Research in Economics 73 (2019) 35–52

Contents lists available at ScienceDirect

Research in Economics

journal homepage: www.elsevier.com/locate/rie

Research Paper

The political economy of public education

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ARTICLE INFO

Article history:
Received 5 February 2019
Accepted 5 February 2019
Available online 6 February 2019

JEL classification:
D72
H42
I21
I22

Keywords:
Probabilistic Voting
Education
Inequality

ABSTRACT

I study the relationship between income inequality and public spending in education in a voting model. Voters collectively choose the uniform quality level of public education, the amount of a public good, and the tax rate on labor income. Parents can decide to opt-out of the public education system by purchasing private education at the desired quality level, and children's expected income is assumed to be increasing in the quality of education. I show that higher income inequality is associated with higher governmental spending in education if and only if the expected marginal returns to education are larger for the children of relatively low income parents. In turn, better public education tends to reduce future inequality. These results are consistent with most findings in the empirical literature about public investment in education. Lastly, I show that for other kind of publicly provided goods, such as health care, the relationship with income inequality exhibits an ambiguous or opposite sign.

1. Introduction

What is the effect of an exogenous increase in income inequality on the level of public intervention in public education in a democratic country? Does such effect mitigate income inequality of future generations? This paper attempts to provide a theoretical framework to answer these questions. The relationship between the degree of governmental intervention in the provision of good and services and the features of the population in democratic political systems has been a major topic of research in Political Economy. Traditional models typically imply a positive relationship between the size of the intervention and income inequality (Meltzer and Richard, 1981). The reason is that such policies tend to have redistributive effects[1], thus an increase in the public provision favors the relatively low income part of the voting population. This has important consequence in a voting model because of two factors. First, an increase in income inequality is associated with an increase in the share and the political power of the relatively low income voters. Second, traditional models do not allow voters to access to other redistributive policies such as lump-sum grants because of technical constraints. Thus, unsurprisingly, such models usually imply a positive relationship between income inequality and the size of any kind of policy with redistributive effects. Empirical evidence suggests that this relationship may hold true only for certain kinds of policies, for instance public education, but it may not hold true for other policies with redistributive effects such as social security and public health. In this paper I attempt to disentangle voters' preferences for redistribution from their demand for public education by allowing them to choose both the size of in-cash redistribution -through a flexible tax system- and the quality of public education. This implies that the policy space is multidimensional. In this setting, the specific features of the public provision of education play an important role in determining the relationship between the size of public intervention and the degree of income inequality. Specifically, the presence of private alternatives to public education and the possibility of *opting-out*

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[1] Such effects are typical consequences of in-kind policies and are achieved even if no income redistribution occurs. For a definition see Appendix A.

https://doi.org/10.1016/j.rie.2019.02.001
1090-9443/

of the public sector are crucial in shaping the results. Because of these reasons, both a multidimensional policy space and the possibility of *opting-out* are essential features of this analysis. Unfortunately, both such modeling choices are source of well-know problems of existence of a voting equilibrium in the traditional deterministic Downsian framework. Thus, I adopt a Probabilistic Voting framework that allows one to tackle both issues, and I use it to study voters' behavior in a model of parental investment in education. I find that public intervention in education may be affected by income inequality not because of its redistributive effects, but because of the peculiar way in which the provision is delivered. I derive analytical conditions for a positive relationship between income inequality and quality of the publicly provided education. I find that the sign of this relationship is positive if the expected marginal returns to public education are decreasing in parental income. This is consistent with recent empirical evidence, and can be due to credit constraints that induce relative low income parents to underinvest in their children. Moreover, I show that if this condition is met, then an increase in the quality of public education reduces income inequality in the next generation.

The paper is structured as follows. In Section 2, I describe the findings of the empirical literature about the relationship between public provision of education and income inequality and how the theoretical literature has tackled this question. In Section 3, I present the voting model and the methodology I propose to study the sign of the relationship between the equilibrium level of public provision of a good of interest and the degree of income inequality in the population of voters. In Section 4, I apply these results to a model of parental investment in education in order to provide an answer to the main question of the paper. In Section 5, I compare the predictions in Section 4 with the one that the same framework would deliver for other kinds of publicly provided goods such as pure public goods and health insurance. Section 6 concludes highlighting the achievements and the limitations of this analysis.

2. Facts and literature

There is a large empirical literature about the relationship between income inequality and governmental spending in redistributive policies (see De Mello and Tiongson, 2006 for a review of this literature). On one hand the traditional theoretical literature typically predicts a positive relationship between income inequality and size of redistribution (Meltzer and Richard, 1981). On the other hand, empirical evidence provides mixed results. Perotti (1996) finds no relationship between inequality and redistribution in democracies. Using data from the U.S. General Social Survey, Lind (2007) finds that inequality between different groups reduces redistribution, while within group inequality increases it. A number of papers have found that support for redistribution and public goods provision is weaker in more unequal or more heterogeneous societies (Alesina et al., 1999; Alesina et al., 2001; Goldin and Katz, 1997, Luttmer, 2001). A more recent paper by Boustan et al. (2010) finds that rising inequality in cities and districts is associated with higher local revenue collection and expenditures. The question becomes even more challenging if one in interested in modeling the degree of public intervention in a specific policy with redistributive effects, such as public education. The literature about the relationship between income inequality and public spending in education is limited and provides mixed evidence. A majority of empirical studies find evidence of a positive correlation between income inequality and public intervention in schooling in cross-sectional studies about U.S. states. Easterly and Rebelo (1993) using cross sectional country data show that high level income inequality tend to be associated with future high level of public spending in education in the period 1970–1988. Sylwester (2000) also finds a weak but significant positive correlation between income inequality and future public spending in education, even if the issue of reverse causality in the relationship is not completely addressed in his paper. Conversely, Corcoran and Evans (2010) using a panel of U.S. school districts spanning 1970–2000 find a negative relationship between inequality and local spending in public education. Fig. 1 shows a small positive correlation between the pre-tax Gini index of income inequality in 2013 and the public expenditure per capita in education in the 50 U.S. States (American Community Survey, 2013).

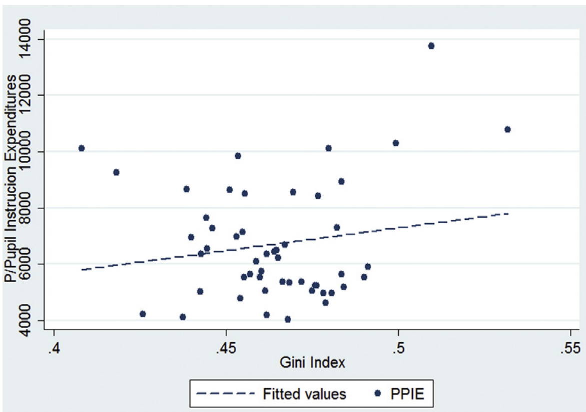

Fig. 1. Income Inequality vs Spending in Education in US States.
Data Sources: 2013 Annual Survey of School System Finances, 2013 American Community Survey

The observed correlation is weak and may be due to several sources of endogeneity. In particular, one that has been suggested in the literature is reverse causality. On one hand Political Economy models typically imply an important role for income inequality in shaping the degree of public intervention in certain policies. On the other hand, part of the literature in public economics suggests that a uniform public investment in education may induce a fall in the degree of future income inequality (see Coons et al., 1970; Sylwester, 2002). Nevertheless, the theoretical foundations of a positive effect of uniform public education on future income inequality have been challenged by the theoretical literature (Glomm and Ravikumar, 2003), and the empirical evidence about this second channel is mixed (see Abdullah et al., 2015 for a review of this literature).

The theoretical literature in Political Economy has relied mostly on a specific theoretical framework to answer this question. Specifically, unidimensional Downsian models (Downs, 1957) are usually adopted to study the questions of this paper. Such theoretical framework has the advantage that the policy chosen in a majority voting equilibrium corresponds to the ideal policy of a specific individual (*pivotal voter*). This implies that comparative static results are easy to derive (see Appendix A for an example). The effect of a rise in income inequality on public investment in education in Downsian models typically depends on some feature of voters' preferences and on how public intervention interacts with market choices. For instance, Ferná ndez and Rogerson (1995) show that in a model in which education is partially subsidized, poorer individuals may be excluded from obtaining an education and that increased inequality in the income distribution makes this outcome more likely. Glomm (2004) adopt the Downsian framework and finds that the relationship between inequality and the amount of redistribution through public education services depends on the elasticity of substitution between consumption and the quality of education in the parents' utility. He argues that for empirically relevant value of this parameter, higher inequality generates less redistribution. Stiglitz (1974) has pointed out that the use of Downsian models to study this question may be prone to some relevant theoretical issues. Namely, he has shown that if consumers are allowed to *opt-out* of the public service in presence of private alternatives, then a *Condorcet Winner* may fail to exist. In detail, the existence of a Condorcet Winner relies on the assumption that individual preferences satisfy some ordinal condition, such as *single peakedness*. Such condition usually fails to apply if the opting-out occurs or if the policy space is multidimensional. Moreover, even when this approach is successful in characterizing a Political equilibrium (see Epple and Romano, 1996a; Epple and Romano, 1996b; Glomm and Ravikumar, 1998; Gouveia, 1997; Naito and Nishida, 2012), it may be unsuitable to study comparative statics and deliver some paradoxical results. For instance, a change in income inequality typically has non-zero effect on the equilibrium level of public education even if, in absence of public intervention, all voters choose exactly the same level of education on the private market. The reason is that in order to achieve single peakedness such models assume a unidimensional policy space. This means that the degree of redistribution provided by the tax system is assumed to be exogenous. The uniform provision of a good financed by tax revenues has redistributive effects.[2] Thus, low-income individuals support a larger amount of public provision relative to the high-income simply because the model does not allow for other endogenous forms of redistribution. In other words, a relatively poor voter can only achieve redistribution through the provision of the good, thus she votes for larger level of provision relatively to the high income individuals. Lastly, higher income inequality increases the political power of the less well-off, and this translates into larger governmental intervention in education. Another way to think about the same mechanism is to notice that that in pivotal voter models -abstracting from possible externalities- the collective demand for public provision of the good is equal to the private demand for the good of an individual characterized by an income level and by a specific marginal tax-price for the good[3] If the identity of the pivotal voter changes, the marginal tax-price faced by the pivotal individual also changes. Thus, the sign of the overall effect depends on the relative size of income and price elasticities. This example suggests that in those models the relationship between income inequality and degree of public intervention in education is driven -at least to some extent- by the redistributive effects of the provision rather than by the specific features of the good. An intuitive way to tackle this problem is to include in the analysis at least another endogenous redistributive policy variable (for instance a uniform in-cash grant). This would allow one to disentangle the social demand for redistribution from the one for public intervention in education. Such modeling choice implies an increase in the dimensionality of the policy space. Unfortunately, the theoretical literature has shown that the conditions for the existence of a *Condorcet winner* are extremely restrictive if the policy space is multidimensional (See Grandmont, 1978; Plott, 1967). Because of this issue, a vast majority of Political Economy papers dealing with questions of similar nature adopt a unidimensional choice space.

The literature in Political Economy accounts for different classes of model that can tackle a less restrictive choice domain. Citizen-candidate models (Besley and Coate, 1991; 1997) allow for a multidimensional policy space but they are not suitable for studying the problem of interest because they do not usually deliver sharp prediction about the equilibrium policy. This is due to the multiplicity of equilibria that is a usual outcome in this models. More recent equilibrium concepts such as the Party Unanimity Nash Equilibrium (Roemer, 1999) also typically imply a large multiplicity of equilibria. Moreover, such models lack of a useful characterization of the policy chosen in equilibrium that can be used to derive analytical comparative statics results. Within this class of papers, the most relevant for this analysis is the work by Levy (2005). She proposes a new equilibrium concept based on coalitions in order to study how democratic societies choose the level of public

[2] In the sense that low-income individuals pay a lower tax-price for the good relatively to its market price. The tax-price is defined as total taxes paid by the individual divided by the size of the public provision.

