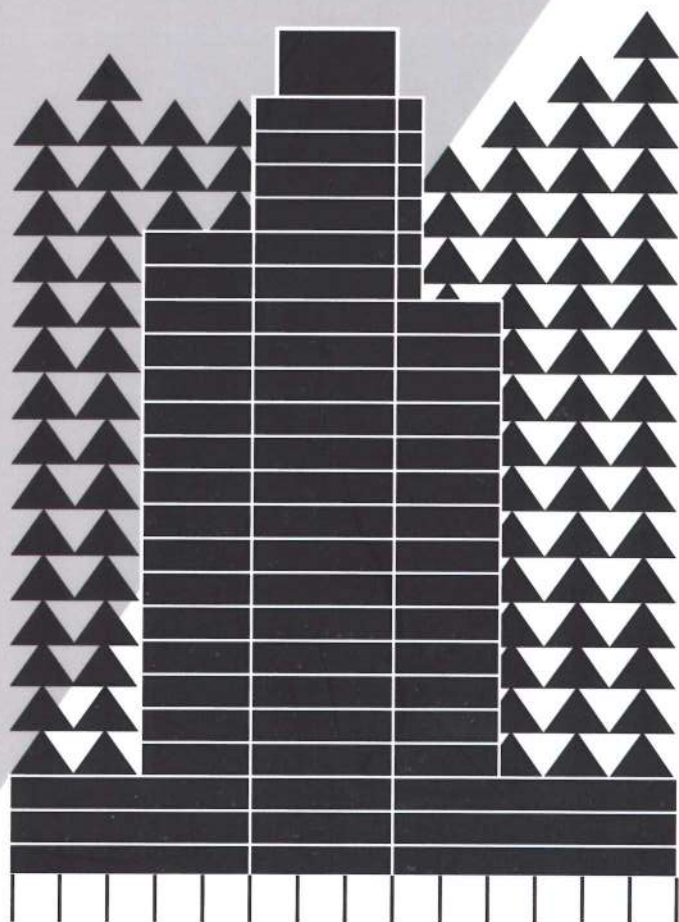


NEXUS

Architecture and
Mathematics

Conference Book



KIM WILLIAMS BOOKS

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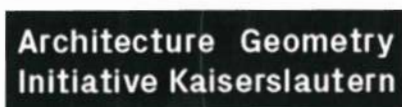


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CONTENTS

Session 1

Representation and Mediation

- 1 RICHARD TALBOT. Thinking, Imagining, Speculation and Building, In and Through, Drawing
- 7 LUIGI COCCHIARELLA. Orthographic Anamnesis on Piero della Francesca's Perspectival Treatise
- 13 GIUSEPPE D'ACUNTO, ISABELLA FRISO. Piazza San Marco Which Is Not There: How it is, How it Was, and How It Should Have Been
- 19 MARINELLA ARENA. The Domes of the Basilica of SS. Pietro and Paolo in Casalvecchio, Messina

Session 2

Space, Perception and Representation

- 27 MARCO HEMMERLING. Anamorphic Spaces
- 31 JAVIER MARTÍN, DANIEL VICENTE MARTÍN FUENTES. Surface Ornamentation and Spatial Distortion
- 37 GIORGIO BURATTI, GIAMPIERO MELE, MICHELA ROSSI. Bramante's *Mirabile Artificio* in Milan
- 43 TIZIANA PROIETTI, SERGEI GEPSHTEIN. Psychophysics of Architectural Proportion in Three Dimensions

Session 3

Parametric Analysis and Designing

- 51 ROBERTA SPALLONE, MICHELE CALVANO. Parametric Experiments on Palladio's Villas
- 57 ELISABETTA CATERINA GIOVANNINI. Making Palladio Digitally Explicit: Geometrical Parameters in the First Book of the *Quattro libri dell'Architettura*
- 63 FEDERICO ANDRES GARRIDO. Understanding 19th-Century Architecture through Parametric Design Strategies
- 69 NIELS-CHRISTIAN FRITSCHÉ. Modelling exceptions. Sense-free Transitions by Guerrilla Design: Cheat on Habits and Software

