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Abstract
To contain bankers' risk-shifting behavior, policymakers use a variety of tools. Among them, mandating the use of default-linked (i.e., debt-like) pay features prominently, typically in the form of bonus deferrals. In our model, a risk-neutral manager is in charge of choosing bank-level asset risk, receiving in exchange a compensation package consisting of a bonus and a default-linked component. In the spirit of existing regulation and widespread industry practices, we give the manager discretion over the allocation of the personal default-linked account between own bank's shares and an alternative asset. The possibility for the manager to tie the value of default-linked pay to equity weakens its debt-like feature and, in the same way, its ability to rein in excessive risk-taking. Bank leverage and bailout expectations appear to exacerbate these effects, which may be further aggravated by the endogenous shareholders' choice to design a more convex bonus as a response to mandatory default-linked pay. Our analysis raises concerns on the robustness of the theoretical foundations of some recent regulatory efforts.

Keywords
Bank Risk-Taking, Banking Regulation, Default-Linked Compensation

JEL Codes
G21, G28, G34, M12

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When Does Linking Pay to Default Reduce Bank Risk?*

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Abstract

To contain bankers' risk-shifting behavior, policymakers use a variety of tools. Among them, mandating the use of default-linked (i.e., debt-like) pay features prominently, typically in the form of bonus deferrals. In our model, a risk-neutral manager is in charge of choosing bank-level asset risk, receiving in exchange a compensation package consisting of a bonus and a default-linked component. In the spirit of existing regulation and widespread industry practices, we give the manager discretion over the allocation of the personal default-linked account between own bank's shares and an alternative asset. The possibility for the manager to tie the value of default-linked pay to equity weakens its debt-like feature and, in the same way, its ability to rein in excessive risk-taking. Bank leverage and bailout expectations appear to exacerbate these effects, which may be further aggravated by the endogenous shareholders' choice to design a more convex bonus as a response to mandatory default-linked pay. Our analysis raises concerns on the robustness of the theoretical foundations of some recent regulatory efforts.

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1 Introduction

Fostering financial stability is a key goal of banking regulation and supervision. In the aftermath of the Great Recession, regulatory restrictions on bankers' pay have become a prominent device in the toolbox of policymakers—in combination with traditional reserve and capital requirements, and with more innovative macroprudential policies—, triggering an active scholarly debate on the their un(intended) consequences (e.g., Bebchuk and Spamann, 2009; Bhagat, Bolton, and Romano, 2014; Hoffmann, Inderst, and Opp, 2022; Murphy, 2013; Thanassoulis, 2012). Regulators around the world introduced a variety of rules that banks must follow when designing their key employees' compensation packages, along the lines of the recommendations contained in the Principles for Sound Compensation Practices issued in 2009 by the Financial Stability Board. The European Union (EU), with Capital Regulation Directives (CRD) III, IV, and V of 2010, 2013, and 2019, has been at the forefront of compensation regulation in banking. One rule in CRD requires EU banks to defer at least 40% of the variable remuneration of their material risk takers (MRTs), a group including, among other employees, the members of the executive team.[1] Such a regulatory focus on mandatory deferrals motivates our analysis.

Common wisdom (and standard theory) predicts that a manager will take more risk as equity-based remuneration increases, and less as the debt-like component grows.[2] Hence, as part of a general effort to tame risk-shifting, policymakers advocate deferrals, which are supposed to make pay packages less convex—and bankers less short-termist. However, a link exists between the payoff of deferred compensation and bank equity performance. This is the result of both regulatory constraints—e.g., CRD establishes that EU banks must pay out at least 50% of deferred variable remuneration in the form of equity-linked instruments—and managers' discretion to invest their deferred compensation accounts into own bank equity.[3] We investigate the risk-taking consequences of this—largely overlooked by the theoretical literature—equity-like feature of deferrals. In the model, we shut down any possible effects of deferrals on decision horizon, to focus, instead, on the extent to which the link with equity performance hinders their purported goal to ren-

[1]See Article 94 of CRD IV and Deutsche Bundesbank (2021). Similar rules apply to banks from the UK. In the US, Section 956 of the Dodd-Franck Act deals with the same issues, but it has not been implemented yet.

[2]Admittedly, stock options can per se make things more nuanced (Ross, 2004).

[3]Jackson and Honigsberg (2014) and Franco and Urcan (2022) show that US public companies, including those in the financial sector, allow their executives to use nonqualified deferred plans to acquire own company stock. Over a similar sample, Cambrea, Colonnello, Curatola, and Fantini (2019) argue—and show empirically—that executives have some degree of discretion in the way their deferred compensation plans are invested and actively reduce their exposure to own firm's stock in bad times.

der bankers' payoff schedules more akin to debt claims. For clarity, we call this type of remuneration—which provides a payment contingent on the bank not defaulting—*default-linked* pay. In brief, we illustrate that in our setting the impact of default-linked pay on managerial risk-taking incentives becomes ambiguous, especially in the presence of government guarantees to banks, thus raising concerns on the robustness of one of regulators' tools of choice.

More specifically, building on the work of Hakenes and Schnabel (2014), we examine the bank-level risk choice of a risk-neutral manager receiving both a bonus and default-linked pay. The bank benefits both from explicit and implicit guarantees in the form of a deposit insurance and of the possibility for uninsured lenders to be bailed out by the government in bankruptcy, respectively, which may loosen market scrutiny and favor risk-shifting. Looking at default-linked pay in a static framework helps us flesh out the risk-taking incentives stemming from the oft-assumed debt-like feature of deferrals, abstracting from their actual postponement of payout relative to a standard bonus contract.

In this setting, mandating a plain form of default-linked pay—insensitive to bank performance over the non-default region—achieves the objective to curb managerial risk-taking. However, the picture turns clouded if, more realistically, we allow the manager to endogenously choose how to invest the personal default-linked account between own bank's shares and an alternative asset, thus making it de facto more akin to equity-linked pay. This weakens the ability of default-linked pay to mitigate the risk-taking incentives arising from bank leverage, bailout expectations, and bonuses, in particular when bank valuations are high. Thus, our analysis raises a doubt about an underexplored facet of remunerating bankers with debt-like instruments, namely that they can (or must) invest them in own bank's equity. When it comes to preventing excessive risk-taking, this feature can be self-defeating, especially if we consider that, in equilibrium, the bank shareholder responds to exogenously-imposed higher default-linked pay by making the bonus contract more convex.

The policy implications emerging from our setting are threefold. First and foremost, if the regulator's goal is to align the manager to public interest, it seems preferable to opt for basic debt-like remuneration forfeited in default and with no performance-sensitivity outside of default. Second, if this last condition is not met, it may be needed to couple mandatory default-linked pay with a cap on bonus convexity, in the spirit of the restrictions on pay curvature advocated by Thanassoulis and Tanaka (2018). Third, enhanced disclosure both on the allocation of the default-linked pay and on its status in default (e.g., security and priority) could be important both for banking supervisors and market participants.

Finally, it is worth discussing two points about the scope of our results. First, whereas the model we develop provides insights into debt-like remuneration, it is silent about how delaying the payout of variable compensation—a quintessential feature of deferrals—impacts the risk and effort choices of bankers. Put differently, the degree of applicability of the analysis to real-world regulation mandating deferrals is limited by the static framework. Nonetheless, already such a model unearths a host of intricacies that question the robustness of existing and proposed bonus deferral rules for bankers. Second, in our model the shareholder delegates the bank-level risk choice to the manager, who can be interpreted as the CEO or, collectively, as the executive team. However, we argue that our results are relevant also for other employees (e.g., some traders) that can significantly and single-handedly impact the bank risk profile—the MRTs in the jargon of CRD.

This paper contributes to the growing literature on the financial stability implications of compensation regulation in the banking sector.[4] The traditional corporate governance nexus between the management and the shareholders of a firm, in the case of a bank, must be augmented to account for wide-reaching supervisory activity as well as for the presence of safety nets. In turn, the latter could lead to negative risk externalities in the form of heightened systemic risk (e.g., Becht, Bolton, and Röell, 2011; De Haan and Vlahu, 2016). Because of this multifaceted agency problem, traditional measures aimed at realigning managers' and shareholders' interests—such as "say on pay"—may not be sufficient in the banking industry, where the risk-shifting problem generally trumps the effort one (e.g., Bebchuk and Spamann, 2009). Consistently, restrictions on bankers' pay—such as bonus caps, clawbacks, deferrals, prolonged vesting schedules—are mainly targeted at reducing their risk-taking incentives.

A number of theoretical studies have investigated the effectiveness of regulatory restrictions on bankers' compensation as a device to prevent excessive risk-taking.[5] For instance, Thanassoulis (2012), based on a setting with multiple banks facing a competitive bankers' labor market, makes the case for regulating their compensation. Hakenes

[4]It is worth noting that our paper also speaks to the wider—and typically focused on nonfinancial firms—literature on inside debt started by the seminal works of Jensen and Meckling (1976) and Edmans and Liu (2011) on the theoretical side, and by those of Sundaram and Yermack (2007) and Wei and Yermack (2011) on the empirical one. In fact, deferred compensation is (together with pension plans) a component of inside debt, i.e., of debt-like claims of managers on the firm's assets. Restricting the attention to banks, Van Bekkum (2016) and Bennett, Güntay, and Unal (2015) find that asset risk during the Great Recession was lower at banks more reliant on inside debt to remunerate their managers.

[5]A growing body of empirical work exists in this area as well (e.g., Cerasi, Deininger, Gambacorta, and Oliviero, 2020; Colonnello, Koetter, and Wagner, 2023; Kleymenova and Tuna, 2021).

and Schnabel (2014) demonstrate that a bonus cap can limit inefficient bank risk-taking in a single-bank model featuring both risk-shifting and effort problems between management and shareholders, as well as the presence of a government safety net protecting the bank. Thanassoulis and Tanaka (2018) evaluate possible endogenous reactions of bank managers and shareholders to specific forms of regulations, among which imposing debt-like pay. Bolton, Mehran, and Shapiro (2015) consider a single-bank framework in which the manager chooses asset risk, showing that it could be optimal to link pay to bank debt by an indexation to credit default swap spreads. Albuquerque, Cabral, and Guedes (2019) illustrate how relative performance evaluation ingrained in compensation packages of bankers may make regulation in the form of bonus cap ineffective at limiting risk-taking. Castiglionesi and Zhao (2023) focus on banks' portfolio diversification choices and show that regulating the mix of relative and absolute performance evaluation can actually enhance the efficiency of the banking sector. Hoffmann et al. (2022)—based on the modeling techniques proposed by Hoffmann, Inderst, and Opp (2021) to deal with persistent effects of managerial actions—illustrate that the manager's outside option is crucial to determine the bank-level risk-management implications of deferred compensation. We add to this literature by investigating how the possibility given to bank managers to modify the sensitivity of debt-like remuneration to equity performance can impact their risk-taking incentives.

2 Model setup

The model extends the framework of Hakenes and Schnabel (2014) to study how linking compensation to default risk influences a bank manager's risk-taking incentives.[6] In addition to a basic bonus contract, we consider an exogenously imposed default-linked pay component reminiscent of recent regulation on mandatory deferrals for bankers. Moreover, we allow the manager to endogenously choose how to invest the latter pay component up to its liquidation, consistently with commonly observed features of deferred compensation plans.

Before delving into the details of this compensation arrangement, we sketch the key features of the setting. The model has one period, with time going from $t = 0$ to $t = 1$. A representative shareholder owns the bank and sets the manager's compensation contract at $t = 0$. The bank has an exogenously-defined capital structure comprising deposits

[6] Where possible, we follow the same notation as Hakenes and Schnabel (2014) for clarity. Note that we focus on the manager's risk choice and abstract from the effort problem, which Hakenes and Schnabel (2014) incorporate in an extension of their model.

with value d (insured with a premium of δ per unit), uninsured liabilities (e.g., unsecured bonds, interbank loans, etc.) with value l, and equity with value k. The manager as well as the bank financiers—the representative shareholder, the depositors, and the uninsured lenders—are risk-neutral. The bank's assets—which can be thought of as a portfolio of loans, securities, derivatives, and the like—have the following distribution of payoffs Y:

$$Y = \begin{cases} Y_h, & \text{with probability } p_h; \\ Y_m, & \text{with probability } p_m; \\ Y_l, & \text{with probability } p_l. \end{cases}$$

We assume that $Y_h > Y_m > Y_l = 0$ and that default only occurs if Y_l realizes. In this case, the bank's creditors (lenders and depositors, but not shareholders) are bailed out by the government with probability ϕ.

The manager selects the unobservable riskiness of bank's assets (a), which, in turn, determines the probabilities of the bank's future payoff:

$$\begin{aligned} p_h(a) &= p_h^0 + \frac{a}{Y_h(Y_h - Y_m)}; \\ p_m(a) &= p_m^0 - \frac{a}{Y_m(Y_h - Y_m)}; \\ p_l(a) &= 1 - p_h - p_m = 1 - p_h^0 - p_m^0 + \frac{a}{Y_m Y_h}. \end{aligned}$$

where $\{p_i^0\}_{i=h,m}$ are the probabilities associated to the "zero risk-shifting" policy, i.e., the social optimum (as explained below). An increase in a rises the probability of obtaining the highest and the lowest payoff and produces a mean-preserving spread.[7] In this way, we can interpret a as the bank's risk-taking. To change a, the manager sustains a private non-monetary cost given by $C(a) = \alpha\frac{a^2}{2}$ with $\alpha > 0$, which could be interpreted as the cost of the effort needed to modify the risk profile of the bank (e.g., the time spent seeking for investment opportunities capable of increasing the risk-return profile of the bank's portfolio).

Uninsured lenders receive a promised payment L in the absence of default. In case of default, they recoup their initial investment l if there is a bailout, otherwise they forfeit it in full. We can express the break-even condition as $(p_h + p_m)L + p_l\phi l = (1 + r_l)l$,

[7]Formally, $\mathbb{E}[Y] = p_h Y_h + p_m Y_m = p_h^0 Y_h + p_m^0 Y_m$ is independent from a, whereas $\mathbb{V}ar[Y] = a + p_h^0 Y_h^2 + p_m^0 Y_m^2 - (p_h^0 Y_h + p_m^0 Y_m)^2$ is increasing in a.

therefore

$$L(a,\phi) = l\frac{(1+r_l) - p_l(a)\phi}{p_h(a) + p_m(a)}, \tag{1}$$

where r_l is the return demanded by uninsured lenders. Note that, keeping other things constant, $\frac{\partial L}{\partial \phi} < 0$: as the bailout probability increases, the risk borne as well as the compensation required by lenders decline.

Depositors receive a riskless return equal to r_d, paid either by the bank or by the deposit insurance scheme.[8] Let $\bar{L}$ denote the promised payment to depositors, their breakeven condition is

$$\bar{L} = (1 + r_d)d.$$

We define the price of (levered) equity as given by the shareholder's expected payoff divided by the number of outstanding shares (normalized to one for simplicity):[9]

$$P_e(a,\phi) = p_h(a)\left(Y_h - L(a,\phi) - \bar{L}\right) + p_m(a)\left(Y_m - L(a,\phi) - \bar{L}\right) - \delta. \tag{2}$$

In the analysis of the model, we contrast the case of full anticipation of the manager's risk choice by creditors with that of no anticipation. Hakenes and Schnabel (2014) assume that the risk choice (a) can be fully anticipated by creditors. In this case, one can replace the promised payment to lenders defined in equation (1) into equation (2) to obtain

$$P_e(a,\phi) = p_h(a)Y_h + p_m(a)Y_m - l(1 + r_l - p_l\phi) - (p_h + p_m)\bar{L} - \delta. \tag{3}$$

When the risk choice cannot be anticipated, creditors need to form rational expectation

[8] r_d and r_l capture the time preference of risk-neutral depositors and uninsured lenders, respectively. Any difference between the two can be motivated by the presence of segmentation across retail and wholesale funding markets. By contrast, the default premium is captured by $L(a,\phi)$ in the break-even condition in equation (1).

[9] Consistent with Bolton et al. (2015), we ignore discounting. This choice has no qualitative consequences for our results. As we shall see, when the manager has the opportunity to invest default-linked pay between own bank's equity and an alternative asset x, the allocation choice changes depending on whether $P_e > P_x$ or not. Because in our analysis both expected return on equity at $t = 0$ and P_x are taken as given, scaling the shareholder's payoff by an exogenously-given discount factor only shrinks the parametric region over which $P_e > P_x$ but the risk-taking implications in the two regions are not affected. Differently from Bolton et al. (2015), we assume—as in Hakenes and Schnabel (2014)—that the non-monetary cost is paid by the manager rather than by the shareholders.

(a^E) about the bank's risk. The price of equity is then given by

$$P_e(a,\phi) = p_h(a)\left(Y_h - L(a^E,\phi) - \bar{L}\right) + p_m(a)\left(Y_m - L(a^E,\phi) - \bar{L}\right) - \delta. \tag{4}$$

In a rational expectation equilibrium, we must have $a = a^E$. All proofs are in Appendix A.

3 The manager's risk choice

To analyze the manager's optimal risk choice, we initially assume that the compensation contract is exogenous and comprises both a bonus and a default-linked component.[10] The asset payoff is verifiable, therefore the manager's overall payoff depends on the realized outcome.

Defining the bonus payment in state i as z_i with $i = h, m, l$, this pay component gives the manager a strictly positive payoff z_h in the good state of the world, and zero otherwise (i.e., $z_m = z_l = 0$). Indeed, as argued by Hakenes and Schnabel (2014), the shareholder as a residual claimant is interested in increasing the probability of the good state of the world, meaning that no payment is made to the manager for reducing the volatility of asset payoffs ($z_m = 0$); the assumption of managerial limited liability together with the zero asset payoff in the bad state implies a zero bonus payment in such a state ($z_l = 0$). Thus, the bonus has a more convex payoff profile than regular equity and constitutes a high-powered incentive akin to stock options.[11] Whereas such a bonus structure is optimal from the shareholder's perspective and can be thus expected to arise endogenously in the managerial labor market, in this section we assume the level of z_h to be exogenous.

Concerning the default-linked pay, the manager receives an amount Θ in the no-default states only (i.e., with probability $p_h + p_m$). The forfeiture of Θ in default—even in the case of a bailout—makes it debt-like, thus aligning the interests of the manager with those of creditors. We consider two alternative designs: (i) a *non-discretionary* arrangement under which Θ is constant and (ii) a *discretionary* arrangement under which the manager can choose to tie Θ to the value of own bank's equity or to the value of alternative assets.

It is important to draw a comparison between our default-linked pay and how deferred compensation is traditionally modeled. Relative to the bonus scheme, deferred compen-

[10] We assume that the manager receives no fixed salary without loss of generality. A non-zero salary would be irrelevant for the risk choice of a risk-neutral manager.

[11] Similarly, other papers analyze contracts in which (either by assumption or as a result of the shareholder's optimal choice) the manager receives a bonus payment only in good states of the world (e.g., Gietl and Haufler, 2018; Gietl and Kassner, 2020).

sation features both delayed and less convex payoffs that make it debt-like.[12] In our one-period model, by contrast, default-linked pay is deferred relative to the manager's choice of bank risk, but not to the realization of risk, making it timing-wise equivalent to the bonus payment. Whereas this simplification makes the model silent about managerial short-termism, default-linked pay captures the essential debt-like nature of deferred compensation and may be interpreted accordingly. At the same time, it allows us to single out the consequences of this feature, whose analysis in standard deferred compensation models is potentially complicated by the accompanying delay of payoffs. Moreover, the discretionary arrangement captures a second important characteristics of deferred compensation, namely the fact that its liquidation value is often tied to company equity, as discussed above. Finally, default-linked pay differs from the bonus scheme in terms of its contracting rationale. As already pointed out, a simple argument can explain the shareholder's decision to award the manager a highly convex bonus paying off in state h alone, but less so for a flat payment Θ made in states m and h. Hence, below we endogenize the choice of the bonus payment z_h but hold the design of default-linked pay exogenous, interpreting it as a restriction in the spirit of some recent efforts to regulate the banking industry.[13] We instead abstract from the design of optimal regulation and from the political economy of it.