[3] Defined as the increase in taxes induced by a marginal increase in the provision.

intervention in education if redistribution in-cash is also available to voters. She allows individuals to differ in their income and age and show that positive levels of provisions are possible in equilibrium. Nevertheless, such model is unsuitable for the aims of this paper because of two reasons. First, in order to keep the problem tractable, the model assumes only two income levels, thus only a limited class of comparative statics exercises can be performed. Second, the analysis does not allow for *opting-out*, which is one of the main aspects of our analysis and it is crucial in order to understand how individuals with different income levels are affected by the public provision of education. This paper adopts a Probabilistic Model because of three appealing features. First, such models deliver existence and uniqueness of a voting equilibrium under relatively mild restrictions even in multidimensional choice domains (Banks et al., 2005; Enelow and Hinich, 1989; Lindbeck and Weibull, 1987). Secondly, the probabilistic nature of voters' choice helps to smooth out the potential non-convexities in individual preferences induced by opting-out in our problem. Third, departing from a pivotal voter equilibrium in favor of a concept in which the equilibrium policy depends - in principle - on the entire distribution of voters' preferences, allows one to link the predictions of the model directly to some measure of income inequality, such as the variance of the income distribution. The latter aspect differs from traditional deterministic voting models, in which the feature of income distribution that is relevant for comparative statics is a measure of *skewness*, such as the mean-to-median ratio. The main shortcoming of this class of models in analysing problems similar to the one that is the object of this paper is the lack of analytical tools to study the effects of changes in the distribution of voter's characteristics and the policy chosen in equilibrium. If one excludes some specific cases in which the predictions collapse to some version of a pivotal voter result (Banks et al., 2005), the equilibrium policy depends on the preferences of all voters, thus analytical comparative statics results are not as straightforward to derive as in Downsian models. In two more recent papers de la Croix and Doepke (2009) and Arcalean and Schiopu (2016) exploit a probabilistic voting framework to study the relationship between income inequality and public intervention in education. They assume a parametric specification of the income distribution and of consumer preferences and a unidimensional policy space. They find that higher inequality decreases public spending per student and increases enrollment in public schools in poor economies, while the opposite holds in the rich ones. In this paper I propose a more general analytical result that links the variance of the income distribution and the equilibrium level of public intervention in a publicly provided good. I do not impose strong parametric restriction on voters' direct utility function other than quasilinearity in consumption of a composite private good and additive separability in other goods. Moreover, I allow for a more general income distribution, namely income is the sum of a continuously distributed variable with no parametric specification and a uniform i.i.d. component. Details about the voting models are described in the next section.

3. Probabilistic voting with non-convex preferences

In this section I will present a relatively simple model of Probabilistic Voting that is substantially similar to the ones that are prevalent in the literature, such as the one proposed by Lindbeck and Weibull (1987),Enelow and Hinich (1989) and Banks et al. (2005). The key feature of these models is that the vote of every individual (or type of individual) is not deterministic. This assumption eases dramatically the conditions for the existence of a Political Equilibrium when the policy space is multidimensional in comparison with Downsian models. The shortcoming is that the characterization of the equilibrium outcomes is not as simple as in Downsian models. In the next subsections I describe the setup of the voting model and I provide sufficient conditions for existence and uniqueness of a political equilibrium in the case in which the interaction between public and private provision of a good leads to possible non-convexities in voters' preferences. Then I derive the sign of comparative statics of interest in such environment.

3.1. Setup

The voting population consists of a continuum of size 1 of consumer-voters. They differ from each other only in a unidimensional parameter w that is continuously distributed with c.d.f. $\hat{R}(\theta, w)$ and p.d.f. $\hat{r}(\theta, w)$ for some parameter $\theta \in [0, 1]$. A feasible policy is a N-dimensional vector $x \in X$ where $X \in R^N$ is a convex set such that $X := \{x \in R^n\colon B(x) \le 0\}$ and $B(x) \le 0$ is a constraint that ensures the feasibility of the policy. There are 2 parties: A and B. Before the election the two parties simultaneously choose a feasible policy x^A and x^B, respectively. Denote with $v(x, w)$ the indirect utility induced by policy x to an individual with parameter w. Following Banks et al. (2005), I define the expected vote share of type w voters for party A given policies x^A, x^B as follows:

$$P^A(x^A, x^B, w) = \mathbb{P}[v(x^A, w) - v(x^B, w)]$$

where $\mathbb{P}(\cdot)$ is an increasing C^2 function. Hence the expected vote share for party A is:

$$V^A(x^A, x^B, \theta) = \int_{\underline{w}}^{\overline{w}} [\mathbb{P}(v^n(x^A, w) - v(x^B, w))]\hat{r}(\theta, w)dw$$

The expected vote share for party B given policies x^A, x^B is simply $V^B(x^A, x^B, \theta) = 1 - V^A(x^A, x^B, \theta)$. Each party maximizes the expected share of votes[4] Notice that a large numbers of voters implies that the actual vote share is equal to the expected

[4] Aranson et al. (1974) have shown that in Probabilistic Voting models this is equivalent to maximizing the expected plurality and, as the number of voters approaches infinity, it is also equivalent to maximize the probability of winning the elections.

share. So far, this setting is relatively standard and resembles the one in Banks et al. (2005). In the next paragraph I am going to impose additional restrictions to this model to allow for the interaction of public and private provision of a good.

3.1.1. Interaction between public and private provision

Denote with x_i the ith element of the policy vector x and suppose that x_i represents the degree of uniform public provision of a good that is also available on the private market. Examples of such goods are Education, Health Care, social security, etc. Publicly provided goods may differ in the way in which the provision is delivered. In particular one can distinguish the following cases. (i) Exclusive provision (socialization of commodities). The publicly provided good is not available on the private market. This is typical in the case of some pure public goods such as national defense (an example is Usher, 1977). (ii) *Top-up* goods. For this kind of goods the nature of the consumer choice is *quantitative*. Individuals receiving a certain level of public provision can decide to supplement this quantity with private purchases. A typical example is Health insurance (see Epple and Romano, 1996a; Gouveia, 1997). (iii) *Opting-out* goods. The nature of the consumer choice is *qualitative* in this case, meaning that individuals can either enjoy the publicly provided good or purchase a different level of quality on the private market (no supplementation occurs). This case is often claimed to represent a good description of the way in which public education is provided in several countries (Epple and Romano, 1996b; Stiglitz, 1974), although some supplementation may occur. In this section I propose a general setting that allows for the interaction of Public and Private provision of a good in the Probabilistic Voting Model described in the previous section. This setting applies for all cases (i); (ii) (iii) mentioned above. In Sections 4 and 5, I describe the different implications of these three cases. First, consider the indirect utility of an individual with income w:

$$v(x,w) = \max\left[v^n(x,w), v^m(x,w)\right]$$

where $v^m(x, t)$ is the indirect utility if the individual decides to purchase some positive amount of the good on the private market and $v^n(x, t)$ is the indirect utility of an individual that does make any private purchase for the good of interest. Notice that, even if $v^n(x, w)$, $v^n(x, w)$ are concave functions, the function $v(x, w)$ may be neither differentiable in all the points of his domain nor concave. In order to keep the problem tractable I will assume that $v^m(x,w) - v^n(x,w)$ is monotone weakly increasing in w $\forall x, w$. This assumption implies that for each vector of policies x either $v^m(x, w) \leq v^n(x, w)$ for all z -i.e. no opting-out occurs-, or $v^m(x, w) > v^n(x, w)$ for all w -i.e. all individuals opt-out-, or there exists$\hat{w}(x)$ such that

$$v(x,w) = \begin{cases} v^n(x,w) & if \quad w \leq (\geq)\hat{w}(x) \\ v^m(x,w) & if \quad w > (\leq)\hat{w}(x) \end{cases}$$

Party A's objective function becomes:

$$V^A(x^A, x^B, \theta) = \int_{\underline{w}}^{\hat{w}(x^A)} [\mathbb{P}(v^n(x^A, w) - v(x^B, w))]\hat{r}(\theta, w)dw + \int_{\hat{w}(x^A)}^{\overline{w}} [\mathbb{P}(v^m(x^A, w, m) - v(x^B, w))]\hat{r}(\theta, w)dw$$

Following Banks et al. (2005), in order to show that the voting game as a unique Nash equilibrium in pure strategies, one has to show that (i) X is compact and convex, (ii) for each w, $\mathbb{P}[v(x^A, w) - v(x^B, w)]$is jointly continuous in (x^A, x^B), (iii) for each x^A and x^B, $V^A(x^A, x^B, \theta)$ is strictly concave in x^A and $V^B(x^A, x^B)$ is strictly concave in x^B. In the setting proposed in this paper, (ii) is ensured because $\mathbb{P}$ is continuous and $v(x^A, w)$, $v(x^B, w)$ are jointly continuous in (x^A, x^B). So one need to show the condition under which (i) and (iii) are satisfied. Regarding (i) the condition is not trivially satisfied if one of the good is publicly provided. Specifically, X may not be a convex set because $B(x)$ may fail to be a convex function. This can be the case, for example, for opting-out goods (see Epple and Romano, 1996b). In the next sections, I am going to show that convexity holds in the applications of this paper. Regarding (iii), $V^A(x, x^B)$ ($V^B(x^A, x, \theta)$) is strictly concave in x for all $x \in X$ if the Hessian matrix $H_V^A(x)$ ($H_V^B(x)$) is negative definite. Lindbeck and Weibull (1987) have shown in a slightly different setting that in the case of concave indirect utility function this condition is satisfied if the distribution of $\mathbb{P}$ is such that $p'[v(x^A, w) - v(x^B, w)] \leq \bar{p}'$ for some positive $\bar{p}'$, where p' denotes the second derivative of the function $\mathbb{P}$. This result simply means that the function $\mathbb{P}$ is sufficiently "flat". Here we have an additional condition to be satisfied. One can show the following.

Theorem 1. *(Existence and Uniqueness). If there exist positive $\bar{r}$ and $\bar{p}'$ such that the distributions $\hat{R}(\theta, w)$ and $\mathbb{P}(d)$ satisfy $\hat{r}(\theta, w) \leq \bar{r}$ for all w and $p'[v(x^A, w) - v(x^B, w)] \leq \bar{p}'$, then there is a unique equilibrium in pure strategies. The unique electoral equilibrium is such that the two parties choose the same policy.*

Proof. See Appendix B.1. □

The additional condition simply states that the distribution of the individual parameter w does not have peak with excessively high density. Intuitively, this additional condition is required because a marginal change in the choice vector x in this case has an additional consequence relatively to the standard case described in Lindbeck and Weibull (1987). On one hand, there is a direct effect of the change in x due to the change in the expected voting behavior of each type w, similar to the one of the standard analysis. On the other hand, there is also an indirect effect. Namely, the threshold $\hat{w}(x)$ may change as a consequence of the change in policy, and the size of the effect of such change on the objective function depends on the density of the distribution in a neighborhood of $\hat{w}(x)$. If such density is sufficiently low, then the effect of the change in $\hat{w}(x)$ is dominated by the direct effect. If the additional condition $\hat{r}(\theta, w) \leq \bar{r}$ for all w is satisfied, not only existence and uniqueness of a Nash equilibrium in pure strategies are ensured, but also other properties of the standard framework hold. Specifically, it is possible to show that an important welfare result holds in this setting as much as in the convex case. Denote with $V(x,\theta) = \int_{\underline{w}}^{\overline{w}} v(x,w)\hat{r}(w,\theta)dw$ the utilitarian social welfare function and with $x^*(\theta)$ the policy vector that maximizes $V(x, \theta)$ subject to the governmental budget constraint, i.e. $x^*(\theta) = \arg\max_{B(x)\leq 0} V(x,\theta)$. One can state the following Theorem.

Theorem 2. *(Utilitarian outcome). The policy chosen by both parties in equilibrium is the same as the policy that would be chosen by an omnipotent Benthamite government, i.e.* $x^A = x^B = x^*(\theta)$.

Proof. See Appendix B.2. □

This result is very useful for the purposes of this paper because it reduces the study of the comparative statics of the equilibrium policy outcome to the one of the utilitarian social optimum.

