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Creditworthiness Assessment

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An MCDA-based Approach for Creditworthiness Assessment*

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Abstract. In this paper we propose a deterministic methodology for creditworthiness evaluation based on the Multi-Criteria Decision Analysis (MCDA) method known as Multi-criteria RANking MEthod (MURAME). This approach allows to rank the firms according to their credit risk characteristics and to sort them into a prefixed number of homogeneous creditworthiness groups. Moreover, the methodology allows to estimate ex-post proxies of the probabilities of default and of the probabilities of transition. Then, we apply the proposed approach to check its capability to evaluate the creditworthiness in real cases; in particular, we consider the case of an important north eastern Italian bank.

Keywords. Multi-Criteria Decision Analysis (or MCDA), MUlti-criteria RANking MEthod (or MURAME), credit risk assessment.

JEL Classification Numbers: C02, G20, G32.

MathSci Classification Numbers: 90C99.

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1 Introduction

Creditworthiness assessment of debtors and loan applicants is one of the main activities of financial institutions like banks and regulatory authorities. In short, it provides quantities for measuring credit features like the rating of obligor quality, the probability that a debtor does not fulfil her obligations in accordance with agreed terms, and so on (for more details see [3] and [17]).

Because of that, creditworthiness assessment constitutes the first step of any credit risk analysis, and credit risk in its turn is considered *«one of the fundamental factors of financial risk»* (see [3] at page V). Although this kind of risk plays all along an important rôle within the granting credit industry, mainly in the last 10–15 years it has been object of an increasing attention by academicians, operators, regulatory authorities and supervising institutions because of the rapid worldwide spread of financial derivatives and financial structured products able to improve the efficiency of its transfer among investors.

In this paper we propose a new deterministic approach for creditworthiness assessment based on the Multi-Criteria Decision Analysis (MCDA) method known as MUlti-criteria RAnking MEthod (MURAME) (for details on MURAME see section 2 and the references therein).

As known, MCDA methods provide support to various kind of decisions concerning with a discrete set of alternatives when the multidimensional nature of real-world problems and the decision maker's preferences have to be taken into account (for example see [20]). Therefore, their use in economic and financial contexts is fully appropriate.

MCDA methods have been applied to financial decision-making problems since the late 1970. Mainly, they tackled questions regarding analysis, selection and management of portfolio, asset analysis and asset evaluation, bond and loan rating, and assessment of various type of risks (for details see [19], [20], [18] and the references therein).

With respect to the topic considered in this paper, the contributions presented in literature are not so numerous as the importance of such a subject could suggest. Among the significant ones we mention [11], [7], [12], [8] and [2]. In [11], the authors propose a system based on the MCDA method ELimination Et Choix Traduisant la REalité (ELECTRE) TRI for rating the credit quality of firms and for sorting them into homogeneous creditworthiness groups. In [7] and [12], the authors investigate the applicability of the MCDA method Multi-group Hierarchical DIScrimination (MHDIS) in classifying firms applying loan into homogeneous groups of creditworthiness, and compare the obtained clustering with some standard classification technique results. In [8], the authors propose a model based on the MCDA method UTilités Additives DIScriminantes (UTADIS) for using publicly available data in order to replicate the assessment of firm creditworthiness assigned by regional rating agencies. In [2], the authors explore the potential of the MCDA methods UTADIS in developing systems for assessing credit risk of financial institutions by using publicly available data. All the quoted papers present real-world financial applications whose results are satisfactory.

Also our approach allows to rate the credit quality of firms and to sort them into a prefixed number of homogeneous creditworthiness groups. More, with respect to the mentioned methodologies, ours is characterized by at least two novelty elements:

- first, it is based on a MCDA method, the MURAME, which has never been used before for creditworthiness assessment;
- second, it allows to estimate ex-post proxies of the probabilities of default and of the probabilities of transition¹.

Development and use of credit risk modeling based on MCDA methods leave open a question that, to the best of our knowledge, has never been dealt with before. As well-known, the theoretical framework of reference for credit risk modeling is the stochastic one of modern quantitative finance (for instance see [3] and [17]). Therefore, why develop and use credit risk modeling based on methods like those taken into account in this paper? In our opinion there exist at least three reasons:

