

# Science in Court Society

Giovan Battista Benedetti's

*Diversarum speculationum mathematicarum et physicarum liber*  
(Turin, 1585)

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Pietro Daniel Omodeo and Jürgen Renn

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## **Benedetti: Life, Work, Contexts**



## Introduction

Giovanni Battista Benedetti is today a lesser known figure in the history of early modern science. This relative oblivion is in striking contrast to the fame he enjoyed during his lifetime as a prominent mathematician and mathematical philosopher of Venetian origin and reputable courtier to the Savoy dukes in Turin. Among his admirers, the astronomer Johannes Kepler regarded him as one of the few Italians to significantly contribute to the advancement of mathematics in his time: “The Italians are asleep with the sole exceptions of Commandino and Giovanni Battista Benedetti. And in fact, Clavius is German.”<sup>1</sup> For his part, the mathematician of the *Collegio Romano*, Christopher Clavius, extolled Benedetti’s scientific merits in the 1589 edition of his reputed commentary of Euclid, *Euclidis Elementorum Lib[ri] XV*. In the dedicatory letter to Carlo Emanuele I of Savoy (*Serenissimo Principi ac Domino D. Carolo Emmanuelli Sabaudiae Duci*), he praised “his court mathematician” Benedetti as “very expert in mathematics” (*scientissimus rerum Mathematicarum*).<sup>2</sup> In contrast to the eulogies of the Imperial mathematician Kepler and the most prominent astronomers of the Jesuit order, the Urbino school gathered around the mathematical purist Federico Commandino was rather reluctant to acknowledge his achievements, probably due to rivalry, reciprocal misunderstandings, and different philosophical and cultural choices. Among Commandino’s pupils, Guidobaldo Del Monte severely criticized Benedetti’s approach to mechanics and his claim to originality, as documented by his manuscript annotations on mathematical issues.<sup>3</sup> Another member of the Urbino school, Bernardino Baldi, gave an extremely negative and reductive portrait of Benedetti in his collection of short biographies of mathematicians from all epochs, *Cronica de matematici*.

The Venetian Giovanni Battista Benedetti occupied himself with mathematics, a field in which he served the dukes of Savoy. He wrote a book on gnomonics, which dealt with many proofs belonging to this discipline. However, he has been criticized by the most exquisite scholars for not having respected the method and the purity of explanation which mathematics requires and which was respected by the Greek masters and by their followers. He also wrote some light things of no great import.<sup>4</sup>

Such a harsh judgment can only be explained on the basis of a profound enmity held by Commandino’s followers against Benedetti. This deserves attention since it also influenced the reception of his work. Therefore, we will offer a reconstruction of Benedetti’s

<sup>1</sup>Kepler 1937–2001, 390: “Itali somniant (preter unum Commandinum et Joh[annem] Baptistam Benedictum, Clavius enim Germanus est).”

<sup>2</sup>Clavius 1589, ff. \*4r–\*5r. The list of Benedetti’s admirers also includes the Pisa philosopher Jacopo Mazzone, the Venetian intellectual leader Paolo Sarpi, and the French scholar Marin Mersenne. See Cappelletti 1966, 262.

<sup>3</sup>See Renn and Damerow 2012 and Renn and Omodeo 2013.

<sup>4</sup>Baldi 1707, 140: “GIO[VANNI] BATTISTA Benedetti veneziano attese alle matematiche, nelle quali servi i Duchi di Savoia. Scrisse un libro di gnomonica, il quale toccò molte cose appartenenti alle dimostrazioni della detta disciplina, se non che viene ripreso da più esquisiti di non haver’osservato quel metodo, e quella purità dell’insegnare, che ricercano le matematiche, et è stato osservato dagl’ottimi Greci, e dagl’imitatori loro. Scrisse anco alcune altre cose leggere, e di nessun momento.” Here and in the subsequent pages, Italian and Latin grammar (e.g., capitalization and punctuation) has been modernized.

cultural environment and the scientific culture of Renaissance Italy in order to understand his work, its roots, and legacy.

Although Benedetti was recognized by his contemporaries, in many respects the theoretical and historical relevance of his work remains obscure. The obscuration of Benedetti is also the result of the prominence of the Urbino school and their bias towards mathematical purism. Moreover, the hagiographic historiography of science sought out the precursors to Galileo and therefore almost naturally adopted the criticism of Benedetti by Galileo's patron Guidobaldo del Monte. Intrinsic factors also played a role in the eclipse of Benedetti's fame, principal among which is the relative rarity of his major work, *Diversarum speculationum mathematicarum et physicarum liber* (Book Including Various Mathematical and Physical Speculations, 1585), not to mention that of most of his earlier publications. Benedetti's fragmentary style is a special difficulty faced by the reader, a defect that was overemphasized by Baldi and denounced by early modern scholars such as Claude-François Milliet Dechales. Dechales dealt with Benedetti's geometry in his *Cursum seu mundus mathematicus* (1690), observing that "the end of his work [the *Diversae speculationes*] comprises many miscellaneous geometrical remarks, some of which are good, in particular with regard to their special concern, but [they are] disordered."<sup>5</sup>

Following historiographic commonplaces and their nineteenth-century crystallization, recent historians of science have tended to neglect Benedetti's work due to the lack of extensive translations of his writings into modern languages, with the exception of the excerpts included in Drake and Drabkin, *Mechanics in Sixteenth-Century Italy* (1969). This anthology isolated passages that were exclusively devoted to what the editors saw, in hindsight, as the most relevant contributions for the progress of mechanics. Due to its selective nature, this publication did little justice to the complexity and richness of Benedetti's stature in the history of science. In fact, isolating certain results elides recognition of the multilayered architecture of a book such as the *Diversae speculationes*, which is characteristic of Renaissance science. Benedetti's showcase of mathematical erudition and scholarship is thus omitted and obscured. Drake and Drabkin's selection could only yield a reductive and rather misleading image of Benedetti as a scientist and thinker. As we will argue, entire paragraphs or chapters expunged from Drake and Drabkin's translation, for instance those concerning philosophy and cosmology, are relevant for an understanding of the author's general conception of mechanics and physics. From a historical viewpoint, it is hardly possible to trace disciplinary boundaries in the Renaissance that fit those established today. Mechanics was an emerging discipline at the crossroads of mathematics, engineering, and natural philosophy. Hence, a reassessment of Benedetti's work in its entirety is necessary not only to understand his personality but also to grasp the scientific culture of his age as the result of interdisciplinary controversies.

This open access edition makes the *Diversarum speculationum mathematicarum et physicarum liber* accessible to a large scholarly readership. Benedetti's volume is a major contribution to Renaissance science, especially due to its insights into mechanics, the mathematization of (or geometrical approach to) natural investigation, and the connection of celestial and terrestrial dynamics in a post-Copernican perspective. The first edition of this work was an elegant folio, which included heterogeneous writings on technical and philosophical issues as well as on mathematics and physics. Benedetti presented them as short treatises (*tractatus*) or letters (*epistolae*) addressed to gentlemen, courtiers, scholars, engineers, and practitioners of different arts. The volume was printed by Niccolò Bevilac-

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<sup>5</sup>Ventrice 1985, 188: "in fine sui operis multa habet miscellanea geometrica, quorum nonnulla ad sectiones praesertim pertinentia bona sunt, sed inordinata."

qua's heir (*apud haeredem Nicolai Bevilacuae*), who was the owner of the main printing house in Turin, which was directly supported by the dukes of Savoy. The *Diversae speculationes* appeared in a series of prestigious volumes aimed at celebrating the magnificence of the court and the capital, including the works of the court historian Emanuele Filiberto Pingone: *Augusta Taurinorum* (1577), on Turin, *Inclytorum Saxoniae Sabaudiaeque principum arbor gentilitia* (1581), on the genealogy of the ruling family, and *Sindon evangelica* (1581), on Christ's shroud, which had been recently transferred from Chambéry to Turin. The *Diversae speculationes* exhibits the same celebratory intention. The volume aimed to make the quality of the court mathematician's research and skills publicly appreciable. It also bore witness to the intensity of the cultural debates going on in Turin, and connected this city with other centers of learning, especially Venice.<sup>6</sup> Its miscellaneous and epistolary form was suitable for displaying the variety of the author's interests and for praising his patrons, friends, and colleagues by naming them as dedicatees or addressees of the different sections and letters.

The structure of the *Diversae speculationes*—its occasional and fragmentary character, its celebratory purpose, and the epistolary display of a network of personal connections—mirrors the socio-cultural embedment of Benedetti's work. We regard it as exemplary of "science in court society." As Norbert Elias has pointed out, the *höfische Gesellschaft*, or court society, is a particular social configuration (*gesellschaftliche Figuration*) characteristic of the transitional phase to an industrial and capitalist Europe, which we conventionally refer to as the early modern period.<sup>7</sup> During the Renaissance and the Ancien Régime, the court was (or became) a hegemonic center from which powerful elites mediated between the ruler and the socio-political environment. Benedetti's activities as a court mathematician exemplify such a function. In his role he was expected to interact with the upper classes and respect aristocratic etiquette, and to act as the Savoy "expert" on a wide range of pedagogical and technical issues linked to his profile as a mathematician and mathematical philosopher. He was required to teach geometry to the offspring of the ducal family, to supervise engineering and architecture projects, to produce instruments or machines for practical purposes, warfare, and recreation (such as fountains, sundials, or nautical instruments). He had to adhere to shared court values, norms, and behaviors, primarily those linked to honor and prestige. These courtly principles are reflected in the epistemic values permeating his scientific production, for instance in the value of scientific disinterestedness that marks his theoretical approach to practical as well as to speculative problems. In a hierarchical and aristocratic society, his theoretical attitude marked at once the continuity and the distance between his role as a court mathematician and those involved in practical activities. Moreover, the primacy of courtly interests over those of science as a purely scholarly endeavor (as it was pursued at universities and academies) is evident from Benedetti's networking strategies, which were aimed at not so much exchange with other scholars as at giving advice to a wide range of people, beginning with the ruling elites of the country. In other words, he was not primarily concerned with establishing a *réseau*, as was typical for the Republic of Letters. As we will show, he did not regard himself and his activity as part of a learned network but rather as the center of courtly interaction. This center-periphery structuring of his network mirrors—in two senses—the "knowledge economy" his work is embedded in. Sociologically, the central-

<sup>6</sup>Cecchini and Roero 2004.

<sup>7</sup>As Norbert Elias put it (Elias [1969] 2002, 73): "Durch das Bemühen um die Struktur der höfischen Gesellschaft und damit um das Verständnis einer der letzten großen nicht-bürgerlichen Figurationen des Abendlandes eröffnet man sich also mittelbar zugleich einen Zugang zum erweiterten Verständnis der eigenen berufsbürgerlich-städtischen-industriellen Gesellschaft."

izing character of court society is reproduced in scientific policies through Benedetti's function as a technocrat; epistemologically, the fluid style and fragmentary form of the *Diversae speculationes* is an expression of the expert-advice character of his scientific work. Thus, in order to properly understand his work, we deem it necessary not only to investigate the technical and theoretical dimensions of Benedetti's science, but also to analyze evidence concerning the ties between these dimensions and the social and cultural environment.

Among the studies on Benedetti, Giovanni Bordiga's monograph *Giovanni Battista Benedetti: filosofo e matematico veneziano del secolo XVI* (1926) still stands out as one of the most important references as far as prosopographical information is concerned.<sup>8</sup> The excellent research accomplished by Carlo Maccagni and the proceedings of the conference on Benedetti held in Venice in 1985 at the *Istituto Veneto di Scienze, Lettere ed Arti* investigated many aspects of Benedetti's contribution.<sup>9</sup> In spite of the accuracy of these relatively recent Italian studies, Alexandre Koyré's evaluation of Benedetti's role in the first stages of modern science, in the *Études galiléennes* (1939), has had a more direct influence on his international reception. Koyré emphasized the link between the incipient mathematical science of motion and heliocentrism in Benedetti's speculations. On account of this, Paul Lawrence Rose regarded him as a herald of the "Italian Renaissance of mathematics".<sup>10</sup> Koyré's grand narrative of the Scientific Revolution, which he conceived as a development with a "prologue" (Copernicus) and "epilogue" (Newton) in the heavens, included Benedetti as a precursor to Galileo because of the interconnection of mathematical and physical themes in the former's work. Koyré's main thesis was that classical physics (the mathematical science of nature of Galileo, Descartes, and Newton) emerged as a direct consequence of Copernicus's geokinetic system, which undermined the traditional (Aristotelian and Ptolemaic) worldview. Although Benedetti's relevance as a source for Galileo cannot be denied, scholars now view Koyré's narrative as dubious due to its abstract treatment of the history of science, conceived of as an internal development of ideas. In the years of the Cold War (or shortly before it), this viewpoint embodied the ideological reaction to Marxist or materialist-oriented accounts, which stressed the technological, empirical, and social roots of modern mechanics, as was the case with Leonardo Olschki and Edgar Zilsel.<sup>11</sup> Benedetti's approach to mechanics and post-Copernican astronomy therefore appears to be an appropriate case study for reconsidering this general historical *problématique*, beginning with a reassessment of the relation between mechanics and astronomy in early modernity. This implies a reconsideration of the basic questions of the historiography of science and of historical epistemology, such as the role of material and intellectual factors in the so-called Scientific Revolution.

In this edition of the *Diversae speculationes*, we aim to present Benedetti's achievement in its rich complexity. Benedetti is emblematic both of his time and of the non-linearity of the historical process of Renaissance science with its multicentric institutions and scientific networks. We will show that the apparently fragmentary nature of his work is expressive of the peculiar character of science in court society and, in spite of this form, it conceals a fundamental unity of his conception of nature and method, both of which rest on geometry. To be sure, Benedetti regarded mechanics as a model, but he enlarged his

<sup>8</sup>Bordiga 1985.

<sup>9</sup>Bordiga 1926, repr. Bordiga 1985, Maccagni 1967b, Maccagni 1967a, Maccagni 1983, and Istituto Veneto di Scienze 1987.

<sup>10</sup>Rose 1975, 154–156.

<sup>11</sup>The cultural-political intentions of Koyré's approach emerge most vividly from his 1943 article on Galileo's Platonism, see Koyré 1943. Lefèvre stresses it in Lefèvre 2001.



perspective to include the most varied fields of investigation in order to concretely demonstrate the fruitfulness of his approach to universal knowledge about astronomy, physics, meteorology, and even literature and ethics.

## Overview and Acknowledgments

Our first chapter is devoted to Benedetti's biography and comprises an overview of his publications. We consider the various dimensions of his identity, such as his nobility as both a Venetian patrician and a Savoy aristocrat in the retinue of Emanuele Filiberto and Carlo Emanuele at the Turin court. In particular, we discuss his role as a courtier, the tasks he was entrusted with, and the relations he established in this context. We also deal with his extra-academic education and his attitude towards mathematics, which he initially saw as an intellectual instrument to be used against the "bookish" culture of the universities and the "idle" rhetoric of the humanists. Later he softened the polemical tone that characterizes his early writings. Moreover, since he saw himself as a philosopher, more precisely as a court philosopher to the Dukes of Savoy, a discussion of Benedetti's mathematics cannot be separated from his philosophical project. Benedetti claimed for himself the right to discuss in mathematical terms issues of natural philosophy that traditionally belonged to the rather qualitative and conceptual approach of the peripatetics. The *Diversae speculationes* is an altogether magisterial example of this merging of philosophical and mathematical perspectives.

Chapter 2 is a reconstruction of the cultural life of Renaissance Turin, the town in which Benedetti spent his mature years and where he composed his major work. His achievement was embedded in the cultural ferment of the new capital of Savoy, a place of ambitious town planning and civil reforms. It was a time in which the arts, literature, and philosophy received a new impetus. Editorial projects were launched; the university was reopened and illustrious scholars were attracted there. The dukes' religious politics was informed by a sense of pragmatism, which is mirrored in the fluctuating relations between the ruling family, the Jesuits, and Rome. Benedetti's secular attitude towards science and philosophy mirrors the cultural politics of his patrons. In addition we discuss his involvement in various scientific debates divided into courtly conversations, academic controversies, and controversies going beyond the settings of the court and the university. Among such extra-academic public controversies, the most important was Benedetti's public defense of the reliability of astronomical calculation against a polemist, Benedetto Altavilla, who indirectly attacked his and others' astrological practice. Newly discovered documents show that Benedetti's successor as court mathematician, Bartolomeo Cristini, continued that polemic after Benedetti's death. Cristini discredited Benedetti's use of astronomical tables to cast horoscopes, in order to ingratiate himself with the dukes and successfully start a career at court. We trust that this chapter offers new insights into the scientific culture of the Renaissance by bringing Turin into focus, a cultural centre that has so far escaped in-depth consideration by historians of early modern science.

In chapter 3 we offer an overview of the structure of the *Diversae speculationes*. We introduce Benedetti's mathematical sections in general terms, focusing on his geometrical demonstrations for the solutions of problems of arithmetic—which were the result of his private teaching of mathematics to the Savoy prince—his sketchy annotations on the theory of proportions based Book 5 of Euclid's *Elements*, and his considerations on linear perspective aimed at supporting the work of painters and architects. The sections on physics, mechanics, and natural philosophy are not discussed in this chapter as they re-

ceive special treatment in other chapters. The last part of the *Diversae speculationes* was a miscellanea of scientific letters. We discuss them vis-à-vis their significance as a mirror of Benedetti's social capital. His epistles were mostly directed to aristocrats, beginning with his patrons, other courtiers, and diplomats, especially those from Venice. He also corresponded with professors, artists, engineers, and practitioners, some of whom lived north of the Alps. His network was markedly a center-periphery one, in which the court expert shared his views on the most varied topics with others seeking his advice or opinion. Thus, it was not a scholarly network implemented for the sake of exchange and the advancement of knowledge. Rather than a networking activity establishing a Republic of Letters, Benedetti's correspondence reflected court-society centralism.

Chapter 4 addresses Benedetti's epistemology on the basis of passages regarding the certitude of mathematics and his effective use of mathematics in physics. His role as an early champion of what would later become known as "physico-mathematics" is understood here against the background of the philosophy of mathematics in the Renaissance. Moreover, we deem the modal epistemology underlying his science of particular interest: his treatment of nature in mathematical terms did not imply the necessary or deterministic nature of physical processes. Rather, he embraced an ontology and an epistemology of contingency that constituted a bridge between medieval scholastic views on nature and the mathematical physics of the time of Galileo and Descartes. We dedicate an excursus to the vision of nature as the realm of contingency in the period extending from the medieval science of weights to seventeenth century mechanics, and ascribe to Benedetti a central position in this intellectual process.

Chapter 5 deals with the field in which Benedetti has received the most credit from historians: mechanics. Actually, Benedetti himself emphasized the importance of his contribution to mechanics as what would secure his fame in posterity. We summarize his theories on equilibrium and his critical reworking of earlier theories such as those developed by Jordanus Nemorarius and Niccolò Tartaglia. We consider Guidobaldo Del Monte's negative reaction to Benedetti's mechanics in detail, as well as the weaknesses and strengths of both authors. We regard this pluralism of clashing and integrating views as revealing the complex paths of discovery undertaken by students of mechanics in a period of the utmost relevance to its modern systematization. Moreover, the subterranean conflict of views and approaches between Benedetti and Del Monte affected Galileo's work. His mechanics drew from both authors, although he did not acknowledge Benedetti explicitly due to circumstances and opportunity.

Chapter 6 summarizes Benedetti's astronomical work. Although he did not see himself as an astronomer, his contribution is quite interesting. He should be acknowledged for his effort to develop a new mathematical physics in accordance with post-Copernican astronomy. His discussion of astronomical theory against the background of a general philosophical reform was strikingly innovative. His specific polemics on the reliability of astronomical calculation also receive close treatment here. Furthermore, in an appendix Günther Oestmann offers an assessment of Benedetti's astrological calculations on the basis of so-far neglected manuscript sources containing two of his horoscopes.

In chapter 7 we deal with Benedetti's natural philosophy as he presented it in Book 4 of the *Diversae speculationes*. Although he entitled it "Disputations on Some Opinions Held by Aristotle" (*Disputationes de quibusdam placitis Aristotelis*), it was a polemic directed "against" fundamental Aristotelian theses on motion, time, space, matter, and cosmology. This is the section in which Benedetti's commitment to "the system of Aristarchus and Copernicus" most clearly emerges. It is also a fundamental section on the existence of the physical void as the necessary presupposition of any local displacement and on free

fall through different media. We see this book of the *Diversae speculationes* as a major contribution to the Renaissance debate on the foundations of physics, going far beyond the treatment of mechanics and cosmology *strictu sensu*. Hence, we take into consideration Benedetti's definition of space as an "inter-bodily gap" (*intervallum corporeum*), his defense of the possibility of actual infinity in nature against Aristotle's veto, his understanding of time as an absolute frame complementary to space and its place in the philosophical debates of the Renaissance, the revision of the concepts of natural and violent motion, and finally, the surprising conclusion of the "Disputations on Some Opinions Held by Aristotle" with a Copernican note.

This volume is a continuation of an Edition Open Access project aimed at the publication and scholarly reassessment of the fundamental sources of Renaissance mechanics. This project began with Jürgen Renn and Peter Damerow's *Guidobaldo del Monte's Mechanicorum Liber* in 2010. Elio Nenci's open-access publication of Bernardino Baldi's *In mechanica Aristotelis problemata exercitationes* appeared in 2011 and, in 2013, Matteo Valleriani's *Metallurgy, Ballistics and Epistemic Instruments*, including a transcription and an English translation of Nicolò Tartaglia's *Nova scientia*. Ideas that were crucial for the writing of this introduction to Benedetti's *Diversae speculationes* are derived from another volume by Renn and Damerow, *The Equilibrium Controversy: Guidobaldo del Monte's Critical Notes on the Mechanics of Jordanus and Benedetti and their Historical and Conceptual Backgrounds* (2012).

We are very thankful to our colleagues at the Max Planck Institute for the History of Science in Berlin for their insightful remarks and the precious exchanges about early modern science that took place in the many workshops and regular meetings over the past years: Angela Axworthy, Sonja Brentjes, Jochen Büttner, Giulia Giannini, Peter McLaughlin, Matthias Schemmel, Joyce Van Leeuwen, Matteo Valleriani, and many others. We are especially thankful to Antonio Becchi and Elio Nenci for their accurate reading of the manuscript of this book and their expert advice. Rodolfo Garau kindly took pictures of manuscripts and rare books that are preserved in the Biblioteca Nazionale Universitaria di Torino and are published here. Stefan Paul Trzeciok helped us with the translation of several Latin passages. We would like to acknowledge the Collaborative Research Centre "Episteme in Motion" (Freie Universität Berlin), which has constituted an important intellectual framework for this collaboration, and the Deutsche Forschungsgemeinschaft for financing Pietro Daniel Omodeo's research in Berlin for several years as well as a research trip to Turin in 2012. We are grateful to the librarians of the Max Planck Institute for the History of Science for their valuable support with bibliographical research and the digitization of sources, and to Louis Berger, Lindsay Parkhowell, and Charlie Zaharoff for their copyediting work and linguistic support. The last phase of the authors' collaboration on the manuscript of this volume is part of a project based at the Ca' Foscari University of Venice that has received funding from the European's Horizon 2020 Research and Innovation Programme (GA n. 725883 *EarlyModernCosmology*).



# Chapter 1

## Prosopography

### 1.1 The Life and Career of a Renaissance Man

Giovanni Battista de Benedetti came from a patrician family of Venice. Although the title of nobility may appear superfluous to the historian of science, it was not so for him and his contemporaries. Benedetti often noted this in his publications, adding to his name the honorific “Patritius Venetus.” Evidence for Benedetti’s noble origins can be found in a document dated January 14, 1570. This is a patent through which Duke Emanuele Filiberto of Savoy conferred upon Giovanni Battista the privileges of Imperial nobility in addition to his previous titles:

We make, create, and constitute the aforementioned Giovanni Battista Benedetti as a true noble of the Holy Roman Empire and of our Empire forever, alongside all his legitimate and natural sons and daughters (those who are already born and those that will be born). We will call and fully declare them such [nobles of the Holy Roman Empire]—although he and his predecessors are noble and were born from an ancient and noble progeny, as we are very well informed.<sup>1</sup>

In those years, the establishment of the Savoy court in Turin brought about a general transformation of the urban *patriziato* into an aristocratic class gravitating around the dukes.<sup>2</sup> This trend was parallel to the more general political-social shift from the civil humanism of the medieval municipalities toward the courtly culture of centralized territorial States.

On the occasion of the conferral of the patent on Benedetti, the cross of Savoy was added to his heraldic design along with the motto “sic vita veritas.”<sup>3</sup> This motto, which indicated a conduct of a life dedicated to the search for truth, was the acknowledgment of his mathematical and philosophical excellence. In the preamble to the duke’s patent of nobility, it was precisely Benedetti’s devotion to the mathematical disciplines, the *humanae litterae*, and the philosophy that was extolled as an example to be imitated and a reason for the conferral of aristocratic privileges on him and his heirs.<sup>4</sup> In this case, scientific distinction led to higher social recognition and even served as a legitimation for it.

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<sup>1</sup>Bordiga 1985, 752: “Habbiamo creato, fatto et costituito, facciamo creamo et costituiamo il detto Giovan Battista de Benedetti con tutti i suoi figliuoli maschi e femine legittimi, et naturali, nati et che nasceranno, et saranno procreati di legittimo matrimonio, con tutti loro posterì et heredi et successori in perpetuo veri nobili del Sacro Romano Imperio et nostri, et per tali li chiamiamo et dicchiariamo per dabondante (ancora ch’egli insieme coi suoi predecessori siano nobili e nati di antica prole nobili come siamo benissimo informati).”

<sup>2</sup>Stumpo 1998, 138.

<sup>3</sup>Bordiga 1985, 601.

<sup>4</sup>Bordiga 1985, 752: “Emanuele Filiberto per gratia di Dio Duca di Savoia Principe di Piemonte etc. Essendoche le attioni che tendono alla Virtù, come che da quella prendano accrescimento et perfettione, sono ammirate et havute in pregio: così gl’huomini che in quelle di continuo si essercitano vengono da ogniuno istimati et tenuti in particolare consideratione, la onde havendomi sempre fatto conto delle persone che dirizzassero ogni loro pensiero al bene operare, et quanto più si potrà, cercassero col mezo delle scienze, et arti liberali sicure et vere guide alla virtù di venire alla cognizione di esso doppo l’haver noi ricercato che

During the Renaissance, nobility was more important than professional appurtenances or academic titles. For instance, the celebrated Danish astronomer Tycho Brahe, himself an appreciative reader of Benedetti, held aristocratic lineage in higher esteem than any status linked to university professorship, including the position of imperial mathematician—an appointment which, by contrast, raised the status of his fellow countryman and opponent Nicolaus Reimarus Ursus, who was of low extraction.<sup>5</sup> Accordingly, Brahe always emphasized Benedetti’s lineage when citing his work, for instance his letter on the superlunary location of the supernova of 1577. The capitalization as well as the reverence in this passage from the *Astronomiae instauratae progymnasmata* (posthumous, 1602) is telling:

The small star of Cassiopeia would not shine as brightly as this nova over the whole surface of the Earth because of the dry fumes placed in-between, if they had been only under that one, and did not affect in the same manner the other stars next to it and augmented that unusual light. But the most excellent philosopher GIOVANNI BATTISTA BENEDETTI, THE VENETIAN PATRICIAN, eminently and skillfully demonstrated this with geometric arguments, in [his] outstanding work concerning mathematical and physical speculations (around the end of his letters). Writing to Annibale Raimondo [...] he clearly showed the absurdity which necessarily follows from his false assumption [i.e., the sublunary position of the nova].<sup>6</sup>

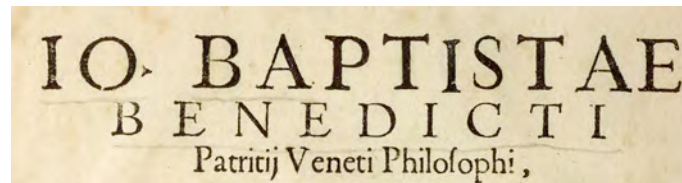


Figure 1.1: An example of the titles Benedetti added to his name in his publications. In the title page of *De gnomonum umbrarumque solarium usu* (1574), he called himself “Venetian Patrician, Philosopher.” (Max Planck Institute for the History of Science, Library)

The prominence accorded to lineage is evident from Brahe’s self-representation in the portrait at the beginning of his *Epistolarum astronomicarum libri* (1596), a collection of

in questo ne sotisfacesse, massime nelle discipline matematiche. Al fine ci è pervenuto nelle mani il nobile messer Giovanni Battista de Benedetti venetiano, nostro mattematico il quale havendo consumato la maggior parte dell’età sua nelle bone lettere et studij di filosofia, et fatto professione delle dette matematiche, et così divinamente et per eccellenza riuscito che si può dire in quelle (tra gl’altri) essere singolare cosa che si porge tal contento, et la sua servitù a noi molto grata tale soddisfattione che lo giudichiamo degno che partecipi de gl’honori dovuti alle sue virtù acciò che gl’accresca l’animo di perseverare et altri siano invitati a seguitare li suoi vestigij.”

<sup>5</sup>This is why Brahe was not and could not desire to be imperial mathematician to Rudolph II, as has often been wrongly thought. See Voelkel 1999.

<sup>6</sup>Brahe 1916, 250: “Accedit et hoc, quod Stellula illa Cassiopeae in toto Orbe Terrarum ob siccas illas fumositates interpositas non tam splendide apparuisset atque haec Nova, si sub hac sola constitissent, et non reliquas illi vicinas pari modo attingissent, lumineque insueto auxissent. Hoc vero ultimum egregie et solerter ex excellentissimo Philosopho IOHANNI BAPTISTA BENEDICTO PATRICIO VENETO in praeclaro illo Opere quod de speculationibus Mathematicis et Physicis inscripsit, circa finem inter Epistolas eius evidenter et dilucide, Geometricis rationibus demonstratur. Ubi ad hunc ipsum Annibalem Raimundum scribens, absurdum, quod ex eius falsa assumptione necessario sequitur, dilucide ostendit.”

epistles that arguably took Benedetti's collection in the *Diversae speculationes* as a model. Brahe's image is encircled by the heraldic designs of the family and makes the signs of his nobility very visible. In the same epistolary, Brahe's letters directed to aristocrats appear more prominently than those addressing "simple" professors or practitioners. He attached greater importance to his correspondence with the patron of sciences, Landgrave William IV of Hesse-Kassel, than to exchanges with the latter's court mathematician Christoph Rothmann.<sup>7</sup> Similarly, in the *Diversae speculationes*, Benedetti published with pride his letters to dukes or to illustrious aristocrats.

Apart from his nobility, we do not know much about Benedetti's origins. According to a horoscope that he cast for himself (Figure 1.2), and was printed by the Neapolitan astrologer Luca Gaurico in *Tractatus astrologicus (Astrological Treatise, 1552)*,<sup>8</sup> Benedetti's father was a learned *Hispanus*, or Spaniard. Based on this thin evidence, his biographer, Giovanni Bordiga, speculated that his family could have been merchants trading with Spain.<sup>9</sup> Other archival documents caused him to speculate about Benedetti's marriage, around 1585, and about the existence of a daughter called Lodovica from an earlier relationship or marriage. She married a certain Domenico Pipino of Racconigi. Benedetti built a sundial for this son-in-law (*magnificus Dominus Dominicus Pipinus generus meus*), as indicated in *De gnomonum... usu* (1574). Lodovica died young, long before her father, in 1580.<sup>10</sup>

For the greater part of his life Benedetti was a courtier. For several years he served duke Ottavio Farnese of Parma, whom he joined in 1558 as "*lettore di filosofia e matematica*."<sup>11</sup> Later, from 1567 up to his death on January 20, 1590, Benedetti served the Dukes of Savoy Emanuele Filiberto and Carlo Emanuele I. His duties were typical for a Renaissance court mathematician and are akin to those of Leonardo da Vinci in Milan, Guidobaldo del Monte in Urbino, Galileo in Florence, and Kepler in Prague, to mention only a few well-known names.<sup>12</sup> Benedetti was required to advise his patrons on issues of mathematical expertise. His fields of competence included engineering and architecture.<sup>13</sup> In Parma and Turin he built sundials (such as the modern one in Figure 1.3). He was also responsible for the construction of a fountain in the ducal park (*Parco di Viboccone*, later *Parco Regio*), which was destroyed by the French army during the siege of 1706.<sup>14</sup> Moreover, he was consulted on astronomy and music, both traditionally considered mathematical disciplines. In Parma he carried out astronomical observations, which he also reported on in the *Diversae speculationes*. In two letters to the Parma choirmaster de Rore, Benedetti explained musical consonance and dissonance of two tones by the ratio of oscillations of waves of air generated by the strings of musical instruments.<sup>15</sup> He claimed that the frequency of two strings of equal tension must have an inverse ratio to the lengths of the strings, and thus proposed to describe the degree of consonance or dissonance of two tones mathematically. In Turin he wrote a proposal for the calendar reform in 1578, *De temporum emendatione*, later reprinted in the *Diversae speculationes* as the

<sup>7</sup>See Mosley 2007.

<sup>8</sup>Gaurico 1552, f. 76r.

<sup>9</sup>Bordiga 1985, 588.

<sup>10</sup>Bordiga 1985, 604–605.

<sup>11</sup>Bordiga 1985, 593–595.

<sup>12</sup>For the broad European context of patronage and the arts in the Early Modern Period, see Bedini 1999, Moran 1981, and Moran 1991.

<sup>13</sup>See Roero 1997 and Mamino 1989.

<sup>14</sup>Maccagni 1967a, 353–354.

<sup>15</sup>Benedetti 1585, 277–278.

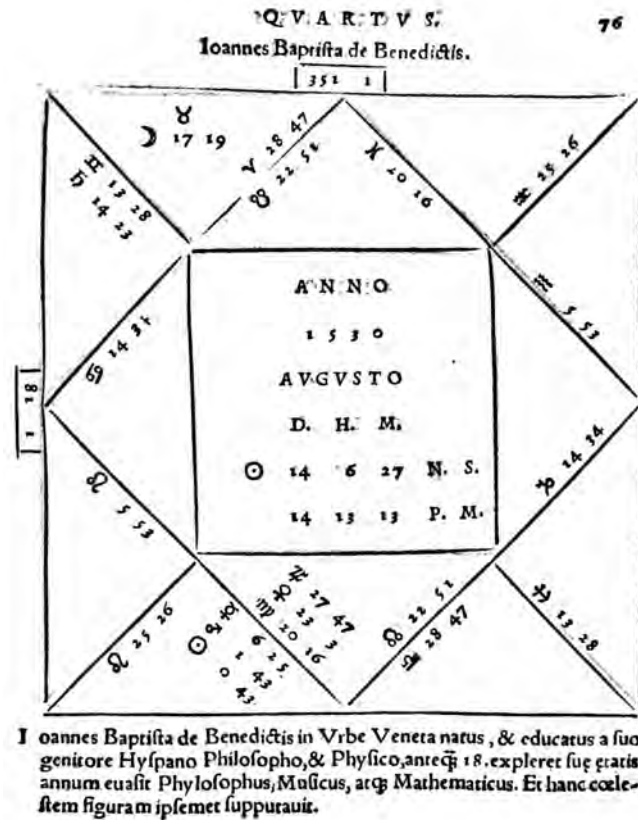


Figure 1.2: Benedetti’s own horoscope, in Luca Gaurico, *Tractatus astrologicus* (1552), f. 76r. (Bayerische Staatsbibliothek)

first of his epistles (to Duke Emanuele Filiberto).<sup>16</sup> This proposal was also sent to Rome and was meant as an aid to Clavius’s efforts to correct the calendar.<sup>17</sup> At the same time, he taught mathematics to Emanuele Filiberto and his son Carlo Emanuele I.

Courtly life included participation in literary culture. Baldassar Castiglione, in his idealization of the court of Urbino in *Il libro del Cortegiano* [*The Book of the Courtier*] (1528), launched the model of a courtier with a refined literary education.<sup>18</sup> Following such cultural dispositions, a courtier versed in mathematics could advocate the usefulness of his expertise for the interpretation and assessment of “scientific” questions raised by classical sources, even poems. This attitude explains the inclusion of a letter on Ovid in the collection of epistles in the *Diversae speculationes*.<sup>19</sup> It was addressed to a certain Pancrazio Mellano, perhaps a courtier, asking Benedetti’s opinion about the astronomical references in Book 2 of the *Metamorphoses*, in which Ovid tells the myth of Phaeton. According to the myth, Phaeton rode his father Apollo’s chariot one day but he was unable to control the horses and keep the sun on its regular path. Finally, he was thrown out of the chariot, took a bad fall, and died. In the poem Ovid described the solar path in some detail but, according to Benedetti, he mixed up daily rotation and annual motion along the ecliptic: “Ovid unduly passes from the daily motion to the annual” (*Quod Ovidius transcurrit*

<sup>16</sup>Benedetti 1585, 205–210.

<sup>17</sup>Benedetti’s advice on the calendar reform is preserved in the Biblioteca Apostolica Vaticana under the signature cod. Vat. lat. 5645, 148r–150r. See Ziggelaar 1983, 211–214.

<sup>18</sup>Baldassar Castiglione, *Il libro del Cortegiano*, ed. Walter Barberis (Torino: Einaudi, 2017).

<sup>19</sup>Benedetti 1585, 417–418.





Figure 1.3: A modern sundial on the Church of San Lorenzo in Turin reminiscent of those designed by Benedetti. (Own photography)

*a motu diurno, ad motum annum praeter rem*). To make his point clear, Benedetti listed the passages dealing with one or other of the two motions ascribed to the sun in ancient astronomy.<sup>20</sup>

As an exponent of the Turin elite, he was himself devoted to poems. For instance, the Milanese painter and writer Giovanni Paolo Lomazzo, who was linked to Savoy's court, celebrated Benedetti in verse as a philosopher, mathematician, and astrologer. In the first lines of a poem dedicated to him, Lomazzo declared himself delighted that Benedetti appreciated his paintings and cast his birth horoscope. Lomazzo's poem paints a vivid picture:

Prudence and knowledge descend  
 From Philosophy into [human] intellects;  
 Which are perfect as far as their disposition is concerned,  
 As each one receives its part of justice and reason.  
 To Benedetti, he so wise  
 And precious in the world,  
 Belongs so much of this [philosophy]  
 That it would be vain to try to equal him:  
 So sublime does his value shine.  
 All the more am I delighted that he appreciated

<sup>20</sup>Omodeo 2012b.

My painting so much so that he considered  
 The time and the point in which I was born in the world.  
 Oh splendor of our time, the sound [of your voice] silenced  
 Every scholar of your art, who had to direct his judgment elsewhere,  
 As it was overshadowed by yours, which is so deep.<sup>21</sup>

Benedetti received no formal or academic education. Like other Renaissance self-taught men (e.g., Niccolò Tartaglia and Tommaso Campanella), Benedetti was even proud of being removed from the academic habitus and training centers. This is particularly evident from the anti-academic tone of some of his polemics. In the preface to his first scientific treatise, *Resolutio omnium Euclids problematum* (1553) (On the solution to geometrical problems using a compass with a fixed opening), the twenty-three year old Benedetti emphasized the fact that he had not had a “common” (*quod vulgus solet*) education at some gymnasium or school. He boldly wrote to his patron, the Dominican abbot and diplomat Gabriel Guzman, that:

Until now I have advanced without any mentor or teacher (under the guidance of God). I have never frequented any gymnasium or school. I have not learned what the vulgar (I mean this word without arrogance) use to estimate erudition, [such as limiting it] to the time spent at school, thus setting an end to learning when the seven years [of regular studies] are ended. As long as I live, I will continue [learning].<sup>22</sup>

It is possible that Benedetti was educated privately by his father, depicted in Gaurico’s *Tractatus astrologicus* as “*Hyspanus, Philosophus, et Physicus*” (see Figure 1.2). For his part, Benedetti acknowledged only one teacher, namely the reputed mathematician and scientist-engineer Niccolò Tartaglia (ca. 1500–1557), for introducing him to the first four books of Euclid’s *Elements*, probably between 1546 and 1548. In the *Diversae speculationes mathematicae et physicae*, Benedetti mentioned Tartaglia again as one of the very few authors of mathematical works whom he deemed worth reading.<sup>23</sup> However, in the

<sup>21</sup>Lomazzo 2006, 177–178, III, 19, “Del Sig. Gio. Battista Benedetti Matematico”:

“De la Filosofia nasce e discende  
 La prudenza e ’l saper de gli intelletti;  
 Co’ quali essendo nel dispor perfetti,  
 A ognuno suo diritto e sua ragion si rende  
 Di questa sì gran parte se ne prende  
 Il saggio e raro al mondo Benedetti,  
 Che d’agguagliarlo in vano è chi s’affetti:  
 Tanto sublime suo valor s’estende.  
 Però tanto godo io che sì gli piacque  
 La mia pittura, e perciò egli volse  
 L’ora et il punto nel qual nacqui al mondo.  
 Splendor di questa etade al tuo suon tacque  
 Ogn’un de l’arte tua, e altrove volse  
 Il suo dir vinto dal tuo sì profondo.”

<sup>22</sup>Benedetti 1553, f. 5r: “[...] huc usque progressus sum (Deo duce) sine monitore praeceptoreque ullo, nullum gymnasium unquam, nullamque scholam frequentavi, neque hoc studui, quod vulgus solet (sed absit verbo arrogantia) pro tempore in scholis transacto, eruditionem estimare, ac septennario finito finem studii imponere, sed dum vivo, illa prosequi.”

<sup>23</sup>One reads in the preface *ad lectorem* of the *Diversae speculationes* the following declaration: “In his autem meditandis, ex arithmetiis authoribus quos inspexi praecipuus fuit Nicolaus Tartalea, quippe quem

*Resolutio omnium Euclidis problematum*, he was quick to add that he had learned the rest of the *Elements* by himself:

As it is honest and right to attribute to everybody his own merit, [I should acknowledge that] Niccolò Tartaglia taught only the first four of Euclid's books to me. I studied the rest alone with effort and diligence. In fact, for the one who wants to know, nothing is [too] difficult.<sup>24</sup>

Bordiga described such self-celebration as a sign of Benedetti's "pride in the assumed independence of his own thinking" (*orgoglio di creduta indipendenza del proprio pensiero*).<sup>25</sup> This is the same pride that would later lead to animosity with other prominent mathematicians such as Del Monte.

Moreover, in the preface to the *Resolutio*, Benedetti contrasted the simplicity of mathematics with the vanity of rhetoric. He went so far as to accuse learned and eloquent doctors of corrupting the sciences.

Furthermore, mathematics does not require much [stylistic] splendor. If some language expert tried to improve its elegance, this would have no value, because a change of the mathematical language and of the scientific terminology could easily confuse the sense [of the reasoning] and render everything obscure. Therefore, I will follow the scholarly tradition and use plain words in my demonstrations, as I disapprove of deceptive elegance. In this respect, I follow the steps of the ancients who taught the sciences and the subjects themselves using plain words. Petty teachers (indeed, charlatans and babblers) corrupted this manner of teaching. Although they do not understand the subject, their babbling obtains the highest praise by the vulgar who regard them as learned scholars. This should not be surprising, considering that the most perfect and distinguished expertise in the sciences is attained by very few—despite the fact that many people write a great deal in all kind of sciences and arts, babbling a lot and capturing the attention of the uneducated with illusions and bombastic words.<sup>26</sup>

The same tone characterized Benedetti's next publication. Its title was intentionally polemical: *Demonstratio proportionum motuum localium contra Aristotilem et omnes philosophos* (1554). In fact, this booklet put forward a novel theory of motion. He

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fere omnia ab aliis scripta collegisse constat, nec alios ex praecipuis quos legere potui omittendos duxi, inter quos sunt Hieronymus Cardanus, Michael Stifelius, Gemma Frisus, Ioanna Novimagus, Cuthbertus Tonstallus, caeterique huiusmodi."

<sup>24</sup>Benedetti 1553, f. 5v: "Caeterum quia cuiusque quod suum est reddi debet, nam et pium et iustum est, Nicolaus Tartalea, mihi quatuor primos libros solos Euclidis legit, reliqua omnia, privato et labore et studio investigavi, volenti namque scire, nihil est difficile."

<sup>25</sup>Bordiga 1985, 588 (4).

<sup>26</sup>Benedetti 1553, f. 5v: "Adde quod Mathematicae disciplinae, neque tantum requirunt splendorem, neque si quis peritus linguarum contendat ad elegantiam rem reducere, egregium quid effecerit, quia mutato usu Mathematicae loquendi, ipsiusque scientiae terminis, sensum facile perturbaverit, et ex nihilo nihil apprehensum obtinuerit. Quare morem scholarum sequutus, obstentatione elegantiae explosa, verbis nudis in demonstrationibus usus sum, hac in parte veterum vestigia sequutus, qui nudis verbis scientias resque ipsas docebant, quem modum docendi, nobis devastarunt scioli vel potius circulatores, garruli, rebus ipsoque iudicio destituti, garrulitate siquidem apud vulgus, laudem summam consequuntur, et pro doctis circumferuntur, nec mirum, cum scientiarum perfecta exquisitaque perita, paucissimis detur, non obstante quod multi permulta de omni generis et scientiis et artibus scribant, permultaque garriant, fucis suis, et ampullis imperitorum oculos perstringentes [...]."

argued that bodies of the same material fall through a given medium with the same speed, and not with speeds proportional to their weights, as Aristotle held. This is the reason for Benedetti's declaration of war "against Aristotle and all philosophers" in the title. Benedetti employed the Archimedean concept of buoyancy to account for the dependence of the motion of fall on their specific rather than absolute weight. As we shall see, these ideas played an important role in the *Diversae speculationes*. The use of Archimedean notions to improve on Aristotle's physics was probably stimulated by Tartaglia's Italian translation (1543) of Book 1 of Archimedes's treatise on bodies in water.<sup>27</sup> Benedetti's challenge to Aristotle must have raised considerable discussion, as is shown by the fact that, in his *Demonstratio*, he discussed Aristotle's views and responded to his critics at length. In the second edition of the *Demonstratio* (13 February 1554 *more veneto*, in fact, 1555), he showed that the resistance encountered by a falling body in a medium depends not on its volume, but on its surface area. Benedetti moreover explained the acceleration of the motion of fall in terms of an increasing impetus of the falling body. He had already outlined his theory of fall in the dedicatory letter of the *Resolutio*, explaining this anticipation as a means of avoiding plagiarism.<sup>28</sup> Still, in spite of his efforts to secure priority for his ideas by repeated publication, they were plagiarized by the Flemish polymath Jean Taisner in 1562 and spread through Europe with no clear acknowledgement of their origin.<sup>29</sup> This prompted Benedetti to express his indignation and rage at Taisner in the dedicatory letter of his *De gnomonum... usu* (1574).<sup>30</sup>

As was to be expected by his irreverent tone, some of the first reactions to Benedetti's early writings were rather critical. As he reports in the preface to the second edition of the *Demonstratio* (1555), some Roman scholars objected that his treatment of motion was in disagreement with Aristotle (*illam [meam propositionem] neutiquam esse iuxta mentem Aristotelis*). Benedetti was informed about their disapproval by a Dominican friend, Petrus Arches, an expert of Hebrew and Greek letters cultivated in philosophy and mathematics.<sup>31</sup> Benedetti replied that those scholars worshipped Aristotle like a pagan god (*veluti coeleste quoddam numen*) and did not admit that their *auctor* could make mistakes. He claimed that he had not misunderstood Aristotle; rather, that he simply disagreed with him.

I remember that he [the very educated Doctor Peter Arches]—after many different conversations on various subjects—told me that many in Rome considered that proposition of mine (which I sent to you, Reverend Mr. Guzman, among other ones) and they mostly reacted with surprise for I did not specify that it was by no means in accordance with Aristotle's mind. Such was the reaction of those who considered my demonstration very attentively.

They could not concede that Aristotle was mistaken in any way, because they do not regard him as a human being. Rather, they confer upon him the celestial condition of a pagan divinity. And they see even slight disagreement as a sin. Therefore [they believe that] I committed (and still commit) heresy if, according to their judgment, I do not follow the pure and authentic mind of Aristotle's doctrine in any manner.

Thus, in order to escape the allegation of such an error or [the rumor] that I am dissimulating and hiding something, especially as far as this issue is con-

<sup>27</sup>Archimedes 1543.

<sup>28</sup>Benedetti 1553, f. 10v. See Maccagni 1967a, 338–340 and Maccagni 1967b, 14–15.

<sup>29</sup>Taisner 1562, see the discussion in Maccagni 1967a, 344–455, n. 13.

<sup>30</sup>Benedetti 1574, f. 4v.

<sup>31</sup>Maccagni 1967b, 20–21, and 20, n. 14.

cerned, I decided to publish this new booklet in which I present my opinion more clearly. In this manner, everybody should become aware that I correctly understood Aristotle and that I disagree with him on a particular issue with considered reason. This is an unpleasant task for me. In fact, it is only unwillingly that I dissent with such a great man. I know nobody who could rival his excellence in all kind of doctrines. Nevertheless, his teaching is to take as true that which is supported by stronger reasons. He himself followed this precept, as he stated in the *Ethics*: “Plato is my friend, Socrates is my friend, but truth is even more friend to me.”<sup>32</sup>

It is evident from these passages that Benedetti regarded mathematics as a support for conclusive rational argumentation in the treatment of natural issues. Therefore, as a *mathematicus* he claimed for himself the right to be called a *philosophus*. Already in the short biographical indication accompanying his birth horoscope, he was said to be a “*Philosophus, Musicus, atque Mathematicus*” (see Figure 1.2). In his publications, Benedetti often stressed his quality as “*philosophus*” or “*filosofo*.” Galileo would later add the title of “philosopher” to that of “court mathematician” in Medici’s Florence.<sup>33</sup> However, in Benedetti’s case, it is evident that adding the title of “*philosophus*” was not part of a strategy aimed at social advance but rather mirrored his cultural and philosophical commitment to a mathematical philosophy of nature with all its consequences, among them that Aristotelian physics was open to critique by means of mathematical reasoning.

Thus, Benedetti not only dealt with fields of mathematical inquiry that traditionally belonged to the domain of mathematics (such as mechanics, optics, mathematical astronomy, and musical theory), but also addressed issues considered beyond the limitations of mathematics, especially terrestrial and celestial physics. The title of the *Diversae speculationes mathematicae et physicae* is itself provocative, as it brings together mathematics and natural philosophy (or *physica*), considered to be separate fields, one dealing with the *quia* (the “phenomena”) and the other with the *propter quid* (the “causes”). In this respect, Benedetti’s methodology is very close to that of Nicolaus Copernicus, whose heliocentric system he admired. In Book 1 of *De revolutionibus orbium coelestium* (1543) and in the *Narratio prima* (1540), Copernicus and his pupil Georg Joachim Rheticus (1514–1574) reversed the Peripatetic hierarchization of physics over mathematics, urging a reform of natural philosophy and celestial and terrestrial physics in order to bring them into accord with the geokinetic and heliostatic innovations in mathematical astronomy. Beyond astronomy, the issue of the status of mathematics and its role in natural investigations was

<sup>32</sup>Maccagni 1967b, 20–21: “Memini eum [eruditissimum Doctorem Petrum Arches], post varia et diversa colloquia utro citroque inter nos habita, mihi retulisse quamplurimos Romae, conspecta mea illa propositione quae ultra reliquas tuae R[everende] D[omine] [Guzman]a me mittebatur, valde mirari solitos me addidisse illam nequitiam esse iuxta mentem Aristotelis, idque ab eis dictum ubi meam demonstrationem attentius considerarunt.

Ne vero Aristotelem ullo modo errasse concederent, cum illum non infra humanae conditionis terminum habeant, sed potius veluti coeleste quoddam numen sibi proponant, censeantque nefas esse si vel latum quidem unguem ab eo quis dissentiat, in hac potius haeresi fuisse, ac etiamnum esse, ut me germanum et genuinum sensum Aristotelicae opinionis nequaquam ex authoris mente assecutum existiment.

Ego vero ne mihi diutius talis impingatur error, neve quid maxime super hac re sentiam, aut dissimulem, aut reticeam, statui, hoc novo libello edito, meam sententiam clarius aperire, ut omnes intelligant me et Aristotelem ipsum antea recte intellexisse, et non temere hoc in loco ab eo discrepare, quod sane quanquam invitatus facio (nec tamen libenter a tanto viro diversum sentio, quippe qui norim quam ille praeclarus extiterit in omni doctrinarum genere), docet tamen maiorem ratione veritatis habere, quo ipsemet facendum censuit, quam inquit in *Ethicis*: ‘Amicus Plato, amicus Socrates, at magis amica veritas.’”

<sup>33</sup>Biagioli 1989, 49–50.

heatedly debated by philosophers and mathematicians during the Renaissance.<sup>34</sup> One ancient predecessor to praise mathematical physics was the Hellenistic “prince of astronomy and geography,” Claudius Ptolemy. In the beginning of the *Almagest*, he pointed out the superiority of mathematics over theology and physics, and even argued for a possible extension of the method of mathematical astronomy to include the treatment of local motion in general, as well as theology and ethics.

Only mathematics can provide sure and unshakeable knowledge to its devotees, provided one approaches it rigorously. For its kind of proof proceeds by indisputable methods, namely arithmetic and geometry. Hence we were drawn to the investigation of that part of theoretical philosophy, as far as we are able to the whole of it, but especially to the theory concerning divine and heavenly things. For this alone is devoted to the investigation of the eternally unchanging. For that reason it too can be eternal and unchanging (which is a proper attribute of knowledge) in its own domain, which is neither unclear nor disorderly. Furthermore it can work in the domains of the other [two divisions of theoretical philosophy, physics and theology] no less than they do. For this is the best science to help theology along its way, since it is the only one which can make a good guess at [the nature of] that activity which is unmoved and separated; [it can do this because] it is familiar with the attributes of those beings which are on the one hand perceptible, moving and being moved, but on the other hand eternal and unchanging, [I mean the attributes] having to do with motions and the arrangements of motions. For almost every peculiar attribute of material nature becomes apparent from the peculiarities of its motion from place to place. [Thus one can distinguish] the corruptible from the incorruptible by [whether it undergoes] motion in a straight line or in a circle, and heavy from light, and passive from active, by [whether it moves] towards the centre or away from the centre.<sup>35</sup>

Even after Copernicus, Ptolemy’s methodological insights maintained their full importance and could guide scholars who intended to expand the realm of the application of mathematics far beyond the limits established by traditional philosophy. In the *Diversae speculationes*, Benedetti deepened the discussion of issues of natural philosophy such as the concepts of space, time, and motion, claiming for a mathematician a better and clearer insight into foundational problems of physics.

Astrology was another area of expertise for Benedetti. During the Renaissance, astronomy and astrology were never separated. Benedetti was expected to cast horoscopes and give astrological advice to his patrons, just as Brahe astrologically advised the King of Denmark, Kepler the Emperor, and Galileo the grand dukes of Tuscany.<sup>36</sup>

In Venice Benedetti frequented celebrated exponents of the astrological culture of the time, among them Annibale Raimondo of Verona and Francesco Giuntini. Raimondo reported about a meeting they had in the residence of the senator and poet Domenico Venier. On that occasion he and Benedetti tested Giuntini’s astrological preparation:

We gathered at Mr. Domenico Venier’s place; his magnificence [came] first, followed by the most excellent Mr. Giovanni Battista Benedetti, many other

<sup>34</sup>De Pace 1993.

<sup>35</sup>Ptolemy 1984, 35–37.

<sup>36</sup>A very informed case study on astrology at Italian Renaissance courts is Azzolini 2013.

gentlemen, myself (Annibale Raimondo), and finally the ex-reverend father Pacifico of Florence (now, as an ex-friar, known as ‘excellent Mr. Francesco Giuntini’). As soon as the latter arrived, he was given the simple astrological chart of the revolution of the magnificent Venier, without any written indication around or below. The good father took countless and endless texts and aphorisms out of his scapular. He related them to the revolution as good as a physician might give prescriptions to sick people by saying ‘God might help you.’ Since the most excellent Mr. Benedetti and myself laughed uncontainably—thereby making the father believe that he could not have better done—the good father, who was already trotting, was spurred by our laughter to gallop so quickly that it became extremely difficult to bring him back to silence and prevent him from telling more stupidities.<sup>37</sup>

An astrological report by Benedetti, cast for Carlo Emanuele I (Turin, 19 October 1589), is still extant and preserved in the Civic Library of Turin (Coss. 4, ff. 1r-2v). It contains a day-by-day personalized astrological forecast for the month of November 1589. The days are qualified with adjectives such as “*buono*” (good), “*mediocre*,” or “*cattivo*” (bad), but some are treated more specifically (the 9th of November is indicated as apt to “*negotii ingeniosi*,” ingenious endeavors, whereas the 10th of November as “*buono in cose femminili ma nel resto cattivo*,” that is, bad except for women’s affairs). Benedetti signed this astrological letter as “*Matematico e Astrologiaro*.”<sup>38</sup> This signature shows that his “professional” profile could vary depending on circumstances, since it depended in part on the kind of advice requested from him.

In the concluding letter of the *Diversae speculationes*, Benedetti envisaged a reform of astrology. He directed this letter to a German correspondent whose name he awkwardly Latinized as *Volfardus Aisestain*.

As for the question whether or not I regard as true all that is written in the books of judicial astrology, I respond that I do not. I even believe that much is wrong [...]. But you will be informed about all this in a special tract of mine, about which I told you on another occasion. In it, you will find many things I have proven through the evidence of many observations. I intend to publish that tract along with some other speculations of mine, if only I will have enough time to do that, before I meet the body of the adverse Mars as indicated by my horoscope. This is going to happen in 1592.<sup>39</sup>

<sup>37</sup>Raimondo 1574: “Ritrovandosi nella camera del Clariss. M. Dominico Veniero prima la sua Mag. [,] lo eccellentissimo M. Gio. Battista Benedetti, molt’altri gentilhuomini, et Annibale Raimondo, che son quel io, vi sopraggiunse al’hora il Reverendo Padre Frate Pacifico Fiorentino de gli bene inculati, adesso per essersi sfratato lo Eccellente M. Francesco Giuntini, alquale, subito giunto, fu dato in mano la figura semplice del cielo della Revolutione del detto Mag. Veniero, senz’altra scrittura intorno, né appresso, il buono padre allora mise mano al suo scapolario et cavò fuori testi, et afforismi senza fine, et senza fondo, allegandoli tanto a proposito della Revolutione, quanto facea quel buon medico le ricette che ’l dava ai suoi infermi, quando le dicea Dio te la mandi buona, et perché lo Eccell. M. Gio. Battista Benedetti et io se smassellavamo dalla risa, ben però in modo di maravigliarsi, come non fusse possibile a dir meglio di quello che dicea sua paternità, il buon padre per il nostro ridere sì come prima andava trotando, si misse a correr’ de modo che fu gran fatica a poterlo tenere et farlo tacere che’l non dicesse più minchionerie.” Cf. Corradeschi 2009, 111, n. 46. On Raimondo and Giuntini, see Ventrice 1989, 140–145.

<sup>38</sup>Roero 1997, 57–58.

<sup>39</sup>Benedetti 1585, 425–426: “Circa vero id de quo me interrogas, scilicet, utrum putem omnia vera esse, ea quae scripta reperiuntur in libris Astrologiae iudiciariae, respondeo quod non, imo puto plurima falsa esse [...]. Sed diffusius haec omnia videbis in meo illo particulari tractatu, de quo tibi alias dixi, in quo multa

This passage concludes his major work. In it, Benedetti predicted, using astrological means, his own death for the year 1592, but he actually died in January 1590.<sup>40</sup> This fact aroused some doubts about his proficiency as an astrologer, especially from his successor as court mathematician, Bartolomeo Cristini.<sup>41</sup>

To sum up, Benedetti's persona and work had various facets, his interests ranging from mathematics to cosmology and from natural philosophy to literature. In a certain sense, he can be seen as a Renaissance polymath. However, his profile can be better encompassed by the title of "*mathematicus*," as long as we do not take it too restrictively. A Renaissance mathematician like Benedetti was an engineer and a technical inventor, as well as a theoretician and a natural philosopher; someone with teaching and civil duties who served as a counsellor, also for astrological matters. Being a court mathematician implied benefiting from high recognition and visibility in society. Thus, this professional and intellectual appurtenance had nothing to do with the rather low acknowledgment that mathematicians received at universities, where physicians, lawyers, and theologians were higher placed and received better salaries.<sup>42</sup> The cultural environment of Turin, with which Benedetti interacted in the most important years of his career, shall be addressed in the next section.

## 1.2 Benedetti's Works and Publications

Benedetti published his first work at the age of 23, the *Resolutio omnium Euclidis problematum* (Resolution to All of Euclid's Problems, Venice 1553), which offered the solution to "all" geometrical problems using a compass with a fixed opening. The work reacted to a challenge that emerged from a controversy between Niccolò Tartaglia and Lodovico Ferrari in the years 1546–1548 and inserted Benedetti into the scientific debates of his time. One year earlier the astrologer Luca Gaurico had already paid homage to him, including in his *Tractatus astrologicus* a horoscope of the promising mathematician cast by Gaurico himself.

In 1554 Benedetti published a *Demonstratio proportionum motuum localium contra Aristotilem et omnes philosophos* (Demonstration Concerning the Proportions of Local Motions against Aristotle and All Philosophers), which is not as famous for its polemical verve as for the presentation of an innovative theory of fall. As we have discussed in the preceding section, in this treatise Benedetti developed a theory of the motion of fall, first proposed in the dedicatory letter of the *Resolutio* of 1553. Benedetti maintained that bodies of the same material fall through a given medium with the same speed and not with speeds in proportion to their weights, as Aristotle and his followers claimed. Benedetti tried to overcome the fallacies of the Aristotelian theory of fall by employing the Archimedean concept of buoyancy, assuming that the motion of fall depends on their specific rather than absolute weight. As we have also discussed above, in the second edition of the *Demonstratio*, published in Venice in 1555,<sup>43</sup> Benedetti argued that the resistance incurred by a

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videbis, quae omnia ab experientia, ex multis a me observatis, comprobata sunt, quem quidem tractatum cum quibusdam aliis meis speculationes in lucem producere cupio, si fieri poterit, antequam ad directionem mei Horoscopi cum corpore Martis Anaeretae perveniam, quae quidem directo circa annum millesimum quingentesimum nonagesimum secundum eveniet."

<sup>40</sup>Benedetti was not the first mathematician who tried to forecast his own death. Among his predecessors are famous the cases of Johannes Stöffler and Girolamo Cardano. Cf. Omodeo 2014b, 3–4.

<sup>41</sup>Vernazza 1783, 16–18.

<sup>42</sup>On the lower status of mathematicians, see Henry 2011.

<sup>43</sup>Benedetti [1554] 1555, see Benedetti 1985.



falling body in a medium depends not on its volume, but on its surface area. This is also the view that he presented in the *Diversae speculationum mathematicarum et physicarum liber*, published in Turin in 1585. He explained the acceleration of the motion of fall in terms of an increasing impetus of the falling body. Such examples show how he dealt with new challenging problems, which were difficult and sometimes impossible to solve using the mainstream theories of his time, by bringing forth and promoting new ideas.

After the *Resolutio omnium Euclidis problematum* and the *Demonstratio proportionum motuum localium*, composed when Benedetti was still in Venice, the next extant works stem from the time when he had already settled in Turin. First, he composed two works on gnomonics, one in Italian and one in Latin. The former is a manuscript preserved in the Civic Library of Carignano (Turin, Italy), entitled *La generale et necessaria instruzione per l'intelligentia et compositione d'ogni sorte [di] Horologij Solari*, which was presumably written between 1567 and 1573. The latter was printed under the title *De gnomonum umbrarumque solarium usu liber* (1574). Here Benedetti dealt at length with the construction of sundials with faces of varying inclinations and also with cylindrical and conical surfaces. At ff. 107r-v one finds a discussion of a sundial that perhaps can still be seen today on a wall of the Royal Palace in Turin.<sup>44</sup>

In 1574 Benedetti also wrote about a trigonometrical measuring instrument of his own invention, *Descrittione, uso, et ragioni del Trigonometro*. It was never printed and is preserved in manuscript form in the Civic Library of Carignano along with the Italian work on sundials, *Intelligentia et compositione d'ogni sorte [di] Horologij Solari*.<sup>45</sup> His next scientific treatise, *De temporum emendatione opinio* (1578), proposed correcting and reforming the calendar. In 1578 the duke initiated a public disputation at the University of Turin where Benedetti argued with Antonio Berga about whether there was more water or more land on the earth, following an argument by Alessandro Piccolomini. The views which Benedetti brought forth against his opponent were published in Turin in 1579 under the title *Consideratione... d'intorno al discorso della grandezza terra et dell'acqua del eccellent[e] sig[nor] Antonio Berga*. This polemic was renowned, as can be seen in the Italian translation and commentary of Sacrobosco's *Sphere* by the theologian, astronomer, and astrologer Francesco Giuntini in Lyon: "The excellent philosopher, Mr. Giovanni Battista Benedetti, mathematician to the serene duke of Savoy, resolved this question very aptly, arguing against the philosopher Berga, a famous reader at the University of Turin. The latter argues against Mr. Piccolomini that there is more water than earth. Benedetti defends the opposite view, which corresponds to truth: that there is less water than earth."<sup>46</sup>

Next came Benedetti's defense of the reliability of the mathematical computations underlying astrological predictions in the context of a heated polemic on this issue that burst out in Turin 1580–1581. Benedetti first communicated his views in Italian, in epistolary form: *Lettera per modo di discorso... all'illustre sig. Bernardo Trotto. Intorno ad alcune nuove riprensioni, et emendationi contra alli calculatori delle effemeridi* (Letter

<sup>44</sup>Roero 1997, 47.

<sup>45</sup>Clara Silvia Roero published Benedetti's letter to Carlo Emanuele I (Turin, 19 October 1589), the index of the manuscript on gnomonics, as well as an excerpt from the manuscript on the mathematical instrument *trigoniometro* as appendices II and III of Roero 1997.

<sup>46</sup>Giuntini 1582, 95–96: "La qual questione ha resoluta molto dottamente lo eccellente filosofo, il signor Giovambattista Benedetti mathematico del serenissimo signor Duca di Savoia, contra il filosofo Berga, famoso lettore nella università di Turino: il quale contra l'opinione del signor Piccolomini defende che l'acqua è maggiore della terra: e il Benedetti defende il contrario in favore della verità: cioè che l'acqua è minore della terra."

in the Form of a Discourse... Addressed to the Illustrious Mr Bernardo Trotto Concerning Some New Criticism and Corrections against the Ephemerides Calculators) (1581). Benedetti later included a Latin translation of this letter in the *Diversae speculationes* (1585).<sup>47</sup> His commitment to astrological practice is testified to by an astrological report he wrote for Carlo Emanuele I, a handwritten letter (Turin, 19 October 1589) preserved in the Civic Library of Turin (Coss. 4, ff. 1r-2v).<sup>48</sup>

Finally, Benedetti had his major work, *Diversarum speculationum mathematicarum, et physicarum liber*, printed in 1585. It was issued again under slightly different titles in Venice in 1586 (*Speculationum mathematicarum et physicarum tractatus*) and, posthumously, in 1599 (*Speculationum liber*).

Two of Benedetti's manuscripts, preserved in the Biblioteca Nazionale Universitaria of Turin until 1904, are irreparably lost due to a fire that burst out in that year, destroying many valuable manuscripts. The first one was a collection of his letters, *Lettere di Giovanni Battista Benedetti, Veneziano, matematico del Duca Emanuele Filiberto e Carlo Emanuele I, in risposta ai quesiti fattigli dal Duca e da altri personaggi intorno alla matematica, fisica, musica e filosofia*.<sup>49</sup> The second one held similar content and was entitled *Lettere di Giovanni Battista Benedetti in risposta a quesiti di fisica e matematica* (Letters by Giovanni Battista Benedetti answering questions on physics and mathematics).<sup>50</sup>

Reprints of Benedetti's works are rather scarce. Excerpts on mechanics from Benedetti's work were included by Stillman Drake and Israel Edward Drabkin in their *Mechanics in Sixteenth-Century Italy: Selections from Tartaglia, Benedetti, Guido Ubaldo and Galileo* (Madison, Wisc.-Milwaukee-London, 1969). Carlo Maccagni's *Le speculazioni giovanili "de motu" di Giovanni Battista Benedetti* (Pisa, 1967) includes excerpts from the dedicatory letter of the *Resolutio omnium Euclidis problematum* and the text of the two editions of the *Demonstratio proportionum motum localium contra Aristotilem et omnes philosophos*.

<sup>47</sup>Benedetti 1585, 228–248, “Defensio ephemeridum.”

<sup>48</sup>See Roero 1997, Appendix I.

<sup>49</sup>Peyron 1904, 73–74, Codex 83, N. II. 50.

<sup>50</sup>Peyron 1904, 95, Codex 94, N. III. 27.

## Chapter 2

### The Subalpine Environment

Benedetti's life, work, and reception are indissolubly linked to Turin and the Duchy of Savoy. As one reads in the *Diversae speculationes*, he chose to live in this town until the end of his life. There he benefited from the patronage of Duke Emanuele Filiberto (Figure 2.1) and, as a consequence, from a prominent social position and recognition. In the dedicatory epistle of his major work, addressed to Filiberto's successor, Carlo Emanuele I, Benedetti extolled the merits of the deceased duke, who had invited him to Turin almost two decades earlier:

Nineteen years have passed since I was sent for by a letter of the most serene [Emanuele Filiberto] father of Your Highness [Carlo Emanuele I] and I moved from the town Parma to this municipality. Upon my arrival, he received me so humanely, and later I met with so much generosity as a reward for my services, that I began to desire vehemently that I could spend the rest of my life under his authority.<sup>1</sup>

As one reads, Benedetti and Emanuele Filiberto were so close that the patron even wanted his court mathematician to accompany him during his periods of residence in the countryside. On such occasions they often discussed scientific matters:

His benevolence toward me, as well as my respect toward him, consolidated through the time we spent together, and our familiarity [grew] to the point that the duke wanted me to accompany him when he resided in the countryside. [He] often [even invited me] to stay with him overnight. In that time he discussed mathematics with me. He used my work in order to learn those sciences, asking questions on arithmetic, geometry, optics, music and astronomy [*astrologia*].<sup>2</sup>

Emanuele Filiberto's passion for mathematics was well known in his day. The Venetian ambassador to Turin, Giovanni Correr, reported on this singular aspect of his personality in 1566:

That Duke is no man of letters but he loves the virtuosi. Hence, he has many of them by him; he likes to listen to their reasoning and he asks them questions. However, there is no subject that delights him more than mathematics, as

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<sup>1</sup>Benedetti 1585, f. A2r: "Agitur nonus decimus annus ex quo litteris Serenissimi patris tuae Celsitudinis, accersitus ex urbe Parmensi in hanc me civitatem contuli. Is advenientem tam humane excepit, tanta deinde liberalitate fuit complexus ego vicissim ei deserviendi, tam vehementi cupiditate fui accensus, ut sub eius ditione quod superesset vitae agere constituerem."

<sup>2</sup>Benedetti 1585, f. A2r: "Cuius in me banignitas, mea in illum observantia mirum in modum mutuo usu, et consuetudine est adaucta, ut idem Dux me secum dum rusticaretur esse vellet, saepe etiam secum pernoctare; quo quidem tempore de Mathematicis scientiis mecum agebat, in quibus perdiscendis mea opera utebatur, quaestiones, Arithmetica, Geometria, Optica, Musica, aut Astrologia spectantes proponens."

this science is not only apt but also necessary to the profession of military commander.<sup>3</sup>



Figure 2.1: Portrait of Emanuele Filiberto from Tonso, *De vita Emmanuelis Philiberti* (1596). (Biblioteca Nazionale Universitaria di Torino)

The duke's passion for science and his special relation with his court mathematician is further confirmed by the Venetian ambassador Giovanni Francesco Morosini, who mentioned Benedetti in a speech delivered to the Senators of Venice in 1570:

The duke of Savoy has a wonderful mind apt to every kind of science. However, he did not learn the sciences [*le lettere*] with the diligence that is necessary to become an expert, as his passion has always been the profession of

<sup>3</sup>Firpo 1983, 123: “Non è quel duca litterato, ma ama li virtuosi, et però ne tiene alquanti appresso di sé, sente piacere a udarli ragionare, egli stesso li fa de quesiti, ma nessun ragionamento più li diletta, che quello delle matematiche, come scientia, che non solo è conveniente ma ancora è necessaria alla professione del capitano.”

war [...]. But since mathematics is very useful and [even] necessary to professional warfare, His Excellency [Emanuele Filiberto] learns [mathematics] with much pleasure and knows more of it than the average man. He is aware that to receive substantial knowledge in any science one has to be in contact with it and learn it continuously; therefore a certain Mr. Giovanni Battista Benedetti of Venice imparts to him a lesson either on Euclid or on another writer of those sciences every day. In my opinion, as well as according to many other gentlemen, he is the most excellent scholar in this discipline in our times. The duke likes him very much. In fact, not only has [Benedetti] mastered this science, but he is also able to transmit it very well to others in his lessons.<sup>4</sup>

However, Benedetti's activities in Turin cannot be fully grasped if we limit our consideration to his relationship with the dukes. Rather, we should consider the wider political and cultural environment in which this relationship was established.

## 2.1 Turin's Economy and Politics between Italy and Europe

From the point of view of economic exchanges as well as of the European balance of power, Turin was located in a delicate and strategic position. It was in fact an obligatory station on the commercial road connecting Italy and France through the *Val di Susa*. For many centuries it had served as a transit point for merchants from Liguria, Lombardy, and Piedmont on their way to Lyon and the French and Flemish markets, and vice versa. Merchants were not the only visitors, as scholars from France, Flanders, and the British Islands began their *iter Italicum* from Turin. Turin was also the first station in Italy of Erasmus of Rotterdam, a key figure of the European Renaissance. On that occasion, on September 4, 1506, he received an "Italian" degree in theology from the University of Turin.

Its intermediate position between Italy and France made the town relevant not only from the point of view of economics and culture but also for military reasons. When Francis I of France and Charles V of Spain fought over Italian and European supremacy, Turin acquired fundamental strategic importance. The French army conquered it in 1536, together with most of Savoy and Piedmont, at the expense of Charles II of Savoy, brother in law to Charles V of Spain. The King of France made Turin the most important center in the region and a bulwark that was fundamental for consolidating his position on the Italian peninsula. Some of the political and administrative reforms promulgated by the new ruler were long-lasting. The most important of them were the creation of a parliament and of a *Camera dei Conti* for the supervision of public finances.<sup>5</sup>

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<sup>4</sup>Firpo 1983, 211: "Ha il signor duca di Savoia un bellissimo ingegno capace d'ogni scienza: ma non ha atteso alle lettere con quella diligenza, che si converria a chi ne volesse sapere, essendo la sua principal professione il mestiero della guerra [...]. E perché la scienza delle matematiche è molto utile e necessaria a chi vuole fare questa professione de l'arme, però se ne diletta assai Sua Eccellenza [Emanuele Filiberto] e di quella sa assai più che mediocrementemente. Con tutto questo sapendo che l'uomo tanto sa di ogni scienza quanto continua in vederla e studiarla, però usa di udire ogni giorno una lezione o d'Euclide o d'altro scrittore di quelle scienze da un messer Giovan Battista Benedetti veneto; uomo, per opinione non solamente mia, ma di molti valentuomini ancora, il maggiore che oggi faccia professione, e di grandissimo gusto del Signor Duca; perché oltre a possedere lui quella scienza eccellentissima sa anco così bene insegnarla ad altri che con molta facilità ne fa restar capacissimo chi lo ascolta."

<sup>5</sup>Merlin 1998, 16.

Emanuele Filiberto, known as “testa di ferro” for his energy and capacity in military affairs, retook Turin on the battlefields. He conducted the campaign against the French as a captain in the service of the Habsburgs. In 1553 he was the supreme commander of Charles V’s imperial army in Flanders and was nominated governor of the Netherlands by Philip II in 1556. His victory in the battle of Saint-Quentin led to the Peace of Cateau-Cambrésis (1559), according to which the Savoy and Piedmontese territories had to be restored. The French agreed to give them back to the dukes of Savoy with the significant exception of five fortified towns, occupied by their troops.

Turin was one of them. Therefore, it took some years before it was eventually returned to Emanuele Filiberto in 1562. In 1563 the duke entered the town and chose it as the new capital of his duchy instead of Chambéry. In this manner, he conferred an Italian identity to his duchy. This transfer set in motion political, social, and economic transformations, which were still in progress when Benedetti arrived in Turin in 1567. Moreover, the Piedmontese territories were politically fragmented. Apart from the centers under French control (Chieri, Pinerolo, Chivasso, and Villanova d’Asti), the region included the *Marchesati* of Monferrato and of Saluzzo. Moreover, the county of Tenda, connecting Piedmont with the Savoy possession of Nice, was an imperial fief. As for Geneva, a former possession of Savoy, it had become the “Jerusalem” of the Calvinists and would never be regained.

Within this difficult territorial and political constellation it was imperative that Emanuele Filiberto reestablish his authority after years of wars and foreign domination. In the European context, this meant striking a balance between the interests of Spain and France, who both wanted to annex the territories of the duchy either as a part of France or as a continuation of the Milanese territories. Piedmont was already split into a faction favorable to the French and one favorable to the Spaniards during the years of the war, and this division would also continue during the reigns of Emanuele Filiberto and Carlo Emanuele I.<sup>6</sup>

International diplomacy was comprised of marriage politics. Emanuele Filiberto received a French spouse, Margret of Valoys, daughter of Francis I of France and sister of King Henry II. This meant a strong political and cultural link to Paris. Margret was well known for her patronage of literati and artists, among them the poets of the Pléiade, Pierre de Ronsard, and Joachim Du Bellay. However, her son Carlo Emanuele I married a Habsburg, the daughter of Philip II of Spain, *infanta* Catherine Michelle, who arrived in Turin in 1585. This liaison was strongly encouraged by the pro-Spain party. Its leader was Andrea Provana of Leynì (1511–1592), with whom Benedetti was well acquainted. Four of the letters included in the epistolary of the *Diversae speculationes* are addressed to him. Benedetti judged the importance of his correspondence with this exponent of the Savoy aristocracy to be second only to those with Emanuele Filiberto (first epistle of his collection) and Carlo Emanuele I (second epistle). We can assume, taking his origins as a guide, that Benedetti supported Provana’s pro-Spain party.

In his relations to other Italian States the duke also followed a politics of balance. He was particularly keen on having good relations with Venice, which he visited in 1566 and 1574. On the latter occasion he was even endowed with the title of *patrizio* of the town. In turn, a Venetian embassy was established in Turin. The Savoy relation with Rome was also cordial. The papacy regarded Turin as a bulwark to stop the dissemination of reformed ideas in Italy, especially from the Swiss cantons. For his part, Emanuele Filiberto saw “heresy” as a danger to the unity of his state and his authority. Therefore, on

<sup>6</sup>Merlin 1998, 33 and Merlin and Stango 1998, 266–267.



Figure 2.2: Portrait of Carlo Emanuele I by Francesco Maria Ferrero di Labriano, *Augustae Regiaeque Sabaudae Domus Arbor Gentilitia* (Turin, 1702), p. 174. (Biblioteca Nazionale Universitaria di Torino)

matters of faith, the Roman interests and his own converged. Against the background of the confessional tensions of those years, his support for the Jesuits is comprehensible. Yet he was no fanatic of orthodoxy. He was influenced by the Imperial policy of mediation, as is shown by his ratification of a compromise with the Valdesans in 1561, in which he accorded to them religious freedom in their valleys.

The ties with Rome and Venice were reinforced through Savoy support for expeditions against the Turks. In 1565 Andrea Provana was sent with three galleys to Malta, as the court historian Pingone recounted in his history of Turin, *Augusta Taurinorum*:

When Malta was besieged by the Turks, in June 1565, duke Emanuele [Filiberto] sent Andrea Provana of Leini with four well-equipped triremes to bring supplies to the isle together with triremes from the Pope, Spain, and other [states]. First, Provana [*Leniacus*] arrived and assessed the difficulties. Then, he conveyed others [to the battle] and broke the siege with divine favor. The holy and vigorous order of the knights of Jerusalem was liberated under the superior command of the French Jean of Valetta. Public demonstrations of

immense joy and pious celebrations of thanks to God for the victory were displayed in Turin.<sup>7</sup>

In 1571 Provana was enlisted to defend Cyprus and contributed to the “holy” victory in the battle of Lepanto.

In 1571, when duke Emanuele [Filiberto] ruled over Turin and a confederation was established between Pope Pius V, the king of Spain and the Venetian Republic, he was asked to command the fleet with everybody’s agreement. But he had to renounce the offer owing to the present danger to his country engendered by local conflicts. [In his place] John of Austria, offspring of emperor Charles V, of great spirit and promising youth, was made commander. Chief Andrea Provana of Leyn  joined this expedition with three triremes. It was fought near Nauplia with the support of the Greeks. The Christians had hardly two hundred triremes and the Ottomans more than three hundred. The battle [*Mars*] was undecided for a long time but finally victory was given to the Christians, with the favor of God or even as a miracle. Provana, who fought bravely in the commanding trireme, was hit by a gun bullet and could hardly escape under the protection of a galley. One of the [Savoy] triremes, named Margara, was scattered and sunk into the depth; [another one], Pedemontana, was saved many times from the enemy. That victory was celebrated in Turin with thanks given to God and holy days set aside for the people.<sup>8</sup>

On these occasions Benedetti served as an advisor to Provana. Three of the four epistles of the *Diversae speculationes* addressed to him deal with mathematical issues related to navigation. As one reads, Benedetti undertook to give Provana suggestions concerning navigation and the employment of navigational instruments.<sup>9</sup> The first epistle is entitled *Per eundem parallelum absque correctione semper navigari non posse ubi notantur Petri Nonii lapsus in correctione erroris navis et alii Petri Medinae errores* (That one cannot always navigate along the same parallel without correction, where an error by Petrus Nonius concerning the correction of the deviation of a ship and other [errors] by Petrus Medina are considered). The second and the third letter deal with a navigation instrument invented by Benedetti based upon the design of Gerardus Mercator (Figure 2.3). They are a description of the instrument accompanied by technical drawings and an explanation of its use. They

<sup>7</sup>Pingone 1577, 85: “Anno Christi 1565 mense Iunio, Dux Emanuel, obsessa a Turcis Melita, Andream Provanam Leniacum cum triremibus quatuor instructissimis mittit, qui una cum Pontificiis, Hispanis et aliis triremibus suppetias insulae afferret. Prior Leniacus applicuit, difficultates exploravit, alios postea advexit, et soluta tandem faventibus superis obsidione, Hierosolymitanorum militum sacer, et strenuus ordo liberatus, Ioanne Valleto Gallo summum magisterium gubernante. Quam ob victoriam Taurini immensae laetitiae publica significatio reddita, et devotae superis gratiarum actiones.”

<sup>8</sup>Pingone 1577, 88: “Anno Christi 1571 Emanuel Dux Taurini agens, confoederatione inita in Turcam Cypri vastatorem, inter Pium quintum Pontificem, Hispaniarum Regem, et Venetam Rempublicam, qui classi praeeset ab omnibus exposcitur: sed ob imminentiam a vicinis discordiis patriae discriminis, excusatus habetur. Ioannes vero Austriacus Caroli quinti Caesaris soboles, magni animi, et expectationis iuvenis praeficitur. At Dux Andream Provanam Leniacum tribus cum triremibus in eam expeditionem adiungit. Apud Naupactum Achaicum concursum, et decertatum. Christianorum vix ducentum triremes: Turcarum vero plussquam trecentum: Mars diu anceps, tandem Deo maximo favente, et quodam potius miraculo ad Christianos inclinavit victoria. Leniacus ex triremi Praetoriam fortiter dimicans sclopeto ictus in capite vix galeae praesidio evasit: triremium una Margaritis nomine dissipata, mersaque penitus, Pedemontana semel atque iterum ab hostibus recepta. Ob eam victoriam, Taurini supplicationes superis, feriae mortalibus indictae.” See also Tonso 1596, 142, 161 and 177–179.

<sup>9</sup>Benedetti 1585, 214–216.



are entitled *De armilla nautica* (On the armillary nautical sphere) and *De usu armillae nauticae* (The utilization of the armillary nautical sphere), respectively.<sup>10</sup> As one reads, the letters follow private discussions with Provana on the difficulties linked to navigation using nautical maps.

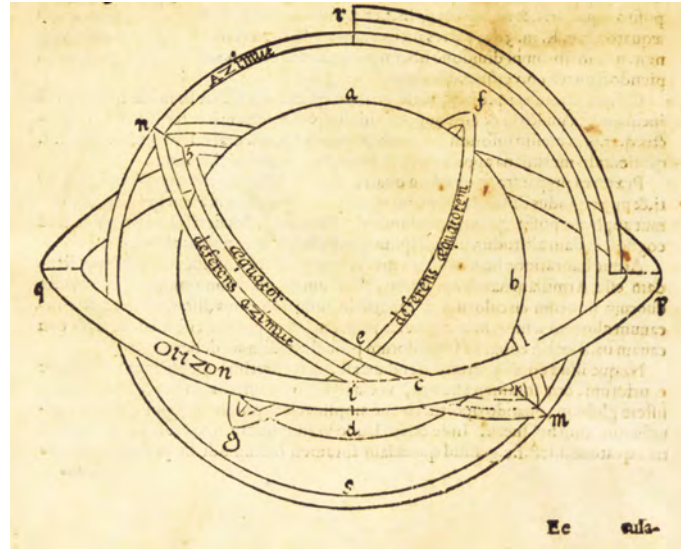


Figure 2.3: An armillary nautical sphere invented by Benedetti for Andrea Provana for navigation purposes, presumably in the Savoy military expeditions against the Turks. (Max Planck Institute for the History of Science, Library)

## 2.2 Civil Reforms and Military Policy

Emanuele Filiberto and his son were very different rulers. While the court of the former has been depicted as “*funzionale, ristretta e popolata di homines novi*” (functional, small and composed of *homines novi*) the latter’s court was “*fastosa, aristocratica, centro propulsore di una politica culturale oramai intensamente barocca*” (pompous, aristocratic, irradiating center of a deeply baroque cultural politics).<sup>11</sup> Their common efforts were directed towards the consolidation of their state. Emanuele Filiberto implemented profound administrative, financial, and military reforms. He issued tax reforms and imposed the use of the vulgar tongue in official documents. As to his military policy, Emanuele Filiberto introduced the obligatory conscription of all men aged between 18 and 50 years. Thanks to this reform, which followed the Swiss example and Machiavelli’s theory, Emanuele Filiberto trained his subjects to defend their territories in case of invasion, disposed of many thousands of soldiers, and limited the use of mercenary troops.<sup>12</sup> Part of his defensive strategy was the erection of new fortifications in Nice, Bourg-en-Bresse, Saint-Julien, and Montmélian (see Figure 2.4). The construction of the *cittadella* of Turin was particularly important and rapid. It was achieved in only two years, between 1564 and 1566, and was celebrated by the official town historian Emanuele Filiberto Pingone in the following terms:

<sup>10</sup>Benedetti 1585, 217–219 and 219–220.

<sup>11</sup>Ricuperati 1998, XXII.

<sup>12</sup>See Stumpo 1993, 561.

In that year [1564], the duke began building a fortification, which is commonly called the citadel, in the most sacred part of the town on the ruins of the temple of the divine Savior.<sup>13</sup>

On March 15, 1566, the citadel of Turin was finished after a few months of work. It was admirable with its five bulwarks, serving all military purposes and built according to the art of architecture. He [the duke] let it be blessed with religious and pious blessings (Archbishop Della Rovere was in charge of the rite). Soon he organized the defenses, entrusting them to Giuseppe Caresana of Vercelli, a subject of his [*benemeritus*] and a man very expert in the military art.<sup>14</sup>

Francesco Horologi was responsible for the construction of the citadel, designed by engineer Francesco Paciotto, whom Emanuele Filiberto recruited in Flanders. Its pentagonal structure, responding to recent developments in warfare, was the model for later fortifications, such as the citadels created by the same Paciotto in Antwerp (1567) and Parma (1591).

As often occurred during the Renaissance, the military-political function of the citadel had two sides. On the one hand, it served to defend the town from possible assaults from outside. On the other, it affirmed the supremacy of the dukes over the new capital and had the function of dissuading the subjects from claiming too much autonomy.<sup>15</sup> As Martha Pollak remarked, “Paciotto proposed a five-sided fortress, with three bastions oriented towards the countryside, defending the approach to the city from the west, and two bastions facing the city, ready to bring it under control in case of riotous uprisings against the duke.”<sup>16</sup> The new urban arrangement transformed Turin along with its political balance of power. During these changes the relations between the dukes and the local *patriziato* were often strained. In fact, all decisions had to be negotiated between civic administrators, state functionaries, and the court. Whereas the town council was eager to keep its medieval privileges, the dukes made the opposite effort of centralizing power in order to grasp control firmly in their hands.

A thorough knowledge of the surrounding territory through cartography, alongside fortification and military reforms, was also seen as an important element of defense. The intensity of mapping efforts in the Savoy in the second half of the sixteenth century is a noteworthy example. A large number of maps of great quality were made, for instance Forlani’s *Savoia* (1552), Boileau de Boullion’s work on the road from Lyon to Turin (1556), Nicolais’s maps *Boulonnais* (1558), *Berry* (1566) and *Bourbonnais* (1569)<sup>17</sup> and, above all, Giacomo Gastaldi’s *Pedemontanae vicinorumque regionum... descriptio* (1574). Many of the maps made in those years are still preserved in Turin, in the *Biblioteca Reale*, the *Biblioteca Nazionale Universitaria*, the *Archivio di Stato*, and in the wide collection of the *Archivio Storico della Città*.<sup>18</sup> Benedetti shared this interest in

<sup>13</sup>Pingone 1577, 85: “Eo anno [1564] Dux in aeditiore parte civitatis, in ipsis templi Divi Solutoris ruinis Acropolis aedificare coepit, Cittadellam vulgo dicunt.”

<sup>14</sup>Pingone 1577, 86: “Anno Christi 1566 idibus Martiis, absoluta paucis mensibus Taurinensi acropoli, quinis propugnaculis admiranda, servata omni rei militaris, et architectonicae artis ratione eam religiosa ac pia benedictione communiri curat, Archiepiscopo Rovereo sacris praeunte: mox praesidiis firmat, eique praeficit Iosephum Caresanam Vercellensem de se benemeritum, ac rei militaris peritissimum.”

<sup>15</sup>Merlin and Stango 1998, 118–119.

<sup>16</sup>Pollak 1991b, 16.

<sup>17</sup>See Broc and Greppi 1989, 113.

<sup>18</sup>The Archivio di Stato preserves Carracha’s maps of Turin: *Augusta Taurinorum* (1577) and *Turino* (ca. 1580)—see Archivio Storico della Città di Torino 1982.



Figure 2.4: Fortification projects in a drawing by Benedetti's follower as court mathematician, Bartolomeo Cristini. (Biblioteca Nazionale Universitaria di Torino)

geography and topography. This especially emerges from some of his epistles, for instance those to the architect Gabriele Busca on topography and measuring instruments, to the imperial land surveyor (*agrimensor*) Anselm Rosenburg (presumably of the Bohemian aristocrat family Rožmberk) on measuring techniques, and to the Turin physician and natural philosopher Giovanni Battista Femello concerning cartographic errors about the position of islands, in particular Iceland.<sup>19</sup>

### 2.3 Engineering and Architecture

Countless engineers worked in Turin under Emanuele Filiberto and Carlo Emanuele I to implement the defenses and the hydraulic system, among them Francesco Paciotto, Ferrante Vitelli, Ascanio Vitozzi, and Vitozzo Vitozzi.

The leading Italian architect of that time, Andrea Palladio, visited Turin between May and June 1568. He might have been the architect behind the park of Viboccone (later known as *Regio Parco*) between the rivers Dora, Po, and Stura. Benedetti is said to have constructed his fountain there. It is also likely that Palladio gave Emanuele Filiberto advice on the organization of his popular militia according to the classical Roman model.<sup>20</sup> Later, he dedicated to the duke of Savoy the third of his four books on architecture, *Quattro libri dell'architettura* (Venice, 1570). This section deals with public constructions, streets, bridges, squares, basilicas, and gymnasia. In the letter to the reader, the author stressed the prestige deriving from public buildings, and the fruitful collaboration between Renaissance princes and architects: “[Public buildings] are bigger and more embellished with rare ornaments than the private ones; and they aim to be used by everybody. Therefore, through them, princes can display the greatness of their generosity to the world and

<sup>19</sup>Benedetti 1585, 271–274, 405–408 and 267.

<sup>20</sup>Tessari 1993.

architects have the occasion to show how capable they are through beautiful and wonderful inventions.”<sup>21</sup> The dedication to Emanuele Filiberto by Palladio was motivated by his “heroic spirit”<sup>22</sup> as well as by his interest in and deep understanding of architecture:

As your Highness is familiar with the most noble arts and sciences related to these issues [concerning architecture], you will have much pleasure and relief by considering the subtle and beautiful inventions of humankind as well as the true science of this art, which you understand very well and which has been brought to the most rare and almost absolute perfection. This is witnessed by the illustrious and royal buildings that have been constructed in many parts of your large and most happy state.<sup>23</sup>

Urban and military developments were accompanied by a flourishing literature on war and defense theory. Emanuele Filiberto was a great supporter and collector of such writings:<sup>24</sup> among other examples, Benedetti’s correspondent Busca authored the treatise *Della espugnazione et difesa delle fortezze* (On the conquest and defense of fortresses, Turin, 1585), which followed the *Istruzione de’ bombardieri* (Education of the bombardiers, Carmagnola, 1584). He would later publish the tract *Architettura militare* (1601) in Milan. Another acquaintance of Benedetti’s, Giacomo Soldati composed *Discorso intorno al fortificare la città di Torino* (Discourse on fortifying the town of Turin).<sup>25</sup>

In this context of military reforms and architectural changes aimed at transforming Turin into the capital of an absolutist state, the skilled engineers implementing the dukes’ vision gained high social recognition. An example is the career of the mathematician Cristini. In 1569 he was courtly librarian and later “calcolatore” or “controllore delle fabbriche,” that is, supervisor of architectural projects. In this capacity, he became closely connected to the celebrated military and civil architect Ascanio Vitozzi. On December 13, 1582, he became “re d’arme dell’ordine dell’Annunziata,” a honorific and administrative title that implied responsibility for the organization of courtly tournaments, feasts, and balls.<sup>26</sup> He entered the court as Benedetti’s successor in the position of ducal mathematician. Cristini’s career shows the enhanced status of Renaissance scientist-engineers bringing together mathematical and natural expertise, technical skills, administrative duties, and courtly honors. Benedetti addressed one of the letters of the *Diversae speculationes* to him (the one dealing with geometrical problems encountered in Ptolemy’s *Geography*) calling him “Bartholomeo Christino Serenissimi Sabaudiae Ducis apparitor.” The term *apparitor* can mean either “servant” or “functionary.” An unknown hand corrected this title in a copy preserved in the Royal Library of Turin, substituting it for the more

<sup>21</sup>Palladio 1570, III, 5: “Ne’ quali [edificii publichi], perché di maggior grandezza si fanno, e con più rari ornamenti, che i privati, e servono a uso, e commodo di ciascun; hanno i Principi molto ampio campo di far conoscere al mondo la grandezza dell’animo loro; e gli Architetti bellissima occasione di dimostrar quanto essi vagliano nelle belle, et meravigliose invenzioni.”

<sup>22</sup>Palladio 1570, III, 3: “Principe, il qual solo a tempi nostri con la Prudenza, e co’l valore s’assimiglia a quelli antichi Romani Heroi, le virtuosissime operationi de’ quali si leggono con maraviglia nell’historie, et parte si veggono nell’antiche ruine.”

<sup>23</sup>Palladio 1570, III, 3: “Delle qual cose [concernenti l’architettura] essendo l’A[ltezza] V[ostra] dotata delle più nobili arti, e scientie; piglierà non poca contentezza, e consolazione considerando le sottili, e belle invenzioni degli huomini, e la vera scienza di quest’arte, da lei molto bene intesa, e ridotta a rara, e perfetta perfezione; come dimostrano gli illustri, e reali edifici fatti fare, e che tuttavia si fanno in diversi luoghi dell’amplissimo, e felicissimo suo stato.”

<sup>24</sup>Pollak 1991a, 18–26.

<sup>25</sup>See Viglino Davico 2005, Pollak 1991a and Signorelli 1969–1970.

<sup>26</sup>Vernazza 1783, 8 and 11.

emphatic “P[rim]o Feciali,” that is to say, “First Herald.”<sup>27</sup> This was in fact the most appropriate title for the “Roy d’armes.”<sup>28</sup> It is possible that this correction was inserted by Benedetti himself.

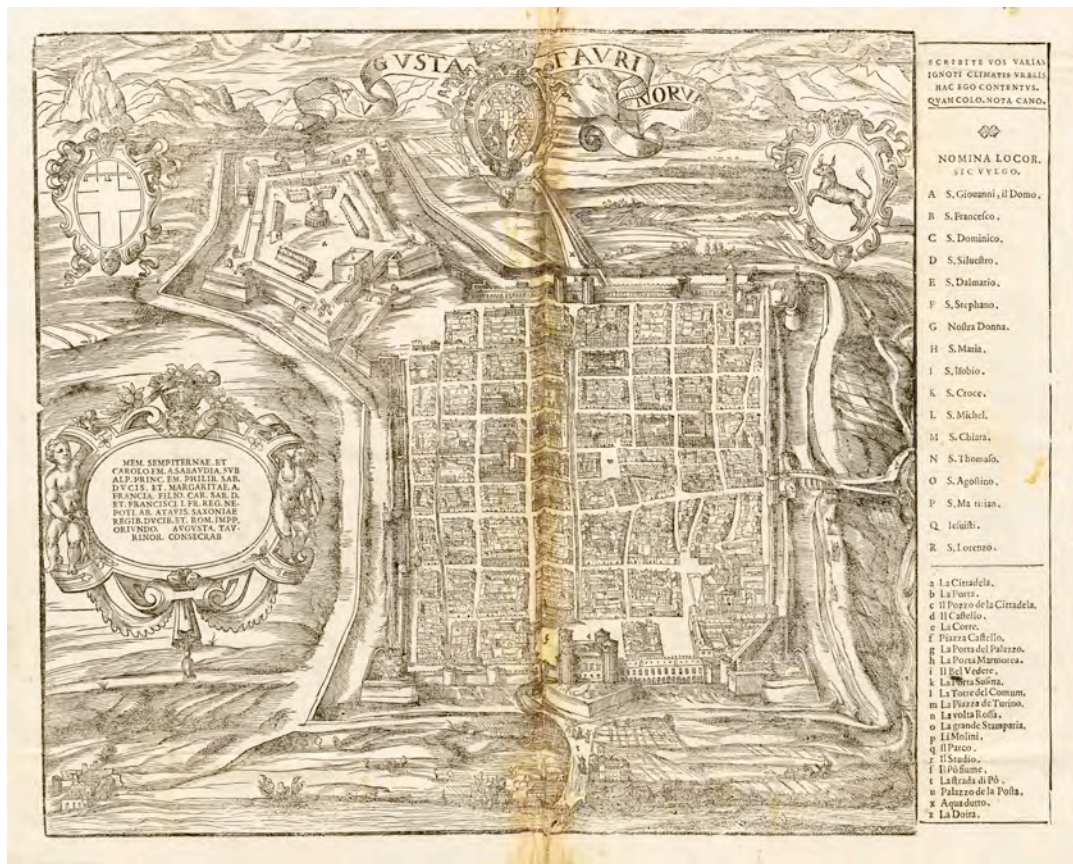


Figure 2.5: Map of Turin in Benedetti’s times, from Pingone’s *Augusta Taurinorum* (1577). (Staatsbibliothek zu Berlin)

Benedetti interacted with architects and engineers, as can be seen in his correspondence. Four of the scientific letters included in the *Diversae speculationes* are addressed to the architect Busca. Their topics, however, are not strictly related to building or engineering. In fact, their topics range from meteorology to instruments, topography, and natural philosophy.<sup>29</sup> Benedetti was closely related to the ducal “architect and cosmographer” (*Architetto e Cosmografo*) Soldati, who had worked as a hydraulic engineer and architect in Milan and in Lombardy, and joined the Turin court in 1576. Benedetti held him in great esteem, judging by the dedication to him in one of the most technical parts of the *Diversae speculationes*, that is, the entire second book, which is entitled “Explanation on Operations of Perspective” (*De rationibus operationum perspectivae*).<sup>30</sup> Besides, Benedetti’s interest in engineering and measuring instruments emerges from many writings, especially from his work on gnomonics (1574) and from a manuscript analysing a measuring instrument that

<sup>27</sup>The letter is to be found in Benedetti 1585, 330–331. The collocation of the volume in the *Biblioteca Reale di Torino* is G 43 8.

<sup>28</sup>Vernazza 1783, 37, n. 31.

<sup>29</sup>Benedetti 1585, 271–277.

<sup>30</sup>Benedetti 1585, 119–140. See Mamino 1989.

is preserved in the *Biblioteca Civica di Carignano*, entitled *Descrittione, uso, et ragioni del Trigonolometro* (1578).<sup>31</sup>

#### 2.4 Intellectual Ferment: Arts, Literature, and Philosophy

Renaissance Turin was as appealing to mathematicians as to all other types of intellectuals, including artists, literati, and scholars in general. One could mention the names of two reputed theorists of art who were closely linked with the court: Federico Zuccari and Lomazzo. The former wrote *L'idea de' pittori, scultori, et architetti* (The idea of the painters, sculptors and architects, 1607) and the latter *Trattato dell'arte della pittura* (Treatise on the art of painting, 1584), dedicated to Carlo Emanuele I. Lomazzo also dedicated to the duke of Savoy his collection of poems, *Rime* (1587), including one for Benedetti. Among the artists appointed by the duke, the Flemish Jan Kraeck or "Carracha," who resided in Chambéry, not only painted; he also worked as a cartographer and made a large map of Turin in 1572 (385x397 cm) as part of a wider cartographic program.

Illustrious scholars came to Emanuele Filiberto's court or to the reopened university, first in Mondovì and later in Turin. Among them, the man of letters from Ferrara, Giovanni Battista Giraldi Cinzio, was appointed to teach humanities. His collection of *novelle*, the *Ecatommiti* (Mondovì, 1565), ended with a long celebratory poem mentioning the most visible intellectuals of the Peninsula. Many of them were linked to the duke of Savoy, either as professors or as courtiers.<sup>32</sup> For instance, Francesco Ottonaio of Florence, who taught mathematics at the university in Benedetti's years and exchanged views with him, is extolled for his expertise in astronomy, meteorology, and astrology:

My Ottonaio moves his feet towards you along with the others.  
He received the gift of scrutinizing the heavens,  
of knowing the reasons for warmth and coldness,  
why the days are short or long,  
and what layer veils the Sun making it dark,  
the manner in which the year becomes adorned of beautiful flowers again,  
what nativity is a sign of honor and merit  
or of shame and disgrace,  
and what is the star presiding over  
a man's state from his birth  
until his vital light is extinguished  
one circle after the other.<sup>33</sup>

<sup>31</sup>Mamino 1989, 432–433 and Roero 1997.

<sup>32</sup>Villari 1988, 93–95 and 107–110. See Doglio 1998, 599ff.

<sup>33</sup>Villari 1988, 93–94:

“Move insieme con lor verso te il piede  
il mio Ottonaio, a cui scorrere il cielo, per grazia, diede.  
Del caldo la cagion saper, del gelo,  
e perché breve sia, sia lungo il giorno,  
e quale offoschi il sole oscuro il velo;  
come ritorni di bei fiori adorno  
l'anno e chi debba aver dal nascimento  
onore e pregio, e qual ingiuria e scorno;  
e da che stella prender de' argomento  
de lo stato suo l'uom, poi ch'egli è nato,  
insin che il suo vital lume sia spento

Giraldi Cinzio did not mention Benedetti as he had not yet come to Savoy, but he included the Turin physician and professor of medicine Antonio Berga, with whom Benedetti would later enter a controversy over the proportion of water and earth in the terraqueous globe:

With his gentle and beautiful works  
he tries to subtract his name from the oblivion,  
defeating the stealing forces of greedy time.  
I refer to my gentle Antonio Berga,  
who shows the way to those who wish to learn  
by writing his papers for the common good.<sup>34</sup>

Two famous authors who visited Turin between 1576 and 1579 are the poet Torquato Tasso and the philosopher Giordano Bruno. Both arrived in the town as fugitives and both enjoyed their stay. Tasso interacted with the cultivated elites. The Turin philosopher and physician Agostino Bucci appears as *persona dialogans* in three dialogues of his (*Il Forno ovvero della nobiltà, Della dignità, and Della precedenza*). His connection with the court is further confirmed by the dedication of the 1581 edition of the *Gerusalemme liberata* to the prince of Savoy.<sup>35</sup>

As for Bruno, he did not establish lasting contacts in town. He visited Turin in 1576 (or at the beginning of 1577) for the first time after abandoning the Dominican cloister of Naples, where he was accused of heresy. However, as he stated later in his Inquisition trial, “non trovando trattenimento a mia satisfattione, venni a Venezia per il Po [as I did not find sufficient means, I came to Venice along the Po].”<sup>36</sup> He visited Turin again in 1578 and went to Chambéry, where he spent the winter of that year as a guest of the Dominicans. On that occasion he possibly carried a booklet, now lost, entitled *De’ segni de’ tempi*, that he had printed in Venice and that probably dealt with the comet of that year.<sup>37</sup> We will refer to his possible involvement in some polemics on the comet of 1577–1578 later. It is hard to say whether Bruno and Benedetti ever met or were informed of each other’s views. In spite of the fact that they belonged to very distant milieus, there is some affinity between their outlooks. Both shared an aversion to Aristotle, the project of reforming natural philosophy, the support for the Copernican system, and other cosmological views.

The case of the philosophical poet Pandolfo Sfondrati also deserves our attention. He was active in Turin as a poet at the same time as Benedetti and authored poems that were often inserted in the first pages of books printed by the Bevilacqua printing house. Pandolfo made himself visible in Turin with celebratory poems that were included in important local publications, in particular in the works of the court historian Emanuele Filiberto Pingone: *Augusta Taurinorum* (1577), *Inclytæ Saxoniae Sabaudiaequæ principum arbor gentilitia*

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di cerchi in cerchio.”

<sup>34</sup>Villari 1988, 95:

“E quel che, con gentil opre, e leggiadre,  
tenta che il nome suo da l’oblio s’erga,  
vinte del tempo avar le forze ladre,  
i’ dico il mio gentile Antonio Berga,  
che addita, a chi imparar cerca, la strada,  
mentre, ad util comun, le carte verga.”

<sup>35</sup>Doglio 1998, 621 and 625.

<sup>36</sup>Firpo 1993, 159. See Ricci 2000.

<sup>37</sup>Ernst 1992.

(1581) and *Sindon evangelica* (1581).<sup>38</sup> Hence, Sfondrati frequented the same courtly and cultural environment as Benedetti. It is likely that they discussed natural issues together, especially when considering that Sfondrati composed philosophically minded poems such as the Copernican *Inferiora regi dum syderis omnia motu*, which opens the *Animadversiones in Ephemeridas* by Benedetto Altavilla (Turin, 1580). There is also evidence that Benedetti was familiar with the Sfondratis, in particular with Paolo Sfondrati, who was a senator of Milan and an ambassador of Filippo II in Turin.<sup>39</sup> Moreover, Pandolfo Sfondrati authored the atomistic poem *Democriti prohibent nosci corpuscula formas* and a treatise on the tides, which he explained in mechanical terms as the result of the interaction of water particles warmed up by solar rays.<sup>40</sup>

## 2.5 Religious Policy

### 2.5.1 Pragmatic Counter-Reformation

The relics were moved from the old to the new capital: Christ's shroud traversed the Alps together with the court. Religion was an essential stabilizing factor. According to the report of the political thinker Giovanni Botero, Emanuele Filiberto declared that piety was essential to guarantee his authority in the state: "Those people who are zealous in their devotion [to religion] are more moderate: in consequence, they obey their Prince better than licentious people."<sup>41</sup> His religious politics were characterized by pragmatism. He undertook measures against the dissemination of the Reformation in his country and repressed the reformed communities only insofar as they jeopardized the integrity of the state or its relations with Rome or with other Catholic countries. The persecution of the Valdesans in the Alpine valleys, between 1559 and 1560, was part of a wider endeavor aimed at establishing a Catholic league that could favor his conquest of Geneva. However, when the prospect of an anti-Protestant confederation vanished, Emanuele Filiberto interrupted the aggression. The resistance of the Valdesans had been strong and persistent. It was a destabilizing factor for the duchy. A compromise was reached on June 5, 1561, when Emanuele Filiberto promulgated an edict, known as the Edict of Cavour, conceding to the Valdesans freedom of worship in their mountains. In exchange, he imposed the construction of new fortifications and strict military control of the Alpine passes.<sup>42</sup>

The other pole of Savoy religious politics was external. It concerned the regulation of jurisdictional issues with the Roman Church. A reason for friction was the taxation of the ecclesiastics, which Rome was reluctant to grant. The economic stake was high, since the estates belonging to the Church amounted to a third of the land; in some areas, it even reached values comprised between 40 and 70 percent. The ancient privileges of the dukes to select the candidates for the principal ecclesiastical positions had to be negotiated anew. It was only in 1573, under Gregory XIII, that these were confirmed. Finally, the introduction and the reinforcement of the Inquisition in the duchy fostered confessional uniformity but interfered with secular jurisdiction.<sup>43</sup>

<sup>38</sup>See Griseri 1998.

<sup>39</sup>In a letter to Benedetti, Francesco Patrizi asked him to give his regards to Baron Sfondrati. See Patrizi 1975, 42–43.

<sup>40</sup>See Omodeo 2008b and Omodeo 2012a.

<sup>41</sup>"La gente infervorata di devotione è molto più regolata: e per conseguenza più ubidiente al Suo Prencipe, che la dissoluta." Botero 1608, 241.

<sup>42</sup>De Simone 1958.

<sup>43</sup>See Merlin 1995, 238–267, chap. IX, "Tra Controriforma e Ragion di Stato."



The new Inquisition, established in the wake of the Council of Trent in order to counter the Reformation and reaffirm Catholic hegemony in Italy, had a centralized structure with ramifications for the entire Peninsula. The Holy Office lay at its heart. The various Italian states accepted it as a preventative measure against public disorder, especially against confessional conflicts such as the civil wars affecting France and other European areas. The Inquisition was a repressive control system binding Rome and the local powers. It was a compromise in which, in Adriano Prosperi's words, "l'aiuto era offerto e richiesto in nome della conservazione del potere, quello politico dei principi e quello della corporazione ecclesiastica" (The aid was offered and requested in order to preserve the power—the political one of the princes as well as that of the ecclesiastical corporation.)<sup>44</sup> Nonetheless, the jurisdiction problem remained acute: what was the legitimacy of a foreign tribunal—the Roman one—trying and condemning the subjects of other countries? In Piedmont, the interests of Turin and Rome were guaranteed through the mediation of the Papal nuncio, who was obliged to inform both the duke and the Holy Office in Rome about Inquisition trials.<sup>45</sup> For their part, these authorities could intervene in the trials and had the right to give their consent concerning the opportunity to carry them out. However, the opposition to the establishment of the Inquisition was strong, particularly in the French-speaking areas. Relying on its Gallican tradition, the local church in Savoy did not accept a form of direct control from Rome.<sup>46</sup>

The relations between Emanuele Filiberto and the Papacy were not always calm, and became strained after the Cavour edict of tolerance in 1561. Its promulgation provoked the protests of Rome and the commission of the intransigent Cardinal Inquisitor, Michele Ghisleri, to the diocese of Alexandria as Bishop of Mondovì, with the aim of reinforcing religious control. The relations between this champion of orthodoxy and the pragmatic politics of the Savoy dukes were tense, including after Ghisleri was elected pontiff as Pius V in 1566.

### 2.5.2 Jesuit Colleges in Piedmont

In 1561 Emanuele Filiberto supported the foundation of the first Jesuit college in his territories, in Mondovì, where the university was located at that time. A second college was established in Chambéry (1564), which was the former capital of the duchy. Among the Jesuit teachers, Antonio Possevino is one of the most renowned. The future compiler of the *Bibliotheca selecta* (1593) sojourned in Piedmont between 1560 and 1562.<sup>47</sup> From 1564 to 1567 the future cardinal and inquisitor Robert Bellarmine was also there but could not be appointed professor of philosophy in the Jesuit college of Turin, opened in 1567, due to the intransigent opposition of the university against the attempts to transfer this chair from the university to the Jesuit institution.<sup>48</sup> Both in Mondovì and in Turin the Jesuits took over the so-called "public schools," which essentially taught the rudiments of grammar.<sup>49</sup>

In those years, the chair of letters belonged to the Ferrara humanist Giovanni Battista Giraldi Cinzio, whose religious tendencies could be called Erasmian.<sup>50</sup> At the moment of the establishment of the Jesuit college in Turin, he sided with the humanistic legacy

<sup>44</sup>Prosperi 1996, 57–58.

<sup>45</sup>See Prosperi 1996, III, "Inquisizione romana e stati italiani" and Black 2013, 30.

<sup>46</sup>Prosperi 1996, 103–105.

<sup>47</sup>Longo 1998, 475 and Scaduto 1959, 52.

<sup>48</sup>Grendler 2002, 42.

<sup>49</sup>Vallauri 1846, 19.

<sup>50</sup>For instance, Giraldi Cinzio defended the famous commentator on Aristotle's *Poetics*, Ludovico Castelvetro, who was excommunicated in 1560 as "eretico fuggitivo e impenitente" for his alleged bias towards Melancthon. On this occasion Giraldi Cinzio argued that violence and coercion could only produce the

against their pedagogy. He paid the price of their expansion, as his chair was suppressed and transferred to the Jesuit college.<sup>51</sup> From 1567 to 1574, the Jesuits received 200 *scudi* per year to teach Greek and Latin grammar, humanities, and rhetoric to the youth (half of Giraldi's salary). Thus, the humanist had to abandon Turin for Pavia in 1569. In a letter to the Florentine philologist Pietro Vettori (Pavia, March 20, 1569), he lamented his expulsion, although he expressed his gratitude to the dukes for the donation of 500 *scudi* for his leave.<sup>52</sup> He particularly protested that his teaching as a learned humanist was being substituted by the teaching of elementary grammar.

The opening of the Turin college set off enduring hostilities between the Jesuits, on the one side, and the university and the municipality on the other. According to Grendler's reconstruction, between 1570 and 1572 the duke and the rector, Achille Gagliardi, made an agreement according to which nine chairs would be given to Jesuit professors. However, the project did not succeed owing to obstruction on the part of the town and the university.<sup>53</sup> In these struggles, the Archbishop Gerolamo della Rovere was among the most strenuous opponents of the Jesuits. The position of Emanuele Filiberto fluctuated. Initially, he supported the "reverend fathers" but later distanced himself from their educational projects. In 1575 he even reintroduced the teaching of humanities in the university. There is indirect evidence that Benedetti sided with the humanists in this anti-Jesuit controversy. In 1583 Francesco Patrizi, who belonged to the same Ferrara cultural environment as Giraldi Cinzio, asked him to support the candidacy of his friend Giovanni Giacomo Orgiazzo for the position of professor of humanities in 1583.<sup>54</sup>

Apart from the political interests at stake (the privileges of the town and of the university), the professors' resistance concerned the contents of the teaching, as one reads in a document from 1593, "Raggioni perché non sia bene che gli Rev[erendi] Padri Gesuiti leggano la filosofia tutta, et la logica nel loro Comento, et si lasci a leggerli nello Studio et pubbliche scuole, come sempre insino a qui si è fatto" (Reasons why it is not good that the Jesuit Reverend Fathers teach all philosophy and logic in their commentary and are allowed to teach at the university and in public schools, as has been the case until now).<sup>55</sup> According to the academics, philosophy should be imparted to students as the fundamental tenet of the study of medicine. Therefore, the focus should be set on Aristotle's natural philosophy and not on metaphysics and logic, as was the case with the Jesuits. Metaphysics, as one reads in the document, is the "last" and not the "first" part of philosophy. By contrast, the Jesuits began their teaching with the most abstract issues, e.g., the statute of ideas and universals, and divine ineffability ("*utrum Deus sit in praedicamento*" or "*utrum Deus sit infinitus*").

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opposite effects than those wished for by the defenders of orthodoxy. See Cinzio 1996, Letter n. 101, 371, n. 3.

<sup>51</sup>Vallauri 1846, 19 and Grendler 2002, 42–43.

<sup>52</sup>Cinzio 1996, Letter n. 127, 425: "Sed Taurino iam menses quatuor absum, Ticinique publice profiteor. Nam, praeter iacturam valetudinis, quam ibi quotidie faciebam, me ad abeundum urgentem, natio illa haec nostra studia nihil quidem facit. Hinc Princeps ille, qui oratoriam ac poeticam facultatem profiteretur, in Academia sua habere constituit neminem, quod satis esse censuerit Iesuitas nescio quos, suo in collegio, hoc muneris cum puerilis ac infantibus obire; qui, cum Deuspaterio quodam, barbaro plane auctore, mollia ingenia, obscurissima, ne dicam foedissima, imbuunt barbarie. Me tamen abeuntem, praeter annuam quadrigentorum aureorum nummum stipe, quam liberaliter exsolvit, centum etiam scutatis aureis donavit."

<sup>53</sup>Grendler 2002, 42–44.

<sup>54</sup>Patrizi to Benedetti (Ferrara, 21 March 1583), Patrizi 1975, 39.

<sup>55</sup>Archivio di Stato di Torino, Istruzione Pubblica/ Regia Università di Torino/ Mazzo 1 (1267–1701), Fascicolo 7/2. The document is included as an appendix to Omodeo 2014d.

### 2.5.3 Benedetti and the Counter-Reformation

What can be said about Benedetti's attitude toward the culture of the Counter-Reformation emerging after the Council of Trent? We can assume that his scorn for Aristotelian philosophy was not only a dispute with the university professors of his day, but also with the theology-oriented Aristotelianism propagated through the cultural apparatus of the post-Tridentine Church. We have no evidence that Benedetti frequented exponents of the Jesuit order, in spite of their presence in Turin. Rather, we have evidence of his connections with scholars who were not in the mainstream of the official Catholic culture of those years.

Among others, he corresponded with Francesco Patrizi of Cherso, whose Platonism was regarded with suspicion and even censured in Rome. The closeness between them is witnessed by the fact that Benedetti acted as a mediator between the philosopher and the duke of Savoy on at least one occasion. When Patrizi issued his *Della nuova geometria libri XV* (Fifteen books on the new geometry, Ferrara, 1587) with a dedication to Carlo Emanuele I, Benedetti passed on a copy to his patron. Patrizi expressed his gratitude with the following words (Ferrara, April 6, 1587):

Very Magnificent and Excellent Signore,

I rejoice with your Lordship that you recovered from sickness quicker than believed. And I am very thankful to you for presenting my book to the very serene Prince and promising to inform me about his remarks after he has read it. If by chance the book will be forgotten, due to his many duties [negozi], I hope at least that you will remember me. If his High Serenity will give some sign that he appreciated it [my book], I will be very glad and I will be particularly grateful to your Lordship for your benevolence.<sup>56</sup>

In exchange, Benedetti sent him a copy of his discussion on the relative sizes of the elements of earth and water, as witnessed by a letter from Patrizi (Ferrara, 18 January 1588).<sup>57</sup>

The two scholars shared views on cosmology that were to be censured by the Inquisition in the 1590s. It is thus expedient to briefly recall Patrizi's natural and cosmological views, as they are close to those Benedetti expressed in his writings, especially in the *Diversae speculationes*. Already in his *De rerum natura libri I priores. Alter de spacio physico, alter de spacio mathematico* (First Two Books on Nature, One on Physical Space and One on Mathematical Space, 1587), Patrizi embraced the neo-Stoic doctrine of the fluidity of the heavens, the infinity of space beyond the sphere of the stars, and the free motion of planets through cosmic space following an inner drive. He later expanded on that in *Pancosmia*, which is the cosmological section of his philosophical masterwork, *Nova de universis philosophia* (New Universal Philosophy, 1591). In it, he did not limit himself to asserting the infinitude of luminous space beyond the visible stars, to repeating the thesis of planetary self-motion, and to rejecting the existence of celestial spheres responsible for the transportation of the heavenly bodies. He also ascribed to Earth the daily motion around its axis without renouncing its central position in the cosmos and in

<sup>56</sup>Patrizi 1975, XXVII, 53: "Molto Magnifico et Eccellentissimo Signore, mi rallegra con Vostra Signoria, che più tosto che non credea si è rilevata dal male, e li rendo moltissime gratie dell'haver presentato il mio libro a quel Serenissimo Principe, e ricevuto il favore, che Ella mi avvisi ciò che haverà detto, dopo che l'havrà letto. Et se per sorte per li molti negozii il libro andasse in oblio, spero da Lei il rimedio di un poco di ricordanza, la quale, se partorirà alcun segno che Sua Altezza Serenissima l'habbia havuto caro, mi sarà carissimo e tutto l'obbligo l'haverò a Vostra Signoria e all'amor suo verso me."

<sup>57</sup>Patrizi 1975, 57–58. In the letter Ottonaio is also mentioned as a common acquaintance and an intellectual partner.

the planetary system.<sup>58</sup> All of these theses—which, as we shall see, are also present in Benedetti's *Diversae speculationes*—were censured by the Inquisition in the 1590s, in the course of an attack directed against Patrizi's *Nova de universis philosophia*.

