CREDIT FRICTIONS AND THE CLEANSING EFFECT OF RECESSIONS*

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Recessions are conventionally considered as times when the least productive firms are driven out of the market. How do credit frictions affect this cleansing effect of recessions? We build and calibrate a model of firm dynamics with credit frictions and endogenous entry and exit to investigate this question. We find that there is a cleansing effect of recessions in the presence of credit frictions, despite their effect on the selection of exiting and entering firms. This result holds true regardless of the nature of the recession: average firm-level productivity rises following a negative aggregate productivity shock, as well as following a negative financial shock.

Recessions are times of rising bankruptcies and business closures. During the Great Recession, the US annual establishment exit rate increased from 11.8% to 13.5% between March 2008 and March 2009, as shown in Figure 1.[1] The increase in the firm exit rate during periods of economic downturns has motivated the view that recessions cleanse the economy: as they become not profitable enough, less efficient firms are scrapped, thus allowing resources to be reallocated towards more productive firms.

This conventional view of recessions, emphasised in Caballero and Hammour (1994), is based on the implicit assumption that markets select the most productive firms. This assumption has however been challenged by several studies showing that the firms' probability to exit depends not only on their productivity but also on their access to credit. Holtz-Eakin *et al.* (1994) show that liquidity constraints raise the likelihood of entrepreneurial failure. In a similar vein, using a panel data of French manufacturing firms over the 1996–2004 period, Musso and Schiavo (2008) find that financial constraints significantly increase the firms' probability of exiting the market. These findings suggest that, in the presence of credit frictions, highly productive but financially vulnerable firms may be forced to exit the market. Credit frictions may therefore alter the productivity-enhancing effect of recessions.

[1] As 95% of firms are single-establishment firms (BLS statistics), the establishment exit rate is likely to be a good proxy of the firm exit rate.
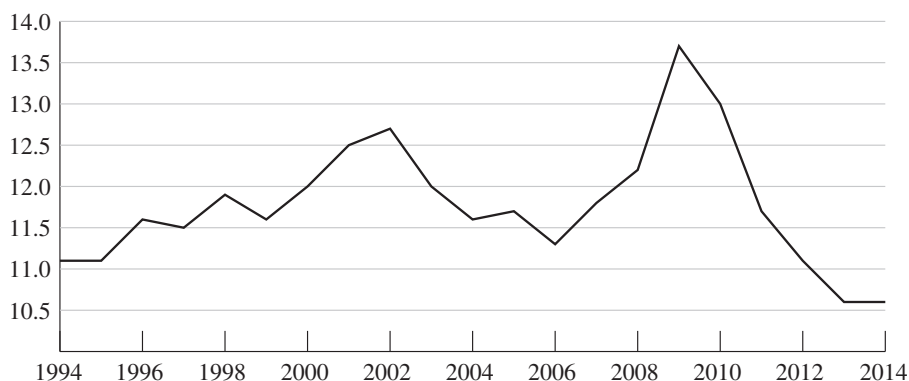
Fig. 1. *Private Sector Establishment Exit Rate (%), March 1994–March 2014*
Source. Bureau of Labor Statistics, Business Employment Dynamics (annual data). The data for each year t refers to the period between March of year $t − 1$ and March of year t. The data are publicly available at http://www.bls.gov/bdm/bdmann.htm.

In this article, we study the effects of credit frictions on the cleansing effect of recessions in a model of firm dynamics with credit frictions and endogenous exit.[2] We find that there is a cleansing effect of recessions even in the presence of credit frictions. Average productivity increases following an adverse aggregate productivity shock as the shock predominantly raises the net exit rate of low productivity firms. These firms are more vulnerable to the aggregate productivity shock than high productivity firms despite facing fewer credit frictions. We find that average productivity also rises when the economic downturn is driven by a negative financial shock. While the cleansing effect of recessions holds true regardless of the presence of credit frictions or the nature of the shock, we show that the intensity of the cleansing effect is dampened in the presence of credit frictions, especially when the recession is driven by a financial shock.

In the model, credit constraints endogenously arise from asymmetric information and costly state verification. As in Cooley and Quadrini (2001), we embed a one-period financial contract *à la* Bernanke and Gertler (1989) into a model of firm dynamics.[3] The financial contract determines the amount the firm can borrow and the interest rate charged by the financial intermediary as a function of the firm's levels of productivity and net worth. When firms are hit by an adverse productivity shock, they may be unable to repay their debt, they default and are left with zero net worth. After default, most firms are excluded from the credit market and are therefore forced to exit. However, default is not the only motive for exit as firms also decide to leave the market when their expected profits are too low. This happens when firms are not sufficiently productive, as in the frictionless economy, but also when their balance

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[2] For the sake of simplicity, our model abstracts from employment as it is outside the scope of this article to study the implications of changes in the exit rate for employment fluctuations.

[3] Cooley and Quadrini (2001) show that the introduction of credit frictions is able to account for the negative correlation between firm growth and firm age and size. We adopt a similar modelling of credit frictions and augment their model by introducing an endogenous exit decision.

sheets are too weak. Firms with a low net worth face tighter credit constraints and higher borrowing costs, which raises their probability to exit. As the firms' exit decision depends on their net worth, exiting firms are not necessarily the least productive ones. Credit frictions therefore modify the selection of exiting firms: some high productivity firms are forced to exit in case of financial distress while some low productivity firms may survive. Credit frictions have similar effects at the entry margin since the entry decision is symmetric to the exit decision, and potential entrants effectively enter the market if their net worth and their productivity are sufficiently high.

We calibrate the model to match the observed exit rate, the productivity distribution and the level of credit frictions in the US economy, and analyse the consequences of credit frictions on the cleansing effect of recession. We find that, though credit frictions modify the selection of entering and exiting firms, average firm-level productivity rises after a negative aggregate productivity shock as the shock disproportionately raises the net exit rate of low productivity firms. While the positive correlation between the end-of-period net worth and productivity contributes to the magnitude of the increase in average productivity, it is worth noting that it is not the only determinant behind the cleansing effect of recessions.[4] In fact, even if the end-of-period net worth were not correlated to productivity, the shift in the productivity threshold and the larger increase in the net worth exit threshold at low levels of productivity would still lead to an increase in average productivity.

We show that the magnitude of the cleansing effect of recessions depends on two offsetting effects. On the one hand, credit frictions lead to an increase in the exit rate of firms across a larger set of productivity levels. Because the exit rate rises mostly for firms below average productivity, this tends to amplify the increase in average productivity. On the other hand, the steady-state exit rate of low productivity firms is higher in the presence of credit frictions; low productivity firms then account for a smaller share of firms in the credit frictions economy. This shift in the productivity distribution tends to dampen the increase in average productivity as it reduces the number of firms vulnerable to the aggregate shock. In our benchmark calibration, the distribution effect prevails and the increase in average productivity is smaller than in the frictionless economy. We show that, depending on the strength of the two offsetting effects, credit frictions may possibly amplify the increase in average productivity but they systematically reduce the intensity of the cleansing effect of recessions. In fact, we find that the increase in average productivity implied by a percentage point increase in the net exit rate is smaller in the credit frictions economy whatever the effects of credit frictions on the steady-state productivity distribution. We then study the response of the economy to a financial shock and investigate how the nature of the shock shapes the response of average productivity. We find that the decline in total net worth, calibrated to match the decline observed in the 2008 recession, also affects predominantly low productivity firms and leads to an increase in

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[4] While the empirical correlation between productivity and net worth has not been studied in the literature, many studies have documented the link between productivity and firm size, which can be used as a proxy for the end-of-period net worth. This correlation has been found to be positive: in Foster *et al.* (2008) the within-industry correlation between establishment-level output and TFP in the US manufacturing sector is 0.19; using Compustat data, Imrohoroglu and Tuzel (2014) reports a correlation between firm size and TFP of 0.38.

average productivity. As the financial shock leads to an increase in the net exit rate that is more similar across productivity levels, the intensity of the cleansing effect is weaker than in the case of a negative aggregate productivity shock.

This article contributes to the literature on the cleansing effect of recessions. This literature, which goes back to Schumpeter (1942), suggests that recessions are important times of restructuring that lay the ground for future expansions. Several theoretical articles have contested this view. Using a job search model, Barlevy (2002) shows that recessions impede the reallocation of workers from low to high productivity jobs and may thereby exacerbate the misallocation of resources. Ouyang (2009) argues that recessions may lower average productivity by increasing the exit of young and potentially productive firms before they learn their productivity. Kehrig (2015) proposes a model in which the fall in factor prices during recessions dampens the impact of the aggregate profitability shock and may hence mitigate the cleansing effect. In Lee and Mukoyama (2015), the exit rate and the productivity of exiting firms are both similar across booms and recessions, consistently with their empirical findings on the manufacturing sector. Our model accounts for the countercyclical exit rate in the overall economy shown in Figure 1 and derives its implication for average productivity in the presence of credit frictions. In an empirical contribution, Foster *et al.* (2014) show that reallocation has been less productivity-enhancing during the Great Recession. Although they do not directly address why the Great Recession is different, they argue that the financial collapse could have played a relevant role. In line with their findings, we find that financial shocks modify the patterns of reallocation and we show that they lead to a lower cleansing intensity.

Barlevy (2003) also investigates the consequences of credit frictions on the allocation of resources during recessions. Using a stylised model in which the most efficient firms are also the most financially vulnerable, Barlevy (2003) shows that credit frictions may reverse the cleansing effect of recessions. We find instead that average productivity rises with the net exit rate whatever the level of credit frictions. Our conclusion differs, not because of the correlation between financial constraints and productivity – in both models, high productivity firms require more borrowing and hence face higher levels of frictions–but because of the modelling of the exit decision. Contrary to Barlevy (2003), where the firm's exit decision is governed by the participation constraint of the bank, the participation constraint of the firm is crucial in our model: most firms exit when they are not sufficiently profitable. Our article shows that once we account for the role of profitability, which has been found to be a key determinant of exit, credit frictions do not reverse the cleansing effect of recessions.

Our article also contributes to the literature on the exit decision of firms (McDonald and Siegel, 1985; Dixit, 1989; Hopenhayn, 1992) as we analytically characterise the exit decision of firms that face credit constraints and incur a fixed cost of production and show how credit constraints modify the selection of exiting and entering firms.[5] Finally, our article is related to the literature that explores the aggregate implications of firm dynamics. In a recent paper, Clementi and Palazzo (2013) show that firm entry and exit account for about one fifth of output growth in the aftermath of a positive

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[5] All these papers consider perfect financial markets and, to the best of our knowledge, the properties of the exit decision under credit constraints had not been derived analytically.

productivity shock. In contrast with Clementi and Palazzo (2013), our article focuses on the role of credit market frictions for the dynamics of average firm-level productivity.

The article is organised as follows. In Section 1, we describe the model of firm dynamics and credit constraints, and show analytically how the exit decision differs from the frictionless economy. In Section 2, we first analyse numerically the properties of the steady-state economy. We then show how credit frictions affect the response of average productivity to a fall in aggregate productivity. Section 3 investigates the determinants of the magnitude of the cleansing effect and studies the response of the economy to a financial shock. Section 4 concludes.

1. A Model of Firm Dynamics and Credit Market Frictions

In this Section, we describe the model of firm dynamics with credit market imperfections. We first describe the production technology and the timing of the firms' decisions and then present the frictionless economy and the economy with credit market frictions.

1.1. *Technology and Timing of Decisions*

The economy is constituted of risk neutral firms with a constant discount factor $0 < \beta < 1$. Firms are heterogenous with respect to their productivity and their net worth, and have access to a production technology with capital as the only input and decreasing returns to scale. Each period, firms incur a fixed operating cost to start production. After production, firms determine the amount of dividends to distribute and the amount of profits to reinvest. Firms can decide to stay in the market and reinvest their profits in production or invest in a risk-free asset. When the value from investing in the safe asset is higher than the value from producing, firms choose to exit and never enter again. Exiting firms lose the opportunity to receive future profits from production but also avoid paying the fixed cost. Firms therefore exit when their expected income from production is not sufficiently high to compensate the fixed cost. In what follows, for any generic variable x, we adopt the notation x' to define the next period value of the variable x.

After paying the fixed operating cost c, the firm produces output $Z(\theta + \epsilon)k^\alpha$ where $0 < \alpha < 1$. The capital k used for production depreciates at rate $0 < \delta < 1$. Z is the stochastic aggregate productivity common across firms. Every period, firms are also hit by a persistent firm-level productivity shock θ and a non-persistent firm-level productivity shock ε. The non-persistent component ε is independently and identically distributed across time and across firms, from the distribution Φ with zero mean and standard deviation σ. The persistent component θ follows a Markov process independent across firms with conditional distribution $F(\theta'|\theta)$. The conditional distribution $F(\theta'|\theta)$ is assumed to be strictly decreasing in θ: the higher is the productivity shock at time t, the more likely are high shocks in period $t + 1$. This assumption ensures that the value of the firm is an increasing function of the current productivity θ.