- first, within the framework of modern quantitative finance, the most important approaches to credit risk modeling, namely the structural and the reduced-form ones, are based on hypotheses about the financial markets which are, although classical, sometimes unrealistic. Typically, the structural approach *«assumes a Black-Scholes type frictionless market»* (see [17] at page 50), and the reduced-form approach assumes *«the framework of an arbitrage-free financial market model»* (see [3] at page 223). Therefore, credit risk modeling based on MCDA methods shows some usefulness in case real financial market do not satisfy these assumptions. Moreover, with specific reference to the structural approach, it is particularly based on *«the evolution of the firm's value and of the firm's capital structure»* (see [3] at page 26). In general, the firm's value is not directly observable, and it has to be *«derived from observable equity value[...]»* (see [17] at page 53). In other words, firm has to be listed. But, credit is very often applied by firms that, given their small or medium size, are not listed². Therefore, credit risk modeling based on MCDA methods shows significant usefulness also in case of no listed and, more generally, small and medium sized firms;
- second, also if real financial markets satisfy the posed classical assumptions, and if the firms applying loan were listed, the decision maker could have at her disposal information which are precluded to the financial markets themselves, and consequently which could not be used by the above-mentioned approaches (in other terms, the considered financial markets could be characterized by some kind of inefficiency). On the contrary, credit risk modeling based on MCDA methods can easily manage this kind of information in order to provide better credit risk analysis results or, when already existing, to improve them;
- third, credit risk modeling based on MCDA methods is very compliant with the new Basel capital accord (for instance see [4] and [1]). In fact, recalling that this accord

¹The probability of default is the probability that a debtor does not fulfil her obligation in accordance with agreed terms. The probability of transition is the probability that an obligor migrates over time from a creditworthiness group to another one.

²Notice that the wide majority of enterprises are small or medium sized. For example, they represent more than the 97% of the firms of the OECD member countries.

introduces the possibility for the banks to develop owner internal credit risk models to use together with those developed by external specialized agencies (like, for instance, Moody's Investors Service, or Standard and Poor's, or Fitch IBCA), this permits the decision maker to use in her credit risk analyses information which are not available to the external agencies themselves.

- finally, notice that credit risk modeling based on MCDA methods is easiest to understand by professional operators than the corresponding modeling based on refined concepts like stochastic process, martingale measure and so on. Because of that, its use can be particularly fruitful.

The remainder of the paper is organized as follows. In the next section we describe the MURAME. In section 3 we present our methodology for evaluating the creditworthiness of firms. In section 4 we check its capabilities when applied to real cases; in particular, we consider an important north eastern Italian bank. Some final remarks are given in the last section.

2 The MURAME

In this section we describe the MURAME, which has been originally proposed in [10] as a method for project ranking.

Let us starting by considering a set of m alternatives $\{a_1, \dots, a_i, \dots, a_m\}$ to be evaluated, and a set of n criteria $\{c_1, \dots, c_j, \dots, c_n\}$. In problems of creditworthiness evaluation, the alternatives are the debtors on loan applicants (individuals or firms), and the criteria are the various features according to which the credit risk may be evaluated.

Based on a preference structure of the decision maker which is properly modeled in a realistic way, the MURAME is implemented in two phases which take inspiration from two well known multicriteria methods: the ELECTRE III, proposed by B. Roy (see [15] and [16]) and the PROMETHEE II, proposed by J.P. Brans and P.V. Vincke (see [5]).

In the first phase, following the ELECTRE III method, an outranking relation is constructed in order to indicate the degree of dominance of a given alternative a_i with respect to another given alternative a_k , with $i \neq k$. Afterwards, according to the PROMETHEE II method, the outranking relation is used in the second phase to produce a total preorder of the considered alternatives.

In the following of this section we first analyze the modeling of the preferences, which leads to a double threshold preference structure, and then we briefly describe the two phases which provide as final result a complete ranking of the alternatives.

2.1 The double threshold preference structure

Let us compare two generic alternatives a_i and a_k , with $a_i \neq a_k$, according to the criterion c_j , and let us denote by $g_{i,j}$ the score of the alternative a_i evaluated by the criterion c_j .

As known, the classical preference structure assumes the following two relations for the pair of alternatives (a_i, a_k) when evaluated by the same criterion c_j :

$$\begin{aligned}
a_i \mathbf{P} a_k &\iff g_{i,j} > g_{k,j} \\
a_i \mathbf{I} a_k &\iff g_{i,j} = g_{k,j}
\end{aligned}
\tag{1}$$

where $\cdot \mathbf{P} \cdot$ and $\cdot \mathbf{I} \cdot$ indicate the preference and the indifference relation, respectively.

Nevertheless, this preference structure may often be too restrictive and may exclude some realistic cases. In fact, the preference between two alternatives is not often clearly defined, and consequently not so easy to identify, as the following very often cited example shows (for instance see [6]). Let us assume that a person wants to choose between two cups of tea, the first cup contains 10 mg of sugar and the second one contains 10 mg plus a grain of sugar. Since the quantity of sugar in the cups is different, according to the traditional preference model, person liking sweet beverages would prefer the second cup. But is able a normal person to perceive such a little difference?

