour levels of equation (11), when firm A has a stronger brand awareness ($J_A = 4, J_B = 1$), but the price of firm B is more competitive ($p_A = 1.5, p_B = 1$). The black dots represent the solution points (x^*, s^*) of (10) depicted at points $(0.35, 0.23)$, $(0.66, 0.38)$ and $(0.99, 0.99)$. The top right corner displays the equilibrium point $(0.99, 0.99)$, which illustrates the strong effect of brand awareness and social interaction, where firm B is practically taken out of market. Moreover, even at the most favorable equilibrium point for firm B, $(0.35, 0.23)$, firm A still possesses an important market share and still competes on the market, although the total market share (s) is considerably reduced.

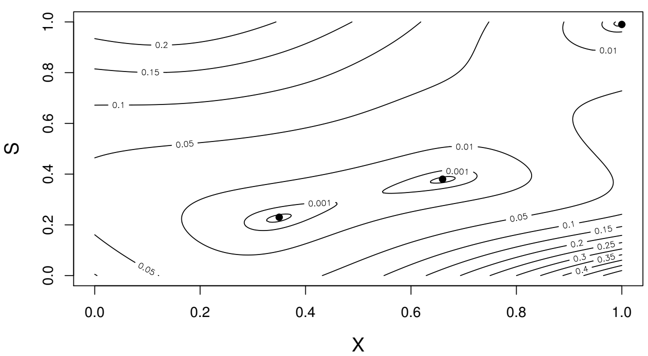


Figure 4.2: Contour levels and equilibrium points (x^*, s^*) of the function $\phi(x, s)$.

In Table 1 we collect the values of (x^*, s^*) emerging at the equilibrium for different specifications of the parameters. The first row shows a baseline full symmetric situation where the two firms share equal characteristics and

Table 1: Equilibrium values of (x, s) for different values of the parameters.

Parameters				*Equilibrium Points*
J_A	J_B	p_A	p_B	(x^*, s^*)
2	2	0.5	0.5	(0.90, 0.95), (0.50, 0.91), (0.10, 0.95)
2	2	1	0.5	(0.04, 0.94)
4	2	0.5	0.5	(1.00,1.00)
4	2	1.5	0.5	(0.99, 0.99), (0.54, 0.9), (0.01, 0.94)
1	1	1	1	(0.50, 0.28)
4	1	1.5	1	(0.99, 0.99), (0.66, 0.38), (0.35, 0.23)

where three equilibria emerge.[11] When p_A increases (second row), there is only one equilibrium, at which the market share of product B is dominant, when J_A increases (third row) the market share of product A is dominant. The fourth row shows how the negative effect of p_A can be attenuated when J_A is high. Additionally, we can see that when brand devotion is high, the total market size s increases. The fifth row shows a scenario similar to the first one, where both products share equal characteristics; however, the prices are twice higher, yet brand awareness is cut by half. In this case the positive effect of brand awareness is not strong enough to offset the negative effect of higher prices. Consequently, there is only one equilibrium where firms share equal market share but where the total market size is considerably reduced. Finally, the last row shows exactly the scenario depicted on Figure 4.2, which is similar to row 4, except that p_B is higher and J_B is lower. As a result, both the medium equilibrium and the one where product B is dominant show a smaller total market size, as well a stronger market share for firm A.

4.3 A case study: The smartphone industry

We apply our model to the case of the smartphone industry. We used the Gartner iDC data[12] and extract quarterly market shares (Q4 2012-Q4 2014), illustrated in Figure 4.3.

[11]The presence of multiple equilibria is due to the non-linearity of the system (7). Recall that functions f and g depend on P_A and P_B which, in turn, involve exponential terms. This fact has significant consequences on the strategic behavior of firms (see, for instance, Pellizzari et al. (2015)). In standard random utility models, it is shown that for $\{J_A, J_B\}$ large enough, multiple equilibria may appear. In our setting, the picture is less clear because of the presence of the two competitors. For the sake of brevity, we leave this issue to future further investigation.

[12]Data taken from Gartner, iDC at http://www.gartner.com

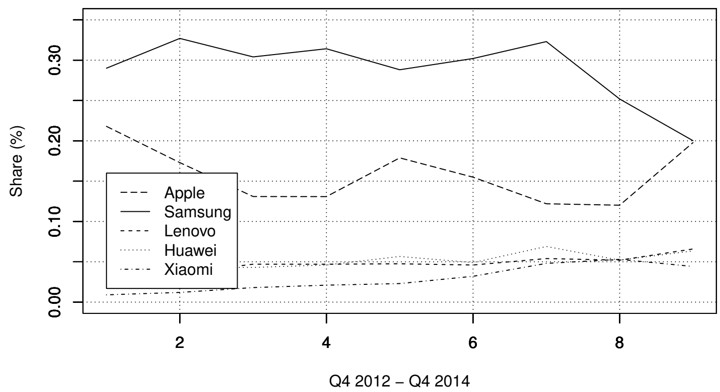


Figure 4.1: Smartphone manufacturer industry market share, taken from Gartner iDC.

