18. Information sharing in oligopoly

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1 INTRODUCTION

Oligopolistic firms face obvious incentives to coordinate their output and price strategies, in order to collude on otherwise contested markets. A seemingly related question is whether oligopolists face incentives to disclose or even share their private information on either market or technological conditions *before* engaging in market competition. These incentives to "collaborate" with rival firms do not stem from the softening (or even avoidance) of competition, but rather from the modification of the informational structure under which the upcoming competition will take place. It has been argued that understanding such incentives has strong policy relevance, as it can guide regulative intervention by suggesting whether evidence of information sharing should or should not be interpreted as evidence of market collusion (see Kuhn and Vives's 1995 report on the EU industry).

These considerations have motivated vast attention in the theoretical industrial organization (IO) literature, where game-theoretic models of incomplete information have been employed to disentangle the forces that finally result in the incentives to disclose or share one's private information. Most papers have dealt with situations where information is shared prior to the realization of uncertainty (the *ex ante* case), so that the decision to disclose does not signal anything about one's own private information. There have also been a few attempts to deal with the *interim* case, where firms receive their private information prior to taking action. The *interim* case provides firms' strategies with a signalling content, and is therefore more complex.

In this survey we discuss the main insights from the vast body of research on the subject, with special attention to what is now understood about the role of the various aspects of the oligopolistic model and of the informational structure in generating incentives to share. While the existing literature has studied the forces behind multilateral sharing (disclosure of private information to all other firms in the market), we devote a considerable part of this survey to recent developments of the model that encompass targeted and bilateral sharing agreements, where pairs of firms decide to exclusively share their private information. Using economics terminology, shared information is here a "club" good, compared to the "public good" property of shared information in the traditional multilateral model.

1.1 The Basic Oligopoly Model

This section is based on Raith's (1996) general model of information sharing in oligopoly. Consider a stochastic oligopoly model with n firms; each firm i's profit is affected by a random variable τ_i, distributed normally with zero mean and variance t_i. The covariance between τ_i

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and τ_j is $t_n \geq 0$ for all i, j. Depending on the specific application, this may represent deviations from the mean of either marginal costs or of the intercept of the demand function. We refer to the vector $\tau = (\tau_1, \tau_2, \ldots, \tau_n)$'s as the "state of the world". Each firm i plays a strategy $s_i \in R_+$ (a quantity in Cournot competition and a price in Bertrand competition). The following expression describes the relation between firm i's profit and the ith component τ_i of the state of the world, firm i's strategy and the strategies of all other firms j:

$$\pi_i = \alpha_i(\tau_i) - \sum_{j \neq i} \varepsilon s_i s_j + (\beta + \gamma_s \tau_i - \delta s_i) s_i. \tag{18.1}$$

In the above expression, the term $\alpha_i(\tau_i)$ is a function of τ_i, δ is assumed to be positive, and $\varepsilon \in \left(-\frac{\delta}{n-1}, \delta\right]$. Expression (18.1) fits a large set of oligopolistic models. Uncertainty on a common demand intercept corresponds to the case of perfectly correlated states of the world (the τ_is) and $\gamma_s = 1$. Uncertainty about costs corresponds to the case where $\gamma_s = -1$ (in which case the demand intercept is given by β alone). A positive and small ε (relative to δ) expresses a high degree of product differentiation (or a quickly increasing marginal cost as in Kirby, 1988); a negative ε expresses strategic complementarity in firms' strategies.

Firms do not observe the state of the world τ. However, each firm i privately observes a noisy signal y_i about τ_i, with $y_i = \tau_i + \eta_i$, where the noise η_i is normally distributed with zero mean, variance u_{ii} and covariance $u_n \geq 0$. We assume that $t_i = t$ and $u_{iii} =$ for all i, and denote by $p_s = (t + u)$ the variance of signals and by $p_n = (t + u_n)$ the covariance.

The following classification of informational structures have been shown by Raith (1996) to be key in determining the incentives of firms to disclose and share information (we will discuss Raith's work in the next sections):

- **Common value (CV)**: $t_n = t$. In this case, all τ_is are perfectly correlated. This is the case, for instance, of firms facing a common demand intercept, on which each firm receives a private noisy signal, or of firms producing with perfectly correlated costs.
- **Independent values (IV)**: $t_n = u_n = 0$. In this case, each firm i's profit is affected by a state of the world τ_i whose distribution is independent of the distribution of all other τ_js. In addition, this condition requires that firms' signals are conditionally independent, that is, that the noise of each firm's signal is independent of the noise of the other firms' signals. So, correlation is ruled out both in the market or technological conditions faced by firms, and in the informational channels that firms use to acquire information of their own τ.
- **Perfect signals (PS)**: $u = 0$. This assumption requires that each firm i gets to know with infinite precision its own state τ_i.

1.2 Modelling Information Disclosure and Sharing

In addition to observing their own private signals, firms are allowed to modify the market information structure by disclosing and/or sharing private information with other firms. We will first discuss the two prevailing models used in the literature to represent the technology of information disclosure and sharing.

In the *strategic* model, each firm decides whether to unilaterally disclose its own information to other firms, and receives the information of all other disclosing firms irrespective of its own disclosure decision. This model is well described as a game in which each firm's strategy is whether to disclose or not its information to either all or a subset of firms, and firms' expected payoffs depend on the disclosure strategy of all firms in the market.