The censure of Patrizi's philosophy occurred after Benedetti's death—he was lucky enough to avoid witnessing the prohibition of theses to which he subscribed. On October 8, 1592, the Master of the Sacred Palace, Bartolomé de Miranda, and his fellow, Pedro Juan Saragoza, wrote a document of censure, attacking many passages and doctrines of the *Nova philosophia*. The same Saragoza would later be one of the two censors of Bruno's work during his Inquisition trial in Rome.<sup>59</sup> The two censors of Patrizi did not limit their criticism to theology (especially Patrizi's theses on the Trinity) but also scrutinized natural philosophy and cosmology. They rejected the idea that there is only one unique heaven and judged this to be an opinion at odds with accepted philosophical doctrines and against theology (*In lib. 13 Pancosmias tenet unum tantum esse caelum*). Furthermore, Patrizi was accused of following Copernicus, whose doctrine of terrestrial motion was seen as incompatible with the Bible:

In *Pancosmia*, Book 17, f. 103, p. 1, column 2a, he [Patrizi] states 'that the motion of the Earth is by far in better agreement with reason than the motion of the heavens or the uppermost celestial bodies.' And he refers to Nicolaus Copernicus's sentence according to which the sidereal heaven is immobile, along with the stars, while the Earth moves.<sup>60</sup>

Further theses to be censured were his vitalistic concept of celestial bodies and celestial infinity. The criticism of the latter point goes as follows:

This [to sustain this view] is to dream in very deep obscurity and fall down a precipice after abandoning the common way. In fact, the best and greatest God created everything according to weight, number, and measure. Therefore, everybody agrees that no infinite body is possible in act and no existing multiplicity can be infinite in act. On the empyrean heaven see the Fathers and Thomas Aquinas.<sup>61</sup>

Patrizi replied with an *Emendatio in libros suae novae philosophiae* (Correction of the Books of His New Philosophy), written before Christmas 1592. As far as Earth's motion is concerned, he clarified that he advocated its motion but not its eccentricity (*Non tamen dixi, eam de medio suo et naturali loco exire*). Furthermore, he stressed that terrestrial motion was supported by many arguments and several philosophers, and claimed that this view does not contrast with theology. However, he declared himself ready to erase passages referring to terrestrial motion, if necessary (*Delebo tamen si iubetis*). He also defended his views about the infinity of space but was ready to renounce this thesis as well, if he was ordered to do so: "Delebo hanc etiam si iubetis."<sup>62</sup>

<sup>58</sup>Seidengart 2006, 116–124 and Omodeo 2014a, 174–175.

<sup>59</sup>See Bruno 2000b, doc. 45, 225.

<sup>60</sup>Baldini and Spruit 2009, Vol. I, 3, 51, doc. 1, 2216: "Lib. 17 Pancosmias fol. 103, pag. 1, col. 2.a ait *quod Terram revolvi longe videtur esse rationi consonantius, quam Coelum, vel suprema astra moveri. Et refert sententiam Nicolai Copernici dicentis Coelum sydereum stare simul cum stellis, Terram vero moveri.*"

<sup>61</sup>Baldini and Spruit 2009, Vol. I, 3, 51, doc. 1, 2219: "Hoc est somnare per altissimas tenebras, et a via communi declinando in praecipitia ruere, nam cum Deus opt. Max. omnia in pondere, numero, et mensura produxerit, nullum infinitum corpus actu dari nullamque rerum subsistentium multitudinem actu infinitam omnes viri fatentur. De Coelo empyreo consultat Patres, et sanctum Thomam."

<sup>62</sup>Baldini and Spruit 2009, Vol. I, 3, 51, doc. 7, 2231.

The Jesuit Benedetto Giustiniani proved a more open-minded censor of Patrizi's natural views in 1593.<sup>63</sup> The same person, however, would be one of the eleven theologians who decided that the Copernican theory was not reconcilable with the Catholic religion, in 1616. As to Patrizi's work, it was downright (*omnino*) prohibited in 1594, and placed in the Clementine Index of 1596 as well as in later Indexes.<sup>64</sup>

Not only did Benedetti correspond with intellectuals engaged against the mainstream in Rome, but he himself had differences with Roman Aristotelians, as he mentioned in the preface to the second edition of the *Demonstratio motuum localium contra Aristotelem* (1555). On some occasions, Benedetti even allowed himself to be ironical about theological dogmas. For instance, in one of the letters of the *Diversae speculationes*, he accompanied his adherence to methodological Pythagoreanism (a mathematical approach to the investigation of nature) with a joke about reincarnation and his earlier life:

If the souls' transmigration imagined by the father of Italian wisdom, Pythagoras, were true, I believe that your soul and mine were once the souls of hunting dogs.<sup>65</sup>

Another indicator of Benedetti's attitude towards the Counter-Reformation and the confessional quarrels of his time emerges from his approach to the calendar reform. This was a very divisive issue. The pope imposed upon all Christianity an emendation of the calendar in an age when it was affected by profound divisions. In this climate, the pope's political and religious legitimacy and his authority in such matters was cast into doubt by many, especially in the reformed countries. Reputed Lutheran astronomers such as Michael Maestlin opposed the calendar reform implemented by Christopher Clavius and promulgated by Gregory XIII in 1582. The elimination of ten days to make the spring equinox date correspond to its date at the time of the Council of Nicaea was particularly controversial, since it symbolically legitimized the universality of the Roman Church through Constantine I. Benedetti wrote a reform proposal on April 1, 1578, and Emanuele Filiberto sent it to Rome on May 31. The text was printed two times, first in 1578, and then reprinted in 1585 as the first of the letters in the epistolary section of the *Diversae speculationes*. The most striking feature of Benedetti's proposal is its technical radicalism and extreme rationalism. August Ziggelaar's perspicuous description of Benedetti's proposal here follows: "Benedetti prefers the Eastern date to be fixed according to the true motions of Sun and Moon rather than by cycles. He finds that the Prutenic tables are sufficiently exact for this purpose. Furthermore not ten days, not even 14, but 21 days should be left out in order to make the first day of January the winter solstice. The lengths of the months are to be adjusted so that they coincide with the presence of the Sun in each of the twelve zodiacal signs. Surely, these time intervals change their lengths in the course of time because of the motion of the perihelion of Earth, but Benedetti assures us that only after 24,000 years will an adjustment be necessary. The proposal is not only interesting and original but also very rational because, by eliminating all reference to the Moon, it makes the year entirely solar."<sup>66</sup> The extreme technicality of such a proposal, one can argue, was

<sup>63</sup>Baldini and Spruit 2009, Vol. I, 3, 51, doc. 10.

<sup>64</sup>For a reconstruction of the anti-Platonic reaction also affecting the reception of Patrizi, see Rotondò 1982. On the censure of 1616, see Bucciantini 1995, Bucciantini, Camerota, and Giudice 2011 and Omodeo 2014a, chap. 7.

<sup>65</sup>Benedetti 1585, 285: "Si vera esset animorum illa transmigratio quam sibi Italicae sapientiae Pater Pythagoras effinxerat; tuam, meamque existimarem animam canis, quandoque venatici fuisse."

<sup>66</sup>Ziggelaar 1983, 211 and 214.

also a means to de-ideologize the issue. Therefore, it was not only rejected for its “scientific radicalism” but also for its rationalistic distance from confessional struggles. This positioning is in line with late-humanistic universalism and signals Benedetti’s distance from the Counter-Reformation and the militant cultural production of those years.<sup>67</sup>

## 2.6 Cultural Institutions: University, Academies, Collections, and the Press

The reformation of the *Studio* was a cornerstone in Emanuele Filiberto’s and his successor’s cultural policy. It was reopened in Mondovì in 1560 after the French closed it as a potential center of dissent and revolt. It was transferred to the new capital on October 12, 1566, after long discussions and a dispute between Turin and Mondovì. According to the new statutes, issued in 1571, the direction of the university was conferred to nine *reformatores*, among whom were the Archbishop, the ducal chancellor, the first and second presidents of the Senate of Turin, and the court physician. These men were responsible for the scientific and administrative direction, and for academic discipline.<sup>68</sup>

Most of the professors (about thirty people) were jurists. Among them, the most reputed was the professor of civil law Guido Panciròli. The physicians Francesco Valleriola and Giovanni Argenterio were also illustrious professors, known and appreciated by Conrad Gesner and Michel de Montaigne. The reopening of the university offered Argenterio the opportunity to come back to Piedmont after many years of practice as a physician in Lyon, and of teaching in Antwerp, Bologna, Pisa, Rome, and Naples.<sup>69</sup> Teodoro Rendio of Chio was appointed to teach Greek grammar and, as mentioned before, the poet Giraldo Cinzio became professor of humanities. At the University of Turin, the teaching of philosophy on the basis of the *corpus Aristotelicum* received high recognition, as the professors in this discipline had a better salary than their colleagues of medicine. Giacomo Castagneri taught on Aristotle’s *Physica*, *De generatione et corruptione*, and *De Anima*. Other celebrated scholars in philosophy were Antonio Berga and Agostino Bucci, both Piedmontese educated at Padua.<sup>70</sup>

One of the most reputed professors appointed in Mondovì was Francesco Vimercato of Milan, with whom Benedetti later corresponded. Vimercato was a thoughtful Renaissance commentator on Aristotle, whose work he read in the original language. He published commentaries on *De anima* (1543), on *Metaphysics* (1551), in particular on book lambda and on *Meteorologica* (1556). However, his commentary on *Physics* has to be seen as his magnum opus. After his studies in renowned Italian universities, he was appointed in Paris as the first royal lector in philosophy. There, he was first involved as a judge in the Ramist disputes and later was a colleague of Petrus Ramus. In 1561 he came to Piedmont to serve the Savoy family as a university professor, a councillor, a tutor to Carlo Emanuele I, and, from 1567 to 1570, as diplomat in Milan.<sup>71</sup>

Bordiga argued that Benedetti might have taught at the reopened university, first in Mondovì and then in Turin, by relying on some indirect sources. In fact, the information about Benedetti’s teaching activity at Mondovì was derived from the sixteenth-century biographer of Emanuele Filiberto, Giovanni Tonso, who included his name on the list of

<sup>67</sup>Steinmetz 2011.

<sup>68</sup>See Bonino 1824–1825, Naso 1993, and Catarinella and Salsotto 1998.

<sup>69</sup>On Argenterio, see Temkin 1974, 141–144 and 149–152 and Mammola 2012, 185–193.

<sup>70</sup>On the philosophical culture of Turin of those years, in particular on Bucci, see Mammola 2013.

<sup>71</sup>See N. W. Gilbert 1965.

those who made that university eminent (*Qui viri insignes publice docuerint*).<sup>72</sup> Another contemporary of Benedetti, the mathematician Cristini, mentioned him as one of those whom Emanuele Filiberto called to Turin in order to increase the prestige of his university.<sup>73</sup> Still, the role that he could have played in the academic life of Turin remains obscure.<sup>74</sup>

We could find no direct evidence that Benedetti served as a professor in the documents preserved at the *Archivio di Stato di Torino*. We considered the acts gathered under the signature “Istruzione Pubblica/ Regia Università di Torino/ Mazzo 1 (1267–1701),” which include the statutes of 1571 and other precious sources concerning the first years of the university. A dossier entitled “1571, Costituzione de’ Riformatori dell’Università dello Studio di Torino, coll’Istruzione da osservarsi da medesimi, colle distribuzioni delle ore per la Lettura, e Rolli de’ Stipendi de’ Lettori” (*fascicolo 7 primo*) includes decrees concerning the reform of the university, the names of those responsible for accomplishing it, and a list of the chairs with the corresponding salaries and the names of the professors. These documents indicate that the professor of mathematics was Francesco Ottonaio of Florence, who had occupied the same chair at Mondovì. Other documents testify that the same person held the chair of mathematics in 1573 and in 1585/6. It is therefore hard to believe that Benedetti was professor in this subject before 1586, as several secondary sources contend.<sup>75</sup> As to his alleged position at Mondovì, that university was opened by Emanuele Filiberto in 1560 and then transferred to Turin in 1566, that is, before Benedetti’s arrival at the Savoy court. Thus, it must be excluded as a possibility that he ever taught there, contrary to the claim made by Tonso. It is possible though, even likely, that he acted as an external advisor on matters concerning the university.<sup>76</sup>

Turin had fewer academies than other centers such as Rome, Naples, and Florence, although it was a typical Renaissance phenomenon to establish academies, circles of learned men who met to discuss issues pertaining to literature, the arts, or the sciences, and who published works representative of their common intellectual efforts. Apart from two minor academies (“*de’ Solinghi*” and “*degl’Impietriti*”), Carlo Emanuele I conceived the project of forming an academy guided by the Jesuits around 1585. Bonifacio Vannozi, a man of letters from Pistoia, described it as follows:

His Highness, the very serene [duke] of Savoy, had the wish to found an academy in this august town of Turin. He charged three Jesuit Fathers of the renowned College with the task. Although they are generally sober of mind, in this case they were so intemperate as to entrust myself [with this endeavor] although the overwhelming responsibility [*machina da incurvar le spalle*] would be excessive for even the most competent person. His Highness has made himself Prince, Protector, and Head [of the academy], in order to attract a good deal of his courtiers [into the academy] who are so cultivated

<sup>72</sup>Tonso 1596, 141: “Neque vero liberalium disciplinarum omniumque artium colendarum quam suscepit cogitationem unquam deposuit: nam et publicum earum Gymnasium pro tempore in oppido Montereali instituit: et qui viri in quacunque scientia excelleret undique conquisuit. [...] Mathematicos illustres Franciscum Othonarium, et Io. Baptistam Benedictum Venetum.”

<sup>73</sup>Bordiga derived this information from a manuscript of Cristini’s preserved in the Biblioteca Marciana in Venice. See Bordiga 1926, 596–597.

<sup>74</sup>The historian of Piedmontese Universities Silvio Pivano complained already in the 1920s about the lack of relevant documents. Pivano 1928, 19–22.

<sup>75</sup>See, e.g., Bauer 1991, 156–157.

<sup>76</sup>Roero 1997, 65, n. 5. Evidence for Benedetti’s role as an advisor in university matters can be found in Patrizi’s correspondence, as already mentioned.

and refined that, if one adds to it the splendor of the arts [lettere], there will be no court in Europe more illustrious than this one. Our name is 'Incogniti.'<sup>77</sup>

In spite of the initial impetus, this academy was not particularly successful and did not leave significant traces of its activities. Perhaps it was negatively affected by the fluctuating relations between the Crown and the Jesuit order.

Emanuele Filiberto also created a *Theatrum omnium disciplinarum*, which was realized for him by Ludovic Demoulin de Rochefort. It is not completely clear what this theater contained. It was probably a *Wunderkammer*, as Mamino argued.<sup>78</sup>

In March 1572, duke Emanuele [Filiberto] established in Turin a museum [*theatrum*] of all disciplines [organized] in marvelous order and at a very high cost. Archbishop Gerolamo della Rovere and the philosopher Ludovic Demoulin de Rochefort, the most educated men in all fields, cared for it.<sup>79</sup>

Moreover, the dukes supported editorial activities. Emanuele Filiberto first called the Flemish printer Laurens Leenaertsz van der Beke, better known as Torrentinus, to Piedmont. Torrentinus had already initiated a printing house in Florence under Cosimo I., but he died shortly after his arrival in Mondovì (1561). Hence, in 1570, Emanuele Filiberto employed another printer, Niccolò Bevilacqua. This pupil of Manuzio founded the *Compagnia della Stampa* (Printing Company), which benefited from ducal privileges (a sort of editorial monopoly). The activity of Bevilacqua and his heirs culminated in 71 editions produced between 1578 and 1580, most of them on juridical subjects. Scientific issues were not neglected in Turin. Among the publications issued during the years of Emanuele Filiberto, between 1563 and 1580, books on scientific and natural subjects constitute about one fifth of the entire production, that is, almost as much as literary publications.<sup>80</sup> They also printed several books by Benedetti, including the *Diversae speculationes*.

## 2.7 Scientific Debates

### 2.7.1 Courtly Conversations

Renaissance Turin was a center of lively cultural and scientific debates taking place in different institutional settings, at court, at the university, and in town. The printing press was a powerful means for public discussion, dissemination of ideas, and criticism. Printed sources are evidently our main source of information about the intellectual debates or polemics that animated Turin in Benedetti's time.

<sup>77</sup>Tiraboschi 1824, 289–290: “L’Altezza di questo Serenissimo di Savoia ha desiderato, che si dia principio a fondar un’Accademia in questa sua Augusta città di Turino, et n’ha data la cura a tre Padri del Gesù di questo insigne Collegio, i quali, non so da che allucinati, soliti però a non s’abbagliare, hanno fatto gran fondamento nella persona mia, caricandomi d’una macchina da incurvar le spalle, quantunque gigantesche. S.A. se n’è fatto Principe, e Protettore, e Capo, per tirarvi buon numero de’ suoi Cortigiani, tanto culti e fioriti nel resto, che, se vi si aggiugne l’ornamento delle belle e delle pulite lettere, non sarà Corte in Europa più rilucente di questa. Il nostro nome è degli Incogniti [...]”

<sup>78</sup>Mamino 1992 and Mamino 1995. By contrast, Cibrario thought that it was an encyclopedic project. See (Cibrario 1839).

<sup>79</sup>Pingone 1577, 88: “Anno Christi 1572 mense Martio, Emanuel Dux Taurini theatrum omnium disciplinarum miro ordine, nec minimis sumptibus instituit, curantibus Hieronymo Ruvereo Archiepiscopo, et Ludovico Molineo Rochefortio Philosopho, viris in omni doctrinae genere absolutissimis.”

<sup>80</sup>On Renaissance publications in Piedmont, see Bersano Begey 1961, especially vol. 1. See also Merlotti 1998.



First of all, we should consider courtly debates. A circle of intellectuals gathered around the Savoy family. The professor of jurisprudence, Bernardo Trotto, depicted the relationship between the rulers and their learned courtiers as follows:

These learned men, played by the Prince like well-tuned musical instruments, immediately give out their specific sounds with words. And they give it their best to be clearly understood in conversations, to please the others with good arguments and to convince them of their opinions. It is like the consonance of truth. In fact, everyone says what one knows or, at least, considers to be true. Hence they discuss natural issues and at times moral ones and mathematical ones. In conclusion, one can regard him [the prince] as Apollo surrounded by the Muses near the water spring that was born from the hoof of Pegasus.<sup>81</sup>

A reflection of the intellectual climate and the topics addressed in such informal meetings is a poem by the court physician Arma, *Proposte tenute co'l Serenissimo Prencipe* (Issues Discussed with the Very Serene Prince), printed in Turin in 1580. In this tiny book, addressed to Emanuele Filiberto, Arma reported a discussion on meteorological problems that took place between Carlo Emanuele I, Benedetti, Ottonaio, Berga, and himself during the prince's meal for three successive days. On the first day, Carlo Emanuele I asked the reason why the sun heats. Arma used Plato to argue that its body is not warm but that it heats through the percussion of its rays on terrestrial elements.<sup>82</sup> The next day, Arma had a quarrel with Benedetti, Berga, and Ottonaio concerning "solar attraction."

The day after, Mister Benedetti  
And Mister Berga, along with Ottonaio  
Expressed opinions that are far from mine:  
That the Sun attracts everything to itself with its great brightness  
As if it had hands.<sup>83</sup>

Arma objected that, if the sun attracts other bodies, this would be very inconvenient for its inhabitants, the solarians (*solari*). The argument is not clear and the reader is only informed about the fact that Benedetti agreed with him.

During the conversation, Ottonaio mentioned the vapors brought upward by the sun's action. Accordingly, the next topic of discussion concerned natural places and elemental displacements with reference to the behavior of vapors.

<sup>81</sup>Trotto 1625, 2–3: "[...] questi huomini saputi, tocchi dal Prencipe, come instrumenti musici bene accordati, subito rendono ciascuno il suo suono con le parole et quanto meglio possono procurare d'essere intesi discorrendo, e di dar diletto con le buone ragioni, et anco di tirare gli altri al suo parere, come ad una consonanza della verità: perché ognuno dice quello ch'egli sa o crede almeno sia vero. E quindi si veggono trattare hor cose naturali, hor morali, hor mathematiche. Sì che egli quasi come uno Apolline si può dire, che sta fra le Muse, intorno al fonte, che uscì dal colpo del piede del cavallo alato." On Trotto's teaching, see Vallauri 1846, 28 and 48–49.

<sup>82</sup>Arma 1580a, f. A2r: "Scalda co raggi [...] / Sbattendo la Terra di caldo priva. Si com' il martel che bate l'incudine, / Riscalda l'un e l'altr' in certitudine."

<sup>83</sup>Arma 1580a, f. A2v:

"Il Signor Benedetti l'indomani  
Col signor Berga, insiem' a l'Ottonaglio  
Forn' in pensier' a me d'assai lontani,  
Che'l Sol tirass'a sé com grand'abbaglio  
Ogni cosa si com'havesse mani."

On the third day, the prince asked about the origin of lightning, and why we perceive their light before we hear the thunder. Arma answered that our sight is much quicker than our hearing, but this remark was received with skepticism by his colleagues. No details are reported about the objections that should have concerned the theory of perception, light, and optics.

The next issue was colors and the rainbow; Benedetti asked about the center of the rainbow's arc and Arma offered the following answer:

Benedetti, as an expert master of his art,  
 Asked me about the center of the arc [of the rainbow].  
 I answered that it was on the vertical line  
 Descending downward from the center of the heavenly body,  
 As was the opinion of Zoroaster.  
 And with this answer I got rid of him.<sup>84</sup>

At the end of this three-day conference, all opinions were written down for the prince and signed by the ducal advisors:

All of this was presented in written form  
 To His Highness, reporting all speeches.  
 Dr Berga confirmed it [the rightness of the report].  
 Benedetti did the same.  
 After that we discussed other issues,  
 Occult things and their effects.<sup>85</sup>

Other publications also mention such table talks at court. For instance, the physicist and philosopher Bucci wrote in the preface to his book on physiology, *Disputatio de principatu partium corporis* (Disputation on the Superiority of Parts of the Body, 1583), that his discussion about whether the heart or the brain held superiority in the body, and about the localization of the soul, originated from a table talk. On a certain occasion, in fact, Carlo Emanuele I had gathered his learned courtiers and addressed these issues. Among the participants, including several physicians, Bucci also mentions the “mathematicians,” Ottonaio and Benedetti.<sup>86</sup>

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<sup>84</sup>Arma 1580a, f. A4r:

“Il Benedetti, come degno maestro,  
 Mi dimandò d’il centro di tal arco.  
 Dissi, che gliera col centro de l’Astro,  
 Ne la medema linea giù scarco.  
 Si com’anchora volse Zoroastro.  
 E con tal dire di lui mi discarco.”

<sup>85</sup>Arma 1580a, f. A4r:

“E tutto quest’ in scritti fu donato  
 A Sua Altezza, con tutti soi detti.  
 E fu dal Dottor Berga conformato.  
 Il che fece’l signore Benedetti.  
 Fu poi d’altre proposte ragionato  
 E de gl’occolte cose, e soi effetti.”

<sup>86</sup>Bucci 1583, 7–8. See Mammola 2013, 6–8.

### 2.7.2 Academic and Scholarly Controversies

Scholarly controversies and polemics on various issues and with very different tones were printed in Benedetti's years. While courtly debates had a polite and entertaining character, academic disputes could be more vehement. However, the two contexts were not always neatly divided. In 1572 two professors of philosophy, Berga and Bucci, held divergent opinions concerning the interpretation of Aristotle's *De anima*.<sup>87</sup> As Simone Mammola has argued, their disputes on that issue should be understood against the background of the Averroistic-Simplician debates on Aristotle's psychology started at Padua and continued at Turin by scholars such as Filateo and Vimercato. The Turin controversy over Aristotle's soul doctrine, which developed through academic disputations and publications, ranged from cognitive problems linked to the functions of the soul (such as the problem of the relationship between imagination and cogitation) to metaphysical and theological issues, e.g., the legitimacy of a reading of Parmenides's theory of being as a form of *prisca theologia*. The real issue at stake was the correct approach to the Aristotelian corpus based on different commentators.<sup>88</sup>

The court physician Arma was at the center of several public controversies as well. In 1575, he defended the scientific status of medicine, traditionally considered "only" an art, in the programmatic book *Quod medicina sit scientia et non ars* (That Medicine is a Science and Not an Art). This writing was considered worthy of a second edition in 1585. In 1579, Arma entered a dispute against the professor of medicine Giovanni Costeo over the healthiness of bread made out of rice ("*pane fatto col decotto di riso*"). The two parties then issued a series of publications on this controversial topic.<sup>89</sup>

Another polemic opposed Berga and Benedetti regarding the proportion between the earthly and the watery element of our globe. The debate was initiated by some questions Carlo Emanuele I asked his courtiers. While Berga stuck to the Scholastic view that the elements have an increasing quantity proportional to their distance to the center, Benedetti favored the merging of empirical and mathematical arguments as proposed by Alessandro Piccolomini. Piccolomini had come to the conclusion that the quantity of the earthly element is superior to that of the water element, as one can read in his *Della grandezza della terra e dell'acqua* (Venice, 1558).<sup>90</sup> Although Benedetti regarded Piccolomini's arguments as conclusive, Berga undertook to compose a Scholastic refutation of them, *Discorso... della grandezza dell'acqua e della terra contra l'opinione di S. Alessandro Piccolomini* (Turin, 1579). Part of his strategy was to eliminate arguments derived from the "misura dei cieli e della terra, dalla Scuola dei matematici immaginate," that is, from mathematical and empirical methods applied to this issue ("measurements of the heavens and the earth imagined by the school of the mathematicians"). Benedetti reacted with his *Considerazione... d'intorno al discorso della grandezza della Terra, et dell'Acqua. Del Eccellent. Sig. Antonio Berga Filosofo nella Università di Torino* (Consideration... on the discourse concerning the dimensions of earth and water by the excellent Mr. Antonio Berga, philosopher of the University of Turin, Turin, 1579). In it, he applied mathematics to show that, even if the entire earth were covered by water, the volume of the terrestrial

<sup>87</sup>Bucci 1572 and Berga 1573.

<sup>88</sup>See Mammola 2013.

<sup>89</sup>Merlotti 1998, 585: "Come s'è visto per la polemica fra Costeo e Arma [...] non si trattava di isolati testi a stampa che generavano dibattiti destinati a rimanere manoscritti e chiusi nell'ambito degli eruditi, ma semmai del contrario: di discussioni, cioè, sorte in circoli ristretti di medici e scienziati, prima affidate a manoscritti e poi trasportate a stampa a vantaggio d'un più vasto pubblico."

<sup>90</sup>For an accurate reconstruction of the polemic and its cultural and scientific context, see Ventrice 1989, 103–145 and Mammola 2014.

element would not be inferior to that of water since the depth of seas and oceans is small in comparison to the terrestrial radius. The oceanic navigations, the geographical explorations, and the European colonization of the globe during the fourteenth and fifteenth centuries had indisputably demonstrated that water and earth constitute one single globe, a unique *globus terraqueus*.<sup>91</sup> This cosmographical advance had been the basis for Piccolomini's considerations, which met with considerable success and agreement among scholars. On such issues, as Benedetti stressed, experience and mathematical reasoning should be joined:

Very serene Prince, the discovery, after two thousand years, that the [element] earth is much more than the [element] water (for which we are greatly indebted to the very learned Mr. Alessandro Piccolomini) very much pleased the spirits of the most renowned philosophers of our time. In the past, they did not dare to depart from the false doctrine they had imbibed for many centuries, although it was sustained by implausible reasons. Today they are glad to embrace the opposite opinion [concerning water and earth], because both the senses and reason are in accordance with the [new] demonstration of the truth. The ancient mistake has been unveiled by the mathematical school with very certain proofs that offer a firm foundation of the measurement of the heavens and the Earth.<sup>92</sup>

The dispute continued with the Latin translation of Berga's writing by Francesco Maria Vialardi (1580) and a skeptical intervention by Arma. The latter was a poem dedicated to Carlo Emanuele I, entitled *Stanze del dottore Arma al serenissimo Carolo Emanuele di Savoia et Piemonte Principe, etc. suo signore sempre osservandissimo. Che l'acqua e la terra non si possono a modo alcuno misurar* (Doctor Arma's Stanzas to the Most Serene Prince Carlo Emanuele of Savoy and Piedmont, His Perpetually Honored Lord, 1580). The composition is poor both from a stylistic viewpoint and a scientific one. The courtly physician could only point out the uncertainty of human knowledge and the wisdom in measure gifted by God to creation, although he also stated we cannot grasp the latter: "The Creator gave it a measure,/ Which cannot be grasped by any creature."<sup>93</sup> Benedetti addressed the issue again in one of the letters of the *Diversae speculationes*.<sup>94</sup>

### 2.7.3 Astronomical-Astrological Polemics

In Renaissance Turin, astronomical and astrological issues were at the center of intense debates and even polemics. In 1578 the protophysician Arma was the target of a denigrating pamphlet in twenty points. An anonymous author attacked a booklet of his on the comet that had just appeared. As we know from indirect evidence, Arma had written one or two treatises on this issue. One was entitled *De significatione stellae crinitae*.<sup>95</sup> Perhaps it has

<sup>91</sup> Vogel 1993.

<sup>92</sup>Benedetti 1579, 3: "[...] l'essersi doppò due mila e più anni scoperto con trionfo della verità, che la terra è molto maggiore dell'acqua, (del che si ha da haver grande obligo tra gl'altri al dottissimo Signor Alessandro Piccolomini) ha non poco rasserenato, Serenissimo Principe, l'animo de' più famosi Filosofi di nostra età; i quali, sì come prima non intendeano dipartirsi dalla già imbevuta falsità, e per molti secoli adietro, benché con inefficaci ragioni difesa, così hora si lasciano volentieri persuadere il contrario; poiché il senso, e la ragione s'accorda alla dimostrazione del vero. E nella scuola de Mathematici per certissime prove si scuopre l'antico errore, puotendosi far fondamento stabile delle misure de cieli, e della terra."

<sup>93</sup>Arma 1580b: "Il Creator gli diede tal misura./ Che saper non si può da creatura."

<sup>94</sup>Benedetti 1579, 397–405, "Defensio nostra contra Antonium Bergam, et Alexandrum Piccolomineum." See Ventrice 1989, 131–134.

<sup>95</sup>This information stems from Bonino 1824–1825.

to be identified with a composition in verses, referred to as *πωγωνία* (bearded comet) in the anonymous pamphlet. The only remaining evidence of the cometary polemics is a defence of Arma, *La Stravagantographia del Sig. Filosofo stravagante, in difesa de la πωγωνία d'il Dottore Arma* (The Stravagantography of Mr. Stravagant Philosopher in Defence of Dr Arma's Bearded Comet).<sup>96</sup> The apologist was one Monsignor Della Torre linked to the court. His apology offers an insight into the controversy: It concerned astronomical and meteorological issues relating to the nature of the comet, its location below or above the sublunary sphere, the criticism of astrological interpretations of the celestial phenomenon, as well as personal insults. The provocateur who had published against Arma is depicted as follows:

I cannot stop wondering who this person is. I cannot understand why he sometimes presents himself as a scholar, sometimes as a cook, as a Roman courtier, or as a practicing friar [*frate osservantino*] (as he speaks about the *osservantini*). I cannot believe that he is a practicing [man of religion], as the ecclesiastics speak in a correct manner and not heedlessly like him (who behaved heedlessly). Moderation has always been praised. Therefore, moderate people will always damn this person. I will never believe that he is a scholar. In fact, today's scholars are well-educated and would never indulge in such excesses, especially against such a man [Dr. Arma] from whom they did not receive anything but pleasure, honor, and courtesy. Although he seems to come from the area of Rome, in the end he shows himself to be a dishwasher because even a cook would behave better than him. Whoever the hell he is, if he will not control himself better in the future, I will repay him as he deserves.<sup>97</sup>

Possibly the identity of this mysterious denigrator was the philosopher Giordano Bruno, who was in Turin in 1578 on his way to Chambéry, in Savoy. At that time he wore the Dominican habit and had just published, as mentioned above, a booklet on meteorology in Venice entitled *De' segni de' tempi* (On the Signs of the Times) that presumably dealt with the comet.<sup>98</sup> The reference to the provocateur as a Roman courtier could correspond to an episode of Bruno's life. In Paris, in 1585, he told the librarian Guillaume Cotin that he had been once received at the Roman court by Pius V and the Cardinal Rebiba to whom he demonstrated his technique of the art of memory.<sup>99</sup> Moreover, the Turin episode resembles a *querelle* that burst out in Geneva in 1579, after the publication of a pamphlet in twenty points ridiculing a professor of philosophy. The detractor, Giordano Bruno, was discovered and condemned to exclusion from communion. As one reads in a document (6 August 1579), "Philippe Jordan, dit Brunus, Italien [était] détenu pour avoir fait imprimer

<sup>96</sup>It is preserved in the Biblioteca Reale of Turin, coll. G 25–67.

<sup>97</sup>Della Torre 1578: "[...] non mi posso quietare pensando chi possi esser costui. Non posso capire, perché quando fa d'il scolaro, quando del cuogo, quando del corteggiano di Roma, quando del frate osservantino, poi che di osservantini parla. Di esser osservante, nol posso pensare, perché li religiosi parlano correttamente, e non si s governano nel parlare, come ha fatto costui, il quale mattamente si è s governato. Fu sempre lodata la modestia. Sarà dunque dalli modesti dannato costui. Che sij scolaro, non lo crederò mai, perché hoggi di li scolari sono ben creati e non farebbono tale scappate specialmente contra di un'huomo tale da cui mai hebbero altro che apiacere, honor e cortesia. Par bene che habbi del Romanesco nel principio, ma il fine dimostra più presto haver del sguattero, perché il cuogo si sarebbe meglio deportato che non fa costui. Sij chi diavol esser si voglia. Se esso per avanti meglio non si governerà, tale e tanto mi ritrovarà, quale e quanto mi ricercherà."

<sup>98</sup>See Omodeo 2008a. On Bruno's lost meteorological-cometary work, see Ernst 1992.

<sup>99</sup>Spampanato 1921, 654–655 and Ricci 2000.

certaines responses et invectives contre Mr. de la Faye, cottans 20 erreurs d' iceluy en une de ses leçons.”<sup>100</sup>

Only one year later, between 1580 and 1581, Benedetti was involved in an astronomical-astrological quarrel with a certain Benedetto Altavilla of Vicenza concerning the reliability of ephemerides and astrological prognostication. The controversy was sparked by the publication of Altavilla's *Animadversiones in ephemeridas* (Remarks against Ephemerides, 1580) and was continued with two further publications by the same author.<sup>101</sup> They cast the reliability of ephemerides' calculations and astrological forecasting into doubt, based on the alleged theoretical flaws of mathematical astronomy. Since they appear to have been directed against scientists linked to the court, Benedetti eventually intervened with a printed letter, *Lettera per modo di discorso... intorno ad alcune nuove riprensioni, et emendationi, contra alli calculatori delle effemeridi* (Turin, 1581). He later translated it into Latin as *Defensio ephemerides* and included it in the miscellanea of epistles of the *Diversae speculationes*. We will expand on this polemic later, in the section on Benedetti's astronomy.

#### 2.7.4 Posthumous Criticism: Cristini on Benedetti

Benedetti died on January 20, 1590, two years before his own astrological prediction. This untimely death did not leave him the time to complete the astrological work that he announced at the end of the *Diversae speculationes*. What is worse, the fact that his own prediction was wrong awakened doubts and rumours about his scientific talent. The mathematician Cristini was quick to compose a critical essay, entitled “Examination of the mistake and emendation of the nativity of the very excellent mathematician, Mr. Giovanni Battista Benedetti, now deceased, to account for his [wrong] prognostication of his own death two years later than it in fact occurred” (*Essaminatione dell'errore, della rettificatione de tempo della natività del fu S[ignor] Gio[vanni] Battista Benedetti mathematico eccellentissimo, per cagion del quale esso s'era pronosticato morte due anni appresso in circa al tempo che gl'è avvenuta*). Although the manuscript was lost in the fire of the Biblioteca Nazionale of Turin in 1904,<sup>102</sup> a transcription of significant parts by the eighteenth-century biographer of Cristini, Giuseppe Vernazza, is still extant in the Biblioteca Reale of Turin among the documents that Vernazza gathered for the composition of his *Notizie di Bartolommeo Cristini* (Notes on Bartolomeo Cristini, 1783).<sup>103</sup>

Cristini started his examination of Benedetti's errors with specific reference to his mistaken self-prognostication in the *Diversae speculationes*. He also reported that Benedetti himself acknowledged an error in his nativity, amounting to four minutes. After that, Cristini took upon himself the task of recalculating that horoscope on the basis

<sup>100</sup>Spampanato 1921, 132.

<sup>101</sup>On astronomical-astrological quarrels in Renaissance Italy and Turin, see Omodeo 2008a and Tessicini 2013.

<sup>102</sup>Bordiga 1985, 609, n. 6. See Omodeo 2014c.

<sup>103</sup>Vernazza 1783. Two manuscript copies of Vernazza's biography of Cristini are still extant. One is preserved in the Turin State Archive (Archivio di Stato di Torino, coll. Miscellanea J.b.VIII. 9), the other is kept in the Biblioteca Reale of Turin (Vernazza manuscript, misc. 67.5). The latter is a good copy, ready for the printer. It contains an appendix of “documents” for the personal use of the author. These are transcriptions or translations of significant passages of documents by Cristini that were lost or seriously damaged after the fire at the Turin National Library in 1904. They comprise the dedication and table of contents of the *Revoluzione trentesimaterza del Ser[enissimo] Sig[nor] il Signor Carlo Emanuele duca di Savoia* (1596), notes from various astrological diaries, an Italian version of the beginning of *La rithmomachia o sia gioco di Pithagora* and, most importantly, a long extract from the *Essaminatione dell'errore... della natività del fu S[ignor] Gio[vanni] Battista Benedetti mathematico*.

of the figure published by Gaurico and came to the conclusion that the mistake was even bigger. It amounted to eight minutes.

Benedetti published his prognostication of the moment of his death in the work entitled *Diversarum speculationum mathematicarum et physicarum liber* (published in 1585), in a letter to the most illustrious Wolfhard Eisenstein [Volfardus Aisenstain], which is to be found at the end of this work. After a brief assessment of those things of the judicial art that he regarded as vain or false, and after announcing to Wolfhard that he would expand [on astrology] in that tract with his astrological observations, which he wished he could publish before his death, he added the indication of the time in which, according to him, [his death] was to happen (that is, [the date] before which he wished he could publish the aforementioned tract). These are his words: “antequam ad directionem mei horoscopi cum corpore Martis anaeretae perveniam, quae quidem directio circa annum millesimum quingentesimum nonagesimum secundum evenienti” [as indicated by my horoscope, before I meet the body of the adverse Mars. This is going to happen in 1592].

As we can see, he was certain that he would die when the direction of his ascendant and Mars would meet. He calls [Mars] “anaereta,” that is, giver or announcer of his death. He confirmed this when [...], just before his death, he felt that the disease was attacking him and declared that he made a mistake of four minutes in the rectification of the time of his birth horoscope [*natività*]. This is as if he would say that, by augmenting by four minutes the time of his birth horoscope, he would have predicted the direction [of his ascendant sign meeting Mars] at about the time when he became sick. Hence, he believed he was dying, and this [his death] in fact occurred at the end of the ongoing year 1590, at 17:00 of 20th January according to [the calendar of] Gregory, which corresponds to the 10th of the old [calendar]. I had to know the time in which he believed he was born in order to assess by how much time he was mistaken in the rectification of his birth horoscope, so that the direction of his horoscope relative to Mars corresponded to the days when he left this world. Therefore, at Benedetti’s death, I immediately began to compute the error of the aforementioned time, though only approximately, because I did not know Mars’s latitude. And I found that it [the mistake] amounted to eight minutes [...]. Later, when the same person who told me that Benedetti had acknowledged a mistake of only four minutes according to his calculations, openly accused me of not being able to do this calculation, as my mistake was two times [that of Benedetti], I began the calculation in the following manner. First, I determined the time attributed to his birth [...] Etc.<sup>104</sup>

<sup>104</sup>From Vernazza’s papers accompanying his manuscript of his *Notizie di Bartolommeo Cristini*. Biblioteca reale di Torino, Misc. 67.5, *Vita di Bartolomeo Cristini con documenti*, “M.S. L. 1.10, 11.493, di pag. 42.” See Omodeo 2014b: “Ha pubblicato il Benedetti, il pronostico fattosi del tempo di sua morte nell’opera sua titulata *Diversarum speculationum mathematicarum et physicarum liber* stampata dell’anno ’85 in una lettera scritta all’ill.mo Volfardo Aisestain, posta nel fine d’ess’opera, perciocché appresso haver brevemente dichiarato quali cose egli stimava vere nella giudiziaria e quali vane o false, et detto com’esso Volfardo potrà veder poi meglio in quel trattato dell’osservationi sue astrologiche, quale sperava dar in luce avanti la sua morte, soggiunge il tempo il quale giudicava essa doverli avvenire, o sia avanti al quale desiava publicar detto trattato, con queste istesse parole: “antequam ad directionem mei horoscopi cum corpore Martis anaeretae perveniam, quae quidem directio circa annum millesimum quingentesimum nonagesimum secundum evenienti.” Donde appare ch’esso teniva per certo d’haver a morire, quando giungerebbe alla

In his transcription of Cristini's *Essaminatione dell'errore*, however, Vernazza omitted numbers and calculations. These can be found in another astronomical-astrological assessment and criticism of Benedetti entitled "Thirty-first revolution of the very serene Sir, Duke Carlo Emanuele of Savoy, for the year 1592, very diligently and reliably calculated and explained by Bartolomeo Cristini, scholar of mathematical disciplines in the service of His Highness, according to the best opinion of the main judiciary astrologers" (*Revolutione trentesima prima del Serenissimo Signore il Signor Carlo Emanuel Duca di Savoia corrente dell'anno 1592 con ogni diligenze et fedeltà calculata et decchiarata secondo le migliori intelligenze de più principali autori dell'astrologia giundiciaria per Bartolomeo Crestino studioso delle mathematiche discipline in servizio di Sua Altezza*). Although the manuscript was damaged by the fire of the Turin library in 1904, it is still readable. The dedicatory letter shows that, at the time of its composition (June 8, 1592), Cristini was striving to obtain a stable appointment at court:

But your very generous Highness awoke in my spirit the desire of mathematical virtues and of undertaking the present endeavor. Your request woke up and unveiled in me the desire (which is always alive) to serve [Your Highness]. However, my desire has been impeded by the difficulties of my continuous poverty and adverse times owing to the fact that no treasurer (or any monetary and financial administrator) regards me as an ordinary servant of Your Highness. [I have been acknowledged as a servant] only in exceptional cases, when my capacity, readiness and knowledge in making calculations has proved useful—as has happened several times, when I was required to serve Your Highness. [...]

Therefore, I place growing hope only in Your Highness the more [you] require my services, the more efforts I make for You and the fewer are the number of [benefactors] by whom I can hopefully be supported<sup>105</sup>

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direttione del suo ascendente al corpo di Marte, quale chiama anaereta cioè datore, o promissore de la morte sua. Il che pare habbi volsuto confirmare quando che, come dice, poco avanti la sua morte ei si sentì carrigar dal male, disse d'essersi fallato di quattro minute nel rettificare il tempo di sua natività, perché questo è come s'havesse detto che quando egli havesse accresciuto tempo di sua natività per quattro minute havrebbe conosciuto la direttione predetta essere minore di quello [che] l'haveva fatta, et per ciò il tempo della sua morte caggionata da essa direttione dover essere circa questo tempo, ch'egli s'era infermato, et credeva di morire come è pur avvenuto, essendosi occorso ciò fare dell'anno presente 1590 circa le 17 hore del 20 giorno di genaro secondo Gregorio, che viene ad essere il dieci dell'anno antico. Perciò volendo io esaminare di quanto tempo egli habbi fallato nella rettificatione di essa sua natività, accioché giustamente la direttione predetta dell'horoscopo suo al corpo di Marte venisse a cadere nel giorno istessi ch'egli partì da questo secolo, m'è stato necessario sapere il tempo ch'egli havea presupposto fosse quando nacque [...]. Perciò mi posi subito seguita la morte del Benedetti a far conto dell'errore del tempo predetto, così alquanto alla grossa, per non haver nota la sopradetta latitudine di Marte, et ritrovai detto errore essere di minute otto in circa di hora [...] Ma perché ho dipoi inteso che chi mi ha riferito il Benedetti haver confessato il detto fallo di min. 4 et haver solamente ritrovato tanto per calcolo ha espressamente detto che io errava del doppio et non sapea far questo conto [...] mi posi a calcolare di questa maniera. Prima ho ritrovato il tempo presupposto della natività [...] Etc."

<sup>105</sup>Cristini, *Revolutione*, Biblioteca Nazionale Universitaria di Torino, N. VII. 10, f. 4r-v: "Ma V[ostra] Alt[ezza] benignissima sì come è stata cagione d'eccitar nell'animo mio il desio delle virtù matematiche, et di farmi fare la presente fatica; così ancora co'l chiamarmela adesso ha risvegliato, o riscoperto le sempre vive brame mie di servirla, le quali erano tenute sepolte dai disaggi che queste carestie et mali tempi mi causano maggiori giornalmente, per ciò che non sono conosciuto per servitore ordinario di V[ostra] Alt[ezza] da Tesoriere alcuno, né da ministro di suoi dinari o finanze; se non ne' casi che la virtù et prontezza, o cognizione mia ne' conti, può reccarli qualche giovamento come ha fatto più volte quando per servizio di



In this case, the allegation against Benedetti is for using the Alfonsine tables to determine the nativity of Carlo Emanuele I, instead of more reliable Copernican tables. Such inaccuracy invalidated his astrological judgments.

I took into account the places where they [the planets] are to be found in the horoscope made according to the true time calculated on the basis of Copernicus, following the teaching of the major authors on astrology. In general, since scholars are in disagreement concerning the employment of different tables to compute their horoscopes [*revolutioni*] and although I have demonstrated (in the calculations at the beginning of my tract) that only one set [of tables] is true, I calculated the astrological figures of the heavens according to both tables—in fact, false ones were also in use by many and in particular by Benedetti—and I offered double astrological judgments depending on the places assigned according to the different figures. In this manner, your Highness will possibly compare them and see which ones are in better agreement with the truth.<sup>106</sup>

The terms of Cristini's polemics echoed those of the controversy of the years 1580–1581 between Altavilla and Benedetti. In fact, Cristini examined two astrological figures calculated by Benedetti: a nativity based on the “Copernican” tables of the German astronomer Erasmus Reinhold, *Prutenicae tabulae* (1551), and a prognostication based on the Alfonsine tables.<sup>107</sup> He pitted these figures against a “figure that is computed from Giovanni Antonio Magini's tables of the second celestial mobiles” (*figura della natività di novo da me calculata con le tavole de secondi mobili celesti di Antonio Magini*) and a “figure of the revolution that I calculated according to the time of the real motion indicated in Magini's's ephemerides” (*figura della revolutione da me calculata sotto il tempo che si trova per il moto vero insegnato nell'effemeridi del Magini*).<sup>108</sup> Finally, he discussed the differences between his and Benedetti's calculations and concluded with an accusation directed against Benedetti for being careless and opportunist:

But I believe that he [Benedetti] followed the calculation of Alfonso X rather than the true one only owing to its simplicity. In fact, before [the publication of] the ephemerides of Magini it was very difficult to establish the true time of the revolution. Before him, nobody calculated the Sun up to the seconds in any ephemerides, which is the presupposition for more exact and true computations [...]. It is only in consideration of Benedetti's authority that I did not omit to compare his horoscope with the other one.<sup>109</sup>

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V[ostra] Alt[ezza] sono stato da loro richiesto [...].

Et per questo sempre cresce maggiore la speranza mia, in solo vostra Altezza quanto ch'essa più m'incita a servirla, et che maggior è fatica che faccio per lei, et minor il numero di quelli in quali posso haver speranza di soccorso.”

<sup>106</sup>Cristini, *Revolutione*, Biblioteca Nazionale Universitaria di Torino, N. VII. 10, f. 8r: “[Ho] havuto riguardo ancora ai luoghi ne' quali cadono essi [pianeti] nella figura della revolutione fatta secondo il vero tempo dato dal Copernico, come è insegnato da principalissimi scrittori dell'astrologia. Et nell'universal giudicio perché ho conosciuto tra scrittori essere certa diversità seguendo alcuni un tempo et altri un altro nel fare delle revolutioni delli quali ancor ch'io provi (come per i calculi di ciascuno posti al principio di questa opera) l'uno solo essere il vero, ho fatto le figure del cielo che si mostrano sotto ambi essi tempi (atteso che ancor la falsa era seguita da diversi et particolarmente dal Benedetti), ho radopiato essi giudici per i luoghi che diversi significati fanno havere esse figure. Accioché V[ostra] Alt[ezza] provandole ambidue conosca ancor lei quale meglio secondi la verità.”

<sup>107</sup>Cristini, *Revolutione*, Biblioteca Nazionale Universitaria di Torino, N. VII. 10, ff. 11v–12r.

<sup>108</sup>Cristini, *Revolutione*, Biblioteca Nazionale Universitaria di Torino, N. VII. 10, ff. 12v–13r.

<sup>109</sup>Cristini, *Revolutione*, Biblioteca Nazionale Universitaria di Torino, N. VII. 10, f. 16v–17r: “Ma io tengo ch'egli seguisse più tosto il calculo d'Alfonso che il vero; solo per causa della facilità d'esso perciòché avanti

In this second criticism, Cristini continued to discredit Benedetti. This time he cast his capacity as both an astrologer and a mathematical astronomer into doubt. Cristini suggested, in fact, that Benedetti misused his prestige to disguise the lack of accuracy in his astrological computations. Mistakes affected not only the horoscope he carried out for himself but also those cast for his patrons.

Once he had established himself as an expert in the field, Cristini continued to prepare prognostications for the ruling family from 1592 to 1595, as testified to by the titles of several manuscripts, such as “*diari*” and “*revolutioni*,” which are for the most part lost.<sup>110</sup> He obtained the position at court that he desired in 1594. Carlo Emanuele I designated him “as our and our princely children’s mathematician, follower of Giovanni Battista Benedetti, who has recently passed away” (*per mathematico nostro et dei principii nostri figliuoli in luogo del fu Gio[vanni] Battista Benedetti ultimamente defonto*). He moreover accorded to the new court mathematician a “reasonable stipend” (*un ragionevole stipendio*) of three hundred scudi per year, “so that, according to our wish, he will cover the efforts of his studies, and will be in condition to serve us with more ease and comfort in all the duties we will entrust him” (*acciò che possi comportar alle fatiche delli studi, et trattenersi al servizio nostro più agevolmente et commodamente come desideriamo in tutti li carighi che ha da noi*).<sup>111</sup>

## 2.8 Strengths and Limitations of the Institutional Framework of Benedetti’s Science

Benedetti’s life, career, and work, as well as his legacy, fortunes, and misfortunes should be understood against the background of the Renaissance world he was part of, in particular the Italian and Turin environments. His case is paradigmatic of both the strength and the limitations of Renaissance science. On the one hand, the cultural and economic flourishing of centers such as Turin, new and challenging engineering and architectural projects, and the establishment of a court and of a modern state apparatus with its need for technical advice and cultural grandeur created an exceptional environment, favorable also to the pursuit of science and philosophical speculations. The constraints of Counter-Reformation culture did not affect the speculative freedom of Benedetti. This is especially due to the pragmatic cultural and religious politics of the Savoy dukes, who were trying to establish a balance between their state and international diplomacy and confessional tensions. On the other hand, however, the fragility of Renaissance knowledge institutions also comes into view. Universities were teaching institutions instead of research centers. Professors were concerned with the transmission of knowledge rather than with the implementation of new knowledge and theories. The intended mission of early-modern universities was preservation, namely the transmission of traditional knowledge to future generations, not producing change. Thus, epistemic processes in institutional settings were often imperceptible and transformations of knowledge often occurred against the explicit intentions of the historical actors. Lectures and commentaries on authoritative sources—the teaching of which was sanctioned by academic statutes and curricula—were not expected to alter the knowledge preserved in the classics and in the textbooks. The Savoy dukes tried to

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l’effemeridi del Magini molto difficil cosa era trovar il tempo vero della revolutione perciocché nissuno avanti lui havea nell’effemeridi calculato il Sole sino alle seconde onde ne seguono i calculi più sottili e veri [...]. Con tutto ciò solo per l’autorità d’esso Benedetti non ho volsuto lasciar del tutto la consideration delle figura sua con l’altra come vedevasi.”

<sup>110</sup>See Peyron 1904, 617–618.

<sup>111</sup>Vernazza 1783, 20–21.

attract prestigious professors to Turin and also supported, at least initially, the teaching of humanities according to the new standards set by humanistic philology. However, the place for free inquiry and innovation was outside universities. Benedetti's works, marked by original and unorthodox conceptions in physics, mathematics, and other disciplines, emerged from a courtly environment. Yet, this institutional frame proved ephemeral as it was dependent on patronage. It also had a strongly personal character, as it depended on informal exchanges within a system of unsystematic patronage. In Turin, modern scientific academies, with a stable body of investigators and statutes, had not yet made their appearance. Not even literary and artistic academies met with a particularly favorable environment.

Given this context, Benedetti's scientific activity, accomplished outside university and institutionalized settings, cannot but appear as occasional. In fact, it was linked to the contingency of courtly life, for instance to the requests for advice by the Savoy rulers or other patrons. This is the case with all of Benedetti's letters and with other publications, such as his writing on the calendar reform. He appears to have given expert advice on issues of cultural policy, such as university appointments, as well as on technical issues, and not least on matters of astrology. His construction of a fountain and of sundials, as well as his writings on technologies and gnomonics are directly connected to his role as a mathematical expert at the court. In the same function, he also entered debates and polemics animating Turin. Some of his interventions were friendly, for instance his exchanges on meteorology with other courtiers in the presence of members of the ruling family. His controversy with Professor Berga over the quantity of water and earth in our globe was more vehement but never harsh. Astrological polemics were the most virulent ones, as evidenced by Benedetti's publications against the critic of astrology, Altavilla. A constant feature of Benedetti's scientific work remains its occasional character. This is also reflected in the lack of systematic order in his *magnum opus*, the *Diversae speculationes*. Benedetti probably saw himself primarily as a courtier, participating in the cultural life of Turin as an exponent of the Savoy elite, and not as a scientist pursuing the immaterial glories of scholarly achievements. As a matter of fact, he did not primarily take upon himself the burden of a scientific effort going beyond the deliverance of brilliant booklets, short judgments, and advice on specialistic issues.

One astonishing aspect of Benedetti's intellectual activities is the lack of an enduring and explicit legacy. On the one hand, his conceptions clearly influenced contemporaries and followers in Italy and abroad. Among others, his impact is reflected in the positive opinions of Brahe and Kepler, in Galileo's reception of several insights of his mechanics and physics, and in the European circulation of his ideas on physics through Taisner's plagiarism. On the other hand, the lack of an enduring acknowledgment of his work is equally evident and seems to be linked to the fact that he was not able, and perhaps not even willing, to establish a school like the one set up by Commandino in Urbino, or by those later set up by Galileo in Padua and Tuscany. Not even in Turin did he benefit from lasting recognition. As we have seen above, his immediate successor as court mathematician, Cristini, even saw the denigration of his astrological and astronomical skills as an opportunity to obtain a visible position in town and start a courtly career.

In many ways, Benedetti is the mirror of his world, in particular of the courtly society he belonged to. His work can be seen as the embodiment of this context. His case is different from that of many other Renaissance scholars, who strongly identified themselves with their scientific work and output. For scholars like Galileo, for instance, the publication of their works had a functional aim in accessing the courtly milieu. In the case of Benedetti, he was already part of the patrician and aristocratic milieu for many reasons. His work is the product of courtly life rather than his entry ticket to it. The author disap-

pears (or almost vanishes) and leaves in his place a sort of collective author, which is not the scientific Republic of Letters, but rather the court itself, its institutions, its elites, its participants, and its networks. In this respect, Benedetti differs from the great protagonists of Italian Renaissance science. He is very far from the self-celebration of intellectuals like Cardano and Galileo. His work is no monument to himself but rather to his environment, ranging beyond the local boundaries of Piedmont and the Savoy. The Urbino school was also populated by scholars less concerned with their own ego than with science. However, in contrast with this school Benedetti conceived of himself as an innovator, rather than as a restorer of antiquity and classicism.

## Chapter 3

### Structure of the Book and Main Issues

#### 3.1 Benedetti Introduces His Physico-Mathematical Speculations

The *Diversarum speculationum... liber* is composed of six books, indicated as follows in the table of contents:

1. “Arithmetic Theorems” (*Theoremata arithmetica*);
2. “Explanation of the Operations of Perspective” (*De rationibus operationum perspectivae*);
3. “Mechanics” (*De mechanicis*);
4. “Disputations on Some Opinions Held by Aristotle” (*Disputationes de quibusdam placitis Arist[otelis]*);
5. “On Euclid’s Fifth Book [of the *Elements*]” (*In quintum Euclidis librum*);
6. “Epistolary Answers on Physics and Mathematics” (*Physica et mathematica responsa per epistolas*).

It should be noted that this partition of themes does not mirror the relative relevance of the issues according to the author. In fact, the length and the importance of the books does not exactly correspond to the subdivision indicated in the table of contents. Two sections are quite short: Book 2 on perspective and Book 5 on Euclid. By contrast, the first one, on arithmetic, and the last one are much longer. Chapter 6 is an extremely diverse collection of letters on the most different subjects.

The dedicatory epistle of the *Diversae speculationes* begins with an acknowledgment of the generosity of Emanuele Filiberto of Savoy, with whom Benedetti had particularly good relations. As one reads, they often talked about mathematical issues pertaining to arithmetic, geometry, optics, music, and astrology.<sup>1</sup> Since the *Diversae speculationes* appeared after Emanuele Filiberto’s death, it was dedicated to his successor, Carlo Emanuele I. Benedetti reports that both dukes encouraged his inquiries and their questions motivated his investigation of specific questions. This is the reason why Benedetti’s *Speculationes* have an occasional character and are not ordered in a systematic manner. The *Theoremata arithmetica* (book one) clearly originated from conversations with Carlo Emanuele I as the theorems are presented as answers to the patron’s questions. For instance, the first one is introduced as follows: “The very serene duke of Savoy asked me to prove by means of science and speculation (as one says) that the product of two fractions is inferior to either factors.”<sup>2</sup> The curiosity the dukes held for mathematical matters was not idle but rather rooted in a deep comprehension of the importance of practical mathematics in military and civil affairs. However, courtly etiquette required that serious matters be

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<sup>1</sup>Benedetti 1585, f. A2r. Note that the pagination A1–A4 is doubled: the first installment A1–A4 contains frontispiece, table of contents, dedicatory epistle, and preface to the reader, whilst the second installment includes the first eight pages of the *Theoremata arithmetica*. Since the latter has also a pagination number (1–8) we will quote from the first fascicle indicating the folio and from the second giving the page.

<sup>2</sup>Benedetti 1585, 1: “Interrogavit me Serenissimus Dux Sabaudiae, qua ratione cognosci posset scientificae et speculative (ut dicitur) productum ex duobus fractis numeris, quolibet producentium minus esse.”

concealed under the mask of aristocratic detachment and disinterestedness. The court was a refined cultural center that also functioned as a political headquarters, where the most important decisions had to be taken. “Reality and imagination—as has been remarked—prescribed that at court the weight of the duty of government be mirrored by the lightness of amusement.”<sup>3</sup>

Following his example [that of the Duke] [...] many asked my advice either in person or by mail on those mathematical problems. As I never avoid work in support of friends, it happened that, after so many years, looking at my paper boxes [*scrinia*] I found so many solved problems that they could be gathered in a fairly big volume.<sup>4</sup>

Benedetti expressed his admiration for the stimulating intellectual environment made possible by the magnificence of Carlo Emanuele I and the legacy of his father. The wide range of scientific interests shared by both dukes is emphasized in the concluding remark of the dedicatory letter:

Therefore, the glory [of your Highness] will equal that of the ancient Persian kings, and we can expect great happiness in this century if Plato’s prophecy is correct: the future State in which princes philosophize will be blissfully happy.<sup>5</sup>

Benedetti’s preface to the reader,<sup>6</sup> following the dedicatory epistle, provides some more information related to his mathematical-physical work. The author repeats that the *Diversae speculationes* are a miscellanea of thoughts on various subjects brought about by his own curiosity or by that of patrons and friends. He declares himself confident that, in spite of their disordered format, his speculations will be considered useful, and makes a further claim for the absolute originality of his ideas: “*non dubitans quin illis [meis scriptis] in illis scientiis aliquid commodi atque utilitatis allatura sint, praesertim cum in eiusmodi quaestionibus investigandis atque perpendendis, nemo (quod sciam) hactenus elaboraverit*” (as I have no doubt that these [writings of mine] will bring something pleasant and useful, especially because (to my knowledge) nobody has so far sufficiently investigated and pondered such issues).<sup>7</sup> The enthusiasm for novelty that was to motivate the supporters of modernity in the *querelle des anciens et des modernes* can be sensed in these words. Moreover, Benedetti does not exclude the possibility that different people, at different times and in different places, could have made the same discoveries by treating similar problems:

<sup>3</sup>Barberis 2017, xvii.

<sup>4</sup>Benedetti 1585, f. A2r: “Illiusque imitatione [...] non pauci aut praesentes, aut per litteras me de his, atque illis mathematicis quaestionibus consuluerunt. Cumque ego nunquam laborem amicorum causa defugerim, evenit ut post tot annorum curricula, mea scrinia scrutatus, invenerim tot absolutas quaestiones, ut ex eis corpus mediocre effici posse videretur.”

<sup>5</sup>Benedetti 1585, f. A2v: “Quare, et veterum Persarum Regum gloriam [tua celsitudo] aequavit, et nos veluti in spem certam faelicitatis huius saeculi induxit, si verum est Platonis vaticinium, beatam eam futuram Republicam in qua Principes philosophentur.”

The comparison between the Duke of Savoy and the ancient Persian kings is in line with an established Renaissance topos derived from classical sources, in particular Xenophon’s *Life of Cyrus*, as discussed by Vester 2007, 228-229.

<sup>6</sup>Benedetti 1585, “Ad lectorem,” ff. A3r–A4r.

<sup>7</sup>Benedetti 1585, f. A3r.

In fact, I included [*traditum est*] nothing in these books that, as far as I remember, I read or heard from others. If I picked up on the suggestions of others, either I offered a somehow different demonstration or I wrote on the same subject more clearly. In the case that somebody else wrote the same ideas [*eadem tradidit*], either I was not informed about this person's speculations or the memory of these readings has vanished. Aristotle himself remarked that it can easily happen that many come to the same ideas. It can even happen that, writing so much, I repeat a certain issue having forgotten that I already wrote on it. Indeed, this has happened to me a few times.<sup>8</sup>

Benedetti adds that very few people ever wrote books which are entirely and solely the fruit of their own mind, except perhaps for Archimedes. Following in the footsteps of this illustrious predecessor, he presents the results of his personal investigations as independent from any authority. He only acknowledges some influence from Tartaglia and a few other authors on whose works he relies: “Hieronymus Cardanus, Michael Stifelius, Gemma Frisus, Ioannes Novimagius, Cuthebertus Tonstallus, caeterique huiusmodi.” The omission of Del Monte as a source on mechanics is striking and telling about the enmity between the two men, which is well documented and will be discussed later.

Benedetti envisages possible criticism of his work and seeks to anticipate objections with a remark inspired by a humanistic sense of relativism: “Quot capita, tot sententiae (As many heads, so many opinions).”<sup>9</sup> Since a book will never receive universal approbation, he writes, it must suffice to provide fruitful insights which will encourage others to undertake further investigations. He mentions only three ancient models: Ptolemy, Euclid, and Pythagoras. Whereas the reference to the first two men can be seen as an appreciation of the most valuable ancient sources on geometry, mathematical astronomy (and astrology), and geography, the mention of Pythagoras explicitly refers to astronomy and should be understood as a reference to Copernicus's “restoration” of heliocentrism.<sup>10</sup>

The letter to the reader ends up with a reflection on and a rebuttal of the principle of authority. Benedetti claims that mathematical and natural investigation should only be inspired by the love for truth. All considerations which do not derive from an open-minded philosophical attitude ought to be dismissed as unfounded:

In order to establish the truth, I occasionally had to oppose the opinions of others in many places, but I do not want you to ascribe it to some vice of mine nor to call me a malevolent and a sycophant as I display the errors of others. They should rather be thankful to me, since I aim to erase wrong opinions while I am dealing with the same issues—according to Antisthenes, it is indeed necessary “to begin by unlearning errors.” I show the truth, which all philosophers, beginning with Aristotle, should hold in higher esteem than any human authority or favor. As you will encounter something of this sort in my volume, I beg you to abandon all passions in your judgement, keeping in mind Sallust's admonishment: “Those who assess controversial issues should avoid all hate, friendship, rage, and compassion.” Thus, always favor truth,

<sup>8</sup>Benedetti 1585, f. A3r: “Nihil enim his libris a me traditum est, quod aut legisse, aut ab aliis audivisse meminerim, nam si aliena attigi, ea, aut cum aliqua differentia demonstrationis, aut dilucidius scripsi, quod si forte alius eadem tradidit, aut eius lucubrationes ad me non pervenerunt, aut earum perlectionis memoria excidit. Ut etiam Aristoteles ipse sensit facile fieri potest, ut pluribus eadem opiniones in mentem veniant. Immo multa scribendi evenire potest, ut cum iamdiu aliquid scripserit, iam oblitus, idem repetat, quod mihi etiam nonnunquam accidit.”

<sup>9</sup>Benedetti 1585, f. A3v.

<sup>10</sup>Omodeo 2014a, “The Invention of the Pythagorean Cosmology,” 167ff.

which is worthy of the greatest efforts, instead of some person, as too many do. Hopefully you will benefit from my work and, in case you will pick some fruit, in the first place you should be thankful to Him from Whom all sciences descend.<sup>11</sup>

### 3.2 Mathematical Sections

Benedetti proved his capacity as a mathematician in his early writings, especially in the *Resolutio omnium Euclidis problematum* (1553). The relevance of this publication for the ongoing debates of the time, involving Ferrari, Cardano, and Tartaglia has often been stressed. For instance, Moritz Cantor, in his classic history of mathematics, *Geschichte der Mathematik*, regarded Benedetti as “*ein wirklicher Geometer*” on account of his treatment of geometrical problems using a compass with fixed opening.<sup>12</sup> Special mathematical problems scattered in the *Diversae speculationes* have been summarized by Bordiga. We will deal with some of these problems in other sections, in connection with other aspects of Benedetti’s work such as mechanics, natural philosophy, and astronomy. For the time being, we will limit our treatment to the two books of the *Diversae speculationes* specifically dealing with mathematics.

#### 3.2.1 Geometrical Demonstrations for the Solutions of Arithmetic Problems

Two books of the *Diversae speculationes* are devoted to mathematics in the strict sense, Book 1 to arithmetics and Book 5 to proportions. Additionally, several letters in Book 6 are dedicated to mathematical problems and some metaphysical and physical issues are also treated as geometrical problems. In Book 4 Benedetti discusses examples such as the perfection of the circle, i. e., whether the circle, owing to its properties, has to be seen as the “first” or the “last” of the geometrical figures, and the possibility of a never-ending motion on a finite line or the possibility of an infinite motion on a finite line.

Book 1 of the *Diversae speculationes* has the form of a collection of mathematical exercises. Although Benedetti did not give them a systematic order, he claimed that his readers might find many useful explanations and remarks.<sup>13</sup> He chose to call his propositions “theorems” instead of “problems” to stress their originality, as Benedetti believed they deserved higher recognition than mere problem-solving. He also apologized for the brevity of his treatment by saying that he had confidence in the intelligence and expertise of his readers (the same remark can be generalized for many other sections of the work). In Book 1 he approached arithmetic problems geometrically following Euclid’s example in Book 2 of the *Elements*. This geometrical approach finds a legitimization in epistemological considerations about the role of geometrical visualization for the comprehension

<sup>11</sup>Benedetti 1585, ff. A3v–A4r: “Quoniam vero multis in locis accidit, ut veritatis iudicandae causa necesse mihi fuerit quorundam sententiis adversari nolim te hoc mihi vitio tribuere, meque hoc nomine carptorem maledicumque habere quod alienos errores aperiam, cum potius habenda sit mihi gratia, quod in iis interdum laborans (quae Antisthenes in disciplinis magis necessaria esse dixit, *ut mala scilicet prius dediscantur*) falsas opiniones evellere studeam, veritatemque ostendere, quam omnis philosophus, Aristotelis exemplo, pluris quam cuiusvis hominis auctoritatem, aut gratiam facere debet. Cumque in hoc volumine aliquid eiusmodi legeris te oratum volo, ut in iudicando, affectum omnes exuas, Sallustianum illud prae oculis habens. *Omnes qui de rebus dubiis consultant, ab odio, amicitia, ira, atque misericordia vacuos esse decet*. Hinc fiet, ut non personae (ut multi solent) sed veritati, quae summo studio dignissima est, semper potius faveas. Vale nostrisque laboribus utere, si quem inde fructum, sicuti spero tuleris, illi precipue habes gratiam a quo omnes fluunt scientiae.”

<sup>12</sup>Cantor 1892, 521–525.

<sup>13</sup>Benedetti 1585, f. A3v.



of truth: “images are necessary to the intelligent one in order to speculate” (*quoniam oportet intelligentem phantasmata speculari*). In Benedetti’s eyes, the process of learning and thinking requires images (*phantasmata*).

Moreover, Benedetti informs his reader about the process through which knowledge is acquired. In other words, his treatment is not restricted to theory but intentionally expands on heuristics. He complains that ancient mathematical knowledge was often transmitted in a very concise manner. It often lacked demonstration and definitions, or a clarification of the fundamental concepts. For this reason modern readers are often forced to investigate the hidden reasons of “the numbers” and “their effects,” with huge effort:

As we know, ancient mathematician-philosophers discovered many properties of the numbers but transmitted them to posterity either without reasons or with too few ones. Hence, several mathematical problems emerged, which have been addressed by the duke of Savoy. I consider some of the ensuing reflections on the ancients’ propositions to be worth transmitting to posterity. In this manner, my speculations will not fall in oblivion. Rather, I will offer to many an occasion to investigate abstruse themes which are entailed in problems and theorems and could hardly find an explanation so far.<sup>14</sup>

In order to examine Benedetti’s geometrical approach to arithmetic problems, let us consider theorem 120. Here is the problem:

The ancients already addressed this problem: three associates have an amount of money. The sum of the [money of] the first and the second is known, as well as the sum of the first and the third and the sum of the second and the third. From such three aggregates [the ancients] derived the particular [amount of money] of each one of them.<sup>15</sup>

Following a method of resolution, which Benedetti ascribes to Gemma Frisus, he offers the solution to a case chosen arbitrarily:

Gemma Frisus solves this problem applying the *regula falsi* [rule of the false]. I will follow the same path. Suppose that the addition of the first with the second is 50, that of the second with the third 70 and that of the first with the third 60. From those sums take any two, for instance 50 and 70, whose addition is 120. Subtract from this sum the other one, that is, 60. The result is 60. Its half is 30. This is the amount of money of the second associate. If you subtract this number from 70 (which is the addition of the second with the third) you will get 40. This is the amount of the third associate. Finally, from this number taken away from 60 you will be able to derive the amount of the first associate.<sup>16</sup>

<sup>14</sup>Benedetti 1585, 1: “Praeclare multa veteres mathematici philosophi de numeris eorum effectibus excogitata posteris tradiderunt, quorum cum vix ullam rationem reddiderint, aut certe per exiguam, occasione diversorum problematum mihi Serenissimo Sabaudiae Duce propositorum praebita, de iis quae ab antiquis proposita fuerunt contemplanda nonnulla occurrerunt, quae posteritate commendare non inutile arbitratus sum, ne heae meae cogitationes interciderent, et occasionem praeberem quamplurimis abstrusa haec indagandi, quae problematibus et theorematibus involuta, vix aliquem qui evolveret nacta sunt.”

<sup>15</sup>Benedetti 1585, 81: “Supponunt etiam antiqui tres socios nummos habere, quorum summa primi et secundi cognita sit, item summa primi et tertii cognita et summa secundi et tertii item cognita, atque ex huiusmodi tribus aggregatis veniunt in cognitionem particularem uniuscuiusque illorum.”

<sup>16</sup>Benedetti 1585, 81: “Gema Frisus solvit hoc problema ex regula falsi. At ego tali ordine progredior. Sit verbi gratia, summa primi cum secundo 50 et secundi cum tertio 70 et primi cum tertio 60; harum trium

If we express the problem in modern form, this is the set of equations Benedetti is dealing with:

$$x + y = 50$$

$$y + z = 70$$

$$x + z = 60$$

The algebraic solution devised by Benedetti is the following:

$$2y = 50 + 70 - (x + z)$$

$$y = \frac{120 - 60}{2} = 30$$

Thus,

$$z = 70 - 30 = 40$$

and

$$x = 60 - 40 = 20$$

After this solution, Benedetti offers a geometrical demonstration of the validity of this procedure, referred to as “Gemma Frisus’s *regula falsi*.” For this purpose he draws a triangle with an inscribed circle (Figure 3.1) and supposes that the three sides correspond to the sums that we have expressed as a set of equations.

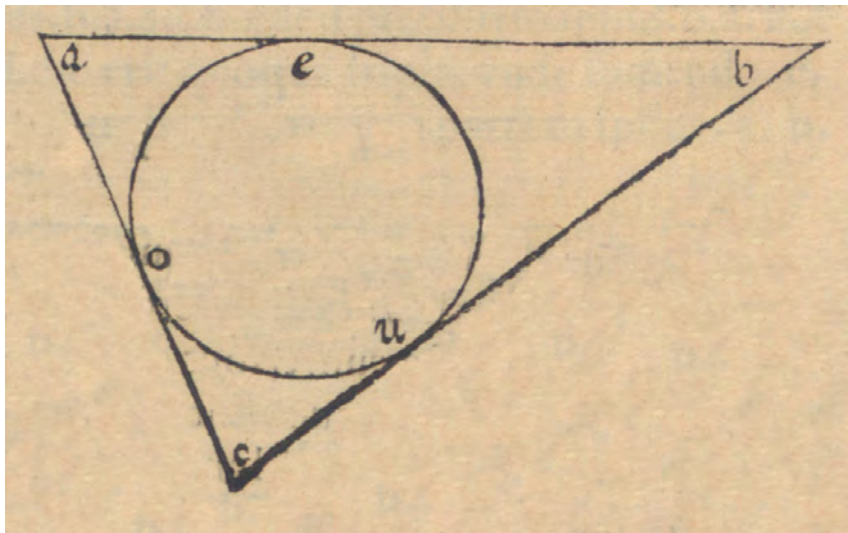


Figure 3.1: Geometrical demonstration for the solution of a particular set of equations. Benedetti, *Diversae speculationes*, Book 1, Theorem 120, p. 82. (Max Planck Institute for the History of Science, Library)

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summarum accipiantur duae quaevis, ut puta 50 et 70, quae coniunctae simul dabunt 120 a qua summa detrahatur reliqua, idest 60. Et restabit nobis 60, cuius medietas ergo 30, hoc est numerus nummorum secundi socii; quo numero detracto a 70 (hoc est a summa secundi cum tertio) remanebit 40, hoc est numerus tertii socii; et adhuc numerus desumptus a 60 residuus ergo numerus primi socii.”

Benedetti's demonstration here follows:

In order to grasp this problem, let us consider the triangle here indicated as  $ABC$  each side of which corresponds to the addition of [the money of] two associates—for instance, assume that the side  $AB$  corresponds to the addition of the first with the second whereas the side  $BC$  to the addition of the second with the third and the side  $AC$  to the addition of the first with the third. Thereby  $AE$ , or  $AO$ , should be the number [i.e., the amount of money] of the first associate;  $EB$ , or  $BU$ , should be the second associate; and  $CU$ , or  $CO$ , should be the third. Furthermore, since  $AE$  is equal to  $AO$ ,  $BE$  to  $BU$  and  $CU$  to  $CO$ , according to premises, if we derive the sum or side  $AC$  resulting from the addition of the side  $AB$  with  $BC$  corresponding to the other sums, we will know the addition of  $BE$  with  $BU$ . Hence, we will also know the half, which is  $BE$  or  $BU$ . Once we subtract it from the sum  $BA$  the resultant will give us the number [corresponding to]  $AE$ . By subtracting the number of  $AE$ , that is  $AO$ , from the sum (or side)  $AC$ , and [by subtracting]  $BU$  from  $BC$  we will know as a result  $OV$  and  $CU$ .<sup>17</sup>

We can (anachronistically) formalize Benedetti's reasoning in the following manner.

$$\begin{aligned} ab &= x + y \\ bc &= y + z \\ ac &= x + z \\ ae &= ao = x \\ eb &= bu = y \\ cu &= co = z \end{aligned}$$

The geometrical visualization of Frisus's method, the so-called *regula falsi*, corresponds to the addition of two sides of the triangle and the subtraction of the third one:

$$ab + bc - ac$$

This corresponds to the following addition:

$$\begin{aligned} ab + bc - ac &= \\ &= (ae + eb) + (bu + cu) - (co + ao) = \\ &= ae + eb + eb + cu - cu - ae = 2eb = 2y \\ &\quad c.v.d. \end{aligned}$$

<sup>17</sup>Benedetti 1585, 82: "Pro cuius ratione consideremus triangulum adhuc subnotatum  $abc$  cuius unumquodque latus significet summam duorum sociorum, ut puta latus  $ab$  significet summam primi cum secundo, latus vero  $bc$  summam secundi cum tertio, latus autem  $ac$  summam primi cum tertio, et  $ae$  seu  $ao$  sit numerus primi socii, et  $eb$  vel  $bu$  sit secundi socii, et  $cu$  seu  $co$  sit tertii, cum autem  $ae$  aequalis sit  $ao$  et  $be$  aequalis  $bu$  et  $cu$  aequalis  $co$  ex supposito si de  $\langle su \rangle$  mpta fuerit summa seu latus  $ac$  datum ex aggregato laterum  $ab$  cum  $bc$  reliquarum summarum, relinquet nobis cognitum aggregatum ex  $be$  cum  $bu$ . Quare et eius medietas  $be$  sive  $bu$  nobis cognita erit, qua detracta ex summa  $ba$  relinquetur nobis cognitus numerus  $ae$ , detracto vero numero  $ae$  hoc est  $ao$  ex  $ac$  summa, seu latus, aut  $bu$  ex  $bc$  remanebit  $oc$  seu  $cu$  cognitus."

### 3.2.2 Theory of Proportions

Book 5 of the *Diversae speculationes* deals with the fifth book of Euclid's *Elements*, on proportions. This book of the *Elements* was a notorious case in medieval translations, as the definitions were often misunderstood, inconsistent interpolations had been added, and flaws introduced in the demonstrations. In his Italian version, Tartaglia tried to emend these flaws. Commandino's and Clavius's attempts followed suit. Moreover, a certain number of Euclidian definitions (especially the fifth and the seventh) could not be simply taken for granted, but required analysis and justification.<sup>18</sup>

Hence, Benedetti proposed to introduce new postulates which he considered to be clearer (*intellectui commodiora*) in order to render the theorems that rely upon them more easily understandable (*quandoquidem iis nostri postulatis admissis, sequentia Theorematum perfacillima reddentur*). Actually, what he undertook was a revision of Euclid's theory of proportions, based on a reorganization of postulates and theorems. As Enrico Giusti stressed, despite its brevity, this booklet by Benedetti stands out as "the first modern attempt to accomplish an organic reform of the fifth book of Euclid's elements."<sup>19</sup> The difficulty that he and his contemporary readers of Euclid had to face was not technical but rather foundational, as it resided in the clarity of the definitions and the internal coherence of the theory. For the sake of intelligibility, Benedetti thus inserted two preliminary axioms and reversed the relation between the most simple of the Euclidean theorems and the less intuitive of the axioms in order to establish a gradual progression from the most simple to the more complex propositions. The first two postulates are derived from Euclid's common notions II and III concerning the addition and subtraction of equal parts to equal quantities.<sup>20</sup>

Benedetti rephrases Euclid's common notions translating the addition (or subtraction) of parts as the composition of ratios:

[First:] All additions [*composita*] of an equal number of equal parts are equal. Similarly, all proportions are equal that result from the addition of an equal number of other proportions, which are equal among themselves. Euclid tries to demonstrate this in [*Elements*] V 22 and 23.

Second: If one subtracts equal parts from equal wholes, the remaining parts are equal among them. Conversely, if one adds equal parts to equal wholes, the additions will be equal among them. One should consider these considerations as referred to proportions.<sup>21</sup>

<sup>18</sup>In Commandino's Italian translation (Euclid 1575, ff. 63r–v), these definitions go as follows:

"V. Le grandezze si dicono essere nella medesima proporzione, la prima alla seconda, et la terza alla quarta, quando le ugualmente molteplici della prima, et della terza, o vero insieme avanzano le ugualmente molteplici della seconda, et della quarta secondo qual si voglia moltiplicatione, o vero insieme le pareggiano e vero insieme sono avanzate da loro."

"VII. Quando delle ugualmente molteplici, la moltiplice della prima vanterà la moltiplice della seconda, et la moltiplice della terza non avanterà la moltiplice della quarta, allhora la prima alla seconda si dirà haver maggior proportione che la terza alla quarta."

<sup>19</sup>Giusti 1993, 22.

<sup>20</sup>In Commandino's Italian translation, these definitions go as follows. See Euclid 1575, f. 7r:

"II. Se alle cose uguali, si aggiungono cose uguali, tutte sono uguali fra loro.

III. Se dalle cose uguali si traggono cose uguali, etiamdio le rimanenti sono uguali tra loro."

<sup>21</sup>Benedetti 1585, 198: "[Primum.] Quod tota composita ex aequali numero partium aequalium, sunt invicem aequalia. Ut si quis diceret omnes proportiones quae compositae sunt ex aequali numero aliarum proportionum invicem aequalium, sunt etiam invicem aequales, quod Euclides conatur demonstrare in 22. et 23. quinti libri.

The postulates that follow (3–9) are Euclid’s propositions 7–13 with minimal changes in the order (the inversion of Euclid’s propositions 8 and 9). Benedetti adds three additional postulates (10–12) before he tackles an explanation of Euclidian theorems on proportions. What is lacking is an explicit treatment of the definitions underlying Benedetti’s “reform” of the theory of proportions.<sup>22</sup>

Bordiga underscored the didactic character of Benedetti’s treatment of Book 5 of the *Elements* by remarking that his concern about clarity and simplicity might have emerged from the teaching of mathematics to his princely pupils in Turin.<sup>23</sup> This might be true; however, the theoretical relevance of this section and of Benedetti’s effort to reform the theory of proportions should not be neglected. During the sixteenth century, geometry, in particular the theory of proportions, was gradually becoming the fundamental tool for the treatment of physics in the process toward a mathematical inquiry and explanation of natural phenomena. In this context, Book 5 of the *Elements* acquired a higher theoretical status by offering a sort of “meta-geometry” or *mathesis universalis*.<sup>24</sup>

The universal meaning of geometry as the foundation of rationality itself had been emphasized in the generation before Benedetti in an emphatic *Encomium geometriae* (*Eulogy of Geometry*) delivered by Girolamo Cardano at the *Academia Platina* of Milan in 1535. In this talk he presented geometry as the highest science, or as a sort of *prisca scientia*, by contending that geometrical rationality, based on quantity and proportion, is the source for all arts and disciplines. Indeed, the *modus geometricus* is the essence of rationality and even the *a priori* of God’s Creation. In Cardano’s eyes, geometry was also a practical discipline insofar as it included statics, mechanics, and architecture as subordinate disciplines. Actually, Cardano’s list of arts and sciences that depend on geometrical rationality (committed to the study of quantities and proportions) is long. Geometry, as one reads in his *Encomium*, is fundamental for arithmetic, music, astronomy, and optics. It is the *conditio sine qua non* of architecture (*non aedificare sine illa licet*) and of the plastic arts, painting and sculpture. Geometry is necessary for the construction and understanding of clocks and machines (*horologiorum, machinarumque structura*). It is further presupposed by natural magic, by the science of weights, by aesthetics (*pulchritudo... tota geometrica ratione constat*), and by countless other fields of human activity and knowledge.<sup>25</sup>

Benedetti agreed on the fundamental relevance of geometry as the cornerstone of natural inquiry. The fact that his treatment of proportions, although it was very short and condensed, was printed as a book in its own right in the *Diversae speculationes* bears witness to the relevance he attached to this part of mathematics. In fact, it was crucial for his treatment of weights and thus a close examination of Book 5 of the *Elements* was an indispensable premise of his mechanics. As Giusti emphasized, Benedetti’s treatment of composite propositions (relating to Euclid’s proposition 17) was the most significant

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Secundum. Quod si a totis aequalibus detractae fuerint aequales partes, quae remanent eruntque partes invicem aequales. Et e converso si aequalibus aequalia addas composita erunt invicem aequalia. Quod in ipsis proportionibus hoc loco semper intelligendum est.”

<sup>22</sup>Cf. Giusti 1993, 27 quoting from Antonio Nardi: “Il Benedetti, Geometra insigne non si accorse, che volendo riformare il 5° libro di Euclide, trascurò la definizione della uguale, e disuguale ragion, quale principio e fondamento dell’opera. Stupiscomi certo di tale inavvertenza.”

<sup>23</sup>Bordiga 1985, 629.

<sup>24</sup>Giusti 1993, 22.

<sup>25</sup>Cardano 1966, vol. 4, 440–445.

aspect of his theory insofar as it offered an elegant computational tool.<sup>26</sup> As a matter of fact, Galileo and his school would follow the same track by applying composite propositions to the analysis of functional relations in physics, in particular to motion. Hence, from the viewpoint of natural inquiry, Benedetti's Book 5 is not exclusively motivated by an abstract interest in pure mathematics but by the challenging problems of contemporary physics.<sup>27</sup>

### 3.3 The Geometrical Theory Underlying Linear Perspective

Book 2 of the *Diversae speculationes*, entitled *De rationibus operationum perspectivae* (Reasons of Perspective Operations), is a short treatise on linear perspective, that is, the optical discipline dealing with the construction of perspective to give the illusion of depth. Its main task was to recreate the "cone of vision." Historically, it originated from the practical problems of three-dimensional representation in the fine arts but also had relevance for architecture (e.g., in surveying or in theoretical treatises), the military art (e.g., the derivation of the structure of the enemy's fortification from a scout's sketches), and found special application in theatrical stage scenery.

Benedetti's book on perspective begins with a claim of originality: "To my knowledge nobody has so far taught the true and inner causes of the operations in perspective in an accomplished manner. Thus, I deemed it to be worth undertaking some reflection [*disputationem*] in this field."<sup>28</sup> The fact that "nobody" (*nullus*) has adequately treated perspective before is attested to by the great number of misunderstandings and widespread errors in this discipline: "In fact, many of those who prescribe the rules of such operations ignore the implications of the true causes, therefore they make various mistakes, as for instance in the following plane figure A [etc.]."<sup>29</sup> Benedetti's treatment was indeed different from the tradition of practical treatises (from Piero della Francesca's *De prospectiva pingendi* onwards) explaining *how* to construct a perspective picture owing to its higher mathematical sophistication and theoretical depth. Moreover, it took a different angle than Federico Commandino's discussion of perspective in connection with mathematical astronomy and geography in his edition of Ptolemy's *Planisphaerium* (1558), which treats it as mathematically equivalent to stereographic projection. Benedetti's focus was the explanation of the reasons underlying perspective, as well as errors that might occur in theory and practice. In a manner that is similar to his treatment of problems of arithmetic, geometry offers the conceptual tools to formalize knowledge embedded in practice.

Like most parts of the *Diversae speculationes*, this book on perspective originated from extemporary sources of inspiration. In fact, it begins *ex abrupto* with the examination of an error in linear perspective which leads Benedetti to more general considerations. Moreover, the dedication of its seventh chapter to the architect Giacomo Soldati strengthens the impression of the occasional character of the writing.<sup>30</sup>

<sup>26</sup>Giusti 1993, 33: "D'altronde, l'interesse della teoria di Benedetti non sta nel risultato globale, ma soprattutto nel ruolo chiave della proporzione composta, e in particolare nella creazione di un algoritmo di calcolo agile ed elegante." See Benedetti 1585, 202.

<sup>27</sup>See Giusti 1993, chap. 2, section 2.

<sup>28</sup>Benedetti 1585, 119: "Cum nullus adhuc (quod sciam) veras internasque causas operationis perspectivae perfecte docuerit, operaeprecium existimavi aliqua de iis disputationem suscipere."

<sup>29</sup>Benedetti 1585, 119: "Multi enim eorum, qui huiusmodi operationis regulas praescribunt, cum eius effectuum veras causas ignorent, varios diversosque errores committunt, ut exempli gratia in subscripta figura superficiali A [etc.]." See Figure 3.2.

<sup>30</sup>Benedetti 1585, 133: "Superioribus diebus non diu postquam de perspectivis inter nos sermonem habuimus, dum animus totus adhuc in his esset. Illud in mentem venit quod eximius ille vir, et profundissimae

The error discussed in the *incipit* of the *De rationibus operationum perspectivae* is represented in a two-dimensional figure (Figure 3.2).

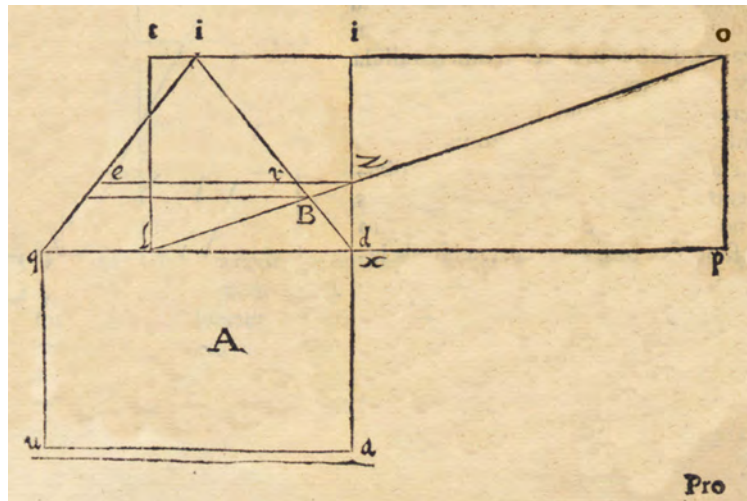


Figure 3.2: *Figura superficialis*, or plane diagram, pointing out an error of linear perspective. From Benedetti, *Diversae speculationes* II, p. 119. (Max Planck Institute for the History of Science, Library)

Benedetti explains how to correctly construct the linear perspective “inscribed” in the given triangle  $IDQ$ :

[...] in the plane figure  $A$  (here included), in order to ‘degradate’ (as one says) [i.e., to put it in perspective] the rectangle  $QA$  [ $QXAU$ ] in the triangle  $IDQ$ , they [the practitioners] draw a line parallel to  $QD$  from point  $B$  (at the intersection of line  $OL$  with the side  $ID$  of the triangle) or, indifferently, they draw it from point  $Z$  (the intersection of the same  $OL$  with the perpendicular  $XI$ ). They are not aware that the latter method is the only correct one, not the former. Conversely, if the former were correct, the latter could not be true. They might excuse themselves by arguing that one draws the aforesaid parallel line from  $B$  supposing that the plane  $IDQ$  is inclined horizontally relative to the rectangle  $QA$  according to the angle  $IDQ$ . But this excuse cannot be accepted because, according to their supposition, if one assumes that  $IDQ$  is inclined, the inferior angles of the rectangle put in perspective should not be as acute as  $IDQ$  and  $IQD$ . This can be easily understood considering their construction [*ratio*], which I show in the plane figure  $A$  here included. However, if one wants to see the rectangle in perspective, one should locate this plane relative to the eye in the same manner as the line  $ID$  relative to  $O$ . But this is too difficult [to represent].

doctrinae, nec unquam satis laudatis Daniel Barbarus se accepisse profitetur a Ioanne Zamberto patritio Veneto, qui ad verbum omnia desumpserat a Ioanne Cusino Parisiense. Nec parum mirabar peritissimus illum Cusinum, quod in capite quarto secundae partis perspectivae, ut quod piam planum quadrilatam composuisset. [...]” See Mamino 1989.

To summarize, the correct manner [to put a rectangle in perspective] is to draw a line  $ER$  parallel to  $QD$  from point  $Z$ , which is common to  $OI$  and  $XI$  (perpendicular to  $LP$ ).<sup>31</sup>

In order to visualize the construction, Benedetti produces an additional diagram (Figure 3.3) offering the tridimensional correspondent of the plane diagram he has just examined. Note that point  $O$  is the point of departure of the cone of sight. The observer is thus lying with his feet in point  $P$ .

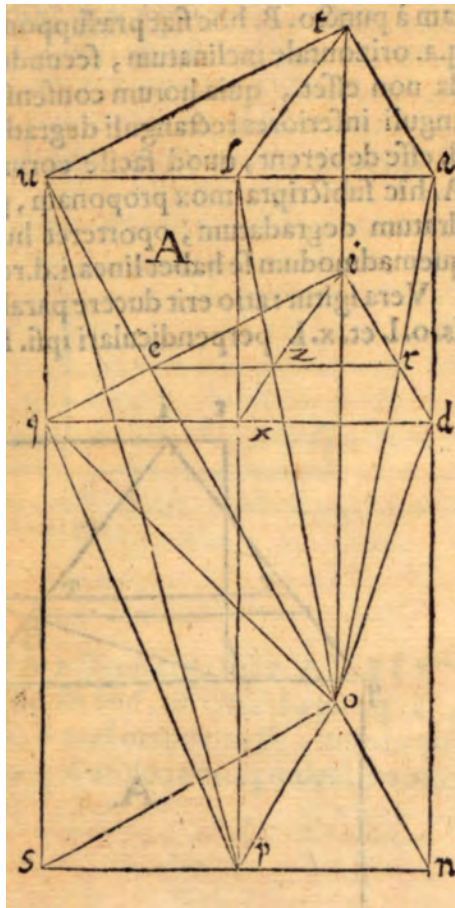


Figure 3.3: *Figura corporea*, or tridimensional diagram, visualizing the same problem of linear perspective as in Figure 3.2. From Benedetti, *Diversae speculationes* II, p. 120. (Max Planck Institute for the History of Science, Library)

Departing from these considerations, Benedetti treats various cases (including the one in which the projection plane is not perpendicular to the observer's line of sight) using the same approach. That is to say, he first draws bi-dimensional diagrams showing the construction and then offers tridimensional geometrical explanations providing an insight into the physical reality underlying the construction. He even offers a sort of virtual instrument

<sup>31</sup>Benedetti 1585, 119: “[...] in subscripta figura superficiali  $A$  volentes degradare (ut dicunt) rectangulum  $qa$  in triangulo  $idq$  ducunt parallelam ipsi  $qd$  a puncto  $B$  intersectionis lineae  $ol$  cum latere  $id$  trianguli, et (idem) indifferenter, eandem quoque a punctoque  $Z$  intersectionis ipsius  $ol$  cum perpendiculari  $xi$  ducunt, nescientes hunc solum esse verum modum, non item alium, quia si alius, talis esset, hic, verus non existeret, nam si vellent sese excusare, quod ducendo dictam parallelam a punctoque  $B$  hoc fiat praesupponendo planum ipsius  $idq$  versus rectangulum  $qa$  horizontale inclinatum, secundum angulum  $idq$ , haec excusatio accipienda non esset, quia horum consensu, praesupponendo planum  $idq$  inclinatum, anguli inferiores rectanguli degradati, non tam acuti, quae sunt duo  $idq$  et  $iqd$  esse deberent, quem facile eorum ratione innotescet, quae de figura corporea  $A$  hic subscripta mox proponam, praeter id, quod volentes deinde aspicere quadratum degradatum, oporteret huiusmodi planum respectu oculi ita collocare, quemadmodum se habet linea  $id$  respectu  $o$  quem factu nimis arduum esset.

Vera igitur ratio ergo ducere parallelam  $er$  ad  $qd$  a puncto  $Z$  communi ipsis  $ol$  et  $xi$  perpendiculari ipsi  $lp$ .”



to guide constructions in linear perspective.<sup>32</sup> As Judith V. Field has argued, Benedetti “shows the applied mathematician’s concern with particular problems, rather than the pure mathematician’s interest in theoretical developments. The significance of his work lies not only in its mathematical insight but also in the fact that it shows us innovative work in the purest of sciences, namely pure mathematics, being carried out within an applied tradition.”<sup>33</sup> As has been argued, it is plausible that Benedetti’s constructions were included in later works on perspective, in particular by Simon Stevin and Guidobaldo del Monte, although none of them explicitly referred to Benedetti. Apart from Del Monte’s enmity towards him, it was typical of Renaissance mathematicians who wrote on optics to leave their sources unmentioned.<sup>34</sup>

### 3.4 Sections on Physics: Mechanics and Natural Philosophy

Book 3 of the *Diversae speculationes* deals with mechanics. Benedetti remarks that many learned people have already written extensively on this issue, but that their achievements are not sufficient because nature and practice (*usus*) awaken many doubts concerning the validity and the completeness of previously acquired knowledge. New problems have emerged. His intention is to present many novelties and to propose theses that either have never been treated or have never been adequately demonstrated (*nusquam ante hac tentata, aut satis exacte explicata*). In the final words of his introduction to Book 3, we can see that he attaches great importance to his speculations on mechanics; he even states that he will be especially remembered after his death for his contribution to the advancement of this discipline:

Many man have written a great deal about mechanics, and they have written most ably. But nature and experience are always wont to bring to light something new or previously unknown. And it is therefore incumbent upon a high-minded and grateful individual, if he happens to discover something previously shrouded in darkness, not to begrudge it to posterity. For he himself gained a great deal from the work of others. Now it is my desire to publish a few items that will, I believe, prove not unwelcome to those who concern themselves with mechanics, items which have never before been dealt with or have not been sufficiently well explained. I may thus either show my desire to be helpful or at least give some evidence of possessing a bit of talent and industry. And perhaps in this way alone may I leave behind me proof that I ever lived at all.<sup>35</sup>

Book 4 essentially deals with Aristotle’s opinions on local motion in *Physica* and *De caelo*, although critical references to *Metaphysica* and *Meteorologica* are also present. In this section, Benedetti seeks to provide new foundations to the theory of motion from a

<sup>32</sup>Benedetti 1585, 131.

<sup>33</sup>Field 1987, 247.

<sup>34</sup>See Andersen 2007, 152

<sup>35</sup>Translation from Drake and Drabkin 1969, 166. Cf. Benedetti 1585, 141: “Scripserunt multi multa, et quidem scitissime, de mechanicis, at cum natura ususque, aliquid semper vel novum, vel latens in apertum emittere soleant, nec ingenui aut grati sit animi, posterius invidere, si quid ei contigerit comperuisse prius tenebris involutum: cum tam multa ipse ex aliorum diligentia sit consequutus. Paucula quaedam futura, ut reor, non ingrata his qui in hisce mechanicis versantur, nusquam ante hac tentata, aut satis exacte explicata in medium proferre voluit: quo vel iuvandi desiderium, vel saltem non ociosi ingeniosi argumentum aliquod exhiberem: atque vel hoc uno modo me inter humanos vixisse testatum reinquerem.”

philosophical perspective based on a mathematical approach to nature (*inconcussa mathematicae philosophiae basis*). He revises basic concepts of physics, such as place and time, as well as natural issues, for example the existence of a physical void. Eventually, he deals with cosmological aspects linked with the theory of motion, including a defense of the Copernican system.

We will treat books 3 and 4 of the *Diversae speculationes* separately and with particular attention (in chapters 5 and 6), owing to their historical and theoretical relevance.

### 3.5 The Epistles: Miscellanea Mirroring a Scientific Network

The last book of the *Diversae speculationes* is a large collection of letters, “Physica et mathematica responsa” (Epistolary Answers on Physics and Mathematics). The autonomy of this section is underscored by the insertion, at its beginning, of a second preface.

To the reader.

Nothing is more suited to virtue than being active and present through constant motion. Just like a shining star shimmering for the eyes of the spectators. It occurred to me that this or that person invited me with words or stimulated me with letters to dispute on those issues in which I am versed. In fact, I was devoted to mathematics and to highly philosophical speculations while I sojourned in the most splendid princely courts and illustrious cities, where many intelligent people displayed admirable curiosity, desire for knowledge and conversation. I collected part of those disputes and answers, because I judged them to be worth communicating. I planned to reread and revise them, if I had some spare time, [having in mind two goals]: on the one hand, that science itself circulates as much as possible and grows; on the other hand, that the names of those excellent men, who stimulated me with their questions, are made immortal, as far as it is in my power to do so, and that I can lead others to follow their example, abandon the idle sloth (which is able to corrupt even the most talented) and start investigating, exchanging, and discussing serious subjects that could prove useful at some point and worth disseminating. For the time being, please, approach our efforts with a benevolent and judicious attitude. Regards.<sup>36</sup>

Significant elements emerge from this passage: the courtly environment influencing Benedetti’s activity, the interest in applied knowledge, the recognition of the importance

<sup>36</sup>Benedetti 1585, 204: “Ad lectorem. Ut nil magis virtutis est proprium, quam agitari, et incessabili motu prodesse. Ac veluti fulgidum sydus ante oculus spectantium commicare. Ita mihi mathematicis iisque maxime philosophicis speculationibus dedito, saepissime, ut in principium summorum aulis, et amplissimis civitatibus degenti, ubi multa semper Nobilium mira curiositate, sciendi desiderio, et conferendi cupiditate referta, versantur, ingenia, contigit, modo ab his, modo ab illis, aut verbis tentari, aut literis provocari ad disserendum, de his, in quorum studiis versamur. Quarum concertationum et responsionum, quoniam non omnino indigna existimavi, quae memoria comendarentur, partem aliquam apud me conseruivi. Ubi vero per ocium licuit, relegi, ac tandem de manu mittere decrevi. Tum ut scientia ipsa quo magis diffundetur, crescat; et quicquid valeo, sine invidia in communem utilitatem conferam. Tum ut virorum praestantissimorum, qui me suis interrogationibus excitaverunt, quantum in me erit, gratitudine ergo, nomina reddam immortalia, et eorum exemplo alios, ocio sordidiore abiecto, quod solet iurialium praecipue excelsa ingenia corrumpere, ad sciscitandum conferendum, et disserendum, de rebus seriis, et quae usui aliquando esse possint, et quandoque evulgari mereantur, alliciam. Tu interim nostris laboribus fruire, et nostram diligentiam boni, et aequi consule, et Vale.”

of dissemination, as well as the celebratory intention of the work (to render the dedicatees immortal). For us, this epistolary is the most tangible evidence of a network of social relations and its scope.

The epistles are not organized chronologically (actually the dates are almost always omitted) but rather according to the importance of the addressees, some of whom were already dead at the time of publication. The first letter was directed to the duke Emanuele Filiberto, the second to his son Carlo Emanuele I, and the following four letters to the powerful nobleman Andrea Provana de Leyn . The topics are linked to Benedetti's role as court mathematician and mathematical advisor.<sup>37</sup> The first letter is an expert report on the reform of the calendar, the second deals with a geometrical problem (the determination of the circle circumscribing a given square) that probably emerged from the lessons imparted to the prince, and the letters from three to six address practical problems of navigation (such as the use of astronomical instruments at sea).

Although it is not possible to identify all of Benedetti's correspondents, one can say with certainty that most of the epistles are tied to the northern Italian environment, specifically to Turin and to Venice. Many correspondents were in fact Subalpine or Venetian patricians or courtiers and artists linked to one of these two centers. Sometimes they were linked to both, as was the case with ambassadors such as Domenico Morosini (*Dominicus Moresinus*), Pietro Pizzamano (*Petrus Pizzamanus*), and Francisco Mendoza. There is great variety in the status and professions of the correspondents. Michela Cecchini and Clara Silvia Roero, in their accurate reconstruction, came to the following assessment: "The variety of themes that were discussed and of the professions of the participants in the debates shows that Giovanni Battista Benedetti was a man of culture and practice. He was ready to engage in a fruitful debate with exponents of the scientific world in the broadest sense (such as mathematicians, physicians, jurists, and philosophers) and with politicians, diplomats, and ambassadors, as well as with experts of military art and religion. Moreover, he did not dislike architects, artisans, constructors of instruments and fortifications, surveyors, and astrologers."<sup>38</sup>

Among his correspondents, the list of Savoy patricians and courtiers is long. Those who emerge most prominently are the orator Francesco Barbaro, who sojourned in Turin between 1578 and 1581, the Turin professor of jurisprudence Bernardo Trotto, the court historian Emanuele Filiberto Pingone, and the functionary and artillery general Giuseppe Cambiani. Benedetti advised Cambiani on ballistics, commenting upon Tartaglia to determine the most effective inclination for a cannon.<sup>39</sup> Another member of the Savoy milieu was Giovanni Battista Femello, philosopher, physician, and ordinary professor of practical medicine at Turin. Benedetti wrote to him on a very special geographical issue: the determination of the position of Iceland. By contrast, the mathematical advice directed to his friend, the jurist Francesco Ferrari, concerned ethics and justice. Benedetti explained

<sup>37</sup>"Mathematics" has to be here understood in the wide and interdisciplinary Renaissance meaning. It comprised arithmetic and geometry, as well as practical mechanics, architecture and engineering, astronomy and meteorology, optics, physics, and even metaphysical and epistemological issues.

<sup>38</sup>Cecchini and Roero 2004, 32: "Dal quadro variegato dei temi discussi e dalle professioni dei protagonisti dei dibattiti emerge la figura di Giambattista Benedetti come uomo di cultura e di pratica, disposto ad instaurare un dialogo proficuo sia con esponenti del mondo scientifico in senso lato, come matematici, medici, giuristi e filosofi, sia con politici, diplomatici e ambasciatori, come pure con uomini d'arme e di religione, non disdegnando neppure architetti e artigiani, costruttori di strumenti e di fortificazioni, agrimensori e astrologi." In the following we will often rely on Cecchini and Roero for an overview of Benedetti's correspondents.

<sup>39</sup>Benedetti 1585, 258–259.

to him how to make use of proportions in order to explain “commutative justice.”<sup>40</sup> An epistle on the quantification of the time necessary to spill the water out of a vase through three tubes, alongside another two epistles on geometry and the application of geometry to the study of solar radiation, are directed to the Savoy secretary Ludovico Niccolò Calusio. The letters on mechanics addressed to Giovanni Paolo Capra of Novara are better known to historians of science. They intermingle considerations on turning wheels with others on astronomy and the boundaries of the cosmos. Other noblemen belonging to the Turin milieu who corresponded with Benedetti were Franchino Trivulzio and Demoulin de Rochefort, responsible for the ducal *Wunderkammer*.

Benedetti had intense exchanges not only with patricians but also with artists, engineers, and practitioners. Four letters are addressed to the architect Gabriele Busca and their issues range from natural philosophy and meteorology to the use of instruments and topography. The Savoy clock-maker Jacopo Mayeto (perhaps Maletto) shared Benedetti’s interest in gnomonics, sundials, and mathematical instruments, as witnessed by one of the letters in the collection.<sup>41</sup> Furthermore, a letter to the Savoy surveyor Angelo Ferrario is at once technical and playful: it is a discussion of the curve described by a hanging rope that Ferrario and Benedetti observed together in the park of the ducal castle of Lucento by Turin.<sup>42</sup> Additionally, Benedetti wrote on music to the Flemish madrigalist Cipriano de Rore, who had been appointed in Venice, Ferrara, and Parma and whom Benedetti had probably already met in his hometown Venice. Among the artists, the Venetian poet Girolamo Fenarolo is the addressee of two letters on geometry.

Some epistles tackle the philosophical and epistemological issues underlying mathematics and their cognition. For instance, those to his Venetian friend Domenico Pisani deal with the issues “de philosophia mathematica” (on mathematical philosophy) and “de imaginatione specierum” (on the imagination of the species).<sup>43</sup> Another philosopher corresponding with him was the Turin professor Francesco Vimercato to whom several letters are addressed. While these mainly deal with optics, theses on natural philosophy and cosmology are discussed in a brief letter to the theologian Gerolamo Cordero (*Hieronimus Condrumerius*). It is entitled “Quod recte Arist[otelis] senserit coelum casu non esse productum” (Aristotle correctly saw that the heavens are not produced in a casual manner).<sup>44</sup> Benedetti moreover corresponded with the Paduan professor Pietro Catena, known as a keen supporter of the *certitudo mathematicarum* (see the next chapter) and the mathematical method applied to natural inquiry.

The identity, biographies, and work of many of Benedetti’s correspondents are still obscure. This is especially true for foreign scholars, presumably Germans, whose names were latinized as follows: Theodosius Raisestaim, Paulus Aemilius Raisestaim, Volfardus Aisestain, and Conradus Neubart. Benedetti does not give these names any titles. An exception to this is the correspondent Anselm Rosenburg, who is designated as Imperial surveyor (*agrimensor Cesareus*).

<sup>40</sup>Benedetti 1585, 284.

<sup>41</sup>Benedetti 1585, 423–425.

<sup>42</sup>Benedetti 1585, 361–363.

<sup>43</sup>Benedetti 1585, 298 and 298–299.

<sup>44</sup>Benedetti 1585, 298 and 298–299. For the identification of this person, see Cecchini and Roero 2004, 58–59.

## Chapter 4

### Epistemology

One of the most challenging aspects of Benedetti's endeavor was his attempt to merge mathematical and physical speculations, as is clearly stated in the title of the *Diversae speculationes mathematicae et physicae*. In order to understand his way to "physico-mathematics," we will discuss his mathematical epistemology starting from some statements scattered in his major work and then look at the premises implicit in his treatment of nature. We will briefly review the Renaissance reflections on mathematics linked to practical developments in technological fields as well as to eclectic reassessments of Pythagorean and Aristotelian debates on the certainty of mathematics and their applicability to natural philosophy. Focusing on the epistemological premises underlying Benedetti's mechanics, we will discuss medieval and early modern approaches to natural knowledge, which, in spite of their mathematical rigor, rested on a physics and metaphysics of contingency. For many centuries, it was assumed that the mathematical regularity of the phenomena does not imply their causal necessity.

#### 4.1 The Certainty of Mathematics

In the letter to the Venetian patrician Domenico Pisani included in the collection of the *Diversae speculationes* and entitled *De philosophia mathematica* (On Mathematical Philosophy), Benedetti emphasized the philosophical dignity of his discipline, placing it at the same rank as physics, metaphysics, and ethics—if not higher than them, considering the certainty of its demonstrations (*certitudo suarum conclusionum*):

I am surprised that, although you are well-versed in Aristotelian philosophy, nonetheless you make a distinction between the philosopher and the mathematician in your writings, as if the mathematician were not as much a philosopher as the naturalist and the metaphysician. In fact, as far as the certainty of his conclusions is concerned, he deserves the title of philosopher much more than them.<sup>1</sup>

This reference to mathematical *conclusiones* reveals Benedetti's methodological focus on the dignity and validity of his discipline. In his connection of mathematical and physical speculations, he seems to put the emphasis on the method rather than on ontology and to seek for the certainty of mathematics and its applications by way of its specific logic. This was the position of his correspondent, the Paduan professor Pietro Catena.<sup>2</sup> Along with him, Benedetti maintained that the certainty of mathematics has an extra-sensible and intelligible character.<sup>3</sup> As Benedetti added in his letter to Pisani:

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<sup>1</sup>Benedetti 1585, 298: "Miror quod cum in Aristotele sis versatus, in tuis tamen scriptis philosophum a Mathematico separe, quasi mathematicus non sit adeo philosophus, ut est naturalis, et metaphysicus, cum multo magis quam ii philosophus sit appellandus, si ad veritatem suarum conclusionum respiciamus."

<sup>2</sup>Benedetti includes a letter to Catena in Benedetti 1585, 371.

<sup>3</sup>See on this De Pace 1993, 228–229.

Actually, you are not the only one who makes this mistake, but this is more grave in consideration of the fact that, although you [Aristotelians] even label ethics as a philosophical discipline, you do not acknowledge that the divine mathematical sciences also should be adorned with the name of philosophy. In fact, if we consider this name more attentively we will clearly see that it is in itself more suited to the mathematician than to anyone else, since none of the others is more certain in his affirmations than the mathematician. And no one is more driven by the love of science in his cognition. This is evident. In fact, [the mathematician] does not rely on the senses nor accepts any presupposition that is not so true and evident to the intellect that no power whatsoever could show that it is false.<sup>4</sup>

Benedetti was acquainted with scholars quarreling over the status of mathematics, its demonstrative methods, and its legitimacy in the treatment of natural issues.

In his time such debates on the foundations and status of mathematics were intense. As an instance of epistemological reflections on the philosophy of mathematics, historians often mention the controversial theses by the Paduan professor of philosophy Alessandro Piccolomini, with whose work Benedetti was familiar. Piccolomini authored, among other writings, a treatise *De certitudine mathematicarum* (On the Certainty of Mathematics, 1547) affixed to his paraphrases of pseudo-Aristotelian mechanics, *In mechanicas questions Aristotelis paraphrasis*. As one reads in this sort of appendix, one ought not to cast into doubt the certainty of mathematics. However, this does not depend on demonstrative methods but rather on the subject of inquiry: “Mathematical disciplines are certain not due to the force of their demonstrations but rather to their subject matter itself.”<sup>5</sup> Their special subject is quantity, connected to matter. Hence, the certainty of mathematics, for an Aristotelian such as Piccolomini, rests on the fact that it deals with universal properties of nature that can be extracted from concrete reality by means of abstraction (*res mathematicae sunt ex abstractione*).

The cause of the certainty of mathematics is evident from Aristotle’s statements. Simplicius is of the same opinion when he states (in *De anima* I 11) that the cause of the certainty of mathematics is due to the fact that they refer to quantity. In fact, as he argues, quantities are sensible things, they have sensible causes and they are known to us as such.<sup>6</sup>

This consideration led Piccolomini to argue that motion can become a mathematical object, if one abstracts from materiality:

<sup>4</sup>Benedetti 1585, 298: “Verum quidem est, te in huiusmodi errore solum non versari; sed gravius est, quod cum vos videatis etiam res morales sub philosophiae appellationem cadere, non animadvertatis divinas scientias mathematicas etiam philosophiae nomine ornandas esse. Quod si eiusdem nomen penitus considerare velimus, inveniemus aperte, mathematico magis illud ipsum quam cuilibet alio convenire, cum nullus ex aliis tam certo sciat id quem affirmat quam mathematicus, neque aliquis sit, qui in cognitionis, et scientiae cupiditatem magis ducantur, ut aperte patet, cum nec etiam ipsi sensui det locum, neque aliquid praesupponat, quem non sit ita verum et intellectui notum, ut nulla quaevis potentia, illud esse falsum ostendere queat.”

<sup>5</sup>Piccolomini 1565, f. 107v: “Mathematics disciplines esse certas non vi demonstrationis, sed ex subjecti ipsius ratione.”

<sup>6</sup>Piccolomini 1565, 106v: “Patet igitur ex dictis Aristotelis causa certitudinis mathematicae. Hoc idem sensit Simplicius, qui primo de Anima 11. dicit causam certitudinis mathematicarum esse, quia versantur circa quantum. Quantitates enim ut dicit ipse, sunt res sensatae, et causas sensatas habent, et ideo nobis notas.”

One could argue that, just like magnitude, motion is a common sensible, too. Moreover, it has its effects and causes (see *Physics* V and VI). Thus, there can be a science of motion (a natural one), which is certain, similar to the science of quantity, that is, mathematics.

We can answer to this [apparent objection], that if we consider motion in general, as separated from matter and insofar as it is a continuum [...], our consideration will be mathematical. This is not in contrast with our principles.<sup>7</sup>

The “ontological” and not only “epistemological” dimension of mathematical physics would concern later scholars such as Kepler and Galileo, going beyond the shared Aristotelian discourse in their investigations of the mathematical properties of material processes.<sup>8</sup> Benedetti was rather concerned with mathematics as an intellectual tool, a sort of “logic of scientific inquiry.” In the above-mentioned letter to Pisani on his mathematical philosophy, he stressed the certainty of mathematical reasoning rather than that of its “objects.” Nonetheless, he was interested in the question raised by Piccolomini as to the usefulness of mathematics in the study of motion. As we will discuss, Benedetti’s insight concerning the generalization of the methods already in use in mechanics, in the science of weights, established the premises for the conceptualization of problems in dynamics.

Benedetti’s interest in mathematics as a conceptual instrument accords with the interest in the demonstrative power of mathematics shown by many scholars entering the debates about mathematical certainty. The publication of Piccolomini’s *De certitudine mathematicarum* led to a series of negative or sympathetic reactions, among them the criticism made by the translator of Proclus’s *Commentary* on Euclid, Francesco Barozzi, as well as those by the Paduan professors Pietro Catena and Giuseppe Moletti. Barozzi, in his 1560 *Quaestio de certitudine mathematicarum*, and Catena, in his 1563 *Oratio pro idea methodi*, argued in favor of the demonstrative certainty of mathematics, contra Piccolomini’s exclusive focus on mathematical objects. The theoretical discussion regarding the status of mathematics, the certainty of its demonstrations, their applicability to the investigation of nature, and the hierarchy between natural philosophy and mathematics continued for a while. It also produced frictions among Jesuit scholars such as the philosopher Benito Pereira and the mathematician Clavius, who were inclined to assign different levels of importance to the study and teaching of mathematics in the colleges of their order.<sup>9</sup>

As far as the institutional side of the defence of mathematics is concerned, it opposed scholars and intellectuals benefiting from varying social status, such as mathematicians, philosophers, and theologians. Benedetti’s self-perception and, later, Galileo’s self-presentation as “philosophers” involved polemical stances. They claimed for their math-

<sup>7</sup>Piccolomini 1565, 107r: “Si vero adhuc replicaretur, quod motus etiam est sensibile quoddam commune, sicut magnitudo; habet autem motus suas passiones, et suas causas, ut patet 5. et 6. Phys. ergo ita erit certa de motu scientia, naturalis scilicet, sicut scientia de quantitate, quae Mathematica est. Ad hoc respondere possumus, quod si motum considerabimus, in communi, abstracto a materia quatenus continuum quoddam est, [...] tunc consideratio erit mathematica, et nihil contra nos.”

<sup>8</sup>As Ofer Gal and Raz Chen-Morris recently stressed: “It is not epistemology that worries the two court mathematicians here, but ontology. Neither of them questions the power of mathematics to provide the knowledge they seek; it is the objects that mathematics can be true about that they both feel forced to establish.” See Gal and Chen-Morris 2013, 118–119.

<sup>9</sup>The literature on the Renaissance debates on the philosophical status of mathematics is wide. Among other sources, see Giacobbe 1972, Giacobbe 1973, Carugo 1983, Jardine 1990, 693–697, De Pace 1993, Cozzoli 2007, and Axworthy 2016, chap. 2. For the Jesuit debates on mathematics, see Romano 1999. For the seventeenth century, cf. Mancosu 1996, 8–33.

ematical and physical investigations a wide cultural meaning against critics who downplayed such investigations as merely technical and specialistic.

Early polemics over the viability of the *mos geometricus* were not purely intellectual and academic but were also rooted in the rising recognition of the practical import of mathematics in engineering, architecture, mechanics, and warfare. A new class of intellectuals was emerging composed of “scientist-engineers,” so to speak, both expert in practical disciplines and trained in letters.<sup>10</sup> Edgar Zilsel already remarked that the Renaissance exaltation of mathematics went far beyond purely Platonic and Pythagorean influences. At that time new mathematical writings were composed and published dealing with the practical problems of commerce, topography, architecture, and the arts.<sup>11</sup> Moreover, the emergence of mathematical and natural conceptions dependent on the advance of technology was reinforced by the growing self-consciousness of new social groups.<sup>12</sup> As an example of the awareness of the status of the practical arts one could mention Filippo Pigafetta’s introduction to the Italian edition of Del Monte’s work on mechanics. Here he reversed the assessment of craftsmen and practical knowledge, which had been marked by the contempt of aristocrats and traditional intellectuals, as follows:

‘Mechanic’ is a very honored title. According to Plutarch it refers to a profession linked with warfare. It is suited to a man of high rank who is also capable of using his hands and his intelligence to realize wonderful works of rare usefulness and pleasure for human life.<sup>13</sup>

This judgment well expresses the shifting opinion on practical knowledge which also marked Benedetti’s environment. We have already stressed the centrality of practical mathematics for the Savoy dukes, in particular Emanuele Filiberto, in their construction of the new capital, Turin.

## 4.2 Physico-Mathematics

As a direct consequence of this mathematizing epistemology Benedetti dismissed the well-established separation between physics and mathematics in cosmology, that is, he refused to separate the investigation of “causes” and calculation.<sup>14</sup> This anti-fictionalist perspective implied a realist commitment related to the Copernican system and its embedding within a renewed cosmology. As we will discuss in the section on Benedetti’s views on the universe, he praised the system “of Aristarchus and Copernicus” as it avoided the absurdities of an anthropocentric conception according to which the immensity of the firmament was created only for us. Rather, all planets are like Earth or, better, like moons reflecting the solar light. Among the direct consequences of the Copernican view was accepting that the fixed stars do not rotate around the center of the world within one day; rather, they are immobile.<sup>15</sup>

<sup>10</sup>See Valleriani 2010 and Valleriani 2013.

<sup>11</sup>Zilsel 1942.

<sup>12</sup>See Lefèvre 1978.

<sup>13</sup>Pigafetta in Del Monte 1581, *Ai lettori*: “Mechanico è vocabolo honoratissimo, dimostrante, secondo Plutarco, mestiero alla Militia pertinente, et convenevole ad huomo di alto affare, et che sappia con le sue mani et co’l senno mandare ad esecuzione opre maravigliose a singulare utilità et diletto del vivere humano.”

<sup>14</sup>Hypotheses on conventionalism already emerged from the debate on the conflict between Ptolemy’s geometrical models and Aristotle’s homocentric cosmology. See Di Bono 1990 and Granada and Tessicini 2005.

<sup>15</sup>Most of these cosmological views are discussed in Benedetti 1585, Book 4. We deal with the details in chapter 6 as well as, partly, in chapter 7.



