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Atoms, combs, syllables and organisms

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Abstract
Mereological atomism is the thesis that everything is ultimately composed of atomic parts, i.e., parts without proper parts. Typically, this thesis is characterized by an axiom stating that everything has atomic parts. The present paper argues that the success of this standard characterization depends on how the notions of sum and composition are defined. In particular, we put forward a novel definition of mereological sum that: (i) is not equivalent to existing definitions in the literature, if no strong decomposition principle is assumed; (ii) can be used to argue that the standard characterization of atomism fails, because having atomic parts is not sufficient to be a sum of atoms; and (iii) provides a purely mereological distinction between structured and unstructured wholes, contributing to the ongoing debate on this crucial topic.

Keywords Atomism · Sum · Composition · Matter · Structured entities · Extensionality

1 Introduction

Atomism is roughly the thesis that everything is ultimately composed of atoms. It is typically considered as a thesis concerning the mereological composition of concrete entities, and characterized in terms of an axiom stating that everything has atomic parts (see Cotnoir & Varzi, 2021; Pietruszczak, 2005; Varzi, 2017). Let us introduce the following propositions:

A1 (*Atomic parts*): Everything has atomic *parts*.
A2 (*Atomic sum*): Everything is a *sum* of its atomic parts.

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A3 (*Atomic composition*): Everything is *composed* by its atomic parts.

While **A1-A3** feature different notions, namely parthood, sum, and composition, **A1** is the axiom that is standardly used to characterize Atomism. In a recent paper (see Shiver,2015), Shiver has argued that this characterization is flawed and that **A1** falls short of capturing Atomism. In essence, Shiver argues that there are models that satisfy **A1** but contain items that are not ultimately composed of atoms, thus failing to meet both **A2** and **A3**. In response, Varzi (see Varzi, 2017) defended the traditional characterization of Atomism.[1]

We believe that Varzi is actually right, and yet Shiver is not entirely wrong. How could that be? This is because we are about to argue that the success of the standard characterization depends on how the notion of sum is defined. That is, in the presence of **A1**, whether **A2** holds or fails depends on the notion of sum used in its formulation. Given that the notion of composition is defined in terms of sum, the success of the standard characterization depends also on the notion of composition. That is, in the presence of **A1**, whether **A3** holds or fails to hold depends on the notion of composition used in its formulation.

Our overall take on the issue contrasts with recent developments in the literature. In effect, the usual response to the Shiver-Varzi debate has been that of considering different ways of cashing out Atomism beyond **A1**.[2] By contrast, in this paper, we want to take a closer look to the notions of sum and composition that feature in **A2** and **A3**. In particular, we provide a novel notion of *mereological sum* that is philosophically interesting for a number of reasons:

(i) provided we work within basic mereological theories,[3] the notion is *inequivalent* to traditional ones in the literature—in effect, this notion of sum is independent from different decomposition principles;
(ii) given that notion of sum, we can distinguish different notions of composition;
(iii) some such notions of composition are such that one can distinguish the notions of "being the sum of" and "being composed of" in **A2** and **A3**;
(iv) given that notion of sum, we can reassess the Shiver-Varzi debate by showing that there is a sense in which **A1** falls short of capturing Atomism— if this is meant to at least entail the thesis that everything is the sum of, or composed by its atoms.

This vindicates our claim. Indeed, Varzi uses a very specific notion of sum (to be precisely characterized below) in his reply to Shiver. If one sticks to that notion, Varzi is right. Yet, Shiver is not completely wrong. There is another notion of sum that can be used to underpin his main claim.

[1] See also Cotnoir and Varzi (2021).

[2] For example in Uzquiano (2017) and Dixon (2020). See also Cotnoir (2013) It is a substantive question whether these stronger formulation of Atomism capture some historically significant theses, such as the atomistic picture of the world Kant discusses in the first Critique, or Leibniz discusses in the *Monadology*. Thanks to [Redacted] here.

[3] We will provide details in due course.

Before moving on, we should register that the philosophical significance of our discussion goes well beyond atomism. In particular, as we will see:

(v) it offers a way to distinguish between structured and unstructured entities;
(w) this distinction can in turn be used to provide a novel understanding of classical (alleged) cases of composition such as Aristotle's notorious syllable case, or Armstrong's account of composition of states of affairs.[4] It also captures concrete hierarchical cases of composition, e.g., that of an organism.

2 The framework

Before entering into the discussion, let us introduce the basic conceptual framework we are going to assume.

2.1 Basic notions and systems

We will mostly work with three mereological systems:

Minimal mereology (MM), where the relation of generic parthood, that is proper or improper parthood, is simply a relation of partial order[5]

Quasi supplemented mereology (QSM), that is, MM plus *Quasi supplementation*. The principle states that if something has a proper part, it has *disjoint* proper parts;

Strongly supplemented mereology (SSM), that is MM plus *Strong supplementation*. The principle states that if something is not part of something else, then the first thing has a part disjoint from the second.[6]

[4] We do not intend to endorse the philosophical lessons that Aristotle and Armstrong draw from such cases. Nor we need to. The thought is that those who endorse such lessons for independent reasons can make significant use of the results of the present paper.

[5] Notice that MM; does not coincide with the system proposed under the same name in Casati and Varzi (1999); Cotnoir and Varzi (2021), which includes among its axioms the *Weak Supplementation* principle. The assumption of the partial ordering axioms is not completely uncontroversial. This is true in particular for Antisymmetry. For instance, Cotnoir (2010) shows that dropping Antisymmetry and re-defining proper part as "x is a proper part of y:= x is part of y but y is not part of x" one can endorse supplementation principles as strong as Strong Supplementation and yet not buy into e.g., extensionality of proper parthood. And indeed, Antisymmetry plays a crucial role in some of the following proofs. We will flag such role in due course. Thanks to an anonymous referee here.

[6] See Cotnoir and Varzi (2021), Gruszczynski and Pietruszczak (2016) and Varzi (2016) for a general introduction to the basic concepts and for the definition of the main systems of mereology, and Gilmore (Forthcoming) for the idea underlying Quasi supplementation.

MM, QSM and SSM are here embedded in a two-sorted first-order logic, containing constants and variables for individual entities (lowercase letters) and plural entities, or pluralities (uppercase letters).[7] In addition, plural entities are characterized by two axioms[8]:

Extensionality: $\forall x(x : A \leftrightarrow x : B) \rightarrow A = B$
Plural comprehension: $\exists x\phi(x) \rightarrow \exists X\forall x(\phi(x) \leftrightarrow x : X)$

where $x : X$ is interpreted as saying that x is one of the Xs,[9] and $\phi(x)$ is an expression containing x, but not X, free.

The following notation is used throughout:

$b \leq a$:	Primitive	(b is a part of a)
$b \ll a$:	$b \leq a \wedge b \neq a$	(b is a proper part of a)
$X \leq a$:	$\forall x(x : X \rightarrow x \leq a)$	(all Xs are parts of a)
$X \ll a$:	$\forall x(x : X \rightarrow x \ll a)$	(all Xs are proper parts of a)

Furthermore, the following notions are employed[10]

$a \circ b$:	$\exists x(x \leq a \wedge x \leq b)$	(a overlaps b)
$a \parallel b$:	$\neg(a \circ b)$	(a is separated, or disjoint from b)
$a \circ X$:	$\exists x(x : X \wedge a \circ x)$	(a overlaps some of the Xs)
$a \parallel X$:	$\neg(a \circ X)$	(a is separated from all the Xs)

Finally, an entity a is said to be *composite*, $C(a)$, just in case it has at least a proper part and *atomic*, $A(a)$, just in case it has no proper parts.

2.2 Notions of sum and composition

Three commonly adopted notions of sum and a standard notion of mereological composition can be now defined as follows.[11]

Definition 1 Notions of sum and composition.

[7] We use pluralities to introduce generalized notions of sum. This can be done in different ways, but nothing in what follows depends on the specific way we have chosen.

[8] See e.g. Oliver and Smiley (2013). The axiom of extensionality states that the identity of a plurality is determined by its elements, so that no two pluralities can share the same elements, whereas the axiom of plural comprehension states that all non-empty descriptions determine a plurality. These are standard axioms in plural logic.

[9] Similarly, $X : Y$ stands for "the Xs are some of the Ys".