The value of the persistent firm-level shock and that of the aggregate shock are revealed at the end of each period. Therefore, at the beginning of the period, firms choose their capital knowing their persistent firm-level shock θ, the value of aggregate productivity Z and their net worth e. At the beginning of the period, firms do not know their transitory shock ε. They observe the realisation of ε after production and must then reimburse their debt over the capital borrowed and the fixed operating cost $(c + k - e)$. They are left with the end-of-period net worth q. At the end of the period, a firm with net worth q observes the productivity shocks θ' and Z', and decides its next period net worth e' (or equivalently the amount of dividends $(q - e')$ to distribute) and whether to exit or stay in the market. A firm decides to exit when its value from producing is lower than the value from investing in the safe asset, which is equal to $q_t + \Sigma_{s=0}^{+\infty}\beta^s[\beta(1 + r) - 1]e_{t+s+1}$. Note that if $\beta(1 + r) \leq 1$, the value from investing in the safe asset simplifies to q_t. In that case, the firm is either indifferent about the timing of dividends $[\beta(1 + r) = 1]$ or prefers to distribute its end-of-period net worth as dividends $[\beta(1 + r) < 1]$.

1.2. *The Frictionless Economy*

In the frictionless economy, firms borrow $(c + k - e)$ at the risk-free interest rate $r = 1/\beta - 1$. The value of a firm at the beginning of the period is:

$$V_{FL}(e, \theta, Z) = \max_k \mathbb{E} \int \max\{q, \max_{e'}[q - e' + \beta V_{FL}(e', \theta', Z')]\}\mathrm{d}\Phi(\epsilon),$$

where the end-of-period net worth is equal to

$$q = Z(\theta + \epsilon)k^\alpha + (1 - \delta)k - (1 + r)(c + k - e),$$

and $\mathbb{E}$ denotes expectations conditional on the current values of θ and Z. The value of the firm depends on the expected outcome of its investment. The firm exits when the value from investing in the safe asset is higher than the value from investing in production. As $r = 1/\beta - 1$, the firm is indifferent about the timing of dividends and the value from investing in the safe asset is then equal to its end-of-period net worth q. Furthermore, the Modigliani–Miller theorem holds and the value of the firm is independent of its financing decision. In particular, the exit and capital decisions of the firm do not depend on its level of equity. It can be shown that, conditional on surviving, the programme of the firm is equivalent to maximising its expected profits:

$$\widehat{V}_{FL}(\theta, Z) = \max_k \mathbb{E} \int [Z(\theta + \epsilon)k^\alpha - (r + \delta)k - (1 + r)c]\mathrm{d}\Phi(\epsilon) + \beta \max[0, \widehat{V}_{FL}(\theta', Z')].$$

When credit markets are perfect, firms exit when they are not productive enough: they exit if $\theta' < \underline{\theta}_{FL}(Z')$, where $\underline{\theta}_{FL}(Z')$ is defined by $\widehat{V}_{FL}(\underline{\theta}_{FL}, Z') = 0$.

1.3. *The Economy with Credit Market Frictions*

As in Bernanke and Gertler (1989) and Carlstrom and Fuerst (1997), credit constraints arise from asymmetric information between the firm and the financial intermediary. After production, the transitory shock ε is privately observed by the firm,

whereas the financial intermediary can observe ε only at a cost μk^{α}.[6] We consider a one-period debt contract in which the firm defaults when the shock is too low, and the financial intermediary monitors the firm's income only when the firm defaults. The terms of the financial contract depend on the value of the firm's net worth e, on its current productivity θ, and on the value of aggregate productivity Z, all observable by the financial intermediary and the firm at zero cost.

ASSUMPTION 1.   *The risk-free interest rate is such that:* $\beta < \dfrac{1}{1+r}$.

As in Cooley and Quadrini (2001), this Assumption implies that the risk-free rate is lower in the economy with credit frictions than in the frictionless economy, and guarantees that firms will not always choose to reinvest all their profits, thus giving an upper bound to their net worth. This condition can be interpreted as a general equilibrium property of economies with financial constraints. As it goes beyond the scope of this article to analyse the impact of credit frictions on the risk-free rate, we choose to leave aside this general equilibrium effect when comparing the results in the credit constrained economy with the frictionless case. In the following, we compare the credit constrained economy with the same economy without credit frictions but with the same risk-free rate r.

The firm finances its capital using its equity e, and if $c + k > e$, the firm borrows $(c + k - e)$ at rate $\widetilde{r}$ from the financial intermediary. When a firm is not able to reimburse its debt, it defaults. In this case, the financial intermediary pays a cost to verify the firm's income and confiscates all the firm's income. The default threshold $\bar{\epsilon}$ is given by:[7]

$$Z(\theta + \bar{\epsilon})k^{\alpha} + (1 - \delta)k = (1 + \widetilde{r})(c + k - e). \tag{1}$$

Default leads to a zero net worth but does not necessarily lead to the exit of the firm, as observed empirically. Depending on its persistent productivity component θ, the firm could find it profitable to stay in the market with zero net worth.

The financial intermediary lends $(c + k - e)$ to the firm only if its expected income from the loan is equal to the opportunity cost of the funds. The break even condition reads:

$$(1 + \widetilde{r})(k + c - e)[1 - \Phi(\bar{\epsilon})] + \int_{-\infty}^{\bar{\epsilon}} [Z(\theta + \epsilon)k^{\alpha} + (1 - \delta)k - \mu k^{\alpha}] \, \mathrm{d} \, \Phi(\epsilon) \geq$$
$$(1 + r)(k + c - e).$$

The expected income of the financial intermediary is equal to the repayment of the loan if the firm does not default ($\epsilon \geq \bar{\epsilon}$) and to the firm's income net of monitoring

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[6] If ε was observed before production, the nature of the credit frictions would be substantially different. Firms with a bad ε shock may borrow with no intention of repaying the bank. In that case, the asymmetry on ε would generate adverse selection: the bank would have to charge an even higher interest rate, which may discourage some firms with high ε from investing and lead to credit rationing.

[7] Note that the debt is never renegotiated after default. The financial intermediary could agree to reduce the debt to $(1 + \widetilde{r})(k + c - e) - D$, with $0 \leq D \leq (1 + \widetilde{r})(k + c - e) - [Z(\theta + \epsilon)k^{\alpha} + (1 - \delta)k]$. This would leave the firm with end-of-period net worth $q = -D$. However, the renegotiation is never mutually profitable. Since there are no additional cost related to default, the firm always prefers to default and start the next period with zero net worth ($q = 0$) than to renegotiate the debt and have a negative net worth ($q = -D$).

costs when the firm defaults ($\epsilon < \bar{\epsilon}$). Using the default condition (1), we can rewrite the participation constraint of the financial intermediary as:

$$Z[\theta + G(\bar{\epsilon})]k^{\alpha} + (1 - \delta)k - \mu k^{\alpha}\Phi(\bar{\epsilon}) \geq (1 + r)(k + c - e),$$

with

$$G(\bar{\epsilon}) \equiv [1 - \Phi(\bar{\epsilon})]\bar{\epsilon} + \int_{-\infty}^{\bar{\epsilon}} \epsilon d\Phi(\epsilon).$$

As it is more convenient to write the problem of the firm as a function of the default threshold $\bar{\epsilon}$, we characterise the financial contract by the couple $(k, \bar{\epsilon})$ and then derive the implied interest rate $\widetilde{r}$ charged by the financial intermediary from the default condition. Given Z, θ and e, the participation constraint indicates the default threshold $\bar{\epsilon}$ required by the financial intermediary to lend a given amount. The problem is well defined if the firm incurs a higher default rate when borrowing a larger amount. Assumption 2 gives the regularity condition on the distribution Φ that ensures a positive correlation between the amount the firm can borrow and the default threshold $\bar{\epsilon}$.[8] In the end, a higher level of net worth relaxes the financial intermediary's participation constraint and reduces the firm's borrowing costs.

ASSUMPTION 2. *The distribution function of the transitory shock is such that* $\dfrac{\Phi'(\epsilon)}{1 - \Phi(\epsilon)}$ *is monotone in* ε *and* $\lim\limits_{\epsilon \to -\infty} \Phi'(\epsilon) < Z/\mu$.

For some firms, their net worth is too low for the participation constraint of the financial intermediary to be satisfied. In fact, given θ and Z, there is a unique threshold $\underline{e}_b(\theta, Z)$ below which the financial intermediary refuses to lend any fund.[9] This threshold is defined as:

$$Z[\theta + G(\bar{\epsilon}_b)]k_b^{\alpha} + (1 - \delta)k_b - \mu k_b^{\alpha}\Phi(\bar{\epsilon}_b) = (1 + r)(k_b + c - \underline{e}_b), \tag{2}$$

where $(\bar{\epsilon}_b, k_b)$ maximise the expected income of the financial intermediary. When the net worth of the firm is below $\underline{e}_b(\theta, Z)$, the financial intermediary would rather invest in the safe asset than lend to the firm. Note that this minimum level of net worth does not bind for firms for which $\underline{e}_b(\theta, Z) \leq 0$ as the option to default bounds the net worth at zero. When the net worth of the firm is below $\underline{e}_b(\theta, Z)$, the net worth of the firm is not sufficiently high to cover the fixed cost of production and the firm is therefore forced to exit the market.[10]

After production, the firm's end-of-period net worth is equal to:

$$q = \begin{cases} Z(\theta + \epsilon)k^{\alpha} + (1 - \delta)k - (1 + \tilde{r})(c + k - e) & \text{if } \epsilon > \bar{\epsilon} \\ 0 & \text{if } \epsilon \leq \bar{\epsilon}. \end{cases}$$

Using again the definition of the default threshold (1), the end-of-period net worth reads:

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[8] This condition will be necessary to prove the continuity of the value function. It implies that the income of the financial intermediary is either increasing in $\bar{\epsilon}$, or is an inverted U-shaped curve.

[9] The characterisation of the financial intermediary threshold $\underline{e}_b(\theta, Z)$ is reported in Appendix A.1.

[10] See Appendix A.2 (Proposition 1).

$$q = \max[Zk^{\alpha}(\epsilon - \bar{\epsilon}); 0].$$

1.3.1. *The firm's problem*

Define V as the value of the continuing firm at the beginning of the period, before choosing its level of capital. The value of the firm depends on the outcome of its investment and on its exit decision. At the end of the period, the firm learns its next period productivity θ' and, depending on its end-of-period net worth, decides which fraction of its profit to distribute as dividends and whether to stay or exit the market. When its end-of-period net worth is too low $q < \underline{e}_b(\theta', Z')$, the participation constraint of the financial intermediary is not satisfied. As explained in the previous Section, in that case the firm cannot finance the fixed cost of production and must therefore exit the market. When $q \geq \underline{e}_b(\theta', Z')$, the firm decides whether to stay in the market or exit by comparing the value from producing with the outside opportunity. As the discount rate is higher than the safe asset return r, the firm always prefers to distribute its end-of-period net worth as dividends rather than invest it in the safe asset. The firm therefore exits when its continuing value is lower than its end-of-period net worth q. We prove in Appendix A.3 that the value function of the firm exists and is unique. The problem of the firm reads:

$$V(e, \theta, Z) = \max_{(k, \bar{\epsilon})} \mathbb{E}\left(\int I(q)q + [1 - I(q)] \max\{q, \max_{e'}[q - e' + \beta V(e', \theta', Z')]\} d\Phi(\epsilon) \right)$$

with:

$$I(q) = \begin{cases} 0 & \text{if } q \geq \underline{e}_b(\theta', Z') \\ 1 & \text{if } q < \underline{e}_b(\theta', Z') \end{cases}$$

subject to:

$$Z[\theta + G(\bar{\epsilon})]k^{\alpha} + (1 - \delta)k - \mu k^{\alpha} \Phi(\bar{\epsilon}) \geq (1 + r)(k + c - e), \tag{3}$$

$$q = \max[Zk^{\alpha}(\epsilon - \bar{\epsilon}); 0], \tag{4}$$

$$\underline{e}_b(\theta', Z') \leq e' \leq q. \tag{5}$$

The firm maximises its expected dividends subject to the participation constraint of the financial intermediary defined by (3). Equation (4) describes the end-of-period net worth q, while (5) imposes that the firm's net worth has to be sufficiently high for the participation constraint of the financial intermediary to be satisfied. Also, the firm cannot issue new shares and can then increase its net worth only by reinvesting its profits.[11] The firm faces a trade-off when deciding its level of capital. On the one hand, if the firm is solvent, a higher level of capital increases its next period level of production. On the other hand, it increases its probability to default as the default threshold required by the financial intermediary increases with the amount borrowed.

We assume that the value function is differentiable. This allows us to derive analytical results on the firm's exit decision. It also permits to characterise the dividend decision.

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[11] Allowing $e' > q$ makes the financial constraints irrelevant as firms would finance all their investment with equity.

Because the discount rate is higher than the risk-free rate (Assumption 1), the firm will not always choose to reinvest all its profits. It will distribute dividends if its end-of-period net worth is above the dividend threshold $\bar{e}(\theta, Z)$ defined by $\beta[\partial V(\bar{e}, \theta, Z)/\partial e] = 1$.