From this viewpoint, Benedetti's understanding of mathematics is not too removed from that of a mathematician such as Copernicus, who, in Book 1 of *De revolutionibus*, indicated that the mathematical superiority (simplicity and intelligibility) of his own planetary system was such that natural philosophy had to be subordinated to mathematical astronomy and not vice versa. The theologian who wrote the anonymous introduction to Copernicus's work, Andreas Osiander, tried to reaffirm the hypothetical character of mathematical astronomy, and its subordinate position as a discipline relative to physics and theology. By contrast, Renaissance scholars who appreciated the physical meaning of the Copernican system called it "Pythagorean" to underscore at once its natural philosophical and mathematical character.<sup>16</sup> As an extreme case one could mention Bruno's declarations during his Inquisition trial. In order to defend his cosmological views, and in particular the motion of Earth, he did not mention Copernicus but the ancient philosophical school of Pythagoras: "I affirmed [the existence of] infinite individual worlds [i.e., planetary systems] similar to that of the Earth. Following Pythagoras, I regard the latter as a celestial body. The Moon is similar to it, as well as other planets and stars, which are infinite [in number]."<sup>17</sup> Pythagorean cosmology was regarded with suspicion by the Inquisitors and the doctrine of the plurality of worlds became one of the allegations against Bruno, who would be eventually executed as a heretic in Rome. In the same years in which Bruno was a prisoner of the Holy Office in Rome and his works were examined for censure, the censors also attacked Patrizi for his natural views, including the doctrine of terrestrial motion. Although Benedetti shared similar views about the plurality of worlds and the possibility of terrestrial motion, he did not incur any censure. We dare say that he was one of the last Renaissance authors who could freely speculate on nature in Italy before natural philosophy became a highly ideological issue in the religious repression escalating in the 1590s.

Benefiting from his subalpine freedom, Benedetti reflected on Pythagorean cosmology in a section entitled *Pythagoreorum opinionem de sonitu corporum coelestium non fuisse ab Aristotele sublatam*, where he excluded the possibility that the "sound of celestial bodies" is the production of any physical sounds. Rather, he identified the Pythagorean doctrine of the world harmony with divine providence:

As to motions, dimensions, distances, and influences there is nothing that corresponds to such proportions, but, since all of them depend upon the infinite Divine Providence of God, these velocities, those dimensions, distances, and influences must have the most perfect order and relations among them and relative to the universe.<sup>18</sup>

According to Benedetti's outlook, the harmony of the heavens does not correspond one to one to musical harmony in the strict sense. From this viewpoint, Kepler's later effort to translate heavenly geometries into musical melodies in the *Harmonices mundi libri V* (1619) can be seen as a radicalization of similar "Pythagorean premises."

Most significantly, Benedetti and Kepler shared a commitment in favor of the fusion of mathematical and physical accounts of nature in the frame of an early modern transfor-

<sup>16</sup>Omodeo 2014a, 167–170.

<sup>17</sup>Bruno 2000b, doc. 13, 67: "Ho dechiarato infiniti mondi particolari simili a questo della Terra; la quale con Pittagora intendo uno astro, simile alla quale è la Luna, altri pianeti et altre stelle, le qual sono infinite."

<sup>18</sup>Benedetti 1585, 191: "Quod autem attinet ad motus, ad magnitudines, ad distantias et ad influxus, nihil est, quod hisce proportionibus conveniat, sed quia haec omnia dependent ab infinita et divina providentia Dei, necessario sit ut istae velocitates, eae magnitudines, distantiae et influxus, talem ordinem et respectum inter se ipsa et universo habeant, qualis perfectissimus sit."

mation of natural science in which the methods of the physico-mathematical disciplines gained a paradigmatic status. The epistemological shift also involved well-established disciplines such as astronomy. Kepler's astrophysics, first illustrated in the *Astronomia nova* (1609), was a significant step toward the derivation of celestial geometries from physical forces. Kepler translated a geometrical discipline (Ptolemaic and Copernican mathematical astronomy) into a physico-mathematical one. In fact, he explained the elliptical path of planetary orbits as the effect of interactions of forces. He emphasized the double bound of his astronomy, inseparably intertwining physics and mathematics, in the title of the work: *Astronomia nova aitiologitós seu physica coelestis de motibus stellae Martis* (New Astronomy Investigating the Causes, or Celestial Physics Concerning the Motions of Mars). As Kepler announced in the introduction: "In this work I mixed celestial physics with astronomy."<sup>19</sup> He meant to launch a new discipline, "celestial physics," that merged mathematical modeling with causal physics.<sup>20</sup> Kepler remarked that the ignorance of physical causes compels scholars to settle for conjectures since no choice can be made between mathematically equivalent hypotheses. By contrast, physical arguments are decisive in deciding between mathematically equivalent models. Therefore, celestial physics and astronomy should be unified. The result was a mixed science (*scientia mixta*) whose data came from the senses and whose demonstrations are expressed in mathematical terms. This physicalization is well shown in Kepler's physico-mathematical concept of "orbit" (*orbitae*) substituting that of orbs (*orbis*) (that is, the material spheres transporting celestial bodies). According to him orbit is "the path together with its physical causes—expressed as physical laws."<sup>21</sup> Shape and velocity of astronomical orbits depend on the force (*vis*) emanating from the sun, that is, on a physical cause of geometrical effects.<sup>22</sup>

Descartes's *Traité du monde et de la lumière* (completed in 1632–1633, but printed posthumously, in 1664) and the *Principia philosophiae* (1644) marked a culminating point in the move toward the reduction of natural disciplines (such as optics and astronomy but also physiology) to material interactions of corpuscles in motion. Descartes's philosophy was particularly influential as it legitimized a mathematical treatment of nature with the advances of physics in his time. At the same time, he connected his explanations to views on matter and causality irreconcilable with the qualitative, essentialist, and teleological accounts of the Scholastic tradition. In particular, his mechanization elevated the results of Renaissance mechanics to a higher and more generalized level.

Benedetti's place is rather at the beginning than at the end of this process. As the title of his major work hints, he was committed to a mathematical-physical investigation of nature. He did not limit his application of a mathematical method to those fields where this approach was already established, but extended it to the treatment of all realms of natural inquiry.

### 4.3 The Contingency of Nature and Mechanics

Benedetti's mathematical approach to nature did not lead him to the belief that physical phenomena are ruled by necessity. Rather, he shared a medieval and early-modern ontology and epistemology of contingency enabling a particular cohabitation of mathematized physics and indeterminism (in other words, formal determination without causal neces-

<sup>19</sup>Kepler 1937–2001, vol. 3, 19.

<sup>20</sup>Gingerich 1975, 261–278.

<sup>21</sup>Goldstein and Hon 2005, 76.

<sup>22</sup>On Kepler's discovery, see Donahue 1988, Donahue 1993 and Wilson 1968.

sity). In order to better understand it one has to look at Scholastic motives informing his physics, in particular his mechanics, and the scientific and philosophical work of his successors. This will require a short excursus.<sup>23</sup>

#### 4.3.1 Scholastic Treatments of Nature as the Realm of Contingency

It would be misguided to think that a mathematical approach to nature in Renaissance science implies the assumption that natural causation is ruled by necessity. This was indeed not the case for well-established medieval and Renaissance views. Only in the course of the seventeenth century would contingency be banned from the realm of natural causation in the developments of post-Cartesian mechanism. For philosophers such as Baruch Spinoza and Gottfried Wilhelm Leibniz contingency marked the limitations of our knowledge and not an ontological limitation of nature. As one reads for instance in Spinoza's *Ethica ordine geometrico demonstrata* (Ethics, demonstrated in geometrical order) I 29: "There is no contingency in nature. All natural beings are determined by divine necessity to exist and operate in a special manner." (*In rerum natura nullum datur contingens, sed omnia ex necessitate divinae naturae determinata sunt ad certo modo existendum et operandum*). By contrast, in the Renaissance a mathematical treatment of natural phenomena underlaid no principle of sufficient reason, hence it did not imply the necessity of natural causation. In particular, mixed mathematical disciplines that had received a Scholastic embedment or systematization rested on a well-established Aristotelian conception, according to which sublunary phenomena are determined without necessity.

Historically, *contingentia* is the Latin variant translation of the Aristotelian concept of "possibility," both as modal logical *endechomenon* as well as physical-metaphysical *dynamis* within a hylemorphic framework. In the context of the Christian reception, this terminus received an onto-theological connotation in a frame of creationist theology. In late Scholasticism, *contingentia* came to signify the worldly reality, or nature as Creation. Nature was deemed to be contingent. It exists *de facto* but could also not exist because it depends on God's will. As John Duns Scotus put it,

So then, the first issue has become clear: how there is contingency in things—because it comes from God—and what is in God which is the cause of this contingency—because it is his will.<sup>24</sup>

In Aristotle, there was a tension between two meanings of "possibility." According to *Analytica Priora* (13: 32 a 18–20) the possible is that which is "neither necessary nor impossible," whereas according to *De interpretatione* (13: 22 a 14–13 a 26) possibility is exclusively that which is opposed to "impossibility" and therefore includes also that which is necessary. As a reminiscence of this original tension, one can find in Scholastic philosophy two different definitions of contingency either as "quod est nec impossibile nec necessarium" (that which is neither impossible nor necessary) or "quod non est impossibile" (that which is not impossible).<sup>25</sup> Both meanings were kept in the Latin rendering of the Aristotelian possibility as *contingentia* by Gaius Marius Victorinus (III–IV cent.

<sup>23</sup>We have first discussed contingency and mechanics in the Renaissance in Omodeo and Renn 2015. A volume entirely devoted to ontological and epistemological contingency in the natural debates of early modernity is Omodeo and Garau 2019.

<sup>24</sup>Duns Scotus 1994, 140: "Sic igitur apparet primum, quomodo est contingentia in rebus, quia a Deo, – et quid est in Deo quod est causa huius contingentiae, quia voluntas eius."

<sup>25</sup>Cf. Vogt 2011, 52. The entire first chapter is relevant for a historical overview of the reception and transformation of the Aristotelian concept of "possibility" as "contingency" in the Latin tradition.

CE) and Boethius (IV–V cent. CE), but the Latin expression also suggested affinity between that which is contingent (*contingit*) and that which occurs (*evenit* or *accidit*).<sup>26</sup> This third connotation would eventually prevail through the late-Scholastic differentiation between *contingentia* and *possibilitas* and its reception in the philosophical systems of the seventeenth century (and most notably by Leibniz).<sup>27</sup> Unlike abstract (purely logical) possibility, contingency referred only to that which is real but not so by necessity: “id, quod [est sed] potest non esse” (that which [is but] could not be). In the Christian perspective of the Almighty’s Creation, contingency happened to include all that is not God himself, that is to say, nature, or the universe.

This background is fundamental to understand not only theological disputes but also natural philosophical and scientific developments during the Middle Ages and the Early Modern Period. The connotation of nature as contingent—as that “which could not be”—is theological and metaphysical in its essence, since it points to the dependency of the world on God. However, from the point of view of natural conceptualizations, not only the “vertical” dimension of metaphysics is relevant but also the “horizontal” dimension of causality within nature. On the horizontal plane of the interrelation among finite beings, contingency refers to a degree of indetermination, and a certain unpredictability in the connection between causes and effects. Moreover, whereas a theological perspective focuses on the radical contingency of that which exists as created being, natural philosophy addresses the *relationship* between contingency and necessity within nature, that is, between divine order and phenomenal imperfection. This relationship between that which is not necessary and that which is necessary *had to be* conceptualized and indeed was conceptualized as the relationship between the *absolutum* and the *conditionale* or *secundum quid*.

In Book 1 of the *Summa contra gentiles*, Thomas Aquinas defined contingency through its distinction from necessity. In the case of the contingent beings, as one reads in *Summa contra gentiles* I 67, a cause can produce its effect or not, whereas in the case of necessary beings, their cause cannot not produce them:

The contingent differs from the necessary according to the way each of them is found in its cause. The contingent is in its cause in such a way that it can both not-be and be from it; but the necessary can only be from its cause. [...] Just as from a necessary cause an effect follows with certitude, so it follows from a complete contingent cause if it be not impeded.<sup>28</sup>

A contingent cause, as one reads, will fulfill its tendency to produce a certain effect “si non impediatur,” that is, if no impediment hinders its realization.

In Book 2 of the *Summa contra gentiles*, Thomas dealt extensively with the contingent being (“*omne quod est possibile esse et non esse*” and “[*id quod*] *ad utrumlibet se habet*”).<sup>29</sup> According to him, the world is contingent insofar as it is created. In this general sense, “God is to all things the cause of being” (*Summa contra gentiles* II 15).<sup>30</sup>

<sup>26</sup>Vogt 2011, 50.

<sup>27</sup>Schepers 1965.

<sup>28</sup>Aquinas 1975, 221f: “Contingens a necessario differt secundum quod unumquodque in sua causa est: contingens enim sic in sua causa est ut non esse ex ea possit et esse; necessarium vero non potest ex sua causa nisi esse. [...] Ex causa necessaria certitudinaliter sequitur effectus, ita ex causa contingenti completa si non impediatur.”

<sup>29</sup>Thomas, *Summa contra gentiles* II,15. Cf. Aquinas 1975, 48: “everything that can be and not-be” and “it is indifferent to either.”

<sup>30</sup>Aquinas 1975, 46: “Deus est omnibus causa essendi.”

In particular, God's free will is the origin of this world. Nonetheless, Thomas does not exclude that natural reality is populated by both necessary and contingent beings. Absolute necessity (*necessitas absoluta*), he writes in *Summa contra gentiles* II 29, does not pertain to God, since His decision and action is independent from any constriction (*debitum*). Rather, absolute necessity pertains to the immaterial, or "separated" beings as well as to those bodies in which the form fulfills all potentialities of their matter, as is the case with the heavenly bodies transported in circles. As for terrestrial (sublunary) bodies, their forms are imperfectly realized. Matter, as the potentiality to take different forms, is at the origin of their contingency, that is, it is the source of the possibility to realize or not to realize a certain effect: "But in things whose form does not fulfill the total potentiality of the matter, there still remains in the matter potentiality to another form."<sup>31</sup> For the low realm of birth, corruption, and change, Thomas speaks of conditional necessity (*necessitas conditionalis*). In the sublunary sphere, contingency cohabits with absolute necessity (e.g., the inevitability of death for all animals and the hylemorphic composition of all bodies). Whereas necessity pertains to the formal determinations of natural phenomena, contingency is the partial fulfillment of necessary tendencies.<sup>32</sup>

According to Scholastic terminology, there is always a "quid" producing the deviation of material phenomena from their formal rule. We will call this outlook an "ontology and epistemology of contingency."<sup>33</sup> The Pythagoreanism of many Renaissance scholars such as Benedetti did not depart from a view stressing the contingent character of natural phenomena in general. As we will argue, one encounters in Benedetti's physics and mechanics a systematic use of theoretical tools implying natural contingency in the form of a distinction and interrelation between formal mathematical necessity and its material realization. In order to understand Benedetti's mathematical treatment of contingency it is useful to consider the medieval approaches to contingency, especially the science of weights (*scientia de ponderibus*) he relied upon.

The idea of contingency informing physics and mechanics was related to its use in other disciplines, even ethics. Whereas there can be no obstacle impeding the realization of God's will, which is therefore "absolute" (*voluntas absoluta*), human will, or *voluntas secundum quid*, is conditioned by circumstances. In other words, the realization of the highest aims of humankind is intrinsically contingent, as Dante expressed in the *Divine Comedy*:

But utterance and feeling among mortals,  
For reasons which are evident to you,  
Have different feathers making up their wings.  
I, too, as man feel this disparity [...].<sup>34</sup>

<sup>31</sup> *Summa contra gentiles* II 30: "In quibus [rebus] vero forma non complet totam potentiam materiae, remanet adhuc in materia potentia ad aliam formam." Cf. Aquinas 1975, 87.

<sup>32</sup> *Summa contra gentiles* II 23: "Omnis enim agentis per necessitatem naturae virtus determinatur ad unum effectum. Et inde est quod omnia naturalia semper eveniunt in eodem modo, nisi per impedimentum: non autem voluntaria. Divina autem virtus non ordinatur ad unum effectum tantum [...]. Deus non agit per necessitate naturae, sed per voluntatem." Cf. Aquinas 1975, 68: "For the power of every agent which acts by natural necessity is determined to one effect; that is why all natural things invariably happen in the same way, unless there be an obstacle; while voluntary things do not. God's power, however, is not ordered to one effect only [...]. Therefore, God acts, not out of natural necessity, but by His will."

<sup>33</sup> Omodeo and Renn 2015.

<sup>34</sup> Alighieri 1984, 94; Dante Alighieri, *Paradiso* XV 79–83:

"Ma voglia e argomento ne' mortali,  
per la cagion ch'a voi è manifesta,

Apart from ethical contingency, Scholastic authors also used *secundum quid* in logic. For instance, Petrus Hispanus explained the meaning of the so-called *secundum quid* fallacy in his *Tractatus sive summule logicales*, commenting on Aristotle's *On Sophistical Refutations* V (166b36–167a14).<sup>35</sup>

In logic, *secundum quid* meant either a “diminution” of a concept through restriction of its definition (*secundum quid et simpliciter*), or the designation of a subject through one of its parts or characteristics (*denominatio totius per partem*). A *secundum quid* fallacy occurs if an identity is established between something considered in a particular respect and the same thing considered absolutely (or *simpliciter*). For instance, the existence of a depicted animal does not imply the existence of the animal *simpliciter*. Thus, the argument “est animal pictum, ergo est animal” is not correct. In this case, there is a *quid* signaling the gap between universal necessity and particular or concrete contingency.

### 4.3.2 Contingent Causation in the *scientia de ponderibus*

The *scientia de ponderibus* heavily drew on the idea of the conditional limitation of natural necessity depending on circumstances (*secundum situationem*, also literally meant as “depending on the position”). In particular, the concept of *gravitas secundum quid*, or positional heaviness, had a powerful explanatory function, most notably in the Aristotelian treatment of weights by Jordanus Nemorarius, and continued to be essential during the Renaissance in the reflections on mechanics by scholars such as Tartaglia, Cardano, and Benedetti himself.<sup>36</sup>

In mechanics the “limitation” or “determination” *secundum quid* implied that the dynamic tendency of a body was reduced or enhanced depending on intervening constraints or circumstances, in particular mechanical ones. The rotations of a lever around a pivot or of a balance around its fulcrum were conceptualized as constrained motions. In such displacements, the inherent (“necessary”) vertical tendency of a weight resulted in a circular motion due to external constraints. Similarly, the heaviness (*gravitas*) of the bodies suspended at the extremities of a simple machine varied in relation to their changing positions within the system. In such cases, a “necessary” straightforward motion in accordance with natural order resulted contingently in a circular one. The implicit mental model for this kind of displacement was that “circular motion is constrained rectilinear motion.” This means that, in the sublunary sphere of contingency, straightforwardness and rectilinear tendency had a higher onto-epistemological status than circularity since straightforwardness was necessarily rooted in natural order. By contrast, circularity, as the deviation from such order, had to be explained. As a consequence, circularity (in the elementary sphere) was allotted a derived and subordinated onto-epistemological status. In other words, circularity was an instance of nature departing from necessity owing to some rather elusive factor or *secundum quid*. From this viewpoint, it was seen as a deviant realization of given potentialities similar to moral deviation from the necessary laws of uprightness. In order to stress that the mechanical treatment of the *scientia de ponderibus* was embedded in the framework of contingency, we could also formulate the principle in this way: “circular motion is rectilinear motion modified by a contingency.”

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diversamente son pennuti in ali;  
 ond' io, che son mortal, mi sento in questa  
 disagguaglianza [...].”

Also, see *Paradiso* IV, 87, IV, 109, IV, 113, and *Purgatorio* VII, 57.

<sup>35</sup>Hispanus 1972, 157–158.

<sup>36</sup>See Renn and Damerow 2012, especially the sections from 3.6 to 3.8.

Almost at the beginning of his small treatise “on the weights,” Nemorarius stressed his Aristotelian commitment. In fact, his approach was based on the opposition between the *natural* vertical motion of the elements and the *violent* hindrances producing circular deviation. At the same time, he introduced the key concept of *gravitas secundum quid* (in some cases, also *levitas secundum quid*), which we will refer to as “positional heaviness.”

[...] if equal arcs are taken on a greater circle, and on a smaller one, the chord of the arc of the greater circle is longer. From this I can then show that a weight on the arm of a balance becomes lighter, to the extent that it descends along the semicircle. For let it descend from the upper end of the semicircle, descending continuously. I then say that since the longer arc of the circle is more contrary to a straight line than is the shorter arc, the fall of the heavy body along the greater arc is more contrary to the fall which the heavy body would have along the straight line than is a fall through a shorter arc. It is therefore clear that there is more violence in the movement over the longer arc than over the shorter one; otherwise the motion would become heavier. Since something moves with more violence in the ascent [along the arc], it is apparent that there is more positional heaviness [*gravitas secundum situm*] and, as it is like that depending on position [*secundum situationem*], one can aptly call it ‘positional heaviness’ [*gravitas secundum situm*].<sup>37</sup>

In its circular descent along a circular path, a weight deviates from its natural tendency, or *intentio*, the more the arm of the balance departs from the horizontal position. Therefore, the “violence” is greater when the arc of displacement is longer, while the weight progressively loses its weight insofar as the vertical component in its motion is reduced.

According to Nemorarius, a weight that reaches the bottom of the circular arc described by the arm in its displacement is not “at rest” but only “lighter.” In fact, a natural being is at rest only if it is fully accomplished, that is, once it has realized the aim, or act, toward which its power is directed teleologically. By contrast, a body is always in motion, or striving to move, until it has reached its end: “All motion strives toward its aim—indeed the whole nature strives towards actuality and is realized [in it]—hence the opposition occurs against [a displacement] contrary [to the natural tendency].”<sup>38</sup>

A body on one arm of the balance becomes lighter during its downward motion than an equal one located on the other extremity. Thus, as Nemorarius assumes, or tries to demonstrate, a balance removed from its state of equilibrium will tend to restore the original state. As one reads in the *propositio secunda* (with reference to the diagram in Figure 4.1), which is the second of a series of propositions developing the details of Nemorarius’s doctrine of weights,

Suppose now that the descent occurs on the side *B* and the ascent on the side *C*. I say that both will go back to the [horizontal] position of equality. In fact,

<sup>37</sup>Nemore 1533, f. A3v (emphasis added): “[...] si sumantur de circulo maiori et minori arcus aequales, corda arcum maioris circuli longior est. Propeterea posset ex hoc ostendi, quod pondus in libra tanto sit levius, quanto plus descendit in semicirculo. Incipiat igitur mobile descendere a summo semicirculi, et descendat continue. Dico tunc quod maior arcus circuli plus contrariatur rectae lineae quam minor, et casus gravis per arcum maiorem, plus contrariatur casui gravis, qui per rectam fieri debet, quam casus per arcum minorem. Patet ergo maior est violentiam in motus secundum arcum maiorem, quam secundum minorem. Aliter enim fieret motus magis gravis. Cum ergo plus in ascensu aliquod movetur violentiae, patet, quam maiore est *gravitas secundum situm*, et quia secundum *situationem* talium sic sit, dicatur *gravitas secundum situm*.”

<sup>38</sup>Nemore 1533, ff. A3v–A3r: “In termino enim cuiscunque motus intenditur, intenditur et viget tota natura in actu, qui in motu sit quasi in potentia, secundum quem fiebat contrarietatis suae oppositio.”

*B* will not further descend, because its descent towards *D* is more oblique than the ascent of *C* towards the [horizontal position of] equality; in fact, *B* and *C* are equidistant from the place of equality.<sup>39</sup>



Figure 4.1: Diagram accompanying proposition two in Apianus's 1533 edition of Nemorarius's *Liber de ponderibus* (1533, f. B2r). (Bayerische Staatsbibliothek)

Nemorarius's reasoning becomes clearer in light of propositions four and five:

Fourth [proposition]: It is positionally heavier, insofar as its descent, in the same position, is less oblique.

Fifth [proposition]: But a more oblique descent partakes less of the straight [descent], for the same quantity [of the path].<sup>40</sup>

In proposition five, it is suggested that the vertical components of the potential descents of the two beams could be identified and compared. This was the source of the idea that the variation of heaviness could also be determined by comparing the straightness of the descents. A similar procedure was later taken up and explained in detail in Niccolò Tartaglia's considerations in the *Questiti et inventioni diverse* (1546) about the manner of ascertaining the positional heaviness of two weights on the basis of the so-called angles of contact. These are the "curvilinear" or "mixed" angles between the circular path of the

<sup>39</sup>Nemore 1533, ff. B2r–v: "Ponatur nunc, quod fiat descensus a parte B, et ascensus a parte C, dico quod redibunt ad situm æqualitatis. Non enim ulterius descendet B, eo quod descensus eius versus D magis obliquus est, quam ascensus C ad æqualitatem; B enim et C iam æqualiter distant a situ æqualitatis."

<sup>40</sup>Nemore 1533, f. A3r: "Quarta [propositio]: Secundum situm gravius esse, quanto in eodem situ minus obliquus est descensus. Quinta [propositio]: Obliquiorem autem descensum minus capere de directo, in eadem quantitate." Translation from Renn and Damerow 2012, 63. For proposition four, see Nemore 1533, f. B3v–B4r and, for proposition five, Nemore 1533, f. B4r–C2v.



arms of a balance and the vertical lines connecting the weights to the cosmological center of gravity (see Figure 4.2). Tartaglia compared the angles of contact of two equal weights located on the extremes of a balance, and argued that the lifted one is always smaller than the lowered one. Thus, the lifted weight would face a descent that is more oblique. It would acquire a greater positional heaviness than its lowered counterweight and, as a further consequence, the inclined system would reestablish its horizontal balance, if not hindered to do so.

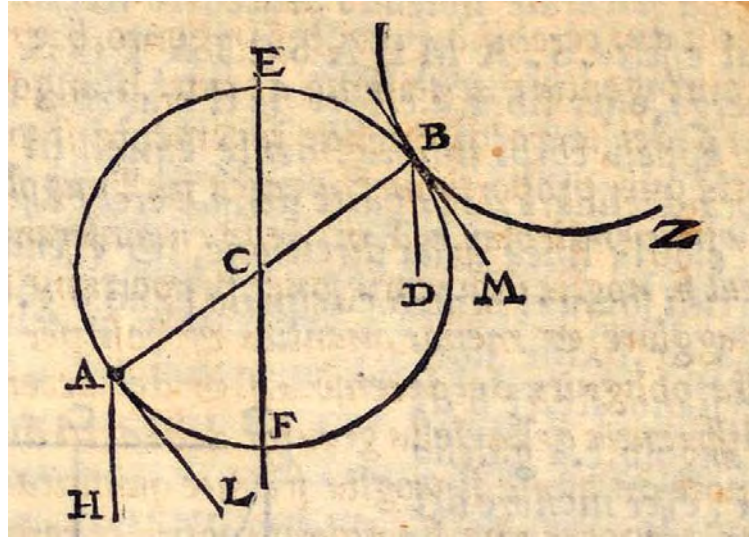


Figure 4.2: In the *Quesiti et invenzioni diverse*, Tartaglia argued that the relative positional heaviness of the weights A and B on a balance could be determined on the basis of the “mixed” angles of contact HAF and DBF. Since it is argued that  $DBF < HAF$ , the weight B will be heavier than A. Thus, the inclined system will strive toward the restoration of a horizontal equilibrium. (Max Planck Institute for the History of Science, Library)

In spite of his attempt to quantify the *quid* accounting for the alleged restorative motion of the inclined balance, Tartaglia’s geometrical quantification maintained a margin of indeterminacy. As he stated, the ratio between the two mixed angles is less than that between any determined quantities. Therefore, it is impossible to stabilize the system in its inclined position by adding a small (no matter how small) weight on the lowered side of the balance. According to Tartaglia, it is impossible to counterbalance the positional heaviness of the lifted weight. Quite on the contrary, any additional weight added to the lowered side would make the balance rotate and reach the vertical position.<sup>41</sup>

#### 4.4 The Epistemological Import of Benedetti’s Generalization from Weights to Forces

As we have argued so far, in the medieval *scientia de ponderibus* circular motion is conceived of as constrained linear motion. Yet, within an Aristotelian cosmology, this mental model is restricted to the sublunary sphere, where motions cannot fulfill their nature. This is indeed the sphere of contingency, where a gap is to be witnessed between the necessary

<sup>41</sup>Tartaglia’s approach was controversial, already in his time. See Renn and Omodeo 2013, sec. 3.6.

order of things (or “nature” as actuality) and the effective phenomena (subjected to “violence” or to external constraints). The four elements naturally tend toward their places through a straightforward descent or ascent. Heavy bodies, for instance, strive toward the center of gravity, which is, at the same time, the center of the cosmos. If their motion is hindered, as is the case with mechanical constraints, a certain factor or *quid* has to be taken into account, which explains the deviation from the rule. In this theoretical context, contingency is the concept expressing the relationship between the natural law and phenomenal reality, which follows a norm while deviating from it. The *secundum quid* is that which explains this deviation. Possibly, it has to be expressed through geometrical means, although it might prove unintelligible or infinitesimal, as was the case with Tartaglia’s ratio between mixed angles accounting for the *gravitas secundum quid* of the weights of a balance. In the treatment of weights, in particular of those on a balance, Nemorarius and his followers made a limited use of the mental model of curvilinear motion as constrained linear motion. In fact, they employed it to account for phenomena linked to gravity (i.e., the vertical fall of bodies explained in Aristotelian terms). It was Benedetti who made the decisive step toward the generalization of this model in the direction of inertial dynamics. Let us consider his application of it first to balances and then to centrifugal forces.

In the section on mechanics of the *Diversae speculationes*, Benedetti picked up and revised the Scholastic concept of *gravitas secundum quid*. Guidobaldo del Monte had already criticized Nemorarius’s and his followers’ conclusion that an inclined balance hinged on its fulcrum as its center of gravity would return to the horizontal position, but his criticism went so far as to renounce the concept of positional heaviness altogether.<sup>42</sup>

Relying on the Archimedean concept of the center of gravity of a body, Del Monte concluded that an equal-arms balance hinged on its fulcrum would remain stable in any position (a correct conclusion only if it is assumed, in modern terms, that the gravitational field is homogeneous): “*Propositio IV: Libra horizonti aequidistans aequalia in extremitatibus, aequaliterque a centro in ipsa libra collocato, distantia habens pondera; sive inde moveatur, sive minus, ubicunque relictā manebit.*” (Fourth Proposition: Take a balance that is equidistant from the horizon and that has weights in its extremities which have the same weight and equally distant from the center (the latter being located in the balance itself). Whether it is displaced or not, it will remain in the same position in any position.)<sup>43</sup>

Benedetti shared the criticism of Nemorarius and Tartaglia with regard to their specific argumentation about the tendency of such an inclined balance to reach the horizontal position but based his judgement on a novel treatment of positional heaviness. The first chapter of Benedetti’s *De mechanicis* begins with the statement: “Every weight placed at the end of an arm of a balance has a greater or a lesser heaviness depending on differences in the position of the arm itself.”<sup>44</sup>

Hence, he clearly committed himself to a mechanical theory of equilibrium based on positional heaviness. Benedetti’s technical terms are not always employed in a rigorous and consistent manner. He treats the *pondus* at times as the varying quantity to be taken into consideration, as is shown by expressions like “*proportio ponderis in C ad idem pondus in F*” and “*unde fit... pondus magis aut minus grave,*” in *De mechanicis* II (Benedetti 1585, 142). Given these semantic fluctuations, we will translate *pondus* as “body” or as

<sup>42</sup>Renn and Damerow 2012, 86–92. We will discuss the divergent interpretations of Benedetti and Del Monte later, in chapter 5.

<sup>43</sup>Damerow and Renn 2010, 65.

<sup>44</sup>Drake and Drabkin 1969, 166. Benedetti 1585, 141: “*Omne pondus positum in extremitate alicuius brachii librae maiorem, aut minorem gravitatem habet.*”

“weight” and *gravitas* as “heaviness” or as “weight,” depending on the context. At the beginning of chapter 1 of his book on mechanics, Benedetti talks of a varying quantity of heaviness, or gravity (*gravitas*), belonging to a weight (*pondus*) or a body placed on a balance beam. Hence, he makes a terminological distinction between *pondus*, as a kind of absolute weight or heavy thing, and *gravitas*, as a downward tendency that can act with more or less force on the body (depending on the inclination of the beam). In this case (as in most cases in the text), *pondus* has the essentialist meaning of a substance (a substratum or ὑποκείμενον). It is the body or weight on the balance, whose special property of being heavy, namely the *gravitas*, varies depending on a *quid*. This *quid* is the position, or *situm*.

Benedetti seeks to quantify it by means of a method he invented. He considers the line, which he calls *linea inclinationis* or *linea itineris*, connecting a weight on an inclined balance beam to the cosmological center of gravity. Note that Benedetti calls the elementary downward tendency an *iter* from a merely kinematic viewpoint, but also an *inclinatio* from a physical and more proper one. According to him, the major or minor heaviness of the weight can be assessed through the projection of the *linea inclinationis* on the horizontal line passing through the fulcrum (Figure 5.1). The more distant it is from the fulcrum, the heavier the positional heaviness becomes. Thus, the weight reaches a maximum of heaviness when the balance is horizontal, and its minimum when it is vertically resting (*nititur*) on the fulcrum or hanging (*pendet*) from it. Notably, this approach anticipates the one based on the determination of the torque in classical physics, and comes to the same conclusions.<sup>45</sup>

Additionally, Benedetti equates the heaviness to a *virtus*, *vis*, or *vigor*, i.e., a force, which might also act in different directions (in *De mechanicis*, Ch. 3) and is applied to the extremity of a constrained mechanical system, like a lever or a balance. This is a significant generalization from weights to forces, but for our present discussion the most important generalization relates to rectilinear tangential tendencies in systems set in circular motion.<sup>46</sup>

The relevant treatment is the epistle to Capra and is included in the *Diversae speculationes*. It deals with the rotation of a millstone and the question of whether its motion could be perpetual. Benedetti denies this by arguing that the rotation is impeded first by the friction of the air and, second and more importantly, by the resistance of the millstone’s parts. The latter have a straightforward tendency, an *inclinatio recte eundi*, along the tangential lines of their rotation (Figure 4.3). As one reads, this rectilinear inclination or impulse (*impetus*) can be bent only by violence. Moreover, the centrifugal tendency grows in proportion to the augmentation of the velocity, as witnessed by other cases, among them the rotation of a catapult or a sling (*machina missilis*). A centrifugal tendency is seen as a rectilinear natural inclination (*naturalis inclinatio recte eundi*).

You ask me this question in your letter. Suppose a millstone rested on a virtually mathematical point and was set in circular motion, could that circular motion continue without end, assuming that the millstone is perfectly round and smooth?

I answer that this kind of motion will certainly not be perpetual and will not even last long. For apart from the fact that the wheel is constrained by the air which surrounds it and offers resistance to it, there is also resistance from

<sup>45</sup>Renn and Damerow 2012, 138. We will deal with the details of Benedetti’s mechanics in the next section.

<sup>46</sup>Cf. Büttner 2008.

the parts of the moving body itself. When these parts are in motion, they have by nature a tendency [*impetus*] to move along a straight path. Hence, since all the parts are joined, and any one of them is continuous with another, they suffer constraint in moving circularly and they remain joined together in such motion only under compulsion. For the more they move, the more there grows in them the natural tendency to move in a straight line, and therefore the more contrary to their nature is their circular motion. And so they come to rest naturally: for, since it is natural to them, when they are in motion, to move in straight line, it follows that, the more they rotate under compulsion, the more does one part resist the next one and, so to speak, hold back the one in front of it.<sup>47</sup>

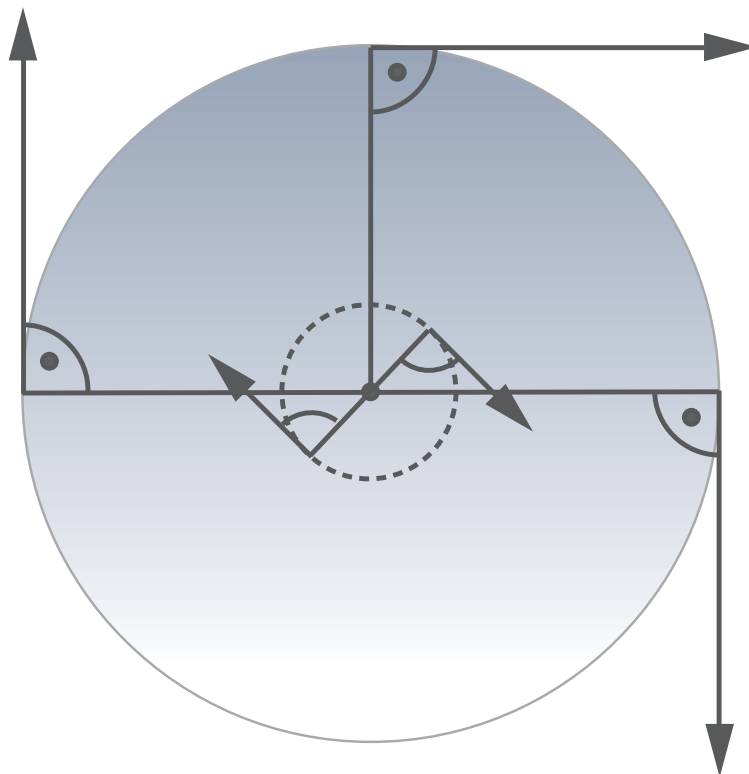


Figure 4.3: A diagram showing Benedetti's considerations on the rotating millstone stressing the centrifugal tendencies of its parts. (Drawing by Irina Tupikova)

The mental model of circular motion as constrained straight motion receives in Benedetti's treatment a higher degree of generalization. In this case, he argues that, since it contrasts

<sup>47</sup>Drake and Drabkin 1969, 229. Benedetti 1585, 285 (emphasis added): "Quaeris a me literis tuis, an motus circularis alicuius molae molendinariae, si super aliquod punctum, quasi mathematicum, quiesceret, posset esse perpetuus, cum aliquando esset mota, supponendo etiam eandem esse perfecte rotundam, et laevigatam. Respondeo huiusmodi motum nullo modo futurum perpetuum, nec etiam multum duraturum, quia praeterquam quem ab aere qui ei circumcirca aliquam resistentiam facit stringitur, est etiam resistentia partium illius corporis moti, quae cum motae sunt, natura, impetum habent efficiendi iter directum, unde cum simul iunctae sint, et earum una continuata cum alia. Dum circulariter moventur patiuntur violentiam, et in huiusmodi motu per vim unitae manent, quia quanto magis moventur, tanto magis in iis crescit naturalis inclinatio recta eundi, unde tanto magis contra suamet naturam volvuntur, ita ut secundum naturam quiescant, quia cum eis proprium sit, quando sunt motae, eundi recta, quanto violentius volvuntur, tanto magis una resistit alteri, et quasi retro revocat eam, quae antea reperitur habere."

with a natural inclination, it cannot be eternal. Note that this assumption (violent motion cannot be eternal) is Aristotelian but emerges in a context in which this legacy is meant to be rejected.<sup>48</sup>

Another Aristotelian echo looms over Benedetti's statement that the linear tendency makes a body "lighter," since if it were freed from the constraint hindering its projection, it would not fall vertically but rather travel through a more or less rectilinear trajectory tangent to the circular motion of the constrained rotation. In the conclusion of his reflection on the natural rectilinear striving of the parts of a body set in circular motion, Benedetti stressed the originality of his treatment "without precedents" and its opposition to Aristotelian dynamics (according to which the projection of a body through a medium presupposes the support of the medium itself).

But if you wish to see this truth more clearly, imagine that while the body, i.e., the top, is spinning around very rapidly, it is cut up or divided into many parts. You will observe not that those parts immediately fall toward the center of the universe, but that they move in a straight line, and, so to speak, horizontally. No one, so far as I know, has previously made this observation on the subject of the top.

From such motion of the top or of a body of this kind it may be clearly seen how mistaken are the Peripatetics on the subject of the forced motion of a body. They hold that the body is driven forward by the air which enters [behind it] to occupy the space left by the body. But actually the opposite effect [that is to say, resistance] is produced by the air.<sup>49</sup>

We have so far observed two instances in Benedetti's work on mechanics in which a tension between mathematical laws of nature and their empirical realization emerges: his treatment of the rotation of a beam about its pole and that of a turning wheel. In both cases, natural straightforward tendencies are constrained and deviated into violent circular ones. The epistemological meaning of these concepts lies in the possibility of a geometrical treatment of natural contingency seen as the connection between the *necessity* of the rules and of the principles and their *necessitation*, that is, their deviation, as witnessed by the empirical reality of curvilinear motions.

#### 4.5 From *inclinatio* to *inertia* and Beyond: Mechanistic Perspectives

René Descartes generalized the insights implicit in the idea that curvilinear motion is contingent rectilinearity at an epistemic level (through the expansion of their realm of application) as well as at an epistemological and ontological level (giving them a foundational meaning). In *Le Monde*, circular motion is treated as a deviation from rectilinear motion.

<sup>48</sup>On Benedetti's anti-Aristotelianism, see Maccagni 1983.

<sup>49</sup>Drake and Drabkin 1969, 229–230. Benedetti 1585, 285: "Sed si clarius, hanc veritatem videre cupis, cogita illud corpus, trochum scilicet, dum velocissime circunducitur secari, seu dividi in multas partes, unde videbis illas omnesque, non illico versus mundi centrum descendere, sed recta horizontaliter, ut ita dicam, moveri. Id quem a nemine adhuc (quem sciam) in trocho est obseruatum. Ab huiusmodi motu trochi, aut huius generis corporis, clare perspicitur, quam errent peripatetici circa motum violentum alicuius corporis, qui existimant aerem qui subintrat ab occupandum locum a corpore relictum, ipsum corpus impellere, cum ab hoc, magis effectus contrarius nascatur."

Descartes develops a general theory of the world in which circularity is the main characteristic of the motions of both the particles of matter as well as of planets revolving about the centers of their orbits.<sup>50</sup>

[...] when a body is moving, even if its motion most often takes place along a curved line and, as we said above, it can never make any movement that is not in some way circular, nevertheless each of its parts individually tends always to continue moving along a straight line. And so the action of these parts, that is, the inclination they have to move, is different from their motion.<sup>51</sup>

This is the third of Descartes's three laws of nature (*loix or règles de la Nature*) as exposed in chapter 7 (“*Des loix de la nature de ce nouveau Monde*”). It follows the inertial law of conservation of the state of the bodies and that of the conservation of the quantity of motion. The third law is particularly relevant from the viewpoint of our epistemological inquiry into mathematics without necessity, since it clearly expresses the gap between law and effective reality, between the straightforward tendency of all bodies and their real circular motions, in a manner that is akin to medieval and Renaissance predecessors such as Benedetti. Note that Descartes calls the rectilinear tendency “*inclination*” just as Benedetti called it “*inclinatio recte eundi*.” This terminological choice is apt to express its character as a natural inner tendency. The examples that Descartes chooses to illustrate his claim are familiar to readers of Renaissance sources on mechanics: the wheel (*une roue*) and the sling (*fronde*) (Figure 4.4).

In the *Études galiléennes*, Koyré affirmed the complete independence of the law of inertia, which is only *in nuce* in Galileo's physics, from experience, since rectilinear motion is never observed in nature. “*Contrairement à ce qu'on affirme bien souvent, la loi d'inertie n'a pas son origine dans l'expérience du sens commun et n'est ni une généralisation de cette expérience, ni même son idéalisation. Ce que l'on trouve dans l'expérience, c'est le mouvement circulaire ou, plus généralement, le mouvement curviligne. On n'est jamais—sauf le cas exceptionnel de la chute, qui n'est justement pas un mouvement inertial—en présence d'un mouvement rectiligne.*”<sup>52</sup>

In light of our reconstruction, this statement proves quite inaccurate. As we have seen, the vertical fall of a heavy body is not the only observable straight motion: the beginning of the trajectory of a projectile thrown with great speed also looks rectilinear. Slings and catapults are in fact the instruments with which turning wheels and rotating millstones were compared, and it was from these instruments that Benedetti, Descartes, and also Galileo in the Second Day of the *Dialogo sopra i massimi system del mondo*, derived the centrifugal tendencies of the parts of rotating objects. Is this not a generalization from experience? Such generalization went so far as to include the explanation of the behavior of bodies on a rotating Earth, in the case of Galileo, and the conceptualization of corpuscular and planetary motions, as was the case for Descartes. Moreover, before the classical law of inertia was defined, what took center stage was the observation of rectilinear motions—either the vertical fall or centrifugal tendencies—and of their circular deviations. A major physical problem faced by Scholastic and post-Scholastic mechanics

<sup>50</sup>On the Cartesian cosmos, see Aiton 1972, 30–64 and Gaukroger 2006, 304–317.

<sup>51</sup>Descartes 1998, 29. Descartes 1986, 43–44: “*Lors qu' un corps se meut, encore que son mouvement se fasse plus souvent en ligne courbe, et qu' il ne s' en puisse jamais faire aucun, qui ne soit en quelque façon circulaire [...], toutesfois chacune de ses parties en particulier tend toujours à continuer la sien en ligne droite. Et ainsi leur action, c' est à dire l' inclination qu' elles ont à se mouvoir, est differente de leur mouvement.*”

<sup>52</sup>Koyré 1986, 206.

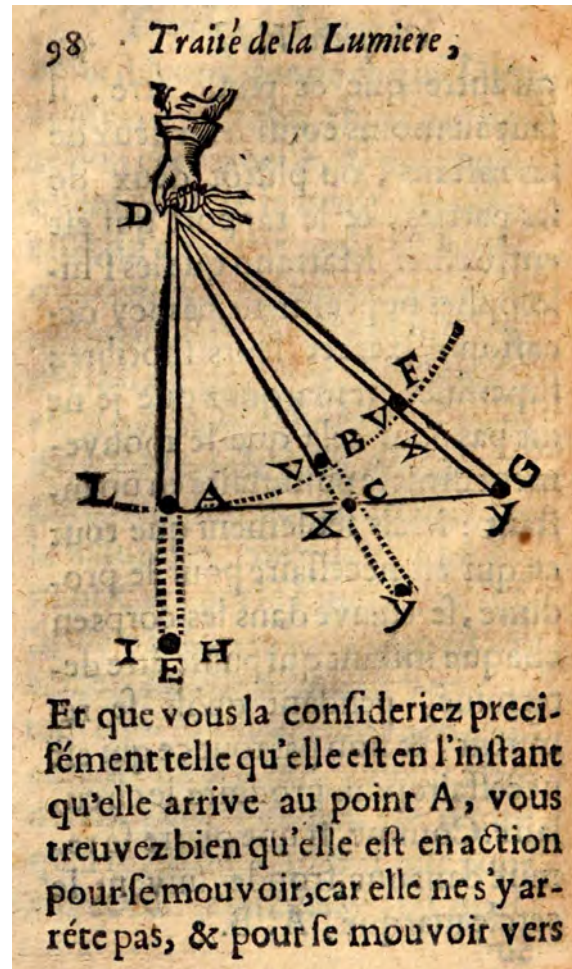


Figure 4.4: Descartes's visualization of the centrifugal tendency of bodies thrown by a sling, in *Le Monde*, Ch.7. (Bayerische Staatsbibliothek)

was precisely that of conceptualizing the relationship between curves and straight lines. In particular, against the backdrop of Aristotelian philosophy, curvilinear motion appeared as constrained. It was a derived displacement resulting from a *violent* external intervention bending the straightforward *natural* tendency of a moving body. In such an Aristotelian and post-Aristotelian context, circular motion was seen as contingent. That is to say, it was the deviation from natural order depending on an obstacle which was called the “*secundum quid*.” As we have argued, the concept of “*secundum quid*” is embedded in the Scholastic reflections upon natural necessity, order, and contingency. It was referred to as a model of causality in which the observed phenomena represent a partial fulfillment of an underlying order, or of natural laws. Accordingly, elementary bodies express their necessary laws in a limited manner, that is, they have to be explained through the so-called *necessitas conditionata* or *necessitas secundum quid*. Contingency is the relation between necessary order and phenomenal reality. The gap has to be explained, and was explained with a *quid*, a factor, or a determination. Accordingly, a *quid* was introduced into mechanics to account for circular motions in terms of mechanical constraints.

In the medieval *scientia de ponderibus*, two determinations were considered for the equilibrium of a balance: first, the circle resulting from the inclusion of the vertical motions of the weights in a mechanical system, and second, the *situm* (location) of the weights in a mechanical system determining a variation in heaviness. The reflection on *gravitas secundum situm* (positional heaviness) from Nemorarius to Benedetti presupposes this twofold *quidditas* and focuses on the latter aspect (the variation of the heaviness).

The conviction that circular motion, as a violent motion, requires an explanation is based on the mental model that “circular motion is constrained (or *contingented*) straight motion.” Although they were embedded in the medieval discourse on contingency, the several attempts to quantify the *quid* accounting for the deviation testify to the common effort to overcome the qualitative and indeterminable characterization of contingency as a form of causality. What was maintained, for instance in Descartes, was the idea of a gap between law and phenomenon. Yet, if the deviation from the law can be perfectly quantified, then the separation between the order of nature and its realization is virtually eliminated, that is, the fracture between absolute necessity and conditional necessity is recomposed. To be sure, this step toward the necessitation of nature, resulting from the abandonment of contingency in both senses (causal and epistemological), was accomplished only later, in the course of the seventeenth century.

The work of Benedetti and his onto-epistemology of contingency are representative of an age of transition from Scholastic and Renaissance natural philosophies to the various instantiations of the classical science of the next century. Benedetti’s Pythagorean commitment to mathematics, seen as the most powerful logical means applied to all fields of knowledge and to nature in particular, is an illustrative case of the complex and non-linear history of scientific thought. His efforts to overcome Aristotelian conceptions could not really renounce the crucial assumption of the Aristotelian outlook under attack. This particularly concerns the ontology and epistemology underlying his scientific theories and practices. Mathematical determination, both in science and nature, did not imply necessity, neither at the level of material causation nor of explanation. The gap between the laws of nature and the effective processes reflected a Scholastic distinction between formal necessity and material imperfection. Such philosophical assumptions underpinned medieval treatments of phenomena, including statics, and Renaissance developments, especially in the line connecting Tartaglia and Cardano to Benedetti and Descartes. The distinction between formal necessity and phenomenal contingency offered them a horizon within which they could conceptualize general laws as well as their empirical instantiation. In particular, Benedetti could extend the area of application for the mental model that circular motion is a constrained (violent) deviation from the law of rectilinear motion. He did this by applying a model originating from statics to the area of dynamics, thus paving the way for the classical concept of inertia. However, we should not neglect the practical roots of his work in a Scholastic-embedded science of weights, which generalized observations of mechanical systems in order to make universal statements about nature.



## Chapter 5

### Mechanics

The book on mechanics, *De mechanicis*, the third of the *Diversae speculationes*, is divided into twenty-five chapters. Mechanical issues and references to mechanics can also be found in the epistles. As to the discussion of the motion of fall through media and of hydraulic problems, these are not part of this book. *De mechanicis* begins with a brief preamble in which Benedetti claims that he treats topics that have never been dealt with before or have not been sufficiently explained. In this section we will discuss the positioning and controversies implicit in this strong statement in an age when mechanical studies were very lively in the Italian peninsula and abroad. We will first offer an overview of Book 3 of the *Diversae speculationes*. Second, we are going to look more closely at the first foundational chapters of the treatise. Third, we will consider the rivalry with Del Monte, emerging from the latter's harsh criticism of Benedetti and, in part, his misunderstanding of some crucial elements of Benedetti's theory. The context of these lively disputes is the reaction to the publication of Tartaglia's eclectic work on this subject, the *Quesiti, et inventioni diverse* (1546), and his re-issue of the medieval classic on the science of weights. Benedetti, as a critical pupil of Tartaglia, could not sympathize with the absolute rejection of Tartaglia and the medieval tradition his approach rested upon. At the same time, he felt the need to distance himself from several aspects of Tartaglia's treatment, as we will reconstruct in detail in this section. The debates between Benedetti and Del Monte arguably culminated with Galileo's work, which stands out as a sort of synthesis of earlier positions. Understanding these historical developments, as well as the intellectual triangle Benedetti-Del Monte-Galileo, is fundamental in order to trace Benedetti's influence on his contemporaries and on the young Galileo.<sup>1</sup>

#### 5.1 An Overview of *De mechanicis*

##### 5.1.1 The Foundations of the Theory of the Balance

Chapters 1 to 6 of *De mechanicis* contain a systematic account of the foundation on which Benedetti built his mechanics. Chapter 1 clarifies qualitatively how the variable weight changes depending on the obliqueness of the balance beam. While a body attached to the end of the beam has a maximum weight if the beam is in a horizontal position, it vanishes when the beam is in a vertical position. Benedetti explained this behavior as a consequence of the different extent to which the attached weight rests on the center of the balance. If the position of the beam is close to the vertical, the weight of a body attached to the end of the beam is close to zero since it rests nearly completely on the center of the balance.

Chapter 2 clarifies the positional changing of the weight quantitatively. Benedetti related the balance with an oblique position of the beam to a bent lever with one horizontal and one oblique arm, thus providing the precondition for a generalization of his result. A

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<sup>1</sup>Section 5.1 is derived from Renn and Damerow 2012, chap. 6.1–6.3 and section 5.2 from Renn and Omodeo 2013.

generalization of this kind is indeed required if the lines of inclination of the bodies at the end of a balance are conceived as being directed to the center of the earth and hence no longer as being parallel to each other. Benedetti mentioned this possibility at the end of this chapter, but considered the angle between the two directions as being too small to be measured and thus not necessary to be taken into account.

In chapter 3 Benedetti generalized from the downward inclination of a body attached to the balance beam to forces acting upon the body not vertically but making an acute or obtuse angle with the horizontal beam. Accordingly, he replaced the bodies at the end of the balance beam with two weights or two moving forces (*duo pondera, aut duae virtutes moventes*), as he formulated somewhat ambiguously. His derivation of their quantities was based on a reinterpretation of the horizontal distances between the center of the balance and the vertical projections of the bodies at the end of a beam in an oblique position (Figure 5.1). He interpreted these distances as perpendicular distances from the center of the balance to the lines of inclination, and was thus also able to apply the result he achieved for vertically descending weights to lines of inclination caused by forces that are not vertical.

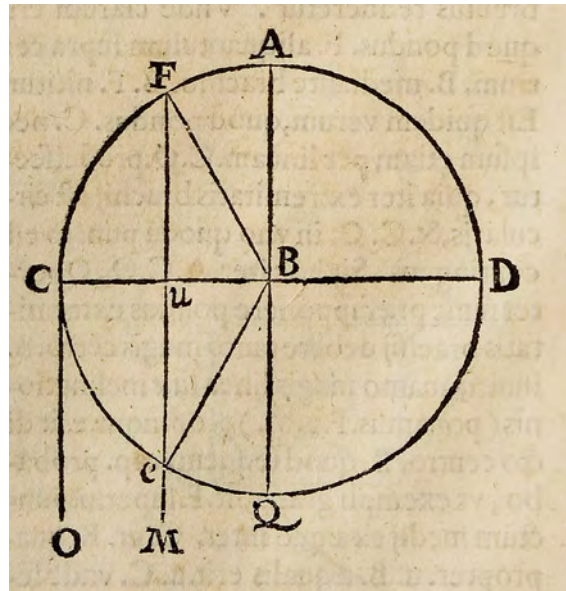


Figure 5.1: Benedetti's diagram showing a balance CBD or FBD. The lines CO and FUEM are the so-called lines of inclination connecting the weights C and F with the center of the elements. The length of the projection on the horizontal is proportional to the positional heaviness. (Max Planck Institute for the History of Science, Library)

Benedetti maintained that his arguments in chapters 1 to 3 clarify all the causes operating on balances and levers. To demonstrate this, he discussed in chapters 4 and 5 the validity of his results if applied to material balances and levers, taking into account that they have a beam with finite extension. This, however, does not imply that he calculated the influence of the weight of the beam itself. His discussion was rather restricted to a justification of his claim that the geometry of a rectangular beam does not require a modification of his propositions. In chapter 5 he treated the case of a lever whose fulcrum is at one of its ends.

Finally, in chapter 6, Benedetti added the description of an instrument used in bakeries for treating the dough. He explained the function of the instrument by applying his proposition from chapter 3.

The systematic approach used by Benedetti in this first part of his treatise is complemented by chapter 9, in which he explained the division of the scale of a steelyard into equal intervals.

### 5.1.2 Criticism of Tartaglia and Nemorarius

In chapters 7 and 8 Benedetti criticized the theorems of his former teacher Tartaglia, in particular those that Tartaglia adapted from Jordanus Nemorarius. Both chapters deal exclusively with some propositions of Book 8 of Tartaglia's *Quesiti, et inventioni diverse*,<sup>2</sup> which is concerned with the science of weights and is entitled, accordingly, *Sopra la scientia di pesi*. In those cases in which Tartaglia's propositions are adapted from Nemorarius, Benedetti mentioned explicitly the corresponding proposition in the edition of Nemorarius' *De ratione ponderis*, corrected and illustrated by Tartaglia, and published under the title *Iordani opusculum de ponderositate Nicolai Tartaleae studio correctum novisque figuris auctum*.<sup>3</sup>

Chapter 7 starts with some brief critical remarks on Tartaglia's propositions 2 to 5. Tartaglia's proposition 2 essentially paraphrases and modifies the Aristotelian claim that the speed of moving bodies is proportional to the driving force. Following Nemorarius, Tartaglia maintained that the velocities of descending heavy bodies of the same kind are proportional to their power (*potentia*), while in the case of ascending bodies their velocities are inversely proportional to their power. For bodies of the same kind their power is conceived here as proportional to their sizes, that is, to their weights. Descending bodies are thus simply falling bodies with velocities proportional to their weights, while in the case of ascending bodies their weight acts as a resistance. Tartaglia's proposition 3 generalizes proposition 2 for bodies with equal weights but unequal positional heaviness. His proposition 4 maintains that in the latter case the power of bodies attached to a balance is proportional to the distances from the center.

Benedetti's critical remarks are somewhat eclectic. He argues that Tartaglia, in his second proposition, does not take into account the quantity of external resistance (*quantitatis momenti sint extrinsecae resistentiae*). With regard to Tartaglia's third proposition, Benedetti points to its assumptions, namely that the bodies have to be homogenous and must have the same shape. He criticizes Tartaglia's proof as it does not actually require these assumptions, but would also be true for heterogeneous bodies or for bodies with differing shapes. Concerning the fourth proposition, he criticizes Tartaglia for not proving what he claimed to prove. Instead, he should have followed Archimedes's proof of the law of the lever.

Benedetti's chapter 7 continues with a detailed discussion of the second part of Tartaglia's proposition 5 and the following two corollaries and is thus directly concerned with the equilibrium controversy, that is, the controversy about whether or not a balance in equilibrium removed from its horizontal position will automatically return to this position. Tartaglia maintained in this proposition that a balance that is in equilibrium in a horizontal position will necessarily return to this horizontal position when moved into an oblique position. In a first corollary, he claimed that the more the balance beam is brought into an oblique position, the more the bodies attached to it become positionally lighter. In a second corollary, he claimed that while both bodies in this case become positionally lighter, the lifted body loses less of its positional heaviness than the body moving down. He concluded that the beam will return to a horizontal position. Benedetti

<sup>2</sup>Tartaglia 1546.

<sup>3</sup>Nemore 1565.

questioned Tartaglia's approach by referring to the first three chapters of his own treatise, arguing in particular that Tartaglia's second corollary must be wrong. He discussed once more the balance beam in an oblique position, but now without the assumption that the lines of inclination of bodies attached to the balance beam are parallel. Rather, he considered the case that these lines are directed to the center of the world, showing, as we have discussed, that it is not the lifted body, but rather the body that is moved down, which loses less of its positional heaviness.

Benedetti continued in chapter 8 with critical comments on Tartaglia's propositions 6, 7, 8, and 14. Tartaglia's proposition 6 contains the proof of his fallacious claim that the lifted body of an oblique balance beam loses less of its positional heaviness than the body moving down, now modified by the further claim that the difference is smaller than any finite quantity. Tartaglia claimed:

[...] that the differences between the heaviness of these two bodies is impossible to give or find between two unequal quantities.<sup>4</sup>

Like Del Monte had done before him, but with different results, Benedetti criticized Tartaglia for not taking into account that the lines of inclination are not parallel.

Tartaglia's proposition 7 contains the simple statement that if the arms of a balance are unequal and bodies with equal weights are attached to the ends of the beam the balance will tilt on the side with the longer arm. Benedetti criticized Tartaglia again for not taking into account that the lines of inclination are not parallel, and claimed that in any case Tartaglia did not give the correct cause of the effect.<sup>5</sup>

Tartaglia's proposition 8 formulates, following Nemorarius, the law of the lever in terms of positional heaviness, stating that if the lengths of the parts of the balance beam with unequal arms are inversely proportional to the weights of the bodies attached to them, their positional heaviness will be equal. Benedetti criticized that this proposition is much better demonstrated by Archimedes.

Finally, Tartaglia's propositions 14 and 15 concern Nemorarius's proof of the law of the inclined plane, which from a modern perspective is essentially correct. Benedetti criticized Tartaglia's argument by attributing to it an interpretation of the inclined plane as a balance, with the top of the plane being its center. His criticism, based on the propositions of his chapters 1 to 3, thus completely missed the point of Tartaglia's argument.

### 5.1.3 Criticism of Aristotle's Mechanics

Benedetti's treatise on mechanics continues mainly with critical notes on the Aristotelian *Mechanical Problems*.<sup>6</sup> His notes are as diverse as the Aristotelian *Mechanical Problems* themselves.

Before he embarked on this criticism, Benedetti dealt with the problem of why a steelyard carries a linear gradation in chapter 9.<sup>7</sup> He took into account the weight of the beam and that of the scale by postulating the equilibrium of the balance when no extra weight is added. Then he added weights of one pound on both sides, arguing that, by

<sup>4</sup>Tartaglia 1546, 91r: "[...] che la differenza ch'è fra le gravità de questi dui corpi egli è impossibile a poterla dar, over trovar' fra due quantità inequali." Translation in Drake and Drabkin 1969, 130.

<sup>5</sup>We will discuss Benedetti's criticism in more detail later.

<sup>6</sup>Aristotle 1980. See Rose and Drake 1971 and also the introduction to Nenci 2011.

<sup>7</sup>Benedetti 1585, 152. See Drake and Drabkin 1969, 178.

common science (*scientia communis*),<sup>8</sup> the balance stays in equilibrium if they are placed at equal distances from the fulcrum. He had thus found the mark on the beam that indicates a magnitude of one pound. He then successively placed further weights onto the scale, now arguing from the law of the lever that they must be compensated by distances proportional to their number. He thus avoided the problem of applying the law of the lever directly to a material steelyard, just as one does in practice when gauging such a balance.<sup>9</sup>

In chapters 10 and 11 Benedetti started with critical remarks on Aristotle's first problem. Aristotle asked why larger balances are more accurate than smaller ones.<sup>10</sup> Actually, this concrete physical question is not the focus of the extensive answer the author gave to this problem. Instead, he provided a long proof of the basic explanatory principle which plays a major role in the whole treatise. At the end of the proof Aristotle argued that the same load will move faster on a larger balance, thus making such balances more accurate.<sup>11</sup>

The criticism Benedetti applied to Aristotle's argument has two parts. In chapter 10 Benedetti began by rejecting Aristotle's claim that the circumference of a circle combines concavity with convexity. He then argued against a specific part of Aristotle's proof of his principle which involves the superposition of motions. In this part Aristotle showed that:

[...] whenever a body is moved in two directions in a fixed ratio it necessarily travels in a straight line.<sup>12</sup>

He concluded:

[...] if a body travels with two movements with no fixed ratio and in no fixed time, it would be impossible for it to travel in a straight line.<sup>13</sup>

For the Aristotelian author this proposition served as a means to describe circular motion as a result of two movements with no fixed ratio. Benedetti, however, did not relate his criticism to this context. He argued only that Aristotle's inference concerning movements in two directions is not sufficient since a straight movement can result from two quite different motions. This criticism does not really relate to the Aristotelian argument, other than by showing that his entire attempt to derive the behavior of a balance from a principle of circular motion is misguided.

In the same vein, Benedetti's criticism in chapter 11 then deals directly with Aristotle's answer to the question of why larger balances are more accurate than smaller ones. He argued that Aristotle's argument is not well founded since the greater accuracy has nothing to do with the motion of the balance beam but only with the geometrical constellation.<sup>14</sup> To conclude he added a consideration of material balances, arguing according to his own principles that a weight on the larger balance will be positionally more effective.

<sup>8</sup>In the sixteenth century the term *scientia communis* was used to designate knowledge common to all mathematical sciences, its core being the Euclidean theory of proportions. See Sepper 1996, 153–154.

<sup>9</sup>See the discussion in Damerow, Renn, et al. 2002.

<sup>10</sup>Aristotle 1980, 1, 848 b 1–850 a 2 (337–347).

<sup>11</sup>Aristotle 1980, 1 (347).

<sup>12</sup>Aristotle 1585, 507: “Quandoquidem igitur in proportione fertur aliqua id, quod fertur, super rectam ferri necesse.” Translation in Aristotle 1980, 1, 848 b 11–848 b 13 (337).

<sup>13</sup>Aristotle 1585, 508: “Si autem in nulla fertur proportione secundum duas lationes nullo in tempore, rectam esse lationem est impossibile.” Translation in Aristotle 1980, 339.

<sup>14</sup>Benedetti 1585, 153; Drake and Drabkin 1969, 180–182.

Benedetti's chapter 12 concerns problems 2 and 3 of the Aristotelian *Mechanical Problems*.<sup>15</sup> Problem 2 raises the question that forms the starting point of the equilibrium controversy:

If the cord supporting a balance is fixed from above, when after the beam has inclined the weight is removed, the balance returns to its original position. If, however, it is supported from below, then it does not return to its original position. Why is this?<sup>16</sup>

Aristotle implicitly assumed that the balance beam has a certain thickness and weight. It follows as a result of the geometry of the balance in an oblique position that if the beam is fixed from above, a greater part of the beam is on the lifted side of the perpendicular line across the suspension point. Consequently the beam will move back by itself into the horizontal position. The opposite is true for a beam fixed from below. In this case, the greater part of the beam is on the lower side so that it cannot move back into a horizontal position by itself.

Benedetti criticized the first case by arguing that it is not only the weight of the beam that causes it to return to the horizontal position, but also the different distances of the weights in an oblique position from the vertical through the point where the beam is fixed. According to his theory of the dependency of the weight on the obliqueness of the beam, the weights must be different on both sides. Benedetti thus generalized Aristotle's argument to the case of a balance without a material beam carrying weight itself.

In the second case of a beam supported from below, he argued that Aristotle is completely mistaken. Benedetti maintained that the beam will not remain in its oblique position, but that the lower part will move down until the beam is in the vertical position.

Problem 3 of the Aristotelian *Mechanical Problems*<sup>17</sup> concerning an explanation of the effect of a lever is, for Benedetti, not worth the effort of a detailed criticism. He only briefly notes that Aristotle did not give the true cause, which one will find in his own theory presented in chapters 4 and 5.<sup>18</sup>

In the very short chapter 13, Benedetti criticized problem 6 of the Aristotelian *Mechanical Problems*:

Why is it that the higher the yard-arm, the faster the ship travels with the same sail and the same wind?<sup>19</sup>

The Aristotelian answer provided in the *Mechanical Problems* is based on an interpretation of the yard-arm as a lever that has its base at the point where the yard-arm is fixed as the fulcrum. Benedetti maintained that this interpretation of the yard-arm as a lever:

[...] does not give the true explanation. For on this kind of explanation the ship would have to move more slowly rather than more swiftly. For the higher

<sup>15</sup>Aristotle 1980, 347–355; Drake and Drabkin 1969, 182–183.

<sup>16</sup>Aristotle 1585, 511: “Cur siquidem sursum fuerit spartum, quando deorsum lato pondere, quispiam id admovet, rursum ascendit libra: si autem deorsum constitutum fuerit, non ascendit, sed manet?” Translation in Aristotle 1980, 347–349.

<sup>17</sup>Aristotle 1980, 353–355.

<sup>18</sup>Benedetti 1585, 154; Drake and Drabkin 1969, 183.

<sup>19</sup>Aristotle 1585, 515: “Cur quando antenna sublimior fuerit, iisdem velis, et vento eodem celerius feruntur navigia?” Translation in Aristotle 1980, 361.

the sail that is struck by the force of the wind, the more the ship's prow will be submerged in the water.<sup>20</sup>

Benedetti added one sentence with his own explanation, according to which the ship with a higher sail moves more swiftly because the wind blows more strongly in the higher region.

Chapter 14 provides a long discussion of problem 8 of the Aristotelian *Mechanical Problems*. The question posed in this problem is why round and circular bodies are easiest to move. Three examples are mentioned and later discussed: the wheels of a carriage, the wheels of a pulley, and the potter's wheel. Benedetti claimed that Aristotle's answer to the question he posed is not sufficient. Nevertheless, Benedetti himself argued essentially in a similar manner, only somewhat more extensively. Both of them argued that the circle, contrary to differently shaped bodies, touches a plane only at one point which can be considered as the fulcrum of a lever. But Benedetti added a further argument which is not given by Aristotle. He argued that a circle can be pulled along a plane without difficulty and resistance:

[...] because in such a case the center will never change its position by moving upward from below, i.e., will never change its position with respect to the distance or interval which lies between it and line *AD*.<sup>21</sup>

At the end of the chapter, Benedetti discussed the question of why a potter's wheel set into motion by an external force will continue to rotate for a time but not forever. In his response he took into account the friction with the support of the wheel and with the surrounding air. But he also discussed reasons that are more deeply concerned with the nature of such motion, as we have discussed above. He claimed, in particular, that the rotational motion is not a *natural motion* of the wheel, evidently making reference to the Aristotelian distinction between natural and violent motions. He also claimed that a body moving by itself because an *impetus* has been impressed upon it by an external force has a natural tendency to move along a rectilinear path. This statement seems to come close to the principle of inertia of classical physics, but it actually deals with rectilinear motion as a forced motion and does not involve any assertion about its uniformity. Benedetti seems to suggest, in any case, that this natural tendency is in conflict with the forced rotational motion of the wheel, which in turn slows it down. The smaller the wheel and the more its parts are constrained to deviate from the rectilinear path, the greater the decrease in speed will be.<sup>22</sup>

In chapters 15 and 16 Benedetti dealt with issues of scale as they are brought up by the Aristotelian *Mechanical Problems*. In chapter 15, consisting merely of one short sentence, Benedetti referred to his own earlier treatment of Aristotle's question of why larger balances are more exact (erroneously citing chapter 10 instead of chapter 11 of his treatise) in order to deal with the ninth problem of the Aristotelian *Mechanical Problems*, which reads:

<sup>20</sup>Benedetti 1585, 155: "[...] verum non est. Huiusmodi enim ratione navis tardius potius, quam velocius ferri deberet, quia quanto altius est velum, vi venti impulsum, tanto magis proram ipsius navis in aquam demergit." Translation in Drake and Drabkin 1969, 183.

<sup>21</sup>Benedetti 1585, 155: "[...] quia huiusmodi centrum ab inferiori parte ad superiorem, nunquam mutabit situm respectu distantiae seu intervalli, quae inter ipsum lineamque *AD* intercedit." Translation in Drake and Drabkin 1969, 184.

<sup>22</sup>For the historical context, see Büttner 2008.

Why is it that we can move things raised and drawn more easily and more quickly by means of greater circles?<sup>23</sup>

In chapter 16 he discussed the tenth problem of the Aristotelian *Mechanical Problems*, which reads:

Why is a balance moved more easily when it is without a weight than when it has one?<sup>24</sup>

In his detailed response to this problem—indeed much more detailed than the one found in the Aristotelian text—Benedetti compared balances that are alike with different sets of weights on their scales, one with two weights of one ounce, the other with two weights of one pound. He then added a half-ounce weight on one side of each balance and observed that the balance with the smaller weights moves more rapidly. He explained this effect by referring to the dynamical assumption that one always has to consider *the ratio of the moving force to the body moved*.

In chapter 17 Benedetti addressed the twelfth problem of the Aristotelian *Mechanical Problems*, which reads:

Why does a missile travel further from the sling than from the hand?<sup>25</sup>

Benedetti's response is based on the concept of *impetus*, conceived as an intrinsic cause of motion originally acquired by the action of an external force that then gradually decreases after separation from the original mover. He argued that a greater impetus can be impressed by the sling due to the repeated revolutions which evidently lead to an accumulation of this intrinsic force. He observed that the impetus would lead, if not impeded by the sling or the hand, to a straight motion of the projectile along the tangent to the circle of its forced motion. He also noted—distancing himself from a claim made by Tartaglia—that the motion due to the impressed force can mingle with the projectile's natural motion downward, thus leading to a curved trajectory. It may well be the case that it was this claim that later convinced Galileo and Del Monte to perform their experiment on projectile motion from which they drew the conclusion that such a mixture of motions indeed takes place.<sup>26</sup>

In chapter 18 Benedetti considered problem 13 of the Aristotelian *Mechanical Problems* dealing with the question of why larger handles can be moved more easily around a spindle than smaller ones.<sup>27</sup> In his short response Benedetti simply referred to the fourth and fifth chapters of his own treatise, stressing that everything depends on the lever. He was evidently convinced that the Aristotelian reduction of such problems to properties of the circle is superfluous, if not misguided.

In chapter 19 he handled problem 14 of the Aristotelian *Mechanical Problems* in the same way. It reads:

<sup>23</sup>Aristotle 1585, 517: "Cur ea, quae per maiores circulos tolluntur et trahuntur, facilius et citius moveri contingit [...]?" Translation in Aristotle 1980, 365.

<sup>24</sup>Aristotle 1585, 517: "Cur facilius quando sine pondere est, movetur libra, quam cum pondus habet?" Translation in Aristotle 1980, 365.

<sup>25</sup>Aristotle 1585, 518: "Cur longius feruntur missilia funda, quam manu missa [...]?" Translation in Aristotle 1980, 367.

<sup>26</sup>See the discussion in Renn, Damerow, and Rieger 2001.

<sup>27</sup>Aristotle 1980, 367.



Why is a piece of wood of equal size more easily broken over the knee, if one holds it at equal distance far away from the knee to break it, than if one holds it by the knee and quite close to it?<sup>28</sup>

Again, Benedetti just referred to the earlier chapters of his treatise.

In chapter 20 Benedetti reconsidered problem 17 of the Aristotelian *Mechanical Problems*, which reads:

Why are great weights and bodies of considerable size split by a small wedge, and why does it exert great pressure?<sup>29</sup>

In the Aristotelian text, the answer is based on interpreting the wedge as two levers opposite to each other, their fulcra being placed at the entry points of the wedge into the wood. Benedetti, however, disagreed with the identification of the two levers allowing the action of the wedge to be interpreted in terms of force, fulcrum, and resistance. He claimed that the fulcrum is actually placed just underneath the deepest point of the opening produced by the wedge entering a block of wood.

In chapter 21 Benedetti claimed to provide the true explanation of compound pulleys. He reduced a compound pulley to a chain of balances by appropriately identifying forces and fulcra, each wheel of the pulley corresponding to one balance.

In chapter 22 Benedetti discussed Aristotle's wheel, i.e., problem 24 of the Aristotelian *Mechanical Problems*, which reads:

A difficulty arises as to how it is that a greater circle, when it revolves, traces out a path of the same length as a smaller circle, if the two are concentric.<sup>30</sup>

While the author of the *Mechanical Problems* referred to dynamical reasons in explaining this apparent paradox, Benedetti resorted to a kinematic argument, a pointwise reconstruction of the trajectory of the motion of a point on the circumference, arguing that it results from a superposition of two motions. In the case in which the motion is controlled by the larger circle, a point on the circumference of the smaller circle traverses a path resulting from an *addition* of two motions. In the case in which the motion is controlled by the smaller circle, a point on the circumference of the larger circle traverses a path resulting from a *subtraction* of two motions.

Chapter 23 of Benedetti's treatise does not exist.<sup>31</sup> In chapter 24 Benedetti discussed problem 30 of the Aristotelian *Mechanical Problems*, which reads:

Why is it that when men stand up, they rise by making an acute angle between the lower leg and the thigh, and between the trunk and the thigh?<sup>32</sup>

In his response Benedetti suggested that the reason for this behavior is to create an equilibrium of the body with regard to the line that serves as support underfoot.

In chapter 25 Benedetti addressed the last problem, problem 35 of the Aristotelian *Mechanical Problems*, which reads:

<sup>28</sup>Aristotle 1585, 518: "Cur eiusdem magnitudinis lignum facilius genus frangitur, si quispiam aequi diductis manibus extrema comprehendens fregerit, quam si iuxta genu?" Translation in Aristotle 1980, 369.

<sup>29</sup>Aristotle 1585, 520: "Cur a parvo existente cuneo magna scinduntur pondera, et corporum moles, validaque sit impressio?" Translation in Aristotle 1980, 371.

<sup>30</sup>Aristotle 1585, 525: "Dubitatur quam ob causam maior circulus aequalem minori circulo convolvitur lineam, quando circa idem centrum fuerint positi." Translation in Aristotle 1980, 387.

<sup>31</sup>In Drake and Drabkin 1969, 193; chapter 22 is erroneously numbered as chapter 23.

<sup>32</sup>Aristotle 1585, 532: "Cur surgentes omnes, femori crus ad acutum constituentes angulum, et thoraci similiter femur, surgunt?" Translation in Aristotle 1980, 403–405.

Why do objects which are travelling in eddying water all finish their movement in the middle?<sup>33</sup>

Benedetti's answer simply referred to the fact that whirlpools are depressed in their middle without giving an explanation of this phenomenon. He could thus restrict himself to arguing that the motion of an object to the center of such a whirlpool is simply its natural downward motion. The final comment by Benedetti is a remarkable conclusion to his criticism of Aristotle as well as his treatise on mechanics:

But in the case of all those other problems that I have omitted, Aristotle's explanations are correct.<sup>34</sup>

## 5.2 The Beginning of Benedetti's Mechanics

After our overview of Benedetti's book on mechanics, we concentrate on the theses he expounded in the first chapters because they have a foundational character and proved particularly controversial, at least in light of Del Monte's criticism, which we are aware of from the comments he made in one of his notebooks and from marginal notes in his own copy of Benedetti's book.

### 5.2.1 *De mechanicis* I: "On the different positions of balance beams"

In chapter 1, Benedetti notes that "a body (*pondus*) [...] acquires a larger or smaller weight (*gravitas*) depending on the different ratio of the beam's position" (*pondus... maiorem, aut minorem gravitatem habet, pro diversa ratione situs ipsius brachii*). According to him, a body has the greatest heaviness when the beam at whose extremity it is loaded is in the horizontal position. His idea is based on a simple common-sense intuition: if one considers an equal-arms balance suspended at its center, the weight of a loaded body is:

- borne entirely by the fulcrum when resting vertically upon it,
- entirely hanging on the fulcrum when suspended vertically below it,
- not supported in any way by the fulcrum when the beam is in the horizontal position.

In the first case, the body completely rests or leans on the center (*nititur*), and the center in turn hinders (*impellet*) the downward tendency of the weight. In the second case, the body is suspended vertically (*pendet*) and the center "attracts" it (*attrahet*), in the sense that it hinders its natural tendency to fall down (*inclinatio*). Hence, the body attains its maximum weight in the third case. If the balance beam moves upward, departing from the horizontal position, the weight slowly decreases and reaches its minimum at the top when the beam is in the vertical position. If the rotatory motion around the fulcrum continues, now downward, the weight increases again until it reaches its maximum in the horizontal position. It then diminishes until it is suspended entirely below the fulcrum. Benedetti visualizes these variations of weight depending on the position (*situs*) in a diagram comparing the lines connecting the weight to the center of the world in different cases, more specifically if the beam is:

- horizontal,

<sup>33</sup>Aristotle 1585, 533: "Cur ea quae in vorticosis feruntur aquis, ad medium tandem aguntur omnia?" Translation in Aristotle 1980, 409.

<sup>34</sup>Benedetti 1585, 167: "[...] a quo aliarum omnium quaestionum, quas ego omisi rationes sunt bene propositae." Translation in Drake and Drabkin 1969, 196.

- raised upward, or
- moved downward with the same angle as in the second case (which is equivalent to 2).

The parallel lines, called *lineae inclinationis* or *lineae itineris*, indicate the direction in which a body would fall if it were free. The closer these lines are to the center of the beam, Benedetti says, the “less heavy” the body becomes.

In his own copy of Benedetti's book, Del Monte wrote a brief annotation in the margin of chapter 1: “this first chapter is derived entirely from our treatise on the balance in the *Mechanicorum liber*.”<sup>35</sup> Clearly, he sought to assert the relevance of his treatise for Benedetti's speculations, in spite of the latter's claims of originality. It should be remarked, however, that Del Monte's treatment of the balance, based on the concept of center of gravity, was significantly different from Benedetti's, which was based on an original reworking of *positional heaviness*. Del Monte merely reassessed a concept received from authors such as Jordanus Nemorarius, Tartaglia, and Cardano, all of whom he personally opposed. In his book on mechanics, Del Monte had in fact criticized the concept of *positional heaviness*. Downplaying Benedetti's theory as a repetition of his predecessor's theories, he could therefore claim that his own treatment already included a summary (as well as a criticism) of Benedetti's approach.

### 5.2.2 *De mechanicis II: On the proportion of weights at the extremities of a balance beam in a position other than the horizontal*

In chapter 2, Benedetti deals with the proportions of a weight placed at the extremity of a balance beam if its position is not horizontal (*De proportione ponderis extremitatis brachii librae in diverso situ ab horizontalis*). The thesis to be demonstrated is the following: “The proportion between [the weight of] a body (*pondus*) at *C* and [the weight of] the same body (*pondus*) at *F* corresponds to that between the whole beam *BC* and its part *BU*, which is [set on the beam *BC* and is] delimited by the fulcrum and the [intersection between the beam and the] inclination line *FUM* that connects the weight at *F* to the center of the world” (Benedetti 1585, 142). For the sake of simplicity, we will represent these relations symbolically in modern terms:

$$C : F = BC : BU$$

where *C* is the weight in the horizontal position and *F* in the inclined position; *BC* is the beam and *BU* the part of the beam *BC* between the center *B* and the perpendicular line drawn from *F*.

Benedetti's demonstration is as follows. He imagines placing a weight *D* on the other extremity of the balance that has the same proportion to *C* as *F*, that is, the following proportion expressed in modern terms:

$$D : C = BU : BC.$$

In accordance with Archimedes's *De ponderibus* I. 6, the balance will be stable if the weight *C* is loaded at *U*, since weights and distances from the fulcrum are proportional by supposition.

<sup>35</sup>“Hoc primum caput to[tum] desumptum est a n[ostro] *Mechanicorum libri* tractatu de lib[ra].”

The next step is to show that  $F : C = BF : BU$  (where  $BF$  is the beam, hence  $BF = BC$ ). In order to demonstrate this, Benedetti resorts to the mental model (*imaginemur*) of a string hanging vertically from  $F$ , to which a weight equal to  $C$  is suspended. He claims that it is visually evident that the weight has the same effect at  $F$  as at  $U$ . The same is valid for the case in which the weight is suspended from  $U$  and intersects the circumference described by the rotation of the beam at a point  $E$ . In both cases, the balance would remain horizontal since the weight  $C$  at  $F$ ,  $U$ , or  $E$  would balance the weight at  $D$ . Benedetti further argues that the balance under consideration can be treated like a bent lever with a horizontal and an inclined arm ( $FBD$  or  $EBD$ ): “si brachium  $BE$  consolidatum fuisset [...]” (If the beam  $BE$  was made solid [...]).

The author concluded that his reasoning has satisfactorily demonstrated his thesis: “A body (*pondus*) is more or less heavy (*grave*) the more or less it hangs from (*pendet*) or rests on (*nititur*) the fulcrum” (Benedetti 1585, 142). And he deems this resting on or hanging from the fulcrum to be the most direct cause (*haec est causa proxima, et per se*) of the positional changing of a weight.

As an additional commentary, Benedetti remarks that in his diagram he supposes the inclination line  $CO$  to be perpendicular to  $CB$  and parallel to  $BQ$ , whereas  $CO$  and  $BQ$  in fact converge at the center of the sphere of the elements (*centrum regionis elementaris*), that is, the earth. But for the sake of his present argumentation, this angle is negligible and one may simply assume perpendicularity and parallelism. Benedetti thus developed a method to quantify positional heaviness that corresponds to the modern concept of “torque.”

### 5.3 Del Monte's Criticism Concerning the Non-Negligibility of the World's Center

As will be shown in the following section, it was only in his initial treatment of the inclined balance, in chapter 1 of *De mechanicis*, that Benedetti neglected to consider the convergence of the inclination lines to the center of the elements. This omission gave rise to criticism. Del Monte severely criticized both this assumption and Benedetti's reasoning in general in *De mechanicis*, in his handwritten notes on scientific and technical matters known as *Meditatiunculae de rebus mathematicis*. In his notes he assessed Benedetti's arguments from his perspective, relying on the concept of the center of gravity as it was developed in his own book on mechanics.

In a marginal note to the *Diversae speculationes* (Figure 5.2), Del Monte expressed his disagreement with Benedetti's conclusion: “Thus, in this manner, a weight (*pondus*) more or less hangs from or rests on the center; this is the next cause and the [cause] in itself [of the variation in heaviness].”<sup>36</sup> His disagreement reads as follows:

because that [that is, the greater or smaller extent to which a weight rests at the center] is neither the next [cause] nor the [cause] in itself. For the weight at  $F$  of the arm  $BF$  is not equally heavy as the weight  $U$  of the arm  $BU$ ; nor is the weight at  $E$  of the arm  $BE$  equally heavy as the weight at  $U$  of the arm  $BU$ . Thus, this entire demonstration is false.<sup>37</sup>

<sup>36</sup> “[...] unde fit ut hoc modo pondus magis aut minus a centro pendet aut eidem nititur: atque haec est causa proxima, et per se [...]”

<sup>37</sup> See Renn and Damerow 2012, 207: “non est neque proxima neque per se; nam [pond]us in  $F$  brachii [ $BF$ ] non est euegrave ut pondus in  $U$  brachii  $BU$ ; [nec] pondus in  $E$  brachii  $BE$  est euegrave ut pondus [in]  $U$  brachii  $BU$ . Unde tota haec demonstratio falsa est.”

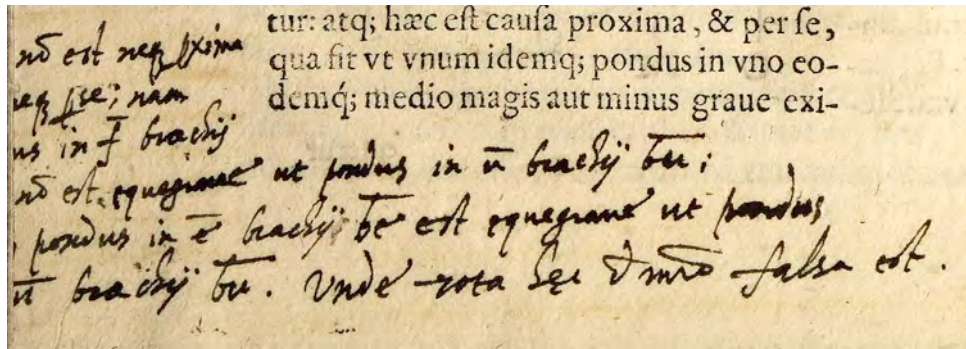


Figure 5.2: Del Monte's marginal note to *De mechanicis*, II. (Max Planck Institute for the History of Science, Library)

This means that Del Monte did not accept the claim that a weight is equally heavy in different positions on the balance beam, provided the projections of the beam along the horizontal are the same length or rather, as Benedetti writes, the distances between the projections of the beam on the horizontal and the center have the same lengths.

To find Del Monte's counter-arguments, one must look to the *Meditatiunculæ*, f. 145, *Contra Cap. 2 Jo. de Benedicti de Mechanicis*. As mentioned, he basically rejected Benedetti's perspective by objecting that he did not take into due account the finite distance of the weights from the center of the world and hence the fact that the plumb lines are not parallel to each other, as Benedetti assumed in this part of his treatise.

In his diagram (Figure 5.3) Del Monte compared the line  $LUS$  (parallel to the line  $AQ$ , connecting the fulcrum  $B$  of the balance with the center of the world  $M$ ) with the line  $FM$  (connecting the upper weight  $F$  and the lower weight  $E$  with the center of the world  $M$ ).  $S$  is the point at which the line  $LUS$  meets the circle that the beam makes around the fulcrum, which is above the position of the lower weight  $E$ .

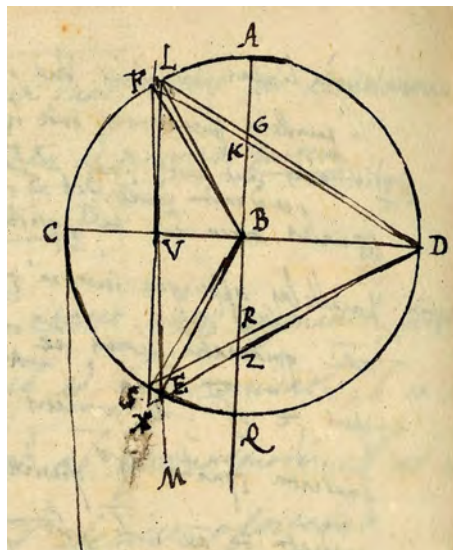


Figure 5.3: Del Monte's critical reworking of Benedetti's diagram in *Meditatiunculæ*, f. 145. (Bibliothèque Nationale de France)

Next, he considered a bent lever made of the oblique arm  $BS$ , rigidly connected to the straight arm  $BD$ , assuming that  $BU$  is half  $BD$ . If a weight is now placed at  $S$  that is double the weight at  $D$ , the bent lever will be in equilibrium, as Del Monte showed with reference to his book, because the center of gravity of the weights at  $S$  and at  $D$  will be at the point  $R$ , which will be in its lowest place on the vertical line  $BQ$ . He therefore concluded that it is the weight at  $S$ , but not the lower weight  $E$ , that will be equally heavy as the weight at  $U$ .

He proceeded to demonstrate this in greater detail by considering the proportions into which the line connecting the two weights is cut by the perpendicular  $BQ$  for the two cases, that is, the weight placed at  $S$  and the weight placed at  $E$ . Del Monte concluded that the same weight is heavier at  $S$  than at  $E$ . He then turned to a closer consideration of the upper weight  $F$ . Again he constructed a bent lever  $LBD$  in equilibrium in order to compare it with the bent lever formed with the upper weight  $F$ . Again he showed that the weight is heavier at  $L$  than at  $F$ .

Del Monte concluded by summarizing that the entire fallacy is due to Benedetti assumption that the weight at  $F$  would gravitate in the same way as at  $U$ , which would only be the case, according to Del Monte, if it were to hang freely.

#### 5.4 Benedetti on Weights and Forces Acting on a Balance

Chapter 3 of Benedetti's *De Mechanicis* contains a generalization of the results of chapter 2 or, rather, presents a general rule concerning the action of forces (*virtutes*) on balance beams, including in the case that they do not act vertically downward but also with an acute or obtuse angle. Benedetti moves forward from the result of the previous chapter as follows: the length of the line perpendicularly connecting the center to the line of inclination (the line  $BU$  in the diagram) allows the quantity of the positional force (*quantitas virtutis... in... situ*) of a weight ( $F$  in the diagram) to be established. Thus, Benedetti calls the positional weight a force, and this is the presupposition that allows him to generalize from *gravitas* the action which he calls *virtutes moventes*, or "moving forces." The thesis of this chapter is summarized in its title: "That the quantity of any given weight (*pondus*) or moving force in relation to another quantity can be determined thanks to the perpendicular projections connecting the center of the balance to the line of inclination."

Benedetti draws two diagrams showing a balance at whose extremities two weights or forces act in different directions (Figure 5.4). At the left extremity  $B$ , a weight  $E$  has a downward tendency, while at the right extremity, a weight  $C$  acts making an acute or an obtuse angle. According to Benedetti, the length of the perpendicular projection drawn from the center to the inclination line,  $OT$ , permits the determination of the distance  $OI$  on the beam at which the same force acting vertically downward produces the same effect. Given this equation, Benedetti can determine how much the force acting in a non-perpendicular direction has to be augmented in order to balance an equal weight acting perpendicularly on the opposite beam. This measure is given according to the following proportion (expressed in modern terms):

$$E : C = BO : OI$$

where  $E$  is the weight acting vertically on the extremity  $B$ ;  $C$  is the *virtus movens* acting on the opposite extremity  $A$  at an angle;  $BO$  is the left beam and  $OI$  the part of the right beam  $OA$  determined as explained above.

In his argumentation, Benedetti thus equates a balance ( $BOI$ ) with a bent lever ( $BOT$ ). Accepting this equation, he concluded that, according to commonly shared knowledge (*communi quadam scientia*), the weights or forces that are required to obtain a perfect balance can easily be calculated.

The chapter ends with a cosmological corollary: "The closer the center  $O$  of the balance is to the center of the elementary sphere, the less heavy (*minus grave*) it becomes." In fact, the angles between the beam and the inclination lines become progressively smaller.

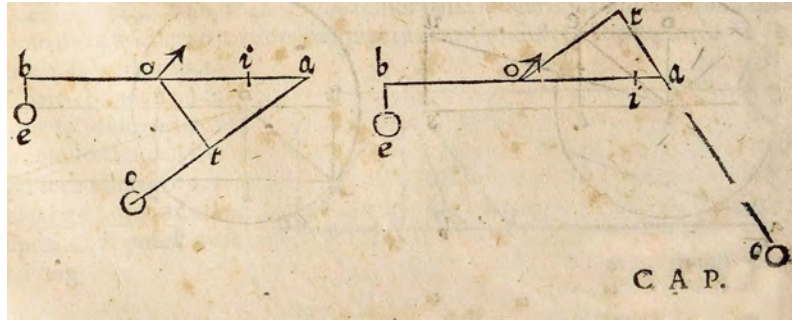


Figure 5.4: Benedetti's representation of forces acting on a balance in arbitrary directions. (Max Planck Institute for the History of Science, Library)

### 5.5 Del Monte's Misunderstanding

In his notes on folio 146 of the *Meditatiunculae*, Del Monte grappled with Benedetti's instructions on how to determine positional heaviness in the case of forces acting in an arbitrary direction. These he refuted at length under the erroneous assumption that Benedetti had claimed forces can be indiscriminately replaced by weights. Like Benedetti, Del Monte considered a bent lever  $BOAC$  with fulcrum  $O$ , weights  $E$  and  $C$ , a straight arm  $BO$ , and a bent arm  $OAC$  to discuss the two cases of an acute and an obtuse angle  $BAC$  (Figure 5.5).

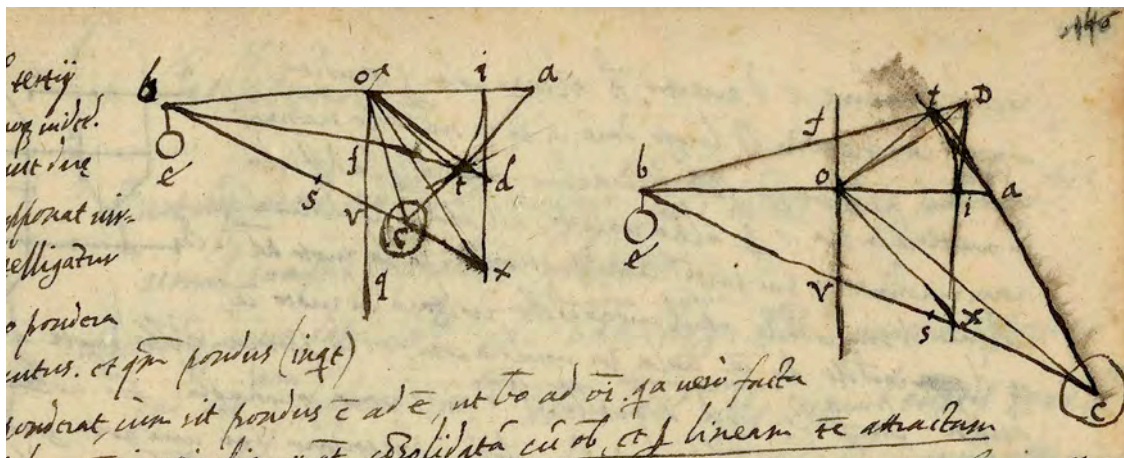


Figure 5.5: Del Monte's critical reworking of Benedetti's representation of forces acting on a balance in arbitrary directions. (Bibliothèque Nationale de France)

He first recapitulated Benedetti's procedure, assuming that a vertical line  $OT$  drawn from the fulcrum to the line  $AC$  represented the oblique arm of the bent lever. He stated that when the weight  $C$  is placed at the end of the horizontal line  $OI$ , whose length is the same as that of the perpendicular  $OT$ , according to Benedetti it will be in equilibrium with the weight  $E$  if the weight  $C$  is to the weight  $E$  as is  $BO$  to  $OT$  or  $OI$ . Del Monte then summarized Benedetti's claim that when a force represented by the weight  $C$  acts along the line  $TC$ , the bent lever formed by the straight arm  $BO$  and the oblique arm  $OTC$  will also be in equilibrium, which he doubted.

Del Monte reformulated this claim by stating that the same weight  $C$  will be in equilibrium with the weight  $E$  whether it is placed on the straight balance  $BOI$  or on the broken bent lever  $BOTC$ . He thus replaced Benedetti's conception of a force acting along an oblique line with that of a weight always tending downward and as a result arrived at absurd conclusions.

Del Monte then showed that the same weight will be heavier on the horizontal at point  $I$  than along the bent lever at  $T$ , demonstrating that the bent lever  $TOB$  will not be in equilibrium if the straight lever  $BOI$  is in equilibrium. To show this, Del Monte again proceeded by finding the center of gravity of the weights  $E$  and  $C$  placed at  $T$ . More precisely, Del Monte determined a position for the weight  $C$  where the bent lever is in equilibrium, a position, however, that is distinct from  $T$ . Thus it follows that  $T$  cannot be the position of equilibrium. For this purpose, he extended the line  $BT$  to  $D$ , just beneath  $I$ , so that it is immediately evident that if the weight  $C$  is placed at  $D$ , the center of gravity of the two weights will be just beneath the fulcrum.

Using the same pattern, he continued by showing that the bent lever  $BOC$  cannot be in equilibrium because its center of gravity  $S$  can never fall on the perpendicular line  $OU$  through the fulcrum. Finally, he applied this argument to the broken bent lever  $BOTC$ . Del Monte next addressed the case in which the bent lever is characterized by an obtuse angle  $BAC$ , showing that the weight at  $T$  is lighter than the weight at  $I$ . In his concluding remarks, however, he began to waver. Once again, he stated that Benedetti is completely mistaken when applying his procedure to weights. But he did admit that this may be true when dealing with a force.

As an afterthought, Del Monte once again criticized Benedetti's appeal to common sense: he did not feel this to be worthy of an expert mathematician. And as a second afterthought, he constructed an extreme case in which it is immediately clear that the broken bent lever cannot be in equilibrium if weights are attached to it rather than forces.

The following considerations enable Del Monte's marginal annotations to Benedetti's *De Mechanicis* III to be understood. These are not perfectly legible, but nonetheless their meaning becomes clear in light of the *Meditatiunculæ*:

If we understand that a weight is at  $C$ , as we can assume from his own words, then  $CT$  must also be understood as being solid [and connected with] the solid lines  $TO$  [...] If we hence understand that  $C$  is a weight and not moving, [the proposition] is false. If it is understood that  $C$  moves as [...] of a man, it can be true, since what moves is not a weight. [But] if he himself assumes in the following that [this] can be demonstrated [also for a weight], nothing [...] therefore as is evident in chapter 7. All demonstrations of the author are founded on these two chapters inasmuch as they are the first fundamentals of mechanics; once their falsity is recognized, everything is rejected.<sup>38</sup>

<sup>38</sup>See Renn and Damerow 2012, 213: "si intelligamus p[ondus] in  $C$ , ut supponi p[otest] ex verbis ipsius, intelligendum est  $C[T]$  quoque consolidatam consolidatis  $TO$  [...]. Unde si intelligamus  $C$  pondus et non



## 5.6 Diverging Approaches to Tartaglia

Del Monte's and Benedetti's criticisms of Tartaglia's conception of positional heaviness help us to understand where these two scholars converge and diverge on the issue of the equilibrium (or lack of equilibrium) of a balance deflected from its horizontal position, and also the reasons for the presumed equilibrium or tendency to restore it. Moreover, their arguments reveal a different attitude toward the medieval tradition of the *scientia de ponderibus* and the *gravitas secundum situm*.

### 5.6.1 The Tradition of Nemorarius, Tartaglia, and Cardano

The concept of *gravitas secundum situm*, or positional heaviness, was extensively employed in Jordanus Nemorarius's *Liber de ponderibus*. Del Monte owned and annotated a sixteenth-century Nuremberg edition of the book, commented, and illustrated by Petrus Apianus. Del Monte's handwritten annotations document his general disagreement with the approach of this medieval scholar, who did not know the Archimedean concept of the center of gravity and therefore tried to develop a deductive science of weights relying solely on the Aristotelian theory of motion and its development in the Arabic tradition of the science of weights. We have already hinted at the Aristotelian framework underlying the concept of *gravitas secundum situm*. In his book, Jordanus stated that a deflected balance would return to the horizontal position (his second proposition) (Nemore 1565, B2 *r*). According to Jordanus, the upper weight acquires more positional heaviness than the lower one due to the fact that its descent is less oblique. In fact, he postulated that positional heaviness depends on the obliqueness of descent of a weight (his fourth postulate) and that "a more oblique descent partakes less of the straight [descent] for the same quantity [of path]" (fifth postulate) (Nemore 1533, A4 *r*). The determination and possibly the quantification of obliqueness was therefore essential to establish the behavior of a deflected balance.

In the sixteenth century, Tartaglia in *Quesiti, et inventioni diverse* (1546), and Cardano in Book 1 of *De subtilitate* (first edition, 1550) and in *Opus novum de proportionibus* (1570), expounded their own versions for determining descent and reinforced Jordanus's second proposition (that the deflected balance returns to the horizontal position). A brief account of three ways to determine positional heaviness is given in the following pages. The first two are derived from Tartaglia and the last from Cardano.

*Descent:* A first method of dealing with positional heaviness consisted in comparing the lengths of the projections of the equal arcs described by the motion of opposite balance beams—one ascending and one descending—on the vertical line of descent to the center of the world.

As Tartaglia's diagram in Figure 5.6 shows, the vertical component of descent of the upper weight is always larger than that of the lower. Thus, the former acquires more heaviness (*secundum situm*) than the latter and the balance returns to the horizontal position.

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movens, falsa est i[ta]que si intelligatur *C* movens ut homi[...] vera esse pote[st] quod [deleted: non] moveat non esse pondus s[i...] ipse [vero] in sequenti accipiat [hoc atque ponderi?] posse demonstratum quare nihil [...] ut patet in 7 cap. In his duobus cap. fundantur omnes authoris demonstrationes ita ut sunt praecipua mechanicorum fundamenta quorum cognita falsitate omnia rem[oventur]."

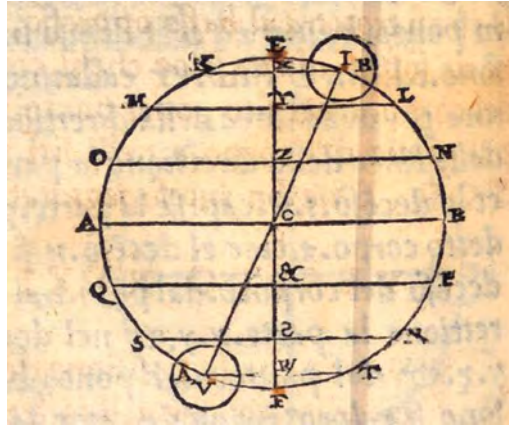


Figure 5.6: According to Tartaglia, the body at  $I$  is positionally heavier than the body at  $V$ , since the projection of the arc  $IL$  on the vertical  $XY$  is greater than the projection of  $VF$ ,  $WF$ . (Max Planck Institute for the History of Science, Library)

*Angle of contact:* Tartaglia's second method of determining positional heaviness consists in comparing the angles between the circular path of the beams and the perpendicular lines connecting the weights to the center of the elements (as already mentioned in chapter 4). These angles "of contact" are also called "curvilinear angles" or "mixed angles" since they result from the intersection of a straight line downward and a curved line, that of the circle circumscribing the balance (Figure 5.7).

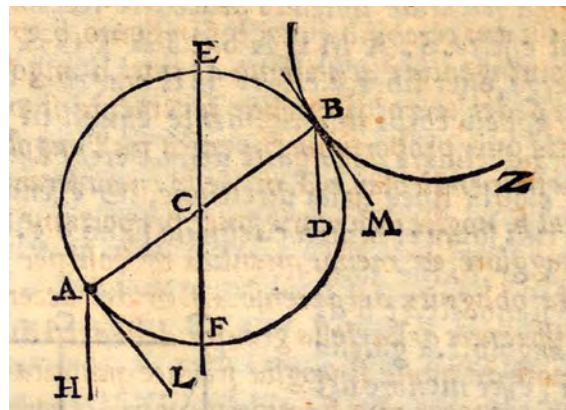


Figure 5.7: Tartaglia's representation of the angle of contact for the determination of positional heaviness. (Max Planck Institute for the History of Science, Library)

By comparing the angles of contact of the two weights, Tartaglia could establish that the higher angle is always smaller than the lower; therefore, the higher weight has a straighter descent and is positionally heavier. The inclined balance would therefore return to the horizontal position. It should be noted that Tartaglia perceived the comparison of curvilinear angles as problematic. He considered the ratio of two such angles to be less than any ratio between determined quantities. As a consequence, no weight placed on the positionally lighter side of the deflected balance could compensate for the other weight and keep the balance inclined. On the contrary, any additional weight—no matter how small—would have produced an opposite displacement of the balance beam toward the vertical.

*The angle between the support and the beams:* We have so far considered two ways of determining positional heaviness on the basis of Tartaglia's *Quesiti*. Assuming that positional heaviness depends on the obliquity and straightness of descent, positional heaviness can be determined either from the projections of the descents on the vertical, or the curvilinear angles that are produced by the intersection of the descent arcs and the lines connecting the weights to the center of gravity. Cardano considered three criteria for establishing positional heaviness which he mistakenly regarded as equivalent: first, the distance of the beam from the vertical; second, its distance from the horizontal; and third, an angle that he called *meta*. This was the angle between the support of the balance and the beam. Commenting on the diagram that is reproduced here as Figure 5.8, he explained:

Aristotle says that this happens when the support is above the balance, because the angle  $QBF$  of the *meta* is larger than the angle  $QBR$ . And similarly, when the support is  $QB$ , the *meta* will be  $AB$ , and thus the  $RBA$  will be larger than the angle  $FBA$ , but the larger angle will render the weight heavier. [...] The general reason is hence this: the more the weights are removed from the *meta* or from the line of descent along a straight or an oblique line, that is, [as measured] by an angle, the heavier they are.<sup>39</sup>

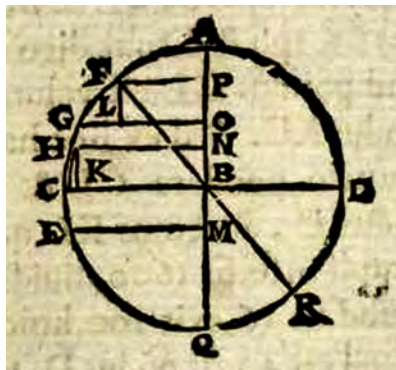


Figure 5.8: According to Cardano, there are three ways to determine positional heaviness. The positional heaviness at point  $F$ , for instance, may be determined by the horizontal  $FP$ , by the vertical  $FL$ , or by the angle  $QBF$ . (Max Planck Institute for the History of Science, Library)

Given these premises, Cardano contended that a weight will reach its maximum positional heaviness in the horizontal position. He therefore shared Nemorarius's and Tartaglia's opinion about the return of an inclined balance to the horizontal position.

### 5.6.2 Del Monte's Critical Remarks on Positional Heaviness

Del Monte's criticism of Benedetti, in the *Meditatiunculae* as well as in the marginal remarks of his copy of *Diversae speculationes*, are closely related to his criticism of Nemorarius, Cardano, and Tartaglia in his *Mechanicorum liber* (1577). Here he dealt

<sup>39</sup>Cardano 1550, 17–18: “Aristoteles dicit hoc contingere, quum trutina est supra libram, quia angulus  $QBF$  metae, maior est angulo  $QBR$ . Et similiter quum trutina fuerit  $QB$ , erit meta  $AB$ , et tunc angulus  $RBA$ , maior erit angulo  $FBA$ , sed maior angulus reddit gravius pondus. [...] Generalis igitur ratio haec sit: pondera quo plus distant a meta seu linea descensus per rectam aut obliquum, id est, per angulum, eo sunt graviora.”

extensively with the balance and provided a detailed discussion of the theories of these scholars which he judged to be irremediable. These theories supported the idea that an inclined balance returns to the horizontal and were thus at odds with his own treatment of the matter, which he based on the Archimedean concept of center of gravity. Del Monte believed that an ideal balance would remain in any position as long as it had equal arms, was hinged on its fulcrum and was loaded with equal weights. The only difficulty in testing this theory, he asserted, was the technical difficulty in constructing a perfect balance. It should be noted, moreover, that he assumed that a center of gravity meeting the requirement of his (and Pappus's) definition of the center of gravity always exists:

The center of gravity is a certain point within it, from which, if it is imagined to be suspended and carried, it remains stable and maintains the position which it had at the beginning, and is not set to rotation by that motion.<sup>40</sup>

Apart from the conceptual irreconcilability between his own approach and that of the Nemorarius school, Del Monte tried to demonstrate the inconsistencies of positional heaviness also within the conceptual framework of his adversaries. One of his main objections was based on a consideration of the cosmological context, which he considered relevant to correctly treat the inclined balance, at least with regard to positional heaviness. Of course, this aspect indeed matters when considering Tartaglia's remark that the difference in positional heaviness is infinitesimally small and cannot be compensated by any finite weight resulting from the infinitesimal difference between curvilinear angles.

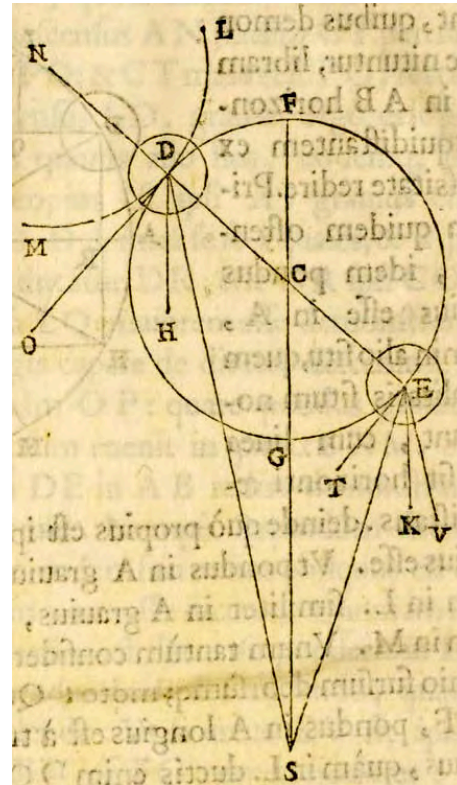
Contrary to the assumptions of Nemorarius and his successors, Del Monte noted that the downward tendencies of the weights are not parallel but converge at the center of the world. Since the directions toward the center of the world from different points on the circular path of the end of the beam cannot be parallel, they are inappropriate for representing positional heaviness. From the fact that those lines converge, he argued further that the lower weight should actually become positionally heavier than the higher one. His idea is clearly illustrated by the diagram in Figure 5.9.

Del Monte objected that, from the point of view of positional heaviness, it is not in the horizontal position that a body weighs the most but at that point where a straight line drawn from the center of the world touches the circle described by the balance arm. Certainly, if the center of the world were infinitely distant and all lines of direction converging at it were perpendicular and parallel to each other, then the extreme point would mark the horizontal position of the balance arm. Still, for a finite distance from the center of the world, the point where the weight is heaviest lies instead slightly below the horizontal through the fulcrum. Del Monte even demonstrated that the closer the balance is to the center of the world, the further this "extreme point" (where the weight is heaviest) will lie from the horizontal position of the balance arm (as seen from the fulcrum).

Del Monte's crucial objection to the Nemorarius school was that one should not consider both weights separately, but rather in terms of their connection by the balance beam. He drew attention to the fact that one must not compare two descents, but rather a descent on one side with a rise on the other. With regard to their positional heaviness the two weights are then equal. Thus Del Monte could claim, using the premises of his adversaries, that the deflected balance does not return to the horizontal.

<sup>40</sup>Del Monte 1577, 1r: "Centrum gravitatis uniuscuiusque corporis est punctum quoddam intra positum, a quo si grave appensum mente concipiatur, dum fertur, quiescit; et servat eam, quam in principio habebat positionem: neque in ipsa latone circumvertitur." Translation in Drake and Drabkin 1969, 259, revised in Damerow and Renn 2010, 57.

Figure 5.9: According to Del Monte, if  $S$  represents the center of the world, then the mixed angle  $SEG$  between the circular path of the weight at  $E$  and the direction to the center of the world is less than the mixed angle  $SDG$ . Thus, contrary to what his adversaries claim, by their own suppositions the weight placed at  $E$  must be heavier than that at  $D$ . (Max Planck Institute for the History of Science, Library)



### 5.6.3 Benedetti on Tartaglia's and Nemorarius's Shortcomings

Benedetti addressed the ideas of Tartaglia and Nemorarius on positional heaviness in section seven of his *De mechanicis*. There, Benedetti stressed that his approach to positional heaviness, focusing on the distance from the fulcrum to the line of inclination, was distinct from and superior to Tartaglia's approach in the Jordanus tradition of straightness of descent.

More specifically, Benedetti refuted several of Tartaglia's claims. In particular, he disputed the central thesis that when a balance is moved from its horizontal position, it will return to this position because the body that has moved upward will attain greater positional heaviness than the body which has moved downward. As we have seen above, Jordanus's and Tartaglia's arguments were based on a comparison of the descents of the two weights. In other words, the balance would have to break in the middle to visualize these descents. Benedetti now pointed to the simple fact, already emphasized by Del Monte, that when one weight descends, the other must ascend, and that the corresponding arcs will always be similar to each other and positioned in the same way. He concluded that no positional difference in heaviness can be produced in the way that Tartaglia argued.

Nevertheless, Benedetti did not believe in an indifferent equilibrium of such a balance when considered in a cosmological context. In the continuation of his argument, he came to the conclusion that when such a balance in equilibrium is displaced from its original horizontal position, the weight that has been lowered will actually assume a greater positional heaviness than the one that has been lifted up:

Therefore the weight of  $A$  in this [lower] position will be heavier than the weight of  $B$ .<sup>41</sup>

<sup>41</sup>Benedetti 1585, 148: "Pondus igitur ipsius  $A$  in huiusmodi situ, pondere ipsius  $B$  gravius erit." Translation in Drake and Drabkin 1969, 176.

He reached this conclusion by taking into account that the lines of inclination of the two weights are not parallel to each other but must converge at the center of the elements. The effective lever arms of the two weights must hence be determined by perpendicular lines drawn from the center of the balance to these lines of inclination. It now turned out that the perpendicular line corresponding to the weight that had been lowered is longer than the line corresponding to the weight that had been lifted. Consequently, the lower weight had become heavier positionally, so that one would expect the balance to tilt into a vertical position.

Benedetti added some more critical remarks on Tartaglia's consideration of positional heaviness. As we have seen, Tartaglia had argued in *Quesiti* that the upper weight attains a greater positional heaviness than the lower one, but that this difference is arbitrarily small and can therefore not be compensated by any finite weight. This conclusion was reached by comparing curvilinear *angles of contact* on each side of the balance. In his analysis of this argument, Benedetti again emphasized that the lines of inclination are not parallel to each other but must converge toward the center of the elements, just as Del Monte had done before him. Clearly, since Tartaglia's argument hinges on angles of contact, which are infinitesimally small compared to ordinary angles, even such a small deviation from the parallel must be relevant. Taking this into account, Benedetti was able to construct a contradiction, thus refuting Tartaglia's argument. He concluded:

Now the whole error into which Tartaglia and Jordanus fell arose from the fact that they took the lines of inclination as being parallel to each other.<sup>42</sup>

In summary, Benedetti introduced a way of determining the positional effect of a weight or a force that, in the cases he considered, essentially produces the same results as the application of the modern concept of torque. In particular, Benedetti had managed to go beyond the consideration of weights tending downward to include forces acting in an arbitrary direction. In this way, he was also able to take into account the fact that, on a spherical earth, the lines of inclination of weights on a balance are not parallel. He did not manage, however, to successfully apply his measure of positional heaviness to challenging objects such as the inclined plane.

### 5.7 The Triangulation Benedetti-Del Monte-Galileo

In this chapter, we have dealt with Del Monte's and Benedetti's different approaches to mechanics emerging from their reflection on the balance and their treatment of earlier authors. Relative to the issue of positional heaviness, Del Monte's self-positioning was essentially external whereas Benedetti positioned himself (albeit critically) within the tradition of the Nemorarius school. He explicitly mentioned Tartaglia and Cardano as relevant sources for his treatment, whereas he omitted any mention of Del Monte.<sup>43</sup> In spite of their opposite intentions and mutual suspicion, Benedetti and Del Monte shared several opinions and sometimes reached the same conclusions, albeit following different paths: both considered the cosmological center of gravity relevant for an evaluation (and criticism) of Tartaglia's concept of positional heaviness, and both remarked that one cannot treat the two balance beams separately, but rather emphasized that they must be considered simultaneously. Moreover, both stressed the ambiguity of the concept of a mixed angle

<sup>42</sup>Benedetti 1585, 150: "Omnis autem error in quem Tartalea, Iordanusque lapsi fuerunt ab eo, quod lineas inclinationum pro parallelis vicissim sumpserunt, emanuit." Translation in Drake and Drabkin 1969, 177.

<sup>43</sup>Benedetti 1585, f. A3r.

and the difficulty of its determination. Nevertheless, their approaches were quite different. As mentioned, Benedetti still worked within the framework of the *gravitas secundum situm*, while Del Monte renounced it in favor of the concept of *centrum gravitatis*. For Del Monte, the displacement of the balance toward the vertical position was an absurdity that revealed the untenability of Tartaglia's premises. Benedetti deemed this vertical tilt to be the consequence of a correct analysis of the balance based on a concept close to the modern idea of torque, in consideration of the cosmological context. Furthermore, one should stress the importance of Benedetti's attempt to determine the quantity of positional heaviness, a fact that distinguishes him from his predecessors. Additionally, unlike Del Monte, he treated the balance by also taking into consideration the general case of forces acting arbitrarily on the beams.

In conclusion, it may be useful to recall the problems linked to the triangulation Benedetti-Del Monte-Galileo, on which the equilibrium controversy sheds new light. The remarkable proximity of these authors on several issues is well known in the history of mechanics. Nevertheless, recent accounts tend to neglect or even deny a possible influence of Benedetti on Galileo.<sup>44</sup> By contrast, the influence of Benedetti on Galileo was assumed and underscored by earlier scholars like Raffaello Caverni, Pierre Duhem, Emil Wohlwill, and Ernst Mach.<sup>45</sup> It is helpful to mention the most important issues common to these authors: the attempt at a theory of motion based on Archimedean hydrostatics, the treatment of the acceleration of fall and its causes, the formulation of what in hindsight appear as proto-inertial principles, a similar treatment of the bent lever, the analysis of the relation between vibrating strings and musical tones, their views on the irradiation of surfaces and on thermal and hydrostatic phenomena, and, last but not least, their support of the Copernican world system.<sup>46</sup> Although many of these themes and ideas belonged to the shared knowledge of preclassical mechanics, in some respects the agreement of their approaches is so striking that one may suspect that this is not mere coincidence.<sup>47</sup> Another potential intermediary was Galileo's friend Paolo Sarpi who discussed Benedetti's theory of fall in *Pensieri naturali e metafisici*. In any case, the strongest evidence of Galileo's acquaintance with Benedetti's insights is provided by Del Monte's *Meditatiunculae*.

<sup>44</sup>See the discussion by Ventrice in Bordiga 1985, 732–736. He mentions Drake, Drabkin, Fredette, and Galluzzi among those who are skeptical about a concrete influence of Benedetti on Galileo. Notable exceptions are the commentaries by Carugo and Geymonat in their edition of Galileo's *Discorsi*, see Carugo and Geymonat 1958. Bertoloni Meli even considers the possibility of Del Monte and Galileo discussing Benedetti, but nevertheless rejects any substantial influence by the latter on Galileo's thinking because that influence supposedly would have arrived too late, see Bertoloni Meli 2006, 61–65.

<sup>45</sup>Cozzi and Sosio 1996. For an overview of such potential connections, see the discussion in Bordiga 1985, 732–736 who also mentions Mersenne, Clavius, and Cardinal Michelangelo Ricci as possible intermediaries.

<sup>46</sup>For an overview, see Bordiga 1985.

<sup>47</sup>See, for instance, Drake and Drabkin 1969, 36. Yet, the question of Benedetti's direct impact on Galileo remains unclear, in particular as Benedetti's work was never mentioned by Galileo.

There are several possible connections between Benedetti and Galileo that have been considered in the past. For instance, Benedetti is referred to by Galileo's Pisan colleague Jacopo Mazzoni in *In universam Platonis et Aristotelis philosophiam praeludia* from 1597. See Mazzoni 1597. He is often mentioned in the Galileo Studies as the addressee of a famous letter by Galileo arguing for the Copernican system (May 30, 1597). See Galilei 1968, vol. 2, 194–202. In his book Mazzoni referred to Benedetti's discussion of the possibility that motion along a straight line can be continuous. See Benedetti 1585, 183–184. For a historical discussion of the context of this argument in contemporary technology, see Freudenthal 2005, a theme that was later taken up by Galileo in chapter 20 of *De Motu*, which also refers explicitly to Copernicus. See Mazzoni 1597, 193 and Galilei 1960, 326. It is conceivable that such issues had been discussed, inspired by Benedetti's work, between Galileo, Mazzoni, and Del Monte during Del Monte's stay in Tuscany in 1589. We would like to thank Pier Daniele Napolitani for drawing our attention to this possibility and to the above-mentioned passages.





with Del Monte in 1588, three years after the publication of Benedetti's *Diversae speculationes* and shortly before he embarked on the writings that later became known as *De Motu*.<sup>50</sup> Galileo first wrote a dialogue version of *De Motu* and then an essay in twenty-three chapters. Only the second essay version of these writings contains his proof of the law of the inclined plane, the argument about continuity of motion along a straight line, and a mention of Copernicus. This version was most likely written after Galileo became familiar with Benedetti's work. His treatise on mechanics, which for the first time discussed explicitly the problem of the effective lever arm, was written much later, certainly after he had visited Del Monte in 1592 during his journey to Padua. Hence, it seems most likely that Galileo was already familiar with Benedetti's key ideas at the time of writing these works.