In the *contractual* model, firms share information with competitors on a *quid pro quo* basis: by refusing to disclose its own information, a firm also loses the information of the other disclosing firms. Almost all papers in the literature (with the exception of Kirby, 1988 and Malueg and Tsutsui, 1996) have focused on the comparison between the total absence of sharing and the universal sharing of information (an industry-wide agreement), interpreting the difference in expected payoffs as the incentives to form a trade association for the industry.

The *contractual* model naturally leads itself to a more extensive analysis, based on games of coalition formation and of network formation, where firms can form information-sharing coalitions or bilateral agreements, and exclude other firms from their private information. Malueg and Tsutsui (1996) have focused on the formation of small coalitions of sharing firms, adopting the concept of "coalition-proof equilibrium", based on the robustness of a coalition to "credible" deviations of sub coalitions (see Bernheim and Whinston, 1987).

In a recent paper, Currarini and Feri (2015) have studied the incentives of firms to form bilateral sharing agreements. In the spirit of the contractual model, they have maintained the assumption of *quid pro quo* exchanges: firm i is not allowed to observe firm j's signal unless it reveals its own signal to firm j. Differently from the multilateral case, transitivity of sharing agreements may fail, in the sense that information sharing between firms i and j and between firms j and k need not imply information sharing between firms i and k. An "information structure" is therefore given by a non-directed *network* g, in which each link ij denotes a bilateral information-sharing agreement between firms i and j. We denote by $N_i \equiv \{j : ij \in g\} \cup \{i\}$ the set of neighbours of i in g (including i) and we denote by $n_i = |N_i|$ the number of such neighbours. The information available to firm i in the information structure g is therefore $I_i(g) \equiv \left\{y_j : j \in N_i\right\}$, that is, the set of signals observed by the neighbours of i. We will use the notation $g + ij$ to denote the network obtained by adding to g the link $ij \notin g$, and $g - ij$ to denote the network obtained by severing the link $ij \in g$ from g.

The basic and least stringent notion of equilibrium that is in line with this approach is that of "pairwise stability", first introduced by Jackson and Wolinsky (1996). A *pairwise stable* network g satisfies two conditions: no firm has an incentive to sever any of its links in g and no pair of firms have an incentive to add a new link to g. More formally, let $E\pi_i(g)$ denote the expected profit of firm i if the information structure underlying market competition is described by the network g.

Definition 1 *The information structure g is* ***pairwise stable*** *if:*

(1) $E\pi_i(g) \geq E\pi_i(g - ij)$ *for all* $ij \in g$;
(2) $E\pi_i(g + ij) > E\pi_i(g) \rightarrow E\pi_j(g + ij) < E\pi_j(g)$ *for all* $ij \notin g$.

The above definition implicitly rules out the possibility of side payments between firms that are contingent on the sharing of information. In the presence of such transfers, the two conditions of definition 1 would be replaced by the following (see Jackson and Wolinsky, 1996):

Definition 2 *The information structure g is* ***pairwise stable with transfers*** *if:*

(1′) $E\pi_i(g) + E\pi_j(g) \geq E\pi_i(g - ij) + E\pi_j(g - ij)$ *for all* $ij \in g$;
(2′) $E\pi_i(g + ij) + E\pi_j(g + ij) \leq E\pi_i(g) + E\pi_j(g)$ *for all* $ij \notin g$.

A stronger notion of stability allows each firm to revise any subset of its links (instead of only one link), and any pair of firms to form a new one. This notion of *pairwise Nash stability* has been sometimes used in the literature (see Bloch and Jackson, 2006). Formally, point (1) in definition 2 is replaced as follows: (1″) $E\pi_i(g) \geq E\pi_i(g - L)$ for all i and subsets L of links maintained by i in g.

In the next sections we discuss the incentives of firms to share information in the various models and approaches discussed above. We start in Section 2 with the traditional multilateral model, to then report in greater detail the more recent contributions on the bilateral model in Section 3.

2 MULTILATERAL INFORMATION SHARING

In this section we discuss the incentives to either disclose or share information with all other firms in a common market. Multilateral information sharing has been the object of a large body of literature, pioneered by Novshek and Sonnenschein (1982), Clarke (1983), Vives (1985), Fried (1984), Gal-Or (1985; 1986), Li (1985), Sakai (1985), Shapiro (1986), Kirby (1988), Sakai and Yamato (1989). More recently, Raith (1996) has provided a general and insightful analysis, encompassing all previous models and sheding light on apparent weaknesses of the theory. The merit of Raith's work is that is has uncovered the primitive forces that are behind all results in the literature, independently of the details of the model of market competition and of technological assumptions.

In a nutshell, the effect of information sharing on competition and profits is the result of: (1) a finer information on market and/or technological conditions (one's own profit function); and (2) a change in the correlation of firms' market strategies, due to the "more similar" information sets available to firms. Early contributions have suggested that information sharing prior to market competition is profitable when it concerns private cost parameters, and when it concerns market demand parameters as long as firms' strategies are complements. When firms, strategies are substitutes, the increased correlation of preferences, due to a widespread better knowledge of demand conditions, makes unilateral disclosure of information unprofitable; moreover, pooling of information in an industry-wide agreement becomes profitable only when products' differentiation is high. In his 1996 paper, Raith shed further light on these early results by stressing the role played by the precision of signals, and the complex interplay between strategic structure of the oligopoly game and the induced correlation of strategies in equilibrium.[1]

[1] All results discussed here refer to the *ex ante* model of information sharing, in which firms set their disclosure and sharing rules prior to being informed via a private signal. There have been a few contributions considering the