The timing of the model can be summarized as follows.

- ▶ $t = 0$: shareholders offer a compensation contract to the manager. Then, conditional on this contract:
  - Lenders and depositors define interest rates on their claims;
  - Shareholders provide equity capital, pay the insurance premium, and fund the financing gap left by depositors and uninsured lenders;
  - The manager sets asset risk level and the investment strategy of the default-linked account;
  - The liquidation value of default-linked pay (when linked to company equity or other assets) is determined.
- ▶ $t = 1$: the bank's asset payoff realizes. If $Y = Y_h$ or $Y = Y_m$, no default occurs and creditors are repaid. If $Y = Y_h$, the manager receives the bonus z_h. If $Y = Y_l$,

[12] For instance, Bolton et al. (2015), to draw a comparison between their proposed CDS-linked compensation and deferred compensation, consider a two-period setting.

[13] Besides banking-specific regulations, another form of institutional friction that could motivate our analysis is the preferential tax treatment of deferrals (e.g., non-qualified deferred compensation (NQDC) plans in the US).

the manager loses default-linked compensation and the bank is bailed out with probability ϕ; with probability $1-\phi$, insured depositors are reimbursed by the deposit insurance company.

3.1 The first-best risk choice

To characterize the manager's risk choice, we start by finding the first-best level of a, which obtains under the maximization of the net expected payoff

$$\mathbb{E}[Y] - \alpha \frac{a^2}{2}. \tag{5}$$

Because $\mathbb{E}[Y]$ does not depend on a (see footnote 7 for details), the optimal first-best choice is $a = 0$. Accordingly, we refer to any risk choice $a > 0$ as risk-shifting. We now go back to the manager's choice under the two alternative default-linked compensation schemes.

3.2 Non-discretionary scheme

Under the non-discretionary arrangement, the manager receives a constant amount $\Theta = \bar{\Theta}$ if the bank is solvent at $t = 1$, and zero otherwise. Therefore, the expected value of default-linked compensation is $(p_h + p_m)\bar{\Theta}$, making this form of remuneration akin to unsecured debt owed by the bank to the manager. The manager selects the riskiness of the bank portfolio by solving the problem

$$\begin{aligned} &\max_a \Pi_M^{ND} \equiv p_h z_h + (p_h + p_m)\bar{\Theta} - \alpha \frac{a^2}{2} \\ &\text{s.t.} \quad \underline{a} \leq a \leq \bar{a}, \end{aligned} \tag{6}$$

where Π_M^{ND} is the manager's personal expected payoff under the non-discretionary (ND) scheme. The feasibility constraint $\underline{a} \leq a \leq \bar{a}$ ensures that the probabilities p_h, p_m and p_l defined above lie between 0 and 1: $\underline{a} = -\min[p_h^0 Y_h(Y_h - Y_m), (1-p_m^0)Y_m(Y_h - Y_m), (1 - p_h^0 - p_m^0)Y_h Y_m]$ and $\bar{a} = \min[(1-p_h^0)Y_h(Y_h - Y_m), p_m^0 Y_m(Y_h - Y_m), (1-p_l^0)Y_h Y_m]$.

Proposition 1 (Optimal risk-taking under the non-discretionary scheme)**.** *Let*

$$\tilde{a}(z_h, \bar{\Theta}) = \frac{z_h}{\alpha Y_h(Y_h - Y_m)} - \frac{\bar{\Theta}}{\alpha Y_h Y_m}$$

be the risk-taking policy when the feasibility constraint is slack. The optimal constrained risk policy $\hat{a}_{ND}$ is:

$$\hat{a}_{ND}(z_h, \bar{\Theta}) = \begin{cases} \underline{a}, & \text{if } \tilde{a}(z_h, \bar{\Theta}) \leq \underline{a}; \\ \bar{a}, & \text{if } \tilde{a}(z_h, \bar{\Theta}) \geq \bar{a}; \\ \tilde{a}, & \text{if } \underline{a} < \tilde{a}(z_h, \bar{\Theta}) < \bar{a}. \end{cases}$$

Proposition 1 summarizes the standard results on the risk-taking incentives stemming from bonus payments and default-linked pay. First, the optimal risk level is in general larger than zero (i.e., the first best), hence involving some degree of risk-shifting. Second, risk-taking incentives increase with bonus payments z_h and decrease with default-linked pay $\bar{\Theta}$ (in line with Jensen and Meckling, 1976; Edmans and Liu, 2011; Hakenes and Schnabel, 2014). Finally, regardless of the assumption on whether the manager's risk choice can be anticipated by creditors or not, the stock price P_e increases with a. In this case, managerial compensation does not depend on P_e and, thus, the possibility to anticipate bank risk does not affect the choice of risk.

3.3 Discretionary scheme

To characterize the discretionary default-linked pay scheme, we take real-world deferred compensation plans as a benchmark. Specifically, we posit that such a scheme allows the manager to choose between own bank's equity and a different asset (e.g., an index fund, a basket of stocks, etc.) with price P_x.[14] Investments of deferred compensation are typically notional and work in a similar way as phantom shares. In practice, the value of phantom shares—or units—of own equity or of the alternative asset is usually calculated using the average price over a pre-determined time window preceding the settlement date.[15] For simplicity, we abstract from the potential price wedge between phantom and actual shares, and just consider P_e from equation (2) and P_x as the reference price. The value of the default-linked account is then $\Theta = \tilde{\Theta} = \pi_e P_e + \pi_x P_x$, where π_e and π_x represent the number of shares of own equity and asset X, respectively. Such an amount is forfeited in case of default.[16] As a result the expected value of the default-linked account is

[14]One may imagine an asset paying off X_i with probability q_i, with $i = h, m, l$. Assuming that $x_l = 0$, the price of the asset is then $P_x = q_h X_h + p_m X_m$. The manager has no impact on the probabilities q_i, thus it may favor interpretation to assume $q_h = p_h^0$ and $q_m = p_m^0$, where $\{p_i^0\}_{i=h,m}$ are the "zero risk-shifting" probabilities. This construction facilitates the comparison between the price of bank equity and that of the alternative asset, but it is not needed for model solution.

[15]For non-listed companies, the plan needs to specify a different valuation method (e.g., the multiple method applied to balance sheet information or a third party appraisal).

[16]This assumption, for instance, is consistent with the treatment of NQDC plans in bankruptcy in the US. The notional assets in NQDC plans are generally backed by the firm's general assets and therefore

$(p_h + p_m)\tilde{\Theta}$. Finally, we assume that the total number of shares awarded cannot exceed the upper bound π. We take the total number of shares in the manager's default-linked account (π) as given, in line with the presence of a regulation imposing the use of deferred compensation and study the endogenous response of managers (and later shareholders) to changes in π.

To maximize personal expected payoff Π_M^D under the discretionary (D) scheme, the manager solves

$$\max_{a,\pi_e,\pi_x} \Pi_M^D \equiv p_h z_h + (p_h + p_m)\tilde{\Theta} - \alpha\frac{a^2}{2} - \beta\frac{\pi_e^2}{2} \tag{7}$$
$$\text{s.t.} \quad \underline{a} \leq a \leq \bar{a}, \quad \pi_e + \pi_x = \pi,$$

where $\beta\frac{\pi_e^2}{2}$ (with $\beta > 0$) represents the cost of changing the investment policy of default-linked compensation (e.g., transaction costs and commissions paid through the brokerage account). The possibility to invest in own equity makes default-linked compensation more similar to equity-based pay to a degree depending on π_e. In other words, under this scheme, default-linked pay—which is essentially inside debt—becomes an hybrid instrument that combines characteristics of debt (i.e., the payoff is flat with respect to bank performance in non-default states as long as $\pi_e = 0$) and equity (i.e., the payoff increases with bank performance in non-default states as long as $\pi_e > 0$) to a varying degree depending on the manager's choice of π_e, which is determined jointly with the choice of bank risk.[17]

Problem (7) does not require the manager's economic exposure to own bank's equity to be positive. However, managerial hedging is a often criticized practice (e.g., Dunham and Washer, 2012), in particular for its ability to undo the manager's incentive compatibility constraint (e.g., Schizer, 2000). As a result, firm-level bans on hedging transactions by managers are widespread (e.g., Colonnello, Curatola, and Xia, 2022); one example in the banking sector, among others, is that of UBS, which fully prohibits hedging to members of its "group executive board" (UBS, 2021). To rule out the possibility that the manager obtains a negative exposure to bank equity, in several cases below we augment problem (7) with the short-selling constraint $\pi_e \geq 0$.[18]

exposed to bankruptcy risk. See, e.g., https://www.callan.com/blog-archive/nqdc-investment-menu/.

[17]By setting $Y_l = 0$, we de facto assume a zero recovery rate on bank debt in the absence of bailout and deposit insurance.

[18]Besides outright short sales of own equity, to limit their exposure to the firm, managers could implement a variety of hedging strategies—differing in how easily they can be detected—such as pledging shares, entering options or variable prepaid forward contracts, investing in assets negatively correlated

3.3.1 A simplified model: Secured default-linked pay

To obtain a closed-form solution for the optimal choice of risk, it useful to first consider the special case of problem (7) in which $p_h + p_m = 1$ and creditors can anticipate bank risk. This can be obtained by assuming that the manager is overconfident (Gietl and Kassner, 2020), and thus overestimates the probability of receiving default-linked compensation, or when the bank provides informal funding on the default-linked account—for instance, through a Rabbi trust or corporate-owned life insurances (Walker, 2019)—to which we refer as the "secured & discretionary" scheme.

Proposition 2 (Optimal risk-taking with secured & discretionary default-linked pay)**.** *Assume that* $P_e(\underline{a}, \phi) < P_x < P_e(\bar{a}, \phi)$ *and* $\alpha > \frac{(l\phi+\bar{L})^2}{\beta Y_h^2 Y_m^2}$. *Let* $\Gamma = P_e(0, \phi)$, *i.e., the equity price corresponding to the first-best risk level, with* P_e *defined as in equation* (3) *(case with full anticipation of risk). We then have two relevant cases:*

1. $\Gamma > P_x$. *The short-selling constraint* ($\pi_e \geq 0$) *is slack and the optimal solution* ($a^*(slack)$, π_e^*) *is*

$$a^*(slack) = \frac{\frac{z_h}{Y_h(Y_h - Y_m)} + (\Gamma - P_x)\frac{l\phi+\bar{L}}{\beta Y_h Y_m}}{\alpha - \frac{(l\phi+\bar{L})^2}{\beta Y_h^2 Y_m^2}} > 0,$$

$$\pi_e^* = \frac{P_e(\alpha^*, \phi) - P_x}{\beta} > 0.$$

with $\frac{\partial \alpha^*}{\partial \phi} > 0$, $\frac{\partial \pi_e^*}{\partial \phi} > 0$, $\frac{\partial \alpha^*}{\partial z_h} > 0$, $\frac{\partial \pi_e^*}{\partial z_h} > 0$.

2. $\Gamma < P_x$. *Let* $\bar{z} = \alpha(P_x - \Gamma)\frac{Y_h^2 Y_m(Y_h - Y_m)}{l\phi+\bar{L}}$, *then we have two sub-cases:*

   (a) $z_h < \bar{z}$. *The short-selling constraint is binding and the optimal solution* ($a^*(bind)$, π_e^*) *is*

$$a^*(bind) = \frac{z_h}{\alpha Y_h(Y_h - Y_m)} > 0,$$

$$\pi_e^* = 0.$$

with bank equity (e.g., Gao, 2010; Bettis, Bizjak, and Lemmon, 2001; Bettis, Bizjak, and Kalpathy, 2015; Fabisik, 2022; ProPublica, 2023). In banking, derivatives-based hedges seem uncommon among executives, but may be more prevalent below executive level (New York Times, 2011; Fahlenbrach and Stulz, 2011). It may be even easier for managers to achieve a negative economic exposure to the firm via negatively correlated assets. Exploring the implications of these elusive forms of hedging goes beyond the scope of this paper.

(b) $z_h \geq \bar{z}$. *The short-selling constraint is slack and the optimal solution is given in point 1 above.*

When the equity price corresponding to the first-best risk level exceeds the value of alternative investments ($\Gamma > P_x$), the manager has the incentive to invest in bank equity even in the absence of risk shifting ($a = 0$). But personal exposure to bank equity provides the manager with an incentive to risk-shift. Because $\Gamma = P_e(0, \phi) > P_x$ and P_e is increasing in a, then $P_e(a, \phi) > P_x$ in the presence of risk-shifting in any degree, thus making the short-selling constraint always slack in this case. As evident from Proposition 2, the manager risk-shifts even for a zero bonus payment ($z_h = 0$). This implies that a bonus cap would lead to lower risk-shifting but would not induce the socially optimal risk choice (i.e., $a = 0$). Similarly, ruling out the possibility to invest the default-linked account in bank equity would reduce risk-taking incentives (see the proof of Proposition 2 for details).

When $\Gamma = P_e(0, \phi) < P_x$, bank risk a needs to be sufficiently high to have $P_e(a, \phi) > P_x$ and, thus, induce the manager to invest the default-linked account in bank equity.[19] As the risk choice increases with the bonus payment, there exists a bonus threshold $\bar{z}$ such that the manager invests in bank equity only for $z_h > \bar{z}$. When $z_h < \bar{z}$, the short-selling constraint binds, the value of default-linked pay does not depend on bank equity, and the optimal risk choice $a = \frac{z_h}{\alpha Y_h (Y_h - Y_m)}$ is equivalent to that obtained under the non-discretionary contract with $\bar{\Theta} = 0$, or, in other words, in the absence of default-linked pay (cf. $\tilde{a}(z_h, \bar{\Theta})$ in Proposition 1).[20]

More intuitively, in the presence of a short-selling constraint, a low bonus payment coupled with valuable investment alternatives makes the discretionary contract equivalent to a non-discretionary one. In this situation, the social optimum can be achieved by setting $z_h = 0$. This equivalence also implies that the value of alternative investment options only affects risk-taking incentives when the short-selling constraint is slack—i.e., $\pi_e > 0$—, with increases in P_x reducing the optimal risk choice $a^*(slack)$. The crucial feature in this setting is the link between π_e and the sensitivity of the manager's expected payoff with respect to risk. Higher values of P_x reduce the exposure to bank equity and make default-linked pay less sensitive to bank risk, thus reducing risk-taking incentives.

[19] For simplicity, here we still use the expression "default-linked pay" even though its payoff is de facto unrelated to default because $p_h + p_m = 1$.

[20] Because $p_h + p_m = 1$ here, changes in the risk choice do not affect the probability of receiving the default-linked pay component, and higher values of the latter do not induce the manager to take on less risk, unlike under the non-discretionary contract (see Proposition 1).

3.3.2 The complete model: Unsecured default-linked pay

We consider now the complete model in which the manager fully forfeits default-linked pay in the case of bankruptcy. We start from the general problem in which the manager can use the default-linked account to attain a negative economic exposure to own bank's equity, to then move to the case with banned short sales on which we will focus in the main analysis.

To ease the exposition of results, we define the marginal gain and cost —in terms of default-linked account value—of changing the bank's risk profile:

$$\text{Marginal gain} = \chi(a) = (p_h + p_m)\pi_e \frac{\partial P_e}{\partial a},$$
$$\text{Marginal cost} = \gamma(a) = -\frac{\partial(p_h + p_m)}{\partial a}\tilde{\Theta} = \frac{[\pi_e(a)P_e(a,\phi) + (\pi - \pi_e(a))P_x]}{Y_h Y_m}.$$

The marginal gain χ captures the change in the default-linked account value induced by an increase in the riskiness of assets, conditional on the bank not defaulting (i.e., with probability $p_h + p_m$). The marginal cost γ is the reduction in the expected value of default-linked account value driven by the decline in the probability of receiving a non-zero payoff (i.e., $p_h + p_m$) that comes together with more risk-taking.

The total marginal cost of increasing a also accounts for the cost of changing bank risk that is not related to default-linked compensation, namely $\alpha\frac{a^2}{2}$. The total marginal cost is thus defined as $K = \alpha a + \gamma$. Similarly, the total marginal gain also accounts for the positive impact of bank risk on the expected bonus payment ($p_h z_h$). The total marginal gain is defined as $M = \frac{\partial p_h}{\partial a} z_h + \chi$.

All-equity bank. It is useful to first consider an all-equity bank (i.e., $l = d = \delta = 0$), which still allows us to obtain a closed-form solution for a. The price of equity in this case is $P_e^0 = p_h\left(Y_h - L - \bar{L}\right) + p_m\left(Y_m - L - \bar{L}\right) - \delta = p_h^0 Y_h + p_m^0 Y_m - \delta$ and depends neither on the manager's risk choice (i.e., $\frac{\partial P_e}{\partial a} = 0$) nor on whether the latter can be anticipated or not by creditors. The marginal gain and cost take the form:

$$
\begin{aligned}
\chi &= (p_h + p_m)\pi_e \frac{\partial P_e}{\partial a} = 0, \\
\gamma &= \frac{(P_e^0 - P_x)^2 \left(p_h^0 + p_m^0 - \frac{a}{Y_h Y_m}\right)}{\beta Y_h Y_m} - \frac{\pi P_x}{Y_h Y_m}, \\
M &= \frac{z_h}{Y_h(Y_h - Y_m)}, \\
K &= \alpha a + \gamma.
\end{aligned}
$$

Since $\chi = 0$ the optimal risk choice can be computed in closed-form for an all-equity bank.

Proposition 3 (Optimal risk-taking under the discretionary scheme in an all-equity bank)**.** *Let $d = l = \delta = 0$ and assume that $\alpha - \frac{(P_e^0 - P_X)^2}{\beta Y_h^2 Y_m^2} > 0$. The manager's optimal risk choice under the discretionary scheme and a slack feasibility constraint is*

$$
\tilde{a}_{DE}(z_h) = \frac{z_{DE}}{Y_h(Y_h - Y_m)} \times \frac{1}{\alpha - \frac{(P_e^0 - P_X)^2}{\beta Y_h^2 Y_m^2}}, \tag{8}
$$

where $z_{DE} \equiv z_h - (p_h^0 + p_m^0)\frac{(P_e^0 - P_X)^2}{\beta Y_h Y_m} - \frac{\pi P_X}{Y_h Y_m}$ and $\frac{\partial \tilde{a}}{\partial \pi} < 0$. The optimal constrained risk policy $\hat{a}_{DE}$ is

$$
\hat{a}_{DE}(z_h) = \begin{cases} \underline{a}, & \text{if } \tilde{a}_{DE}(z_h) \le \underline{a}; \\ \bar{a}, & \text{if } \tilde{a}_{DE}(z_h) \ge \bar{a}; \\ \tilde{a}_{DE}, & \text{if } \underline{a} < \tilde{a}_{DE}(z_h) < \bar{a}. \end{cases}
$$

The closed-form expression in equation (8) allows to isolate the risk-mitigating impact of the number of deferred shares (π). Increasing π rises the value of the default-linked account and, thus, curbs risk-taking incentives. This mechanism holds only when default-linked pay is effectively at risk in bankruptcy (and the manager correctly internalizes the probability of default). Differently, if secured, changing π has no effect on risk-taking incentives[21] (Proposition 2). Hence, our models suggests that regulating any form of default-linked pay—such as deferred compensation—requires precise knowledge on its security and priority in the event of bankruptcy.[22]

[21]The negative relationship between the number of deferred shares π and managerial risk-taking incentives does not exist with secured compensation. When $p_h + p_m = 1$ changes in the risk choice do not affect the probability of receiving the default-linked pay component, and higher values of the latter do not induce the manager to take on less risk.