1.3.2. Exit thresholds

By contrast with the frictionless economy, productivity is not the only determinant of the firms' exit decision. In the presence of credit frictions, firms exit if they are not sufficiently productive $[\theta < \underline{\theta}(Z)]$ but they may also exit if their net worth is too low. The level of net worth determines whether or not firms can borrow and the rate at which they can borrow. Net worth therefore affects the firms' profitability and hence their exit decision. In fact, firms may exit because their level of net worth is not high enough either for their participation constraint $[q < \underline{e}_f(\theta, Z)]$ or for the participation constraint of the financial intermediary to be satisfied $[q < \underline{e}_b(\theta, Z)]$.[12]

As both net worth and productivity matter for the exit decision of firms, we represent the exit thresholds in the (θ, q) plane in Figure 2.[13] All firms with a couple (θ, q) below the downward sloping frontier (in the hatched area) exit the market, whereas all firms
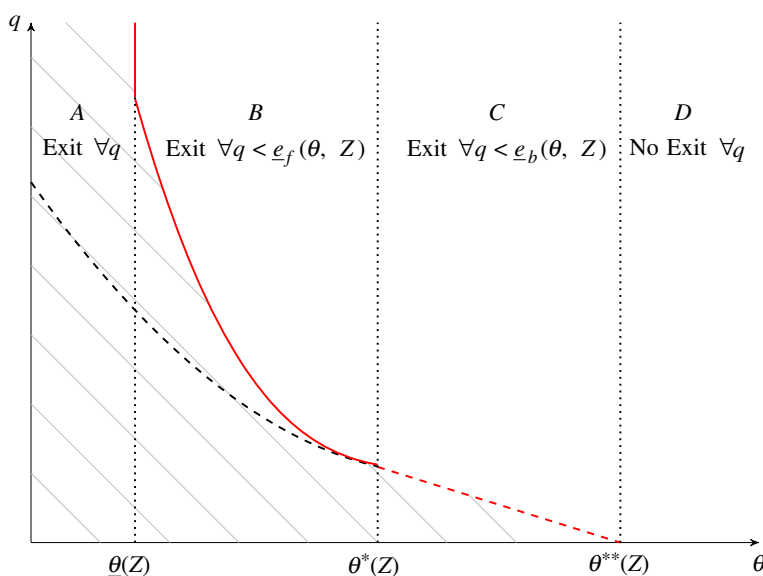

Fig. 2. *Exit Thresholds*

Notes. The solid line reports the firm's participation threshold $\underline{e}_f(\theta, Z)$, and the dashed line reports the financial intermediary's participation threshold $\underline{e}_b(\theta, Z)$.

[12] The bank threshold $\underline{e}_b$ is defined in (2) and the firm threshold $\underline{e}_f$ is defined by $\beta V(\underline{e}_f, \theta, Z) = \underline{e}_f$. The exit thresholds can also be defined in terms of the transitory shock ε, with the transitory shock thresholds equal to $\underline{\epsilon}_f = \underline{e}_f/k^\alpha + \bar{\epsilon}$ and $\underline{\epsilon}_b = \underline{e}_b/k^\alpha + \bar{\epsilon}$. Note that the transitory shock ε does not have a direct effect on the firm's exit decision: because ε is i.i.d., its current value gives no information on the firm's expected profits and therefore does not intervene in the firm's exit decision. The transitory shock ε only plays an indirect role via its impact on the firm's end-of-period net worth q.

[13] The formal characterisation of the exit thresholds is reported in Appendix A.4.

with a couple (θ, q) above the exit frontier are profitable and stay in the market. In region A $[\theta < \underline{\theta}(Z)]$, firms are not sufficiently productive and exit whatever their level of net worth. In region B $[\underline{\theta}(Z) \leq \theta < \theta^*(Z)]$, firms exit when their participation constraint is not satisfied. A low level of net worth raises the borrowing costs of the firm, which may then not be sufficiently profitable to stay in the market. Firms with a higher level of productivity always find it profitable to stay in the market. However, in region C $[\theta^*(Z) \leq \theta < \theta^{**}(Z)]$, firms can be forced to exit the market when their net worth is too low for the participation constraint of the financial intermediary to be satisfied. High productivity firms $\theta \geq \theta^{**}(Z)$ (region D) are not required to have a minimum level of net worth, they therefore never exit because of an insufficient level of net worth.

As illustrated in Figure 2, the exit thresholds $\underline{e}_b(\theta, Z)$ and $\underline{e}_f(\theta, Z)$ are both decreasing functions of the persistent component of productivity θ.[14] This implies that low productivity firms have a higher probability of exiting the market.[15] Firm productivity is therefore an important determinant for exit decision even in the presence of credit market frictions.

1.3.3. *Firm Distribution, Entry, Exit and Average Productivity*

We assume that the mass of potential entrants is constant. Despite this assumption, the actual number of entrants is endogenous as firms enter the market only when their expected profits are sufficiently high. The net worth e and productivity θ of potential entrants are characterised by the joint distribution v. The distribution v of potential entrants, the distributions Φ and F of the productivity shocks, together with the firms' decision rules on capital, default, dividends and exit generate an endogenous joint distribution of productivity and net worth ξ. More specifically, these conditions give rise to a mapping Ω that indicates the next period joint distribution of net worth and productivity given the current distribution and the current and next values of the aggregate shock: $\xi' = \Omega(\xi, Z, Z')$.[16] The stationary joint distribution is the fixed point of the mapping $\xi^* = \Omega(\xi^*, Z, Z)$. We can now use the joint distribution of firms to write the average firm-level productivity in the economy:

$$\int_e \int_\theta \theta \, \mathrm{d}\xi(e, \theta). \tag{6}$$

2. Credit Frictions and the Cleansing Effect of Recessions

In this Section, we analyse numerically how credit market frictions affect the response of average productivity to a negative aggregate productivity shock. The numerical simulations allow us to analyse the interaction between the aggregate shock, credit market frictions and exit, as well as to take into account the impact of the endogenous distribution of net worth. Our objective is not to give a precise quantification of the

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[14] See Appendix A.4.
[15] They have a higher probability of having a net worth below the bank threshold $\underline{e}_b(\theta, Z)$ or their participation threshold $\underline{e}_f(\theta, Z)$, as well as of drawing a productivity shock below $\underline{\theta}(Z)$.
[16] See Appendix B.2.

cleansing effect of recessions but to provide a clear analysis of the consequences of credit frictions on the cleansing effect of recessions. We solve the model using value function iteration.[17] We first present the benchmark calibration and describe the firm capital and exit decisions at the steady state. Then, we analyse the change in average productivity after a negative aggregate productivity shock.

2.1. Calibration

The model period is one year. We calibrate the parameters on the steady state of the credit frictions economy, with Z normalised to 1.[18] Consistently with the business cycle literature, we set the risk-free rate r to 4%, the discount rate β to 0.956 and the depreciation rate δ to 7%. Following the estimates of Hennessy and Whited (2007), the returns to the scale parameter of the production function α is set to 0.7. We assume that firms' persistent productivity follows an AR(1) process:

$$\ln \theta' = \rho_\theta \ln \theta + (1 - \rho_\theta)\eta_\theta + \varepsilon_\theta, \text{with } \varepsilon_\theta \sim \mathcal{N}(0, \sigma_\theta).$$

We approximate this process with a Markov chain over 50 grid points in $[\theta_{\min}, \theta_{\max}]$ using the same method as Tauchen (1986), amended to allow for more grid points in the middle of the distribution.[19] We normalise the mean of θ to 0.3, which implies $\eta_\theta = \ln(0.3) - 0.5 \sigma_\theta^2/(1 - \rho_\theta^2)$ and set the autocorrelation coefficient ρ_θ to 0.9. We then calibrate σ_θ to obtain an *ex post* interquartile ratio of 1.35. This value is line with Del Gatto *et al.* (2008), whose estimates of the intra-industry shape parameter of total factor productivity in the Italian economy imply a interquartile ratio between 1.3 and 1.44, as well as with the estimates of Syverson (2004), who reports an average interquartile ratio that ranges between 1.3 and 1.6 in the US manufacturing sector. We assume that the transitory shock ε is drawn from a normal distribution with mean zero and standard deviation σ:

$$\epsilon \sim \mathcal{N}(0, \sigma).$$

We make this process discrete over 10 grid points following Tauchen's (1986) method. We set the standard deviation to $\sigma = 0.3$ to match a default probability of 1%, in line with Carlstrom and Fuerst (1997).[20] We set the monitoring costs μ to match an average bankruptcy cost equal to 10% of the capital. This value is meant to include not just direct costs such as administrative and legal fees but also indirect costs of bankruptcy linked to the efficiency of debt enforcement. Andrade and Kaplan (1998) estimate these costs to be between 10% and 20% of the value of the firm's capital.

The remaining parameters are chosen to match firm dynamics statistics in the US economy computed from Business Employment Dynamics (BLS) data over the period

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[17] For a detailed description of the numerical method, see Appendix B.

[18] Note that though the targets are computed on the steady state with $Z = 1$, the firms' decision rules take into account the stochastic process for Z.

[19] It is crucial to use a high enough number of grid points for aggregate shocks to affect the exit rate. To limit the computation length, we modify Tauchen's procedure to increase the number of grid points in the middle of the distribution.

[20] They follow Fisher (1999), who finds a default probability of 1% using Dun and Bradstreet data over 1984–94.

Table 1

Benchmark Calibration

Parameter	Symbol	Value
Discount factor	β	0.956
Risk-free rate	r	0.04
Depreciation rate	δ	0.07
Returns to scale	α	0.70
Aggregate productivity	Z	1
Persistent productivity, mean	η_θ	-1.2630
Persistent productivity, volatility	σ_θ	0.1498
Persistent productivity, persistence	ρ_θ	0.9
Fixed cost	c	0.7
Idiosyncratic volatility	σ	0.3
Monitoring cost	μ	0.25
Entrants net worth upper bound	$\overline{e}_{\text{entry}}$	9.7

1994–2012. The fixed cost c is set to match the average establishment exit rate of 11.6%.[21] We assume that entering firms draw their level of productivity and their level of net worth independently. Their productivity is drawn from the stationary exogenous distribution of θ. Their level of net worth is drawn from a uniform distribution over $[0, \overline{e}_{\text{entry}}]$, where the upper bound is set at the lowest dividend threshold $\overline{e}_{\text{entry}} = \min_\theta \overline{e}(\theta, Z)$. Assuming a correlation of zero between the potential entrants productivity and net worth allows us to provide a clear analysis of the endogenous mechanisms of the model. Section 3 discusses the implications of alternative assumptions on the distribution of the entrants' net worth.

We report the set of values of the benchmark calibration in Table 1. The model interquartile ratio is equal to 1.37, the monitoring cost is 10.6%, the steady-state exit rate is 11.4% and the default rate 0.92%.

2.2. *Credit Frictions and Steady-state Exit Decisions*

Before studying the response of the firms' exit decisions to aggregate shocks, let us first analyse how credit frictions shape the firms' exit and capital decisions at the steady state. In the presence of credit frictions, the net worth becomes an important determinant of the firms' decisions. As illustrated in the left panel of Figure 3 for a firm with the median level of productivity, the exit probability declines with net worth. A firm with a high initial net worth is less likely to fall below the net worth exit threshold since its net worth after production is likely to be high as well.[22] The right panel of Figure 3 shows that net worth also affects the level of capital chosen by the firm. A higher net worth relaxes the firm's credit constraint and thereby allows the firm to

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[21] This establishment exit rate is somewhat higher than values reported for the firm exit rate in previous studies. Dunne *et al.* (1989) report a five-year exit rate of 36% in the US manufacturing sector, which induces a 7.2% annual exit rate, assuming that the number of firms remains constant during these five years.

[22] For a given level of capital, the end of period net worth q is an increasing function of initial net worth e: $q = (1 + \widetilde{r})e + Z(\theta + \varepsilon)k^\alpha - (\delta + \widetilde{r})k - (1 + \widetilde{r})c$. Firms with a high initial net worth are more likely to have a high end-of-period net worth also because a high initial net worth allows them to increase their level of capital and expand their production scale.
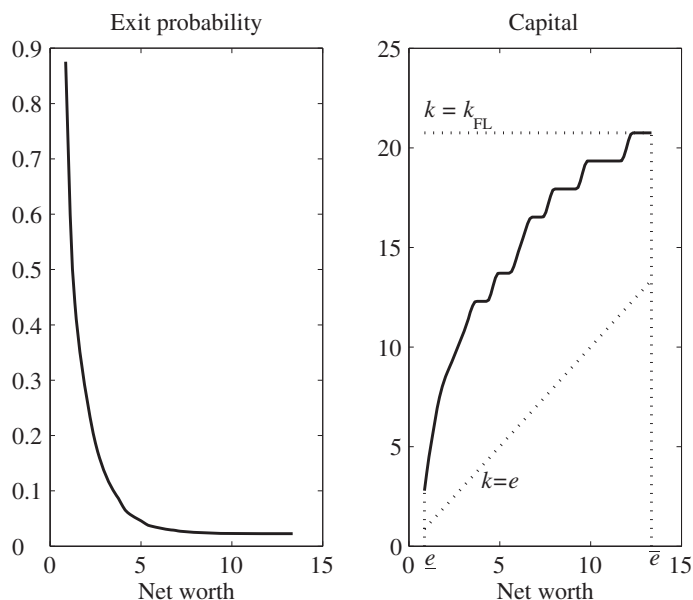
Fig. 3. *Exit Probability and Capital as a Function of Net Worth*
Note. The Figure displays the exit probability and capital as a function of net worth for a firm at the median level of productivity.

increase its level of capital. Firms with a sufficiently high level of net worth can actually invest as much as in the frictionless economy.

