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Keywords
Agglomeration, videoconferencing, innovation, disparities

JEL Codes
J24, O31, O41, R12

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The Day After Covid-19: Implications for Growth, Specialization, and Inequality

Stefano Magrini* Alessandro Spiganti†

Abstract

In the post-pandemic world, digital communication will be integral part of daily working to a higher extend than before, with a disproportionally strong impact on knowledge-based activities, like innovation and research. We present a multi-area endogenous growth model where abstract knowledge flows at no cost across space but tacit knowledge arises from the interaction between researchers and hence is hampered by distance. Digital communication reduces this "cost of distance" for flows of tacit knowledge and reinforces productive specialization. This increases the system-wide growth rate, but at the cost of an increase in inequality within and across areas.

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1 Introduction

Since the end of the 1990s, broadband technology and high-speed connections have allowed near-instant communication, through e.g. electronic mail, instant messaging, voice over Internet Protocol telephone calls, and videoconferencing. This had a strong impact on the way people lived and worked over the last two decades, but the Covid-19 pandemic is likely to dramatically accelerate these trends. For example, the proportion of US employees who primarily work from home tripled in approximately 30 years from 0.75% in 1980 to 2.4% in 2010 (Bloom et al., 2015), but this number was an order of magnitude larger in 2020.[1] Even if some of these jobs will go back to be performed in offices, it is likely that working remotely will still be part of the new reality: for example, Dingel and Neiman (2020) estimate that 37% of jobs in the United States can be performed entirely at home, many tech giants have already made working from home a permanent option for employees,[2] and the share of working days spent at home is expected to triple after the Covid-19 crisis ends compared to before the pandemic hit.[3] Therefore, in the world that will emerge when the lockdown period is eventually over, digital communication in general, and videoconferencing in particular, will most probably be integral part of daily working to a much higher extent than before. There will be significant variation across workers and industries, but the impact will be disproportionally strong on all those activities in which knowledge and information are fundamental for production, like research and innovation. How will this change the relative productivity of researchers and their ability to innovate? Which impact will this have on the spatial distribution of these activities and their contribution to growth? What will the repercussions be on per capita income and inequality levels?

To start investigating these questions, we construct an endogenous growth model, where we allow for different urban areas and various knowledge spillovers. The economy features two urban areas, each with three sectors: a research sector producing patents using knowledge and skilled labour, an intermediate sector producing differentiated inputs using patents, and a manufacturing sector using skilled labour, unskilled labour, and intermediate inputs. Workers are free to move across areas, and skilled workers can

[1] In March 2020, 42% of respondents to a survey of American adults who earned at least $20,000 in labour income in 2019 were working from home (https://voxeu.org/article/covid-19-and-labour-reallocation-evidence-us).

[2] See the article on Business Insider by Aaron Holmes, https://www.businessinsider.com/how-tech-companies-plan-to-reopen-facebook-google-microsoft-amazon-2020-5?IR=T.

[3] See the article by Altig et al. for the Federal Reserve Bank of Atlanta's Policy Hub: Macroblog, https://www.frbatlanta.org/blogs/macroblog/2020/05/28/firms-expect-working-from-home-to-triple.

also decide in which sector to work; location and sector decisions are evaluated solely in terms of wage rates. Knowledge takes two forms in the model: abstract and tacit. As in the endogenous growth literature originating from Romer (1986), the former represents codifiable knowledge created during the research effort, which spreads freely throughout the system enhancing the productivity of every researcher. Tacit knowledge is instead all that body of knowledge that cannot be codified, being the non-written heritage of individuals or groups (Polanyi, 1967). This form of knowledge can be transmitted and positively affects the productivity of the researchers, but the flows of tacit knowledge occur essentially through direct, face-to-face, contacts rather than through impersonal means such as patent documents or scientific papers. This difference introduces a distinction between system-wide and bounded external spillovers on the basis of the type of knowledge being transmitted.

We assume that one urban area is endowed with a more productive research sector, which may parsimoniously reflect a more developed absorptive capacity, i.e. a higher ability to assimilate new knowledge, recognize its value, and apply it to commercial use (Cohen and Levinthal, 1990), or a richer network capital, defined as an area's capacity and capability to access economically beneficial knowledge (Huggins and Thompson, 2014). As a consequence of this productivity gap, geographical specialization arises in equilibrium: the more productive research sector attracts a larger share of researchers and thus the related area specializes in research activities; conversely, the other area attracts a larger share of skilled and unskilled workers producing the final good, thus specializing in manufacturing activities. Since skilled workers command a higher wage than unskilled ones, the area with a more productive research sector is characterized by higher income per capita; if skilled workers are relatively scarce in the entire population, this area also exhibits a more unequal income distribution. However, the growth rate is the same across areas, since the presence of spillovers means that this only depends on the aggregate flows of new knowledge generated in a period.

We then model a boost to near-instant communication technologies as a fall in the "cost of distance", i.e. a facilitation of the informal interactions among researchers. First, this has a positive effect on the growth rate of the economy, since both areas benefit from an increase in the effectiveness of their research effort. Second, a skilled worker becomes relatively more productive if employed in the research sector than in the manufacturing sector, causing a reallocation of skilled workers from manufacturing to research activities. Third, since the more productive research sector is better equipped to exploit these additional interactions (consistently with the interpretation of the productivity of a research sector as its absorptive capacity or network capital), it attracts a larger share of these new researchers, strengthening the previously existing patterns of specialization. As a consequence, this shock increases the previously existing disparities in income per capita and Gini coefficients between areas, as well as the Gini coefficient of the entire system.

Finally, we consider a negative shock to the productivity of the system, perhaps as a result of confinement and social distancing policies. In general, such a shock determines a fall in the growth rate of the overall economy and a decrease in the total number of researchers. If the shock is symmetric to both areas or if it hits more severely the more productive one, this results in a weakening of the previously existing pattern of specialization and a subsequent decrease in the previously existing differences across areas. Conversely, a shock which increases the productivity gap between areas makes them more heterogeneous, both within and relatively to one another.

The remainder of this paper is organised as follows. Section 2 reviews previous literature. Section 3 presents the model and Section 4 describes its balanced growth path. Section 5 carries out the comparative statics and presents a numerical example. Finally, Section 6 concludes.

2 Previous Literature

Our paper is connected to three strands of literature. First, our model is based on the endogenous growth literature originating from Romer (1986), that stresses the role of knowledge as a key driver of productivity and economic growth. In particular, we provide an expanding variety model with knowledge spillovers à la Romer (1990b), where current researchers "stands on the shoulders of past giants". Whereas Romer (1990b) focuses on a single research sector, we modify the model to allow for different areas, so that the growth rate of the entire economy results from the R&D decisions of all areas. In terms of modelling, our paper is similar to models of endogenous technological change with knowledge spillovers across countries, such as Howitt (2000), Acemoglu (2008, Chapter 18), and Acemoglu et al. (2017). However, differently from these papers, we allow our researchers to move freely across areas and sectors, thus endogenising the spatial distribution of human capital.

Second, this paper is related to the new economic geography literature, that studies the link between agglomeration and economic integration. Its canonical setting is the so-called core-periphery model (Krugman, 1991), which was merged with Romer's (1990b) endogenous growth model by Baldwin and Forslid (2000). Among the numerous subsequent core-periphery growth models, the paper closest to ours is Bond-Smith and

McCann (2020), with whom we share a focus on innovation, the presence of multiple sectors, and footloose skilled workers (i.e. freely choosing location in response to wage pressure). Whereas they parsimoniously capture knowledge spillovers across geographical and technological spaces through exogenous parameters, we introduce gravity-type spillovers based on the endogenous allocation of workers across sectors and areas. Related is also a literature that tries to understand urban dynamics using endogenous growth theory, following the seminal contribution of Black and Henderson (1999); whereas the focus of this literature is on how local authorities can foster efficient investment in knowledge, we share an interest in the effect of agglomeration on income inequalities.

Finally, this paper connects to the literature on innovation and agglomeration, which studies how they relate to economic performance and growth (see Carlino and Kerr, 2015, for a literature review). This literature suggests that population and economic activity are spatially concentrated, and that R&D activities are more concentrated than manufacturing activities (e.g. Audretsch and Feldman, 1996, Buzard et al., 2017). One of the underlying explanation for this phenomenon, which dates back to Marshall (1890), is that geographic proximity facilitates the transfer of knowledge, especially through serendipitous interactions among workers and firms. However, there is a growing base of evidences suggesting that knowledge is increasingly being shared across geographic clusters, but through more selective routes that require conscious investments, absorptive capacity, and network capital (see e.g. Huggins and Thompson, 2014, for a review). In this paper, we take as given that one area is endowed with a research sector relatively more effective at exploiting the knowledge spillovers and analyse theoretically the resulting spatial allocation of innovative activities.

3 The Model

We consider an infinite-horizon economy in continuous time. This is inhabited by a continuum of infinitely-lived agents comprising a constant mass H of skilled workers and a constant mass L of unskilled workers. The economy features two urban areas, i and j. Each area has three sectors: a research sector which produces patents using knowledge and skilled labour, an intermediate sector producing differentiated intermediate inputs using forgone final good and patents, and a manufacturing sector producing a homogeneous good using skilled labour, unskilled labour, and intermediate inputs. Unskilled workers are employed in the manufacturing sector and are free to move across areas; a skilled worker is employed in either the research sector or the manufacturing sector, and can freely move across areas and sectors. Locations and sectors are evaluated solely in terms of wage rates.

3.1 The Agents

Agents, indexed by z, are infinitely-lived and have an instantaneous constant elasticity of substitution utility function, meaning that they each maximize, subject to a budget constraint,

$$\int_{t=0}^{\infty} e^{-\rho t} \frac{c_z(t)^{1-\sigma} - 1}{1-\sigma} dt, \tag{1}$$

where $c_z(t)$ is the consumption of agent z at time t, $\rho > 0$ is the subjective discount rate, and $1/\sigma > 0$ measures the willingness to substitute intertemporally. Agents inelastically supply one unit of labour and own equal shares of all the firms in the area; they use their income to consume and save.

Agents consume a unique final good that can be transported between the two areas at no cost; therefore, all consumption arising from the system can be aggregated in the system-wide variable $C(t)$. The maximization problem of the agents results in the usual consumption Euler's equation, which relates the interest rate $r(t)$ to the rate of growth of consumption according to

$$\frac{\dot{C}(t)}{C(t)} = \frac{r(t) - \rho}{\sigma}. \tag{2}$$

Here, we concentrate on the case in which the growth rate of consumption is positive, which implies $r(t) > \rho$. To ensure that the integral in (1) converges, the rate of growth of current utility is assumed to be smaller than the rate of time preference, i.e.

Assumption 1. $(1-\sigma)\,\dot{C}(t)/C(t) < \rho.$

3.2 The Manufacturing Sector

The final good is produced competitively by a representative firm using unskilled labour, skilled labour, and a set of intermediate inputs. The available variety of intermediate inputs in a urban area at any point in time is taken as given by the firm and consists of the summation of inputs produced in the same area and inputs imported from the other urban area (as in e.g. Rivera-Batiz and Romer, 1991, Rivera-Batiz and Xie, 1993). The intermediate inputs depreciate fully after use.[4] Below and in the next

[4] This is a standard assumption in the expanding variety models. Indeed, it simplifies the exposition considerably, since the past amounts of these inputs are not additional state variables. However, results without this assumption are identical.

subsections, we describe i's sectors, but the same applies to j's; for ease of reading, we drop the time index.