[22]See also Anantharaman, Fang, and Gong (2014) and Colonnello, Curatola, and Hoang (2017).

Levered bank. We then consider a levered bank when the manager can engage in short sales of own equity through the default-linked account and the creditors cannot anticipate the risk choice.

Proposition 4 (Optimal risk-taking under the discretionary scheme in a levered bank)**.** *Assume that the net marginal gain of changing risk using the default-linked account (namely $\chi-\gamma$) decreases with a. Let then the unconstrained risk choice $\tilde{a}_D$ which satisfies the optimality condition (i.e., the total marginal benefit of changing the bank's risk profile equal to total marginal cost)*

$$\begin{aligned}\frac{z_h}{Y_h(Y_h-Y_m)} &+ p_{hm}(\tilde{a}_D)\pi_e^*(\tilde{a}_D)\frac{\partial P_e(\tilde{a}_D,\phi)}{\partial a} \\ &- \frac{[\pi_e^*(\tilde{a}_D)P_e(\tilde{a}_D,\phi)+(\pi-\pi_e^*(\tilde{a}_D))P_x]}{Y_hY_m} - \alpha\tilde{a}_D = 0,\end{aligned} \tag{9}$$

where

$$p_{hm}(a) = p_h^0 + p_h^0 - \frac{a}{Y_hY_m}, \quad \pi_e^*(a) = p_{hm}(a)\frac{(P_e(a,\phi)-P_x)}{\beta},$$
$$\frac{\partial P_e}{\partial a}(a,\phi) = \frac{L(a,\phi)+\bar{L}}{Y_hY_m} > 0, \quad \frac{\partial^2 P_e}{\partial a\partial\phi} = \frac{\partial L/\partial\phi}{Y_hY_m} < 0, \quad \frac{\partial\tilde{a}}{\partial z_h} > 0, \quad \frac{\partial\tilde{a}}{\partial\pi} < 0.$$

Then, the risk choice that solves the optimization problem (7) is

$$\hat{a}_D(z_h) = \begin{cases} \underline{a}, & \text{if } \tilde{a}_D \le \underline{a}; \\ \bar{a}, & \text{if } \tilde{a}_D \ge \bar{a}; \\ \tilde{a}_D, & \text{if } \underline{a} < \tilde{a}_D < \bar{a}. \end{cases}$$

Finally, assume that the value of default-linked account under the discretionary scheme $\pi_e^*P_e + (\pi-\pi_e^*)P_x$ equals its value under the non-discretionary scheme. Then, the optimal constrained risk policy under the discretionary contract exceeds (is smaller than) the optimal risk choice under the non-discretionary contract when $P_e > P_X$ ($P_e < P_X$).

Under the discretionary contract, the manager compensation depends directly on the value of bank equity. Therefore, the manager has to take into account the effect of risk and bailout expectations on the price of equity.[23] The direct link of compensation to bank equity price produces several key implications.

[23] Under the non-discretionary scheme—or in the benchmark model of Hakenes and Schnabel (2014)—the value of bank equity affects the manager's risk choice only indirectly, through its effect on the shareholders' optimal choice of the bonus payment z_h.

First, the possibility to invest deferred compensation in bank stock is an additional tool to align the manager's interests to the shareholder's. The alignment mechanism and its relationship with the manager's risk appetite depends on $P_e - P_x$. When the optimal risk choice is such that $P_e > P_x$ (i.e., alternative assets have lower expected payoffs and, thus, lower price) incentives are aligned: the manager finds it optimal to have a positive exposure to company equity (through the default-linked account) whose value, in turn, is increasing in equity price and, thus, in bank risk. These results are broadly consistent with Franco, Ittner, and Urcan (2017) and Franco and Urcan (2022), who find that executives and directors convert their cash pay into deferred company equity at a higher rate before the release of good earning news or when growth opportunities are higher. In this case, there is alignment between the manager and the shareholder, and the manager's risk appetite rises. When, instead, $P_e < P_x$, the manager takes a negative exposure to bank equity: the manager and the shareholder become disaligned, and the manager risk appetite thus declines.[24]

Second, consider two default-linked accounts with the same current value, one following the discretionary scheme, the other the non-discretionary one. If $P_e > P_x$ ($P_e < P_x$), the discretionary case is associated with higher (lower) risk-taking incentives. This suggests that bank creditors—and the public at large in the presence of a non-zero bailout probability—cannot rationally anticipate the manager's risk choice just by looking at the level of default-linked pay at a given point in time. One should also pay attention to the investment strategy of deferred compensation and the investment options available.

Finally, when the number of shares in the default-linked account (π) increases, the manager takes less risk, as before. It is important to stress that when the default-linked account is not invested in bank equity (non-discretionary contract) or when the price of bank equity does not depend on the risk choice (all-equity bank), the effect of π coincides with that of the price of the alternative assets P_x: both of them reduce risk-taking incentives.[25] When the value of the default-linked account depends on the price of bank equity this equivalence ceases to hold. Risk-taking incentives still decrease in π but the effect of P_x becomes ambiguous when $\pi_e < 0$. The result is intuitive: the value of the default-linked account is given by $\pi_e(P_e - P_x) + \pi P_x$, with π_e decreasing in P_x (details in the proof of Proposition 4). Therefore, an increase in P_x unambiguously increases the value of the default-linked account only when $P_e - P_x < 0$ (because in this

[24]To see this consider that $\frac{\partial P_e}{\partial a} > 0$ and that P_x is a constant. Take an optimal risk choice a_1 such that $P_e > P_x$ and an alternative choice a_2 such that $P_e < P_x$. Then, it must be the case that $a_1 > a_2$.

[25]It is immediate to verify that $\text{sgn}\left(\frac{\partial a}{\partial \pi}\right) = \text{sgn}\left(\frac{\partial a}{\partial P_x}\right)$.

case the manager reduces the negative exposure to the bank equity, which is worth less than alternative investments). Consistent with the benchmark non-discretionary case the increase in the value of default-linked compensation reduces risk-taking incentives. When $P_e - P_x > 0$ the increase in P_x has two opposite effects on the value of the default-linked account. On the one hand, it induces the manager to reduce the exposure to bank equity which is now worth more than alternative investments. On the other, it increases the value of the alternative investment making the overall effect on the value of default-linked account unclear.

Levered bank with a short-selling constraint. We now augment the problem analyzed in Proposition 4 with a short-selling constraint $\pi_e \geq 0$, again assuming that the creditors cannot anticipate the manager's risk choice.

Proposition 5 (Optimal risk-taking under the discretionary scheme with a short-selling constraint)**.** *Assume that $P_e(\underline{a}, \phi) < P_x < P_e(\bar{a}, \phi)$ and that the net marginal gain of changing risk using the default-linked account (namely $\chi - \gamma$) decreases with a. Let $\bar{z}$ (defined in the proof) be the bonus payment such that $P_e = P_x$ with $\frac{\partial \bar{z}}{\partial \phi} < 0$. Hence, for any $z_h \geq \bar{z}$, $P_e \geq P_x$, which makes the manager willing to hold own bank's equity (i.e., the short-selling constraint is slack) and the manager's optimal choices (a and π_e) are described in Proposition 4. When $z_h < \bar{z}$, the short-selling constraint is binding and $\pi_e = 0$. Define*

$$\dot{a}(z_h) = \frac{z_h}{\alpha Y_h (Y_h - Y_m)} - \frac{\pi P_x}{\alpha Y_h Y_m},$$

with $\frac{\partial \dot{a}}{\partial z_h} > 0$ and $\frac{\partial \dot{a}}{\partial \pi} < 0$. Then, the optimal risk policy ($\hat{a}_D$) is

$$\hat{a}_D(z_h) = \begin{cases} \underline{a}, & \text{if } \dot{a}(z_h) \leq \underline{a}; \\ \dot{a}, & \text{if } \underline{a} < \dot{a}(z_h) < \bar{a}. \end{cases}$$

Similarly to the simplified case above (Proposition 2), there exists a threshold for the bonus payment ($\bar{z}$), such that $P_e > P_x$ and the short-selling constraint is slack for any $z_h \geq \bar{z}$. When instead $z_h < \bar{z}$, the constraint is binding and the manager zeroes out the personal exposure to bank equity ($\pi_e = 0$), fully investing the default-linked account in the alternative asset and making its final value effectively independent of bank performance. As a result, a discretionary scheme featuring a short-selling constraint and low bonus payments is equivalent to a non-discretionary one.

Moreover, a comprehensive assessment of the consequences of default-linked compensation for managerial risk-taking incentives cannot abstract from its interactions with moral hazard stemming from bailout expectations. This form of implicit guarantees to the bank are key to grasp the government's position in the problem at hand, in which policy-makers can intervene by tying pay to default events. A further observation on the role of bailout expectations is therefore in order. When the constraint is binding, the chosen risk level decreases in πP_x—the part of deferred compensation whose value is outside the manager's control. When P_x represents the price of other banks' equity or the price of equity of firms in the rest of the economy, it is plausible to assume it is also increasing in the bailout probability ϕ. In this constellation, an increase in the bailout probability will have a risk-mitigating effect through its positive impact on P_x. The prediction changes when the short-selling constraint is slack. As highlighted above, in this case the increase in P_x has unclear effects on the value of the default-linked account and, thus, on risk-taking incentives.

Numerical illustration. We then analyze the model numerically, focusing on the unsecured and discretionary default-linked pay scheme when creditors cannot anticipate the manager's risk choice. To satisfy the requirement $P_e(\underline{a}, \phi) \leq P_x \leq P_e(\bar{a}, \phi)$ set forth in Propositions 2–5, we assume that $P_x(\phi) = \frac{P_e(\underline{a},\phi)+P_e(\bar{a},\phi)}{2}$. This assumption implies that P_x is increasing in ϕ and can be rationalized by assuming that the manager seeks to diversify the personal portfolio by investing in the equity of other banks whose price also depends (positively) on the bailout probability. Castiglionesi and Zhao (2023) show how such a behavior may arise endogenously in the presence of compensation contracts containing a relative performance evaluation component.[26] If, instead, we expect the manager to more broadly diversify away from the banking sector, one can think about the positive link between bailout probability and the price of alternative assets as capturing the long-run positive impact of bailouts on the rest of the economy—for instance, by fostering faster recoveries from banking crises (see Modena, 2023, and references therein).[27]

In Figure 1, we study the implications of the bonus payment z_h for the manager's risk-portfolio choices and the price of bank equity. In Panel (a), we observe that the optimal risk choice a is increasing in the bonus payment, leading to risk-shifting behavior

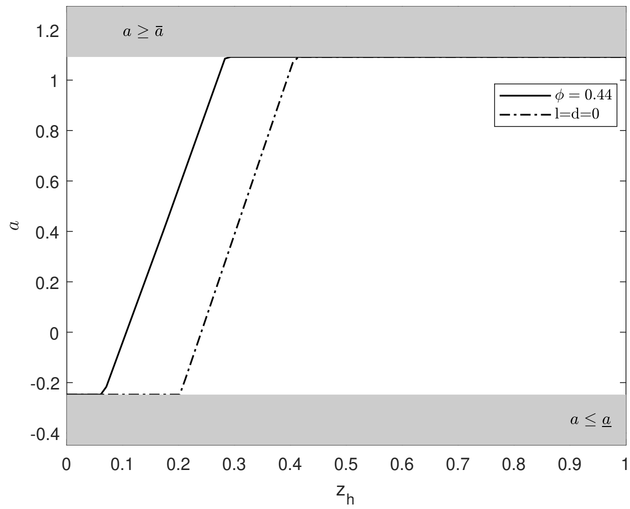
[26] This assumption has two desirable implications: i) it is always possible, although maybe not advantageous for the manager, to find a risk level such that $P_e > P_x$; ii) at the bonus payment $z_h = \bar{z}$ (the one that gives $P_e = P_x$), the manager's optimal choice of risk is $\underline{a} < a < \bar{a}$.

[27] The case of constant P_x is analyzed in Appendix B, together with further details on the numerical procedure.

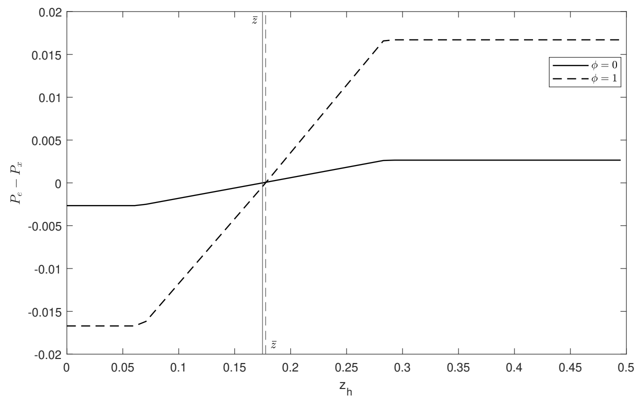
($a > 0$) for most numerical values of z_h. For low enough bonus payments, the manager becomes excessively conservative and finds it optimal to reduce risk relative to the first best (i.e. $a < 0$). The presence of leverage increases managerial risk-taking incentives for any bonus payment z_h. In Panel (b), we look at the bank price of equity. If $z_h < \bar{z}$, the short-selling constraint binds and $P_e < P_x$. In this situation, $\pi_e = 0$ and the value of the default-linked account does not depend on the equity price. As a result, the optimal risk choice, and thus equity price, is higher under $\phi = 0$ (no bailout in default) than $\phi > 0$ (positive bailout probability in default) because of the risk-mitigating effect of the higher value of alternative assets, which, under the assumed specification for P_x, benefit from a higher bailout probability. Differently, if $z_h > \bar{z}$, the short-selling constraint is slack. In such a case, the manager invests default-linked compensation in the bank equity and the effect of bailout expectations is inverted: an increase in ϕ induces the manager to take on more risk (see the discussion of Proposition 5 above) and increases the equity price P_e.

Panel (c) shows the manager's optimal investment in bank equity (π_e) through the default-linked account. For a low bonus payment, the level of risk is such that $P_e < P_x$ and the short-selling constraint is binding ($\pi_e = 0$). Higher bonus payments increase risk-taking incentives and, thus, $P_e - P_x$ rises. As a result, the manager wishes to buy more shares of the bank equity. For large enough bonus payments, the feasibility constraint binds, making the level of risk, $P_e - P_x$, and, thus, π_e constant with respect to z_h.

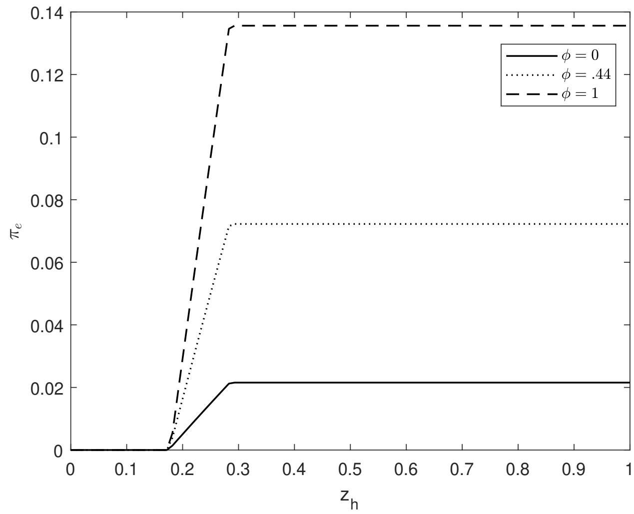
From a policy perspective, it is also relevant to investigate the impact of default-linked compensation. Through the lenses of our model, the recent regulatory efforts mandating deferred compensation would amount to an increase in π, the number of shares in the default-linked account. Figure 2 analyzes the effect of measures that intervene on this dimension of bankers' compensation structure, for different levels of the bonus payment. As we would expect, an increase in π rises the value of default-linked compensation and reduces managerial risk-taking incentives irrespective of whether the short-selling constraint binds or not (Panel (a)). As a result, the price of equity declines (Panel (b)) and the manager finds it optimal to invest less in bank stock for any bonus payment z_h (Panel (c)).



(a) Optimal risk choice. The optimal a as a function of the bonus payment z_h.

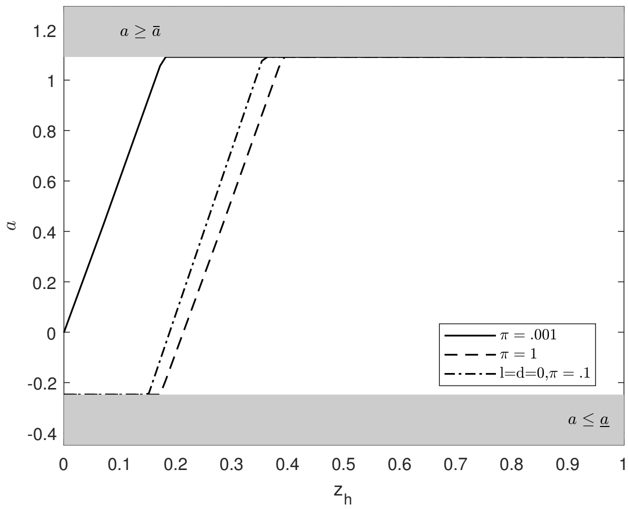


(b) Price of bank equity. $P_e - P_x$ as a function of the bonus payment z_h.

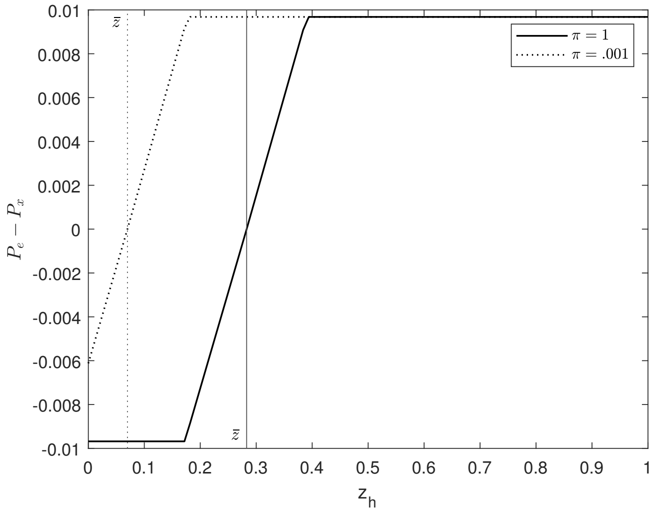


(c) Optimal investment. Optimal π_e as a function of z_h.