Session 4

Mathematics of Natural Forms and Structures

- 77 DMITRI KOZLOV. Mathematics of Forms and Structures of Animate Nature in the Theory and Practice of Architectural Bionics
- 83 CLAUDIA REDENBACH. Random Tessellations: How to Build Random Cell Structures
- 87 SABINE BRINITZER. How Organic Architecture was Realized at the Beginning of the 20th Century

- 93 ANTONIO CALANDRIELLO, MARCO FASOLO, VALERIA TALARICO. The Geometric Language of the Brouillon Project: An Original Interpretative Proposal

Session 5

Design Theory and Analysis

- 101 ATHANASSIOS ECONOMOU. Durand Redrawn: A Formal Description of Durand's *Précis of the Lectures on Architecture*
- 107 JU HYUN LEE, MICHAEL J. OSTWALD, MICHAEL J. DAWES. Syntactical and Mathematical Measures of Visitor and Inhabitant Relations in Palladian Villas
- 113 NIKI SIDIROURGOU, GIORGOS KOKKALIS, TATIANA ZOUMPOULAKI. Designing with Space Syntax: An Open-Frame Computational Methodology for Spatial Analysis and Design
- 119 CRISTINA CÂNDITO, ALESSANDRO MELONI. Anne Tyng's Unrealized Projects between Ideal Geometry and Real Architecture

Session 6

Architectural Education

- 127 AYÇE DÖŞEMECILER, ANDRÉE SONAD KARAVELI KARTAL. Overview of Geometry of Design Course: Integration of Mathematics with Design in Architectural Education
- 133 KAROLINA OSTROWSKA-WAWRYNIUK, MARCIN STRZALA, JAN SLYK. Form Follows Parameter: Algorithmic Form-Finding Methods in Architectural Education
- 139 DEENA EL-MAHDY. Making to Learn: A Pedagogical Teaching Model from Manual to Fabrication in Architectural Education Curriculum

Session 7

Short Presentations and Exhibitions

- 145 WERNER BÄUMLER, CORNELIE LEOPOLD. Stairs Like Walking on a Smooth Hill
- 151 PAU NATIVIDAD-VIVÓ. Relationship between Geometry and Stereotomy in Spanish Renaissance Ashlar Sail Vaults
- 157 FEDRICO PANAROTTO. From Topology Optimization to BIM Model: An Interoperable Approach for Building Elements
- 163 KWACHIT SHETH. Timbrel Vaulting Technique: Gauging Skill Acquisition of Unskilled Labourers
- 169 URSULA ZICH, CATERINA CUMINO, MARTINO PAVIGNANO. About Architectural Paper Models: Geometry as a Tool, from Design to Engineering

Session 8

Design Methods

- 177 PAOLO BORIN, ANDREA GIORDANO, RACHELE A. BERNARDELLO, CARLO ZANCHETTA, CRISTIANO GUARNERI, CRISTINA CECCHINI. Configuration vs. Structure: Form Follows Functions

- 183 DANIEL LORDICK, MARTIN EICHENAUER. Line Geometry Applied to Form Finding
- 187 LISS C. WERNER. Prototype Component for a Data-Driven 2.5-Dimensional Façade to Increase Visual Comfort
- 193 MARIA DA PIEDADE FERREIRA. Embodied Emotions: A Methodology for Experiments in Corporeal Architecture

Session 9

Design History

- 201 IZUMI KUROISHI. *Madori* and the Mathematics of Planning during Housing Modernization in Japan
- 207 NICK MOLS. Numeric Intersections between Serlio's and Palladio's Room Ratios
- 213 TERESA BELO RODEIA, JOÃO MIGUEL COUTO DUARTE. The Singular Application of the Modulor to the Oliveira do Hospital *Pousada*
- 219 PETER WILSON. Geometry and Astronomy in Vitruvius's Fanum Basilica

Session 10

Historical Design Analysis

- 227 ROBERT BORK. Geometrical Analysis of Gothic Buildings: Changing Challenges and Opportunities
- 233 BRUNO SCHINDLER. Built Mathematical Complexity: The Towers of Laon
- 239 LUCAS FABIAN OLIVERO, ANTÓNIO BANDEIRA ARAUJO, ADRIANA ROSSI. Reconstruction of St. Michael's Church, Hildesheim, in Cubical Perspective
- 245 LAURA FLORIANO, MARIANGELA LIUZZO, GIUSEPPE MARGANI. Geometric Characterization of Late-Baroque Domes in Sicily