Define A_i and A_j as the number of intermediate inputs designed and produced in i and j, respectively. Let the quantity of any intermediate input produced in i and employed in the same urban area be $x_i(a_i)$, with $a_i \in A_i$; analogously, the quantity of any intermediate input produced in j and employed in i is $x_i(a_j)$, with $a_j \in A_j$. The overall production structure in i's final sector is represented by the following additively separable function:

$$M_i = L_i^{\alpha} H_{m,i}^{\beta} \left[\int_0^{A_i} x_i(a_i)^{\gamma} da + \int_0^{A_j} x_i(a_j)^{\gamma} da \right] S_{m,i}, \tag{3}$$

where M_i is the final good produced in i, $H_{m,i}$ represents skilled labour employed in i's manufacturing sector, and $S_{m,i}$ reflects the size of spillovers arising from the interaction between skilled workers employed within the same urban area.[5] Formally, these intra-area spillovers are parametrized through the following gravity-type function,

$$S_{m,i} = (H_{m,i} H_{r,i})^{\phi}, \tag{4}$$

where $H_{r,i}$ represents skilled labour employed in i's research sector and $0 \leq \phi < 1$ determines the strength of the economies arising from the agglomeration of skilled workers; when $\phi = 0$, there are no local spillover effects in the manufacturing sector.

The Cobb-Douglas formulation of the production function in (3) leads to iso-elastic demand curves; in particular, the demands of intermediate inputs by the final good producer in area i are

$$x_i(a_i) = \gamma^{\frac{1}{1-\gamma}} L_i^{\frac{\alpha}{1-\gamma}} H_{m,i}^{\frac{\beta}{1-\gamma}} S_{m,i}^{\frac{1}{1-\gamma}} p_i(a_i)^{-\frac{1}{1-\gamma}} \tag{5a}$$

$$x_i(a_j) = \gamma^{\frac{1}{1-\gamma}} L_i^{\frac{\alpha}{1-\gamma}} H_{m,i}^{\frac{\beta}{1-\gamma}} S_{m,i}^{\frac{1}{1-\gamma}} p_i(a_j)^{-\frac{1}{1-\gamma}}, \tag{5b}$$

where $p_i(a_i)$ and $p_i(a_j)$ are the prices of an intermediate good sold in i but produced in i and j, respectively. Implicitly, we are assuming the absence of transportation costs for intermediate goods across areas.

The final sector operates in a perfectly competitive setting, hence $\alpha + \beta + \gamma = 1$. To ensure that the wage rate earned by skilled workers is higher than the wage rate earned by unskilled workers, we assume

5 This type of local spillovers have a long-tradition in economics, see e.g. Jacobs (1970).

Assumption 2. $H_{m,i}/L_i < \beta/\alpha$.

For simplicity, we assume that the final good is traded freely within the system in the absence of any transportation cost. As a consequence, in equilibrium its price must be the same in both urban areas, and we normalize it to one.

3.3 The Research Sector

Following the large literature originated from Romer (1990a,b), the research sector produces knowledge in the form of designs for new intermediate inputs, using skilled labour and existing knowledge. Formally, the flow of new knowledge, i.e. the number of new designs, created in urban area i at any point in time is given by:

$$\dot{A}_i = \delta_i H_{r,i}^{\eta} S_{r,ij} A, \tag{6}$$

where $H_{r,i}$ is the number of *researchers* in i, $0 \leq \eta < 1$ is a parameter inducing decreasing returns in its stock (similarly to Kortum, 1993, Jones, 1995), $\delta_i > 0$ is an exogenous parameter characterizing the productivity of the local research system, A is an index of the economy technology frontier (which will be endogenized below), and $S_{r,ij}$ reflects inter-area spillovers in research. This form of the innovation possibility frontier implies that new knowledge in i results from the effort of the researchers in the area, but the effectiveness of these efforts more generally depends on the research done in the entire economy.

Indeed, equation (6) introduces two types of spillovers. First, there is a positive a-spatial spillover coming through the economy technology frontier, A. This is assumed to be given by

$$A \equiv A_i + A_j, \tag{7}$$

meaning that A simply represents the aggregate number of designs already existing, or, equivalently, the overall level of abstract knowledge created so far and available to all researchers.[6] Second, there is a positive network effect between the researchers in the two areas; this represents the flow of tacit knowledge, which occurs essentially through informal interactions and exchange of ideas. We assume that these inter-area spillovers

[6] The qualitative results are unaffected as long as the economy technology frontier is a linearly homogeneous function of the number of intermediate inputs in the two areas, e.g. equal to the technology level of the most advanced area or an average of the two.

have the following gravity-type representation:

$$S_{r,ij} = \left(H_{r,i} H_{r,j} \nu_i\right)^{\psi}, \tag{8}$$

with the parameter ψ governing the strength of their impact on researchers in i and the function ν_i expressing the effectiveness of the interaction to the benefit of i. For ease of exposition, we take the following assumption:

Assumption 3. $0 \leq \psi \leq 1 - \eta$.

While not strictly necessary, this assumption eases calculations since, as clarified in Appendix A.1, it is a sufficient condition for the stability of the equilibrium allocation of researchers across urban areas.

It is well-known that any sort of distance, d, between the researchers of the two areas, being geographical or technological, may make these informal interactions more difficult (see e.g. Jaffe et al., 1993); however, a natural assumption is that a higher productivity of the local research system, which may partly be intended as its absorptive capacity (Cohen and Levinthal, 1990) or its network capital (Huggins and Thompson, 2014), may not only facilitate the exploitation of these interactions but also (partly) compensate for the distance. As a consequence, we let $\nu_i \equiv \nu(\delta_i, d)$ and $\nu_j \equiv \nu(\delta_j, d)$ and we take the following assumption:

Assumption 4. *The function $\nu(\delta, d)$ is twice differentiable in δ and d, and satisfies*

$$\frac{\partial \nu(\delta, d)}{\partial \delta} \geq 0, \qquad \frac{\partial \nu(\delta, d)}{\partial d} \leq 0, \qquad \frac{\partial^2 \nu(\delta, d)}{\partial d \partial \delta} \leq 0, \qquad \frac{\partial}{\partial \delta} \left| \frac{d}{\nu(\delta, d)} \frac{\partial \nu(\delta, d)}{\partial d} \right| > 0,$$

where the last condition ensures that the d-elasticity of $\nu(\delta, d)$ increases with δ.

3.4 The Intermediate Sector

The intermediate sector in area i is composed of an infinite number of firms on the interval $[0, A_i]$. Each of these firms has purchased a patent from the research sector and can then produce the related intermediate input at marginal cost equal to $\kappa > 0$ units of the final good, as long as it is manufactured in the same region in which the relative patent has been developed. We assume that this marginal cost is strictly higher if the intermediate input is manufactured in the other area, thus excluding the existence of an inter-area trade of patents.

In line with the endogenous technological change literature, an intermediate producer acts as a monopolist in the production of its particular intermediate input. An intermediate firm in i faces the demand $x_i(a_i)$ in (5a) from the final producer in i with the corresponding price $p_i(a_i)$ and the demand $x_j(a_i)$ at price $p_j(a_i)$ from the final producer in j; let aggregate demand faced by an intermediate firm in i be $X(a_i) \equiv x_i(a_i) + x_j(a_i)$. Since demands are iso-elastic, the monopoly price is a constant mark-up over marginal cost. Without loss of generality, we normalise the marginal cost of machine production to $\kappa \equiv \gamma$, so that

$$p \equiv p_i(a_i) = p_j(a_i) = \kappa\gamma^{-1} = 1. \tag{9}$$

As usual in this kind of models, each intermediate firm sets the same constant price p. Here, equation (9) also means that intermediate inputs all have the same price across areas, since the marginal cost is the same. Intermediate inputs depreciate fully after use, and so p can also be interpreted as a rental price or the user cost of the input.

Substituting (9) into (5) shows that i's manufacturing firm demands the same quantity x_i of each intermediate input, irrespective of their origin; similarly, the final firm in j demands the same quantity x_j of each intermediate input. In particular,

$$x_i = \gamma^{\frac{1}{1-\gamma}} L_i^{\frac{\alpha}{1-\gamma}} H_{m,i}^{\frac{\beta}{1-\gamma}} S_{m,i}^{\frac{1}{1-\gamma}} \tag{10a}$$

$$x_j = \gamma^{\frac{1}{1-\gamma}} L_j^{\frac{\alpha}{1-\gamma}} H_{m,j}^{\frac{\beta}{1-\gamma}} S_{m,j}^{\frac{1}{1-\gamma}}. \tag{10b}$$

As a consequence, the intermediate input producers located in the two different areas all face the same aggregate demand, $X = x_i + x_j$, and enjoy the same instant profits, $\pi = X(1-\gamma)$. Hence, final good production simplifies to

$$M_i = AL_i^{\alpha} H_{m,i}^{\beta} x_i^{\gamma} S_{m,i}. \tag{11}$$

The decision about undertaking the production of a new intermediate input is taken comparing the discounted value of the flow of future profits to the cost of the initial investment in acquiring a patent from the research sector. With this knowledge, the monopolistically competitive research sector sets the price of a patent equal to the present value of the stream of future profits of the intermediate sector's monopolist. Therefore, the cost of a patent, irrespective of its location, is $P = \int_{t=0}^{\infty} \pi(t)e^{-rt}dt$. Patents are infinitely lived; hence, if the interest rate is constant,

$$P = \frac{X\,(1-\gamma)}{r}. \tag{12}$$

4 The Equilibrium

In this section, we characterize the equilibrium of the model; when necessary to avoid any confusion, we reintroduce time indexes. An allocation is defined by time paths of consumption levels $[C(t)]_{t=0}^{\infty}$, aggregate spending on intermediate inputs $[X_i(t), X_j(t)]_{t=0}^{\infty}$, labour allocations $[H_{m,i}(t), H_{m,j}(t), H_{r,i}(t), H_{r,j}(t), L_i(t), L_j(t)]_{t=0}^{\infty}$, available intermediate input varieties $[A_i(t), A_j(t)]_{t=0}^{\infty}$, and time paths of interest rates $[r(t)]_{t=0}^{\infty}$, wage rates in the research sectors $[w_{r,i}(t), w_{r,j}(t)]_{t=0}^{\infty}$, wage rates for skilled and unskilled workers in the manufacturing sectors $[w_{m,i}(t), w_{m,j}(t), w_{l,i}(t), w_{l,j}(t)]_{t=0}^{\infty}$, quantities of each intermediate input $[x_i(t), x_j(t)]_{t=0}^{\infty}$, and patent costs $[P(t)]_{t=0}^{\infty}$. An equilibrium is an allocation in which final good producers, research firms, and intermediate good producers choose, respectively, $[H_{m,i}(t), H_{m,j}(t), L_i(t), L_j(t), x_i(t), x_j(t)]_{t=0}^{\infty}$, $[H_{r,i}(t), H_{r,j}(t), P(t)]_{t=0}^{\infty}$, and $[x_i(t), x_j(t)]_{t=0}^{\infty}$ as to maximize (the discounted value of) profits, the evolution of wages and interest rate is consistent with market clearing, agents make savings and consumption decisions as to maximize their lifetime utility, and the evolution of $[A_i(t), A_j(t)]_{t=0}^{\infty}$ is determined by free entry.

In particular, we focus on a balanced growth path, i.e. an equilibrium in which aggregate variables, like consumption $C(t)$ and output $M(t)$, grow at the same constant rate as system-wide abstract knowledge, $g \equiv \dot{A}(t)/A(t)$ for all t. This is possible, from equation (2), only if the interest rate is constant: we thus look for an equilibrium in which $r(t) = r$ for all t.