2.1 Incentives to Disclose Information

We start by considering the incentives of firms to unilaterally disclose their private information. We have referred to this case as the “strategic model”. By disclosing private information, a firm is refining the knowledge of rival firms about its own profit function; when states and/or signals are correlated, it also refines rival firms’ knowledge about their respective states of the world (the τ_js). In any case, after disclosure there is more “shared” information in the system, and this affects the correlation of strategies in equilibrium. Intuition would suggest that when states are positively correlated, firms take advantage of the increased correlation of strategies when these are complements. The following result in Raith (1996) gives the full account of firms’ incentives to disclose for all possible scenarios:

Proposition 1 *Under “independent values”, “perfect signals” and “common value” with strategic complements, disclosing information is a dominant strategy. Under “common value” with strategic substitutes, concealing information is a dominant strategy.*

This result is best understood by considering the effects of disclosing private information on one’s expected profits. Raith (1996) has shown how:

1. Letting rival firms refine their knowledge about their own respective profit functions has a positive effect on expected profits under strategic complements, and a negative effect on profits under strategic substitutes.
2. Letting rival firms refine their knowledge about one’s own profit function always has a positive effect on one’s own expected profits.

By disclosing one’s own private information, a firm affects its rivals’ knowledge about their respective payoff functions only if signals are correlated and imperfectly observed. This implies that point (1) above does not apply under IV and PS. Only point (2) applies in those cases, and disclosure is always profitable. Under CV and complements, both (1) and (2) work in favour of disclosure. Under CV and substitutes, the incentives to disclose result from the trade-off between points (1) and (2). The results by Fried (1984), Li (1985) and Shapiro (1986) follow as corollaries of the above results for “perfect signals”. Also, results by Vives (1985) and Gal-Or (1985) on CV situations come as special cases of the result above. Interestingly, the key categories driving the incentives to disclose are not whether uncertainty is about demand or costs, or (not only) whether there are private or common values, but rather about how precise and correlated the signals are, since these aspects of information will determine to what extent disclosure improves rivals, knowledge about their own profit functions, and, ultimately, to what extent correlation of strategies will increase as a result.

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interim model, in which disclosure and sharing occur after the signals are privately observed. When information is verifiable, the decisions of whether to share takes on a signalling power. If uncertainty is about costs, each firm would like to be perceived as low cost; it follows that low-cost firms reveal their type, and information revelation unravels to the whole market (Okuno-Fujiwara, Postlewaite and Suzumura, 1990; Van Zandt and Vives, 2006). When there is uncertainty about whether the firm is indeed informed about its cost, then unravelling may fail even if information is verifiable (Jansen, 2005).

2.2 Incentives to Share Information

We then turn to the richer case of exclusive information-sharing contracts, which we referred to as the “contractual model”. Here, firms disclose their private information to all rival firms and receive in return all private information held by rival firms. The main exercise consists therefore in comparing the expected payoff of firms in two opposite scenarios: no information sharing and universal (or complete) pooling of information:

Proposition 2 *Under “independent values” and “perfect signals”, complete pooling is always profitable. In the “common value” case, pooling is profitable if:*

$$\frac{\epsilon}{\delta} < \frac{2}{n+1}.$$

The fact that under independent values and perfect signals firms prefer to pool information comes almost as a corollary of Proposition 1. Here, in addition to disclosing one’s own private information (which has a positive effect on profits), firms receive additional information about rivals’ signals and, therefore, behaviour that, under these conditions of IV and PS, is beneficial. Note that here receiving information from rivals does not improve a firm’s information about its own state of the world (in Raith’s terminology, there is no “direct adjustment” of strategies after sharing.

The more interesting result here is about common value situations. Here, the final effect of sharing on profits comes as a result of the two effects discussed for the case of disclosure, plus the positive effect of refining one’s own information about other firms’ behaviour and about one’s state of the world (through the correlation induced by the common state of the world). The main insight here is that pooling becomes profitable when the effect of the increased correlation of strategies in equilibrium is weak enough, and this happens when market competition is not too harsh – that is, when the level of product differentiation (here measured by the inverse of ϵ) is strong enough (Kirby, 1988, has shown that the same effect as product differentiation is replicated by steeply increasing marginal costs of production).

Malueg and Tsutsui (1996) have raised the issue of smaller-scale agreements. They show that not only industry-wide agreements can be profitable and immune to individual defections (when products are differentiated), but also that a coalitional agreement by a subset of firms can be stable to defections (more precisely, can be a coalition-proof Nash equilibrium). Their result is obtained in the framework of a three-firm model, and fails to predict information sharing of any kind when products are strongly homogeneous. Smaller-scale agreements are not therefore conducive to information sharing when goods are homogeneous.

As we will discuss in some detail in the next section, a recent contribution by Currarini and Feri (2015) can be used to show that small-scale sharing agreements between firms (bilateral agreements) can generate positive amounts of information sharing in equilibrium even when products are perfectly homogeneous and strategies are substitutes. This result rests on the effect of the conditional correlation of private signals on firms’ incentives to share information in small coalitions ($u_n > 0$). The basic intuition behind this result goes as follows. When firms’ private information is (conditionally) correlated, the exchange of information within a small coalition of firms has the effect of refining these firms’ expectation about all outside firms’ signals (and behaviour). This refinement results from the assumed conditional correlation of signals, and comes at "no cost", since it does not imply any additional correlation of strategies with the outside competitors who do not receive the information about coalitional members' signals. The result is therefore due to the "strategic adjustment" mentioned in Raith (1996). The magnitude of the resulting increase in expected profits is larger the larger the number of firms outside the sharing coalition.