Session 11

Design and Construction History

- 253 ANA LÓPEZ-MOZO, ENRIQUE RABASA-DÍAZ, JOSÉ CALVO-LÓPEZ, MIGUEL ÁNGEL ALONSO-RODRÍGUEZ, ALBERTO SANJURJO-ÁLVAREZ. Geometry and Actual Construction in Brick Vaults by Slices: The case of Carranque in Spain
- 259 MACARENA SALCEDO GALERA, RICARDO GARCÍA BAÑO. The Vault of the Bab El Nasr Gate: An Early Case of Groin Vault
- 265 GIZEM EFENDIOĞLU. Muqarnas Construction Techniques with Geometrical Background
- 271 MARA CAPONE, EMANUELA LANZARA. Search for Geometric Rules: Procedural Modelling for Distribution of Majolica Tiles on Domes

CONFIGURATION VS. STRUCTURE: FORM FOLLOWS FUNCTIONS

Paolo Borin,¹ Andrea Giordano,² Rachele A. Bernardello,³
Carlo Zanchetta,⁴ Cristiano Guarneri,⁵ Cristina Cecchini⁶

Introduction

This contribution is inspired by a quote of the engineer Tony Guéritte (1932): “*The hyperboloid of revolution, in its simplicity as a mathematical body of pure lines, gives a feeling of strength, stability and industrial beauty*”. It is reiterated that a surface makes clear an architecture, being its skin, the exterior part, thus giving it a formal connotation. The research therefore aims to analyse a hyperbolic hyperboloid, a cooling tower in Marghera now dismissed and converted.

The research intends to clarify first the occurred analytical-geometric-historical process of the tower and its construction, then to demonstrate the geometric-mathematical importance of the hyperboloid of revolution to maximize the structural and thermodynamic performance of the tower. Through the simultaneous interface between parametric models and physical simulation environments, the study shows how the tower of the case study represents one of the possible optimization values in the domain of realizable hyperboloid forms.

The Research

This research highlights the role that surfaces and the related geometric-mathematical knowledge has for architecture not only in formal terms, but also in structural terms. It tries to overcome a reductive use of the surface concept, connecting it to adjectives as ‘superficial’, ‘external’ and, in the worst cases, as ‘trivial’. In 1999, in the book ‘*Cupole, volte e altre superfici*’, it was reported: “*In architecture this surface (hyperboloid) was adopted in a specific period, when the as called tense structures were discovered. Also Le Corbusier uses it in Chandigar, Kisho Kurokawa in Abu Dabi and, more unimportantly, it is applied to the external structure of thermal power plants*”. Twenty years later this claim appears inadequate. In fact, after the research on the case study of the cooling tower in Marghera, it was also overturned the contrast between the concept of “engineering structure” and “architectural design”.

Apart from this remark – or perhaps because of it – a deep study of the concept of surface related to any engineering and architectural work seems necessary. For this reason, for each of them a rigorous geometric analysis is fundamental, it expects to identify its own configurative genesis. In fact the genesis knowledge of each surface

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allows a first definition according to the mathematical common characteristics to other surfaces of the same family. Only by referring to the canonical classification of surfaces, it is emphasised how complex can be the 'superficial' study of an architectural artefact. Just considering the same surface generated in different ways - changing the generator element and the law of movement- makes it essential to have a critical "attitude", not only in the representation phase, but also in the one of project, planning, construction and maintenance.

Subsequently a correct awareness of the geometric figures makes it possible to easily cross the border of that dense network between ideas and reality, becoming outstanding 'messengers' inside that network. Marsilio Ficino, in one of his numerous demonstrations on the incorporeal nature of beauty (involving colour, light and numbers as formal and order principles) required his reader to visualize a building, and then to try to neutralize in their mind its material nature, leaving visible only the internal geometric order:

<p><i>Sforzati un poco a trarne la materia, se tu puoi: tu la puoi trarre col pensiero. Orsù trai a lo edificio la materia e lascia sospeso lo ordine: non ti resterà il corpo materiale.</i></p>	<p><i>Struggle a little to draw the substance, if you can: you can draw it with thought. Now bring the substance of the building and leave the order suspended: you won't have the material body.</i></p>
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Following Ficino's "instructions", we can un-materialize the form. But it is essential to reverse this process, so that these forms can be made by a material precision to satisfy our imagination, freed from the pure ideal location. This is the primary purpose of the link between architectural concept and geometric / mathematical genesis. This relationship must promote and stimulate the originality of the designer, who in an initial state in which the idea is born, thinks and generates the architectural work - *in fieri* - in configurative terms of the spaces. Above all it makes those works realizable, considering their structural and constructive features: the semantic value of a surface therefore becomes fundamental for its structure. As Guéritte states, this is the case of the hyperbolic hyperboloids.