[10] Given these definitions *Quasi supplementation*: is: $x \ll y \rightarrow \exists z\exists w(z \ll y \wedge w \ll y \wedge z \parallel w)$, whereas *Strong supplementation* is: $x \nleq y \rightarrow \exists z(z \leq y \wedge z \parallel x)$.

[11] See Cotnoir and Varzi (2021) and Gruszczynski and Pietruszczak (2016) for an analysis of the various ways of defining the notion of sum in mereology and van Inwagen (1990) and Varzi (2017) for the notion of composition. Sum_i correspond to the notion of Goodman's fusion, Lesniewski's fusion and Algebraic fusion respectively in Cotnoir and Varzi (2021).

1. $Sum_1(a, X) := \forall x(x \circ a \leftrightarrow x \circ X)$
a is a Sum_1 of the items in *X* if and only if *a* overlaps all and only the items that are overlapped by at least one item in *X*.
2. $Sum_2(a, X) := X \leq a \wedge \forall x(x \leq a \rightarrow x \circ X)$
a is a Sum_2 of the items in *X* if and only if all the items in *X* are parts of *s* and every part of *a* overlaps at least one item in *X*.
3. $Sum_3(a, X) := X \leq a \wedge \forall y(X \leq y \rightarrow a \leq y)$
a is a Sum_3 of the items in *X* if and only if all the items in *X* are parts of *a* and *a* is part of all the items all the items in *X* are parts of.
4. $Com_i(X, a) := Sum_i(a, X) \wedge \forall x, y : X(x \neq y \rightarrow x \parallel y)$
a is *i*-composed by the items in *X*—or, equivalently, the items in *X* are the components of *a*—if and only if *a* is a Sum_i of the items in *X* and such distinct items are pairwise separated.

Note that, since atoms are pairwise separated, having no proper part, and so no part in common, composition by atoms just amounts to sum:

Proposition 1 *Let* $\mathbb{A}$ *be a plurality of atoms.*

Then, provided $X : \mathbb{A}$, $Com_i(X, a) \leftrightarrow Sum_i(a, X)$.

While it is well-known (see Pietruszczak, 2005) that the previous definitions of sum are equivalent in SSM, we will show in Sect. 4 that they are not equivalent in QSM, and so in minimal mereology MM. It is also worth noting that Sum_3 coincides with the least upper bound of the *X* with respect to the order induced by the relation of parthood. As a consequence, if $Sum_3(a, X)$ then *a* is the unique Sum_3 of the *X*.[12] Thus, if composition is defined in terms of a notion of sum that is stronger than Sum_3, then every entity that is composed of a plurality of atoms is the unique entity that can be composed of that plurality.

Assuming that $\mathbb{A}$ is the plurality of all the atoms, the initial theses **A1**-**A3** can be now formulated as follows:

A1 (*Atomic parts*): $\forall x \exists y(y : \mathbb{A} \wedge y \leq x)$
A2 (*Atomic sum*): $\forall x(C(x) \rightarrow \exists X(X : \mathbb{A} \wedge Sum_i(x, X)))$
A3 (*Atomic composition*): $\forall x(C(x) \rightarrow \exists X(X : \mathbb{A} \wedge Com_i(x, X)))$

where *i* varies on the indices of the notions of sum previously introduced. We are now ready to address the Shiver-Varzi debate.

[12] Uniqueness follows from the second conjunct of Sum_3 and Antisymmetry.

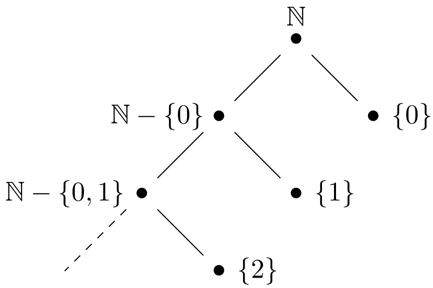
3 On the notion of atomism

The Shiver–Varzi debate can be characterized as a contrast about whether *having atomic parts* captures Atomism, in that it is sufficient for *being ultimately composed of atoms*. As we understood things here, this amounts to the question as to whether **A1** captures Atomism insofar as it entails **A2** and **A3**.

3.1 The *Infinite Comb* model

Shiver contends that the aforementioned entailment from **A1** to **A2** and **A3** fails on the basis of the following *Infinite Comb* model:

Infinite Comb:



There are two kinds of items here:

(i) *Atomic items* (the tips of the teeth):
Sets of kind $\{n\}$, where $n \in \mathbb{N}$;
(ii) *Composite items* (the joints along the shaft):
Sets of kind $\{i \in \mathbb{N} \mid n \leq i\}$, where $n \in \mathbb{N}$.

The relation of generic parthood is the relation of inclusion between sets. Note that *Infinite Comb* is a model of SSM,[13] but it is *not* a model of *every* extensional mereology. This is clear from the following:

Proposition 2 *The principle of unrestricted composition, intended as a principle of unrestricted sum, where sum is interpreted as union of sets, fails in the previous model.*[14]

[13] It is not difficult to see that M satisfies the axioms of SSM First, the relation of inclusion is a partial order. Second, Strong Supplementation is provable as follows: suppose $x, y \in M$ and x is not a subset of y; then, in case x is an atom, x has a part, x itself, that is separated from y and, in case x is composite, x has a part, the first atomic part in x that contains an n that is not in y, that is separated from y.

[14] In effect, it violates even weaker principles of composition such as the *Remainder* principle. See (Cotnoir and Varzi, 2021).

Proof Indeed, while all the singletons containing elements of $\mathbb{N}$ are in the domain, being the atoms of the model, not all entities composed of singletons are in the domain, for instance $\{0, 1, 2\}$. □

Shiver's basic idea is that, even if everything in M has atomic parts, thus satisfying **A1**, any plurality of things which compose any composite entity in M includes at least one *composite* proper part, thus failing to satisfy both **A2** and **A3**. Therefore, so the thought goes, it should not be true that every composite entity is a sum of, or composed by its atomic parts—at least if no further problematic principle, such as the unrestricted composition principle in Proposition 2, is assumed. In turn, this is supposed to show that **A1** falls short of capturing Atomism.

3.2 The Atomism of the *Infinite Comb* Model

In his Varzi (2017), Varzi proves that, provided one of the notions of sum in Sect. 2 is used, and the notion of composition is defined either in terms of sum or in terms of sum of separated items, then (i) the previous model is actually atomistic in that it satisfies **A3**, and (ii) **A1** implies **A3**. Given Proposition 1, the same argument establish that **A1** entails **A2**.[15]

Let us show, by way of illustration, that this is true with respect to the notion of Com_2.[16] We offer a somewhat streamlined proof.

Proposition 3 *Infinite Comb satisfies* **A3** (*composition is intended as* Com_2), *that is, Infinite Comb satisfies* $\forall x(C(x) \rightarrow \exists X(X : \mathbb{A} \wedge Com_2(x, X)))$.

All we have to show is that, given a composite set x, there is a plurality of atoms $X : \mathbb{A}$ such that $Com_2(x, X)$.

Proof Suppose $C(x)$. Then x is a set of kind $\{i \in \mathbb{N} \mid n \leq i\}$, since these are the only composite sets. Let X_n be the plurality of atoms $\{i\}$ such that $n \leq i$. Then X_n is the set of atoms composing $\{i \in \mathbb{N} \mid n \leq i\}$. To be sure, every item in X_n is a part of $\{i \in \mathbb{N} \mid n \leq i\}$, since $\{i\} : X_n \leftrightarrow n \leq i$, by the definition of X_n, and every part of $\{i \in \mathbb{N} \mid n \leq i\}$ overlaps some element of X_n, since every part of $\{i \in \mathbb{N} \mid n \leq i\}$ contains a number i such that $\{i\} \in X_n$. □

As Varzi himself notices, Varzi (2017), the model is still "disturbing", but this depends on the fact that in *Infinite Comb* there are entities that cannot be possibly *decomposed in their atomic parts*, even if they are *composed by their atomic parts*. However, as he points out—rightly we believe— Atomism is a thesis about *composition*, not *decomposition*. Furthermore, Varzi also proves Proposition 4 below (where composition is again assumed to be Com_2). Once again, our proof is streamlined:

[15] See also (Cotnoir and Varzi, 2021).

[16] The reader can check this is true for Com_1 and Com_3 as well.