3 BILATERAL CONTRACTS AND INFORMATION-SHARING NETWORKS

We now turn to the bilateral model, in which firms agree to share information in pairs. We restrict our attention to the case of uncertainty on a common demand intercept. This is therefore the case of common value: $\tau_i = \tau$ for all i. For a more general analysis that covers the whole class of quasilinear games, see Currarini and Feri (2015). This section is based on published and unpublished results of the authors. In particular, all results in Sections 3.2 and 3.4 are unpublished, and can be found in working paper versions of the paper "Bilateral Information Sharing in Oligopoly" by the authors (Currarini and Feri, 2007). We omit formal proofs of the propositions, some of which involve long algebraic expression and use of computation software. All proofs are available from the authors on request.

3.1 Equilibrium Use of Information

With each possible information structure g we associate the Bayesian Nash equilibrium of the game in which each firm i sets its strategy s_i in order to maximize its profit, given its available information determined by i's links in g, and given the optimal strategies of other firms. Formally, a Bayesian Nash equilibrium associated with g is a vector $s^*(g)$ of function mapping, for each $i \in N$, the available information $I_i(g)$ into a choice s_i, and such that for each firm i, the function $s_i^*(g)$ solves the following problem for all $I_i(g)$:

$$s_i^*(g)(I_i(g)) = \arg\max_{s_i \in R_+} E_{\tau,\eta}\left[\pi_i\left(s_i, s_{-i}^*(g)\right)|I_i(g)\right]. \tag{18.2}$$

The reaction function of firm i as a function of i's information structure is:

$$s_i^*(g)\,(I_i(g)) = \frac{1}{2\delta}\left(\beta + \gamma_s E\left[\tau_i|I_i(g)\right] - \varepsilon \sum_{j\neq i} E\left[s_j|I_i(g)\right]\right). \tag{18.3}$$

Firms' equilibrium strategies are affine in the observed signals:[2]

$$s_i^*(g)\,(I_i(g)) = a_i^g + \sum_{j\in I_i(g)} b_{ij}^g y_j, \quad i = 1, 2, \ldots n. \tag{18.4}$$

[2] Standard results (see Radner, 1962 and Proposition 3.1 in Raith, 1996).

The a_i^g and b_{ij}^g coefficients can be computed by solving the following system, which immediately points to the main forces at work within a given information structure:

$$a_i^g = \frac{1}{2\delta}\left(\beta - \varepsilon \sum_{j \neq i} a_j^g\right); \tag{18.5}$$

$$b_{ih}^g = \frac{1}{2\delta}\left(\gamma_s k_1^{ig} - \varepsilon\left(\sum_{j \in N_h \setminus \{i\}} b_{jh}^g + \sum_{z \notin N_i}\sum_{j \in N_z} k_2^{ig} b_{jz}^g\right)\right),\ \forall h \in N_i$$

The coefficients $k_1^{ig} = \frac{t}{p_s + (n_i^g - 1)p_n}$ and $k_2^{ig} = \frac{p_n}{p_s + (n_i^g - 1)p_n}$ describe the way in which a firm $i \in N$ in a network g uses its observed signals to update its beliefs. In particular, k_1^{ig} is applied to all $y_j \in I(g_i)$ to take the expectation of τ, while k_2^{ig} is applied to all $y_j \in I(g_i)$ to take the expectation of the signals y_h, for all $h \notin N_i$.

The β coefficients measure the sensitivity of equilibrium actions to the information received from a given source. For the above expression, we learn that under strategic substitutes (complements) the reaction of firm i to signal h is stronger (weaker) the less signal h is used by other firms. In the case of demand uncertainty, this can be understood as a local congestion effect: the more other firms use a signal, the less a firm wishes to use it. This echoes results from Morris and Shin's (2002) study of the use of information when both private signals and public signals are available to players. In our case, a signal is public only to agents in the neighbourhood of the firm that acts as the source of that signal. From the expression from the β coefficient we also learn that a firm reacts less to a signal that is used by other firms to infer something about the information held by firms they are not linked with. This effect goes through the correlation of signals, and is stronger the larger the k_2 coefficient.

3.2 Exchange of Information and Equilibrium Networks

The incentives of firms to form and maintain links are measured by the *ex ante* value of the equilibrium profits in the various networks that may form as a consequence. For a given network g and firm i, these are given by the expectation $E\pi_i(g)$ of the interim profits taken over all possible realization of the information $I_i(g)$ observed by i in g. Following Proposition 3.4 in Raith (1996), we can write the following:

$$E\pi_i(g) = E\left(\alpha_i(\tau)\right) + \delta\left(a_i^g\right)^2 + \beta_n \sum_{j \neq i} a_j^g + \delta Var\left(s_i^*\left(g\right)\right) \tag{18.6}$$

It can be shown that the difference in firm i's expected profit in the two information structures g and g' can be expressed as:

$$E\pi_i(g) - E\pi_i(g') = \delta\left[var(s_i^*(g) - var(s_i^*(g'))\right] \tag{18.7}$$

So, a firm's incentive to move from network g to network g' is measured by the change in the variability of its own equilibrium strategy. Since network structures can be highly asymmetric and complex in nature, the analysis of the incentives to form and delete links is conceptually and computationally very complex. In the next section we report on some results that can be obtained in the simplified framework of independent signals.