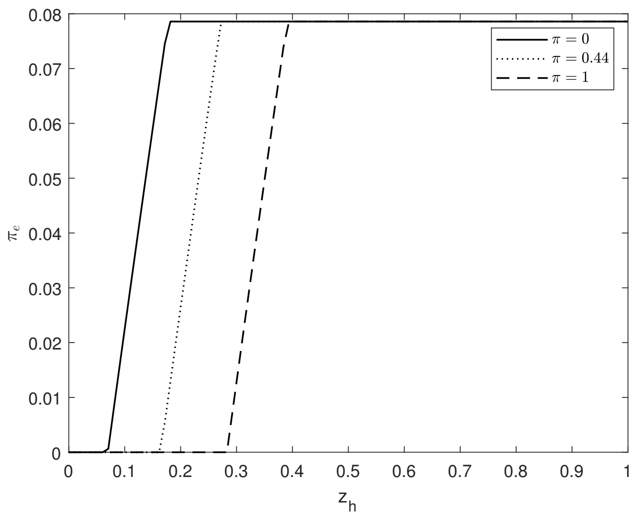
Figure 1: Optimal manager's risk-portfolio choices and corresponding equity price as a function of the bailout probability under the unsecured and discretionary scheme for default-linked compensation. Calibration: $Y_h = 5.5$, $Y_m = 5.2$, $Y_l = 0$, $p_h^0 = 0.15$, $p_m^0 = 0.7$ $p_l^0 = 0.15$, $\alpha = 0.1$, $\beta = 0.1$, $\pi = 0.5$, $\delta = 0.1$, $r_d = 0.02$, $r_l = 0.04$, $l = 0.6$, $k = 0.3$, $d = (1 - l - k)/(1 - \delta)$, $P_x = (P_e(\underline{a}, \phi) + P_e(\bar{a}, \phi))/2$.



(a) Optimal risk choice. The optimal a as a function of z_h.



(b) Price of bank equity. $P_e - P_x$ as a function of z_h.



(c) Optimal investment. The optimal π_e as a function of z_h.

Figure 2: Optimal manager's risk-portfolio choices and the corresponding equity price as a function of the bailout probability under the unsecured and discretionary scheme. Calibration: $Y_h = 5.0$, $Y_m = 5.5$, $Y_l = 5.2$, $p_h^0 = 0.15$, $p_m^0 = 0.7$ $p_l^0 = 0.15$, $\alpha = 0.1$, $\beta = 0.1$, $\delta = 0.1$, $r_d = 0.02$, $r_l = 0.04$, $l = 0.6$, $k = 0.3$, $d = (1 - l - k)/(1 - \delta)$, $\phi = 0.5$, $P_x = (P_e(\underline{a}, .5) + P_e(\bar{a}, .5))/2$.

In summary, the following results arise. i) The discretionary scheme may increase or decrease managerial risk-taking incentives as compared to the standard non-discretionary case depending on the characteristics of default-linked compensation (e.g., investment options or informal funding). ii) The risk-shifting behavior linked to the discretionary scheme tends to be more severe when alternative investment options have low value relative to bank equity. iii) The risk-taking effects of different compensation components are intertwined, which complicates the analysis of the potential consequences of policy measures aimed at reining in risk-shifting incentives (e.g., a bonus cap, a change in the bailout probability, or mandating default-linked compensation). In the next section, we characterize the endogenous response of the shareholder to such measures when designing the manager's compensation structure.

4 Optimal compensation contract

So far we have taken the compensation contract as given. In this section, we allow the bank's shareholder to set the remuneration structure with the goal of maximizing the expected payoff (i.e., share price net of deposit insurance premia compensation), taking into account the manager's i) optimal risk choice (a) and the associated investment strategy π_e (in case of discretionary contract), ii) participation constraint ($\Pi_M \geq 0$), and iii) limited-liability constraint ($\{z_h, \bar{\Theta}\} \geq 0$).

4.1 Non-discretionary scheme

We start by illustrating the benchmark case of the default-linked non-discretionary contract. The shareholder chooses the bonus payment in the good state of the world (z_h) and the value of default-linked compensation ($\bar{\Theta}$) to maximize the expected payoff (Π_e):[28]

$$\begin{aligned} \max_{z_h \geq 0, \bar{\Theta} \geq 0} \Pi_e(z_h, \bar{\Theta}) \equiv\; & p_h \left(Y_h - L - \bar{L} - z_h - \bar{\Theta}\right) + p_m \left(Y_m - L - \bar{L} - \bar{\Theta}\right) \\ & - \delta - \nu[\bar{\Pi} - \Pi_M^{ND}]. \end{aligned} \tag{10}$$

The first two terms capture the contingent equity payoff in the non-default states $\{h, m\}$, i.e., the asset payoff net of the promised payments to uninsured creditors, depositors, and the manager. The third term denotes the non-contingent insurance premium paid by

[28] P_e in equation (2) is inclusive of managerial pay and thus akin to *inside* equity. Instead, here we take the perspective of the *outside* shareholder designing the manager's incentive scheme, hence the expected payoff Π_e is net of managerial pay.

the bank. Together, the first three terms describe the shareholder's expected net payoff Π_e, which depends on the risk choice of the manager, a. The last term of problem (10) captures the participation constraint (i.e., $\Pi_M^{ND} \geq \bar{\Pi} = 0$) with Lagrange multiplier ν.

Proposition 6 (Optimal contract under the non-discretionary scheme)**.** *The shareholder rewards the manager only if $Y = Y_h$ and, thus, $\bar{\Theta} = 0$. Concerning the bonus payments, we have the following cases.*

- *If creditors cannot anticipate the manager's risk choice, the optimal bonus payment, z_h^*, satisfies*

$$- p_h^0 - \frac{2z_h^*}{\alpha Y_h^2 (Y_h - Y_m)^2} + \frac{L(a_{ND}(z_h^*)) + \bar{L}}{\alpha Y_h^2 Y_m (Y_h - Y_m)^2} = 0. \tag{11}$$

  Moreover, $\Pi_M^{ND} > (=)0$, if $z_h > (=)0$ and $\frac{\partial z_h}{\partial \phi} \lesseqgtr 0$. If $l = d = 0$, the limited-liability constraint is binding and $z_h^* = 0$.

- *If creditors are able to anticipate the manager's risk choice—the case analyzed by Hakenes and Schnabel (2014)—and $l\phi + \bar{L} - \alpha p_h^0 Y_h^2 Y_m (Y_h - Y_m) > 0$, we have*

$$z_h^* = \frac{Y_h - Y_m}{Y_m} \frac{l\phi + \bar{L} - \alpha p_h^0 Y_h^2 Y_m (Y_h - Y_m)}{2}, \tag{12}$$

  and $\Pi_M > 0$. If $l\phi + \bar{L} - \alpha p_h^0 Y_h^2 Y_m (Y_h - Y_m) \leq 0$, then $z_h^* = \Pi_M^{ND} = 0$.

More intuitively, the shareholder has no incentive to align the manager's interests with those of bank creditors or of the government providing explicit or implicit guarantees to the bank, and, thus, sets $\bar{\Theta} = 0$. In other words, non-discretionary default-linked compensation in bankers' contracts would arguably emerge only in the presence of a binding regulatory constraint (or government's moral suasion) to adopt it.

4.2 Discretionary scheme

Policymakers may also decide to mandate a default-linked account that gives the bankers some leeway in how to allocate it—our discretionary scheme. In turn, the shareholder can endogenously respond by adapting the design of the bonus contract. In particular, under the discretionary scheme with a short-selling constraint, the shareholder maximizes the expected payoff taking into account that the expected value of default-linked pay depends on the manager's investment strategy π_e (i.e., the optimal risk level a and the associated investment strategy π_e satisfy the manager's first order conditions):

$$\max_{z_h \geq 0} \quad \Pi_e(z_h) \equiv p_h\left(Y_h - z_h\right) + p_m Y_m - (p_h + p_m)\left(\tilde{\Theta} + L + \bar{L}\right) - \delta - \nu[\bar{\Pi} - \Pi_M^D]$$
$$\text{s.t.} \quad \tilde{\Theta} = \pi_e P_e(a) + (\pi - \pi_e) P_x.$$

To build intuition, as above, we first consider the simplified model with secured default-linked compensation, to then move to the more general case of an unsecured scheme.

4.2.1 Secured default-linked pay

The optimal bonus design chosen by the shareholder in the presence of secured default-linked compensation and bank leverage is the following.

Proposition 7 (Optimal bonus payment with secured & discretionary default-linked pay)**.** *Let* $A = \frac{1}{Y_h(Y_h - Y_m)}$, $B = (\Gamma - P_x)\frac{l\phi + \bar{L}}{\beta Y_h Y_m}$, *and* $C = \alpha - \frac{(l\phi + \bar{L})^2}{\beta Y_h^2 Y_m^2} > 0$, *where* Γ *is defined in Proposition 2 above.*

- *When* $\Gamma > P_x$ *the optimal bonus is*

$$z_h^*(slack) = \frac{A\frac{\phi l + \bar{L}}{C Y_h Y_m} - p_h^0 - \frac{B}{C y_h (Y_h - Y_m)} - \frac{2A\left(\Gamma - P_x + B\frac{\phi l + \bar{L}}{C Y_h Y_m}\right)(l\phi + \bar{L})}{\beta Y_h Y_m}}{\frac{2A}{C Y_h (Y_h - Y_m)} + \frac{2A^2(\phi l + \bar{L})^2}{\beta C^2 Y_h^2 Y_m^2}}$$

  provided that the limited-liability constraint ($z_h^*(slack) > 0$) *is satisfied. Otherwise* $z_h^*(slack) = 0$.
- *Assume* $\Gamma < P_x$ *and let*

$$z_h^*(bind) = \frac{Y_h - Y_m}{Y_m} \frac{l\phi + \bar{L} - \alpha p_h^0 Y_h^2 Y_m (Y_h - Y_m)}{2} \tag{13}$$

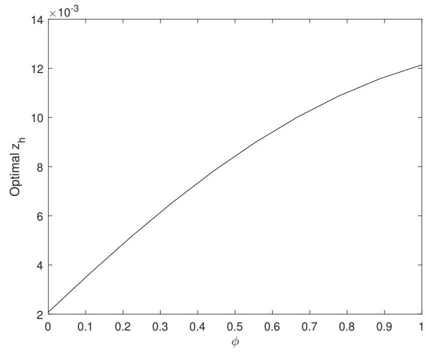
  for $l\phi + \bar{L} - \alpha p_h^0 Y_h^2 Y_m (Y_h - Y_m) \geq 0$ *(otherwise the limited-liability constraint binds and* $z_h^*(bind) = 0$*). Assume parameters are such that* $0 < z_h^*(bind) < \bar{z} < z_h^*(slack) > 0$*. Then, the optimal solution is* $z_h^*(bind)$ $(z_h^*(slack))$ *if* $\Pi_e(z_h^*(bind)) >$ $(<)$ $\Pi_e(z_h^*(slack))$.

Already in this simplified setting, default-linked compensation introduces nonnegligible nuances in the interaction between the probability of bailout and bonus design by the shareholder. Indeed, the optimal bonus coincides with that obtained in the non-discretionary case only if $\Gamma < P_x$ and when the manager's short-selling constraint is binding. In this case, the optimal bonus unambiguously increases with the bailout probability ϕ because the latter increases the value of the debt repayment shifted to the government.

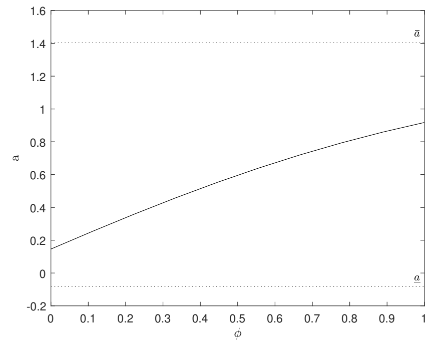
By contrast, the effect of the bailout probability is more complex, and possibly ambiguous, if $\Gamma > P_x$, i.e., when the short-selling constraint is slack. The first reason is that, when default-linked compensation is invested in bank equity, ϕ also drives up the value of equity (keeping P_x constant) and induces the manager to take on more risk (see Proposition 2). In this case, the shareholder can obtain the same level of risk-shifting with a lower bonus payment. Second, the value of the alternative investment option affects the optimal bonus only when the short-selling constraint is slack. An increase in P_x (induced by higher probability of bailout) makes the manager less willing to pursue risky policies (see Proposition 2). To compensate for the risk reduction, the shareholder needs to increase the bonus payment. Figure 3 visualizes these patterns[29]. The optimal bonus payment is increasing in the bailout probability (Panel (a)). A higher bonus payment induces the manager to take on more risk (Panel (b)), increases the price of bank equity (Panel (c)). As a result, the manager buys more own bank's shares (Panel (d)).

When P_x is constant (Figure A.5) the optimal bonus payment is increasing in the bailout probability only for ϕ smaller than a given threshold. After this point the manager becomes sufficiently exposed to bank equity that shareholders can conveniently reduce bonus payment without curbing risk-taking incentives (Panel (a)-(b)). As a result the optimal z_h because an inverted u-shaped function of ϕ.

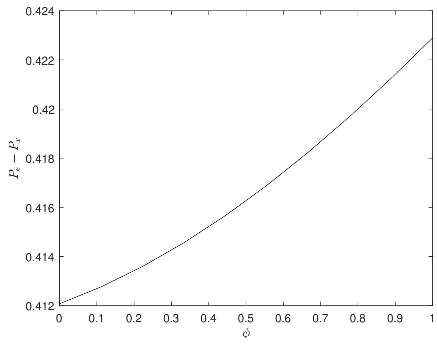
[29]Note that, to avoid the limited liability constraint to bind we need to slightly modify the parameters of the model, such as the zero-risk probabilities p_i^0. In addition, in this case the optimal risk choice and, thus, the optimal bonus payment does not depend on the number of deferred shares π (Proposition 4 and related discussion).



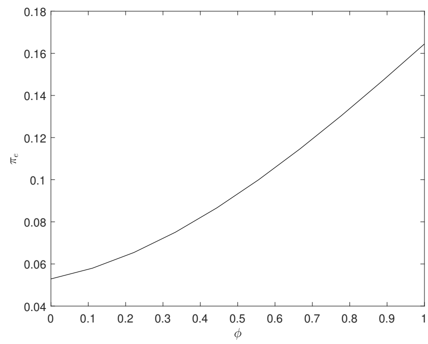
(a) Optimal bonus payment. The optimal bonus payment z_h from the shareholder's perspective as a function of ϕ.



(b) Optimal risk choice. The manager's optimal risk choice a as a function of ϕ.



(c) Price of bank equity. $P_e - P_x$ as a function of ϕ.



(d) Optimal investment. Optimal π_e as a function of ϕ.

Figure 3: Optimal shareholder's bonus payment and corresponding manager's risk choice as a function of the bailout probability when the short-selling constraint is slack, under the secured and discretionary scheme for default-linked compensation. Calibration: $Y_h = 5.5$, $Y_m = 5.2$, $Y_l = 0$, $p_h^0 = .05$, $p_m^0 = .9$ $p_l^0 = 0.05$, $\alpha = 0.01$, $\beta = 0.2$, $\delta = 0.1$, $r_d = 0.02$, $r_l = 0.04$, $\pi = 0.5$ $l = 0.6$, $k = 0.3$, $d = (1 - l - k)/(1 - \delta)$, $\Gamma > P_x = P_e(0, \phi) - .1$.

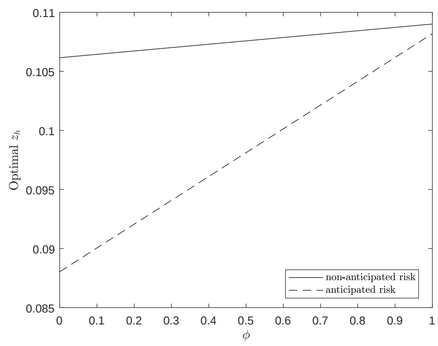
4.2.2 Unsecured default-linked pay

In this section, we characterize the equilibrium in the case of an unsecured and discretionary default-linked pay scheme when creditors cannot anticipate the manager's risk choice and the bank is levered. Because of the difficulty of obtaining closed-form expressions in this setting, we resort to a numerical analysis to analyze the optimal shareholder's choice.

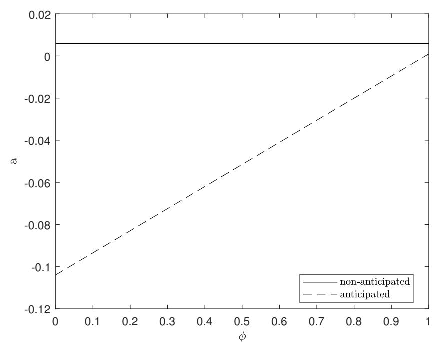
We proceed in two steps: i) we compute the first-order condition (FOC) of the maximization problem and, then, ii) we show numerically that the shareholders objective function is concave (see Appendix B for details), which implies that the FOC characterizes the shareholder's optimal behavior. Let $\hat{a}_D(z)$ be the manager's risk choice under the discretionary scheme as a function of the bonus payment. Hence, under the assumption that creditors cannot anticipate the manager's risk choice, the optimal bonus (z_h^*) has to satisfy the following equation:

$$\left[-\frac{z_h^*}{Y_h(Y_h - y_m)} + \frac{\tilde{\Theta} + \bar{L}}{Y_h Y_m} + \frac{L(\hat{a}_D(z_h^*))}{Y_h Y_m}\right] \frac{\partial \hat{a}_D}{\partial z_h} - (p_h + p_m)\frac{\partial \tilde{\Theta}}{\partial z_h} - p_h = 0. \tag{14}$$

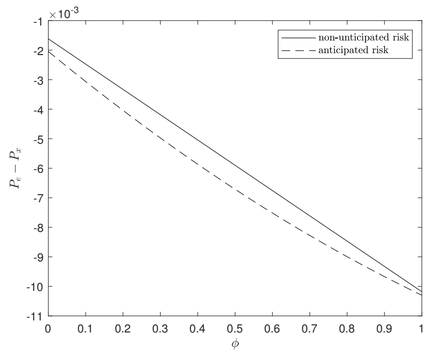
By inspection of Figure 4, we observe that the optimal bonus payment increases with the bailout probability and is larger when creditors cannot anticipate the risk choice (Panel (a)). The same applies to the optimal risk choice induced by the bonus payment (Panel (b)).[30] The price differential $P_e - P_x$ (Panel (c)) is negative and decreases with the bailout probability (because the positive effect of ϕ on P_x dominates the positive effect of risk on P_e for this calibration) and, as a result, the short-selling constraint binds (Panel (d)).



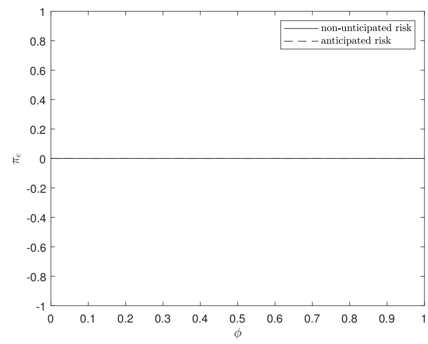
(a) Optimal bonus payment. The optimal bonus payment z_h from the shareholder's perspective as a function of ϕ.



(b) Optimal risk choice. The manager's optimal risk choice a as a function of ϕ.



(c) Price of bank equity. $P_e - P_x$ as a function of ϕ.



(d) Optimal investment. Optimal π_e as a function of ϕ.

Figure 4: Optimal shareholder's and manager's and the bailout probability when the short-selling constraint is binding, under the unsecured and discretionary scheme for default-linked compensation. Calibration: $Y_h = 5.5$, $Y_m = 5.2$, $Y_l = 0$, $p_h^0 = 0.15$, $p_m^0 = 0.7$, $p_l^0 = 0.15$, $\alpha = 0.1$, $\beta = 0.1$, $\pi = 0.1$.