Looking at Figure 4.3, representing the market shares of the major competitors in the phone industry, it is easy to spot the predominance of the Samsung-Apple duo, holding under their control 40% of the global market, with their best performing competitors lagging more than 13 percentage points behind. For this reason and for the importance that socio-psychological dynamics assumed in the choice of our next smartphone, we believe that the smartphone industry represents a perfect case study to test the model being presented.

Samsung and Apple have adopted different strategies when it comes to their offered portfolio of products. Apple offers a small variety of mobiles phones compared to the considerably wider collection of products offered by Samsung. In our analysis, we focus on the S5 and S6 Samsung's smartphones, and the iPhone 6 family plus the iPhone 5S Apple's collection.[13] The average price[14] for a Samsung model is €550, while for Apple's is €729. We let Apple represent product A and Samsung product B. Being the model of comparative nature, we normalize $p_B = 1$ and $J_B = 2$.[15] Then, we set

[13]Taking into account the product's release date and Apple's higher prices, the Samsung's S5 and S6 in all of their different versions represent the main competitors of Apple's products.

[14]Samsung: S6 Edge Plus (€839), S6 Edge (€739), S6 (€739), S5 Neo (€330), S5 (€410), S5 Mini (€240). Apple: iPhone 6S Plus (€779), iPhone 6S (€889), iPhone 6 Plus (€779), iPhone 6 (€669), iPhone 5S (€529).

[15]The value of $J_B = 2$ has been chosen after a careful preprocessing of the model. We have tested it using J_B set equal to $\{1, 2, 3, 4\}$. When $J_B = 1, J_A^*$ results to be negative (not feasible); while when $J_B > 2$, the value of β^* falls between 5 and 50 which we

$p_A = \frac{729}{550} = 1.325$. Finally, J_A and β will be calibrated. To this aim, we rely on equation (11)[16]. Figure 4.2 shows the estimated brand awareness ratio J_A^*/J_B, for the different quarters under analysis.

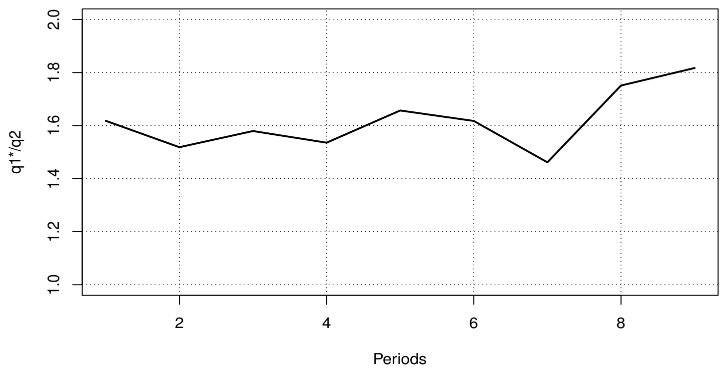


Figure 4.2: Estimated brand awareness J_A^*/J_B, when $J_B = 2$.

Our model shows a consistent higher level of Apple brand awareness compared to Samsung. Brand awareness and brand image has a significant impact on customer behavior; see Zhang (2015). Moreover, brand image is perceived as an important driving force of customer loyalty (see Saeed et al. (2013)). It is no hyperbole stating that Apple consumers make religious references to Steve Jobs and Apple's products, to the point that some of them are used to queue for days waiting for the release of the latest iPhone. Our model thus corroborates the well-known fact that the brand awareness of Apple is higher than the one of other firms. The novelty brought about by this model consists in its capacity to provide estimates of the ratio between the two main competitors' brand awareness; moreover, the values we find are consistent with price and market shares data. The maximum points of the ratio of Apple's to Samsung's awareness degree correspond to the quarters in which Apple introduced its new products on the market: i.e., period

considered too high. Indeed, the parameter β is related to the variance of the population's taste ($\sigma^2 = \frac{\pi^2}{3\beta^2}$). β in $[5, 50]$ correspond to variances between $[0.1, 0.001]$, meaning that population preferences are not dispersed enough.

[16]We used the statistical computing software R and its optimization package *optim* to minimize equation (11). For this purpose, we let J_A and β be the parameters to be estimated, so the function to be minimized with its argument can be described by $\phi(J_A, \beta) = f(J_A, \beta)^2 + g(J_A, \beta)^2$. We also let $p_B = 1$, $p_A = 1.325$, $J_B = 2$, while x and s where taken from the quarterly market shares (Q4 2012-Q4 2014) as it was shown in Figure 4.3.