In order to make the modeling of a preference structure more consistent with the real behaviour of the decision maker, some multi-criteria-based methods (*in primis* the ELECTRE family) consider the following “double threshold preference” structure:

$$\begin{aligned}
a_i \mathbf{P} a_k &\iff g_{i,j} > g_{k,j} + p_j \\
a_i \mathbf{Q} a_k &\iff g_{k,j} + q_j \leq g_{i,j} \leq g_{k,j} + p_j \\
a_i \mathbf{I} a_k &\iff |g_{i,j} - g_{k,j}| \leq q_j
\end{aligned}
\tag{2}$$

where $\cdot \mathbf{P} \cdot$ and $\cdot \mathbf{I} \cdot$ indicate, as before, the preference and the indifference relations, respectively; $\cdot \mathbf{Q} \cdot$ denotes a weak preference relation; p_j and q_j indicate a preference and an indifference thresholds, respectively, with $p_j \leq q_j$.

The introduction of such thresholds allow to take into consideration, beyond the case in which the decision maker is perfectly sure to prefer a given alternative with respect to another one and the case in which he/she is surely indifferent between two given alternatives, also an hesitation area in which he/she is not completely sure to prefer a given alternative with respect to another one (it permits the introduction of the concept of weak preference).

In fact, a crucial rôle in the multicriteria-based methods is played by these thresholds, whose values have to be fixed in advance by the decision maker.³ Of course, in credit risk evaluation problems, the presence of these threshold levels introduces some kind of flexibility in incorporating the preference structure of the credit managers.

By adopting the above described preference structure, the MURAME method (likes the ELECTRE-based ones) deals with an outranking relation which describes that also when the alternative a_i does not surely dominate the alternative a_k , the decision maker can still consider a_i better enough than a_k .

The following subsection leads to the construction of the outranking relation.

2.2 Phase I

The objective of this first phase is to build an outranking relation in order to evaluate the strength of the assertion “the alternative a_i is at least as good as the alternative a_k ”, for each pair of alternatives (a_i, a_k) .

³Notice that this could not be always an easy task.

The outranking relation is obtained in the steps which follow through the calculation of proper concordance and discordance indexes.

For each pair of alternatives (a_i, a_k) , a local concordance index $C_j(a_i, a_k)$ is constructed, in order to specify the dominance of a_i over a_k according to a given criterion c_j . Moreover, a discordance index $D_j(a_i, a_k)$ is also constructed in order to measure how much the hypothesis that a_i dominates a_k according to c_j is not satisfied.

In particular, the local concordance index is specified as follows:

$$C_j(a_i, a_k) = \begin{cases} 1 & \text{if } g_{k,j} \leq g_{i,j} + q_j \\ 0 & \text{if } g_{k,j} \geq g_{i,j} + p_j \\ \frac{g_{i,j} - g_{k,j} + p_j}{p_j - q_j} & \text{otherwise} \end{cases} . \quad (3)$$

Notice that:

- if $g_{k,j} \geq g_{i,j} + p_j$, then the decision maker prefers the alternative a_k to the alternative a_i (this is the case of strict preference). This entails that the local concordance index of the pair (a_i, a_k) reaches its minimum value as the alternative a_i is dominated by the alternative a_k ;
- on the contrary, if $g_{k,j} \leq g_{i,j} + q_j$, then the alternative a_k is not preferred to a_i (this is again the case of strict preference). This implies that the local concordance index of (a_i, a_k) reaches its maximum value;
- in the intermediate preference region, where the decision maker is not sure if he/she prefers a_k or a_i (the case of weak preference), the local concordance index takes values in the interval $(0,1)$.

Similarly, the discordance index $D_j(a_i, a_k)$ is specified as follows for each pair of alternatives (a_i, a_k) :

$$D_j(a_i, a_k) = \begin{cases} 0 & \text{if } g_{k,j} \leq g_{i,j} + p_j \\ 1 & \text{if } g_{k,j} \geq g_{i,j} + v_j \\ \frac{g_{k,j} - g_{i,j} - p_j}{v_j - p_j} & \text{otherwise} \end{cases} \quad (4)$$

where v_j is a veto threshold, with $v_j \geq p_j$, that is used to reject the fact that the alternative a_i is at least as good as the alternative a_k ; in this case the discordance index reaches its maximum value.

In particular, the veto threshold allows to treat the situation in which an alternative with a bad performance in a given criterion will receive a bad final score.