3.2.1 Independent signals

The case of independent signals was studied in Gal-Or (1985), where each firm receives an imperfect signal of one piece of the demand intercept. This case can still be viewed as a special case of Raith's model, in which the states of the world τ_is are perfectly correlated but the (conditional) correlation of signals exactly compensate the natural correlation of signals through the state of the world.

The equilibrium parameters of firm i (see (18.5)) take the following simple form:

$$a_i^g = \frac{\beta}{2\delta + \varepsilon(n-1)}; \tag{18.8}$$

$$b_{ih}^g = \frac{\gamma_s t}{p_s(2\delta + n_h^g - 1)}, \ \forall h \in N_i$$

From (18.8) we note that for each signal j we have $b_{ij} = b$ for all i and h in N_j^g. Also, from (18.8) $a_i = a_h$ for all $i, h \in N$.

The next proposition provides necessary and sufficient conditions for a network to be pairwise stable when signals are independent:

Proposition 3 *A network g is pairwise stable if and only if both of the following conditions are verified:*

For all $ij \in g$:

$$\frac{1}{(2\delta + \varepsilon(n_j^g - 1))^2} \geq \frac{1}{(2\delta + \varepsilon(n_i^g - 2))^2} - \frac{1}{(2\delta + \varepsilon(n_i^g - 1))^2}; \tag{18.9}$$

$$\frac{1}{(2\delta + \varepsilon(n_i^g - 1))^2} \geq \frac{1}{(2\delta + \varepsilon(n_j^g - 2))^2} - \frac{1}{(2\delta + \varepsilon(n_j^g - 1))^2}; \tag{18.10}$$

For all $ij \notin g$:

$$\frac{1}{(2\delta + \varepsilon n_j^g)^2} > \frac{1}{(2\delta + \varepsilon(n_i^g - 1))^2} - \frac{1}{(2\delta + \varepsilon n_i^g)^2} \tag{18.11}$$

$$\textit{implies } \frac{1}{(2\delta + \varepsilon n_i^g)^2} < \frac{1}{(2\delta + \varepsilon(n_j^g - 1))^2} - \frac{1}{(2\delta + \varepsilon n_j^g)^2} \tag{18.12}$$

Note that the congestion effect discussed at the end of Section 3.1 translates here into simple and stark incentives to form a link ij, which only depends on the degrees of the nodes i and j, and on no other features of the network. In particular, when $\epsilon > 0$ (strategic substitutes), the gain in profit due to a link with node j decreases with the degree of j. It is indeed possible to determine two thresholds in the degree of a node j: the value $F(n_i)$ above which a node i of degree n_i would not maintain the link ij, i.e. if $n_j > F(n_i)$ inequality (18.9) is not satisfied; the value $f(n_i)$ above which a node i of degree n_i would not form the new link ij; i.e. if $n_j > f(n_i)$ inequality (18.11) is not satisfied. It can also be shown that F and f are increasing in n_i, meaning that the incentives of node i to link with a given node j increase with the degree of i (see Lemma 1 in Currarini and Feri, 2015).

Define now $\mu = \frac{\varepsilon}{\delta}$ the degree of products' differentiation; the next proposition fully characterises the set of pairwise stable networks:

Proposition 4 *Let $n \geq 3$. If $\mu < 0$ the unique stable network is the complete one. If $\mu > 0$ the set of pairwise stable networks contains: the empty network, the complete network, and all networks made of $s \leq n-3$ isolated nodes and $p \geq 1$ completely connected components of size $n_1 \geq 3, n_2, \ldots, n_p$ such that $n_i > f(n_{i-1})$ for all $i = 2, \ldots, p$.*

Remark 1 *The set of pairwise stable networks characterized in Proposition 4 is very large. However, Proposition 4 provides two precise qualitative predictions on how information is shared in equilibrium. First, information sharing is essentially organized in groups (the completely connected components), within which the transmission of information is equivalent to one in which firms publicly disclose their signal to all other firms in the group. This type of public disclosure, characterizing the traditional "contractual approach", is here obtained endogenously as a result of private and bilateral arrangements. Second, information-sharing groups must be of different size, to make sure that firms in different groups do not form links (in fact, it can be shown that firms with similar degrees link together) (Figure 18.1).*

We obtain a more narrow prediction for the case in which firms can agree on side payments that are contingent on information sharing. In this case, the formation of links that bridge two components is made easier by the sharing of individual gains, and at most one component of information-sharing firms can be compatible with stability:

Proposition 5 *If side payments are possible, the set of pairwise stable networks contains: the empty network, the complete network, all networks g made of one completely connected component h of size $n(h) \geq 3$ and $n - n(h)$ isolated nodes.*

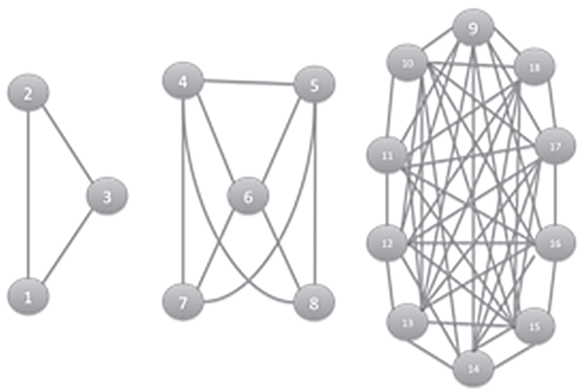

Figure 18.1 A pairwise stable information-sharing network: $\epsilon = \delta = 1$, $n = 18$

3.3 The Role of Signals' Correlation

One of the conclusions from the case of independent signals is that the empty network, characterized by no sharing of information, is always a stable outcome in Cournot competition with homogeneous goods and demand uncertainty. In this proposition we discuss the role of signals correlation in generating incentives to share information and, at the same time, to exclude some of the rivals from sharing.