Proposition 4 **A1** *implies* **A3**, *that is* $\forall x(C(x) \rightarrow \exists a(a : \mathbb{A} \wedge a \ll x))$ *implies* $\forall x(C(x) \rightarrow \exists X(X : \mathbb{A} \wedge Com_2(x, X)))$.

Proof Suppose $C(x)$. Then, $\exists a(a : \mathbb{A} \wedge a \ll x)$. Let X be the plurality of atomic parts of x, whose existence is guaranteed by *Plural Comprehension*. Then, $Com_2(x, X)$. To be sure, every atom in X is a part of x, by the definition of X, and every part of x overlaps some atom in X, since every part of x has an atomic part by **A1**, and this atomic part is a part of x, by the transitivity of parthood. □

Thus (Varzi, 2017 pp. 10–11) concludes that:

> *[N]o matter how we understand the notion of sum*, the thesis that everything has atomic parts turns out to imply the thesis that everything is a sum of atoms. *Insofar as being composed of atoms amounts to being a sum of atoms* [...], it follows therefore that the standard way of characterizing mereological atomicity implies precisely the thesis that it is meant to capture: everything is ultimately composed of atoms (italics added).

In an atomistic mereology everything is ultimately composed of atoms. Still, for what follows, we want to note that Varzi's conclusion consists of two different, yet related, theses—it is actually worth having a name for both:

Varzi 1. *No matter how we understand the notion of sum*, the thesis that everything has atomic parts turns out to imply the thesis that everything is a sum of atoms, provided that the relation of generic parthood is reflexive and transitive (and, in the case of Sum_3, strongly supplemented). In other words, **A1** entails **A2**.[17]

Varzi 2. *Insofar as being composed of coincides with being a sum of disjoint entities*, the thesis that everything has atomic parts turns out to imply the thesis that everything is composed of atoms, provided that the relation of parthood is reflexive and transitive (and, in the case case of Com_3, strongly supplemented). In other words, **A1** entails **A3**.

3.3 Reassessing the debate

As we pointed out above, we believe that there is indeed a sense in which both Varzi and Shiver are (partly) right. In order to see this, we will establish the following.

1. The fact that **A1** entails **A2** *crucially depends* on the notion of sum used in the proof.

[17] In context, it is clear that with "no matter how we understand the notion of sum", Varzi means "no matter whether we understand the notion of sum as Sum_1, Sum_2, or Sum_3".

2. Indeed, what we shall call *General Sum* allows us to construct models where everything has atomic parts even if something is not a sum of atoms.
3. The fact that **A1** entails **A3** *crucially depends* on the notion of composition used in the proof.
4. Indeed, what we shall call *General Composition* allows us to construct models where everything that has atomic parts even if it is not composed of atoms.

As we formulated them (1) and (2) threaten **Varzi 1**, whereas (3) and (4) threaten **Varzi 2**. We will provide arguments for (1) and (2) in the next section and arguments for (3) and (4) in Sect. 5.3.

4 On the notion of sum

As we saw **Varzi 1** can be proven if the notion of sum used in the proof is any of the Sum_i in Sect. 2— in the case of Sum_3, the strong supplemention principle is to be assumed. However, it is not clear if these notions exhaust all plausible notions of mereological sum. To answer this question, we first outline some desiderata a notion of sum *could* be required to meet,[18] and then show that there is a notion of sum that meets these desiderata and is not equivalent to any of the Sum_i in Sect. 2—as long as *no strong mereological principle is assumed*. This is crucial in evaluating the validity of **Varzi 1**, as we will demonstrate that it fails under this new notion of sum.

4.1 A general notion of sum

Let us then consider the following conditions, which one *may* put forward as desiderata on any notion of sum:

1. S (*Success*):
   $Sum(a, X) \rightarrow X \leq a$.
   The *X*s are parts of their sum.
2. NJ (*No Junk*):
   $Sum(a, X) \rightarrow \forall x(x \parallel X \rightarrow x \parallel a)$.
   What is separated from *X*s is separated from their sum.
3. M (*Minimality*):
   $Sum(a, X) \rightarrow \forall x(X \leq x \rightarrow a \leq x)$.
   What includes the *X*s includes their sum.

The first condition, S, simply requires that a sum of the *X*s contains *all of them*, that is, no *X* is left behind. The second one, NJ, requires that a sum of the *X*s contains *no more* than the *X*s. In other words, the first two conditions require that a sum is inclusive enough, but not too inclusive, i.e., that it includes just the right amount of items. Finally, the third condition, M, requires a sum to be minimal, that is, to be included

18 Some might go as far as claiming that it *should* be required to meet.

in everything that includes the plurality it sums. It also implies that the identity of the sum of the *X*s is *determined* by the *X*s and nothing else, meaning that no additional structure is required to fix or determine the identity of the sum of the *X*s, other than that it is indeed their sum.[19] It is then not difficult to show that the following propositions hold in the system MM of minimal mereology.[20]

Proposition 5 (operations satisfying **S**)

(i) **S** is satisfied by Sum_2 and Sum_3 (By definition).
(ii) Sum_1 does not satisfy **S** (see Model 1 below).

Proposition 6 (operations satisfying NJ)

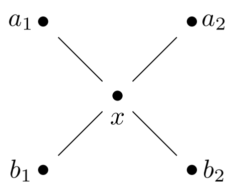
(i) NJ is satisfied by Sum_1 and Sum_2 (By definition).
(ii) Sum_3 does not satisfy NJ (see Model 2 below).

Proposition 7 (operations satisfying M)

(i) M is satisfied by Sum_3 (By definition).
(ii) Sum_1 and Sum_2 do not satisfy M (see Model 3 below).

Propositions 5–7 show that, provided no strong principle governing the relation of parthood is assumed, no notion of sum satisfies all the proposed desiderata. This will play a crucial role in suggesting a new notion of mereological sum. Before turning to such suggestion, it is instructive to consider a few mereological models, which provide some support for our desiderata and a proof of claim (ii) in propositions 5–7.

Model 1:



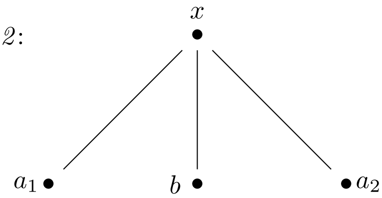
In this model, x turns out to be a Sum_1 of a_1 and a_2, for an item is separated from x if and only if it is separated from a_1 and a_2. Since the notion of Sum_1 does not include—nor it entails—**S**, there is no need for the items that compose a sum *to be parts* of the whole they compose. For instance, it is allowed for atoms to be sums of non-atomic entities. Faced with these consequences, one reaction would be to

[19] This is true in MM since the axiom of antisymmetry is necessary in order to prove that sums of the same entities are the same sum. Since in the present context antisymmetry is not put into question and MM is assumed as the basic system of mereology, the uniqueness of the sum is a direct consequence of this condition.

[20] And in effect, as we will argue, in QSM as well.

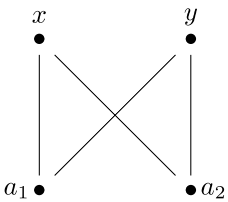
require **S** to be satisfied by any *reasonable* notion of sum. One should then arguably reject Sum_1 as an appropriate notion of sum.[21]

Model 2:



In this model, x is a Sum_3 of a_1 and a_2, and yet it has b as a part. In this case, the sum of two items is something that has a part that is separated from these two items. Sum_3 fails to satisfy NJ. Hence, there is no need for the items that compose a sum *to be the only parts* of the whole they compose. If one maintains that sums should not contain parts that are separated from the summands, one should require NJ to be satisfied by any *reasonable* notion of sum, thus rejecting Sum_3.[22]

Model 3:



In this model, x_1 is *a* Sum_2 of a_1 and a_2, and y is *a different* Sum_2 of a_1 and a_2. In this case, no sum is minimal, and the sum of two items is not uniquely determined by its parts. The notion of Sum_2 does not satisfy M. Hence, there is no need for the item that coincides with a sum *to be the only sum* of the parts it is composed of. If one holds that it is a sensible requirement on the notion of sum to be *minimal* and *uniquely determined* by its summands one could require M to be satisfied by any *reasonable* notion of sum, and therefore reject Sum_2.[23]

Where does that leave us? We saw from Propositions (5–7) that no notion of sum actually satisfies all the three requirements we discussed. As a result, we could

[21] Note that we don't need to go as far here. In mereologies including specific supplementation principles models like the one here considered can be excluded without problems.