The indexes (3) and (4) are computed for each pair of alternatives (a_i, a_k) according to each considered criterion c_j . As a final result of phase I, the local concordance index and the discordance one are used in order to obtain an outranking index that indicates how much the alternative a_i outranks the alternative a_k jointly considering all criteria. Such an outranking index $O(a_i, a_k)$ for each pair of alternatives (a_i, a_k) is computed as follows:

$$O(a_i, a_k) = \begin{cases} C(a_i, a_k) & \text{if } D_j(a_i, a_k) \leq C(a_i, a_k) \quad \forall j \\ C(a_i, a_k) \prod_{j \in K} \frac{1 - D_j(a_i, a_k)}{1 - C(a_i, a_k)} & \text{otherwise} \end{cases} \quad (5)$$

where $C(a_i, a_k)$ represents a proper weighted mean of local concordance indexes; K denotes the subset of deponents of the criteria for which $D_j(a_i, a_k) > C(a_i, a_k)$.

In particular, this weighted mean is specified as:

$$C(a_i, a_k) = \frac{\sum_{j=1}^n w_j C_j(a_i, a_k)}{\sum_{j=1}^n w_j} \quad (6)$$

where w_j indicate the relative importance of each criterion.

Notice that the outranking index coincides with the aggregate, or global, concordance index if $C(a_i, a_k)$ is greater than $D_j(a_i, a_k)$ for all criteria. Otherwise, if there exist at least one criterion for which the discordance index exceeds the global concordance index, then the score results reduced to

$$C(a_i, a_k) \prod_{j \in K} \frac{1 - D_j(a_i, a_k)}{1 - C(a_i, a_k)}, \quad (7)$$

in which $C(a_i, a_k)$ is multiplied by as many factors as there are the criteria for which the discordance index is greater than the aggregate, or global, one.

Moreover, notice also that, if there exists even only one criterion for which there is maximum discordance (*i.e.* $D_j(a_i, a_k) = 1$), then the outranking index is equal to zero. This entails that, if for one of the given criteria the alternative a_i is “worse” than the alternative a_k , then it is not more possible consider a_i at least as “good” as a_k although this were true for all the remaining criteria. Under this point of view, the above described outranking approach is considered prudential.

2.3 Phase II

The outranking index (5) is used in this second and last phase in order to obtain a complete ranking of the alternatives. Following the PROMETHEE II methodology, for each alternative a_i , a leaving flow $\phi^+(a_i)$ and an entering flow $\phi^-(a_i)$ are computed as follows:

$$\phi^+(a_i) = \sum_{k \neq i} O(a_i, a_k), \quad (8)$$

$$\phi^-(a_i) = \sum_{k \neq i} O(a_k, a_i), \quad (9)$$

which indicate the strength and the weakness of a_i over the remaining alternatives, respectively.

Then, in order to obtain a total preorder of the alternatives, and not only partial ones (see [9]), for each alternative a_i , a new index measuring the net flow is computed as the difference between the leaving and the entering flows:

$$\phi(a_i) = \phi^+(a_i) - \phi^-(a_i). \quad (10)$$

In such a way the alternatives are ranked in a descending order, according to their net flows (10).

3 Our MURAME-based methodology for creditworthiness evaluation

In this section we develop a MURAME-based integrated evaluation methodology to rate the credit quality of firms and to sort them into a prefixed number of homogeneous creditworthiness groups. This methodology can be articulated in two steps which are described in the following subsections.

3.1 Step 1: the rating assignment phase

This phase is mainly addressed to the determination of the rating classes to which the credit applicants have to be assigned. In its turn, this phase is articulated in the following various subphases.

1.a) Specification of the kind of credit risk to adopt in the assessment process. At the beginning of the assessment process it is important to carefully define what kind of credit risk “idea” has to be considered in the analysis. In our approach we propose to divide the investigated firms in 10 rating classes (notice that the new Basel capital accord obliges to take into account at least 7 classes to which is added the default one). Within this frame, we consider as bankrupt firms the ones whose respective debts are “bad”/doubtful. Moreover, we also estimate ex-post proxies of the probabilities of default and of the probabilities of transition.

1.b) Subdivision of the firms in homogeneous classes with respect to their size. It is possible to divide the firms in classes by using the definition of Small and Medium Enterprises (SMEs) provided by the new Basel capital accord committee, following which the SME retails, the SME corporates and the large corporates can be classified on the basis of their sales proceeds, of their number of employees and so on (for example see [4]).

1.c) Detection of the evaluation criteria. This subphase plays a very crucial rôle: it is devoted to choose the evaluation criteria which can really affect the firms’ credit risk conditions. It is known in the business principles that many factors can determine the bankruptcy of a firm. Therefore, in order to perform an as better as possible assessment process, it is suitable to take into account all possible informative criteria: beyond the usual quantitative accounting ones (related, for instance, to the profitability, to the funding, to the liquidity of the firms), also qualitative criteria (related, for instance, to the management quality, to

the operational efficiency, to the generational replacement of the firms), and also macroeconomic quantities (related, for instance, to the economic sector, to the regional areas in which act the firms).