The next proposition shows that in all common value situations (that is, independently of the degree of products differentiation), the empty network is not a pairwise stable structure (and, therefore, not a strongly pairwise stable structure), provided the number of firms in the market and the (conditional) correlation of signals is not too small:

Proposition 6 *Consider Raith's model of oligopolistic competition:*

- *(i)* *If $\mu < \frac{2}{3}$ the empty network is not pairwise stable.*
- *(ii)* *If $\frac{2}{3} < \mu < \frac{2}{1+\sqrt{2}}$ then there exists a p_n^{**} such that for all $p_n < p_n^{**}$ the empty network is not pairwise stable; otherwise (when $p_n > p_n^{**}$) there exists a finite number of firms $n^*(p_n)$ such that for all $n > n^*(p_n)$ the empty network is not pairwise stable.*
- *(iii)* *If $\mu > \frac{2}{1+\sqrt{2}}$ there exists p_n^* and a finite value $n^*(p_n)$ such that for all $p_n > p_n^*$ and $n > n^*$ the empty network is not pairwise stable.*

Let us compare Proposition 6 with Raith's (1996) results for the contractual model (note here that when only two firms are in the market our model and Raith's model are equivalent). In point *(i)*, values of $\mu < \frac{2}{3}$ are such that two duopolists would always pool their private information ($\frac{2}{3}$ is in fact $\frac{2}{n+1}$ for $n = 2$). Our result shows that these incentives remain when more firms are in the market. Points *(ii)* and *(iii)* cover situations in which two duopolists may or may not have the incentive to share information, depending on the level of the covariance of signals p_n. Point *(ii)* shows that when these incentives exist (low p_n), they do not vanish as we add firms to the market. More interestingly, when such incentives to bilaterally share information in a duopoly are absent (high p_n), they appear as we add more firms in the market. Finally, point *(iii)* refers to the range of parameters for which two duopolists would never share information, for any value of p_n. Here, it is shown that by adding firms in the market we can generate incentives for bilateral information sharing, provided the covariance p_n is large enough.[3]

To understand the forces at work in Proposition 6, consider again the incentives of two Cournot duopolists to share information. These are determined by two opposite effects on expected profits: the increased accuracy of firms' expectations (a positive effect) and the increased correlation of equilibrium strategies (a negative effect since strategies are substitutes). Unless products are very differentiated (ε positive but small), the second effect dominates the first. The crucial new element of the present proposition is that as we increase the number of firms, the bilateral exchange of information between firms i and j has the additional positive effect of improving the accuracy of the expectation of these two firms on the signal observed by the other firms in the market (and thereby on

[3] Note that the threshold levels of μ in Raith's paper are decreasing in n. Therefore, it is not possible that by adding firms in the market we pass from a situation where the empty network dominates the complete graph to a situation where the opposite is true.

their equilibrium behaviour). This improved accuracy comes without the disclosure of any additional information to any of these other firms and, in this sense, at no cost. Moreover, this positive effect on profits is larger the larger the number of other firms in the market (from which the requirement on n in Proposition 6).

The result of Proposition 6 rules out the complete absence of information sharing in equilibrium (at least under certain conditions on p_n and n), but leaves open the question of whether stable networks exist in general. Proposition 7 below shows that the complete network is always pairwise stable, for all values of the parameters:

Proposition 7 *Let* $n \geq 3$*. The complete network is always a pairwise stable information structure.*

We conclude that some positive amount of information sharing is always compatible with pairwise stability (Proposition 7), and is always a feature of pairwise stable networks when p_n and n are large enough (in the sense made clear in Proposition 6). The result of Proposition 7 does not fully extend to the notion of strong pairwise stability. The explicit expression of expected profits when multiple links are severed from the complete network is quite complex and does not allow for a closed-form result for all parameters' values. However, numerical simulations suggest that there exist a threshold level of signals' correlation above which the complete network is strongly pairwise stable, and below which it is not. This result is obtained algebraically in the two polar cases of the common value: perfect substitutes ($\mu = 1$) and high differentiation ($\mu = 0$):

Proposition 8 *Let* $n \geq 3$*. If* $\mu = 0$ *the complete network is always strongly pairwise stable. If* $\mu = 1$ *and* p_n *is large enough the complete network is strongly pairwise stable.*

One final issue we wish to address is whether stable networks can be incomplete, with some, but not all, private information being shared. Example 1 presents a common value problem with four firms and homogeneous goods where, for a certain range of parameters, in a strongly pairwise stable network three firms exchange information, and a fourth firm is excluded (again, the algebraic derivations behind Example 1 are omitted and are available from the authors upon request):

Example 1 *Let* $n = 4$ *and* $\varepsilon = \delta$*. For* $p_n > 0.53 \cdot p_s$ *the complete network is strongly stable. For* $p_n < 0.62 \cdot p_s$ *and for* $p_n > 0.71 \cdot p_s$ *the network consisting of a fully connected component of three nodes and an isolated node is pairwise stable, and it is Nash pairwise stable for* $0.58 \cdot p_s < p_n < 0.62 \cdot p_s$ *and for* $p_n > 0.71 \cdot p_s$ *(Figure 18.2).*

We end this section by comparing the insight from Example 1 with Malueg and Tsutsui's (1996) results on stable sharing coalitions in the case of three firms. With only three firms on the market, the empty network would be a pairwise stable architecture and, for all $p_n < 0.68 \cdot p_s$, there are no empty strongly pairwise stable architecture.[4] This is in line with the results by Malueg and Tsutsui (1996), where no information sharing ever occurs when products are homogeneous. By adding a fourth firm in the market, we increase the profitability of bilateral

[4] For greater values of p_n the complete network is strongly stable.