Recent research into Del Monte's biography has shown that Del Monte and Galileo must have met as early as 1589 in Tuscany.<sup>51</sup> They might even have met jointly with Galileo's teacher, Mazzoni, who, as mentioned earlier, cited Benedetti in his work. Thus, Del Monte, Mazzoni, and Galileo may have discussed Benedetti's *Diversae speculationes*, leading Galileo to reconsider his work in progress on motion and, in particular, his treatment of motion along inclined planes, making use of Benedetti's theory of the bent lever that was mentioned in Del Monte's notebook. But Benedetti's impact on Galileo probably went even further than that. Galileo may have started taking the Copernican hypothesis much more seriously after his encounter with Benedetti's work, discussing this as well as other subjects with Mazzoni. In the above-mentioned letter of 1597, Galileo praised Mazzoni for his *Praeludia* and reminded him of the controversial issues on which they had meanwhile reached an agreement, and also tried to press him on the Copernican hypothesis.

In particular, Galileo's concept of *momento*<sup>52</sup> and his analysis of the bent lever—crucial to both his mechanics and his theory of motion—evidently emerged from the midst of the controversy about positional heaviness. In that debate, Galileo took a position much closer to Benedetti than to Del Monte. Rather than *gravitas secundum situm*, Galileo used the concept of *momento* or *momentum* that Del Monte had introduced in his book by quoting Commandino's definition of the center of gravity. But while Del Monte made no further use of this in his mechanics, Galileo took this concept from the respected Urbino school, gave it a new meaning that was taken from Benedetti, and made it a pillar of his own conception, which included Commandino's definition of the center of gravity:

Center of gravity is defined as that point in every heavy body around which parts of equal moments are arranged.<sup>53</sup>

The evidence for this claim concerning Benedetti's legacy in Galileo's work derives from the marginal notes Del Monte made in his copy of Benedetti's book, as well as from his entries in the *Meditatiunculae* which contain traces of Galileo's intervention in this controversy.<sup>54</sup>

According to Benedetti and Galileo (and contrary to Tartaglia and Del Monte), the effective length of the lever arm, obtained by drawing a perpendicular from the fulcrum

<sup>50</sup>Galilei 1960. For a thorough discussion of the chronology of these writings, see Giusti 1998.

<sup>51</sup>Menchetti 2012.

<sup>52</sup>See the extensive discussion in Galluzzi 1979.

<sup>53</sup>Galilei 1968, vol. 2, 159: "Centro della gravità si diffinisce essere in ogni corpo grave quel punto, intorno al quale consistono parti di eguali momenti." Translation in Galilei 1960, 151. See also Galilei 2002.

<sup>54</sup>Del Monte 1587.

of the balance to the line of inclination, determines the effectiveness of a weight or a mechanical constellation. In his *Mechanics*, Galileo later stressed how important it is to carefully define the effective distances of weights from their support:

There is one thing that must be considered before proceeding further, and this concerns the distances at which heavy bodies come to be weighed; for it is very important to know the sense in which equal and unequal distances are to be understood, and in what manner they must be measured.<sup>55</sup>

In his analysis of the inclined plane using the bent lever, Galileo also made clear that this procedure is critical for determining the *momento* of a given weight.<sup>56</sup> As discussed earlier, in his *Diversarum speculationum [...] liber*, Benedetti convincingly demonstrated the efficacy of this method for determining the magnitude of a force or weight according to its position.

In conclusion, the very existence of Del Monte's annotations on his copy of Benedetti's *Diversae speculationes* provides a definitive answer to the question of whether Del Monte had read this book or not.<sup>57</sup> It is also difficult to imagine that he did not discuss his views on Benedetti's mechanics with Galileo, views that he considered both misguided and profoundly challenging, as is made evident in his handwritten notes. It was most probably Del Monte, Benedetti's fervent opponent in matters of mechanics, who served as a conduit to Galileo. At the same time, he also made it virtually impossible for Galileo to openly admit to Benedetti's influence if he did not also want to jeopardize the protection of the most important patron of his early career.

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<sup>55</sup>Galilei 1968, vol. 2, 164: "Un'altra cosa, prima che più oltre si proceda, bisogna che sia considerata; e questa è intorno alle distanze, nelle quali i gravi vengono appesi: per ciò che molto importa il sapere come s'intendano distanze eguali e diseguali, ed in somma in qual maniera devono misurarsi." Translation in Galilei 1960, 156–157.

<sup>56</sup>See Galilei 1968, vol. 2, 181. Translation in Galilei 1960, 173.

<sup>57</sup>The knowledge that he had read it, however, is not entirely new. See Renn, Damerow, and Rieger 2001, 74.

## Chapter 6

### Astronomy

#### 6.1 Benedetti as an Astronomer

Benedetti's astronomical considerations are not systematic. They are scattered throughout the volume in different sections. In spite of the difficulty of ordering them and obtaining an overview, they were very much appreciated among his contemporaries. Apart from Kepler's eulogy of Benedetti's ingenuity, the broad European success of the astronomical parts of this work is documented in other references. A few years after the publication of the *Diversae spaeculationes*, Brahe must have had a copy of it in Denmark, as he quoted it extensively and accurately on two occasions. In his correspondence with Landgrave William IV and the Hesse-Kassel court mathematician Christopher Rothmann, he referred to Benedetti's observation of the light of Venus reflected on the part of the lunar disc not presently enlightened by the sun:

In fact, I sometimes saw that Venus illuminated in a rather sensible manner that part of the Moon that was most distant and opposed to the Sun, although the Moon is by far more distant from Venus's circuit than the comet. I remarked that the Venice patrician Giovanni Battista Benedetti, the most excellent philosopher and mathematician, noted something similar in that erudite work which he wrote on mathematical and physical speculations. At the end of an epistle to a certain Savoy baron, Filiberto, he says: "[...] that the part of the Moon which is deprived of the Sun's light is sometimes partially illuminated by Venus's light. I observed this often and showed it to many people."<sup>1</sup>

Brahe quotes this passage correctly from Benedetti's letter to Baron Emanuele Filiberto Pingone "*De Luce, Lumine, et Colore, De obiectu oculi, De lumine Lunae, et Rubedine nubium*" (On light, lumen, and color; on the eye's object, on the lunar lumen, and the redness of the clouds).<sup>2</sup>

A second long direct quotation of Benedetti can be found in Brahe's book on the nova of 1572, which was part of the *Astronomiae Instauratae Progymnasmata*, posthumously published in Prague in 1602.<sup>3</sup> The Danish astronomer here praised Benedetti as a "philosophus et mathematicus inprimis excellentem," and his work as "praeclarum Opus." He entirely reproduced Benedetti's letter and diagrams on the star in Cassiopeia.<sup>4</sup> This letter

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<sup>1</sup>Brahe 1919, 172: "Veneris enim Stella, visa est mihi aliquando eam partem Lunae, quae a Sole aver-sa erat, et ipsi obiecta, satis sensibiliter illuminare, utut Luna longe remotius a Veneris circuitus distiterit, quam Cometa. Simile quid Ioannem Baptistam Benedictum, Patricium Venetum Philosophum et Mathematicum inprimis excellentem, animadvertisse reperio, in erudito illo Opere, quod de Mathematicis et Physicis speculationibus inscripsit. Sic enim in fine Epistolae, ad Baronem quendam Sabaudarum Philibertum scribens, ait: '[...] quod pars Lunae lumine Solis destituta, a lumine Veneris aliquantulum illustratur, quod ego saepe vidi, et multis ostendi.'"

<sup>2</sup>Benedetti 1585, 256–257.

<sup>3</sup>Brahe 1916, 251–253.

<sup>4</sup>Benedetti 1585, 371–374.

was directed against Annibale Raimondo—an author whom Brahe also criticized—and demonstrated that the nova appeared above the sublunary sphere. Brahe commented:

Here follows the epistle which I referred to. It is taken from the aforementioned book by [Giovanni] Battista Benedetti alongside the demonstrative diagrams offered by the same author. Afterwards I will consider others, who discussed that star [i.e., the nova of 1572] in an extraordinarily incompetent manner. This [quotation from Benedetti] (as mentioned) will cast light on these issues through a synthetic and wise geometrical truth, so that no significant doubt will survive.<sup>5</sup>

Another reader of the *Diversae speculationes* was the English scholar of magnetism William Gilbert. In *De mundo nostro sublunari philosophia nova* (New Philosophy on Our Sublunary World, written about 1600 but published long after the author's death, in Amsterdam in 1651), he in fact discussed Benedetti's views on the spots on the surface of the moon, in a chapter trying to determine which parts of it were seas and continents.<sup>6</sup> It is evident that the *Diversae speculationes* had a wide European circulation, and that the astronomical part attracted the attention of many scholars dealing with mathematical and physical issues.

Benedetti's treatment of astronomical matters ranges from the calendar reform to the nova of 1572, sundials, and astrology. We would like to focus on a special issue: Benedetti's defense of ephemerides, *Defensio ephemerides*, and the quarrel that motivated its writing. This defense of ephemerides figures as one of the epistles of the *Diversae speculationes*. It is the Latin translation of an Italian letter, *Intorno ad alcune nuove riprensioni... contra alli calculatori delle effemeridi* (Letter in the Form of a Discourse... Addressed to the Illustrious Mr Bernardo Trotto Concerning Some New Criticism and Corrections against the Ephemerides Calculators, Turin, 1581), addressed to Trotto, which Benedetti had already published when a heated quarrel on the reliability of ephemerides burst out in Turin between 1580 and 1581. In the following pages we will give an account of these facts.<sup>7</sup>

## 6.2 The Controversy over the Reliability of Ephemerides

The ephemerides controversy began with the publication of Altavilla's *Animadversiones in ephemeridas* (Remarks against Ephemerides, Turin, 1580). This lesser-known author from Vicenza intended to denounce the inexactitude of all existing astronomical computations.<sup>8</sup> For this purpose he compared predictions and horoscopes cast using different sets

<sup>5</sup>Brahe 1916, 251: "Nunc igitur epistolam, quam pollicitus sum, subiungam, verbotenens e praedicto Baptistae Benedicti libro desumptam, una cum demonstrationum delineationibus, quas ipse autor assignavit. Deinde ad caeteros qui de hac stella nimis incompetenter, sententiam tulerunt, calamum dirigam. Ex quo (uti dixi) haec adeo succinte et scite geometricam veritatem redoleant, ut nullum, quod alicuius sit momenti, super esse queat, dubium."

<sup>6</sup>W. Gilbert 1651, 173: "Luna maculas quasi ostendit substantiae et peripheriae differentia: ita Tellus erga Lunam maculas repraesentat, terrarum continentium minus relucens; aquarum vero et Oceani, propter laeviore et luminis apprehensivam naturam magis splendentem. [...] Non enim maculae Lunae existunt a partibus Lunae magis perspicuis, ut Iohannem Benedictus contendit, in quibus lumen non reflexum sed penetrans nobis occultatur." See Pumfrey 2011, 193–203.

<sup>7</sup>Section 6.2 is a revision of Omodeo 2014a, chap. 3.8–9 and chap. 6.3 of Omodeo 2014a, chap. 4.7.

<sup>8</sup>This Benedetto Altavilla could be the same person involved many years later, in 1606, in a gunpowder plot in Venice; he pretended to have discovered it by astrological means and was tortured by the Venice authorities in order to obtain information about the perpetrators. Cf. L. P. Smith 1907, vol. 1, 364–365.

of tables and ephemerides. In particular, he pointed out that ephemerides diverged from each other even more than the astronomical tables, Alfonsine or Copernican, from which they were derived. In his opinion, this fact undermined the reputation of astronomy in general, regardless of whether its cause was the inaccuracy of the compilers (*calculatores*) or the inexactitude of the tables themselves: “We consider nothing to be more odious than an unreliable person who is regarded by many as trustworthy.”<sup>9</sup> Altavilla declared himself unwilling to decide between Alfonsine or Copernican computations. However, he himself was probably interested in the cosmological issue, judging by the fact that the *Animadversiones* were introduced with a poem by Pandolfo Sfondrati in favor of a new world system with the earth in motion.<sup>10</sup>

Altavilla had established by observation that both Alfonsine ephemerides and Johannes Stadius's Copernican computations were in disagreement with the heavens. Still, Stadius's computations proved to be in better agreement with the heavens. The reference to Stadius is not casual, since the Flemish astronomer had been a protégé of duke Emanuele Filiberto of Savoy, as one can read in the *Ephemerides novae* of 1556, where the author gave himself the title “mathematician to the King [of Spain] and the Duke of Savoy” (*Regius et Ducis Sabaudiae mathematicus*). Altavilla listed predictive errors of Ptolemaic astronomers (Regiomontanus, Stöffler, Leowitz) as well as those of post-Copernican ephemerists (Stadius and Giuntini). This led him to skepticism toward predictions in general: “You see, dear reader, how reliable ephemerides are.”<sup>11</sup> Altavilla invited scholars (*magistri*) to trust only their eyes and to correct astronomy through observational campaigns with no regard for any authority: “Posterity should learn how dangerous it is to blindly adhere to the opinions of the ancients without [perfecting the art through] daily observations of the heavens, and to prefer their opinions to truth.”<sup>12</sup>

The *Animadversiones* were soon followed by a second publication in Italian: *Breve discorso intorno gli errori dei calculi astronomici* (Brief discourse on the mistakes of astronomical calculations, 1580). A poem by a certain Francesco Onto of Pinerolo, inserted as a preface to the *Breve discorso*, made its polemical target explicit: “Altavilla has unveiled the astrologers' fallacy, as they think to cast certain [astrological] judgments about our lives relying on flawed ephemerides.”<sup>13</sup> Altavilla's criticism was directed mainly against astrology, whose validity he considered to be doubtful due to the inaccuracy of predictions. His argumentative strategy was no different than that of Pico della Mirandola in books 8 and 9 of the *Disputationes in astrologiam divinatricem* (Disputations against divinatory astrology, 1496): an attack on mathematical astronomy aimed to discredit astrological forecasting. Altavilla even claimed that astrologers and ephemerists should renounce their activity, as they were not capable of superseding the flaws of their discipline: “Since it is impossible for the scholars in those sciences (especially those who are not capable of using the tables) to renounce ephemerides, and they know that they will encounter irremediable errors, they should be forced to abandon their studies.”<sup>14</sup>

In his second publication, the *Discorso*, Altavilla complained that many scholars (who were not named) pretended to ignore his criticism. He explained that the decision to write another booklet, this time in Italian instead of Latin, originated from the desire to reach readers outside academic and scholarly circles, probably also at the Savoy

<sup>9</sup>Altavilla 1580a, f. A2r.

<sup>10</sup>See Omodeo 2008b and Omodeo 2012a.

<sup>11</sup>Altavilla 1580a, *Conclusio*.

<sup>12</sup>Altavilla 1580a.

<sup>13</sup>Altavilla 1580b, 2.

<sup>14</sup>Altavilla 1580b, 4–5.

court: “In these few pages, I aimed at demonstrating not only to the learned man, but also to everybody else, that the errors [of the ephemerides] are worthy of consideration.”<sup>15</sup> He first reassessed the inadequacy of Alfonsine tables and Alfonsine ephemerides (those of Peurbach, Prugnerus, Bianchini, Regiomontanus, Stöffler, Schöner, Gaurico, Pitati, Simi, Carelli, Moletti, Leowitz, and others). He moreover stressed the superiority of the Copernican tables in order to show the inconsistency of some unnamed Turin ephemerists who used Alfonsine ephemerides for their predictions although they claimed to prefer Copernicus. To illustrate this inconsistency, he analyzed some astrological figures on the basis of Stadius's and Giuntini's tables. In the last section Altavilla turned on the Copernican ephemerists, denouncing the excessive difference between computations based on Stadius and Giuntini: “And the difference between one computation and the other is really great and monstrous.”<sup>16</sup>

This attack on the reliability of astronomical computations and astrology provoked negative reactions both at the university and at the court. Altavilla thus felt compelled to challenge his critics to an academic debate on August 14 and 15, 1581, announcing it through a broadside that is still preserved in the libraries of Turin, along with copies of his *Animadversiones*.<sup>17</sup> The public dispute concerned the theory of Mars for which, as one reads, some scholars blamed him. He maintained, in fact, that Mars cannot stay in a zodiacal sign for more than two months, considering that its entire revolution lasts twenty-four months. He argued that ephemerides are wrong if they forecast that it would spend six or even seven months in the same zodiacal constellation. This incorrect opinion presented the court mathematician and philosopher Benedetti with an occasion to intervene and criticize Altavilla on this and other issues related to astronomical theory, computation, and astrological prediction.

Soon after Altavilla's public dispute, Benedetti published an epistle “on some recent remarks and emendations directed against ephemerists” (Turin, 1581). At the beginning, Benedetti indicated Altavilla's intentions: “I assume [...] that his intention was only to demonstrate that [different] ephemerides assigned a different place to the planet at the same point of time [...] and that, as a consequence, they offer no certain ground on the basis of which the future can be judged or predicted.”<sup>18</sup> In his account, Benedetti rejects Altavilla's complaint that Copernican and Alfonsine ephemerides diverge from each other more than the tables from which they are derived. He assures the reader that “the people who calculated have been very accurate and trustworthy” (*i calcolatori sono stati diligentissimi e fedeli*) and they are exact in their calculations, although some minor and accidental mistakes can occur.<sup>19</sup>

Moreover, he accuses Altavilla of misunderstanding Ptolemy's astrology, interpreting it in light of Abu Ma'shar and Al-Qabisi (*Alcabitus*). In particular, Altavilla draws from these sources the rule of the “triplicity” of the conjunctions of Jupiter and Saturn, according to which these planets meet four times in the same three astrological signs, or trine, before they can meet in the next trine. However, although the mean motions of two planets should meet in the triplicity sign, nonetheless their “real” motions (those observed and calculated by the ephemerides upon which astrological predictions rely) may meet elsewhere. This is an obvious consequence of planetary theory. In fact, it distinguishes between “mean” motions, which correspond to the revolutions of the deferents, and “real” motions, which

<sup>15</sup>Altavilla 1580b, 3.

<sup>16</sup>Altavilla 1580b, 6.

<sup>17</sup>In Turin: Biblioteca Nazionale di Torino, coll. Q.V.191, and Biblioteca Reale di Torino, coll. G.25.12.

<sup>18</sup>Benedetti 1581, 5.

<sup>19</sup>Benedetti 1581, 6.

correspond to observable phenomena and are the product of moving epicycles. Benedetti calculates the period of triplicity to be 794 years and 138 days, whereas the Arabs on whom Altavilla relies overestimated it at 960 years.<sup>20</sup> He furthermore remarks that Altavilla neglected planetary theory by criticizing those who let Mars run too fast or too slowly along the signs of the zodiac. Simple observations would show the correctness of the theory according to which the planet can remain in the same sign for six or even seven months. Benedetti explains that the amplitude of Mars's epicycle accounts for its complex phenomenology, in particular the long period of retrograde motion. On this account, he reports an observational campaign accomplished between 1565 and 1566 in order to check Stadius's ephemerides:

Yet, he [Altavilla] dared too much, seeking to reprimand so many talented ancient and modern men who, as is required by diligent observers of the heavens, checked with their own eyes these appearances of Mars as well as of the other [planets]. From those [observations], they were forced to "imagine" such a large [Martian] epicycle. By contrast, he has never observed the motions of either this or any other planet, but rather limited himself to look at what is written in the ephemerides. In fact, if he had at least said that he observed Mars's journey for a certain period, and that he found that the others' opinion was false, he would have at least given some "color" to his opinion. In my assessment, however, if he had made an observation of the path of Mars, he would not have held the contrary view. In fact, the truth is the following: in every revolution of its epicycle, Mars in the lower part of its epicycle always stays many months (six or seven, or more) in a twelfth [*duodecatemerio*] of the zodiac. I observed this many times, for instance, in the years 1565 and 1566. First, consulting Stadius's ephemerides, I found that Mars would finish its retrograde motion on about 12 January 1566, in 16° of Gemini, and that, equally, Mars would be in the same place on the last day of August 1565, before it began its retrograde motion. Second, I found that, after that retrograde motion, on 11 April 1566, Mars would be in 16° of Cancer, so that it would take [Mars] seven months and eleven days [to move] those thirty degrees, from 16° of Gemini to 16° of Cancer. After these computations, I took the instruments and got ready to make a test. And I found that the last night of August of the year 1565 Mars was in the aforesaid 16° of Gemini, as Stadius had noted. I then made observations every week, in order to see the retrograde motion, and I saw that, at about the end of October, the [planet] began its retrograde motion and that retrograde motion lasted until January (or about January) 1566. I later observed the position of that planet on 11 April, and I found it in 16° of Cancer, that is, the place where Stadius had located it. Thus, my experience confirmed Stadius's computations and I found that he was not mistaken. In the same manner, everybody can ascertain the truth every two years by carrying out observations.<sup>21</sup>

Benedetti thus demonstrated not only the theoretical incompetence of his opponent, but also his lack of empirical verification. Altavilla's appeal to base astronomy using observation backfired. Benedetti challenged his opponent to observe Mars's backward motion in Cancer which, according to Stadius's tables, would begin on November 20, 1582 and

<sup>20</sup>See Bonoli 2012, 49–55.

<sup>21</sup>Benedetti 1581, 17–19.

last until the end of February 1583. He furthermore observed that everyone familiar with planetary theory would understand the reasons for the orbit of Mars and other planets. For the theory, he added, it did not matter whether one relied on Ptolemy's *Almagest* or on the "*Rivolutioni de gl'orbi celesti dell'eccellentissimo Copernico.*"<sup>22</sup> Of course they were only equivalent as far as the understanding of a system of deferents and epicycles was concerned, but not in their general hypotheses, since Benedetti himself tended toward heliocentrism.

As to the difference between Leowitz's and Stadius's computations, Benedetti traced this back to the contrast between the theories underlying the Alfonsine and the Copernican tables. Nonetheless, he ensured that ephemerides never diverged by more than three degrees. Thus, if Altavilla detected greater discrepancies, this was due only to false computations. Benedetti added that Stadius's superiority over Leowitz was a consequence of him employing better parameters. He advised Altavilla to always rely on the most recent observations and tables.<sup>23</sup> In fact, he judged the progress of astronomy to be such that more recent tables would inevitably be superseded by new ones, augmented and perfected through new observations, just as Copernicus had superseded Alfonso's astronomers. Divergence between ephemerides was not a shortcoming, but a necessary and desirable sign of the advancement of knowledge and predictive accuracy.

As a courtier expert of mathematics, Benedetti defended the validity of some astrological figures that Altavilla criticized in his second published work, *Breve discorso*. These horoscopes had probably been cast by somebody that he knew well. Altavilla complained that some astrological figures had not been calculated on the basis of Copernican tables. Benedetti replied that it was not always necessary to use the best tables for predictions, especially if a generic horoscope was expected and if the astrologer had no Copernican tables to consult. He showed, moreover, that Altavilla himself was not able to employ Giuntini's tables properly and made mistakes of computation. He concluded: "And such monsters [those denounced by Altavilla] are not generated by different tables or ephemerides but, instead, they are the offspring of this author."<sup>24</sup> He added as a remark: "As to the difference of the Sun according to Copernicus and Alfonso, no learned man, [expert] in these sciences, ignores it, and, as a consequence [everybody knows] the different place [assigned to it] in the heavens during the annual revolutions."<sup>25</sup> In 1581, the general views of *De revolutionibus* were so well known in Benedetti's environment that he deemed it unnecessary to expand on them in the context of a polemic on the accuracy of heavenly computations. The cosmological implications of these different hypotheses were not addressed explicitly in this dispute. However, the defense of mathematical astronomy could not avoid a reference to Copernicus as a source for tables (Reinhold, Stadius, Giuntini) and theory. In this context, "Copernican" and "not Copernican" are expressions that merely mean "based on Copernican tables" or not. Altavilla's criticism would have been more effective if it had been directed against astrological beliefs as such, rather than attempting to show the inconsistency of the mathematical basis of astrology without sufficient preparation. On the other hand, Benedetti, in his *Lettera*, focused on the mathematical aspects and cautiously avoided expanding on ethical issues related to astrology.

Altavilla never responded to the court mathematician who had rebutted his arguments so forcefully. The epilogue to their quarrel was the inclusion of a Latin translation of the

<sup>22</sup>Benedetti 1581, 20.

<sup>23</sup>Benedetti 1581, 32–33.

<sup>24</sup>Benedetti 1581, 37.

<sup>25</sup>Benedetti 1581, 37–38.



*Lettera, as Defensio ephemeridum* (A defense of Ephemerides), in Benedetti's *Diversae speculationes*.<sup>26</sup>

### 6.3 The System of the World

Benedetti did not limit himself to considering astronomy from a computational point of view, but also expanded on cosmological aspects. The epistle “De fine corporum coelestium, et eorum motu” (On the Aim of Celestial Bodies, and their Motions),<sup>27</sup> addressed to Pingone, bears witness to his interest in cosmology and his realist interpretation of Copernicus's hypotheses. Benedetti remarks that it is not reasonable (*si [...] humanam rationem sequi volueris*) to believe that the heavens were created only for the sake of terrestrial life, “as these [celestial] bodies are divine, uncountable, and endowed with the greatest dimensions” (*cum ea corpora sunt divina, in numero incompraehensibilia, maximis magnitudinibus, et motibus velocissimis praedita*).<sup>28</sup> This absurdity can be avoided, as Benedetti claims, if one accepts the planetary doctrine of Aristarchus and Copernicus:

[...] this will hardly be believed by those who embrace the doctrine of Aristarchus of Samos and Nicolaus Copernicus. Following their approach it is impossible to make them believe that the rest of the universe has no other aim than to rule over this center of the lunar epicycle [the earth] (to use their way of speaking).<sup>29</sup>

Although he speaks in the third person, as if he were reporting the views of someone else, these are his own views. He is inclined to accept the Copernican system or some variation of it, as the following pages of the letter and the force of the arguments show. Firstly, he assumes a principle of cosmological homogeneity according to which there is no reason why other planets should not be subjected to alterations (*ab ortu, et interitu*), as the Aristotelians suppose. The peripatetic argument that no change in the heavens was ever observed is not valid, because the distance does not permit verification of whether there is any life or alterations on distant bodies (*unde etiam fieri potest, ut in coelo sint particulares alterationes, quae a nobis tamen, qui ab illis longe distamus, non compraeherentur*).<sup>30</sup> Benedetti even surmises that other planets are moons reflecting the solar light to dark planets invisible to us.<sup>31</sup> He ascribes this opinion to the followers of Copernicus. This is a free interpretation on his part. Perhaps he aimed to explain the epicyclic motions of other planets through an analogy with the lunar epicycle around the earth. Benedetti also rejects Ptolemaic and Aristotelian arguments against terrestrial motion. Following Copernicus (*De revolutionibus* I 8), he stresses that the axial rotation avoids the otherwise enormous motion of the fixed stars: “which is eliminated by the rotation of the Earth about its axis (as they say) as it is sufficient to receive the light and the influences of the [celestial] bodies.”<sup>32</sup> Moreover, the annual revolution respects the dignity of the “divine body of the

<sup>26</sup>Benedetti 1585, 228–248, “Defensio ephemeridum.”

<sup>27</sup>Benedetti 1585, 255–256.

<sup>28</sup>Benedetti 1585, 255.

<sup>29</sup>Benedetti 1585, 255: “[...] id etiam minus putabunt hii, qui opinionem Aristarchi Samii, et Nicolai Copernici sequuntur, quorum ratione fieri non potest, ut credant eius, quod ex universo reliquum est, alium finem non habere, quam regimen huius centri [Tellus] epicycli Lunaris, ut illorum more loquar.”

<sup>30</sup>Benedetti 1585

<sup>31</sup>The same thesis is presented in Benedetti 1585, 195–196.

<sup>32</sup>Benedetti 1585, 255–256: “quae quidem omnia [phaenomena], cum simplici gyro terrae circa suum axem (ut dicunt) tolluntur, quod sufficit ad recipiendum lumen, et influentias illorum corporum.”

Sun" (*divinum corpus solare*), which stands still at the center of the planetary circles.<sup>33</sup> Note Benedetti's astrological concern. In the final passage of his letter, he reassesses Copernicus's objection to Ptolemy's view of how bodies suspended in the air are affected by terrestrial motion:

Ptolemy's objections are not valid for them [astronomers who assume that the earth moves]. As they say, every part maintains the nature of the whole, apart from the fact that the air and water circumscribing the earth receive the same natural impulse of motion [*impetum motus*]. This is slower the further the air is distant from the earth. According to the same doctrine, there is no necessity that the place of the fixed stars has (either convex or concave) superficial boundaries.<sup>34</sup>

According to this passage, the air close to the earth is transported by the motion of the planet and slows down the more it is distant from it. The fixed stars are placed in a motionless air whose place (*locus*) has no boundaries, either convex or concave.

In a letter to the courtier Capra, Benedetti confronts the issue of the form of the heavens.<sup>35</sup> This is said to be a sphere encompassed by infinite space. Accordingly, Benedetti distinguishes between *spacium* (space) and *coelum* (heavens), a distinction that can be traced back to Stoic cosmology or to the more recent views of Marcellus Palingenus Stellatus. The idea of the infinity of space beyond the starry vault can be found also in Patrizi's *Nova de Universis Philosophia* (1591).<sup>36</sup>

Furthermore, Benedetti rejects the existence of material spheres with the role of transporting the planets:

That you do not accept that distinction of spheres, which was well-established in the past, but rather that you believe that the whole is a continuum accommodating the stellar bodies, this is not new. In fact, some philosophers of solid doctrine were of the same opinion.<sup>37</sup>

The motion of celestial bodies is accompanied by that of transparent bodies similar to vapors (*fumi*). Their motion is the cause of the apparent sparkling of the most distant stars.<sup>38</sup> The sparkling of the new star in Cassiopeia in 1572 bears witness to its great distance above the moon, which Benedetti also demonstrates through geometry.<sup>39</sup>

<sup>33</sup>Benedetti 1585, 256.

<sup>34</sup>Benedetti 1585: "Rationes autem a Ptolomeo in contrarium adductae apud ipsos, nullae sunt, quia quaelibet pars (ut inquit) retinet naturam totius, praeterquam quod aer, et aqua, quae ipsam terram circundant, plane eundem naturalem impetum motus obtineant, qui tanto lentior est, quanto longius distat aer, ab ipsa terra, secundum etiam talem opinionem, nulla necessitas, ut locus fixarum terminaretur aliquibus superficiebus, convexa scilicet, et devexa."

<sup>35</sup>Benedetti 1585, 285–286, "De motu molae, et trochi, de ampullis, de claritate aeris, et Lunae noctu fulgentis, de aeternitate temporis, et infinito spacio extra Coelum, Coelique figura."

<sup>36</sup>For Benedetti's correspondence with Patrizi, see Claretta 1862.

<sup>37</sup>Benedetti 1585, 411: "Quod eam distinctionem orbium, quae iam invaluit, non teneas, sed putes totum esse quoddam continuum excipiens corpora stellarum, novum non est, nam nonnulli solidae doctrinae philosophi idem confuerunt."

<sup>38</sup>Benedetti 1585, in the section entitled "Disputationes de quibusdam placitis Arist[otelis]," n. 38: "Occultam fuisse gravissimo Stagiritae causam scintillationis stellarum," 186: "Scintillatio ergo stellarum, neque aspectus nostri ratione, neque alicuius mutationis earundem stellarum, sed ab inaequalitate motus corporum diaphanorum mediorum nascitur, quemadmodum clare cernitur, quod si inter aliquod obiectum, et nos, aliquis fumus, qui ascendat, intercesserit, videbimus obiectum illud quasi tremere. Hoc autem tanto magis fiet, quanto magis distabit obiectum ab ipso fumo; unde admirationi locus non erit, si stellas fixas magis scintillare, quam errantes cernamus. Lumen stellae ad oculus nostrum accedens, perpetuo per diversas diaphaneitates penetrat, medio continuorum motuum corporum mediorum, unde continuo eorum lumen variatur, et hoc in longitudinis magis, quam in propinquis stellis apparet."

<sup>39</sup>Benedetti 1585, 371–374.

One of the books of the *Diversae speculationes* entails a discussion and a refutation of Aristotelian physical and celestial theses *de motu*. It has the rather neutral title *Disputationes de quibusdam placitis Arist[otelis]* (Disputations on Some Opinions Held by Aristotle) but it is indeed an attempt to revise basic concepts of natural philosophy such as *locus* (place) and *tempus* (time). We shall deal with this issue in detail in the next section. For now, it is important to anticipate that this anti-Aristotelian section entails Benedetti's most explicit defense of Copernican planetary hypotheses. Another remarkable thesis of these *Disputationes* on Aristotle is the statement of a principle of relativity according to which planets appear to us as we appear to them:

Aristotle did not consider that one could affirm the same about the Earth as seen from great distance. There is no doubt that, even if the Earth had the light of the Sun and somebody tried to observe it from the eighth sphere, he would not be able to perceive it. In fact, those celestial bodies that are said to be of the first magnitude and that are believed to be more than a hundred times bigger than the Earth look just like points.<sup>40</sup>

Benedetti supports the plurality of worlds as well (*Minus sufficienter explosam fuisse ab Aristotele opinionem credentium plures mundos existere*). Every planet should be regarded as another Earth with its elements and natural places: "If those worlds existed, each of them would have its own center and its own circumference and the earths and fires would have an inclination towards the centers and the circumferences of their worlds, respectively."<sup>41</sup>

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<sup>40</sup>Benedetti 1585, 197, "Disputatio XXXIX, Examinatur quam valida sit ratio Aristotelis de inalterabilitate Coeli: Aristo[teles] non consideravit, quod similiter de terra dici posset, quando ipsa ita eminens prospiceretur, imo absque dubio putandum est, quod si terra luce Solis praedita esset, et aliquis ipsam ab octavo orbe vellet videre, nullo pacto cerneret, cum sidera illa quae primae magnitudinis vocantur, et quae plusquam centies maiora ipsa terra putantur non nisi ut puncta videantur."

<sup>41</sup>Benedetti 1585, 195: "Si essent dicti mundi, eorum quilibet suum proprium centrum, suamque propriam circumferentiam haberet, terraeque et ignes haberent inclinationem ad centra circumferentiasque suorum mundorum."

#### 6.4 Appendix: An Assessment of Benedetti's Horoscopes (by Günther Oestmann)

For the recalculation of a historical horoscope, the same methods and means the author had at his disposal must be employed, that is, the use of modern parameters or tables is not allowed.<sup>42</sup> In the following disposition, planetary positions are rendered in ecliptic longitude (degrees ; minutes) for each zodiacal sign (0–30°), geographical coordinates likewise in degrees ; minutes, and time in hours ; minutes. Latitude is denoted as  $\varphi$ .

##### 6.4.1 Nativity Cast by Benedetti for Duke Carlo Emanuele I of Savoy

January 11, 1562 (Julian date), 16;23 p.m.,  $\varphi = 45^\circ$ ; Planetary positions according to the *Prutenicae Tabulae* by Erasmus Reinhold (1551).

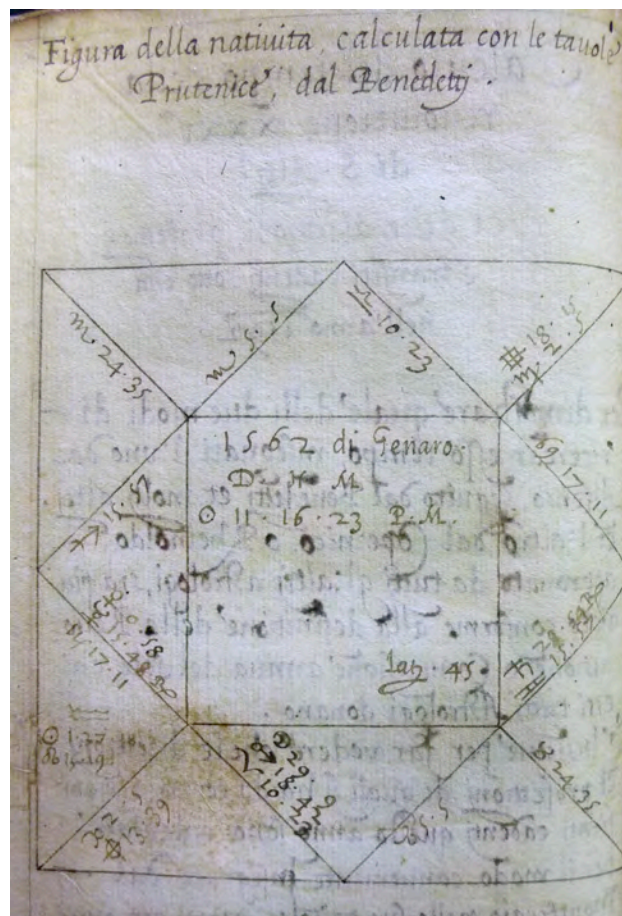


Figure 6.1: The horoscope cast by Benedetti for Duke Carlo Emanuele I of Savoy. This was calculated with the Prutenic tables, as transcribed by Bartolomeo Cristini in *Revoluzione trentesima prima del Serenissimo Signore il Signor Carlo Emanuel Duca di Savoia corrente dell'anno 1592 con ogni diligenza et fedeltà calculata et decchiarata secondo le migliori intelligenze de più principali autori dell'astrologia giundiciaria*, Turin. (Biblioteca Nazionale Universitaria: Coll. N VII 10, f. 11v)

<sup>42</sup>Here a convenient, unfortunately little-known computer program created by Peter Schiller especially for the needs of historians has been used. See Schiller 2001. There is not sufficient room here for a detailed analysis of the choice of appropriate historical parameters; for a concise description, see Oestmann 2002 and Eade 1984, 1–103.

Carlo Emanuele I of Savoy was born in the Castle of Rivoli (c. 15 km west of Turin) on January 12, but here the “noon epoch” is indicated in the manner commonly used by astronomers/astrologers: the date changes at 12:00 local time, and the hours are counted from there to 24—contrary to civil use, where sunrise or sunset often marked the change of day. With the proliferation of mechanical clocks in the late Middle Ages, the date change at midnight gradually became predominant and hours were counted from 1–12.<sup>43</sup>

The geographical coordinates of the nearest town to Turin listed in the *Catalogus locorum* in Reinhold's *Prutenicae Tabulae* is Venice, which is 0 h 50 m (12;30) west of Königsberg, the reference meridian of the Prutenic tables. In Petrus Apianus's *Cosmographicus liber* (1533), the following specifications are given: Turin 30;30 and Königsberg 42;16 east of the island Porto Santo near Madeira (f. XLIIr, XXXIXr) → the difference of longitude is 11;46 (modern value: 12;46). In the following recalculation, a longitude of 12;00 west of Königsberg has been assumed:

Table 6.1: Planets

Planets	Original Source	Recalculation
Sun	1;27 Aq	1;27 Aq
Moon	29;09 Ar	29;16 Ar
Saturn	28;54 Ge retrograde	28;55 Ge
Jupiter	[missing]	21;02 Ta
Mars	18;42 Ar	18;41 Ar
Venus	0;58 Cp	0;58 Cp
Mercury	15;48 Cp retrograde	15;48 Cp
Lunar node (asc.)	15;19 Aq	15;16 Aq

Table 6.2: Houses

Houses (Regiomontanus)	Original Source	Recalculation
X	10;23 Li	10;16 Li
XI	5;05 Sc	4;59 Sc
XII	24;35 Sc	24;32 Sc
I	15;57 Sa	15;54 Sa
II	17;11 Cp	17;07 Cp
III	2;05 Pi	1;58 Pi
Lot of Fortune (Night)	18;15 Vi	18;05 Vi
Lot of Fortune (Day)	13;39 Pi	13;43 Pi

The Lot of Fortune (*Pars Fortunae*; named for the Roman goddess of luck and wellbeing) is calculated in diurnal charts by subtracting the ecliptic longitude of the sun from the

<sup>43</sup>For details, see Bilfinger 1888, 262–286 and Ginzel 1914, 94–96.

longitude of the moon. Then the difference is added to the longitude of the Ascendant: Lot of Fortune = Ascendant + Moon – Sun. For nocturnal charts, the calculation is Ascendant + Sun – Moon.

Although this is a night-time birth chart, Benedetti has marked the Lot of Fortune for night and day.

The sign and degree occupied by the moon when crossing the ecliptic from southern to northern latitude is the ascending node (*Caput Draconis*). When the moon is moving in the opposite direction (crossing the ecliptic from north to south), the point of intersection is called the South Node (*Cauda Draconis*). The nodes are not fixed, but have a retrograde movement (a complete revolution of the nodes in the ecliptic takes 6798 days/18.61 years). To both points (which are important in the interpretation of a chart), the strength of a planet has been assigned. The Dragon's Head is considered beneficial, the Dragon's Tail malefic. (In Hindu astrology, the ascending node is called *Rāhu* and the descending node *Ketu*; both are considered malefic planets.<sup>44</sup>.)

Benedetti forgot to inscribe Jupiter. Apart from this flaw everything has been calculated accurately.

#### 6.4.2 Revolution or Solar-Return Horoscope

January 21, 1592 (Gregorian Date), 23 h 15 m 30 s p.m.,  $\phi = 45^\circ$ .

Geographical coordinates of Turin according to Petrus Apianus in *Cosmographicus liber* (1533): f. XXXVr – Toledo 9;04 East of Porto Santo; f. XLIIr – Turin 30;30 → 21;26 East of Toledo (the reference meridian of the Alfonsine tables).

Table 6.3: Planets

Planets	Original Source	Recalculation (Alfonsine tables)	Recalculation (Prutenic tables)
Sun	1;27 Aq	2;08 Aq	1;12 Aq
Moon	9;27 Ta	11;20 Ta	8;12 Ta
Saturn	7;29 Ca	10;30 Ca	7;30 Ca
	retrograde		
Jupiter	11;44 Sa	10;31 Sa	11;42 Sa
Mars	3;05 Ar	4;23 Ar	3;00 Sa
Venus	25;09 Sa	23;00 Sa	25;08 Sa
Mercury	9;36 Aq	5;43 Aq	9;25 Aq
Lunar node (asc.)	4;54 Ca	5;05 Ca	4;53 Ca

<sup>44</sup>See Hartner 1938, 131–134

Table 6.4: Houses

Houses (Regiomontanus)	Original Source	Recalculation I ( $\phi = 45;00; 23;15$ p.m.)	Recalculation II ( $\phi = 45;00, 23;17$ p.m.)
X	21;00 Ca	20;30 Cp	20;59 Cp
XI	1;00 Aq	10;36 Aq	11;11 Aq
XII	17;00 Pi	16;26 Pi	17;18 Pi
I	10;49 Ta	9;56 Ta	10;43 Ta
II	15;00 Ge	14;36 Ge	15;08 Ge
III	5;00 Ca	4;18 Ca	4;45 Ca
Lot of Fortune (Day)	18;49 Le	16;55 Le	17;44 Le

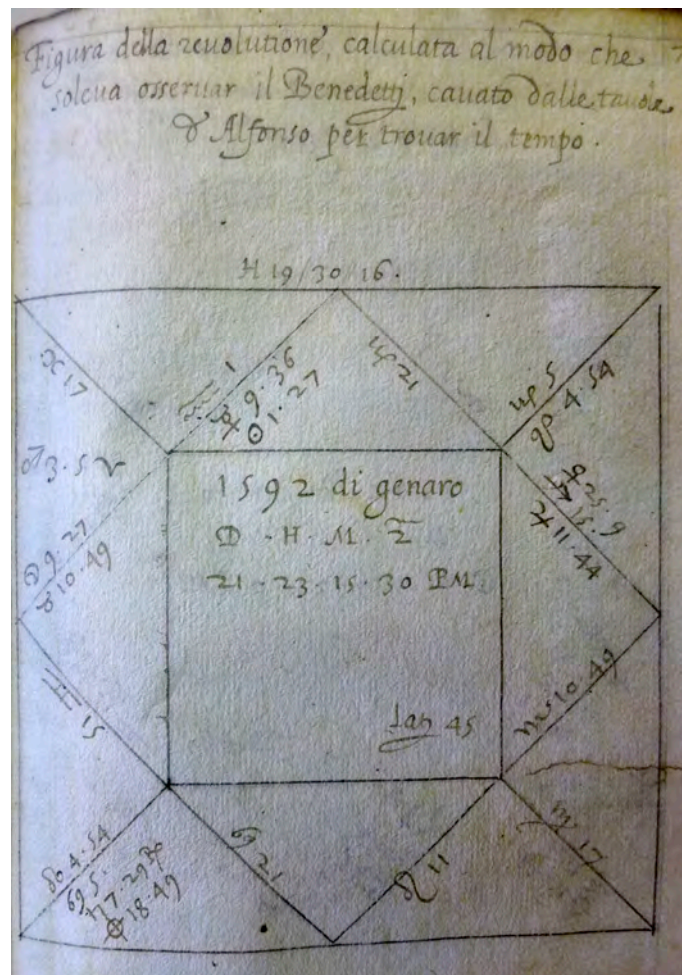


Figure 6.2: Benedetti's horoscope for Carlo Emanuele I, calculated with the Alfonsine tables, as transcribed by Cristini in *Revolutione trentesima prima* (1592), f. 12r. (Biblioteca Nazionale Universitaria di Torino, coll. N VII 10)

The second of Benedetti's horoscopes to be considered is also taken from Bartolomeo Cristini, *Revolutione trentesima prima del Serenissimo Signore il Signor Carlo Emanuel Duca di Savoia corrente dell'anno 1592 con ogni diligenza et fedeltà calculata et decchiarata secondo le migliori intelligenze de più principali autori dell'astrologia giudiciaria*, Turin, Biblioteca Nazionale Universitaria: Coll. N VII 10, f. 12r (Figure 6.2).

This is a chart constructed for the moment in which the sun returns to the degree and minute of its longitude at nativity (i.e., transiting the position of the "natal" sun) for the respective location. A revolution horoscope indicates the course of events during the ensuing year.

Contrary to Benedetti's caption (*Figura della revolutione [...] cavato dalle tavole d'Alfonso per trovar il tempo*), he has obviously used the Prutenic tables for calculating the planetary positions of this chart. But the moon's position is off by c. 1°, and the cusps of the houses deviate somewhat. Calculating with a time of 23;17 p.m. gives a reasonably good compliance, however. The cusp of house XI (1;00 Aq instead of 11 Aq) is most likely a scribal error.

It is noteworthy that minutes for an arc are only provided for the first house (i.e., the ascendant). For the other cusps, only whole degrees are noted. Benedetti simply cut off the minutes, which was a common rounding procedure at his time.

### 6.4.3 Natal Horoscope of Giovanni Battista Benedetti

August 14, 1530 (Julian Date), 13 h 13 m p.m., Venice; planetary positions according to the Alfonsine tables. Geographical coordinates of Venice according to Petrus Apianus (1533): Toledo 9;04 East of Porto Santo (f. XXXVr); f. XLIIr: Venice 32;30, Latitude  $\phi = 44;50 \rightarrow 23;26$  East of Toledo (the reference meridian of the Alfonsine tables).

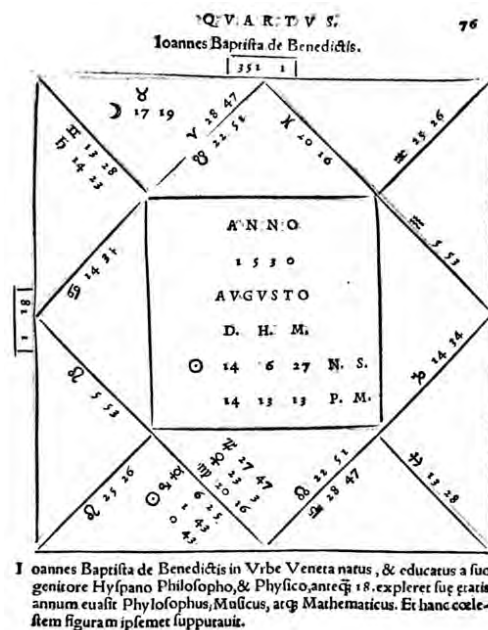


Figure 6.3: Benedetti's own horoscope, detailed in Luca Gaurico's *Tractatus astrologicus* (1552, f. 76r). (Bayerische Staatsbibliothek)



Table 6.5: Planets

Planets	Original Source	Recalculation
Sun	0;43 Vi	0;42 Vi
Moon	17;19 Ta	15;02 Ta
Saturn	14;23 Ge	14;23 Ge
Jupiter	27;47 Vi	27;46 Vi
Mars	1;43 Vi	1;45 Vi
Venus	23;03 Vi	23;02 Vi
Mercury	6;25 Vi	6;25 Vi
Lunar node (desc.)	22;51 Ar	22;52 Ar

Table 6.6: Houses

Houses (Regiomontanus)	Original Source	Recalculation
X	20;16 Pi	20;14 Pi
XI	28;47 Ar	28;36 Ar
XII	13;28 Ge	13;17 Ge
I	14;34 Ca	14;23 Ca
II	5;53 Le	5;46 Le
III	25;26 Le	25;23 Le

Apart from the moon's position (which is about  $2^\circ$  off) the horoscope is correct. In all three horoscopes the houses have been constructed according to the so-called "rational method," commonly—but erroneously—attributed to Regiomontanus:<sup>45</sup> Circles of position joining in the north and south point of the observer's horizon are laid at distances of  $30^\circ$  through the celestial equator, thus giving unequal sections of the ecliptic. This method of house division was widely used by astrologers during the sixteenth and seventeenth centuries.

<sup>45</sup>It was already known in the Maghreb in the eleventh century, see Kennedy 1996, 543. For a profound treatment of the history of house division, see North 1986, although the way this text coins new designations is awkward and may lead to confusion.



## Chapter 7

### Foundations of Physics

In this chapter we analyze Book 4 of the *Diversae speculationes*, entitled *Disputationes de quibusdam placitis Aristotelis* (Disputations on Some Opinions Held by Aristotle). We will refer to this section as *Physical Disputations*. Benedetti here developed his theory of motion and clarified his physical conceptions by means of a discussion and criticism of Aristotle's physics. He dealt with fundamental concepts such as place and time. Moreover, it is here that the interdependency of physics and cosmology in his conception most clearly emerges.

#### 7.1 Sections of the *Physical Disputations*

The *Physical Disputations* are a discussion of and an objection to Aristotle's theses on local motion and cosmology as presented in *Physics* and *De caelo*, and partly also in *Meteorologica* and *Metaphysica*. Benedetti does not limit himself to criticism but rather seeks to provide a new approach to and foundation of physics and cosmology, beginning with the theory of motion. He calls his approach mathematical (*inconcussa mathematicae philosophiae basis*). In actual fact, his conceptual tools for the treatment of motion are principally derived from Archimedes's *Floating Bodies* and Euclid's Book 5 on proportions. As we have seen, the reader of Benedetti's *Diversae speculationes* is provided with a brief book dedicated to Book 5 of Euclid's *Elements*, directly following the *Physical Disputations*.<sup>1</sup> Benedetti moreover reworks and transforms basic concepts of physics, such as place and time, and propounds anti-Aristotelian natural views such as spatial infinity and the existence of the void. His treatment culminates with cosmological speculations, including a defense of the Copernican system.

One can conveniently subdivide the *Physical Disputations* into five sections (see table 7.1 below). The first twelve chapters are a lengthy discussion of Aristotle's *Physics* IV 8. This section deals with the ratio of velocities of bodies moving through different media or the void. Secondly, from chapter 13 to chapter 18, Benedetti challenges *Physics* VII 5 on further problems linked with the theory of motion. The third subdivision (chapters 19 to 22) deals with basic philosophical matters (the void, infinity, place, and time). This section is extremely important for an overarching understanding of Benedetti as it connects the investigation of motion with a general reform of natural philosophy. Whereas Drake and Drabkin, in *Mechanics in Sixteenth-Century Italy*, translated the first two subdivisions (chapters 1–18), they neglected the foundational one, except for chapter 19 on the void. Their choice to expunge this part of the *Physical Disputations* deprived the English-speaking readership of some of the most daring pages in Benedetti's work.

Another subdivision (chapters 23–26, entirely translated in Drake and Drabkin 1969) deals with local motion and the shortcomings of the Aristotelian theory of natural places. The fifth and last subdivision, only partly included in the English edition, presents cosmological ideas. It deals with the "sphere" as a geometrical-cosmological figure, as well

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<sup>1</sup>Benedetti 1585, 198ff.

as with the (apparent) motion of the sun, with stars, meteorological aspects linked to the sun, the propagation of light in the cosmos, and other issues connected with astronomy in a broad sense. The Copernican system is discussed in the second part of this last section (chapters 35 to 39), along with other innovative theses such as the plurality of inhabited worlds akin to the earth and the reciprocity of the observational points in the universe.

We will concentrate on sections 1 (on motion), 3 (on the foundations of physics), and 5, part 2 (on cosmology). As one sees from our overview (table 7.1), the *Physical Disputations* are no less complex and heterogeneous than the volume of which they are part, the *Diverse speculationes*. Therefore, we will review Benedetti's arguments in enough depth to enlighten the thematic interdependency and his approach in general, deliberately leaving aside an excessive analysis of details that would not help to understand his intellectual endeavor as a whole.

Table 7.1: An overview of the *Physical Disputations* and of their English translation in Drake and Drabkin 1969.

Sections and their subjects	Chapters	Details	Presence in Drake and Drabkin 1969
1. Phys. IV 8	§1–12	Discussion on the void and down- and upward motion in different media	✓
2. Phys. VII 5	§13–18	On the proportions of velocities, volumes, and surfaces	✓
3. Foundations of physics revised	§19–22	On the void (XIX ≈ Phys. IV 8), place (XX ≈ Phys. IV 4), the infinite (XXI ≈ Phys. III 5, De Caelo I,9 etc.), and time (XXII)	Only §19
4. Violent and natural motion	§23–26	Rejection of the Aristotelian theory of natural places	✓
5. The sphere (geometrical and cosmological)	§27–34	On the sphere (§29), on starry light (§30), on infinite motion (§31), on the Sun, its warmth and seasonal changes (§30–31 and §34), celestial music, and harmony (§32–33)	Only §28 and §29
	§35–39	On the composition of circular and rectilinear motion and Copernicus's hypotheses (§35), plurality of worlds (§36), cosmic propagation of light (§37), geometrical aspects relative to elements (§38), and relativity of the point of observation (§39)	✓ (only §38 missing)

## 7.2 An Archimedean Theory of Motion

The Renaissance rediscovery of Archimedes's work can be seen as a crucial contribution to the theoretical advance of modern physics.<sup>2</sup> The wide dissemination of this ancient work produced a renewed interest in mathematical methods for the investigation of nature. In the *Questiti et inventioni diverse* of 1554, Tartaglia's Archimedean perspective implied a critical approach to Aristotle's mechanics. By contrast, Del Monte had a rather syncretistic approach merging Archimedean and Aristotelian elements. He even argued for the possibility of deriving terrestrial motion from Aristotelian premises (probably referring to geological motions rather than to an astronomical phenomenon).<sup>3</sup> Benedetti's feelings toward Aristotelian philosophy are rather hostile; nonetheless, it is clear that his anti-Aristotelian theory of motion is embedded in the Renaissance discourse on natural philosophy, mechanics, and motion among mathematically-trained scholars.<sup>4</sup>

### 7.2.1 Aristotle's *Physics* IV 8

Benedetti's considerations on motion are presented as a criticism of Aristotle's *Physics* IV 8. In that section Aristotle argued against the existence of the void and infinity of space and presented some reflections on the motion of bodies through different media. According to Aristotle, the void and infinity would undermine any reasonable theory of natural motion (indeed, his own theory of the natural and violent motions). His polemic is directed against "those," probably Democritus and his followers, who held the void to be the condition of motion. Aristotle claims that in an infinite and void space there would be no absolute directions; an up- or downward motion would be conceptually impossible. By contrast, he keeps to the "observation" that the elements display natural tendencies upward or downward (water and earth downward, air and fire upward), which, according to him, falsifies the thesis of an infinite and void space. Additionally, this assumption would lead to "absurd" consequences:

But in vacancy [...] nothing could go on moving unless it were carried. Nor (if it did move) could a reason be assigned why the projectile should ever stop—for why here more than there? It must therefore either not move at all, or continue its movement without limit, unless some stronger force impedes it.<sup>5</sup>

As far as motion is concerned, Aristotle remarks that the difference of speeds between two bodies depends either on their different weight or lightness or on the different density of the media through which they travel. He adds that the ratio of the speeds is reciprocally proportional to that of the densities, whereas it is directly proportional to that of the times.

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<sup>2</sup>Paolo D'Alessandro and Pier Daniele Napolitani have emphasized the impact of its sixteenth-century reedition in their studies on the Latin reception of Archimedes: "Nel 1544 usciva a Basilea l'*editio princeps* greco-latina di gran parte dei testi di Archimede oggi noti. Senza tema di esagerare, si può sostenere che questo avvenimento – al pari della pubblicazione del *De revolutionibus orbium coelestium* di Niccolò Copernico, apparso l'anno precedente a Norimberga – abbia fornito impulso alla nascita della scienza moderna." D'Alessandro and Napolitani 2012, 9.

<sup>3</sup>See Omodeo 2015.

<sup>4</sup>As has been argued in *Exploring the Limits of Preclassical Mechanics*, the wider conceptual framework of Archimedean theories of motion was in fact deeply rooted in Aristotelianism. See Damerow, Freudenthal, et al. 2004.

<sup>5</sup>Aristotle 1995, IV 8, 215a 17–22 (351).

For the sake of simplicity, we could represent these relations symbolically, in a modern way. Heath, in his study on *Mathematics in Aristotle*, put it as follows:<sup>6</sup>

$$\frac{\text{speed in } B}{\text{speed in } D} = \frac{\text{density of } D}{\text{density of } B}$$

Furthermore:

$$\frac{\text{time taken in } B}{\text{time taken in } D} = \frac{\text{density of } B}{\text{density of } D}.$$

On this basis, Aristotle concludes that motion in the void would be impossible, since “a body would move through the void with a speed beyond any ratio.”

According to Aristotle, differences depending on the weights and on the dimensions of the bodies would disappear *in vacuo* with “very unpleasant” consequences:

What reason can be assigned for this greater velocity [of a heavier falling body]? If the passage is through a medium, there must be such a difference [in the velocity of fall between heavier and lighter bodies]; for when there is anything there to cleave, the body superior in force of its thrust will necessarily cleave the medium faster, since either its more suitable shape or the natural thrust it exercises, whether following its natural movement or being thrown, makes it cleave the better. Where there is nothing to cleave, therefore, all bodies will move at the same velocity; which is impossible.<sup>7</sup>

Aristotle assumes that the speed of falling bodies is proportional to their weight (or dimension). For instance, one reads in *De caelo*:

His must surely be a careless mind who does not wonder how it is that a small particle of the earth, if raised to a height and then set free, should refuse to remain where it was but begin to travel, and travel the quicker the bigger it is, whereas if one held the whole Earth in the air and let it go, it would not move. But in fact, for all its weight, it is at rest.<sup>8</sup>

This argument was repeated by Ptolemy in *Almagest* I 7: “If the Earth had a single motion in common with other heavy objects, it is obvious that it would be carried down faster than all of them because of its much greater size.”<sup>9</sup>

In summary, *Physics* IV 8 provides Benedetti with a series of entangled problems involving the void, infinity, and motion. He begins his reform of physics by dealing with the last issue and then moving to a treatment of the other ones. In relation to motion through a medium, he enlists some commonly accepted assumptions (*primo verissima et obiecta intellectus per se cognita*) in chapter 2:

Therefore, whenever two bodies are subjected to or receive one and the same resistance to [the motion of] their surfaces, [the speed of] their motions will turn out to be to each other in precisely the same proportion as their motive forces. And, conversely, whenever two bodies have one and the same heaviness or lightness, but are subject to different resistances, [the speed of] their

<sup>6</sup>Heath 1949, 116.

<sup>7</sup>Aristotle 1995, IV 8, 216a 17–21 (357).

<sup>8</sup>Aristotle 1986, II 13, 294 a (223–225).

<sup>9</sup>Ptolemy 1984, 44.

motions will have the same ratio to each other as the inverse ratio of the resistances.<sup>10</sup>

The cases that have to be considered are basically two: first, different bodies in the same medium, and second, equal bodies in different media. A decisive difference between two bodies is their *gravitas* (gravity, weight) or *levitas* (lightness). For Benedetti, *gravitas* and *levitas* act as moving forces. He calls them *virtutes moventes* or *facultates moventes*. For the sake of brevity, we will refer to them as *virtus/virtutes*.

If we use V like *velocitas* for speed, P like *pondus* for *virtus*, and R like *resistentia* for resistance, we can formalize the previously mentioned general assumptions in the following way:<sup>11</sup>

I. Case in which R is constant:

$$\frac{V_1}{V_2} \sim \frac{P_1}{P_2}$$

II. Case in which P is constant

$$\frac{V_1}{V_2} \sim \frac{R_2}{R_1}$$

### 7.2.2 Speed in Different Media

Benedetti regards the Aristotelian theory of motion as inadequate and prefers to rely on Archimedes. He conceives of motion as analogous to the behavior of bodies in water. Following the *Floating bodies*, he holds that weight and lightness are not absolute but relative properties because they depend on the medium: “*quia in medio se densiore si poneretur, non grave esset, sed leve, quemadmodum Archimedes ostendit*” (in fact, if it is put in a denser medium it is not heavy but light, as Archimedes demonstrates).<sup>12</sup> Thus, the direction of a vertical motion and, as we will see, the speed depend on the matter of a body as well as on the fluid (air, water, or whatever) in which it is merged.

In the second *disputatio*, Benedetti declares that the *virtus* (weight or lightness)<sup>13</sup> of a body varies depending on the *densitas* (density) of the medium. Here he has in mind the three different cases considered by Archimedes in Book 1 of the *Floating bodies*: a body the weight of which is equal to that of the fluid, lighter, or heavier. Archimedes’s seventh proposition, concerning the last case, might illustrate Benedetti’s mental model for motion in a medium:

<sup>10</sup>Drake and Drabkin 1969, 198. Cf. Benedetti 1585, 169: “Quotiescunque igitur duo corpora unam eandemque resistentiam ipsorum superficiebus, aut habebunt aut recipient, eorum motus inter seipsos eodem plane modo proportionati consurgunt, quo erunt ipsorum virtutes moventes: et e converso, quotiescunque duo corpora unam eandemque gravitatem, aut levitatem, et diversas resistentias habebunt, eorum motus inter seipsos eandem proportionem sortientur, quam habebunt eorum resistentiae converso modo [...]”

<sup>11</sup>It should be remarked that this algebraic rendering of Benedetti’s thoughts does little justice to contemporary efforts to represent functional dependencies among different magnitudes with the help of the available mathematical techniques, such as compound proportions, as will become clear from the following discussion; see also Damerow, Freudenthal, et al. 2004.

<sup>12</sup>Benedetti 1585, 170.

<sup>13</sup>In this passage, Benedetti employs the word *pondus*, but in the discussion he also talks of “lightness”; therefore, we prefer to use the term *virtus movens* or, more briefly, *virtus*, which is the term used by Benedetti.

Solids heavier than the fluid, when thrown into the fluid, will be driven down as far as they can sink, and they will be lighter in the fluid by the weight of a portion of the fluid having the same volume as the solid.<sup>14</sup>

The actual *virtus* (either weight or lightness) of a body results by subtracting from the total virtue (*virtus totalis*) a quantity which Benedetti calls *resistentia extrinseca* (resistance, for instance in disp. 9) and depends on the *densitas* (density, for instance in disp. 3), which we could regard as an expression indicating the specific weight:<sup>15</sup>

$$virtus\ in\ medio = virtus\ totalis - resistentia$$

Accordingly, he holds the position against Aristotle that the ratio of the speeds in different media is not inversely proportional to the densities of the media but directly proportional to the *virtutes* in different media.

$$\frac{speed\ in\ A}{speed\ in\ B} = \frac{virtus\ totalis - resistance\ of\ A}{virtus\ totalis - resistance\ of\ B}$$

Given this equation, Benedetti shows (chapters 3–4) that the thesis of *Physics* IV 8, namely that the ratio of the speeds is reciprocally proportional to that of the densities of the media, is only true in a special case, which can be constructed *ad hoc*: if the ratio of an object’s weight (or the corresponding speed) in one medium (for instance air) to its weight in another medium (for instance water) is equal to the ratio of the first medium (air) to the second (water). However, it is incorrect to claim that the ratio of the speeds of all bodies moving through different media remains the same. In chapter 6, Benedetti demonstrates moreover that the ratios of the weights (or speeds) of a body through different media are not maintained as constant in different media (*Quod proportione ponderum eiusdem corporis in diversis mediis proportiones eorum mediorum densitatum non servant. Unde necessario inaequales proportiones velocitatum producuntur*).<sup>16</sup>

We should now add some considerations on Benedetti’s argumentative strategy. He begins chapter 2 with a reference to principles generally taken for granted: *primo verissima et obiecta intellectus per se cognita*. The adverb “primo” can have two meanings in this context: it could indicate either that the author is proposing the “first” principles of the motion theory or, quite on the contrary, that he is presenting theses that are universally true “only at a first glance.” These “very true assumptions,” these *primo verissima*, are in fact the Aristotelian statements concerning the proportion of the ratios of velocities and weights as well as the proportion of the ratios of velocities and resistances. In the second part of chapter 2, Benedetti brings forward his own conception of the proportion between the ratio of velocities and that of weights in a medium. He begins the paragraph on his theory with these words: “Aliud quoque supponendum est.” This *incipit* can be interpreted in two different and rather opposite ways: as “Another proposition must also be presupposed” (which is Drake and Drabkin’s translation) or as “We must presuppose a different proposition.” As a matter of fact, the thesis that follows these words, that on the subtraction of the resistances from the total *virtutes*, is meant as a general truth. Benedetti himself intends to show that the Aristotelian laws are not true universally but only for

<sup>14</sup>Dijksterhuis 1956, 376.

<sup>15</sup>See Benedetti 1585, 174: “dictis corporibus subtrahitur proportio resistentiarum extrinsecarum.” For a discussion of Benedetti’s employment of the termini ‘pondus,’ ‘densitas,’ and ‘resistentia,’ see Helbing 1987, 155–168.

<sup>16</sup>The details, the text, and the notes in the English edition by Drake and Drabkin are clear enough, so we will not expand on these aspects here.



special cases. Benedetti presents his theory of motion in opposition to that of Aristotle, so we tend to interpret chapter 2 as an opposition between two contrasting perspectives. Yet a certain ambiguity in his words cannot be denied. Perhaps it is a rhetorical strategy consciously employed by Benedetti: he first presents to the reader some propositions with which natural philosophers should be familiar, and then leads his reader to reject these common statements as not universally true rather than absolutely false.

### 7.2.3 Motion *in pleno* and *in vacuo*

After these considerations on motion in different media, Benedetti deals with the motion of bodies with different dimensions (but of the same material) through a medium as well as through the void. Chapter 7 considers the case in which two bodies A and O, made of the same matter and having the same figure, traverse the same medium. According to Aristotle the ratio of their speeds should be directly proportional to that of their weights. Benedetti demonstrates that this is false, since one could imagine a third body U, made of a different material, with the same *virtus* (here: *gravitas*) as O and the same dimensions (*magnitudo et figura*) as A. If V is the speed, M the *magnitudo* corresponding to a certain weight (or more in general, a *virtus*), then:

$$\frac{VA}{MA} = \frac{VO}{MO}$$

(according to Aristotle)<sup>17</sup>

$$\frac{VA}{MA} = \frac{Vu}{Mu}$$

(case of two bodies with equal dimensions and different weights).

Since by assumption  $MO = Mu$ , it follows that  $Vu = VO$ , but this is not consistent with the assertion that two bodies of equal weights and different dimensions will move with different speeds in a medium because a bigger body needs more “force” to penetrate the medium than a smaller one:

The body which, when compared to the other, is of equal weight or lightness, but is subject to smaller resistance, moves [in natural motion] more swiftly than the other in the same proportion as its surface is subject to a smaller resistance than that of the other body, by reason of its being able more readily to divide the continuity of the air or water.<sup>18</sup>

Therefore, the ratio of the velocities of two bodies having equal material composition and figure but different dimensions is untenable, *pace* Aristotle.

Chapter 9 introduces the issue of motion through the void, asserting, against Aristotle, that the ratio of two bodies *in pleno* and *in vacuo* cannot be the same:

In a plenum the ratio of the external resistances in the case of these bodies is subtracted from the ratio of the weights, so that the ratio of the speeds

<sup>17</sup>It corresponds to the above-mentioned equation:  $\frac{V_1}{P_1} = \frac{V_2}{P_2}$ .

<sup>18</sup>Drake and Drabkin 1969, 198. See Benedetti 1585, 169: “Corpus illud quod alteri comparatum, aequalis erit ponderis, aut levitatis, sed minoris resistentiae, existet velocius altero, in eadem proportione, cuius superficies resistentiam suscipit minorem ea quae alterius est corporis, ratione facilioris divisionis continuitatis aeris, aut aquae [...]”

remains. And this last ratio would be annulled if the ratio of these resistances were equal to the ratio of the weights.<sup>19</sup>

The argumentation is not immediately intelligible to the modern reader. Contrary to appearance, in fact, this passage does not describe the following subtraction:  $\frac{P_1}{P_2} - \frac{R_1}{R_2}$  (V is the speed, P the weight, and R the resistance).

$$\frac{V_1}{V_2} \neq \frac{P_1}{P_2} - \frac{R_1}{R_2}$$

According to the theory of proportions, the addition and subtraction of a ratio to or from another ratio can be conveniently represented in a modern fashion as follows:<sup>20</sup>

I. Meaning of ratios addition:

$$\frac{a_1}{a_2} + \frac{b_1}{b_2} \sim \frac{a_1 b_1}{a_2 b_2}$$

II. Ratios subtraction:

$$\frac{a_1}{a_2} - \frac{b_1}{b_2} \sim \frac{a_1 b_2}{a_2 b_1} \sim \frac{a_1}{a_2} \frac{b_2}{b_1}$$

Given this, the relation indicated by Benedetti in the above-mentioned passage can be rendered through the following symbolic expression:

$$\frac{V_1}{V_2} \sim \frac{P_1 R_2}{P_2 R_1}$$

This relation expresses in a concise form the Aristotelian perspective concerning the relation of velocities, weights, and resistances, as already taken up by Benedetti at the beginning of the *disputatio* number two. Benedetti notices that the ratio of the velocities is annulled (*nulla esset*) if the ratio of the *virtutes* is equal to that of the resistances ( $\frac{P_1}{P_2} \sim \frac{R_1}{R_2}$ ). In fact, in this case (and only in this case) the speeds of bodies with different weights would be the same. As a consequence, there would be “no proportion” between different quantities but rather an equality of speeds. Benedetti indicates that this special case of motion *in pleno*, in which bodies with different weights have equal speeds, is the rule for motion *in vacuo*.

Chapter 10 expands on motion *in vacuo*. It presents the famous thesis that bodies of the same material move with the same speed *in vacuo*, although they might have different dimensions. Benedetti remarks that a body O can be divided into halves A and E, each of the same weight (or *virtus*). If an ideally weightless bar connects them, the weight at the center of the connection should be the sum of the partial weights and thus equal O. Therefore, bodies composed of the same material will fall with the same speed, independently of their weight. In chapter 11, one reads that motion *in pleno* is different as a consequence of the friction of the medium, except for the special case in which the parts travel through

<sup>19</sup>Drake and Drabkin 1969, 205. See Benedetti 1585, 174 (emphasis is ours): “In pleno dictis corporibus *subtrahitur* proportio resistentiarum extrinsecarum a proportione ponderum, ut velocitatum proportio remaneat, quae nulla esset, si dictarum resistentiarum proportio, ponderum proportioni aequalis esset, et hanc ob causam diversarum velocitatum proportionem in vacuo haberent ab ea, quae est in pleno.”

<sup>20</sup>The same concept of addition and subtraction of ratios can be found in the manuscripts of Thomas Harriot, who called them “compositio” and “subductio” (or “compositio contraria”). See for instance Schemmel 2008, 635, reference letters *g* and *o*.

media whose resistances have “the inverse proportion of the weights” (*si duo corpora... suas resistentias ad invicem proportionatas haberent, ut sunt eorum pondera*).<sup>21</sup>

#### 7.2.4 A Note on the Historical Relevance of Benedetti’s Reflections on Motion

Our analysis of the first part of the *Physical Disputations* should be sufficient to understand Benedetti’s approach to motion and the scope of his investigation. Subdivision two tackles Aristotle’s *Physics* VII 5 and deals with the proportions of velocities, volumes, and surfaces.<sup>22</sup> We are not going to analyze in further detail Benedetti’s views on motion. Instead, we deem it important to consider the “philosophical” part of the *Physical Disputations* (which we have indicated as section 3), as it introduces novel prospects on the foundations of physics. It extends from chapter 19 to chapter 22 and is an attempt to revise basic concepts of physics from an anti-Aristotelian perspective.

Before we tackle these foundational aspects, we would like to recount the influence that Benedetti’s theory of motion exerted on the young Galileo. We have already hinted at Benedetti’s triangulation with Galileo via Del Monte. At a theoretical level, one of the strongest pieces of evidence of this influence is the affinity between Galileo’s early speculations on motion and the theses that Benedetti propounded in the *Diversae speculationes*. This link is evident and well known, beginning with the hydrostatic analogy to explain the motion through a medium, the relativity of heavy and light, and the subtraction of resistance from weight, which allows motion through a vacuum to be accepted and makes it physically plausible.<sup>23</sup> For instance, several chapters of Galileo’s first manuscript among those gathered by Favaro under the title of *De motu* are very close to Benedetti’s treatment; above all Galileo’s chapter 8, “in which it is shown that different bodies moving in the same medium maintain a ratio [of their speeds] different from that attributed to them by Aristotle”; chapter 10: “in which, in opposition to Aristotle, it is proved that, if there were a void, motion in it would not take place instantaneously, but in time”; chapter 12: “in which, in opposition to Aristotle, it is concluded that the absolutely light and the absolutely heavy should not be posited; and that even if they existed, they would not be earth and fire, as he believed”; and chapter 15: “in which, in opposition to Aristotle, the conclusion is reached that rectilinear and circular motions have a ratio to each other.” This connection between the work of the young Galileo and Benedetti’s insights on motion is significant, the more so since Galileo’s patron Del Monte was skeptical about the possibility of quantifying motion.<sup>24</sup> Actually, in his manuscripts, Del Monte took some annotations on falling bodies in different media. This might have been an issue of discussion between him and Galileo.<sup>25</sup>

Moreover, it should be remarked that the Copernican cosmological element is absent from Galileo’s early manuscript *De motu*, although this would become a crucial aspect of

<sup>21</sup> See Drake and Drabkin 1969, 206, n. 119: “Benedetti asserts that the speeds are equal *only in the void*, on the ground that in a plenum there would be an additional frictional resistance that would disturb this equality (unless this resistance itself were proportional to the weights of the bodies: Ch. 11).”

<sup>22</sup> See Helbing 1987, 162.

<sup>23</sup> Apart from Koyré 1986, see also Drabkin, “Introduction” to Galilei 1960, 9–10.

<sup>24</sup> The ongoing debate between Galileo and Del Monte on the possibility of developing a mathematical dynamics is seen in Galileo’s letter of November 29, 1602: “V.S. Ill.ma scusi la mia importunità, se persisto in voler persuaderle vera la proposizione dei moti fatti in tempi uguali nella medesima quarta del cerchio; perché essendomi parsa sempre mirabile, hora viepiù mi pare, che da V.S. Ill.ma vien reputata come impossibile: onde io stimerei grand’errore e mancamento il mio, s’io permettessi che essa venisse repudiate dalla di lei speculazione, come quella che fusse falsa.” See Galilei 1968, 97.

<sup>25</sup> See Tassora 2001, 281–283.

his later investigations. Also, the alliance of mechanics and Copernican astronomy, which emerged only later in Galileo, bears witness to Benedetti's influence on his work.<sup>26</sup>

### 7.3 On the Void: Atomistic Prospects

Benedetti's considerations on motion are followed by a treatment of the void which, from the perspective of Aristotle's *physics*, is directly connected with the former issue. His theory of motion through media and through the void is the basis upon which he casts Aristotle's rejection of the void into doubt. Chapter 19 of the *Physical Disputations*, *Quam sit inanis ab Aristotle suscepta demonstratio quod vacuum non detur*, is a transition from the Archimedean theory of motion (chapters 1–18) to the reconceptualization of natural philosophy in general. The discussion on the void is directed against Aristotle's *Physics* IV 8, in which the theory of motion serves to reject the physical void and infinity. Benedetti claims that the void is conceptually possible as a consequence of his previous demonstration that the Aristotelian assumptions on the relation between speed and density of the medium are wrong or, at least, not universally valid. "*Ex iis, quae superius demonstravimus facile cognosci potest irritam esse eam rationem, quam Aristoteles 8 cap. lib. 4 Physicorum ad destruendum vacuum, confinxit*" (From the demonstrations above it is easy to see that the argument that Aristotle devised in *Physics* VIII 4 is vain).<sup>27</sup> According to Benedetti's Archimedean dynamics, motion through a void is not absurd at all. He explains that such a motion is simply quicker than one taking place through any medium, because no resistance has to be subtracted from the body's *virtus*.

Hence, Benedetti reintroduces the concept of the void into physics, which was excluded by Aristotle as a criticism of Democritean atomism.<sup>28</sup> Benedetti's move is in line with the rehabilitation of atomistic philosophy during the Renaissance and the early modern period. The void and atoms are the two ingredients of the same matter theory inspired by the rediscovery of Lucretius and a reassessment of Epicureanism.

Among Benedetti's contemporaries the most committed natural philosopher supporting atomism was Bruno. Although they probably developed their theories independently of each other, they have many points of contact. In his Italian philosophical dialogues (published in London in the years 1584–1585), Bruno widely drew on ancient atomistic doctrines. He called Democritus and Epicurus those "who contemplated nature with open eyes" (*con occhi più aperti han contemplata la natura*).<sup>29</sup> He derived from them (often relying on Lucretius) a vision of the universe as infinite, the idea of a countless number of worlds (similar to the solar system), as well as the principle of cosmological homogeneity and the atomic theory of matter and of minima moving through a void. In *De l'infinito universo e mondi*, he celebrated Epicurus's theory of matter, summarizing it as follows:

Epicure similarly nameth the whole and the universe a mixture of bodies and of the void; and in this universe and in the capacity thereof to contain the void and the empty, and furthermore in the multitude of the bodies contained therein he maintaineth that the nature of the world, which is infinite, doth exist.<sup>30</sup>

<sup>26</sup>Drabkin 1964, Drake 1976, and Renn and Damerow 2012, 142–155.

<sup>27</sup>See Benedetti 1585, 179.

<sup>28</sup>On the medieval debates on the void and on the historical place of Galileo, and Benedetti before him, see Grant 1981, 60–66.

<sup>29</sup>Bruno 2000a, 374.

<sup>30</sup>Bruno 1968, 272–273.

However, the concept of the void assumes in Bruno a very special meaning since, in his philosophical terminology, the terms ‘space,’ ‘void,’ and ‘aether’ are used as synonyms. The ethereal void is the medium that makes the motion of bodies possible.<sup>31</sup> In the second dialogue of *De l’infinito*, he recapitulates the following three meanings of *vacuo*:

- First, the void refers to all which is not bodily and thus does not have the property of resisting penetration. In this sense, there is a “boundless ethereal region” (*eterea regione immensa*), in which the worlds are plunged. The countless worlds populating the universe are themselves composed of matter and the void.<sup>32</sup>
- Second, the void has the meaning of infinite space (*spacio infinito*). In it everything is included. It is the container that cannot be included in anything else.
- Third, space can be assumed to be nothingness, in accordance with Aristotle. Bruno calls this a metaphysical meaning, or rather lack of meaning. According to him, this extra-physical meaning was a sophism that served Aristotle to refute the possibility of the void in nature. For Bruno, only the first and the second meaning of the void make sense. They could and should be embraced in natural philosophy. By contrast, the metaphysical void has to be rejected as meaningless.

Bruno’s reflections on the physical void are part of his program to revive an atomistic conception of nature. He combined it with cosmological infinity and the post-Copernican idea that a plurality of worlds exists, each one of them a solar system centered on a star similar to ours. This vision, which shows similarities to Benedetti’s, paved the way for seventeenth-century cosmologies, in particular for the Cartesian multi-centric universe.

Other Renaissance scholars supported combinations of atomism and heliocentric astronomy as well. A case in point is the circle of scientists gathered around Henry Percy of Northumberland in London. Just like Benedetti and Bruno, they brought together heliocentrism, atomism, an empirical and mathematical approach to nature, Renaissance naturalism, and anti-Aristotelianism.<sup>33</sup> Thomas Harriot, for example, was an atomist and a supporter of the infinity of the universe.<sup>34</sup> Nicolas Hill, another member of the “Northumberland circle,” authored an apology of Epicureanism entitled *Philosophia Epicurea* (1601), which was directly inspired by Bruno. It included Copernican arguments in favor of terrestrial motion and many others in favor of the earth’s magnetism, in accordance with Gilbert.<sup>35</sup> Following ancient and modern atomists, Hill affirmed the boundlessness of the universe and the plurality of worlds.<sup>36</sup>

Atomism was to be embraced by celebrated exponents of sixteenth-century science and philosophy such as Galileo and Pierre Gassendi. At the same time, corpuscular alternatives were spread by Descartes and his followers.<sup>37</sup> Atomism and, more generally, corpuscular theories of matter encountered much censorship, especially owing to theological difficulties, since they appeared to be irreconcilable with Scholastic accounts of the

<sup>31</sup> See Del Prete 1999, 61 and Michel 1962.

<sup>32</sup> Bruno 2000a, 348: “[...] perché questo spirito, questo aria, questo etere non solamente è circa questi corpi, ma ancora penetra dentro tutti, e viene insito in ogni cosa.”

<sup>33</sup> Kargon 1966.

<sup>34</sup> On his intellectual stature and achievement, see Schemmel 2008, who stressed that Harriot’s unpublished manuscripts reveal that his research activity was similar to that of Galileo in the same years. Moreover, Harriot’s reflections on infinity and the minimum bear witness to his familiarity with the work and ideas of Bruno. See Fox 2000 and Henry 1982.

<sup>35</sup> N. Hill 2007, 155–157. See Plastina 1998, Plastina 2004 and Omodeo 2014a, 372–377.

<sup>36</sup> For an overview of English science in that period, see C. Hill 1997, 15–76 and 118–200.

<sup>37</sup> On medieval and early-modern corpuscular matter theories, see Lüthy, Murdoch, and Newman 2001. On Galileo, see Galluzzi 2011.

Eucharist.<sup>38</sup> Nonetheless, many modern scholars were attracted, just as Benedetti was, to the prospect of connecting the advances of mechanics with a physics and a theory of motion rooted in corpuscularism alongside a post-Copernican cosmological outlook. As Galluzzi has recently pointed out, this was a guiding idea for Galileo already in his *Iuvenilia*.<sup>39</sup> Theories of motion, atomism, and Copernican astronomy are three interconnected issues in his as well as Benedetti's work. In *De motu*, Galileo was committed to the homogeneity of matter and reflected on it in connection with motion, in particular with the Benedettian thesis that “*si vacuum esset, motuum in instant non contingere, sed in tempore*” (If the void exists, motion does not occur instantaneously but rather in time).<sup>40</sup> In the tenth section of this manuscript, Galileo followed in Benedetti's footsteps and came to the same conclusion: “Hence it follows, not that motion in a void is instantaneous, but that it takes place in less time than the time of the motion in any plenum.”<sup>41</sup>

Benedetti shared many ideas with contemporary and later scholars in his criticism of Aristotle's natural philosophy as well as in his effort to construct a new physics. Whereas he developed his conceptions on matter and the cosmos independently of Bruno, and probably did not directly influence the English circle of Harriot and Hill, nonetheless he must have influenced the views of Italian scholars such as Galileo who were familiar with the *Diversae speculationes*. Benedetti showed them that a revision of dynamics could not be fulfilled independently of a broader program of philosophical reform.<sup>42</sup>

#### 7.4 On Place: Space as *intervallum corporeum*

Chapter 20 of Benedetti's *Physical Disputations* deals with the concept of place. Aristotle, in *Physics* IV 4, defines it as the surface in which a body is included and denies that it could be conceived as an *intervallum*. By contrast, Benedetti defines it precisely as *intervallum corporeum*, an expression which could be roughly translated as an inter-bodily gap. Aristotle says that *locum* (the place) and *locatum* (the placed) must be coextensive but, as Benedetti notices, a spherical surface contains more “extension” (*intervallum*) than any other figures with an equal surface. As a consequence, one can imagine two places (in the sense given to the term by Aristotle) occupied by bodies of different dimensions or, the other way round, two bodies of different dimensions which suit the same surface. By contrast, an *intervallum* of space contains only bodies of equal volume, thus respecting the bijective relation between the volume of the place and that of the placed body: “But equal bodily extensions [*intervalla*] delimited by any figure will always contain equal bod-

<sup>38</sup>See Redondi 1983, chap. 7, 203–226. Also see Ariew 2013.

<sup>39</sup>Galluzzi 2011, 9.

<sup>40</sup>Galilei 1968, vol. I, 276.

<sup>41</sup>Galilei 1960, 47. See Galilei 1968, vol. I, 282: “[...] et ita non est necessarium, motum in vacuo fieri in instanti, sed in tempore minori quam sit motus in quovis pleno.”

<sup>42</sup>Thus, Galluzzi's remarks on the scientific production of the young Galileo could be conveniently extended to Benedetti (Galluzzi 2011, 19): “Il laboratorio del *De motu antiquiora* servì a Galileo per prendere definitivamente coscienza che la cosmologia e la fisica aristoteliche non potevano essere riformate con interventi limitati ad ambiti di indagine circoscritti. In particolare, lo scritto pisano evidenziava la raggiunta consapevolezza dell'importanza strategica di una radicale riforma della concezione aristotelica del movimento. Era questo, tra l'altro, l'ostacolo più serio da superare perché potesse essere accolta la visione copernicana, intesa non come semplice ipotesi matematica, ma come dottrina fisica. [...] Né si può escludere che abbia tratto anche da esse [da tali problematiche] l'impulso per dedicarsi, con l'impegno proprietario esibito dai documenti dell'attività giovanile, all'impresa ambiziosa di una radicale riforma della concezione tradizionale del movimento e della struttura della materia.”

ies.”<sup>43</sup> The definition of place as *intervallum* has the advantage that it allows us to assign a place to every body, “in the world or outside it, *pace* Aristotle.”<sup>44</sup> This remark anticipates the cosmological treatment of infinite space, or of the possibility of space beyond our worldly system (*mundus*). In chapter 20, on place, Benedetti remarks that Aristotle is wrong when he rejects the concept of place as *intervallum* because it could lead to infinity. Benedetti objects that this is not inconvenient at all, since one could conceive of *infinita loca*. To grasp this infinity is no more difficult than to understand the never-ending process which Aristotle calls “potential infinity” (*infinitum in potentia*) and concerns the division of a body *ad infinitum*:

That infinite places must exist, if place is [conceived of as] *intervallum*, is not inconvenient. In fact, just as any body can be [divided into] infinite bodies (which he [Aristotle] would call “*in potentia*”), so can any *intervallum* be [divided into] infinite *intervalla* as well.<sup>45</sup>

Benedetti is always brief in his treatment of philosophical issues and leaves much implicit. Therefore, the reader is challenged to reconstruct the complete vision implied in his short statements and critical points. Reference to Renaissance philosophical sources discussing the same theses from an Aristotelian viewpoint can help us to better understand Benedetti’s conception. In this case, Alessandro Piccolomini’s refutation of the conception of space as a receptacle of bodies can shed light on the opposite viewpoint defended by Benedetti. The following quotation is taken from the first part of Piccolomini’s *Della filosofia naturale* (On natural philosophy), a very successful introduction to natural philosophy in Italian, which Benedetti might have known:

Other philosophers (and not unimportant but very reputed ones) asserted that there is void space among natural beings and that it is not only distinguished in essence [*per sua natura*] from the bodies it accommodates, but also it is so separated from them that it can remain completely naked and really deprived of them. Furthermore, many supposed that such a space (which is completely void, without any bodies) is mixed and situated between the beings of this world and their parts. Additionally, they believed that it infinitely extends outside the heavens. Thus, these philosophers considered the void to be that being which not only can be deprived of all bodies and substances [...] but also is apt and capable of receiving the bodies, giving them a place (something which does not occur to lines, points, and other accidents). One of the main reasons for holding such doctrines was, as they said, that, if there is no space in nature (or a void place without bodies), the motion from place to place would be impossible, although the motion of alteration would be possible. In fact, all beings, in order to move to some other place, need some in-between space through which they can move. If this space were not void, it would be necessary that, as it is filled with other bodies, different bodies

<sup>43</sup>Benedetti 1585, 180: “Sed intervalla corporea aequalis a quavis figura terminata, continebunt semper corpora aequalia.”

<sup>44</sup>Benedetti 1585, 181: “Et hoc modo nullum est corpus, quod in mundo aut extra mundum (dicat autem Aristoteles quicquid voluerit) locum suum non habeat.”

<sup>45</sup>Drake and Drabkin 1969, 198. Cf. Benedetti 1585, 180: “Quod si locus intervallum esset, infinita loca existerent [...] inconveniens [non] existit, quia eodem plane modo quo aliquod corpus potest esse infinita corpora (quod ipse diceret in potentia), sic etiam intervallum aliquod posset esse infinita intervalla.”

would penetrate each other while crossing that full space. And this is impossible.<sup>46</sup>

Thus Piccolomini presented the views of the supporters of a natural void in order to reject them. Yet his discussion is helpful as it indirectly presents us with the views of such thinkers as Benedetti who defended void and infinity. Benedetti was in fact favorable to the notions of the void and infinity. For him, space is prior to and independent of bodies. It should be remarked that, in Benedetti's perspective, infinite space does not imply cosmological infinity or the boundlessness of the material universe, precisely because space is independent of matter. In this respect, his conception is different from the one that Bruno defended in those years. Rather, it is closer to that of his correspondent Patrizi. As De Risi has pointed out, Patrizi's conception of space is marked by its ontological autonomy from bodies. It is in fact an "incorporeal and immaterial extension, tridimensional and infinite, which receives and precedes all created bodies."<sup>47</sup> Patrizi saw space as a sort of Plotinian *hypostasis* (*extensio hypostatica*), that is, a pre-corporeal quantity quantifying reality. He saw this idea as the lever to be employed in order to radically reform Scholastic metaphysics, a project that he developed in a series of publications, *De spacio physico et mathematico* (On physical and mathematical space, 1587), *Della nuova geometria* (On the new geometry, 1587), and eventually in the *Nova de universis philosophia* (1591).<sup>48</sup> It also served him to set the foundations of an epistemology of mathematics according to which the ancient science dealing with figures had to be substituted for a new science of space itself. This is the concept of his *New Geometry*. Patrizi sent a copy of this book to Benedetti, probably before the publication, to gain the favor of Carlo Emanuele I, to whom the book was dedicated.

To sum up, Benedetti defines the *locus*, against Aristotle, as *intervallum corporeum*, or bodily extension. He regards it as an invariable empty extension capable of being occupied by material bodies, an idea that becomes clearer from the related discussion on time. He basically rejects the Aristotelian definition of place as the *superficies corporis ambientis* (surface of the containing body), remarking that the place is a measure, actually an *intervallum*, and not a surface. Moreover, he explains that only his understanding of locus as a kind of objective space guarantees that two bodies of equal material constitution necessarily occupy the same quantity of "place" as *intervallum* or empty extension, independently of their form. By contrast, the definition given by Aristotle is inconsistent with the assumption of a bijective relation between *locus* and *locatus*. In fact, as geometry shows, if one accepts Aristotle's definition "a great absurdity [*maximum inconveniens*]

<sup>46</sup>Piccolomini 1585, I 3, chap. 5, f. 44r: "Altri filosofi sono stati poi, et non già minimi, ma di gran fama, che han voluto che tra le cose della natura, si trovi spatio voto, non solamente distinto per sua natura dai corpi ch'egli riceve; ma intanto separato da quelli, che ignudo in tutto, et attualmente spogliato ne può restare. Né son mancati molti, che cotale spatio, voto in tutto d'ogni corpo, han posto, non solo meschiato, et interposto tra le cose che sono al mondo, e tra le parti loro; ma ancora fuor dei Cieli, infinitamente han creduto che si distenda. Questi filosofi adunque per il voto intendevano quella cosa, che non solo fusse privata d'ogni corpo et d'ogni sostanza [...] ma fosse ancora atta, et capace, a ricevere i corpi, et dar lor luogo, il che delle linee, et de punti, et altri accidenti, non avviene. Tra le ragioni principali, che gli spingeva a credere una tal cosa, l'una era questa nella qual dicevano, che se non si potesse trovare tra le cose della natura spatio alcuno, o luogo voto d'ogni corpo; allora, se bene il movimento dell'alteratione potrebbe forse restare al mondo, tuttavia il movimento di luogo a luogo, non potrebbe senza 'l voto restar giammai. Conciosia c'havendo bisogno quella cosa, che ha da muoversi ad alcuno altro luogo, di alcuno spazio di mezo, per il quale si muova; se cotale spatio non fosse voto, saria forza che essendo pieno di altro corpo, nel passar per quello spatio pieno, venissero a penetrarsi più corpi insieme; adunque è cosa al tutto impossibile."

<sup>47</sup>De Risi 2014, 282.

<sup>48</sup>De Risi 2014, 276–277.



would follow, namely that equal places can contain unequal bodies or equal bodies can occupy unequal places.”<sup>49</sup> Additionally, Benedetti remarks that his concept of place admits an infinite universe, since it is capable of containing *infinita corpora* (infinite bodies).

### 7.5 In Defense of Infinity

Chapter 21 is devoted to infinity: *Utrum bene Aristoteles senserit de infinito* (Whether Aristotle judged correctly about infinity). Needless to say, Benedetti holds that Aristotle’s arguments on this issue are idle. The Greek philosopher rejects the possibility of an infinite body (*infinitum corpus*), that is to say, the infinity of the universe because—as the argument goes—there is no *locus infinitus* which could include it. Benedetti protests that this is a *petitio principii* since this rejection is based on a questionable definition of *locus*: “*cum Aristoteles debuerit beneficio loci destruere infinitum, ordine perverso de infinito prius*” (Since Aristotle had to destroy infinity with the benefit of [a correct understanding of] place, he wrongly started with [a discussion of] infinity).<sup>50</sup> By contrast, Benedetti’s redefinition of place as *intervallum* entails no conceptual hindrances to the acceptance of the infinity of the universe. As one reads: “*hoc modo nullum inconueniens sequeretur, quod extra caelum reperiri possit corpus aliquod infinitum*” (In this manner it is not inconvenient to assume that one can find an infinite body outside the heavens).<sup>51</sup>

Additionally, Aristotle writes (*Physics* VIII 8 and *De caelo* I 9) that a continuum can include infinite parts only *in potentia* (potentially) and not *in acto* (in act). Benedetti does not agree with him. He argues that, if the continuum is *in acto*, its infinite parts should be *in acto* as well, because it is foolish (*stultum*) to believe that something actually existing could be composed of parts which exist only potentially (*quae potentia existunt*).

The weakest argument brought forward by Aristotle is, according to his critic, that the infinite cannot be considered to be a quantity (*Physics* III 5), because only a finite quantity is thinkable, for instance the dimensions of a square or a cube. Benedetti objects that the definition of number (which he does not provide, however) does not include finitude (*necessitas terminorum*). Numbers are not necessarily terminated (*determinati*) and one can conceive an infinite multitude exactly like a finite one:

Aristotle’s arguments in the same part of *Physics* III 5 are even worse. He denies that infinity can be regarded as a quantity by saying that only one defined quantity is intelligible, such as a cubit, a three-cubit, etc. Thereby, he does not consider that in the same manner the quantity of infinite cubits is intelligible as well. Moreover, the definition of quantity does not imply limitation by necessity. For instance, the definition of number does not imply the necessity of any determined number, since an infinite multitude is not less intelligible than a finite.<sup>52</sup>

<sup>49</sup>Benedetti 1585, 180: “maximum inconueniens sequeretur, scilicet aequales locos capere inaequalia corpora, aut corpora aequalia, locos inaequales occupare.”

<sup>50</sup>Benedetti 1585, 181.

<sup>51</sup>Benedetti 1585, 181.

<sup>52</sup>Benedetti 1585, 181: “Sed peius etiam sensit Aristoteles eodem loco capituli quinti lib. 3 Physicorum, negando infinitum posse connumerari inter quantitates, dicens unam aliquam quantitatem intelligi ut cubitum, tricubitum, et cetera; ubi non considerat eadem etiam ratione intelligi posse aliquam quantitatem infinitorum cubitorum, et in quantitatis definitione nullam esse necessitatem terminorum, ut exempli gratia in definitione numeri, non est necessitas alicuius determinati numeri, quia multitudo, non minus infinita, quam finita, intelligi possit.”

The last false Aristotelian assumption on infinity is the equation of *infinitum* and *vacuum* in *Physics* IV 8. Benedetti's commentary is harsh: "Later, in *Physics* IV 8, he says that there is no difference between infinity and the void. Indeed, he could not assert and imagine anything more absurd than this."<sup>53</sup>

Like Bruno, the infinitist *par excellence*, in *De l'infinito universo e mondi* (1584), Benedetti remarks that Aristotle's arguments are not compelling. Aristotle denied the possibility of an unbounded space on the basis of a finite cosmology (i.e., the theory of the spherical and geocentric heavens along with the theory of the natural places), which excludes cosmological infinity by definition. Yet his assumptions are not self-evident.

In summary, in chapter 21 of the *Physical Disputations*, Benedetti defends the mathematical and physical possibility of the infinite. The question of the title "Whether Aristotle Judged Correctly about Infinity" is rhetorical. It calls for a negative answer.

## 7.6 On Time: Toward an Absolute Frame for Physics

Benedetti's definition of *tempus* (time) is closely connected with that of *locus* (place). He deals with it in *Physical Disputations*, chapter 22, *Exagitatur ab Aristotele adducta temporis definitio* (Rejection of Aristotle's Definition of Time). Benedetti questions the definition of time as *motus mensura numerusque* (the measure and number of motion) and offers an alternative conceptualization. But before we discuss his opinion, we will recount standard conceptions of time from antiquity.

It should be noted as a preliminary remark that the understanding and definition of time was regarded as a natural as well as a psychological issue. This should not be surprising, since the doctrine of the soul, or *psychologia*, was an integral part of natural philosophy, or *physica*, in standard university curricula. A standard reference on time was the *Timaeus*, where Plato called time, rather poetically, the "moving image of eternity" (εἰκὼ κινητός αἰῶνος).<sup>54</sup> According to his myth, the divine Demiurge created time together with the heavens (οὐρανός), making them inseparable. Plato's pupil Aristotle, in Book 4 of the *Physics*, then proposed the definition of time as the "dimension of movement in its before-and-afterness."<sup>55</sup> He also noticed that χρόνος (time) had generally been connected with the motion of the celestial sphere and was referred first and foremost to the heavens because all measurements of motion and time depend on celestial regularities:

Neither qualitative modification nor growth nor genesis has the kind of uniformity that rotation has; and so time is regarded as the rotation of the sphere, inasmuch as all other orders of motion are measured by it, and time itself is standardized by reference to it.<sup>56</sup>

For both philosophers, Plato and Aristotle, the tie between time and cosmic order was unquestionable. This interconnection was later refuted by a third authoritative source. At the downfall of the ancient world, Augustine, in Book 11 of the *Confessiones*, denied the commensurability of time and local motion: "The motion of a body is one thing, and that by which we measure how long it is, another thing."<sup>57</sup> He pointed out the transient

<sup>53</sup>Benedetti 1585, 181: "Ubi postea cap. 8 lib. 4 Physicorum ait nullam esse differentiam inter infinitum, et vacuum, revera nihil absurdius hoc dicere fingere poterat."

<sup>54</sup>Plato *Timaeus* 37C–39E

<sup>55</sup>Aristotle 1995, IV 11, 220 a 25–26 (395).

<sup>56</sup>Aristotle 1995, IV 14, 223 b 21–22 (425).

<sup>57</sup>Augustine 1988, XI 14, 265: "aliud sit motus corporis, aliud quo metimur quamdiu sit."

character of the temporal flux and noticed that the three dimensions of time are a product of our mind (*animus*). In the mind, time is constituted as the memory (*memoria*) of the past, the intuition (*attentio*) of the present, and the expectation (*expectatio*) of the future.<sup>58</sup> In other words, Augustine underscored the subjectivity of time, conceived of as a *distensio animi*, a “stretching” of the mind independent of heavenly cycles.

According to Benedetti’s criticism of Aristotle, the definition of time as “the measure and number of motion” is intrinsically wrong because measuring presupposes commensurability. But because time and motion are heterogeneous, they cannot be compared. A line is measured by a line, a surface by a surface, and a *corpus*—that is, a three-dimensional body—by a *corpus*. Similarly, motion is measured by motion and not by time: “Time cannot be the measure of motion, but only motion can measure motion, precisely a faster one [measures] a less fast one, and a shorter one [measures] a longer one, whereas a number is measured by a number, and time by time inasmuch as it is long or short, and not inasmuch as it is fast or slow.”<sup>59</sup> Hence, time can only be measured extrinsically (*per accidens*) through motion, as is the case with common expressions like “two hours, or two days, or two years,” referring to astronomical displacements. These are only metaphors that refer to heavenly motions as “placed” in the interval of time that corresponds to their motion. In the following passage, presenting these reflections, time is called the “place of motion” (*locus motus*):

It could seem to somebody that, to indicate [*significare*] a certain quantity of motion, one has to assume as much time as if one says, for instance, that a certain operation has been carried out in the space of two hours, or two days, or two years. Still, it should be remarked that this is not literally [*simpliciter*] true, since the imagination conceives the interval of two hours, days, or years as the motion of celestial bodies without which neither years, not days, not hours would exist, even though time is placed, so to speak, in time, just as a body in a place. Thus, motion is measured by motion and time by time, and not the one by the other.<sup>60</sup>

Benedetti claims that time, unlike motion, is eternal: “Necessarily, from a philosophical point of view, time is eternal and motion not.”<sup>61</sup> Whereas a motion can be extinguished and a body can be at rest after a displacement, time goes on inexorably. It is always present to our senses and always escapes them because it is the never-ending passing of a single instant. Benedetti makes clear that he intends the *instans* to mean “one in species” (*unum in specie*), i.e., the essence underlying the vanishing flux of time that cannot be experienced in itself as a given and measurable succession (*non in numero*).<sup>62</sup> This Augustinian

<sup>58</sup>Augustine 1988, XI 28.

<sup>59</sup>Benedetti 1585, 183: “Tempus non erit mensura motus, sed motus quidem potest mensurare motum, videlicet velocior minus velocem, et brevior longiorem; et numerus mensuratur numero, et tempus tempore in quantum longum est, aut breve, non in quantum velox, aut tardum.”

<sup>60</sup>Benedetti 1585, 182: “Si alicui videtur, quod ad significandam aliquam quantitatem motus, dicere huiusmodi operationem duarum horarum, aut duorum dierum, aut duorum annorum spatio completam esse, sit ponere tantum tempus, animadvertere debet hoc simpliciter non esse verum, quia horarum, dierum, et annorum intervalla, imaginatione concipiuntur ut motus corporum caelestium, sine quibus neque anni, neque dies, neque horae existerent, etiam si omnis motus sit (ut ita dicam) locatus in tempore, ut corpore in loco, unde motus motu, et tempus tempore, non autem aliud ab alio mensuratur.”

<sup>61</sup>Benedetti 1585, 182: “Tempus ex necessitate, philosophice tamen loquendo, res est aeterna, motus non item.”

<sup>62</sup>Benedetti 1585, 182: “Tempus igitur potius locus motus erit dicendum, quam numerus aut mensura eius, et tale est, ut consumatum videatur a continuo quodam fluxu unius instantis [...] et cum dico ab uno instanti,

paradox helps Benedetti to stress the heterogeneity of time and motion. Still, he does not renounce an objective meaning, which is essential to his dynamics. Motion and time, he writes, are *continua successiva*, continuous and successive quantities. Their relation can be conveniently described as that between a place and the bodies that it contains. His explanation sheds light on the concept of *locus* as well as on that of *tempus*:

Just as a dense body occupies a lesser interval of place than a less dense [*rarus*] body, similarly a quick motion is accomplished [*peragetur*] in a shorter space of time than a slower motion.<sup>63</sup>

From this passage it is clear that Benedetti's idea of place as *intervallum corporeum* goes in the direction of an empty homogeneous space which can be occupied by bodies of greater or lesser density. Time has the same absolute character as place. It can contain more or less rapid motions without being affected by them. Space and time or, more precisely, *intervallum corporeum* and *tempus* are objective and independent coordinates of natural phenomena.

In order to understand Benedetti's considerations on time, it is useful to delve into the views of his immediate forerunners, in particular Cardano and Scaliger.

### 7.6.1 Cardano's Subtleties on Time

In the letter to the reader of the *Diversae speculationes*, Benedetti indicated Cardano as one of his principal sources.<sup>64</sup> However, concerning the concept of time their opinions are not in agreement. Cardano tackles the issue *tempus quid sit* (What is time?) in Book 18 of *De subtilitate*. Unlike Benedetti, he does not explicitly question Aristotle, but rather quotes his definition as a common truth: "in fact, motion is the measure of time" (*motus enim tempus est mensura*).<sup>65</sup> Still, he reworks the Aristotelian concept within a rather Augustinian perspective, which leads to original results.

He introduces the problem of time in connection with his treatment of the dream (*somnium*), beginning with the question: "Why does time appear much longer while dreaming than in reality?"<sup>66</sup> He reports that once he dreamed that he visited an unknown city far away from his home in Milan. He traveled through mountains, valleys, and fields. In order to cover that distance six days of travel would be barely sufficient. Thus, when he woke up, he thought that he had slept for a long time but in actual fact his nap had taken less than one hour. The reason for this expansion of time, Cardano explains, is due to the fact that dreamed activities (*operationes*) are accomplished independently of any bodily effort (*absque corporis labore*) and therefore very rapidly. A correct judgment about time depends on bodily movement. This is why the perception of time is not altered in our mind when we imagine long-lasting processes while awake. "During sleep, time is contracted in the opposite manner than if we are awake: in fact, motion is the measure of time."<sup>67</sup> Surprisingly, Cardano's Aristotelian conclusion, "motion is the measure of time," does

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unum in specie, et non in numero intelligo, quod a sensibus nostris percipi non potest, neque etiam notari, quia novum semper instans nobis occurrit."

<sup>63</sup>Benedetti 1585, 183: "Quemadmodum corpus densum occupat minus intervallum loci, contra quam fiat in corpore raro; sic etiam motus velox breviori temporis spatio peragetur, quam tardus."

<sup>64</sup>Benedetti 1585, *Ad lectorem*, f. A3r.

<sup>65</sup>Cardano 1966, vol. 3, 651.

<sup>66</sup>Cardano 1966, vol. 3, 651: "Cur somnium tempus longius multo ostendat quam sit."

<sup>67</sup>Cardano 1966, vol. 3, 651: "Contraria ratione tempus in somnio contrahitur, cum vel non somniamus [...]: motus enim tempus est mensura."

not refer to heavenly motions, but to corporeal activity. In other words, he extrapolates and isolates the peripatetic definition from its original context. In fact, from an objective, physical, and cosmological meaning, time acquires a rather subjective meaning, connected with physiology and perception. Time, as Augustine would say, is an “expansion of the mind.”

It should be added that there is a passage of the *Physics* where Aristotle also hinted at the subjective dimension of time, even at how the soul grasps it. This passage might have been a source of inspiration for Cardano:

Time cannot be disconnected from change; for when we experience no changes of consciousness, or, if we do, and are not aware of them, no time seems to have passed, any more than it did to the men in the fable who ‘slept with the heroes’ in Sardinia, when they awoke; for under such circumstances we fit the former ‘now’ to the later, making them one and the same and eliminating the interval between them, because we did not perceive it. So, just as there would be no time if there were no distinction between this ‘now’ and that ‘now,’ there appears to be no time between two ‘nows’ when we fail to distinguish between them. Since, then, we are not aware of time when we do not distinguish any change (the mind appearing to abide in a single indivisible and undifferentiated state), whereas if we perceive and distinguish changes, then we say that time has elapsed, it is clear that time cannot be disconnected from motion and change.<sup>68</sup>

These considerations are not aimed at isolating a subjective meaning of time by eliminating any objective references. Still, it is plausible that Cardano took into account such passages in order to freely speculate on time, in Book 18 of *De subtilitate*, from a perspective that owes more to Augustine than to Aristotle.

This Augustinian influence is particularly evident from the following passage, in which Cardano tries to better define time:

But what is time? Although nothing of it is ever [given], nonetheless everything is in it and it accompanies [*assistit*] everything always. It itself generates and destroys everything; it is the source [*auctor*] of life and death. Its expectation is always very long, while its memory is always very short. Although it is always with us, we never grasp it. Even though there is such an abundance of it, nonetheless no restoration [*reparatio*] of it is ever conceded, thence the waste of no other thing is greater or worse.<sup>69</sup>

In this passage, Cardano brings together ideas derived from erudite lectures, as well as from commonsense, experience, and even trivial commonplaces. Additionally, he recounts the Augustinian paradoxes: time is everywhere and nowhere, it is made out of expectation and memory, and it is for us the most familiar and unknown mental presence. To quote from the *Confessiones*: “What is time then? If nobody asks me, I know: but if I

<sup>68</sup>Aristotle 1995, IV 11, 218 b 20–219 a 1 (383).

<sup>69</sup>Cardano 1966, vol. 3, 651: “Sed quid tempus est? Cuius cum nihil unquam sit, omnia tamen in illo sint et semper omnibus assistit. Illud idem omnia generat et occidit, auctor vitae ac mortis. Utque illius expectatio longissima, ita semper memoria brevissima. Cumque nos semper comitetur, nunquam ipsum tamen agnoscimus. Nec cum eius tanta sit copia, reparatio tamen ulla conceditur: unde fit, ut nullius alterius rei iactura sit maior et vilior.”

were desirous to explain it to one that should ask me, plainly I know not.”<sup>70</sup> Cardano adds to this paradox a popular sense of the caducity of life, according to which a discourse on time is a kind of *memento mori*. Time itself is said to be the cause of life and death.

Hence, for Cardano, the assumption that “time is the measure of motion” does not mean maintaining the cosmological dependency of time on astronomical cycles. Quite to the contrary, time transcends motions and changes. In fact, we do not perceive it in itself but rather that which happens in it. What we know about time is a product of the mind, precisely of its imaginative faculty:

Thus, we do not comprehend time but rather that which occurs, or occurred, and endures in it. But time itself per se is out of reach [*ignotum*] to the senses. In fact all that we know [about it] is constituted through imagination.<sup>71</sup>

Like Benedetti, Cardano denied the interdependency of cosmological space and time, although from a different standpoint. He was not concerned with reformulating the physical space and time framework of motion in mathematical terms. Instead, he concentrated on the psychological and physiological dimension of time as a construction of the *imaginatio*. Therefore, this treatment was connected with that of the mind and was introduced by considerations on sleeping and dreaming. This even led to considerations on altered states of mind such as ecstasies and hallucinations, as well as the divine and demoniac visions of the hermits—Cardano went so far as to report some hallucinations that he had when he was a child. His mental treatment of time, as well as this excursus on altered states of mind, were both harshly criticized by the Flemish humanist Julius Caesar Scaliger, as we shall now discuss.

### 7.6.2 Scaliger’s Aristotelian Restoration

Scaliger conceived of his *Exoticae exercitationes* as a critical confrontation with Cardano’s *De subtilitate*, although he formally presented them as a reverent discussion of some points that arose from the lecture on the “subtleties” of that *doctissimus vir*, as one reads in the dedicatory epistle, “who will never be praised enough” (*nunquam satis laudatus*). Among the most notable theses indicated in the *Index acutiorum sententiarum* at the end of the book, one is directly relevant to our discussion: the *exercitatio* number 352.2, which explains why “time is not the measure of motion.” That *exercitatio* deals with the passage of *De subtilitate* on time that we have discussed, but reverses Cardano’s viewpoint.

Scaliger tries to answer the question of “whether time is the measure of time,” remarking that most people just repeat this definition without properly understanding its meaning.<sup>72</sup> The allusion is clear: Cardano repeats a commonplace without any thorough reflection on its meaning. In fact, even though time might be regarded as the measure of “our motion,” it is definitely not the measure of worldly motion, in particular not of the first motion of the heavens, i.e., the daily one.<sup>73</sup> Scaliger therefore denounces Cardano’s

<sup>70</sup> Augustine 1988, XI 14, 238: “Quid est ergo tempus? Si nemo ex me querat, scio; si querenti explicare velim, nescio.”

<sup>71</sup> Cardano 1966, vol. 3, 651: “Ergo nos non tempus, sed quod in eo fit, factumque est, atque manet, comprehendimus. Tempus vero ipsum per se est sensui ignotum: nam imaginatione constat quod a nobis cognoscitur.”

<sup>72</sup> Scaliger 1557, f. 458v: “Hoc quidem ab omnibus iactatur: pauci introspectant.”

<sup>73</sup> Scaliger 1557, f. 458v: “Nostri sane motus mensura sit: primi motus mensura non erit.”

subjective reading of the Aristotelian definition. According to him, the objective dimension of time cannot be dismissed. From a cosmological-ontological perspective it is in fact a dependent (*affectus*) of heavenly motion;<sup>74</sup> to be precise, it descends from the “first motion,” or the daily rotation of the starry sphere. The first body (*primum corpus*), that is, the heaven of the fixed stars, is the measure of all bodies. Similarly, its motion, the *primus motus*, is the yardstick of all motions. In accordance Book 12 of the *Metaphysica*, where Aristotle advocates a spherical and geocentric cosmology of concentric spheres, Scaliger states that “time is either the same thing as motion or its affection.”<sup>75</sup> In other words, he intends to restore an Aristotelian objective conception of time in accordance with a metaphysical perspective that attaches an ontological priority to motion over time.

Scaliger acknowledges that Aristotle ambiguously defined the interrelation between time and motion by accepting both possibilities: “time is the measure of motion and, in turn, motion is the measure of time.”<sup>76</sup> In his attempt to dispel this paradoxical interdependency, Scaliger distinguishes between two fundamental dimensions of time: the objective and the subjective dimension or, as one reads, the understanding “according to nature” (*a Natura*) and that “according to us” (*mensura nostra*). In nature, motion is the measure of time as well as its source. By contrast, time is the measure of motion only for us, which is an aspect that Cardano allegedly overemphasizes. This is clearly documented by expressions like “the equinoxial circle accomplishes its motion in this much time.”<sup>77</sup> In actual fact, things are the opposite of what our way of speaking suggests: time is derived from the translation from “here” (*ubi*) to “there” (*ubi*) in space (*in loco*). The *ubi* (where), to which people improperly refer in order to express a quantity of time, is a naive way of thinking that reduces time to certain spatial determinations. As to the definitions: time is a transient “now” (*nunc*), whereas place (*locus*) is a continuous “where” (*ubi*):

Since the quantity of time corresponds to that of a motion between a ‘here’ [*ubi*] and a ‘there’ [*ubi*], the ‘where’ [*ubi*] that we use for time is deduced [*transumptum est*], without inconvenience, from that ‘where’ [*ubi*] which, in fact, pertains to [a determination of] place [*locus*]. Actually, time is a transient *now*, whereas place is an enduring *where* [*ubi*].<sup>78</sup>

Scaliger’s criticism of Cardano is not limited to a vindication of the objective meaning of time, its interconnection with space (or more accurately, place), and the priority of cosmological motions over time, i.e., time as something derived from celestial motions. He additionally criticises Cardano’s hint that time is the cause of generation and corruption, dealing with the question of “Whether time generates and corrupts [things]” (*An tempus generet et corrumpat*). In Scaliger’s assessment—and according to the Aristotelian authority on which he relies—this is impossible. Time cannot generate or corrupt anything, since it is not a substance but a quantity, that is, a property of a substance. Therefore, it cannot produce any effects.<sup>79</sup> Scaliger opposes his “very subtle” (*subtilissime*) opinion to Cardano’s superficiality: “Our life is the act of the soul. In it, time has neither jurisdiction

<sup>74</sup> Scaliger 1557, f. 458v: “At tempus est posterius motu primo. Est nimirum affectus eius, ab illo pendens.”

<sup>75</sup> Scaliger 1557, f. 458v: “Tempus, inquit, aut est idem, quod motus, aut affectus eius.”

<sup>76</sup> Scaliger 1557, f. 458v: “Tempus esse mensuram motus, et vicissim motum mensuram temporis.”

<sup>77</sup> Scaliger 1557, f. 458v: “[...] dicimus tot partes aequinoctialis tanto moveri tempore.”

<sup>78</sup> Scaliger 1557, f. 458v (punctuation and emphases have been standardized and modernized): “Quia tempus tantum est, quantus est motus inter ‘ubi’ et ‘ubi’ in loco. Iccirco ‘ubi’ quod pro tempore usurpamus, transumptum est, haud iniuria, ab eo ‘ubi’ quod est vere loci. Erit ergo tempus nunc fluens; et locus erit ubi continuatum.”

<sup>79</sup> Scaliger 1557, f. 458v: “Cum enim tempus sit quantitas, nihil agit.”

nor power.”<sup>80</sup> Scaliger adds that Cardano’s words “are suited to the vulgar” (*vulgo prior*), not to philosophers. What’s more, his references to visions and alternative states of mind should be regarded as only insignificant phenomena which concern children and *melancholici* like Cardano himself.<sup>81</sup>

### 7.6.3 Benedetti and the Renaissance Concepts of Time

Our excursus on Renaissance concepts of time is far from exhaustive, but it helps us to grasp the scientific context out of which Benedetti’s own position emerged. It also permits us to point out some major problems in the conceptualization of time, in particular its subjective and objective dimensions. The interrelation between *tempus* and *locus* was at the center of the reflections, the debates, and even the polemics of scholars investigating nature from various angles. The debate on time and on its relation to motion has meanings that are, at the same time, physical (Benedetti), psychological-physiological (Cardano), and cosmological-metaphysical (Scaliger). Finally, for a more traditional cosmological perspective on time as the measure of celestial motions and of the motion of the first mobile as the standard measure for all other measurements of time, one can refer to Alessandro Piccolomini’s *Della filosofia naturale*, where one finds the following definition:

Hence, time, which is the measure of all movements, mainly has to measure a motion that is the most regular and enables the measurement of all other [motions] that do not have the same regularity in their components. As this motion is that of the first heaven, one has to conclude that time (which is like [a property] of a substance [*in proprio soggetto*],) is first of all measured by it through that motion of the first mobile. Through it all other motions are ruled. Hence, although time can be called the rule and measure of any motion, it will be reasonable not to pluralize it alongside the plurality of motions. Rather, it will remain one and the same for the whole world, just as the first motion, which is its proper and true subject [*soggetto*], is singular.<sup>82</sup>

In the generation before Benedetti, Cardano affirmed that time is independent of cosmological space on the basis of his assumption that time is a “stretching of the mind” connected with the perception of corporeal activity. Cardano formally accepted the traditional Aristotelian definition of time as the measure of motion, but only as the consciousness of physiological motion. Scaliger criticized this perspective and accused Cardano of misunderstanding Aristotle. In the *Exercitationes* he tried to restore a peripatetic metaphysical conception of time as a product of celestial motions. In a sense, his conception of time has a “conservative” character for his commitment to the Aristotelian tradition. However, the distinction of the subjective and objective dimensions of the issue permits us to highlight a problematic aspect of the Renaissance definition of time. According to Scaliger, time can

<sup>80</sup> Scaliger 1557, f. 458v: “[...] vita nostra est actus animae: in quem nihil habet tempus aut iuris, aut imperii.”

<sup>81</sup> Scaliger 1557, f. 459r, *Exercitatio* 353, “De tua visione.”

<sup>82</sup> Piccolomini 1585, I 3, chap. 7, ff. 52v–53r: “Di qui è, che’l tempo, c’ha da esser la misura di tutti i movimenti, bisogna che principalmente si applichi a misurare alcun movimento regolato sopra tutti gli altri, mediante il quale possa poi far da misura de gli altri tutti, che tal regola [...] non hanno nelle parti loro. E tal movimento [...] essendo [...] quello del primo Cielo: si deve concludere, che’l tempo sia come in proprio soggetto da lui primamente misurato, nel detto movimento del mobil primo: mediante il quale, regolandosi tutti gli altri movimenti: ragionevol cosa sarà, che’l tempo, se ben si può chiamar regola, et misura d’ogni movimento, come di sopra habbiam lungamente detto, tuttavia egli non si moltiplichi con la moltiplicatione delli movimenti, ma resi uno stesso per tutto il mondo, sì come uno solo è quel movimento primo, che è il proprio, et vero soggetto suo.”



be regarded as the measure of motion only from a subjective viewpoint, not from a natural one. According to nature, the relation between time and motion is the reverse: motion is the measure and time the measured thing.

For Benedetti, as a mathematician advancing a new Archimedean theory of motion and a post-Aristotelian physics, time is the *locus motus*, the place of motion, that is to say, an objective—we dare say, absolute—measure independent of its content and of spatial determinations. He shared with Cardano the independence of time from matter but not his subjectivism, while he maintained with Scaliger the objectivity of time without assuming the Aristotelian dependency of time on motion.