1.d) Determination of the reference profiles. The reference profiles⁴ can be determined following different approaches. For instance, they can be determined by a panel of experts and/or professionals. The approach we propose here does not need external assistance. In short: first, for each alternative we determine its sample deciles⁵; then, we construct the generic l -th reference profile by putting together the l -th deciles of all the alternatives. One of the merits of this approach consists in the fact the reference profiles are obtained by using only the actual data.

1.e) Determination of the threshold, of the veto parameter and of the weight values. As in the previous subphase, also the values of the preference and indifference thresholds, of the veto parameters and of the weights values can be determined by a panel of experts and/or professionals. Moreover, they can be also determined in some a subjective way.

1.f) Application of the MURAME. In this last subphase the MURAME is performed in relation to the data related to the investigated alternatives (*i.e.* the firms and the reference profiles). This permits to compute the outranking index by means of which to rank the firms applying for credit. Notice that the use of the reference profiles permits to separate contiguous creditworthiness classes and, consequently, to classify the firms in homogeneous rating classes.

3.2 Step 2: the rating quantification phase

This second phase is addressed to the estimation of ex-post proxies of the probabilities of default and of the probabilities of transition. Of course, it is evident that, in general, ex-post estimates have mainly descriptive importance. But, just as much evidently, once a suitably sized time series of ex-post estimates has been collected, they can be used for making (ex-ante) forecasts.

2.a) Ex-post estimation of the probabilities of default. For each homogeneous rating class, it is possible to estimate the ex-post proxy of the related probability of default in terms of relative frequency by computing the ratio:

$$\frac{\#f_{l,B}}{\#f_l}$$

where $\#f_{l,B}$ indicates the number of bankrupt firms belonging to the l -th creditworthiness class; $\#f_l$ indicates the number of all the firms belonging to the l -th creditworthiness class.

2.b) Ex-post estimation of the probabilities of transition. In order to estimate the ex-post proxies of the probabilities of transition we have to take into account only the not bankrupt

⁴A reference profile is a fictitious alternative, *i.e.* a fictitious firm, which is used as benchmark and, consequently, separates contiguous creditworthiness classes (for example see [11], [7], and [12]).

⁵Notice that we use deciles since we consider 10 creditworthiness classes. Of course, one has to use as many quantiles as the homogeneous rating classes are.

firms, and we need their data concerning with two consecutive business period. As in the previous subphase, it is possible to estimate these ex-post proxies in terms of relative frequency by computing the ratio:

$$\frac{\#f_{l,\overline{B},t}}{\#f_{m,t+1}}$$

where $\#f_{l,\overline{B},t}$ indicates the number of not bankrupt firms belonging to the l -th creditworthiness class in the first period; $\#f_{m,t+1}$ indicates the number of firms belonging to the m -th creditworthiness class in the second period (of course, l may, or may not, be equal to m).

Notice that, by construction, $\sum_{l=1}^{10} \#f_{l,\overline{B},t} = \sum_{m=1}^{10} \#f_{m,t+1}$.

4 A stress-testing analysis

In this section we use our MURAME-based integrated evaluation methodology in order to test its capabilities when applied to a meaningful real case. We utilize data provided by an important north-eastern Italian bank whose name is not given for reserve reasons.

The analysis regards the years 2001 and 2002, and for each year it takes into account 1000 firms which have obtained funding from the bank. In table 1 we report for each year the number of the firms that have been “healthy” (marked by a *flag* equal to 0), and of the firms whose debts have been “bad”/doubtful or non performing (marked by a *flag* equal to 1). Notice that, given this kind of labeling, it is not possible to distinguish the firms which have been strictly bankrupt (*i.e.* the ones whose debts have “bad”/doubtful) from the firms which have “simply” gone through a difficult period (*i.e.* the ones whose debts have been non performing). This makes harder than usual the task for our MURAME-based integrated evaluation methodology.

Moreover, the bank provided also information about the dimension of the firms: it considers as *small business* a firm whose sales proceeds belong to $[0.5, 3]$ millions of euros, and as *middle business* a firm whose sales proceeds belong to $(3, 100]$ millions of euros.

	2001	2002
0	856	839
1	144	161
#	1000	1000

Table 1: Number of firms which have obtained funding from the bank in the years 2001 and 2002, classified in “healthy” and not “healthy”.

In the following subsections we illustrate the criteria we use, present the way by which we determine the values of the thresholds, of the veto parameters, of the weights and of the reference profiles, and report the results.