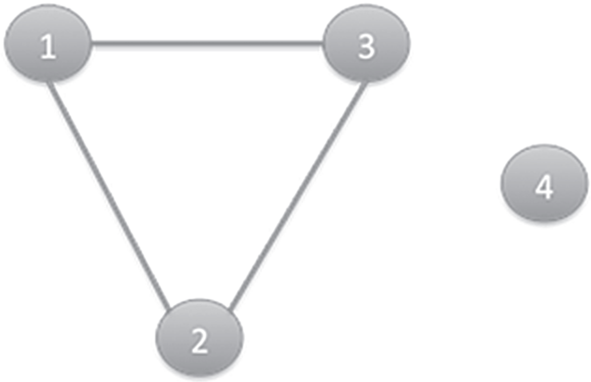

Figure 18.2 A pairwise stable information-sharing network: $n = 4$, $\varepsilon = \delta, p_n < 0.62 \cdot p_s$ *and* $p_n > 0.71 \cdot p_s$

agreements through the externality effect discussed after Proposition 6, so that the empty network becomes unstable for $p_n > 0.75 \cdot p_s$. Consider then the network consisting of a three-firm fully connected component and an isolated node; within the fully connected component, no firm has an incentive to sever one of its links and, for $p_n > 0.58 \cdot p_s$, no firm has an incentive to sever both its links.[5] Moreover, these firms have an incentive to link to the fourth firm if and only if $p_n < 0.71 \cdot p_s$; otherwise none of them has an incentive to link, having acquired enough information on the private signal of the fourth firm through the existing bilateral agreements. The fourth firm, instead, has an incentive to acquire additional information by forming a link if and only if $p_n > 0.62 \cdot p_s$. Therefore for high values of p_n the fourth firm is excluded from the information-sharing group.

3.4 Asymmetric Firms and the Emergence of Core–Periphery Structures

In this section we wish to discuss the role of asymmetry in the information-structure on the incentives to share and on the equilibrium information-sharing networks. To keep things simple, we limit the analysis to the case of independent signals, as we did in Section 3.2.1 for the symmetric case, and work with homegeneous goods, setting δ to 1.

We relax the assumption that signals are identically distributed, and allow the variances of signals p_s^i to differ across firms. The stability conditions of Proposition 3 are modified to account for this new source of heterogeneity: the network g is pairwise stable if and only if:

for all $ij \in g$:

$$\frac{p_s^j}{(n_j^g + 1)^2} \geq \frac{p_s^i}{(n_i^g)^2} - \frac{p_s^i}{(n_i^g + 1)^2} \tag{18.13}$$

$$\frac{p_s^i}{(n_i^g + 1)^2} \geq \frac{p_s^j}{(n_j^g)^2} - \frac{p_s^j}{(n_j^g + 1)^2} \tag{18.14}$$

[5] Note, that as shown above, with only three firms in the market the complete network is strongly stable for $p_n > 0.68 \cdot p_s$. By adding a fourth firm this threshold becomes smaller because is more convenient to share information (or more costly to defect fron the information-sharing group).

for all $ij \notin g$:

$$\frac{p_s^j}{(2+n_j^g)^2} > \frac{p_s^i}{(1+n_i^g)^2} - \frac{p_s^i}{(2+n_i^g)^2} \tag{18.15}$$

$$\text{implies } \frac{p_s^i}{(2+n_i^g)^2} < \frac{p_s^j}{(1+n_j^g)^2} - \frac{p_s^j}{(2+n_j^g)^2} \tag{18.16}$$

We see that, given the degrees n_i^g and n_j^g, the incentive of *i* to sever the link *ij* increases with the ratio of variances $\frac{p_s^i}{p_s^j}$ (conditions (18.13)–(18.14)) and the incentive of *i* to form the link *ij* decreases with $\frac{p_s^i}{p_s^j}$ (condition (18.15)). This effect can be understood in terms of the additional variability of *i*'s equilibrium quantity coming from the link *ij*, remembering that (expected) profits are correlated with the variability of own equilibrium strategy. The higher the term p_s^j, the higher the additional variability of *i*'s quantity due to the link *ij*, and the higher the informational "value" of *j*'s signal for firm *i*. Similarly, the higher the term p_s^i, the lower the incentive of firm *i* to form the link *ij*; this is because it is more costly to share a signal with higher variance with one additional firm. Again, a high value of p_s^i therefore reflects a high informational value of *i*'s signal.