Assuming for a moment that the labour market is characterized by a stable allocation of both unskilled and skilled labour across areas and sectors, then it is clear from equations (10) that the equilibrium demands of intermediate inputs would also be constant, $x_i(t) = x_i$ and $x_j(t) = x_j$ for all t; as implied by (12), in such an equilibrium, also the price of a patent is constant over time, $P(t) = P$ for all t. Under such a constant allocation of resources, equation (11) ensures that the output in both urban areas, $M_i(t)$ and $M_j(t)$, grows at the same rate as system-wide abstract knowledge, g. As a consequence, aggregate output, $M(t)$, also grows at g. Therefore, in an economy characterized by a constant allocation of unskilled and skilled labour across areas and sectors, a balanced growth path allocation exists in which

$$\frac{\dot{M}(t)}{M(t)} = \frac{\dot{M}_i(t)}{M_i(t)} = \frac{\dot{M}_j(t)}{M_j(t)} = \frac{\dot{C}(t)}{C(t)} = \frac{\dot{A}(t)}{A(t)} \equiv g.$$

To solve the model for this balanced growth equilibrium it is therefore necessary to determine the equilibrium allocation of workers across areas and sectors.

4.1 The Equilibrium Allocation of Workers

In this section, we characterize the allocation of skilled and unskilled workers across areas and sectors.

4.1.1 The Inter-Area Allocation of Researchers

For the time being, take the aggregate number of researchers, $H_r \equiv H_{r,i} + H_{r,j}$, as given; this will be endogenized below. From the maximization problem of a firm in the research sector, the wage rate for a researcher in urban area i, $w_{r,i}$, must satisfy the first order condition $w_{r,i} = \partial(P\dot{A}_i)/\partial H_{r,i}$. Using equations (6) and (12), the wage rates of a skilled worker in the two research sectors are, respectively,

$$w_{r,i} = AX\eta\delta_i H_{r,i}^{\eta-1} S_{r,ij} \frac{1-\gamma}{r} \tag{13a}$$

$$w_{r,j} = AX\eta\delta_j H_{r,j}^{\eta-1} S_{r,ji} \frac{1-\gamma}{r}. \tag{13b}$$

Any skilled worker is free to enter either research sector: in equilibrium, researchers must receive the same compensation across the two areas, i.e. $w_{r,i} = w_{r,j} \equiv w_r$. The following equilibrium allocation ensues:

$$\frac{H_{r,i}}{H_{r,j}} = \left(\frac{\delta_i}{\delta_j}\right)^{\frac{1}{1-\eta}} \left(\frac{\nu_i}{\nu_j}\right)^{\frac{\psi}{1-\eta}}. \tag{14}$$

For given distance and research productivities, the equilibrium spatial allocation of skilled labour in research is thus constant. Moreover, since $\partial\nu_i/\partial\delta_i \geq 0$ by Assumption 4, there is a positive relationship between productivity in research and the relative concentration of research activities: an urban area characterized by a relatively higher productivity of the research sector will attract a larger share of researchers.

4.1.2 The Inter-Area Allocation of Workers in the Manufacturing Sector

The manufacturing sectors are competitive, hence the wage rates of a unskilled worker employed in area i or j are, respectively,

$$w_{l,i} = \frac{\partial M_i}{\partial L_i} = \alpha L_i^{\alpha-1} H_{m,i}^{\beta} A x_i^{\gamma} S_{m,i} \tag{15a}$$

$$w_{l,j} = \frac{\partial M_j}{\partial L_j} = \alpha L_j^{\alpha-1} H_{m,j}^{\beta} A x_j^{\gamma} S_{m,j}. \tag{15b}$$

Similarly, the wage rates of the skilled workers in the manufacturing sector are

$$w_{m,i} = \frac{\partial M_i}{\partial H_{m,i}} = \beta L_i^{\alpha} H_{m,i}^{\beta-1} A x_i^{\gamma} S_{m,i} \tag{16a}$$

$$w_{m,j} = \frac{\partial M_j}{\partial H_{m,j}} = \beta L_j^{\alpha} H_{m,j}^{\beta-1} A x_j^{\gamma} S_{m,j}. \tag{16b}$$

Since workers can freely move between the two manufacturing sectors, in equilibrium unskilled and skilled workers must receive the same compensation across areas, i.e. $w_{l,i} = w_{l,j} \equiv w_l$ and $w_{m,i} = w_{m,j} \equiv w_m$. Using equations (15) and (16), this implies $L_i/L_j = H_{m,i}/H_{m,j}$; consequently, $H_{m,i}/L_i = H_{m,j}/L_j = H_m/L$, where $H_m \equiv H_{m,i} + H_{m,j}$ is the aggregate number of skilled workers employed in the manufacturing sector. Moreover, by combining this result with (10), we prove in Appendix A.1 that the equilibrium allocation of skilled workers in manufacturing between i and j is simply the inverse of the corresponding allocation of researchers, $L_i/L_j = H_{m,i}/H_{m,j} = H_{r,j}/H_{r,i}$. This, in turn, implies the endogenous equalization of external effects, $S_{m,j} = S_{m,i}$, and that the equilibrium ratios between the areas' endowment of production factors are the same. Summarizing, in equilibrium,

$$\frac{H_{r,j}}{H_{r,i}} = \frac{H_{m,i}}{H_{m,j}} = \frac{L_i}{L_j} = \frac{x_i}{x_j}. \tag{17}$$

These ratios are constant along the balanced growth path given condition (14).

4.1.3 The Inter-Sector Allocation of Skilled Workers

Finally, the intra-area equilibrium requires inter-sectoral wage equalisation for skilled workers, $w_{m,i} = w_{r,i}$ and $w_{m,j} = w_{r,j}$. Given the inter-area equilibrium allocation of researchers, these conditions become $w_m = w_r \equiv w_h$, where w_h is the unique wage paid to a skilled worker across sectors and areas. We show in Appendix A.1 that this condition is met when

$$\frac{H_r}{H_m} = \frac{\eta(1-\gamma)\gamma}{\beta} \left(\frac{r-\rho}{r\sigma} \right). \tag{18}$$

Condition (18) maintains that the equilibrium allocation of the given stock of skilled labour depends on parameters (i.e. factors' shares in final good production, the strength of the diminishing returns in knowledge creation, and consumers' preferences) and the endogenous interest rate. Since the interest rate is constant along the balanced growth path, the proportional allocation of skilled workers in the research sector and in the final good sector also remains constant along the balanced growth path.

4.2 The Equilibrium Growth Rate

We showed in Section 4.1 that the system is characterized by a constant allocation of workers across sectors and urban areas. Given that such a constant allocation exists, the economy exhibits a balanced growth path. To complete the characterization of the balanced growth path, note that free entry into research implies

$$\eta\delta_i S_{r,ij} A H_{r,i}^{\eta-1} \frac{X(1-\gamma)}{r} = w_h, \tag{19}$$

where the left hand side is the private return from hiring one more researcher, and the right hand side is the related flow cost. Together with (16), this implies that the equilibrium interest rate must be $r = \eta(1-\gamma)\delta_i\gamma\beta^{-1}S_{r,ij}H_{r,i}^{\eta-1}H_m$, which is constant under the constant allocation of workers.

Proposition 1. *The system exhibits a globally stable balanced growth path equilibrium in which output, consumption, physical capital, aggregate abstract knowledge, abstract knowledge in each area, and wages grow at the same constant rate given by*

$$g = \delta_j S_{r,ji} H_{r,j}^{\eta-1} H_r = \delta_i S_{r,ij} H_{r,i}^{\eta-1} H_r. \tag{20}$$

Along the balanced growth path, the price of a patent, the price of each intermediate input, the price of the final good, the interest rate, and the labour allocations across sectors and areas are constant.

Proof. Equation (20) is obtained by substituting (6) and (8) in $g \equiv (\dot{A}_i + \dot{A}_j)/A$ and then using condition (14). The preceding discussion establishes most of the claims in the proposition, except that abstract knowledge grows at the same rate in both urban areas and that the equilibrium is stable. To determine the growth rates of abstract knowledge within each urban area, $g_i(t) \equiv \dot{A}_i(t)/A_i(t)$ and $g_j(t) \equiv \dot{A}_j(t)/A_j(t)$, it will be convenient to define $\mathcal{A}_i(t) \equiv A_i(t)/A(t)$ as an inverse measure of the proportional abstract knowledge gap between area i and the overall economy. Applying logs to both sides and taking derivatives with respect to time, we obtain $g_i(t) = \dot{\mathcal{A}}_i(t)/\mathcal{A}(t) + g$, or, equivalently,

$$g_i(t) = \delta_i S_{r,ij} H_{r,i}^{\eta} \frac{A(t)}{A_i(t)}. \tag{21}$$

Since along the balanced growth path $g_i(t)$ must be constant, $A_i(t)$ must grow at the constant rate g; with an identical reasoning, also $A_j(t)$ grows at g. Alternatively, one can use (14) to show that $\dot{A}_i/\dot{A}_j = H_{r,i}/H_{r,j}$ in equilibrium. Applying L'Hôpital's rule, $\lim_{t\to\infty} A_i/A_j = H_{r,i}/H_{r,j}$ and $\lim_{t\to\infty} A/A_i = H_r/H_{r,i}$. Substituting this latest result into (21), one obtains $g_i(t) = g$. Finally, the stability of this equilibrium is proved in Appendix A.1. □

4.3 Income, Inequality, and Growth in Urban Areas

In this section, we evaluate whether differences in income per capita levels and growth rates arise between the two urban areas along the balanced growth path. Without loss of generality, we assume that area i is endowed with a more productive research sector, i.e.

Assumption 5. $\delta_i > \delta_j$.

Our first result characterizes the relative specialization of skilled labour between the two areas.

Proposition 2. *Along the balanced growth path, the urban area with a relatively more productive research sector is characterized by a relative specialization in research activities.*

Proof. From condition (14), Assumptions 4 and 5 imply $H_{r,i} > H_{r,j}$. Condition (17) implies that the opposite occurs for manufacturing, $H_{m,i} < H_{m,j}$, and unskilled labour, $L_i < L_j$. □

Having established the relative productive specialization of the urban areas, we can turn our attention to disparities in income levels. The level of income in each urban area, its GDP, is calculated as the summation of the wages of its workers, since profits are driven down to zero by competition or free entry. Thus, the overall GDP level in i and j can be expressed as, respectively,

$$Y_i = w_l L_i + w_h H_{m,i} + w_h H_{r,i} \tag{22a}$$

$$Y_j = w_l L_j + w_h H_{m,j} + w_h H_{r,j}. \tag{22b}$$

Corollary 2.1. *Along the balanced growth path, a relative specialization in research activities is a sufficient condition for a constantly higher level of GDP per worker.*

Proof. See Appendix A.1 □

In area i, there are $H_{r,i} + H_{m,i}$ skilled workers earning w_h and L_i unskilled workers earning w_l. With two income levels, the Gini coefficient, G_i, is simply the difference between the proportion of all income accruing to the high income group and the proportion of agents in the high income group, i.e.

$$G_i = \frac{(H_{r,i} + H_{m,i})\, w_h}{Y_i} - \frac{H_{r,i} + H_{m,i}}{L_i + H_{r,i} + H_{m,i}}. \tag{23}$$

Corollary 2.2. *If skilled workers are sufficiently scarce (as made explicit in the proof), a relative specialization in research activities is a sufficient condition for a constantly higher Gini coefficient.*

Proof. See Appendix A.1 □

Finally, we consider the effect of the relative specialization of the urban areas on the growth rates of their income levels.

Corollary 2.3. *Along the balanced growth path, GDP per worker grows in both urban areas at the constant rate g, irrespective of the areas' specialization.*

Proof. Along the balanced growth path, wages grow at g whereas labour allocations are constant. Thus, the areas' GDP levels in (22) must also grow at rate g. Since labour allocations are constant, GDP per worker also grows at g in both areas. □

Therefore, the urban area whose research system is more productive features a relative specialization in research activities compared to the other urban area and enjoys a permanently higher level of GDP per worker but, possibly, a more unequal society. However, the growth rates are the same.