### 7.7 Natural and Violent Motions Revisited

After his revision of the physical concepts of the void, infinity, space, and time, Benedetti moves on to discard the Aristotelian theory of natural and violent motions in the section from chapter 23 to chapter 26. Chapter 23, “*Motuum rectum esse continuum, vel dissidente Aristotele*” (Rectilinear motion is continuous although Aristotle is in disagreement), shows that continuity is not exclusive to circular motion, therefore Aristotle’s distinction between circular and rectilinear motion must be corrected. In *Physics* VIII 8 one reads that “the motion of a body moving on a finite straight line cannot be continuous.”<sup>83</sup> By contrast, Benedetti demonstrates that the projection of a circular motion has the same continuity as the circular motion itself, although it is a forward and backward motion on a line. This is the case with planetary appearances produced by the displacement onto an epicycle as seen from the earth.<sup>84</sup> In a way, this section is a rehabilitation of the epicyclic model against Averroist-Aristotelian criticisms concerning the physical tenability of a non-concentric model of heavenly circles. This contrast between mathematical and physical astronomy received much attention from sixteenth-century Aristotelian scholars who received their education at Padua. In the 1530s Giovan Battista Amico and Girolamo Fracastoro promoted a fleeting rebirth of homocentric astronomy through the publication of *De Motibus corporum coelestium iuxta principia peripatetica, sine eccentricis et epicyclis* (Venice, 1537 and Paris, 1540) and *Homocentrica* (Venice, 1538), respectively.<sup>85</sup> More directly, Benedetti’s assertion that a continuous rectilinear motion is possible seems to be directed at the opposite statement in Piccolomini’s Aristotelian *Della filosofia naturale* (Book 2, Part 2, chapter 3, “*Come esser non possa infinito corpo alcuno di quelli che per natura loro si muovano per retta linea*” (There can be no infinite body among those that move straightforward following their own nature)). Just as in the *Diversae speculationes*, this section is complemented with diagrams.<sup>86</sup>

<sup>83</sup>Aristotle 1995, VIII 8, 261b 32–33 (369).

<sup>84</sup>Benedetti 1585, 183: “[...] ut accidit lineae directionis, & retrogradationis planetarum, unde circulus uan erit ut epicyclus et b ut terrae centrum.”

<sup>85</sup>Pierre Duhem pointed out the tension between physical and mathematical astronomy, albeit in a very inadequate way. See Duhem 1908. For a better understanding of the matter, see Di Bono 1990.

<sup>86</sup>Piccolomini argues as follows: “Non è a punto manco sconvenevole il concedere cotale infinità in qual si voglia corpo semplice, che o salendo, o scendendo si muova per retta linea; che si sia veduto disopra esser di quello che in circolo si rivolge. Percioche essendo il partirsi salendo dal mezo dell’universo, e’ l’venir discendendo a quello, che movimenti di luogo a luogo tra di lor contrarii: e ricercando li contrarii movimenti, luoghi contrarii parimenti; confessaremo esser opposti tra di loro il luogo disopra e questo qua giù da basso. E perché sempre tra due contrarii, se l’uno e per natura sua finito, e determinato, non sopporta il giustissimo governo della natura, che l’altro sia infinito e senza termine, secondo che discorrendo per tutte le contrarietà si può vedere.” See Piccolomini 1585, II 2, chap. 3, f. 24v.

Chapter 24 is a refutation of a series of Aristotelian assumptions on natural and violent motion. In the title of this chapter, Aristotle is called *vir gravissimus*, but this attribution sounds quite ironic, since the theses of this “very grave man” are here refuted. The first criticism concerns the idea that a projectile is transported by air once separated from its thrower. According to Benedetti, it is rather the contrary: air is a hindrance to motion because it resists the penetrations. Secondly, Aristotle writes in *De caelo* I 8 that a body accelerates the closer it is to its aim. Instead, one should say that a body moves (e.g., falls) quicker the further it is from its aim (e.g., the ground). In fact, the longer the distance it covers, the more it is pushed (*maior fit semper impraessio*) by its inner *impetus*, which is due to the spontaneous inclination toward its proper place (*inclinatio ad locum suum eundi*). Chapter 26 is a rejection of Aristotle’s statement that a body is not “heavy” in its proper place. In fact, air in air, or water in water, has no weight at all, except for when one artificially compresses an element (for instance inflating air in a bladder). The difference in density of the compressed element produces a difference in weight. Chapter 25 denies that vertical motion could legitimately be called natural. In fact, only perpetual circular motion is natural. An entire (i.e., spherical) body and its parts spontaneously move in circles. By contrast, rectilinear motion is that of a part separated from its whole. The cosmological significance of these remarks should be stressed. It is implicit but can be demonstrated by comparison with Copernicus’s *De revolutionibus* I 8, where the same distinction between the motion of the whole and that of its parts accounts for the difference between the natural circular motion of a planet, basically the earth, and the vertical fall of heavy bodies.<sup>87</sup>

Copernicus presented his considerations on natural and violent motions in *De revolutionibus* I 8, which is the chapter aimed at rejecting Ptolemaic and Aristotelian arguments against terrestrial motion. There Copernicus attacked the Aristotelian theory of natural and violent motion and sought to abandon the doctrine of natural places. Benedetti’s undertaking is very close, even though the cosmological theme has not emerged yet.

## 7.8 The Cosmological Perspective of the *Physical Disputations*

The cosmological dimension of Benedetti’s anti-Aristotelian discussion is documented in the last part of the *Physical Disputations*.

### 7.8.1 Physico-Mathematical Astronomical Issues

The astronomical-cosmological section begins (chap. 28) with a reflection on the sphere that goes against the opinion of Aristotle, ironically called *Princeps Peripateticorum*. Whereas the ancient philosopher regarded the circle as the “first plane figure” (*prima figurarum superficialium*) and the sphere, the form of the heaven, as the “first body” (*prima corporearum figurarum, that is, the first three-dimensional figure*), Benedetti claims that they are the “last” figures. In fact, they can be regarded as polygons with infinite sides:

<sup>87</sup>One can compare the text of Benedetti, *Diversae speculationes*, disp. XXV, with that of Copernicus, *De revolutionibus*, I 8. The latter wrote: “Igitur quod aiunt, simplicis corporis esse motum simplicem (de circulari in primis verificatur) quamdiu corpus simplex in loco suo naturali, ac unitate sua permanserit. In loco, siquidem non alius, quam circularis est motus, quo manet in se totus quiescenti similis. Rectus autem supervenit iis, quae a loco suo naturali peregrinantur, vel extruduntur, vel quomodolibet extra ipsum sunt. Nihil autem ordinationi totius et formae mundi, tantum repugnat, quantum extra locum suum esse. Rectus ergo motus non accidit, nisi rebus non recte se habentibus, neque perfectis secundum naturam, dum separantur a suo toto, et eius deserunt unitatem.”

“the triangle is the first plane figure and the circle the last one.”<sup>88</sup> He adds that the *principium*, the beginning and the origin of everything, is its center and those figures which equally encircle it in all directions can be said to be perfect. The author concludes as follows: “That which is perfect, although it is [qualitatively] first as to its essence [*natura*], is last as to its generation.”<sup>89</sup> The circle, according to him, is perfect because it is, in a sense, an “infinite figure.” If one considers it as a polygon of infinite sides, one can say that the sum of its angles is equal to an infinite number of right angles. One can interpret this statement as follows: every polygon inscribed in a circumference can be divided into several triangles whose vertices touch the center of the circumference and whose bases coincide with the sides of the polygon. In the case of the circle, taken as the “last” polygon, the triangles decomposing it are infinite in number. Since the angles at the vertices are zero and the sum of all internal angles must be  $180^\circ$ , it follows that the angles at the base must be two square angles. Thus, Benedetti feels vindicated: “The circle and the sphere are not constituted of one single angle, as Aristotle believes [...]. Rather, these are figures of infinite right angles. For that reason I call them last and perfect, because one cannot add anything to infinity.”<sup>90</sup> To sum up this reasoning, Benedetti shares Aristotle’s opinion that the sphere is the perfect figure, but adduces geometrical-metaphysical reasons. For both authors the sphere is the form of the world (Benedetti would say, “of the *caelum*” surrounded by an infinite empty space) for aesthetic and metaphysical reasons.

In the following chapters, Benedetti reviews a series of astronomical and meteorological issues on which he accused Aristotle of being wrong. Chapter 28 deals with the sparkling of the stars, which is, according to Benedetti, the effect of the motion of heavenly transparent media (*ab inaequalitate motus corporum diaphanorum mediorum nascitur*).<sup>91</sup> Among other things, Benedetti denies (chap. 30) that the warmth of the sun can be produced by its motion rather than by its light and subsequently (chap. 31) explains the seasonal variations. Chapter 33 reassesses, against Aristotle, the plausibility of the Pythagorean doctrine of celestial harmony. This has nothing to do with the production of sounds, nor with any harmonic proportions between the “aspects” of the heavenly bodies. Rather, it is the secret order imparted by to the world by divine providence.<sup>92</sup> Chapter 33 comprises a lengthy discussion on meteorology, in which atmospheric phenomena are essentially explained through the variations of air density.

## 7.8.2 The Copernican Conclusion of the *Physical Disputations*

Chapter 35, *Motum rectum curvo posse comparari* (Straight and curvilinear motions are comparable), is a crucial chapter for our analysis, since it is here that Benedetti, almost

<sup>88</sup>Benedetti 1585, 186.

<sup>89</sup>Benedetti 1585, 185: “Quod autem perfectum est, licet natura sit primum, est tamen ultimum generatione.”

<sup>90</sup>Benedetti 1585, 185–186: “Circulus sphaeraque non ex uno solo angulo recto constat, ut idem Aristoteles putat [...] sed sunt figurae infinitorum angulorum rectorum, et hanc ob causam a me dicuntur ultimae et perfectae, quia infinito nihil addi potest.” The authors would like to thank here Irina Tupikova for suggesting this interpretation of Benedetti’s argument.

<sup>91</sup>Benedetti 1585, 189.

<sup>92</sup>The Pythagorean harmony was ridiculed by Alessandro Piccolomini among others. See Piccolomini 1585, II 4, chap. 12, “Del suono, et armonia, che i pitagorici pensavano, che nascesse per li movimenti de’ corpi celesti,” f. 105v: “[...] Quando i corpi celesti movendosi facesser suono avvenir dovrebbe a noi poscia che da si alto, et soverchio strepito, et sproportionato all’odito nostro quasi assordati, né quello né altro suono che qua giù si faccia, odire in modo alcuno dovremmo giamai. Per la qual cosa essendo manifesto che tante diversità di strepiti, che tra questi corpi inferiori si fanno, ancora che piccolissimi sieno, son da noi odite distintamente, è forza dire, per la ragione ultimamente fatta che né suono, né armonia, non può causarsi per li movimenti de’ gli Orbi, o delle stelle che volgendosi faccin la su in cielo.”

at the end of his *Physical Disputations*, introduces the Copernican theory. Although we have already dealt with Benedetti's astronomical views in the previous chapter, it is useful to recount here the most important features of his cosmology in the context of the philosophical section of the *Diversae speculationes* (IV) and add some more details.

The subject matter is the comparability of rectilinear and circular motion: “[Aristotle] is wrong when he says that straight motion cannot be compared to the curvilinear (*Physics* VII 4), where he mistakenly also says that one cannot find any lines equal to the circumference of a circle.”<sup>93</sup> It is directed against Aristotle's denial that a straight and a circular motion could be compared, thus hinting at the qualitative difference between celestial circular motions and the vertical tendency of the elements in the sublunary sphere. From a Copernican perspective, Aristotle's words could be considered to be an implicit rejection of terrestrial motion. In fact, if the earth rotates, one should assume that the trajectory of a falling body is rectilinear for an observer on the earth but has a circular component as well, if considered in relation to the outside world.

Benedetti first appeals to Archimedes's *De quadratura circuli* (On the quadrature of the circle) to argue that the circle and the straight line are comparable: “If, then, this quadrature can exist, there can also exist, for the reason already given, a straight line equal to the circumference of that circle.”<sup>94</sup> Thus, a geometrical problem, the squaring of the circle, attains a direct cosmological meaning. If the issue at stake is the distinction between celestial and elementary motions, they are of course different, but this difference does not lie in the circularity of the former and the straightness of the latter, but rather in the uniformity of speed opposed to acceleration.

These considerations offer Benedetti the occasion to expand on the velocity of celestial motions. According to the commonly held opinion (*secundum opinionem communem*), the heavens would have to cover an immense distance within the 24 hours of the daily rotation. Close to the equator, the sun would cover 1,000 Italian miles per minute and Saturn 260,000 miles per minute, not to speak of the rapidity of the fixed stars. The assumption of this inconvenient velocity would of course be avoided if one assumed “the most beautiful theory” (*pulcherrima opinio*) of Aristarchus, “divinely” restored by Nicolaus Copernicus:

And as for the speed of the fixed stars situated near the equator, one may make one's estimate, and, in fact, this will seem very difficult to some. But this difficulty does not occur in the most beautiful system of Aristarchus of Samos that has been so divinely expounded by Nicolaus Copernicus.<sup>95</sup>

From a Copernican perspective, the sun would cover “only” 48 miles per minute and Saturn 24, whereas the heavens would be stationary.

In the subsequent chapter (chap. 36), Benedetti reworks the doctrine of the *doctissimus Aristarchus*. It is entitled “*Minus esse explosam ab Aristotele opinionem credentium plures mundos existere*” (The view of those who hold that many worlds exist was not adequately refuted by Aristotle) and deals with the plurality of worlds. According to Aristotle,

<sup>93</sup>Benedetti 1585, 194: “[Aristoteles] recte dicere non potest motum rectum ad curvum comparabilem non esse 4. cap. lib. 7 *Physicorum* ubi errat quoque dicens reperiri non posse lineam aliquam rectam alicuius circuli circumferentiae aequalem.”

<sup>94</sup>Drake and Drabkin 1969, 220. Cf. Benedetti 1585, 194: “Si igitur dicta quadratura dari potest, potest etiam dari una recta linea aequalis circumferentiae eiusdem circuli.”

<sup>95</sup>Drake and Drabkin 1969, 221. Cf. Benedetti 1585, 195: “Et amplius de stellis autem fixis circa aequatorem positis quivis cogitet; quod revera difficillimum quibusdam videbitur, quod quidem non occurrit secundum pulcherrimam Aristarchi Samii opinionem, divinitus a Nicolao Copernico expressam, contra quam nil plane valent rationes ab Aristotele, neque etiam a Ptolomeo propositae.”

a universe with a plurality of worlds similar to the earth would be unstable and eventually collapse, since the earthly parts of the other worlds would fall toward the cosmological center and the fiery parts would eventually become part of the fiery sphere of our sublunary world. This Aristotelian objection is based on an *a priori* assumption of the theory of the natural places. It is therefore easy for Benedetti to contradict him by arguing that all worlds (that is, planets) would have their elements and their places.<sup>96</sup>

Apart from that, as we have already seen, Benedetti proposes a bizarre transformation of the Copernican system based on an analogy between the moon and the other planets. Like our satellite, all these light-mirroring and wandering bodies are supposed to turn around dark earths which, in turn, spin about their axis:

If the system of the learned Aristarchus is correct, it will be perfectly logical for that which takes place in the case of the Moon to take place also in the case of any of the five other planets. Thus, just as the Moon with the help of its epicycles revolves around the Earth as if on the circumference of a certain other epicycle of which the Earth is like a natural center (i.e., in the middle), carried around the Sun by the sphere of annual motion, so too may Saturn, Jupiter, Mars, Venus and Mercury revolve about some body situated in the center of their major epicycle. And this body, also having some motion about its axis, may be opaque, possessing conditions like those of the Earth, with conditions on the epicycle in question similar to those on the lunar epicycles described.<sup>97</sup>

This conception could provide an explanation for the existence of planetary epicycles, whose physical tenability has been already stressed in the *disputatio* 23. We could also regard these views of Benedetti as a cosmological reading of Copernicus focused on possible cosmological and physical consequences of the planetary theory. The plurality of worlds and the analogy between the moon and the planets are not the only innovative elements in comparison with the theses of *De revolutionibus*. After a section on the motion of light through the cosmic void (chap. 37) and one on the geometry of the elements (chap. 38), the conclusive section of the *Physical Disputations* (chap. 39) attacks a Peripatetic dogma: the unalterability of the heavens. In *De caelo* I 22 Aristotle remarked that no change in the heavens was ever observed. This is, according to Benedetti, not a valid argument. One should rather assume a principle of relativity of the point of observation. In fact, the earth would be invisible from the eighth heaven (that of the fixed stars), even though, by supposition, it was endowed with a light equal to that of the sun. The distance hinders us from perceiving changes that occur on other worlds.<sup>98</sup>

With this rejection of the distinction between a sublunary and a heavenly realm, Benedetti's criticism of Aristotelian physics is complete. It should be noticed that this final objection hits at the core of the Peripatetic natural philosophy, since it is a denial of the fundamental distinction between a terrestrial and a celestial physics, on which the entire physics and cosmology of the Aristotelians relies.

<sup>96</sup>Benedetti 1585, 195.

<sup>97</sup>Drake and Drabkin 1969, 222. Cf. Benedetti 1585, 195–196: “Si doctissimi Aristarchi opinio est vera, rationi quoque consentaneum erit maxime, ut quod Lunae contingit, cuilibet etiam ex aliis quinque planetis eveniat, idest, ut quemadmodum Luna suorum epicyclorum ope circum terram volvitur, quasi per circumferentiam alterius cuiusdam epicycli, in quo terra sit instar centri naturalis (idest sit in medio) delati ab orbe annuo circa Solem; sic etiam Saturnus, Iupiter, Mars, Venus, atque Mercurius, circum aliquod corpus in medio sui epicycli maioris, situm habens, volvantur; quod quidem corpus, et aliquem quoque habeat motum circa suum axem, sit opacum, iis conditionibus, quae terrae sunt similes, praeditum existat, et in dicto epicyclo sint res similes istis lunaribus.”

<sup>98</sup>Benedetti 1585, 197.

### 7.9 An Evaluation: Benedetti's Path to Natural Philosophy

The *Disputationes de quibusdam placitis Aristotelis* is a complex book within the larger book. It concerns at least three main fields of investigation: motion, the foundations of physics, and astronomy, in particular cosmology. It begins with a rejection of the theory of the natural places (violent and natural motion) based on an Archimedean relativization of heaviness and lightness as well as on a mathematical approach derived from the Euclidean theory of proportions. It deals subsequently with basic concepts of physics. It defines space and time anew as an absolute framework for the investigation of natural phenomena, in particular motion. This part of the *Physical Disputations* also aims at demonstrating actual infinity and the void, which are Democritean theses rejected by Aristotle in *Physics* and *De caelo*. The astronomical part then follows, which confronts many special issues and illustrates what we shall call a "post-Copernican cosmology." Benedetti advocates the heliocentric system (albeit modified relative to the model of Copernicus's *De revolutionibus*), the plurality of worlds, the inter-changeability of the observational viewpoint in the universe, and, last but not least, the homogeneity and continuity of the sublunary and the heavenly realm, contrary to one of the most fundamental assumptions of Aristotelian philosophy.

Let us summarize the Copernican considerations that could have influenced Benedetti and consider the extent to which he went beyond them. First of all, Copernicus abandons the theory of natural and violent motions because, "if anyone believes that the Earth rotates, surely he will hold that its motion is natural not violent."<sup>99</sup> Additionally, the daily rotation of the heavens is more absurd than that of the earth because it would be excessive compared to that required of the relatively small earth. A third Copernican remark concerns the infinity of space. It is directed against the Aristotelian assumption that there is nothing, "no space, no body, no void," outside the heavens (*dicunt quod extra caelum non esse corpus, non locum, non vacuum*). Copernicus remarks that the axial rotation of the earth undermines the strongest argument in favor of cosmological finiteness: "For the chief contention by which it is sought to prove that the universe is finite is its motion."<sup>100</sup> As to the objections against the earth's motion based on considerations of the effects to be expected for flying and thrown objects, Copernicus assumes, against Aristotle's claim for the simplicity of motion, that things on Earth participate in the planetary motion and, therefore, the vertical displacement of light and heavy bodies (*cadentium vero et ascendentium*) is a composite motion (*duplicem*) relative to the whole (*mundi comparatione*), with a rectilinear and a circular component. Copernicus holds that only circular motion is natural and it does not only pertain to celestial bodies but also to the elements in their natural place. As we have seen, he defines rectilinear motion as the tendency of bodies to reach their whole if they have been separated from it. This vertical appetency is not uniform but accelerated. Copernicus also criticizes Aristotle's opinion that bodies are heavy (or light) in their proper place, since weight depends exclusively upon the tendency of the part towards their whole.

Many of these Copernican ideas and suggestions appear also in Benedetti's *Physical Disputations*: the rejection of the theory of natural places and of violent and natural motions, the excessive rapidity of the rotation of the heavens, the void, infinity, the naturalness of circular motion against the unnaturalness of the vertical motion of the parts separated from their whole, and the criticism of Aristotle's assertion about the weight of

<sup>99</sup>Copernicus 1978, 15

<sup>100</sup>Copernicus 1978, 15–16.

the bodies in their natural place. However, it should be remarked that Copernicus does not expand on these ideas for the most part and cursorily presents them only for the sake of his apology for terrestrial motion. Benedetti's treatment is much more explicit and, what is more, his motivations and presuppositions appear to be quite different. His Archimedean and Euclidean treatment of motion is the basis of his rejection of the distinction between natural and violent motions. No consideration of this kind is present in Copernicus's work. Moreover, the reference to spatial infinity in *De revolutionibus* is limited to a remark. Copernicus himself does not explicitly support this thesis and leaves the discussion to the natural philosophers or, as he calls them, the *physiologi*. Actual infinity receives a substantially different treatment in Benedetti since it is closely related to the attempt to define space anew as *intervallum corporeum*. It is precisely this broad, natural philosophical dimension which is absent in Copernicus's work and which, in our opinion, Benedetti did not derive from his reading of *De revolutionibus* or from general astronomical concerns. It seems, by contrast, that he was primarily interested in the physical issue of a mathematical treatment of motion and that the criticism of the Aristotelian philosophy led him in a quite natural way to also confront cosmology. Nor could issues like the void and atomism be reasonably derived from Copernicus. Even the planetary theory of Benedetti departs from *De revolutionibus* as it includes theses like the plurality of worlds and the corruptibility of the heavens. However, it is clear that the heliocentric and geokinetic theories fit perfectly into Benedetti's worldview. In light of his general theory, as he writes, Aristotelian and Ptolemaic arguments against Copernicus's theory appear extremely weak: "contra quam [doctrina] nil plane valent rationes ab Aristotele, neque etiam a Ptolomeo propositae."<sup>101</sup> Koyré wrote that Bruno's *La cena de le Ceneri* (London, 1584) was the best defense of the Copernican system from a natural and physical point of view before Galileo's *Dialogo sopra i due massimi sistemi del mondo* (Florence, 1632). However, this statement seems to underestimate the force of Benedetti's *Physical Disputations*, which are perhaps less speculative than Bruno's dialogues but should nonetheless be regarded as an extremely strong apology for the physical tenability of the Copernican system. A reciprocal influence between Bruno and Benedetti cannot be excluded, since the wandering philosopher from Nola stayed for a period in Turin and the Savoy around 1578 and probably participated in a debate concerning the comet of 1577.<sup>102</sup> At any rate, the *Diversae speculationes* encountered much more acknowledgment among astronomers of the time than Bruno's work. As we have seen, Brahe extensively quoted Benedetti both in his *Epistolae astronomicae* of 1596 and in his book on the nova of 1572. Kepler's admiration for Benedetti was no less and was only equaled by his admiration for Commandino and Clavius.<sup>103</sup> The proximity of many themes of the *Diversae speculationes* and those of the young Galileo are a well-known issue in the history of mechanics; in light of our discussion, it is plausible to assume that Benedetti's influence on Galileo also concerned the insight into the close relation between the heliocentric theory and a new mechanics.<sup>104</sup>

Our analysis has shown that the heliocentric system is not the main issue at stake in the *Physical Disputations*, although that theory becomes part of a general program of reform for natural philosophy. Far from being a mere "Copernican enterprise," Benedetti's visionary project is much more complex. It is an ambitious attempt to build a new physics, in the wide Renaissance meaning of the term, out of a criticism of Aristotelian physics. Concerning Aristotle, it is clear that the *princeps peripateticorum* provides him with a

<sup>101</sup>Benedetti 1585, 195.

<sup>102</sup>Omodeo 2008a.

<sup>103</sup>Kepler 1937–2001, 390.

<sup>104</sup>Damerow and Renn 2010.

model, albeit a negative one, in which the theory of motion, cosmology, astronomy, meteorology, natural philosophy, and metaphysics are closely interrelated. Benedetti's undertaking is precisely a revision and a restructuring of these interdisciplinary ties on the basis of new insights and a mathematical approach. Although his investigation intentionally and explicitly departs from Peripatetic physics, it is historically possible only in the form of a thorough confrontation with Aristotelian themes. Indeed, the *Physical Disputations* have the form of a dispute on Aristotelian places. Benedetti's familiarity with Aristotle's *Physics*, *De caelo*, and *Meteorologica* should also be underscored. This apparently contradictory aspect of early modern physics in its ambiguous relation to Aristotelianism has already been stressed by Anneliese Maier in her studies on the medieval contributions to classical science.<sup>105</sup> In a sense, the development of a new physics required a thorough confrontation with Aristotle and his concepts, as also the examples of Bruno and Galileo bear witness to in different ways.

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<sup>105</sup>Maier 1951, 304–305.



## Concluding Remarks

Giovanni Battista Benedetti, the Renaissance scientist, has received ambivalent historical judgements by scholars in the past. The historian of medieval science and philosophy Anneliese Maier, for one, viewed him with mixed feelings. To her, Benedetti appeared to be a sort of intellectual companion of Galileo Galileo, at the same time his “forerunner” in mathematical physics and an epigone who was disrespectful to his own medieval *Vorläufer* or predecessors. Maier wrote that Parisian scholastics such as Nicole Oresme and Jean Buridan had provided Benedetti and Galileo with the concepts they needed to inquire into physics—she particularly had the concept of *impetus* in mind—which they did not acknowledge in their fierce attacks on Aristotelian philosophy.<sup>1</sup> Maier shared Koyré’s view that modern mechanics was constructed around a few central concepts and authors relevant for Newton’s *Principia mathematica*. They were perplexed by the concomitant reception and rejection of medieval physics by Renaissance scientists. In our view, however, it is too narrow a point of view to just consider individuals and sets of ideas and their genealogies. Instead, one should consider the wider intellectual currents and the shared knowledge they generated. The incipient *querelle des anciens et des modernes*<sup>2</sup> is an example of a debate transcending specific questions and problems, even approaches and methodologies, towards a larger reflection on the relation between past and present. The problems inherent in this gap between the individual perception of change and the intellectual transitions of the time are exhibited by the astronomy of Nicolaus Copernicus, a sort of “unaware revolutionary,”<sup>3</sup> who saw himself (or at least presented himself) as a Renaissance restorer of planetary theories defended in antiquity by the legendary Pythagoreans. By contrast, his scholastic counterpart, the Padua-trained physician and natural philosopher, Girolamo Fracastoro, presented his homocentric reform of mathematical astronomy as a radical innovation, comparable with Amerigo Vespucci’s discovery of the New World.<sup>4</sup> Fracastoro’s work was based on the modeling of all celestial motions through concentric spheres (in line with a well-established Aristotelian tradition). In Benedetti’s case, the rejection of the *philosophia naturalis* taught in the universities was achieved with intellectual means descending from that very philosophical tradition. Rather than viewing this fact as a paradox, it should be regarded as a sign of a profound tension in Renaissance science between past and present and a hallmark of what we have called preclassical mechanics.<sup>5</sup> The *intention* to outdo traditional authorities in order to move beyond their legacy had to rely on the shared knowledge of the time, which was marked by Aristotelian thought. In our introduction we delved into Benedetti’s conceptions and reconstructed their socio-cultural coordinates, characterized by the Renaissance tension between conceptual heritage and novelty. Maier’s perplexity thus rests upon a lack of reflection on the embedment of in-

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<sup>1</sup>Anneliese Maier established a connection between Benedetti’s treatment of motion and that of Galileo in Maier 1951, 304–305.

<sup>2</sup>Lehner and Wendt 2017.

<sup>3</sup>Copernicus’s revolutionary role *malgré soi* already puzzled Thomas S. Kuhn, who called him at once “radical” and “conservative” and regarded *De revolutionibus orbium coelestium*, the book propounding the first modern heliocentric theory in mathematical astronomy, “revolution-making” rather than “revolutionary.” Cf. Kuhn 1959, 135 and 148.

<sup>4</sup>Goddu 2010 and Granada and Tessicini 2005. Also see Omodeo 2017.

<sup>5</sup>See Renn, Feldhay, et al. 2018.

lectuals and their theories in socio-cultural processes. Benedetti in particular ought not to be seen as a link in a chain, but rather as one representative of a complex and comprehensive knowledge economy.<sup>6</sup>

In order to correctly locate Benedetti in the knowledge economy of the Renaissance, it is expedient to consider him against the background of the material and intellectual conditions of early-modern science, and as a figure between the intentions and identities of a new genre of intellectuals who formed the archetype for modern scientists. Benedetti's case helps us to reflect upon the social position and intellectual identity of these new types of scholars as well as on the way socio-cultural coordinates penetrated science, as far as its demarcation, content, form, and justification are concerned. With social coordinates we refer to the institutional setting involving Benedetti's role as a courtier and thus to his function as a court mathematician, which, in turn, was linked to the wider socio-economic interests of a Renaissance territorial state.<sup>7</sup>

In his seminal work on the sociological roots of modern science, Zilsel discussed the scientific relevance of the social transformations taking place in the late Middle Ages and the Renaissance. In particular, he argued that the emergence of modern science depended on the rise of capitalism. We could aptly refer to this phase as a pre-capitalistic or early-capitalistic "knowledge society." Technical knowledge proved to be a key element in the organization of life and production while the status of the artisans, those whom Zilsel called the "artist-engineers," increased and received high recognition among civil and political authorities. The town of Florence is prototypical for these changes, as Leonardo Olschki has forcefully demonstrated in his studies on science and vulgar literature.<sup>8</sup> A wide range of artist-engineers transformed Florentine society and its mentalities. Filippo Brunelleschi, most representatively and symbolically, forever changed the skyline of the same town in which, at the end of the Italian Renaissance, Galileo composed works that irreversibly modified the landscapes of science and scientific culture.<sup>9</sup> In Florence and Europe more generally, in the passage from the Middle Ages to early modernity the "artisan-practitioners" were confirmed as a new class. The codification of their experience and knowledge profoundly changed epistemology and science, most evidently in mechanics. This practical art was first codified as a physico-mathematical discipline, and then as a science in its own right, and was later adopted as a methodological and ontological point of reference in the shift toward the mechanistic world views of the seventeenth century.<sup>10</sup>

According to a corollary of the Zilsel thesis about the social origins of modern science, scientific culture was reshaped by the merging of three intellectual strands: the artisanal/technical, the scholastic/logical, and the humanistic/rhetorical. This fusion was accomplished by mediators, who were social actors with an in-between status bridging different intellectual and social realms. "Hybrid experts" became increasingly necessary because of their capacity to bring together the technical and the theoretical dimensions of knowledge. Their socio-cultural relevance would never diminish from the late Middle Ages to the Industrial Revolution and beyond.<sup>11</sup>

<sup>6</sup>For a recent study accomplished in this vein, see Trzeciok 2016.

<sup>7</sup>For further considerations on Benedetti in light of a discussion on methodological and historiographical approaches, see Renn, Feldhay, et al. 2018.

<sup>8</sup>Olschki [1919–1927] 1965.

<sup>9</sup>On the Florentine prototype, see Renn 2014. Cf. Zilsel 2000, 941.

<sup>10</sup>On artisanal knowledge and its codification, see P. Smith 2004 and Long 2001. On practical knowledge, see Valleriani 2017. On the elevation of mechanics to a worldview, see Renn and Damerow 2010.

<sup>11</sup>Ursula Klein has made this point most forcefully in Klein 2015.

During the Renaissance, this mediation was secured by a new group of “scientist-engineers,” a series of court mathematicians of which Galileo is the best-known figure and which also included his protector, Guidobaldo Del Monte. Actually, the description of the Renaissance figure of the “scientist-engineer” suits the intellectual and social profile of Benedetti very well.<sup>12</sup> Galileo and his like were well versed both in the technical as well as in the intellectual dimensions of knowledge production. Renaissance “scientist-engineers” underwent a period in apprenticeship of practical mathematics, in some field of application like architecture or the art of war, but later distanced themselves from artist-practitioners as they aspired to gain higher social recognition, especially as courtiers. They had a high degree of education, as they mastered theoretical mathematics, the language of the learned, Latin, as well as the courtly language, for instance by acquainting themselves with the elegant Italian of the literature of the time. Scientist-engineers thus acted as mediators connecting the centers of power and decision on the one hand and the workshops and building sites on the other. As was the case with Benedetti, these experts could supervise artisanal work or give advice on technical issues.<sup>13</sup> As courtiers they were additionally required to participate in the refined dialogical and literary culture of the elite, to serve as educators as well as to use their astronomical expertise to cast horoscopes for the rulers.

The most specific socio-political aspect of Benedetti’s time is the affirmation of court society as a particular social formation whose features show continuities and fractures both with the earlier aristocratic setting of the feudal society and the later capitalist one. A distinguishing feature is the centralization of power and administration around the court. As Norbert Elias argued, this formation culminated in the absolutism of the Ancien Régime but was preceded, on a smaller scale, by early attempts at territorial centralization.<sup>14</sup> Although such social formations apparently gravitated around an individual sovereign who made all decisions (as much of the literature of the time on the *Principe* and its privileges boasted), it was in fact a hierarchical system in which the group of experts surrounding the princely ruler constituted an oligarchy who operated the complex organization of modern states. The Duchy of Savoy is one such case. The dukes strove to create a “modern” capital city partly by following the model of Florence, insofar as culture and prestige are concerned, but also the Spanish and French models, insofar as the suzerainty of the ruling family is concerned. Other models played a role, too, for instance the Netherlands for military technology and Switzerland for military conscription and discipline.

Benedetti shared the enthusiasm of his patrons (especially Emanuele Filiberto) for mathematics and its perceived powerfulness as an instrument for successful navigation in war and peace. He also shared the aristocratic values of the court such as disinterest and prestige. Adherence to these values largely explains his bias toward theory despite the practical origins of his knowledge and the fields of application of his mathematics (ranging from mechanics to navigation, architecture, and perspective). He also ventured into the most general fields, such as cosmology and philosophy (as seen through his criticism of Aristotelian natural conceptions, his favorable opinion on Copernican astronomy and post-Copernican cosmology, and his remarks on “Pythagorean” philosophy of mathematics).

<sup>12</sup>The figure of the “scientist-engineer” has been introduced into the history of science by Renn 2001, particularly in the contributions by Lefèvre (Lefèvre 2001) and Renn, Damerow, and Rieger (Renn, Damerow, and Rieger 2001). Valleriani discusses it in detail in Valleriani 2010, chap. 6.

<sup>13</sup>Valleriani 2010, 208: “Except for the period of the apprenticeship, an engineer-scientist was almost never personally employed in workshops or building sites, but he was aware of the work procedures followed in these locations and was able therefore either to commission craftsmen or other persons involved with practical activities, to supervise or teach them, or simply be consulted to evaluate their works.”

<sup>14</sup>As already discussed in the introduction. The reference work is Elias [1969] 2002.

Actually, he did not hesitate to call his wide and unsystematic work “speculations,” an expression that stresses the theoretical character of the endeavor.

From the perspective of a court scientist such as Benedetti, mathematics was the key to practice and theory. It was his specific field of expertise among the Turin courtly elites; through it he acquired a central epistemological status in line with the exaltation of the *certitudo mathematicarum* by many of his contemporaries, among them his correspondent Pietro Catena. At the same time, the practical context surrounding the mathematical approach in many fields such as mechanics led him to emphasize the contingent element of natural phenomena. Thus, the centrality of mathematics in Benedetti’s work has a multilayered meaning, including the theoretical, practical, epistemological, and social. The limits of validity and applicability of Benedetti’s mathematical science mirror the boundaries of his field of competence in the division of intellectual labor within his courtly environment. Although he used geometry as a sort of universal key, he could not impose his views on other courtiers who were experts in fields such as philosophy and medicine. In this context of enforced openness, Benedetti’s criticisms of Aristotelianism appear as a sort of defense of his professional position in the framework of a courtly dialogical pluralism. Such an environment explains the occasional (and fragmentary) character of the *Diversae speculationes*, which brings together occasional materials such as texts for private teaching, letters, short treatises, expert advice, and polemical essays (among others), in which Benedetti made his mathematical expertise manifest and showed its usefulness.

The intellectual distribution of labor in the Renaissance ensured that Benedetti was at the heart of the courtly milieu by virtue of his family’s social status and not through his ambition alone. His work exhibits many similarities with the work of other Italian court mathematicians, most eminently that of the aforementioned Del Monte and Galileo, as far as the range of their interests and the overall approach are concerned. Benedetti’s most daring passages, which open up unconventional solutions to technical and theoretical problems, and his remarkable disregard for authority qualify him as one of the Italian *novatores*, although he did not make explicit his natural conception as an all-encompassing alternative to the well-established Aristotelian philosophy. His fierce attacks on crucial aspects of the Aristotelian conception—relating to motion, the void, infinite space, time, infinity, and planetary theory—did not result in a systematic new natural philosophy. Rather, he limited himself to collecting results in different areas and to working on the most varied aspects without finding their common denominator. He also made elliptical references to Pythagoreanism and implicitly rehabilitated some aspects of atomist and stoic conceptions, for instance the plurality of worlds and the fluidity of the heavens. Cardano, whom he appreciated, went much further in the inquiry of the common foundations of the sciences (specifically mathematics, practical arts, and medicine) while Benedetti’s correspondent Patrizi advanced a systematic natural philosophy inspired by neo-Platonism. In the same years in which Benedetti finished and published his physico-mathematical speculations in Turin, Bruno published philosophical dialogues in London expounding a natural philosophy and an anthropology that led to far more radical consequences for the premises of cosmology, similar to those reached by Benedetti. Another contemporary of Benedetti, Telesio, had offered the first modern attempt to build up a conception of nature on new principles. His *Natura iuxta propria principia* paved the way for the next generations of scholars searching for new foundations in natural science. Among them was his direct follower Campanella, who brought his philosophy to France in the seminal years of the mechanical philosophies of Pierre Gassendi and René Descartes. Benedetti participated in this wide cultural transformation; he contributed to advancing the mathematical and phys-

ical disciplines and discarding consolidated theories—but without offering a systematic alternative.

To summarize the most evident features of Benedetti's endeavor: it was courtly, secular, anti-Academic, unsystematic, occasional, elitist, learned, abstract, pleasant, and useful. It was *secular*, that is, non-theological, as it was linked to the interests of the ruling class and the state. It was a *useful and pleasant* science: on the one hand, it was practice-oriented but not purely empirical; on the other hand, it proved witty and fit for courtly sociability. It was *abstract and disinterested*: superior to the vulgar and tuned to aristocratic values. *Learned*: fit to be exhibited at court alongside the other arts. *Elitist*: Benedetti elevated mathematics from a practical discipline of scientist-engineers to a refined cultural activity. *Occasional*: linked to the variegated political and cultural interests of the court. *Unsystematic*: fragmented, lacking the inner coherence of scholasticism. *Anti-Academic*: free from concerns about respect for university scholarly traditions. All of these characteristics of Benedetti's science were the hallmark of court science: it was technical and abstract without losing contact with practice and experience—a mathematical-empirical science *in nuce*; it was (relatively) free from bookish tradition and theology but not from the contingencies of courtly life.

What is the common denominator of the great variety of subjects dealt with by Benedetti? What is the center around which they all gravitate? Is there one unifying principle behind the apparent disorder and heterogeneity? It should be emphasized that Benedetti first established his fame *as a mathematician*. His early treatment of motion by mathematical means was explicitly directed “against Aristotle and all philosophers” (*contra Aristotilem et omnes philosophos*). In his time “mathematics” had a wide scope. It comprised arithmetic and geometry, astronomy and astrology, as well as music, but also reached far beyond the boundaries of the *quadrivium* by encompassing optics, practical mechanics, architecture, and engineering. The expansion of mathematics into the fields of physics, natural philosophy, meteorology, and even metaphysics and epistemology was a crossing of the disciplinary boundaries. Benedetti's time bears witness to several attempts to expand the boundaries of mathematics. Cardano, for one, claimed that geometry had the function of a universal logic fundamental to rational thought, and that the practical disciplines including statics, mechanics, and architecture were its subordinate fields of inquiry.<sup>15</sup>

Benedetti's intellectual identity, however, proves much more complex than his corporate identity as a mathematician.<sup>16</sup> His pronounced titles vary. In a short biographical note accompanying the birth horoscope published by Gaurico, he was referred to as “Philosophus, Musicus, atque Mathematicus”; on October 19, 1589, he signed an astrological report cast for Carlo Emanuele I as “Matematico e Astrologiaro”;<sup>17</sup> contemporary admirers of his such as the Milan painter and poet Lomazzo and the Danish astronomer Brahe called him “matematico” and “philosophus et mathematicus in primis excellentem,” respectively.<sup>18</sup> Probably, Brahe's designation of Benedetti as both philosopher and mathematician best captures the poles of his intellectual activity. Intriguingly enough, Benedetti generally dropped the title of “mathematician,” keeping only that of “philosopher” in his

<sup>15</sup>Girolamo Cardano, *Encomium geometriae recitatum anno 1535 in Academia Platina Mediolana* in Cardano 1966, vol. 4, 440–445.

<sup>16</sup>By “corporate” we refer here to the *esprit de corps* of a group that considers itself a bounded entity whose interests are marked as separate from other groups. The guild culture of the Middle Ages originated this particular meaning of corporation, which precedes the modern sense of a professional group or legal body.

<sup>17</sup>Roero 1997, 57–58.

<sup>18</sup>Lomazzo 2006, 177: “Del Sig. Gio. Battista Benedetti Matematico” Brahe 1916, 251–253.

publications. On the title page of his *magnum opus* of 1585, the *Diversae speculationes*, he appears as “patritius Venetus philosophus,” exactly the same epithet that appears in *De gnomonum umbrarumque solarium usu liber* (1574). In the publications in the vernacular, he correspondingly appears as “filosofo del sereniss. duca di Savoia,” e.g., in the *Consideratione... d’intorno al discorso della grandezza terra et dell’acqua* (1579). In the last publication, his self-presentation as court philosopher is interestingly opposed to the designation of his intellectual opponent, Antonio Berga, as “*filosofo nella Università di Torino*,” that is, “university philosopher”—which is equivalent to *scholastic* philosopher. These references are telling for Benedetti’s self-perception or, to use an in-vogue expression, his *self-fashioning*.<sup>19</sup> In both cases, the image of court philosopher was his intended identity, whether reflected or purposely constructed (or a mixture of both). As was the case with Galileo, the Florentine courtier, the philosopher’s social status and reputation was higher than that of the mathematician. This is why, among the conditions for Galileo’s appointment as a courtier to the Medicis, he regarded the designation “philosopher” as relevant.<sup>20</sup>

As for the epistemological debates mirroring the disciplinary and social divides and hierarchies of the time, heated controversies began over the “certainty of mathematics.” The determination of the degree of certainty of mathematics also concerned the legitimacy of using mathematics in physics. In the case of Benedetti, the tension between his function as court mathematician and his identity as philosopher—and *patrizio*—lies beneath his science. While philosophical legitimacy was essential for the acknowledgment of the intellectual dignity of his endeavor, the practical dimension of mathematics remained fundamental for the social justification of his function as a court expert.

One could single out the social and the political-cultural coordinates of Benedetti’s science as two complementary drives. On the one hand, his position as a court mathematician directly determined much of the content of his writings, occasioned by the requests addressed to him as a court *expert* in technical issues pertaining to mathematics. His position also determined formal aspects of his work, in particular its occasional character and fragmentation. On the other hand, Benedetti’s identity as a philosopher was directly related to his cultural ambitions and his engagement aimed to affirm *mathematical philosophy* in the intellectual arena against scholastic thinkers and humanistic literati. His political identity as a lay aristocrat made him an organic part of the centralizing project of the court and marked his distance from Counter-Reformist drives which sought to impose Roman universal interests over territorial states’ autonomy. His support for a sort of party of the *politiques* resulted in treatises advising on politically relevant technical and cultural issues (e.g., navigation on the occasion of the battle of Lepanto or the calendar reform). His activity as a lay educator, e.g., his arithmetic teaching to the prince, Carlo Emanuele I, is found in his pedagogical writings, some of which were published in his scientific miscellanea. In summary, both content and form, as well as the demarcation of the fields of his scientific competence as a mathematician and philosopher, depended on social settings and cultural engagement.

The fact that Benedetti never established a scientific school around himself can be seen as an indication of the precarity of patronized science, linked to the person of a particular ruler and not institutionalized at the level of an academic body. In the course of the seventeenth century, these limitations of early court society would be solved by securing scientific continuity for patronized science through the foundation of scientific societies.

<sup>19</sup>Greenblatt 1980.

<sup>20</sup>Biagioli 1993. Also see Biagioli 1989.

These societies constituted an improvement over the volatility of Renaissance patronage, which depended on the humors and interests of a prince, by replacing him with a corporative *persona ficta* deputed to protect, credit, and promote science. This did not imply a diminution of the political relevance of science. As has been argued, the institution of the Académie Royale des Sciences as a means to patronize all of the sciences also meant the conquest of a new kingdom, *la république des lettres tout entière*, for Louis XIV.<sup>21</sup>

Montesquieu was a perspicacious observer of the courtly society in which Benedetti lived and worked. In his opinion, the “courtly air,” or the ethos of the ruling elites of a monarchic state, “consists in putting away one’s own greatness for a borrowed greatness. This greatness is more flattering to a courtier than is his own.”<sup>22</sup> Such *grandeur empruntée*, or borrowed greatness, was a function of a person’s distance from the ruler. Benedetti’s greatness could have solely consisted in his mathematical acumen, in his mechanical insights and demonstrations, or in his philosophical discernment; these are the virtues that the historian of science is inclined to observe as principal. However, Benedetti saw himself as a court gentleman, and only valued his capacity as a mathematician as subordinate. He presented himself as a *court* intellectual, more precisely, as a “philosopher to the Dukes of Savoy.” He “borrowed his greatness” (in Montesquieu’s words) from his proximity to the rulers. In the courtly milieu, it was honor and rank, together with their corollary, ambition, rather than skill, diligence, and measure that marked the character of a nobleman who belonged to the hegemonic class of the new state. Greatness is a major motivation for Benedetti’s science, which cannot be confined to technical demonstrations or the solution of specific problems. Rather, his treatment of details never departed from concerns about the big picture; in his work, special issues were constantly elevated and received their meaning on the level of a grand overview, natural and epistemological.

Greatness is not the only courtly quality to enter Benedetti’s science. As Montesquieu further observed: “At court one finds a delicacy of taste in all things, which comes from continual use of the excesses [*superfluités*] of a great fortune, from the variety, and especially the weariness, of pleasures, from the multiplicity, even the confusion, of fancies, which, when they are pleasing, are always accepted.”<sup>23</sup> To be sure, one cannot say that Benedetti’s knowledge was superfluous in the sense that it had no concrete application. In the Renaissance, it was evident to anybody how closely mathematics was connected to practical realms ranging from war technology to fortification, navigation, and administration. Benedetti’s work and activities related to these realms; even his astrological consultancies can be appreciated for their practical orientation—as astrology notably coincided with the so-called *astronomia practica*, as opposed to mathematical astronomy, or *astronomia theorica*. Still, Benedetti insisted on his lineage as a “philosopher” (connected with his claims about the Pythagorean universality of his method and the fragmentation of its applications) despite the attention given to practice and concreteness in Renaissance mathematics. Such a contention was aimed at confirming his superiority over the immediate application of knowledge or the material origin of arts such as mechanics.

His stress on theory—on “speculation”—is well attuned to the spirit of court society, which was centered on nobility, that is, on disinterest and rank, rather than efficacy. The “superfluity of Benedetti’s science” corresponds to the leisure character of knowledge in general, due to fact that its bond with materiality and practice was sublimated. Whereas corporative and merchant societies like those of the Italian Quattrocento (or, more gen-

<sup>21</sup> Biagioli 1995, 1418 and 1438.

<sup>22</sup> Montesquieu 1989, 33.

<sup>23</sup> Montesquieu 1989, 33.

erally, bourgeois and democratic ones like those emerging in the seventeenth century) would emphasize the practical origin and meaning of science, a court society stresses its symbolic value rather than direct usefulness and economical importance.

Besides the *superfluité* (which applies to Benedetti only if it is not taken too literally), all of the other qualifications Montesquieu attached to the court atmosphere suit his endeavor: good taste (we can add, “wit”), variety, pleasure, multiplicity, even confusion. The main virtue of a court society rested on the sense of honor and ambition: “Honor, meddling in everything, enters into all the modes of thought and all the ways of feeling and even directs the principles.”<sup>24</sup> Norbert Elias, who agreed with this assessment, also pointed out the fatal consequences for budgetary issues of a mentality that is so distanced from a modern bourgeois economy. From an economic viewpoint, court society was intrinsically flawed. It was destined to bankruptcy because form, ritual, and etiquette counted more than parsimony. Similarly, courtly science displayed detachment from monetary return. Elias has also emphasized the centrality of etiquette for this detachment. At court, formal etiquette was decisive, as it served to maintain and reinforce distances and hierarchies.<sup>25</sup>

The sense of honor and superiority typical of such social formations appears in Benedetti’s intended distance (social, intellectual, moral, and epistemological) from artisanal practice and the erudition of university professors. He appropriated the results and methods of both fields, in particular those of the practical arts, but at a higher level of generalization. He particularly envisaged a reformed natural philosophy as the most cherished fruit of his “mathematical-physical speculation.” Such theoretical distance from immediacy is the epistemological parallel of the sense of honor and social distance and, as such, it became an essential ingredient of Benedetti’s science and added symbolic value. As a court intellectual, he did not identify himself with traditional forms of higher culture such as Scholastic Aristotelianism or humanistic rhetoric. He proudly affirmed himself as a courtier, free to think and philosophize in the protected space of the court, independent of the most immediate material needs, of academic constraints dictated by tradition, and concerns about systematicity and completeness. Ambition, the companion of aristocratic honor, “meddled in everything” and directed Benedetti’s search for the most general principles of a new vision of nature, both mathematical and physical. The court protected and promoted a science and philosophy in which disinterestedness was foremost. In its favorable womb, a daring mind could venture out to explore new realms beyond established disciplinary boundaries. The speculative freedom of the court also determined the specific form of Benedetti’s work, its occasional character, and the amazing variety exhibited by his *diversae speculationes mathematicae et physicae*.

The economy of honor in the court society left an enduring epistemological imprint on the social fabric of science. Symbolic capital governed modern science long after it became coupled with economic capital and, in many ways, it still significantly influences science and research. The legacy of courtly ingenuity and leisure has to be acknowledged as a lasting influence upon scientific practice as well. Moreover, the topos of a protected space, so attractive to the emergent category of philosopher-scientists in the sixteenth and seventeenth centuries, contributed to creating the myth of the independence of pure science. Constant claims and controversies about scientists’ autonomy have accompanied the modern path to science in its migration from the court to the scientific academy and from the scientific academy to the laboratory. The connections of modern science to the

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<sup>24</sup>Montesquieu 1989, 33.

<sup>25</sup>Elias [1969] 2002, 173.



economy and society at large, politics, and cultural structures can be appreciated by considering the complex historical ties that link knowledge with its material and cultural conditions reaching far beyond the perception of the individual historical actors. The spirit of Benedetti's science can be seen as typical of an age of profound social transformation and political reconstitution, which is reflected in the exceptional re-structuring of knowledge and the transition to novel forms of scientific acquisition, legitimation, and transmission.



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***Diversarum speculationum mathematicarum, et physicarum liber***  
**(Turin 1585)**





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BENEDICTI

Patritij Veneti Philosophi.

*DIVERSARVM SPECVLATIONVM  
Mathematicarum, & Physicarum*

Liber.

Quarum seriem sequens pagina indicabit.

---

AD SERENISSIMVM CAROLVM EMANVELEM  
ALLOBROGVM, ET SVBALPINORVM  
DVCEM INVICTISSIMVM.



---

TAVRINI, Apud Hæredem Nicolai Beuilaquæ, MDLXXXV.

*Superioribus permissum.*

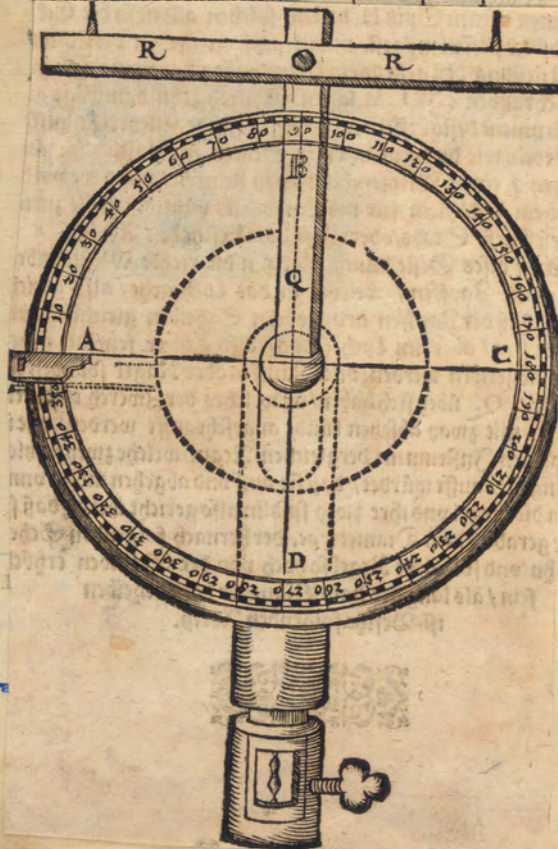
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Philosophi  
DIVERSARUM SPECULATIONUM  
Libri.

TRACTATUS QVI IN HOC  
volumine continentur.

- Theoremata Arithmetica.
- De rationibus operationum perspectiua.
- De Mechanicis.
- Disputationes de quibusdam placitis Arist.
- In quintum Euclidis librum.
- Physica & Mathematica responsa per Epistolas.



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S E R E N I S S I M O  
C A R O L O E M A N V E L I  
S a b a u d i æ D u c i , & c .



G I T V R nonusdecimus annus ex quo litte-  
ris Serenissimi patris tuae Celsitudinis, ac-  
cerfitus ex urbe Parmensi in hanc me ciui-  
tatem contuli. Is aduenientem tam humanè  
exceptit, tanta deinde liberalitate fuit com-  
plexus ego vicissim ei deseruiendi, tam vehe-  
menti cupiditate fui accensus, ut sub eius ditione quod super-  
esset vita agere constituerem. Cuius in me benignitas, mea  
in illum obseruantia mirum in modum mutuo usu, & consue-  
tudine est adaueta, ut idem Dux me secum dum rusticaretur  
esse vellet, saepe etiam secum pernoctare; quo quidem tempo-  
re de Mathematicis scientijs mecum agebat, in quibus perdi-  
scendis mea opera utebatur, quaestiones, Arithmeticae, Geo-  
metriam, Opticam, Musicam, aut Astrologiam spectantes  
proponens. Cui ut quod in me esset satisfacerem, acrius  
quam antea in ea studia (ad quae tamen semper fui propensissi-  
mus) incubui. Illiusque imitatione (ut ferè ceteri Principum  
studia imitantur) non pauci aut praesentes, aut per litteras me  
de his, atque illis Mathematicis quaestionibus consuluerunt.  
Cumque ego nunquam laborem amicorum causa defugerim,  
euenit ut post tot annorum curricula, mea scrinia scrutatus,  
inuenerim tot absolutas quaestiones, ut ex eis corpus mediocre  
effici posse videretur. Quas, cum rationibus in epistola sub-  
sequenti allatis edere constituissem, non sub cuiusque alte-  
rius nomine, & auspicijs quam tuae Celsitudinis volui apparere;  
tum quod patri debitum libellum filio reddere par erat, tum  
A 2 quòd

quòd in tua Celsitudine paternam in me fouendo, & augendo  
 benignitatem inesse semper sum expertus, tum quòd tua Celsi-  
 tudinis interrogationibus excitatus non pauca qua hoc volu-  
 mine continentur, elucubraui. Accessit, quod ego semper in  
 his dedicationibus spectandum putavi, tuam Celsitudinem tan-  
 tos progressus in Mathematicis fecisse, ut vel idonea aestima-  
 trix mearum vigiliarum esse possit. Quare, & veterum Per-  
 sarum Regum gloriam aquauit, & nos veluti in spem certam  
 felicitatis huius saeculi induxit, si verum est Platonis va-  
 tadinium, beatam eam futuram Rempublicam in qua  
 Principes Philosophentur. Tua igitur celsi-  
 tudo libellum tot ei nominibus debitum,  
 ea qua solet humanitate accipe-  
 re ne grauetur. Deus tuas  
 omnes cogitationes,  
 & conatus ad  
 felicissi-  
 mos  
 semper exitus perducas,  
 teq; diutissime ser-  
 uet incolu-  
 mem.

## A D L E C T O R E M .



V M Varijs temporibus permulta in diuersis disciplinis contemplatus sum , partim à præstantibus viris patronis ac amicis meis excitatus , qui super eis sententiam meam exquirebant , partim , ab ingenito mihi desiderio , rationem , & causam eorum percipiendi , committendum non putavi , quin qualiacunque mea scripta in illis scientijs , studiosis impartirer , non dubitans quin illis aliquid commodi atque vtilitatis allatura sint , præfertim cum in eiusmodi quæstionibus inuestigandis atque perpendendis , nemo ( quod sciam ) hæctenus elaborauerit . Nihil enim his libris à me traditum est , quod aut legisse , aut ab alijs audiuisse meminerim , nam si aliena attingi , ea , aut cum aliqua differentia demonstrationis , aut dilucidius scripti , quod si forte alius eadem tradidit , aut eius lucubrationes ad me non peruenerunt , aut earum perlectionis memoria excidit . Vt enim etiam Aristoteles ipse sensu facile fieri potest , vt pluribus , eadem opiniones in mentem veniant . Immo multa scribenti euenire potest , vt cum iam diu aliquid scripserit , iam oblitus , idem repetat , quod mihi etiam nonnunquam accidit . In his autem libris non suscepi munus integræ alicuius scientiæ tradendæ , ne , quæ ab alijs iam tradita sunt , ipse inutiliter repeterem , mihiq; viderer ex alienis laboribus laudem voluisse comparare . Singularum enim scientiarum volumina , iam ab alijs collecta , atque in ordinem sunt digesta , & si paucissimi sunt libri quorum omnes sententiæ , omniaq; inuenta vnius sunt authoris , excipio Archimedis volumina . Cumque multi sint , qui vel vnã rem à se inuentam in publicum proferre non dubitent , multo magis mihi qui multa excogitavi , & si inter se hætereogena , atque vtcunque expressa , idem licere sum arbitratus . In his autem meditandis , ex Arithmetiis authoribus quos inspexi , præcipuus fuit Nicolaus Tartalea , quippe quem ferè omnia ab alijs scripta collegisse constat , nec alios ex præcipuis , quos legere potui omittendos duxi , inter quos sunt Hieronymus Cardanus , Michael Stifelius , Gemma Frisius , Ioannes Nouiomagus , Cuthebertus Tonstallus , cæteriq; huiusmodi . Quorundam tamen volumina illorum qui à Tartalea citantur , vt Leonardi Pisani , Prosdocimi , Ioannis Infortunati , Fratris Lucæ , Petri Borgi , aliorumq; aliquot inspiciendorum ,

facultas

facultas mihi non fuit . Præterea , licet in his libris nonnullæ inueniantur  
 propositiones , quæ disiunctam ab alijs habeant rationem , eæ non sper-  
 nendæ tamen sunt , viam fortasse alicui aperient vltèrius progrediendi .  
 Quemadmodum enim , exempli gratia , ex sub contraria coniectione ,  
 sumpta postea fuit diuina illa Planisferij delineatio , quæ sub Ptolomæi no-  
 mine legitur , & sicuti ex penultima primi Euclidis , quam Pythagoras  
 excogitauit propè innumeræ pulchræ consequentiæ in Astronomia , in  
 Architectura , in multisq; alijs scientijs desumptæ sunt , immo quemad-  
 modum ex singulis propositionibus à nostris maioribus excogitatis mul-  
 ta egregia sunt deducta , ita fortasse continget , vt ex mearum inuentio-  
 num aliqua , nõnihil in posterum vtilitatis desumatur . Si quid verò , hic in-  
 ueneris , quod tuo genio non arrideat , illa prudentissimi hominis sen-  
 tentia in mentem veniat . *Quot capita , tot sententia* , ac per raro con-  
 tingere , vt idem omnibus probari , atque placere queat , & perdifficiliter  
 inueniri hominem cui placeant omnia quæ alteri satisfaciunt . Nec te mo-  
 ueat , quod hæc Theoremata siue excogitationes non videas ordine illo di-  
 spositas , quò collocari debere existimaueris , tum in Arithmetiis , tum in  
 cæteris . Cum enim in huiusmodi rebus ordo non sit necessarius , vi-  
 sum est mihi posse me , sine repræhensione , illum negligere , cum spe-  
 culationi , siue inuentioni præcipuè adeo mihi incumbendum decreuerim  
 vt in collocatione operam ponere , & tempus absumere operæpretium  
 non duxerim , quod idem in epistolarum collocatione feci , in quibus per-  
 sonarum ad quas scribo nullus ferè graduum ordo seruatus est , nec tem-  
 poris , quò sunt scriptæ , quæ sitorum tantummodo ratione habita . Nec  
 admirari quenquam velim , quod in speculandis numerorum passio-  
 nibus , figuris vtar geometricis , ita enim in .2. libr. fecit Euclides , qui mo-  
 dus , eo magis mihi arridet , quò minus est abstractus , *quoniam oportet in-*  
*telligentem phantasmata speculari* , cum præterea perspicuum sit , discretum  
 omne , ex continui diuisione aliquo modo oriri , siue actu , siue potentia .  
 Deinde si forte meis in demonstrationibus tibi videbor aliquando bre-  
 uior , illud in causa fuisse scias , quod ibi ad viros scribebam in his discipli-  
 nis exercitatos , quibus satis fuit rem significare . Libuit autem mihi om-  
 nes voluminis Arithmetici propositiones potius vocabulo theoremata-  
 rum appellare , quam problematum , quia pars earum speculatiua tan-  
 tum mea est , & si ex varijs eiusmodi propositionibus etiam operatiuam  
 adinuerim . Quoniam verò multis in locis accidit , vt veritatis iudi-  
 candæ causa necesse mihi fuerit quorundam sententijs aduersari nolim te  
 hoc

hoc mihi vitio tribuere, meq; hoc nomine carptorem maledicumq; ha-  
 bere quod alienos errores aperiam, cum potius habenda sit mihi gratia,  
 quod in ijs interdum laborans (que Antisthenes in disciplinis magis ne-  
 cessaria esse dixit, *ut mala scilicet prius dediscantur*) falsas opiniones eucl-  
 lere studeam, veritatemq; ostendere, quam omnis philosophus, Aristo-  
 telis exemplo, pluris quam cuiusvis hominis auctoritatem, aut gratiam  
 facere debet. Cumque in hoc volumine aliquid eiusmodi legeris  
 te oratum volo, ut in iudicando, affectum omnem exuas,  
 Sallustianum illud præ oculis habens. *Omnes qui de re-  
 bus dubijs consultant, ab odio amicitia, ira, atque  
 misericordia vacuos esse decet.* Hinc fiet, ut  
 non personæ (ut multi solent) sed  
 veritati, que summo studio di-  
 gnissima est, semper po-  
 tius faueas. Vale  
 nostrisque  
 labo-  
 ribus utere, si quem inde fructum,  
 sicuti spero tuleris, illi præ-  
 cipuè habeas gratiam à  
 quo omnes fluunt  
 scientiæ.





IO. BAPTISTAE  
 BENEDICTI  
 PATRITII VENETI  
 SERENISS. CAR. EM.  
 ALLOBROGVM DVCIS  
 PHILOSOPHI.

*Theorematum Arithmetica.*



RAECLARE' multa veteres mathematici philosophi de numeris eorumque effectibus excogitata posteris tradiderunt, quorum cum vix vllam rationem reddiderint, aut certe per exiguam, occasione diuersorum problematum mihi a Serenissimo Sabaudia Duce propositorum praebita, de ijs quae ab antiquis proposita fuerunt contemplanda nonnulla occurrerunt, quae posteritati comendare non inutile arbitratus sum, ne haec mea cogitationes intercederent, & occasionem praerberem quamplurimis, abstrusa haec indagandi, quae problematibus & theorematibus inuoluta, vix aliquem qui euolueret nacta sunt.

Inter caetera vero a me quaesita, hoc fuit theorema

**T H E O R E M A P R I M V M.**

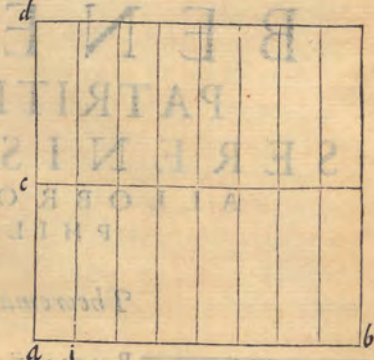
**I**NTERROGAVIT me Serenissimus Dux Sabaudia, qua ratione cognosci posset scientificae & speculatiuae (vt dicitur) productum ex duobus fractis numeris, quolibet producentium minus esse. Cui respondi, mente & cogitatione concipiendum esse fractos producentes cum fractis productis, non vnus eiusdemque naturae esse, imò longè diuersa.

Exempli gratia, fractis numeris propositis . a. i. et . a. c. quorum integri sint . a. b. et . a. d. qui tanquam lineae cogitentur, apertum sanè esset productum . c. i. superficiale futurum, quod nomen caperet à producto superficiale. d. b. generato ex vno in aliud totorum linearium, nam si constitueretur . a. i. octauum ipsius . a. b. et . a. c. dimidium . a. d. multiplicato . a. i. cum . a. c. produceretur sextumdecimum ipsius . d. b. Quare . d. b. esset totum relatiuū ipsius . c. i. non aliquod totum producentium. Mirum itaque non est si productum . c. i. minus videatur suis producentibus, cum toto, diuersae naturae à primis conferatur, fractum siquidem ab integro eiusdem naturae, linearis, superficiale, aut corporeae denominatur.

Quòd si amplioris cognitionis gratia ex scientiae praecipis speculari voluerit aliquis

A quis

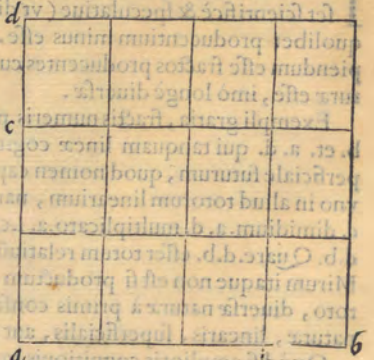
quis, qua ratione fractus numerus . c. i. minor sit in suo integro . d. b. fracto. a. i. in suo integro . a. b. aut fracto . a. c. in suo integro . a. d. consideret is quo pacto proportio. c. i. ad. d. b. minor sit proportione. a. i. ad. a. b. et. a. c. ad. a. d. hac ratione, Manifestum est ex prima sexti de quantitate continua, aut. 18. septimi Euclidis de discreta, proportionem ipsius. d. i. ad. d. b. esse sicut. a. i. ad. a. b. & cum. c. i. minor sit. d. i. velut pars suo toto, proportio, c. i. ad. d. b. minor erit proportione. d. i. ad. d. b. ex. 8. quinti, quare minor erit pariter proportione. a. i. ad. a. b. ex. 12. eiusdem vnà etiam proportio. c. i. ad. d. b. minor erit. a. c. ad. a. d. ex eisdem causis, medio. c. b. Ex quibus patet ratio, cur fracti diuersarum denominationum ad vnicam reducantur. Cur etiam numeros integros in partes fractis similes frangere liceat, quæ omnia ex subsequenti figura facile cognosci possunt.



T H E O R E M A I I.

Q VAE sit ratio, cur hi, qui numeros, fractos diuersarum denominationum colligere volunt, & in summam redigere, multiplicent vnum ex numerantibus per denominatorem alterius, & postmodum denominatores adinuicem, quorum vltimum productum, commune est denominans duorum priorum productorum, quæ collecta in summam efficiunt quod querebatur.

Qua in re sciendum est, denominantes considerari tanquam partes vnus eiusdemque magnitudinis quantitatis continuæ, linearum (verbigratia) a. b. et. a. d. æqualiū in longitudine, quarū. a. b. in quatuor partes diuidatur, et. a. d. in tres. Quare si colligere voluerimus duo tertia cum tribus quartis, multiplicabimus. a. c. duo tertia, cum. a. b. diuisa in 4. partes, producetur quæ. c. b. octo partium superficialium, dehinc multiplicando. a. i. tres quartas cum. a. d. diuisa in. 3. partes producet. i. d. primis singulis æqualis, nouem partium superficialium, multiplicata deinde. a. b. diuisa in. 4. partes per. a. d. in. 3. diuisa, producet. quadratum. d. b. in continuo, in 12. partes diuisum, quod erit totum commune singulis productis, quorum primum erat. c. b. Quare. c. b. ita se habet ad totum. d. b. sicut. a. c. ad. a. d. ex prima sexti in continuis, aut. 18. septimi in discretis quantitibus, et. d. i. ad. d. b. sicut. a. i. ad. a. b. ex eisdem propositionibus. Collectis deinde partibus producti. c. b. cum partibus producti. d. i. manifestè deprehendetur eiusmodi summam componi ex partibus vnus totius communis singulis earum.

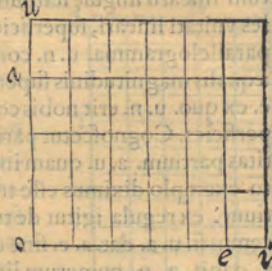


T H E O -

THEOREMA III.

CVR reperitur qualis sit fractus aliquis numerus respectu alterius, multiplicare debeant numeratores adinuicem & ita etiam denominatores, ex quo productum ex numeratoribus nomen capiat à producto denominatorum.

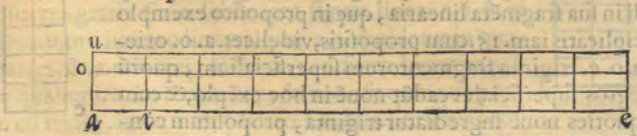
Huius si causam nosce vis, sume. o. i. & o. u. pro totis denominatoribus, tum. o. e. & o. a. pro numeratoribus (exempli causa) sit. o. i. fenarius. o. u. quaternarius. o. e. quaternarius. o. a. ternarius. Si nosce vis quæ sint tres quartæ partes quinque sextarum, patet ex regulis practicis oriri quindecim vigesimaquartas. Id quomodo fiat, ex subscripta figura deprehendetur, memores tamen esse oportet, quodlibet productum considerari tanquam superficiem, producentia autem tanquam lineas. In hac igitur figura productum ex totis linearibus est. u. i. aggregatum ex. 24. partibus, & u. e. productum aggregatum ex. 20. Quod ita se habebit ad productum totale. u. i. sicut. o. e. ad o. i. ex prima texti aut. 18. septimi, ita. u. e. erunt quinque sextæ partes. u. i. quarum in proposito exemplo, tres quarta quaruntur. Si itaq; multiplicabitur. o. e. cū. o. a. oriatur productum. a. e. ita proportionatū ad. u. e. sicut. o. a. ad o. u. reperitur, ex prædictis rationibus. Quod si statutum est. o. a. tres quartas partes esse ipsius. u. o. etiā. a. e. tres quartæ partes erūt. u. e. sed. u. e. quinque sextæ sunt ipsius. u. i. ex quo sequitur bonum esse huiusmodi opus.



THEOREMA IIII.

CVR multiplicaturi fractos cum integris, rectè multiplicent numerantem fracti per numerum integrorum, partianturque productum per denominantem fracti, ex quo numerus quaesitus colligitur.

Propter quod merito concipiamus in subsequenti figura, numerum integrorum tanquam lineam. a. e. qui, verbigratia, sit denarius, quorum vnusquisque sit æqualis a. i. cogiteturque productum ipsius. a. e. in. a. i. sitque. u. e. quod quidem erit denarius superficialis, constituta prius. a. u. æqualis. a. i. & a. o. sint duæ tertiæ. a. u. quarum duarum tertiarum productum in numerum. a. e. fit. o. e. pariter. u. i. vnitas sit superficialis prout. a. i. vnitas est linearis, quam. u. i. respicere debet productum. o. e. ex quo integer superficialis. u. i. erit tanquam ternarius, & productum. o. i. tanquam binarius, & quia quælibet pars è viginti ipsius. o. e. æqualis est tertiæ parti. u. i. vnitatis superficialis; si cupiamus scire quot integræ vnitates sint in partibus. o. e. consultum est easdem diuidere per denominantem. u. i. compositum ex tribus partibus superficialibus, & cum tam linea u. a. quam superficies. u. i. diuidatur in 3. partes equales nosce per oportunitatem est eiusmodi partitionem numeri. o. e. fieri per numerum ipsius. u. i. non. u. a. ex prædictis causis.

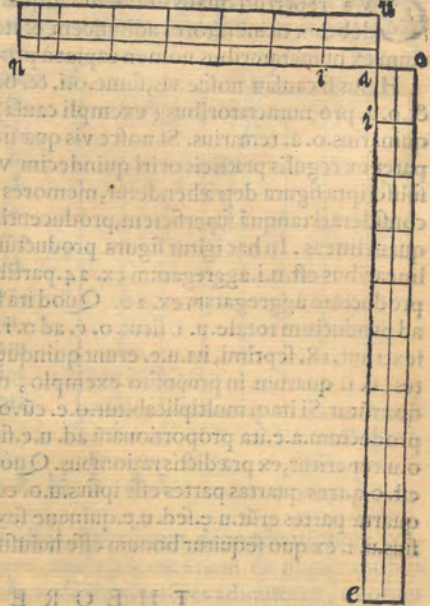


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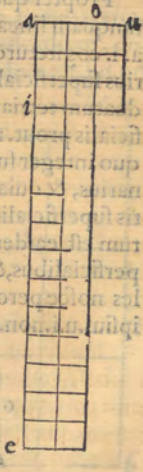
THEOREMA V.

**A**LIA quoque via prædicti effectus causa, speculando innotescere potest, cuius rei gratia formetur sequens figura. e. o. a. u. n. eiusmodi, vt. a. e. sit numerus linearis integrorum, & o. e. productum numerantis ipsorum factorum in integris, ex quo. a. o. erunt duæ tertiæ, verbigratia, a. i. aut a. u. quarum linearum singule statuuntur æquales vnitati linearis, superficies autem parallelogramma. u. n. constituatur æqualis magnitudinis superfici. o. e. ex quo. u. n. erit nobis cognita superficies. Cognoscetur pariter quantitas partium. a. u. quam in proposito exemplo diximus esse trium partium. ex regula igitur de tribus, dicemus si. u. a. dat. a. e. sine dubio. o. a. dabit. a. n. numerum linearem. quæ regula ex. 15. sexti in continuis, & ex 20. septimi in discretis, depro-mittitur. rectè igitur multiplicatur fracti numerantes cum integris, & productum diuiditur per denominantem factorum.



THEOREMA VI.

**I**TEM & alia speculatione cognosci potest hoc rectè fieri multiplicantes enim has duas tertias per decem, debemus considerare quantitatem duarum tertiarum decies produci, ex quo oriuntur 20. tertia, quandoquidem singula vnitates, nunc pro duobus tertijs sumuntur, sed cum quilibet integer tria fragmenta contineat, ideo ex ratione partiendi quoties ternarius ingrediatur viginti, statim cognoscemus quod optabamus. Id ipsum accideret si integri in eiusmodi specie factorum diuiderentur. quo facto hi multiplicandi essent cum numerante proposito, & partiendum productum per quadratum denominantis. Cuius rei hæc est speculatio. Sit linea. a. e. constans ex quinque integris numeris, quorum vnusquisq; æqualis sit. a. u. vel. a. i. & a. o. sint duo tertia vnitatis integra linearis. cogitemus nunc hos quinque integros diuidi in sua fragmenta linearia, quæ in proposito exemplo erunt. 15. multiplicatis iam. 15. cum propositis, videlicet. a. o. oriatur productum. o. e. triginta fragmentorum superficialium, quorum in singulos integros superficiales cadunt nouè in hoc exemplo, & cum notauerimus quoties nouè ingrediatur triginta, propositum consequemur.



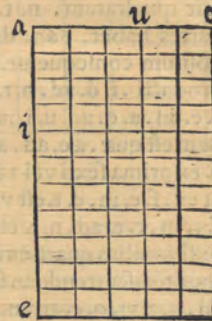
THEO-

THEOREMA VII.

**C**Vm multiplicaturi integros numeros & fractos, cum integris & fractis, debeant integros reducere ad species fractorum, eos colligendo cum fractis: deinde multiplicare hos vltimos numerantes adinuicem & productum partiri per productum denominantium.

Vt (exempli causa) si volumus multiplicare vnum & duo tertia, per duo & tria quarta, reducentur omnia in fractos, ex quo vna ex parte essent quinque tertia, multiplicanda cum vndecim quartis ex altera, quo facto oriretur productum quinquagintaquinque fractorum, quod diuisum per productum ternarij in quaternarium, videlicet per duodecim, quatuor integri proferentur cum septem duodecimis fractis vnus integri.

Detur subsequens figura in qua linea a. i. æqualis sit lineæ u. a. quarum vnaquæq; cõsideretur pro integro numero: cogiturq; a. i. valere quatuor in præfenti exẽplo, & a. u. tria: detur deinde linea. a. o. æquipollens vni integro cū duobus tertijs, & a. e. æquipollens duobus integris & tribus quartis. Iam si hæ duæ lineæ in suos fractos reducantur, multiplicata (vt in sequenti figura apparet.) a. o. cū a. e. oriatur productum. o. e. fractorum superficialium quinquagintaquinq; quorum integer superficialis valet duodecim, scilicet. u. i. vt cuique manifestum est, ex quo, quærenti mediâ partitione, quoties duodecim ingrediatur quinquagintaquinque, citra errorem, quæsitum occurret.

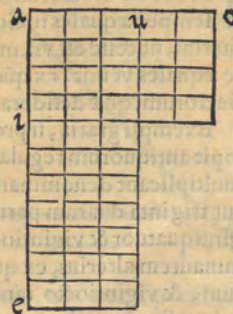


THEOREMA VIII.

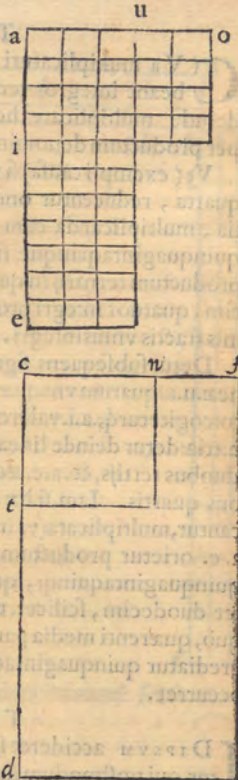
**I**DIPSVm accideret si fracti ad vnam eandemq; denominationem reducerentur, qui postmodum simul multiplicarentur, productumq; partiremur per quadratum denominantis communis.

Exempli causa, sint eadem quinque tertia, & vndecim quarta adinuicem multiplicanda, quæ si reducantur ad vnam & eandem denominationem quaternarius numerans vnus, multiplicabitur cum quaternario denominante alterius, & vndenarius secundi cum ternario denominante primi. ex quo vna ex parte essent viginti, ex altera 33. numerantia vnus cõmunis denominantis, quod esset productum ternarij in quaternarium, videlicet duodecim, vt ex veteri regula patet. Iam si multiplicentur viginti cum trigintatribus, dabuntur. 660. fracti, quorum integer erit quadratum duodenarij, nempe. 144. quibus quidem. 660. diuisis per. 144. proferentur quatuor integri & septem duodecimi.

Cuius rei gratia sit in subscripta figura linea. a. i. & ei æqualis. a. u. pro integro lineari, quæ. a. i. diuidatur in quatuor partes, & a. u. in tres, & linea. a. e. sit vndecim partiũ talium qualium. a. i. est quatuor, & a. o. sit quinque productum. a. u. est trium. nunc multiplicato. a. o. & a. i. oriatur productum. o. i. viginti partium superficialium. tum multiplicato



cato . a . e . per . a . u . dabitur productum . u . e . trigintiū  
 partium . ad hæc quadratum . u . i . constabit ex duode-  
 cim partibus eiusdem rationis cum reliquis duobus  
 productis , quod quadratum . u . i . vnitas est superficia-  
 lis , & communis denominans duorum productorum .  
 quod si in præsentiarum cogitabimus lineam . c . d . tri-  
 gintatium partium æqualium , et . c . t . duodecim simi-  
 lium , et . c . f . viginti . c . n . duodecim , multiplicato . c .  
 d . cum . c . f . dabitur superficies . f . d . 660 . fractorum  
 superficialium , quorum vnitas integra superficialis  
 erit quadratum . n . t . 144 . partium cuiusmodi . f . d .  
 partes habet . 660 . diuiso itaque . f . d . per . n . t . pro-  
 positum consequetur . eo quod eadem proportio erit  
 producti . f . d . ad . n . t . quæ producti eius quod fit ex  
 a . e . in . a . o . ad . u . i . nam proportio . c . d . ad . c . t . ea-  
 dem est quæ . a . e . ad . a . i . & c . f . ad . c . n . vt . a . o . ad . a .  
 u . ex prima sexti vel 18 . septimi , sed vt . f . d . ad id quod  
 fit ex . f . c . in . c . t . est vt . c . d . ad . c . t . & vt eius quod  
 fit ex . f . c . in . c . t . ad . n . t . est vt . f . c . ad . c . n . ex dictis pro-  
 positionibus quare ex æqua proportionalitate , eodem  
 modo discurrendo in figura . o . a . e . ita se habebit . f . d .  
 ad . n . t . vt . o . e . ad . u . i . Porro ex ijs , quæ hæcenus de  
 fractorum multiplicatione considerata fuerunt , apertè  
 ratio deprehenditur , cur productum , singulis producen-  
 tibus semper minus sit , cum producta sint superficialia  
 producentia verò semper linearia , omiſſis productis  
 corporeis , quæ omnia ad superficialia reducuntur .



T H E O R E M A I X.

**I**N IPSA fractorum diuisione , animaduertendum est , denominantes numeros  
 semper æquales inuicem esse debere , vnus scilicet speciei , quod si æquales non  
 fuerint , necesse est via multiplicationis ipsorum denominantium adinuicem effice-  
 re æquales vt sint , ex quo productum oritur eiusmodi , vt aptum sit habere partes  
 fractorum , quæ desiderabantur .

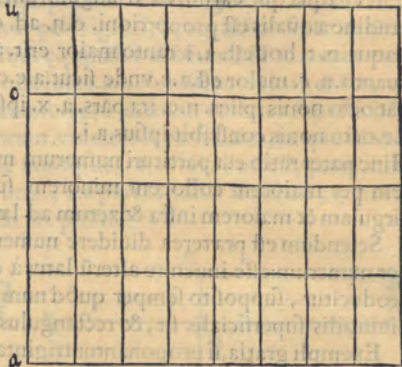
Exempli gratia , si proponerentur diuidentia septem octaua per tria quarta præ-  
 cipit antiquorum regula , vt ad vnã tantum denominationem reducantur . quare  
 multiplicat denominantes inuicem . ex quo productum in materia propofita ori-  
 tur triginta duarum partium commune denominans , cuius duo numerantes sunt vi-  
 gintiquatuor & vigintiocto , producti ex multiplicatione vnus numerantis in deno-  
 minantem alterius , ex quo dantur vigintiquatuor tamquam tria quarta trigintadu-  
 rum , & vigintiocto tanquam septem octaua particularum vniformium , prout ope  
 primæ sexti aut decimæ octauæ septimi in subscripta figura cognosci potest .

Sit

THEOR. ARITH. I

7

Sit itaque linea .a.i. diuifa in partes octo, & ei æqualis in longitudine .a.u. in quatuor, productum verò vnus in alteram fit .u. i. trigintaduorum particularum superficialium fimilium & æqualiũ adinuicem. fit deinde .a. e. feptem partiũ lineæ .a. i. & .a. o. trium partium .a. u. tunc productum .a. e. in .a. u. erit .u. e. particularum superficialium vigintiocto & productum .a. o. in .a. i. erit .o. i. particularum superficialiũ vigintiquatuor eiusdem naturæ cum partibus trigintaduabus totius denominantis communis. vnde diuifo numerante vigintiocto pernumerantem vigintiquatuor, dabitur vnum cum sexta parte illius vnus.

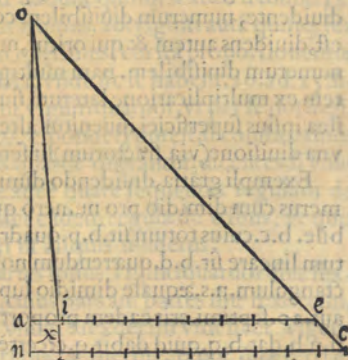


THEOREMA X.

**P**ARTIRI seu diuidere vno numero alium numerum, est etiam quodammodo cuiusmodi partem numeri diuisibilis inuenire respectu totius numeri diuisibilis, cuiusmodi est vnitas in diuidente respectu totius diuidentis, partem inquam numeri diuisibilis sic se habentem ad totum numerum diuisibilem sicut vnitas ad totum diudentem, quod similiter ex regula de tribus præstamus dicentes, si tantus numerus diuidens dat vnitatẽ, quid dabit numerus diuisibilis, quemadmodum ex .15. sexti seu .20. septimi licet speculari, Idcirco quotiescunque minorem numerum per maiorem diuidimus, sc̃mp̃er qui prouenit fractus est.

Exempli gratia, si cogitaremus lineam .a. e. diuisam in octo partes æquales, quarum vna scilicet vnitas esset .a. i. & cupere-  
mus eam diuidere in nouem partes, ac scire  
quanta sit nona illius pars, manifestum esset,  
nonam partem ipsius .a. e. minorem futuram  
ipsa .a. i. cum .a. i. diminiui debeat à sua inte-  
gritate eadem proportione, qua .a. e. minor  
reperitur vna linea nouem partium æqualium  
singularum .a. i.

Quod vt dilucidẽ cuius innotescat, hoc  
etiam modo licebit videre sit linea .n. c. no-  
nupla ad .a. i. & parallela ad .a. e. dubium non  
est quin .n. c. maior futura sit ipsa .a. e. iam si  
earum extrema coniungantur med. js duabus  
lineis .n. a. et .c. e. quæ simul concurrant in  
puncto .o. (quod est probata facillimum) da-  
buntur certẽ duo trianguli similes .a. o. e. et .n. o. c. Sit deinde .n. t. vna è partibus  
ipsius .n. c. quæ .n. t. æqualis erit .a. i. ex præsupposito. ducatur deinde .o. t. quæ  
intersecet .a. e. in puncto .x. dico .a. x. tanto minorem futuram .a. i. quanto .a. e.  
minor est. n. c. neque enim dubium esse potest quin proportionẽ .n. t. ad .a. x. et .



n. c.

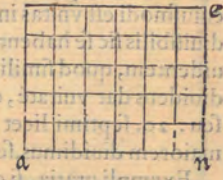
n. c. ad. a. e. sint æquales inuicem quandoquidem vnaquæque earum ex triangulorum similitudine æqualis est proportioni. o. n. ad. o. a. itaque. n. t. hoc est. a. i. tanto maior erit. a. x. quanto. n. c. maior est. a. e. vnde sicut. a. e. constat octo nonis ipsius. n. c. ita pars. a. x. ipsius. a. e. octo nonis constabit ipsius. a. i.

Hinc patet ratio cur partituri numerum minorem per maiorem collocent minorem supra virgulam & maiorem infra & zerum ad laeuam.

Sciendum est præterea diuidere numerum per numerum: esse inuenire alterum latus à quo producitur, supposito semper quòd numerus diuisibilis superficialis sit, & rectangulus.

Exempli gratia, si proponantur triginta diuidenda per quinarium, nihil aliud erit hæc diuisio, quam inuentio alterius numeri, qui multiplicatus per quinarium producat triginta superficies rectangulas, huiusmodi verò est senarius, cuius singula vnitates superficiales erunt.

Cuius rei gratia sit subscriptum rectangulum. a. e. triginta vnitatum superficialium, cuius latus o. n. sit quinque vnitatum. hinc latus. a. n. erit sex vnitatum; ita diuidentes rectangulum. e. a. nihil aliud faciemus, quam vt inueniamus quantum valeat latus. a. n. quod erit sex vnitatum. Sin verò diuiserimus per latus. a. n. quaeremus latus. e. n. quinque vnitatum. ex quo, proportio totius numeri diuisibilis ad numerum qui oritur, erit sicut diuidentis ad vnitatem, ex prima sexti, aut. 18. vel. 19. septimi, & permutata ita se habebit diuisibile ad diuidentem, sicut numerus qui oritur ad vnitatem.



Partiri igitur nihil aliud est, quam inuenire latus rectanguli, quod productum in diuidente, numerum diuisibilem compleat, ex quo numerus diuisibilis superficialis est, diuidens autem, & qui oritur, numeri lineares & latera producentia huiuscemodi numerum diuisibilem. nam multiplicare & diuidere opponuntur inuicem, cum autem ex multiplicatione laterum siue linearum generatur superficies, ex diuisione postea ipsius superficiei inuenitur alterum latus. quare mirum non est, si proueniens ex vna diuisione (via factorum) sit semper maius numero diuisibili.

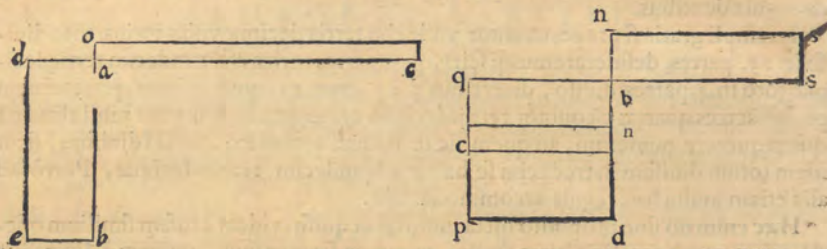
Exempli gratia, diuidendo dimidium per tertiam partem, resultat vnus integer numerus cum dimidio pro numero qui oritur. Sit itaque dimidium superficiale diuisibile. b. c. cuius totum sit. b. p. quadratum. tertium verò lineare diuidens, b. n. cuius totum lineare sit. b. d. quaerendum nobis est latus. b. s. quod cum latere. b. n. producat rectangulum. n. s. æquale dimidio superficiali proposito. b. c. quod si fiat, ex. 15. sexti, aut. 20. septimi, erit eadem proportio. b. n. ad. b. q. quæ est. q. c. ad. b. s. dicemus itaque si. n. b. dat. b. q. quid dabit, q. c. certè. b. s. sed. n. b. est tertium lineare et. b. q. lineare integrum, & b. s. proueniens lineare. & quia. b. c. dimidium superficiale, producitur à. q. c. dimidio lineari in. q. b. integro lineari, quare cum. n. s. sit æqualis. b. c. & productum ex b. n. minor. q. c. necesse est, vt producat in. b. s. maiore. q. b. quod. q. b. maius est. q. c. quod quidem. q. c. ita appellatur sicut. b. c. quare mirum non est si proueniens per fractos numeros ex diuisione, maior sit numero diuisibili.

Hinc



T H E O R. A R I T H.

Hinc manifeste patet quamlibet diuisionē aut partitionem oriri ex regula de tribus, quandoquidem singuli diuidentes æquipollent vni integro, & loco illius sumuntur. Perinde enim est diuidere centum per viginti, ac regulā obseruare de tribus dicētes, si viginti æquipollent vni, quibus æquialebūt cētum? Hoc autem ex subsequenti figura facile deprehendetur, in qua linea. a. b. significat viginti, et. a. o. vnitatē linearē, et. a. c. vnitates lineares centū: o. c. verò centum vnitates superficiales, et. a. d. quinq; vnitates lineares, et. d. b. centum vnitates superficiales, ex quo manifestè deprehenditur quòd quemadmodum multiplicare, nihil aliud est, quam inuenire productū ex duobus lateribus propositis, ita partiri nihil aliud est, quam dato vno latere inuenire aliud latus producti propositi.



Nam quotiescunq; ratiocinātes dicimus tantundem numeri, immediate produci mus superficiem, mediāte vnitare in huiusmodi numero, qui numerus antequā producat in vnitatem, mente concipiendus est tanquam linearis, tanquam linea inquam diuisa in totidem particulas lineares, singulas continuas & æquales vnitati propositæ. Cū verò productus fuerit numerus in vnitare superficialis, erit ac si tot essent vnitates quadratæ, quod si ita non esset, nulla mentio faciendā esset quorumuis fractorū. Ex eadē regula de tribus reduci potest ad praxim tertiu theorema.

Quare cupientes scire quæ sint illæ partes, quæ sunt tres quartæ, ipsarum quinque sextarum, dicemus si quatuor dant tria, quid dabunt quinq; sextæ? dabunt. 15. vigesimas quartas, quæ quindecim sunt tres quartæ ipsius. 20. viginti autē quinq; sextæ viginti quatuor, quandoquidem nos numerum querimus, cui ita proportionentur quinq; sextæ alterius numeri, sicut quatuor ad tria, vnde sic se habent. 20. ad. 15. sicut. 4. ad. 3. ipse autem. 20. quinq; sextæ partes sunt viginti quatuor, vt per se notū est.

Ex eadem regula de tribus, huiusmodi quæsito responderi potest, si constituamus prædictas quinq; sextas esse numerum, cuius tres quartæ querantur, dicentes, si vnus integer dat tres quartas, quid dabunt quinq; sextæ? quare sequentes regulam de tribus, dabuntur quindecim vigesimæ quartæ. Valet eadem regula de tribus, vt quis scire possit, quæ pars aut partes numeri propositi sit aliquis numerus.

Exempli gratia, scire cupienti, quæ pars aut partes ipsius viginti quatuor sint sexdecim, constituentur. 24. tanquam vnum totum, cuius pars aut partes sint sexdecim, dicemus igitur si. 24. dant sexdecim, quid dabit vnum? sexdecim videlicet vigesimas quartas, quæ cum ad primos numeros reductæ fuerint, erunt duæ tertiæ. Eadem ratione qui scire uellet, quæ partes aut pars essent tres quartæ, octo nonarum, diceret, si octo nonæ dant tres quartas, quid dabit vnum? prouenient. 27. trigesima secundæ.

Subseruiit pariter ad sciendū naturā partiū numeri propositi. Exempli causa, si quis querat, cuius numeri, duodecim sint duæ tertiæ partes. Dicit si duo dant tria, quid

B dabunt

dabunt duodecim ? nempe dabunt decem octo, numerum quæsitum scilicet, Tunc autem nil aliud præstamus quam quòd quærimus numerum ad quem ita se habeant duodecim, sicut duo ad tria. Ita etiam si quis quærat, cuius numeri duo tertia sint tres quintæ, dicet, si tria dant quinq;, quid dabunt duo tertia ? nempe dabunt integrum cum fracto nono. Hoc erit itaq; querere numerum ad quem sic se habeant duo tertia sicut tria ad quinq;, quod manifestum est per se.

Eadem ratione qui scire vellet, cuius numeri duæ septimæ, essent octo integrarum cum duabus quintis, diceret, si duo dant septem quid dabunt octo integra cum duabus quintis ? nempe dabunt. 29. integra cum duabus quintis numerum quæsitum. Sic etiam qui transferre vellet fractum numerum in fractum, id perficeret ex regula de tribus.

Exempli gratia si proponerentur vnde cim tertiædecimæ vnius totius, toto diuiso in. 13. partes, desiderarem; scire, quot partes totius esset vnde cim tertiædecimæ, toto in. 4. partes diuiso, diceremus si. 13. dant. 11. quid dabunt quatuor ? nempe dabunt tres quartas cum quinq; tertijsdecimis vnius quartæ, hoc verò nihil aliud est quam querere numerum, ad quem sic se habeat totum in 4. partes diuisum, sicut idem totum diuisum in tredecim se habet ad undecim tertiasdecimas, Porro ad alia etiam multa hæc regula accommodata est.

Hæc enim nõ sine proposito dicta sunt, sed ut quisq; videat causam similibus operationum, quæ à practicis circa fractos numeros scriptæ sunt, omnem à diuina illa regula de tribus originem trahere ut etiam in sequentibus videbimus.

## T H E O R E M A X I.

**C**Vr productum ex eo quod oritur in diuidente, semper æquale est numero diuisibili si queras ita accipe.

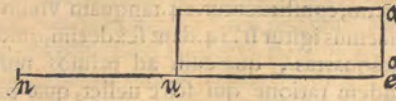
Sit numerus diuisibilis. b. quod oritur sit. c. diuidens. d. & vnitas diuidentis. t. cum igitur, vt in præcedenti theoremate dictum fuit, eadem sit proportio . b. ad. c. quæ est. d. ad. t. manifeste deprehenditur ex. 20. septimi, productum ex. b. in. t. æquale esse producto . c. in d.

$$\frac{b}{d} = \frac{c}{t}$$

## T H E O R E M A X I I.

**I**D ipsum alia ratione contemplari licet.

Numerus diuisibilis significetur per lineam. n. e. diuidens verò per lineam. a. e. quod oritur linea. u. e vnitas diuidentis. o. e. quæ cogitamus esse vnitatem linearem; ad hæc productum ex. u. e. in. a. e. sit superficies. u. a. Dico superficiem. u. a. componi ex tot vnitatibus superficialibus quot linearibus constat linea. n. e. nam ex ijs quæ diuidendi ratione notauimus, constituitur eandem proportionem esse . n. e. ad. u. e. quæ est. a. e. ad. o. e. At ex prima sexti aut 18. septimi sic se habet totale productum. u. a. ad partiale. u. o. sicut. a. e. ad. o. e. quare sic se habebit. u. a. ad. u. o. sicut. n. e. ad. u. e. sed. u. e. et. u. o. numero non differunt, cum sint vnius & eiusdem speciei, (tamen si numerus. u. o. sit superficialis et. u. e. linearis). Itaq; ex nona quinti numerus. u. a. æqualis erit numero. n. e.



THEO-

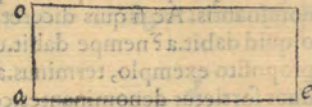
THEOREM. ARITH.

II

THEOREMA XIII.

CVR diuidentibus numerum diuisibilem per proueniens, oritur numerus diuidens?

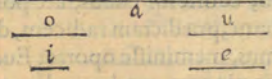
Sit subscriptus rectangulus. o. e. numerus diuisibilis, qui producitur, tam ex a. o. in a. e. quam ex a. e. in a. o. quare si a. o. diuidens fuerit. a. e. proueniens erit, si vero a. e. diuidens extiterit, a. o. proueniens erit futurum.



THEOREMA XIII.

Hoc ipsum, alia quoq; uia licebit speculari.

Sit linea. a. denotans numerum diuisibilem, et. o. primi proueniens linea. e. primi diuidentis. u. secundi proueniens idest cum. o. pro diuidente sumetur. Iam ex indicata definitione diuisionis nono theoremate huius libri, dabitur proportio. a. ad. o. prout datur. e. ad unitatem significatam linea. i. & permutarim. a. ad. e. sicut. o. ad. i. sed. a. ad. u. sic se habet prout. o. ad. i. ex eadem definitione diuisionis, itaq; sic se habebit. a. ad. u. sicut. a. ad. e. unde. u. aequalis erit. e. ex. 9. quinti



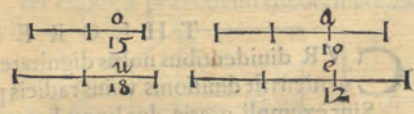
THEOREMA XV.

Vnde prouenit, ut qui velit cognoscere cuius numeri quatuor quinta partes, sint, dua tertie, aut quid simile, cōsultissime faciat, si ad unam eandemq; denominationem reducerit.

Prout in proposito exemplo, cū denominas cōmunis sit quindecim, cuius dua tertia sunt decē, & quatuor quinta duodecim, cōmunis autē denominans. 15. multiplandus sit per quatuor quintas, scilicet duodecim, & productum diuidendum per duas tertias, hoc est decem, ex quo oriantur decem octo quēsitus numerus?

Quod ad reductionē numeratorū ad vnam & eandem denominationem attinet, ea de causa fit quo uti possimus regula de tribus, qua tribus tantummodo notis terminis indiget, quo quartus à predictis dependens, inueniri possit, quandoquidem bini illi respectus, tribus terminis comprehendi possūt. At quod ad multiplicationem spectat denominantis cōmunis cū numerante denominantis in cogniti & diuisionem producti per numerantem cognitū illē nihil aliud sunt, quam quartū terminū inuenire, ita proportionatum tertio, ut secundus primo.

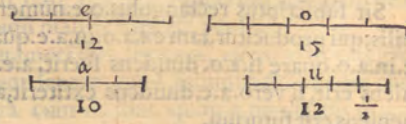
Exempli gratia, sit. a. denotans numerantem denominantis cogniti, qui significetur linea. o. et. e. sit denominantis incogniti numerans, denotati linea. u. imò verò & cognitū. o. nempe quatuor quinta, Iam si. o. cum. e. multiplicemus, & productum per. a. diuidemus dabitur. u. sic se habens ad. e. sicut. o. ad. a. ex. 20. septimi.



B 2 Inue-

## T H E O R E M A X V I.

**I**nuenire autem cupienti cuius numeri, duæ tertiæ, sint quatuor quintę partes, multiplicandę essent duæ tertiæ per denominantem communem, & productum diuidendum per quatuor quintas ipsius denominantis. Ac si quis diceret si. e. dat. o. quid dabit. a? nempe dabit. u. nam in proposito exemplo, terminus. a. locõ. e. duos sortietur denominantes; cognitum videlicet. o. et. u. incognitum quod postea cognitum oritur ex regula de tribus, vt dictum est.

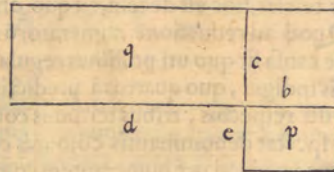


## T H E O R E M A X V I I.

**Q**ua ratione cognosci poterit proportionem quantitatis censicę censicę ad similem quantitatem quadruplam esse ad eam, quę est suarum radicum; proportionem autę primarum relatarum esse quintuplam, atq; ita deinceps?

Cuius rei gratia, sciendus est modus, pductionis harũ dignitatũ qui oritur ex productione primę radicit in seipsam, prout qui cubũ requirit, ducat radicę in suo quadrato, & orietur cubus, hæc postea ducta in cubum, quantitatem censicam censicę, et in hanc, prædictam radicem, dabit quantitatem primam relatarum. Quod vbi sciuerimus, meminisse oportet Euclidem decima octaua sexti aut. i. octaua docere, proportionem quadrati ad quadratũ, duplam esse proportioni suarum radicum, &. 36. vndecimi aut. i. octaua, cubi ad cubũ triplam esse, ego verò nunc assero, censicę censicę ad radicem proportionem quadruplam esse, primi verò relati ad primum relatum quintuplam atq; ita gradatim.

Cuius speculationis gratia, detur linea. d. quę cubum maiorem significet. et. b. minorem. c. verò sit radix ipsius. d. et. e. ipsius. b. ita ordinate adiuicem, vt in subscripta figura cernitur. Iam. c. cum. d. producatuŕ proueniatq; . q. censicum censicum, tum producatuŕ. e. cum. b. et dabitur. p. alterum censicum censicum. Dico igitur proportionem. q. ad. p. quadruplam esse proportioni. c. ad. e. hac de causa quòd proportio. q. ad. p. componatur ex proportione. d. ad. b. et. c. ad. e. prout facile ex. 24. sexti, aut quinta octaua deprehenditur. Quare cũ proportio. d. ad. b. proportioni. c. ad. e. tripla sit, patet proportionem. q. ad. p. quadruplam esse proportioni. c. ad. e. Idem de ceteris dignitatibus dico, sumptis semper. d. et. b. pro duobus censibus censuum, aut duobus primis relatis, aut alio quouis axiomate.

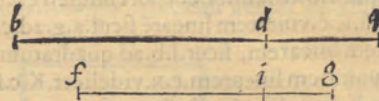


## T H E O R E M A X V I I I.

**C**ur diuidentibus nobis dignitatem, per dignitatem, radix prouenientis: proueniens sit diuisionis vnius radicit per alteram?

Sint exempli gratia duę lineę. b. q. et. f. g. quę significant duas radices cuiusuis dignitatis; demusq; esse radices duorum quadratorum, quadratumq; ipsius b. q. per quadratum ipsius. f. g. diuidatur; quadrataque radix prouenientis sit. d. q. vnitas verò linearis sit. i. g. Dico ipsam. d. q. esse proueniens ex diuisione. b. q. per. f. g. Patet enim ex definitione diuisionis nono theoremate tradita quadratum

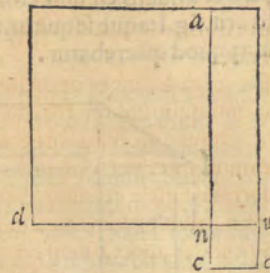
rum ipsius. d. q. talem esse partem quadrati ipsius. b. q. qualis quadratum ipsius. g. i. est quadrati ipsius. f. g. Scimus præterea ex. 19. sexti, aut vndecima octavi, proportioné quadrati ipsius. b. q. ad quadratū ipsius. d. q. duplam esse proportioni. b. q. ad. d. q. suarum radicum (cuborum enim tripla esset & censuum censuum, quadrupla, atq; ita deinceps ex præcedenti theoremate) Id ipsum dico de dignitatibus ipsius. f. g. et. i. g. respectu radicum. f. g. et. i. g. Vnde cum proportio dignitatis ipsius. b. q. ad illam. d. q. equalis sit proportioni dignitatis ipsius. f. g. ad illam. g. i. ex communi scientia apertè cognoscemus simplices proportionés esse inter se æquales, nempe eam quæ est. b. q. ad. d. q. æqualem esse ei, quæ est. f. g. ad. i. g. itaq; sequitur ex definitione diuisionis. d. q. esse proueniens ex diuisione. b. q. per. f. g.



THEOREMA XVIII.

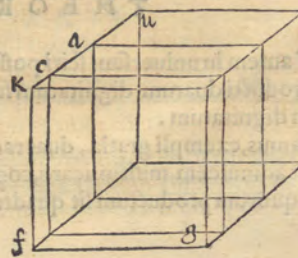
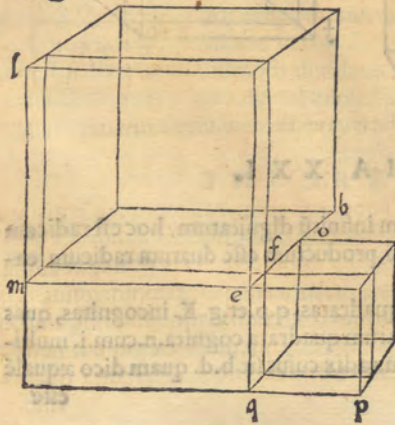
**C**VR productum ex duabus radicibus quadratis, est quadrata radix, producti suorum quadratorum simul?

In cuius rei gratiam, sint duo quadrata. d. a. et n. o. coniuncta simul, prout in subscripta figura apparet, ita tamen vt angulus. a. n. u. sit rectus, quare ex quartadecima primi, duo latera. n. c. et. n. a. directe coniungentur adinuicem, prout etiam reliqua duo latera. n. u. et. n. d. Cogitato deinde. a. u. producto ipsius. a. n. in. n. u. duarum videlicet radicum quadratarum simul, dabitur ex prima sexti, aut decima octaua septimi, productum. a. u. medium proportionale inter quadratum. a. d. et. u. c. quod si cogitemus has tres superficies, tres numeros esse, patebit ex vigesima prima septimi productum. a. u. in se ipsum, quadratum scilicet. a. u. æquale esse producto. a. d. in. u. c. ex quo propositi euidencia consequetur.

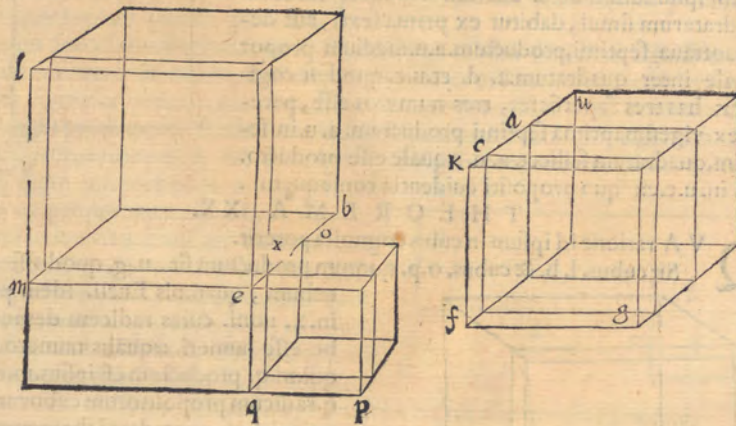


THEOREMA XX.

**Q**VA ratione id ipsum in cubis cognosci poterit. Sit cubus. l. b. & cubus. o. p. quorum productum sit. u. g. quod assero esse cubum, quamuis Eucli. idem probet in. 4. noni. cuius radicem demonstrabo esse numeri æqualis numero. m. q. qui. m. q. productum est ipsius. m. e. in. e. q. radicem propositorum cuborum. Patet enim ex præcedenti theoremate. m.



q. radicem esse quadratam producti .l. e. in. e. p. quod productum sit quadratum corporeum. c. g. cogitemus pariter duo quadrata. l. e. et. e. p. esse pariter corporea, tantę profunditatis, quantam, vnitas linearis radicum. m. e. et. e. q. requirit. Hęc duo corpora producentur à superficie in vnitatem, vocenturq; .l. x. et. x. p. quo factō, cogitemus corpus. a. g. tamquam productum cubi. l. b. in quadratum. e. p. Vnde ex decima octaua, aut decimanona septimi, eadem erit proportio. a. g. ad. c. g. quę est. l. b. ad. l. x. corporeum, sed ex. 25. vndecimi & prima sexti, ita se habet. a. K. ad. K. c. vnitatem linearem sicut. a. g. ad. c. g. & ex eisde ita se habebit. b. e. ad. e. x. vnitatem linearem, sicut. l. b. ad quadratum. l. x. corporeum. Itaque sic se habebit. b. e. ad vnitatem linearem. e. x. videlicet. K. c. sicut. a. K. ad ipsam. K. c. Vnde ex nona quinti. a. K. æqualis erit. e. b. & consequenter æqualis. m. e. Iam verò sit. u. g. productum. l. b. cubi, in cubum. o. p. vt supra dictum est. Hinc patebit ex quauis duarum propositio- num, decima octaua, aut decimanona septimi, eadem futuram proportionem. u. g. ad. a. g. quę est. o. p. ad. x. p. quadratum corporeum. Quare ex postremis, dictis ratio- nibus, eadem erit proportio. u. K. ad. a. K. quę est. o. e. ad vnitatem linearem. e. x. at ex dictis decima octaua & decimanona septimi, ita se habet numerus. m. q. ad numerum superficialē. m. e. qui pducitur à lineari. m. e. in vnitatem linearem ipsius. e. q. sicut nume- rus. q. e. ad suam vnitatem, sed cū numerus. a. K. æqualis sit numero. m. e. vt probatū est erit ergo ex vndecima & nona quinti, numerus. u. K. æqualis numero. m. q. At. f. g. pariter æqualis est numero. m. q. ex præcedenti theoremate, vnde. K. u. pariter æqua- lis erit. f. g. Itaque sequitur. u. g. cubum esse, & f. g. radicem ipsius, æqualem numero. m. q. quod querebatur.



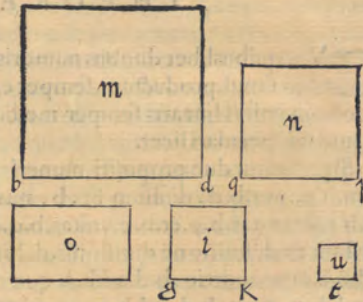
## T H E O R E M A X X I.

**V**T autem in uniuersum sciri possit totum infinitum dignitatum, hoc est radicem producti duarum dignitatum similiarum, productum esse duarum radicum earundem dignitatum.

Ponamus, exempli gratia, duas radices quadratas. q. p. et. g. K. incognitas, quas qui velit adinuicem multiplicare, cogatur earum quadrata cognita. n. cum. i. multiplicare, quorum productum sit quadratum. m. radix cuius sit. b. d. quam dico æqualē esse

THEOREM. ARIT.

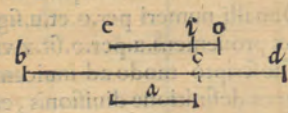
esse pducto. q. p. in. g. k. qđ autē sit. o. Patet enim proportionē. o. ad. q. p. eandē esse cum proportionē. g. k. ad suam vnitatem linearem, ex decima octaua, aut decima nona septimi, hæc vero vnitatis linearis sit. t. cuius superficialis sit. u. vnitatis scilicet toties in seipsam multiplicata quoties proposita dignitas patitur, tamēti in præsen ti exemplo quadrata dignitas sumatur. Itaq; ex eisdem propositionibus decima octa ua aut decima nona, sic se habet. m. ad. n. sicut. i. ad. u. Scimus præterea proportionē. m. ad. n. (eo quod in proposito exemplo sint quadrata) duplam esse proportioni. b. d. ad. q. p. et ipsius. i. ad. u. pariter duplam proportioni. g. k. ad. t. iam autem dictum fuit sic se habere. m. ad. n. sicut. i. ad. u. Itaq; b. d. sic se habebit ad. q. p. sicut. g. k. ad. t. quandoquidem sic se habeat totum ad totū, sicut pars ad partē, dū similes sint, probatū autē est superius ita se habere. o. ad. q. p. sicut. g. k. ad. t. itaq; o. sic se habebit ad. q. p. sicut. b. d. ad. q. p. vnde. o. æqualis erit. b. d. Hoc ipsum ceteris dignitatibus conueniet, mutatis tantummodo proportionibus. m. n. ad proportionem. b. d. q. p. sic proportionibus duarum dignitatum. i. u. ad proportionem suarum radicum. g. k. t.



THEOREMA XXII.

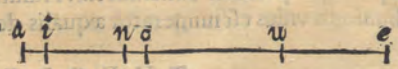
DOCENT veteres, quod si quilibet numerus in duas partes inæquales diuisus fuerit, totumq; diuisum per vnā partium, & per eandem pars altera diuisa fuerit: differentia proueniens semper vnitatis erit. quod quidem verissimum est.

Detur enim. b. d. propositus numerus in duas partes inæquales diuisus. b. c. et. c. d. & in primis totū. b. d. per. c. d. diuidatur, ex quo oriatur e. o. vnitatis autem. p. i. o. significetur, tum pars ipsa. b. c. p. eadem. c. d. diuidatur, sitq; proueniens. a. Sanē ex definitione diuisionis, eadem erit proportio. b. d. ad. e. o. quæ est. c. d. ad. i. o. et ita. b. c. ad. a. sicut. c. d. ad. i. o. Ex. 19. autem quinti, ita se habet. b. c. ad. e. i. sicut. b. d. ad. e. o. at. b. d. ad. e. o. sic se habet sicut. c. d. ad. i. o. hoc est sicut. b. c. ad. a. Quare ex. 11. quinti sic se habebit. b. c. ad. e. i. sicut. ad. a. ex quo ex. 9. prædicti. a. æqualis erit. e. i. sed. e. i. minor est. e. o. per. i. o. Quare sequitur propositum verum esse. Quod ipsum paucissimis verbis sic defini potest, si dixerimus, eiusmodi diuidens. in parte diuisibili, quā in toto, semel minus ingredi, quandoquidem altera pars est, ex qua totum integrum perficitur.

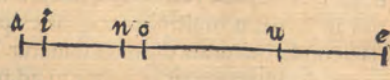


THEOREMA XXIII.

Hoc ipsum alia ratione contemplari poterimus. a. i. n. u. e. Significetur enim totalis numerus per. a. e. in duas partes diuisus. a. u. et. u. e. totius autem diuidens sit. u. e. & partis alterius. a. u. totius verò proueniens sit. a. c. partis autē, sit proueniens. a. n. tum differentia sit. n. c. vnitatis



tas vero cui differentiā.n.c.æquari dico,fit.a.i.Patet enim in primis,eandem propor-  
tionem esse. a.e. ad.a.c. quæ est. u. e.ad.a.i. ex definitione diuisionis, et eandem  
esse.a.u.ad.a.n.quæ est.u.e.ad.a.i.vnde ex.  
11. quinti sic se habebit.a.e.ad.a.c.ficut.a.  
u. ad.a.n. et ex.19. eiusdem sic se habe-  
bit.u.e.ad.n.c. ficut. a.e.ad. a.c. sed. sic se  
habebat.u.e.ad.a.i. Itaq; ex prædicta. 11. quinti, sic se habebit. u.e. ad.n.c. ficut ad.a.  
i. Quare ex.9. eiusdem.n.c.æqualis erit.a.i.et idcirco.n.c.pariter vnitas erit.



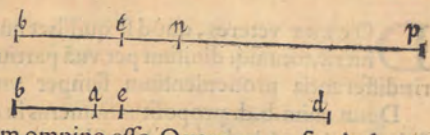
T H E O R E M A X X I I I I.

**C**Vr quibuslibet duobus numeris diuisis adinuicem,multiplicatisq; prouenien-  
tibus simul,productum,semper est vnitas superficialis?Nempe ex.20. septimi,  
quoniam vnitas linearis semper media proportionalis est inter bina prouenientia.  
Quod ita speculari licet.

Significetur duo propositi numeri per.b.p.et.b.d. mutuo diuisi, proueniens au-  
tem.b.p. per.b.d. diuisum fit. b. n.tum proueniens.b.d. diuisum per.b.p.fit.b.a.  
et.b.t.fit vnitas.b.p.et.b. e.vnitas.b.d.ex quo.b.t.æqualis erit.b.e.

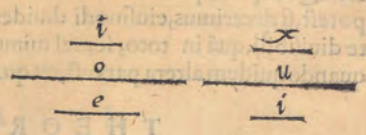
Tam ex definitione diuisionis,dabitur eadem proportio.b.p.ad.b.n.quæ est.b.d.  
ad.b.e.et proportio .b.d.ad.b.a. quæ est. b. p. ad. b. t. Sed cum sic se habeat.b.  
p.ad. b.n. ficut .b.d. ad.b.e. permutando sic se habebit.b.p.ad.b.d.ficut.b.n.ad.b.  
e.hoc est ad.b.t.et cum sic se habeat.b.d.ad.b.a, ficut. b.p.ad.b.t: permutando sic se  
habebit. b.d.ad. b.p. ficut.b.a. ad. b.t.

Quare euersim sic se habebit.b. p.ad. b.d.ficut. b.t.ad.b.a.fed. b.n.ad. b.t. sic  
se habebat vt. b.p.ad.b.d. Itaq; ex.11.  
quinti sic se habebit.b.n.ad.b.t.ficut.b.  
e.ad.b.a.Dictum autem est.b.e.et.b.t.idem omnino esse. Quare ex.20.septimi pro-  
positi veritas innotescet.



T H E O R E M A X X V.

**I**Dipsum & hac altera uia patebit .  
Duo illi numeri per.o.et.u.significentur mutuo diuisi,proueniens autē.o. per.  
u.fit.e.et proueniens.u.per.o.fit.x.vnitas uerò per.i.significetur, quas tamen quanti-  
tates subscripto modo ad inuicem disponi-  
to. Itaq; ex definitione diuisionis, eadem erit  
proportio.o.ad.e. quæ est.u.ad.i.et.o.ad.i. quæ  
est.u.ad.x. Quare ex æqualitate proportionū.  
e.ad.i. sic se habebit ficut.i.ad. x. erit enim.i.  
media proportionalis inter.ç.et.x.ex.20. autē  
septimi propositum concludetur. Huiusmodi rei causa etiam est, quod proueniens  
diuisionis vnus est numerator æqualis denominatori diuisionis alterius.



T H E O R E M A X X V I.

**C**Vr duobus numeris mutuo diuisis,sūptis deinde prouenientibus simul et adinuicem,& per hanc summam,diuisa summa quadratorum dictorum propositorū  
nume-



THEOREM. ARITH.

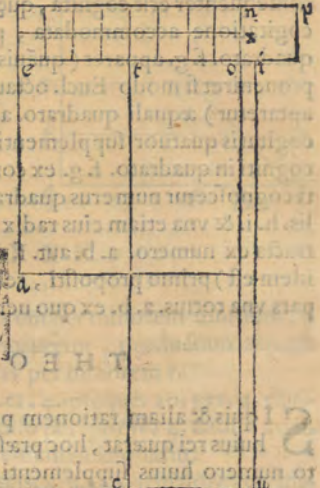
num erorum, proueniat numerus æqualis numero producti duorum primorum numerorum simul.

Sint exempli gratia propositi numeri. 2. et. 8. qui mutuo diuisi in primis dent prouenientia quatuor integra, tum quartam partem pro altero proueniente, hæc collecta dabunt summam quatuor integrorum et quartæ partis vnus, summa autem quadratorum binarij & octonarij erit. 68. qui quidem numerus per quatuor & quartam partem vnus diuisus dabit. 16. pro proueniente, quæ. 16. æqualia erunt producto binarij in octonarium.

Cuius rei hæc erit speculatio, sint duæ lineæ. o. e. et. o. n. quæ duos numeros propositos significant, inuicem ad angulum rectum. o. coniuictæ, quarum quadrata sint. o. a. et. o. p. ipsorum productum fit. n. e. tum. o. t. fit proueniens ex diuisione. o. e. per. o. n. Hæc singulatim consideremus (nã si in partibus simplicibus quod dicimus acciderit, id ipsum in compositis consequenter eueniet) quamobrem ex definitione diuisionis dabitur eadem proportio. o. e. ad. o. t. quæ est. o. n. ad unitatem, quæ sit. o. x. Nunc cogitemus superficiem rectangulã. o. c. æquale quadrato. o. a. tunc numerus. c. t. proueniens erit, ut patet, ex diuisione numeri quadrati. o. a. per numerũ. o. t. eritq; eadẽ proportio. c. t. ad. o. e. quæ est. o. e. ad. o. t. ex secunda parte quintæ decimæ sexti, aut. 20. septimi. Iã autẽ dictum est. o. e. ad. o. t. sic se habere sicut. o. n. ad. o. x. Itaq; ex. 11. quinti sic se habebit. c. t. ad. o. e. sicut. o. n. ad. o. x. Sed ex prima sexti, aut. 18. vel. 19. septimi, sic se habet productum. n. e. ad. e. x. sicut. o. n. ad. o. x. quare denuo sic se habebit numerus. c. t. ad numerum. o. e. sicut numerus. n. e. ad numerum. x. e. Sed numerus. o. e. cum numero. x. e. specie idem est, igitur ex. 9. quinti numerus. c. t. numero. n. e. æqualis erit.