5 (Q4-2013) and 9 (Q4-2014). The decline after the fifth period could reflect the disappointment of the consumers who would have preferred to see a completely new smartphone, instead of an updated version of the old one (i.e., the iPhone 5S Apple release). Such a result shows that, no matter how strong Apple's brand is, there is still a chance for other brands to capture a slice of its market share. Finally, our calibration exercise also provides an estimate $\sigma^2 = 1$ for the variance of the logistic distribution characterizing the spread of taste among the buyers.[17] This figure, often used in random utility models, is rarely calibrated to real data.

[17]The estimated average β^* across the evaluated periods is 1.81.

4.4 Final remarks

Social interaction plays an important role in order to determine the success of goods or services, specially when consumers react differently to brand images that firms promote to capture their attention (see Asch (1951), Virvilaite et al. (2015), Wood and Hayes (2012)). In this chapter we have modelled consumer decision making under the assumption that brand awareness plays the role of a social component in the utility. We suggest a different way to look at and use random utility models. A large number of agents faces two subsequent choices: adopt or not a new technology and, eventually, which one between the two releases of the technology to buy. Dealing with a duopoly, a peculiarity of our model is the presence of two bunches of parameters characterizing the two competitors. Once provided the equations needed to determine the market shares at the equilibrium, we calibrate the model with real data related to the major players on the smartphones industry, obtaining a quantitative measure of the brand awareness ratio of the two competitors. Our results support the fact that Apple's brand awareness is higher than the one of other firms.

Appendix

4.A Proof

The existence of Nash equilibria and the convergence of (x_N^*, s_N^*) to the points solving (7) can be deduced from arguments developed in Dai Pra et al. (2013). We now concentrate on the shape of the functions $f(x,s)$ and $g(x,s)$, hence on P_A and P_B. From (3) and (4), it follows that

$$P(U_A(i) > 0) = P(-p_A + J_A xs + t_A(i) > 0) = P(t_A(i) > X_A) \tag{12}$$

$$P(U_B(i) > 0) = P(-p_B + J_B(1-x)s + t_B(i) > 0) = P(t_B(i) > X_B) \tag{13}$$

where $X_A = p_A - J_A xs$, $X_B = p_B - J_B(1-x)s$. Therefore, (8) can be derived as follows:

$$\begin{aligned}
P_A = P(U_A(i) > U_B(i), U_A(i) > 0) &= P(t_B(i) - t_A(i) < X_0, t_A(i) > X_A) \\
&= P(t_B(i) < X_0 + t_A(i), t_A(i) > X_A) \\
&= \int_{X_A}^{\infty} \eta\left(X_0 + \xi\right) d\eta(\xi) \\
&= \int_{X_A}^{\infty} \left(\frac{1}{1+\exp(-\beta(X_0+\xi))}\right) \frac{\beta \exp(-\beta\xi)}{\left(1+\beta\exp(-\beta\xi)\right)^2} d\xi \\
&= \int_{X_A}^{\infty} \frac{\beta}{(\exp(-\beta\xi)+\exp(-\beta X_0))(1+\exp(-\beta\xi))^2} d\xi \\
&= \frac{\exp(\beta X_0)}{\left(\exp(\beta X_0)-1\right)^2} \cdot \log\left(\frac{\exp(-\beta X_0)+\exp(\beta X_A)}{\exp(\beta X_A)+1}\right) \\
&\quad + \frac{\exp(\beta X_0)}{(\exp(\beta X_0)-1)(\exp(\beta X_A)+1)},
\end{aligned}$$

where $X_0 = p_B - p_A - J_B s + (J_A + J_B)sx$.

We used the convolution formula for independent random variables to derive the third line and the form of the logistic distributions of η_A and η_B to derive the fourth. The latter expression follows by direct integration: differently from the normal random variable, an explicit expression for the logistic distribution can be provided.

Similarly, (9) is derived as follows:

$$\begin{aligned}
P(U_B(i) > U_A(i), U_B(i) > 0) &= P(t_B(i) - t_A(i) > X_0, t_B(i) > X_B) \\
&= P(t_A(i) < t_B - X_0(i), t_B(i) > X_B) \\
&= \int_{X_B}^{\infty} \eta\left(\xi - X_0\right) d\eta(\xi) \\
&= \int_{X_B}^{\infty} \left(\frac{1}{1 + \exp(-\beta(\xi - X_0))} \right) \frac{\beta \exp(-\beta\xi)}{\left(1 + \beta \exp(-\beta\xi)\right)^2} d\xi \\
&= \int_{X_B}^{\infty} \frac{\beta}{(\exp(\beta\xi) + \exp(\beta X_0))(1 + \exp(-\beta\xi))^2} d\xi \\
&= \frac{-\exp(\beta X_0)}{\left(\exp(\beta X_0) - 1\right)^2} \cdot \log\left(\frac{\exp(\beta X_B) + 1}{\exp(\beta X_0) + \exp(\beta X_B)} \right) \\
&\quad - \frac{1}{(\exp(\beta X_0) - 1)(\exp(\beta X_B) + 1)}.
\end{aligned}$$

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