Interestingly, for this calibration the manager finds it optimal to reduce bank risk

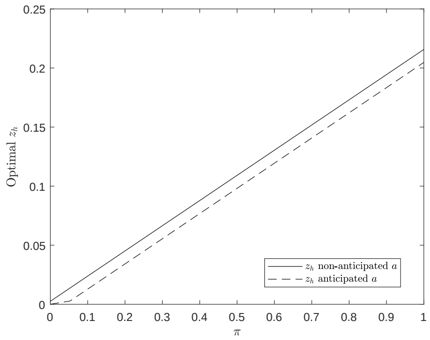
[30] See Bolton et al. (2015)) for a similar impact of observability on managerial risk-taking.

relative to the first-best (i.e., the optimal a is negative) when lenders can anticipate the risk choice. This result hinges on the risk-reducing effect of default-linked compensation (Proposition 1) and appears to be consistent with Duran and Lozano-Vivas (2014), who find no evidence of risk-shifting after the Great Recession. In this situation, a bonus cap, aiming at inducing the first-best level of risk, would thus be ineffective. However, the risk-taking implications of default-linked compensation also depend on the ability of lenders to anticipate the manager's risk choice. When they are able to do so, the manager always engages in (limited) risk-shifting, irrespective of the probability of bailout.

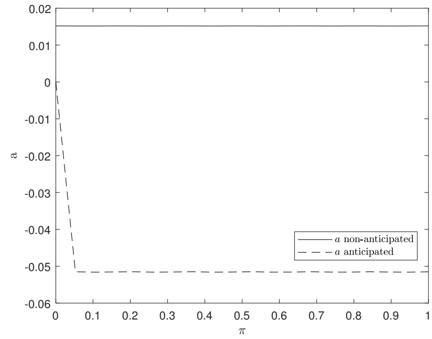
Figures A.6 and A.7 in the Appendix consider the case of constant P_x. For a low level of the bailout probability ϕ (roughly below 0.6), we obtain $P_e < P_x$ and, thus, the manager's short-selling constraint is binding. As a result, the impact of ϕ is the same as in Figure 4, namely the bonus depends positively on the bailout probability. For larger values of the bailout probability, we have that $P_e > P_x$ (because of the positive effect of the bailout probability on the price of bank equity) and the effect of ϕ is inverted (see the discussion of Proposition 7 above). In this case, higher ϕ and lower z_h have opposite effects on risk-taking, which may become flatter as a function of ϕ. Finally, whereas the size of the default-linked account does not appear to qualitatively affect the aforementioned patterns, it does so quantitatively for the optimal bonus payment, especially when risk is not anticipated by lenders. The shareholder reacts to higher mandatory default-linked compensation by awarding the manager a higher bonus payment.

Moreover, the manager's incentives to take risk are sensitive to the specification of bank-level asset payoffs (i.e., Y_h and Y_m), which, in turn, affect the price of bank equity. For this reason we also study the optimal bonus payment and the corresponding risk choice for a calibration featuring lower bank-level asset payoffs outside of default (Appendix Figure A.8). In this case the lower upside potential reduces the price of equity and, ceteris paribus, the value of default-linked compensation, whose risk-mitigating role is thus reduced. At the same time, the lower equity price curbs the economic cost of granting bonuses borne by the shareholder, who now accepts a higher risk to raise bank equity price, in particular when the regulator mandates a large default-linked account (governed by the parameter π). Hence, in this case risk-shifting behavior becomes optimal and is exacerbated by a high bailout probability.

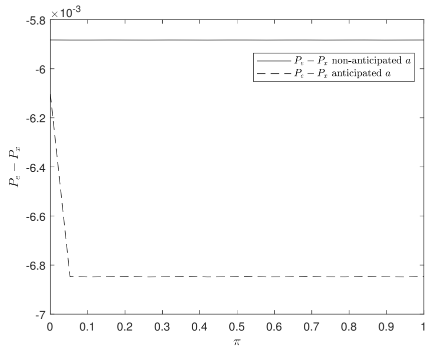
Figure 5 shows the implications of the level of mandatory default-linked compensation π. As shown above, when π increases, the optimal risk is reduced. This is not optimal for the shareholder, who reacts by increasing the bonus payment (Panel (a)) in such a way to maintain the risk choice unchanged (Panel (b)).[31] Consequently, the price differential $P_e - P_x$ is negative and insensitive to changes in π (Panel (c)), and the short-selling constraint is binding for the manager (Panel (d)). In other words, the shareholder can offset the risk-mitigating effect of default-linked pay by increasing bonus payments, which points to the possible effectiveness of a bonus cap.



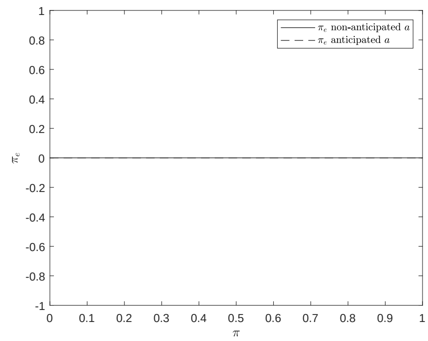
(a) Optimal bonus payment. The optimal bonus payment z_h from the shareholder's perspective as a function of π.



(b) Optimal risk choice. The manager's optimal risk choice a as a function of π.



(c) Price of bank equity. $P_e - P_x$ as a function of π.



(d) Optimal investment. Optimal π_e as a function of π.

Figure 5: Optimal shareholder's and manager's and the size of the default-linked account when the short-selling constraint is binding, under the unsecured and discretionary scheme. Calibration: $Y_h = 5.5$, $Y_m = 5.2$, $Y_l = 0$, $p_h^0 = 0.15$, $p_m^0 = 0.7$, $p_l^0 = 0.15$, $\alpha = 0.1$, $\beta = 0.1$, $\phi = 0.5$.

To sum up, these numerical exercises highlight a potential lack of robustness of regulations mandating default-linked compensation in its discretionary form. Even considering a model parametrization such that the short-selling constraint always binds—which makes default-linked compensation less performance-sensitive in the non-default region—, the manager still has an incentive to risk-shift, in particular when lenders cannot observe risk. In addition managerial risk choice and, thus, the optimal bonus payment as a function

[31]The risk choice is decreasing for low values of π. In this case, the limited-liability constraint is binding and shareholders are forced to select $z_h = 0$ and, thus, cannot compensate for the risk-mitigating effect of higher π.

of the bailout probability crucially depend on the investment options available (i.e., their price compared to the price of bank equity and whether or not their price also depend on the bailout probability).

5 Discussion

The key goal behind post-Great Recession regulations concerning bankers' pay—already implemented or just proposed—is to rein in risk-taking incentives and favor financial stability. In the presence of implicit and explicit government guarantees to banks, in principle this can be achieved by aligning managers' interests to those of depositors and other bank creditors, moving away from standard equity governance provisions. In practice, policymakers developed a menu of regulatory restrictions—ranging from clawbacks and malus clauses to bonus caps and deferrals—to tie bankers' pay to bank default, making it less performance-sensitive in non-default states of the world, and curb their myopic behavior.

We develop a theoretical framework, in which a risk-neutral manager selects bank-level asset risk and receives a pay package comprising a bonus and a default-linked component. Incorporating the latter in the model is relevant to improve our understanding of the risk-taking incentives induced by mandatory bonus deferrals, which are commonly expected to reduce short-termism as well as default risk. The highly stylized, static setting allows us to abstract from managerial decision horizon to focus on the consequences for risk-taking of linking pay to default.

We start from a standard finding in the literature. Complementing a bonus contract with a pure debt-like pay component—namely performance-insensitive over the non-default region—reaches the goal of curbing risk-taking, which nonetheless remains above its first-best level. However, the shareholders would not grant this type of default-linked remuneration endogenously. Put differently, a simple regulation mandating truly debt-like pay could go a long way in stymieing risk-shifting behavior by bankers.

This result weakens once the manager is given discretion in the allocation of the personal default-linked account between own bank's shares and an alternative asset—the main innovation in our model. This feature makes, to a varying degree, default-linked pay performance-sensitive in the non-default region. Hence, such a discretionary scheme appears to be ineffective at holding back risk-shifting incentives stemming from bonuses, bailout expectations, and leverage, especially at times of high bank valuations, when the manager invests in own bank's shares through the default-linked account. These patterns become more nuanced once we endogenize the bonus contract. Yet, the optimal shareholders' response to larger, exogenously imposed discretionary default-linked pay is to convexify even more the bonus design. This may point policymakers to combining discretionary default-linked pay with a bonus cap, under the caveat that model parametrization appears to matter a lot in our setting.

Our model speaks to recent efforts to regulate bankers' pay. Most prominently, CRD mandated EU banks to defer at least 40% of their MRTs' variable remuneration. The deferrals are to be paid in annual installments, composed for at least 50% of equity-linked instruments (e.g., Deutsche Bundesbank, 2021). This link between deferred compensation and own bank's equity is reminiscent of our discretionary scheme, although our analysis is admittedly silent about any risk-taking effect working through a modification of bankers' decision horizon.[32] It is also worth noting that while EU regulation forbids bankers from zeroing out the equity exposure of their deferred compensation account, it still grants them and the bank substantial discretion on the weight of own equity. This is exactly the case in which default-linked is least effective at curbing risk-shifting in our framework. Enhancing disclosure requirements on how deferred compensation is invested and paid out—as well as on its security and priority in the event of bank insolvency or resolution—could improve our understanding of how it shapes managerial risk-taking incentives.

In a nutshell, the message of this paper is that the consequences for risk-taking of linking bank managers' remuneration to default are not obvious when such a pay component may be awarded in equity instruments, especially if this choice is (partially) under the control of the management. The ambiguous impact on risk-taking attains even in a highly stylized framework like ours, which abstracts from dynamic contracting and realistic risk preferences of bankers.

[32] We also abstract from the clawback mechanism imposed by CRD.

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Appendix for
"When Does Linking Pay to Default Reduce Bank Risk?"

A Proofs

In this section, we show the proofs behind the results of the paper.

Proof of Proposition 1. The manager solves

$$\begin{aligned}\max_a \quad & \Pi_M^{ND} + \lambda_1[a - \bar{a}] - \lambda_2[a - \underline{a}] \\ &= p_h z_h + (p_h + p_m)\bar{\Theta} - \alpha\frac{a^2}{2} + \lambda_1[a - \bar{a}] - \lambda_2[a - \underline{a}].\end{aligned} \tag{A.1}$$

The optimality conditions are

$$\frac{z_h}{Y_h(Y_h - Y_m)} - \frac{\bar{\Theta}}{Y_h Y_m} - \alpha a + \lambda_1 - \lambda_2 = 0,$$
$$\lambda_1[a - \bar{a}] = 0, \quad \lambda_2[a - \underline{a}] = 0, \quad \lambda_1, \lambda_2 \geq 0,$$

where λ_1 and λ_2 are the Lagrange multipliers attached to the feasibility constraints. The optimal solution follows from the complementary-slackness conditions. □

We are now in the position to derive several intermediate results. The unconstrained solution $\tilde{a}$ is available in closed-form

$$\tilde{a}(z_h, D) = \frac{z_h}{\alpha Y_h(Y_h - Y_m)} - \frac{\bar{\Theta}}{\alpha Y_h Y_m}.$$

From this expression, we obtain

$$\begin{aligned}
p_h(\tilde{a}) &= p_h^0 + \frac{\tilde{a}}{Y_h(Y_h - Y_m)} = p_h^0 + \frac{z_h}{\alpha Y_h^2(Y_h - Y_m)^2} - \frac{\bar{\Theta}}{\alpha Y_h^2 Y_m(Y_h - Y_m)}, \\
p_m(\tilde{a}) &= p_m^0 - \frac{\tilde{a}}{Y_m(Y_h - Y_m)} = p_m^0 - \frac{z_h}{\alpha Y_h Y_m(Y_h - Y_m)^2} + \frac{\bar{\Theta}}{\alpha Y_h Y_m^2(Y_h - Y_m)}, \\
p_h(\tilde{a}) + p_m(\tilde{a}) &= p_h^0 + p_m^0 - \frac{\tilde{a}}{Y_h Y_m} = p_h^0 + p_m^0 - \frac{z_h}{\alpha Y_h^2 Y_m(Y_h - Y_m)} + \frac{\bar{\Theta}}{\alpha Y_h^2 Y_m^2}, \\
p_l(\tilde{a}) &= 1 - p_h(\tilde{a}) - p_m(\tilde{a}) = 1 - p_h^0 - p_m^0 + \frac{z_h}{\alpha Y_h^2 Y_m(Y_h - Y_m)} - \frac{\bar{\Theta}}{\alpha Y_h^2 Y_m^2}.
\end{aligned}$$

Next, we can compute the effect of bonus payment on the probabilities of the bank's future asset payoff:

$$\frac{\partial p_h}{\partial z_h} = \frac{1}{\alpha Y_h^2 (Y_h - Y_m)^2} > 0, \quad \frac{\partial p_h}{\partial \bar{\Theta}} = -\frac{1}{\alpha Y_h^2 Y_m (Y_h - Y_m)} < 0$$
$$\frac{\partial (p_h + p_m)}{\partial z_h} = -\frac{1}{\alpha Y_h^2 Y_m (Y_h - Y_m)^2} < 0, \quad \frac{\partial (p_h + p_m)}{\partial \bar{\Theta}} = \frac{1}{\alpha Y_h^2 Y_m^2} > 0.$$

Then, the lenders' minimum required payment for a risky choice a is

$$L(a) = l\frac{(1 + r_l) - p_l\,(a)\,\phi}{p_h\,(a) + p_m\,(a)}, \tag{A.2}$$

with

$$\frac{\partial L}{\partial \phi} = -\frac{l p_l}{p_h + p_m} < 0,$$

$$\frac{\partial L}{\partial a} = l\frac{-\frac{\partial p_l}{\partial a}\phi(p_h + p_m) - (1 + r_l - \phi p_l)\frac{\partial (p_h + p_m)}{\partial a}}{(p_h + p_m)^2} = l\frac{1 + r_l - \phi}{(p_h + p_m)^2 Y_h Y_m} > 0,$$

$$\frac{\partial L}{\partial z_h} = \frac{\partial L}{\partial a}\frac{\partial a}{\partial z_h} = l\frac{1 + r_l - \phi}{(p_h + p_m)^2 Y_h Y_m}\frac{\partial a}{\partial z_h} > 0, \quad \frac{\partial L}{\partial \Theta} = \frac{\partial L}{\partial a}\frac{\partial a}{\partial \Theta} = l\frac{1 + r_l - \phi}{(p_h + p_m)^2 Y_h Y_m}\frac{\partial a}{\partial \Theta} < 0.$$

Finally, the reaction of the equity price to the manager's risk choice depends on the assumption about the possibility to anticipate the bank's risk. When the risk choice cannot be anticipated, we have

$$\begin{aligned} P_e(a, \phi) &= p_h\left(Y_h - L - \bar{L}\right) + p_m\left(Y_m - L - \bar{L}\right) - \delta \\ &= p_h^0 Y_h + p_m^0 Y_m - (p_h + p_m)\left(L + \bar{L}\right) - \delta \\ &\Rightarrow \frac{\partial P_e^{unobs}}{\partial a}(a) = \frac{L(a) + \bar{L}}{Y_h Y_m} > 0. \end{aligned} \tag{A.3}$$

Since $L(a)$ is decreasing in ϕ, then $\frac{\partial^2 P_e}{\partial a \partial \phi} < 0$.

Differently, when the lenders can anticipate the manager's risk choice, we have

$$\begin{aligned} &P_e(a, \phi) = p_h\left(Y_h - L - \bar{L}\right) + p_m\left(Y_m - L - \bar{L}\right) - \delta \\ &= p_h Y_h + p_m Y_m - (p_h + p_m)L - (p_h + p_m)\bar{L} - \delta \\ &= p_h Y_h + p_m Y_m - l(1 + r_l - p_l\phi) - (p_h + p_m)\bar{L} - \delta = \Gamma + a\frac{l\phi + \bar{L}}{Y_h Y_m} \\ &\Rightarrow \frac{\partial P_e^{obs}}{\partial a} = \frac{l\phi + \bar{L}}{Y_h Y_m} > 0. \end{aligned} \tag{A.4}$$

where $\Gamma = P_e(0, \phi)$ is the equity price corresponding to the first-best risk level. As a result, $\frac{\partial^2 P_e}{\partial a \partial \phi} > 0$.

Proof of Proposition 2. The manager solves

$$\max_{a,\pi_e} \quad p_h z_h + \pi_e P_e + (\pi - \pi_e)P_x - \alpha\frac{a^2}{2} - \beta\frac{\pi_e^2}{2} - \lambda_1[a - \bar{a}] - \lambda_2[\underline{a} - a] + \lambda_3 \pi_e.$$

Taking the FOCs and focusing first on the interior solution, we obtain

$$a: \quad \frac{z_h}{Y_h(Y_h - Y_m)} + \pi_e \frac{\partial P_e}{\partial a} - \alpha a = 0, \tag{A.5}$$

$$\pi_e: \quad P_e - P_x - \beta\pi_e = 0 \Rightarrow \pi_e = \frac{P_e - P_x}{\beta}. \tag{A.6}$$

Substituting π_e and $\frac{\partial P_e}{\partial a}$ into the FOC for a, we get

$$\begin{aligned}
\frac{z_h}{Y_h(Y_h - Y_m)} + \frac{P_e - P_x}{\beta}\frac{\bar{L} + l\phi}{Y_h Y_m} - \alpha a = \\
\frac{z_h}{Y_h(Y_h - Y_m)} + \frac{\Gamma - P_x}{\beta}\frac{\bar{L} + l\phi}{Y_h Y_m} + a\left(\frac{(\bar{L} + l\phi)^2}{\beta Y_h^2 Y_m^2} - \alpha\right) = 0 \\
\Rightarrow a^* = a^*(slack) = \frac{\frac{z_h}{Y_h(Y_h - Y_m)} + (\Gamma - P_x)\frac{l\phi + \bar{L}}{\beta Y_h Y_m}}{\alpha - \frac{(l\phi + \bar{L})^2}{\beta Y_h^2 Y_m^2}},
\end{aligned}$$

where $\alpha\beta - \frac{(l\phi + \bar{L})^2}{Y_h^2 Y_m^2}$ is the determinant of the Hessian matrix, which has to be positive to ensure that the second-order condition is satisfied. The sign of $\frac{\partial\alpha^*}{\partial\phi}$, $\frac{\partial\pi_e^*}{\partial\phi}$, $\frac{\partial\alpha^*}{\partial z_h}$ and $\frac{\partial\pi_e^*}{\partial z_h}$ follows. To introduce the short-selling constraint, it is useful to assume that $P_e(\underline{a}, \phi) < P_x < P_e(\bar{a}, \phi)$. Consider first the case $\Gamma > P_x$. In this case, $a^* > 0$. Moreover, Γ represents the price of equity corresponding to $a = 0$. Using the fact that $\frac{\partial P_e}{\partial a} > 0$, we conclude that $P_e(a^*, \phi) > P_x$ and the short-selling constraint must be slack.