Trying to consider the generative variety of a hyperbolic hyperboloid (or one-sheeted), it is:

1. a ruled surface generated by a straight line, which moves in space, constantly leaning on three lines, skewed two by two.
2. a surface generated by an ellipse (or a circumference) of variable size, which moves by leaning on two points of a fixed hyperbola, the plane always keeps the same position, and running its centre-point on a line overlapping with the diameter of the hyperbola, conjugated to the direction of the strings, corresponding to the support points of the ellipse (or of the circumference);
3. a surface of rotation, generated by the rotation of a branch of hyperbola around one of its non-transverse axis (hyperbolic hyperboloid of rotation) or of a straight line

around an axis, not coplanar with it, related to an axis of rotation (getting, again, a ruled surface).

It is therefore easy to understand that the actual feasibility of a hyperbolic hyperboloid cannot ignore the multiple and complex geometric genesis.

The study proposed, implementing an innovative research methodology by a multidisciplinary work group, highlights the described strong bond through the study of the natural cooling tower in Marghera. The applied method in fact is based on the cohabitation and the relation of a historical archive / document research, of a high precision digital survey and of a digital model.

All of this has allowed not only to understand the evolution of the building - from the construction to the actual state, up to a subsequent restoration works - but also to implement the interactions between advanced analytical methods, including the geometric / mathematical, structural and fluid dynamics. The investigation, elaborated in several phases, had in fact the focus on the link between every disciplinary characteristic with the formal development of the tower, identifying the connection between the geometry and the performances that the cooling tower had to satisfy during the operation phase.

From the analysis of the historical beginning of the cooling towers, it was possible to repeat the fundamental and following steps that allow to identify, in the hyperbolic reinforced concrete hyperboloid, the perfect balance between geometry and performance. There were two significant elements in the experimentation process on cooling towers: the shape and the material. The first examples were made of wood on rectangular plants, characterized by a limited duration and a poor circulation of water volumes. Then there are the metallic cylindrical towers, based on frame structures, with even worse results. From the earliest years of the twentieth century, however, the engineers successfully tested the use of cylindrical reinforced concrete cooling towers, a construction technique patented by the French François Hennebique in 1882. The use of this material and the circular base section had multiple advantages from multiple points of view. Reinforced concrete had the advantage of being resistant to both the action of water steam and fire, it did not require maintenance and offered perspectives that persisted much longer than wood and metal. Moreover, the cylindrical towers guaranteed a better distribution of water inside them and an excellent resistance to the action of the wind. Once the suitable material and the optimal planimetric plant, reinforced concrete and the circular one respectively, were found, the engineers began to work on the volumetric development of the cooling towers looking for the maximum efficiency. Problems were soon highlighted also for the cylindrical towers, especially when they reached large diameters, more and more required by industrial plants.

The director of the Dutch State Mines, Frederik van Iterson, with the help of Gerard Kuypers developed, a cooling tower based on completely new principles, that transform this type of plant: the hyperbolic tower. Starting from the invention of the Dutch engineer, a flourishing debate was soon run and an intense construction activity of hyperbolic towers of different sizes and different profiles took place.

Around the section of the *Société des Ingénieurs Civils de France* gathered the most innovative engineers in the field, including the same Van Iterson and Guéritte.

The hyperbolic refrigerant of Vetrocoke in Porto Marghera also was born from this pioneering environment in the field of applications of reinforced concrete in the industrial field. Built between 1937 and 1938, it comes from a French model belonging to the *Réfrigérants hyperboliques* of Paris, as indicated on the project tables.