5 Instant Communication, Productivity, and Specialization

The Covid-19 pandemic is likely to yield effects that extend well above the short term. In this paper, we focus on the following possible shocks: i) a boost to near-instant communication technologies, that in terms of the model translates into a reduction in the distance involved in inter-urban relations among researchers, d, and ii) a general fall in the productivity of a system, perhaps as a result of confinement and social distancing policies, that could be either specific to an area, e.g. on the parameters δ_i or δ_j, or common to the system, e.g. on ψ. We start from the former.

5.1 The Diffusion of Videoconferencing

A firm in the research sector needs knowledge and information, in addition to labour: the flow of tacit knowledge, which occurs through informal interactions and exchange of ideas, not only allows to keep up with scientific and technological advancements, but also to gain timely access to problems, needs, and requests that may direct its activity. In this regard, the diffusion of near-instant communication technologies and videoconferencing certainly plays an important role. Their importance, however, is likely to depend on the features of the network of relations in which they are employed: their effectiveness is probably stronger when these tools are adopted within an already established network (Cohen and Levinthal, 1990, Huggins and Thompson, 2014).[7]

Consistently with this interpretation, we assume that it is within the inter-area networks of relations that these tools are more likely to be successful in reducing distances, possibly giving a boost to the pre-existing phenomenon towards a digitalization of communications. In terms of the model, this takes the form of a permanent fall in the cost of distance between the two areas d, which implies a strengthening of inter-area spillovers between researchers that the more productive area is more able to exploit. This has the following long-term effects on the balanced growth path:

Proposition 3. *A permanent reduction in the distance between areas, d, determines an increase in the growth rate of the system along the balanced growth path, an increase in the total number of researchers, and a strengthening of the previously existing pattern of specialization.*

Proof. See Appendix A.1 □

Not surprisingly, an improvement in the flow of tacit knowledge has a positive effect on the growth rate of the economy, since both areas essentially benefit from an increase in the effectiveness of their own research efforts. Moreover, a skilled worker becomes relatively more productive if employed in the research sector than in the manufacturing

[7] The positive effect of the diffusion of communication services on growth is well documented in the literature, at least since Hardy (1980); see Kolko (2012) and Castaldo et al. (2018) for studies focusing on the effect of broadband adoption on growth, Gómez-Barroso and Marbán-Flores (2020) for a literature review on telecommunications more generally, and Xu et al. (2019) who instead focus more specifically on access to the internet as a determinant of innovation. In line with this paper, Mack and Rey (2014) report a generally positive relationship between broadband adoption and the level of knowledge intensive activities across US metropolitan areas but also that specialization in traditional manufacturing has a negative impact on this relationship; Chen et al. (2020) find that high-speed internet significantly increases productivity, but the effect is stronger for the more educated workers (but see Maurseth, 2018, who finds the opposite effect by extending the period of analysis).

sector, thus causing an influx of these workers from manufacturing to research. However, the relatively more research-intensive area was already best equipped to exploit these increased interactions and thus attracts a larger share of these added researchers. In equilibrium, this same area must also experience a relatively greater reduction of skilled workers in the manufacturing sector and an outflow of unskilled workers towards the relatively more manufacturing-intensive area. This reallocation of workers across sectors and areas strengthens the previously existing patterns of specialization in research and manufacturing, with important repercussions in terms of inter-area inequality.

Corollary 3.1. *A permanent reduction in the distance between areas, d, increases the previously existing differences in the levels of GDP per worker.*

Proof. This follows directly from Corollary 2.1 and Proposition 3. □

Corollary 3.2. *If skilled workers are sufficiently scarce, a permanent reduction in the distance between areas, d, increases the previously existing differences in the areas' Gini coefficients.*

Proof. This follows directly from Corollary 2.2 and Proposition 3. □

Finally, we look at the overall level of inequality, as measured by the Gini coefficient of the entire system,

$$G = \frac{Hw_h}{Y} - \frac{H}{L+H} = \frac{H}{wL+H} - \frac{H}{L+H}, \tag{24}$$

where $w \equiv w_l/w_h = (\alpha/\beta)(H_m/L)$. The permanent reduction in the distance between the areas modifies the relative marginal productivity of the workers in the different sectors to the advantage of the researchers (and thus of the skilled workers in general). Together with the strengthening of the previously existing patterns of specialization, this implies the following:

Corollary 3.3. *A permanent reduction in the distance between areas, d, increases the Gini coefficient of the entire system.*

Proof of Corollary 3.3. The Gini coefficient in (24) is clearly decreasing in $w \equiv w_l/w_h = (\alpha/\beta)(H_m/L)$. From Proposition 3, $\partial H_m/\partial d > 0$ and thus $\partial G/\partial d < 0$. □

5.2 A Productivity Shock

Here, we turn our attention to the long-term effects of a variation in the productivity of a research sector, i.e. δ_i or δ_j.

Proposition 4. *A permanent reduction in the productivity of an urban area, δ_i or δ_j, determines a decrease in the growth rate of the system along the balanced growth path and a decrease in the total number of researchers. A relative reduction in the productivity of the more (cf. less) research-intensive area, δ_i (cf. δ_j), determines a weakening (cf. strengthening) of the previously existing pattern of specialization.*

Proof. See Appendix A.1 □

We now look at the long-term effects of a variation in the productivity of both research sectors through a weakening of the inter-area spillovers in research, i.e. a fall in ψ.[8] For ease of exposition, we only consider the case $H_{r,i}H_{r,j} \times \min(\nu_i, \nu_j) \geq 1$.

Proposition 5. *A permanent weakening of the inter-area spillovers in research, ψ, determines a decrease in the growth rate of the system along the balanced growth path, a decrease in the total number of researchers, and a weakening of the previously existing pattern of specialization.*

Proof. See Appendix A.1 □

Finally, we turn to the implications on the overall level of inequality.

Corollary 5.1. *A permanent reduction in the productivity of an urban area, δ_i or δ_j, or in the inter-area spillovers in research, ψ, decreases the Gini coefficient of the entire system.*

Proof of Corollary 5.1. The Gini coefficient in (24) is clearly decreasing in $w \equiv w_l/w_h = (\alpha/\beta)(H_m/L)$. From Proposition 4, the derivative of H_m with respect to either δ_i or δ_j is negative, making the derivative of G with respect to either δ positive. From Proposition 5, $\partial H_m/\partial\psi < 0$ and thus $\partial G/\partial\psi > 0$. □

5.3 A Numerical Example

Here, we report the results of a simple quantitative example: our aim is to highlight the effects of the possible shocks analysed above on the equilibrium allocation, rather than providing a comprehensive quantitative evaluation.

[8] Given the endogenous equalization of the external effects, $S_{m,i} = S_{m,j}$, a change in the economies arising from the agglomeration of skilled workers, i.e. a change in ϕ, does not change the equilibrium allocation of workers within and across areas.

5.3.1 Parameter Choices

A period in our model corresponds to one year. We take $\alpha = \beta = 1/3$, so that the shares of unskilled and skilled labour in production are approximately 33% and the share of income spent on machines is approximately equal to the share of capital. The constant relative risk aversion parameter is taken to be $\sigma = 2$ (see e.g. Kaplow, 2005) and the concavity parameter of the innovation production function is $\eta = 0.5$ (Hall and Ziedonis, 2001). We set $\psi = 1 - \eta = 0.5$. The fraction of skilled workers is chosen such that it equals the percentage of individuals in the U.S. with at least a postgraduate degrees i.e. $H/L = 13\%$ (U.S. Census, 2018). We normalise the size of the entire population to ten and $d = 1$. We calibrate the function $\nu_i = d^{-\delta_i/\delta_j}$, which respects Assumption 4 but makes it explicit that what matters is the relative productivity of a research sector rather than its absolute productivity. We start with a productivity gap between the two research sectors of $\delta_j/\delta_i = 75\%$. Finally, we set δ_i as to target a long-run annual growth rate equal to 2%; by setting the annual subjective discount rate equal to $\rho = 0.01$, we obtain a long-run annual interest rate equal to $r = 5\%$.[9]

5.3.2 Results

The balanced growth path values resulting from the above calibration are shown in the first column of Table 1. Consistently with the results from the theoretical model, area i, which is endowed with a relatively more productive research sector, hosts a larger share of researchers than area j (approximately 1.8 times as much), which is instead specialized on manufacturing. As a consequence, area i enjoys a higher level of output per capita but a relatively more unequal society.

In the second column, we show how the balanced growth path values change after a permanent negative shock to d, such that the cost of distance between the two areas is reduced by one fourth. Consistently with the theoretical results above, the annual growth rate grows by 20%, since both areas benefit from an increase in the effectiveness of their own research efforts. The shock means that researchers are now relatively more productive than before, and thus the percentage of skilled workers employed in research increases by half a percentage point. However, area i is more equipped to take advantage of this increase, and this strengthens the pre-existing agglomeration dynamics: the share of researchers employed in area i sharply increases, whereas the reverse happens for skilled

[9] For what concerns this exercise, the value of ϕ is irrelevant, given the endogenous equalization of the external effects in the manufacturing sector: we set $\phi = 0$ and thus $S_{m,i} = S_{m,j} = 1$. Moreover, we normalize the initial level of the technology frontier to $A = 1$.

Table 1: Balanced Growth Path Values Under Different Parameters

	Baseline	$\Delta d = -25\%$	$\Delta \delta_j/\delta_i = -25pp$
g	2.00%	2.40%	2.32%
$H_{r,i}/H_{r,j}$	177.78%	210.26%	400%
H_r/H	11.76%	12.12%	12.06%
$H_{r,i}/H_r$	64.00%	67.77%	80.00%
L_i/L	36.00%	32.23%	20.00%
y_i/y_j	106.05%	108.38%	119.02%
G_i/G_j	107.87%	110.79%	123.42%
G	0.416	0.417	0.417

and unskilled workers in the manufacturing sector. The mass of unskilled workers moving from the research-intensive area to the manufacturing-intensive area is relatively bigger than the mass of skilled workers moving in the opposite direction, causing a relative increase in the level of GDP per worker and the Gini coefficient in area i with respect to area j, and a rise in inequality in the economy at large.

In the third column, we summarise the changes following a permanent increase in the productivity of area i's research sector, without any change to the productivity of area j, such that the productivity gap between the two research sectors passes from $\delta_j/\delta_i = 75\%$ to $\delta_j/\delta_i = 50\%$. Consistently with Proposition 4, this has a direct positive effect on the annual growth rate of the economy, but also an indirect positive effect through a reallocation of skilled workers from research to manufacturing (such that the fraction of researchers out of all skilled workers increases by a quarter of a percentage point). Given the asymmetric nature of the shock that increases the productivity gap between the research sectors, the economy experiences a radical strengthening of the previously existing patters of specialization, with an increase in the share of researchers employed in area i of 16 percentage points, and a corresponding outflow of unskilled workers. This reallocation stretches the gap in the levels of GDP per worker and Gini coefficients; a more heterogeneous composition of the populations in the two areas also leads to an increase in inequality in the overall economy.

5.3.3 Transitional Dynamics

It is straightforward to see that our expanding variety model does not exhibit transitional dynamics, as the economy always grows at the constant rate given in Proposition 1. Therefore, following an exogenous shock as those considered in this section, the economy immediately moves to the new balanced growth path. We introduce transitional dynamics into this numerical example by assuming that workers relocate across sectors and areas according to a logistic function, which is commonly used to model e.g. population growth (since Verhurst, 1845), migration patterns (e.g. à la Bass, 1969, even if the Bass model was originally built to study the diffusion of new durable products), and the diffusion of innovations (e.g. Griliches, 1957).