Next, consider the case $\Gamma < P_X$, for which π_e could be negative, even if $a > 0$. When this happens, the short-selling constraint binds and we have

$$\begin{aligned}
a^* &= a^*(bind) = \frac{z_h}{\alpha Y_h(Y_h - Y_m)} > 0, \\
\pi_e^* &= 0.
\end{aligned}$$

When $\underline{a} < a^*(bind) < \bar{a}$, there exists a risk level ω such that $P_e(\omega, \phi) = P_x$ (because $P_e(\underline{a}, \phi) < P_x < P_e(\bar{a}, \phi)$). Correspondingly, let $\bar{z}$ be the bonus such that $\frac{\bar{z}}{\alpha Y_h(Y_h - Y_m)} = \omega \Rightarrow \bar{z} = \omega\alpha Y_h(Y_h - Y_m)$. Then, we have the following results i) $P_e = P_x \Leftrightarrow a = \omega = (P_x - \Gamma)\frac{Y_h Y_m}{l\phi + \bar{L}}$ (the last equality follows from Eq A.4); ii) for any $z_h \leq \bar{z}$, $a^* = \frac{z_h}{\alpha Y_h(Y_h - Y_m)}$ and $\pi_e^* = 0$ is a solution to the manager's problem; iii) for any $z_h > \bar{z}$, the solution has to satisfy equation (A.5). Moreover, for $z_h = \bar{z}$ equation (A.5) becomes

$$\frac{z_h}{Y_h(Y_h - Y_m)} + \frac{P_e - P_x}{\beta}\frac{\bar{L} + l\phi}{Y_h Y_m} - \alpha a = \alpha(\omega - a) + \frac{P_e - P_x}{\beta}\frac{\bar{L} + l\phi}{Y_h Y_m} = 0,$$

with solution $a = \omega$, which implies $P_e = P_x$. For a bonus $z_h > \bar{z}$, we have

$$\frac{\bar{z} + \Delta}{Y_h(Y_h - Y_m)} + \frac{P_e - P_x}{\beta}\frac{\bar{L} + l\phi}{Y_h Y_m} - \alpha a = 0,$$

for some $\Delta > 0$. Given that a^* increases with the bonus payment, we conclude that the solution in this case, say $a^*(slack)$, satisfies $a^*(slack) > \omega$ and, thus, $P_e(a^*(slack), \phi) > P_x$.

When $\underline{a} > a^*(bind)$, then $\underline{a}$ is the optimal risk choice and the short-selling constraint is binding. If instead $a^*(bind) > \bar{a}$, then $a^*(bind)$ cannot be the solution and the optimal risk choice must satisfy equation (A.5).

Finally, we compare the risk levels ω, $a^*(slack)$ and $a^*(bind)$. First, it is useful to re-write $a^*(slack)$ as follows

$$\begin{aligned} a^*(slack) &= \frac{\frac{z_h}{Y_h(Y_h - Y_m)} + (\Gamma - P_x)\frac{l\phi + \bar{L}}{\beta Y_h Y_m}}{\alpha - \frac{(l\phi + \bar{L})^2}{\beta Y_h^2 Y_m^2}} \\ &= \frac{a^*(bind)\alpha - \frac{\omega}{\beta}\frac{(l\phi + \bar{L})^2}{\beta Y_h^2 Y_m^2}}{\alpha - \frac{(l\phi + \bar{L})^2}{\beta Y_h^2 Y_m^2}} \end{aligned}$$

which implies

$$a^*(slack) \geq a^*(bind) \Leftrightarrow \frac{1}{\beta}\frac{Y_h^2 Y_m^2}{(l\phi + \bar{L})^2}\left[\omega - a^*(bind)\right] < 0.$$

When $\Gamma > P_x$, $\omega < 0$ and the previous condition is always satisfied. Thus, we have $\omega \leq a^*(bind) \leq a^*(slack)$ (with $a^*(slack)$ being the optimal solution).

When $\Gamma < P_x$, $\omega > 0$. Therefore,

$$\begin{aligned} & a^*(slack) \geq a^*(bind) \Leftrightarrow \omega < a^*(bind) \\ & \Leftrightarrow (P_x - \Gamma)\frac{Y_h Y_m}{l\phi + \bar{L}} < \frac{z_h}{\alpha Y_h(Y_h - Y_m)} \\ & \Leftrightarrow z_h > \bar{z} = \alpha(P_x - \Gamma)\frac{Y_h^2 Y_m (Y_h - Y_m)}{l\phi + \bar{L}}. \end{aligned}$$

Hence, for $z_h > \bar{z}$ we obtain again $\omega \leq a^*(bind) \leq a^*(slack)$ (with $a^*(slack)$ being the optimal solution). Instead, for $z_h < \bar{z}$, $a^*(slack) \leq a^*(bind) \leq \omega$, with $a^*(bind)$ being the optimal solution in this case. In this situation, a bonus cap—which implies a bonus payment smaller than $\bar{z}$—would force the manager to take a level of risk smaller than

ω. Similarly, a ban on own equity investment would force the manager to implement the level of risk $a^*(bind)$ (when $\Gamma > P_x$ or $\Gamma < P_x$ and $z_h > \bar{z}$), which is smaller than $a^*(slack)$.

$\square$

Proof of Proposition 3. For an all-equity bank, $L = \bar{L} = \delta = 0$ and therefore

$$\begin{aligned} P_e(a, \phi) &= p_h\left(Y_h - L - \bar{L}\right) + p_m\left(Y_m - L - \bar{L}\right) \\ &= p_h^0 Y_h + p_m^0 Y_m \\ &\Rightarrow \frac{\partial P_e}{\partial a} = 0. \end{aligned}$$

The manager solves

$$\begin{aligned} \max_{a,\pi_e} \quad & \Pi_M^D - \lambda_1[a - \bar{a}] - \lambda_2[\underline{a} - a] \\ &= z_h p_h + (p_h + p_m)\left[\pi_e P_e + (\pi - \pi_e)P_x\right] - \alpha\frac{a^2}{2} - \beta\frac{\pi_e^2}{2} - \lambda_1[a - \bar{a}] - \lambda_2[\underline{a} - a]. \end{aligned} \tag{A.7}$$

The FOCs are given by

$$\begin{aligned} a: \quad & \frac{z_h}{Y_h(Y_h - Y_m)} - \frac{\left[\pi_e P_e + (\pi - \pi_e)P_x\right]}{Y_h Y_m} - \alpha a + \lambda_1 - \lambda_2 = 0, \\ \pi_e: \quad & (p_h + p_m)(P_e - P_x) - \beta\pi_e = 0 \Rightarrow \tilde{\pi}_e = (p_h + p_m)\frac{P_e - P_x}{\beta}. \end{aligned}$$

Under the assumption that the determinant of the Hessian matrix is positive ($\alpha\beta > \frac{(P_e^0 - P_x)^2}{\beta Y_h^2 Y_m^2}$) we can replace $\tilde{\pi}_e$ into the FOC for a and solve for the optimal risk level. This gives the optimal unconstrained risk choice $\tilde{a}_{DE}$. The constrained solution follows from the application of the complementary-slackness conditions. $\square$

Proof of Proposition 4. Under the discretionary contract, the manager solves

$$\begin{aligned} \max_{a,\pi_e} \quad & \Pi_M^D - \lambda_1[a - \bar{a}] - \lambda_2[\underline{a} - a] \\ &= z_h p_h + (p_h + p_m)\left[\pi_e P_e + (\pi - \pi_e)P_x\right] - \alpha\frac{a^2}{2} - \beta\frac{\pi_e^2}{2} - \lambda_1[a - \bar{a}] - \lambda_2[\underline{a} - a]. \end{aligned} \tag{A.8}$$

In this case we also consider the possibility that lenders cannot anticipate the risk choice. When the risk choice cannot be anticipated, the optimal risk is determined by the intersection of the manager's and lenders' best response functions. The manager's best response function is obtained from the FOCs of problem (A.8) (taking L as given), while the lenders' best response function is given by equation (A.2). As a result, the FOCs of the manager's problem are

$$
\begin{aligned}
a: \quad & \frac{z_h}{Y_h(Y_h - Y_m)} + (p_h + p_m)\,\pi_e \frac{\partial P_e^{unobs}}{\partial a} \\
& \qquad - \frac{[\pi_e P_e + (\pi - \pi_e) P_x]}{Y_h Y_m} - \alpha a + \lambda_1 - \lambda_2 = 0,
\end{aligned} \tag{A.9}
$$

$$
\pi_e: \quad (p_h + p_m)(P_e - P_x) - \beta \pi_e = 0 \Rightarrow \tilde{\pi}_e = (p_h + p_m) \frac{P_e - P_X}{\beta},
$$

where $\tilde{\pi}_e$ (the optimal portfolio choice) depends on a through p_h, p_m and P_e. Therefore, the unconstrained risk choice ($\tilde{a}$) follows by substituting $\tilde{\pi}_e$ into equation (A.9) and finding $\tilde{a}$ such that

$$
\begin{aligned}
& \frac{z_h}{Y_h(Y_h - Y_m)} + (p_h(\tilde{a}) + p_m(\tilde{a}))\,\tilde{\pi}_e(\tilde{a}) \frac{\partial P_e^{unobs}}{\partial a}(\tilde{a}) \\
& \qquad - \frac{[\tilde{\pi}_e(\tilde{a}) P_e(\tilde{a}, \phi) + (\pi - \tilde{\pi}_e(\tilde{a})) P_x]}{Y_h Y_m} - \alpha \tilde{a} = 0.
\end{aligned} \tag{A.10}
$$

The constrained risk choice follows from the complementary slackness conditions. Using the definition of marginal gains and marginal costs, equation (A.10) boils down to

$$
\frac{z_h}{Y_h(Y_h - Y_m)} + \chi(a) - \gamma(a) - \alpha a = 0. \tag{A.11}
$$

The assumption of decreasing net marginal benefits ($\chi - \gamma$ decreasing in a) implies that equation (A.10) is monotonically decreasing in a and changes sign (from $-\infty$ to $+\infty$). Therefore, it exists a unique solution to equation (A.10).

We are now in the position to derive several properties of the optimal solution. First, we show that $\frac{da}{dz_h} > 0$. By applying the implicit function theorem to equation (A.10), one obtains

$$
\begin{aligned}
& \frac{dz_h}{Y_h(Y_h - Y_m)} + \left[\frac{\partial(\chi - \gamma)}{\partial a} - \alpha \right] da = 0 \\
& \Rightarrow \frac{da}{dz_h} = -\frac{Y_h(Y_h - Y_m)}{\underbrace{\left[\frac{\partial(\chi - \gamma)}{\partial a} - \alpha \right]}_{<0}} > 0.
\end{aligned}
$$

Second, we compare the solution obtained under the discretionary contract with that obtained under the non-discretionary one, focusing on the interior solution. The optimal risk under the non-discretionary scheme satisfies equation (A.10) with $\frac{\partial P_e^{unobs}}{\partial a}(a) = \pi_e = 0$ and for a constant value of default-linked compensation $\pi P_x = \bar{\Theta}$. In particular, the optimal risk choice in this case satisfies

$$\frac{z_h}{Y_h(Y_h - Y_m)} - \frac{\bar{\Theta}}{Y_h Y_m} - \alpha\tilde{a} = 0.$$

Let now $\tilde{\Theta} = \tilde{\pi}_e(\tilde{a})P_e(\tilde{a}) + (\pi - \tilde{\pi}_e(\tilde{a}))P_x$ be the value of default-linked compensation under the discretionary contract and assume that $\tilde{\Theta} = \bar{\Theta}$, namely its value under the non-discretionary scheme. According to equation (A.10)—for simplicity, we suppress the arguments of p_i, P_e and $\frac{\partial P_e^{unobs}}{\partial a}$—, the optimal risk choice satisfies

$$\frac{z_h}{Y_h(Y_h - Y_m)} + (p_h + p_m)\,\tilde{\pi}_e \frac{\partial P_e^{unobs}}{\partial a} - \frac{\overbrace{\tilde{\pi}_e\,(P_e - P_x) + \pi P_x}^{\tilde{\Theta}}}{Y_h Y_m} - \alpha\tilde{a} = 0, \tag{A.12}$$

where the third term is always negative, whereas the sign of the second term depends on π_e. When $P_e < P_x$, $\tilde{\pi}_e < 0$ and, thus, the optimal risk choice satisfying equation (A.12) (assuming $\tilde{\Theta} = \bar{\Theta}$) must be smaller than the non-discretionary risk choice. The opposite is true when $P_e > P_x$.

Finally, we show some comparative statics results. First, to show that $\frac{\partial a}{\partial \pi} < 0$, we apply the implicit function theorem to equation (A.10):

$$\begin{aligned}
&\frac{z_h}{Y_h(Y_h - Y_m)} + \overbrace{(p_h + p_m)\,\tilde{\pi}_e \frac{\partial P_e^{unobs}}{\partial a} - \frac{\tilde{\pi}_e(P_e - P_x)}{Y_h Y_m}}^{\lambda} - \frac{\pi P_x}{Y_h Y_m} - \alpha\tilde{a} = 0 \\
&\Rightarrow -\frac{P_x d\pi}{Y_h Y_m} + \left[\frac{\partial \lambda}{\partial a} - \alpha\right] da = 0 \\
&\Rightarrow \frac{\partial a}{\partial \pi} = \frac{P_x}{Y_h Y_m} \underbrace{\left[\frac{\partial \lambda}{\partial a} - \alpha\right]^{-1}}_{<0} < 0,
\end{aligned}$$

where the last line inequality descends from the fact that the sign of $\frac{\partial \lambda}{\partial a}$ equals the sign of $\frac{\partial(\chi - \gamma)}{\partial a}$ (because λ and $\chi - \gamma$ are equal except for a an additive constant which does not depend on a).

Second, we evaluate the sign of $\frac{\partial a}{\partial P_x} < 0$:

$$\frac{z_h}{Y_h(Y_h - Y_m)} + \overbrace{(p_h + p_m)\,\tilde{\pi}_e \frac{\partial P_e^{unobs}}{\partial a} - \frac{\tilde{\pi}_e(P_e - P_x)}{Y_h Y_m}}^{\lambda} - \frac{\pi P_x}{Y_h Y_m} - \alpha\tilde{a} = 0$$
$$\Rightarrow \underbrace{\left[-\frac{\pi}{Y_h Y_m} + (p_h + p_m)\frac{\partial \tilde{\pi}_e}{\partial P_x}\frac{\partial P_e^{unobs}}{\partial a} + 2\frac{(p_h + p_m)\,(P_e - P_x)}{\beta Y_h Y_m}\right]}_{\rho} dP_x + \left[\frac{\partial \lambda}{\partial a} - \alpha\right] da = 0$$
$$\Rightarrow \frac{\partial a}{\partial P_x} = -\rho \underbrace{\left[\frac{\partial \lambda}{\partial a} - \alpha\right]^{-1}}_{<0} < 0.$$

Because $\frac{\partial \tilde{\pi}_e}{\partial P_x} < 0$ and $\frac{\partial P_e^{unobs}}{\partial a} > 0$, we conclude that $\frac{\partial a}{\partial P_x}$ is unambiguously negative when $P_e < P_x$. Otherwise, the effect of P_x remains unclear. When the risk choice is anticipated, lenders are able to adjust L immediately and the manager has to take $\partial L/\partial a$ into account when deciding the optimal risk. Precisely, the manager will use $\frac{\partial P_e^{unobs}}{\partial a}(a)$ into the first order condition. □

Proof of Proposition 5. Consider now the short-selling constraint $\pi_e \geq 0$ and assume that $P_e(\underline{a}, \phi) < P_x$ and $P_e(\bar{a}, \phi) > P_x$. If the solution of equation (A.10), say a_1, is such that $\bar{a} < a_1$ or $P_e(a_1, \phi) > P_x$ for some $\underline{a} < a_1 < \bar{a}$, then $P_e(a_1, \phi) > P_x$ and the short-selling constraint is slack. If $\underline{a} > a_1$ or $P_e(a_1, \phi) < P_x$ for some $\underline{a} < a_1 < \bar{a}$, the constraint binds and the optimal portfolio must be $\pi_e = 0$. In this case, the corresponding risk choice (say a_2) satisfies

$$\frac{z_h}{Y_h(Y_h - Y_m)} - \frac{\pi P_x}{Y_h Y_m} - \alpha a_2 = 0$$
$$\Rightarrow a_2 = \frac{z_h}{\alpha Y_h(Y_h - Y_m)} - \frac{\pi P_x}{\alpha Y_h Y_m}$$
$$\Rightarrow \frac{\partial a_2}{\partial z_h} > 0, \quad \frac{\partial a_2}{\partial \pi} < 0.$$

Assume that $\underline{a} < a_2 < \bar{a}$. Since P_e is a continuous increasing function of a and $P_e(\underline{a}, \phi) < P_x < P_e(\bar{a}, \phi)$, there exists a level of risk, say $\underline{a} < \omega < \bar{a}$, such that $P_e(\omega, \phi) = P_x$, $P_e(a, \phi) > P_x$ for any $\omega < a$ and $P_e(a, \phi) < P_x$ for any $\omega < a$.

Moreover, $a_2 \geq \omega \Leftrightarrow z_h \geq \bar{z} = \omega\alpha Y_h(Y_h - Y_m) + \frac{\pi P_x(Y_h - Y_m)}{Y_m}$. As a consequence, we obtain the following results. i) For a bonus payment $z_h = \bar{z}$, $P_e(a_2, \phi) = P_x$ and $a_2 = \omega$ solves the manager's problem. In addition, when $z_h = \bar{z}$, equation (A.10) becomes

$$\alpha(\omega - a) + (p_h + p_m)\,\tilde{\pi}_e(a)\frac{\partial P_e^{unobs}}{\partial a}(a) - \tilde{\pi}_e(a)\frac{(P_e(a, \phi) - P_x)}{Y_h Y_m} = 0, \qquad \text{(A.13)}$$

which can be solved for $a = \omega$ and, thus, $P_e = P_x$. Given the assumption of decreasing marginal benefits, this is the only solution. In other words, a_1 and a_2 coincide for $z_h = \bar{z}$. ii) For a bonus $z_h > \bar{z}$, a_2 cannot be a solution to the manager's problem (because it would imply $P_e > P_x$, thus violating the condition that the short-selling constraint should be binding in this region), and the optimal solution must satisfy equation (A.10). Let $z_h = \bar{z} + \Delta z$ with $\Delta z > 0$. The optimal risk choice solves

$$\alpha(\omega - a) + \frac{\Delta z}{Y_h(Y_h - Y_m)} + (p_h + p_m)\,\tilde{\pi}_e(a)\frac{\partial P_e^{unobs}}{\partial a}(a) - \underbrace{\tilde{\pi}_e(a)\frac{(P_e(a,\phi) - P_x)}{Y_h Y_m}}_{>0} = 0. \tag{A.14}$$

Given that $\frac{da}{dz_h} > 0$ (see results above), the optimal risk that solves the previous equation has to be bigger than the solution associated to $z = \bar{z}$, which implies $P_e > P_x$ for any $z > \bar{z}$. iii) Assume $z_h = \bar{z} + \Delta z$ for some $\Delta z < 0$. In this case, $P_e < P_x$, the short-selling constraint is binding, and the optimal solution is a_2. Taken together, these results imply that the optimal solution is a_2 (a_1) when $z < \bar{z}$ ($z \geq \bar{z}$). If $a_2 < \underline{a}$ (which happens when $z < \bar{z}$) then the only solution is $a = a_2$ and $\pi_e = 0$. If $a_2 > \bar{a}$ (which happens when $z > \bar{z}$) then a_2 cannot be the optimal solution and we need to solve equation (A.10). Previous results imply that in this case $a_2 > \bar{a}$ and the risk choice is binding. □

Proof of Proposition 6. The shareholder selects the optimal bonus compensation to maximize expected profits subject to the manager's participation constraint ($\Pi_M^{ND} \geq \bar{\Pi} = 0$) and the limited liability constraint

$$\begin{aligned}\max_{z_h \geq 0, \bar{\Theta} \geq 0} \quad & \Pi_e - \nu[\Pi_M^{ND} - \bar{\Pi}] \\ & = p_h\left(Y_h - L - \bar{L} - z_h - \bar{\Theta}\right) + p_m\left(Y_m - L - \bar{L} - \bar{\Theta}\right) - \delta + \nu[\Pi_M^{ND} - \bar{\Pi}],\end{aligned}$$

where ν is the Lagrange multiplier attached to the participation constraint. Note that the contract offered to the manger is equivalent to a contract such that the manager receives $\hat{z} = z_h + \bar{\Theta}$ if the good outcome is realized and $\bar{\Theta}$ if the medium outcome is realized. This is the same problem analyzed by Hakenes and Schnabel (2014) and we can therefore conclude—using the same argument—that the optimal $\bar{\Theta}$ is equal to zero (i.e., shareholders reward the manager for the good outcome only).