3.2. Comparative statics

In this section I describe the way in which a change in income inequality is defined in this paper. I assume that individual productivity is given by the sum of two independent random variables $Z \perp \Sigma$ such that $w = z + \epsilon$. One can interpret this as the sum of a component due to parental and public investment in early life plus an idiosyncratic i.i.d. component. Variables z and ϵ have joint p.d.f. $r(z, \epsilon, \theta)$ in the form[5]:

$$r(z,\epsilon,\theta) = \{f(z) + \theta[g(z) - f(z)]\}\sigma(\epsilon)$$

In order to impose an exogenous variation to the degree of inequality, I adopt the concept of Mean Preserving Spread (MPS). Consider the marginal distributions of z at $\theta = 0$ and $\theta = 1$, given by $g(z)$ and $f(z)$ respectively. Denote with $G(z)$, $F(z)$ the correspondent marginal c.d.f.s. Distribution G is a MPS of F if and only if $E_g(z) = E_f(z)$ and $VAR_g(z) > VAR_f(z)$. Notice how this concept is much more general and easy to interpret in comparison with the mean-to-median ratio that typically drives the comparative statics in Downsian models. A mean preserving spread is imposed as follows: F, G are two c.d.f.s such that $\int_{\underline{z}}^{z} [G(z) - F(z)]dz \geq 0$ for all $z \leq \bar{z}$ (i.e. the distribution F Second Order Stochastically Dominates G), and $\int_{\underline{z}}^{\bar{z}} [G(z) - F(z)]dz = 0$ (i.e. z has same mean under G and F). The expected value of productivity is given by:

$$E(w) = \int_{\underline{z}}^{\bar{z}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} (z+\epsilon) r(z,\epsilon,\theta) d\epsilon dz = (1-\theta)\int_{\underline{z}}^{\bar{z}} z f(z)dz + \theta \int_{\underline{z}}^{\bar{z}} z g(z) dz + \int_{\underline{\epsilon}}^{\bar{\epsilon}} \epsilon\sigma(\epsilon) d\epsilon dz = E_f(z) + E_\sigma(\epsilon)$$

Notice that a change in θ preserves the average of w. Moreover, independence implies that $Var(z+\epsilon) = Var(z) + Var(\epsilon)$. Thus, the effect of moving θ in a neighborhood of $\theta = 0$ corresponds to the effect of increasing the variance of w keeping the mean constant. Moreover, the derivative of the equilibrium value x_i^* of a policy dimension i with respect to θ at $\theta = 0$ corresponds to the comparative statics of interest. Lastly, in order to understand the effect of a marginal mean preserving spread in the distribution of z on the equilibrium level of one policy variable, say x_i, one can use the simple monotone comparative statics result that follows. Consider a subset of policy dimensions with index $i \leq L < N$. Suppose that the utilitarian social welfare function can be written in the form in the form $V(x,\theta) = a(x,\theta) + \sum_{i<L} e_i(x_i,\theta)$ and the government budget constraint $B(x,\theta) = b(x,\theta) + \sum_{i\in L}\delta_i(x_i)$ for some twice differentiable functions $a, b, \{e_i, \delta_i\}_{i=1}^L$. Suppose that a, b are constant functions of x_i for all $i \leq L$.

Lemma 3. *(Monotonicity): If there exists at least one x_j with $N \geq j > L$ such that (i) the solution of the maximization problem is interior for x_j, (ii) $b(x, \theta)$ is such that $\frac{\partial b(x,\theta)}{\partial x_j} = \alpha\frac{\partial a(x,\theta)}{\partial x_j}$ for some constant α, and if (iii) $\frac{\partial^2 e_i(x_i,\theta)}{\partial x_i \partial\theta} \geq 0$ (≤ 0), then x_i is weakly increasing (decreasing) in θ in a neighborhood of $x^*(\theta)$.*

Proof. See Appendix B.1. □

This result is relatively restrictive, but it will prove useful for the purposes of this paper. In the next section I show that a simple model of public provision of a good financed by tax revenues satisfies the conditions of Lemma 3.

3.2.1. General publicly provided good

Suppose that w is a scalar individual parameter capturing some characteristics that are positively related to income, such as productivity. Income is a weakly increasing function y of w. P_i^m represents the unitary cost for the public sector of a level of public provision of good x_i for each individual that purchase a positive amount of the good on the private market. P_i^n is the cost for an individual that consume exclusively the public provision. Notice that if an individual cannot supplement the public provision, then $P_i^m = 0$. Conversely, if the public provision can be perfectly supplemented and the government also purchase the good on the private market, then $P_i^m = P_i^n$. Intermediate cases are possible (see Section 4.2). Denote with $\pi(x) \in [0, 1]$ the share of individuals that enjoys exclusively the public provision. Notice that $\pi(x)$ is not affected by θ under the assumptions stated in Section 3.2. Lastly x_l is the amount of another good that is provided by the government at price

[5] Notice that the distribution of w would have the following p.d.f. $\hat{r}(w,\theta) = \int_{-\infty}^{+\infty} \sigma(w-z) r(z,\theta)dz$.

P_l per unit. This captures other public spending, such as provision of public goods. The choice of opting-out is endogenous, thus the governmental budget constraint can be modeled in the form proposed here:

$$B(x) = -E_w[\tau(x,w) - \lambda(x,w)] + P_i^n x_i \pi(x) + P_i^m x_i (1-\pi(x)) + P_l x_l \leq 0$$

where x is a $N \times 1$ vector of policy variables and $\tau(x,w) - \lambda(x,w)$ is strictly convex in x. In this setting $\tau(x, w)$ represents the amount of taxes paid by an individual with productivity w under policy x and $\lambda(x, w)$ is a function capturing losses from taxation with $E_w[\lambda_{x_j}(x,w)] = \gamma E_w[\tau_{x_j}(x,w)]$ for some $j \neq i$, l and some constant γ. Lastly x_i is the level of provision of the private good of interest, and x_l represents the public spending in other goods and services. For instance, one may consider a tax system with a linear component and a lump-sum tax in the form $\tau(x,w) = x_1 w - x_2$ for $x_1 \in [0, 1]$ and $x_2 \in [0, E(w)]$, and loss function in the form $\lambda(x,w) = \hat{\lambda}(x_1 w) + \alpha x_2$ for some convex function $\hat{\lambda}$. Because of the presence of $\pi(x)$ the budget constraint may not be linear. I study a simple model with quasilinear utility in the form $U^n(c, x_i, x_l, w) = c + u(x_i, w) + d(x_l, w)$ for an individual that chooses to consume only the public provision, and in the form $U^m(c, x_i, x_l, w) = c + \upsilon(x_i, w) + d(x_l, w)$ for and individual that purchase some positive amount on the private market, in which c is the consumption of a composite private good. Notice that u, υ are (possibly non-constant) functions of the parameter w the reason of this assumption will become clear in the next sections. The corresponding indirect utility functions conditional on choice of provision are $v^n(x,w) = y(w) - \tau(x,w) + u(x_i, w) + d(x_l, w)$ and $v^m(x,w) = y(w) - \tau(x,w) + \upsilon(x_i, w) + d(x_l, w)$ respectively. Thus, the indirect utility of a voter is given by:

$$\nu(x,w) = \max[v^n(x,w), v^m(x,w)]$$

Lastly, assume that in the neighborhood of the equilibrium, either (i) $\bar{z} - \underline{\epsilon} \leq \hat{w}(x) \leq \underline{z} + \bar{\epsilon}$ or (ii) $\hat{w}(x) \in \{\underline{w}, \overline{w}\}$. Now in case (i) one can define a threshold level $\tilde{\epsilon}(x,z)$ that satisfies $v^n(x, z + \tilde{\epsilon}(x,z)) = v^m(x, z + \tilde{\epsilon}(x,z))$. In case (ii) one gets $\tilde{\epsilon}(x,z) = \bar{\epsilon}$ if $v^n(x, z+\epsilon) < v^m(x, z+\epsilon)$ for all ϵ; $\tilde{\epsilon}(x,z) = \underline{\epsilon}$ if $v^n(x, z+\epsilon) >< v^m(x, z+\epsilon)$ for all ϵ. The utilitarian social welfare is given by:

$$V(x,\theta) = E[y(z+\epsilon) - \tau(x, z+\epsilon)] + \int_{\underline{z}}^{\bar{z}} \int_{\underline{\epsilon}}^{\tilde{\epsilon}(x,z)} u(x_i, z+\epsilon) k r(z,\theta) d\epsilon dz$$
$$+ \int_{\underline{z}}^{\bar{z}} \int_{\tilde{\epsilon}(x_i,z)}^{\bar{\epsilon}} \upsilon(x_i, z+\epsilon) k r(z,\theta) d\epsilon dz - P x_i \pi(x,\theta) + \int_{\underline{z}}^{\bar{z}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} d(x_l, w) k r(z,\theta) d\epsilon dz$$

Assume that $r(z, \theta)$ and k are such that the conditions of Theorem 1 are satisfied. Theorem 2 implies that the unique equilibrium of the voting game corresponds to the vector of policies that maximizes $V(x, \theta)$. The quasilinearity of the utility function simplifies the analysis because it implies that the threshold $\tilde{\epsilon}(x,z)$ is a constant function of x_j for all $j \neq i$. Moreover it is easy to show that under the assumption stated $\pi(x) = k \int_{\underline{z}}^{\bar{z}} [\tilde{\epsilon}(x,z) - \underline{\epsilon}] r(z,\theta) dz$ is also a constant function of x_j for all $j \neq i$. An immediate consequence of Theorem 4 is that if the solution to the optimization problem is internal for x_i, then the sign of the effect of a marginal increase in θ on the equilibrium level of x_i is given by the following Theorem.

Theorem 4. *If the solution to the utilitarian social welfare maximization problem is interior x_k, and (ii) $\int_{\underline{z}}^{\bar{z}} [\upsilon_{122}(x_i, z+\bar{\epsilon}) - u_{122}(x_i, z+\underline{\epsilon})] \left(\int_{\underline{z}}^{z} [G(s) - F(s)] ds \right) dz \geq 0$, then x_i is weakly increasing in θ in a neighborhood of $x^*(\theta)$.*

Proof. Notice that $V(x, \theta)$ and $G(x, \theta)$ satisfy all the assumptions of Lemma 3 for $a(x,\theta) = E[y(z+\epsilon) - \tau(x, z+\epsilon)]$, $b(x,\theta) = -E[\lambda(x,w)]$, $e_i(x_i,\theta) = \int_{\underline{z}}^{\bar{z}} \int_{\underline{\epsilon}}^{\tilde{\epsilon}(x,z)} u(x_i, z+\epsilon) k r(z,\theta) d\epsilon dz + \int_{\underline{z}}^{\bar{z}} \int_{\tilde{\epsilon}(x_i,z)}^{\bar{\epsilon}} \upsilon(x_i, z+\epsilon) k r(z,\theta) d\epsilon dz$, $\delta_i(x_i) = P_i x_i \pi(x,\theta)$, $e_l(x_l,\theta) = \int_{\underline{z}}^{\bar{z}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} d(x_l, w) k r(z,\theta) d\epsilon dz$ and $\delta_l(x_l) = P_l x_l$. Thus then x_i is weakly increasing in θ in a neighborhood of the equilibrium policy x^* if $\frac{\partial^2 e_i(x_i,\theta)}{\partial x_i \partial \theta} \geq 0$, and weakly decreasing if $\frac{\partial^2 e_i(x_i,\theta)}{\partial x_i \partial \theta} \leq 0$. □

Notice that $\int_{\underline{z}}^{z} [G(s) - F(s)] ds$ is weakly positive by assumption, therefore the sign of the above depends on $\upsilon_{122}(x_i, z + \bar{\epsilon}) - u_{122}(x_i, z + \underline{\epsilon})$.

3.2.2. Interpretation as weighted average

Define function h over the support $z \in [\underline{z}, \bar{z}]$. Specifically, $h(z) = \frac{2 \int_{\underline{z}}^{z} [G(s) - F(s)] ds}{VAR_g(z) - VAR_f(z)} \geq 0 \ \forall z \in [\underline{z}, \bar{z}]$. It is easy to show that $h(z)$ is weakly positive for all $z \in [\underline{z}, \bar{z}]$, it integrates to 1 and it is inverse U-shaped. Thus, it can be interpreted as the p.d.f. of a distribution with support $[\underline{z}, \bar{z}]$ and with higher density for central values of z. Denote with E_h the expectation under such distribution and with $\tilde{c} = 0.5[VAR_g(z) - VAR_f(z)] > 0$. One can rewrite $\int_{\underline{z}}^{\bar{z}} [\upsilon_{122}(x_i, z+\bar{\epsilon}) - u_{122}(x_i, z+\underline{\epsilon})] \left(\int_{\underline{z}}^{z} [G(s) - F(s)] ds \right) dz = \tilde{c} E_h[\upsilon_{122}(x_i, z+\bar{\epsilon}) - u_{122}(x_i, z+\underline{\epsilon})]$, thus the condition above can thus be restated as follows.

Corollary 5. *If (i) the solution to the utilitarian social welfare maximization problem is interior for x_k, and (ii) $E_h[\upsilon_{122}(x_i, z+\bar{\epsilon}) - u_{122}(x_i, z+\underline{\epsilon})] \geq 0$, then x_i is weakly increasing in θ in a neighborhood of x^*.*

Proof. Straightforward from Theorem 1 and the definition of h. □

Corollary 5 delivers the conditions for local monotonicity of the outcome of interest. In the next section I show how these conditions have a useful economic interpretation if the model is applied to the study of public intervention in education.