Conclusion

It was therefore possible to understand, through historical research, survey and digital modelling, the authentic structural realization of the tower, highlighting its close link with the geometric / mathematical genesis. But, as stated, the double-curved shape not only guarantees surprising structural behaviour, but also optimizes its function of cooling water and expelling steam. In fact, through computational fluid dynamics algorithms the model of the envelope surface has shown the increase in velocity and temperature inside the hyperboloid form, compared to the primitive form, the cylindrical one. The double curvature of a hyperbolic tower therefore, on one hand it gives a remarkable structural strength and further improves the aerodynamics compared to the cylinder, allowing a uniform weight discharge on the ground. On the other hand it increases the draft of the air and evenly distributes the water reaching its full cooling function.

References

- Krätzig, B., P.L. Gould, R. Harte. 2001. Power technology shell structure: cooling and solar updraft towers. In: *Fifty years of progress for shell and spatial structures, Proceedings of the IASS*, I. Mugnan, ed. Madrid: J.F. Abel.
- McKelvey, K.K., M. Brooke. 1959. *The industrial cooling tower: with special reference to the design, construction, operation and maintenance of water cooling towers*. Amsterdam-London-NewYork-Princeton: Elsevier Publishing Company.
- Mungan, I., Wittek U., eds. 2004. *Natural Draught Cooling Towers. Conference Proceedings*. London: Taylor & Francis.
- Giordano, A. 1999. *Cupole, volte e altre superfici. La genesi e la forma*. Torino: UTET.
- Guéritte, T.-J. 1932. Les réfrigérants a cheminee a contre-courant. *Le Génie civil. Rev. générale des Ind. françaises étrangères* 7: 158-161.
- Guerritte, T.-J. 1936. Hyperbolische Kuehltuerme bei Hams Hall in England. *Beton und Eisen* 35: 213-219.
- Van Iterson, F.K.T. 1946. Hyperboloïdical cooling towers. *Koninklijke Nederlandse Akademie Van Wetenschappen. Conference Proceedings* 49: 113-122.

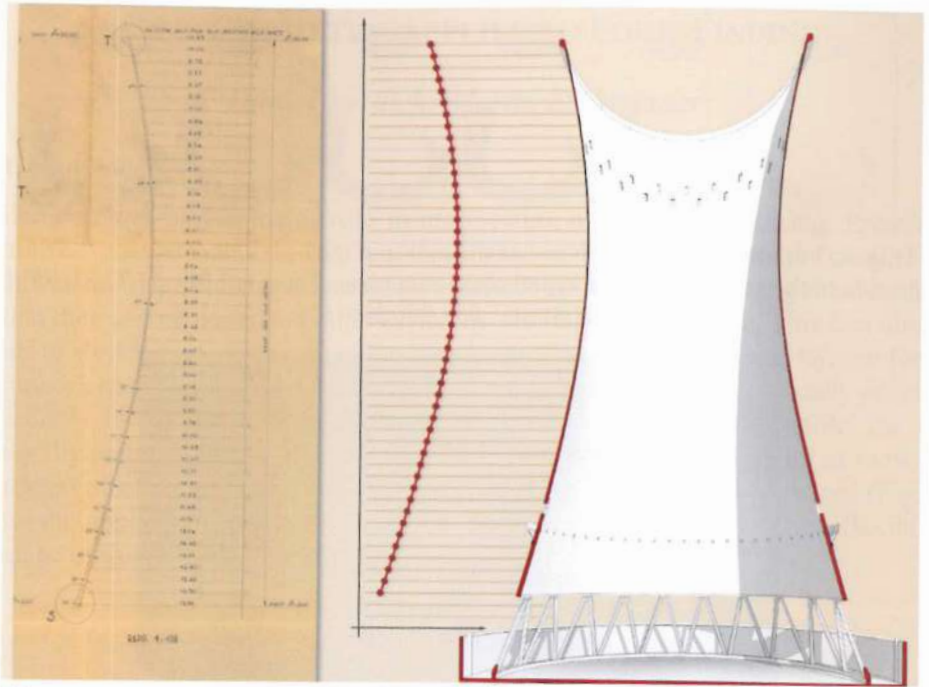


Fig. 1. Hyperbole equation and graphic. Project information and as built. Comparison