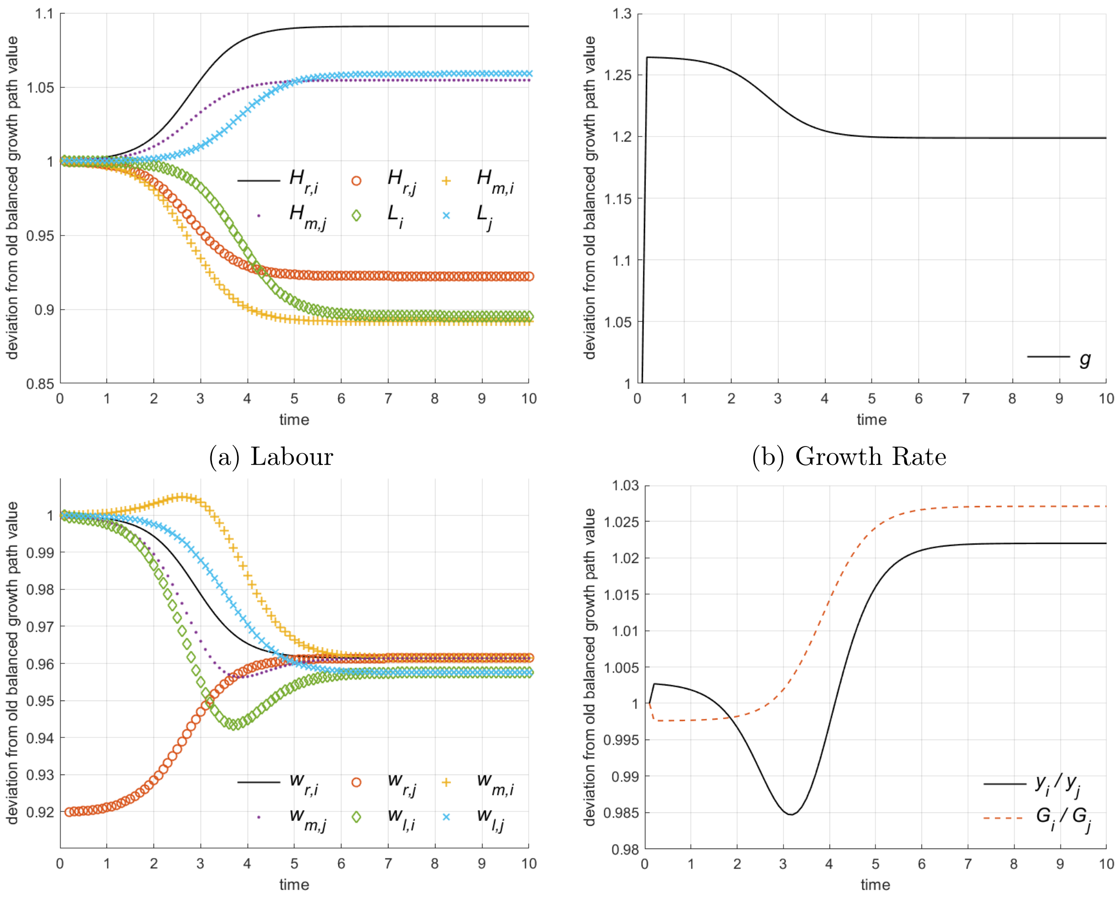
In particular, we assume that, following a shock, the stock of workers in a given sector, say $H_{r,i}$, evolves according to

$$H_{r,i}(t) = \frac{H^{\star\star}_{r,i} - H^{\star}_{r,i}}{1 + e^{a-t}} + \min\left(H^{\star\star}_{r,i} - H^{\star}_{r,i}\right), \tag{25}$$

where $H^{\star}_{r,i}$ is the old balanced growth path value, $H^{\star\star}_{r,i}$ is the new balanced growth path value (the *carrying capacity* of the sector), and a is a parameter defining the sigmoid's midpoint. Whereas the initial and final values for each sector correspond to the different balanced growth paths from the numerical examples above, the parameter a remains to be set. To facilitate the interpretation of the results, we assume only two possible values for this parameter: $a = 0.3$ for skilled workers and $a = 0.4$ for unskilled workers, thus assuming that unskilled workers move more sluggishly in response to shocks (see e.g. Wozniak, 2010, Notowidigdo, 2020). We assume that, along these dynamic paths between balanced growth equilibria, the remaining endogenous variables evolves following the changes in labour stocks according to their respective equations in Sections 3 and 4.

Imagine our economy in period $t = 0$ in the balanced growth path described by the first column of Table 1 being hit by a permanent shock such that $\Delta d = -25\%$; as we already know, this economy will converge to the balanced growth path described by the second column of Table 1. The transitional dynamics following this shock are given in Figure 1, where panel 1a presents the evolution of the stocks of labour as deviation from their old balanced growth path values (these are s-shaped as typical of the logistic function). These stocks monotonically converge to their new values, characterized by a strengthening of the previously existing patterns of specialization, but unskilled workers are more sluggish in their response to the shock: by assumption, both $L_i(t)/L^{\star}_i$ and $L_j(t)/L^{\star}_j$ take longer to converge to their new balanced growth path values than the curves for skilled workers. Panel 1b shows the resulting evolution of the growth rate of the economy, with a discontinuous jump at the time of the shock and a subsequent smooth descent towards its new value.

Panel 1c shows the evolution of wages: the stocks of labour adjusts to the shock at different speed and the composition of the workforce in each area changes between



(a) Labour

(b) Growth Rate

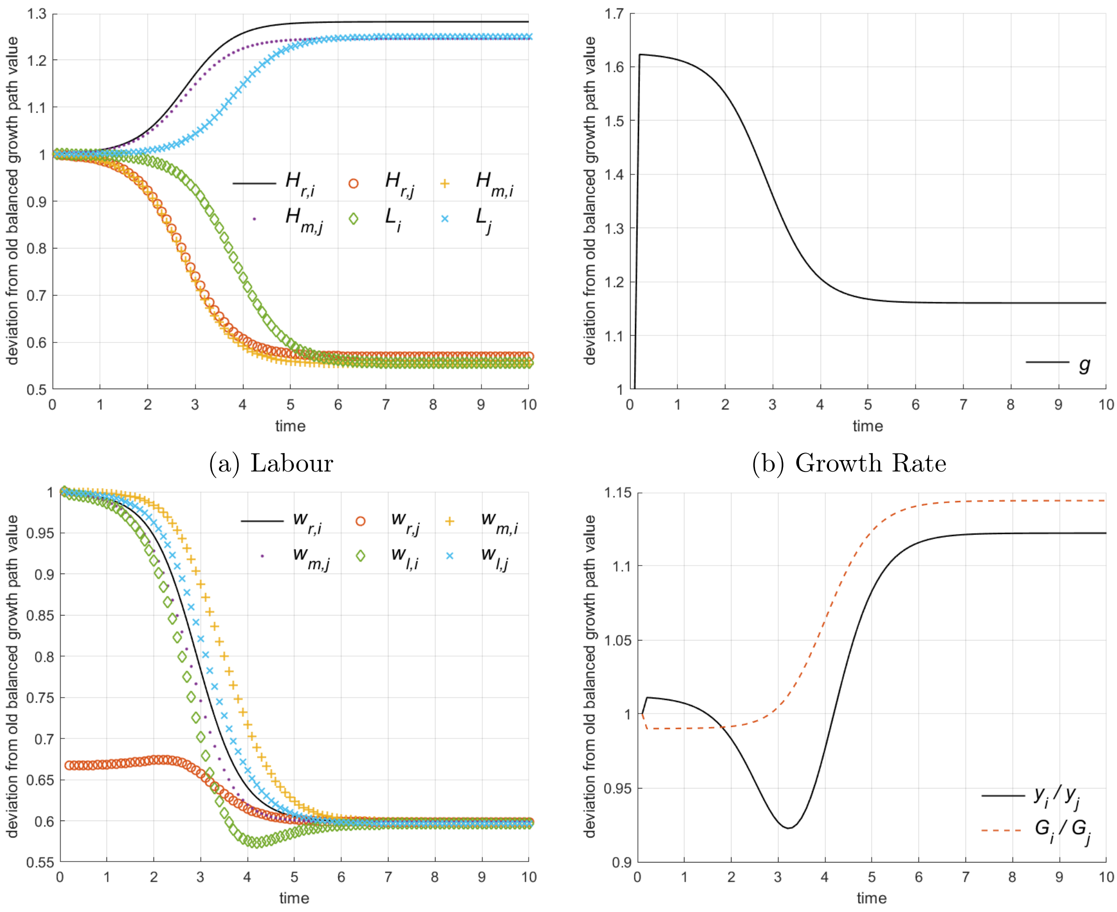
(c) Wages

(d) Inequalities

Figure 1: Transitional dynamics following a shock $\Delta d = -25\%$

periods during the transition. This is reflected in wage transitions that are not necessarily monotonic. Adjusting workforce composition and non-monotonic wages translate in inequality dynamics that may exhibit cycles, as shown in panel 1d.

The transitional dynamics following our second shock, where δ_i suddenly and permanently increases as in the third column of Table 1, are given in Figure 2. The results are similar to the one above but with a greater magnitude, since this shock directly changes the relative productivity of the two areas' research sector. The agglomeration effects are more accentuated, and thus the change in inequality is more pronounced and the cycle more evident.



(a) Labour

(b) Growth Rate

(c) Wages

(d) Inequalities

Figure 2: Transitional dynamics following a shock $\Delta\delta_i/\delta_j = +25pp$

6 Conclusions

Broadband technology and high-speed connections have steadily changed the way people lived and worked over the last decades, and the pandemic is likely to dramatically accelerate the adoption and use of digital communication in general and videoconferencing in particular. The impact will be disproportionally strong on those activities where knowledge and information are essential for production, like innovation and research. In this paper, we proposed an endogenous growth model with two geographical areas to investigate how this could change the spatial distribution of research activities and their contribution to growth, and the subsequent repercussions on per capita income and inequality levels.

We showed that, when one area is endowed with an higher ability to assimilate new knowledge and apply it to commercial use, specialization arises in equilibrium, as this area attracts a larger share of researchers; conversely, the other area specializes in manufacturing activities. Since researchers are scarcer in the entire population and command a higher wage than the average manufacturing worker, specialization in research translates into a higher income per capita level but a more unequal distribution. In this context, a boost towards a digitalization of communications, while it increases the growth rate of the overall economy, also strengthens the previously existing patterns of specialization, thus increasing the existing disparities in income per capita and Gini coefficients between areas, as well as the Gini coefficient of the entire system.

We have made many simplifying assumptions to keep the model tractable. For example, we have assumed that one area is exogenously endowed with a more productive research sector, and is thus more able to exploit a more intense transmission of knowledge; it would instead be interesting to analyse the case in which this is the outcome of conscious investments in network capital and absorptive capacity (as suggested by e.g. Huggins and Thompson, 2014). Moreover, to focus primarily on the knowledge externality, we have assumed zero transport costs and no differences in the areas' amenities; however, one could include those to analyse how workers and firms balance these factors in making location decisions. We leave these extensions to future research.

References

Acemoglu, D. (2008). *Introduction to Modern Economic Growth.* Princeton University Press, Princeton.

Acemoglu, D., Robinson, J. A., and Verdier, T. (2017). Asymmetric growth and institutions in an interdependent world. *Journal of Political Economy*, 125(5):1245–1305.

Audretsch, D. B. and Feldman, M. P. (1996). R&D spillovers and the geography of innovation and production. *The American Economic Review*, 86(3):630–640.

Baldwin, R. E. and Forslid, R. (2000). The core-periphery model and endogenous growth: Stabilizing and destabilizing integration. *Economica*, 67(267):307–324.