4. Publicly provided opting-out good: public education

In this section I assume that the good of interest is an opting-out good, such as public education. It is publicly provided at a uniform quality level x_i and is also available on the private market at a continuum of different quality levels $q \in \mathcal{Q}$ at price Pq, with $\mathcal{Q} = [0, \bar{q}]$ for sufficiently large $\bar{q}$ (one may assume a discrete number of quality levels with no changes in the results, see Appendix C.1). Notice that with opting-out $P_i^m = 0$, i.e. an opting-out individual has no cost for the government. For simplicity I assume $P_i^n = P$, i.e. the price of quality on the private market is equal to the one faced by the government. The first consequence of the opting-out assumption is that the governmental budget set may not be linear. In order to understand why this is the case, consider the following way of modeling uniform provision of an opting out good. The total cost of providing the quality x_i is equal to the price per unit of quality $P = P^n$ times the quality x_i times the number of individuals that use the public service and the price per unit of quality P. Because the consumers-voters are a continuum of size 1, this means that the total spending in public education is given by $Px_i\pi(x_i) = x_iP\int_{\underline{z}}^{\bar{z}}\int_{\underline{\epsilon}}^{\tilde{\epsilon}(x,z)} kr(z,\epsilon,\theta)d\epsilon dz$. Lastly assume that w is the marginal productivity of a worker and income $y(w) = w$. This would be the case in a model with labor supply if the labor market is perfectly competitive and labor supply is perfectly inelastic. The governmental budget constraint is also more complex in comparison with the convex utility case because of the endogeneity of the threshold $\tilde{\epsilon}(x, z)$ as I described in Section 3.1. The budget constraint has form:

$$\int_{\underline{z}}^{\bar{z}}\left(\int_{\underline{\epsilon}}^{\bar{\epsilon}} -[\tau(x, z+\epsilon) - \lambda(x, z+\epsilon)]kd\epsilon + Px_i\int_{\underline{\epsilon}}^{\tilde{\epsilon}(x,z)} kd\epsilon\right)r(z,\theta)dz + P_lx_l \le 0$$

Consider the following simplified version of the Becker and Tomes (1986) model of parental investment in children's education, in which the utility of a parent with income w is a function of parents' consumption c, of public spending in public goods x_l, and of the expected income of their children w^s, i.e.:

$$U(c, x_i, w) = c + d(x_l, w) + \beta E[w^s(x_i, w)|w]$$

Suppose a child's future productivity w^s is a function of the quality of education and of the endowment $e(w)$ she receives from her parents, plus an idiosyncratic i.i.d. ability ν^s. For simplicity, I assume $e = w$. This formulation describes a simple transmission mechanism of human capital from parents to children. Education is provided by the government at uniform quality level x_i but other levels of quality q are available on the private market. Lastly, one may want to allow for positive spillovers of education. This can be the case, for instance, if there are *peer effects*. The formula for the productivity of a child with parents of type w and public education of quality x_i is given by the following formula:

$$w^s(x_i, w) = \begin{cases} \rho_{t+1}[H(x_i, w) + s(x_i, \phi) + \nu^s] & if\ \ public \\ \rho_{t+1}\left[\check{H}(\check{q}(w), w) + s(x_i, \phi) + \nu^s\right] & if\ \ private \end{cases}$$

where $\check{q}(w)$ represents the quality of private education chosen among the levels available in the set $\mathcal{Q}$, and H, $\check{H}$ and s are twice differentiable. The human capital production function of the child is allowed to differ between a child that attends public school (H) relative to one that receive private education ($\check{H}$). Assume that the first derivative $H_1(x_i,\ w)$ is finite for all x, w and that ν^s is independent of w, x and of the choice of the kind of education. Lastly, $s(x_i,\ \phi)$ represents the spillovers from other children's education. For instance, $s(x_i,\ \phi)$ could be the average human capital of other children $s(x_i, \phi) = \phi E_{w,\nu}[w^s(x_i, w)]$ and therefore $s(x_i, \phi) = \frac{\phi}{1-\phi}\left[\int_{\underline{z}}^{\bar{z}}\int_{\underline{\epsilon}}^{\tilde{\epsilon}(x_i,z)} H(x_i, z+\epsilon)kd\epsilon r(z,\theta)dz + \int_{\underline{z}}^{\bar{z}}\int_{\tilde{\epsilon}(x_i,z)}^{\bar{\epsilon}} \check{H}(\check{q}, z+\epsilon)kd\epsilon r(z,\theta)dz\right]$ for some $\phi \in [0,\ 1)$. The objective functions of a parent choosing public education or private education respectively have form:

$$\nu^n(x, w) = \xi w - \tau(x, w) + d(x_l, w) + \beta\rho_{t+1}[H(x_i, w) + s(x_i, \phi)]$$
$$\nu^m(x, w) = \max_{q\in\mathcal{Q}} \xi w - \tau(x, w) - Pq + d(x_l, w) + \beta\rho_{t+1}\left[\check{H}(q, w) + s(x_i, \phi)\right]$$

This corresponds to $u(x_i, w) = H(x_i, w) + s(x_i, w)$ and $\upsilon(x_i, w) = \check{H}[\check{q}(w), w] + s(x_i) - P\check{q}(w)$. Notice that because H is differentiable, then $\upsilon(x_i, w)$ is continuous and differentiable with respect to x_i, w under the assumption that $\mathcal{Q}$ is a continuum. The conclusions are unchanged if one allows for a discrete number of alternatives, see Appendix C.1. First of all, one can notice that equilibria with a positive level of public intervention are possible even if there are no externalities in consumption, i.e. $\omega = 0$ and the tax system allows for uniform lump-sum transfers. The reason is that, if some opting out occurs, the amount that can be rebated to the voters if the government stop providing the good is lower than the cost of purchasing the same amount on the private market (if available). Nevertheless, if the tax system fully flexible, a positive level of provision may emerge in equilibrium only if there are positive externalities in consumption. If a positive level of provision is chosen in equilibrium, then the sign of the comparative statics of interest can be derived using Corollary 5. It is straightforward to show that in this case $\upsilon_{12}(x_i, z+\bar{\epsilon}) = 0$, thus the condition (ii) in Corollary 5 reduces to $E_h[H_{12}(x_i, z+\underline{\epsilon})] \le 0$. Again, the sign of the comparative statics depends on the sign of H_{12}. An interesting interpretation of this condition follows. Define the Expected Individual Marginal Returns to Public Education: $MRE(x_i, w) = \frac{\partial E[w^s(x_i,w)|w]}{\partial x_i} = \rho_{t+1}[H_1(x_i, w) + \omega s(x_i)]$. Moreover, define the public spending per capita in education as the total spending divided by the size of the population, i.e. $PCE(x_i, \theta) = Px_i\pi(x_i, \theta)$. One can show the following.

Proposition 6. *If MRE(x_i, w) is decreasing in income for all x, w, then (i) the quality of public education x_i and (ii) the public spending per capita in education PCE(x_i, θ) are weakly increasing in θ in a neighborhood of $x^*(\theta)$.*

Proof. *(i)* One can derive the Expected Individual Marginal Returns to Public Education: $MRE(x_i, w) = \rho_{t+1} [H_1(x_i, w) + \omega s(x_i)]$, hence $\rho_{t+1} H_{12}(x_i, z + \underline{\epsilon}) = \frac{\partial MRE(x_i, w)}{\partial w}$. Substituting in condition (ii) of Corollary 5 one gets $E_h[u_{12}(x_i, z + \underline{\epsilon})] = E_h\left[\frac{\partial MRE(x_i, z+\underline{\epsilon})}{\partial w}\right]$, which is negative if $MRE(x_i, w)$ is decreasing in w for all x_i, w. *(ii)* The derivative including the political equilibrium change is: $\frac{dPPE(x_i,\theta)}{d\theta} = \frac{\partial PPE(x_i,\theta)}{\partial\theta} + \frac{\partial PPE(x_i,\theta)}{\partial x_i}\frac{dx_i}{d\theta} = P\frac{\partial x_i}{\partial\theta} \left[\int_{\underline{z}}^{\bar{z}}\left(\int_{\underline{\epsilon}}^{\tilde{\epsilon}(x,z)} kd\epsilon r(z,\theta)dt + \frac{\partial\tilde{\epsilon}(x,z)}{\partial x_i}k\right) r(z,\theta)dz\right] + Px_i\frac{\partial\pi(x_i,\theta)}{\partial\theta} = P\frac{\partial x_i^*}{\partial\theta}(\pi(x_i,\theta) + \frac{\partial\tilde{\epsilon}(x,\bar{z})}{\partial x_i})$. Result (i) implies $\frac{\partial x_i^*}{\partial\theta} \geq 0$. Moreover, $\frac{\partial\tilde{\epsilon}(x,z)}{\partial x_i} = \frac{H_1(x_i, z+\tilde{\epsilon}(x,z))}{\check{H}_2(q^*, z+\tilde{\epsilon}(x,z)) - H_2(x_i, z+\tilde{\epsilon}(x,z))} \geq 0$ because the denominator is positive by assumption. Thus, $PPE(x_i, \theta)$ is weakly increasing in θ in a neighborhood of $x^*(\theta)$. □

The result in Proposition 7 has an intuitive interpretation. That is, an increase in the political weight of relatively low income individuals imply a rise in the quality of public education if a better education reduces the intensity of the transmission mechanism of income from parents to children. This implication is potentially testable and can be the object of future empirical research. There are both theoretical and empirical argument in favor and against a negative value for such derivative. Some are related to aspects not explicitly modeled in this paper, such as credit constraints, partial supplementation, complementarities between parental education and children's returns to education, etc. In particular, one aspect that is important is the question if education and parental endowment are complement or substitutes in the creation of human capital. As explained in Becker et al. (2015), the question is strictly related to the one if government spending and parental investments are substitutes or complements in the production of human capital. Another aspect that is highlighted in the literature is the possibility that parents that wish to supplement public education with private spending face credit constraints. To understand why this may be important, it is useful to show a simple example. Suppose parents can supplement public education with private spending $s \in [0, \bar{s}]$, which is a substitute (perfect or imperfect) of the public spending in education in the form: $H(x, w) = \max_{s \in [0,\bar{s}]} \tilde{H}[x_i + a(s)] - \tilde{p}s$ subject to $w - \tau(x, w) - s \geq 0$. Say X is such that $\tau(x, w) = -\underline{b}$ for all $w \leq w^{\min}$ and some $\underline{b} \geq 0$ which means that low income households do not pay taxes, but they may receive a grant that ensure a minimum level of private consumption. Denote with $s^{int}(x_i)$ the optimal level of private spending of a parent that is not credit constrained. Lastly, suppose that $s^{int}(x_i) \leq \underline{b} + w^{\min}$, which means that all liquidity constrained individuals are not positive taxpayers under any policy $x \in X$. These two assumptions imply that the optimal level of private investment $s^*(x_i, w) = s^{int}(x_i)$ for $w + \underline{b} - s^{int}(x_i) \geq 0$ and $s^*(x_i, w) = w + \underline{b}$ otherwise. Then $H_{12}(x_i, w) = \tilde{H}''[x_i + a(w)]a'(w) < 0$ for all $w < s^*$ and $H_{12}(x_i, w) = 0$ for all $w \geq s^{int}(x_i)$. Because there are theoretical arguments in both directions, the question if the marginal returns to education are decreasing in income can only be addressed by empirical analysis. The estimation of returns to education is a classical exercise in applied economics that involves several issues, the analysis of which is beyond the scope of this paper. Nevertheless, it is interesting to mention the results of a few recent papers that have tried to disclose the relationship between returns to education and parental income in the data. Brenner and Rubinstein (2012) estimate a model of returns to a year of additional education and find that controlling for individual and family characteristics the returns to educations are decreasing in the quintile of parental income. Their findings suggest that individuals from low-income families have lower levels of educational attainment because they face higher costs of schooling, not because they cannot gain from further education. In particular a strong and statistically significant difference between the 1st and the 5th quintile is observed in all specifications, such that the returns to educations at the top quintile are less than 50% the ones in the lowest quintile. Although the concept of returns to education in their paper is not directly comparable with the one implied by this analysis, this result suggests that the sign of the above may be indeed negative. Conversely, other papers (Altonji and Dunn, 1996) find a positive relationship between parental education and marginal returns to education. Lastly, Card (2001) reviewing the literature about returns to education, find some support for the partial supplementation hypothesis by comparing OLS and IV estimates of the returns to education in several studies.

4.1. Effects on future generations

It is interesting to analyse how a change in the level of public intervention in education affects the income levels and income inequality of the future generations. This analysis provides a further possible interpretation of the condition derived in the previous section. In this section, I derive results about how the expected income, the variance and the coefficient of variation of the distribution of w^s change if there is a marginal increase in x_i. I can state the following.