[22] See the previous footnote.

[23] Once again, we need not go as far here, provided we are willing to assume some specific supplementation principles. It is worth noting that M is typically rejected in mereological frameworks in which it is allowed for the same items to be part of a non-structured whole and a structured whole, a chunk of clay and a statue for instance. We do not want to discuss this position now, but only point to the fact that distinguishing the notion of sum from the notion of composition, as per the final section, allows for a more intuitive solution of the problem of accounting for the distinction between non-structured and structured wholes. For similar considerations regarding the notions of Sum_1 and Sum_2 see (Hovda, 2009), especially Sect. 2.1.

construct problematic Models (1–3) in which certain items are indeed sums of a given plurality, even if there is pressure to resist such a claim. This leads to the unsurprising suggestion of defining a general notion of sum by simply taking the conjunction of S, NJ, and M above:

Definition 2 General Sum.
$Sum(a,X) := X \leq a \wedge \forall x(x \parallel X \rightarrow x \parallel a) \wedge \forall y(X \leq y \rightarrow a \leq y)$

In plain English, *a* is the *Sum* of the items in *X* if and only if all the *X*s are parts of *a*, *a* is separated from any thing which is separated from all the *X*s, and *a* is part of any things which has all the *X*s as parts.

It is immediately clear that our proposed notion of *Sum* is not equivalent to any of the Sum_i. Indeed, *provided no further mereological principle is assumed*, it turns out that the *Sum* of the *X*s is also a Sum_i of the *X*s for any *i*, while the converse does not hold. In effect, in each of Model (1)-(3), *x* is *not* a *Sum* of a_1 and a_2—contrary to what happened for at least some Sum_i. As we are about to see, this has interesting consequences on the debate on Atomism. Still, before coming to that, let us consider a possible objection.

4.2 An objection

We defined *Sum* in terms of the notion of parthood and insisted that our results hold in MM *provided we do not assume further mereological principles* regimenting that basic notion. Nevertheless, the thought goes, some such principles are required to fix the very meaning of 'part", and one cannot simply be silent about this. As we pointed out, the notions of Sum_i and the general notion of *Sum* are not extensionally equivalent in MM. Thus, if one were to stick to MM, all of our arguments would go through. Yet, there is a well-known complaint that MM is too weak. In fact, the general thought is that the partial ordering axioms are too weak to single out a genuine relation of parthood[24] and the usual "fix" is to require that parthood obeys some sort of *decomposition* principle.[25] Which principle of decomposition one should assume is a matter of dispute. Arguably the most cited example is the principle of *Weak Supplementation*: *WSP*.[26] Simons, in his Simons (1987), goes as far as claiming that *WSP* is analytic with respect to the notion of parthood. If *WSP* is assumed, then Sum_1, Sum_2 and *Sum* turn out to be equivalent. Now, the inequivalence of *Sum* and Sum_i fuels (at least partly) the significance of the present discussion. Therefore, we need to address the issue at hand here.

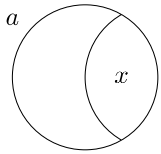
There are at least four considerations to note in response.

24 The *locus classicus* is Simons (1987).

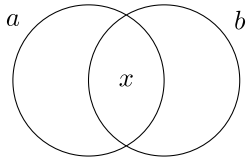
25 The terminology is taken from Varzi (2016). See also (Cotnoir and Varzi, 2021).

26 This principle has it that, if *y* has *x* as a proper part, then *y* has another part which is separated from *x*: $x \ll y \rightarrow \exists z(z \leq y \wedge z \parallel x)$.

1. First, one can push the point that the philosophical significance of *Sum*—as detailed in (i)-(vi) in Sect. 1—is reason enough to explore notions of sum independently of any decomposition principles. Indeed, while developing a system of mereology, the introduction of a *composition principle* should be kept separate from the issue of what *decomposition principle* is to be adopted, since the characterization of the operation of sum is independent on the standpoint we take concerning what parts a thing has.
2. Second, one can put into question that *WSP* is analytic. In fact, the analytic status of *WSP* is at least controversial, as witnessed e.g. in Cotnoir (2018). Now, one of the main reasons to assume this principle is given by considering diagrams like this[27]:



Here we have that $x \ll a$ and we can "see" that there is another part of a which is disjoint from x. However, the same visual evidence is also at work in a case like this:



In this case we have that $a \nleq b$ and we can "see" that there is another part of a which is disjoint from b. Hence, if we acknowledge that it is impossible to visualize $x \ll a$ without visualizing a as having a disjoint part, then we should also acknowledge that it is impossible to visualize $a \nleq b$ without visualizing a as having a part that is disjoint from b. Therefore, it seems that any "visual" support we have for *WSP* (first case) also supports Strong Supplementation (second case). Still, Strong Supplementation is not considered analytic.

3. Third, and relatedly, it has been argued in the literature that much of the support in favor of *WSP* should really be re-directed towards a weaker decomposition principle, namely the *Quasi Supplementation* principle we mentioned in Sect. 2 (See Gilmore Forthcoming). And, in QSM, *Sum* and Sum_i turn out to be *not equivalent*. The argument is straightforward: just note that *all* models (1–3) are quasi-supplemented.[28]
4. Finally, there are several metaphysical theses that are indeed committed to violations of *WSP*, ranging from Whitehead's mereotopology to Brentano's theory of accidents, from Fine's *qua-objects* to the conjunction of backward time-travel

[27] We consider another reason in Sect. 6.2.

[28] For a critical discussion of *Quasi Supplementation* see (Cotnoir and Varzi, 2021), and Cotnoir (2016).

> and endurantism. Therefore, working in a framework where *WSP* is not imposed as an analytic principle makes room for different metaphysical projects,[29]

In any event, even the supporters of the analyticity of *WSP* can read the arguments in the rest of the paper as conditional arguments to the effect that, *provided we do not work with a mereological theory that is stronger than* QSM, then the intended conclusion of such arguments follows.

5 Building things from atoms

We turn now to discuss the consequences of what we have been exploring so far for the question of Atomism, thus showing how the notion of sum just introduced can be used to shed light on the Shiver–Varzi debate. To do so, we will first take a closer look at one particularly disputable passage of Aristotle, where the model of a syllable is introduced to highlight the distinction between heaps and wholes. Then, we will go back to the notion of composition and advance a new definition of composite entity.

5.1 Aristotle's syllable

In *Met* (Z.17, 1041b11–33) Aristotle discusses *the composition of a syllable*, which constitutes a paradigmatic case of a structured whole.[30]

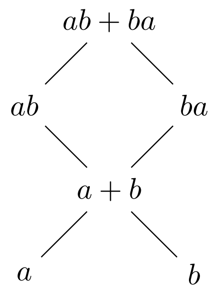
> As regards that which is compounded out of something so that the whole is one—not like a heap however, but like a syllable—the syllable is not its elements, *ba* is not the same as *b* and *a*, nor is flesh fire and earth [...] The syllable, then, is something—not only its elements (the vowel and the consonant) but also something else; and the flesh is not only fire and earth or the hot and the cold, but also something else. (*Met*. Z.17, 1041b11-33; Ross's translation).

Without entering exegetical details, we suggest that Aristotle's Syllable Model could be thought of as follows (where + is the operation of binary sum):

[29] See Cotnoir and Varzi (2021) 4.3.1. One needs not to endorse such projects to claim that they are not conceptually wrong-headed, as it is entailed by the failure of an alleged analytic principle.

[30] A way of framing the notion of structured whole in an extensional mereology is proposed in Canavotto and Giordani (2020), where it is argued that structured wholes are best understood in terms of a distinction between actual and potential parts. Here we will not discuss the connections with the present accounts, but we observe that the notion of matter that emerges in our framework is structurally identical with the one introduced in Canavotto and Giordani (2020).

Aristotle's Syllable Model:



The idea on which this model is based—discussed in the passage above—is that syllables are "more" than the letters they are composed of. Indeed, according to Aristotle's own analysis, a syllable is a whole consisting of elements and a form.[31] Thus, in the previous model, *ab* is composed of *a* and *b* in this order, while *ba* is composed of *a* and *b* in the opposite order. Hence, while being composed of the same letters, *ab* and *ba* differ as to the order of composition, and both of them also differ from the sum of *a* and *b*. A similar idea is proposed by Armstrong in his account of the composition of states of affairs. According to Armstrong, Romeo's loving Juliet and Juliet's loving Romeo are states of affairs composed by he same constituents, i.e., Romeo, Juliet, and the universal relation of loving, but they are not the same state of affairs and they both differ from the sum of Romeo, Juliet, and the universal relation of loving (See Armstrong,1997).