Fig. 2. Point cloud of the hyperboloid hyperbolic as built

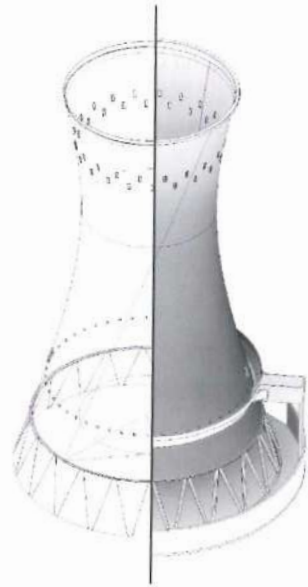


Fig. 3. Geometric schema of the hyperboloid hyperbolic and the obtained BIM model

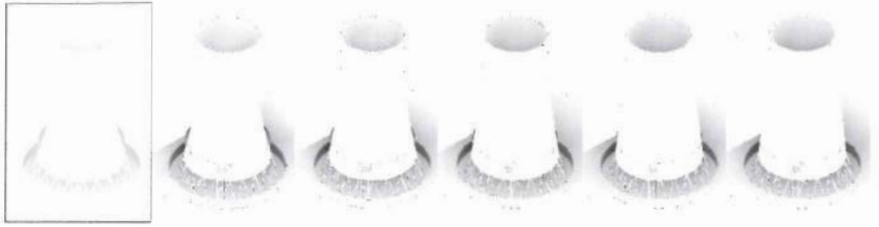


Fig. 4. Variation of the angle between the radius of the base circumference and the projection of the generatrix on the hyperboloid base plane. From left $\beta_1=91,7^\circ$ $\beta_2=75^\circ$ $\beta_3=60^\circ$ $\beta_4=45^\circ$ $\beta_5=30^\circ$ $\beta_6=15^\circ$

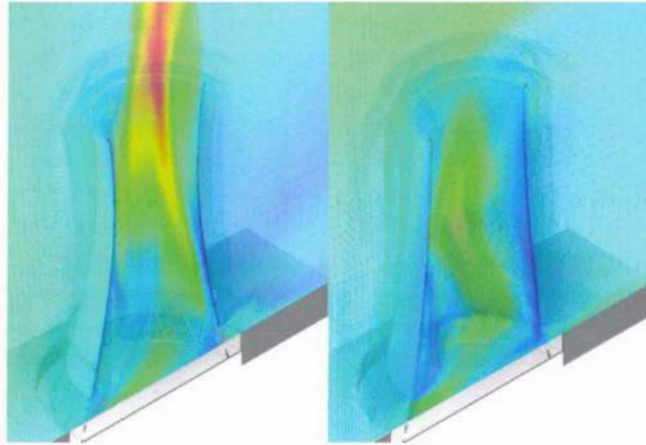


Fig. 5. CFD simulation of the air flow velocity of the air (m/s). Analysis made through the variation of the angle between the radius of the base circumference and the projection of the generatrix on the hyperboloid base plane. The two represented towers have an angle of $\beta_1=91,7^\circ$ $\beta_2=15^\circ$ respectively

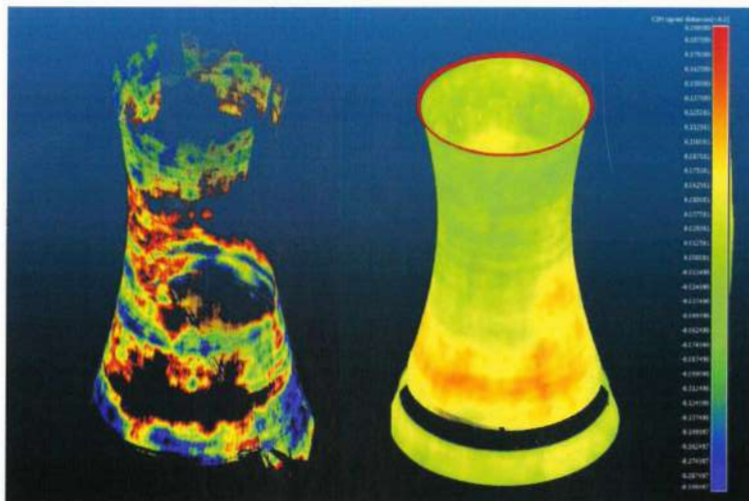


Fig. 6. Deviance analysis between the point cloud and the BIM model. Interior surface on the left and exterior surface on the right