Proposition 7. *If Var(Σ) is large enough, then (i) the expected income of the next generation E(w^s), (ii) the variance Var(w^s, θ) and (iii) the coefficient of variation CV(w^s, θ) of the income distribution of the next generation are all weakly increasing in x_i. Moreover, if the expected marginal returns to public education are weakly decreasing in parental income, then (iv) he total effect of a marginal increase in θ on Var(w^s) and CV(w^s) is ambiguous.*

Proof. See Appendix C.1. □

The interpretation of result (i) is simple. A marginal increase in the quality of public education has two effects in this setting. On one hand, it causes an increase in the future productivity of the individuals that choose public education for their children. On the other hand, it implies that some individuals with ϵ close to the cutoff $\tilde{\epsilon}$ may switch from private to public education, reducing the future income of their children. If the marginal density k of ϵ is sufficiently low, then the share of switching parents is low and the first effect dominates. Results (ii), (iii), (iv) imply that if a society has sufficient income inequality, then an exogenous shock that further increases such inequality is going to be mitigated in its effects on future generations. The intuition is that a rise in current income inequality has two opposite effects. On one hand, the mechanism of transmission of human capital imply that the direct effect may increase the inequality of the next generation. On the other hand, if marginal returns to education are higher - on average - for children from poorer families, then the increase in the degree of public intervention in education is going to cause a decrease in future income inequality.

4.2. Vouchers

Suppose that the government provides vouchers that cover the cost of education in public institutions, but can be also spent to partially cover the fees of private schools if parents decide opt-out of the public service. This setting is similar to the one proposed by Ireland (1990). In this case the indirect utility of an opting-out individual becomes $v^m(x,w) = w - \tau(x,w) - P(\check{q}(w) - x_i)1[\check{q}(w) - x_i \geq 0] + d(x_l, w) + \check{H}(\check{q}(w), w) + s(x_i, \phi)$, while the one of an individual that does not opt-out is unchanged. The government budget constraint is now simplified in the form $\int_{\underline{z}}^{\bar{z}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} [-\tau(x,z) + \lambda(x,z)] k d\epsilon r(z,\theta) dz + P_i x_i + P_l x_l \leq 0$ because the voucher is provided to everybody. Notice that in this case the presence of positive spillovers $s(x_i, \phi)$ is crucial to ensure a positive level of public provision. Specifically, for any positive level of public provision x_i, if (i) $s(x_i, \phi) = 0$, (ii) the tax system allows for uniform lump-sum transfers with no losses and if (iii) the same level of provision is also available on the private market, i.e. $\bar{q} \geq x_i$, then each individual can be weakly better off with a policy x' such that $x'_i = 0$ and $\tau(x', w) = \tau(x, w) - Px_i$. This implies in turn that no positive level of public intervention is chosen by voters in equilibrium[6]. If uniform lump-sum transfers are not allowed, or they are costly, or the private market does not provide all the quality levels that can be provided by the public sector, then the equilibrium may exhibit positive x_i even in absence of positive spillovers. This result suggests that the introduction of a voucher system may have unexpected effects and lead to a fall in public spending in education per capita. Regarding the comparative statics induced by an increase in θ, it is easy to show that, if $x_i > 0$ at the equilibrium, then the sign of the comparative statics is the sign of $-E_h[H_{12}(x_i, z + \underline{\epsilon})]$, as in the baseline case.

5. Comparison with other kinds of publicly provided goods

In this section I compare the results in Section 4 with the ones that this voting framework delivers if used to analyse other kinds of publicly provided goods. The aim is to show that the way in which the good is provided and consumed plays a crucial role in determining how income inequality affects the degree of public intervention in the provision of such good.

5.1. Exclusive public provision (pure public good)

Consider the case in which the provision of a certain good is exclusively public, either because of legal restrictions or because of a market failure. A typical example is National Defense. Following the structure of the previous section, one can model this case as follows. The individual indirect utility $v(x, w)$ is given by (a) $v(x,w) = v^n(x,w) = w - \tau(x,w) + u(x_i, w) + d(x_l, w)$ and $\tilde{\epsilon}(x,z) = \bar{\epsilon}$ for all z, x. The government budget constraint is simply linear in x_i in the form $P_i x_i + P_l x_l - E_w[\tau(x,w)] \leq 0$. Substitute (a) and (b) into condition (ii) of Corollary 5. The condition for a positive sign for the comparative statics of interest becomes (b) $E_h[u_{12}(x_i, z + \bar{\epsilon}) - u_{12}(x_i, z + \underline{\epsilon})] \geq 0$. Thus, one can state the following result.

Proposition 8. *The effect of a marginal increase in income inequality on the equilibrium level of a publicly provided good with exclusive public provision is ambiguous. If $u(x_i, w) = a(x_i)c(w)$, then the effect is weakly positive if c is concave, and weakly negative if c is convex.*

Proof. Straightforward from Corollary 5. □

Proposition 8 suggests that the strong relationship between income inequality and size of public spending in public goods usually implied by traditional Downsian models may not survive if one departs from the traditional deterministic framework. Moreover, It shows that - differently from traditional models - imposing restrictions on the sign of the cross derivative u_{12} is not sufficient to deliver a monotone comparative statics. As an example, compare this result with the one of the correspondent unidimensional Downsian prediction, and for simplicity set $\tau(x,z) = x_j \check{\tau}(w)$. It is easy to show that if $\check{\tau}$ is such that an individual with median income is a positive taxpayer and $u_{12}(x_i, w) \leq 0$ for all x_i, w, then $v(x, w)$ satisfies the *Spence-Mirrlees* condition and the level of public provision of the good would be weakly increasing in the median income, and strictly increasing if $x_j \check{\tau}(w)$ is not a lump-sum tax. This means that the skewness of the income distribution

[6] A small positive level equal to the lowest optimal private purchase at $x_i = 0$ may still prevail only if rising revenues does not imply net losses.

increases (i.e. the median income decreases at constant mean), this would translate into an increase in public spending in the good in equilibrium. Lastly, notice that the function u may exhibit different features depending on the kind of public good considered. For instance policing and other services that improve the protection of property rights may be more desirable by individuals with high productivity, who are likely to accumulate larger wealth. Conversely, for other public goods the direction of the relationship may be reversed.

5.2. Top-up goods

Now consider the case of a good that is uniformly provided by the government and such that consumers can supplement the public provision with private purchases from a set of available market options $\mathcal{Q}$. The set $\mathcal{Q}$ can have a discrete number of elements or it can be a continuum. In the case of *top-up* goods, the interaction between the public provision and private purchases of the good has a *quantitative* nature. Specifically, consumers care only about the total quantity of the goods they can consume, independently on the source of provision. Because of this, I am going to assume in this section that the (direct) utility from consuming the top-up good is solely a function of the quantity consumed. Thus, the indirect utility of individuals that does not supplement the public provision is given by $v^n(x, w) = w - \tau(x, w) + \hat{u}(x_i, w) + d(x_l, w)$, while the one of an individual that does supplement the public provision with a positive amount of public purchases is $v^m(x, w) = \max_{q \in \mathcal{Q}} w - \tau(x, w) - Pq + \hat{u}(x_i + q, w) + d(x_l, w)$, for some twice differentiable function $\hat{u}$ that is increasing and concave in its first argument. Because no opting-out occurs, the government budget constraint is linear in x_i, in the form $-E_w[\tau(x, w) - \lambda(x, w)] + P_i x_i + P_l x_l \leq 0$. Thus, the condition (ii) in Corollary 5 for a positive sign of the comparative statics becomes $E_h\left[\hat{u}_{12}(x_i + q, z + \bar{\epsilon}) - \hat{u}_{12}(x_i, z + \underline{\epsilon})\right] \geq 0$. In the next subsections I will analyse the consequences of top-up for the level of provision and the comparative statics at the political equilibrium.

5.2.1. Pure private goods: undifferentiated consumption good

If the good of interest is a pure private good, then each consumer's utility is affected only by its own consumption. Denote with $\bar{x}_i$ the maximum level of quality that can be provided by the government and with $q^*(x_i, w)$ the optimal quantity purchased on the private market by an individual with productivity w facing public provision x_i. Define $\check{q}(w) = \min_{w \in [\underline{z}+\underline{\epsilon}, \bar{z}+\bar{\epsilon}]} q^*(0, w)$.

Proposition 9. *If (i) $[0, \bar{x}_i] \subseteq \mathcal{Q}$ and (ii) the tax system allows for costless lump-sum transfers, then in equilibrium $x_i \leq \check{q}(w)$. Moreover, if (iii) rising tax revenues is costly, then no positive public provision of a pure private good occurs.*

Proof. Consider any level of spending Px_i that would emerge if a feasible policy vector $x \in X$ with $x_i > 0$ is chosen. Under assumption *(ii)*, a policy vector such that $\tau(x', w) = \tau(x, w) - Px_i$ for all w and $x'_i = 0$ is feasible. Such policy makes all voters weakly better off relative to x because all of them can choose a bundle that implies the same amount of consumption of both the *numeraire* and the pure private good. Thus all consumers are as well off as under policy x' only if $x_i \leq \check{q}(w)$. This implies that if x is such that $x_i > \check{q}(w)$, then $V(x', \theta) > V(x, \theta)$, thus x cannot be the policy chosen in a voting equilibrium. Lastly, if *(iii)* also applies, then $x_i > 0$ implies $V(x', \theta) > V(x, \theta)$, thus no positive provision occurs. □

This result implies that in this setting a private good is provided by the government only if the private market is not capable of providing all the level of consumption per capita that would be enjoyed if the good is provided by the public sector in some positive amount and/or the tax system does not allow for costless lump-sum transfers. In other words, if the private market is effective in providing the good and the tax system is sufficiently flexible, no pure private good should be publicly provided. If inefficiencies in the private provision or an excessively restrictive tax system imply a positive level of provision, notice that the sign of the comparative statics would be positive if and only if (a) $E_h\left[\hat{u}_{12}(x_i, z + \underline{\epsilon})\right] \leq 0$ in the case in which a continuum of alternatives is available on the private market (i.e. $\mathcal{Q} = [0, \bar{q}]$ for sufficiently high $\bar{q}$) and if and only if (b) $E_h\left[\hat{u}_{12}(x_i + \check{q}, z + \bar{\epsilon}) - \hat{u}_{12}(x_i, z + \underline{\epsilon})\right] \geq 0$ in the case of a discrete number of alternatives on the private market. About case (a), notice that the sign of the comparative statics is weakly negative if the publicly provided private good is a normal good. Regarding case (b), because the sign of $\hat{u}_{12}(x_i + \check{q}, z + \bar{\epsilon}) - \hat{u}_{12}(x_i, z + \underline{\epsilon})$ does not have a straightforward economic interpretation. Thus, one can conclude that the sign of the comparative statics is ambiguous. These results suggests that, if there is governmental intervention in the provision of a pure private good, one should not expect the size of this provision to be increasing in the degree of income inequality.

5.2.2. Imperfect public good with supplementation: public health insurance

It is well known that a positive level of public intervention in the provision of a good may be socially desirable if the good of interest exhibits positive externalities in consumption or it is an imperfect public good. The reason is that in such cases the level of consumption that is chosen by self-interested private agents tends to be suboptimally low on a social welfare point of view. Thus, if the loss due to underprovision is large relatively to the one induced by a uniform public provision, then a positive level of public intervention in the provision of the good characterizes the political equilibrium. A typical example of a good that is described in the literature as a top-up (see, Epple and Romano, 1996a; Gouveia, 1997) is Health insurance. Typically, the government can provide a certain level of service or insurance coverage, and consumer can purchase additional insurance on the private market. In this case the availability on the private market may influence the comparative statics, thus it is worth to analyse two cases. The first possibility is that a discrete number of options is available on the private market, i.e. $\mathcal{Q} = \{q_1, q_2, \dots, q_n\}$. In such case, similarly to what shown for a opt-out good, the sign of the comparative statics is the same as in the case in which only one private option is available, provided that such option is selected from $\mathcal{Q}$ as the one that maximize the objective function of an individual with income $w = \hat{w}$ when the policy chosen is the equilibrium policy (see Appendix C.1). In this case, I assume that individual utility is given by $U(c, x_i, w) = c + d(x_l, w) + \beta E[Health(x_i, w, \phi)|w]$, where *Health* is a function of the level of health insurance, of the individual parameter w, of the externality produced by the health of other individuals and by an i.i.d shock. It has form: $Health(x_i, w, \phi) = I(x_i + q, w) + s(x_i, \phi) + \nu$ where $q = 0$ if an individual does not top-up. I is a function that captures the effect of insurance on voters' health. The interaction between the level of health insurance and the parameter capturing productivity in the function I may be due to various reasons. For instance, individual with higher productivity may have different costs of illness relative to low productivity ones, or they may have a different probability of getting sick. Lastly, $s(x_i, \phi)$ represents the average level of health in the whole population, i.e. $s(x_i, \varphi) = \frac{\varphi}{1-\varphi}\left[\int_{\underline{z}}^{\bar{z}} \int_{\underline{\epsilon}}^{\tilde{\epsilon}(x_i, z)} H(x_i, z+\epsilon) k d\epsilon r(z, \theta) dz + \int_{\underline{z}}^{\bar{z}} \int_{\tilde{\epsilon}(x_i, z)}^{\bar{\epsilon}} H(x_i + \check{q}, z+\epsilon) k d\epsilon r(z, \theta) dz\right]$. The indirect utilities of individuals that top-up and do not top-up the public provision are given by

$$v^n(x, w) = w^f - \tau(x, w^f) + I(x_i, w) + s(x_i, \varphi)$$
$$v^m(x, w) = \max_{q \in \mathcal{Q}} w^f - \tau(x, w^f) - pq + I(x_i + q, w) + s(x_i, \varphi)$$

respectively. The condition (ii) in Corollary 5 for a weakly positive sign of the comparative statics of interest reduces to $E_h[I_{12}(x_i + q, z + \bar{\epsilon}) - I_{12}(x_i, z + \underline{\epsilon})] \geq 0$ if a discrete number of options is available on the private market (i.e. $\mathcal{Q} = \{q_1, q_2, \dots, q_n\}$) and to $E_h[I_{12}(x_i, z + \underline{\epsilon})] \leq 0$ if there is a continuum of private insurance available (i.e. $\mathcal{Q} = [0, \bar{q}]$ for sufficiently large $\bar{q}$).