The first interesting result we are now able to derive is that, in Aristotle's Syllable Model, (i) every plurality of entities has a *unique Sum*,[32] (ii) every entity has at least an atomic part—*a* and *b* are assumed to be atomic, but (iii) not every entity having an atomic part is the *Sum* of its atomic parts.

In effect, it is not difficult to see that:

1. *a* is the *Sum* of *a*
2. *b* is the *Sum* of *b*
3. $a + b$ is the *Sum* of *a*, *b*
4. *ab* is the *Sum* of *ab*
5. *ba* is the *Sum* of *ba*
6. $ab + ba$ is the *Sum* of *ab* and *ba*
7. Any plurality which includes *ab* but not *ba* has *ab* as *Sum*
8. Any plurality which includes *ba* but not *ab* has *ba* as *Sum*

[31] We do not want to enter exegetic discussions. Suffice it to say that there is a substantive debate about whether the form is itself a proper part of a whole: Koslicki (2006) argues that it is, whereas Rotkale (2018) argues it is not. Here we present a model which is consistent with the idea that a form is a principle of unity, instead of a part, of a substance.

[32] Interestingly, the model satisfies *Extensionality of Sum* but *Extensionality of Proper Parthood* fails—*ab* and *ba* have the same proper parts and yet they are distinct. For a discussion of the distinction see (Varzi, 2008).

The crucial thing to note is that *ab* and *ba* are entities having *a* and *b* as atomic parts. Yet, none of them is a *Sum* of *a* and *b*. To see this, just note that the third conjunct in the definition of *Sum* fails. In effect, the only *Sum* of *a* and *b* is $a + b$ which is distinct from both.[33] Hence, both *ab* and *ba* satisfy **A1** without satisfying **A2**, *contra* **Varzi 1**. In fact, it is plain that, in the present case, it is not true that everything is a sum of atoms, even if any entity in the model has atomic parts. Hence, the main upshot of having isolated a supplementation-independent notion of sum is that Atomism, understood as **A1**, i.e., as the assumption that everything has atomic parts, does no longer entail **A2**, that is the thesis that every composed entity is a sum of its atomic proper parts.

5.2 A difference without difference-making parts

In studying the mereological relations involved in the syllable model we note two peculiar facts (and, in effect, the peculiarity of the model is precisely that it allows for such facts):

Fact 1: $a + b \ll ab$, but there is no entity that grounds (mereologically) the difference between $a + b$ and *ab*.
Fact 2: $ab \neq ba$, but there is no entity that grounds (mereologically) the difference between *ab* and *ba*.

This is as expected. The first fact witnesses the failure of *WSP*, whereas the second one witnesses the failure of *Extensionality of Proper Parthood*. The following question then arises: how should we account for the existence of entities which are different while sharing the same atoms, or entities which are different while sharing the same proper parts? It seems that, in these cases, we are confronted with differences without difference-makers. Still, on a closer look, what we get are not cases of differences without difference-makers, but of differences without *difference-making parts*, and this distinction is crucial. When learning logic and philosophy of language, we have been told again and again that the sense of a composite expression is determined by both its components and the way of composition, but we have been never told that the way of composition is a part of the expression. We want to advance the same idea here: *ab* and *ba* are composed from the same components but according to a different way of composition. Thus, the difference between *ab* and *ba* is accounted for in terms of ways of compositions. A detailed investigation of what a way of composition is goes beyond the scope of this paper. Yet, we should note that the proposal would allow us to distinguish between *ab* and $a + b$ without invoking a difference in proper parts: this might be important e.g. in the discussion of Aristotle's substances or Armstrong's states of affairs.

[33] This also suggsets that $a + b$ is to be considered a proper part of *ab* but not viceversa. The same goes for *ba*.

5.3 On the notion of composition

The second interesting result to be discussed concerns the notion of *composition*. We noted that ab, ba, and $a+b$ are all distinct and they are all Sum_1 and Sum_2 of a and b. The general definition of *Sum* introduced in Sect. 4 allows us to distinguish between those Sum_1 and Sum_2 that are not *Sum* because they do not meet the M condition, namely ab and ba, from those that are also *Sum* because they do satisfy M. This is helpful in order to provide a purely mereological distinction between structured and unstructured wholes. The basic idea is as follows. The relation between $a+b$ and ab is *sui generis*, since $a+b$ has as parts all and only the proper parts of ab, while being different from ab.[34] This difference—we submit—provides a distinction between a non-structured whole and a structured whole, both composed of the same parts, in particular of the same atomic parts.

Our general strategy, as we shall see, is to define the notion of composition in terms of the notion of *matter* (of a given entity), which is in turn defined in terms of *Sum*. As of now, we do not have any principle about the *existence* of *Sum*-s and we want to be as ecumenical as possible in this respect. Therefore we will simply suggest different *existence axioms* that are enough for the purpose of the paper, in that they all guarantee that the relevant *Sum* exists, while leaving the choice between them open. Consider:

Unrestricted Sum: $\exists x(x : X) \rightarrow \exists y(Sum(y, X))$
Restricted Sum 1: $X \ll x \rightarrow \exists y(Sum(y, X))$
Restricted Sum 2: $X \ll x \land \forall y(y \ll x \rightarrow y : X) \rightarrow \exists y(Sum(y, X))$

According to the first axiom every non empty plurality has a *Sum*.[35] According to the second one every plurality of proper parts of x has a *Sum*. Finally, according to the last axiom, *the* plurality of proper parts of x has a *Sum*. It is not difficult to see that all the downstream entailments go through, whereas none of the upstream entailment holds.

We are now ready to introduce the notion of *matter* of a composite entity.

Definition 3 Matter of a: $m(a)$.

Let X be the plurality of the proper parts of a.

$$m(a) = \begin{cases} \text{the unique } s \text{ such that } Sum(s, X), & \text{if } X \text{exists} \\ a, & \text{otherwise} \end{cases}$$

Thus, x is the matter of a when either a has proper parts and x is their sum or a is an atom and $x = a$. As an illustration, $a+b$ is the matter of ab, ba (Case 1), and a is the matter of a (Case 2). Importantly, all the *existence axioms* for *Sum* introduced above—even the weakest one—guarantee that, for any x, the matter of x exists.

[34] The same goes for *ba*.

[35] As we noted in Proposition 2, this is violated in the Infinite Comb Model.

Let us spend a few words on this disjunctive definition of matter and some of its consequences.[36] First, note that we do not pretend to furnish a comprehensive account here, since the consequences of this definition can be thoroughly appreciated only once a definite mereological system is specified, which we do not provide. In the present context, we will assume QSM as our basic system, and we remain neutral with respect to the three *Sum*-existence axioms above.

1. The matter of a non-atomic entity is the *Sum* of its proper parts.
2. This notion allows us to distinguish between *structured* and *unstructured* wholes. Indeed, structured wholes, like the syllable, can be identified with the ones that are *distinct* from their matter, whereas unstructured wholes are the ones that are identical with it. Hence, we will say that a is structured provided that $a \neq m(a)$ and that it is unstructured otherwise.
3. As a consequence, we get that atoms, sums of atoms and, in general, all proper sums—defined as sums of pluralities whose members are different from the sum itself—are unstructured wholes.[37]
4. The matter of an entity a is a very *sui-generis* part of a: it is either its only improper part or the *maximal* proper part of it, being such that no other proper part of a has the matter of a as a proper part.
5. The matter of an entity is an *unsupplemented* part: every other proper part of the entity overlaps it. This is trivial for unstructured entities, a little less so for structured ones.[38]
6. In light of the above, it seems we have a thoroughly *mereological understanding* of the relation between an entity and its matter: either the entity is its matter, or its matter is the unique maximal proper part of that entity.[39] This is by no means a small feat, since we are now in a position to avoid the introduction of controversial notions, such as the notion of *constitution*, to cash out the problematic relation between an entity and its matter in the case of structured wholes.[40]

Before moving on to composition, let us address an issue about our framework raised by an anonymous referee. Aristotle's Syllable Model in Sect. 5.1 is such that it allows for the existence of an object $x = ab$, a plurality X, with members a, b, $a + b$, and a plurality Y, with members a, b, satisfying—abusing terminology—both of the following: $x \nleq Sum(X)$ and $Sum(Y) = Sum(X)$. The issue is that one may have no idea of what kinds of reasons can support the belief that there is an object x, having the Xs as proper parts, such that

[36] The proofs of these claims are not difficult and are left to the reader.