Bass, F. M. (1969). A new product growth for model consumer durables. *Management Science*, 15(5):215–227.

Black, D. and Henderson, V. (1999). A theory of urban growth. *Journal of Political Economy*, 107(2):252–284.

Bloom, N., Liang, J., Roberts, J., and Ying, Z. J. (2015). Does working from home work? Evidence from a Chinese experiment. *The Quarterly Journal of Economics*, 130(1):165–218.

Bond-Smith, S. C. and McCann, P. (2020). A multi-sector model of relatedness, growth and industry clustering. *Journal of Economic Geography*, 20(5):1145–1163.

Buzard, K., Carlino, G. A., Hunt, R. M., Carr, J. K., and Smith, T. E. (2017). The agglomeration of American R&D labs. *Journal of Urban Economics*, 101:14–26.

Carlino, G. and Kerr, W. R. (2015). Agglomeration and innovation. In Duranton, G., Henderson, J. V., and Strange, W. C., editors, *Handbook of Regional and Urban Economics*, volume 5 of *Handbook of Regional and Urban Economics*, chapter 6, pages 349–404. Elsevier.

Castaldo, A., Fiorini, A., and Maggi, B. (2018). Measuring (in a time of crisis) the impact of broadband connections on economic growth: An OECD panel analysis. *Applied Economics*, 50(8):838–854.

Chen, S., Liu, W., and Song, H. (2020). Broadband internet, firm performance, and worker welfare: Evidence and mechanism. *Economic Inquiry*, 58(3):1146–1166.

Cohen, W. M. and Levinthal, D. A. (1990). Absorptive capacity: A new perspective on learning and innovation. *Administrative Science Quarterly*, 35(1):128–152.

Dingel, J. I. and Neiman, B. (2020). How many jobs can be done at home? *Journal of Public Economics*, 189:104–235.

Gómez-Barroso, J. L. and Marbán-Flores, R. (2020). Telecommunications and economic development – The 21st century: Making the evidence stronger. *Telecommunications Policy*, 44(2).

Griliches, Z. (1957). Hybrid corn: An exploration in the economics of technological change. *Econometrica*, 25(4):501–522.

Hall, B. H. and Ziedonis, R. H. (2001). The patent paradox revisited: An empirical study of patenting in the U.S. semiconductor industry, 1979-1995. *RAND Journal of Economics*, 32(1):101–128.

Hardy, A. P. (1980). The role of the telephone in economic development. *Telecommunications Policy*, 4(4):278–286.

Howitt, P. (2000). Endogeous growth and cross–country income differences. *American Economic Review*, 90(4):829–846.

Huggins, R. and Thompson, P. (2014). A network-based view of regional growth. *Journal of Economic Geography*, 14(3):511–545.

Jacobs, J. (1970). *The Economy of Cities*. Vintage Books, New York.

Jaffe, A. B., Trajtenberg, M., and Henderson, R. (1993). Geographic localization of knowledge spillovers as evidenced by patent citations. *The Quarterly Journal of Economics*, 108(3):577–598.

Jones, C. I. (1995). R&D-based models of economic growth. *Journal of Political Economy*, 103(4):759–784.

Kaplow, L. (2005). The value of a statistical life and the coefficient of relative risk aversion. *Journal of Risk and Uncertainty*, 31(1):23–34.

Kolko, J. (2012). Broadband and local growth. *Journal of Urban Economics*, 71(1):100–113.

Kortum, S. (1993). Equilibrium R&D and the patent–R&D ratio: U.S. evidence. *The American Economic Review*, 83(2):450–457.

Krugman, P. (1991). Increasing returns and economic geography. *Journal of Political Economy*, 99(3):483–499.

Mack, E. A. and Rey, S. J. (2014). An econometric approach for evaluating the linkages between broadband and knowledge intensive firms. *Telecommunications Policy*, 38(1):105–118.

Marshall, A. (1890). *Principles of Economics*. Macmillan, London.

Maurseth, P. B. (2018). The effect of the Internet on economic growth: Counter-evidence from cross-country panel data. *Economics Letters*, 172:74–77.

Notowidigdo, M. J. (2020). The incidence of local labor demand shocks. *Journal of Labor Economics*, 38(3):687–725.

Polanyi, M. (1967). *The tacit dimension.* An anchor book: Philosophy. Doubleday, New York.

Rivera-Batiz, L. A. and Romer, P. M. (1991). Economic integration and endogenous growth. *The Quarterly Journal of Economics*, 106(2):531–555.

Rivera-Batiz, L. A. and Xie, D. (1993). Integration among unequals. *Regional Science and Urban Economics*, 23(3):337–354. Special Issue European Regional Economic Integration.

Romer, P. M. (1986). Increasing returns and long-run growth. *Journal of Political Economy*, 94(5):1002–1037.

Romer, P. M. (1990a). Capital, labor, and productivity. *Brookings Papers on Economic Activity. Microeconomics*, 1990:337–367.

Romer, P. M. (1990b). Endogenous technological change. *Journal of Political Economy*, 98(5):71–102.

Verhurst, P. F. (1845). Recherches mathématiques sur la loi d'accroissement de la population. *Nouveaux Mémoires de l'Académie Royale des Sciences et Belles Lettres de Bruxelles*, 18:1–38.

Wozniak, A. (2010). Are college graduates more responsive to distant labor market opportunities? *The Journal of Human Resources*, 45(4):944–970.

Xu, X., Watts, A., and Reed, M. (2019). Does access to internet promote innovation? A look at the U.S. broadband industry. *Growth and Change*, 50(4):1423–1440.

A Appendix

A.1 Proofs

Proof of equation (17). We first prove that $w_{l,i}/w_{l,j} = w_{m,i}/w_{m,j} = 1$ implies $L_i/L_j = H_{m,i}/H_{m,j}$. Substitute equations (15) and (16) into $w_{l,i}/w_{l,j} = w_{m,i}/w_{m,j} = 1$ to obtain

$$\frac{\alpha L_i^{\alpha-1} H_{m,i}^{\beta} A x_i^{\gamma} S_{m,i}}{\alpha L_j^{\alpha-1} H_{m,j}^{\beta} A x_j^{\gamma} S_{m,j}} = \frac{\beta L_i^{\alpha} H_{m,i}^{\beta-1} A x_i^{\gamma} S_{m,i}}{\beta L_j^{\alpha} H_{m,j}^{\beta-1} A x_j^{\gamma} S_{m,j}} = 1.$$

Trivial algebraic steps lead to

$$\frac{L_j}{L_i} = \frac{H_{m,j}}{H_{m,i}} = \left(\frac{L_j}{L_i}\right)^{\alpha} \left(\frac{H_{m,j}}{H_{m,i}}\right)^{\beta} \left(\frac{x_j}{x_i}\right)^{\gamma} \frac{S_{m,j}}{S_{m,i}}. \tag{A.1}$$

We now prove that equation (A.1) leads to $L_i/L_j = H_{r,j}/H_{r,i}$. Substitute $S_{m,j}/S_{m,i}$ using equations (4), rearrange, and notice that $\alpha + \beta = 1 - \gamma$, to obtain

$$\frac{L_j}{L_i} = \left(\frac{L_j}{L_i}\right)^{\alpha+\beta} \left(\frac{x_j}{x_i}\right)^{\gamma} \left(\frac{H_{m,j}}{H_{m,i}}\right)^{\phi} \left(\frac{H_{r,j}}{H_{r,i}}\right)^{\phi} = \left(\frac{x_i}{x_j}\right)^{\frac{\gamma}{\phi-\gamma}} \left(\frac{H_{r,i}}{H_{r,j}}\right)^{\frac{\phi}{\phi-\gamma}}. \tag{A.2}$$

The ratio of the demands function of intermediate input can be computed from (4) and (10) as

$$\frac{x_i}{x_j} = \left(\frac{L_i}{L_j}\right)^{\frac{1-\gamma+\phi}{1-\gamma}} \left(\frac{H_{r,i}}{H_{r,j}}\right)^{\frac{\phi}{1-\gamma}}. \tag{A.3}$$

Substituting this into (A.2), after a few algebraic steps one obtains the desired result $L_i/L_j = H_{r,j}/H_{r,i}$. Substituting this back into (A.3) entails $L_i/L_j = x_i/x_j$. As a corollary, the above results also ensure that $S_{m,i} = S_{m,j}$ in equilibrium. □

Proof of equation (18). The ratio of the wages of skilled workers across sectors is

$$\begin{aligned}\frac{w_m}{w_r} &= \frac{\beta L_i^{\alpha} H_{m,i}^{\beta-1} A x_i^{\gamma} S_{m,i}}{A X \eta \delta_i H_{r,i}^{\eta-1} S_{r,ij} (1-\gamma) r^{-1}} = \frac{\beta L_i^{\alpha} H_{m,i}^{\beta-1} x_i^{\gamma} S_{m,i} r}{X \eta g H_r^{-1} (1-\gamma)} = \frac{\beta H_{m,i}^{-1} x_i r \gamma^{-1}}{X \eta g H_r^{-1} (1-\gamma)} = \\ &= \frac{\beta x_i H_r r}{X \eta g \gamma (1-\gamma) H_{m,i}} = \frac{\beta x_i H_r r}{x_i H_m H_{m,i}^{-1} \eta g \gamma (1-\gamma) H_{m,i}} = \frac{\beta H_r r}{H_m \eta g \gamma (1-\gamma)},\end{aligned}$$

where the first equality in the first line comes from simply replacing wages with their definition in (13b) and (16), the second from applying (20) to the denominator, and the third by applying (10a) to the numerator. The first equality in the second line comes from rearranging the previous expression, the second from using $X = x_i + x_j$ combined with $x_i/x_j = H_{m,i}/H_{m,j}$ to substitute for X, and the third from simply rearranging the previous one. Finally, it is enough to notice that the w_m/w_r must be equal to one in equilibrium to obtain expression (18). □

Proof of the Stability of the Steady State. To check the stability of the equilibrium, it is sufficient to see whether the wage gap declines as researchers move towards the area offering a higher wage. The condition for the stability of the equilibrium in equation (14) thus is

$$\frac{\partial w_{r,i}}{\partial H_{r,i}}\bigg|_{H_{r,i}=H^{\star}_{r,i}} < 0, \tag{A.4}$$

where $H^{\star}_{r,i}$ makes it explicit that we are evaluating the labour allocation along the balanced growth path. Using equation (13a), the definition of the spatial spillovers in equation (8), and taking note that $H_{r,j} = H_r - H_{r,i}$, equilibrium stability requires

$$\left(\frac{\psi+\eta-1}{H_{r,i}} - \frac{\psi}{H_{r,j}}\right)\left(H_{r,i}^{\psi+\eta-1}H_{r,j}^{\psi}\right)\nu_i^{\psi}\eta\delta_i X\frac{1-\gamma}{r}A < 0.$$

The sign of the left hand side depends on the sign of its first term; hence, the stability condition simplifies to:

$$\frac{\psi+\eta-1}{\psi} < \frac{H_{r,i}}{H_{r,j}}. \tag{A.5}$$

This condition is always satisfied given Assumption 3. It must be noted, however, that this restriction is sufficient but not necessary to ensure stability. For instance, when $\delta_i > \delta_j$ as in the rest of the analysis, the equilibrium allocation leads to $H_{r,i}/H_{r,j} > 1$ and a less demanding restriction $0 \leq \psi$ would suffice. It is only when $\delta_i < \delta_j$ and $H_{r,i}/H_{r,j} < 1$ that $\psi < 1-\eta$ might be necessary to ensure stability. ☐