Id ipsum de quadrato ipsius. o. n. videlicet. p. o. dico. Nam si proueniens. o. n. diuiso per. o. e. id est. o. i. proportionale respondens ad. o. t. cum. o. t. coniuictũ fuerit, et per hæc summam diuisa summa quadratorum. o. a. et. o. p. patet per se proueniens futurum eiusdem numeri. c. t. ipsumq; c. t. proueniens semper futurum.

Quo autem lucidius res hæc innotescat. Cogitemus proueniens quadrati. o. p. diuisi ab. o. i. respondentisq; o. t. esse. i. u. quod via prædicta inuenitur æqualis esse numero. n. e. ex quo consequenter æquale. c. t. cogitato deinde rectangulo. o. u. æquali. o. p. coniuictõ. o. c. totum. t. u. æquale erit composito duorum quadratorum. o. a. et. o. p. cum in nullo numerus. c. t. mutetur, tam ex composito. t. n. quæ ex simplici. o. c. ex quo propositi se se ueritas profert.



THEOREMA XXVII.

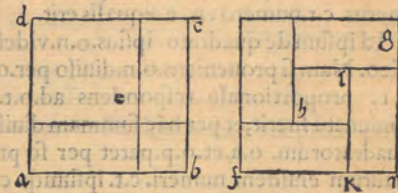
PROPOSUERUNT veteres nobile quidem problema, sed quod tamen citra algebraicam effectiõnem, aut nescierunt, aut noluerunt dissoluere, quod nihilominus facillimum est.

C Propo-

Proponunt hi numerum in binas eiusmodi partes diuidentum, vt summa quadratorum dictarum partium, alteri numero possibili proposito æqualis fit, possibili inquam, etenim si eiusmodi numerus propositus, minor esset producto totius primi in suum dimidium, esset huiusmodi factum impossibile. Quod nos exequi cupientes, sumamus primum numerum propositum, quem in se ipsum multiplicemus. ab hoc quadrato deducamus secundum numerum propositum, tum quod remanserit duplicemus, quod duplum denuo iubeo ex eodem primo quadrato detrahi, accepta postea radice quadrata residui & dempta ex priori numero proposito, tunc dimidium residui vna pars erit ex duabus primi numeri quæ sita.

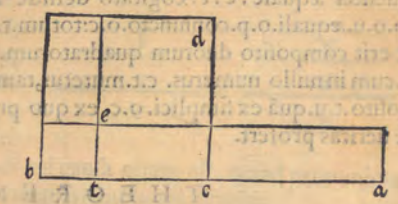
Exempli gratia proponantur. 20. diuidentur in duas eiusmodi partes, vt summa quadratorum ipsarum partium æqualis sit. 272. qui numerus maior est. 200. maior inquam dimidio quadrati. 400. ipsorum. 20. hic autem numerus. 272. è quadrato. 400. deducatur, remanebunt enim. 128. quod duplicari iubeo, producetur siquidem. 256. quæ pariter deducta è quadrato totali, remanebunt. 144. cuius radicem sumi volo, quæ erit. 12. & dempta ex. 20. priori numero dato remanebit. 8. cuius dimidium erit. 4: pars vna ex quæ sitis, quæ ex primo numero proposito. 20. detrahetur, remanebitque. 16. pro altera parte.

Cuius demonstrationis causa, in primis cogitemus quadratum. a. c. cognitum numeri. a. b. primo proposito, qui cogitur diuisus in duo quadrata. d. e. et. e. b. duoque supplementa. a. e. et. e. c. numerus autem summæ duorum quadratorum. d. e. b. pro secundo proposito datur; ex quo, summa duorum supplementorum. a. e. c. consequenter erit cognita, quæ cum duplicata fuerit, & quatuor hæc supplementa cogitatione accommodata, prout in quadrato. f. g. apparet (quæuis id ipsum proueniret si modo Eucl. octaua secundi aptaretur) æquali quadrato. a. c. ita vt cogitatis quatuor supplementis numeri cogniti in quadrato. f. g. ex consequenti cognoscetur numerus quadrati partialis. h. i. & vna etiam eius radix qua detracta ex numero. a. b. aut. f. n. (quod idem est) primo propositi, relinquetur numerus cognitus duplum. x. n. aut. t. b. pars vna totius. a. b. ex quo verum erit hoc meum problema.



T H E O R E M A X X V I I I.

**S**I quis & aliam rationem perficiendæ huius rei quarat, hoc præstet inuento numero huius supplementi, cum in præcedenti theoremate dictum fuerit, qua ratione manifestetur duplum supplementi ipsius.



Cogitemus in subscripta figura lineam. a. b. tanquam primum numerum propositum, & productum. a. e. supplemento. a. e. primæ præcedentis figuræ æquale fit, ac deinde ordine ab antiquis tradito procedatur, ad quadratum reducto dimidio. a. b. videlicet. b. c. quod erit. b. d. ex quo detrahatur deinde. a. e. quare remanebit

THEOREM. ARI T.

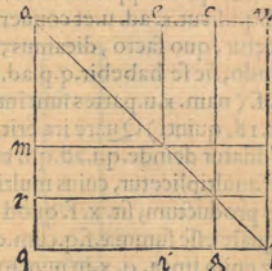
bit quadratum. e. d. cognitum, cuius radix æqualis erit. c. t. qua coniuncta dimidio. c. a. ex quinta secundi Eucli. dabit quod propositum erat.

THEOREMA XXIX.

QVID causæ est, cur subtracto duplo producti duorum numerorum ad inuicem multiplicatorum ex summa suorum quadratorum, semper quod super est duorum numerorum quadratum differentia sit?

Exempli gratia si proponerentur duo numeri. 16. et. 4. duplum producti eorum esset. 128. quod detracto ex summa suorum quadratorum, nempe ex. 272. remaneret. 144. cuius quadrati radix esset. 12. tanquam differentia inter. 4. et. 16.

Id ut sciamus, duo numeri propositi, duabus lineis significentur, maiore. q. g. et minore. g. p. directè coniunctis, super quas, totale quadratum extruatur. a. p. in quo cogitetur diameter. a. p. et à puncto. g. ducatur parallela. g. n. c. et à puncto. n. parallela. n. s. r. ex quo duo producta dabuntur. q. n. et. n. u. singula æqualia producto. q. g. in g. p. et. a. n. et. n. p. duo quadrata dictorum numerorum propositorum, quod satis superq. probatur quarta secundi Eucli. Cogitemus deinde. n. o. æqualem. n. p. et à puncto. o. ducatur. o. m. t. parallela. r. s. et. o. e. ad. n. c. quare ex allatis ab Eucli. octaua secundi, dabuntur quantitas. m. n. æqualis. q. n. producto. q. g. in g. p. et quantitas. o. c. minor ipso producto, ex quantitate quadrati. n. p. ex quo quantitas. m. n. e. vna cum quadrato. n. p. æqualis erit duplo producti. q. g. in. g. p. sed hæc duæ quantitates, sunt partes duorum quadratorum dictorum, & quæ super est. m. e. quadratum differentia vnus numeri propositi ab altero, prout in subscripta figura licebit cui libet considerare. Itaque veritas hæc manifesta erit.



THEOREMA XXX.

CVR ij qui ex duobus numeris propositis maiorem per minorem diuidunt, si proueniens per maiorem numerum multiplicauerint, productum æquale erit proueniens ex diuisione quadrati maioris numeri per minorem?

Exempli gratia si proponantur duo numeri. 20. et. 4. ipseque. 20. per. 4. diuidatur, dabit quinque, tum. 400. quadrato. 20. diuiso per priorè. 4. dabit. 100. quod proueniens, producto ex. 20. in. 5. primo proueniens adæquatur.

Cuius speculationis causa, sint duo numeri, qui lineis. x. u. et. x. s. maiore atq; minore significetur, tum. u. x. numerus per. s. x. diuidatur, sitque proueniens. x. n. postmodum quadratum. u. x. fit. x. o. et productum ex. n. x. in. u. x. fit. x. e. quod æquale esse dico proueniens ex diuisione quadrati. o. x. per. s. x. quod fit. m. Patet enim ex definitione diuisionis, talem futuram proportionem. u. x. ad. n. x. qualis est. s. x. ad vnitatem, & quadratum. o. x. ad rectangulum. e. x. ita se ha-



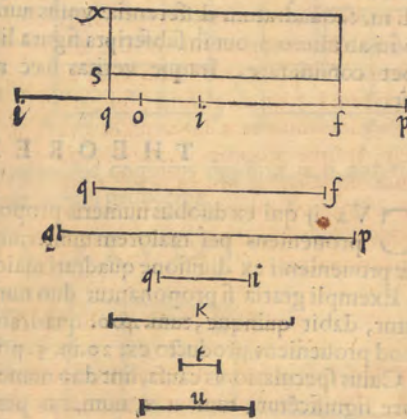
biturum, sicut. u. x. ad. n. x. ex prima sexti aut. 18. vel. 19. septimi, quare ex 11. quinti ita se habebit. o. x. ad. e. x. sicut. s. x. ad vnitatem; sed sicut se habet. s. x. ad vnitatem, ita se habet pariter. o. x. ad. m. vnde ex. 11. prædicta ita se habebit. o. x. ad. m. sicut idipsum. o. x. ad. e. x. itaque ex. 9. prædicti quinti. m. æqualis erit. o. x.

T H E O R E M A XXXI.

**C** V R proposito aliquo numero in duas partes inæquales diuiso, si rursus per quamlibet ipsarum diuidatur, prouenientia tantumdem coniuncta quantum multiplicata efficiant.

Exempli gratia, sit denarius propositus numerus, per binarium & octonarium diuisus, prouenientia erunt quinque & vnum cum quarta parte, quæ coniuncta erunt. 6. cum quarta parte lineari, quæ simul multiplicata, pariter erunt. 6. cum quarta parte superficiali.

Cuius speculationis causa, totalis numerus, linea. q. p. significetur, eius duæ partes, per. x. maiorem et. u. minorem, ipsa vnitas per. t: proueniens ex diuisione. q. p. per. x. fit. q. i. proueniens autem ipsius. q. p. per. u. fit. q. f. quare ex definitione diuisionis ita se habebit. q. p. ad. q. i. sicut. x. ad. t. et. q. p. ad. q. f. sicut. u. ad. t. hoc est. q. f. ad. q. p. sicut. t. ad. u. vnde ex æqualitate proportionum sic se habebit. q. f. ad. q. i. sicut. x. ad. u. et conuersim. Ad hæc in linea. q. p. vnitas, per lineam. q. o. significetur, quo facto, dicamus, si. q. p. ad. q. i. sic se habet vt. x. ad. q. o. itaque permutando, sic se habebit. q. p. ad. x. sicut. q. i. ad. q. o. hoc est. x. u. ad. x. sicut. i. q. f. ad. q. f. (nam. x. u. partes sunt integrales totius. q. p. et. x. u. ad. x. est sicut. i. q. f. ad. q. f. ex. 18. quinti) Quare ita erit. i. q. f. ad. q. f. sicut. q. i. ad vnitatem. q. o. ex. 11. quinti Addatur deinde. q. i. ad. q. f. et. q. i. per. q. f. multiplicetur, cuius multiplicationis productum, fit. x. f. quod probabo æquale esse summæ. f. q. cum. q. i. Secetur enim linea. q. x. in puncto. s. ita. vt. q. s. æqualis sit. q. o. signeturque productum. s. f. quare eadē erit proportio quantitatis. x. f. ad. s. f. quæ est. q. x. ad. q. s. ex prima sexti, aut. 18. vel 19. septimi, hoc est, sicut. q. i. ad. q. o. et ex. 11. quinti (vt dictum est) sicut. i. q. f. ad. q. f. sed numerus. s. f. superficialis tantus est, quantus linearis. q. f. quare ex. 9. quinti tantus erit (superficialiter) numerus. x. f. quantus (lineariter) . f. q. i. quod erat propositum.



T H E O R E M A XXXII.

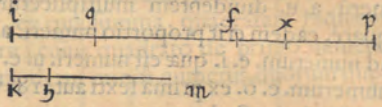
**C** V R numero aliquo in duas partes inæquales diuiso, si rursus diuidatur per singulas partes, summa duorum prouenientium per binarium, semper maior sit summa prouenientium ex diuisione vnus partis per alteram.

Exempli gratia, si proponeretur numerus. 24. qui in duas partes inæquales diuide retur.

THEOREM. ARIT.

retur. 20. scilicet et. 4. certè. 24. per singulas partes diuiso, daretur vnum proueniens sex integra, & alterum vnum & quinta pars, quorum summa esset septem integra cum quinta parte, tum altera parte per alteram diuisa, daretur vnum proueniens quinque integrorum & alterum vnus quinti tantum, quorum summa esset quinque integra, & vna quinta pars, minor prima reliquorum duorum prouenientium per binarium.

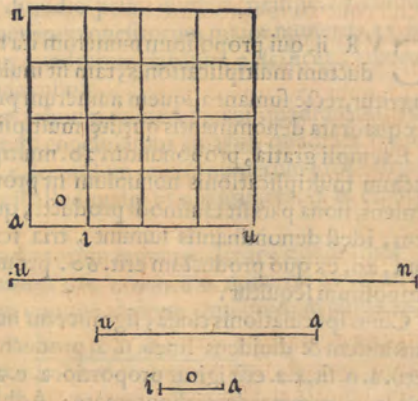
Cuius considerationis causa, propositus numerus linea. q. p. significetur, eius due partes lineis. q. x. et. x. p. r. u. q. f. sit proueniens ex diuisione totius. q. p. per. x. p. et. q. i. sit proueniens ex diuisione eiusdem. q. p. per. q. x. adhac. h. m. sit proueniens, ex diuisione. q. x. per. x. p. et. h. k. proueniens ex diuisione. p. x. per. q. x. pater igitur ex. 22. theoremate huius libri proueniens. h. m. minus esse proueniente. q. f. per vnitatem, & proueniens. h. x. minus proueniente. q. i. per alteram vnitatem. Itaque. f. q. i. maior erit. m. h. k. per numerum binarium, quod erat propositum.



THEOREM A. XXXII

QUILIBET numerus, medius est proportionalis inter numerum sui quadrati & vnitatem.

Detur enim numerus propositus, qui linea. a. u. significetur, cuius quadratum sit. u. n. vnitatis linearis sit. i. a. et superficialis. o. patebit ex. 18. sexti aut. 11. octavi proportionem. u. n. ad. o. futuram duplam proportioni. u. a. ad. i. a. sed. i. a. et. o. eadem (specie) res sunt, tanta scilicet. a. i. quanta. o. vnitatis est, Itaque proportio numeri. u. n. ad. u. a. aequalis erit proportioni. u. a. ad. i. a. Quare numerus. u. a. inter numerum. u. n. & vnitatem, medius erit proportionalis.



THEOREM A XXXIII.

Hoc ipsum quod diximus & alia ratione speculari licebit. Propositus numerus, nunc etiam per. a. u. significetur, eius quadratum per. u. n. vnitatis linearis per. a. i. productum q. a. u. in. a. i. terminetur, sitq. n. i. quare n. i. constabit numero superficiali aequali numero lineari. a. u. & ex prima sexti aut. 18. vel. 19. septimi, eadem erit proportio. u. n. ad. i. n. quae est. a. u. ad. a. i. sed numerus. a. u. cum numero. n. i. idem specie est. Itaque medius est proportionalis inter. u. n. & vnitatem.

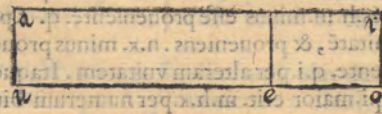
THEO-

T H E O R E M A X X X V.

Q V I V I S numerus per alterum multiplicatus, & diuifus, medius est propor-  
tionalis inter productum multiplicationis, & proueniens diuifionis.

Exempli gratia, si. 20. multiplicetur per quinque & inde per quinque diuidantur  
productum erit. 100. proueniens. 4. inter quos numeros. 20. medius est propor-

tionalis. Hoc ut speculemur, proponatur numerus multiplicandus & diuidendus, qui si-  
gnificetur linea. u. e. multiplicans autem & diuidens linea. a. u. multiplicationis  
productum fit. e. a. proueniens ex diuifione fit. o. e. Nunc proueniens. e. o. per nu-  
meru. a. u. diuidentem multiplicetur, cuius multiplicationis productum fit. e. i.  
quare, eadem erit proportio numeri. a. e. ad numerum. e. i. quæ est numeri. u. e. ad  
numerum. e. o. ex prima sexti aut. 18. vel 19. septimi. Sed cum numerus. u. e. ex  
i. theoremate presentis libri, numero. e. i. aequalis fit. verum esse, quod propofi-  
tum fuit consequetur.

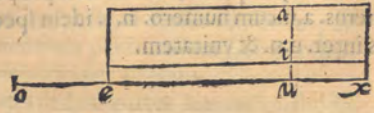


T H E O R E M A X X X V. I.

C V R ij, qui propositum numerum ita multiplicare & diuidere cupiunt, vt pro-  
ductum multiplicationis, tam fit multiplex proueniens ex diuifione, quam  
queritur, rectè sumant aliquem numerum pro multiplicante & diuidente, qui fit ra-  
dix quadrata denominantis quæ sitæ multiplicatis.

Exempli gratia, proponuntur. 20. multiplicanda atque diuidenda, ita vt pro-  
ductum multiplicationis nonuplum fit proueniens ex diuifione, nempe, vt pro-  
ueniens, nona pars fit eiusmodi producti, quare quadratam radicem ipsorum no-  
uem, idest denominantis sumunt, tria scilicet, multiplicant igitur & diuidunt  
data. 20. ex quo productum erit. 60. proueniens autem. 6. cum duabus tertijs. &  
propositum sequitur.

Cuius speculationis causa, significetur numerus propositus linea. u. e. multipli-  
cans autem & diuidens linea. u. a. productum fit. e. a. proueniens. e. o. quadratum  
verò. a. u. fit. x. a. erit igitur proportio. a. e. ad. e. o. dupla proportioni. a. e. ad nume-  
rum. u. e. ex precedenti theoremate: Adhæc, cogitemus in linea. u. a. vnitatem,  
u. i. terminenturq; duo producta. e. i. et. x. i. quare eadem erit proportio. a. e. ad. e. i.  
quæ est. a. e. ad. u. e. numerus enim. e. i. (quantus superficialis) idem est cum nume-  
ro lineari. u. e. sed. a. e. ad. e. i. sic se habet sicut. a. u. ad. u. i. ex prima sexti aut. 18.  
vel. 19. septimi, (quod ipsum dico de. a. x. ad. x. i.) quare proportio, a. x. ad. x. i. hoc  
est. x. u. equalis erit, proportioni. a. e. ad. u. e. at trigefimoterrio & trigefimo quarto theo-  
remate probatum est proportionem numeri. a. x. ad vnitatem, duplam esse propor-  
tioni eiusdem numeri. a. x. ad. u. x. sequitur  
igitur cum dimidia sint æqualia, tota etiam  
æqualia esse: hoc est proportionem numeri.  
a. e. ad numerum. e. o. æqualem esse propor-  
tioni numeri. a. x. ad vnitatem. Itaque rectè  
sumitur numerus. a. u. eiusmodi vt quadratū  
ipfius.



ipſus. a. x. tam fit multiplex ad vnitatem, quam cupimus numerum. a. e. numero. e. o. multiplicem eſſe.

THEOREMA XXXVII.

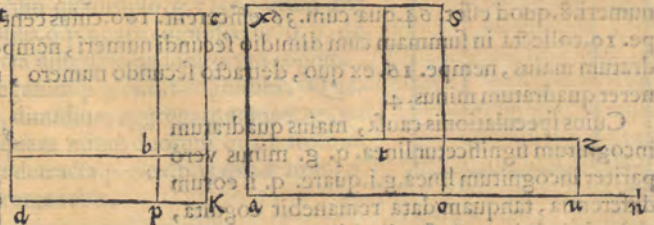
CVR inuenire cupientes duos numeros, quorum quadrata in ſummam collecta, æqualia ſint numero propoſito, & iſdem numeris multiplicatis adinuicem, productum alteri numero propoſito fit æquale, rectè ſumant dimidium primi numeri propoſiti, cui ſumma quadratorum æquari debet, hocq; dimidium in ſeipſum multiplicent, vnà etiã alterum numerum propoſitum in ſeipſum multiplicent, quod quadratum detrahunt de primo, & reſidui quadratam radicem, dimidio primi numeri propoſiti coniungunt, ex qua ſumma, quadratam radicem eruunt, quæ duobus quaſitis numeris maior erit, cuius quadrato de primo numero detracto, & ex reliquo eruta radice quadrata, detur minor numerus, duorum quaſitorum.

Exempli gratia, ſi proponerentur. 34. pro primo numero cui æquari deberet ſumma duorum quadratorum, quorum radicum productum æquale eſſe deberet alteri numero, verbi gratia. 15. iubet antiquorum regula, dimidium primi numeri in ſeipſum multiplicari, cuius dimidij quadratum erit. 289. è quo ſi detrahas quadratum ſecundi numeri, nempe. 225. remanebit. 64. atq; huius ſi quadratam radicem ſumas nempe. 8. quam dimidio primi numeri, nempe. 17. coniungas, dabitur duorum quadratorum numerorum quaſitorum maior numerus. 25. hac deinde radice è dimidio detracta, minus quadratum dabitur. 9. ſcilicet, quorum radices. 5. et. 3. eſſent ij numeri, qui quaeruntur.

Cuius speculationis gratia, cogitemus primum numerum, cui quadratorum ſumma æquari debet, ſignificari linea. a. n. tum concipiamus quaſita quadrata ſignificari, coniungiq; modo ſubſcripto. t. b. k. ſecundum porrò numerum propoſitum, ſignificari producto. d. b. Iam nil ſuper eſt aliud quam vt quantitates. d. p. et. b. p. quaeramus.

Itaque cum in linea. a. n. ſumma quadratorum numerus detur, quadratum dimidij. o. a. fit. s. a. quod nobis erit cognitum; ſit etiã. a. u. numerus quadrati maioris, et. u. n. minoris, et. a. z. productum vnus in alterum; qui quidem numerus. a. z. æqualis erit

quadrato numero  
t. b. d. b. ex. 19.  
theoremate huius  
libri. Itaq;  
a. z. cognitum  
erit, cum eius  
radix. d. b. fit ſe-  
cundus numerus  
propoſitus, qua

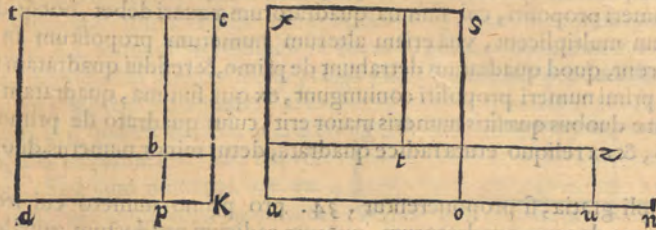


minor erit. a. s. ex quinta ſecundi, aut ſeptima conſequentia poſt. 16. noni Euclidis. Iam ſubtracta quantitate. z. a. è quadrato. a. s. cognoscetur quadratum. t. x. cuius radix æqualis erit. o. u. ex poſtremo adductis, Itaque cognoscemus. o. u. qui numerus coniunctus dimidio. o. a. cognito, dabit quadratum. a. u. cognitum; atque ita. u. n. pariter cognoscetur, & eorum radices conſequenter.

Hoc

Hoc ipsum & alia ratione perfici potest, nempe, iuncta summa.  $\kappa. b : b. d : et. b. r.$  alteri rectangulo æquali.  $b. d.$  quod sit.  $b. c.$  ex quo totum quadratum lineæ.  $d. \kappa.$  cognitum erit, atq; ita etiam consequenter eius radicem.  $d. \kappa.$  cognoscemus, cuius ope ac producti.  $d. b.$  cognoscemus.  $d. p. et. p. k.$  prout ex theoremate quadragesimo quinto huius libri patebit.

Michael Stifelius, vndecimo cap. tertij libri, problema eiusmodi proponit, quod tamen ipse via algebræ dissolvit.

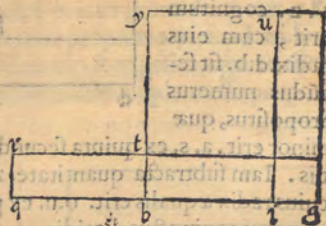


**T H E O R E M A X X X V I I I.**

**C** V R ij, qui duos numeros inuenire volunt, quorum productum alicui numero proposito æquetur, & quadratorum eorundem differentia alteri numero proposito æqualis fit. Rectè dimidium secundi numeri propositi in seipsum multiplicent, cui quidem numero differentia quadratorum æquari debet; porro huic quadrato primi propositi numeri, cui æquandum est productum numerorum quæsitum, quadratum adiungant; tum radicem quadratam huius summe copulet dimidio secundi numeri propositi, ei inquam, cui differentia quadratorum æqualis esse debet, ex quo quadratum maius confurgit, à quo, detracto secundo numero, superest quadratum minus.

Exempli gratia, si proponeretur primo loco numerus. 8. cui æquandum est productum numerorum quæsitum, tum proponeretur numerus. 12. cui, detracto minore à maiore, differentia quadratorum vtriusque quæsitum numeri æqualis esse debet, oportet huius ultimi numeri. 12. dimidium in seipsum multiplicare, fietque. 36. quadratum dimidij, vnde in summam colligeremus quadratum primi numeri. 8. quod esset. 64. quæ cum. 36. efficerent. 100. cuius centenarij radice, nempe. 10. collecta in summam cum dimidio secundi numeri, nempe. 6. daretur quadratum maius, nempe. 16. ex quo, detracto secundo numero, nempe. 12. remaneret quadratum minus. 4.

Cuius speculationis causa, maius quadratum incognitum significetur linea.  $q. g.$  minus verò pariter incognitum linea.  $g. i.$  quare.  $q. i.$  eorum differentia, tanquam data remanebit cognita, vna etiam.  $b. i. et. q. b.$  sua dimidia; tunc cogitatur quadratum.  $y. g.$  super.  $b. g.$  et parallelogrammum rectangulum.  $g. r.$  designatum, & ita etiam gnomon.  $u. g. r.$  prout sexta secundi Euclidis proponitur, ex quo quadratum.  $b. i.$  nempe.  $u. t.$  cognitum erit, sed gnomon æqualis est rectangulo.  $g. r.$  ex prædicta, aut ex. 8. post. 16. noni,





THEOREMA A ROIT.

noni, hocq; rectangulum. g. r. quadratum est primi numeri propositi ex. 19. theoremate huius libri; itaq; cognitum erit; vna etiam gnomon. u. g. r. cognoscetur; quare totum quadratum. g. y. eiusq; radix. b. g. manifesta erit; cui coniuncta. q. b. data, maius quadratum. q. g. cognoscetur, ex qua. b. g. detracta. b. i. data; cognoscetur. i. g. quadratum minus consequenter, etiam eorum radices notæ erunt.

THEOREMA XXXIX.

ALIA etiam ratione idipsum definiiri potest, prætermiffa antiquorum via; nempe multiplicatis in femetipsis primo & secundo, numeris propositis, quadruplicatoq; quadrato primi, qua summa coniuncta cum quadrato secundi numeri, & ex hac altera summa eruta radice quadrata, ex qua detracto secundo numero, & è reliquo sumpto dimidio, quod erit quadratū minus, quo detracto ex radice postremo iuncta, supererit quadratum maius.

Exempli gratia, si proponeretur numerus. 8. cui productum duorum numerorum quaesitorum æquandum est, proponeretur idem. 12. cui differentia quadratorum duorum numerorum æqualis esse debet. Tubeo primum numerum, nempe. 8. in se ipsum multiplicari, ex quo exurget. 64. pro numero sui quadrati, quod quadruplicari volo, eritq; productum. 256. quod cenfeo coniugendum cum quadrato secundi numeri propositi, nempe. 144. eritq; summa. 400. ex qua sumetur radix, sci licet. 20. & ex hac detrahetur secundus numerus. 12. residuiq; dimidium, nempe; 4. pro quadrato minore, quo in summam collecto cum. 12. dabit quadratum maius. 16.

Cuius speculationis causa, quadratum maius per lineam. q. g. minus per. g. p. significetur: super integram autem. q. p. erigatur quadratum integrum. d. p. diuisum, vt quadratum. f. g. vigesimiseptimi theorematiss huius libri, (idipsum accideret diuiso quadrato modo octauæ secundi Euclidis) quæ quidem diuisio, est via quatuor productorum. q. g. in. g. p. è quibus vnum sit. g. r. quod erit cognitum ex. 19. theoremate cum sit quadratū primi numeri propositi, ex quo illa quatuor cognita erūt. Iam verò si cogitemus. q. p. sectam in puncto. t. ita vt. q. t. æqualis sit. p. g. dabitur differentia. t. g. cognita, vt radix quadrati. e. o. cum ex præsupposito. r. n. æqualis sit. q. g. et. r. e. g. p. ex quo etiam. q. t. ita pariter. e. n. t. g. æqualis erit. Collecto itaq; quadrato. e. o. ipsius. t. g. cum quadruplo. g. r. cognitum erit quadratum. d. p. ipsius. q. p. quare cognoscetur. q. p. de quo numero detracta differentia quadratorum cognita. t. g. supererit aggregatum. p. g. et. q. t. cognitum. Quare ex consequenti, dimidium aggregati, nempe. g. p. cognoscetur, tanquam minus duorum quadratorum. cui iuncta. g. t. aut detracta. p. g. ex. p. q. quadratum. q. g. maius cognitum remanebit.



THEOREMA XL.

CVR ijs, qui volunt duos eiusmodi numeros inuenire, vt eorum maior minore, numero proposito superet, & productum vnus in alterum, alteri numero proposito adæquetur, consultiſſimum sit dimidium primi numeri propositi; D numerum

numerum inquam, cui differentia duorum quæstorum æquanda est, in seipsum multiplicare, atque huic quadrato, secundum numerum propositum iungere, cui productum numerorum quæstorum æquale esse debet, & ex hac summa eruere quadratam radicem, quæ coniuncta dimidio primi numeri propositi, dabit maiorem duorum numerorum & ex eadem radice detracto dimidio primi numeri, minorem numerum duorum quæstorum.

Exempli gratia, si proponeretur. 12, cui differentia vnus numeri ab altero æquari deberet, tum proponeretur. 64, cui productum multiplicationis duorum quæstorum simul æquandum esset. Dimidium primi numeri in seipsum multiplicarem, proueniretq; quadratū. 36. cui coniuncto secundo, nempe. 64. totum esset. 100. ex quo detracta quadrata radice. 10. et ipsi coniuncto senario, dimidio primi numeri, & ex eadem detracto eodem dimidio. 6. pro maiore numero. proueniret. 16. & pro minore. 4.

Cuius rei speculatio hæc est. Sit. e. o. differentia cognita duorum incognitorum numerorum. a. o. et. a. e. quorum productum datum siue cognitum sit. a. s. consideremus nunc. e. i. dimidium. e. o. datæ differentiæ, & ex composito. a. i. imaginetur quadratam. a. x. in quo protracta sit. t. u. æquidistans lateri. a. i. & tam ab ipsa. a. i. remota, quam. x. i. ab. s. e. vnde. t. e. quadratum erit. e. i. dimidiæ scilicet differentiæ datæ, e. o. et. t. n. rectangulum æquale erit rectangulo. n. e. vt cuilibet licet per se considerare, vnde sequitur gnomonem. e. r. t. æqualem esse producto. a. s. ideo cognitum, qui quidē gnomon, si coniunctus fuerit quadrato. e. t. cognitio ex radice. e. i. cognita ( vt dimidia totalis differentiæ. e. o. datæ) habebimus quadratarum totale. a. x. cognitum, & ita eius radicem. a. i. cognitam & reliqua omnia consequenter quæ quidem speculatio eadem est quæ. 6. secundi seu. 8. noni Euclidis.

Porēis tamen ex modo & rationibus præcedenti theoremate. allatis, hoc ipsum concludere.

### T H E O R E M A X L I.

**C**VR ij, qui aliquo proposito numero, inuenturi sunt duos numeros inter se differentes, quorum quadratorum summa altero numero proposito æqualis sit, rectè primum numerum propositum in seipsum multiplicant, quod quadratum, ex secundo numero detrahūt, & dimidium residui sumunt, quod productum erit multiplicationis duorum numerorum inter se, in reliquis præcedentis theorematum ordinem sequuntur.

Exempli gratia, si proponeretur. 12. tanquam numerus, cui differentia duorum numerorum quæstorum æquanda est, proponerentur præterea. 272. quibus summa quadratorum duorum numerorum quæstorum æquari deberet, oporteret sanè primum numerum, nempe. 12. in seipsum multiplicare, cuius quadratū hoc loco esset. 144. atque hoc detrahere ex secundo numero, superesset. 128. sumpto deinde dimidio huiusce numeri, nempe. 64. producto inquam duorum numerorum quæstorū. Cum hoc. 64. postea et duodenario primo proposito numero, præcedentis theorematum ordinem sequeremur.

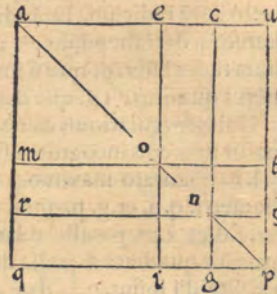
Quod



**T H E O R E M . A R I T .**

27

Quod ut speculemus, consideremus subscriptam figuram, vigesimoini theorematis figuræ similem, in qua numeri quæsitæ duabus lineis directè coniunctis. q. g. et. g. p. significantur, horum quadrata erunt. r. c. et. g. s. quorum summa iterum proponitur, quare etiam cognita. Differentia autem duorum numerorum primo proposita sit. q. i. eius verò quadratum. m. e. quod cognitum est ex sua radice. q. i. quare gnomon. e. n. m. simul cum quadrato minori. g. s. cognitus erit, quæ summa æqualis est duplo. g. r. producto datorum numerorum. Itaque & ipsa. g. r. cognoscetur, nunc si præcedentis theorematis speculationem in reliquis consuluerimus propositum consequemur.



**T H E O R E M A X L I I .**

**A** Dhuc etiam & alia ratione id ipsum consequi possemus, non consulto quadragesimo theoremate. Nam subtracto quadrato differentie, numeri primi (inquã) propositi, ex summa duorum quadratorum, nempe ex secundo numero proposito colligendum esset residuum in summam cum prædicto secundo numero, & ex summa hac defumenda quadrata radix, quæ duorum numerorum summa erit, de qua detracto primo numero, remanebit duplum minoris numeri quæsitæ, cuius dimidio addito primo numero proposito, aut detracto minore inuento ex radice postremo inuenta, dabitur numerus maior, qui quæritur.

Exempli gratia, cum super fuerint. 128. hæc si cum secundo numero nempe. 272. iunxerimus, dabunt. 400. quorum radix erit. 20. de quo numero detracto primo proposito, nempe. 12. supererunt. 8. quorum dimidiũ erit. 4. quo ex. 20. detracto aut coniuncto. 12. maior numerus orietur.

Cuius rei contemplatio, præcedenti figura aperitur. Nam residuum detractionis quadrati. m. e. ex summa duorum quadratorum. r. c. et. g. s. numerum præbet æqualem duobus supplementis. q. n. et. n. u. ex. 8. secundi Euclidis. qui coniunctus duobus quadratis (quorum summa secundo proposita fuit) cognitionem profert quadrati. q. u. & eius radice. q. p. de qua, detracto primo dato numero, scilicet. q. i. superest. i. p. cuius dimidium nempe. g. p. minor est numerus qui quæritur; residuum verò totius. g. q. maior scilicet.

**T H E O R E M A X L I I I .**

**C** VR ij, qui volunt duos numeros inuenire, quorum summa æqualis proposito alicui numero futura sit, & summa quadratorum maior eorum producto per quantitatem alterius propositi numeri, rectè dimidium primi dati numeri in seipsum multiplicant, quod quadratum ex secundo dato numero detrahunt, sumunt quæ tertiæ partis residui quadratam radicem, quam dimidio primi numeri coniungunt, ex quo maior numerus duorum quæsitõrum datur, quo ex toto primo detracto, supererit minor.

Exempli gratia, proposito numero. 20. cui æquanda est summa duorum numerorum quæsitõrum, datoq; secundo numero, 208. qui semper maior esse debet

D 2 quadrato

quadrato dimidij, prout ex speculatione huiusmodi operis cognoscetur, cui æquãda est differẽtia inter summã quadratorũ duorũ qui quærũtur numerorũ, simul cũ producto eorũ radicũ. Dimidium numeri. 20. in seipsum multiplicandum esset, quadratumq; detrahendum ex. 208. vt remanerent. 108. quorum. 108. tertiã partis quadrata radix esset. 6. quæ si iuncta fuerit dimidio. 20. nempe. 10. daretur maior numerus quæsitus. 16. quo detracto è. 20. darentur. 4.

Cuius speculationis causã, datus primus numerus significetur linea. g. h. in qua maior numerus incognitus sit. g. h. minor verò. b. h. quorum quadrata sint. y. t. et. b. l. in quadrato maximo. g. p. tum productum. g. b. in. b. h. fit. g. c. cogitenturq; duo diametri. q. h. et. g. p. diuisi per medium in puncto. o. per quod duẽ lineã ducantur. f. d. et. k. m. parallelæ lateribus maximi quadrati. Hæ dictum quadratum in quatuor quadrata æqualia diuident, quorum vnumquodq; æquale erit quadrato. g. f. dimidij ipsius. g. h. datẽ, quare eorum vnumquodq; cognitum erit. Iterum cogitemus. s. x. per. e. parallelã. g. k. tantum distantem à. g. k. quantum. y. l. ab. g. h. distare inuenitur. Cogitetur pariter. z. i. a. per punctum. i. parallelã. d. p. quare. a. t. æqualis erit. f. c. et. y. x. æqualis. f. e. et. y. s. b. l. æqualis. Ita subtractis è duobus quadratis superius dictis. a. t. y. x. et. b. l. producto. y. b. æqualibus, supererunt. k. d. et. a. c. x. cognita, tanquam æqualia dato secundo numero, sed. k. d. quadratum est medietatis. g. f. cognita, cognoscetur igitur residuum. a. c. x. vnã etiam singulã tertiã partes nempe quadrata. o. i. o. c. et. o. e. & radix. b. f. vel. f. s. singularum, qua coniuncta dimidio. g. f. rursumq; ab eodẽ detracta, propositum consequemur.



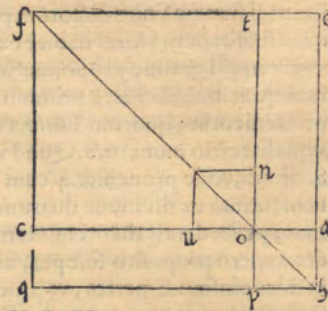
### T H E O R E M A X L I I I.

**C**VR si quis cupiat numerum propositum in duas eiusmodi partes diuidere, vt quadratum maioris, quadratum minoris superet quantitate alterius numeri propositi, rectẽ primum numerum in seipsum multiplicabit, & ab eodem secundum numerum detrahet, residuum verò per duplum primi diuidet, ex quo proueniens primi pars minor erit, quæ ex illo primo detracta, partem maiorem proferet.

Exempli gratia, si proponantur. 20. diuisa in duas eiusmodi partes, vt quadratũ maioris superet quadratum minoris numero æquali ipsi. 240. oportebit primum numerum, qui quadratus cum fuerit, erit. 400. in seipsum multiplicare, & ex hoc quadrato secundum numerum nempe. 240. detrahere, tunc remanebunt. 160. quẽ diuisa per. 40. numerũ duplũ primo, dabuntur quatuor pro minori numero, à residuo verò. 20. detractis quatuor, erunt. 16. pro maiori numero.

Quod vt exactẽ consideremus, primus numerus propositus significetur linea. q. h. diuidendus in duas partes. q. p. et. p. h. tales quales quærimus. Postmodum erigatur quadratum. q. c. diuisum diametro. f. h. ductisq; p. o. t. et. a. o. c. parallelis lateribus quadrati, dabuntur imaginaria quadrata. c. t. et. p. a. duarum partium. q. p. et. p. h. incognitarum. Ad hæc cogitemus quadratum. u. n. æquale quadrato. p. a. è quadra

to maiore. c.t. extractum quare residuum quadrati. c.p. cognitum erit, quam quantitatem cognitam, cum sit secundo loco data, cogitemus detrahi è toto quadrato cognito. q. e. ex quo summa duorum supplementorum. q. o. et. o. e. cognoscetur, vnà cum quadratis. u.n.et.p.a. duplo scilicet. q. a. quo diuiso per duplum. q. h. aut simplex. q. a. per. q. h. simplicem, dabitur. a. h. nempe. p. h. minor numerus quæsitus.

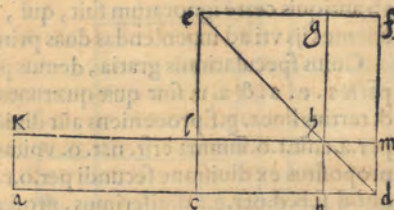


T H E O R E M A X L V.

**C**VR volentes diuidere numerum propositum in duas eiusmodi partes, vt productum vnus in alteram, alteri numero proposito æquetur, rectè dimidium primi dati numeri in seipsum multiplicat, ex quo quadrato secundum datum numerum detrahunt, residuq; radicem sumunt, qua coniuncta vni dimidio primi numeri, pars maior datur, ex altero verò dimidio detracta, minorem manifestabit.

Exempli gratia, si numerus partiendus esset. 34. alter verò numerus esset. 64. cui productum vnus partis in alteram æquale esse deberet. Dimidium primi numeri, in seipsum multiplicarem, cuius quadratum esset. 289. de quo detracto secundo numero nempe. 64. remaneret. 225. cuius quadrata radix nempe. 15. coniuncta. 17. dimidio. 34. proferet. 32. maiorem partem, detractoq; ex. 17. superesset. 2. pars inquam minor.

Cuius speculationis causa, primus numerus propositus significetur linea. a. d. cuius dimidium. c. d. cognitum erit, vnà etiam eius quadratum. c. f. quo diuiso per diametrum. e. d. supponantur partes ignote ipsius. a. d. esse. a. b. et. b. d. & à puncto. b. duci lineam. b. h. g. parallelam. d. f. et. m. h. k. parallelam. d. a. extructa figura simili figuræ quintæ secundi Eucli. quare dabitur gnomon. l. d. g. æqualis producto. b. k. & proinde cognitus, quo detracto è quadrato. c. f. remanebit quadratum. g. l. cuius radice æquali. c. b. coniuncta. a. c. & detracta ex. c. d. partes. a. b. et. b. d. quæsitæ dabuntur.



T H E O R E M A X L V I.

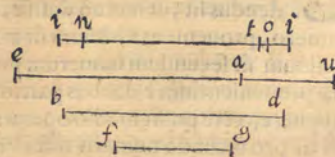
**C**VR propositis tribus numeris, quorum prior in duas eiusmodi partes diuidendus sit, ut mutuò diuisæ, & per summam prouenientium diuiso secundo numero, proueniens vltimum sit æquale tertio numerorum propositorum. Consul tissimum sit secundum numerum per tertium diuidere, ex quo proueniens sit summa prouenientium è duabus partibus mutuò diuisis, quam summam si quis velit distinguere, rectè possit medio operationis præcedētis theorematissimæ supra vnitatem superficiali pro secundo numero distinctis postmodam prouenientibus, rectè meo iudicio operabimur per regulam de tribus (quod fuit ab antiquis prætermissum) Si dixerimus,

rimus, si summa vnus dictorum prouenientium cum vnitate dat primum numerum, quid ipsa eadem vnitas dabit? ex quo propositum oriatur.

Exempli gratia, proponuntur tres numeri, primus. 20. secundus. 34. tertius. 8. Iam querimus diuidere primum. 20. in duas partes quæ mutuò diuisæ præbeant duo prouenientia, quorum summa tanta sit vt per eam diuiso. 34. proueniat numerus æqualis tertio numero. 8. Quod vt præstemus iubet regula secundum. 34. per tertium. 8. diuidi, vnde proueniet. 4. cum vna quarta parte, quod proueniens erit summa prouenientium ex diuisione duarum partium quæ sitare, quæ si distinguere voluerimus, præcedentis theorematis methodum sequemur, vnitate superficiali pro secundo numero proposito sumpta, ac si diceremus, diuidatur. 4. cum vna quarta parte in duas eiusmodi partes, vt productum vnus in alteram sit vnitas superficialis, certè fractis integris cum quarta parte coniungendis, darentur vnitatis decemseptem quartæ lineares, verum cum necesse sit, ex præcedenti theoremate, dimidium in seipsum multiplicare, essetq; dimidium. 8. quartarum partium cum octaua, commodius totum constituetur. 34. octauarum, quarum dimidium, nempe decemseptem octauæ, in seipsum multiplicatum erunt. 289. sexagesimæ quartæ vnus integri superficialis, quandoquidem integrum superficiale, cuius vnitas linearis in. 8. partes diuiditur est. 64. vt ex primo theoremate huius libri deprehendi potest. Nunc vnitate hac superficiali, nempe. 64. ex. 289. detracta, supererit. 225. cuius radix quadrata, scilicet. 15. coniuncta dimidio dictorum prouenientium, nempe. 17. dabit maius proueniens. 32. detracta q; ex altero dimidio, dabit proueniens minus. 2. hoc est pro maiore proueniente. 32. octauas, & pro minore duas, quatuor scilicet integros pro maiore, & quartam partem vnus integri pro minore. Nunc si ex regula de tribus dixerimus, si. 4. iuncta vni, nempe. 5. dant. 20. primum numerum, quid dabunt. 4. integra (proueniens inquam maius) dabunt certè. 16. partem maiorem. Tum si dixerimus, si quarta pars coniuncta vnitati dat. 20: quid dabit quarta illa pars (hoc est proueniens minus) dabit pfectò quatuor scilicet minorem partem, quod ab antiquis certè ignoratum fuit, qui, inuentis prouenientibus quieuerunt, nescientes ijs vt ad inueniendas duas primi numeri partes.

Cuius speculationis gratia, demus primum numerum significari lineam. e. u. cuius partes. e. a. & a. u. sint quæ queruntur, alter verò numerus significetur lineam. b. d. tertius lineam. g. f. proueniens aut diuisionis. e. a. per. a. u. sit. n. t. diuisionis aut. a. u. per. a. e. sit. t. o. summa erit. n. t. o. vnitas verò. n. i. et. o. i. Iam si numerus. f. g. tertio propositus ex diuisione secundi per. o. t. n. proferri debet. Ex. 13. theoremate patet, quod si. b. d. per. g. f. diuiserimus, proferetur. o. t. n. qui cum fuerit inuentus, summam esse oportet duorum prouenientium, ex diuisione mutua duorum numerorum, nempe. a. e. per. a. u. et. a. u. per. a. e. deinde manifestum est ex. 24. aut. 25. theoremate eorum productum (multiplicatis prouenientibus adinuicem) vnitatem superficiale futuram esse. Haec tenus igitur, totum. o. n. ex doctrina præcedentis theorematis diuiditur in puncto. t. ita vt productum. o. t. in. t. n. solam vnitatem superficiale contineat, quo facto, si, vt antedictum est, cogitauerimus. n. t. proueniens esse ex diuisione. e. a. per. a. u. et. t. o. proueniens ex diuisione. a. u. per. a. e. patebit ex definitione diuisionis, quod eadem erit proportio. a. e. ad. n. t. quæ est. a. u. ad vnitatem. n. i. et. a. u. ad. o. t. eadem quæ est. e. a.

ad



THEOREM. ARITH.

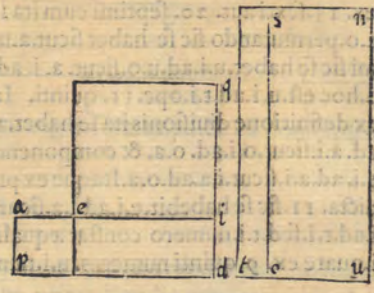
ad unitatem. o. i. permutando q. e. a. ad. a. u. sicut. t. n. ad. n. i. & componendo. e. a. u. ad. a. u. sicut. t. n. i. ad. n. i. & euerfim. e. a. u. ad. e. a. vt. t. n. i. ad. t. n. Quare, ex. 20. septimi, recte utimur regula de tribus. Idem & de altera parte dico, quamuis qui vnam teneat, alteram quoque habiturus sit. Non mirum tamen si huiusmodi problema ab antiquis definitum non fuerit, qui hanc ultimam partem non cognouerunt.

THEOREMA XLVII.

**C**VR duobus numeris mutuò diuisis, si per summam prouenientium, productum vnus in alteram multiplicetur, vltimum productum, summæ quadratorum duorum numerorum æquale futurum sit.

Exempli gratia, propositis. 16. et. 4. mutuò diuisis, summa prouenientium erit. 4. integrorum cum quarta parte, qua summa multiplicata cum producto primorū numerorum, nempe. 64. dabuntur. 272. integri superficiales, qui summæ quadratorum duorum numerorum æquantur.

Hoc vt consideremus, duo numeri partibus. a. e. et. e. i. in linea. a. i. significantur, quorum productum sit. e. d. & quadratū ipsius. a. e. sit. e. p. ipsius verò. e. i. sit. e. q. proueniens aut ex diuisione. e. i. per. a. e. sit. o. u. proueniens aut. a. e. per. e. i. sit. o. t. quorum summa sit. o. u. t. tum productum. e. d. linea. u. n. significetur ad angulum rectū coniuncta in puncto. u. extremo ipsius. o. u. t. productum aut. u. o. t. in. u. n. sit. n. t. Iam probandum nobis est. n. t. æqualem esse summæ duorum quadratorum. q. e. p. Quod singillatim probo, & afferò productum. o. n. æquale esse quadrato. q. e. & productū. s. t. quadrato. e. p. Nam ex. 35. theoremate patet numerum. e. i. medium esse proportionale inter. e. d. et. o. u. cum numerus. e. i. ex præsupposito ab. e. a. multiplicetur & diuidatur, cuius multiplicationis productum est. d. e. nempe. u. n. & proueniens ex diuisione est. o. u. quare ex dicto theoremate. e. i. media proportionalis est inter. u. n. et. u. o. Itaq; productum. o. n. æquale est quadrato. e. q. ex. 16. sexti vel. 20. septimi. Idem dico de producto. s. t. nempe æquale esse quadrato. e. p. quandoquidem numerus. a. e. ab. e. i. multiplicatur ac diuiditur, cuius multiplicationis productum est. d. e. nempe o. s. & proueniens ex diuisione. o. t. inter quæ ex. 35. theoremate. a. e. media proportionalis est. Quare ex allatis propositionibus productū. s. t. æquale est quadrato. e. p. sed totū productum. n. t. summa est duorum productorum. o. n. et. s. t. ex prima secundi Eucli. Itaque verum esse quod dictum est, consequitur.

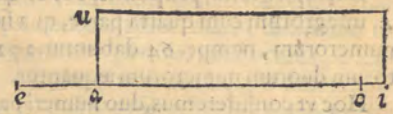


THEOREMA XLVIII.

**C**VR si quis maiorem duorum numerorum sola unitate inter se differentium, per minorem diuidat, maioremq; per proueniens multiplicet, productum summæ ipsius maioris cum eodem proueniente æquale erit.

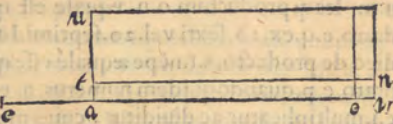
Exempli gratia. 10. per. 9. diuiso, datur vnum cum nona parte, quo multiplicato per proueniens, ipso nempe. 10. datur productum. 11. cum nona parte, tantum scilicet

licet quanta summa est maioris cum proueniente .  
 Cuius speculationis causa, maior numerus significetur . a . i . et minor linea . a . o . ex  
 quo ex praesupposito . o . i . vnitas erit . Sit autem proueniens ex diuisione . a . i . per . a . o .  
 a . e . quod . e . a . directe coniungatur ipsi . a . i . et productum . a . i . in . a . e . fit . u . i . Probabo  
 numerum superficialem . u . i . aequalem esse lineari . i . a . e . quare meminisse oportet,  
 decimotertio theoremate probatum fuisse , quod si numerus diuisibilis per pro-  
 ueniens diuidatur , proueniens futurus sit numerus diuidens , quare . a . o . erit pro-  
 ueniens ex diuisione . a . i . per . a . e . & ex definitione diuisionis ita se habebit . e . a . ad .  
 a . i . sicut . o . i . ad . o . a . & componendo ita . e . i . ad . a . i . sicut . i . a . ad . o . a . quare . a . i . erit me-  
 dia proportionalis inter . e . i . et . a . o . sed . a . i . non modò diuisa nunc cogitatur ab . e . a . ex  
 quo sit proueniens . a . o . sed etiam per eandem . e . a . multiplicata , ex quo produ-  
 ctum oriatur . u . i . Itaq; ex . 25 . theorema-  
 te . a . i . media est proportionalis inter . u . i .  
 i . et . a . o . Quare . ex . 11 . quinti . eadem erit  
 proportio . u . i . ad . a . i . sicut . e . i . ad eandem .  
 a . i . Igitur ex . 9 . praedicti numerus . u . i .  
 aequalis erit numero . e . i . quod erat propositum .



THEOREMA XLIX.

Dipsum etiam alia ratione considerari potest .  
 Linea . u . a . fecetur in puncto . t . ita vt . a . t . aequalis sit vnitati . o . i . & media paral-  
 lela . t . n . terminetur productum . t . i . quod constabit aequali numero , quamuis superfi-  
 ciali , numero . a . i . tamen si lineari . Tum parallela ducatur à puncto . o . ipsi . a . u . termi-  
 neturq; productum . o . u . ex quo bina producta dabuntur . u . o . et . t . i . inter se aequalia  
 ex . 15 . sexti aut . 20 . septimi cum ita se habeat . a . i . ad . a . u . sicut . a . o . ad . a . t . sed . a . i . ad .  
 a . o . permutando sic se habet sicut . a . u . ad . a . t . & ex prima sexti aut . 18 . vel . 19 . septi-  
 mi sic se habet . u . i . ad . u . o . sicut . a . i . ad . a .  
 o . hoc est . u . i . ad . t . i . ope . 11 . quinti . Iam  
 ex definitione diuisionis ita se habet . a . e .  
 ad . a . i . sicut . o . i . ad . o . a . & componendo .  
 e . i . ad . a . i . sicut . i . a . ad . o . a . Itaque ex praedicta . 11 . sic se habebit . e . i . ad . i . a . sicut . u . i .  
 i . ad . t . i . sed . t . i . numero constat aequali . a .  
 i . quare ex . 9 . quinti numerus . u . i . numero . e . i . aequalis erit .



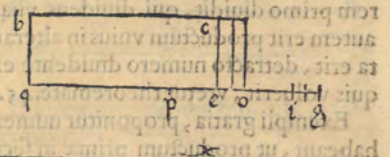
THEOREMA L.

VR diuidentes numerum propositum in duas eiusmodi partes , vt productū  
 vnus in alteram cum ipsarum differentia in summam collectum , aequale sit  
 alicui alteri numero maiori primo . Rectè primum ex secundo detrahunt , residuum  
 verò conseruant , tum ex primo semper binarium desumunt , dimidiumq; conser-  
 uant , alterum verò dimidium in seipso multiplicant , & ex quadrato numerum con-  
 seruatum eruunt , residuiq; radicem ex dimidio conseruato , quod vltimum resi-  
 duum propositi numeri quaesita pars minor est .  
 Exempli gratia , si proponatur numerus . 20 . ita diuidendus , vt productū vnus partis  
 in alteram , cum partium differentia collectum in summam , aequale sit proposito  
 nume-



numero, verbi gratia. 92. præcepit regula detrahi primum numerum ex secundo, nempe. 20. ex. 92. cuius residuum, scilicet. 72. conseruetur, tum detrahi iubet binarium ex primo, sic in proposito exemplo remanebunt. 18. huius autem. 18. dimidium in seipsum multiplicari iubet, quod cum sit. 9. datur numerus. 81. ex quo. 81. primum numerum conseruatum, nempe. 72. vult regula detrahi, sic remanebit. 9. tum huius. 9. quadrata radix detrahenda est ex dimidio ipsius. 18. quod fuit ante quadratum, sic supererit. 6. hoc est. 9. excepta radice quadrata, qui. 6. erit minor pars quaesita, maior verò. 14. quarum productum. 84. coniunctum cum partium differentia præbet exactè. 92.

Cuius rei hæc est speculatio. Primus numerus minor, qui proponitur diuisibilis significetur linea. q. g. maior vero linea. x. rum cogitemus. q. g. diuisam, cuius maior pars sit. q. o. minor. o. g. differentia. q. p. ex quo. p. o. æqualis erit. o. g. sit autem productum. b. o. Oportet igitur, ut. b. o. simul cum differentia. q. p. æquale sit numero. x. secundo proposito, qui notus est, quare etiam summa producti. b. o. cum differentia q. p. cognita erit, ex qua detracto primo numero. q. g. residuum cognitum erit, nunc igitur quodnam erit hoc residuum? attendamus qua ratione. ex summa. b. o. et. q. p. detrahenda sit. q. g. In primis si subtraxerimus ex dicta summa. q. p. que pars est. q. g. supererit detrahenda. p. g. ex. b. o. pars inquam ipsius. q. g. quod fiet quotiescunque cogitauerimus. q. o. duabus vnitatibus diminutam, et per. o. g. multiplicatam, sit autem productum. b. e. nam cum. o. g. toties. b. o. ingrediatur, quot sunt in. q. o. vnitates ex prima sexti aut. 18. vel. 19. septimi, detrahendaq; sit. p. g. ex. b. o. qua. p. g. dupla est. o. g. patebit. o. c. æqualem esse. p. g. supererit itaque. b. e. productum. q. e. in. e. i. cognitum, eritis autem ex. q. g. iisdem duabus vnitatibus, remanebit. q. i. nobis nota, ex quo. e. i. æqualis erit. e. c. Cum igitur productum. q. e. in. e. i. cognoscamus simul cum. q. i. Si voluerimus partes. q. e. et. e. i. cognoscere, vtemur. 45. theoremate huius libri, & propositum obtinebimus, nam cognoscemus. e. i. & ex consequenti. o. g. eius æqualem.



THEOREMA L. I.

**D**IVIDERE numerum in duas eiusmodi partes, quæ pro medio proportionali alterum numerum propositum recipiant, primi dimidio minorem, aliud nihil est, quàm binas primi numeri partes inuenire, quæ inter se multiplicatæ quadrato secundi numeri numerum æqualem proferant, ex. 16. sexti aut. 20. septimi, quod tamen. 45. theoremate fuit à nobis speculatum.

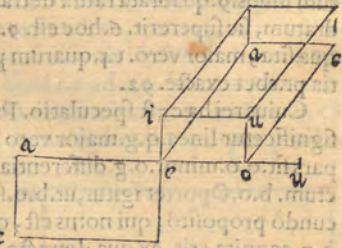
THEOREMA L. II.

**C**VR. propositis tribus numeris quibuscunque, si productum primi in secundum per tertium multiplicetur, atque secundum hoc productum corporeū, per primum numerum diuidatur, proueniens erit numerus æqualis producto secundi in tertium.

Exempli causa, proponantur hi tres numeri. 10. 11. 12. multiplicenturq;. 10. cu.

E 11.

11. dabuntur. 110. quo producto multiplicato cum. 12. dabuntur. 1320. hoc pro  
 ueniens per primum nempe. 10. diuisum dabit. 132. numerum æqualem producto  
 secundi in tertium numerorum propositorum, scilicet. 132. *i. b. i.*  
 Hoc ut speculemur, primus numerus significetur linea. o. u. secundus. e. o. tertius.  
 e. a. productum verò. o. u. in. o. e. fit. o. i. ipse uero. o. i. per. e. a. productum corporeum fit. i. c. tum  
 productum. e. o. in. e. a. fit. e. c. Dico nunc quod di-  
 uiso numero corporeo. i. c. per primum. o. u. proue-  
 niens æquale erit numero producti. e. c. Qua-  
 re in primis cogitandum est, quod cum produ-  
 ctum. i. c. ortum fuerit ex multiplicatione. o. i.  
 in. e. a. dictum. o. i. toties ingreditur. i. c. quo-  
 ties unitas reperitur in. e. a. eadem ratione, to-  
 ties. e. c. in. i. c. quot unitates erunt in. o. u. Itaq;  
 sequitur quod diuiso. i. c. per. o. u. proueniens fit  
 e. c. corporeum, æquale nihilominus producto. e. c. superficiali.



THEOREMA LI. I.

**C**VR diuidens propositum numerum in tres partes sic se habentes ut produ-  
 ctum primi in secundam, in tertia multiplicatum, præbeat numerum alteri nu-  
 mero proposito æqualem. Rectè secundum numerum per quemcunque alium mino-  
 rem primo diuidit, qui diuidens vna erit ex tribus partibus quæ sitis, proueniens  
 autem erit productum vnus in alteram reliquarum duarum, quarum summa cogni-  
 ta erit, detracto numero diuidente ex primo dato, quam quidem si distinguere  
 quis voluerit, vteretur theoremate. 45.

Exempli gratia, proponitur numerus. 20. in tres partes diuidendus, quæ sic se  
 habeant, ut productum primæ in secundam in tertia multiplicatum det. 90. itaque  
 sumenda erit pro prima vna pars ipsius. 20. quæcunque illa sit, verbi gratia. 2. qua  
 secundus numerus, nempe. 90. diuidatur, dabitur igitur. 45. quod erit productum  
 cæterarum partium inter se, quarum summa est. 18. quam summam si distinguere  
 volueris in cæteris duabus partibus separatis, vteris. 45. theoremate, ut quàm citif-  
 sumè quod cupis exequaris, erunt autem partes. 3. et. 15.

In cuius speculationis gratiam nihil aliud occurrit, quàm quod præcedenti theo-  
 remate, & superiore. 45. allatum est.

THEOREMA LI. II.

**D**IVIDERE numerum in. 3. cuiusmodi partes, ut quadratum vnus sit æquale  
 producto reliquarum duarum inter se, idem omnino est cum. 51. theoremate.  
 Nam qui sumet quamlibet partem propositi numeri, quæ tertia parte maior tamen  
 non sit, residuumq; in duas tales partes diuiderit, ut prima sumpta, media proportio  
 nalis sit ex probatione. 51. theoremate allata, propositum consequetur.

THEOREMA L. V.

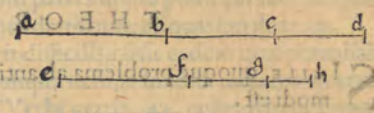
**I**Dipsum alia ratione ab ea diuersa quâ. 51. theoremate adduximus, p̄fici potest.  
 Suman-

THEOREM. ARIT.

Sumantur enim tres numeri continui proportionales, cuiuscunque denique proportionalitatis, qui in summam colligantur, ac postmodum, regula de tribus dicamus. Si summa hæc primo numero proposito in tres partes diuidendo respondet, cui respondebit una ex tribus partibus huiusce summe? idem de reliquis duabus partibus dico.

Exempli gratia, si proponatur numerus. 57. diuidendus in tres continuas partes proportionales proportionem sesquialtera, tres numeros in eiusmodi proportionalitate distinctos sumemus, ut potest. 4. 6. 9. qui in summam collecti dabunt summam. 19. dicemusque si 19. dant. 4. quid dabunt. 57? unde proueniens unius partis erit. 12. Tum si dicamus, si 19. dat. 6. quid dabit. 57? nempe dabit. 18. Postremo, si 19. dat. 9. quid dabit. 57? nempe. 26. atque ita dabitur. 18. cuius quadratum æquabitur producto reliquarum duarum partium inter se.

Quod ut sciamus, numerus propositus in tres quaslibet partes diuidendus significetur linea. a. d. tres autem numeri dictæ proportionalitatis, lineis. c. f. f. g. et. g. h. directè inter se coniunctis denotentur. Cogitemus pariter lineam. d. a. in tres partes diuisam. a. b. b. c. et. c. d. eadem cum cæteris proportionalitate, tunc eadem erit proportio. a. d. ad quamlibet suarum partium, quæ est. e. h. ad respondentem ipsius in. a. d. Verbi gratia respondentem. a. b. ipsi. e. f. et. b. c. f. g. et. c. d. g. h. Dico enim quod ita se habebit. a. d. ad. c. d. sicut. e. h. ad. g. h. Nam cum sic se habeat. a. b. ad. b. c. sicut. e. f. ad. f. g. ex præsupposito, permutando sic se habebit. a. b. ad. e. f. sicut. b. c. ad. f. g. & eadem ratione sic se habebit. c. d. ad. g. h. sicut. b. c. ad. f. g. & cõsequenter sicut. a. b. ad. e. f. ex quo ex. 13. quinti sic se habebit tota. a. d. ad totam. e. h. sicut. e. d. ad. g. h. aut. b. c. ad. f. g. aut. a. b. ad. e. f. permutando itaque propositum manifestum erit, ipsum autem productum. a. b. in. c. b. æquale erit quadrato. b. c. ex. 15. sexti aut. 20. septimi.



THEOREMA LVII

VETERES aliud quoque problema indeterminatum proposuerunt, quod ex more ratione à me definitur, est autem eiusmodi.

Quomodo propositus numerus in tres eiusmodi partes diuidatur, ut quadratum unius æquale sit summæ quadratorum reliquarum duarum partium.

Hoc ut efficiamus tria quadrata separata sumamus, quorum unum æquale sit reliquis duobus, eorum autem radices in summam simul colligantur, tum regulam de tribus sequemur, ratione præcedenti theoremate demonstrata, & rectè ut infra docebimus, quod autem dico de quadratis, etiam de cubis, & quibusvis dignitatibus assero.

Exempli gratia, si numerus diuisibilis proponatur. 30. in tres eiusmodi partes diuidendus, ut quadratum unius æquale sit summæ quadratorum reliquarum duarum partium, in primis radices trium quadratorum sumemus, sic quomodocunque se habentes, ut maius ipsorum æquale sit summæ reliquorum duorum, verbi gratia. 25. 16. et. 9. nempe. 5. 4. et. 3. quæ si colligantur in summam efficiunt. 12. Tum ex regula de tribus dicemus, si 12. responder. 30. cui. 5. radix maior respondebit? nempe. 12. cum dimidio.

Deinde si dixerimus si. 12. valet. 30. quid valebit. 4. radix media? nempe valebit. 10. tertia autem minor. 7. cum dimidio. Itaque tota summa erit. 30. & quadratum.

E 2 tum.

tum. 12. cum dimidio erit. 155. quod æquale erit summæ quadratorum duarum partium, nempe. 100. cum. 55.

Hoc ut demostremus, numerus diuisibilis propositus significetur linea. a. d. & summa radicum, nostro modo sumptarum, linea. e. h. quarum prima & maior sit. e. f. secunda. f. g. tertia. g. h. cogitemus etiam lineam. a. d. ea ratione diuisam esse qua. e. h. patebit enim ex modo præcedentis theorematum vnamquamque partium. a. d. ita se habituram ad suum totum sicut se habent singulæ. e. h. ad suum. Quod ideo dico, ut intelligamus rectè nos dicere. Si. e. h. dat. a. d. ergo. e. f. dabit. a. b. atq; ita de cæteris. Quare permutando sic se habebit. a. b. ad. b. c. sicut. e. f. ad. f. g. idem dico de reliquis. Igitur ex. 18. sexti aut. 11. octauæ, eadem erit proportio quadrati. a. b. ad quadratū. b. c. quæ quadrati. e. f. ad quadratum. f. g. tota enim sunt æqualia, cum eorum partes similes inter se sunt æquales. Idem dico de proportione quadrati. a. b. nempe ita se habere ad. c. d. sicut quadratum. e. f. ad quadratum. g. h. ex quo ex. 24. quinti proportio quadrati. a. b. ad summam quadratorum duarum partium. b. c. et. c. d. sic se habebit ut quadrati. e. f. ad summam quadratorum. f. g. et. g. h. At quadratum. e. f. æquale est summæ quadratorum. f. g. et. g. h. igitur sic etiam se habebit quadratum. a. b. nempe æquale quadratis. b. c. et. c. g. Id ipsum de cæteris dignitatibus dices, ut erisq; 21. theoremate huius libri.

#### T H E O R E M A L V I I.

**S**IMILE quoque problema ab antiquis indeterminatum proponitur, quod eius modi est.

An numerus aliquis in tres eiusmodi partes diuidi possit, ut quadratum vnus æquale sit summæ quadratorum cæterarum duarum partium simul cum producto vnus in alteram.

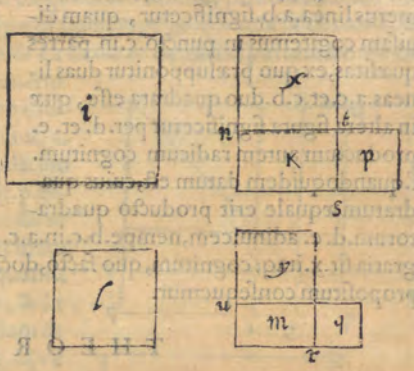
Exempli gratia, si proponatur numerus. 50. vt iam dictum est diuidendus, repertendus erit alius quilibet numerus, qui tamen summa sit trium radicum sic se habentium, ut quadratum vnus æquale sit summæ quadratorum duarum partium simul cum producto vnus in alteram, cum autem qui primò occurrit sumamus, ut potè. 30. qui summa est numerorum. 6. 10. 14. partium sic se habentium, ut quadratum ipsius. 14. æquale sit summæ quadratorum cæterarum partium simul cum producto vnus in alteram, agamusque regula de tribus, ac dicamus, si. 30. valeret. 50. quid valeret. 14. nempe. 23. cum tertia parte. Idem efficiemus in cæteris partibus, quarum vna erit. 16. cum duabus tertijs, altera vero. 10. absque fractis, ex quo quadratum primæ erit. 544. cum. 4. nonis, secundæ. 277. cum septem nonis, tertiæ. 100. & productum secundæ in tertiã. 166. cum. 6. nonis, quod productum, cum quadratis secundæ & tertiæ collectum erit. 544. cum. 4. nonis.

Huius rei speculario eadem est, que fuit præcedentis theorematum, vsque quo noveris eandem proportionem esse quadrati. a. b. ad summam quadratorum. b. c. et. c. d. quæ quadrati. e. f. ad summam quadratorum. f. g. et. g. h. Sed cum hic non demus quadratum. e. f. æquale summæ quadratorum. f. g. et. g. h. sed maius ex producto. g. h. in. f. g. aut quod idem est, e contrario, subsequentes figuræ cogitandæ erunt, quarum. i. sit quadratum. a. b. l. sit quadratum. e. f. x. quadratum. b. c. y. quadratum. f. g. p. quadratum. c. d. q. quadratum. g. h. k. sit productum. b. c. in. c. d. m. sit productum. f. g. in

g. in

THEOREM. VARI TH.

g.in.g.h.Nunc ex speculatione præcedentis theorematis, eadem erit proportio. n. t.ad.o.u. quæ est.n.s.ad.o. n. quare productum. k. ex definitione simile erit producto. m. cum vtraque sint rectangula, vnde proportio. k. ad. m. ad proportionem.n.t.ad.o.u. ex. 18. sexti dupla erit. Igitur proportio. k. ad. m. æqualis erit proportioni. x. ad. y. et. p. ad.q. et.i.ad.l.& permutando sic se habebit. x.ad.i.sicut.m.ad.l.fed.x.p.ad.i. sic se habere probatum est vt.y.q.ad.l. Quare ex eadem. 24. quinti sic se habebit.x.p.k.ad.i.sicut.y.q.m.ad.l.fed.y.q.m.æqualis est.l. Itaque.x.p.k.pariter. æqualis erit.



THEOREM. ALVIIII.

**A**LIIII. quoque problema, nec tamen definitum, veteres proposuerunt, nempe an aliquis numerus in.4. eiusmodi partes diuidi possit, vt summa quadratorum duarum partium dupla sit summæ quadratorum reliquarum duarum.

Verum huius effectio & speculatio non erit difficilis, cū sit eadem quæ præmissis proximè duobus theorematis allata fuit, sumpta nempe summa radicum quarumcunque sic se habentium, prout dictum fuit. Verbi gratia. 44. cuius partes erunt. 16. 12. 14. 2. tūc progrediemur regula de tribus dicentes. Si 44 numerum propositum valet, quid. 16. pars maior? nempe valebit partem maiorem numeri propositi respondentem. 16. idem de cæteris dico.

Porrò speculatio eadem est cum superioribus.



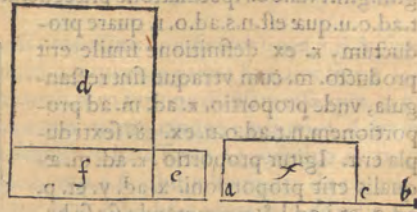
THEOREM. ALIX.

**C**VR diuidens propositum numerum in duas eiusmodi partes, vt productum radicum quadratarum ipsarum partium æquale sit alteri numero proposito, cuius tamè quadratum maius nō sit quadrato dimidij primi numeri propositi. Rectè secundum numerum propositum in seipsum multiplicat, & eundē ex quadrato dimidij primi detrahit, residuūq. quadratarum radicem subtrahit ex dimidio ipsius primi, ex quo datur minor pars quaerita, qua ipsi dimidio coniuncta, maior pars habetur.

Exempli gratia, si proponatur numerus. 20. proposito modo, in duas partes eiusmodi diuidendus, vt productum radicum æquale sit (verbi gratia) 8. Dimidium primi numeri in seipsum multiplicabimus, cuius quadratum erit. 100. ex quo quadratum secundi numeri, nempe. 64. detrahemus, remanebitq. 36. cuius radice quadrata coniuncta. 10. dimidio inquam primi numeri propositi, dabitur numerus. 16. pars maior, & subtracta à dimidio, dabitur minor pars, nempe. 4.

Hoc

Hoc ut demonstremus, primus numerus linea. a. b. significetur, quam diuisam cogitemus in puncto. c. in partes quæstas, ex quo præsupponitur duas lineas. a. c. et. c. b. duo quadrata esse, quæ in altera figura significetur per. d. et. e. productum autem radicum cognitum. f. quandoquidem datum est, cuius quadratum æquale erit producto quadratorum. d. e. adiuicem, nempe. b. c. in. a. c. ex. 19. theoremate huius. Quod verbi gratia sit. x. itaq; cognitum, quo facto, doctrinam. 45. theorematis libri huius secuti, propositum consequemur.

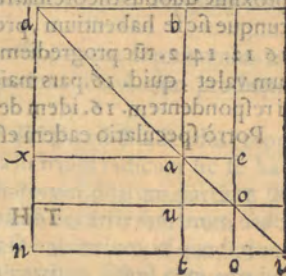


T H E O R E M A L X.

**C** V R productum differentia duarum radicum in summam ipsarum, semper differentia sit quadratorum ipsarum radicum.

Exempli gratia, quoslibet duos numeros pro radicibus sumpserimus, ut potè. 3. et. 5. quorum differentia est. 2. certè si differentiam hanc per summam radicum scilicet. 8. multiplicauerimus; dabitur numerus. 16. quod productum differentia est suorum quadratorum, nempe inter. 9. et. 25.

Hoc ut speculemur, duæ radices in linea. n. i. significentur, quarum una sit. n. c. & altera. c. i. ipsarum autem differentia. n. t. ex quo. t. c. æqualis erit. c. i. Tum cogitato toto quadrato. d. i. cum diametro. d. i. ductaq; parallela lateri. n. d. à puncto. c. & altera à puncto. t. & à puncto. o. tertia ipsi. n. i. & à puncto. a. quarta. x. a. e. parallela ipsi. o. inueniemus. b. n. productum esse differentia. n. t. in summa radicum. n. i. & cum. d. o. et. a. o. sine quadrata radicum prædictarum: b. c. æquale erit. n. u. cum vtrunque horum productorum æquale sit. x. u. ex quo gnomon. e. d. ut æqualis erit producto. b. n. quod scire cupiebamus.



T H E O R E M A L X I.

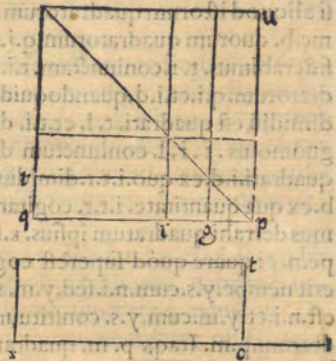
**C** V R propositum aliquem numerum diuisuri in duas eiusmodi partes, ut differentia radicum quadratarum æqualis sit alteri numero proposito, cuius tamen quadratum dimidij primi quadratum non excedat. Rectè secundum numerum in seipsum multiplicant, productum verò ex primo numero detrahunt, rursusq; dimidium residui quadrant, & quadratum hoc ex quadrato dimidij primi subtrahunt, atque ita radice quadrata residui, dimidio primi coniuncta, pars maior datur, quæ ex ipso dimidio detracta, pars minor relinquitur.

Exempli gratia, proposito numero. 20. ita ut propositum est, diuidendo; nempe ut differentia radicum quadratarum dictarum partium æqualis sit binario; binarium hoc in seipsum multiplicabimus, cuius quadratum. 4. è primo numero. 20. detrahemus,

**T H E O R E M A A R I T .**

trahemus, supereritq; numerus. 16. cuius dimidium scilicet. 8. in seipsum multipli-  
cabitur, dabiturq; numerus. 64. qui cum ex quadrato dimidij primi detractus fue-  
rit, nempe ex. 100. & residuo. 36. radix quadrata nempe. 6. coniuncta denario, di-  
midio primi, dabit. 16. partem maiorem, & ex denario detracta, partem minorem.

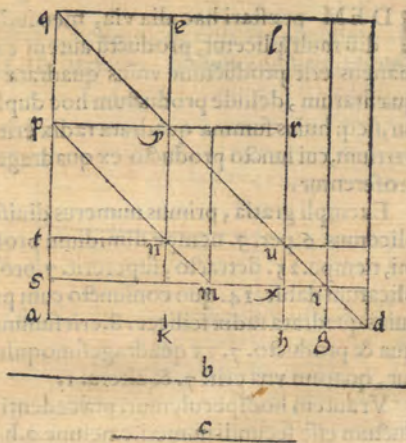
Cuius speculationis causa, primus numerus  
propositus significetur linea. x. y. pro voto diui-  
sa in puncto. c. et. x. t. productum sit ipseus. x.  
& in. c. y. pariter etiam. q. p. sit summa radicum  
quadratarum, nempe. q. g. ipsius. t. c. et. g. p. ip-  
sius. e. y. Tum super. q. p. extruatur & diuidatur  
quadratum. q. u. ea ratione qua. 41. theoremate  
aut. 29. diuisimus, in quo sane quadrato, quadra-  
tum ipsius. q. i. cernemus datae differentiae, & in  
eo collocata quadrata. x. c. et. c. y. ita etiam &  
rationem, qua cognoscimus productum. g. r. (vsi  
modo. 29. theoremaris) cuius quidem. g. r. qua-  
dratum, ex. 19. theoremate aequale erit produ-  
cto. x. t. ideo etiam cognitū, ac proinde cum no-  
uerimus. x. y. si rationem sequemur. 45. theore-  
mate cognoscemus non solum ratione. 41. theoremate allata hoc recte perfici, sed  
hac etiam alia ratione.



**T H E O R E M A L X I I .**

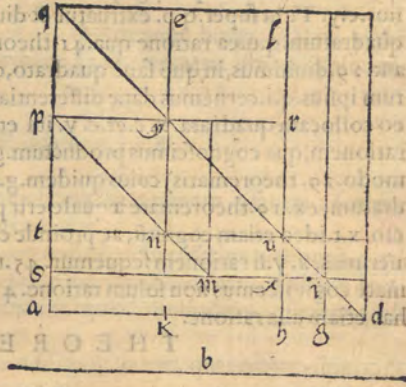
**C** V R propositum numerum diuisuri in duas eiusmodi partes, vt differentia  
suarū radicū quadratarū aequalis sit alteri numero proposito. Cuius tamē qua-  
dratū maius non sit quadrato medietatis ipsius primi propositi numeri. Recte etiā  
quadratū dimidij secundi numeri ex dimidio primi detrahūt, residuiq; radicem per  
secundum multiplicant, & productum ex dimidio primi detrahunt, vt residuum  
pars quaesita minor sit, & illud alterum totius residuum, pars maior.

Exempli gratia, si numerus. 50. in  
praedictas duas partes diuidendus pro-  
poneretur, & alter etiam. 6. quadratum  
dimidij secundi numeri esset. 9. eo detra-  
cto ex dimidio primi, remaneret. 16. cu-  
ius radix. 4. scilicet per totum secundum  
nempe. 6. multiplicata, proferet. 24.  
quo producto ex dimidio primi detra-  
cto, nempe. 25. dabitur. 1. pars minor,  
maior autē erit residuū. 50. hoc est. 49.  
radices autem erunt. 1. et. 7. differentes  
inter se, numero senario.



Hoc vt sciamus, duo numeri lineis si-  
gnificetur, primus linea. b. secundus linea.  
c. duae autem partes. b. duobus quadra-  
tis. q. i. et. i. d. notentur, eorum verò radi-  
ces lineis. a. g. et. g. d. differentia porro ip-  
si. c. aequalis & cognita sit. a. h. ex quo. h.  
g. aequa-

g. æqualis erit: g. d. tum productum. a. g. in. g. d. fit. a. i. et. t. i. æqualis. a. i. et. l. i. pariter  
 fecetur æqualis. t. i. quæ omnia ex diametro. q. d. cogitari possunt: erit igitur. u. i. æ-  
 qualis. i. d. supereritq; quadratum. q. u. differentia. a. h. cognitum, hoc verò cogi-  
 temus diuisum esse in. 4. partes æquales medijs diametris. p. r. et. n. e. quare vnaqueq;  
 partium cognoscetur, & quadratū erit ipsius. a. x. aut ipsius. x. h. dimidij. a. h. Quòd  
 si aliquod istorum quadratorum detrahere voluerimus, nempe. n. r. ex dimidio sum-  
 mæ. b. duorum quadratorum. q. i. et. i. d. cognita, hac via procedemus, primum con-  
 siderabimus. t. r. coniunctam. t. i. quæ quantitates erunt summa dimidij duorū qua-  
 dratorum. q. i. et. i. d. quandoquidem. t. r.  
 dimidiū est quadrati. t. l. et. t. i. dimidiū  
 gnomonis. t. i. l. coniunctum dimidio  
 quadrati. i. d. ex quo. i. t. r. dimidium erit.  
 b. ex qua quantitate. i. t. r. cogitare debe-  
 mus detrahi quadratum ipsius. x. h. nem-  
 pe. n. r: quare quod superest cognitum  
 erit nempe. y. s. cum. n. i. sed. y. m. æqualis  
 est. n. i. et. y. m. cum. y. s. constituunt qua-  
 dratum. p. m. Itaq; p. m. quadratum &  
 consequenter. p. s. eius radix cognosce-  
 tur, ita etiam & productum huius. p. s. in.  
 s. x. æqualis. c. nempe. p. x: estq; produ-  
 ctum huiusmodi semper minus quantita-  
 te. r. t. i: per. u. i. æquale quadrato minori.  
 i. d. quare. i. d. cognoscetur, consequen-  
 ter. i. q. tanquam residuum ex. b. & co-  
 rum radices quadrata cognoscuntur. a.  
 g. et. g. d.



T H E O R E M A L X I I I.

I D E M præstari hac alia via, meo iudicio potest. Secundus numerus in suū dimi-  
 diū multiplicetur, productū autem ex dimidio primi detrahatur, ex quo re-  
 manens erit productum vnius quadratae radices in alteram partium primi numeri  
 quæsitaram, deinde productum hoc duplicetur, & primo numero dato coniunga-  
 tur, sicq; huius summæ quadrata radix erit summa radicum quadratarum dictarum  
 partium, cui iuncto producto ex quadragesimo quinto theoremate singulae radices  
 proferentur.

Exempli gratia, primus numerus diuisibilis erat. 50. alter verò. 6. Iam si multi-  
 plicemus. 6. per. 3. nempe dimidium proferetur numerus. 18. quo ex dimidio pri-  
 mi, nempe. 25. detracto, supererit. 7. productum vnius radices in alteram, quod du-  
 plicatum dabit. 14. quo coniuncto cum primo numero. 50. dabitur numerus. 64.  
 cuius quadrata radix scilicet. 8. erit summa radicum duarum partium quæsitaram,  
 qua & producto. 7. ex quadragesimo quinto theoremate dictæ radices distinguen-  
 tur, quarum vna erit. 7. & altera. 1.

Vt autem hoc speculemur, præcedenti figura uti poterimus, in qua patet. t. r. pro-  
 ductum esse secundi numeri. c. nempe. a. h. hoc est. t. u. in dimidio. a. e. scilicet. p. r. re-  
 siduum autem dimidij primi. b. esse. t. i. nempe. a. i. productum radicum, quod supple-  
 men-



THEOREM ARITH. I

41

mentum est quadrati. q. d. totalis. Quare duplicato. a. i. & coniuncto. b. cognoscimus totum. q. d. & consequenter. a. d. suam radicem, hoc est summam duarum radicum. a. g. et. g. d. quæ medio. a. i. cognito, & quadagesimo quinto theoremate singulæ cognoscuntur.

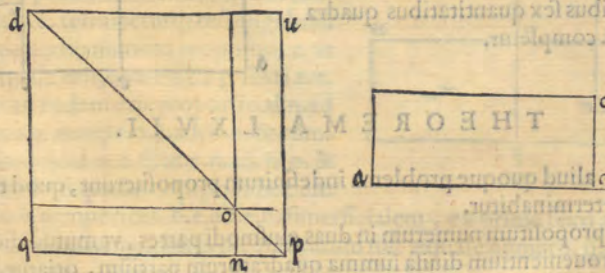
THEOREMA LXXIII.

**C**VR propositum aliquem numerum in duas eiusmodi partes diuisuri, vt summa radicum dictarum partium æqualis sit alteri numero proposito. Rectè secundum numerum in seipsum multiplicant, ex quo quadrato, primum datum numerum detrahunt, rursusq; residuum in seipsum multiplicant, & ex eo quadrato quartam partem desumunt, quæ ex quadrato dimidij primi numeri detrahunt, radicemque quadratam residui cum iunxerint, & ex dimidio primi numeri detraxerint, partes quasitæ proferuntur.

Exempli gratia, si proponeretur primus numerus. 20. diuidendus et. 6. secundus pro summa radicum, hunc secundum. 6. in seipsum multiplicabimus, dabiturq; numerus. 36. ex quo quadrato primus numerus detrahetur, supereritq; numerus. 16. qui quadratus dabit. 256. cuius numeri quarta pars sumetur, nempe. 64. quæ ex quadrato dimidij primi numeri detrahetur, nempe. 100. supereritq; 36. cuius radix quadrata. 6. coniuncta & detracta ex. 10. dabit. 16. partem maiorem et. 4. minorem.

Cuius rei hæc speculatio, primus numerus diuisibilis significetur linea. a. b. diuisa in puncto. e. in partes adhuc incognitas, et. a. c. sit productum. a. c. in. e. b. item. q. p. secundum numerum significet, æqualem summæ radicum, quæ puncto. n. distinguantur. Postmodum totum quadratum. p. d. erigatur (quod nobis est cognitum) in duo quadrata diuisum. o. p. et. o. d. quorum summa. a. b. cum detur, cognita remanet summa duorum supplementorum. o. u. et. o. q. qua quadrata cū fuerit dabit quadruplū quadrati supplementi. o. q. nempe quadruplū producti. a. c. etenim. a. c. ex. 19. theoremate huius libri quadratum est ipsius. q. o. sicq; poterant etiam veteres quadrare dimidium differentiæ. a. b. ab. p. d. nempe quadrato tantummodo supplemento. q. o. Tunc habito. a. c. eius ope tanquam producti. a. e. in. e. b. ex. 45. theoremate singulæ partes cognoscuntur.

Quod alia etiam ratione præstari poterat, nempe cognito supplemento. q. o. distinguendæ radices q. n. et. n. p. ex. 45. theoremate, quibus cognitis, eorum etiam quadrata cognoscuntur.



F THEORE-

T H E O R E M A L X V.

**C**V R proposito numero in tres qualescunque partes diuiso, si prima in tertiam multiplicetur, & huic producto, secundæ in primam productum coniungatur, itemq; secundæ in tertiam, hæc summa duplicata æqualis sit summæ productorum singularum in cæteras duas.

Exempli gratia, si proponatur. 20. diuisus in tres partes nempe. 12. 5. 3. multiplicato primo. 12. per. 3. tertiam partem dabitur. 36. secunda verò multiplicata per reliquas duas, hoc est. 5. per. 12. et. 3. in primis dabitur. 60. postea. 15. quoræ triū productorum summa erit. 111. quæ duplicata dabit. 222. qui numerus æqualis esse dicitur summæ productorum singularum partium in reliquas duas, nempe summæ. 60. 36. 60. 15. 36. 15. hoc est ipsi. 222.

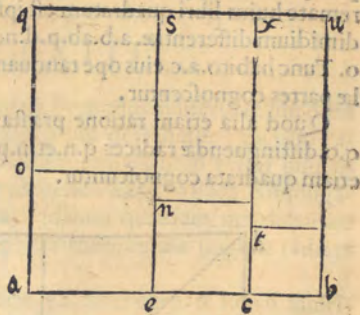
Cuius rei per se patet speculatio, cum in his sex vltimis productis, singula tria prima duplicentur.

T H E O R E M A L X V I.

**C**V R proposito numero in. 3. qualescunque partes diuiso, si in reliquas duas singulæ multiplicentur, & hæc producta cum summa suorum quadratorum coniungantur, tota summa hæc vltima æqualis erit quadrato totali propositi numeri.

Exempli gratia, si fuerit idem numerus. 20. in. 3. partes diuisus. 12. 5. 3. Si. 12. in. 5. et. 3. producat, summa productorum erit. 96. at. 5. in. 12. et. 3. erit. 75. postmodum. 3. in. 12. et. 5. erit. 51. nempe in vniuersum. 222. quadratorum porrò summa erit. 178. quæ coniuncta. 222. dabit. 400. quadratum ipsius. 20.

Erit autem huiusce rei facillima speculatio, si sequentem figuram mente conceperimus, in qua. a. b. propositum numerum significet, cuius partes distinctæ sint medio. e. et. c. Ipsi autem. q. b. sit quadratum totale parallelis. e. s. et. c. x. diuisum, quæ quadratum in tria rectangula diuident, quorum primum erit. q. e. compositum ex producto. a. e. in semetipsam, nempe quadratum. o. e. & ex producto eiusdem. a. e. in. e. b. quod erit rectangulum. o. s. ex quo tria rectangula. o. s. et. n. x. et. t. u. tria producta erunt singularum partium in cæteras duas, et. e. o. c. n. b. t. tria quadrata erunt: quibus sex quantitibus quadratum totale. q. b. completur.



T H E O R E M A L X V I I.

**V**ETRES aliud quoque problema indefinitum proposuerunt, quod tamen à nobis determinabitur.

Cur diuisuri propositum numerum in duas eiusmodi partes, vt mutuò diuisis, & per summam prouenientium diuisa summa quadratorum partium, oriatur proueniens alter numerus propositus.

Proposito deinde tertio quolibet numero diuidendo per singulas partes primi, ita

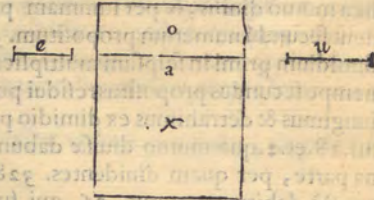


## T H E O R E M A L X V I I I.

**C** V R numero per numerum diuiso, productoq; duorum numerorum per proueniens multiplicato, quod vltimò productum est, diuisi numeri semper quadratum existat.

Exempli gratia, si diuidamus. 10. per. 2. proueniens erit. 5. quo producto ex duobus numeris multiplicato, nempe. 20. habebimus. 100. quadratum numeri diuisi.

Cuius gratia duo numeri sint. a. et. e. porro. a. per. e. diuiso detur. u. tum. o. productum. a. in. e. esse constituatur, quo per. u. multiplicato dabitur. x. quadratum. a. propterea quòd. a. medium est proportionale inter. o. et. u. ex. 35. theoremate. itaque ex. 16. sexti aut. 20. septimi, propositi veritas clucescet.



## T H E O R E M A L X I X.

**C** V R numero aliquo per duos alios multiplicato & diuiso, si per horum duorum productum, summa duorum primorum productorum diuisa fuerit, vltimum proueniens, summæ duorum primorum proueniens aequale sit.

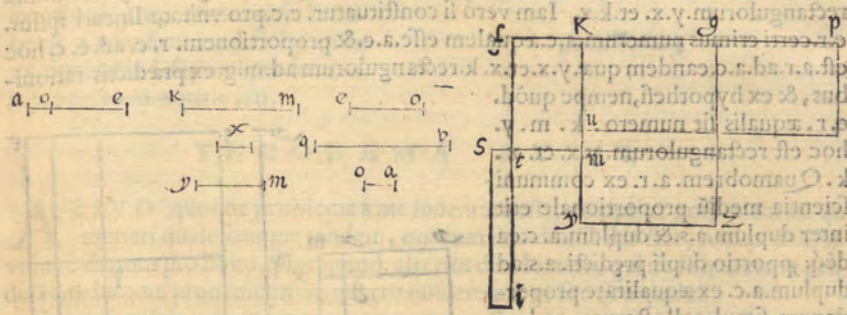
Exempli gratia, proponitur numerus. 24. per. 8. et. 6. multiplicandus & diuidendus summa productorum erit. 336. proueniens autem. 7. si igitur summam. 336. productorum per productum duorum secundorum numerorum nempe. 48. diuiserimus, proueniens pariter erit. 7.

In cuius gratia primus numerus significetur linea. q. b. multiplicandus & diuidendus numeris designatis per. k. m. et. y. m. productorum summa sit. k. z. proueniens autem. a. e. et. a. o. ex. k. m. et. o. e. ex. y. m. tum productum. k. m. in. m. y. fit. f. m. Dico quòd si. k. z. per. f. m. diuiserimus proueniet. a. e. Quod cum sic fuerit, erit quoque verum quòd diuisa. k. z. per. a. e. proueniet. f. m. numerus scilicet æqualis numero. f. m. ex. 13. theoremate huius. Itaque quotiescunque probauerò quòd diuisa. k. z. per. a. e. proueniat numerus æqualis ipsi. f. m. propositum verum esse consequetur. ex. 13. theoremate. Quòd si proueniens ex diuisione. k. z. per. a. e. æquale fuerit. f. m. patet ex. 7. quinti quòd eadè erit proportio numeri. k. m. y. ad ipsum proueniens, quæ ad numerum. f. m. Cogitemus itaq; k. u. æqualem. a. e. super quam mente concipiamus rectangulum. u. p. æqualem. k. z. ex quo eadem erit proportio k. p. ad. k. y. quæ. g. k. ad. k. u. ex. 15. sexti, aut. 20. septimi, numerus autem. k. p. erit proueniens, quod probandum est æquale esse. f. m.

Probabitur autem sic, ex. 9. quinti, nempe demonstrato quòd numerus. k. p. eandem proportionem habeat ad numerum. k. y. quam habet numerus. f. m. ad eundem k. y. Sed probatum est sic se habere. k. g. ad. k. u. sicut. k. p. ad. k. y. sufficit igitur probare sic se habere. k. g. ad. k. u. sicut. f. m. ad. k. y. Sed. k. g. dicitur æqualis esse. q. b. et. k. u. a. e. satis erit igitur probare ita se habere. q. b. ad. a. e. sicut. f. m. ad. k. y. Scimus autem quòd eadem est proportio. q. b. ad. a. o. quæ. m. k. ad. vnitatem, quæ sit. x. & quod proportio. o. e. ad. q. b. eadem est, quæ. x. ad. m. y. ex definitione diuisionis. Quare ex æqualitate proportionum eadem erit proportio. k. m. ad. m. y. quæ. e. o. ad. o. a. &

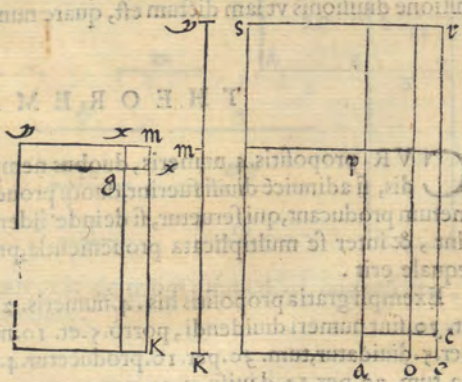
THEOREM. ARIT.

componendo sic se habebit.  $k.y.ad.m.y.$  sicut  $e.a.ad.o.a.$  & permutando  $k.y.ad.e.$   
 $a.$  sicut  $m.y.ad.o.a.$  & ex 19. quinti ita  $k.m.ad.e.o.$  sicut  $k.y.ad.e.a.$  & permutando  
 $x.m.ad.k.y.$  sicut  $e.o.ad.e.a.$  Nunc producat  $f.r.$  donec  $t.i.$  æqualis sit  $k.y.$  produ-  
 ctus  $m.t.$  donec  $t.s.$  æqualis sit unitati  $x.$  termineturq; rectangulum.  $s.i.$  ex quo da-  
 bitur proportio numeri  $f.m.$  ad numerum  $s.i.$  composita ex  $m.t.ad.t.s.$  et  $f.t.ad.t.i.$   
 ex 24. sexti, aut quinta octavi, sed ita etiam proportio  $q.b.$  ad  $a.e.$  componitur ex  
 eisdem proportionibus, nempe ex  $q.b.$  ad  $o.e.$  æquali,  $m.r.$  ad  $t.s.$  & ex proportione  
 $o.e.$  ad  $a.e.$  æquali.  $f.r.$  ad  $t.i.$  itaque proportio numeri  $f.m.$  ad  $s.i.$  hoc est ad numerum  
 ipsius  $k.y.$  æqualis est proportioni numeri  $q.b.$  ad  $a.e.$  nempe  $k.g.$  ad  $k.u.$  hoc est  $k.p.$  ad  
 $k.y.$  ex quo sequitur  $k.p.$  constare numero æquali  $f.m.$  proueniens igitur ex diuisione  
 numeri  $k.z.$  per  $f.m.$  æquale est numero ipsius  $a.e.$



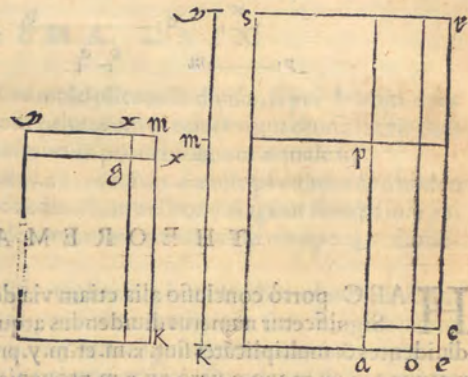
THEOREMA LXX.

**H**ÆC porrò conclusio alia etiam via demonstrari potest.  
 Significetur numerus diuidendus atque multiplicandus linea.  $b.a.$  Deinde  
 diuidentes & multiplicates sint.  $k.m.$  et  $m.y.$  prouenientia ex diuisione sint  $a.o.$  et  $o.$   
 $e.$  atque  $a.o.$  ex  $m.y.$  o.e. verò ex  $k.m.$  proueniat, quorum summa sit  $a.e.$  productum  
 autem  $b.a.$  in  $k.m.$  sit  $b.p.$  et  $p.s.$  productum  $b.a.$  in  $m.y.$  ad hæc rectangulum  $k.y.$  sit  
 productum  $k.m.$  in  $m.y.$  quo to-  
 tum productum  $a.s.$  diuidatur, pro-  
 ueniensq; sit  $a.c.$  cui,  $a.c.$  productū.  
 $a.s.$  eadē proportionē seruabit, quā  
 $k.y.$  rectangulum ad unitatem ex  
 definitione diuisionis, hoc autem  
 proueniens  $a.c.$  constare numero æ-  
 quali assero summa.  $a.e.$  Primum  
 enim ex dicta definitione diuisionis  
 habemus eandem esse propor-  
 tionem  $b.a.$  ad  $a.o.$  quæ  $m.y.$  ad  
 unitatem, & quod sic se habet  $b.a.$   
 ad  $o.e.$  sicut  $k.m.$  ad eandem unita-  
 tē. Itaque unitas hæc linearis si-  
 gnificetur per  $m.x.$  in singulis late-  
 ribus  $k.m.$  et  $m.y.$  producentibus rectangulum  $k.y.$  superficialis autem unitas sit.



g.m.

g.m. cogiteturq; rectangulum. y.x. & rectangulum. k. x. Itaque dabitur eadem pro-  
 portio. k.m.ad.m.x.nempe.k.x.rectanguli ad.m.g.quæ est.b.a.ad.o.e.et.y.x.ad. m.  
 g.quæ.b.a.ad.a.o.fed ex prima sexti aut. 18.vel. 19.septimi, sic se habet rectangu-  
 lum. k.y.ad.x.y.ficut.k.m.ad.m.x. quare sicut.b.a.ad.o.e.ex. 11.quinti,& eiusdem  
 rectanguli. k.y.ad rectangulum. k.x.ficut.y.m ad.x.m.nempe.b.a.ad.a.o. Quare  
 ex communi scientia, sic se habebit duplum rectanguli. k.y.ad summam. y.x. cum.  
 k.x.rectangulorum, sicut duplum. b.a.ad summam.a.o.e. et proportio summæ re-  
 ctangulorum. y.x.et.k.x.duplo.g.m.ficut duplum.b.a.ad.a.o.e. Igitur summa duo-  
 rum rectangulorum.y.x.et.k.x.media proportionalis erit inter duplum rectanguli.  
 k.y.& duplum vnitatis superficialis. g.m.Nunc terminetur rectangulum.a.r.ex quo  
 dabitur eadem proportio dupli.a.s.ad.a.r.ficut dupli.b.a.ad.a.e.ex propofitioni-  
 bus notatis, sexti aut septimi. Quare etiam sicut dupli rectanguli. k.y. ad summam  
 rectangulorum.y.x.et.k.x. Iam verò si constituatur. e.c.pro vnitare lineari ipsius.  
 e.r.certi erimus numerum.a.c.æqualem esse.a.e.& proportionem. r.e.ad.e.c.hoc  
 est.a.r.ad.a.c.eandem quæ.y.x.et.x.k.rectangulorum ad.m.g.ex prædictis rationi-  
 bus, & ex hypothesi,nempe quòd.  
 e.r.æqualis sit numero. k.m.y.  
 hoc est rectangulorum. y.x.et. x.  
 k. Quamobrem. a.r.ex communi  
 scientia mediū proportionale erit  
 inter duplum.a.s.& duplum.a.c.ea  
 deq; pportio dupli prædicti. a.s.ad  
 duplum.a.c.ex æqualitate propor-  
 tionum simul collectarum, eadem  
 erit quæ proportio dupli rectangu-  
 li.k.y.ad duplum.m.g.hoc est. a.s.  
 simplicis ad simplicem,a.c.qua sim-  
 plicis rectanguli.k.y.ad simplicem  
 vnitatem.g.m.fic enim se habet sim-  
 plex ad simplex, sicut duplum ad  
 duplum. Sed pariter ita se habet.a.s.ad.a.c.cogitato. a.c. tamquam proveniente  
 ex diuisione. a.s.per rectangulum.k.y.vt constitutum est, sicut.k.y.ad.m.g.ex defi-  
 nitione diuisionis vt iam dictum est, quare numerus.a.c.æqualis erit numero. a.o.e.



T H E O R E M A L X X I.

**C**VR propositis.4.numeris, duobus nempe diuidentibus ac duobus diuiden-  
 dis, si adinucè diuisi fuerint,duoq; proueniētia inuicè multiplicata quouis nu-  
 merum producant,qui seruetur, si deinde ijdem numeri versa vice mutuo diuisi fue-  
 rint, & inter se multiplicata proueniētia,productū hoc, primo seruato numero  
 æquale erit.

Exempli gratia propositis his. 4.numeris. 20.30.5. 10. duo autem. 20. scilicet  
 et. 30.sint numeri diuidenti, porro. 5.et. 10.numeri diuidentes, nepe vt primo. 20  
 per. 5. diuidatur, tum. 30. per. 10. producet. 4. et. 3. qui simul multiplicati proferēt.  
 12. tum. 20. per. 10. diuiso et. 30. per. 5. proueniētia erunt. 2.6. quæ inter se multi-  
 plicata producent etiam. 12.

m.g

Cuius

Cuius rationem si quæris, significantur. 4. numeri lineis. a. e. o. u. diuidaturq; a. per. o. & oriatur. s. & per. u. oriatur. y. et. e. diuiso per. o. oriatur. z. & per. u. proueniat. f. tum. n. fit productum. z. in. y. et. m. productum. s. in. f. Dico n. futurum æquale. m. Sit deinde. x. vnitas, quare ex definitione diuisionis eadem erit proportio. s. ad. a. et. z. ad. e. quæ. x. ad. o. Sed ita se habet. a. ad. y. et. e. ad. f. sicut. u. ad. x. ex quo sic se habebit. s. ad. a. sicut. z. ad. e. et. a. ad. y. sicut. e. ad. f. Itaque ex æqualitate proportionum sic se habebit. s. ad. y. sicut. z. ad. f. Igitur ex 15. sexti aut. 20. septimi productum, n. producto. m. æquale erit.



THEOREMA LXXII,

**A**LIVD quoque problema à me inuentum est, nempe vt proponantur. 4. numeri qualescunque tandem, quorum duo diuisibiles sint, tertius diuisor vnus è duobus pro libito, quæramusq; alterius diuidentem, qui sic se habeat vt productum duorum prouenientium quarto numero proposito sit æquale.

Exempli gratia, proponuntur. 4. numeri. 20. 48. 5. 12. porro. 20. et. 48. numeri sint diuisibiles et. 5. diuides vnus, ut potè. 20. Quæredus nunc erit diuidens alterius nempe. 48. eiusmodi vt productum prouenientium æquale sit. 12. Diuidam itaque. 20. per. 5. prouenietq; 4. quem per. 48. multiplicabo, nempe per alterum diuisibilem, sicq; proueniet. 192. quod productum per quartum numerum nempe. 12. diuisum dabit. 16. qui erit diuidens quæsitus, quo diuiso. 48. proueniet. 3. secundum scilicet proueniens, quo per alterum hoc est. 4. multiplicato producet quartus numerus. 12.

Quod vt sciamus, primus numerus diuisibilis significetur rectangulo. a. i. secundus rectangulo. o. u. primus diuidens latere. a. e. quartum numerum rectangulo. i. o. primum proueniens latere. e. i. secundus diuidens latere. e. u. (hic autem est quem quærimus) tum alterum proueniens significetur latere. e. o. Iam eadè erit proportio. e. i. ad. e. u. quæ. o. i. ad. o. u. Sed cum cognitæ sint tres quantitates. e. i. i. o. et. o. u. quarta quoque. e. u. ex regula de tribus immediatè cognoscetur, cætera in subscripta figura facillimè patebunt.

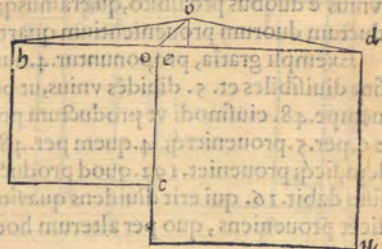


## THEOREMA LXXIII.

**H**OC etiam problema à me inuentum est, nempe si duæ radices quadratæ in summam collectæ fuerint, & ex dimidio eiusmodi summæ detracta fuerit minor radix, residuq; quadratum duplicatum eiq; summæ coniungatur duplum producti ipsius residui in dimidium summæ radicum, atque huic summæ duplum producti eiusdem residui in radicem minorem coniunctum fuerit; vltima hæc summa differentia erit duorum quadratorum propositorum.

Exempli gratia duæ radices quadratæ sint. 5. et. 11. harum summa erit. 16. & dimidium. 8. differentia minoris ab ipso dimidio erit. 3. duplum quadrati huius differentia erit. 18. duplum producti huius differentia in dimidium summæ radicum erit. 48. item & huius differentia duplum in minorem radicem erit. 30. quarum omnium summa erit. 96. tantaque erit differentia suorum quadratorum, quorum vnum erit. 25. alterum verò. 121.

Pro cuius rei scientia, duæ quadratæ radices sint. h. o. et. o. d. directæ inter se coniunctæ, quæ summa per medium in puncto. e. diuidatur, tum cogitetur. e. b. æqualis o. e. perpendicularis. h. d. ducanturq; lineæ. b. h. b. o. et. b. d. Iam ex. 4. primi. b. h. æqualis erit. b. d. & quadratum. b. h. æquale quadrato. h. o. & quadrato. o. b. simul cum duplo producti. o. e. in. o. h. ex. 12. secundi Eucli. Sed ex. 13. eiusdē quadratum. b. d. minus est quadrato. o. d. cum quadrato. o. b. ex duplo producti. o. e. in. o. d. at duplum eiusmodi producti æquale est duplo quadrati. o. e. & duplo producti. o. e. in. e. d. ex tertia eiusdem, itaque duo quadrata scilicet. o. b. et. o. d. maiora erunt duobus quadratis, nempe. o. b. et. o. h. collectis cum duplo producti. o. e. in. o. h. ex duplo quadrati. o. e. vñ cū duplo producti. o. e. in. e. d. Quare differentia summæ duorum quadratorum o. b. et. o. d. à summa duorum. o. b. et. o. h. duplum erit quadrati. o. e. cum duplo producti. o. e. in. e. d. & duplo producti. o. e. in. o. h. Quod si ex singulis duabus summis quadratorum demptum fuerit quadratum. o. b. eadem producta & quadrata ipsius. o. e. remanebunt, tanquam differentia duorum quadratorum. o. u. et. h. c.



## THEOREMA LXXIIII.

**C**VR summa duorum extremorū quatuor terminorum proportionaliū arithmetice, æqualis est summæ duorum mediorum, vbi nota hac in re necessarium non esse proportionalitatem continuam existere.

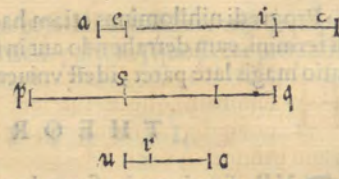
Exempli gratia, si darentur hi quatuor termini. 20. 17. 9. 6. quorum proportio eadem esset primi ad secundum quæ tertij ad quartum, summa primi cum quarto esset 26. tantaq; secundi cum tertio.

Cuius speculationis causa, primus maiorq; numerus significetur linea. e. o. secundus. s. q. tertius. u. c. quartus. g. t. differentia porrò inter. e. o. et. s. q. fit. i. o. quæ æqualis erit differentia. r. c. qua quartus à tertio superatur ex hypothesi. Itaque assero summam. e. o. cum. g. t. nempe. a. o. æqualem esse summæ. q. s. et. u. c. sitq; q. p. Nam in. a. o. Secundus



THEOREM. ARIT.

Secundus tertiusque terminus reperiuntur, est enim secundus. e. i. tertius. i. o. et. e. a. quandoquidem ex præsupposito. e. i. æqualis est. s. q. et i. o. æqualis. r. c. et. a. e. cum sit æqualis. g. t. cui pariter æqualis est. r. u. ex quo. a. e. æqualis est. u. r. Itaque illud sequitur. a. o. ipsi. q. p. æqualem esse.

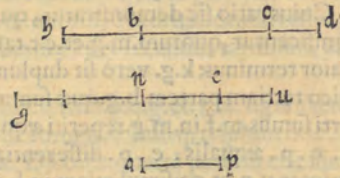


THEOREMA LXXV.

**C**VR summa duorum terminorum extremorum imparium arithmetica proportionalitatis semper duplo medij termini æqualis est.

Exempli gratia, sunt hi tres termini proportionalitatis arithmeticae, 20. 15. 10. summa duorum extremorum erit. 30. quæ duplum est medij termini. 15.

Quod ut speculemur, tres termini, tribus lineis. b. d. n. u. et. q. p. significetur. Dico nunc quòd summa. b. d. cum. q. p. nempe. h. d. semper duplo. n. u. scilicet. g. u. æqualis erit. Tum differentia. b. d. ad. n. u. fit. c. d. quæ æqualis erit. e. u. differentia inter n. u. et. q. p. patet enim in linea. h. d. b. c. æqualem esse. n. u. sed. n. u. ex. n. e. componitur æquali. q. p. et ex. e. u. æquali. c. d. cum itaq; in. h. d. partem. h. b. reperiamus æqualem. n. e. gratia. q. p. & partem. c. d. æquale. m. e. u. manifestum erit h. d. æqualem esse. g. u.



BINA PROBLEMAT A EX DVOBVS PRÆDICTIS  
THEOREMATIBVS DEPENDENTIA.

**E**X duobus prædictis theorematibus duo problemata oriuntur, quorū primum est. Datis tribus quantitatibus cognitiss, si quis quartam inuenire voluerit, quæ eiusmodi sit respectu tertiæ, qualis est secunda respectu primæ, secunda cum tertia in summam colligenda erit, ex qua detracta prima, supererit quarta.

Exempli gratia, cognitis tribus quantitatibus. 20. 17. 9. si quartam inuenire voluerimus eiusmodi proportionem cum tertia arithmetice seruantem, quam secunda cum prima, secundam cum tertia in summam colligemus, dabiturq; summa. 26. ex qua detracta prima quantitate, quarta relinquetur nempe. 6. quod ex. 74. theoremate dependet.

Idipsum tamen proueniret si quis ex tertio termino differentiam primi atque secundi detraheret; hæc tamen via non tam vniuersalis est quàm illa. N si quartus terminus incognitus tertio maior esse deberet, dictam differentiam cum tertio termino in summam colligere oporteret.

Alterum problema est, quòd inuentis duobus terminis, si tertius requiratur, secundus duplicandus erit, ex qua summa detracto primo, statim tertius profereatur, quod problema ex præcedenti theoremate dependet.

Progredi nihilominus etiam hac in re possemus per differentiam primi & secundi termini, eam detrahendo aut in summam cum secunda colligendo, attamen prior ratio magis latè patet, idest vniuersalior est.

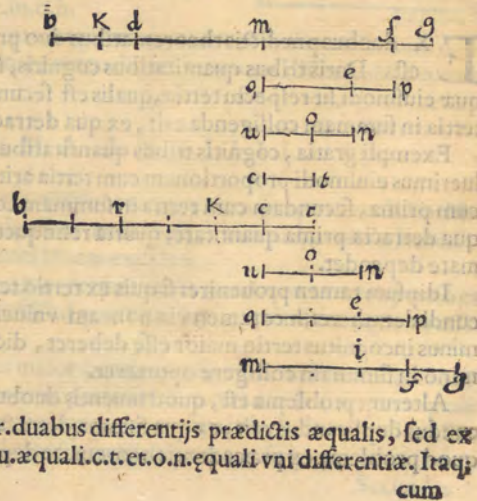
## T H E O R E M A L X X V I.

**C**VR si quis cupiat secundum terminum inuenire, quatuor terminorum arithmetice proportionalis continuæ, quorum nobis duo extrema proponantur. Rectè primum duplicabit coniungetq; vltimo termino, nempe quarto, ex qua summa tertiam partem desumet, quæ erit secundus terminus quæsitus.

Exempli gratia, si horum quatuor terminorum. 12. 9. 6. 3. duo nobis extrema proponantur. nempe. 12. et. 3. quorum secundus inueniendus sit, sumpto quolibet pro primo, sit autem. 3. primus numerus, quartus verò. 12. quare duplicato. 3. vtpotè primo, & coniuncto. 12. quarto, summa erit. 18. cuius est tertia pars. 6. secundus numerus scilicet sumpto principio à minimo. Id ipsum euenit sumpto principio à maximo. Nam si datur secundus à minimo aut à maximo, illico tertius datur differentia inter hunc & primum, secundo coniuncta, aut ex eodem detracta.

Cuius ratio sic demonstratur, quatuor termini quatuor lineis. m. g. q. p. u. n. c. t. significantur, quorum. m. g. et. c. t. tantummodo cognoscantur. sitq; m. g. primus ac maior terminus: k. g. verò sit duplum primi. m. g. cui coniungatur. b. k. æqualis. c. t. Dico tertiam partem. b. g. quæ summa totalis est, æqualem esse. q. p. In primis enim certi sumus. n. f. in. m. g. reperi i æqualem. q. p. superestq; f. g. differentia inter. m. g. et. q. p. æqualis. e. p. differentia inter. q. p. et. u. n. & æqualis. o. n. differentia inter. u. n. et. c. t. simul etiam in. k. m. habemus. d. m. æqualem. m. f. quare etiam. q. p. et. k. d. æqualem. f. g. nempe. e. p. aut. o. n. Hactenus in. k. g. reperimus duplum. q. p. simul cum. f. g. et. k. d. æqualibus. e. p. et. o. n. & quia. b. k. æqualis. c. t. fuit coniuncta. considerandum est an hæ tres quantitates. f. g. k. d. et. b. k. simul æquales sint. q. p. quod tamen per se manifestum est. nam. q. p. superat. u. n. per. e. p. et. u. n. excedit. c. t. per. o. n. æqualem. e. p. quare. q. p. per duplum differentie. f. g. superat. c. t. itaque. f. g. k. d. et. k. b. ipsi. q. p. sunt æquales, ex quo sequitur. q. p. tertiam partem esse. b. g. Hæc quæ hactenus dicta fuerunt, in genere maioris inæqualitatis probata fuerunt.

At in genere minoris, sumpto ordinis principio à minimo terminorum, duplicetur. e. t. sitq; duplum hoc. k. t. cui. k. b. æqualis. m. g. coniungatur, quæ summa sit. b. t. Dico. u. n. tertiam esse partem ipsius. Nam in primis in. b. t. datur terminus. b. k. æqualis vltimo. m. g. in quo semel reperitur. u. n. vnà cum duabus differentijs, nempe. i. g. in ipsa autem. b. t. u. n. significetur primo loco per. r. x. ex quo supererit. b. r. duabus differentijs prædictis æqualis, sed ex præsupposito. u. n. componitur ex. o. u. æquali. c. t. et. o. n. æquali vni differentia. Itaq;



cum

THEOREM. ARIT.

cum in .b.t. præter .r.x. bis detur .c.t. nempe .x.t. et .b. r. duabus differentiis æquipol-  
lens, illud efficitur . u.n. pariter ipsius .b.t. esse tertiam partem, quod erat propositū.

THEOREMA LXXVII.

**C**VR si quis velit secundum quinque continuorum proportionalium termi-  
num inuenire, solis extremis cognitis. Rectè vltimū triplo primi coniunget,  
ex qua summa quartam partem detrahet, quæ erit secundus terminus quæsitus.  
Quod ipsum faciet qui inuenire vult secundum terminum senarij septenarij, octo-  
narij aut alterius cuiuscunque, crescente tamen multiplicatione primi, vltimoq;  
coniuncto.

Exempli gratia, dantur duo extremi termini, horum quinque numerorum . 18.  
16. 14. 12. 10. nempe .18. et .10. si .18. primus erit, hoc est, si à genere maioris inæ-  
qualitatis progrediemur, triplicabimus terminum .18. dabunturq;. 54. cui numero  
coniuncto quinto termino. 10. dabitur numerus .64. cuius quarta pars erit .16. vtpo-  
tè secundus terminus gratia, aut secundi sex terminorum, quadruplicandus esset pri-  
mus .18. deinde adiuncto vltimo, quinta pars summæ esset secundus terminus, atq;  
ita deinceps.

Cuius speculationis gratia, dicti termini lineis .z.h.:f.s.:u.p.:e.g. et .r. x. significetur.  
In primis ex genere maioris inæqualitatis, triplicabimus .z.h. sitq; triplum hoc .x.  
h. cui cōiungatur .b.k. equalis vltimo termino .r.x. Dico .f.s. quartā partem esse sum-  
mæ .b.h. Nam in .k.h. secundus terminus .f.s. ter cum tribus differentiis æqualibus .n.h.  
reperitur. Probandum nunc est tres has differentias .n.h.:a.c. et .d. x. simul cum .b.  
x. equales esse .f.s.

quod in dubiū re-  
uocari nō potest,  
cum .f. s. superet  
.r.x. per .o. s. t. p. et  
.i. g. At in genere  
minoris inæquali-  
tatis, triplum .r.x.  
sit .x. a. et .a. b. sit  
æqualis .z.h. & cū  
z.h. tribus differē-  
tijs .n.h.:o.s.t.p. su-  
peret .e.g. quæ in  
.a. b. sint .b. k. x. d.  
d.c. ex quo .a. c.  
æqualis erit .e. g.  
et .a. x. cum .b. c. tripla .e. g. Itaque tota summa .b. x. quadrupla erit .e. g.



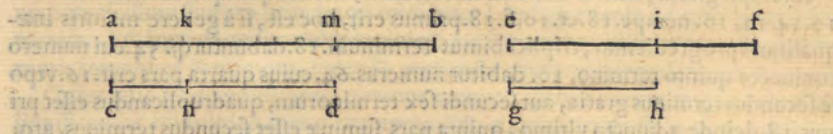
THEOREMA LXXVIII.

**Q**uantitates quæ fuerint inuicem in proportionalitate arithmetica proportio-  
nales, permutando quoque proportionales erunt.

G 2 Sint

Sint exempli gratia. 4. quantitates. a. b. c. d. e. f. e. t. g. h. inuicem proportionales in  
 proportionalitate arithmetica. Hoc est vt quæ proportio (licet improprie dicta)  
 est ipsius. a. b. ad. c. d. eadē sit ipsius. e. f. ad. g. h. Tunc permutando dico eandem pro  
 portionem fore ipsius. a. b. ad. e. f. quæ ipsius. c. d. ad. g. h.