[37] Interestingly, in the Infinite Comb Model, every entity is unstructured

[38] Whether it is the only *unsupplemented* proper part depends on which further mereological principles one endorses.

[39] We should register an alternative. One can define a different notion m^* of matter by simply taking the first case in the foregoing. It would then follow that only structured entities have matter and that this matter is the *Sum* of their proper parts.

[40] Arguably, the *loci classici* are Baker (1997), Wiggins (1968), and Wiggins (2001). See also (Lowe, 2009). For an insightful critique see (Smid, 2017a). Smid (2017a) goes as far as calling Constitution theory, the "Standard View".

- (i) x is not a part of every object having all of the Xs as parts (and so is a structured object);
- (ii) the sum of a sub-plurality of the Xs is identical to the sum of the Xs (and so is the matter of X).

We agree that this is a crucial point.[41] To shed some light on it, let us note that point (*i*) should be endorsed by anyone who claims that the whole is more than just a sum of its parts, while point (*ii*) should not be problematic since, in that case, the Xs are precisely the Ys together with their sum, so that the sum of the Ys is just the sum of the Xs. To be sure, as the sum operation is associative and idempotent, we have that $Sum(Y) = (a + b) = (a + b) + (a + b) = Sum(a, b, a + b) = Sum(X)$. Furthermore, it is worth noting that our aim was not to present a knock-down argument in favor of either (*i*) and (*ii*), but simply to put forward an appropriate mereological framework for those who endorse one or both of them.

We are now ready to provide different notions of *composition* in terms of the notion of matter. First, we simply have:

Definition 4 General composition
$Com(X, a) := \forall x, y(x : X \land y : X \rightarrow x \parallel y) \land Sum(m(a), X)$
The Xs compose a if they are pairwise and their Sum is the matter of a.

It should be clear that this first notion is equivalent to the one given in definition 1.4—where Sum_i is replaced by Sum, and Com_i by Com. This is because the matter of an entity is the Sum of its proper parts. Still, one common complaint against this definition is that it is "blind to natural divisions" of a given whole into parts. Take an organism: you can divide it into its organs, cells, and atoms. A different divide is as follows: its heart, exactly 2 cells in its liver, exactly 8 atoms in its spleen, and the mereological remainder of those.[42] One can claim that a division into organs, cells, molecules and atoms is more natural than the gerrymandered division we envisage—we shall return to this shortly.

We can remedy this situation by defining a notion of conditioned composition.

Definition 5 ϕ-Composition
$Com_\phi(X, a) := Com(X, a) \land \forall x(x : X \rightarrow \phi(x))$
The Xs compose$_\phi$ a when they are ϕ and compose a.

[41] There are however some considerations in the literature on the metaphysics of concrete objects that one can point to. As far as we can see, one of the most telling examples comes from the literature on metaphysical emergence, where *emergent wholes*, that is composite objects with emergent properties, are usually characterized as being more than just a sum of their parts—see Wilson (2021). In such a case, we would say, emergent wholes are not mere sums, and the sums of their parts are their matter. In other words, emergent properties might provide us with one of the possible independent considerations we were looking for in our classification of objects. Indeed, in the literature on emergence, a layered picture of mereological composition similar to the one in Fig. 1 below is usually implicitly assumed —even if such a picture is not indispensable. The exploration of this fascinating topic is left for a further work.

[42] The sum of all the parts of the organism that are disjoint from the one listed before. Note that one needs a suitable mereology to ensure of the existence of such mereological remainder.

The notion of ϕ-composition is not blind to the *structure* that ϕ induces, so to speak. For example, suppose ϕ is "being a cell". Then, the organism in question will be "naturally" divided into its cells. One example of ϕ-Composition that is of particular importance in the context at hand is when ϕ is "being an atom". This gives us the notion of *Atomic Composition*:

Definition 6 Atomic Composition
$Com_A(X, a) := Com(X, a) \wedge \forall x(x : X \rightarrow \neg\exists y(y \ll x))$

It can now be proved that there are entities, namely *structured entities*, that are not the *Sum* of their components:

Proposition 8 *Suppose a is a structured entity. Then, if s is the Sum of a's components, then* $a \neq s$.

Proof Since a is a structured entity, $a \neq m(a)$. Since $m(a)$ is part of a, $m(a) \ll a$, and so a is a non-atomic entity. Since s is the *Sum* of a's components, $m(a) = s$, so that $a \neq s$. □

What goes for *Composition* goes for *Atomic Composition*. That is, an entirely similar argument—that builds on **Proposition 1**—establishes that structured entities are not the *Sum* of their *atomic components*.

This should be enough to fulfill the promises we made in Sect. 1. First, the notion of general sum helps us to distinguish between having atomic parts and being a sum of atoms, as witnessed by Aristotle's model, contra **Varzi 1**. Second, given the notion of ϕ-composition, we can distinguish between being the sum of and being composed by, insofar as, for suitable conditions, $Sum(a, X) \rightarrow Com_\phi(X, a)$ does not hold.[43] Proposition 8 establishes then that having atomic parts is not sufficient for being composed of atoms—exactly because a structured entity with atomic parts is not the *Sum* of its atoms. This simply means that, in the present context, **A1** does not entail **A3**, contra **Varzi 2**.

5.4 Sums, matter, structures: a concrete application

The discussion so far has been conducted at a fairly abstract level. In this section we propose a discussion of how the new notion of sum affects substantive questions about the mereological structure of concrete objects in the world, so to speak.[44] In

[43] This last distinction does not depend on our notion of *Sum*, since it is available also with respect to each of the Sum_i. Still, the present framework enables us not only to account for the difference between composition and sum, but also to highlight some specific cases that instantiate that difference, i.e., structured wholes and their matter.

[44] Let us be upfront in declaring that the section is not meant to provide an exhaustive account. Such an account deserves a paper in its own right. Here we will be content in sketching the first steps leaving an extended treatment of the problem for a future work. Thanks to an anonymous referee for pressing us on this point.

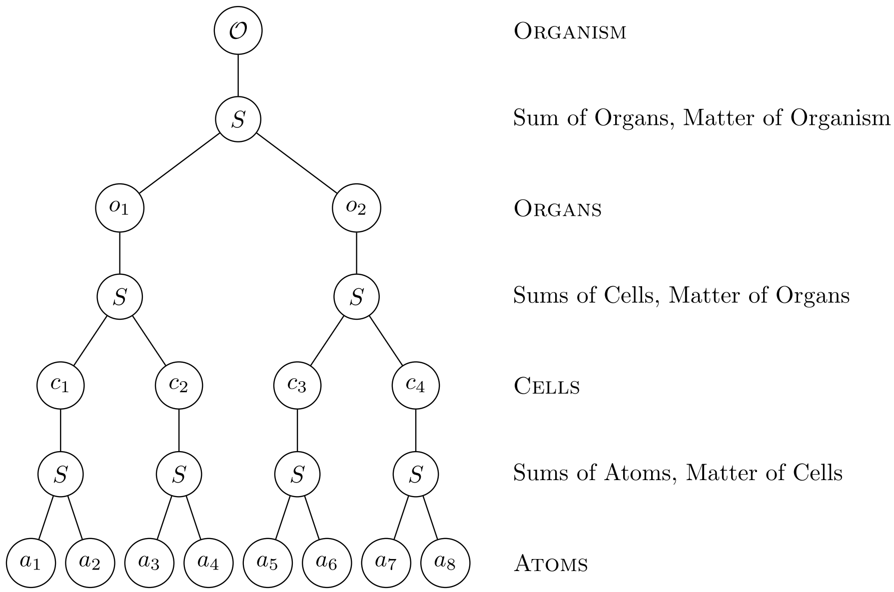


Fig. 1 Mereological structure of an organism

particular, we will try and show that, once this new notion is available, we are in a position to define *in purely mereological terms* the distinction between structured and unstructured wholes, and that structured wholes so defined are key in understanding *natural joints in the mereological hierarchy*. In order to clarify what's at stake, let us first consider how complex, concrete wholes are usually modeled in extant literature:

1. Wholes are "nothing over and above their parts".[45] Therefore there exists no principled distinction between structured and unstructured wholes, and there is no substantive mereological hierarchy. Mereology is not to blame for not being able to draw what turn out to be metaphysically shallow distinctions.
2. Wholes are "something over and above their parts". There exists a principled distinction between structured and unstructured wholes, and a substantive notion of mereological hierarchy. Mereology fails in both respects. It is able neither to draw the unstructured/structured distinction, nor to provide a satisfactory account of mereological hierarchy.
3. Wholes are "something over and above their parts". There exists a principled distinction between structured and unstructured wholes, and a substantive notion of mereological hierarchy. Mereology itself has the conceptual resources to account for both.