4.1 The criteria

The criteria we use in the evaluation process are the same ones used by the bank and consist in a set of balance indicators. The choice to adopt the same criteria considered by the bank has been wilful in order to permit to compare under the same conditions our results with the ones of the bank. Any case, notice that our MURAME-based integrated evaluation methodology allows to take into account also qualitative and macroeconomic-based criteria that, if they were available, could improve the results.

The balance indicators we use as criteria are mainly related to the following aspects: profitability; funding and leverage; liquidity; growth; size. In detail:

Profitability related criteria

- I₁**: ratio between *interest expenses* and *total debt*. This indicator measures the firm's capability to remunerate the external capital.
- I₂**: ratio between *turnover* and *total debt*. This indicator evaluates the capability of the external capital to produce sales proceeds.
- I₃**: ratio between *operating income* and *total assets*. This indicator is a measure the auto-financing capacity of the firm, and is a reflection of the probability of the capital invested into the business;

Funding and leverage related criteria

- I₄**: ratio between *permanent capital* and *fixed assets*. This indicator measures the firm's capability to cope with the financial requirement resulting from immobilization investments through the use of permanent capital.
- I₅**: ratio between *permanent capital* and *total assets*. This indicator evaluated the portion of total assets financed by long term funding – shareholders' funds included –. To be significative, it should take values around 80% (for details see [13]).
- I₆**: ratio between *debt* and *shareholders funds*. This indicator is a measure of the firm's financial leverage.
- I₇**: ratio between (*capital – intangibles*) and (*assets – intangibles*). This indicator evaluates the degree of financial autonomy of the firm. To be significative, it should take values around 50% (for details see the web site <http://agevolazionifinanziarie.consrag.it/>).

Liquidity related criteria

- I₈**: ratio between *cash* and *total assets*. This indicator is a measure of the firm's liquidity. To be significative, it should take values around 20% (for details see [13]).
- I₉**: ratio between *cash* and *current liabilities*. This indicator is another measure of the firm's liquidity. If it assumes values greater than 1, it indicates a good liquidity level.

I₁₀: ratio between (*financial income + financial expenses*) and *cash flow*. This indicator indicates how much the financial management affects the total cash flow produced by firm’s management. Its ideal values should be 0 since it should imply that the firm is able to cover its financial expenses.

Growth related criteria

I₁₁: *year-over-year total assets difference*. This indicator allows of monitoring over time the variation of the invested capital.

I₁₂: *year-over-year net sales difference*. This indicator allows of monitoring over time the variation of the sales proceeds.

Size related criterion

I₁₃: *firm’s size*. This indicator classify the firm as *small business* or as *middle business*.

Generally speaking, an “healthy” firm should be characterized by high values of the indicators **I₂**, **I₃**, **I₄**, **I₉**, **I₁₁**, **I₁₂**, and **I₁₃**, and by small values of the indicator **I₁**. Therefore, in our stress-testing analysis we maximize the former indicators and minimize the latter one.

As far the indicators **I₅**, **I₇**, **I₈** and **I₁₀** concern, as above described it is preferable that they take values around predetermined optimal values. So, in order to penalize their divergences from their respective optimal levels, in our stress-testing analysis we minimize the following related criteria:

$$\tilde{\mathbf{I}}_5 = |\mathbf{I}_5 - 0.8|, \tilde{\mathbf{I}}_7 = |\mathbf{I}_7 - 0.5|, \tilde{\mathbf{I}}_8 = |\mathbf{I}_8 - 0.2|, \text{ and } \tilde{\mathbf{I}}_{10} = |\mathbf{I}_{10}|.$$

At least, a further different situation regards the indicator **I₆**. In fact, since it is computed as the ratio between the firm’s total liabilities – shareholders’equity excluded – and the shareholders’equity explaining the funding composition, considering the cost of funding, the firm amplifies the potential gain from an investment or project, but also increases the potential loss. In such a context it is fundamental to maintain a sound equilibrium among funding sources. In particular, if **I₆** takes values around 2 then the firm’s financial equilibrium is generally assured, whereas, if **I₆** assumes values meaningfully lower and higher than 2 then the firm’s financial equilibrium could become critical, although not in a symmetric way. So, following the relevant literature, in our stress-testing analysis we minimize the following related criterion:

$$\tilde{\mathbf{I}}_6 = f(\mathbf{I}_6) = \begin{cases} 0 & \text{if } \mathbf{I}_6 < 1.0 \\ 125\mathbf{I}_6 - 125 & \text{if } 1.0 \leq \mathbf{I}_6 < 1.8 \\ 100 & \text{if } 1.8 \leq \mathbf{I}_6 < 2.2 \\ -50\mathbf{I}_6 + 210 & \text{if } 2.2 \leq \mathbf{I}_6 < 4.0 \\ 10 & \text{if } 4.0 \leq \mathbf{I}_6 \end{cases} \quad (11)$$

Notice that also this criterion penalizes the departure from its optimal value.