When lenders can anticipate the manager's risk choice, we can plug their compensation L defined in equation (1)) into the shareholder's problem:

$$\begin{aligned}&\max_{z_h \geq 0} \Pi_e + \nu[\Pi_M^{ND} - \bar{\Pi}] \\ &= p_h\left(Y_h - z_h\right) + p_m Y_m - (p_h + p_m)L - (p_h + p_m)\bar{L} - \delta + \nu[\Pi_M^{ND} - \bar{\Pi}] - \delta \\ &= p_h\left(Y_h - z_h\right) + p_m Y_m - l(1 + r_l - p_l\phi) - (p_h + p_m)\bar{L} - \delta + \nu[\Pi_M^{ND} - \bar{\Pi}] - \delta.\end{aligned}$$

After replacing p_h, p_m and p_l with their expressions derived above and differentiating with respect to z_h, we obtain the FOC (we focus on the case with $\nu = 0$ first)

$$
\begin{aligned}
& - p_h^0 - \frac{z_h}{\alpha Y_h^2 (Y_h - Y_m)^2} + \frac{Y_h}{\alpha Y_h^2 (Y_h - Y_m)^2} - \frac{z_h}{\alpha Y_h^2 (Y_h - Y_m)^2} - \frac{Y_m}{\alpha Y_h Y_m (Y_h - Y_m)^2} \\
& + \frac{l\phi + \bar{L}}{\alpha Y_h^2 Y_m (Y_h - Y_m)} = 0 \\
& \Rightarrow z_h^* = \frac{Y_h - Y_m}{Y_m} \frac{l\phi + \bar{L} - \alpha p_h^0 Y_h^2 Y_m (Y_h - Y_m)}{2},
\end{aligned} \tag{A.15}
$$

and therefore $\frac{\partial z_h^*}{\partial \phi} > 0$. By replacing the optimal bonus into the manager's expected compensation ($\Pi_M^{ND} = p_h z_h - \alpha \frac{a^2}{2}$), we obtain

$$
\begin{aligned}
\Pi_M^{ND} &= z_h^* p_h - \frac{\alpha}{2} \left(\frac{(z_h^*)^2}{\alpha^2 Y_h^2 (Y_h - Y_m)^2} \right) \\
&= z_h^* \left(p_h - \frac{z_h^*}{2\alpha Y_h^2 (Y_h - Y_m)^2)} \right) \\
&= z_h^* \left(p_h^0 + \frac{z_h^*}{2\alpha Y_h^2 (Y_h - Y_m)^2} \right).
\end{aligned}
$$

Thus, $\Pi_M^{ND} = 0$ only when $z_h^* < 0$, which violates the limited liability constraint, or for $z_h^* = 0$. In other words, $\Pi_M^{ND} > 0 \Leftrightarrow z_h^* > 0 \Leftrightarrow l\phi + \bar{L} - \alpha p_h^0 Y_h^2 Y_m (Y_h - Y_m) > 0$. When $l\phi + \bar{L} - \alpha p_h^0 Y_h^2 Y_m (Y_h - Y_m) < 0$, $z_h^* = 0$ and the manager's participation constraint is binding.

When lenders cannot anticipate the manager's risk choice, the shareholder solves

$$
\begin{aligned}
\max_{z_h \geq 0} \quad & \Pi_e + \nu[\Pi_M^{ND} - \bar{\Pi}] \\
& = p_h (Y_h - z_h) + p_m Y_m - (p_h + p_m) L - (p_h + p_m) \bar{L} - \delta + \nu[\Pi_M^{ND} - \bar{\Pi}] - \delta.
\end{aligned}
$$

In this case, the shareholder takes L as given when selecting the optimal z_h. The FOC is

$$
\begin{aligned}
& - p_h^0 - \frac{2z_h}{\alpha Y_h^2 (Y_h - Y_m)^2} - \frac{\partial (p_h + p_m)}{\partial z_h} (L + \bar{L}) = \\
& - p_h^0 - \frac{2z_h}{\alpha Y_h^2 (Y_h - Y_m)^2} + \frac{L + \bar{L}}{\alpha Y_h^2 Y_m (Y_h - Y_m)} = 0.
\end{aligned} \tag{A.16}
$$

In equilibrium, the optimal bonus scheme must be determined by the intersection of the shareholder's and lender's optimal responses: z_h^* must satisfy equation (A.16) together with L from equation (A.2).

We can determine the effect of the bailout probability by applying the implicit function theorem to equation (A.16):

$$-\frac{2}{\alpha Y_h^2(Y_h-Y_m)^2}\frac{\partial z_h^*}{\partial \phi}+\frac{\overbrace{\frac{\partial L}{\partial \phi}}^{<0}+\overbrace{\frac{\partial L}{\partial a}\frac{\partial a}{\partial z_h}}^{>0}\frac{\partial z_h}{\partial \phi}}{\alpha Y_h^2 Y_m(Y_h-Y_m)}=0, \tag{A.17}$$

which implies that $\frac{\partial z_h^*}{\partial \phi}>0$ when $\frac{l(1+r_l-\phi)}{(p_h+p_m)^2Y_h^2Y_m^2(Y_h-Y_m)^2}-2>0$, and vice versa.

Finally, by setting $L=\bar{L}=0$ into equation (A.16) and solving for z_h, we obtain the optimal bonus for an all-equity bank:

$$z_h^*=-\frac{p_h^0\alpha Y_h^2(Y_h-Y_m)^2}{2}<0.$$

The limited liability constraint is binding and the optimal bonus is thus $z_h^*=0$. □

Proof of Proposition 7. The shareholder maximizes

$$\begin{aligned}\max_{z_h\geq 0}\quad & \Pi_e(z_h)-\nu[\Pi_M^D-\bar{\Pi}]\\ &=p_h\left(Y_h-L-\bar{L}-z_h\right)+p_m\left(Y_m-L-\bar{L}\right)-\tilde{\Theta}-\delta+\nu[\Pi_M^D-\bar{\Pi}]-\delta.\end{aligned}$$

Assume first that $\Gamma>P_x$. Based on the results above (Proposition 2), the manager always selects $a^*(slack)$ in his case. Hence,

$$\begin{aligned}&p_h(a^*(slack))=p_h^0+\frac{Az_h+B}{CY_h(Y_h-Y_m)},\quad p_m(a^*(slack))=p_m^0-\frac{Az_h+B}{CY_m(Y_h-Y_m)},\\ &p_{hm}(a^*(slack))=p_h(a^*(slack))+p_m(a^*(slack))=p_h^0+p_m^0-\frac{Az_h+B}{CY_hY_m}\\ &p_l(a^*(slack))=1-p_h(a^*(slack))-p_m(a^*(slack)).\end{aligned}$$

where $A=\frac{1}{Y_h(Y_h-Y_m)}$, $B=(\Gamma-P_x)\frac{l\phi+\bar{L}}{\beta Y_hY_m}$ and $C=\alpha-\frac{(l\phi+\bar{L})^2}{\beta Y_h^2Y_m^2}$. Assuming a slack participation constraint, the shareholder's problem becomes

$$\max_{z_h\geq 0}$$
$$p_h(a^*(slack))(Y_h-z_h)+p_m(a^*(slack))Y_m-\tilde{\Theta}-l(1+r_l-p_l(a^*(slack))\phi)-p_{hm}(a^*(slack))\bar{L}-\delta.$$

The FOC is

$$-p_h^0+A\frac{l\phi+\bar{L}}{CY_hY_m}-\frac{2Az_h}{Y_h(Y_h-Y_m)}-\frac{B}{CY_h(Y_h-Y_m)}-2\frac{P_e-P_x}{\beta}A\frac{l\phi+\bar{L}}{CY_hY_m}=0.$$

Replacing the expression for P_e (equation (3)) and solving for z_h yields

$$z_h^*(slack) = \frac{A\frac{\phi l+\bar{L}}{CY_hY_m} - p_h^0 - \frac{B}{Cy_h(Y_h-Y_m)} - \frac{2A\left(\Gamma-P_x+B\frac{\phi l+\bar{L}}{CY_hY_m}\right)(l\phi+\bar{L})}{\beta Y_hY_m}}{\frac{2A}{CY_h(Y_h-Y_m)} + \frac{2A^2(\phi l+\bar{L})^2}{\beta C^2Y_h^2Y_m^2}}.$$

When instead $\Gamma < P_x$, the short-selling constraint binds only for $z_h < \bar{z}$ (with $\bar{z}$ always positive when $\Gamma < P_x$), and the manager selects $a_h^*(bind)$ (Proposition 2). The shareholder's objective function then changes depending on the bonus payments. For $z_h \le \bar{z}$ the shareholder solves (assuming a the participation constraint of the manager is satisfied)

$$\max_{0\le z_h\le\bar{z}}$$
$$p_h(a^*(bind))(Y_h - z_h) + p_m(a^*(bind))Y_m - \tilde{\Theta} - l(1 + r_l - p_l(a^*(bind))\phi) - p_{hm}(a^*(bind))\bar{L} - \delta.$$

In this region of bonus payment the short-selling constraint binds. As a result $\tilde{\Theta} = \pi P_x$ does not depend on the bonus payment and the optimal bonus is given by z_h^*, as defined in Equation (A.15). We label this solution $z_h^*(bind)$ (and assume $l\phi + \bar{L} - \alpha p_h^0 Y_h^2 Y_m(Y_h - Y_m) >$ to rule out the non-interesting solution $z_h^*(bind) = 0$).

For bonus payments larger than $\bar{z}$, the manager selects $a^*(slack)$. Accordingly the shareholder's problems becomes

$$\max_{\bar{z}\ge z_h}$$
$$p_h(a^*(slack))(Y_h - z_h) + p_m(a^*(slack))Y_m - \tilde{\Theta} - l(1 + r_l - p_l(a^*(slack))\phi) - p_{hm}(a^*(slack))\bar{L} - \delta.$$

The solution is $z_h^*(slack)$, as defined above, assuming again that parameters are such that the participation constraint is satisfied. The shareholders will then select the bonus which maximizes their expected payoff. The assumption $z_h^*(bind) < \bar{z} < z_h^*(slack)$ produces the solution illustrated in the Proposition. Without imposing that $z_h^*(bind) < \bar{z} < z_h^*(slack)$, we may have the following additional cases:

- If $\bar{z} < z_h^*(bind) < z_h^*(slack)$ the optimal solution is $\bar{z}$ if $\Pi_e(\bar{z}) > \Pi_e(z_h^*(slack))$. Otherwise, $z_h^*(slack)$ is the optimal solution.
- If $z_h^*(bind) < z_h^*(slack) < \bar{z}$ the optimal solution is $\bar{z}$ if $\Pi_e(\bar{z}) > \Pi_e(z_h^*(bind))$. Otherwise, $z_h^*(bind)$ is the optimal solution.
- If $z_h^*(bind) > \bar{z}$ and $z_h^*(bind) < \bar{z}$, the optimal solution is $\bar{z}$.

Finally, consider the manager's payoff corresponding to the optimal choice of the bonus payment.

$$\begin{aligned}\Pi_M^D &= p_h z_h^* + \left[\pi_e\left(P_e(a(z_h^*), \phi) - P_x\right) + \pi P_x\right] - \frac{\alpha}{2}a(z_h^*)^2 - \frac{\beta}{2}\pi_e(a(z_h^*))^2 \\ &= p_h z_h^* - \frac{\alpha}{2}a(z_h^*)^2 + \pi P_x + \frac{\pi_e^2}{2\beta} \qquad \text{(A.18)}\end{aligned}$$

Consider first the case of a binding short selling constraint (i.e., $\pi_e = 0$). In this case the optimal bonus payment is given in Proposition 6. In addition, the shareholders find it optimal to to set $\bar{\Theta} = 0$. As a result, the first two terms on the right hand side of $A.18$ correspond to the manager's payoff under the non-discretionary scheme for default-linked compensation and we can write

$$\Pi_M^D = \Pi_M^{ND} + \pi P_x$$

Since $\Pi_M^{ND} \geq 0$ (Proposition 6) we conclude that $\Pi_M^D > 0$ when the short selling constraint binds. We then move to the other possible equilibria. When $\Gamma > P_x$, the manager selects $a^* = a^*(slack)$ for any z_h^* yielding

$$\begin{aligned}\Pi_M^D &= p_h(a^*(slack))z_h^* + \pi P_x + \frac{\pi_e^2}{2\beta} - \frac{\alpha}{2}a^*(slack)^2 \\ &\geq p_h(a^*(bind))z_h^* + \pi P_x - \frac{\alpha}{2}a^*(bind)^2 = \Pi_M^{ND} + \frac{\pi_e^2}{2\beta} + \pi P_x = \Pi_M^{ND} + \pi P_x > 0,\end{aligned}$$

where the last inequality follows from the fact that any risk choice different from $a^*(slack)$ would be sub-optimal in this case. When $\Gamma < P_x$, the manager chooses $a^*(slack)$ only for $z_h^* \geq \bar{z}$ yielding

$$\begin{aligned}\Pi_M^D &= p_h(a^*(slack))z_h^*(slack) + \pi P_x + \frac{\pi_e^2}{2\beta} - \frac{\alpha}{2}a^*(slack)^2 \\ &\geq p_h(a^*(bind))z_h^*(slack) + \pi P_x - \frac{\alpha}{2}a^*(bind)^2 \\ &> p_h(a^*(bind))z_h^*(bind) + \pi P_x - \frac{\alpha}{2}a^*(bind)^2 = \Pi_M^{ND} + \frac{\pi_e^2}{2\beta} + \pi P_x = \Pi_M^{ND} + \pi P_x > 0.\end{aligned}$$

$\square$

The case of the unsecured and discretionary compensation In this case the shareholder's problem can only be solved numerically. In particular, the shareholder selects the optimal bonus compensation to maximize the expected profits (Π_e)

$$\Pi_e = p_h\left(Y_h - z\right) + p_m Y_m - (p_h + p_m)\left(\tilde{\Theta} + L + \bar{L}\right) - \delta,$$

where $\tilde{\Theta} = \pi_e P_e(a, \phi) + (\pi - \pi_e)P_x$.

When the lenders can anticipate the manager's risk choice, we can replace the lenders' compensation (L) into Π_e, obtaining

$$\Pi_e = p_h\left(Y_h - z_h\right) + p_m Y_m - (p_h + p_m)(\tilde{\Theta} + \bar{L}) - l(1 + r_l - p_l\phi) - (p_h + p_m)\bar{L} - \delta.$$

The FOC for the optimal bonus is

$$\left[-\frac{z_h}{Y_h(Y_h-y_m)}+\frac{\tilde{\Theta}+\bar{L}}{Y_hY_m}+\frac{\phi l}{Y_hY_m}\right]\frac{\partial a}{\partial z}-(p_h+p_m)\frac{\partial\tilde{\Theta}}{\partial z}-p_h=0,$$

where a is the manager's risk choice of Proposition 4.

When the lenders cannot anticipate the manager's risk choice, shareholders maximize their objective function for a given L. The FOC is thus

$$\left[-\frac{z_h}{Y_h(Y_h-y_m)}+\frac{\tilde{\Theta}+\bar{L}}{Y_hY_m}+\frac{L}{Y_hY_m}\right]\frac{\partial a}{\partial z}-(p_h+p_m)\frac{\partial\tilde{\Theta}}{\partial z}-p_h=0, \tag{A.19}$$

and the optimal z_h must satisfy equation (A.19), with L given in equation (A.2).

For an all-equity bank, the optimal z_h is obtained numerically by setting $L=\bar{L}=0$, $\frac{\partial a}{\partial z}=\frac{1}{Y_h(Y_h-Y_m)}\times\frac{1}{\alpha-\frac{(P_e^0-P_x)^2}{\beta Y_h^2Y_m^2}}$ and $\frac{\partial\tilde{\Theta}}{\partial z}=-\frac{(P_e^0-P_x)^2}{\beta Y_hY_m}\frac{\partial a}{\partial z}$ into equation (A.19) and solving for z_h. The analysis of Appendix Section B reveals that the shareholders' objective function is concave.

The manager's payoff corresponding to the optimal choice of the bonus payment is

$$\begin{aligned}\Pi_M^D &= p_hz_h^*+(p_h+p_m)\left[\pi_e\left(P_e(a(z_h^*),\phi)-P_x\right)+\pi P_x\right]-\frac{\alpha}{2}a(z_h^*)^2-\frac{\beta}{2}\pi_e(a(z_h^*))^2\\ &= p_hz_h^*+\frac{\beta}{2}\pi_e(a(z_h^*))^2+(p_h+p_m)\pi P_x-\frac{\alpha}{2}a(z_h^*)^2,\end{aligned}$$

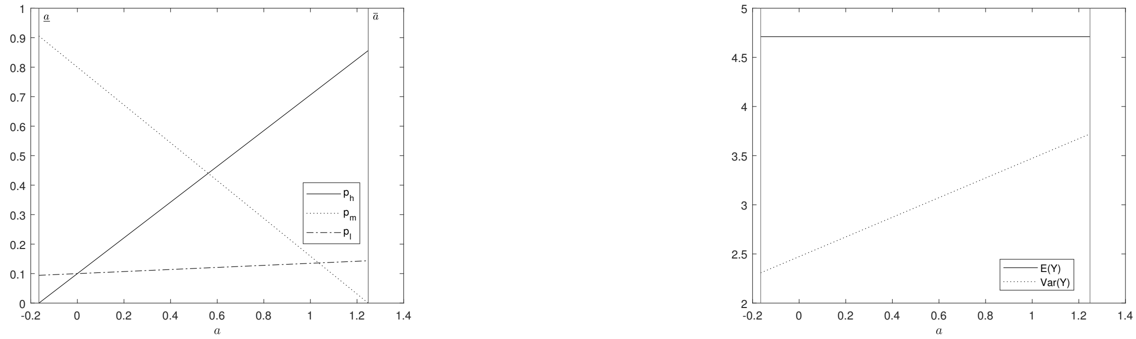
where a is the optimal risk choice of Proposition 4 and the term in square brackets—the value of the default-linked account—is positive because π_e and P_e-P_x have the same sign (see Proposition 4). Finally, computations akin to those in the proof of Proposition 2) reveal that the manager's participation constraint is slack.

B Additional numerical analyses

In this section, we provide more details on the numerical solutions to the manager's and the shareholder's maximization problems under the unsecured and discretionary scheme for default-linked compensation.

B.1 Probabilities and moments of bank asset payoff

Appendix Figure A.1 illustrates how the probability of each state of the world (Panel (a)) and the first two moments of bank asset payoff (Panel (b)) change as the manager's risk choice varies.