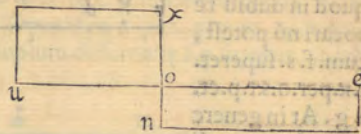
Nam, ex hypothesi, differentia qua. a. b. superat. c. d. (quæ sit. m. b.) æqualis est  
 differentia qua. e. f. superat. g. h. (quæ sit. i. f.) vnde. a. m. residuum ex. a. b. æquale erit  
 c. d. & residuum. e. i. æquale. g. h. Sit igitur exempli gratia. c. d. maior. g. h. per. c. n.  
 vnde. n. d. æqualis erit. g. h. quare. a. n. maior erit. e. i. per. a. k. æqualem. c. n. ex com  
 muni conceptu. b. k. æqualis erit ipsi. f. e. fed. n. d. æqualis est. g. h. vt dictum est.  
 Cum ergo. b. k. æqualis sit. e. f. et. d. n. ipsi. g. h. et. a. b. maior sit ipsa. k. b. per. a. x. æquā  
 lem ipsi. c. n. per quam. c. n. d. c. maior est ipsa. d. n. sequitur verum esse propositū hoc  
 est, quod eadem proportio sit ipsius. a. b. ad. e. f. quæ. c. d. ad. g. h. arithmetice scilicet.



T H E O R E M A L X X I X.

C V R proueniētia duorum numerorum diuidentium eiusdem numeri diuisi  
 bilis, geometricè eandē inter se proportionē seruant, quā ipsimet diuidētes.  
 Exempli gratia si per senarium & octonarium numerus vigintiquatuor diuida  
 tur, proueniētia erunt. 4. et. 3. eadem proportionē, qua diuidētes.

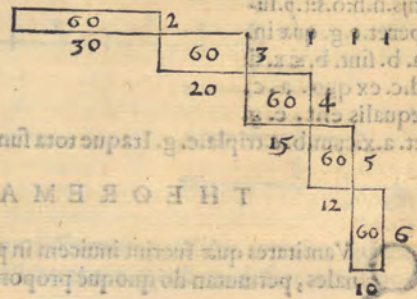
Cuius est ratio numerus diuisibilis significetur rectangulis. u. x. et. n. e. diuidentes  
 autem sint. u. o. et. e. o. quare ex ijs, quæ. 10.  
 theoremate dicta fuerunt. u. x. per. u. o. diui  
 so dabit. x. o. & diuiso. n. e. per. e. o. dabit. o.  
 n. Dicimus itaque eandē esse proportionē  
 o. x. ad. o. n. quæ. e. o. ad. o. u. quod patet sub  
 scriptam figuram considerantibus, in qua,  
 ex. 15. sexti aut. 20. septimi, eadem propor  
 tio cernitur. o. x. ad. o. n. quæ. o. e. ad. o. u.



T H E O R E M A L X X X.

C V R quauis quantitate, tribus  
 aut quatuor aut etiam pro libi  
 to pluribus diuidentibus numeris di  
 uisa, proueniētia eandem prorsus  
 inter se proportionem seruabunt,  
 quam ipsi diuidentes habere compe  
 riuntur.

Exempli gratia, proponitur nu  
 merus. 60. quinque numeris diuiden  
 tis, vt pōt. 30. 20. 15. 12. 10. pro  
 ueniētia erunt. 2. 3. 4. 5. 6. eadem  
 pro-



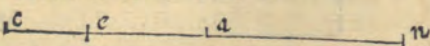
proportione diuidentium, quamuis ex aduerso. Cuius ratio ex. 15. sexti aut. 20. septimi dependet. prout in subscripto ordine facillimè deprehendi potest.

THEOREMA LXXXI.

VR quantitate in tres continuas partes proportionales secta, & per singulas ipsarum diuisa, summa trium prouenientium quadrato medij prouenientis aequalis est.

Exempli gratia, proponitur. 14. diuidendus in tres continuas partes proportionales, nempe. 8. 4. 2. ipseq; numerus. 14. per singulas diuiditur, ex quo tria prouenientia oriuntur, nempe ex prima parte. 8. proueniens erit. 1. cum tribus quartis partibus ex secunda. 4. datur proueniens. 3. cum dimidio vnus, & ex tertia. 2. prouenient. 7. integri, qui in summam collecti dant. 12. integros & vnã quartã partem tantundem, videlicet quantum quadratum prouenientis medij, nempe. 3. cum dimidio.

Cuius speculationis gratia, totalis numerus significetur linea. n. c. qui in tres partes diuidatur. n. a. a. e. et. e. c. quæ sint continuæ proportionales, quarum singulis, numerum. n. c. diuisum esse cogitemus, proueniens autem ex diuisione. n. c. per. n. a. sit. i. d. quod verò prouenit ex diuisione. n. c. per. a. e. sit. d. u. proueniens quoque ex diuisione. n. c. per. e. c. sit. u. o. quorum summa sit. i. o. quæ asseritur esse numeri æqualis numero quadrati. d. u. Quod hac ratione probabo, producatuŕ linea. i. o. donec. o. p. æqualis sit. o. u. erigaturq; .o. m. æqualis. d. i. perpendiculariter. o. p. in puncto. o. quæ producatuŕ donec. o. q. vnitati sit æqualis, terminenturq; duo rectangula. m. p. et. q. i. ex quo habebimus rectangulum, aut productum. m. p. æquale quadrato. d. u. ex. 16. sexti aut. 20. septimi, quandoquidem tria prouenientia. o. u. u. d. et. d. i. ex præcedenti theoremate sunt inter se continua proportionalia, proportionalitate qua partes. n. c. iam verò si probauero. q. i. productum, producto. m. p. æquale esse, propositum quoque probatum erit. Numerus enim producti. q. i. æqualis est numero summa. i. o. Habemus autem ex definitione diuisionis ita se habere. n. c. ad. i. d. sicut. n. a. ad. o. q. Itaque permutando sic se habebit. n. c. ad. n. a. sicut. d. i. hoc est. m. o. ad. o. q. sed sicut se habet. n. c. ad. n. a. ita pariter se habet. i. o. ad. o. u. hoc est ad. o. p. Itaque. i. o. ad. o. p. sic se habebit sicut. m. o. ad. o. q. ex quo ex. 15. sexti aut. 20. septimi. q. i. æqualis erit. m. p. & consequenter quadrato. d. u. Vt autem lector minori labore cognoscere queat. i. o. ad. o. u. sic se habere, vt. n. c. ad. n. a. sciendum est quòd, sic se habet. i. d. ad. d. u. ut. c. e. ad. e. a. ex quo componendo sic se habebit. i. u. ad. d. u. sicut. c. a. ad. a. e. & permutando ita. i. u. ad. c. a. vt. d. u. ad. e. a. sed cum ex præcedenti theoremate sic se habeat. d. u.



ad. u. o. sicut. e. a. ad. a. n. permutando sic se habebit. d. u. ad. a. e. sicut. u. o. ad. a. n. ex quo ex. 11. quinti sic se habebit. i. u. ad. e. a. prout. o. u. ad. a. n. permutando que. i. n. ad. u. o. vt. c. a. ad. a. n. & componendo, ita. i. o. ad. u. o. sicut. c. n.

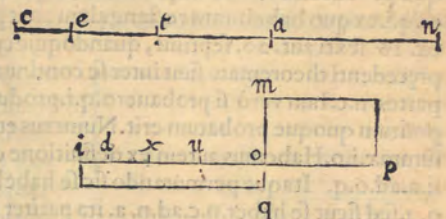
THEOREMA LXXXII.

**C**VR quantitate aliqua in quatuor partes cōtinuas proportionales secta per- que singulas diuisa, summa quatuor prouenientium æqualis sit producto secundi in tertium.

Exempli gratia, si triginta in quatuor partes proportionales secetur, hoc est. 16.8.4.2. perq; harum singulas idem numerus. 30. diuidatur, primum proueniens erit. 1. cum septem octauis partibus. Secundum. 3. cum tribus quartis, tertium. 7. cum dimidio, quartum. 15. integri, quorum summa erit. 28. cum octaua parte, tantumq; erit productum secundi prouenientis in tertium.

Quod vt sciamus, quantitas. n. c. in partes continuas proportionales quatuor secetur. n. a : a. t : t. e. et. e. c. rursusq; per singulas partes illa ipsa diuisa, prouenientia sint. i. d. d. x. x. u. u. o. quorū summa sit. i. o. hanc summā dicimus æqualem esse numero producti. d. x. in. x. u.

Quod hac ratione probo, cogito productam esse lineam. i. o. quousq; o. p. æqualis sit. o. u. erectamq; m. o. æqualem. i. d. perpendiculariter. o. p. & productam donec. o. q. vnitati sit æqualis. Iam terminatis rectangulis. m. p. et. i. q. patebit ex. 15. sexti aut. 20. septimi, productum. m. p. producto. d. x. in. x. u. æquale esse. Ita quod si probauero productum. i. q. producto. m. p. æquale esse, facile patebit propositum. Cuius gratia, sequuti præcedentis theorematis ordinem, primum ex definitione diuisionis, eadem proportio erit. n. c. ad. i. d. quæ. n. a. ad. o. q. ex quo permutando. n. c. ad. n. a. sic se habebit vt. i. d. hoc est. m. o. ad. o. q. & si progrediamur eodem ordine, quo præcedenti theoremate, sumpto principio ab. i. d. et. e. c. versus. d. x. et. e. t. gradatimq; permutando ac coniungendo, inueniemus eandem proportionem esse c. n. ad. n. a. quæ. i. o. ad. o. u. nempe. o. p. ex quo ex. 11. quinti, ita se habebit. i. o. ad. o. p. vt. m. o. ad. o. q. quare ex. 15. sexti aut. 20. septimi productū. i. q. erit producto. m. p. æquale, ex quo etiam æquale erit producto. d. x. in. x. u. Idem ordo in qualibet quantitate in quantasuis partes diuisa seruari poterit, cum huiusmodi sciētia in vniuersum pateat.



THEOREMA LXXXIII.

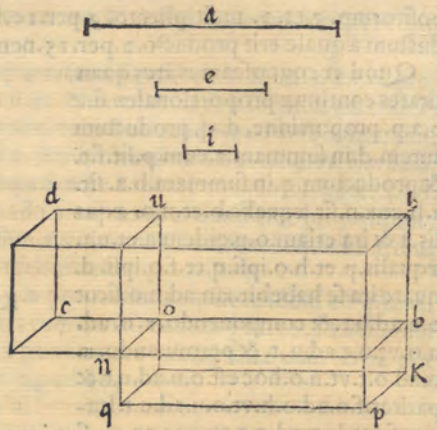
**C**VR termini medij cubus, trium continuè proportionalium, semper producto rectanguli comprehensi à maximo & medio in minimo termino æqualis sit.

Exempli gratia, datis his tribus terminis continuis proportionalibus. 9. 6. 4. si sumpserimus productum maximi in medium nempe. 54. quod per minimū. 4. multiplicemus, dabitur numerus. 216. cubo medij. 6. æqualis.

In cuius gratiam tres numeri cōtinui proportionales tribus lineis. a. e. i. significētur, cubus autem. e. significetur figura. d. n. productumq; a. in. e. fit. b. n. ipsius autēmet in. i. fit. p. o. ita quod. q. p. aut. b. o. cum sint eiusdē speciei, æqualis erit. a. et. o. n.

æqua-

æqualis. e. et. q. n. æqualis. i. Nunc cogitemus absolui corpus. n. h. ita ut. b. o. c. sit vnica recta linea, ex quo ex. 25. vndecimi proportio. n. h. ad. n. k. eadem est quæ. o. h. ad o. k. sed sic se habet. o. h. ad. o. k. vt. h. b. ad. b. k. ex prima sexti aut. 18. vel. 19. septimi itaque. n. h. ad. n. k. ex. 11. quinti sic se habebit. vt. h. b. ad. b. k. sed. n. h. ad. n. d. ex eisdem sic se habet ut. h. u. ad. d. u. et. h. u. ad. u. d. ita ut. h. b. ad. b. k. ex præsupposito. Itaque ex 11. prædicta. n. h. ad. n. k. eadem erit proportio quæ. n. h. ad. n. d. Quare ex. 9. quinti. n. k. æqualis erit. n. d. Quod erat propositum.

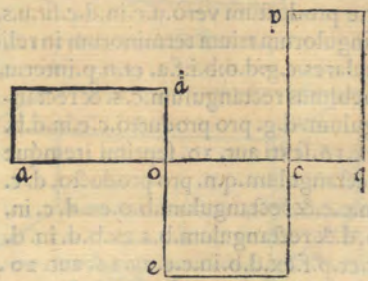


THEOREMA LXXXIII.

**C**VR quadrato vnus quantitatis radice proportionalis, per singulos tres terminos diuiso, prouenientia, singulis dictis terminis sint æqualia.

Exempli gratia, datis tribus terminis continuis proportionalibus. 9. 6. 4. quadratum medij erit. 36. quod per. 9. diuisum dabit. 4: per. 6: 6. per. 4: 9.

Cuius gratia, sint tres termini cõtinui pportionales. a. o. c. et. c. q. quadratũ autẽ medij sit. e. c. Iam si applicetur rectangulũ. a. d. æquale quadrato. e. c. ipsi. a. o. & rectangulum. q. p. æquale eidem quadrato. e. c. ipsi. c. q. si quadratum. e. c. per. a. o. diuiserimus, proueniens erit. o. d. diuisoq; per. c. q. proueniens erit. c. p. quod si per suam radicem. o. c. diuidatur, proueniens erit. o. e. quod sine dubio æquale est. o. c. sed dico. o. d. æqualem esse. c. q. Nam ex. 16. sexti aut 20. septimi eadem est proportio. a. o. ad. o. c. quæ. o. e. ad. o. d. nempe. o. c. ad. o. d. itaque o. d. ex. 9. quinti æqualis est. c. q. quandoquidem ex. 11. sic se habet. o. c. ad. o. d. sicut. o. c. ad. c. q. Applicatis iisdem rationibus ipsi. p. c. probabimus. c. p. æqualem esse. a. o. cum o. c. media sit proportionalis, tã inter. c. p. et c. q. quam inter. a. o. et. c. q. itaque. c. p. æqualis est. a. o.



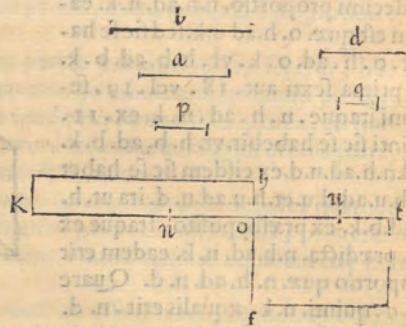
THEOREMA LXXXV.

**C**VR propositis tribus quantitibus continuis proportionalibus proportione aliarum duarum nobis datarum, multiplicata maiori postremarum duarum in summam mediæ cum minima trium primarum, productum æquale sit producto minoris duarum in summam maximæ cum mediâ trium.

Exempli gratia proponuntur quantitates. 9. 6. 4. proportione numeroꝝum propositi-

positorum. 3. et. 2. multiplicato. 3. per. 10. summā. 6. cum. 4. dantur. 30. quod productum æquale erit producto. 2. per. 15. nempe per summam. 9. et. 6.

Quod ut cognoscamus, tres quantitates continuæ proportionales sint b. a. p. proportione. d. q. productum autem. d. in summam. a. cum. p. fit. f. t. & productum. q. in summam. b. a. fit. x. h. et. k. n. fit æqualis. b. et. n. o. æqualis. a. & ita etiam. o. u. eidem. a. et. u. t. æqualis. p. et. h. o. ipsi. q. et. f. o. ipsi. d. quare ita se habebit. k. n. ad. n. o. sicut o. u. ad. u. t. & componendo. k. o. ad. n. o. vt. o. t. ad. u. t. & permutando. k. o. ad. o. t. vt. n. o. hoc est. o. u. ad. u. t. & pariter. f. o. ad. o. h. vt. o. u. ad. u. t. Itaque sicut. k. o. ad. o. t. ex quo ex. 15. sexti aut. 20. septimi. k. h. æqualis erit. f. t.

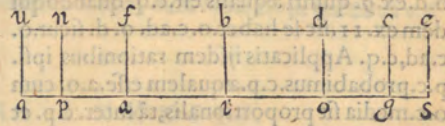


T H E O R E M A L X X X V I.

**C**VR multiplicatis singulis tribus quantitatibus continuis proportionalibus in reliquas duas, sex producta æqualia sint producto dupli summæ ipsarum trium in mediam proportionalem.

Exempli gratia, proponuntur hi tres termini continui proportionales. 9. 6. 4. productum. 9. in. 6. erit. 54. at. 9. in. 4. erit. 36. et. 6. in. 9. 34. et. 6. in. 4. 24. et. 4. in. 9. 36. et. 4. in. 6. 24. quæ producta simul collecta efficiunt numerum. 228 sed tantum est productum dupli summæ trium terminorum in secundum nempe. 38. in. 6.

Cuius intelligentiæ causa, tres termini cōtinui proportionales significantur linea. b. e. nempe. b. d. d. c. c. e. cuius duplum sit. u. e. et. b. f. æqualis sit. b. d. et. f. n. d. c. et. n. u. c. e productum verò. u. e. in. d. c. fit. u. s. cui dico æqualem esse summam productorum singulorum trium terminorum in reliquos duos. Quamobrem ducantur perpendicularares. c. g. d. o. b. i. f. a. et. n. p. inter. u. e. et. q. s. ex quo pro producto. c. e. in. c. d. habebimus rectangulum. c. s. & rectangulum. d. g. pro producto. c. e. in. d. b. ex. 16. sexti aut. 20. septimi itemque rectangulum. q. n. pro producto. d. c. in. c. e. & rectangulum. b. o. ex. d. c. in. b. d. & rectangulum. b. a. ex. b. d. in. d. c. et. p. f. ex. d. b. in. c. e. ex. 16. aut. 20. prædictas. Quare sex producta æquantur inter se, replentq; productum. u. s. ex quo verum est propositum.



T H E O R E M A L X X X V I I.

**Q**VA ratione cognosci possit verū esse proportionem summæ quatuor quantitatū continuarum proportionalium ad summam secundæ & tertiæ, eandem esse, quæ summæ primæ & tertiæ ad secundam simplicem.

Exempli gratia, si inuenirentur hæ quatuor quantitates continuæ proportionales. 16. 8. 4. 2. earum summa erit. 30. summa verò secundæ & tertiæ. 12. tum summa primæ

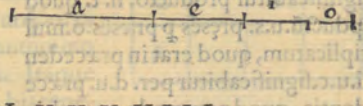


THEOREM ARITH.

primæ cum tertia. 20. ex quo sic se habet. 20. ad. 8. nempe ad secundam, vt. 30. ad. 12.

Quod ut sciamus, quatuor prædictæ quantitates significantur. linea. a. e. i. o. probabo ita se habere. a. e. i. o. ad. e. i. vt. a. i. ad. e. Nam cum sic se habeat. a. ad. e. ut. e. ad. i. & vt. i. ad. o. ex æqualitate proportionum vel permutando ita se habebit. a. ad. i. vt. e. ad. o. & è conuerso ita. o. ad. e. vt. i. ad. a. & cõponendo ita. o. e. ad. e. vt. i. a. ad. a. permutando q̄. o. e. ad. i. a. vt. e. ad. a. nempe. i. ad. e. & componendo ita. o. i. e. a. ad. i. a. vt. i. e. ad. e. & permutando ita. o. i. e. a. ad. i. e. vt. i. a. ad. e. quod erat propositum. Ex quo patet error antiquorum qui idipsum, accidere arbitrati sunt in quantitatibus discretæ proportionalitatis, quod tamen falsum est.

Exempli gratia, si proponantur. 12. 6. 4. 2. proportio. 12. ad. 6. eadem est quæ. 4. ad. 2. Sed à proportione. 6. ad. 4. frangitur, cum non sit eadem quæ. 12. ad. 6. harum autem summa erit. 24. & summa secundæ cum tertia. 10. sed primæ cum tertia erit 16. ex quo. 16. ad. 6. non sic se habebit vt. 24. ad. 10. At in speculatione quatuor quantitarum. a. e. i. o. si proportio. e. ad. i. non esset eadem quæ. a. ad. e. minimè licuisset dicere ita se habere. i. ad. e. vt. e. ad. a.

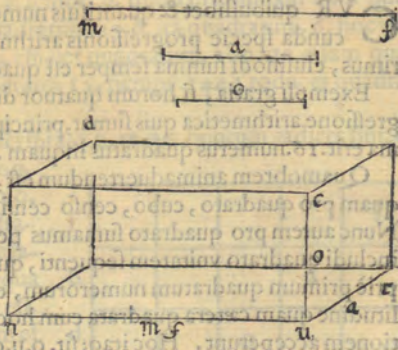


THEOREMA LXXXVIII.

CVR ex tribus quantitatibus quibuslibet, productum duarum in tertiam, vna semper eademq; sit quantitas.

Exempli gratia, proponuntur. 15. 8. 2. si multiplicauerimus. 15. per. 8. tum productum per. 2. tantum eris quantum si quis multiplicaret. 8. per. 2. & hoc per. 15. et. 15. per. 2. rursusq; per. 8.

Quod ut pateat, tres quantitates tribus lineis significantur. m. f. a. et. o. Dico productum. m. f. in. a. multiplicatum per. o. æquale esse producto. a. in. o. multiplicato per. m. f. aut producto. m. f. in. o. multiplicato per. a. Sit enim corpus. d. u. rectangulum, cuius latus. n. u. sit æquale m. f. et. u. t. a. et. u. c. o. patebit manifestè n. t. esse productum. m. f. in. a. quod. n. t. multiplicatum in. u. c. æquali. o. producit corpus. d. u. sed idipsum corpus. d. u. ex multiplicatione producti. c. t. in latus. n. u. æquale. m. f. oritur, & idipsum. d. u. ex multiplicatione. n. c. in latus. u. t. æquale. a. profertur.



THEOREMA LXXXIX.

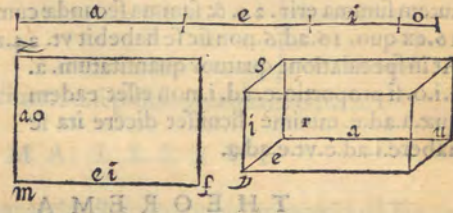
CVR quarumcunq; quatuor quantitarum, si prima in secundam multiplicetur & hoc productum in tertiam, rursusq; hoc alterum in quartam, vltimum productum æquale sit producto producti secundæ in tertiam, in productum primæ in quartam.

H Exempli

Exempli gratia, casu sese offerunt hi quatuor numeri. 8. 5. 3. 2. multiplicato. 8. per. 5. & hoc. 40. per. 3. rursus hoc. 120. per. 2. vltimum productum esset. 240. æquale producto. 15. (quod ex. 5. in. 3. oritur) in productum. 16. quod ex. 8. in. 2. profertur.

Cuius speculationis gratia, cogitemus quatuor numeros quatuor lineis. a. e. i. o. significari, productum autem. e. in. i. esse. m. f. et. r. s. similiter & productum. a. in. o. esse. m. z. et. z. f. productum esse. m. f. in. m. z. cui productum. a. in. e. multiplicatum per i. & hoc tandem per. o. æquari debet.

Sit itaque. u. y. productum. a. in. e. quod. u. y. per. i. multiplicatum proferat. u. s. hocque. u. s. multiplicatum per. o. Dico quod dabit numerum æqualem numero. f. z. Quamobrem. r. s. aut. m. f. quod idem est, in figura præcedentis theorematris significetur linea. n. u. & linea. r. u. huius, nempe. a. significetur per. u. t. præcedentis, ex quo numerus producti. u. s. præsentis, in præcedenti significabitur producto. n. t. quod productum. u. s. præses p. præses. o. multiplicatum, quod erat in præcedenti. u. c. significabitur per. d. u. præcedentis, quod non modo ex multiplicatione. n. t. præcedentis, nempe. u. s. præsentis. in. u. c. præcedentis æquali. o. præsentis oritur, sed etiam ex. c. t. præcedentis æquali. m. z. præsentis in. n. u. præcedentis æquali. m. f. præsentis. Itaque verum est propositum.



### THEOREMA XC.

**C**VR quibuslibet & quantifuis numeris in summam collectis, si ab vnitatem in secunda specie progressionis arithmetice imparium numerorum progressi fuerimus, eiusmodi summa semper est quadratus numerus.

Exempli gratia, si horum quatuor disparium numerorum summam, in dicta progressionem arithmetica quis sumat, principio ab vnitatem sumpto, nempe. 1. 3. 5. 7. summa erit. 16. numerus quadratus inquam. Idem de cæteris.

Quamobrem animaduertendum est, vnitatem, tam sumi pro sui ipsius radicem, quam pro quadrato, cubo, censo censi, primo relato, & alia quavis dignitate. Nunc autem pro quadrato sumamus per. o. significato, cogitemusque quadratum. o. includi quadrato vnitatem sequenti, quod, vt patet, est quatuor vnitatum, ac proprie primum quadratum numerorum, ex quo etiam nomen accepit, vnde ex similitudine quam cætera quadrata cum hoc primo retinent, ex quaternario denominationem acceperunt. Hoc itaque sit. o. u. c. e. ita ex communi scientia quadrato. o. iungitur gnomon. e. c. u. constans tribus vnitatibus, quare primus gnomon, numero impari constat. Scimus etiam ex additione numeri binarij ad imparem, numeris disparibus summam excrecere, cum propius accedere quã binario nequeant, ex quo medio binario, sibi inuicem succedunt. Dico igitur quod quinario ternarium sub sequente, coniuncto quadrato. o. u. c. e. profertur quadratum, quod in numeris, binarij quadratum sequitur, eritque ternarij, quodque significetur per. o. f. patet enim primo non differre ab. o. c. præter quam gnomone. b. f. d. qui coniungitur quadrato. o. c. quique duabus vnitatibus maior est. e. c. u. Iã scimus gnomonem. e. o. u. æqualem esse

THEOREM. ARIT.

esse gnomoni. e. e. u. itemq; gnomonem. b. f. d. æqualem gnomoni. b. o. d. at hic gnomon. b. o. d. ex præsupposito, maior est gnomone. e. o. u. duabus vnitatibus. b. et. d. Itaque etiam gnomon. b. f. d. duabus vnitatibus gnomonem. e. c. u. superabit. Quare. b. f. d. erit impar immediatè sequens ternarium, qui coniunctus quadrato. o. c. quadratum subsequens componet. Eadem ratione probabitur de quadrato. o. n. sequenti. o. f. & gnomone. i. n. a. cum hic ordo speculationis sit vniuersalis. In quo cernitur quemlibet gnomonem sibi contiguū inferiorem semper duabus vnitatibus excedere, cumque quadrata non nisi gnomonibus sibi inuicem succedant. Sed cū primus. e. c. u. dispar fuerit, pculdubio etiā necessarioq; cæteri dispares erūt. Ex qua speculatione, oritur regula ab antiquis tradita inueniendi vltimi numeri disparis cōcurrentis ad cōpositionem alicuius quadrati. Vt si quis scire desideret numerum vltimum disparem, quo mediante quadratum. o. n. constitutum fuit, quod aliud non est quam scire quantus sit numerus vltimi gnomonis. i. n. a. æqualis gnomoni. i. o. a. Itaque vt sciamus hunc gnomonem. i. o. a. patet duplicandam esse radicem. o. e. b. i. dabiturq; o. e. b. i. et. o. u. d. a. vbi bis reperitur. o. nos autem tantummodo quærimus scire gnomonem. i. b. e. o. u. d. a. Itaque minor est vnitare duplo radicis, cum unitas. o. bis repetatur, quæ tamen in gnomone semel tantum sumebatur.

o	e	o	i
u	c		
d		f	
a			n

THEOREMA XCI.

**C**VR summa quadratorum, quorum radices sunt in proportione sesquitertia nempe. 4. ad. 3. quadrata sit.

Exempli gratia, sumemus quadratum. 3. scilicet 9. quod in summam cum quadrato. 4. colligemus, nempe. 16. eritq; quadratum. 25. & ita quadratum. 6. hoc est. 36. collectum cum quadrato. 8. nempe. 64. efficiet quadratum. 100. ita etiam quadratum. 9. hoc est. 81. coniunctum quadrato. 12. nempe. 144. producet quadratum. 225.

In cuius gratiam sint duo quadrata subscripta. q. o. et. q. a. quorum radices sint. q. g. et. q. p. hoc est. q. g. quatuor vnitatum, et. q. p. trium, ex quo. q. a. erit. 16. vnitatum et. q. o. nouem. Ad hæc cogitemus applicari quadrato. q. a. gnomonem. f. s. h. tam amplum siue latum quæ gnomon. b. a. g. nempe vt. h. fit æqualis. g. g. verò differentia sit qua. q. g. maior est. q. p. huncq; gnomonem. f. s. h. dico æqualem esse quadrato. q. o. nam ex præsupposito. g. ter radicem. q. p. ingreditur, & quater. q. g. ex quo, tres partes. q. k. p. inter se æquales sunt vnde etiam quadratum. q. o. nouem partibus superficialibus quadratis constabit, quarum singularum radix æqualis erit. g. cumque præcedenti theoremate didicerimus quemlibet gnomonem quadrati immediatè sequentis æque amplitudinis cum gnomone præcedentis,

f				s
b			a	
d		o		
q	k	p	g	h

H 2 semper

per duab. vnitatibus superficialibus crescere, quarū singularū radix æqualis est. g. ne cessariò sequitur gnomonem. b. a. g. duabus partibus aut vnitatibus gnomonem. d. o. p. superare, ita vt gnomon. b. a. g. septem vnitatibus, aut partibus superficialibus quadratis constet. Quare eadem ratione gnomon. f. s. h. constabit nouem similibus. Ita que æqualis erit quadrato. q. o. Quamobrem verum est, quòd quadrato. q. o. coniuncto quadrato. q. a. proueniet quadratum. q. s. cuius radix ita differet à. q. g. vt. q. g. à. q. p. ex quo tres radices arithmetice inter se continuæ proportionales erunt. Idipsum dico si. q. p. fuerit. 6. et. q. g. 8: tunc enim singulae partes. q. k. p. g. h. æquipollebunt duabus vnitatibus, quæ cogitabuntur in summam collectæ, ut cum patribus. q. k. p. g. h. integris contemplari liceat. Idem accidet si. q. p. erit. 9. et. q. g. 12. singulae enim partes. q. k. p. g. h. tripartitæ erunt. Idcirco dixi gnomonem. f. s. h. tam amplum cogitari debere, quam gnomon. b. a. g. nempè ut. h. æqualis sit. g. Idem occurret si. q. g. erit. 12. et. q. p. quinque, quod cum fuerit patebit ex præcedentis theorematum speculatione, gnomonem f. s. h. 25. vnitatibus constare, cogitatum amplitudinis simplicis vnitatis denominatæ in. q. p. aut. q. g. non amplitudinis gnomonis. b. a. g. qui septem vnitatibus latus esset. Cum igitur. q. p. quinque vnitatibus linearibus constet scimus. q. o. 25. superficialibus constare, collecto itaque in summam quadrato. q. o. cum quadrato. q. a. cognoscetur quadratum. q. s. vna etiam eius radix. Eadem ratione, alia multa quadrata similia contemplari licebit.

f					s
b				a	
d			c		
	q	k	p	g	h

T H E O R E M A X C I I.

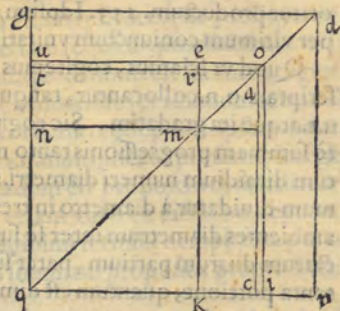
**C** V R. proposito numero pari maiori binario, qui detrahi & in summam colligi debeat ex altero numero quarendo, vt tam residuum quam summa sint quadrata numerorum integrorum. Rectè dimidium propositi numeri in seipsum multiplicamus, & quadrato huic addimus vnitatem, eritq; numerus quæsitus. Exempli gratia proponitur. 12. numerus detrahendus, & coniungendus numero inuestigando, ut residuum detractiois, & summa sint quadrati numeri. Addita vnitatem ipsi. 36. quadrato dimidij, dabitur. 37. numerus quæsitus. Cuius speculationis gratia, subscripta quatuor quadrata cogitemus. g. p. u. i. r. e. n. k. cogitemusq; quadratum. g. p. esse quadratum summæ, k. n. verò residui subtractionis: u. i. aut numerum inuestigandū, ex quo gnomon. u. d. i. cognoscetur ita etiam et. n. o. k. qui inter se sunt æquales. Iam certi erimus. e. i. esse plus quam dimidium gnomonis. n. o. k. Itaque cogitemus rectangulum. r. c. exactum dimidiū esse gnomonis. n. o. k. ex unitatibus superficialibus quarum una erit. m. a. Cuius numeri quadratum sit. t. c. vnde etiam cognitum & cum. k. c. ex communi scientia sit vnitatis linearis, propterea quod. m. a. est superficialis hoc est quadrata, quæ detracta ex. q. c. dimidio gnomonis. n. o. k. (quamuis lineari) supererit. k. q. cognita, numerorum integrorum (nota q. k. i. semper minor erit duabus vnitatibus linearibus & maior vna ex dictis vnitatibus, ut ex te ipso contemplari potes) quare.

T H E O R . A R I T H .

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n. k. ipsius quadratum numerorum integrorum cognoscetur, cui addito gnomone. n. o. k. cognoscemus numerum. u. i. quæsitum.

Sed cum nobis hæc via, tenenda propositum non fuit, hoc est primo loco inveniendi quadrati minoris. n. k. ideo superest probandum gnomonem. t. o. c. unitati æqualem esse, nempe quadratulo. m. a. quod patebit, si consideremus nos sumpsisse rectangulum. r. c. pro dimidio gnomonis. n. o. k. etenim si supplemento etiam. n. r. quadratulum æquale. m. a. adderetur, pateret gnomonem. n. a. k. cum dicto quadratulo collectum, æqualem esse gnomoni. n. o. k. cum duo supplementa. m. t. et. m. c. inter se sint æqualia. Quamobrem inuento quadrato. t. c. ex dimidio gnomonis cognito, additur vnitas, gnomon scilicet. t. o. c. ex quo cognoscitur numerus. u. i. quæsitus. Quod autem quadratum. g. p. numeris integris constet, hac ratione probatur visum enim fuit supra quadratum. n. k. verè quadratum esse, & numeris integris constare, pariter etiam. t. c. seque mutuò consequi (nam. k. c. est vnitas linearis) ex quo gnomon n. a. k. numero dispari constabit, ex ijs quæ. 90. theoremate probata fuerunt. Itaque ex eodem theoremate necesse est gnomonem. t. d. c. etiam numero dispari constare, ita vt à numero. n. a. k. non nisi duabus unitatibus differat, nempe vt. c. p. sit vnitas linearis, sed ita reuera est, numerus enim. u. d. i. ex præsupposito par est, quare numerus. t. d. c. dispar erit, cum alterum unitate superet, videlicet gnomone. t. o. c. unitati æquali, tum. n. a. k. minor est. n. o. k. ex eodem gnomone. t. o. c. unitati æquali. Itaque. n. a. k. minor erit. u. d. i. per unitatem, & minor. t. d. c. per duas unitates, ex quo sequitur. g. p. esse quadratum integrorum ex dicto theoremate ac consequens quadrato t. c. quare. c. p. vnitas erit, & radices. q. k. et. q. p. horum quadratorum numero binario inter se different. Vnà etiam scienda est causa, cur numerus propositus necessariò binario maior esse debeat. Etenim cum ipse sit futurus gnomon. n. o. k. nec possit minor esse numero ternario, vt patet ex. 90. theoremate, idcirco sequitur necessariò maiorem esse binario debere. Quòd si dispar numerus proponeretur, nec forma operis nec speculationis mutanda esset. Non erit tamen necessarium vt ipsa quadrata. n. k. et. g. p. numeris integris constarent. Sæpius enim fractis cõponerentur, quòd ex. 90. theoremate facile erit speculari nihilominus fractis integris, ipsisq; collectis cum suis fractis summæ essent quadrata.



T H E O R E M A X C I I I .

**C**VR propositis duobus numeris altero pari, altero verò dispari, duplo primi minore per unitatem, si alium inuenire numerum voluerimus, cui alterum isto rum coniunctum proferat quadratum, & altero detracto, quadratum superfit. Rectè datos números in summam colligemus, quam summam in duas quam maximas poterimus partes diuidemus, quarum vna pari, altera dispari constet, tum vtraque in seipsam multiplicabimus, & quadrato minori, duorum numerorum propositorum quemuis ademus, ex quo cupimus nobis quadratum minus superesse, & proueniet nobis numerum quæsitum.

Exempli gratia, proponuntur numeri. 11. et. 6. quorum alter alicui numero adden-

dendus, alter ex eodem detrahendus sit, ex quo proferri debeant bina quadrata. Itaque numeri illi in summam collecti dabunt. 17. differentiam minoris quadrati & maioris. Iam si ex hoc. 17. binas partes fecerimas, altera erit. 8. altera. 9. quibus in seipsis multiplicatis alterum quadratum erit. 64. alterum. 81. addito itaque ipsi. 64. 11. aut. 6. pro libito, propositum numerum consequemur. cui addito. 6. vel. 11. dabit nobis. 81. vel ex ipso detracto. 11. vel. 6. relinquet nobis 64. in presenti autem exemplo talis numerus erit, aut. 70. vel. 75. Huius autem theorematis speculatio ex. 90. dependet, quo demonstratum fuit gnomonem proximè quadratum sequentem, vnitatem duplo radice minorem esse.

### THEOREMA XCIII.

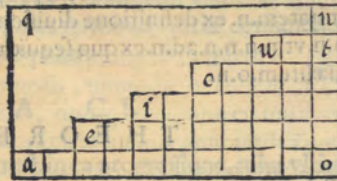
**C**VR si quis cupiat summam progressionis arithmetice quam citissime cognoscere. Rectè coniungat vltimo termino vnitatem primum terminum, huius postea vltimi termini dimidium cum numero terminorum multiplicabit, ex quo multiplicationis productum, erit omnium propositorum terminorum summa, aut eundem vltimum terminum iunctum primo, per dimidium numeri terminorum multiplicabit. Nam id ipsum eueniet.

Exempli gratia, si proponerentur. 17. termini in prima progressionem arithmetica naturali, vltimus esset. 17. cui coniuncta vnitatem primo termino summa erit. 18. cuius dimidium cum numero terminorum, nempe. 17. multiplicatum cum fuerit, oritur productum. 153. Id ipsum eueniet, multiplicato dimidio numeri terminorum per vltimum coniunctum vnitati primo termino.

Quod ut sciamus, cogitemus terminos progressionis collocari, ut in figura subscripta. a. o. n. collocantur, tanquam per gradus, sumpto principio ab vnitatem. n. tum. u. t. atque ita gradatim. Sic cogitato absoluto parallelogrammo. q. o. sciemus apertè summam progressionis tanto maiorem esse dimidio totius parallelogrammi, quantum dimidium numeri diametri. a. e. i. c. u. n. requirit. Nam cum parallelogrammum diuidatur à diametro in tres partes, diameter vnam occupat, reliquæ verò duæ ambientes diametrum inter se sunt æquales. Sumpto itaque diametro cum altera dictarum duarum partium, patet summi plusquam dimidiū totius parallelogrammi. pro tanta portione, quantum est dimidium occupatum à diametro, qui cæ ex discretis respondentibus numero terminorum componatur, constat numero æquali esse dicto numero terminorum. o. n. Iam si quis multiplicet. a. o. per dimidium. o. n. procul dubio, ex prima sexti aut. 18. septimi, oriatur dimidiū numeri parallelogrammi. q. o. quod minus erit summa progressionis dimidio numeri diametri, aut quod idem est dimidio. o. n. sed hoc dimidium. o. n. æquale est producto dimidij vnitatis. n. in. o. n. ex. 20. septimi, cum dimidium. o. n. sit eius productum in vnitatem. Itaque multiplicato. n. o. per dimidium. o. a. coniunctum dimidio vnitatis. n. oritur summa quæ sita propositæ progressionis. Id ipsum accidet multiplicata summa. o. a. & vnitatem. n. p. dimidium. o. n. ex. 20. septimi, cum proportio totius ad totum eadem sit, quæ dimidij ad dimidium, ex causâ permutationalitatis. Patet etiam in progressionibus, quæ ab vnitatem initium ducunt, si fiat ascensus per binarium summa vltimi termini cum primo semper duplam futuram esse numero terminorum, quod sequentes figura

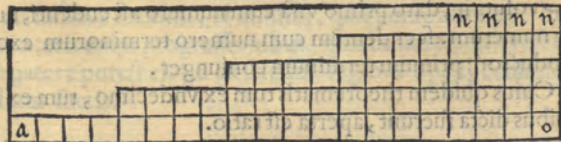
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ras consideranti speculari licebit, Diametros harum figurarum notavi literis siue characteribus. a. e. i. c. u. n.



THEOREMA XCV.

**I**N progressionibus, quæ ab alio termino quam vnitatem incohantur, id ipsum ut monuimus accidit, hoc tamen notato, quod ex consequenti quælibet pars diametri parallelogrammi, minimo termino æqualis erit, prout in progressionibus quæ ab vnitatem originem ducunt, singulæ partes diametri, vnitati sui primi termini æquales sunt. At in reliquis progressionibus, ut in figura patet, eadem est proportio totius diametri ad. o. n. quæ minimi termini ad vnitatem ex. 13. quinti, nempe. a. o. ad. o. n. vt. n. n. n. ad. n. In eiusmodi progressionibus accidit quoque parallelogrammum à diametro in tres partes diuidi, quarum vnam ipse occupat, reliquæ vero inter se æquales ipsum ambiunt. Ex quo illud etiam sequitur, productum. a. o. in dimidium. o. n. æquale esse dimidio parallelogrammi, quod minus est summa progressionis dimidio diametri, quod dimidium si inuenire voluerimus, minimum terminum. n. n. n. n. per dimidium. o. n. multiplicabimus, & ex. 18. aut. 19. septimi ipsum habebimus, quandoquidem minimo termino per totum. o. n. multiplicato profertur integer diameter ex. 20. prædicti. Etenim ut diximus, eadem est proportio totius diametri ad. o. n. quæ minimi termini ad vnitatem. Ita etiam dico ex dicta. 20. septimi. idem dimidium diametri oriri, si quis dimidium minimi termini nempe. n. n. per totum. o. n. multiplicauerit. Quamobrem qui statim summam propositæ progressionis cognoscere voluerit, semper primum terminum. n. n. n. n. cum. a. o. coniungat, qua summa per dimidium. o. n. multiplicata, aut. o. n. per dimidium dictæ summæ, ex prædictis rationibus propositum consequemur.



THEOREMA XCVI.

**C**VR si quis numerum terminorum inuenire velit, cognitis tantummodo primo atque ultimo, rectè vltimum per primum diuidet, ex quo proueniens nume-

numerus quæsitus erit.

Quod intelligendum est tamen quoties primus terminus differentia terminorum est, nempe ascendens ipsorum terminorum.

Cuius ratio manifestè speculari potest in figura præcedentis theorematum. Nam diuisa .a. o. per .n. n. n. eadem proportio erit. a. o. ad proueniens, qua. n. n. n. ad unitatem. n. ex definitione diuisionis. At superius dictum fuit ita se habere. a. o. ad. o. n. vt. n. n. n. ad. n. ex quo sequitur ex. 11. et. 9. quinti proueniens esse numerum quæsitum. o. n.

THEOREMA XC VII.

**V**BI verò primus terminus, reliquorum non erit differentia. Hac de causa necesse est detrahere primum ex ultimo, residuumq; per numerum ascendentem differentiam scilicet, partiri, proueniensq; unitati coniungere, quò numerum terminorum habere possimus. Scimus etenim tam multas unitates esse in ultimo terminorum quot in omnibus intervallis aut differentijs in summam collectis simul cum unitatibus primi termini, totq; sunt termini, quot intervalla simul cum primo termino. Quare si minimus terminus intervallo æqualis fuerit. Ultimo per primum diuiso, ex ad ductis præcedenti theoremate propositum consequemur. Itaq; primo termino ex ultimo detracto residuoq; per intervallum, hoc est numerum differentie diuiso, proueniens erit numerus terminorum absque primo qui vnus est, coniuncto quoque dicto proueniens propositum consequemur.

**C**VR si quis arithmeticae progressionis dato primo & ultimo simul cum numero terminorum, ascendente numerum cognoscere voluerit. Rectè primum ex ultimo detrahet, residuumq; per numerum terminorum excepto vno diuidet. Huius theorematum speculatio ex. 13. theoremate manifesta erit, nam in præcedenti cap. numerus terminorum erat proueniens diuisionis residui subtractionis primi termini ex ultimo.

THEOREMA XC VIII.

**C**VR si quis maximum omnium terminorum dictæ progressionis cognoscere voluerit, dato primo vna cum numero ascendenti, numeroq; terminorum. Rectè numerum ascendente cum numero terminorum excepto vno multiplicabit, productoq; primum terminum coniunget.

Cuius quidem theorematum ex vndecimo, tum ex ijs quæ præcedentibus capitibus dicta fuerunt, aperta est ratio.

THEOREMA XC.

**C**VR veteres cupientes obtinere summam progressionis continuæ naturalis, quæ ab unitate initium ducit, dato ultimo termino tantummodo. Dimidium ultimi termini cū toto sequenti multiplicabant, productumq; summa quæsitæ erat. Exempli gratia, si vltimus terminus eiusmodi progressionis fuerit. 7. multiplicato



to dimidio ipsius nempe. 3. & dimidio, cum numero ipsum terminum sequenti, nempe. 8. summa dictorum terminorum erit. 28.

Huius autem speculatio ex. 94. theoremate dependet, in quo facile deprehendere dicitur ex figura continuæ progressionis naturalis, numerum terminorum maximo termino semper æqualem esse; ex quo tantum est dimidium numeri terminorum, quantum maximi dimidium, tantusque est ultimus terminus unitati coniunctus, quantum numerus is, qui ultimum terminum consequitur.

THEOREMA C. I.

**C**VR anti qui id ipsum, quod iam dictum est, in ea progressionem, cuius ultimus terminus dispar est, scire cupientes, numerum integrorum proximè dimidium maximi sequentem sumebant, quem per maximum multiplicabant, ex quo summa quaesita oriebatur.

Exempli gratia, si dimidium maximi fuisset. 3. cum dimidio, sumebant quatuor, & per maximum. 7. multiplicabant, ex quo pariter proferebatur summa. 28.

Cuius ratio ex. 20. septimi Euclidis oritur, cum eadem sit proportio numeri sequentis maximum ad numerum dimidium maximi sequentem, quæ maximi ad suum dimidium, est enim dupla.

THEOREMA C. II.

**T**Raditum est à nonnullis, à veteribus obseruatam fuisse hanc regulam, qua scire possent summam alicuius progressionis arithmeticæ discontinuæ aut intercisæ, quæ numero pari terminetur. Multiplicabāt enim dimidiū vltimi termini per proximum numerum dimidio dicto maiorem, ex quo inquebāt semper productum summæ quaesitæ æquale esse, subiiciuntque exemplum progressionis, quæ à binario inchoata crescit per binarium. In qua quidem progressionem non per se, sed per accidens regula vera est. Hoc est; non quia ex se vnus ex producentibus numeris dimidium termini maioris futurus sit, alter uerò proximè sequens dimidium, sed quia vt dictum est. 95. theoremate, eadem est proportio maximi termini ad numerum terminorum, quæ minimi ad unitatem. Cumque in præsentem exemplo minimum sit duplum unitati in eiusmodi casu, numerus terminorum, dimidio maximi termini æqualis est, qui terminorum numerus ex se, vt patet, vnus est ex producentibus, alter uerò producens numerus, est proximè dimidium sequens, non ex se, sed quia numerus sequens, dimidium est summæ maximi, & minimi, quæ per se alter esse debet producens numerus. In cæteris enim progressionibus, quæ binario non crescūt regula falsa est, prout facile patere potest ei, qui ex scientiæ legibus ope speculationis. 95. theorematem speculatus fuerit.

THEOREMA C. III.

**A**LIAM quoque tradunt regulam, qua veteres vsos fuisse dicunt, quo summam scire possent progressionis discontinuæ, quæ numero dispari absoluitur. Ea autem est eiusmodi. Vltimum terminum in duas quam maximè poterant maximas partes diuidebant, quarum vna semper altera maior erat, hanc autem maiorem in seipsam multiplicabant, atque quadratum hoc, summam progressionis esse affir-

affirmabant, Quæ sanè regula, non semper, etsi interdum vera sit.

Sumebant hi exemplum progressionis, quæ ab unitate incohata crescit per binarium, in qua per accidens euenit vt numerus dimidium vltimi termini proximè sequens, nempe è duabus partibus vltimi termini maior, æqualis sit numero terminorum, qui per se vnus è producentibus, ex ijs quæ. 94. theoremate diximus, esse debet; alter verò producens, qui per se dimidium summæ primi & vltimi esse debet, per accidens pars maior est duarum vltimi termini; & alteri producenti æqualis.

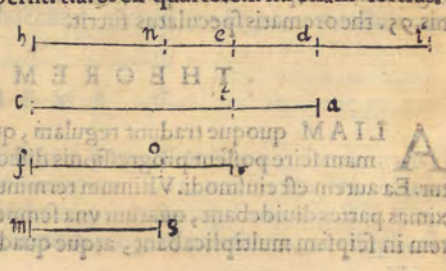
Aut alio modo ratiocinemur, dicentes, in huiusmodi progressionem dimidium summæ vltimi termini cum primo, semper medium proportionale est inter eam summam & dimidium numeri terminorum, etenim huiusmodi summa numero terminorum semper dupla est, prout. 94. theoremate tradimus. Itaque ex. 20. septimi, quadratum partis maioris; producto summæ dictæ in numerum dimidij terminorum æquale erit, quod productum per se summæ progressionis est æquale. At in cæteris eiusmodi progressionibus fallit regula, vt ex supradictis facillè demonstratur.

THEOREMA CIIII.

**P**ermultis terminis ad libitum propositis, dispositis nihilominus progressionem, aut proportionem geometricam continuam, si minimus ex maximo & ex sequenti minimum detrahatur, residuum maximi, eam proportionem ad summam reliquorum omnium terminorum retinebit, quam residuum secundi ad primum.

Proponuntur, exempli gratia, quatuor termini. 3. 12. 48. 192. continui geometricè proportionales, si primum, hoc est minimum, ex secundo, & maximo detrahas, ex secundo supererit. 9. ex maximo. 189. quod si minimum per residuum maximi multiplicaueris, hoc est. 189. orietur. 567. tum si huiusmodi productum per. 9. (residuum secundi) diuiseris, proueniet. 63. quod proueniens æquale erit summæ reliquorum omnium terminorum, maximo excepto. Ex quo inferre licet ex. 20. septimi eandem proportionem esse. 189. ad. 63. quæ. 9. ad. 3. aut si residuum secundi per summam dictorum terminorum multiplicaueris produceretur idem. 567. quare ex. 20. septimi & cætera.

Quod vt sciētificè possimus, & in vniuersum speculari. Quatuor termini propositi, quatuor subscriptis lineis significētur. b. i. c. a. f. r. m. s. (quod aut de his quatuor dico de centum illibus, & eo amplius dicere possum.) Nunc minimus terminus. m. s. ex maximo. b. i. detrahatur, superfitq;. n. i. idemq;. m. s. ex secundo termino. f. r. subtrahatur, superfitq;. o. r. Dico proportionem. n. i. ad summam reliquorum omnium terminorum. e. a. f. r. m. s. eandem esse, quæ. o. r. ad. m. s. Quamobrem ex tertio & quarto secundus. f. r. detrahas, ex tertioq; superfit. t. a. & ex quarto. e. i. ita etiam tertius. c. a. ex quarto. b. i. superfitq;. d. i. sanè sic se habebit. c. a. ad. f. r. vt. c. t. ad. f. o. vt quisq; per se scire potest. Quare ex 19. quinti sic se habebit. a. t. ad. r. o. vt. c. a. ad. f. r. & permutando ita. a. t. ad. a. c. vt. o. r. ad. r. f. & separando sic. a. t. ad. a. c. (hoc est. f. f.) vt. r. o. ad. o. f. vide licet. m. s. Idè dico de. d. i. ad. a. c. nempe sic se habebit. d. i. ad. a. e. vt. a. t. ad. r. f.



THEOREM. A. A. R. I. T.

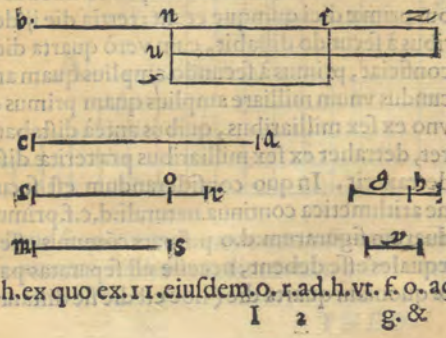
r. f. hoc est . o . r . ad . m . s . ex . i . r . quinti . Itaque ex communi scientia sic se habe-

THEOREMA C V .

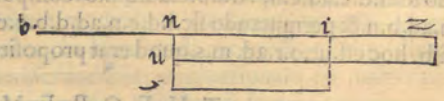
CVR desideranti summam quorumcunque terminorum progressionis conti-

Exempli gratia , si darentur quatuor termini continui proportionales . 8 . 12 . 18 . 27 . primum hoc est minimum . 8 . ex ultimo . 27 . detraheremus : remaneretq ; 19 . qui

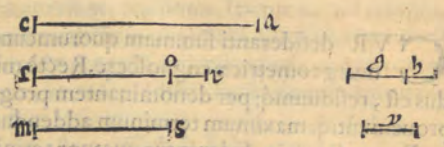
Pro cuius speculatione , quatuor termini significantur , quatuor lineis . m . s . f . r . e . a . b . i . primum autem terminus . m . s . ex ultimo . b . i . detrahatur , residuumq ; sit . n . i . & ex



g. & permutando. o. r. ad. f. o. hoc est ad. m. s. vt. h. ad. g. hoc est . y. Quamobrem eadem erit proportio. o. r. ad. m. s. quæ. n. u. ad. n. x. Absoluantur itaque duo rectangula. x. i. et. u. z. ita tamen vt rectangulū. u. z. cogitetur æquale rectangulo. x. i. cuius. x. i. superficialis numerus ex communi conceptione lineari. n. i. æqualis erit, quare ex eadē communi conceptione, numerus superficialis. u. z. lineari. n. i. æqualis erit, qui quidem numerus in figura rectangula superficialis cogitandus erit, cum diuidendus sit per. h. hoc est per. n. u. ex quo proueniens ex huiusmodi diuisione erit numerus . n. z. ex ijs quæ. 10. theoremate dicta fuerunt.



Sed ex. 15. sexti aut. 20. septimi eadem est proportio. n. i. ad. n. z. quæ. n. u. ad. n. x. hoc est. o. r. ad. m. s. videlicet vt. n. i. ad aggregatum reliquorum omnium terminorum. c. a. f.



r : m. s. ex præcedenti theoremate, & ex. 11. quinti Euclidis. Itaque ex. 9. eiusdem numerus. n. z. æqualis erit summæ trium terminorum. c. a. f. r. m. s. cui coniuncto quarto termino. b. i. propositum obtinetur.

T H E O R E M A C V I.

**P**roposuerē veteres quæ sita nonnulla de itineribus interq; hoc vnum fuit. Ponamus duos iter agere per eandem viam quorum alter quatuor milliaria singulis diebus conficiat, alter verò prima die milliare vnum, secunda duo, tertia tria, atque ita singulis diebus milliare addit; quærimus quot dierum spacio socium consequetur.

Quamobrem numerus milliarium primi viatoris duplicatur, sic sunt. 8. milliaria, ex quo semper vnitas detrahatur, quæ in præsentī exemplo erit. 7. totq; dies erunt quibus socius socium consequetur, & milliarium numerum æqualem absoluerit.

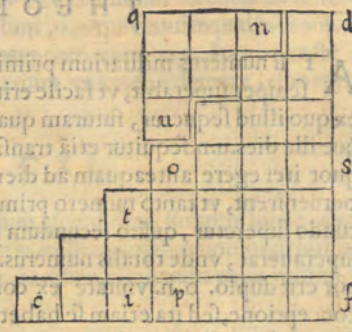
Cuius rei facilis erit speculatio, si subscripta figura diligenter consideretur, in qua primus viator, die prima, quatuor milliaria linea. q. d. significata conficit, atque illa ipsa die alter vnum tantum designatum per. d. perficit, ita vt primus viator tribus milliariis socium antecesserit, altera verò die secundus viator cum duo milliaria conficiat, excedetur à primo duobus milliariis tantummodo, quæ cum tribus primæ diei quinque erunt; tertia die iisdem de causis primus sex tantum milliariis à secundo distabit, cum verò quarta die tot secundus quot primus milliaria conficiat, primus à secundo amplius quam antea non distabit; quinta verò cum secundus vnum milliare amplius quam primus conficiat, propius accedit ad primum vno ex sex milliariis, quibus antea distabat, tum sexta cum duobus primum superet, detrahet ex sex milliariis præteritæ distantia tria, septima tandem illa sex detraherit. In quo considerandum est secundum viatorem iter agere progressionē arithmetica continua naturali. d. c. f. primum autem per rectangulum. q. f. quarū duarum figurarum. d. o. p. f. pars cōmunis esse reperitur, quæ quantitates si inuicem æquales esse debent, necesse est separatas partes. u. q. n. et. t. i. c. inter se æquales esse, & quoniam quarta die ( hoc est die sic distante a primo, nempe numero milliarium primi

primi

THEOREMATA ARITHMETICAE

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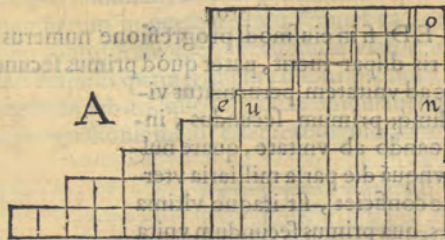
primi viatoris ) tot miliaria absoluat vnus  
 quot alter absque vlla differentia, quæ signi-  
 ficetur per .o. s. necesse est itaque ex communi  
 conceptione tot dies esse post .o. s. quot ante-  
 cesserant, vt excessus æqualis sit defectui, qui  
 simul collecti, iuncta etiam .o. s. duplum erunt  
 d. s. dempta vnitatem, prout facile in subscripta  
 figura qui sique per se scientificè poterit specu-  
 lari. Quamobrem consultum erit duplicare  
 numerum .o. s. & ex duplo vnitatem detrahe-  
 re, quandoquidem dies supra infraq; .o. s. cum  
 die .o. s. minores sunt duplo numeri .d. s. aut .o.  
 s. (quod idem est) vnitatem.



THEOREMA C.VII.

QVOD si secundus viator ordinè secundæ progressionis arithmeticæ seruas iter  
 agat, nempe ea quæ ab vno per binarium ascendit, semper numerus dierum  
 æqualis erit numero miliarium diurnorum primi viatoris.

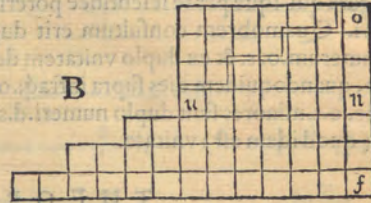
In cuius gratiam animaduertendum est numerus miliarium diurnorum primi  
 viatoris par an impar sit. Etenim si par est, primus viator in fine singulorum die-  
 rum primæ medietatis numeri omnium dierum secundum antecedit numero dispa-  
 ri miliarium; altero verò dimidio numero dierum, à secundo numero etiam dispa-  
 ri præteribitur, vt in sequenti figura patet. Nam prima die, secundus ex primo  
 miliare vnum ex numero pari, qui à primo conficitur detrahit; secunda verò die  
 idem secundus, duo subtrahit miliaria ex dispari, qui primo reliquus fuerat, sicq;  
 perpetuò dispar remanet vsque ad vnitatem, ad quam cum peruenerint, nempe ad  
 illius diei exitum, quo primus secundum vnitatem tantummodò superat, manifestè  
 deprehenderetur subsequente die secundum vnitatem primum superaturum, altera ve-  
 rò tribus vnitatibus, prout penultima die secundus à primo tribus vnitatibus supera-  
 batur. Quare necesse erit, tot diebus secundum cum primo iter agere, inchoan-  
 do ab ea die, qua secundus primam superabit, quot egerat dum à primo superare-  
 tur, vt ex communi conceptione, media figura. A. deprehendi potest. Quod autem  
 singula dimidia dierum, dimi-  
 dia sint numeri miliarium diurno-  
 rum primi, patet ex sequenti fi-  
 gura, cogitato termino. u. n. vlti-  
 mo progressionis superatè à primo  
 vsque ad vnitatem. e. qui terminus  
 u. n. coniunctus primo. o. nempe. e.  
 semper duplè est numeri termino-  
 rum. o. n. vt .94. theoremate circa  
 finem dictum fuit. Sed. u. n. cum. e.  
 numero æquali constat numero  
 miliarium diurnorum primi viatoris,  
 ex quo sequitur totum numerum dierum, quo-  
 rum. o. n. dimidium est, æqualem esse numero miliarium diurnorum primi via-  
 toris.



THEO-

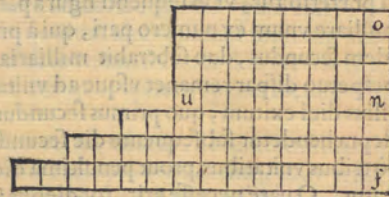
T H E O R E M A C V I I I.

**A**T si numerus milliariū primi viatoris dispar fuerit, secundum numero pari semper superabit, vt facile erit sequentem figuram consideranti intelligere, ex quo illud sequetur, futuram quandam diem, qua paria milliaria conficient. Sitque illa dies. u. n. sequitur etiā transacta ea die, tot diebus vtrique ambulandum esse quot iter egere anteaquam ad diem. u. n. peruenirent, vt tanto numero primus à secundo superetur, quāto secundum primus superauerat, vnde totalis numerus. o. f. minor erit duplo. o. n. vnitatem ex communi conceptione, sed ita etiam se habet terminus. u. n. hoc est minor duplo. o. n. per. o. vt 94. theoremate dictum fuit, itaque. o. f. æqualis erit. u. n. quod erat propositum.



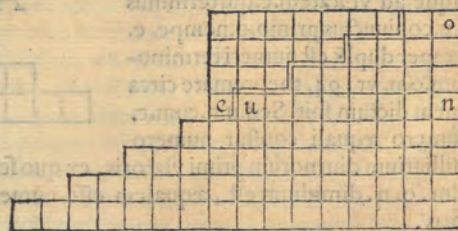
T H E O R E M A C I X.

**S**IN verò progressio secundi viatoris, non ab vnitatem sed à binario inchoata, per binarium quoque ascenderet, numerusq; milliariū diurnorum primi viatoris par esset, absque dubio quadam die paria milliaria vterq; conficeret, quæ significetur. u. n. qua transacta, tot diebus vtrique ambulandum erit, quot fuerūt dū primus secundum superaret, vt totidem alijs primus à secundo superetur, in qua tamen progressionem terminus. u. n. semper duplus est numero terminorum. o. n. ex. 95. theoremate, totq; sunt infra. u. n. termini vsque ad. f. quot supra. ex quo illud sequitur omnes terminos aut dies. o. n. f. pauciores esse. u. n. vnitatem, atque ita præcipit regula de- trahendam esse vnitatem ex numero milliariū diurnorum primi viatoris, si dierum numerum habere voluerimus.



T H E O R E M A C X.

**S**ED si in eiusmodi progressionem numerus milliariū diurnorum primi viatoris dispar fuerit, patet quòd primus secundum numero dispari superabit, donec ad vnitatem perueniatur vicissimq; primum secundum, inchoando ab vnitatem, quare nulla vnquā die paria milliaria vterque conficiet, sit itaque vltima dies, qua primus secundum vnitatem antecedit. u. n. qui terminus duplus est numero terminorum. o. n. & cum illa die primus secundum milliario antecedit, sequente

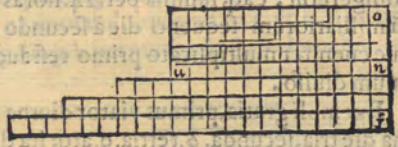


THEOREM. ARIT.

te verò à secundo milliario vno primus antecedatur, ex communi scientia necesse est secundum tot diebus cū primo iter agere quot sunt .o.n. qui simul æquales erunt. u.n. sed. u.n. minor est numero milliariū diurnorum primi vnitatē. e. Itaque rectē sequemur regulam, quæ iubet ex numero milliariū vnitatem demere, quo numerum dierum habere possimus.

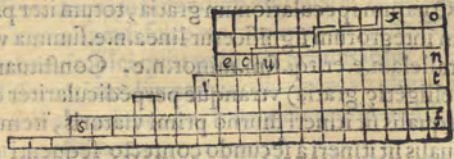
THEOREMA CXI.

SI verò secundi viatoris progressio per ternarium ascenderet, sumpto initio ab ipso ternario, animaduertendum est an numerus milliariū diurnorum primi, ternario mensuretur nec ne; etenim si mensuretur, tandem aliquando paria milliaria conficiet, quæ dies sit. u.n. quare sub u. n. totidem quot supra termini erūt, & cū. o.n. tertia sit pars. u.n. ex. 95. theoremate. Itaque tota. o. f. minor erit duabus tertijs. u. n. vnitatē, vt iam rectē sumendæ sint duæ tertiæ partes. u.n. ex quibus vnitas detrahatur superfitque numerus. o. f. dierum quæsitorum.



THEOREMA CXII.

CVM verò milliariū numerus primi viatoris metiri non poterit à numero ascendente secundi, patet nullam futuram diem qua pari milliaria conficiet, quare illa vltima qua primus secundum antecedit, vno aut duobus milliariis antecedit in præsentī casu. Antecedit itaque duobus milliariis, sitq; dies. u.n. & altera. t. i. secundus primum vno milliari superabit, ita quod sub. t. i. non poterunt plures integros dies iter agere, quam ambulauerunt ante diem. u.n. hoc est vsquequo secundus iunctus sit primo, qui numerus dierum, tertia parte. o.n. ipsius. u.n. vnitatē minor erit, cum ex. 95. theoremate. o.n. sit tertia pars. u.n. ex quo numerus. o. f. terminorum aut dierum integrorum cognitus erit, qui si cum numero ascendente cognoscetur, statim ex. 99. theoremate deueniemus in cognitionem vltimi diei in tegri. s. atque ita etiam totius summæ progressionis ex. 95. theoremate. Iam verò cognito numero milliariū diurnorum primi, simul cum numero terminorum, aut dierum consequenter nouerimus rectanguli summam, hoc est productum à primo viatore formatum, quarum duarum summarum in præsentī casu semper ea, quæ huiusmodi producti est, maior erit, cum constitutum fuerit secundum viatorem à primo superari ipsa die. u.n. vno milliari amplius quam sequente die. t. i. primus à secundo superatur, tum pari gradu iter egerunt sub. t. i. quo supra. u.n. ambulauerant. Hoc animaduertendo, quod si summa progressionis maior esset rectangulo, ex ea summa necesse esset numerū milliariū vltimi termini in summa inclusi detrahere, & residuo operari. Nunc verò summam progressionis ex summa rectanguli à primo viatore facti subtrahi debet, residuumq; seruari voceturq; pri.



primū residuū. Ad hæc numerū milliariorū, quæ secundus viator die sequenti, s. f. conficiet sumat; ex quo numerus milliariorū diurnorum primi detrahatur, residuūq; pariter referuetur, voceturq; secundum residuum, postmodum numerum milliariorum primi vnus diei multiplicetur per primum residuum seruatum, productūq; per secundū residuum diuidatur. a. c. proueniēs (quod erit iter primi in sequenti die) iungatur residuo primo, tot enim erunt millaria conficienda a secundo sequenti die, vt sese consequantur.

THEOREMA CXI  
 Vt autem sciamus quantam partem diei sequentis, singulos itinere agere oporteat, proueniēs per. 24. horas multiplicetur (supposito quod ambulātes nullā requiē nec die nec nocte capiāt) productūq; per numerū milliariorum vnus diei primi viatoris diuidatur, ex quo dabitur quāritas horarū, & pars hora, qua cuiq; illa die ambulandū est. Idem accideret si primum residuum referuatum cum proueniēte in summam colligeretur, eaq; summa per. 24. horas multiplicaretur, productūq; per numerū milliariorum sequenti die a secundo conficiendorum diuideretur. Idipsum quoque eueniret multiplicato primo residuo per. 24. & producto per secundum residuum diuiso.

Exempli gratia, primus viator diurna millaria vndecim conficit, secundus, prima die tria, secunda. 6. tertia. 9. atq; ita deinceps, diuidatur ergo. 11. per. 3. vnde pro numero. 0. n. dabitur. 3. supereritq; 2. quare: u. n. ab. e. n. duobus millariis superabitur, et. i. t. dictum. e. n. vno millario, ex quo ante diem. e. u. n. duobus diebus iter egerunt, totq; diebus ambulandū erit post. t. hoc est. 6. in vniuersum integris. Ad hæc multiplicato. 0. f. hoc est. 6. per. x. 0. hoc est. 3. habebimus. s. f. milliariorū 18. tū cōiuncto. x. 0. primo termino hoc est. 3. cū. s. f. hoc est. 18. ultimo termino, habebimus. 21. quo multiplicato. cū dimidio. 0. f. hoc est. 3. habebimus totam summam progressionis. 63. sex dierum integrorum ex. 94. theoremate, cum multiplicato. 11. nempe numero milliariorū diurnorum primi cum. 6. hoc est. cum. 0. f. habebimus parallelogrammum a primo sex diebus integris confectum, milliariorum. 66. ex quo detracta. 63. summa inquantū progressionis, supererit pro primo residuo. 3. sumpris postea millariis. 21. pro itinere, quod secundus die sequenti. s. f. conficeret, & ex ijs detracto numero milliariorū diurnorum primi, nempe. 11. secundum residuum erit. 10. quod pro diuidenti seruabitur. Iam multiplicato. 11. cum primo residuo. 3. dabitur. 33. qui diuisus per. x. 0. secundum residuum profert. 3. cum tribus decimis, eritq; iter a primo viatore sequenti die conficiendum, hoc etiam ipsum proueniens cum primo residuo. 3. coniunctum, dat. 6. cum tribus decimis, quod est iter secundi viatoris illa sequenti die. Ad inueniendam autem quantitatem diei, qua vtrique ambulandum est, perinde erit multiplicare proueniens. 3. & tres decimas per. 24. horas, & productū per. 11. dimidium iter primi viatoris partiri, ac multiplicare summam. 6. & tres decimas cum. 24. horis, productūq; diuidere per. 24. hoc est per iter secundi viatoris sequentis diei, vtrinque enim semper septem hora cum. 12. minutis proueniens. Idipsum accidet multiplicato per. 24. horas primo residuo. 3. productūq; diuiso per secundum residuum. 10.

Quarum speculationum gratia, totum iter parallelogrammi primi viatoris dierum integrorum significetur linea. n. e. summa verò progressionis secundi linea. f. m. parallela. n. e. eritq; f. m. minor. n. e. Constituamus deinde a termino. f. n. (maioris intelligentie gratia) vtranque perpendiculariter duci, pducatur deinde. n. e. donec. e. d. æqualis sit itineri diurno primi viatoris, item etiam producat. f. m. donec. m. n. æqualis sit itineri a secundo confecto sequenti die vltimum integrum progressionis,

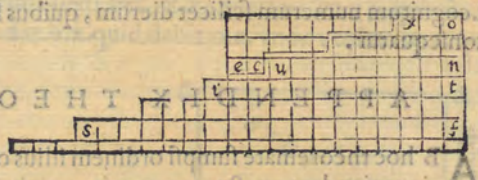


THEOREMA PARIT H.

nis, ex quo. m. k. prolixior erit. e. d. ex præsupposito. Postmodum. m. e. et. k. d. duabus lineis rectis coniungantur, quæ productæ concurrent in puncto. b. ducatur pariter. e. g. à puncto. e. parallela. b. k. et. m. a. e. h. et. b. q. parallela. f. n. ex quo. f. m. æqualis erit. n. a. et. m. h. a. e. et. h. q. e. o. et. g. k. e. d. et. f. q. n. o. ex. 34. primi Eucli. vnde proportio. m. h. ad. h. q. erit vt. m. g. ad. g. k. quando quidem vtraque æqualis est proportioni. m. e. ad. e. b. ex. 2. sexti, sed cum. m. k. et. g. k. notæ sint, pariter cognoscetur. m. g. secundum residuum, cum etiam notæ sint. n. e. et. n. a. Itaque cognoscemus. a. e. hoc est. m. h. cognitis verò. m. g. g. k. et. m. h. ex. 15. sexti aut. 20. septimi cognoscetur. h. q. erit igitur. a. e. aut quod idem est. m. h. primum residuum, et. m. g. secundum, et. h. q. aut. e. o. proueniens, et. n. o. et. f. q. itinera vtriusque viatoris inter se æqualia. Nec verò præmittenda est speculatio vltimæ rationis inueniendæ quantitatæ diei, quæ constat ope diuisionis producti. m. h. in. 24. per. m. g. Ea autem eiusmodi est. Probatum fuit sic se habere. m. h. ad. h. q. vt. m. g. ad. g. k. Itaque componendo sic se habebit. m. q. ad. h. q. vt. m. k. ad. g. k. & permutando. m. q. ad. m. k. vt. h. q. ad. g. k. Sed cum sic se habeat. m. h. ad. h. q. vt. m. g. ad. g. k. permutando sic se habebit. m. h. ad. m. g. vt. h. q. ad. g. k. itaque ex. 11. quinti ita. m. h. ad. m. g. vt. m. q. ad. m. k. ex quo permutando. m. h. ad. m. q. vt. m. g. ad. m. k. sed cū. m. k. sit motus toti diei respondens, securè dicere poterimus, si m. g. talis est respectu horarum. 24. significatarum per. m. k. qualis erit. m. h. & quoræ parti diei respondens: quæ postmodū erit. m. q. quæ, vt dictū fuit, talis est respectu. m. k. qualis. m. h. respectu. m. g. Reliquæ duæ speculationes priorum modorū, vna & eadem est, facilisq; per se mediocriter intelligenti. Eodem modo reliquæ omnes progressionés secundi viatoris cū rectangulo primi conferri ex hoc theoremate poterunt.

THEOREMA CXLII.

Propositur & aliud, primum scilicet viatorem iter incipere diebus aliquot antè secundum, primum tamen lentius, quàm secundum ambulare, & utrunque eorum certa quædam milliaria conficere. Iam si scire voluerimus in quot diebus sese consequentur, vulgari regula iubet, inspicere quot milliaria primus solus iter agens confecerit, tum animaduerti differentiam diurnam motus vnus ab altero, atq; milliarium numerum primi viatoris soli abundantis per hanc differentiã diuidi, proueniens autem erit numerus dierum quaesitus.

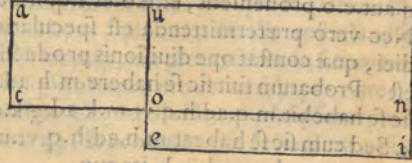


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K Exempli

Exempli gratia, si primus octo diebus antequam secundus iter arripuisset, confecissetq; singulis diebus. 20. milliaria, tum secundus. 25. quotidie perfecisset, multiplicandus esset numerus. 8. cum. 20. ex quo darentur. 160. milliaria à primo solo ambulante confecta, quibus diuisis per. 5. differentiam motuum diurnorum, daretur. 32. numerus quæsitus dierum.

Cuius ratio apertissima est. Sint enim duo rectanguli. a. n. et. u. i. æquales inter se, quibus motus itinerarium significentur, quorum. a. n. sit primi, et. u. i. secundi, præterea. a. c. numerum milliarium diurnorum primi, et. u. e. secundi, ex quo. a. c. minor erit. u. e. per. o. e. atque ita. o. e. cognoscetur. Tum. c. o. numerum dierum primi soli iter agentis denotet, cumq; constituamus. a. n. æqualem esse. u. i. o. i. equalis erit. o. a. atque. o. a. cognitus ex suis producentibus. a. c. et. c. o. itaque. o. i. etiam cognitus, qui diuisus per latus cognitum. o. e. dabit. e. i. cognitum numerum scilicet dierum, quibus secundo ambulandum est, ut primum consequatur.



#### APPENDIX THEOREM. CXIII.

**A**B hoc theoremate sumpsi ordinem illius operationis, numeris mediantibus, ad inueniendam exactam temporis quantitatem, seu interuallum, quod transit, vel intercedit inter vnâ mediocrem coniunctionem & aliam proximam sequentem duorum planetarum, ut patet in epistola nostra ad Illustræm Bernardum Trotium contra Benedictum Altauillam reprehensorem Ephemeridum. Verum tamen est quod cum praxis huiusmodi theorematis sit multiplex, visum fuit vnâ proponere, quæ non ita perspicua sit, sed subobscura, non quod aliquid voluerim latere illum amicum mihi dilectissimum, cui priuatim omnes modos prius ostenderam, sed ut cerebrum illius mei aduersarij in laberintum conijcerem inextricabilem ut feci, quamuis modus ille egregius etiam sit, ut nunc ostendam.

In dicta epistola igitur mente cogitavi medium motum tardioris planetæ, puta saturni, illius temporis quo velocior planeta, scilicet Iupiter, percurrit suo medio motu totum zodiacum, incipiendo ambo eodem temporis puncto, nec non ab vnâ eorum media coniunctione, hoc est ab eodem zodiaci puncto, in quo coniuncte fuerunt eorum lineæ mediorum motuum, vbi inueni vi regulæ de tribus, quod Saturnus spacio dierum vnus mediocris reuolutionis Iouis, qui sunt. 4328. progreditur medio motu gra. 145. min. 4. hoc est min. 8704. posito quod ipse Saturnus perficiat vnâ mediam reuolutionem spacio dierum. 10746. vidi. Incipiendo igitur iterum Iupiter aliam reuolutionem percurrere, reperto Saturno per min. 8704. ante ipsum spacio. 4328. dierum, certus eram hos dies significatos esse à linea. a. u. vel. c. o. (æquales enim inuicem sunt) in figura huiusmodi theorematis, & quod rectangulum. a. o. præbebat summam graduum. 145. min. 4. hoc est min. 8704. et quod. a. c. vel. o. u. significabat iter vnus diei ipsius Saturni, et. u. e. iter vnus diei Iouis. Cogitemus nunc. u. x. significari dies. 30. & à puncto. x. productam esse. x. f. parallelam ipsi. u. o. e. vnde certi erimus rectangulum. e. x. significare iter Iouis spacio temporis dierum. 30. rectangulum verò. o. x. iter Saturni eodem temporis interuallo, vnde rectangulum.



Taurino Patavium. 220. quæ quisque confecerit.

Dum autem hæc specularer attentius, occurrit alius soluendi modus, quamuis prolixior. Is autem est eiusmodi. Accipiat medietas minoris numeri dierum, nempe. 4. cum dimidio, & per. 400. multiplicetur, productumque; per maiorem numerum diuidemus scilicet 11. ex quo dabuntur. 163. cum. 7. vndecimis, quo proueniente è dimidio milliariorum itineris. 200. detracto, & residuum nempe. 36. cum. 4. vndecimis multiplicato productoque; diuiso per summam dimidij itineris. 200. cum primo prouertur. 163. et. 7. vndecimis nempe. p. 363. et. 7. vndecimas partes, pueniet. 16. cum. 4. vndecimis, quo coniuncto primo puenienti, primus. 180. milliaria confecerit, quæ è. 400. detracta supererunt. 220. pro itinere secundi, qui. 9. diebus iter absoluit. Ad hæc si tempus scire velimus eius, qui. 11. diebus appellit, multiplicabimus. 11. cum. 180. productumque; per. 400. partiemur, prouenientque; paulominus, quam quinque dies, nempe. 4. cum. 22. horis et. 48. minutis, quod tempus utriusque viatori inferuiet, quandoquidem id ipsum prouenit multiplicato. 220. per. 9. productoque; per. 400. diuiso.

Huius autem, qui à me prescribitur modi, speculatio talis est. Duo termini duabus rectis lineis æqualibus, & parallelis inter se. b. p. et. d. q. significentur, quæ alijs duabus. b. d. et. q. p. coniungant, quæ parallela & æquales erunt ex. 33. primi, quibus significentur duo itinera. Viator primus quidem lentior à. b. in. d. velocior à. q. in. p. tam sumatur punctum medium. q. p. sitque; k. & ab ipso ad. b. d. ducatur. k. i. parallela. d. q. aut b. p. quod idem est, ex quo. b. i. æqualis erit. p. k. ex. 34. primi, hoc est. q. k. certique; erimus primum viatorem. q. p. in dimidio itineris. q. k. occurrere non potuisse viatori ipsius. b. i. quandoquidem eo tempore, quo is, qui ipsius. q. p. mouetur per. q. k. (cum sit altero velocior) qui per. b. d. nondum peruenerit ad. i. Sit itaque punctum. c. in quo lentior reperitur, dum velocior est in. k. ex quo certum est eos inter. c. et. i. sibi inuicem obuiaturos esse. Cogito deinde rectam lineam ductam. k. c. & ut se habet. i. c. ad. c. b. ita cogito se habere. u. k. ad. k. q. & à puncto. u. ad. i. duco. u. i. quæ, ut manifestum est, lineam. k. c. in puncto. c. interfecabit, à quo cum fuerit ducta. e. o. n. parallela. k. i. habebimus. o. n. ea scilicet puncta, quibus occurrunt sibi ipsis, nam cum sic se habeat. q. k. ad. k. u. vt. b. c. ad. c. i. et. k. u. ad. k. n. vt. c. i. ad. c. o. ex similitudine manifestis triangulorum, ex æqualitate proportionum sic se habebit. q. k. ad. k. n. vt. b. c. ad. c. o. & permutando ita. k. q. ad. b. c. vt. k. n. ad. c. o. & cum. q. k. et. b. c. spatia sint temporibus æqualibus confecta, itaque spatia. k. n. et. c. o. ex communi scientia temporibus æqualibus conficientur.

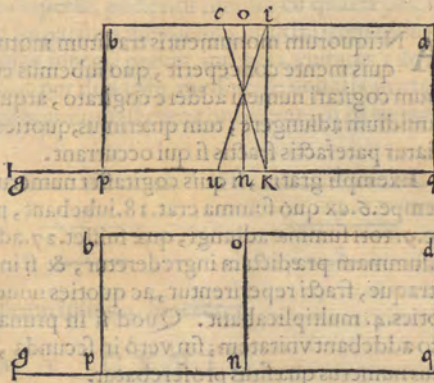
Quare rectè dicimus, si tot diebus à. b. in. d. aliquis puenit, quot milliaria in dimidio temporis alterius viatoris idem conficiet? ex quo ex regula de tribus quam primum iter. b. c. cognoscitur, quo ex dimidio itineris detracto, remanet. c. i. cognitus, sed cum probauerimus. q. k. ad. k. n. hoc est. i. o. (cum sint æquales inter se, ex. 34. primi) ita se habere. vt. b. c. ad. c. o. permutando sic se habebit. q. k. ad. b. c. vt. i. o. ad. o. c. & componendo. q. k. et. b. c. ad. b. c. vt. i. c. ad. c. o. quare rectè dicimus si summa. q. k. cum. b. c. dat. b. c. quid dabit. i. c. nempe dabit. c. o. quo coniuncto cum. b. c. cognoscitur. b. o. quo. b. o. detracto ex. b. d. remanet cognitus. o. d. nempe. q. n. illi æqualis ex. 34. prædicta. Gratia vero temporis patet nos rectè dicere si. b. d. tot diebus absoluitur, aut etiam. q. p. quo. b. o. aut. q. n. absoluetur.

Vt autem ad speculationem regulæ antiquorum deueniamus, cogitemus primum viatorem ipsius. q. p. velociorem eo, qui per. b. d. iter agit, tanto tempore prætergredi. p. quanto alter. b. d. absoluit. Is autem ad. g. pertingat, ex quo eadem proportio spatij. q. g. ad. q. p. hoc est. b. d. dabitur, quæ temporis quo. b. d. absoluitur ab-

eo qui per .b. d. ad tempus quo q. p. solum, qui per .q. p. mouetur ( motus enim continui regulares & uniformes constituuntur ) eadem ratione itaque ea erit proportio q. k. ad .b. c. quæ q. g. ad .q. p. & cum probatum fuerit ita se habere .k. n. ad .c. o. vt .q. k. ad .b. c. itaque sic se habebit .k. n. ad .c. o. vt .q. g. ad .q. p. probatum etiam fuit ita se habere .q. k. ad .k. n. vt .b. c. ad .c. o. ex quo componendo sic se habebit .q. n. ad .n. k. vt .b. o. ad .o. c. & permutando ita .q. n. ad .b. o. vt .k. n. ad .c. o. hoc est .q. g. ad .q. p. nempe vt tempus lenti ad tempus velocis itinerantis, & componendo ita .q. n. cum .o. b. hoc est .b. d. ad .b. o. vt summa dierum vnus & alterius viatoris ad minorem numerum dierum velocioris. Breuiter itaq; obtineremus intentum quod diceremus si summa dierum, quibus iter agitur à viatoribus talis est (20) respectu numeri dierum velocioris (9) qualis & cui respondebit totum spacium .b. d. vnde dabitur spacium .b. o. vnde reliqua omnia nobis cognita emergent.

Cum autem antiquorum regula iubeat numerum dierum vnus, cum numero dierum alterius multiplicari, ac postmodum diuidi productum per summam omnium dierum, recte id quidem fit. Nam cum sic se habeat .b. d. ad .b. o. vt summa omnium dierum ad minorem quantitatem dierum velocioris scilicet. Ideo temporis proportio à mobili per .b. d. absumpti ad tempus mobilis per .b. o. eadem erit, quæ summæ omnium dierum ad numerum dierum velocioris. Quare recte dicemus, si eiusmodi summa talem respectum habet ad minorem numerum dierum, quem numerum respiciet dies ipsius .b. d. ex quo proferentur dies respondentes ipsi .b. o. cætera iam dicta fuerunt.

Huiusmodi verò speculationis amplitudo ad paucissima verba reduci potest, in cuius gratiã sit subscripta figura pars in qua præcedentis, in qua constituam<sup>o</sup>. o. n. locum eum esse quo sibi viatores obuiant, ex quo spacium .q. n. à suo viatore conficietur, eo ipso tempore, quo à suo spacium .b. o. itaque eadem erit proportio .q. n. ad .b. o. quæ .q. g. ad .b. d. eadem erit in qua proportio .d. o. ad .o. b. quæ numeri dierum eius, qui à .b. pergit in .d. ad numerum dierum alterius qui à .q. in .p. proficiscitur, & componendo eadem erit proportio .d. b. ad .b. o. quæ summæ dierum ad minorem numerum ipsorum, & eadem quæ dierum .b. d. ad dies ipsius .b. o.



T H E O R E M A C X V

CIRCA hæc ipsa itinera aliud queritur peruenisse, in quo queritur illud constituitur cognitum esse, nempe interuallum inter duo diuersa loca, à quibus duo viatores eodem instanti vt sibi occurrant proficiscuntur, certa quæ milliaria singulis diebus conficiant, ita tamen, ut unus ordinatè plura altero ambulet, queritur deinde quoto die sibi occurrant. Hoc autem fit diuiso toto interuallo locorum per summam milliariorum quam vterque quotidie absoluit.

Exem-

Exempli gratia, distant loca. 100. milliariibus à se inuicem; vnus autem viator singulis diebus. 15. milliaria; alter. 10. conficit si itaque. 15. cum. 10. coniugamus; summa erit. 25. per quã diuisis milliariibus. 100. totius interualli proferetur. 4. numerus quæsitus dierum quo viatoribus iter agendum erit prius quam sibi obuient.

In cuius speculationis gratiam totum iter significetur linea. a. u. primi autem viatoris iter diurnum sit. a. e. & alterius. u. o. terminus verò. i. sit occurfus ita vt eodem tempore, alter spacium. a. i. alter. u. i. contecerit; spacij autem. a. e. tempus per. b. significetur & tempus spacij. u. o. per. c. quæ tempora erunt inter se æqualia, portio spacij. a. i. tempus per. d. & spacij. u. i. per. f. denotetur, æqualibus inquam; ex quo eadem proportio erit. a. e. ad. a. i. quæ. b. ad. d. et. o. u. ad. u. i. quæ e. ad. f. vnde permutando eadem erit proportio itineris ipsius. b. ad. iter ipsius. c. quæ itineris. d. ad. iter ipsius. f. & componendo itinerum ipsius. b. c. ad. iter. c. vt itinerum. d. f. ad. iter. f. & permutando itinerum b. c. ad. itinera. d. f. vt itineris. c. ad. iter ipsius. f. meritò itaque queritur si itinera. b. c. dat itinera. d. f. quid dabit tempus. c. nempe dabit tempus. f. sed. c. signatum est pro vna die, quare in proposito exemplo. f. significabit 4. dies.

## T H E O R E M A C X V I.

Antiquorum monumentis traditum motum reperimus diuinandi numeri quem quis mente conceperit, quo iubemus eum qui numerum cogitauerit, dimidium cogitari numeri addere cogitato, atque huic summæ; rursus eiusdem summæ dimidium adiungere, tum querimus, quoties noueratus totam eam summam ingrediatur patefactis fractis si qui occurrant.

Exempli gratia, si quis cogitasset numerum. 12. iubebant huic dimidium addi, nempe. 6. ex quo summa erat. 18. iubebant, præterea dimidium huius summæ nempe. 9. toti summæ adijungi, quæ fuisset. 27. adhuc quærebant sibi patefieri quoties. 9. summam prædictam ingrederetur, & si in prima aut secunda diuisione aut etiã vtraque, fracti reperirentur, ac quoties nouem vltimam summam ingrediebatur, toties. 4. multiplicabant. Quod si in prima diuisione fracti erant, vltimo producto addebant vnitatem; sin verò in secunda, binarium adiungebant; ex quo exactus numerus quæsitus proferebatur.

Pro cuius rei ratione sit. a. numerus cogitatione comprehensus et. e. ipsius. a. cum eiusdem medietate summa et. i. ipsius. e. cum eiusdem medietate itidem summa, vnde. i. e. a. tres numeri continui proportionales, in sesquialtera proportione euadent. Sumantur nunc tres numeri. 4. 6. 9. in eadem proportionalitate. Vnde ratione equalitatis proportionum ita se habebit. i. ad. a. quæ admodum. 9. ad. 4. & permutando. i. ad. 9. quemadmodum. a. ad. 4. & ob id. 4. toties ingreditur. a. quoties. 9. ipsam. i. Sed quia sepe contingit, vt in secunda diuisione, aut in ambabus etiam diuisionibus reperiantur numeri fracti, animaduertendum est numerum animo comprehensum. a. scilicet aut parum aut imparum semper futurum. Si par est, aut multiplex erit ad. 4. aut non. Si priori modo se habebit in duabus diuisionibus, nullus numerus fractus admittetur; sed si ad. 4. multiplex non erit, à multiplicibus per duo semper differet, & si per medium diuidatur, eiusdem medietas impar semper erit, vnde prior

quoque summa par nunquam existet, cuius medietatem aliquod medium semper ingreditur, & hanc ob causam posterior summa cum fracto semper erit, & numerum desumptum maiorem esse multiplici ad quatuor per duo significabit.

At verò si inter impares reponatur, aut eorum erit qui superant multiplicem ipsius quatuor per vnum, seu per tria, quod hinc innotescet, nempe, quia si eorum erit qui dictum multiplicem per vnum tantum vincunt, sua medietate ipsi numero addita, & præter hanc medietatem medio etiam integro adiuncto, tota hæc prior summa in numerum parem semper euadet, vnde in posteriori summa nullus numerus fractus, conspicietur, & hanc ob causam multiplici ipsius. 4. vnitas semper addetur.

Sed si numerus desumptus, in serie eorum, qui multiplicem ipsius. 4. per tria superant, collocabitur, hinc comprehendetur, quia primæ summæ numerus cum mediâ vnitate semper impar erit, vnde secunda summa præter integras cum mediâ vnitate nobis semper occurret.

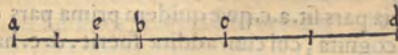
Quod autem nobis proderet faciamus an in prima diuisione, & secunda numerus aliquis fractus consistat, eò tantum nobis inseruit, quò deueniamus in cognitionem an numerus animo conceptus multiplicem ipsius. 4. per vnum, per duo, aut tria superet. Quòd etiam medias eas vnitates ad integros reducere faciamus, eò tantum refertur, vt minori labore eum, qui numerum imaginatione comprehendit, oneremus, quia reuera numerus impar nunquam mente concipi potest, quin aliquis fractus in prima diuisione, aut in secunda sequatur: vnde à numeris imparibus, qui multiplicem ipsius. 4. unitatis tantum excessu superant, posterior summa cū quarta parte unitatis, præter integros numeros, & ab imparibus qui dictum multiplicem ipsius. 4. per tria vincunt, cum tribus quartis vnus integri præter integras vnitates; & à numeris paribus, qui multiplicem ipsius. 4. per duo cum medietate unitatis præter integros semper procedit. Ita cum is qui numerum secum considerat, si in numeris fractis versatus esset, qui eum inter rogat prudenter se gereret, si sibi declarari curaret, quis nam ex fractis super integros secundæ summæ remaneret, quia p quot quarta integros secundæ summæ superaret, per totidē integros numerus mente conceptus multiplicem ipsius. 4. superaret.

T H E O R E M A C X V I I.

V N D E fiat, vt si aliquis quemuis numerum animo comprehendat, eique numero alium etiam quemlibet numerum propositum addat, & à tertia parte huius summæ tertiam partem numeri imaginati detrahat, residuum secundi numeri adiuncti, idest propositi, tertia pars erit.

Vt exempli gratia, si aliquis de numero denario cogitasset, huicq; 24. adderet, vnde triginta quatuor efficerent, detrahendo nunc tertiam partem numeri denarij cogitatione concepti, idest 3. cum tertia parte vnus, à tertia parte huius summa idest ab vndecim. & vna tertia parte remanerent. 8. idest tertia pars numeri additi. Id quod mihi inter iocos in honestorum hominum catu in mentem venit.

Pro cuius ratione, prior numerus imaginatus mediante linea. a. b. et is, qui additus est intercedete linea. b. d. è directo



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coniunctis denotetur, et. b. e. sit tertia pars ipsius. a. b. prioris numeri imaginati, et. b. c. tertia pars ipsius, b. d. secundi numeri propositi, unde coniunctum vnus harum tertiarum partiū cū alia sit. e. c. quod quidem. e. c. esse tertiam partem summæ duorum primorum idest. a. d. assero. Tam manifestum est ipsius. d. b. ad. b. c. esse quemadmodum ipsius. a. b. ad. b. e. unde vicissim ipsius. d. b. ad. b. a. erit quemadmodum ipsius. b. c. ad. b. e. & coniunctim ipsius. d. a. ad. a. b. quemadmodum ipsius. c. e. ad. e. b. & vicissim ipsius. d. a. ad. e. c. quemadmodum ipsius. b. a. ad. b. e. sed proportio ipsius. b. a. ad. b. e. est tripla, ergo ea quæ est ipsius. a. d. ad. e. c. erit quoque tripla; unde sumendo. e. c. pro tertia parte ipsius. a. d. & ab ipsa. e. c. subtrahendo tertiam partem ipsius. a. b. tertia pars ipsius. b. d. remanebit. b. c.

Aut alio hoc modo, supponendo. e. c. tertiam partem ipsius. a. d. et. e. b. ipsius. a. b. exister. Dico. b. c. tertiam partem ipsius. b. d. futuram: quia si totius. a. d. ad totum e. c. ita se habet, quemadmodum. a. b. à toto. a. d. dissecti atque diuulsi ad. e. b. à toto. e. c. distractum, ergo ex. 19. lib. quinti Euclid. residui. b. d. totius. a. d. ad residuum. b. c. totius. e. c. erit, vt totius. a. d. ad totū. e. c. atque hic quidem modus rem proposita speculandi mihi aptior & commodior esse videtur.

**T H E O R E M A C X V I I I.**

**P**ermulta ac varia problemata inuenerunt antiqui, longioribus verò vijs resoluta, propterea quòd nò semper nobis succurrit breuissima in vnaquaque re explicatio. Vt exempli gratia, proponitur numerus. 50. diuidendus in tres tales partes, quod secunda dupla sit primæ, & adhuc eam superet tribus vnitatibus, tertia verò æqualis sit aggregato primæ cum secunda, & amplius ipsum aggregatum superet quinque vnitatibus.

Ad hoc autem quæsitum soluendum antiqui vtebantur regula falsi, quod reuera breuiori modo potest solui, videlicet detrahendo illud secundum excessum, quinque scilicet ex. 50. ita vt nobis. 45. remaneret, cui medietati hoc est. 22. cum dimidia vnitare, si addiderimus illud quinque habebimus. 27. cum dimidia vnitare pro tertia parte quæsitæ ipsius numeri. 50. deinde si ab eodem numero. 22. cum dimidia vnitare detractum fuerit illud. 3. primus excessus datus, remanebit. 19. cum dimidia vnitare, cuius tertia pars, hoc est. 6. cum dimidia vnitare, prima pars, ex tribus quæsitæ erit, quæ quidem si detraxerimus ex. 19. cum dimidia vnitare, reliquum erit. 13. cui cū additus fuerit primus excessus idest. 3. iam propositum resultabit nobis. 16. pro secunda parte quæsitæ.

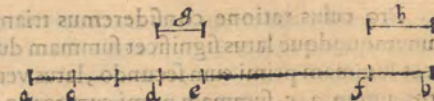
Ratio verò huiusmodi operationis. talis est, sit verbi gratia totalis numerus propositus significatus per lineam. a. b. cuius secundæ partis numerus datus significetur per lineam. g. & numerus tertiæ partis propositus per lineam. h. Nunc dempta. h. ex a. b. nobis cognita, remanebit. f. a. quæ quidem per æqualia imaginatione diuisa in puncto. e. & ipsi. e. f. addita. f. b. tota. e. b. nobis cognita erit, quæ quidem tertia pars quæsitæ ipsius. a. b. erit, propterea quòd. a. e. (quæ æqualis est ipsi. e. f.) erit summa primæ, & secundæ partis. Detrahatur postea. g. ex. e. a. & remanebit. d. a. cuius tertia pars sit. a. c. quæ quidem prima pars quæsitæ erit, & nunc cognita, & ita. e. d. cognita, cui cum addita fuerit. d. e. habebimus secundam partem quæsitam, quæ compo-



THEOREMA ARIOTI

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componitur ex d. c. dupla ad a. c. primam partem, & ex d. e. numero dato. tertia vero pars e. b. composita est ex e. f. aequali a. e. hoc est aequali composito ex prima, & secunda parte, & ex b. numero dato vt proponebatur.

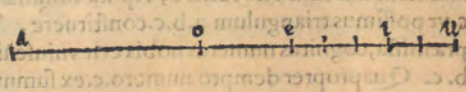


THEOREMA CXXIX

Inter alia problemata ab antiquis inuenta, hoc etiam ponitur. Aliquis interrogat quot sint horæ, alius vero respondit tot esse, quot duæ tertie præteriti temporis simul iuncta cum tribus quintis futuri temporis totius diei naturalis efficiunt. Nunc quaeritur quot sint horæ.

Antiqui, hoc etiam problema soluebant mediante regula falsi, sed mihi alio modo soluendum esse dictum problema videtur. Accipio enim ex quinque, tres vnitates, pro parte futuri temporis, quas quidem in tres vnitates præteriti temporis duco, vnde proueniunt mihi nouem vnitates, quod productum coniungo cū quinque futuri temporis, vnde veniunt. 14. vnitates, ex regula postea de tribus ita dico si ex. 14. mihi prouenit. 9. quid resultabit ex. 24. & prouenient mihi horæ. 15. cum tribus septimis vnius horæ, hoc est minuta fere. 26.

Pro cuius ratione, quinque vnitates, seu partes temporis futuri significentur à linea. e. u. quarum trium significentur à linea. e. i. supra deinde sit linea. e. o. æqualis lineæ. e. i. et. e. a. tripla sit ad. o. e. vel ad. e. i. quod idem est, vnde. a. e. composita erit ex. a. o. (hoc est ex duabus tertijs ipsius. a. e.) & ex. o. e. (hoc est ex tribus quintis ipsius. e. u.) vnde. a. u. ad. a. e. eandem rationem obtinebit, quæ. 14. ad. 9. propterea igitur possumus recte ratiotinari si. 14. nobis dat. 9. quid dabit. 24. qui quidem. 24. nobis dabit. 15. cum min. 26. quod rectè factum est ex. 20. septimi Euclidis.



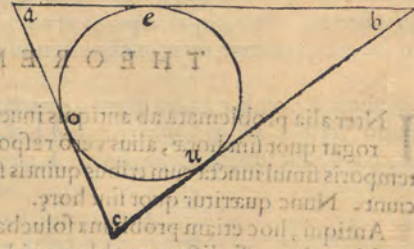
THEOREMA CCXIX

Supponunt etiam antiqui tres focios nummos habere, quorum summa primi & secundi cognita sit, item summa primi & tertij cognita & summa secundi & tertij item cognita, atque ex huiusmodi tribus aggregatis veniunt in cognitionem particularem vniuscuiusque illorum.

Gemafrius soluit hoc problema ex regula falsi. At ego tali ordine progredior. Sit verbi gratia, summa primi cum secundo. 50. & secundi cum tertio. 70. & primi cum tertio. 60. harum trium summarum accipiantur duæ quæuis, vt puta. 50. & 70. quæ coniunctæ simul dabunt. 120. à qua summa detrahatur reliqua, idest. 60. & restabit nobis. 60. cuius medietas erit. 30. hoc est à summa secundi cum tertio remanebit. 40. hoc est numerus tertij focij, & hic numerus defumptus à. 60. residuus erit numerus primi focij.

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Pro cuius ratione consideremus triangulum hic subnotatum . a . b . c . cuius unumquodque latus significet summam duorum sociorum, ut puta latus . a . b . significet summam primi cum secundo, latus verò . b . c . summam secundi cum tertio, latus autem . a . c . summam primi cum tertio, et . a . e . seu . a . o . fit numerus primi socij, et . e . b . vel . b . u . fit secundi socij, et . c . u . seu . c . o . fit tertij, cum autem . a . e . æqualis sit . a . o . et . b . e . æqualis . b . u . et . c . u . æqualis . c . o . ex supposito si depra fuerit summa seu latus . a . c . datum ex aggregato laterum . a . b . cum . b . c . reliquarum summarum, relinquet nobis cognitum aggregatum ex . b . e . cum . b . u . Quare & eius medietas . b . e . siue . b . u . nobis cognita erit, quæ detracta ex summa . b . a . relinquetur nobis cognitus numerus . a . e . detracto vero numero . a . e . hoc est . a . o . ex . a . c . summa, seu latus, aut . b . u . ex . b . c . remanebit . o . c . seu . c . u . cognitus.



## T H E O R E M A C X X I .

HAC etiam methodo hoc facere possumus non solum de tribus socijs, sed etiam de omnibus quotquot volueris, ut exempli gratia, sint sex socij . a . b . c . d . e . f . quorum summa per binos cognita, ut puta summa numeri . a . cum . b . cognita nobis sit, & summa numeri . b . cum . c . & summa . c . cum . d . & summa . d . cum . e . & summa . e . cum . f . necesse est etiam scire summam duorum vno relicto, ut puta summam . a . cum . c ., ut possimus triangulum . a . b . c . constituere, Vnde ex præmissa, cognitus numerus nobis erit vniuscuiusque . a . b . c . Quapropter dempto numero . c . ex summa . c . cum . d . & numero . d . ex summa . d . cum . e . & numero . e . ex summa . e . cum . f . habebimus intentum.



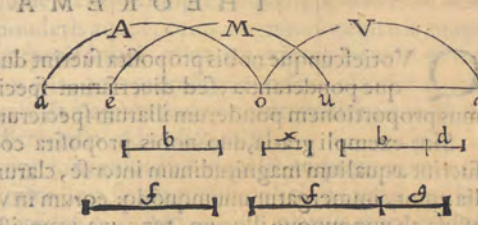
## T H E O R E M A C X X I I .

CVM aliquando, illud quod Archimedes inuenit, ut furtum Regi ab aurifabro in regia corona factum, quemadmodum scribit Vitruuius, proderet, contemplarer, mihi etiam visum est, ut aliquem modum scientificum inuestigarem, quo proportio auri ad argentum, quod in aliquo proposito corpore ex ipsis misto cogniti ponderis cognosci posset. Et cum multos diuersis temporibus excogitarim officio meo deesse nolui in iisdem literarum monumentis mandandis, quorum hic vnus erit: proposita nobis sint tria corpora . A . M . V . æqualia inter se, sed diuersarum specierum materiei, ut puta quod . A . fit argenteum, & omogeneum . V . verò aureum omogeneum, & . M . mixtum ex auro, & argento, id est heterogeneum, cupimus ergo scire iustam quantitatem auri & argenti, quæ est in ipso corpore . M . misto . Ita igitur faciamus. Videamus primum quantum sit pondus vniuscuiusque ipsorum corporum, ponamus autem pondus corporis . V . auri esse vt . 234 . pondus autem

autem corporis. M. misti. vt. 216. argentei verò. A. vt. 156. detrahatur nunc pondus. A. ex pondere. V. Reliquum erit. 78. quod vocetur prima differentia seruanda, dematur etiam pondus. M. ex pondere. V. reliquum erit. 18. pro secunda differentia, etiam seruanda, multiplicetur postea pondus. A. per secundam differentiam, productum verò diuidatur per primam differentiam. Vnde in presenti exemplo prouenit nobis: 36. qui quidem prouentus erit quantitas argenti ipsius corporis misti. M. quo etiam detracto ex pondere totali ipsius. M. reliquum erit quantitas auri eius corporis, hoc est. 180.

In cuius operationis speculatione, aliquid natura sua prius cognitum præcedere oportet hoc est, quod omnia corpora omogenea eandem proportionem obtinent inter quantitates, quam inter pondera. Quo supposito denotetur corpus. A. linea. o. a. corpus autem. V. linea. o. c. & corpus. M. linea. e. u. sed. e. o. significet partem argenti, et. o. u. partem auri in corpore misto. M. vnde ex communi conceptu habebimus. o. e. æqualem. u. c. cum ex hypothese. e. u. æqualis sit. o. e. et. a. o. similiter. Significetur postea pondus. a. o. ab. f. & pondus. e. u. ab. b. x. & pondus. o. c. ab. f. g. pondus verò. o. e. ab. b. pondus autem. o. u. ab. x. pondus enim. u. c. ab. b. d. et. g. sit differentia, qua. f. g. maior est. f. et. d. differentia qua. b. d. maior est. b.

Vnde ex ratione omogeneitatis eadem proportio erit. a. o. ad. e. o. vt. f. ad. b. et. o. c. ad. u. c. quæ. x. b. d. seu f. g. (quod idem est) ad. b. d. Quare ex. 11. quinti cadẽ erit proportio. f. ad. b. vt. f. g. ad. b. d. & permutando ita erit. f. ad. f. g. vt. b. ad. b. d. & separando ita. f. ad. g. vt. b. ad. d. Sed. g. cognita nobis est, vt differentia in ter. f. g. et. f. cognita nobis est etiam. f. cognoscimus itidem. d. vt differentiam inter. x. b. d. et. b. x. quapropter cognoscemus. b. ex. 20. septimi Eucli. & sic. x. residuum. ex. b. x.



THEOREMA CXXIII.

**N**VNC ex methodo præcedentis propositi deuenire possumus in cognitionem veræ quantitatis auri, & argenti confusi in corona Hieronis constituendo primùm duo corpora simplicia æqualia inter se, & coronæ hoc modo videlicet, immergendo coronam, seu corpus mistum in aliquod vas aqua plenum, & diligenter colligere aquam, quæ ex eo effundetur, postea verò oportet aliud vas inuenire præcisæ capax illius aquæ collectæ, in quod demum infundatur tantum auri, & postea tantum argenti, quantum fieri potest, vnde vnumquodque horum duorum corporum simplicium æquale erit mixto, seu coronæ, & sic quod dictum est in præcedenti theoremate exequemur.

THEOREMA CXXIII.

**S**ED vt breuiori methodo idem præstemus, quod in antecedenti proposito dictum est, quædam theoremata præmittenda sunt, videlicet quod quotiescunque fuerint tria corpora, quorum duo inuicem æqualia sint in quantitate, sed diuersarum

rum specierum materiae, tertium verò corpus maius, vel minus sit in quantitate utroque illorum, sed eiusdem materiae vnus quod vis illorum, ponderis verò alterius, seper eadem proportio erit inter pondera aequalium corporum, quæ inter quãtãtatem corporis inæqualis, & eam quæ vnus cuiusuis aequalium.

Exempli gratia, sit. b. corpus aliquod aureum aequale corpori. u. argenteo, sit etiam corpus. a. argenteum maius corpore. b. vel. u. sed ponderis eiusdem, quod auri. b. Tunc dico eandem esse proportionem ponderis. b. ad pondus. u. quæ est magnitudinis. a. ad magnitudinem. u. Quod ratiocinemur hoc modo, nam cum proportio corporeitatis. a. ad corporeitatem. u. eadem sit, quæ ponderis. a. ad pondus. u. ex ratione omogeneitatis, ponderis verò. b. ad pondus. u. ex. 7. quinti, eadem quæ ponderis. a. ad pondus. u. ideo ex 1. eiusdem proportio ponderis. b. ad pondus. u. eadem erit, quæ corporeitatis. a. ad corporeitatem. u. vel ad corporeitatem. b. quæ aequalis est alteri.

T H E O R E M A C X X V

**Q**uotiescunque nobis proposita fuerint duo corpora cuiusuis magnitudinis æque ponderantia, sed diuersarum specierum materiae, cum scire voluerimus proportionem ponderum illarum specierum inter ipsas hoc modo faciemus.

Sint exempli gratia, duo nobis proposita corpora. a. et. b. (vt dictum est) quæ si fuerint aequalium magnitudinum inter se, clarum erit quod queritur, sed inæqualia erunt, immergatur unumquodq; eorum in vas aqua plenum, & collecta sit aqua effusa ab vnoquoque illorum, tunc vnaquæq; istarum aquarum aequalis magnitudinis erit sui corporis impellentis, & proportio ponderositatis illarum eadem erit, quæ earum magnitudinum ex omogeneitate, quapropter si vnamquamque illarum ponderabimus, habebimus propositum ex præcedenti theoremate.

T H E O R E M A C X X V I

**S**ED cum scire voluerimus pondus alicuius magnitudinis aquæ aequalis alicui corpori ponderoso, breuissimus modus erit ponderando ipsum corpus tam in aere, quàm in aqua, & quia semper leuius erit in aqua, tunc differentia ponderum ipsius corporis, erit pondus quæsitum, hoc est vnus corporis aquei aequalis magnitudinis magnitudini corporis propositi ex. 7. propositione. lib. Archimedis de insidentibus aquæ.

Quare ex præmissis quotiescunque immersa fuerint in aquam dicti vasis duo corpora æquè ponderantia, sed diuersarum specierum, vt dictum est, proportio ponderis aquæ maioris ad pondus aquæ minoris magnitudinis eadem semper erit, quæ ponderis minoris corporis ad pondus alicuius corporis eidem aequalis, speciei verò maioris, vel eadem proportio ponderis alicuius corporis aequalis maiori, speciei verò minoris ad pondus ipsius maioris.

Vt puta sit corpus. a. argenteum aequalis ponderis corpori. b. aurei, & corpus. u. argenteum aequalis magnitudinis corpori. b. aurei, corpus verò. n. aureum aequalis magnitudinis corpori. a. argentei, corpus verò. f. aqueum aequalis magnitudinis corpori

THEOREM. ARII.

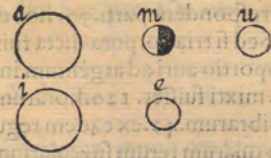
pori. a. argentei, corpus autem. e. aqueū æqualis magnitudinis corpori. b. aurei. Tunc dico proportionem ponderis. f. ad pondus. e. eadem esse, quæ ponderis. b. ad pondus. u. vt in præcedenti theoremate iam dictum est, vel quæ ponderis. n. ad pondus. a. ex 11. quinti Euclidis. Propterea quod ponderis. n. ad pondus. a. est vt ponderis. b. ad pondus. u. eo quod permutando ponderis. n. ad pondus. b. est vt ponderis. a. ad pondus. u. ex corporum omogeneitate, & ex æqualitate magnitudinum corporum antecedentium & consequentium.



THEOREMA CXXVII.

Sic etiam nos oportet, quod quotiescumque fuerint duo corpora aquea, quorum vnum æqualis magnitudinis sit alicui misto, quod quidem mistum graue sit tam in aere, quam in aqua, alterum verò corpus aquem æqualis sit magnitudinis alicui corpori simplici, quod quidem corpus simplex æqualis ponderis sit dicto corpori misto. Tunc proportio ponderis aquei, cuius magnitudo æquatur magnitudini corporis misti, ad pondus corporis aquei, cuius magnitudo æqualis est magnitudini corporis simplicis, eadem erit, quæ proportio ponderis alicuius corporis simplicis, cuius magnitudo æqualis sit magnitudini corporis misti superius dicti, sed speciei corporis simplicis iam dicti, ad pondus dicti misti.

Exempli gratia, sit corpus aqueum. e. magnitudinis æqualis corpori. m. mixto, corpus verò aqueum. i. æqualis magnitudinis sit corpori simplici. a. quod quidem corpus. a. æqualis ponderis sit cum corpore. m. & corpus. u. sit æqualis magnitudinis cum corpore. m. sed speciei corporis. a. Tunc dico proportionem ponderis. e. ad pondus. i. eadem esse, quæ ponderis. u. ad pondus. m. primum nulli dubium est, quin eadem proportio sit magnitudinis. e. ad magnitudinem. i. quæ magnitudinis. m. ad a. sed. m. ad a. est vt. u. ad. a. ex. 7. quinti quare ex. 11. eiusdem proportio. e. ad. i. erit vt. u. ad. a. de ipsius magnitudinibus loquendo, sed proportio ponderis. u. ad pondus. a. eadem est, quæ magnitudinis. u. ad magnitudinem. a. ex omogeneitate. Idem dico de pondere. e. ad pondus. i. Quare proportio ponderis. e. ad pondus. i. eadem erit quæ ponderis. u. ad pondus. a. Sed ponderis. u. ad pondus. m. eadem est quæ ponderis. u. ad pondus. a. ex. 7. quinti, ergo ex. 11. eiusdem proportio ponderis. e. ad pondus. i. eadem erit, quæ ponderis. u. ad pondus. m. quod est propositum.



THEOREMA CXXVIII.

Nunc ad cognoscendam proportionem duarum diuersarum specierum in corpore misto proposito, tribus corporibus aqueis mediantibus, quæ quidē corpora æqualium magnitudinum sint alijs tribus corporibus vnus & eiusdem ponderis, quorum vnum sit mixtum, reliqua verò duo simplicia, sed specierum mixti, hoc ordine procedemus.

Sint

Sint exempli gratia, tria corpora æque ponderantia, & vnumquodque illorum sit quinque librarum, quorum vnum sit aureum, aliud argenteum, reliquum verò mixtum ex ijs duobus metallis, vnde corpus aureum simplex minus erit, & argenteum maius corpore mixto, quod nulli dubium est, sit tunc pondus corporis aquei equalis corpori aureo, librarum. 3. aquei verò equalis misto, sit librarum. 3. cū quarta parte, aquei demum æqualis argenteo, librarum. 4. cum dimidia, vnde ex ijs, quæ in præcedenti theoremate, & in. 126. theoremate diximus, si imaginatione concipiemus alia duo corpora simplicia, auri, & argenti, sed æqualium magnitudinum mixto, habebimus proportionem ponderis aurei ad pondus corporis mixti vt triū librarum cum quarta vnius ad. 3. libras, & proportio ponderis mixti ad pondus argentei erit, vt proportio librarum. 4. cum dimidia ad tres libras cum quarta parte vnius libræ, & proportio ponderis aurei ad pondus argentei vt librarum. 4. cum dimidia ad libras. 3. hoc est aurei ad mixtum, vt. 13. ad. 12. & mixti ad argenteum, vt. 18. ad. 13. & aurei ad argenteum, vt. 3. ad. 2. id est, vt. 18. ad. 12.

Nunc inueniantur duo numeri ita inter se proportionati, vt. 3. ad. 2. habentes tamen inter ipsos numerum ita proportionatum ad maximum, vt. 12. se habet ad. 13. & ita proportionatum ad minimum, vt se habet. 18. ad. 13. quod hoc modo inueniemus, multiplicabimus. 18. per. 12. & proueniet nobis. 216. pro numero medio, postea multiplicabimus. 18. per. 13. & proueniet. 234. pro maximo, demum multiplicando. 12. per. 13. proueniet. 156. pro minimo, ita quod. 234. correspondebit ponderi corporis aurei: 216. verò ponderi mixti, et. 156. ponderi argentei æqualium magnitudinum.

Cum autem proportionem horum trium corporum inuenierimus, si ordinem theorematis. 122. sequemur, habebimus quod quærebamus, & inueniemus in præsentis exemplo proportionem ponderis auri ad pondus argenti in corpore mixto esse, vt. 180. ad. 36. sed quia suppositum fuit corpus mixtum esse quinque librarum, propterea dicemus. Si. 216. hoc est toti corpori mixto correspondent quinque libræ: tunc parti. 180. hoc est auro in ipso corpore mixto, correspondent libræ. 4. cum duabus vncijs, ex regula de tribus, residuum verò quinque librarum, id est vnciæ decem, correspondent parti. 36. hoc est argento in dicto corpore mixto.

Sed si tria corpora dicta fuissent inuicem ita proportionata, vt. 40. 47. 60. tunc proportio auri ad argentum in corpore mixto esset vt. 13. ad. 7. quapropter cū pondus mixti fuisset. 120. librarum, tunc aurum ipsius esset librarum. 78. argentum verò librarum. 42. ex eadem regula.

Pro quarum rerum speculatione nil aliud oportet hunc dicere cum satis dictum à nobis superius fuerit, vno excepto, hoc est rationem reddere, qua motus sui ad inueniendos illos. 3. numeros ita inter se dispositos, vt dictum est, quæ quidem ratio fuit, vt haberemus. 3. numeros ita inter ipsos ordinatè dispositos, vt sunt pondera trium illorum corporum æqualium magnitudinum. Propterea quod quamuis inter primos. 3. numeros ponderum corporum aqueorum eadem fuerint proportionem ponderum corporum metallicorum, nihilominus medijs numeris extra proprium locum, & inordinatè inueniebatur, respectu extremorum, vnde medius numerus in suo vero situ inter. 18. et. 12. fuisset. 16. cū. 8. tertijs decimis, sed vt fractorū incommunitatem veniemus, præcepi, vt multiplicarentur extrema per. 137. vnde producti fuerunt numeri. 234. et. 156. in eadè proportionem, quæ est. 18. ad. 12. ex. 18. septimi, iussi etiã multiplicari. 18. per. 12. vt nobis prodiret. 216. ad quem numerum, numerus. 234. ita se haberet, ut. 13. ad. 12. ex. 19. septimi, quod autem ita sit proportionatus

THEOREMATA ARITH.

tionatus. 216. ad. 156. vt. 18. ad. 13. manifestum est ex iisdem, nam tam. 18. quam. 13. multiplicatus fuit per. 12.

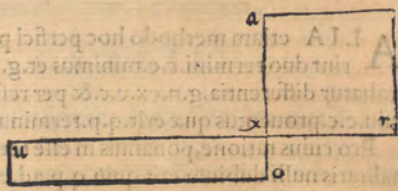
THEOREMA CXXIX.

**A**LIVD proponitur problema hoc modo: supponitur obsidio alicuius loci, vbi alimento ad nutriendos. 10000. homines sufficiunt pro quinque mensibus tantum, sed quia eum locum obsidione non liberari putatur nisi. 18. mensibus exactis, quaeritur, quot homines eo tempore illis alimentis nutriri possint, hoc est. 18. mensibus.

Præcipit regula, vt multiplicetur primus numerus, hoc est hominum. 10000. cum secundo, hoc est mensium quinque, productum verò diuidatur per. 18. hoc est mensium, tunc proueniet. 2777. cum. 7. nonis.

Cuius operationis ratio est hæc, sint exempli gratiã duo hic subscripta producta superficialia. a. n. et. o. u. inuicem æqualia, sed tali figura delineata, vt proportio. u. x. ad. x. o. sit, vt. 10000. ad quinque, & proportio. a. x. ad. x. o. sit vt. 18. ad quinque, et. x. n. sit nobis ignota, quæ quidem est illa, quæ indagatur, ita q̄ vnumquodque istorum productorum significabit alimentum, et. u. x. significabit numerum hominum. 10000. qui quidem homines comederent totum alimentum. u. o. spacio temporis. x. o. quinque mensium, propterea quòd u. o. supponitur productum esse ab. u. x. in. x. o. Deinde supponedo. a. x. tem-

pus esse. 18. mensium, ergo. x. n. significabit numerum hominum, qui eo temporis spacio ali possunt, hoc est. x. a. alimento. n. a. eo quòd. a. n. producit ex. n. x. in. a. x. vnde ex. 15. sexti, seu ex. 20. septimi proportio. x. u. ad. x. n. eadẽ erit, quæ. a. x. ad. x. o. quapropter rectè factum erit accipere productũ. u. o. quod idem est in quantitate, quod productum. a. n. & ipsum diuidere per. a. x. vnde nobis proueniat. n. x.

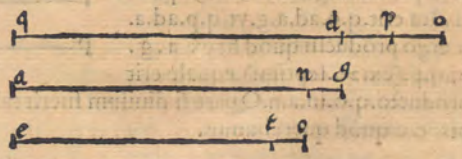


THEOREMA CXXX.

**Q**uotiescunque nobis propositum fuerit inuenire tertium terminum, trium terminorum continuè proportionalium armonicae proportionalitatis, quorum duo nobis cogniti sint, ita agemus.

Sint, exempli gratia, tres termini. q. p. a. g. et. e. c. continuæ proportionalium armonicae proportionalitatis, quorum. q. p. maior et. a. g. medius sint nobis cogniti, cum ergo voluerimus tertium. e.

c. cognitum nobis esse: a. g. detrahatur ex. q. p. differentia verò. d. p. addatur. q. p. quorum summa erit. q. o. cognita, qua mediante diuidatur productum, quod ex. a. g. in. d. p. exurgit, & proueniet nobis. n. g. hoc est minor differentia, eo quòd productum. q. o. in. n. g. æquale est producto



ducto

ducto. a. g. in. d. p. ex. 20. septimi, propterea quod proportio. q. o. ad. o. p. hoc est ad. d. p. est vt. a. g. ad. g. n. coniunctim cum disiunctim ita sit. q. p. ad. p. o. vt. a. n. ad. n. g. permutado eo quod. q. p. ad. a. n. (idest ad. e. c.) ita se het ut. p. o. (hoc est. d. p.) ad. n. g. ex conditionibus armonice proportionalitatis. Deinde si detrauerimus. n. g. ex. a. g. remanebit. e. c. minor terminus.