Proof of Corollary 2.1. Letting $w \equiv w_l/w_h$,

$$\begin{aligned}
\frac{y_i}{y_j} &\equiv \frac{w_l L_i + w_h H_{m,i} + w_h H_{r,i}}{w_l L_j + w_h H_{m,j} + w_h H_{r,j}}\left(\frac{L_j + H_{m,j} + H_{r,j}}{L_i + H_{m,i} + H_{r,i}}\right) = \\
&= \frac{L_i w_h\left(w + H_{m,i}/L_i + H_{r,i}/L_i\right)}{L_j w_h\left(w + H_{m,j}/L_j + H_{r,j}/L_j\right)}\left(\frac{L_j\left(1 + H_{m,j}/L_j + H_{r,j}/L_j\right)}{L_i\left(1 + H_{m,i}/L_i + H_{r,i}/L_i\right)}\right) = \\
&= \frac{w + H_{m,i}/L_i + H_{r,i}/L_i}{w + H_{m,j}/L_j + H_{r,j}/L_j}\left(\frac{1 + H_{m,j}/L_j + H_{r,j}/L_j}{1 + H_{m,i}/L_i + H_{r,i}/L_i}\right) = \\
&= \frac{w + H_m/L + \left(H_r/L\right)\left(L_j/L_i\right)}{w + H_m/L + \left(H_r/L\right)\left(L_i/L_j\right)}\left(\frac{1 + H_m/L + \left(H_r/L\right)\left(L_i/L_j\right)}{1 + H_m/L + \left(H_r/L\right)\left(L_j/L_i\right)}\right) = \\
&= \frac{\left(w + \frac{H_m}{L}\right)\left(1 + \frac{H_m}{L}\right) + \frac{H_r}{L}\frac{L_j}{L_i}\left(1 + \frac{H_m}{L} + \frac{H_r}{L}\frac{L_i}{L_j}\right) + \left(w + \frac{H_m}{L}\right)\frac{H_r}{L}\frac{L_i}{L_j}}{\left(w + \frac{H_m}{L}\right)\left(1 + \frac{H_m}{L}\right) + \frac{H_r}{L}\frac{L_i}{L_j}\left(1 + \frac{H_m}{L} + \frac{H_r}{L}\frac{L_j}{L_i}\right) + \left(w + \frac{H_m}{L}\right)\frac{H_r}{L}\frac{L_j}{L_i}},
\end{aligned}$$

where the first line follows from dividing (22) by total area's employment, the second from aggregating for $w_h L_i$ and $w_h L_j$ and rewriting, the third from simplifying, the fourth from using the following identities which result from (18), $H_{m,i}/L_i = H_{m,j}/L_j = H_m/L$, $H_{r,i}/L_i = \left(H_r/L\right)\left(L_j/L_i\right)$, and $H_{r,j}/L_j = \left(H_r/L\right)\left(L_i/L_j\right)$, and the last by multiplying throughout. A sufficient condition for $y_i > y_j$ then is

$$\frac{H_r}{L}\frac{L_j}{L_i}\left(1+\frac{H_m}{L}\right)+\left(w+\frac{H_m}{L}\right)\frac{H_r}{L}\frac{L_i}{L_j} > \frac{H_r}{L}\frac{L_i}{L_j}\left(1+\frac{H_m}{L}\right)+\left(w+\frac{H_m}{L}\right)\frac{H_r}{L}\frac{L_j}{L_i}$$
$$\text{i.e.}\quad \left(\frac{L_j}{L_i}-\frac{L_i}{L_j}\right)\frac{H_r}{L}\left(1-\frac{\alpha}{\beta}\frac{H_m}{L}\right) > 0,$$

where the second line follows from substituting (15) and (16) into $w \equiv w_l/w_h$ to obtain $w = (\alpha/\beta)(H_m/L)$ and rearranging. This sufficient condition is always satisfied since $\delta_i > \delta_j$ implies $L_j > L_i$ from Proposition 2, and $\alpha H_m/(\beta L) < 1$ by Assumption 2. □

Proof of Corollary 2.2. The Gini coefficient in area i is given in equation (23); equivalently $G_i = H_i/(H_i + L_i w) - H_i/(H_i + L_i)$, where $H_i \equiv H_{r,i} + H_{m,i}$. Then, with some analytical steps,

$$G_i - G_j = \left(\frac{H_i}{H_i+L_iw}-\frac{H_j}{H_j+L_jw}\right)-\left(\frac{H_i}{H_i+L_i}-\frac{H_j}{H_j+L_j}\right)$$
$$= \frac{(w-1)(H_iL_j-H_jL_i)(H_iH_j-wL_iL_j)}{(H_i+L_iw)(H_j+L_jw)(H_i+L_i)(H_j+L_j)}.$$

Since the denominator is positive, $w - 1 < 0$ by Assumption 2, and $H_iL_j - H_jL_i > 0$ by Assumption 5 and Proposition 2, a sufficient condition for $G_i > G_j$ is $H_iH_j - wL_iL_j < 0$. Using $w = (\alpha/\beta)(H_m/L)$ and conditions (14) and (17), this is equivalent to

$$\frac{H_m}{L}\frac{H_m}{L}+\frac{H_m}{L}\frac{H_r}{L}\left(\frac{L_j}{L_i}+\frac{L_i}{L_j}\right)+\frac{H_r}{L}\frac{H_r}{L} < \frac{\alpha}{\beta}\frac{H_m}{L},\ \text{i.e.}$$
$$L > \frac{\beta}{\alpha}\frac{1}{H_m}\left\{H_mH_m + H_mH_r\left[\left(\frac{\delta_i\nu_i^{\psi}}{\delta_j\nu_j^{\psi}}\right)^{-\frac{1}{1-\eta}}+\left(\frac{\delta_i\nu_i^{\psi}}{\delta_j\nu_j^{\psi}}\right)^{\frac{1}{1-\eta}}\right]+H_rH_r\right\},$$

which is always satisfied if L is sufficiently larger than H. □

Proof of Proposition 3. Take the derivative of $H_{r,i}/H_{r,j}$ in (14) with respect to d to obtain

$$\frac{\partial\left(H_{r,i}/H_{r,j}\right)}{\partial d} = \left(\frac{\delta_i}{\delta_j}\right)^{\frac{1}{1-\eta}}\frac{\psi}{1-\eta}\left(\frac{\nu_i}{\nu_j}\right)^{\frac{\psi-1+\eta}{1-\eta}}\frac{1}{v_j^2}\left(v_j\frac{\partial v_i}{\partial d}-v_i\frac{\partial v_j}{\partial d}\right),$$

the sign of which depends on the sign of the last term on the right hand side. This is negative given Assumption 4, implying that a reduction in d determines an increase in $H_{r,i}/H_{r,j}$; given the equilibrium condition in (17), this is associated with an increase in $H_{m,j}/H_{m,i}$ and L_j/L_i. Proposition 2 shows that, in the previous equilibrium, $H_{r,i}/H_{r,j} = H_{m,j}/H_{m,i} = L_j/L_i > 1$ by Assumption 5: the shock thus strengthens the previously existing patterns of specialization.

Along the balanced growth path, the constant growth rate is given by equation (20). Using (8), and depending on which of the two definitions is used, the derivative with respect to d is

$$\begin{aligned}\frac{\partial g}{\partial d} &= g\left[-\frac{1-\eta-\psi}{H_{r,i}}\frac{\partial H_{r,i}}{\partial d}+\frac{\psi}{H_{r,j}}\frac{\partial H_{r,j}}{\partial d}+\frac{\psi}{\nu_i}\frac{\partial \nu_i}{\partial d}+\frac{1}{H_r}\frac{\partial H_r}{\partial d}\right]\\ &= g\left[-\frac{1-\eta-\psi}{H_{r,j}}\frac{\partial H_{r,j}}{\partial d}+\frac{\psi}{H_{r,i}}\frac{\partial H_{r,i}}{\partial d}+\frac{\psi}{\nu_j}\frac{\partial \nu_j}{\partial d}+\frac{1}{H_r}\frac{\partial H_r}{\partial d}\right].\end{aligned} \tag{A.6}$$

Substituting the following results obtained from differentiating (18),

$$\frac{\partial H_r}{\partial d} = \frac{(H-H_r)H_r}{H}\frac{\rho}{rg}\frac{\partial g}{\partial d}, \tag{A.7}$$

into (A.6), and rearranging, one obtains

$$\begin{aligned}\frac{\partial g}{\partial d} &= \frac{rHg}{rH-(H-H_r)\rho}\left[-\frac{1-\eta-\psi}{H_{r,i}}\frac{\partial H_{r,i}}{\partial d}+\frac{\psi}{H_{r,j}}\frac{\partial H_{r,j}}{\partial d}+\frac{\psi}{\nu_i}\frac{\partial \nu_i}{\partial d}\right]\\ &= \frac{rHg}{rH-(H-H_r)\rho}\left[-\frac{1-\eta-\psi}{H_{r,j}}\frac{\partial H_{r,j}}{\partial d}+\frac{\psi}{H_{r,i}}\frac{\partial H_{r,i}}{\partial d}+\frac{\psi}{\nu_j}\frac{\partial \nu_j}{\partial d}\right],\end{aligned} \tag{A.8}$$

which requires an equalisation of the terms inside the brackets. After some manipulations, this accounts to

$$\frac{\partial H_{r,j}}{\partial d}H_{r,j}^{-1} - \frac{\partial H_{r,i}}{\partial d}H_{r,i}^{-1} = \frac{\psi}{1-\eta}\left(\frac{\partial \nu_j}{\partial d}\nu_j^{-1} - \frac{\partial \nu_i}{\partial d}\nu_i^{-1}\right). \tag{A.9}$$

The right hand side of (A.9) is positive by Assumptions 3 and 4. As a consequence,

$$\frac{\partial H_{r,j}}{\partial d} > \frac{\partial H_{r,i}}{\partial d}\frac{H_{r,j}}{H_{r,i}}, \tag{A.10}$$

where the last ratio on the right hand side is lower than one by Assumption 5 and Proposition 2. Since $\partial(H_{r,i}/H_{r,j})/\partial d < 0$ was proven above, there are two possible cases consistent with (A.10):

$$i)\quad \frac{\partial H_{r,i}}{\partial d} < \frac{\partial H_{r,j}}{\partial d} < 0 \tag{A.11}$$

$$ii)\quad \frac{\partial H_{r,j}}{\partial d} > 0 > \frac{\partial H_{r,i}}{\partial d}. \tag{A.12}$$

In case i), both research sectors experience an influx of skilled workers after a decrease in d, but the change is relatively bigger in the more advanced research sector. Since both $H_{r,i}$ and $H_{r,j}$ increase after a negative shock to d, whereas H is constant, it must be the case that some skilled workers move from the manufacturing sector to the research sector. Given condition (17) and the fixed supply of H and L, $\partial H_{m,i}/\partial d > \partial H_{m,j}/\partial d > 0$, and thus both areas also experience a reduction in the number of skilled workers in the manufacturing sector, which is more pronounced in urban area i; at the same time, the relatively less research-intensive urban area j receives an influx of unskilled workers from i, $\partial L_i/\partial d > 0 > \partial L_j/\partial d$. Finally, since $\partial H_r/\partial d < 0$, equation (A.7) implies $\partial g/\partial d < 0$: a permanent negative shock to d causes a permanent increase in the common growth rate g.

In case ii), $H_{r,j}$ decreases after a negative shock to d, whereas $H_{r,i}$ increases. From the second line in (A.8), g increases; from (A.7), so does H_r, which implies a decrease in the number of skilled workers in the manufacturing sectors. Given condition (17) and the fixed supply of H and L, $\partial H_{m,i}/\partial d > \partial H_{m,j}/\partial d$; at the same time, the less research-intensive area j receives an influx of unskilled workers from i, $\partial L_i/\partial \delta_i < 0 < \partial L_j/\partial \delta_i$.