4.2 Determination of the parameters, the weights, and reference profiles

Generally speaking, in the multi-criteria-based methods a tricky point concerns the choice of the values of the thresholds, of the veto parameters, and of the weights. The problem of determining appropriate values for all these parameters is investigated in the specialized literature (for example see [14]).

In our stress-testing analysis, following standard operational indications, for each criterion we determine the values of the parameters p_j , q_j and v_j in the way which follows:

- first, we compute the range $s_j = |\max(\mathbf{I}_j) - \min(\mathbf{I}_j)|$;
- then, we determine the values of the parameters in the following way:

$$p_j = \frac{2}{3}s_j, \quad q_j = \frac{1}{6}s_j \quad \text{and} \quad v_j = \frac{5}{6}s_j^6.$$

As far the values of the weights concern, we set them all equal since the bank has not revealed preference with regard to any of them.

At least, the reference profiles have been determined as described in the subsection 3.1.

	\mathbf{I}_1	\mathbf{I}_2	\mathbf{I}_3	\mathbf{I}_4	$\tilde{\mathbf{I}}_5$	$\tilde{\mathbf{I}}_6$	$\tilde{\mathbf{I}}_7$	$\tilde{\mathbf{I}}_8$	\mathbf{I}_9	$\tilde{\mathbf{I}}_{10}$	\mathbf{I}_{11}	\mathbf{I}_{12}	\mathbf{I}_{13}
r_1	0.02	3.66	0.42	0.43	0.20	2.74	0.21	0.10	86.00	0.20	1.20	0.07	1.00
r_2	0.02	2.43	0.25	0.27	0.10	2.23	0.14	0.14	55.10	0.29	2.00	0.14	1.00
r_3	0.03	1.83	0.18	0.18	0.05	1.97	0.11	0.17	21.30	0.35	2.83	0.22	1.00
r_4	0.04	1.48	0.12	0.12	0.03	1.77	0.08	0.18	10.00	0.41	3.85	0.35	1.00
r_5	0.04	1.25	0.07	0.07	0.02	1.59	0.06	0.19	10.00	0.46	5.14	0.49	1.00
r_6	0.05	1.09	0.03	0.03	0.01	1.45	0.05	0.19	10.00	0.51	7.30	0.70	2.00
r_7	0.05	0.93	0.00	0.00	0.01	1.29	0.04	0.20	10.00	0.57	9.50	0.99	2.00
r_8	0.05	0.78	-0.10	-0.10	0.00	1.14	0.04	0.20	10.00	0.62	13.50	1.50	2.00
r_9	0.07	0.58	-0.10	-0.10	0.00	0.89	0.02	0.20	10.00	0.68	25.20	3.36	2.00

Table 2: The reference profiles.

4.3 The results

We have performed 5 experiments which differ each from other for the criteria that have been taken into account in each of them. In table 3 we synthetically illustrate such experiments.

In the first one we have considered all the criteria. In the second, the third and the fourth experiment we have progressively excluded all the criteria coming from some kind of transformation of an underlying balance indicators (namely: $\tilde{\mathbf{I}}_5$, $\tilde{\mathbf{I}}_6$, $\tilde{\mathbf{I}}_7$, $\tilde{\mathbf{I}}_8$ and $\tilde{\mathbf{I}}_{10}$) in order to avoid possible biasing effects due to the transformation itself. At least, in the fifth and last experiment we have excluded only the last criteria (namely: \mathbf{I}_{13}), *i.e.* the one concerning with the firms' size.

⁶Notice that such a determination of the parameters satisfy the inequalities $q_j < p_j < v_j$.

	I_1	I_2	I_3	I_4	\tilde{I}_5	\tilde{I}_6	\tilde{I}_7	\tilde{I}_8	I_9	\tilde{I}_{10}	I_{11}	I_{12}	I_{13}	#
Exp. 1	•	•	•	•	•	•	•	•	•	•	•	•	•	13
Exp. 2	•	•	•	•	•		•	•	•	•	•	•	•	12
Exp. 3	•	•	•	•					•		•	•	•	8
Exp. 4	•	•	•	•					•		•	•		7
Exp. 5	•	•	•	•	•	•	•	•	•	•	•	•		12

Table 3: Experiments that have been performed. The • under a given criterion indicates that such a criterion has been considered in the experiment. The last column report the number of criteria taken into account in the various experiments.