Note that in this setting of heterogeneous variance, a firm with high variance may not wish to maintain a link (or to form a new one) with another firm with same degree but lower variance. As a consequence, while the empty network is always a pairwise stable information structure (as was proved for the case of independent and identically distributed [i.i.d.] signals), the complete network may fail to be stable when firms have significant heterogeneity in variances. However, as the next proposition shows, this can only happen when the number of firms is small:

Proposition 9 *(i) The empty network is pairwise stable for all distributions of variances, even if side payments are possible; (ii) there exist configurations of variances for which the complete network is not pairwise stable; (iii) for every configuration of variances, there exists a finite number of firms $\bar{n}$ such that for all $n \geq \bar{n}$ the complete network is pairwise stable.*

The intuition of this result is clear: when the degree of two nodes increase, their difference in variances becomes less and less relevant in the stability conditions (18.13)–(18.14).

Since signals with large variance possess higher informational value, the incentive to link to firms observing such signals may remain high even when these firms have already a large degree. We can therefore envisage stable architectures in which firms with large variance have larger degrees than firms with low variance. Among such architectures, we will focus on two classes: networks made of a collection of completely connected components (as in the case of i.i.d. signals) and core–periphery networks.

We next turn to the existence of pairwise stable networks with non–completely connected components. We show that a special class of incomplete architectures, usually referred to as "core–periphery networks", can be pairwise stable for suitable distributions of variances. In more detail, core–periphery networks present a dense set of interconnected nodes – the

Figure 18.3 A core–periphery pairwise stable information-sharing network: $n = 5, p_s^1 = 1, p_s^2 = p_s^3 = \frac{1}{5}, p_s^4 = p_s^5 = \frac{1}{2}$.

core – each linked with all nodes in the network, and sets of peripheral nodes that are internally connected and are linked with the core nodes (see Figure 18.3). Formally, a core–periphery network g consists of a set $\{g_1, g_2, \ldots .g_H\}$ of fully connected subnetworks, such that $i \in g_k$ and $j \in g_m$ implies that $ij \notin g$ for all $k \neq m$ such that $k \in \{2, 3, \ldots, H\}$ and $m \in \{2, 3, \ldots, H\}$, and such that $i \in g_1$ and $j \in g_k$ implies $ij \in g$ for all $k = 1, 2, 3, \ldots, H$. We call the subnetwork g_1 *core* (with size n_c), and the subnetworks $\{g_2, \ldots .g_H\}$ *peripheral planets*. We define a *symmetric* core–periphery network as one in which all peripheral planets have the same size $n_p \geq 1$. Also, we say that planets are *consecutive* in variances if planets can be obtained as a consecutive partition of the set of peripheral nodes ordered with respect to variance.

Proposition 10 provides two qualitative features of pairwise stable symmetric core–periphery networks: peripheral firms are organized in groups that are consecutive in variance, and core firms have larger variance than peripheral firms:

Proposition 10 *Every symmetric pairwise stable core–periphery network is such that peripheral planets are consecutive in variances. Moreover, for each given size* n_p*, there exists a finite* n' *such that if* $n > n'$ *then every symmetric pairwise stable core–periphery network is such that* $\min_{i \in g_1} p_s^i > \max_{j \in g \setminus g_1} p_s^j$.

Intuitively, core firms observe signals that are publicly observed, and therefore have lower informational value. These signals are "desirable" only if they have large variance, from which the second result in Proposition 10. An example of pairwise stable core–periphery network is the following:

Example 2 *Consider a network with five nodes: node 1 is the "core" node, while the two peripheral components are* $\{23\}$ *and* $\{45\}$*. Variances are* $p_s^1 = 1, p_s^2 = p_s^3 = \frac{1}{5}, p_s^4 = p_s^5 = \frac{1}{2}$*. Relevant stability conditions (18.15)–(18.16) (for links 12, 15 and 34, respectively) are satisfied:*

$$\frac{1}{36} + \frac{1}{5}\frac{1}{16} \geq \frac{1}{25};\ \frac{1}{36} + \frac{1}{2}\frac{1}{16} \geq \frac{1}{25};\ \left(\frac{1}{2} + \frac{1}{5}\right)\frac{1}{25} \leq \frac{1}{5}\frac{1}{36}.$$

4 CONCLUSIONS

In this chapter we have reviewed the main results and insights coming from theoretical research on the incentives to share information in oligopolistic markets. While most of the literature has focused on the formation of industry-wide agreements to either disclose or share information, we have devoted a substantial space to recent contributions that consider the possibility that firms establish sharing agreements with selected partners, excluding other competitors from their private information. All the contributions covered in this survey assume that firms agree to share information at the *ex ante* stage, that is, before getting to know their private signals. This approach aims at describing the incentives to establish long-term agreements, in which information is repeatedly shared and in which the decision to share cannot be made contingent on what the realized information is. In reality, however, firms may decide to share information conditionally to the private signal they receive, that is, in certain situations. This case goes under the name of *interim* model, and has not received much attention in the literature. We believe that more research is needed here, that formally studies the signalling role of non-disclosure decisions, to determine under which condition unravelling of information occurs and full disclosure is guaranteed.

Despite the considerable amount of theoretical work on the subject, there is little empirical evidence on information sharing in real world oligopolistic markets. If anything, the theoretical insights have been used as evidence of collusion in quantity and/or prices where direct evidence of collusion was missing. The lack of empirical research is due to the difficulty in obtaining data that serve as good proxy of information sharing in a world where sharing is itself illegal. This motivates future effort in experimental research in a laboratory-controlled environment. While there exists some experimental research on the use of information in games with complementarities and signal with different degrees of publicness (see Cornand and Heinemann, 2014), experiments on information sharing seem like a very promising avenue of research.

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