Proposition 10. *(a) (Continuum of choices the private market). If (i) $\mathcal{Q} = [0, \bar{q}]$ for sufficiently large $\bar{q}$ and (ii) health insurance is a normal good for all income levels, then a marginal rise in income inequality has a weakly negative effect on the equilibrium level of the public provision of health insurance. (b) (Discrete set of choices on the private market). If (i) a marginal rise in income inequality has an ambiguous effect on the equilibrium level of the public provision of health insurance. If (ii) $I(x_i + q, w) = b(x_i + q)c(w)$ for some monotone functions b, c then the effect is weakly positive if c is concave, and weakly negative if c is convex.*

Proof. (a): denote with $q^*(x_i, w)$ the demand of private health insurance of an individual with productivity w given a level of public insurance x_i. Using the F.O.C. of the consumer's optimization problem, an individual that purchases a positive amount on the private market has income elasticity of demand $\frac{\partial q^*(x_i, w)}{\partial w} \frac{w}{q^*(x_i, w)} = -\frac{I_{12}(x_i + q^*(x_i, w), w)}{I_{11}(x_i + q^*(x_i, w), w)} \frac{w}{q^*(x_i, w)}$. This is positive if $I_{12}(x_i + q^*, w) \geq 0$. Thus, if q is a normal good for all w, then $E_h[I_{12}(x_i, z + \underline{\epsilon})] \geq 0$ which implies a weakly negative sign of the comparative statics. (b): (i), straightforward from the condition (ii) Theorem 2; (ii) notice that the condition becomes $E_h[b'(x_i + q)c'(z + \bar{\epsilon}) - b'(x_i)c'(z + \underline{\epsilon})]$ where b must be concave by assumption. Thus the expectation is weakly negative if c is concave. Proposition 10 implies that the sign of the relationship between income inequality and degree of public intervention in health insurance is ambiguous, and that under additional restrictions it is negative. This results shows that, if voters' preferences over a publicly provided good are separated from redistributive motives, then the effect of a shock on income inequality on the degree of public intervention depends solely on the characteristics of the good. Moreover, this result has important consequences for the traditional theoretical literature that analyses link between income inequality and size of public intervention in redistributive policies (e.g., Meltzer and Richard, 1981). If the marginal effect of an increase in income inequality may have different sign for different kinds of public intervention, then the total effect on the size of public spending in policies with redistributive effects may depends on various factors, including the relative size of public intervention in different kinds of policies at the political equilibrium. For instance, one may expect to observe a positive link between inequality and redistribution - as implied by the traditional literature - if in-cash policies and public education absorb a large share of the governmental budget, but this may not hold true if other form of public spending - such as public health insurance - are substantial. □

6. Conclusions

This paper provides a theoretical framework to analyse the effects of marginal shocks in income inequality on public spending in education in democratic countries. In order to separate voters' demand for public provision of education from their preferences for redistribution I assume a multidimensional policy space that includes as choice variables the quality of public education, the parameters of the tax system and the public spending in other policies. Moreover, following the literature, I assume that education is a good that is characterized by the possibility *opting-out*, which means that individuals can either enjoy the quality of public education, or purchase private education at one of the quality levels available on the market. The *qualitative* aspect of the consumer's choice is crucial, because the choice of *opting-out* implies the loss of the private benefits from the public provision. This implies in turn that preferences may exhibit non-convexities and that standard results in deterministic voting models may not hold. I adopt a probabilistic voting model similar to the one in Lindbeck and Weibull (1987) and Banks et al. (2005) to study how a marginal increase in income inequality - at constant mean - affects the equilibrium quality of education. This choice allows one to tackle the problems of existence of a political equilibrium induced both by the multidimensionality of the policy space (Grandmont, 1978) and by the presence of non-convexities in the objective function (Stiglitz, 1974), that are well-known in the theoretical literature. I show that the sign of the effect of a marginal mean preserving spread of the income distribution on the equilibrium quality of public education is positive if the expected marginal returns to public education are larger for children that have relatively low-income parents, which is in line with the findings in the empirical literature. Such literature suggests that this may be the case if public education can be partially supplemented by parental private investment and low income parents face credit constraint. Moreover, this condition is equivalent in the model to the case in which the degree at which income is transmitted across generation is lower if better public education is provided. Unsurprisingly, I can also show that, under the same conditions, better quality of public education implies lower income inequality in the next generation. This result suggests that exogenous shocks on income inequality may be mitigated in future generations by the endogenous adjustment in the degree of public intervention in education. Such predictions rely on the particular way in which this good is publicly provided and do not hold for other kinds of publicly provided goods, such as pure public goods and *top-up* goods. Specifically, I show that for a typical *top-up* good such as health insurance, a marginal increase in income inequality has ambiguous effects on the level of insurance provided by the government, and that under some mild additional assumption such effect is weakly negative. This suggests that the direction of the relationship between income inequality and total size of governmental intervention in redistributive policies may differ from the one implied by traditional models, and may depend on the relative size of the public spending in different kinds of public intervention. In other words, if one can write a sufficiently flexible model, then voters choose to redistribute in the most effective way - through the tax system - and redistribution motives do not strongly affect the size of the public intervention in other policies. Public education is an exception in this framework, because it is a policy that allows relatively low income voters to achieve income redistribution in the generation of their children. Because this kind of redistribution cannot be achieved through the tax system, relatively low income voters support high levels of public education in order to ensure higher consumption levels to their children. If that is the case, then the effects of a positive shock on income inequality are mitigated by the endogenous political choices both in the short run - through the tax system - and in the long run - through better public education, thus the consequences of shocks on income inequality are less dramatic for the society.

Acknowledgments

I would like to thank I. Preston and A. de Paula for the supervision of this work and V. Bhaskar, M. Bassetto, T. König, F. Squintani, and M. Vera-Hernandez for the useful comments and suggestions. I also thank seminar and conference participants at UCL and at Royal Holloway. I gratefully acknowledge financial support from ESRC and UCL.

Appendix A. Simple Downsian model

Consider the following simple Downsian model with n voters. Agents differ only in a unidimensional parameter $w \in W$ (income). There are two choice variables x_1, x_2 related by a convex governmental budget set X such that $X \equiv \left\{(x_1, x_2) | (x_1, x_2) \in R_+^2 \cap \tilde{B}(x_1, x_2, E(w)) \leq 0\right\}$. Individual preferences are represented by the indirect utility function $u(x_1, x_2, w)$, which continuous, increasing in x_1, x_2, w and twice differentiable in each argument. The Spence-Mirrlees condition states $M(x_1, x_2, t) = \frac{\partial}{\partial t}\left(\frac{u_1(x_1, x_2, t)}{u_2(x_1, x_2, t)}\right) > (<) 0 \, \forall x_1, x_2, t$. If such condition is satisfied, then the voting game has a Condorcet Winner, which is the individual with median w. Thus the social choice is given by $(x_1, x_2) = \arg\max_{(x_1, x_2) \in X} u(x_1, x_2, w^m)$ where w^m is the median of w. Because of the Spence-Mirrlees condition the preferred choice of an individual with parameter w is such that x_1 is increasing (decreasing) in w and x_2 is decreasing (increasing) in w. Hence the equilibrium social choice would also change in this way if the distribution of w is changed in such a way that the median voter has higher t and $E(t)$ is unchanged. This implies a monotone link between a measure of the skewness of the income distribution (in this case the difference the mean to median ratio). For instance, suppose $x_2$is a private good that in uniformly publicly provided (no private purchases are allowed in this simple example), after tax income is given by $w - x_1 \hat{\tau}(w)$ and the indirect utility function is $u(x_1, x_2, w) = u(w - x_1 \hat{\tau}(w), x_2))$. The government budget constraint is in the form $-x_1 E(\hat{\tau}(w)) + Px_2 \leq 0$ where $P_2$is the price of one unit of the good. Notice that in an interior solution the budget constraint is binding hence the problem is equivalent to $\max_{x_2 \in X} u(w - p(w, P)x_2, x_2)$ where $p(w, P)$ is the tax-price of the good defined as total amount of tax paid divided by the size of the provision, i.e.: $p(w, P) = \frac{x_1 \hat{\tau}(w)}{x_1 E[\hat{\tau}(w)]/P} = \frac{P\hat{\tau}(w)}{E[\hat{\tau}(w)]}$. The formula shows that the tax-price of the good is lower than the market price for all individuals that pay less taxes than average. If this is the case for the median income voter, this results in a positive level of public intervention in equilibrium. Moreover, this suggest the provision has redistributive effects, in the sense that after a positive provision is implemented, relatively lower income individuals can afford new consumption bundles, while for relatively high income individuals some bundles are not affordable anymore. The total effect of a marginal increase in income on the demand for public provision is given by:

$$\frac{dx_2^*(w, P)}{dw} = \frac{\partial x_2^M(w, \tilde{P})}{\partial w} + \frac{\partial x_2^M(w, \tilde{P})}{\partial P} p'(w, P)$$

where $x_2^*(w, P)$ is the demand for public provision of and individual with income w and $x_2^M(w, \tilde{P})$ is the private Marshallian demand of the same individual at price $\tilde{P} = p(w, P)$. Denote with $\eta_w^M(w, P)$ the income elasticity of the Marshallian demand for an individual with income w at price P, and with $\eta_P^H(w, P)$ the corresponding price elasticity of the Hicksian demand.

Also, denote with $\eta_w^{pub}(w, P)$ the income elasticity of the demand for public provision of and individual with income w. Using Slutsky equation one gets: $\frac{dx_2^*(w,P)}{dw}\frac{w}{x_2^*(w,P)} = \underbrace{\frac{\partial x_2^m(w,\tilde{P})}{\partial w}\frac{w}{x_2^m(w,\tilde{P})}[1 - p'(w,P)x_2^m(w,\tilde{P})]}_{A} + \underbrace{\frac{\partial x_2^h(w,\tilde{P})}{\partial P}\frac{p(w,P)}{x_2^m(w,\tilde{P})}\frac{wp'(w,\tilde{P})}{p(w,P)}}_{B} =$ which rewrites $\eta_w^{pub}(w,P) = \eta_w^M(w,\tilde{P})(1 - p'(w,P)x_2^*(w,\tilde{P})) + \eta_P^H(w,\tilde{P})\frac{wp'(w,P)}{p(w,P)}$. Because $\eta_P^H(w,P) < 0$ by the law of compensated demand, this implies that for any tax system in which the tax paid is increasing in income (i.e. $\hat{\tau}'(w) > 0$ which implies $p'(w) > \geq 0$), the income elasticity of the demand for public provision of the good is strictly lower than the income elasticity of the private demand at price $\tilde{P}$. Thus implies that the demand for public provision is going to be decreasing in w unless the good has sufficiently high income elasticity of private demand. Moreover, the higher is the progressivity of the tax system, the larger $p'(w, P)$. Hence if the tax system is progressive, $\eta_w^{pub}(w,P)$ tends to be negative implying that a decrease in the income of the median voter at constant mean will lead to a higher level of public provision in equilibrium. This may lead to paradoxical results. For instance, suppose that all individuals in the economy demand the same amount on the private market, i.e. $x_2^M(w,P) = \bar{x}_2^M(P)$ for all w. Nevertheless, the demand for public provision of the good will be decreasing in the income of the pivotal voter. The intuition is that a larger public provision is desirable for low income voters because implies more redistribution. Because such voters cannot achieve redistribution in cash because of the restrictions in the tax system, they support a larger public provision.