The left panel of Figure 4 displays the endogenous distribution of net worth implied by the firms' decision rules on capital, dividends and exit. The Figure shows that high productivity firms have a larger fraction of firms at high levels of net worth, consistently with the fact that those firms accumulate net worth faster and have larger financing needs. But high productivity firms are also profitable at lower levels of net worth than low productivity firms and may then have a larger fraction of firms at low levels of net worth as well. As illustrated in the right panel of Figure 4, the positive effect of productivity on net worth prevails and average net worth increases with productivity (after first slightly declining). Note that the positive correlation between net worth and productivity does not imply that high productivity firms are less constrained than low productivity firms. On the contrary, we find that credit frictions have more impact on the level of capital of high productivity firms. As shown in the right panel of Figure 5, the average capital is lower than in the frictionless economy, particularly so for high productivity firms. While high productivity firms face less stringent credit constraints for a given level of capital, they have a larger optimal scale of production and therefore larger financial needs which make them more sensitive to credit frictions. Figure 5 also illustrates how the exit probability varies with productivity. As shown in the left panel, the exit probability declines with the firm's productivity. As in the frictionless economy, high productivity firms have a lower probability of drawing a productivity level below $\underline{\theta}(Z)$ but they also have a lower probability of falling below the net worth exit threshold. They have a lower probability of falling below the net worth exit threshold
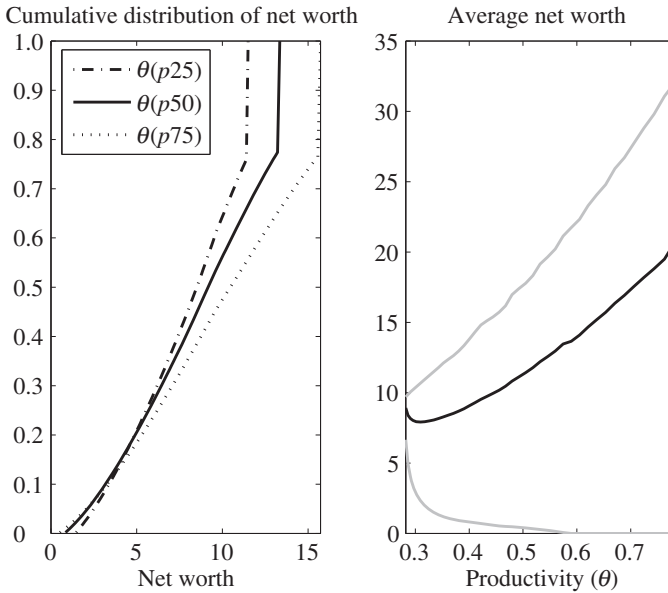
Cumulative distribution of net worth Average net worth

Fig. 4. *Net Worth Distribution*

Notes. The left panel reports the cumulative distribution of initial net worth conditional on productivity for three levels of productivity: the first quartile $\theta(p25)$, the median $\theta(p50)$ and the third quartile $\theta(p75)$. The right panel reports the average net worth (dark line) as well as the exit and dividend thresholds (grey lines).
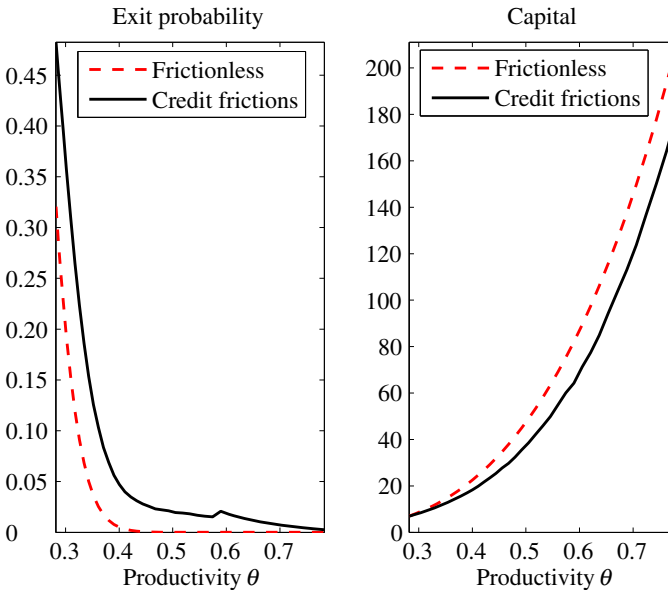
Exit probability Capital

Fig. 5. *Exit Probability and Average Capital as a Function of Productivity*

because the net worth exit thresholds $\underline{e}_b(\theta, Z)$ or $\underline{e}_f(\theta, Z)$ are both declining in θ.[23] In addition, high productivity firms are less likely to fall below the net worth exit threshold as they accumulate net worth faster than low productivity firms. As a result, credit frictions raise the exit rate of low productivity firms more than that of high productivity firms. In contrast with Barlevy's (2003) model in which the most efficient firms are also the most financially vulnerable and the most likely to exit the market, credit frictions do not reverse the relation between productivity and exit.

Since productivity remains an important determinant of firms exit, one may wonder to which extent credit frictions affect the firms' exit decision. We analyse how credit frictions affect the selection of exiting firms by computing the productivity distribution of exiting firms and comparing it to the productivity distribution of continuing firms in both credit frictions and frictionless economies. Figure 6 shows that credit frictions substantially modify the selection of exiting firms. As access to credit becomes an additional determinant of profitability, the exit decision no longer solely depends on productivity and some high productivity firms may be forced to exit. In the credit frictions economy, exiting firms are not the least productive firms. In fact, about 28% of exiting firms have a productivity above the productivity threshold and 8% of exiting firms have a productivity above the median productivity of continuing firms. This may suggest that credit frictions would dampen the cleansing
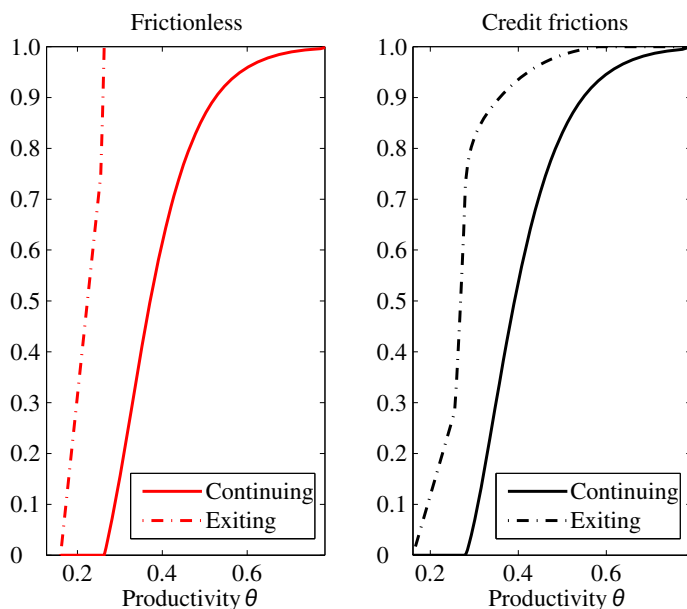

Fig. 6. *Productivity Distribution*
Note. The graph reports the cumulative distribution of productivity for continuing and exiting firms.

[23] See subsection 1.3.2.

effect of recessions by leading to the exit of high productivity firms. In the next Section we study the response of the economy to a negative aggregate productivity shock and show that other factors are at play to explain why credit frictions may dampen the cleansing effect of recessions.

2.3. *The Cleansing Effect of Recessions*

In this Section, we study the response of the economy to a decline in aggregate productivity Z and analyse its implication for average productivity.[24] We assume that Z follows a symmetric Markov chain, and takes two values: 1 and 0.97. As Lee and Mukoyama (2015), we calibrate the transition probabilities such that the average duration of each state is three years, in line with the average length of contractions and expansions reported by the NBER. We suppose that the economy is at the steady state, where aggregate productivity $Z = 1$, and analyse the impact of a one-year decline in aggregate productivity.[25]

Figure 7 displays the impact of the fall in aggregate productivity on the exit and entry rates and on average firm-level productivity in both credit frictions and frictionless economies. As in the frictionless economy, average productivity rises after the negative aggregate shock. On impact, the net exit rate in the credit frictions economy increases by 1.53 percentage points and average productivity increases by 0.43 percentage points. In the frictionless economy, the fall in aggregate productivity raises the net exit rate by 1.44 and average productivity by 0.48 percentage points.

In order to better understand why average productivity rises, we now turn to the response of the net exit rate across productivity levels. Figure 8 makes clear why there is a cleansing effect of recessions even in the presence of credit frictions. The Figure shows that, as in the frictionless economy, the aggregate shock mainly raises the exit threshold of low productivity firms and hence reduces the number of firms at the bottom end of the productivity distribution. Low productivity firms experience a higher increase in their exit rates as they are more likely to become insufficiently productive to stay in the market (increase in θ). In the presence of credit frictions, low productivity firms are also more likely to have an insufficient level of net worth to stay in the market. In fact, they tend to have a low net worth – because of the positive correlation between net worth and productivity ($\mathrm{corr}(q, \theta) > 0$) – and their net worth exit threshold is more affected by the aggregate shock (increase in $\underline{e}_f$). Panel (*a*) of Figure 8 indeed shows that the level of net worth above which firms decide to stay in the market rises all the more so for less productive firms. Even in the presence of credit frictions, the aggregate shock disproportionately affects the participation constraint of low productivity firms. The three mechanisms – the increase in θ, the increase in $\underline{e}_f$ and the positive $\mathrm{corr}(q, \theta)$ – all work in the same direction and average productivity unambiguously rises following the negative aggregate shock. It is worth noting that the

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[24] In our numerical simulations, we leave aside the implications of credit market frictions on the risk-free interest rate. We therefore compare the economy with credit frictions with a frictionless economy characterised by the same risk-free rate.

[25] Contrary to standard impulse response functions, we do not capture the role of the persistence of the aggregate shock (though persistence plays an indirect role via the firms' decision rules).
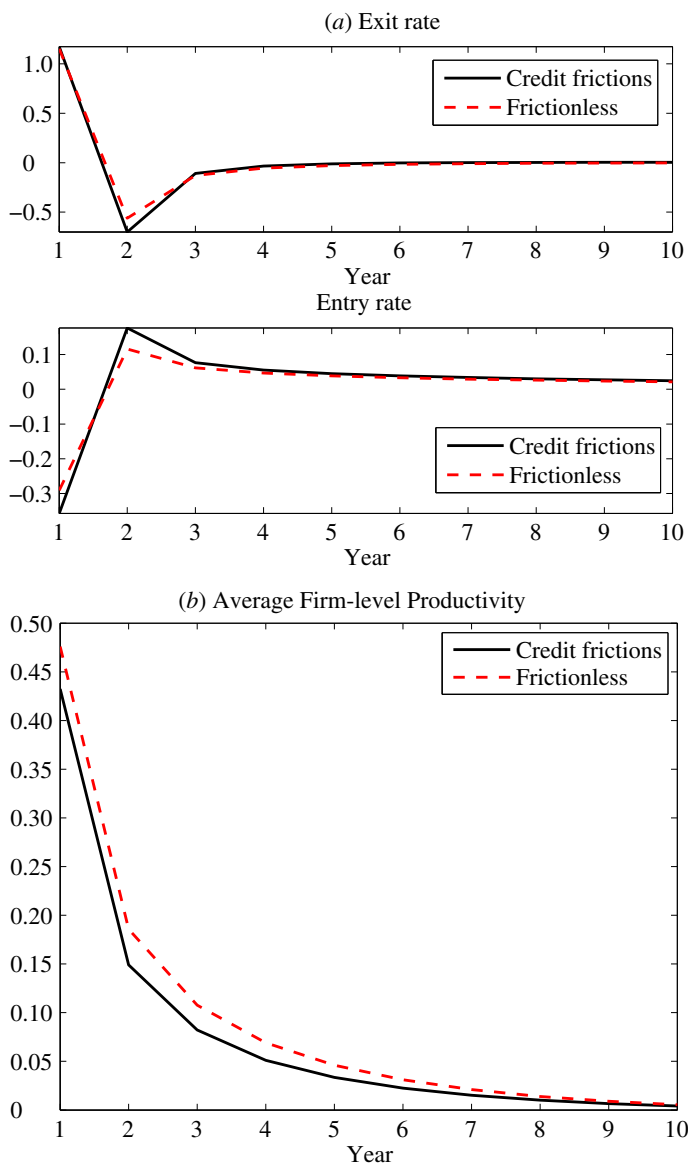
Fig. 7. *The Response to the Aggregate Productivity Shock*
Note. The entry and exit rates and the average firm-level productivity are expressed as percentage points deviations from the steady state.

positive correlation between productivity and net worth is not the only factor behind the increase in average productivity. Even if the firms' end-of-period net worth were not correlated to their productivity, the increase in the productivity threshold and the larger increase in the net worth exit threshold at low productivity levels would still lead to an increase in average productivity.