[45] For some elucidations of the "nothing over and above" locution and its possible different meanings, see Smid (2017b).

To the best of our knowledge, the first strategies are well-represented in the literature about the composition of concrete objects.[46] By contrast, the last strategy has almost no representative.[47] Our suggestion is that the system we developed in the paper with the new notion of *Sum* goes exactly in this direction. Before we provide some details, we should note that the sheer availability of such an account is already philosophically significant. For it shows that one needs not to abandon mereological monism—the view that there is just one notion of parthood—let alone endorse some form of hylomorphism to account for (some) mereological structures and hierarchies. That being said, we will now take a look at one concrete application of our system to a paradigmatic class of structured objects that are typically assumed to display mereological hierarchy: organisms.

We all are familiar with pictures where the levels of organization of an organism are displayed. In what follows we show how far we can go in distinguishing structured and unstructured wholes and capturing such levels, by comparing what we can say about the constitution of an organism in classical extensional mereology and what we can say about that constitution in a minimal system of mereology based on the novel notion of sum.

Classical extensional mereology In systems of classical extensional mereology, what we can say about the constitution of an organism is roughly the following.

1. An organism has parts. Indeed, we can say that an organism like a zebra is a composite object.
2. An organism has organs as parts. Indeed, we can say that the heart of the zebra is part of the zebra.
3. There is no principled mereological distinction between an organ and an arbitrary part of the organism. Indeed, organisms are sums of atoms, organs are sums of atoms, and arbitrary parts of an organism are sums of atoms. That is to say, every composite object, no matter whether it is a cell, an organ, or an arbitrary sum of gerrymandered parts of the organism, is just a sum of atoms. That suggests that one cannot draw a purely mereological distinction between (intuitively) structured objects—organs—and (intuitively) unstructured objects—arbitrary parts of

[46] See Cotnoir and Varzi (2021) Chapter 5 for an overall presentation of the problem and the positions. We just notice that the second point of view gives rise to two principal strategies. According to the first one, see Armstrong (1997), the right way to account for the distinction at issue is to acknowledge the existence of two kinds of composition: a mereological one and a non-mereological one. This allows us to define structured wholes as objects with non-mereological components. According to the second one, see Koslicki (2008), the right way to account for the distinction is to acknowledge the existence of non-concrete unifying parts, such as forms. This allows us to define structured wholes as objects with non-concrete unifying components. Other possible strategies—if less common—have been developed, such as e.g., *slot-mereology*. See Bennett (2013) and Sattig (2019).

[47] The only proposal we are aware of is presented in Canavotto and Giordani (2020). In that paper structured and unstructured wholes are indeed distinguished in mereological terms, but the basic mereological system which allows us to do that is a system which relies on a distinction between *two kinds of parts*: *actual parts* and *potential parts*. What we are interested in here is whether structures and hierarchies can be distinguished in a mereological system that involves *only* one notion of part.

the organism. Granted, some sums satisfy different predicates, such as "being a cell", or "being an organ". Suppose now that the division into (i) atoms, (ii) cells (iii) organs, (iv) organism corresponds to a robust hierarchical structure in the composition of the organism. Then, so the argument goes, classical extensional mereology is unable to account for that substantive hierarchy in mereological terms precisely because there is no principled *mereological distinction* between organisms, organs, and cells: they are all just sums of atoms.

4. This much is well-known: classical extensional mereology is (almost) blind to structure and hierarchy. They would need to be accounted for in non-mereological terms, if at all.

Minimal QS mereology In a minimal system base on *Sum* we can say that a structured whole is a whole which is different from its matter. This is not enough to separate organisms from other kinds of structured wholes, but is sufficient to separate organisms from the structured wholes they are composed of and to account both for the fact that composition is sensitive to levels and for the fact that composite objects are hierarchically structured. In fact, what we can say about the constitution of an organism is the following.

1. *Organisms have parts* and we are able to say that an organism like a zebra is a composite object.
2. *Organisms have organs as parts* and we are able to say that the heart of a zebra is part of the zebra.
3. *Organisms are structured entities* and we are able to say that there is a principled distinction between structured and unstructured entities: the former are distinct from their matter, whereas the latter are identical with it. An organism like a zebra is structured insofar as it is different from its matter—which is the often-raised objection against classical extensional mereology (which identifies them).
4. *Organs are structured parts of organisms* and we are able to say that an organ like the heart of a zebra is structured, insofar as it is different from its matter, and part of the zebra.
5. *The matter of an organism has organs as parts*. Indeed, the matter of a zebra is defined as the sum of its proper parts. The heart of the zebra is a part of that matter, for—as we saw—the heart of the zebra is a proper part of the zebra.
6. As a first consequence, we can say that the matter of an organisms has structured parts, e.g., organs. In general, *unstructured entities can have structured entities as parts*.
7. As a second consequence, we can say that organisms have structured parts, e.g., organs. In general, *structured entities can have structured entities as parts*.
8. And, in effect, in general there is no principled restriction on what kinds of objects (structured and unstructured) can have what kind of parts (structured and unstructured). The following table illustrates all the cases:

	Structured Wholes	Unstructured Wholes
Structured Parts	heart $\leq$ zebra	heart $\leq$ top half zebra
Unstructured Parts	right half heart $\leq$ zebra	right half heart $\leq$ top half zebra

9. As of now, we simply showed how to cash out the distinction between structured ad unstructured entities in mereological terms and we applied it to a paradigmatic case of a structured object, namely an organism. We showed that there are in principal no restriction when it comes to what kind of entities can be parts what kind of wholes. Let us now look at *mereological hierarchies*. Discussing classical extensional mereology, we suggested that it was unable to account for the hierarchical structure of (i) atoms, (ii) cells, (iii) organs and (iv) organism in purely mereological terms, at least insofar as all the "higher-level" composite entities have the same *mereological* status, they are all sums of atoms. We are now going to argue that our proposal fares significantly better. The basic idea is that every point where a difference between an entity and its matter occurs, that is, every time we pass from an unstructured to a structured entity, a new significant joint in the compositional hierarchy is added/reached. Consider the atomistic case, which is the most relevant in the present context. We start off with some atoms. Sums of atoms are the matter of cells. Sums of cells are the matter of organs. Finally the sum of the organs is the matter of the organism. The hierarchical division into (i) atoms, (ii) cells, (iii) organs and (iv) the organism that classical mereology was blind to is now clearly reflected in our mereology. We start from the atomic layer, and sums of entities in the previous layer constitute the matter of the entities in the new layer in the hierarchy. It goes without saying that the identification of the hierarchical structure that characterizes *any given entity* is a task which is left to the appropriate scientific discipline. Still, what is crucial is that *such identification can be related to the mereological structure of composition involving the identity or difference between an entity and its matter*.

They say that a picture is worth a thousand words. Let us look then at Fig. 1 below:

One immediately sees that, apart from the atomic level—the only one where there are no complex entities—every rung in the mereological ladder is represented by the "emergence" of a structured object, so to speak. This, we contend, establishes our claim: the hierarchy of composition is clearly reflected in our mereological account.

In this section we presented a detailed application of our new mereological account. First, we provided a few details on the mereological distinction between structured and unstructured objects we proposed. Then, we provided an application of that distinction to a concrete case, namely the composition of a paradigmatic structured object, an organism. The result is that its hierarchical structure is reflected and captured by our mereology. We do not claim that this constitutes a fully fledged mereological theory of structured entities, but we are confident that it provides an interesting, substantive first step towards a complete theory.