Appendix B. Existence and uniqueness with opting-out or top-up

Recall voters choose a n-dimensional policy vector and have indirect utility $v(x,w) = \max\{v^n(x, w), v(x, w)\}$ where $v^n(x, w)$, $v^m(x, w)$ are two differentiable functions of x, w and concave in x. Suppose $v_w^n(x,w) - v_w^m(x,w) \leq 0 \forall x, w$. Then for given x there is at most one $\hat{w}$ such that $v^n(x,\hat{w}) = v^m(x,\hat{w})$. This implies the existence of an endogenous threshold in w such that Party A's objective function becomes:

$$V(x^A, x^B, \theta) = \int_{\underline{w}}^{\hat{w}(x^A)} [\mathbb{P}(v^n(x^A, w) - v(x^B, w))]\hat{r}(\theta, w)dw + \int_{\hat{w}(x^B)}^{\overline{w}} [\mathbb{P}(v^m(x^A, w) - v(x^B, w))]\hat{r}(\theta, w)dw$$

B.1. Existence

Theorem 1. *(Existence, uniqueness and policy convergence). If there exist $\bar{r}$ and $\bar{p}'$ such that if the distributions $\hat{R}(\theta, w)$ and $\mathbb{P}(d)$ is such that $\hat{r}(\theta, w) \leq \bar{r}$ for all w and $p'[v(x^A, w, n) - v(x^B, w)] \leq \bar{p}'$for some positive $\bar{r}$, $\bar{p}'$, then (i) there is a unique equilibrium in pure strategies. The unique electoral equilibrium is such that (ii) the two parties choose the same policy.*

Proof. (i) Existence and uniqueness. Sufficient conditions for existence and uniqueness imply $V^A(x, x^B)$ being a (strictly) concave function of x and the inequality constraint $B(x, \theta) \leq 0$ is a continuously differentiable convex function. Define $V_{jk}^A(x^A, x^B)$ an element of the Hessian H_V, i.e. $H_V(j,k) \equiv V_{jk}^A(x^A, x^B) = \frac{\partial^2 V^A(x,x^B)}{\partial x_j \partial x_k}$. Denote with $d^l(x^A, x^B, w) \equiv v^l(x^A, w) - v(x^B, w)$ with $l \in \{n, m\}$. Then one can show that:

$$\begin{aligned} V_{jk}^A(x^A, x^B, \theta) = & \frac{\partial \hat{w}}{\partial x_k} p(d(x^A, x^B, \hat{w}))[d_j^n(x^A, x^B) - d_j^m(x^A, x^B, \hat{w})]\hat{r}(\theta, \hat{w}) \\ & + \int_{\underline{w}}^{\hat{w}(x^A)} \frac{\partial^2}{\partial x_j \partial x_k}[\mathbb{P}(d^n(x^A, x^B, w)]\hat{r}(\theta, w)dw + \int_{\hat{w}(x^A)}^{\overline{w}} \frac{\partial^2}{\partial x_j \partial x_k}[\mathbb{P}(d^m(x^A, x^B, w)]\hat{r}(\theta, w)dw \end{aligned}$$

Define a matrix $M(x^A, x^B)$ such that each element is $M_{jk}(x^A, x^B) = \frac{\partial \hat{w}}{\partial x_k} p(d(x^A, x^B, \hat{w}))[d_j^n(x^A, x^B) - d_j^m(x^A, x^B, \hat{w})]\tilde{\epsilon}_j(x,z)\hat{r}(\theta, \hat{w})$ and two matrices $H^l(x^A, x^B, w)$ for $l \in \{n, m\}$ such that each element is $H_{jk}^l(x^A, x^B, w) = \frac{\partial^2 \mathbb{P}(d(x^A, x^B, w))}{\partial x_j \partial x_k}$. Recall that the sum of negative semidefinite matrices is negative semidefinite. Hence one needs $x^T H_V x \leq 0$ for negative semidefiniteness. Using the matrices defined above can be written as:

$$x^T H_V x = x^T M(x^A, x^B)x + \int_{w}^{\hat{w}(x)} \left[x^T H^n(x^A, x^B, w)x\right]\hat{r}(\theta, w)dw + \int_{\hat{w}(x)}^{\overline{w}} \left[x^T H^m(x^A, x^B, w)x\right]\hat{r}(\theta, w)dw \leq 0$$

That can be written as $x^T[M(x^A, x^B) + E_W(H^n | w \leq \hat{w}(x))\pi(x) + E_W(H^m | w \geq \hat{w}(x))(1 - \pi(x))]x \leq 0$. Define $H_v^l \equiv D^2[d^l(x^A, x^B, w)]$ as the Hessian of individual indirect utility and $\nabla v^n(x^A, w)$ the gradient vector. Following Enelow and Hinich (1989) for the second and third component of $x^T H_V x$ we need for any $n \times 1$ vector y:

$$y^T H^n y = \int_{\underline{w}}^{\hat{w}(x^A)} \left(p'(d(x^A, x^B, w))y^T\left[\nabla v^n(x^A, w)\right]\left[\nabla v^n(x^A, w)\right]^T y + p(d(x^A, x^B, w))y^T H_v^n y\right)\hat{r}(\theta, w)dw \leq 0$$

And similarly one can derive $y^T H^n y$. Thus sufficient conditions for uniqueness are

$$\frac{p'(d^l(x^A, x^B, w))}{p(d^l(x^A, x^B, w))} \le -y^T H_{\upsilon}^l y \Big[\big[\nabla d^l(x^A, x^B, w) \big] \big[\nabla d^l(x^A, x^B, w) \big]^T \Big]^{-2}$$

for $l = n, m$ and for all w and $x^T M(x^A,\ x^B) x \le 0$. Notice that as $\mathbb{P}$ becomes close to uniform this condition is equivalent to the matrix $H_v(i)$ to be negative semidefinite, which is equivalent to a concave utility function. But in comparison with Enelow and Hinich (1989) we have an additional element: $M(x^A,\ x^B)$. Consider the definition of $\hat{w}(x^A)$:

$$\upsilon^m(x^A, \hat{w}) = \upsilon^n(x^A, \hat{w})$$

Differentiate this w.r.t. x_k and rearrange to get:

$$\frac{\partial \hat{w}(x^A)}{\partial x_k} = -\frac{\upsilon_k^m(x^A, \hat{w}) - \upsilon_k^n(x^A, \hat{w})}{\upsilon_w^m(x^A, \hat{w}) - \upsilon_w^n(x^A, \hat{w})}$$

Substituting into $M_{jk}(x^A,\ x^B)$ we get:

$$M_{jk}(x^A, x^B) = -p(d(x_A, \hat{t}, n)) \frac{[\upsilon_j(x_A, \hat{t}; n) - \upsilon_j(x_A, \hat{t}; m)][\upsilon_k(x_A, \hat{t}; n) - \upsilon_k(x_A, \hat{t}; m)]}{\upsilon_t(x_A, \hat{t}; n) - \upsilon_t(x_A, \hat{t}; m)} \hat{r}(\theta, \hat{w})$$

Hence

$$M(x^A, x^B) = -\frac{p(d(x^A, x^B, \hat{w}))}{\upsilon_w^n(x^A, \hat{w}) - \upsilon_w^m(x^A, \hat{w})} \big[\nabla \upsilon^n(x^A, \hat{w}) - \nabla \upsilon^m(x^A, \hat{w}) \big] \big[\nabla \upsilon^n(x^A, \hat{w}) - \nabla \upsilon^m(x^A, \hat{w}) \big]^T \hat{r}(\theta, \hat{w})$$

Hence the sufficient conditions for existence of a Political Equilibrium are the same as in Enelow and Hinich (1989), plus the additional condition stated above. Given that $\big[\nabla \upsilon^n(x^A, \hat{w}) - \nabla \upsilon^m(x^A, \hat{w}) \big] \big[\nabla \upsilon^n(x^A, \hat{w}) - \nabla \upsilon^m(x^A, \hat{w}) \big]^T$ is the product of the same vector it is positive semidefinite, hence $x^T M(x^A,\ x^B) x \le 0$ for all x is not satisfied under the assumption $\upsilon_w^n(x^A, \hat{w}) - \upsilon_w^m(x^A, \tilde{}_j(\widehat{x, z})\hat{w}) < 0 \forall x$. i.e. if individuals with relatively high w choose to enjoy the private provision. On the other hand, it is possible that even if $\upsilon_w^n(x^A, \hat{w}) - \upsilon_w^m(x^A, \hat{w}) < 0$ the second and the third elements of $V_{jk}^A(x^A, x^B, \theta)$ are sufficiently concave to guarantee concavity of the whole function. Specifically, notice that as the variance of w increases (with $\overline{w} - \underline{w}$ increasing), $\hat{r}(\theta, \hat{w}) \to 0$, thus the conditions for existence become similar to the ones in Enelow and Hinich (1989). Specifically, under the assumption of Section 4 that $W = Z + \Sigma$, with Σ being a uniformly distributed random variable independent of Z, one can find a threshold k such that if $\bar{\epsilon} - \underline{\epsilon} \ge k$ then the conditions are satisfied.

(ii) Policy convergence. Notice that the game described above can be modeled as a zero sum game because the expected plurality for Party B: is equal to $1 - V(x_A, x_B)$. Suppose $(x^A,\ x^B)$ is an equilibrium strategy with $x^A \neq x^B$ and delivering expected plurality $V(x^A,\ x^B)$ to party A. Party A can always achieve a certain value $\overline{V}^A$ by playing $x^A = x^B$. Hence if $V(x^A, x^B) < \overline{V}^A$ (a) then x^A cannot be a best response for Party A because it can profitably deviate to $\hat{x}^A = x^B$. If $V^A(x^A, x^B) > \overline{V}^A$ (b), then $V^B(x^B, x^A) = 1 - V^A(x^A, x^B)$. Then x^B cannot be a best response for Party B because it can deviate to $\hat{x}^B = x^A$ and get $V^B(\hat{x}^B, x^A) = 1 - \overline{V}^A$. Inequality (b) implies that this deviation is profitable. Hence in equilibrium it must be true that $V^A(x^A, x^B) = \overline{V}^A$ and $V^B(x^B, x^A) = 1 - \overline{V}^A$ and $x^A = x^B$. □

B.2. Utilitarian outcome

Theorem 2. *(Utilitarian outcome). The policy chosen by both parties in equilibrium is the same as the policy that would be chosen by an omnipotent Benthamite government, i.e.* $x^* = \arg\max_{x \in X} V(x, \theta)$ *where* $V(x, w) = \int_{\underline{w}}^{\overline{w}} \upsilon(x, w) \hat{r}(w, \theta) dw$.

Proof. The Lagrangian for this problem is:

$$L = \int_{\underline{w}}^{\hat{w}} [\mathbb{P}(\upsilon^n(x, w) - \upsilon(x^B, w))] r(\theta, w) dw + \int_{\hat{w}}^{\overline{w}} [\mathbb{P}(\upsilon^m(x, w) - \upsilon(x^B, t))] \hat{r}(\theta, w) dw - \mu B(x)$$

First order conditions are

$$[x_i]: \int_{\underline{w}}^{\hat{w}} p\big[d^n(x^A, w)\big] \upsilon_{x_i}^n(x^A, w) r(\theta, w) dw + \int_{\hat{w}}^{\overline{w}} p\big[d^m(x^A, w)\big] \upsilon_{x_i}^m(x^A, w) \hat{r}(\theta, w) dw \le \mu B_{x_i}(x)$$

with respect to each policy dimension x_i and $B(x) \le 0$ with respect to μ. Hence for any $i \neq j$ such that $L_{x_i} = L_{x_j} = 0$ one gets

$$\frac{\int_{\underline{w}}^{\hat{w}} p\big[d^n(x^A, w)\big] \upsilon_{x_i}^n(x^A, w) r(\theta, w) dw + \int_{\hat{w}}^{\overline{w}} p\big[d^m(x^A, w)\big] \upsilon_{x_i}^m(x^A, w) \hat{r}(\theta, w) dw}{\int_{\underline{w}}^{\hat{w}} p\big[d^n(x^A, w)\big] \upsilon_{x_j}^n(x^A, w) r(\theta, w) dw + \int_{\hat{w}}^{\overline{w}} p\big[d^m(x^A, w)\big] \upsilon_{x_j}^m(x^A, w) \hat{r}(\theta, w) dw} = \frac{B_{x_i}(x)}{B_{x_j}(x)}$$