(a) Exit Threshold

(b) Decline in the Number of Firms
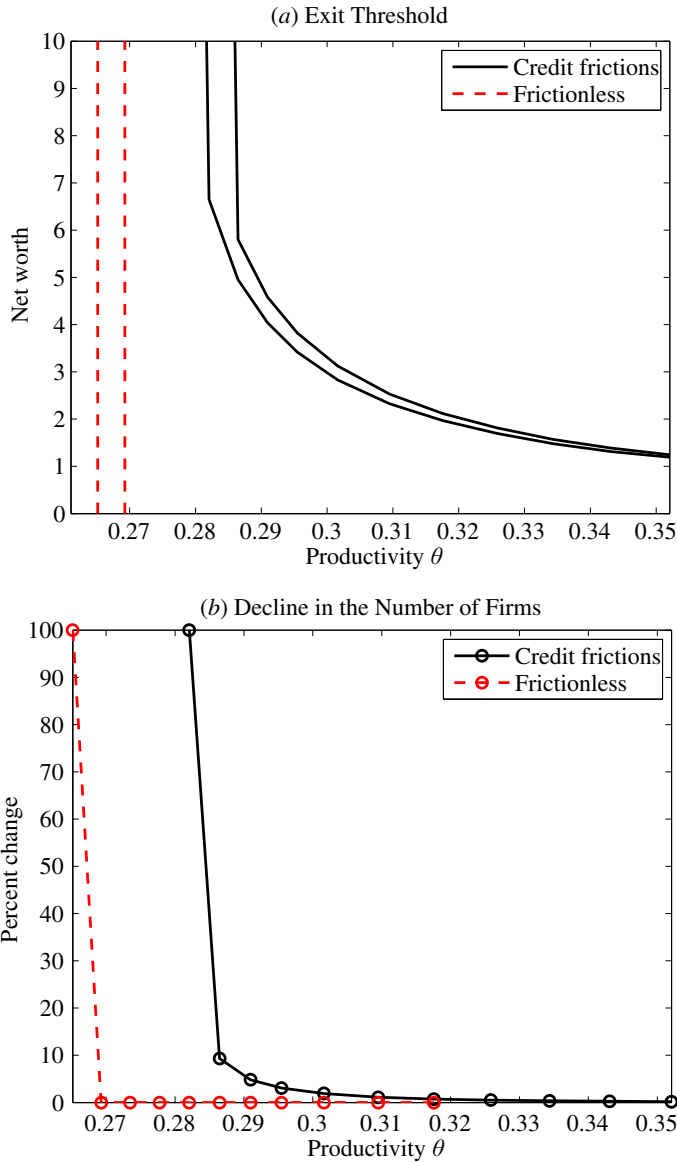
Fig. 8. *The Cleansing Effect of Recessions: Z-shock*
Note. The Figure displays the rightward shift in the exit thresholds and the change in the number of firms following the negative aggregate productivity shock. We restricted the axis to $\theta < 0.35$ for readability.

Figure 8 also sheds lights on why the net exit rate increases somewhat more in the credit frictions economy. While the aggregate shock only affects the least productive firms in the frictionless economy, in the economy with credit frictions the aggregate shock leads to a decline in the number of firms for relatively higher levels of

productivity as well. The negative aggregate shock hence raises the net exit rate of firms across a larger set of productivity levels. As these firms have a below-average productivity, this tends to amplify the increase in average productivity and thus the cleansing effect of recessions. However, credit frictions not only affect the response of the net exit rate to the aggregate shock, they also modify the steady-state productivity distribution. As shown in the previous Section, the steady-state exit rate of low productivity firms is disproportionately higher in the presence of credit frictions and low productivity firms then account for a smaller proportion of firms. The magnitude of the increase in average productivity therefore depends on two offsetting effects. On the one hand, the aggregate shock raises the net exit rate of firms on a wider range of below-average productivity levels and this tends to amplify the increase in the net exit rate and in average productivity. On the other hand, the proportion of low productivity is lower in the presence of credit frictions. As low productivity firms account for the bulk of the increase in the net exit rate, this distribution effect tends to dampen the increase in the net exit rate and in average productivity.

In our simulations, we find that the two effects offset each other and the increase in net exit rate is only slightly larger than in the frictionless economy. Interestingly the larger increase in the net exit rate does not lead to a larger increase in average productivity. Despite the slightly larger increase in the net exit rate (1.53 *versus* 1.44), credit frictions lead to a smaller increase in average productivity (0.43 *versus* 0.48). Credit frictions therefore dampen the intensity of the cleansing effect of recessions, measured by the ratio of the percentage increase in average productivity over the percentage increase in the net exit rate.

3. Discussion

In this Section we investigate in more detail the mechanisms behind the cleansing effect of recessions in an economy with credit frictions. We first analyse how the distributions of net worth and productivity affect the magnitude of the increase in average productivity. We then investigate how the nature of the shock affects the cleansing effect of recessions and consider the response of average productivity to a financial shock.

3.1. *The Determinants of the Cleansing Effect of Recessions*

Do credit frictions always dampen the cleansing effect of recessions? To answer this question we explore the factors that shape the increase in average productivity in the credit frictions economy. The magnitude of the increase in average productivity depends on the proportion of firms affected by the aggregate shock and on the extent to which the shock disproportionately affects low productivity firms. The distributions of net worth and productivity are therefore key to understand the increase in average productivity. We study the impact of credit frictions on the steady-state distribution of productivity and then investigate the role of average net worth and the correlation between net worth and productivity. To illustrate the role of each factor, we simulate the model for alternative distributions of the entrants' net worth. We thereby indirectly modify the incumbents' productivity and net worth distributions.

3.1.1. *The distribution effect*

The results of the benchmark calibration indicate that the increase in average productivity is smaller in the economy with credit frictions than in the frictionless economy. Contrary to the intuition, the smaller increase in average productivity is not due to the fact that firms with higher productivity levels are affected as well. Although the exit rate of some firms above the productivity threshold increases, most of these firms actually have a productivity lower than the average. Hence, the fact that these relatively higher productivity firms are also vulnerable to the aggregate shock tends to amplify the increase in average productivity. The dampening effect comes instead from the impact of credit frictions on the steady-state productivity distribution. As explained in the previous Section, credit frictions reduce the proportion of firms with low productivity and thereby reduce the number of firms affected by the aggregate shock.

To illustrate the role of the distribution effect, we simulate the model with an alternative specification for the entrants' distribution of net worth. In the benchmark specification, potential entrants draw their net worth on $[0, \overline{e}_{\text{entry}}]$ but they actually enter only if their net worth is above the net worth exit threshold $\underline{e}(\theta)$. Since the net worth exit threshold $\underline{e}(\theta)$ declines with productivity, the number of entrants is smaller than in the frictionless economy, particularly so at low productivity levels. Low productivity firms then account for a smaller share of the number of firms. To study the distribution effect, we assume that potential entrants draw their net worth on $[\underline{e}_{\text{entry}}, \overline{e}_{\text{entry}}]$ and vary $\underline{e}_{\text{entry}}$ between 0 and $\underline{e}(\theta)$. The higher is $\underline{e}_{\text{entry}}$, the closer the steady-state productivity distribution is to the frictionless level. When $\underline{e}_{\text{entry}} = \underline{e}(\theta)$, the productivity distribution of actual entrants is identical to that of the frictionless economy.[26] The results, displayed in Table 2, show that the increase in average productivity becomes larger as the productivity distribution gets closer to the

Table 2
The Distribution Effect

Z-shock	Average productivity	Net exit rate	Cleansing intensity
Frictionless	0.48	1.44	0.33
Benchmark	0.43	1.53	0.28
$[0.25 \times \underline{e}(\theta), \overline{e}_{\text{entry}}]$	0.45	1.58	0.28
$[0.5 \times \underline{e}(\theta), \overline{e}_{\text{entry}}]$	0.46	1.64	0.28
$[0.75 \times \underline{e}(\theta), \overline{e}_{\text{entry}}]$	0.49	1.74	0.28
$[\underline{e}(\theta), \overline{e}_{\text{entry}}]$	0.55	1.93	0.28

Notes. This Table gives the percentage change in average productivity, the percentage point change in the exit, entry and net exit rates, and the cleansing intensity after the negative aggregate productivity shock. The cleansing intensity is defined as the ratio of the change in average productivity over the change in the net exit rate. The first line displays the results for the frictionless economy, the other lines are refer to the economy with credit frictions. The first line shows the result of the benchmark calibration (net worth of potential entrants distributed on $[0, \overline{e}_{\text{entry}}]$). The last line gives the result for the case where the productivity distribution of entrants is identical to that of the frictionless economy (net worth of potential entrants distributed on $[\underline{e}(\theta), \overline{e}_{\text{entry}}]$).

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[26] Note that the distribution effect is not fully mitigated in that case as the higher exit rate of low productivity firms still creates a wedge between the credit frictions and the frictionless productivity distributions.

frictionless economy. When the productivity distribution of actual entrants is identical to that of the frictionless economy, average productivity increases by 0.55% – more than in the frictionless economy.

The consequences of credit frictions on the size of the cleansing effect of recessions hence crucially depend on how they affect the steady-state productivity distribution. When the share of low productivity firms is close to the frictionless level, credit frictions raise the proportion of firms vulnerable to the aggregate shock and thereby lead to a larger increase in the net exit rate. The larger increase in the net exit rate in turn leads to a larger increase in average productivity. Depending on how credit frictions modify the steady-state distribution of productivity, the increase in average productivity may then be amplified. However, the intensity of the cleansing effect of recessions is always lower in the presence of credit frictions. The last column of Table 2 shows that for each percentage point increase in the net exit rate, average productivity increases by less than in the frictionless economy (0.28 *versus* 0.33). While credit frictions may amplify the increase in average productivity, they systematically dampen the intensity of the cleansing effect whatever their effects on the productivity distribution.

3.1.2. *The role of average net worth*

The distribution of net worth is likely to be an important determinant of the cleansing effect as it affects the proportion of firms vulnerable to the increase in the net worth exit threshold. Intuitively, a lower average net worth across all productivity levels raises the number of firms affected by the increase in the net worth threshold and would hence amplify the increase in average productivity. Conversely, if average net worth is high, we should expect the increase in the net worth exit threshold to have little effect on average productivity.

To study the role of average net worth, we simulate the model for alternative upper bounds of the net worth distribution of entrants. We raise the upper bound up to 10 times the benchmark value. This in turn leads to higher levels of average net worth in the overall economy. As shown in Table 3, the increase in average productivity becomes larger when average net worth is high. This result, which may seem counterintuitive at first, comes from the effect of the entrants' upper bound on the steady-state productivity distribution. When the upper bound of the net worth distribution of entrants is higher, average net worth increases but the number of low

Table 3

The Role of Average Net Worth

Z-shock	Average productivity	Net exit rate	Average net worth	Cleansing intensity
Benchmark	0.43	1.53	9.71	0.28
$[0, 1.5 \times \bar{e}_{entry}]$	0.46	1.57	10.58	0.29
$[0, 3 \times \bar{e}_{entry}]$	0.49	1.61	11.47	0.30
$[0, 10 \times \bar{e}_{entry}]$	0.50	1.64	12.00	0.31

Notes. Average productivity and net exit rate refer to the percentage change for each variable after the negative aggregate productivity shock. Average net worth refers to the average net worth of all firms at the steady state. The cleansing intensity is defined as the ratio of the change in average productivity over the change in the net exit rate. The second line shows the result of the benchmark calibration (net worth of potential entrants distributed on $[0, \bar{e}_{entry}]$)

productivity firms that actually enters the economy also goes up. Our results indicate that the effect of the upper bound net worth of entrants on the productivity distribution prevails over the reduced sensitivity of firms to the net worth exit threshold. As a result, when average net worth is higher, the increase in average productivity becomes larger. The results also indicate that though the intensity of the cleansing effect increases with average net worth, it remains lower than the frictionless level.

The last row of Table 3 corresponds to the case where the productivity distribution of actual entrants is equal to the frictionless distribution. The increase in average productivity is however smaller than the one reported in the last row of Table 2 (0.50 *versus* 0.55). This indicates that for a given steady-state productivity distribution, a higher average net worth leads to a smaller increase in average productivity, as it reduces the number of firms affected by the increase in the net worth threshold. However, since the factors that raise the average net worth are also likely to increase the proportion of low productivity firms, they would instead tend to amplify the increase in average productivity. The distribution effect is therefore crucial to understand the size of the increase in average productivity.

3.1.3. *The correlation between net worth and productivity*

We now turn to the role of the correlation between net worth and productivity. The magnitude of the increase in average productivity depends not only on the proportion of firms affected by the aggregate shock but also on the extent to which the aggregate shock disproportionately raises the exit rate of low productivity firms. A higher correlation between net worth and productivity is likely to reinforce the increase in the net exit rate of low productivity firms and hence amplify the increase in average productivity.

We study the effects of the correlation between net worth and productivity by varying the correlation among entrants. In the benchmark specification, we assume that the correlation between productivity and net worth among potential entrants is equal to zero. We consider here the effects of a positive and a negative correlation. To isolate the effects of the correlation and avoid changes in the steady-state distribution of productivity, we set the entrants' lower bound to $\underline{e}(\theta)$.[27] For simplicity, we assume the net worth upper bound to be a linear function of productivity. Entrants draw their net worth on $[\underline{e}(\theta), \overline{e}_{\text{entry}}(\theta)]$, with $\overline{e}_{\text{entry}}(\theta) = a\theta + b$. We first simulate the model in the case where the upper bound is positively correlated to productivity and then consider the impact of a negative correlation. As changes in the upper bound may also modify the average level of net worth, we set the upper bound such that the average net worth among entrants stays constant.[28] The results, displayed in Table 4, show the effect of the correlation between net worth and productivity on the cleansing effect of recessions. A positive correlation between net worth and productivity among entrants increases the overall correlation in the economy. As expected, we find that the larger the correlation between net worth and productivity, the larger is the increase in

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[27] See subsection 3.1.1.
[28] For the positive correlation $a = 29.72$ and $b = -1.51$, which yields a correlation among entrants of 0.28. For the negative correlation, $a = -21.45$ and $b = 17.79$, which yields a correlation among entrants of -0.52.