□

Proof of Proposition 4. From the equilibrium condition (14), and using Assumption 4,

$$\frac{\partial\left(H_{r,i}/H_{r,j}\right)}{\partial \delta_i} = \frac{1}{1-\eta}\frac{H_{r,i}}{H_{r,j}}\left[\frac{1}{\delta_i} + \frac{\psi}{\nu_i}\frac{\partial \nu_i}{\partial \delta_i}\right] > 0 \tag{A.13a}$$

$$\frac{\partial\left(H_{r,i}/H_{r,j}\right)}{\partial \delta_j} = -\frac{1}{1-\eta}\frac{H_{r,i}}{H_{r,j}}\left[\frac{1}{\delta_j} + \frac{\psi}{\nu_j}\frac{\partial \nu_j}{\partial \delta_j}\right] < 0. \tag{A.13b}$$

Therefore, a decrease in δ_i (δ_j) determines a decrease (increase) in $H_{r,i}/H_{r,j}$; given the equilibrium condition in (17), this is associated with an increase (decrease) in $H_{m,i}/H_{m,j}$ and L_i/L_j.

Along the balanced growth path, the constant growth rate is given by equation (20). Using (8), and depending on which of the two definitions is used, the derivative with respect to δ_i is

$$\begin{aligned}\frac{\partial g}{\partial \delta_i} &= g\left[\frac{1}{\delta_i} - \frac{1-\eta-\psi}{H_{r,i}}\frac{\partial H_{r,i}}{\partial \delta_i} + \frac{\partial H_{r,j}}{\partial \delta_i}\frac{\psi}{H_{r,j}} + \frac{\partial \nu_i}{\partial \delta_i}\frac{\psi}{\nu_i} + \frac{\partial H_r}{\partial \delta_i}\frac{1}{H_r}\right] \\ &= g\left[-\frac{1-\eta-\psi}{H_{r,j}}\frac{\partial H_{r,j}}{\partial \delta_i} + \frac{\partial H_{r,i}}{\partial \delta_i}\frac{\psi}{H_{r,i}} + \frac{\partial H_r}{\partial \delta_i}\frac{1}{H_r}\right].\end{aligned} \tag{A.14}$$

Substituting the following results obtained from differentiating (18),

$$\frac{\partial H_r}{\partial \delta_i} = \frac{\left(H - H_r\right)H_r}{H}\frac{\rho}{rg}\frac{\partial g}{\partial \delta_i}, \tag{A.15}$$

into (A.14), and rearranging, one obtains

$$\begin{aligned}\frac{\partial g}{\partial \delta_i} &= \frac{rHg}{rH - (H-H_r)\rho}\left[\frac{1}{\delta_i} - \frac{1-\eta-\psi}{H_{r,i}}\frac{\partial H_{r,i}}{\partial \delta_i} + \frac{\partial H_{r,j}}{\partial \delta_i}\frac{\psi}{H_{r,j}} + \frac{\partial \nu_i}{\partial \delta_i}\frac{\psi}{\nu_i}\right] \\ &= \frac{rHg}{rH - (H-H_r)\rho}\left[-\frac{1-\eta-\psi}{H_{r,j}}\frac{\partial H_{r,j}}{\partial \delta_i} + \frac{\partial H_{r,i}}{\partial \delta_i}\frac{\psi}{H_{r,i}}\right],\end{aligned} \tag{A.16}$$

which requires an equalisation of the terms inside the brackets. After some manipulations, this accounts to

$$(1-\eta)\left\{\frac{\partial H_{r,j}}{\partial \delta_i}H_{r,j}^{-1}-\frac{\partial H_{r,i}}{\partial \delta_i}H_{r,i}^{-1}\right\}=-\left\{\frac{\partial \nu_i}{\partial \delta_i}\frac{\psi}{\nu_i}+\frac{1}{\delta_i}\right\}. \tag{A.17}$$

The right hand side of (A.17) is negative by Assumptions 4 and 3. As a consequence,

$$\frac{\partial H_{r,j}}{\partial \delta_i}<\frac{\partial H_{r,i}}{\partial \delta_i}\frac{H_{r,j}}{H_{r,i}}, \tag{A.18}$$

where the last ratio on the right hand side is lower than one by Assumption 5 and Proposition 2. Since $\partial(H_{r,i}/H_{r,j})/\partial\delta_i > 0$ as proven above, there are two possible cases consistent with (A.18):

$$i)\quad \frac{\partial H_{r,i}}{\partial \delta_i}>\frac{\partial H_{r,j}}{\partial \delta_i}>0 \tag{A.19}$$

$$ii)\quad \frac{\partial H_{r,i}}{\partial \delta_i}>0>\frac{\partial H_{r,j}}{\partial \delta_i}. \tag{A.20}$$

In case i), both $H_{r,i}$ and $H_{r,j}$ decrease after a negative shock to δ_i, whereas H is constant: given condition (17) and the fixed supply of H and L, both sectors also experience an increase in the number of skilled workers in the manufacturing sector, which is more pronounced in urban area i, $\partial H_{m,i}/\partial\delta_i < \partial H_{m,j}/\partial\delta_i < 0$; at the same time, the more research-intensive urban area i receives an influx of unskilled workers from j, $\partial L_i/\partial\delta_i < 0 < \partial L_j/\partial\delta_i$. Since $\partial H_r/\partial\delta_i > 0$, equation (A.15) implies $\partial g/\partial\delta_i > 0$: a permanent negative shock to δ_i causes a permanent decrease in the common growth rate g.

In case ii), $H_{r,i}$ decreases after a negative shock to δ_i, whereas $H_{r,j}$ increases. From the second line in (A.16), g decreases; from (A.15), so does H_r, which implies a rise in the number of skilled workers in the manufacturing sectors. Given condition (17) and the fixed supply of H and L, $\partial H_{m,i}/\partial\delta_i > \partial H_{m,j}/\partial\delta_i$; at the same time, the more research-intensive urban area i receives an influx of unskilled workers from j, $\partial L_i/\partial\delta_i < 0 < \partial L_j/\partial\delta_i$. □

Proof of Proposition 5. From the equilibrium condition (14), and since Assumptions 4 and 5 imply $\nu_i > \nu_j$,

$$\frac{\partial\left(H_{r,i}/H_{r,j}\right)}{\partial\psi}=\left(\frac{\delta_i}{\delta_j}\right)^{\frac{1}{1-\eta}}\left(\frac{\nu_i}{\nu_j}\right)^{\frac{\psi}{1-\eta}}\frac{\ln\left(\frac{\nu_i}{\nu_j}\right)}{1-\eta}>0. \tag{A.21a}$$

Therefore, a decrease in ψ determines a decrease in $H_{r,i}/H_{r,j}$; given the equilibrium condition in (17), this is associated with an increase in $H_{m,i}/H_{m,j}$ and L_i/L_j.

Along the balanced growth path, the constant growth rate is given by equation (20). Using (8), and depending on which of the two definitions is used, the derivative with respect to ψ is

$$\begin{aligned}\frac{\partial g}{\partial \psi} &= g\left[-\frac{\partial H_{r,i}}{\partial \psi}\frac{1-\eta-\psi}{H_{r,i}} + \frac{\partial H_{r,j}}{\partial \psi}\frac{\psi}{H_{r,j}} + \ln\left(H_{r,i}H_{r,j}\nu_i\right) + \frac{\partial H_r}{\partial \psi}\frac{1}{H_r}\right] \\ &= g\left[-\frac{\partial H_{r,j}}{\partial \psi}\frac{1-\eta-\psi}{H_{r,j}} + \frac{\partial H_{r,i}}{\partial \psi}\frac{\psi}{H_{r,i}} + \ln\left(H_{r,i}H_{r,j}\nu_j\right) + \frac{\partial H_r}{\partial \psi}\frac{1}{H_r}\right]\end{aligned} \tag{A.22}$$

Substituting the following result obtained from differentiating (18),

$$\frac{\partial H_r}{\partial \psi} = \frac{\left(H - H_r\right)H_r}{H}\frac{\rho}{rg}\frac{\partial g}{\partial \psi}, \tag{A.23}$$

into (A.22), and rearranging, one obtains

$$\begin{aligned}\frac{\partial g}{\partial \psi} &= \frac{rHg}{rH-(H-H_r)\rho}\left[-\frac{1-\eta-\psi}{H_{r,i}}\frac{\partial H_{r,i}}{\partial \psi} + \frac{\partial H_{r,j}}{\partial \psi}\frac{\psi}{H_{r,j}} + \ln\left(H_{r,i}H_{r,j}\nu_i\right)\right] \\ &= \frac{rHg}{rH-(H-H_r)\rho}\left[-\frac{1-\eta-\psi}{H_{r,j}}\frac{\partial H_{r,j}}{\partial \psi} + \frac{\partial H_{r,i}}{\partial \psi}\frac{\psi}{H_{r,i}} + \ln\left(H_{r,i}H_{r,j}\nu_j\right)\right],\end{aligned} \tag{A.24}$$

which requires an equalisation of the terms inside the brackets. After some manipulations, this accounts to

$$(1-\eta)\left\{\frac{\partial H_{r,j}}{\partial \psi}H_{r,j}^{-1} - \frac{\partial H_{r,i}}{\partial \psi}H_{r,i}^{-1}\right\} = \ln\left(H_{r,i}H_{r,j}\nu_j\right) - \ln\left(H_{r,i}H_{r,j}\nu_i\right). \tag{A.25}$$

The right hand side of (A.25) is negative by Assumptions 4 and 5. As a consequence,

$$\frac{\partial H_{r,j}}{\partial \psi} < \frac{\partial H_{r,i}}{\partial \psi}\frac{H_{r,j}}{H_{r,i}}, \tag{A.26}$$

where the last ratio on the right hand side is lower than one by Assumption 5 and Proposition 2. Since $\partial(H_{r,i}/H_{r,j})/\partial\psi > 0$ as proven above, there are two possible cases consistent with (A.26):

$$i)\quad \frac{\partial H_{r,i}}{\partial \psi} > \frac{\partial H_{r,j}}{\partial \psi} > 0 \tag{A.27}$$

$$ii)\quad \frac{\partial H_{r,i}}{\partial \psi} > 0 > \frac{\partial H_{r,j}}{\partial \psi}. \tag{A.28}$$

In case i), both $H_{r,i}$ and $H_{r,j}$ decrease after a negative shock to ψ, whereas H is constant: given condition (17) and the fixed supply of H and L, both sectors also experience an increase in the number of skilled workers in the manufacturing sector, which is more pronounced in urban area i, $\partial H_{m,i}/\partial\psi < \partial H_{m,j}/\partial\psi < 0$; at the same time, the more research-intensive urban area i receives an influx of unskilled workers from j, $\partial L_i/\partial\psi < 0 < \partial L_j/\partial\psi$. Since $\partial H_r/\partial\psi > 0$, equation (A.23) implies $\partial g/\partial\psi > 0$: a permanent negative shock to ψ causes a permanent decrease in the common growth rate g.

In case ii), $H_{r,i}$ decreases after a negative shock to ψ, whereas $H_{r,j}$ increases. From the second line in (A.24) and since $H_{r,i}H_{r,j} \times \min(\nu_i, \nu_j) \geq 1$, g decreases; from (A.15), so does H_r, which implies a rise in the number of skilled workers in the manufacturing sectors. Given condition (17) and the fixed supply of H and L, $\partial H_{m,i}/\partial\psi < \partial H_{m,j}/\partial\psi$,; at the same time, the more research-intensive urban area i receives an influx of unskilled workers from j, $\partial L_i/\partial\psi < 0 < \partial L_j/\partial\psi$. □