Notice that at an equilibrium point $x^A = x^B$ (see proof to Theorem 1) hence $d^n(x^A, w) = d^m(x^A, w) = 0 \forall w$ and $\hat{w}(x^A) = \hat{w}(x^B)$. Given that $p(d(x^A, w))$ is independent of w in this case, i.e. $p(d(x^A, w)) = p(0) \neq 0$, then the previous equation becomes:

$$\frac{\int_{\underline{w}}^{\hat{w}} v^n_{x_i}(x^A, w) r(\theta, w) dw + \int_{\hat{w}}^{\overline{w}} v^m_{x_i}(x^A, w) \hat{r}(\theta, w) dw}{\int_{\underline{w}}^{\hat{w}} v^n_{x_j}(x^A, w) r(\theta, w) dw + \int_{\hat{w}}^{\overline{w}} v^m_{x_j}(x^A, w) \hat{r}(\theta, w) dw} = \frac{B_{x_i}(x)}{B_{x_j}(x)}$$

which is the same condition that one can derive for the problem:

$$\max_{x \in X, G(x) \leq 0} \int_{\underline{w}}^{\hat{w}} v^n(x, w) \hat{r}(\theta, w) dw + \int_{\hat{w}}^{\overline{w}} v^m(x, w) \hat{r}(\theta, w) dw$$

which is the Utilitarian Social Optimum. Notice that for this result to hold it is crucial that the function $\mathbb{P}$ is the same for all income levels w. □

B.3. Comparative statics

Lemma 3. *(Monotonicity): If there exists at least one x_j with $N \geq j > L$ such that (i) the solution of the maximization problem is interior for x_j, (ii) $b(x, \theta)$ is such that $\frac{\partial b(x,\theta)}{\partial x_j} = \alpha \frac{\partial a(x,\theta)}{\partial x_j}$ for some constant α, and if (iii) $\frac{\partial^2 e_i(x_i,\theta)}{\partial x_i \partial \theta} \geq 0$ (≤ 0), then x_i is weakly increasing (decreasing) in θ in a neighborhood of $x^*(\theta)$.*

Proof. The Lagrangian for the maximization problem is $L = a(x, \theta) + \sum_{i \in L} e_i(x_i, \theta) - \mu[\kappa a(x, \theta) + b(x, \theta) + \sum_{i \in L} \delta(x_i)]$. The F.O.C.s with respect of each $i \leq M$ are given by the following: $e_{1i}(x_i, \theta) - \mu\delta'(x_i) \leq 0$ and for each j such that $M < j \leq N$, they are given by $a_{x_j}(x, \theta) - \mu\kappa a_{x_j}(x, \theta) - \mu b(x, \theta) \leq 0$. Lastly, $\kappa a(x, \theta) + b(x, \theta) + \sum_{i \in L} \delta_i(x_i) \leq 0$. If the solution is interior for at least one j, and $b_{x_j}(x, \theta) = 0$, this implies $\mu = 1/\kappa$, i.e. the Lagrangian multiplier is a constant in a neighborhood of x^*. Now either one gets a corner solution for the i policy dimension, in which case x_i is unaffected by marginal changes in θ, or the solution is interior for such dimension. In the latter case, one can differentiate the F.O.C. with respect to θ to get:

$$\frac{dx_i(0)}{d\theta} = -\frac{1}{L_{x_j x_j}(x, \theta)} \left[L_{x_j \theta} + \sum_j L_{x_i x_j} \frac{dx_i(0)}{d\theta} + \sum_j L_{x_i \mu} \frac{d\mu}{d\theta} \right]$$

Notice that additive separability of x_i in the indirect utility and in the budget constraint implies $L_{x_i x_j} = 0$ for all $j \neq i$. Moreover, because μ is constant in a neighborhood of x^*, one gets $\frac{d\mu}{d\theta} = 0$. Lastly, strict concavity of V and convexity of the budget set B imply $L_{x_j x_j}(x, \theta) < 0$, and $L_{x_i \theta} = e_{x_i \theta}(x_i, \theta)$ because B is a constant function of θ. Thus, $sign\left(\frac{dx_i^*(0)}{d\theta}\right) = sign\left(\frac{\partial^2 e(x_i,\theta)}{\partial x_i \partial \theta}\right)$. □

Appendix C. Income inequality and public education

This section presents the proofs about the comparative statics in presence of a discrete set of choices on the private market with $n > 1$ elements and of the effects on changes in the level of public provision of education on the features of the income distribution of the next generation.

C.1. Multiple discrete options on the private market

Suppose there is more than one option available on the private market $\mathcal{Q} = \{q_1, q_2, \ldots, q_n\}$ and that $k \geq 2$ options are chosen with positive probability at an equilibrium. Denote with $\upsilon^{(j)}(x_i, z + \epsilon)$ the indirect utility of an individual that purchase option q_j. Define new thresholds $\tilde{\epsilon}_j(x, z)$ such that $\upsilon^{(j)}(x_i, z + \tilde{\epsilon}_j(x, z)) = \upsilon^{(j+1)}(x_i, z + \tilde{\epsilon}_j(x, z))$ for $j = 1, 2, \ldots, k$. Assume $\upsilon^{(j)}(x_i, w) - \upsilon^{(j+1)}(x_i, w)$ to be decreasing in w, that is, individuals with higher income choose higher levels of quality on the private market. Also, assume that the distribution of ϵ has enough variance to ensure that for any $z \in [\underline{z}, \bar{z}]$ there exists $\tilde{\epsilon}_j(x, z)$ defined as above for all $\underline{k} \leq j \leq \bar{k}$, where $\underline{k}$ and $\bar{k}$ are the lowest and the highest quality levels that are chosen by a positive share of individuals. Then the objective function becomes:

$$\begin{aligned}
\tilde{V}(x, \theta) = & \int_{\underline{z}}^{\bar{z}} \int_{\underline{\epsilon}}^{\tilde{\epsilon}_0(x,z)} u(x_i, z + \epsilon) k d\epsilon r(z, \theta) dz + \sum_{i=\underline{k}}^{\bar{k}} \int_{\tilde{\epsilon}_{j-1}(x,z)}^{\tilde{\epsilon}_j(x,z)} \upsilon^{(j)}(x_i, z + \epsilon) k d\epsilon r(z, \theta) dz \\
& + \int_{\tilde{\epsilon}_{k-1}(x,z)}^{\overline{\epsilon}} \upsilon_k(x_i, z + \epsilon) k d\epsilon r(z, \theta) dz + E_{z,\epsilon}(z + \epsilon) \\
& + \int_{\underline{z}}^{\bar{z}} \int_{\underline{\epsilon}}^{\overline{\epsilon}} d(x_l, w) k r(z, \theta) d\epsilon dz + E_{z,\epsilon}[y(z + \epsilon) - \tau(x, z + \epsilon)]
\end{aligned}$$

And the government budget constraint is unchanged in comparison with the baseline case. In the same way shown in the proof of Theorem 1, one can show that x_i is weakly increasing in θ in a neighborhood of $x^*(\theta)$ if $\int_{\underline{z}}^{\bar{z}}\left[\upsilon_{122}^{(\bar{k})}(x_i,z+\bar{\epsilon})-u_{122}(x_i,z+\underline{\epsilon})\right]\left(\int_{\underline{z}}^{z}[G(s)-F(s)]ds\right)dz\geq 0$. Hence the comparative statics is not affected in the proximity of the political equilibrium if one analyses a simpler problem in which $\widetilde{\mathcal{Q}}=\{q_j\}$ where $q_j\in\arg\max_{q\in\mathcal{Q}}\upsilon^{(\bar{k})}(x_i,z+\epsilon)$. Q.E.D.

C.2. Effects of policy changes on the next generation

Proposition 7. *If $Var(\Sigma)$ is large enough, then (i) the expected income of the next generation $E(w^s)$, (ii) the variance $Var(w^s,\theta)$ and (iii) the coefficient of variation $CV(w^s,\theta)$ of the income distribution of the next generation are all weakly increasing in x_i. Moreover, if the expected marginal returns to public education are weakly decreasing in parental income, then (iv) he total effect of a marginal increase in θ on $Var(w^s)$ and $CV(w^s)$ is ambiguous.*

Proof. (i) Differentiate $E(w^s)$ with respect to θ. $\frac{\partial E[w^s(x,w)|w]}{\partial x_i}=\rho_{t+1}\left\{\int_{\underline{z}}^{\bar{z}}\frac{\partial\tilde{\epsilon}(x,z)}{\partial x_i}k[H(x_i,z+\tilde{\epsilon})-\check{H}(\check{q},z+\tilde{\epsilon})]+\int_{\underline{\epsilon}}^{\tilde{\epsilon}}[H_1(x_i,z+\epsilon)]\right.$ $\left.kd\epsilon r(\theta,z)dz+\omega s'(x_i)\right\}$ which using the formula for $\frac{\partial\tilde{\epsilon}(x,z)}{\partial x_i}$ becomes $\frac{\partial E[w^s(x,w)|w]}{\partial x_i}=\rho_{t+1}\left[\int_{\underline{z}}^{\bar{z}}-kH_1(x_i,z+\tilde{\epsilon})+\int_{\underline{\epsilon}}^{\tilde{\epsilon}}H_1(x_i,z+\epsilon)\right.$ $\left.kd\epsilon r(\theta,z)dz+\omega s'(x_i)\right]$. Notice that $\int_{\underline{\epsilon}}^{\tilde{\epsilon}}[H_1(x_i,z+\epsilon)]kd\epsilon r(\theta,z)dz+\omega s'(x_i)>0$ for all x if $s'(x_i)\geq 0$. Recall $Var(\Sigma)=(\bar{\epsilon}-\underline{\epsilon})^2/12$. As $Var(\Sigma)\to\infty$ it must be true that $k=1/(\bar{\epsilon}-\underline{\epsilon})\to 0$. Thus, because $H_1(x_i,z+\tilde{\epsilon})$ is finite by assumption, then there exists $\hat{k}\in[0,\infty)$ such that if $k\leq\hat{k}$ then $\frac{\partial E[w^s(x,w)|w]}{\partial x_i}\geq 0$.*(ii)* The derivative of the variance the income of the next generation $Var(w^s,\theta)$ is given by the following:

$$\frac{\partial Var(w^s)}{\partial x_i}=\int_{\underline{z}}^{\bar{z}}\underbrace{\frac{\partial\tilde{\epsilon}}{\partial x_i}k\left[H(x_i,z+\tilde{\epsilon})^2-\check{H}(\check{q},z+\tilde{\epsilon})^2\right]}_{A}+2\underbrace{\int_{\underline{\epsilon}}^{\tilde{\epsilon}}\left[H(x_i,z+\epsilon)H_1(x_i,z+\epsilon)-E(u)\frac{\partial E(w^s)}{\partial x_i}\right]}_{B}kd\epsilon g(z)dz$$

$$=\underbrace{\frac{\partial\tilde{\epsilon}}{\partial x_i}k\left[H(x_i,z+\tilde{\epsilon})^2-\check{H}(\check{q},z+\tilde{\epsilon})^2\right]\left[H(x_i,z+\tilde{\epsilon})+\check{H}(\check{q},z+\tilde{\epsilon})-2E(w^s)\right]}_{A}$$

$$+2\underbrace{\int_{\underline{z}}^{\bar{z}}\int_{\underline{\epsilon}}^{\tilde{\epsilon}}H_1(x_i,z+\epsilon)[H(x_i,z+\epsilon)-E(w^s)]kd\epsilon g(z)dz}_{B}=$$

Thus, $B\leq\int_{\underline{z}}^{\bar{z}}\int_{\underline{\epsilon}}^{\tilde{\epsilon}}H_1(x_i,z+\epsilon)][H(x_i,z+\epsilon)-E(w^s)]kd\epsilon g(z)dz$ if $z+\tilde{\epsilon}(x_i,z)\geq E(z+\epsilon)$ and H is concave in w, because of Jensen's inequality. Lastly, recall $H_1(x_i,w)$ is decreasing in w, which implies:

$$\int_{\underline{z}}^{\bar{z}}\int_{\underline{\epsilon}}^{\tilde{\epsilon}}H_1(x_i,z+\epsilon)][H(x_i,z+\epsilon)-E(w^s)]kd\epsilon g(z)dz\leq$$

$$\leq E_{z,\epsilon}[H_1(x_i,z+\epsilon)]\int_{\underline{z}}^{\bar{z}}\int_{\underline{\epsilon}}^{\tilde{\epsilon}}[H(x_i,z+\epsilon)-E(w^s)]kd\epsilon g(z)dz=0$$

Thus B is negative. The sign of A is ambiguous, but the magnitude tend to zero as k becomes large (i.e. the variance increases). Thus the variance of the income distribution of the next generation is decreasing in x_i. (iii) Regarding the coefficient of variation notice that it is defined as $CV(w^s)=\sqrt{Var(w^s)}/E(w^s)$. Recall from Proposition 10 that $E(w^s)$ is increasing in x_i for sufficiently small k. Thus it is decreasing in x_i if $Var(w^s)$ is. (iv) Notice that $\frac{dVar(w^s)}{d\theta}=\frac{\partial Var(w^s)}{\partial\theta}+\frac{\partial Var(w^s)}{\partial x_i}\frac{dx_i}{d\theta}$. The first part $\frac{\partial Var(w^s)}{\partial\theta}$ may be positive, but as the second part has negative sign if $MRE(x,w)$ is decreasing in w for all x, w, then the sign of the total effect is ambiguous. □

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