Table 4

The Role of the Correlation Between Net Worth and Productivity

Z-shock	Average productivity	Net exit rate	Correlation (q,θ)	Cleansing intensity
Negative correlation	0.54	1.92	0.48	0.28
Zero correlation	0.55	1.93	0.52	0.28
Positive correlation	0.60	2.07	0.56	0.29

Notes. Average productivity and net exit rate refer to the percentage change for each variable after the negative aggregate productivity shock; correlation (q, θ) denotes the correlation between the end-of-period net worth and productivity for all firms. The cleansing intensity is defined as the ratio of the change in average productivity over the change in the net exit rate. Each line refers to respectively a negative, zero and positive correlation of the upper bound of the entrants' net worth $\bar{e}_{entry}$ and productivity. The upper bounds are set such that the average net worth is identical in all three cases and the lower bound is set to $\underline{e}(\theta)$ to mitigate the distribution effect.

average productivity. The results also show that the cleansing intensity remains lower than in the frictionless economy.

Our analysis of the determinants of the cleansing effect of recessions points out two important results. First, the size of the increase in average productivity crucially depends on the net worth and productivity distributions. We show that credit frictions actually amplify the increase in average productivity for some of these distributions. Second, regardless of the distributions of net worth and productivity, credit frictions dampen the intensity of the cleansing effect of recessions. We find that the cleansing intensity remains below the frictionless level across all alternative specifications studied above.

3.2. *Financial Shock*

We have focused so far on the response of the economy to an aggregate productivity shock. Do financial shocks have similar effects on average productivity? We investigate this question by studying the response of average productivity to an exogenous decline in the firms' net worth. The decline in net worth reduces the firms' access to credit and raises the cost of borrowing and is then likely to affect firms differently depending on their level of borrowing. In particular, high productivity firms may be more affected by this shock as they operate on a larger scale and are more dependent on external finance.

The end of period net worth is re-written:

$$q = \max[Z(\epsilon - \bar{\epsilon})k^{\alpha}(1 - \gamma_t), 0],$$

where γ_t is the unexpected exogenous financial shock. To illustrate the effects of the financial shock, we calibrate γ_t to the 34% decline in the equity of non-financial businesses observed during the 2008 recession.[29] We consider a one-time shock and assume that the shock affected both entering and incumbent firms.

[29] Source: Series FL143181105, Financial Accounts, Federal Reserve Board of Governors; peak-to-trough percentage change (2007q3-2009q1).

Table 5

Aggregate Productivity Shock Versus Financial Shock

	Average productivity	Net exit rate	Cleansing intensity
Z-shock	0.43	1.53	0.28
e-shock	0.82	3.53	0.23

Notes. Z-shock refers to the negative aggregate productivity shock, and e-shock to the financial shock. Average productivity and net exit rate refer to the percentage change for each variable after the negative aggregate shocks in the economy with credit frictions. Cleansing intensity is defined as the ratio of the change in average productivity over the change in the net exit rate.

The results, which are displayed in Table 5, indicate that the recession generates a cleansing effect regardless of the nature of the shock. In the aftermath of the financial shock, the net exit rate increases by 3.5 percentage points and average productivity by 0.82% – more than after the aggregate productivity shock. The financial shock thus also predominantly affects low productivity firms. Despite relying more on borrowing, high productivity firms are less vulnerable to the financial shock since they have lower net worth exit thresholds and tend to accumulate more net worth. While the financial shock also leads to an increase in average productivity, the intensity of the cleansing effect is lower: if the recession were driven by an aggregate productivity shock, average productivity would have increased by 1% in the credit frictions economy and by 1.17% in the frictionless economy for a similar increase in the net exit rate.

This result indicates that the type of firms affected by the aggregate shock differs according to the nature of the shock. In particular, a lower cleansing intensity implies that the impact of the financial shock is more uniform across productivity levels and that relatively higher productivity firms are affected by the shock as well. The left panel of Figure 9 illustrates the impact of the decline in net worth in the (θ, q) plane, which is equivalent to a proportional increase in the net worth exit threshold.[30] Compared with the aggregate productivity shock (Figure 8), the decline in net worth affects firms more uniformly across productivity levels and this is reflected in a more uniform decline in the number of firms as well, as illustrated in the right panel of Figure 9. All in all, this result indicates that the nature of the shock is important to understand how resources are reallocated during the recession.

4. Conclusion

How do credit frictions affect the cleansing effect of recessions? This article builds a model of firms dynamics and credit frictions to answer this question. We show that credit frictions modify the selection of exiting and entering firms and may lead to the exit of some high productivity firms. Despite the impact of credit frictions on the selection of firms, average productivity rises when the economy is hit by a negative

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[30] $Z(\epsilon - \bar{\epsilon})k^{\alpha}(1 - \gamma_t) < \underline{e}$ can be rewritten as $Z(\epsilon - \bar{\epsilon})k^{\alpha} < \underline{e}/(1 - \gamma_t)$, and the γ_t shock on the end of period net worth is therefore equivalent to a shift in the net worth exit threshold of $\gamma_t/(1 - \gamma_t)$ percentage points.
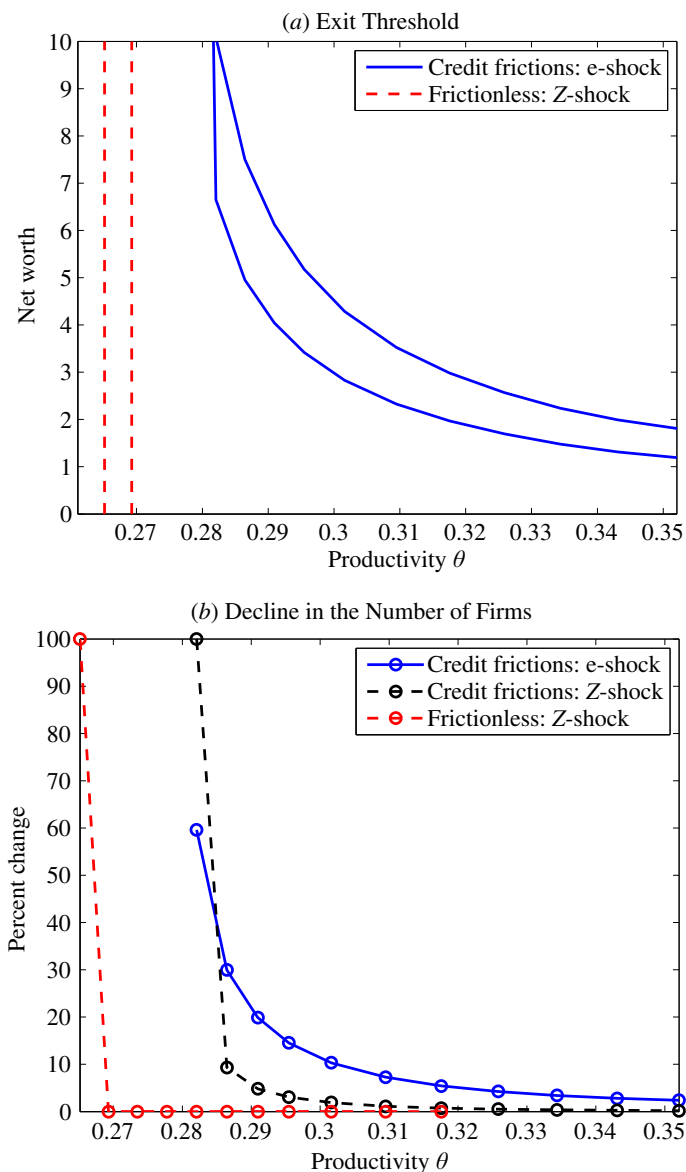
Fig. 9. *The Cleansing Effect of Recessions: e-shock*
Notes. The Figure displays the rightward shift in the exit thresholds and the change in the number of firms following the financial shock (e-shock). We restricted the axis to $\theta < 0.35$ for readability.

aggregate productivity shock. As in the frictionless economy, the negative aggregate productivity shock leads to a higher average productivity because the shock predominantly raises the net exit rate of low productivity firms. We show that the magnitude of the cleansing effect of recessions crucially depends on the steady-state productivity and

net worth distributions and that the increase in average productivity could possibly be larger than in the frictionless economy depending on these distributions. When credit frictions and the productivity distribution of firms are calibrated to plausible levels, we find that credit frictions dampen the increase in average productivity. We then show that credit frictions systematically dampen the intensity of the cleansing effect, that is, the increase in average productivity for a percentage point increase in the net exit rate, whatever the level of credit frictions and the productivity distribution of firms. Finally, we consider how the nature of the shock may affect the cleansing effect of recessions by studying the impact of a financial shock. We find that there is a cleansing effect of recessions even when the economic downturn is driven by a financial shock but the cleansing intensity is lower since the financial shock affects relatively higher productivity firms as well. This result indicates that the nature of the shock matters for the intensity of the cleansing effect of recessions: negative aggregate productivity shocks are more cleansing intensive than financial shocks. In line with the rest of the literature on the cleansing effect of recessions, this article focuses on the impact of entering and exiting firms on average firm-level productivity. While the recession leads to the exit of low productivity firms even in the presence of credit frictions, it is important to note that our results do not imply that recessions improve the efficiency of resource allocation. In fact, exit is inefficient in the credit frictions economy as some of the exiting firms would have survived in the absence of credit frictions. This is particularly striking in the case of a financial shock, as none of the exiting firms would have been scrapped in the frictionless economy.

Appendix A

A.1. *Financial Intermediary Net Worth Threshold* $\underline{e}_b$

Let us define the net income of the financial intermediary as $B(e, k, \bar{\epsilon})$ where:

$$B(e, k, \bar{\epsilon}) = Z[\theta + G(\bar{\epsilon})]k^{\alpha} + (1 - \delta)k - \mu k^{\alpha}\Phi(\bar{\epsilon}) - (1 + r)(k + c - e).$$

The participation constraint of the financial intermediary is not satisfied when the firm's net worth is below $\underline{e}_b(\theta, Z)$ defined by:

$$B(\underline{e}_b, k_b, \bar{\epsilon}_b) = 0,$$

with $(k_b, \bar{\epsilon}_b)$ being the values of capital and default threshold that maximise the income of the financial intermediary. $\bar{\epsilon}_b$ solves:

$$Z[1 - \Phi(\bar{\epsilon}_b)] = \mu\Phi'(\bar{\epsilon}_b),$$

and k_b is given by:

$$k_b = \begin{cases} 0 & \text{if } Z[\theta + G(\bar{\epsilon}_b)] - \mu\Phi(\epsilon) < 0 \\ k_b = \left(\dfrac{\alpha\{Z[G(\bar{\epsilon}_b) + \theta] - \mu\Phi(\bar{\epsilon}_b)\}}{\delta + r} \right)^{\frac{1}{1-\alpha}} & \text{otherwise.} \end{cases}$$

Since the income of the financial intermediary $B(e, k, \bar{\epsilon})$ is strictly increasing in the net worth e, there is a unique net worth threshold $\underline{e}_b$ such that $B(\underline{e}_b, k_b, \bar{\epsilon}_b) = 0$.

A.2. *Exit of Firms Due to Credit Rationing*

PROPOSITION 1. *Firms that are not financed by the financial intermediary cannot cover the fixed cost of production:*

$$\underline{e}_b(\theta, Z) \leq c, \ \forall \theta \in [\theta_{\min}, \theta_{\max}], \ \forall Z.$$

Proof. The firm exits the market when the participation constraint of the financial intermediary is not satisfied, that is when the end-of-period net worth is too low $q < \underline{e}_b(\theta, Z)$. Indeed, we show that $e_b(\theta, Z) \leq c, \ \forall \theta \in [\theta_{\min}, \theta_{\max}]$ and therefore a firm which is rationed from the credit market cannot self finance its fixed operating cost. Recall that $\underline{e}_b(\theta, Z)$ is defined by the following equation:

$$\max_{(k, \bar{\epsilon})} \{ Z[\theta + G(\bar{\epsilon})]k^{\alpha} - (\delta + r)k - \mu k^{\alpha} \Phi(\bar{\epsilon}) \} = (1 + r)(c - \underline{e}_b).$$

Notice that $\max_{(k, \bar{\epsilon})} \{ Z[\theta + G(\bar{\epsilon})]k^{\alpha} - (\delta + r)k - \mu k^{\alpha} \Phi(\bar{\epsilon}) \} \geq 0$ since the financial intermediary can always choose $k = 0$ and have 0. It follows that $c - \underline{e}_b(\theta, Z) \geq 0$.