Although the results related to each experiments are satisfying, in this subsection we report only some results of the last experiment since it is the unique “coherent” with the acting of the bank, in the sense that it takes into account exactly the same criteria considered by the bank’s experts in order to rate the firms applying for credit. In particular, it is important to notice that the results of this last experiment well agree with the ones of the bank. This indicates that our MURAME-based methodology for creditworthiness evaluation is able to incorporate in a satisfactorily way the preference structure of the decision maker.

In table 4 we report the ex-post estimates of the probabilities of default for both the considered years. As one should expect, in each year the default probabilities related to the first rating classes are small, and the default probabilities related to the last rating classes are considerably high. Moreover, one should also expect that these probabilities strictly increase as the rating classes increase. This is not always true for all the intermediate classes. Reasonably, it should depend on the fact that the bank does not distinguish between the strictly bankrupt firms (that generally belong to the last rating classes) and the firms whose debt are “only” non performing (that generally can belong also to intermediate rating classes). This shows the good classifying capabilities of our MURAME-based integrated evaluation methodology.

	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
2001	0.00	2.99	3.49	2.06	7.07	19.64	14.74	27.72	30.39	36.90
2002	0.00	1.58	2.11	8.89	7.84	10.42	24.74	29.31	29.00	44.79

Table 4: Values of the ex-post estimates of the default probabilities for the years 2001 and 2002. The columns represent the rating classes.

At least, in table 5 we report the ex-post estimates of the probabilities of transition. Recalling that the values along the principal diagonal represent the firms’ probabilities of permanence in the same rating classes for both the years, one notice that there exists a notably high probability (from about 25% to about 70%) that from 2001 to 2002 a firm moves from a rating class to the contiguous ones.

	c ₁	c ₂	c ₃	c ₄	c ₅	c ₆	c ₇	c ₈	c ₉	c ₁₀
c ₁	30.43	65.22	4.35	0.00	0.00	0.00	0.00	0.00	0.00	0.00
c ₂	4.84	75.27	11.83	3.23	2.15	0.54	1.08	0.54	0.54	0.00
c ₃	1.22	24.39	43.90	19.51	4.88	3.66	1.22	0.00	0.00	1.22
c ₄	1.09	7.61	21.74	34.78	26.09	2.17	4.35	2.17	0.00	0.00
c ₅	0.00	3.45	12.64	20.69	40.23	17.24	3.45	1.15	1.15	0.00
c ₆	0.00	1.14	2.27	6.82	20.45	39.77	15.91	9.09	2.27	2.27
c ₇	0.00	0.00	1.28	2.56	5.13	20.51	37.18	19.23	10.26	3.85
c ₈	0.00	0.00	0.00	1.43	2.86	15.71	11.43	41.43	17.14	10.00
c ₉	0.00	0.00	0.00	1.43	2.86	1.43	8.57	24.29	50.00	11.43
c ₁₀	0.00	0.00	0.00	0.00	0.00	1.96	5.88	13.73	17.65	60.78

Table 5: Matrix of the values of the ex-post estimates of the transition probabilities from the year 2001 to the year 2002.

5 Some concluding remarks

In this paper we have proposed a methodology for the credit risk assessment which is based on a deterministic multi-criteria approach which is known as MURAME.

It is possible to apply such an evaluation methodology also in case of no listed firms, and in case of small and medium sized ones. In particular, it allows to rank the firms according to their credit risk characteristics and to sort them into a prefixed number of homogeneous creditworthiness groups. Moreover, by means of the use of suitable reference profiles, this methodology allows to estimate ex-post proxies of the probabilities of default and of the probabilities of transition.

The use of a multi-criteria-based approach for creditworthiness assessment provides several advantages: the possibility to incorporate the decision maker preference structure; the possibility to take into account both quantitative and qualitative criteria; the fact that it is compliant with the new Basel capital accord; the fact that it is easy to understand by professional operators.

We have proved the applicability of the MURAME methodology that is able to obtain coherent results with those obtained by the investigated bank.

We have shown the usability of our MURAME-based integrated evaluation methodology when applied to a meaningful real case. In particular, it has been able to obtain results coherent with those obtained by an important north-eastern Italian bank.

It is to notice that other multi-criteria-based methodologies for credit risk assessment have been proposed in the literature. So, it should be interesting to compare the performances obtained by using ours and others of such multi-criteria-based evaluation methodologies.

At least, recalling that the excellence of the results of our integrated evaluation methodology also depend on a suitable choice of the values of the preference and indifference thresholds, of the veto parameters, and of the weights, it should also interesting to perform

sensitivity analyses with respect to these quantities.

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