(a) Probabilities of bank asset payoff. Probability p_i as a function of the manager's risk choice a.

(b) Moments of bank asset payoff. Moments of payoff as functions of the manager's risk choice a.

Figure A.1: Probabilities and moments of bank asset payoff as a function of the manager's risk choice. Calibration: $Y_h = 5.5$, $Y_m = 5.2$, $Y_l = 0$, $p_h^0 = 0.1$, $p_m^0 = 0.8$, $p_l^0 = 0.1$.

B.2 The manager's objective function

Appendix Figure A.2 shows the manager's expected payoff as a function of the risk choice and the allocation of default-linked compensation for the main calibration. Panel (a) consider the case of a low bonus payment, whereas Panel (b) considers a high bonus payment.

(a) Managerial expected payoff under a low bonus payment. Managerial expected payoff as a function of a and π_e, with $z_h = 0.5$.

(b) Managerial expected payoff under a high bonus payment. Managerial expected payoff as a function of a and π_e, with $z_h = 1$.

Figure A.2: The manager's expected payoff, risk choice, and allocation of default-linked compensation under the unsecured and discretionary scheme for different levels of the bonus payment. Calibration: see Section 3.3.2.

B.3 The shareholder's objective function

Appendix Figure A.3 shows the shareholder's objective function as a function of the bonus payment for different levels of the promised payment to lenders and of the bailout probability.

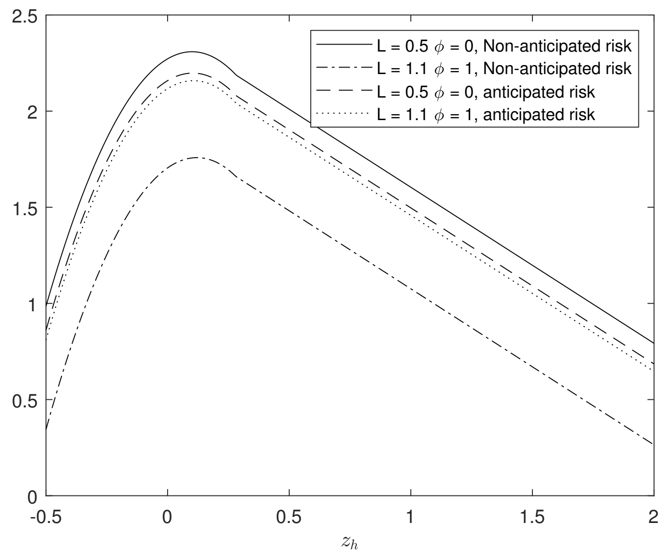


Figure A.3: The shareholder's expected payoff as a function of the bonus payment for different values of the lenders' promised payment (L) and bailout probability (ϕ) under the unsecured and discretionary scheme for different levels of the bonus payment. Calibration: see Section 3.3.2.

B.4 Differences in risk-taking and allocation of the default-linked account

In Appendix Figure A.4, we further characterize the bank-level asset risk and the personal allocation of the default-linked account optimally chosen by the manager. In the top graph, we compare the risk choice under the discretionary contract (a_D^j) against that under the non-discretionary contract (a_{ND}^j), imposing that the value of the default-linked account is the same under the two schemes. The superscript j refers to different levels of the bailout probability: $j = 1$ corresponds to $\phi = 0$, $j = 2$ to $\phi = 0.5$, and $j = 3$ to $\phi = 1$. In the bottom graph, we plot the manager's personal allocation of the default-linked account under the discretionary contract. We observe that $a_{ND}^j - a_D^j < 0$ for moderate levels of the bonus payment z_h—to which the manager's exposure to own bank's equity π_e responds positively. In this region, holding the value of default-linked compensation constant, discretion over the investment strategy induces higher risk-taking incentives. By contrast, when the short-selling (or the feasibility) constraint is binding, the risk choice is the same under the two contracts.

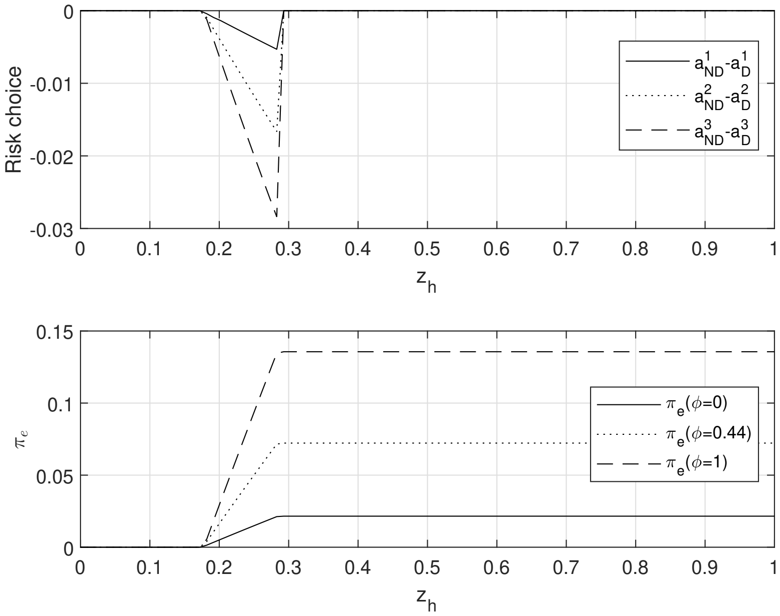
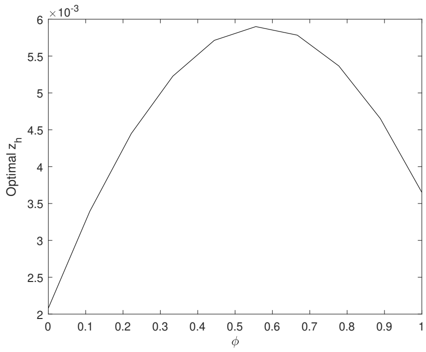


Figure A.4: Impact of the bonus payment on: i) the difference between the manager's risk choice under the discretionary contract (a^j_D) and that under the non-discretionary contract (a^j_{ND}), imposing that the underlying value of default-linked compensation is the same (top graph); ii) the manager's personal allocation of the default-linked account under the discretionary contract (bottom graph). Parameters: Calibration: see Section 3.3.2.

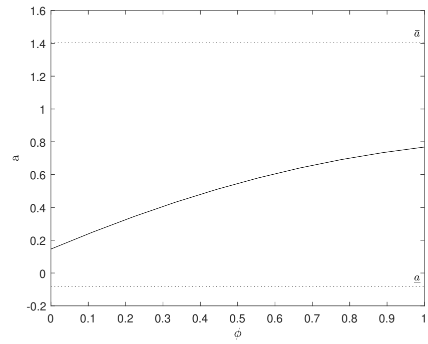
B.5 Optimal risk choice and bonus design with constant P_x

B.5.1 Secured-discretionary compensation

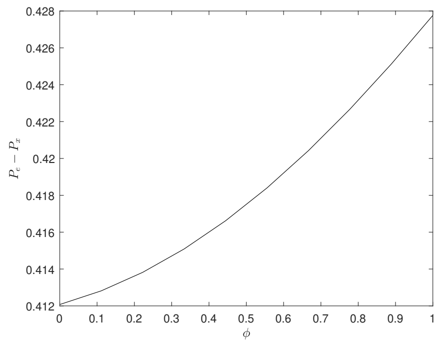
We start by considering the case of secured-discretionary compensation when $\Gamma > P_x$. The optimal bonus payment is increasing in the bailout probability only for ϕ smaller than a given threshold (about .55 for this calibration). After this point the manager becomes sufficiently exposed to bank equity that shareholders can conveniently reduce bonus payment without curbing risk-taking incentives (Panel (a)-(b)). Higher risk taking incentives increase the price of bank equity (Panel (c)) and, as a result, the manager buys more own bank's shares (Panel (d)).



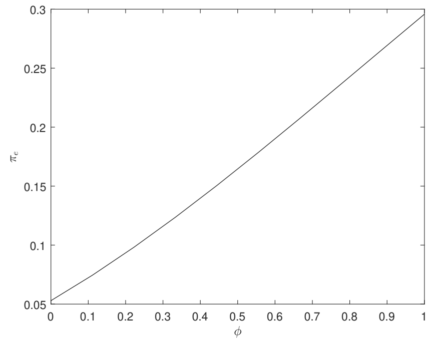
(a) Optimal bonus payment. The optimal bonus payment z_h from the shareholder's perspective as a function of ϕ.



(b) Optimal risk choice. The manager's optimal risk choice a as a function of ϕ.



(c) Price of bank equity. $P_e - P_x$ as a function of ϕ.

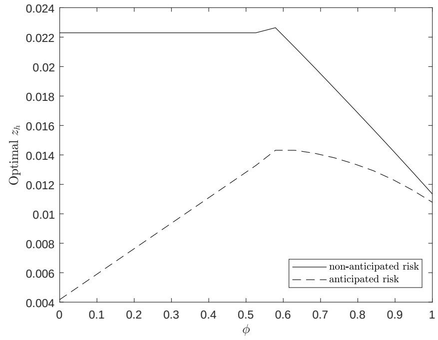


(d) Optimal investment. Optimal π_e as a function of ϕ.

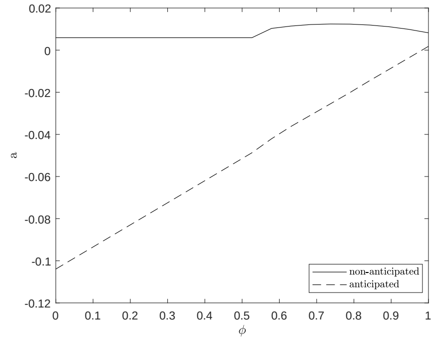
Figure A.5: Optimal shareholder's bonus and corresponding manager's risk choice as a function of the bailout probability when the short-selling constraint is slack, under the secured and discretionary scheme for default-linked compensation. Calibration: $Y_h = 5.5$, $Y_m = 5.2$, $Y_l = 0$, $p_h^0 = .05$, $p_m^0 = .9$ $p_l^0 = 0.05$, $\alpha = 0.01$, $\beta = 0.2$, $\delta = 0.1$, $r_d = 0.02$, $r_l = 0.04$, $\phi = 0.5$ $l = 0.6$, $k = 0.3$, $d = (1 - l - k)/(1 - \delta)$, $\Gamma > P_x = P_e(0, 0) - .1$.

B.5.2 Unsecured-discretionary

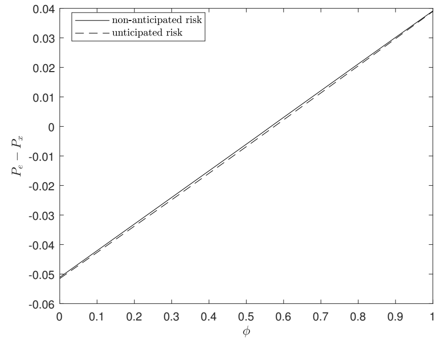
In Figures A.6 and A.7, we repeat the exercise of Section 4.2.2 but assuming a constant P_x. Specifically, we look at the optimal bonus design from the shareholder's perspective (Panels (a)), the corresponding risk choice from the manager's perspective (Panels (b)), the equity price (Panels (c)) and the manager investment in bank equity (Panels (d) distinguishing between the case of a low and that of a high mandatory default-linked compensation π (A.6 and A.7).



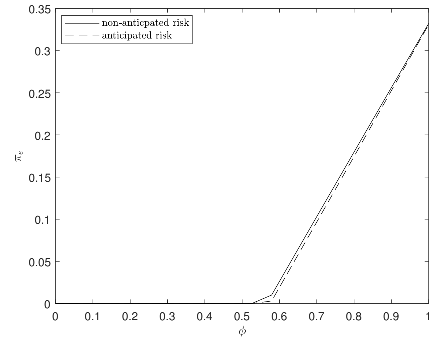
(a) Optimal bonus payment under low default-linked pay. The optimal bonus payment z_h from the shareholder's perspective as a function of ϕ, with $\pi = 0.1$.



(b) Optimal risk choice under low default-linked pay. The manager's optimal risk choice as a function of the bailout probability, with $\pi = 0.1$.

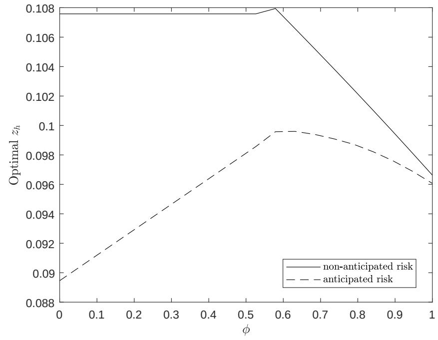


(c) Equity price under low default-linked pay. The optimal bonus payment z_h from the shareholder's perspective as a function of ϕ, with $\pi = 0.15$.

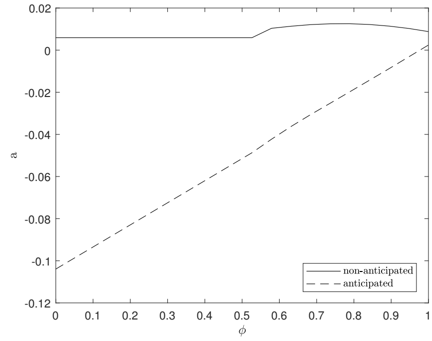


(d) Optimal investment under low default-linked pay. The manager's optimal risk choice as a function of the bailout probability, with $\pi = 0.1$.

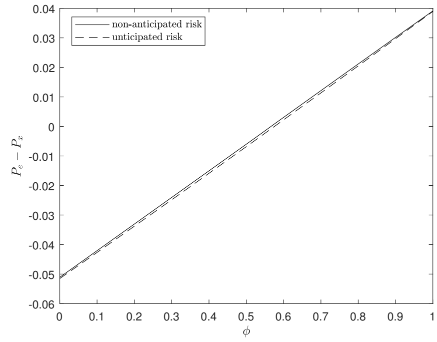
Figure A.6: Optimal shareholder's and manager's and the bailout probability with constant P_x, under the unsecured and discretionary scheme for default-linked compensation. Calibration: $Y_h = 5.5$, $Y_m = 5.2$, $Y_l = 0$, $p_h^0 = 0.15$, $p_m^0 = 0.7$, $p_l^0 = 0.15$, $\alpha = 0.1$, $\beta = 0.1$.



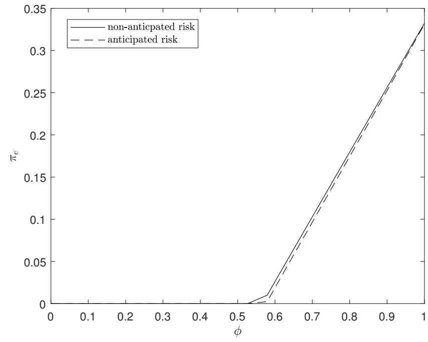
(a) Optimal bonus payment under low default-linked pay. The optimal bonus payment z_h from the shareholder's perspective as a function of ϕ, with $\pi = 0.1$.



(b) Optimal risk choice under low default-linked pay. The manager's optimal risk choice as a function of the bailout probability, with $\pi = 0.1$.



(c) Equity price under under high default-linked pay. The optimal bonus payment z_h from the shareholder's perspective as a function of ϕ, with $\pi = 0.5$.

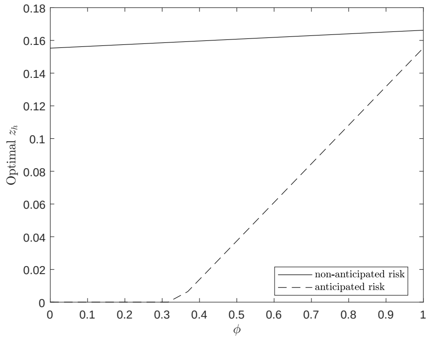


(d) Optimal investment under high default-linked pay. The manager's optimal risk choice as a function of the bailout probability, with $\pi = 0.5$.

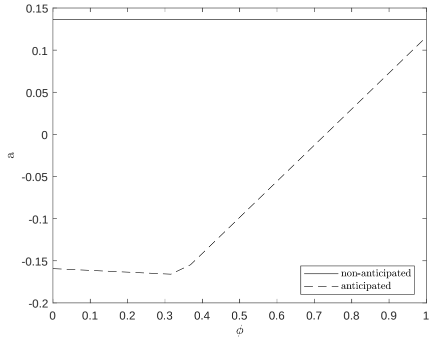
Figure A.7: Optimal shareholder's and manager's and the bailout probability with constant P_x, under the unsecured and discretionary scheme for default-linked compensation. Calibration: $Y_h = 5.5$, $Y_m = 5.2$, $Y_l = 0$, $p_h^0 = 0.15$, $p_m^0 = 0.7$, $p_l^0 = 0.15$, $\alpha = 0.1$, $\beta = 0.1$.

B.6 The role of bank asset payoff

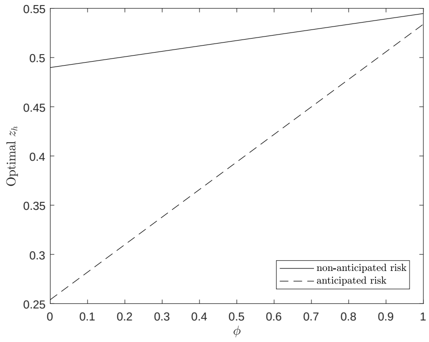
In Appendix Figure A.8, we investigate the sensitivity of our baseline results on optimal bonus design (Panels (a) and (c)) and risk-taking (Panels (b) and (d)) to using lower non-default bank asset payoffs in the calibration. Moreover, we compare the cases of low and high mandatory default-linked compensation (top vs. bottom graphs), showing that the latter induces the shareholder to choose a higher bonus payment.



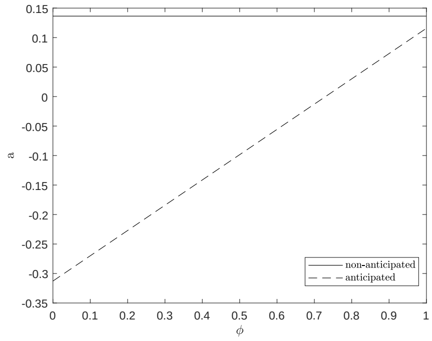
(a) Optimal bonus payment under low default-linked pay. The optimal bonus payment z_h from the shareholder's perspective as a function of ϕ, with $\pi = 0.1$.



(b) Optimal risk choice under low default-linked pay. The manager's optimal risk choice as a function of the bailout probability, with $\pi = 0.1$.



(c) Optimal bonus payment under high default-linked pay. The optimal bonus payment z_h from the shareholder's perspective as a function of ϕ, with $\pi = 0.5$.



(d) Optimal risk choice under high default-linked pay. The manager's optimal risk choice as a function of the bailout probability, with $\pi = 0.5$.

Figure A.8: Optimal shareholder's and manager's choices and the bailout probability with low bank asset payoffs, under the unsecured and discretionary scheme for default-linked compensation. Calibration: $Y_h = 3.5$, $Y_m = 2.0$, $Y_l = 0$, $p_h^0 = 0.15$, $p_m^0 = 0.7$, $p_l^0 = 0.15$, $\alpha = 0.1$, $\beta = 0.1$, $\delta = 0.1$, $r_d = 0.02$, $r_l = 0.04$, $l = 0.6$, $k = 0.3$, $d = (1 - l - k)/(1 - \delta)$.