A.3. *Existence and Uniqueness of the Value Function*

Consider the problem of the firm defined as:

$$T(V)(e, \theta, Z) = \max_{(k, \bar{\epsilon}) \in \Gamma(e)} \mathbb{E} \left(\int I(q)q + [1 - I(q)] \max\{ q, \max_{e' \in \Upsilon(q)} [q - e' + \beta V(e', \theta', Z')] \} d\Phi(\epsilon) \right)$$

with:

$$I(q) = \begin{cases} 0 & \text{if } q \geq \underline{e}_b, \\ 1 & \text{if } q < \underline{e}_b, \end{cases}$$

$$\Gamma(e) = \{ (k, \bar{\epsilon}) \in \mathbb{R}^+ \times \mathbb{R} : Z[\theta + G(\bar{\epsilon})]k^{\alpha} + (1 - \delta)k - \mu k^{\alpha} \Phi(\bar{\epsilon}) \geq (1 + r)(k + c - e) \},$$

$$\Upsilon(q) = \{ e' \in \mathbb{R} : \underline{e}_b \leq e' \leq q \},$$

$$q = \max[Zk^{\alpha}(\epsilon - \bar{\epsilon}); 0].$$

In the following, we prove that there is a unique function V that satisfies the functional equation $V = T(V)$ assuming the firm-level productivity θ and aggregate productivity Z are constant. The proof extends to non-permanent level of $\theta \in [\theta_{\min}, \theta_{\max}]$ and Z.

First note that the participation constraint of the financial intermediary and Assumption 1 ($\beta(1 + r) < 1$) limits the set of net worth for continuing firms to $X = [\underline{e}_b(\theta, Z), \bar{e}(\theta, Z)]$. Because $\beta(1 + r) < 1$, entrepreneurs will not always reinvest their net worth in the firm and will start distributing dividend when their net worth is sufficiently high. In particular, a threshold $\bar{e}(\theta, Z)$ exists above which the firm will stop accumulating net worth.

We then show that the value of the continuing firm $V : X \rightarrow \mathbb{R}^+$ is necessarily bounded. The value of the firm is the discounted sum of the income from production and/or investing in the safe asset. As the decreasing returns to scale technology put an upper bound on the profits of the firm and Assumption 1 limits net worth accumulation, the value of the firm is bounded. This also means that the function resulting from the mapping TV is bounded and T maps the space of bounded functions $B(X)$ into itself. Then, we observe that the operator T is a contraction since it satisfies the Blackwell conditions of monotonicity and discounting. The condition for discounting is verified as $\forall V \in B(X)$,

$$T(V+c) = \max_{(k,\bar{\epsilon})\in\Gamma(e)}\left(\int I(q)q\right.$$

$$+[1-I(q)]\max\{q, \max_{e'\in\Upsilon(q)}[q-e'+\beta V(e',\theta,Z)+\beta c]\}\mathrm{d}\Phi(\epsilon)\Big)$$

$$\leq \max_{(k,\bar{\epsilon})\in\Gamma(e)}\left(\int I(q)q\right.$$

$$+[1-I(q)]\max\{q+\beta c, \max_{e'\in\Upsilon(q)}[q-e'+\beta V(e',\theta,Z)+\beta c]\}\mathrm{d}\Phi(\epsilon)\Big)$$

$$\leq TV + \beta c,$$

where $c > 0$, and $0 < \beta < 1$ by definition.

Since $B(X)$ is a complete metric space (Godement, 2001), the contraction mapping theorem applies and the operator T has a unique fixed point V, which is bounded.

Let us show that V is a continuous function. The participation constraint of the bank generates a discontinuity in the firm's end-of-period outcome. For $q < \underline{e}_b$, the value of the firm is simply q and if $q = \underline{e}_b$, the firm can invest in production and obtains the value $\beta V(\underline{e}_b)$ which has no reason to coincide with q. However, we can show that, though the end-of period value of the firm is discontinuous, the expectation of this value is a continuous function of the threshold $[\underline{e}_b/(Zk^{\alpha})]+\bar{\epsilon}$ below which the firm cannot borrow from the financial intermediary. This appears clearly when rewriting the value function as follows:

$$V(e,\theta,Z) = \max_{(k,\bar{\epsilon})\in\Gamma(e)}\left(\int_{-\infty}^{\underline{e}_b/(Zk^{\alpha})+\bar{\epsilon}} q\mathrm{d}\Phi(\epsilon) + \int_{\underline{e}_b/(Zk^{\alpha})+\epsilon}^{+\infty} \max\{q, \max_{e'\in\Upsilon(q)}[q-e'+\beta V(e',\theta,Z)]\}\mathrm{d}\Phi(\epsilon)\right).$$

Despite the discontinuity at $\underline{e}_b$ in the end-of-period value of the firm, the discontinuity disappears in the objective function of the continuing firm. Furthermore, note that if V is a continuous function, then the objective function of the firm deciding its next period net worth e' is also continuous. Because the correspondence $\Upsilon(q)$ that describes the feasibility constraint for e' is non-empty, continuous and compact-valued, the theorem of the maximum ensures that the maximum exists and the function resulting from this dividend choice is continuous. As the threshold $[\underline{e}_b/(Zk^{\alpha})]+\bar{\epsilon}$ below which the firm cannot borrow from the financial intermediary is a continuous function of e, the objective function of the firm deciding its capital level is the sum of two continuous functions and is therefore a continuous function of e. Using again the theorem of the maximum, we can finally show that the function resulting from the mapping $T(V)(e)$ is continuous since the correspondence Γ that describe the feasibility constraint for k and $\bar{\epsilon}$ is non-empty, continuous and compact-valued.[31] This means that T maps the space of continuous and bounded functions into itself, $T : C(X) \to C(X)$. As $C(X)$ is a closed subset of the complete metric space of bounded functions $B(X)$, the fixed point V is a continuous function by the corollary of the contraction mapping theorem.[32]

Let us now characterise more precisely the value function V. Notice that Υ and Γ are increasing correspondences: $q_1 \leq q_2$ implies $\Upsilon(q_1) \subseteq \Upsilon(q_2)$ and $e_1 \leq e_2$ implies $\Gamma(e_1) \subseteq \Gamma(e_2)$. A higher net worth e relaxes the credit constraint of the firm and allows the firm to reach a higher end-of-period net worth q. This means that the period return function q is strictly increasing in e. This implies that T maps the space of bounded continuous and strictly increasing functions into itself. As this is a closed subset of the space of bounded functions B

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[31] The continuity of Γ derives from the continuity of the participation constraint of the bank $B(e, k, \bar{\epsilon})$ and from the uniqueness of $\bar{\epsilon}_b$ guaranteed by Assumption 2.

[32] See Corollary 1 of Theorem 3.2 in Stokey *et al.* (1989).

(X), V is a strictly increasing function of e. By the same reasoning, we can show that V is also strictly increasing in θ.[33]

To further characterise the value function, let us write the Lagrangian of the firm's problem:

$$\mathcal{L}(k, \bar{\epsilon}, \lambda) = \int_{-\infty}^{\frac{e_b}{Zk^\alpha} + \bar{\epsilon}} [Zk^\alpha(\epsilon - \bar{\epsilon})] d\Phi(\epsilon)$$
$$+ \int_{\frac{e_b}{Zk^\alpha} + \bar{\epsilon}}^{+\infty} \max\{Zk^\alpha(\epsilon - \bar{\epsilon}), \max_{e' \in \Upsilon(q)} Zk^\alpha(\epsilon - \bar{\epsilon}) - e' + \beta V(e', \theta, Z)\} d\Phi(\epsilon)$$
$$+ \lambda g(k, \epsilon, e),$$

with $g(k, \bar{\epsilon}, e) = Z[\theta + G(\bar{\epsilon})]k^\alpha + (1 - \delta)k - \mu k^\alpha \Phi(\bar{\epsilon}) - (1 + r)(k + c - e)$.

In the following, we assume that V is differentiable. We can then compute its second-order derivative to show that V is a concave function. Using the envelop theorem, we can write:

$$\frac{\partial^2 V}{\partial e^2} = \frac{\partial \lambda}{\partial e}(1 + r).$$

We then take the total differential of the financial intermediary's participation constraint with respect to λ and e:

$$\frac{d\lambda}{de} = - \frac{1 + r}{\frac{\partial}{\partial \lambda} g[k(\lambda), \epsilon(\lambda), e]}.$$

Applying the implicit function theorem to the first-order optimality conditions, we can write:

$$\frac{\partial}{\partial \lambda} g[k(\lambda), \bar{\epsilon}(\lambda), e] = - \begin{pmatrix} \frac{\partial g}{\partial \bar{\epsilon}} \\ -\frac{\partial g}{\partial k} \end{pmatrix}' \boldsymbol{H} \begin{pmatrix} \frac{\partial \boldsymbol{g}}{\partial \epsilon} \\ -\frac{\partial \boldsymbol{g}}{\partial \boldsymbol{k}} \end{pmatrix},$$

where $\boldsymbol{H}$ is the hessian matrix of the Lagrangian function.[34] As long as the second-order optimality conditions of the firm problem are satisfied, $\boldsymbol{H}$ is negative definite. Then $\frac{\partial}{\partial \lambda} g[k(\lambda), \bar{\epsilon}(\lambda), e] > 0$ and V is a concave function of e.

The concavity of the firm value function also allows us to characterise the dividend decision of the firm. There exists a unique threshold $\bar{e}(\theta, Z)$ above which the firm decides to distribute some dividends. This optimal threshold is given by:

$$\beta \frac{\partial V}{\partial e}(\bar{e}, \theta, Z) = 1. \tag{A.1}$$

Therefore, the optimal next period net worth is $e' = \min[q, \bar{e}(\theta, Z)]$.

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[33] In the case of stochastic productivity θ, we further assume that the transition function $F(\theta'|\theta)$ is strictly decreasing in θ.

[34] $\boldsymbol{H} = \begin{pmatrix} \frac{\partial \mathcal{L}^2}{\partial \bar{\epsilon}^2} & \frac{\partial \mathcal{L}^2}{\partial \bar{\epsilon} \partial k} \\ \frac{\partial \mathcal{L}^2}{\partial \bar{\epsilon} \partial k} & \frac{\partial \mathcal{L}^2}{\partial k^2} \end{pmatrix}.$

A.4. *Exit Thresholds*

For a given level of aggregate productivity Z, there exist three thresholds $\underline{\theta}(Z) < \theta^*(Z) < \theta^{**}(Z)$ that characterise the exit decision of the firm. These productivity thresholds delimit four exit regions:[35]

- (*i*) the firm exits when $\theta < \underline{\theta}(Z)$ whatever its level of net worth;
- (*ii*) the firm exits when $\underline{\theta}(Z) \leq \theta < \theta^*(Z)$ if its end-of-period net worth is too low for its participation constraint to be satisfied: $q < \underline{e}_f(\theta, Z)$, where $\underline{e}_f(\theta, Z)$ is defined by $\underline{e}_f = \beta V(\underline{e}_f, \theta, Z)$;
- (*iii*) the firm exits when $\theta^*(Z) \leq \theta < \theta^{**}(Z)$ if its end-of-period net worth is too low for the participation constraint of the financial intermediary to be satisfied: $q < \underline{e}_b(\theta, Z)$ where $\underline{e}_b(\theta, Z)$ is defined by (2); and
- (*iv*) the firm never exits when $\theta \geq \theta^{**}(Z)$ whatever its level of net worth.

PROPOSITION 2. *The exit thresholds $\underline{e}_b(\theta, Z)$ and $\underline{e}_f(\theta, Z)$ are both decreasing functions of the persistent component of productivity θ.*

Proof. Under Assumptions 1 and 2, let us first show that for a given Z, the thresholds $\underline{\theta}(Z), \theta^*(Z)$ and $\theta^{**}(Z)$ exist and are unique.