6 On extensional and atomistic mereologies

Before closing, let us address some interesting questions that arise in the light of the above. The first one is how to obtain an extensional system of mereology out of QSM. The second concerns the possibility of introducing a stronger supplementation principle in QSM. The third is how to recover SSM in its entirety. The final one is how to provide characterizations of Atomism.[48]

6.1 Back to an extensional mereology

We can obtain an extensional system of mereology by introducing the following principle.

Everything is Its Matterr: $\forall x(x = m(x))$

It is not difficult to see that this principle basically requires that everything is simply a *Sum*, and so that there are no structured entities. It turns out that this is sufficient to obtain extensionality, thus suggesting the hypothesis that extensionality is a feature that characterizes domains of structureless entities—like regions of space or spacetime. Let ESM be the system obtained by adding *Everything is Its Matter* to QSM. We can prove

Proposition 9 *ESM entails extensionality.*

It is enough to show that the following *Proper Part Principle* is provable in ESM. This is because it is well known that it entails extensionality.[49]

Proper Part Principle (*PPP*): $C(a) \wedge \forall x(x \ll a \rightarrow x \ll b) \rightarrow a \leq b$

Proof Since a is composite, a has proper parts, and so the plurality X of a's proper parts exists, by *Plural Comprehension*. Since $\forall x(x \ll a \rightarrow x \ll b)$, b has proper parts as well, and so the plurality Y of b's proper parts also exists, again by *Plural Comprehension*, and it is such that $X : Y$. Thus, $m(a) \leq m(b)$, by the definition of *Sum*, given that $Sum(m(a), X)$ and $Sum(m(b), Y)$, and finally $a \leq b$, by *Everything is Its Matter*. □

[48] We are indebted to [Redacted] for insightful discussions on these points.

[49] See Simons (1987) Note that the proof depends on Antisymmetry.

6.2 Adjoint supplementation

Let us now address the second question. In doing that, let us note that a principle like *WSP* fails just in light of the exceptional role played by structured wholes. In fact, the only unsupplemented entities are the structured wholes, and only with respect to their matter. This suggests the introduction of the following supplementation principle:

Adjoint supplementation (*ASP*): $a \ll m(b) \rightarrow \exists x(x \leq b \wedge x \parallel a)$

According to *ASP* the matter of a composite entity is the sole part of that entity which is not supplemented. We submit that QSM plus *ASP* is the mereology that better fits a world of structured entities.[50] Let us spend a few words on this. One of the intuitions weak supplementation is supposed to capture is the following. Consider any composite whole. Now "annihilate" one of its proper parts (and the proper parts of that proper part). Something *should* remain of the whole we started with, insofar as there is a mereological distinction between proper parts and whole. *Weak Supplementation* guarantees exactly that, because for every proper part of a whole, there is another that is disjoint from it, so that the disjoint part is surely capable to survive the aforementioned annihilation. There seems to be something here, and yet we already argued that we should not consider *Weak Supplementation* as e.g., analytic. This is exactly where the distinction between structured and unstructured wholes comes in. Our suggestion is that the intuition behind *Weak Supplementation* holds true *only with respect to unstructured wholes*, as previously defined. If a whole is identical to the *Sum* of its proper parts, then it seems that there should be something of the whole left if one annihilates one of its proper parts. By contrast, the intuition misfires when applied to structured objects as we defined them. That is because it seems controversial at best to demand that if one annihilates a particular proper part of the structured whole, namely its *matter*—that is the *Sum* of the proper parts of the whole—then a part of the whole should remain. Consider a simple, paradigmatic case, the statue and the clay, and assume that the clay is the matter of the statue. Why should we expect something of the statue to remain if we annihilate the clay? Or consider an organism. If we annihilate the hunk of matter it is composed of, why should we expect that a part of the organism remains? This discussion provides reasons for our suggestion. To see this take a look at *Adjunct Supplementation*. It is basically *Weak Supplementation* restricted to unstructured entities, namely those entities that are identical with their matter. Indeed, one can simply prove that in the presence of the *Everything is its Matter* principle *Weak Supplementation* and *Adjunct Supplementation* are equivalent.[51] In other words: structured objects are exactly those objects for which *Weak Supplementation* fails. And it fails precisely for a particular proper part of the structured objects, their unique unsupplemented

[50] It is worth noting that *ASP* is not strong enough to imply *Quasi supplementation*. To be sure, consider a model with two entities a and b such that $a \ll b$. In this case *Quasi supplementation* fails, while *ASP* is satisfied, since $a = m(b)$.

[51] Proof is trivial and left to the reader.

proper part, their matter. *Adjunct Supplementation* captures a similar intuition behind *Weak Supplementation* but restricting it to unstructured objects—hence the presence of the matter of an entity rather than the entity itself in its antecedent.

6.3 Back to strongly supplemented mereology

The third question we want to address is how to recover SSM. The crucial claim is that SSM is equivalent to ESM plus *ASP*. First, note that SSM is at least as strong as ESM, since it is stronger than QSM and *Everything is Its Matter* is provable in it.[52] Next, note that the following proposition is provable.

Proposition 10 *ESM plus ASP entails Strong supplementation.*

Proof Straightforward: $a \ll m(b) \rightarrow \exists x(x \leq b \wedge x \parallel a)$, by *ASP*, and so $a \ll b \rightarrow \exists x(x \leq b \wedge x \parallel a)$, since $b = m(b)$ for all b, by *Everything is Its Matter*. □

Therefore, QSM is to ESM as QSM + *ASP* is to SSM, so that extensionality marks the divide between a classical system of extensional mereology like SSM, and a system of mereology that allows for the existence of distinct structured wholes like QSM + *ASP*.

6.4 Atomism

Finally, let us also ask how to cash out different notions of Atomism. The following seems a straightforward suggestion:

$$\textit{Atomism}_1\text{: } \forall x(C(x) \rightarrow \exists X(X : \mathbb{A} \wedge Sum(m(x), X))$$

In plain English, *Atomism* $_1$ requires that for every composite entity, there is a plurality of atoms such that the matter of that entity is the sum of that plurality of atoms. Note that adding *Everything is Its Matter* with *Atomism* $_1$ one gets exactly **A2**, which we can take to provide a further notion of Atomism, to be spent in an extensional context:

$$\textit{Atomism}_2\text{: } \forall x(C(x) \rightarrow \exists X(X : \mathbb{A} \wedge Sum(x, X)) \equiv \mathrm{A}_2.$$

[52] Any atom coincides with its matter, by the definition of matter. Any composite entity coincides with its matter, otherwise the matter would be a unsupplemented proper part of that entity, which is impossible in SSM.

7 Conclusion

Let us sum up. We started off with the Shiver-Varzi debate. We were left with a vague, lingering impression that, while Varzi is provably right in claiming that the *Infinite Comb* is atomistic, this is not the end of the story. We then gave a precise shape to that vague impression by showing that, while Varzi's claims are justified in a mereological setting including suitable decomposition principles, in a framework like QSM it is possible for entities composed of atomic parts to be distinct from the sums of their atoms. This was our first significant result. In order to do that, we introduced a novel definition of sum which is robust, insofar as it coincides with the standard definitions on the market in mereologies with strong decomposition principles, and improves the standard definitions by excluding controversial cases of sum in mereologies where no decomposition principle holds. This was our second and more significant result. Indeed, in doing mereology, we are now given a notion of sum that is untouched by the kind of supplementation we want to adopt, thus being free to develop the composition-part of a system independently from its decomposition-part. Furthermore, we used the new notion of sum to understand Aristotle's syllable model and the distinction between structured and non-structured wholes. In fact, the notion of structured entity was defined in purely mereological terms and exploited in order to provide a novel contribution to the debate on the notion of composition. Finally, we showed how to recover mereological systems of different strengths.

And so we should know more about atoms, and about all the (un)structured things that are built from them. Or perhaps Margaret knew about it all already:

Small Atomes of themselves a World may make,
As being subtle, and of every shape:
And as they dance about, fit places finde,
Such Formes as best agree, make every kinde.

For when we build a house of Bricke, and Stone.
We lay them even, every one by one:
And when we finde a gap that's big, or small,
We seeke out Stones, to fit that place withall.

For when not fit, too big, or little be,
They fall away, and cannot stay we see.
So Atomes, as they dance, finde places fit,
They there remaine, lye close, and fast will sticke.

(M. Cavendish, ***Poems and Fancies, 1653***)

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