- No exit threshold θ^{**}: We have already shown that firms exit the market when the participation constraint of the financial intermediary is not satisfied. However, for some high productivity firms the participation constraint of the financial intermediary is always satisfied. The financial intermediary accepts to lend to firms with $\theta \geq \theta^{**}$ whatever their level of net worth. Therefore, these firms never exit because they are not sufficiently creditworthy. The no exit threshold θ^{**} is characterised by:

$$Z[\theta^{**} + G(\bar{\epsilon}_b)]k_b^\alpha - \mu k_b^\alpha \Phi(\bar{\epsilon}_b) - (\delta + r)k_b - (1+r)c = 0. \tag{A.2}$$

  Let us show that this threshold is unique. Denote $\hat{\theta} = (\mu/Z)\Phi(\bar{\epsilon}_b) - G(\bar{\epsilon}_b)$, the level of productivity below which the net income of the financial intermediary is decreasing in k. The left hand side of the no exit threshold condition is strictly increasing in θ for all $\theta > \hat{\theta}$. Furthermore, as this expression is negative for $\theta = \hat{\theta}$, this implies that the threshold θ^{**} is unique.
- Credit market exit threshold θ^*: Firms also exit if their participation constraint is not satisfied. Using the optimal dividend decision (2), firms exit when:

$$e' > \beta V(e', \theta, Z), \quad \text{with } e' = \min[q, \bar{e}(\theta, Z)].$$

Note that if the firm is sufficiently productive, their participation constraint is always satisfied. Recall $V(e, \theta, Z)$ is defined on $[\underline{e}_b(\theta, Z), \bar{e}(\theta, Z)]$. Then, if $\underline{e}_b(\theta, Z) < \beta V(\underline{e}_b, \theta, Z)$, the firm finds it profitable to stay in the market whatever its level of net worth. The productivity threshold θ^* above which firms always satisfy their participation constraint is characterised by:

$$\underline{e}_b(\theta^*, Z) = \beta V[\underline{e}_b(\theta^*, Z), \theta^*, Z]. \tag{A.3}$$

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[35] The productivity thresholds $\theta(Z)$, $\theta^*(Z)$ and $\theta^{**}(Z)$ are defined by the following equations:

$$\bar{e}(\underline{\theta}, Z) = \beta V[\bar{e}(\underline{\theta}, Z), \theta, Z],$$
$$\underline{e}_b(\theta^*, Z) = \beta V[\underline{e}_b(\theta^*, Z), \theta^*, Z],$$
$$(1+r)(k_b + c) = Z[\theta^{**} + G(\bar{\epsilon}_b)]k_b^\alpha - \mu k_b^\alpha \Phi(\bar{\epsilon}_b) + (1-\delta)k_b.$$

Above this threshold, firms exit only when the participation constraint of the financial intermediary is not satisfied. To show that this threshold is unique, we start with the fact that a firm with productivity $\theta_0 \leq \underline{\theta}_{FL}$ is not profitable and therefore exits the market in the frictionless economy but also in the credit constrained economy (as the financing cost faced by the firm is higher in this case). Therefore: $\underline{e}_b(\theta_0, Z) > \beta V[\underline{e}_b(\theta_0, Z), \theta_0, Z]$. We now need to show that $\underline{e}_b(\theta, Z)$ is a decreasing function and $V[\underline{e}_b(\theta, Z), \theta, Z]$ a strictly increasing function of θ. Recall that the threshold $\underline{e}_b(\theta, Z)$ below which the financial intermediary refuses to loan any funds is defined by:

$$Z[\theta + G(\bar{\epsilon}_b)]k_b^\alpha + (1 - \delta)k_b - \mu k_b^\alpha \Phi(\bar{\epsilon}_b) = (1 + r)(k_b + c - \underline{e}_b).$$

Taking the total differential of this equation indicates that firms with a high productivity are less frequently rationed from the market: $\mathrm{d}\underline{e}_b/\mathrm{d}\theta \leq 0$. We now need to show that $V[\underline{e}_b(\theta), \theta, Z]$ is increasing in θ. Using the envelope theorem, it comes:

$$\frac{\partial V}{\partial e} = \frac{\partial \mathcal{L}}{\partial e} = \lambda(1 + r)$$

$$\frac{\partial V}{\partial \theta} = \frac{\partial \mathcal{L}}{\partial \theta} = \lambda Z k^\alpha + \beta \int_{\frac{\underline{e}_b}{Zk^\alpha} + \bar{\epsilon}}^{+\infty} \frac{\partial V}{\partial \theta}(e', \theta, Z) \mathbb{1}_{e' < \beta V(e', \theta, Z)} \mathrm{d}\Phi(\epsilon)$$

$$+ \frac{1}{Zk^\alpha} \frac{\mathrm{d}\underline{e}_b}{\mathrm{d}\theta}[\underline{e}_b - \beta V(\underline{e}_b, \theta, Z)] \Phi'\left(\frac{\underline{e}_b}{Zk^\alpha} + \bar{\epsilon}\right) \mathbb{1}_{\underline{e}_b < \beta V(\underline{e}_b, \theta, Z)}.$$

Then, it follows that $V(\underline{e}_b(\theta), \theta, Z)$ is strictly increasing in θ,

$$\frac{\mathrm{d}V}{\mathrm{d}\theta}[\underline{e}_b(\theta, Z), \theta, Z] = \frac{\partial V}{\partial e}(\underline{e}_b, \theta, Z)\frac{\mathrm{d}\underline{e}_b}{\mathrm{d}\theta} + \frac{\partial V}{\partial \theta}(\underline{e}_b, \theta, Z)$$

$$= -\lambda(1 + r)\frac{Zk_b^\alpha}{1 + r} + \lambda Z k_b^\alpha + \beta \int_{\frac{\underline{e}_b}{Zk^\alpha} + \bar{\epsilon}}^{+\infty} \frac{\partial V}{\partial \theta}(e', \theta, Z)\mathbb{1}_{e' < \beta V(e', \theta, Z)}\mathrm{d}\Phi(\epsilon)$$

$$+ \frac{1}{Zk_b^\alpha}\frac{Zk_b^\alpha}{1 + r}[\beta V(\underline{e}_b, \theta, Z) - \underline{e}_b]\Phi'\left(\frac{\underline{e}_b}{Zk^\alpha} + \bar{\epsilon}\right)\mathbb{1}_{\underline{e}_b < \beta V(\underline{e}_b, \theta, Z)}$$

$$> 0,$$

where the last line follows from the fact that V is strictly increasing in θ.

• Full exit threshold $\underline{\theta}$: Low productivity firms always exit whatever their level of net worth. We can find a productivity threshold θ below which the participation constraint of the firm is never satisfied. As $e' \leq \bar{e}(\underline{\theta}, Z)$ if $\bar{e}(\underline{\theta}, Z) = \beta V[\bar{e}(\underline{\theta}, Z), \underline{\theta}, Z]$ the firm never finds it profitable to stay in the market. The threshold θ is therefore defined as:

$$\bar{e}(\underline{\theta}, Z) = \beta V[\bar{e}(\underline{\theta}, Z), \underline{\theta}, Z]. \tag{A.4}$$

Firms with productivity $\theta_0 \leq \underline{\theta}_{FL}$ are not profitable and therefore exit the market: $\bar{e}(\theta_0, Z) > \beta V[\bar{e}(\theta_0, Z), \theta_0, Z]$. We complete the proof by showing that the dividend threshold $\bar{e}$ increases with θ less than the value function. Using the dividend decision condition (A.1), we can show:

$$\beta\frac{\mathrm{d}V}{\mathrm{d}\theta}[\bar{e}(\theta, Z), \theta, Z] = \beta\frac{\partial V}{\partial e}(\bar{e}, \theta)\frac{\mathrm{d}\bar{e}}{\mathrm{d}\theta} + \beta\frac{\partial V}{\partial \theta}(\bar{e}, \theta)$$

$$= \frac{\mathrm{d}\bar{e}}{\mathrm{d}\theta} + \beta\frac{\partial V}{\partial \theta}(\bar{e}, \theta)$$

$$> \frac{\mathrm{d}\bar{e}}{\mathrm{d}\theta}.$$
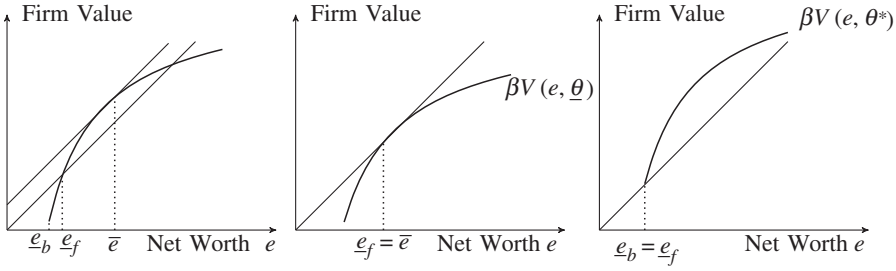
Fig. A1. *Firms' Net Worth Threshold $\underline{e}_f$, Productivity Thresholds $\underline{\theta}$ and θ^**

A.4.1. *Firms' Net Worth Threshold $\underline{e}_f$*

For $\underline{\theta} \leq \theta < \theta^*$, firms exit if their participation constraint is not satisfied: they exit if $e < \underline{e}_f(\theta, Z)$ with $\underline{e}_f(\theta, Z)$ defined by:

$$\underline{e}_f = \beta V(\underline{e}_f, \theta, Z). \tag{A.5}$$

Given θ, we show that this threshold is unique by observing that $\beta[\mathrm{d}V(e, \theta, Z)/\mathrm{d}e] > 1$ as long as $e < \bar{e}(\theta, Z)$ and $\underline{e}_b(\theta, Z) > \beta V[\underline{e}_b(\theta), \theta, Z]$ for any $\underline{\theta} \leq \theta < \theta^*$.

Furthermore we can show that the exit threshold $\underline{e}_f$ is decreasing in θ as

$$\frac{\mathrm{d}\underline{e}_f}{\mathrm{d}\theta} = \beta \frac{\partial V}{\partial \theta} \Big/ \left(1 - \beta \frac{\partial V}{\partial e}\right).$$

A1 illustrates how the thresholds $\underline{e}_f$, $\underline{\theta}$ and θ^* are determined.

Appendix B

This Appendix describes how we solve for the value function and compute the impulse response functions.

B.1. *Solving for the Value Function*

The model is solved, using value function iteration on the discretised state space, using splines to approximate between grid points.

We first rewrite the problem of the firm in a more convenient way:

$$V(e, \theta, Z) = \max_{(k, \bar{\epsilon})} \left(\int \{ \max_{e'} q - e' + \beta \mathbb{E}[1 - I(e')] V(e', \theta', Z') \} \mathrm{d}\Phi(\epsilon) \right),$$

with:

$$I(e') = \begin{cases} 0 & \text{if } e' \geq \underline{e}_b(\theta', Z') \\ 1 & \text{if } e' < \underline{e}_b(\theta', Z') \end{cases}$$

subject to:

$$Z[\theta + G(\bar{\epsilon})]k^\alpha + (1 - \delta)k - \mu k^\alpha \Phi(\bar{\epsilon}) = (1 + r)(k + c - e)$$

$$q = \max[Zk^\alpha(\epsilon - \bar{\epsilon}), 0].$$

    (*i*) We make the productivity shocks ε, θ and Z, discrete with respectively 10, 50 and 2 grid points.

(*ii*) For each (θ, Z), we construct a grid for net worth over $[\max[e_b(\theta, Z), 0], e_{sup}])$. The lower bound is the minimum net worth required by the bank to lend money and the upper bound e_{sup} is set at an arbitrary value.

(*iii*) We construct a grid for capital. The lower bound is set to 0 and the upper bound at the frictionless level.

(*iv*) For each (e, θ, Z), we compute the feasible set for capital. The maximum level of capital that the firm can borrow $k_{\max}(e)$ is defined by:

$$Z[\theta + G(\bar{\epsilon}_b)]k^{\alpha} + (1 - \delta)k - \mu k^{\alpha}\Phi(\bar{\epsilon}_b) < (1 + r)(k + c - e) \text{ for } k \geq k_{\max}(e),$$

where $\bar{\epsilon}_b$ maximises the income of the financial intermediary. The minimum level of capital that the firm can borrow $k_{\min}(e)$ is defined as follows: if $\alpha Z[\theta + G(\bar{\epsilon}_b)] \leq 0$ and $(1 + r)(e - c) < 0$, then $k_{\min}(e) > 0$ and is characterised by

$$Z[\theta + G(\bar{\epsilon}_b)]k^{\alpha} + (1 - \delta)k - \mu k^{\alpha}\Phi(\bar{\epsilon}_b) < (1 + r)(k + c - e) \text{ for } k \leq k_{\min}(e).$$

Otherwise $k_{\min} = 0$

(*v*) We compute the default threshold $\bar{\epsilon}(k, e, \theta, Z)$ for $k_{\min}(e) < k < k_{\max}(e)$ by solving:

$$Z[\theta + G(\bar{\epsilon})]k^{\alpha} + (1 - \delta)k - \mu k^{\alpha}\Phi(\bar{\epsilon}) = (1 + r)(k + c - e).$$

Let $\bar{\epsilon} = 10$ if $k < k_{\min}(e)$ and $k > k_{\max}(e)$.

(*vi*) We take an initial guess for the value function: $V^0(e, \theta, Z)$.

(*vii*) We then solve for the optimal decision rules and compute the corresponding value function:

$$V^1(e, \theta, Z) = \max_{k}\left\{\int [\max_{e'} q - e' + \beta\mathbb{E}[1 - I(e')]V^0(e', \theta', Z')]\mathrm{d}\Phi(\epsilon)\right\}$$

with $q = [\epsilon - \bar{\epsilon}(k, e, \theta, Z)]Zk^{\alpha}$.

(*viii*) We update the guess with the new value function.

(*ix*) We iterate steps (*vii*) and (*viii*) until convergence, i.e. until $V^{n+1} - V^n < 10^{-6}$.

(*x*) We check that the dividend thresholds $\bar{e}(\theta, Z)$ are lower than the upper bound of net worth. If $e_{sup} < \max \bar{e}(\theta, Z)$, start again from step 2 with a higher value for e_{sup}.

B.2. *Stationary Distribution*

Given the decision rules of the firms, we can compute the law of motion of the probability measure on e and θ:

$$\xi' = \Omega(\xi, Z, Z'),$$

with $\Omega(\xi, Z, Z')(A', \theta') = \int\int \mathrm{d}F(\theta'|\theta) \text{ Prob } (e' \in A'|e, \theta, \theta', Z, Z')\mathrm{d}\xi + \mathrm{M}_e \int_{e\in A'} I_e(\theta, e, Z')\mathrm{d}v$ where M_e is the mass of potential entrants, v is the joint distribution of potential entrants over net worth e and productivity θ, and $I_e(\theta, e, Z)$ is the entry indicator.[36]

The stationary distribution is then found by solving for:

$$\xi^* = \Omega(\xi^*, Z, Z).$$

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Additional Supporting Information may be found in the online version of this article:

Data S1.

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