Sed si. e. c. tertius terminus nobis propositus esset simul cum. a. g. medio, & voluerimus maiorem inuenire. q. p. scilicet, oportebit. e. c. ex. a. g. detrahere, differentiam vero. n. g. similiter demeremus

ex. e. c. unde remaneret nobis. e. t.

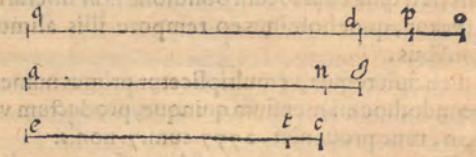
cognitum, quo residuo. e. t. mediante diuidemus productum, q.

furgit ex. a. g. in. t. c. & prouentus.

d. p. erit differentia maior, eo q.

productum quod fit ex. e. t. in. d. p.

æquale est producto quod fit ex. a. g. in. t. c. per 20. septimi Eucli. eo quod. a. g. (idest. q. d.) ad. d. p. est ut. e. t. ad. t. c. disiunctim, cum coniunctim ita sit. q. p. ad. d. p. vt. e. c. ad. t. c. permutando, quia. q. p. ad. e. c. est vt. d. p. ad. t. c. hoc est ad. n. g. ex legibus dictis.



T H E O R E M A C X X X I.

**A**LIA etiam methodo hoc perfici posse comperi. Propositi enim cum nobis fuerint duo termini. e. c. minimus et. g. a. medius, maximus vero quærendus sit, de trahatur differentia. g. n. ex. e. c. & per residuum. e. t. diuidatur productum q. fit ex. a. g. in. e. c. prouentus quæ erit. q. p. terminus quæsitus.

Pro cuius ratione, ponamus in esse terminum. q. p. tunc ex forma huius proportionalitatis nulli dubium erit quin. q. p. ad. e. c. fit vt. d. p. ad. n. g. hoc est ad. t. c. vnde ex 19. quinti vel. 12. septimi ita esset. q. d. ad. e. t. vt. q. p. ad. e. c. quare ex. 20. septimi productum q. nascitur ex. p. d. (hoc est. a. g.) in. e. c. æquale erit producto. e. t. in. q. p. quapropter si diuiserimus id per. e. t. proueniet nobis. q. p.

Sed cū nobis propositi fuerint duo termini. q. p. maximus, et. a. g. medius, si minimum. e. c. voluerim inuenire. Termino. q. p. maximo, iugat. p. o. equalis, p. d. differentie propositæ, diuidatur postea productum q. ex. q. p. in. a. g. generatur per. q. o. prouentus autem fit. e. c. qui quidem erit terminus quæsitus.

Cuius operationis speculatio hæc erit, supponatur terminum. e. c. inuentum esse vnde. n. g. differentia fit inter. e. c.

et. a. g. ex forma igitur armonice

proportionalitatis ita erit. q. p. ad. a.

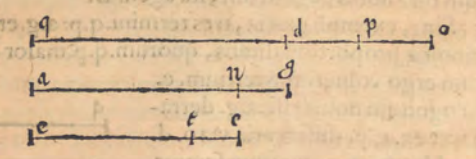
n. vt. p. o. ad. n. g. vnde ex. 13. quinti.

Ita erit. q. o. ad. a. g. vt. q. p. ad. a.

n. ergo productum quod fit ex. a. g.

in. q. p. (ex. 20. septimi) æquale erit

producto. q. o. in. a. n. Quare si diuisum fuerit tale productum per. q. o. proueniet nobis. e. c. quod quærebamus.



T H E O.



THEOREMA CXXXII.

**S**ED quia aliquis posset in dubium reuocare, an possibile sit inuenire tertium terminum rationalem, seu communicantem duobus datis terminis inter se communicantibus in tali proportionalitate, hoc est harmonica. Vt hoc ostendatur. Sint duo termini dati. a. o. et. a. e. inter se communicantes, tertius verò inuentus sit. a. c. qui maximus, primò, sit in ea proportionalitate, quem dico communicantem esse cum primis datis.

Nam ex conditionibus huiusmodi proportionalitatis, habebimus primum eandem proportionem esse. a. c. ad. a. o. quæ est. e. c. ad. e. o. vnde permutando ita erit. a. c. ad. e. c. vt. a. o. ad. o. e. & quia ex. 9. decimi Euclid. a. o. communicat cum. o. e. quare ex. 10. eiusdem. a. c. communicabit cum. e. c. & per. 9. cum. a. e. et per. 8. cum. a. o. quod est propositum.

Sed si datus fuerit maximus. a. c. cum medio. a. e. inter se communicantes minimum verò. a. o. probabo comunicantem cum illis esse. Cogitemus ergo. c. f. æqualem esse differentia. c. e. cognita, vnde habebimus proportionem, a. c. ad. c. f. vt. a. o. ad. o. e. & componendo. a. f. ad. f. c. vt. a. e. ad. e. o. & quia (ex supposito). a. c. communicat cum. e. c. hoc est cum. c. f. quare ex eadem. 9. dicti decimi. a. f. et. f. c. erunt inter se communicantes, & per. 10. a. c. communicabit cum. o. e. & per. 9. a. e. cõmunicabit cum. a. o. vnde per. 8. a. o. communicabit cum. a. c. similiter.

THEOREMA CXXXIII.

**S**ED si nobis duo extremi termini propositi fuerint, & medium inuenire desideremus in dicta proportionalitate, ita faciendum erit.

Sint, exempli gratia, duo termini dati. q. b. et. b. r. minor, b. r. ex maiori. b. q. detrahatur, residuum verò. q. x. multiplicetur per. b. r. productum postea diuidatur per q. r. vnde proueniet nobis. x. l. pro differentia minori, quæ addita cum. b. x. minimo termino, dabit nobis. b. l. medium terminum harmonicum.

Pro cuius ratione cogitemus dictum medium terminum. b. l. iam inuentum esse, vnde ita erit proportio. q. l. ad. l. x. vt. q. b. ad. b. r. ex forma huius proportionalitatis, quare coniunctim ita erit. q. r. ad. r. b. vt q. x. ad. x. l. & propterea ex. 20. septimi productum, quod fit ex. q. r. in. x. l. æquale erit producto. q. x. in. b. r. Rectè igitur fit cum diuiditur hoc productum per. q. r. vt proueniat nobis. x. l. differentia minor.

THEOREMA CXXXIII.

**P**offumus etiam harmonicè diuidere vnam datam proportionem absque aliqua diuisione productorum, ne nobis fractiones proueniant, hoc modo videlicet.

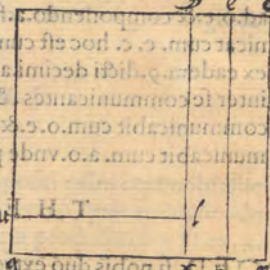
Nobis propositum sit diuidere harmonicè sesquialteram proportionē inueniantur primo minimi termini huius proportionis ut putà. 3. et. 2. quarum summa, hoc est quinque, multiplicetur per minorem idest. 2. vnde proueniet nobis. 10. qui quidem erit minor terminus trium quæsitorum, quorum maximus erit productum summa

M ma

ma iam dicta in maiorem eorum, hoc est quod fit ex quinque in 3. quod erit. 15. Ut autem medium terminum harmonicum inter istos habeamus, accipiatur duplu producti, quod fit ex primis minimis terminis, quod erit. 12.

Cuius rei speculatio est ista: significantur duo termini datae proportionis ab. q. b. et. b. r. quorum summa erit. q. r. cuius quadratum sit. q. o. sic etiam imaginata. b. e. parallela ad. o. r. Sitq. b. x. aequalis. b. r. et. q. u. similiter, & ducatur. x. y. parallela ad r. o. et. u. l. ad. q. x. Tunc habebimus. b. o. aequala ei producto, quod fit ex. q. r. in. b. u. et. b. y. eidem etiam aequala, et. q. e. pro producto, quod fit ex. q. r. in. q. b. et. q. l. pro eo, quod fit ex. q. x. in. b. r. Unde. q. l. cum. b. y. aequala fiet duplo ei, quod fit ex. q. b. in. b. r. Dico nunc. b. o. esse minimum terminum eorum, quos quaerimus, et. y. b. cum. x. u. medium. q. e. vero maximum huiusmodi proportionalitatis.

Primum ergo certi scimus ex prima sexti vel. 18. septimi eandem existere proportionem. q. e. ad. b. o. seu ad. b. y. qua. q. b. ad. b. r. sed. u. y. ad. u. x. est vt. y. l. ad. l. x. hoc est vt. q. b. ad. b. r. id est vt. q. e. ad. b. o. & summa. u. y. cum. u. x. id est. q. y. minor est quam. q. e. maximus terminus per. b. y. minimum terminum. & coniunctim. q. y. ad. q. l. vt. y. x. ad. x. l. hoc est vt. q. r. ad. r. b. Unde ex speculatione praecedentis theorematis, sequitur. u. y. esse differentiam inter maximum & medium terminum, et. u. x. esse differentiam inter medium & minimum dictae proportionalitatis. Nam eadem proportio est. q. e. maximi termini ad. b. o. minimi. qua. u. y. (differentia inter. q. e. & gnomonem. u. b. y.) ad. u. x. (differentia inter dictum. u. b. y. et. b. y. minimum terminum, quia sunt ambae ut. q. b. ad. b. r. vt diximus. Quare, b. y. coniunctum cum. x. u. medius terminus erit, qui quidem (vt dictum est) duplus est ei quod fit ex. q. b. in. b. r.

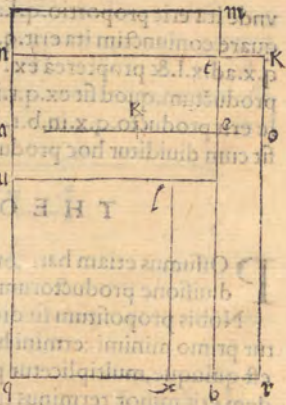


**T H E O R E M A C X X X V.**

**A**LIVM etiam modum ab antiquis traditum ad hoc problema perficiendum inueni, qui talis est. Inueniatur primo inter datos terminos extremos, medius terminus in arithmetica proportione, per quem multiplicetur vnusquisque dictorum extremorum, deinde multiplicentur ipsi extremi inter se, vnde habebimus tria producta eadem proportione inuicem existentia, vt quaerebatur.

Exempli gratia, ponamus duos propositos terminos esse. 3. et. 2. quorum medius arithmetice esset. 2. cum dimidia vnitare, per quem cum vnum quemque priorum multiplicauerimus, emergent nobis duo producta, quorum primum id est maius esset 7. cum dimidia vnitare, reliquum vero esset quinque, productum postea quod ex ipsis extremis prouenit, erit. 6. quod quidem est harmonicè collocatum inter. 7. cum dimidia vnitare, & quinque.

Cuius rei speculatio omnis a praecedenti theoremate dependet. Sint exempli gratia, duo termini



pro

propositi. a. e. maior, et. e. o. minor, Sitq; o. x. medius arithmeticus inter dictos, vade clarè patebit. o. k. esse dimidium summæ dictorum terminorum ex. 75. theoremata huius libri. Sit ergo productum a. r. id quod fit ex. a. e. in. o. k. et. o. r. fit productum quod fit ex. e. o. in. o. k. et. n. m. fit productum quod fit ex. a. e. in. e. o. quorum vnumquodque erit dimidium vniuscuiusque producti præcedentis theorematis, ex. 18. et. 19. septimi Eucli. vnumquodque sui relatiui. Quare argumentando per mutando à conclusionibus præcedentis theorematis ad has præsentis, habebimus productum.

THEOREMA CXXXVI.

**M**EDIUM autem contra harmonicum inuenire cum quis voluerit inter duos propositos terminos, ita faciendum erit, hoc est per summam datorum extremorum diuidatur productum quod fit ex minimo termino in differentiam datorum, prouentus postea erit differentia inter maximum & medium quaesitum.

Vt exempli gratia, si nobis propositi fuerint hi duo termini. 3. et. 2. summa eorum erit quinque, per quam cum diuiserimus productum, quod nascitur ex minimo. 2. in differentiam eorum, quæ est vnum, quod quidem erit. 2. tunc duæ quintæ partes prouenient, quæ si demptæ fuerint ex maximo termino, reliquum erit. 2. cum 3. quintis, hoc est medius terminus contra harmonicus.

Pro cuius ratione cogitemus. u. d. et. x. c. esse duos terminos nobis propositos, inter quos desideremus inuenire. o. s. medium ita illis relatum, vt proportio excessus ipsius supra. x. c. (qui fit. e. n.) ad excessum. u. d. supra. o. s. (qui fit. n. d.) eadem sit quæ. u. d. ad. x. c.

Cogitemus igitur. x. c. coniunctum esse cum. u. d. & hæc summa vocetur. b. d. vnde habebimus proportionem. u. d. ad. u. b. vt. e. n. ad. n. d. Quare componendo ita erit. d. b. ad. u. b. vt. e. d. ad. n. d. sed quia. d. b. u. b. et. e. d. quantitates nobis cognitæ sunt, idè d. n. ex. 20. septimi cognita nobis erit.

THEOREMA CXXXVII.

**S**upponunt antiqui aliquot mercatores dantes pecunias lucro in diuersis vnus anni temporibus, tunc in fine anni summa totius lucri datur cognita, sed quaeritur quantum vnicuique illorum ex ipsa summa debeat.

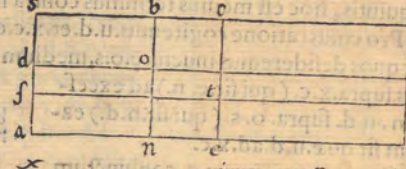
Exempli gratia, primus in principio anni posuit. 100. aureos, secundus verò. 100. diebus post primum posuit. 50. aureos tertius autem. 200. diebus post primum posuit. 25. aureos summa lucri postea in fine anni fuit aureorum. 60.

Nunc vt sciamus quantum huius summæ vnicuique illorum proueniat, præcipit regula, vt faciamus tria producta, quorum primum sit ex numero dierum totius anni in numerum aureorum primi, vnde tale productum in præsentis casu erit. 36500. secundum verò sit ex numero dierum à primo die in quo ipse secundus posuit usque ad finem anni, in numerum ipsorum nummorum, quod erit. 13250. tertium autem productum ex diebus tertij in numerum suorum aureorum, quod quidem erit. 4125. quæ producta simul collecta faciunt. 53875. deinde multiplicetur vnumquodque

ipſorum productorum per ſummam lucri hoc eſt per. 60. vnde multiplicatio primi producti erit. 2190000. multiplicatio verò ſecundi producti erit. 795000. tertij poſtea erit. 247500. quarum multiplicationum vnaquæque diuidatur per ſummam 53875. productorū, & proueniet ex prima diuiſione. 40. cū fractis. 35000. vnius integri diuiſi in partes. 53875. quod erit lucrum primi, prouentus autem ſecundæ diuiſionis erit. 14. cum fractis. 41050. vnius integri diuiſi in partes. 53875. lucrum ſecūdi. prouentus verò quartæ diuiſionis erit. 4. cum fractis. 32000. vnius integri, vt ſupra diuiſi in partes. 53875. hoc eſt lucrum tertij.

Cuius rei ſpeculatio ex ſe in ſub ſcripta figura patet, vbi. a. q. ſignificat numerum dierum totius anni pro primo mercatore. q. n. autem ſignificat numerum dierum ſecundi mercatoris. e. q. poſtea ſignificat numerum dierum tertij ſit etiam. s. a. pro numero denariorum primi, et. o. n. pro numero ſecundi, et. e. t. pro numero tertij, productum autem. q. s. ſignificet valorem primi lucri, et. q. o. ſecundi, et. q. t. tertij. x. y. autem ſignificet ſummam lucri omnium, et. x. i. ſignificet partem primi, et. i. p. ſecundi, et. p. y. tertij. vnde clarè patebit ex communi ſcientia quòd eadem proportio erit. x. y. ad. x. i. quæ aggregati omnium productorum. q. s. q. o. et. q. t. ad. q. s. & ita. x. y. ad. i. p. vt aggregati dicti ad. q. o. et. x. y. ad. p. y. vt dicti aggregati ad. q. t. Rectè igitur ex regula de tribus multiplicatio. q. s. in. x. y. diuiditur per aggregatum omnium

productorum, ita vt ſi aliquis diceret, ſi ex dicto aggregato, prouenit x. y. quid proueniet vnicuique illorū productorū. Nā ſi numerus denariorum ſecūdi æqualis eſſet numero a. s. primi vt putā. n. b. tunc eius lucrū ſignificaretur à rectangulo. q. b. & ita de tertio dico q. ſignificaretur à re-



ctangulo. q. e. vel ſi ſtantibus iſſedē denariorū quantitatis. n. o. et. e. t. omnes ſuas pecunias eodem tempore poſuiſſent, tunc rectangula ſignificantia eorum luca eſſent q. s. q. d. et. q. f. ſed cum nec eodem tempore, nec eandem quantitatem poſuerunt rectè eorum luca ſignificantur à rectangulis. q. s. q. o. et. q. t. q. ex prima. 6. vel. 18. aut. 19. ſeptimi ratiocinando clarè patebit.

T H E O R E M A C X X X V I I I .

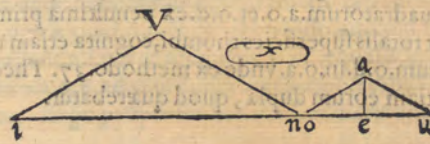
**N**icolaus Tartalea in primo libro vltimæ partis numerorum ad. 35. quaſtum docet inuenire quantitatem laterum vnius propoſiti trianguli, cuius laterum proportio nobis data ſit ſimul. cum area ſuperficiali ipſius trianguli, ſed quia ipſe Tartalea vtitur regula algebræ, mihi viſum eſt breuiori methodo hoc idem facere, & etiam vniverſaliori via. ſi ſupponamus igitur duo triangula, quorum vnum. u. n. i. ſit nobis propoſitū, & cognita ſuperficiæ, proportionales ſimiliter laterum. i. n. ad. n. u. et. u. n. ad. u. i. ſint nobis data; alterū verò triangulū ſit. a. o. u. à nobis tamen ita confectū, vt latera ſint inter ſe proportionata eodem modo, quo latera prioris trianguli, ſed hæc nobis etiā cognita ſint, q. facillimum eſt. Nunc verò ſi demptū fuerit quadratū. a. o. minimi lateris, ex quadrato. o. u. maximi, relinquet nobis duplum producti. o. u. in. u. e. per penultimā. 2. Eucli. ſupponēdo. a. e. perpendiculararem ad. o. u. vnde tale productum quòd ſit ex. o. u. in. u. e. conſequenter nobis cognitum erit, & quia. o. u. nobis cognitum eſt,

THEOREMA ART. I

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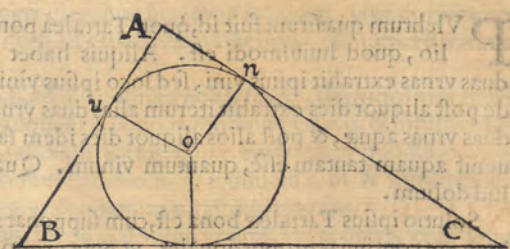
tum est, ideo cognoscemus. e. u. sed cū. e. u. minor sit. a. u. ex. 18. & penultima primi, si demptū fuerit quadratum. e. u. ex quadrato. a. u. remanebit nobis cognitū quadratū. a. e. & sic nota erit nobis perpendicularis. a. e. ex penultima primi, quæ quidem. a. e. si multiplicata fuerit in dimidium. o. u. dabit nobis superficiē trianguli. a. o. u. ex 41. dicti libri. Et quia proportio trianguli. a. o. u. ad triangulum. u. i. n. (propter similitudinem) est vt quadrati. o. u. ad quadratum. n. i. ex communi scientia cum vnaqueque istarum proportionum dupla sit proportioni. o. u. ad. n. i. ex. 17. et. 18. sexti, deinde cum nobis cognita sint tres istarum quatuor quantitatum hoc est superficies trianguli. a. o. u. superficies trianguli. u. i. n. & quadrati. o. u. quare ex regula de tribus cognoscemus etiam quadratum. n. i. & sic. n. i. latus primi trianguli, vnde reliqua latera illico nobis innotescunt ex ipsa regula de tribus, cum dixerimus, si. o. u. dat nobis u. a. tunc. i. n. dabit. u. n. quod etiam infero de. u. i.

Possemus etiam ita hoc perficere, scilicet inuenire. x. quantitatem mediam proportionalem inter duas superficies triangulorum, vnde superficies trianguli. i. a. u. o. ad. x. se haberet vt. o. u. ad. i. n. & ita ex regula de tribus cognoscemus. i. n. Multo tēpore postquam hoc theorema construxi, ipsum conscriptum inueni in decimo secundi libri Ioannis de monte Regio, satis tamen obscure expressum.



THEOREMA CXXXIX.

IN eodem primo libro vltimæ partis numerorum, Tartalea probat, via algebræ quod quælibet duo latera trianguli orthogonij, angulum rectum continentia, sint tertio longiora per diametrum circuli inscripibilis in ipso triangulo. sed hoc breuius geometricè potest demonstrari, quemadmodum in subscriptra hic figura videre est, propterea quod cum anguli. A. o. u. et. n. omnes sint recti et. A. u. æqualis o. n. et. A. n. æqualis. u. o. ipsa. A. u. et. A. n. æquales erunt diametro ipsius circuli. Sed eadem. A. u. et. A. n. sunt superfluum, quo. A. B. et. A. C. sunt maiores. B. C. cum. B. u. et. C. n. sint æquales. B. C. ex penultima tertij Eucli.



THEO. SEQVENS THEO. CXXXIX.

Similiter in nono capite secundi libri nouæ scientiæ poterat ipse Tartalea breuiori methodo absque vlla operatione ipsius Algebræ inuenire. A. H. respectu. A. E. esse vt. 4. cū vno septimo ad vnū. Nā ipse supponit. A. E. decimā partē esse ipsius A. I.

A. I. unde quadratum linea. A. I. erit. 100. idem dico de quadrato linea. I. L. quare ex penultima primi. A. L. erit radix quadrata quadrati. 200. id est. 14. cum vno septimo ferè. quare. A. L. iuncta. A. O. erit. 28. cum duobus septimis. sed. L. O. ex supposito erit. 20. eo quod. L. I. equatur ipsi. A. I. similiter et. I. O. vt ipse etiam probauit. quæ dempta ex. L. A. O. relinquetur. H. A. M. (nam. L. H. cum. O. M. æquatur ipsi. L. O. ex. 35. t. tertij ipsius Eucl. partium. 8. cū duobus septimis. cuius dimidiū hoc est. A. H. erit 4. cum una septima, quod est propositum. Respice figuram ipsius Tartaleæ.

## THEOREMA CXL.

**Q**uadragesimum nonum quæsitum similiter possumus alio modo soluere, vt putà cum vnumquodque latus rhombi simul cum area cognitum, seu datum nobis sit cognitū similiter nobis erit quadratum lateris. a. d. hoc est summa duorum quadratorum. a. o. et. o. d. ex penultima primi Euclid. cūque nobis cognita etiam sit totalis superficies rhombi, cognita etiam nobis erit eius medietas, hoc est productum. o. d. in. o. a. unde ex methodo. 37. Theorematis cognoscemus. a. o. et. o. d. & sic etiam eorum duplā, quod quærebatur.



## THEOREMA CXLI.

**P**lurimum quæsitum fuit id, quod Tartalea ponit pro. 18. noni libri in quarto folio, quod huiusmodi est. Aliquis habet dolium mero plenum, ex quo duas urnas extrahit ipsius vini, sed loco ipsius vini infundit duas urnas aquæ. Deinde post aliquot dies extrahit iterum alias duas urnas illius misti, & iterum infundit duas urnas aquæ, & post alios aliquot dies idem facit, & hac vltima tertia vice inuenit aquam tantam esse, quantum vinum. Quæritur nunc quot urnas capiat illud dolium.

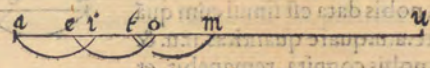
Solutio ipsius Tartaleæ bona est, cum supponat illas quatuor quantitates vini esse inuicem continuas proportionales, vt putà primò totum vinum merum, postea residuum pro secunda quantitate, deinde pro tertia in secunda, & pro quarta in tertia extractione, hoc est quod proportio totius vini meri ad vinum in prima sit, vt huius ad vinum in secunda, & vt huius ad vinum in tertia mitione. Sed quia ipse non probat hanc continuam proportionalitatem ex methodo scientifica, mihi visū est hoc loco illam describere.

Cogitemus igitur. a. u. pro capacitate dolij, et. a. i. pro quantitate duarum urnarum. Nunc uerò supponamus quamlibet partem huius misti omogeneam esse suo toto, quapropter sequetur eandem proportionem esse vini ad aquam in qualibet parte, quæ erit in toto, & ideo imaginemur. e. o. æqualem. a. i. Sed in puncto. i. tali modo diuisam, vt proportio. i. e. ad. i. o. eadem sit quæ. i. a. ad. i. u. Supponamus etiā

c. o.

THEOREM. ARITH.

e.o. esse duas primas vinas misti hoc est primæ mitionis, vnde cum eadem pro-  
 portio sit. a. i. ad. i. u. vt. e. i. ad. i. o. ita erit (ex. 19. quinti). a. e. ad. o. u. ut. a. i. ad. i. u. &  
 componedo ita erit. a. e. cum. o. u. hoc est. i. o. u. (propterea quod. i. o. æqualis est. a. e.  
 vt residua totorum æqualium) ad. o. u. quemadmodum. a. i. u. ad. i. u. Quare. i. u. erit  
 media proportionalis inter. a. u. et. o. u. vnde proportio. a. u. ad. o. u. dupla erit pro-  
 portioni. i. u. ad. o. u. Nunc autem cum extracta fuerit quantitas. e. o. ex primo mi-  
 sto, & postea infusa aqua vsque ad plenitudinem dolij, proportio ingredientium  
 huius secundi misti erit ea, quæ est inter. o. u. et. o. a. eo quod in prima mitione pro-  
 portio ingredientium erat ea, quæ est inter. o. u. et. a. e. vel inter. a. e. et. o. u.  
 vt demonstrauius. Accipiamus ergo. t. m. huiusmodi secundi misti, magnitudi-  
 nis. a. i. vel. e. o. significantis duas vinas, & permitemus eum in tantam aquam,  
 sitq; punctum. o. quod nobis diuidat. t. m. in. o. m. et. o. r. partes simplices, tali propor-  
 tione inuicem relatas, vt sunt. o. u. et. o. a. vnde habebimus ex supradictis rationibus  
 eandem proportionem ipsius. a. i. ad. m. u. vt. a. o. ad. o. u. & componendo. a. r. cum. m.  
 u. hoc est. i. m. u. (eo quod cum. t. m. æqualis sit. a. i. per consequens. i. m. æqualis erit.  
 a. t.) ad. m. u. vt. a. o. u. ad. o. u. sed proportio. a. o. u. ad. o. u. dupla erat proportioni. i. o.  
 u. ad. o. u. quemadmodum supra diximus. Ergo proportio. i. m. u. ad. m. u. erit dupla  
 similiter proportioni. i. o. u. ad. o. u. quapropter. o. u. erit media pro-  
 portionalis inter. i. u. et. m. u. Ecce itur quomodo eadem est pro-  
 portio. a. u. ad. i. u. quæ. i. u. ad. o. u. & quæ. o. u. ad. m. u. qui quidem modus necessarius  
 est vt intellectus acquiescat, id quod experientia non facit.



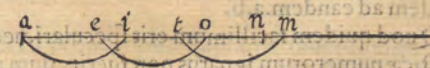
THEOREMA CXII.

Præcedens Tartaleæ questum elegans quidem est, sed pulchrum etiam vide-  
 tur quærare proportionem ingredientium in vltima mitione, cum cognita fue-  
 rit nobis proportio continentis dolij ad capacitatis vine simul cum numero vitium  
 extractionum & impletionum.

Exempli gratia, si proportio. a. u. ad. a. i. cognita nobis fuerit, cognoscemus etiam  
 e. i. ex regula de tribus & per consequens etiam. i. o. residuum ex. e. o. & similiter ag-  
 gregatum. a. i. cum. i. o. & sic. o. u. residuum totius, et. o. r. similiter, eo quod. a. u. ad. a.  
 o. est ut. t. m. ad. o. r. vnde cognoscemus etiam. o. m. vt residuum. t. m. & similiter ag-  
 gregatum. a. o. cum. o. m. hoc est. a. m. & etiam. m. u. residuum totius.

Cognoscere autem proportionem totius dolij ad vineam, vel e contra, cum cog-  
 nita nobis fuerit proportio ingredientium in vltima mitione simul cum numero vi-  
 tium extractionum, & repletionum, quod scribit Tartalea, hoc etiam modo  
 possumus,

Exempli gratia, si proportio. m. u. ad. m. a. cognita nobis fuerit, illico scie-  
 mus proportionem. a. u. ad. m. u. & cum sciuerimus numerum vitium extractionum,  
 & impletionum illico cognosci-  
 mus multipliciter proportio-  
 nis. a. u. ad. m. u. ad proportionem.  
 o. u. ad. m. u. quapropter propor-  
 tio. o. u. ad. m. u. nobis cognita erit  
 hoc est. a. u. ad. i. u. & similiter ea, quæ est. a. u. ad. a. i. & e conuerso similiter.



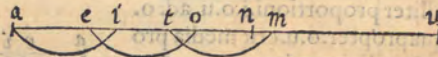
Vnde

Vnde cum aliquis diceret priori modo, dolium habeo vrnarum. 400. vini, & per vices. 25. extrahi & impleui ipsum, vt dictum est. Nunc verò velim scire proportionem vini ad aquam hac vltima vice. Nunc igitur si procedemus iuxta doctrinam primi exempli huius theoremat, obtinebimus quod querebamus.

Sed si diceret iuxta Tartaleæ quæsitum, hoc est dolium habeo, quod ignoro quot nã urnas contineat, volo tamen per. 25. vices extrahere, & implere vt supradictū est, ita vt vltima vice proportio vini ad aquam sit sesquialtera. Tunc si iuxta modum secundi exempli huius theoremat, procedemus habebimus quod cupimus.

Alio etiam modo aliquis quæreere posset, hoc est, habeo doliū quod capit. 400. urnas. Habeo etiam vas trium vrnarum, quo mediante me oportet extrahere, & implere. Vclim tamen scire quoties me hoc facere oporteat, ita vt postrema vice vinum se habeat ad aquam in proportione sesquialtera, vnde multoties accidet vltimam extractionem, & impletionem mutilatam, seu imperfectam, euadere.

Exempli gratia, si proportio vini ad aquam in vltima mitione deberet esse vt. n. u. ad. n. a. ita vt extrema vice fuisset. t. m. quæ quidem. t. m. excederet terminum per. n. m. quæ. n. m. reuera esset nobis cognita, eò quòd ex priori modo hic supra dicto proportio. a. m. ad. m. u. nobis innotesceret, & proportio. n. a. ad. n. u. nobis data est simul cum quãtitate. a. u. quare quantitas. n. u. & m. u. nobis cognita, remanebit, et n. m. eorum differentia similiter, etiam, et. t. n. residuum vasis, quo metimur, vnde necesse erit, quòd vltima vice vas contineret solum. t. n. reliqua uerò per se patent.



## T H E O R E M A C X L I I I.

**H**ieronymus Cardanus in lib. suæ arithmetice cap. 66. quæstione. 56. quam Cardanicam vocat, ita inquit.

Quidam perambulauit prima die certam quantitatem spatij, & secunda die, tantò plus proportionaliter, quantò diameter est maior costa, & tertia die tantò plus secunda, quantò proportionaliter portio lineæ diuisæ secundum proportionem habentem medium, & duo extrema excedit minorem portionem, & quarta die in proportione ad tertiam vt secunda ad primam, & quinta die proportionaliter tantò plus quarta, quantò in tertia plus secunda, & ita alternatis vicibus in diebus nouem peregit nouem milliaria. Quæritur igitur quantum ambulauit die prima.

Hoc autem nihil aliud est, quàm si aliquis diceret, propono tibi, exempli gratia, lineam. a. l. nouem partibus inuicem non æqualibus ita diuisam. a. c. d. e. & cæteris, quarum partium proportionem tibi etiam do, vt putà. a. c. ad. c. d. et. c. d. ad. d. e. et. d. e. ad. e. f. & sic de cæteris vsque ad postremam. k. l. quæ quidem proportionem sint etiam inuicem dissimiles, seu inæquales, do tibi etiam proportionem totius lineæ. a. l. ad. a. b. suam partem, quæ vt in proposito exemplo nonupla est.

Quæro nunc quam proportionem habebit. a. c. ad. a. b. & sic de cæteris partibus eiusdem ad eandem. a. b.

Quod quidem facillimum erit speculari, nec non operari vnicuique, qui omnino practicæ numerorum ignarus non fuerit, dum ab ordine scientifico non discedat.

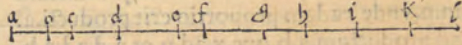
Cum enim cognoscimus proportionem. a. c. ad. c. d. consequenter cognoscemus etiam proportionem aggregati. a. c. d. ad. c. d. cum autem cognouimus proportio-

nem.



THEOREM. ARIT.

nem. c. d. ad. d. e. si. c. d. accipiemus, vt medium inter. a. d. et. d. e. cognoscemus etiam proportionem. a. d. ad. d. e. quare etiam eam quæ. a. e. ad. d. e. collocando postea. d. e. inter. e. f. et. a. e. innotescet ea, quæ est. a. e. ad. e. f. & ita gradatim accedemus ad perfectam cognitionem proportionis totius. a. l. ad. k. l. Nunc autem mediante. k. l. cognoscemus proportionem totius. a. l. ad. i. k. & hac mediante, eam cognoscemus, quæ totius. a. l. ad. g. h. & hac mediante eam quæ totius. a. l. ad. f. g. & sic gradatim, cognita nobis erit proportio totius lineæ. a. l. ad. suam partem. a. c. beneficio postea totius lineæ. a. l. cognoscemus proportionem. m. a. c. ad. a. b. & sic aliarum respectu lineæ. a. b. vt quærebatur, quæ quidem propositio, etsi car danica uocetur leuissima tamen est.



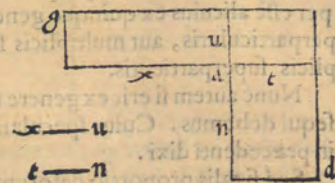
THEOREMA CXLIII.

Quamuis multi de modo in summa colligendi, subtrahendi, multiplicandi, & diuidendi proportionem scripserint, nullus tamen (quod sciam) perfecte, ac scientifice speculatus est has operationes, quapropter hanc rem cum silentio transire nolui, quin aliquid de ipsa conscribam à summa dictarum proportionum incohando.

Quotiescunque igitur volunt duas proportionem inuicem aggregare, simul earum antecedentia multiplicant, & similiter earum consequentia. Tunc proportio terminata ab illis productis euadit in summam illarum duarum propositarum proportionum.

Vt exempli gratia, si uoluerimus colligere proportionem sesquialteram cum sesquitercia, multiplicando. 3. cum. 4. antecedentia scilicet, productum erit. 12. postea multiplicando. 2. cum. 3. consequentia, tunc productum erit. 6. Proportio igitur, quæ inter. 12. et. 6. reperitur. (quæ dupla est) est summa propositarum proportionum.

Cuius rei speculatio erit huiusmodi sint. x. et. u. duo antecedentia quarumuis proportionum. t. u. uerò et. n. sint eorum consequentia, productum autem antecedentium sit. a. g. illud uerò quod consequentium sit. d. a. unde proportio. a. g. ad. a. d. composita erit ex proportione. x. ad. t. & ex ea, quæ est. u. ad. n. per. 24. sexti vel quintam octauum. Patet igitur ratio rectè faciendi, vt supra dictum est.



THEOREMA CXLV.

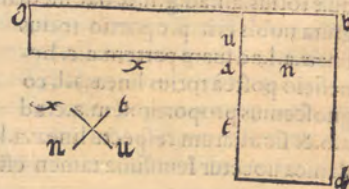
Quotiescunque deinde detrahere uolunt vnã proportionem ex altera multiplicat antecedens vnus cum consequenti alterius. Tunc proportio, quæ inter talia duo producta inclusa reperitur, est residuum, seu differentia illarum duarum proportionum datarum.

Vt exempli gratia, si aliquis uellet ex proportione dupla detrahere sesquialteram, multiplicaret. 2. antecedens duplæ cum. 2. consequenti sesquialteræ, quorum productum esset. 4. pro antecedenti residuæ proportionis. Deinde multiplicaret. 3. antecedens sesquialteræ cum. 1. consequenti duplæ, & productum esset. 3. pro consequenti

N sequenti

sequenti residuę proportionis; quę quidem residua proportio esset vt. 4. ad. 3. hoc est sesquitercia, & sic de cęteris.

Pro cuius ratione, sit proportio. x. ad. n. ea quę (exempli gratia) maior sit, à qua volumus demere proportionem. t. ad. u. minorem scilicet. Nunc autem productum. x. in. u. sit. a. g. illud verò. t. in. n. sit. a. d. Tunc dico proportionem. a. g. ad. a. d. esse residuam quęsitam. Sit. b. a. productum u. in. n. vnde eadem proportio erit producti. a. g. ad productum. a. b. quę. x. ad. n. et. a. d. ad. a. b. quę. t. ad. u. ex prima sexti, feti. 18. vel. 19. septimi, sed proportio. a. g. ad. a. b. hoc est. x. ad. n. componitur ex ea, quę est. a. g. ad. a. d. & ea, quę est. a. d. ad. a. b. hoc est. t. ad. u. ergò ea, quę est. a. g. ad. a. d. erit quàm quęrebamus.



### THEOREMA CXLVI.

**R**ATIO verò, quòd rectè fiat, quotiescunque aliquam proportionem duplicare volentes, quadramus terminos ipsius proportionis, vel si eam triplicare voluerimus, cubamus ipsos terminos, vel si eam quadruplicare voluerimus inuenimus censicos censicos terminorum ipsius proportionis, & sic de singulis, in. 17 Theo. huiusmodi tractatus manifesta est.

### THEOREMA CXLVII.

**Q**uotiescunque nobis propositi fuerint duo numeri ad libitum, desideraremus quę duas proportionales tali relatione inuicem refertas, quali sunt hi duo propositi numeri inter se, ita faciendum erit.

Sciendum primo est proportionem maioris numeri propositi ad minorem semper esse alicuius ex quinque generum, hoc est aut erit generis multiplicis, aut superparticularis, aut multiplicis superparticularis, aut superpartientis, aut multiplicis superpartientis.

Nunc autem si erit ex genere multiplici, iam ab antiquis traditus est modus, quę sequi debemus. Cuius speculatio à me inuenta patet. in. 17. Theo. huius libri, vt in precedenti dixi.

Sed si talis proportio datorum numerorum erit alicuius aliorum generum, ita agemus, si fuerit superparticularis.

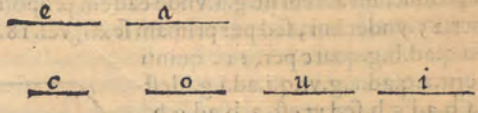
Sit exempli gratia, sesquialtera, tunc sumantur duo numeri inuicem inæquales, quos à casu volueris. o. et. c. qui quidem cubentur, & eorum cubi sint. a. et. e. Inueniatur postea. u. ita proportionatus ad. o. vt. o. est ad. c. ex regula de tribus, hoc est diuidendo quadratum ipsius. o. per. c. vnde nobis proueniat. u. & quia proportio. a. ad. c. tripla est proportioni. o. ad. c. & proportio. u. ad. c. dupla est eidem, quę. o. ad. c. ideo proportio. a. ad. e. sesquialtera erit proportioni. u. ad. c.

Sed si proportio numerorum propositorum fuerit sesquitercia, faciemus. a. et. e. esse censica censica ipsius. o. et. c. tunc sumemus. u. consequentem ad. o. vt dictum est, deinde inueniremus. i. consequens ad. u. ita ut. u. consequens ipsius. o. tunc habebimus proportionem. i. ad. c. triplam, & eam quę est. a. ad. e. quadruplam proportioni. o.

THEOREM. ARTT.

ni.o.ad.c. Idem dico de reliquis proportionibus superparticularibus.

Sed si data proportio numerorum fuerit ex superpartientibus, vt exempli gratia de quinque ad tria, efficiemus, vt.a.et.e.sint prima relata ipsius.o. et. c. vnde proportio.a.ad.e. ita se habebit ad proportionem. o. ad. c. vt quinque ad vnū & proportio.i.ad.c.ut tria ad vnū. Quare proportio. a. ad. e. ad proportionem.i.ad.c. se habebit, vt quinque ad tria, & sic de reliquis.

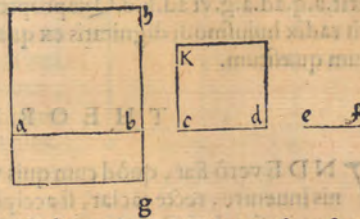


Pro alijs, eundem ordinem seruando, obtinebimus quod volumus.

THEOREMA CXLVIII.

Quamuis in. 16. sexti et. 20. septimi manifestè pateat ratio, quare rectè fiat accipiendam radicem quadratam illius producti, quod fit ex duobus datis terminis, vt medium proportionale geometricè inter ipsos habeamus: nihilominus, quia per aliam methodum hoc idem scire possumus, inconueniens non erit aliquid circa hoc dicere.

Cogitemus igitur exempli gratia, tres numeros continuè proportionales geometricè.a.b:c.d.et.e.f. quorum.a.b.et.e.f.tantummodo nobis cogniti sint, imaginemur etiam.g.a. esse productum quod fit ex.a.b.in.e.f.et.d.k. quadratum.c.d.et. a.h. id quod fit ex.a.b.vnde eandem proportionem habebimus.a.h.ad.a.g. quæ est. h. b. ad. b.g. ex prima. 6. aut. 18. vel. 19. septimi, sed per. 11. octaui ita est quadrati. a. h. ad quadratum. k. d. vt. a.b.ad.e.f. hoc est vt. h. b. ad. b.g. ergo per. 11. quinti ita erit.a.h.ad.a.g. vt ad.k. d. vnde.a.g. æquale erit.k.d. per. 9. quinti. Rectè ergo erit accipere radicem quadratam.a. g. pro.c. d. quod etiam est diuidere vnam datam proportionè per æqualia, hoc est in duas æquales partes, non dubito quin posset aliquis dicere non oportere vt posterioribus Theorematibus ad demonstrandum priora illis, sed hoc. 148. dictum sit ludendi loco.



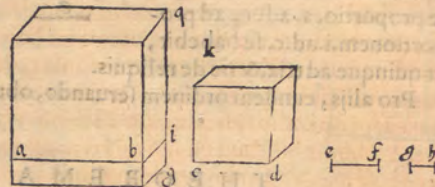
THEOREMA CXLIX.

Vnde fiat qd si quis inuenire voluerit secundum terminum ex quatuor numeris continuè, & geometricè proportionalibus, quorum duo extremi tantummodo nobis cogniti sint, rectè factum sit quadrare primum eorum, & hoc quadratum postea per alium terminum cognitum multiplicare, cuius producti demum accipere radicem cubam pro secundo termino quæsito, hoc loco videbimus.

Imaginemur quatuor terminos continuè proportionales, vt dictum est, esse.

N 2 a.b:

a. b. c. d. e. f. et. g. h. quorum. a. b. et. g. h. nobis tantummodo cogniti sint, sitq; imagine  
 tione descriptus cubus. a. q. primi termini, cubusq; d. k. secundi termini, considere-  
 mus etiam basim. a. i. quadratam ipsius cubi. a. q. hoc est præcedentem dignitatem ip-  
 sius cubi eiusdem radice, quæ quidem basis. a. i. multiplicetur per quartum terminū  
 g. h. productum autem sit. g. a. unde eadem proportio erit. a. q. ad. a. g. quæ. b. q. ad. b.  
 g. per. 25. vnde per primam sexti, vel. 18. aut. 19. septimi ita est. q. i. ad. i. g.  
 vel. b. q. ad. b. g. quare per. 11. quinti  
 ita erit. a. q. ad. a. g. vt. q. i. ad. i. g. id est  
 vt. a. b. ad. g. h. sed vt est. a. b. ad. g. h.  
 sic est. a. q. ad. k. d. per. 36. vnde per. 11. quinti  
 seu per. 11. octavi, vnde per. 11. quinti  
 ita sic erit. a. q. ad. a. g. vt ad. k. d. Qua-  
 re per. 9. eiusdem. a. g. equalis erit. k. d.  
 d. Vnde rectè erit accipere radicem  
 cubam. a. g. pro secundo termino. c. d.  
 id, quod nobis inferuit ad inueniendam tertiam partem vnus propositæ propor-  
 tionis.



## T H E O R E M A C L.

**S**ED vt speculatio ista ita vniuersalis fiat vt ad oēs dignitates applicari possit;  
 Supponamus. a. q. et. k. d. esse duas dignitates quas volueris vnus, sed eiusdem  
 speciei, et. a. i. dignitas præcedens dignitatem. a. q. à cuius multiplicatione in. a. b.  
 eius radix producit dignitas. a. q. & ab ipsius. a. i. multiplicatione in. g. h. resultet. a.  
 g. vnde ex. 18. vel. 19. septimi eadem proportio erit. a. q. ad. a. g. quæ. a. b. ad. g. h. sed  
 eadem etiam est. a. q. ad. k. d. ex ijs, quæ in. 17. theoremate dixi, vnde ex. 11. quinti,  
 ita erit. a. q. ad. a. g. vt ad. k. d. Quapropter. a. g. equalis erit. k. d. & ideo cum inuenta  
 fuerit radix huiusmodi dignitatis ex quantitate. a. g. habebimus. c. d. secundum ter-  
 minum quæsitum.

## T H E O R E M A C L I.

**V**NDE verò fiat, quòd cum quis voluerit dimidium alicuius datæ proportio-  
 nis inuenire, rectè faciat, si accipiat radices quadratas illorum datorum ter-  
 minorum, et si voluerit tertiam partem, accipiat radices cubas: si autem quartam,  
 accipere radices censitas censitas ipsorum, & sic de singulis in. 17. Theoremate om-  
 nia patent.

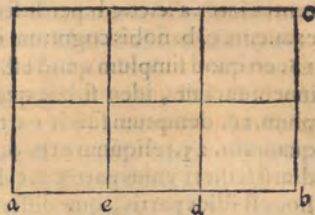
## T H E O R E M A C L I I.

**V**NDE autem fiat, vt cum quis voluerit multiplicare aliquam proportionem  
 per fractos, rectè faciat prius multiplicando eam per numeratorem, dein-  
 de productum diuiserit per denominationem ipsorum fractorum.

Vt exempli gratia, cum aliquis voluerit multiplicare proportionem sesquiquar-  
 tam per duo tertia, multiplicabit prius ipsam proportionem per numeratorem. 2.  
 & productum, erit proportio. 25. ad. 16. qua postea diuisa per. 3. denominatorem,  
 prouentus erit proportio radice cubæ. 25. ad radicem cubam. 16. vel vt proportio.  
 25. ad

25. ad radicem cubam. 1000. quæ quidem proportiones æquales inuicem sunt, cū tam vna, quàm alia, sit tertia pars totius.

Pro cuius ratione cogitemus. a. b. esse aliquod totum, quod multiplicare cupimus per duas tertias, quod quidē nihil aliud est, quàm accipere duas tertias partes vnus totius superficialis, imaginemur igitur hoc totum. a. b. lineare diuisum esse in tertias partes mediantibus. c. et. d. & tunc multiplicando ipsum per 2. tertias lineares productum erit. a. c. sex vnitatum superficialium, quod quidem productum postea diuisum per. 3. dabit. d. c. hoc est duas tertias superficiales (quæ est tertia pars ipsius. a. c.) & æquales numero. c. b. duabus vnitatibus linearibus, id est duabus tertijs ipsius. a. b. Notandum etiam est, quòd cum ferè omnia reducantur ad regulam de tribus, propterea etiam multiplicatio alicuius quantitatis per aliam quantitatem, nihil aliud est quàm quædam operatio ipsius regulæ de tribus, vt exempli gratia volo multiplicare. 25. per 20. hoc nihil aliud est nisi quærere alium numerum ita proportionatum ad. 25. vt 20. se habet ad vnum, vnde multiplicando. 25. cum. 20. & productum diuidendo per vnum ex regula de tribus, prouentus est idem numerus ipsius producti, & propterea cum volumus multiplicare aliquem numerum per fractos hoc nihil aliud est quàm quærere aliquem numerum ita proportionatum ad ipsum numerum datum, vt se habet numerator ad denominatorem, exempli gratia si. 2. 4. aliquis voluerit multiplicare per duo tertia hoc idem est vt si quæreret numerum ad quem. 2. 4. ita se habeat, vt. 3. ad. 2. & idem dico de proportionibus, hoc est quod aliud non est multiplicare aliquam proportionem per fractos, quàm aliam proportionem quærere ad quã data se habeat, vt denominator se hêt ad numeratorem; & hoc ex regula de tribus perficitur, cõstituendo denominatorem in primo loco, qui locus est diuisoris, numeratorem verò in secundo loco, multiplicando postea proportionem per numeratorem, & productum diuidendo per denominatorem, prouentus demum erit proportio, ad quam data se habeat, vt denominator se hêt ad numeratorem ex ratione ipsius regulæ de tribus. Ratio verò methodi diuidendi vnã datam proportionem per fractos, ex se satis patet, cum idem sit modus diuidendi quemlibet numerum integrum per fractos. Quare, quæ vnus, & alterius est ratio.



THEOREMA CLIII.

Nicolaus Tartalea in. 3. lib. quintæ partis numerorum soluit. 24. quæsitum sibi propositum à Hieronymo Cardano, via particulari & non generali. Quæsitum autem tale est quamlibet propositam rectam lineam in duas partes ita diuidere via Euclidis, ut cubus totius lineæ ad cubos partium se habeat in proportione tripla.

Tartalea igitur inquit quòd vt satisfiat speculatiuis ingenijs soluendum sit huiusmodi quæsitum, secando lineam propositam. a. b. in tres æquales partes, quarum vna sit. c. b. vnde problema solutum erit.

Verum dicit, sed hæc non est methodus generalis, propterea, quod cum tale problema alterius fuisset proportionis quàm triplæ, talis methodus nihil valeret.

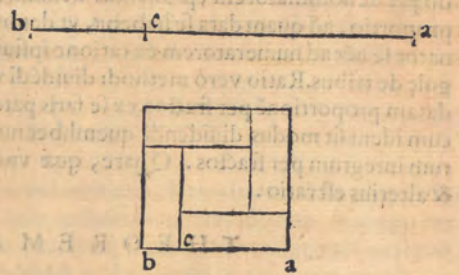
lem

Quapropter non tacebo quod mihi in mentem venit circa hoc problema.

Sit ergo linea .a. b. diuisibilis in puncto .c. ita vt cubum totius dicta .a. b. lineæ ad summam cuborum suarū partium .a. c. et .c. b. oporteat eam proportionem habere, exempli gratia, vt. 125. ad. 65. vt vitemus fracta pro nunc, notantes talem proportionem quadrupla nunquam maiorem esse posse, vt quilibet ex se contemplari potest, constituendo punctum .c. in medio loco inter .a. et .b. vnde proportio totalis cubi ad summam partialium esset omnium maxima quæ possint esse, collocando .c. vbi volueris in dicta linea .a. b. & hæc esset quadrupla.

Sed vt ad propositum reuertamur, considerabimus cubum totalem ipsius .a. b. esse vt. 125. & summam partialium vt. 65. quam detrahemus ex cubo totali & nobis remanebit. 60. pro summa trium solidorum inuicem æqualium, quorum longitudo vniuscuiusque erit tota linea .a. b. nobis cognita vt radix dati cubi totalis, quæ erit in hoc exemplo quinque partium, latitudo verò vniuscuiusque dictorum solidorū erit .a. c. pars maior ipsius .a. b. quæ quidem .a. c. adhuc nobis ignota est, profunditas seu altitudo vniuscuiusque illorum solidorum, erit .c. b. pars reliqua ipsius .a. b. & etiā nobis incognita, sed quia summa horum trium solidorum nobis manifesta superius fuit, quæ erat. 60. propterea nobis cognita erit quantitas vniuscuiusque illorum solidorum, vt tertia pars totius summæ ipsorum quæ erit. 20. in proposito exemplo, deinde cum vnumquodque illorum solidorum producat a superficie contenta seu producta ab .c. a. in .c. b. in tota linea .a. b. sequitur quòd si diuiserimus hoc solidum. 20. per lineam .a. b. quinque partium proueniet nobis cognita superficies producta ab .a. c. in .c. b. quatuor partium, sed cum quadratum totius .a. b. nobis cognitum sit, eo quod .a. b. vt eius latus etiam cognitum est. Tunc dictum quadratum erit. 25. quod quidem æquale est quadruplo illius quod fit ex .a. c. in .c. b. simul cum quadrato differentia inter .a. c. et .c. b. per. 8. secundi Eucli. Vnde quia quadruplum illius quod fit

ex .a. c. in .c. b. nobis cognitum est, vt 16. eo quod simpliciter quod est. 4. iā inuentum fuit, ideo si hoc quadruplum. 16. demptum fuerit ex totali quadrato. 25. reliquum erit. 9. quadratū scilicet vnius partis .a. c. ipsius hoc est illius partis, quæ differentia est inter .a. c. et .c. b. quæ quidem erit. 3. partium quæ differentia cum subtracta fuerit ex .a. b. reliquum erit duplum ipsius .c. b. duo scilicet. Quare



c. b. erit vt. 1. et. 2. c. vt. 4. & productum .a. c. in .c. b. erit. 4. vnitarum superficialium. &c.

IO. BAPT. BENEDE.  
103  
APPENDIX  
DE SPECVLATIONE  
REGVLAE FALSI.



VNC idem ferè mihi accidit, quod & Michaeli Stifelio, à quo cum Petreius Tipographus nuper totam suam Arithmetica[m] recepisset, mox postea per literas petijt explicacione[m] regulæ falsi. Similiter post incisas omnes superiorum Theorematum figuras, opereq[ue] Typographo commisso, amicus quidam omnium scientiarum ornatissimus maxima necessitudine mecum coniunctus monuit me, vt aliquid de regula falsi scribere vellem, cuius

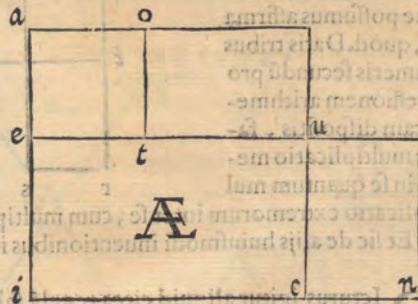
suafu hæc, quæ sequuntur appendicis vice ponere libuit, ne lector, quidpiam quod ad hanc rem pertinet iure merito à nobis desiderare posset; vt autem ad ipsam regulam accedamus Ego sicut, & in alijs multis, ita & in huiuscæ regulæ inuentione cum ipso Stifelio maximè conuenio, putans regulam falsi, seu falsarum positio[n]um inuentam fuisse per paruos numeros in quæstionibus facillimis & cognitissimis, eodem ferè modo, quo ipse monstrat illis duobus exemplis, quæ quamuis ipse appellet theore[m]ata, nihilominus theore[m]ata ego illa non vocarem, nisi adiuncta fuerit speculatio ab ipso præterita, & non experientia tantummodo, vt ipse fecit. Primum eius exemplum est, quod

Quorumcumque duor[um] numeror[um] differentia, si fuerit multiplicata in aggregatum eor[um], producit ipsam differentiam, quæ est inter quadrata eor[um].

Secundum verò exemplum est, quod

Datis tribus numeris secundum progressionem arithmetica[m] dispositis, facit multiplicatio medij in se, quàm multiplicatio extremor[um] inter se cum multiplicacione differentiarum inter se.

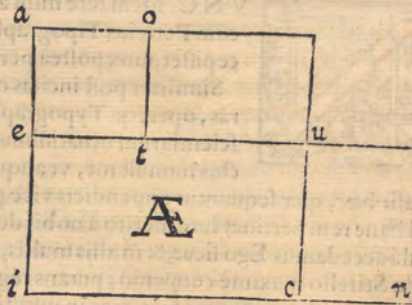
Talia enim exempla ipse aliter non probat nisi experientia in aliquibus numeris, arbitratus ex eo inuentam esse regulam falsi, experientia tantummodo confirmatam, quod quidem etiam & ego credo. At experientia in philosophia mathematica, aut nullam profus facit scientiam, aut omnino superflua fuit Euclides in multis suis propositionibus, & præcipue in eius secundo libro, si sufficeret experientia. Idcirco quo magis ad euidentiam ipsius veritatis, quam profiteor, deuenire possim, accipiam primò primum exemplum ipsius Stifelij hic superius citatum, & pro numero maiori, in prima hic subscripta figura. AE. accipio. a. i. cuius quadratum sit. a. c. pro minori verò numero capio. a. e. partem ipsius a. i. cuius quadratum sit. a. t. differentia autem horum numeror[um] erit. e. i. reliqua pars ipsius. a. i. & differentia ipsor[um] quadrator[um] erit gnomon. e. c. o: Nunc autem protrahe. i. c. latus quadrati maioris quousque e. n. æ qualis sit. a. e. numero minori, perficioq[ue] rectangulum. e. n. quod produ-



producitur ex .i. e. differentia in .i. n. aggregatum amborum numerorum, sed hoc productum excedit productum .e. c. partem gnomonis dicti per .u. n. quod quidem .u. n. æquatur ipsi .u. o. reliquæ scilicet parti ipsius gnomonis, nã .c. u. æqualis est. i. c. quare et .a. i. sed .e. t. æquatur .e. a. unde .t. u. æqualis erit. e. i. quare et .u. c. at cum .c. n. æqualis sit ipsi .a. e. erit etiam æqualis ipsi .o. t. quare .u. n. æqualis erit ipsi .u. o. & tunc intellectus quiescit, & absq; aliqua alia experientia verè scientifi ceq; dicere potest, quòd

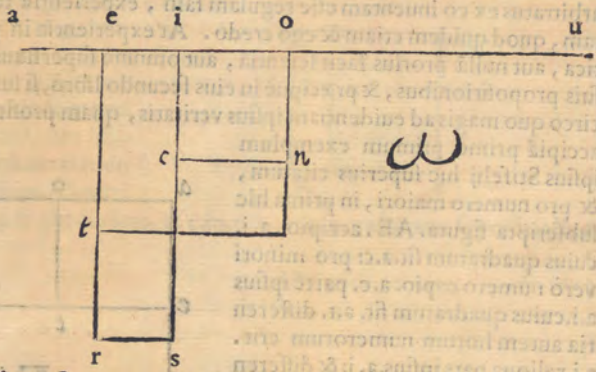
Quorumcumque duorum numerorum differentia, si fuerit multiplicata in aggregatum eorum, producit ipsam differentiã, quæ est inter quadrata eorum.

Hæc autem propositio à me ipso etiam in .60. Theoremate huius libri aliter demonstrata fuit.



DE speculatione autem, et scientia secundi exempli, in secunda hic subscripta figura. . cogitemus lineam .u. a. tribus in partibus arithmetice diuisam, quarum maxima sit .u. o. media .fit. o. e. minima verò sit .e. a. multiplicatio autem mediæ, o. e. in se fit quadratum .o. t. abscindatur deinde ex .o. e. i. æqualis .e. a. tunc .o. i. erit differentia inter .o. e. et .e. a. & æqualis differentia inter .o. e. et .o. u. ex hypotesi, quæ quidem .o. i. in se ducta procreabit quadratum .o. c. quod erit productum ex differentiis ipsarum partium, & erit pars quadrati .o. t. superius dicti, vt ex se patet. Nunc autem dico gnomonem .i. t. n. æqualem esse ei quod fit ex .a. e. in .o. u. Producatur igitur .e. t. quousque .t. r. æqualis sit ipsi .o. i. tunc .e. r. erit æqualis .o. u. quod etiam clarum est. Clauditur ergo rectangulum .i. r. quod erit æquale producto ipsius .e. a. in .o. u.

Nam .e. i. sumpta fuit æqualis .e. a. sed ex rationibus in priori exemplo allatis, pductum .i. r. æquale erit gnomoni .i. t. n. Nunc autem verè, scientificè que possumus affirmare, quòd. Datis tribus numeris secundum progressionem arithmeti cam dispositis, facit multiplicatio medij in se quantum multiplicatio extremorum inter se, cum multiplicatione differentiarum inter se.



Et sic de alijs huiusmodi inuentionibus infero.

Dicturus igitur aliquid circa regulã falsi, videtur mihi nullam oportere facere mentionem de origine huiuscã regulæ, cum in hoc Stifelius satisfecerit, sed potius

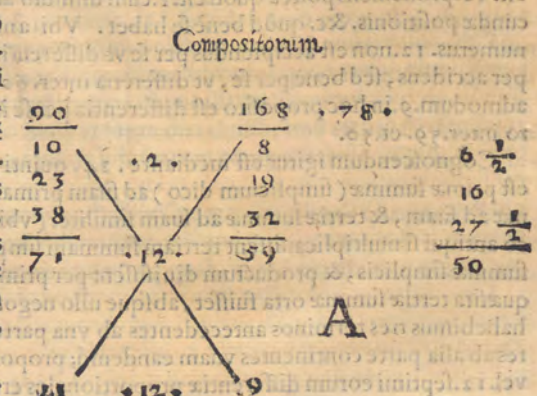


potius veras rationes propriasq; fundamenta huiusmodi operationis ostendere, sumendo eadem exempla proposita ab ipsis practicis, & maxime à Nicolao Tartalea viro accuratissimo, qui vbiunque potuit speculatus est causas ipsarū operationum, et si de huiusmodi falsi regula circa finem cap. 8. lib. 17. promittat postea loqui, nūllibi tamen loquutus est. Monendum etiam censeo, me nihil de rationibus regulæ falsi simplicis dicturum, cum ex seipsis satis appareant, quod non ita est de positionibus duplis. Incipiam ergo à primo problemate lib. 17. ipsius Tartalea, quo etiā ipse vtitur pro exemplo docendi gratia, ipsam regulam duplæ positionis, quod quidem problema aliter à me solutū fuit in. 118. Theoremate huius mei lib. quod similiter ob hanc demum occasionem mihi oblatam, alia etiam via, speculatus sum idem posse fieri, quæ quidem via seu methodus generalis erit, & ita se habet.

Accipio enim propositum numerum diuisibilem, à quo detraho summam datorum numerorum, primo duplicato, eo quod tam in secunda quam in tertia parte reperitur, vt in proposito exemplo, datus numerus est, 50. à quo detraho summam datorum numerorum, quæ est. 11. nam tres, & tres, & quinque sunt vndecim, eo quod primus ingreditur in secunda, & in tertia parte, dempto igitur hoc numero. 11. ex 50. remanet. 39. qui quidem numerus intelligendus est pro summa trium partium simplicium adhuc incognitarum, à quo extrahenda est prima, eo modo quo nunc proponam ex regula de tribus, hoc est aggregando dictas partes simplices sine aliqua additione vtcunque volueris (sed commodius erit in minimis numeris) iuxta propositum, quod quidem propositum est, vt secunda pars dupla sit primæ, tertia verò æqualis sit primæ & secundæ, quæ partes in dictis minimis numeris, ita dispositæ erunt. 1. 2. 3. quarum summa erit. 6. Nunc si ex regula de tribus dixerimus, cum hæc summa proueniat nobis ab vno, à quo proueniet. 39. et veniet nobis. 6. cum dimidio pro prima parte quaesita in proposito numero. 39. cum ergo habuerimus primam partem, reliquas postea illico cognoscemus.

Huiusmodi verò operationis ratio ex se manifesta patet, eo quod proportio summae partium in minimis numeris ad primam eorum partem eadem esse debet, quæ ipsius. 39. ad primam partem quaesitam huiusmodi aggregati partium simplicium, sed quia nemo adhuc, quod sciam, satis animaduertit rationem modorum, qui ab antiquis obseruati sunt, qui quidem modi duo sunt circa hoc Helcataym duplæ falsæ positionis, igitur non prætermittam aliquid de hac re speculari, & primo de primo modo.

In primis igitur sciendū est, quod veritas ita inueniri poterit eorum modo, mediantibus simplicibus partibus, vt etiā mediantibus compositis, ut in presenti exemplo pro primis positionibus acceperunt. 10. et. 8. pro secundis verò compositis cū numero. 3. inuenerūt. 23. et. 19. pro tertijs aut compositis cū quinq; notaerunt. 38. et. 32. vnde prima summa resultauit. 71. secunda verò. 59. ita quod primus error remanebat. 21. secundus aut. 9. vt in figura. A.

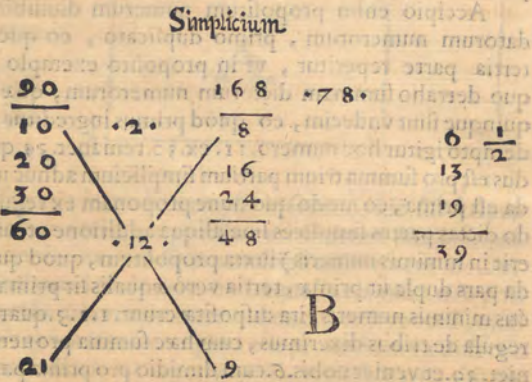


O Sed

**S**ED ijdem errores proueniunt ex summis partium simplicium. Vt exempli gratia, in figura. B. summa proposita partium simplicium est. 39. vt diximus, eo quod ab ipso. 50. detraherimus. 11. summa scilicet numerorum adijciendorum ad efficiendas partes compositas; summa postea simplicium partium primæ positionis, erit. 60. eo quod prima pars erat. 10. secunda autem simplex. 20. tertia verò simplex. 30. iuxta ordinem propositi. Summa deinde simplicium partiũ secundæ positionis esset. 48. quia prima eius pars erat. 8. secunda verò simplex. 16. tertia autem simplex. 24. vnde prima summa excederet datam. 39. per. 21. differentia, secunda verò per. 9. vt supra vidimus de summis compositis à dato. 50. composito, & hoc quidem mirandum non est, quod scilicet tres summæ simplicium partium sint inuicem inæquales, ijdem differentijs mediantibus, quibus differunt dictæ tres summæ compositarũ ablati sit numerus. 11. æqualiter, vnde ex necessitate, permutando, earũ differentia relinquerentur æquales inuicem ex. 78. theoremate huius nostri lib. summæ enim compositæ erant. 71. 59. et 50. simplices verò. 60. 48. et. 39. differentes à primis per. 11. vt dictum est, quare veritas ita manabit à compositis, quemadmodum à simplicibus, sed à simplicibus per se, & à compositis per accidens vt iam iam videbimus.

**A**ntiquorum igitur primus modus vtitur regula de tribus, hoc ordine, multiplicando scilicet secundum errorem, qui est. 9. cum differentia primarum partium positarum, quæ est. 2. & productum diuidendo per differentiam errorum, quæ est. 12. proueniens postea quod est. 1. cum dimidio additur hoc loco primæ parti secundæ positionis. &c. quod benè se habet. Vbi animaduertendum est, quod ille numerus. 12. non est accipiendus per se vt differentia errorum hoc est. 21. et. 9. nisi per accidens, sed benè per se, vt differentia inter. 60. et. 48. simplices summæ, quem admodum. 9. in hoc proposito est differentia per se inter. 48. et. 39. per accidens vero inter. 59. et. 50.

Cognoscendum igitur est mediante. 24. quinti Eucli. quod eadem proportio est primæ summæ (simplicium dico) ad suam primam partem, quæ secundæ summæ ad suam, & tertiæ summæ ad suam similiter (vbi rectè etiam fecissent hoc in loco antiqui si multiplicauissent tertiam summam simplicem cum prima parte prioris summæ simplicis, & productum diuisissent per primam summam, vnde prima pars quæsita tertiæ summæ orta fuisset, absque ullo negotio ipsius plus vel minus) Quare habebimus tres terminos antecedentes ab vna parte, & tres terminos consequentes ab alia parte continentes vnã eandemq; proportionem, vnde ex. 19. quinti, vel. 12. septimi eorum differentia proportionales erunt, hoc est, & eadem proportio



tio erit eius differentia, quæ est inter primam & secundam summam, ad differentiam quæ est inter primas earum partes, quæ illius differentia, quæ est inter secundam & tertiam summam, ad differentiam, quæ est inter primas illarum partes, sed harum. 4. differentiarum, tres nobis cognitæ sunt, idest. 1. 2. 2. et. 9. ergo ex regula de tribus ab Eucli. in. 20. septimi speculata inueniebatur quarta differentia, quæ est. 1. cum dimidio.

A compositis summis idem etiam proueniet, sed non ut ex proprijs causis, & per se, sed per accidens. Nam quamuis eadem differentia sit inter 71. et. 59. quæ inter. 60. et. 48. & eadẽ inter. 59. et. 50. quæ inter. 48. et. 39. Nihilominus non est eadẽ proportio (proprie) ipsius. 71. ad. 59. quæ ipsius. 60. ad. 48. nec ea quæ ipsius. 59. ad. 50. est quæ ipsius. 48. ad. 39. Vnde non erit eadem proportio ipsius. 71. ad. 59. quæ ipsius. 10. ad. 8. nec ea quæ est ipsius. 59. ad. 50. quæ ipsius. 8. ad. 6. cum dimidio. Sed minores illis. Nam ex æqualibus additamentis diminuuntur proportionales maioris inæqualitatis.

A simplicibus igitur summis pendet ratio huiusmodi effectus. Si vero prima pars secundæ positionis esset. 4. tunc secundæ eius pars esset. 8. & tertia. 12. quarum summa esset. 24. (harum simplicium partium scilicet) & minor uera (39.) per. 15. & differens à summa primarum. (60.) per. 36. & differentia primarum partium esset. 6. differentia uero primæ partis secundæ positionis, a prima parte quæ sita esset. 2. cum dimidio. Vnde in huiusmodi exemplo uidere est quare colligantur errores inuicem, quando alter eorum excedit, reliquus uero deficit à numero proposito. Quod quidem ob aliam causam non fit, nisi ut cognoscatur differentia. 36. differentia scilicet simplicium summarum ipsarum positionum.

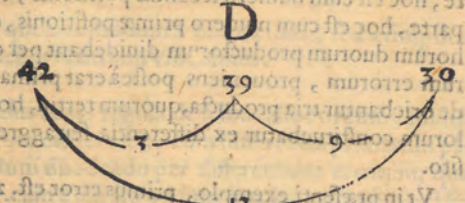
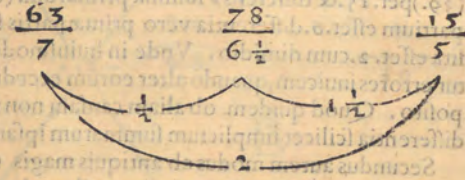
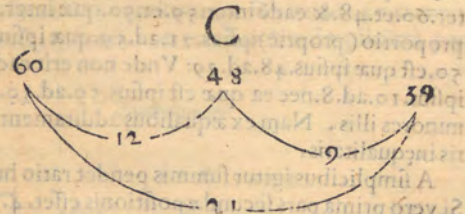
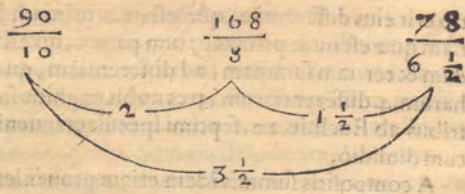
Secundus autem modus ab antiquis magis exercitatus est, quod multiplicabant diametraliter errores cum primis partibus, hoc est primum errorem cum prima parte, hoc est cum numero secundæ positionis, secundum uero errorem cum prima parte, hoc est cum numero primæ positionis, differentiam postea uel aggregatum horum duorum productorum diuidebant per differentiam uel aggregatum dictorum errorum, proueniens postea erat prima pars quæ sita numeri propositi. Vnde oriebantur tria producta, quorum tertium, hoc est differentia, seu aggregatum florum constituebatur ex differentia seu aggregato errorum, & ex numero quæsito.

Ut in præsentis exemplo, primus error est. 21. qui multiplicatus cum prima parte secundæ positionis, quæ est. 8. producit. 168. secundus uero error est. 9. qui multiplicatus cum prima parte primæ positionis producit. 90. differentia autem horum productorum est. 78. quæ diuisa per differentiam errorum, quæ est. 12. dabit. 6. cum dimidio, pro prima parte quæ sita dari numeri diuisibilis, qui erat. 50.

Hæc omnia recte se habent. Sed, ut supra dixi diuisor non est per se differentia errorum, neque etiam differentia per se summarum compositarum, sed bene simplicium.

Pro cuius rei speculatione, accipiendæ sunt summæ simplices, quarum differentia per se uiles sunt in huiusmodi operatione, & quia etiam rationes ueritatis ex istis, & non ex illis fluunt; quamuis tam unæ, quam aliæ sint eadem in quantitate, idest æquales.

Disponantur igitur huiusmodi numeri tali ordine, vt simplex summa, quæ ab vna reliquarum superatur, & aliam superat, medium locum teneat; vt in proposito exemplo summa mediocris est. 48. quæ à summa. 60. superatur, & superat summam. 39. locata igitur sit hæc. 48. inter illas, suæ verò primæ partes similiter constitutæ sint supra dictas summas, cum suis differentijs, & tria producta iam dicta, vt in figuris. C. et. D. arithmetice clarè patet: figura enim. C. est pro exemplo ipsius plus simpliciter: figura verò. D. pro exemplo ipsius plus, & minus. Et sic in figura. C. habebimus tres numeros consequentes. 60. 48. 39. & tres antecedentes. 10. 8. 6. cum dimidio, vnam, & eandem proportionem terminantes, ex. 24. quinti, vt diximus; quæ eorum differentiæ similiter proportionales erunt, quod etiam vidimus. Supponamus nunc nos ignorare æqualitatem maximi producti cum reliquis duobus, accipiendo solum pro hypotefi, quod dicta producta oriuntur ex lateribus iam dictis.



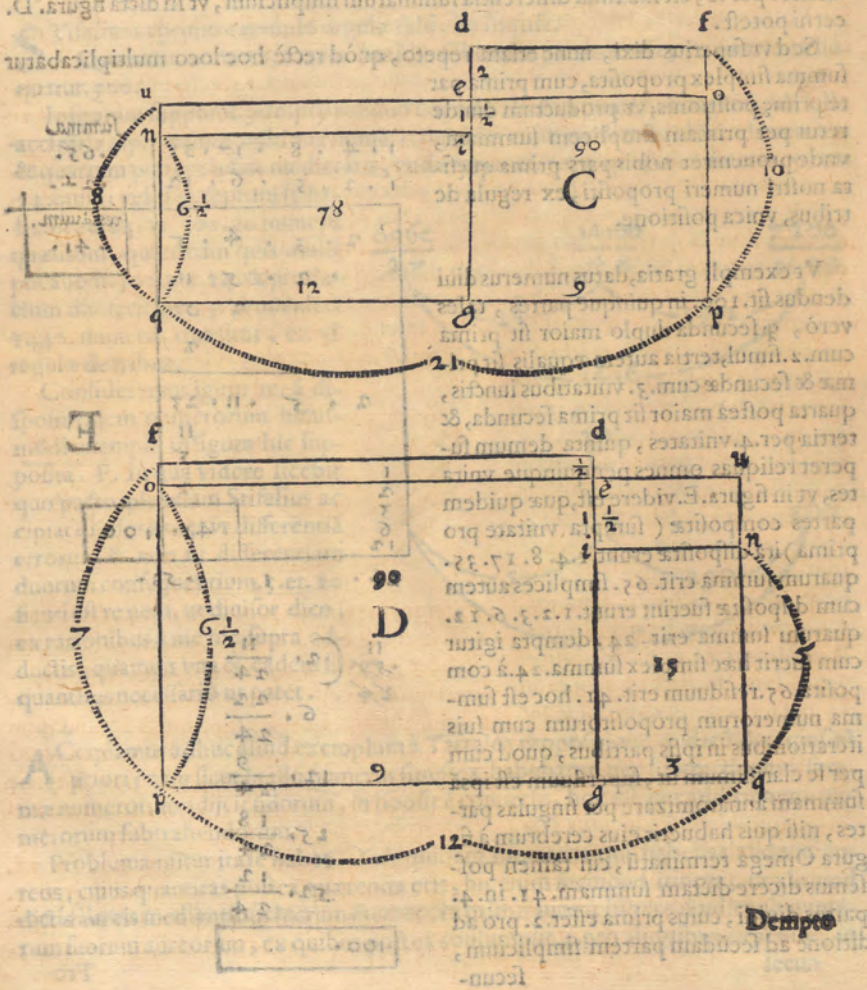
Demonstrandum nobis nunc relinquetur, maximum productum æquale esse reliquis duobus; hoc est productum. 168. æquale esse productis. 90. et. 78. quorum duorum productorum alterum. 90. scilicet, generatur à differentia. 9. quæ est secundæ, & tertie summe, in primum numerum antecedentem, qui est. 10. alterum vero productum. 78. scilicet, generatur à differentia. 12. quæ est prime, & secundæ summe in tertium numerum antecedentem, qui est. 6. cum dimidio, maximum vero productum. 168. scilicet, generatur à differentia maxima. 21. quæ est prime, & tertie summe (& semper equalis prioribus duabus differentijs, 12. et. 9.) in secundum numerum antecedentem, qui est. 8.

Constituatur igitur duo producta simul iuncta equalia duobus. 90. et. 78. lateralibus supra vnam aliquam rectam lineam. q.p. sit q; productum. f. g. æquale. 90. productum verò. g.n. æquale. 78. sit etiam basis. g.p. vt. 9. et. g.q. vt. 12. vnde. g. i. vel. q.n. erit vt. 6. cum dimidio. et. g.d. vel. p.f. vt. 10. & ideo. i. d. differentia erit. 3.

-opli D

cum

cum dimidio, ut in figura. C. geometrica hic subscripta videre licet, et. q. p. erit . 21.  
 Cogitemus nunc differentiam. d. i. diuisam esse in puncto. e. ita vt eadem proportio  
 sit ipsius. d. e. ad. e. i. quæ ipsius. q. g. ad. g. p. hoc est vt. 12. ad. 9. quapropter. d. e. erit.  
 2. et. e. i. erit. 1. cum dimidio, vt in dicta figura. C. arithmetica reperiuntur esse dif-  
 ferentiæ ipsorum antecedentium numerorum, deinde à puncto. e. ducatur imagina-  
 tione. u. e. o. æquidistans ipsi. q. p. & producat. q. n. vsque ad. u. unde ita se habebit  
 u. e. ad. e. o. ut. q. g. ad. g. p. quare vt. d. e. ad. e. i. ideo ex. 15. sexti vel. 20. septimi. n. e.  
 rectangulum æquale erit ipsi. e. f. quapropter rectangulum. q. o. æquale erit duobus  
 rectangulis. f. g. et. g. n. sed cum. g. i. sit vt. 6. cum dimidio, et. i. e. vt. 1. cum dimidio, er-  
 go. g. e. erit ut. 8. qui quidem numerus multiplicatus cum. q. p. 21. producit. 168. ve-  
 rum est igitur quod dictum fuit, hoc est q. maximum productum æquale sit reliquis  
 duobus.



**D** Empro postea quo volueris horum altero productorum ex maximo, diuisoq; reliquo per differentiam consequentium, ipsi diametraliter oppositam, proueniet tibi numerus antecedens correspondensq; illi.

Animaduertendum tamen est, quod si in figura a me ita ordinata, summa simplex proposita medium locum occuparet, vt in figura. D. arithmetica videri potest; tunc vt habeatur eius productum, addenda simul erunt circumstantia producta. eo q; eius secundum latus esset antecedens medio loco constitutum, & prima pars quesita numeri propositi: in qua figura. D. manifestè patet ratio, quare colligendi sint tam errores, quam producta, dum eorum alterum est plus, reliquum verò minus.

Speculatio figure. D. arithmetice videbitur in figura. D. geometrica, eodem sepe modo quo fecimus in figuris. C. mutatis mutandis, respectu ipsius plus, & minus.

Collectio namque erroru similiter accidentalis est, eo quod essentialis numerus diuisor per se, est maxima differentia summarum simplicium, vt in dicta figura. D. cerni potest.

Sed vt superius dixi, nunc etiam repeto, quod rectè hoc loco multiplicabatur summa simplex proposita, cum prima parte primæ positionis, vt productum diuide retur per primam simplicem summam, vnde proueniret nobis pars prima quesita nostri numeri propositi, ex regula de tribus, vnica positione.

Vt exempli gratia, datus numerus diuidendus sit. 100. in quinque partes, tales verò, q; secunda duplo maior sit prima cum. 2. simul, tertia autem æqualis sit primæ & secundæ cum. 3. vnitate iunctis, quarta postea maior sit prima secunda, & tertia per. 4. vnitates, quinta demum superet reliquas omnes per quinque vnitates, vt in figura. E. videre est, quæ quidem partes compositæ ( sumpta vnitate pro prima) ita dispositæ erunt, 1. 4. 8. 17. 35. quarum summa erit. 65. simplices autem cum dispositæ fuerint erunt. 1. 2. 3. 6. 12. quarum summa erit. 24. dempta igitur cum fuerit hæc simplex summa. 24. à composita. 65. residuum erit. 41. hoc est summa numerorum propositorum cum suis iterationibus in ipsis partibus, quod cum per se clarissimum sit, superfluum est ipsam summam anatomizare per singulas partes, nisi quis habuerit eius cerebrum à figura Omega terminatū, cui tamen possemus dicere dictam summam. 41. in. 4. partes diuidi, cuius prima esset. 2. pro additione ad secundam partem simplicium, secun-

1 . 4 . 8 . 17 . 35	Summa	. 65 .
1 . 2 . 3 . 6 . 12		. 24 .
2 . 3 . 4 . 5	residuum.	. 41 .
. 2 . 5 . 11		
2 . 5		
2		
2 . 5 . 11 . 23		
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100 .		

THEOREMATA

Secunda verò effct. 5. pro additione ad tertiam partem simplicium, tertia autem effct. 11. pro additione ad quartam partem simplicium, quarta demum effct. 23. pro additione quintæ partis simplicium, quarum partium. 2. 5. 11. 23. summa est. 41. vt diximus. Hæc igitur summa. 41. subducenda est à numero. 100. proposito, vnde relinquetur. 59. pro summa partium simplicium numeri propositi, quarum prima erit 22. cum vndecim vigesimisquartis ex diuisione huiusmodi. 59. per. 24. summam partium simplicium ex vi regulæ de tribus, dicendo si. 24. prouenit nobis ab. 1. prima partium simplicium, à quo proueniet nobis. 59. vnde proueniet à. 2. cum vndecim vigesimisquartis pro prima parte quæsitâ, secunda verò iuxta propositum, erit. 6. cum. 22. vigesimisquartis, tertia autem. 12. cum nouem vigesimisquartis, quarta postea. 25. cum. 18. vigesimisquartis, quinta demum erit. 52. cum. 12. vigesimisquartis, quarum omnium summa erit. 100.

Stifelius in primo exemplo regulæ falsi, ita inquit. Queratur numerus, à cuius dimidio subtractæ partes tertia, & quarta relinquitur. 300. Ipse enim supponit. 300. pro residuo cognito alterius numeri incogniti, deinde accipit. 24. pro prima positione numeri cogniti, à cuius medietate abscindit tertiam & quartam partem ipsius medietatis, vnde remanet. 5. qui quidem numerus. 5. ex. 22. quinti vel. 15. septimi se habebit ad. 24. vt. 300. ad numerum quæsitum, quare cum quis multiplicauerit. 300. per. 24. & productum diuiderit per. 5. proueniet. 1440. numerus quæsitus, ex vi regulæ de tribus.



Consideremus igitur meâ dispositionem numerorum huiusmodi exempli, in figura hic supposita. F. in qua videre licebit quo pacto ipse etiam Stifelius accipiat diuisorem. 5. vt differentiâ errorum & non ut differentiam duorum consequentium. 5. et. 10. sicuti est re vera, ut diuisor dico, ex rationibus à me hic supra adductis, quamuis vna & eadem sit quantitas necessariò ut patet.

Accipiamus adhuc aliud exemplum à Tartalea propositione. 9. datû, & oppositû priori; nam sicut in illo numerus simplex habebatur per subtractionem summae numerorum adijciendorum, in hoc fit è conuerso, hoc est per additionem numerorum subrahendorum. Problema igitur ita se habet. Fuit quidam mercator qui habebat aliquot aureos, cuius quantitas postea quærenda erit, hic enim fecit duo itinera, ut aliquod dictis aureis mediantibus lacrum faceret; in primo autem itinere duplicauit numerum suorum aureorum, ex quibus postea consumpsit. 4. pro aliquibus expensis, in secun-

secundo itinere iterum duplicavit suos aureos, ex quibus etiam postea consumpsit. 8. numeratis postea pecunijs reperit tantummodo. 24. aureos in eius marsupio, que ritur nunc quot habebat aureos in principio primi itineris.

In tali casu, cum ipse quolibet itinere duplicabat eius pecuniam, nulli dubium est quod in fine secundi itineris ipse habuisset pecuniam suam quadruplicatam, si ex ipsa nihil detractum fuisset, sed quia in fine primi itineris consumpsit. 4. aureos, quibus alios. 4. lucratus esset in secundo itinere, postea consumpsit iterum. 8. aureos, ita quod ex quadruplo suae primae pecuniae, recte dici potest, quod consumpsit. 16. aureos; qui quidem numerus ex communi conceptu erit differentia inter. 24. & quadruplum prioris pecuniae, cum qua profectus fuit in principio eius itineris; quapropter si addiderimus. 16. ipsi. 24. habebimus. 40. pro quadruplo eius prioris pecuniae. Recte igitur dici potest, si. 4. provenit ab vno, a quo numero proveniet. 40.

Videamus igitur nunc quo pacto hoc respondeat cum methodo antiquorum. Ego enim inveni duas positiones scriptas a Tartalea pro prima pecunia hoc est. 12. et. 14. sed a. 12. pro primo errore reperi. 8. more antiquo a. 14. vero pro secundo errore proveniebat. 16. producta autem horum numerorum diametraliter, sunt. 112. et. 192. quorum differentia est. 80. pro tertio producto, quo diviso per differentiam errorum. 8. scilicet, praebet nobis. 10. pro pecunia quaesita, ut etiam ego inveni.

Sed hoc mihi visum est subtilius examinare mea methodo mediante, ut in figura. G. videre est, prius enim suo loco posui tria producta dicta, deinde duas positiones. 12. et. 14. & quia sciebam productum. 112. oriri a multiplicatione. 14. cum. 8. ideo posui talem numerum. 8. suo loco diametraliter opposito ei producto. 112. & quia sciebam etiam productum. 192. nasci ex. 12. et. 16. ideo suo loco posui hunc numerum. 16. qui est maxima differentia inter duos consequentes (ita a me supra nominatos) a qua differentia dempta priori. 8. iam inuenta, reliqua. 8. mihi datur, qua suo loco notavi, suo etiam loco scripsi. 2. differentiam inter 12. et. 14. antecedentium. sed quia sciebam eandem proportionem esse inter hanc differentiam & differentiam. 8. huic suppositam, quae reperitur inter. 12. antecedentem, & suum consequentem; ideo posui. 48. pro dicto consequenti, divisi postea productum. 80. per. 8. differentiam ei diametraliter oppositam, unde provenit mihi 10. cui ita proportionatus est suus numerus consequens. 40. ut. 48. ad. 12. et. 56. ad. 14. ex iisdem rationibus a me supra dictis.

In tali igitur figura videntur numeri naturaliter correspondentes ipsi positionibus, & hac methodo possumus inuenire tales numeros consequentes in omnibus alijs exemplis a nostris maioribus scriptis.



Pro-



T H E O R E M . A R I T .

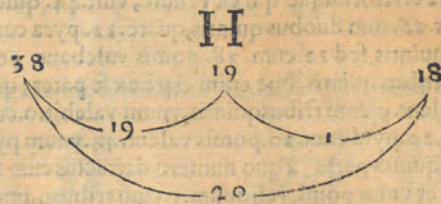
**P**roponitur etiam quoddam vas, cuius pes sit quarta pars totius vasis cum operculo, pars autem sine operculo, sit quinta pars ipsius pedis, operculum verò. 18. libras pendeat. quaritur nunc quantitas dicti pedis.

Ex methodo enim antiquorum inuentus est pes. 4. cum. 14. decimisonis tallium partium, seu librarum, qualium operculus est. 18. Videamus igitur & nos ex nostra figura, quo pacto hoc respondeat veritati.

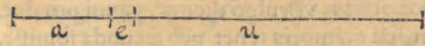
Inuenta enim sunt tria producta, iã orta ex dicta methodo. 10. 100. 90. quæ suis locis notauit, vt in figura. H. subscripti etiam duas illorum positiones. 5. et. 10. cum sua differentia. 5. & cum productum. 10. oriretur ab vno latere. 10. reliquum erat. 1. quod suo loco notauit, similiter quia. 100. productum, pro vno eius laterum erat. 5. reliquum autem. 20. suo loco posui, & quia differentia inter. 20. et. 1. duo latera, quæ est. 19. æqualis est ei, quæ inter duo consequentia duarum positionum, etiam suo loco ipsam constitui, sed quia hæc differentia est vnum laterum producti. 90. ergo reliquum latus quæsitum erit. 4. cum. 14. decimisonis, rectè igitur operatur. sed cum eadem proportio sit inter differentiam. 5. superiorem, et. 19. inferiorem, quæ est vnus antecedens ad suum consequens, quare. 10. antecedens habebit pro suo consequenti. 38. et. 5. habebit. 19. et. 4. cum. 14. decimisonis habebit. 18. rectè igitur dictum fuisset si. 19. prouenit. a. 5. a quo proueniet. 18?



Huiusmodi autem rei ratio ita se hæt, esto linea. a. e. u. cuius pars a. sit quarta reliquarum. e. u. iunctarum, sed. e. sit quinta ipsius. a. Tunc clarum erit quod. e. erit vigesima dictarum. e. u. quare erit decimona ipsius. u. sed cū u. supra sit vt. 18. rectè igitur dici potest, si. u. ut. 19. prouenit ab. a. ut quinque, a quot ipsius. a. proueniet. u. ut. 18.



Quis enim non uidet quod diuisa cum fuerit. u. in partes. 19. quod quinque illarum æquabuntur ipsi. a. cum quælibet fuerit æqualis. e. quintæ parti ipsius. a.



**H**Ac igitur mea numerorum dispositione mediante reperiuntur ipsi numeri inferiores naturaliter consequentes, correspondentesque ipsis superioribus antecedentibus; quamuis multoties cõtingere possit, ut generationes seu compositiones ipsorum ignorentur: & quia tam à differentijs errorum, quam ab illis, quæ sunt inter ueros consequentes numeros (propter eorum æqualitatem) elicitur ipsa ueritas, propterea rectè antiqui illis vsi sunt, quamuis sint potius sensum sequuti, uel experientiam, quam rationem: quæ quidem ratio pendet ab ipsis naturalibus numeris consequentibus (ut supra uidimus) et si incognitis ut plurimum, quod si ipsos inuenire primò nobis datum fuisset, unica tantummodo positio sufficeret

ret, mediante ipsa regula de tribus, vt iā sepius dictū est, quod etiā clarè patet ex diuersis problematibus. 17. lib. ipsius Tartaleæ, vt ex primo, quod assumpsimus pro nostro etiam primo exemplo, ex. 9. 15. 16. 17. 18. 19. 20. 27. 28. 29. 30. 33. & ex alijs multis, vbi facillimè inuenitur consequens ipsius positionis, qui quidem numerus est diuisor producti ipsius numeri propositi in numerum positionis, vnde postea prouenit secundū latus huiusmodi producti, hoc est numerus quæsitus, per regulā de tribus, vt dixi.

Alia verò multa problemata inueniuntur, pro quorum resolutione possumus aliqua methode vtī, in qua manifestè pateant eorū rationes absque regula falsi, cuius regulæ rationes non ita promptè ipsi intellectui se offerunt, vt supra vidimus.

Accipiamus pro exemplo. 21. problema ipsius Tartalæ in dicto. 17. libr. vbi supponit vnum hædum diuisum in. 4. partes, quarum quælibet vendebatur eodem precio, interiora vero. 6. denarijs minus quam quælibet dictarum partium, summa autem omnium istorum denariorum fuit. 127. quæritur nunc precium cuiusque partis.

Tale enim problema hoc etiam alio breuiori modo potest solui, vt rationes magis pateant, quam ex regula falsi.

Nam si illi numero. 127. denariorum, additus fuerit numerus. 6. summa erit. 133. qua diuisa per quinque, illico proueniet. 26. cum tribus quintis pro precio vniuscuiusque quatuor partium, à quo. 26. cum tribus quintis dempto. 6. remanebit. 20. cum tribus quintis pro precio interiorum.

Simili modo in. 24. problemate inquit.

Duodecim pyra cum. 28. pomis venduntur. 36. denarijs, et. 20. pyra. cum. 200. pomis venduntur. 44. denarijs, quæritur nunc, quod nā fuerit precium vniuscuiusque illorum.

Hoc etiam problema, hac alia methode solui potest, dicendo ex regula de tribus, si ex. 20. vtrorunque qui ea vendit, vult. 44. quid volet ex. 127. manifestū erit quod volet. 26. cum duobus quintis, quare. 12. pyra cum. 12. pomis valebunt. 26. cum duobus quintis, sed 12. cum. 28. pomis valebant, 36. ergo. 16. poma sola valebunt. 9. cum tribus quintis, hoc enim clarè ex se patet; quare cum dixerimus, si. 16. poma sola valent. 9. cum tribus quintis, vnum valebit. 0. cum tribus quintis, sed quemadmodum. 20. pyra cum. 20. pomis valent. 44. vnum pyrum, cum vno pomo valebat. 2. cum quinta parte, à quo numero detractus cum fuerit. 0. cum tribus quintis, precio scilicet vnius pomi, reliquum. 1. cum tribus quintis, erit precium vnius pyri.

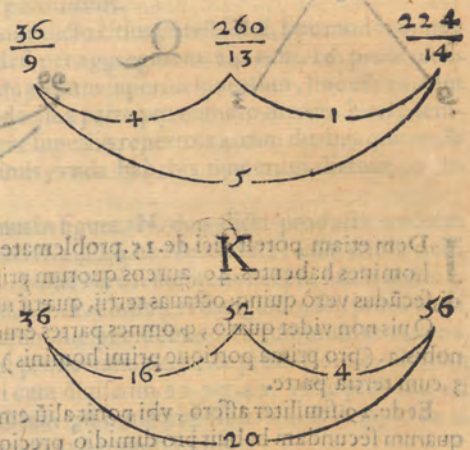
Idem etiam dico de. 28. problemate, vbi supponit quod quidam comparasset quatuor petias, vt vulgo dicitur, panni pro ducatis. 96. quarum primæ precium oblitus sit, sed memoria tenet pro secunda soluisse. 6. plusquam pro prima, & pro tertia soluisse. 8. plusquam pro secunda, & pro quarta soluisse. 10. plusquam pro tertia, quæritur nunc quantum fuerit precium vniuscuiusque illarum.

Quod quidē problema breuius esset ita solui, vt in subscripta figura. I. videri potest, addēdo simul omnes excessus. Nam excessus secundæ supra primam est. 6. sed cum excessus tertiæ supra secundam sit. 8. ergo excessus tertiæ supra primam erit. 14 sed

4.	6	8	10	24.	96
3.	6	8		14.	44
2.	6	8	9	6.	13. 24
1.	6		9	44	19. 24
					27. 34
					37. 48
					96

sed excessus quartæ supra tertiam est. 10. unde supra secundam erit. 18. & supra primam erit. 24. quæ omnia simul addita erunt. 44. & in qualibet harum trium remanebit una pars æqualis primæ quantitati, quare si ex. 96. deductus fuerit numerus. 44. reliquus. 52. erit quadruplus primæ, quare prima pars valebit. 13. secunda. 19. tertia. 27. & quarta. 37. quarum omnium summa est. 96.

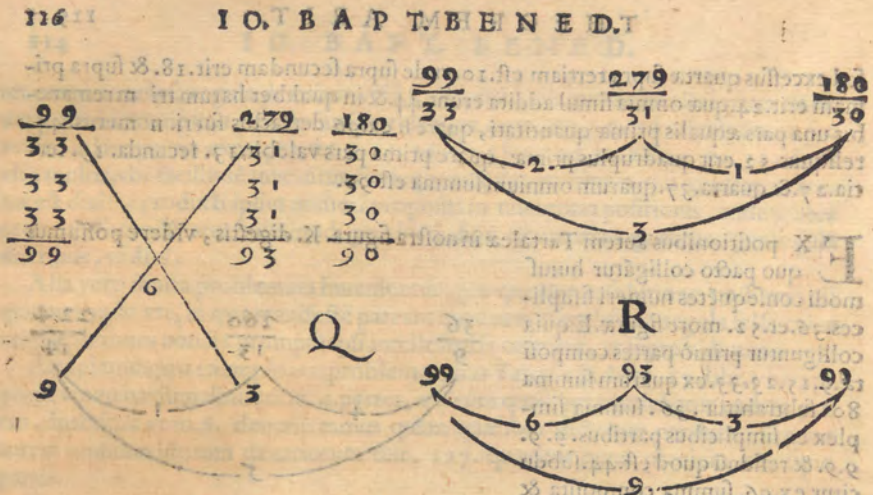
**E**X positionibus autem Tartaleæ in nostra figura. K. digestis, videre possimus quo pacto colligantur huiusmodi consequentes numeri simplices. 36. et. 52. more figura. E. quia colliguntur primò partes compositæ. 9. 15. 23. 33. ex quarum summa 80. subtrahitur. 36. summa simplex ex simplicibus partibus. 9. 9. 9. 9. & residuū quod est. 44. subducitur ex. 96. summa composita & proposita, unde remanet. 52. pro summa simplici, ex numero dato, cuius proportio ad. 13. eadem est quæ. 36. ad. 9. & propterea superfua est secunda positio, quâdo sci mus inuenire tales duos numeros consequentes, ut in hoc exemplo sunt. 36. et. 52. quia ex regula de tribus postea elicitur veritas quæ sita. Idem dico de 33. problemate.



**P**R O quo. 33. problemate accipiuntur positiones primi exempli Tonstalli hoc est. 33. et. 31. ut in figuris hic subiectis. P. Q. facile quis potest videre, ubi in figura. P. videbit numeros compositos, in figura. Q. erunt numeros simplices, à quibus proveniunt rationes per se huiusmodi operationis, in figura. R. videtur meus ordo, & istæ tres figure si miles erunt tribus illis primis. A. B. C. ita quod cum quis illas intellexit, illico etiam istas cognoscat, ubi etiam videbit quam confusè ratiocinetur ij qui ignorant hunc meum ordinem simplicium numerorū, à quibus fluit tota ratio (ut supra dixi) huiusmodi operationis.



P 2 ob Idem



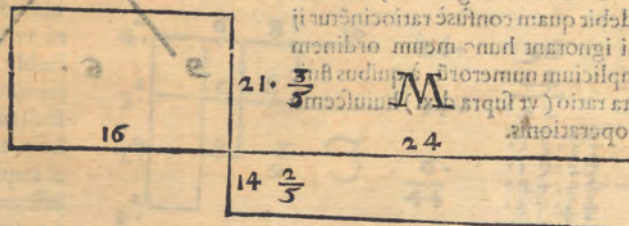
**I**dem etiam potest dici de .15. problemate (sicut de alijs multis.) vbi ponit tres homines habentes. 40. aureos quorum primus habet duas quintas partes secundi, secundus vero quinque octavas tertij, querit nunc quot ducatos habeat vnusquisque. Quis non videt quæso, quod omnes partes erunt. 15. quare cum dixerimus si .15. dat nobis. 2. (pro prima portione primi hominis) quid dabit. 40. vnde nobis proueniet 5. cum tertia parte.

Et de .29. similiter assero, vbi ponit aliū emisse tria frusta panni pro ducatis. 48. quarum secundam habuit pro dimidio precio primæ, tertiam autem pro quarta parte ipsius secundæ, quare omnes partes erunt. 13. quapropter precium tertiæ partæ erit tertiadecima pars ipsius. 48. hoc est. 3. cum .9. tertijs decimis.

Adhuc duo exempla videtur mihi proponere, quorum primum est. 38. eiusdem lib. vbi supponitur operarium quendam velle perficere opus quoddam spacio dierum. 36. tali pacto, quod qualibet die, in qua ipse operaturus sit lucratur solidos. 16. qualibet vero die, in qua nihil agit perdat solidos. 24. Tunc accidit, vt exacto termino perfectoque opere, tantum lucratus sit, quantum perdidit. Queritur nunc quot fuerint dies lucri, quotque perditionis.

Huiusmodi problematis operatio breuissima absque vlla falsa positione ita erit, hoc est diuidendo productum. 36. in .24. per 40. id est per aggregatum ipsius. 24. cum 16. & prouentus erit. 21. cum tribus quintis pro diebus lucri, vnde reliquum ex 36. erit. 14. cum duabus quintis pro diebus perditionis.

Cuius operationis ratio ex se satis patet, cum duo producta, vnus lucri, alterum vero perditionis aequalia esse debeant, vnde ex duodecima & vigesima septimi ex regula de tribus reperiuntur partes ipsius 36. eodem modo

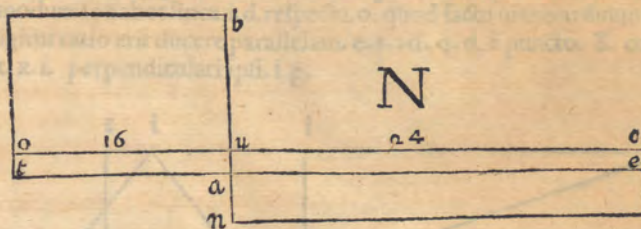


do se inuicem habentes, vt. 24. et. 16. quæ sunt. 21. cum tribus quintis, et. 14. cū duabus quintis, ex quo sequitur, vt quod fit ex. 21. cum tribus quintis, in. 16. quale sit ei quod fit ex. 14. cum duabus quintis, in. 24. & ita reperiuntur duo producta æqualia, vnum lucri, reliquum verò perditionis, vt in figura. M. clarè videtur.

**A**liud verò exemplum est. 39. quod quidem à superiori non differt, nisi quod in fine operationis, operarius dictus lucratus est solidos. 60: queritur nūc vt supra, quot fuerunt dies lucri, & quot perditionis. Hoc etiam absque ulla falsa positione dicto citius potest solui, hoc modo, diuidendo scilicet illos. 60 solidos per. 40. idest per aggregatam. 24. cum. 16. proueniens autem, quod erit. 1. cum dimidio, adde ad latus superius inuentum, hoc est. 21. cum tribus quintis, & summa erit. 23. cum decima parte pro numero dierum lucri, deinde idem prouentum demit ex alio latere superius reperto, 14. cum duabus quintis, & residuum erit. 12. cum nouem decimis, vnde habebis numerum dierum perditionis.

Pro cuius rei speculatione cogitemus in figura: N. duo dicta producta inuicem æqualia, o. b. et. n. c. existente latere. u. c. vt. 24. u. o. ut. 16. b. u. vt. 21. cum tribus quintis, et. u. n. vt. 14. cum duabus quintis. Nunc verò si mente concepta fuerit recta. e. a. t. æquidistans. o. c. ita vt rectangulum. o. e. sit. 60. tunc rectangulum, seu productum b. t. superabit rectangulum seu productum. n. c. per idem. 60. ex communi conceptu, eo quod ex producto. n. c. sublatum est productum. a. c. 24. & producto. o. b. additum est productum. e. a. 16. rectè igitur feci cum diuiserim. 60. per. 40. vnde prouenit mihi. u. a. idest. 1. cum dimidio, quod additum ipsi. b. u. composuit. b. a. & dempto ex. u. n. relinquit. a. n. pro lateribus duorum productorum. b. t. et. n. c.

Sed si idem operator perdidisset. 60. tunc cogitarem parallelam dictam. e. a. r. superius ductam esse ita vt secaret. b. u. & non. u. n. vnde adderet. 24. ipsi producto. n. c. & demeret. 16. à producto. b. o.



**C**IRCA verò talia quæ sita videtur mihi non inutile fore si aliquid notatu dignum aduerterim, hoc est quod sæpe accidere poterit ut casus impossibiles proponantur. Quemadmodum si aliquis diceret, cupio mihi uestimentum conficere ex duobus pannis colore & pretio differentibus, quorum unus exempli gratia.

tia fit albus, rubeus uerò alter, deinde albus fit pretij. 40. solidorum uniuscuiusque cubiti, rubeus uerò precij. 50. uellemque omnes cubitos esse. 8. nec plus nec minus. Vellem etiam soluere solidos. 450. neque minus.

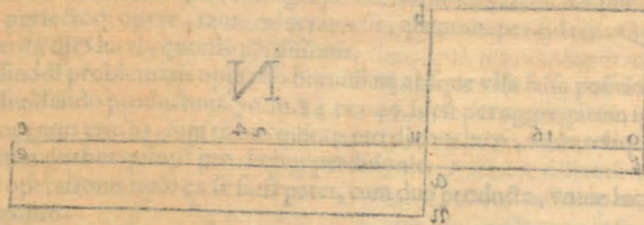
Hic igitur casus impossibilis est, eo quòd. 8. cubiti totius rubei essent precij solidorum. 400. tantummodo, unde ex alio panno albo minoris precij sumere aliquid non possumus.

Idem etiam eueniret si uoluisset soluere solidos. 320. neque plus, eo quòd. 8. cubiti illius minoris precij, hoc est. 40. solidorum, essent ualoris. 320. solidorum tantummodo, quare pro alio panno nullus esset locus. Animaduertendum igitur erit quod numerus possibilis ad soluendum tale quaesitum erit inter. 400. et. 320. & non extra istos terminos, ut unicuique patere potest.

Similiter idem in hoc alio casu accidere poterit, ut si quis diceret. Emi quinque petias panni pro aureis. 55. pretium tamen primae oblitus sum, sed memoria teneo, quòd secunda altioris pretij erat quam ipsa prima per. 4. & tertia preciosior secunda per. 7. et quarta carior tertia per. 9. quinta uerò superabat quartam per. 2.

Hic etiam reperitur impossibilitas quaedam, eo quòd aggregatum omnium harum rerum, dato etiam quòd pro prima nihil solutum esset, superat aureos. 55. quòd quidem nullo pacto fieri potest, ut veri sint supra dicti excelsus, si uerus est numerus totalis aureorum. 55. Nam. 4. cum. 7. faciunt. 11. qui quidem. 11. cum. 9. efficiunt. 20. & hic cum. 2. facit. 22. sed. 22. cum. 20. et. 11. et. 4. dant. 57. qui numerus maior est quam. 55.

**FINIS THEOREM. ARIT.**



**CIRCA** uerò ratio daturus uisum nisi non inuente fore si aliquid notat in  
 Quia aduerit, hoc est quod habet accidere potest in casu impossibilis  
 probonantur. Quae ratio daturus uisum nisi non inuente fore si aliquid notat in  
 fite ex quibus pennis colore & pretio differunt, duorum uisus exemplis gra-

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# DE RATIONIBVS OPERATIONVM PERSPECTIVAE.

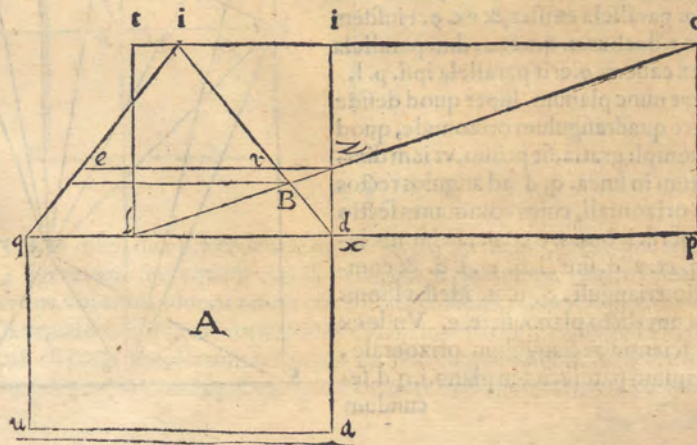
CAP. I.



VM nullus adhuc (quod sciam) veras internasq; causas operationis perspectivæ perfectè docuerit, operæ precium existimaui aliquã de ijs disputationem suscipere.

Multi enim eorũ, qui huiusmodi operationis regulas præscribunt, cum eius effectuum veras causas ignorent, varios diuersosq; errores committunt, vt exempli gratia in subscripta figura superficiali. **A.** volentes degradare (vt dicunt) rectangulum. **q.a.** in triangulo. **i.d.q.** ducunt parallelã ipsi. **q.d.** à puncto. **B.** intersecationis lineæ. **o.l.** cum latere. **i.d.** trianguli, & (idem) indifferenter, eandem quoque à puncto. **Z.** intersecationis ipsius. **o.l.** cum perpendiculari. **x.i.** ducunt. nescientes hunc solum esse verum modum, non item alium, quia si alius, talis esset, hic, verus non existeret, nam si vellent sese excusare, quòd ducendo dictam parallelam à puncto. **B.** hoc fiat præsupponendo planum ipsius. **i.d.q.** versus rectangulum. **q.a.** horizontale inclinatum, secundum angulum. **i.d.q.** hæc excusatio accipienda non esset, quia horum consensu, præsupponendo planum. **i.d.q.** inclinatum, anguli inferioris rectanguli degradati, non tam acuti, quam sunt duo. **i.d.q.** et. **i.d.q.** esse deberent, quod facillè eorum ratione innotescet, quæ de figura corporea. **A.** hîc subscripta mox proponam, præter id, quòd volentes deinde aspicerè quadratum degradatum, oporteret huiusmodi planum respectu oculi ita collocare, quemadmodum se habet linea. **i.d.** respectu. **o.** quod factu nimis arduum esset.

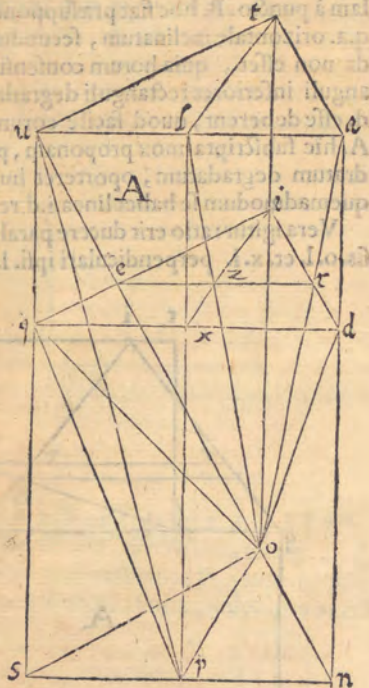
Vera igitur ratio erit ducere parallelam. **e.r.** ad. **q.d.** à puncto. **Z.** communi ipsius. **o.l.** et. **x.i.** perpendiculari ipsi. **l.p.**



Pro

Pro cuius rei speculatione imaginemur in figura corporea. *A*: *q. a.* esse figuram re-  
ctangulam horizontalem; ad degradandam super aliquod planum perpendicularare  
orizonti, & cum eo primum coniunctam in linea. *q. d.* cuius plani triangulum. *i. q. d.*  
pars erit, fit autem oculus respicientis. *o.* cuius altitudo. *o. p.* ab horizonte, qui quide  
conspicit reſtángulum dictum horizontale. *q. a.* in pyramide. *o. q. o. u. o. a. et. o. d.*  
terminata quatuor triangulis. *o. q. u. o. u. a. o. a. d. et. o. d. q.* fit verò primum ita  
collocatus pes. *p.* eius qui respicit, vt linea. *p. l.* perpendicularis ipsi. *u. a.* lateri re-  
ctanguli, medio loco posita fit, inter, *a. n. et. u. s.* Idq; primum nobis erit exem-  
plum.

Imaginemur nunc lineas. *u. q. et. a. d.* indefinitè productas esse, quæ in superficie-  
bus duorum triangulorum. *o. u. q. et. o. a. d.* & reſtánguli horizontalis. *q. a.* ex prima  
vndecimi Euclid. positæ erunt. Imaginemur etiam lineam. *p. s. n.* perpendicula-  
rem ipsi. *p. l.* quæ etiam cum duabus. *u. q. s. et. a. d. n.* ex. 34. primi Euclid. angulos  
rectos constituet, cum ex. 28. duæ. *u. q. s. et. a. d. n.* sint parallelæ ipsi. *p. l.* et. *s. n.* ipsi. *u.*  
*a.* & quia supponitur. *o. p.* perpendicularis plano horizontali, Angulus ergo. *o. p. l.* re-  
ctus erit ex secunda definitione. 11. Euclid. Imaginemur quoque ductas esse  
duas. *o. s. et. o. n.* vnde. *l. p.* ei superficiæ, in qua sunt duæ lineæ. *o. p.* et. *s. n.* ex. 4.  
11. perpendicularis erit, & superficies horizontalis. *a. s.* perpendicularis erit cum dicta  
*o. s. n.* ex. 18. eiusdem lib. vnde ex dicta definitione. *o. s. u. et. o. n. a.* erunt anguli recti  
et. *o. s. et. o. n.* ex communi scientia, in superficiebus duorum triangulorum. *o. u. q. et.*  
*o. a. d.* erunt, si noluerimus cogere aduersariam ad confitendum duas lineas rectas in-  
cludere superficiem, quemadmodum cogere-  
tur facere, si opinaretur duas alias rectas per  
eandem puncta. *o. s. n.* transire, quæ sunt in di-  
ctis superficiebus. Vnde. *o. s. et. o. n.* communes  
erunt sectiones duarum dictarum superficiu  
cum superficie. *o. s. n.* Imaginemur nunc has  
duas superficies. *o. u. et. o. s.* quarum comuni-  
nis sectio sit. *o. t.* (quæ erit linea recta ex. 3. lib.  
11.) quæ erunt perpendiculares superficiæ. *o. s.*  
ex. 4. et. 14. iam dictis. & ex. 19. eiusdem  
*o. t.* perpendicularis eidem superfici. *o. s. n.*  
erit, & ex. 6. eiusdem hæc linea. *o. t.* duabus. *u.*  
*q. s. et. a. d. n.* parallela existet, & ex. 9. eiusdem  
hæc linea. *o. t.* duabus. *u. q. s. et. a. d. n.* parallela  
existet, & ex eadem. 9. erit parallela ipsi. *p. l.*  
Imaginemur nunc planum, super quod deside-  
remus videre quadrangulum horizontale, quod  
planum, exempli gratia, sit primo, vt iam dixi-  
mus, locatum in linea. *q. d.* ad angulos rectos  
cum plano horizontali, cuius communes sectio-  
nes cum superficiebus. *s. t. et. n. t.* visionis la-  
terum. *u. q. et. a. d.* sint. *i. q. et. i. d.* & com-  
munis sectio trianguli. *o. u. a.* idest visionis  
lateris. *a. u.* cum dicto plano, sit. *r. e.* Vnde ex  
communi scientia reſtángulum horizontale,  
oculo. *o.* seipsum patefaciet in plano. *i. q. d.* se-  
cundum



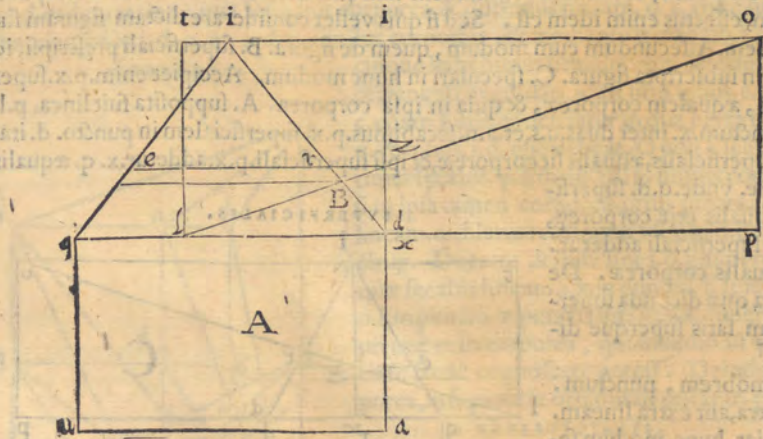


cundum figuram quadrilateram. q. d. r. e. Communis autem sectio superficiæ. p. r. cum dicto plano, fit. i. x. quæ. i. x. perpendicularis erit. s. a. superficiæ horizontali ex 19. lib. 11. quia. p. r. est etiam horizonti perpendicularis ex. 18. eiusdem, cum. o. p. eisdem perpendicularis existat. Vnde. i. x. erit altitudo trianguli. i. q. d. & æqualis ipsi. o. p. ex. 34. primi. Sit deinde. o. l. cõmunis sectio superficiæ triangularis. o. a. u. cõ superficiæ. p. t. quæ. o. l. secando lineam. e. r. in puncto. Z. nobis ostendet quantum distare seu eminentes esse debeat latus. e. r. in plano ab. q. d. medio ipsius. z. x. Et quia præsupposuimus. p. l. in eodem medio, inter. u. s. et. a. n. ideo. x. q. æqualis erit. x. d. & ex. 4. lib. primi. i. q. ipsi. i. d. et. e. r. parallela ipsi. q. d. ex. 6. lib. 11. cum ipsa quoque sit perpendicularis superficiæ. p. t. ex. 19. eiusdem. Hucusque igitur in figura corporea. A. prodeunt in lucem omnes causæ effectuum figuræ superficialis. A. id est unde fiat, vt in ipsa figura superficiali, triangulum. o. p. l. tale consurgat, & quid significet. o. et. o. p. et. p. l. et. o. l. & quam ob causam tale quoque formetur triangulum. i. q. d. atque in tantam altitudinem, quantum obrinet. o. p. & quid sint latera. i. q. et. i. d. & quare erigatur. x. i. parallela ipsi. p. o. ab eadem. p. o. tanto spatio distans, & qua ratione producat a puncto. Z. ipsa. Z. r. e. parallela ipsi. q. d.

Nunc obseruandum est, quòd si planum ipsius. i. q. d. in figura corporea aliquantulum inclinatum esset horizontem versus, anguli. i. q. d. et. i. d. q. maiores existerent, quàm cum idem est ipsi horizonti perpendicularare, quemadmodum clarè demonstratum fuit in. 39. primi Vitelionis.

Non igitur rectè fit si in figura superficiali ducatur a puncto. B. parallela ipsi. q. d. absque maiori a pertura angulorum. i. q. d. et. i. d. q.

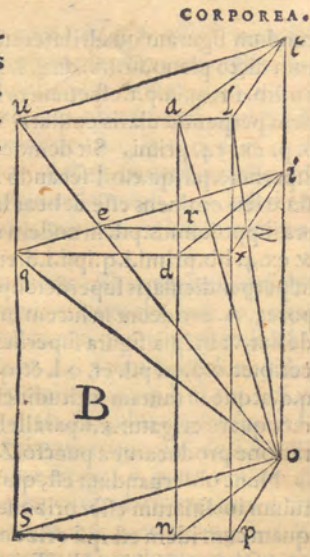
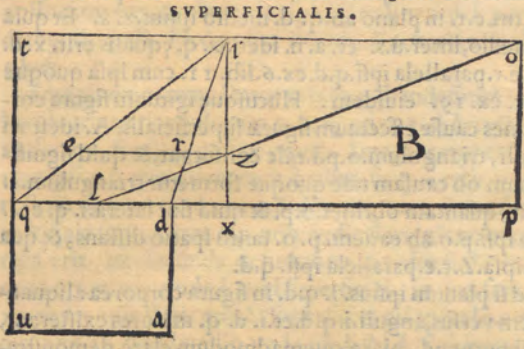
SVPERFICIALIS.



C A P. I I.

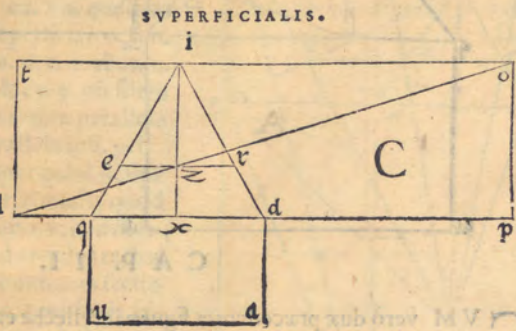
**C**VM verò duæ præcedentes figuræ intellectæ erunt, facilè quoque erit intelligere duas subsequentes. B. B. in corporea quarum. p. l. extra lineas. u. s. et. a. n. reperitur, vbi enim aduertendum erit oportere sumere semper. p. x. figuræ superficialis æqualem ei, quæ est corporeæ, & eidem superficiali, adiungere. x. d. æqualem ei, quæ est corporeæ, & composito. p. d. ex dictis duabus lineis, in figura superficiali, addere. d. q. æqualem ei, quæ est figuræ corporeæ, deinde accipere punctum. l. in superficiali

perfaciali, ita distans à. p. vt in corporea reperitur. Postea. x. i. erigetur æqualis lineæ. p. o. & suis terminis concludetur triangulum. i. q. d. & id quod remanet. Vnde si longius diseredo progrediare, patebit ex. 4. primi. i. d. superficiale, futuram æqualem. i. d. corporeæ. Idem dico de. i. q. & de reliquis.



C A P. I I I.

**M**ODVS hic, proprius est, & vniuersalis, licet in figura superficiali. A. superior posita, secundum communem antiquorum consuetudinem exemplum dederim, effectus enim idem est. Sed si quis vellet considerare dictam figuram superficiale. A. secundum eum modum, quem de figura. B. superficiali prescripti, id poterit in subscripta figura. C. speculari in hunc modum. Accipiet enim. p. x. superficiale, æqualem corporeæ, & quia in ipsa corporea. A. supposita fuit linea. p. l. idest punctum. x. inter duas. u. s. et. a. n. secabimus. p. x. superficiale in puncto. d. ita, vt. d. x. superficiale, æqualis sit corporeæ, & ipsi superficiali. p. x. addetur. x. q. æqualis corporeæ. vnde. q. d. superficiale æqualis erit corporeæ, et. p. x. superficiali addetur. x. l. æqualis corporeæ. De ijs postea quæ dicenda superfunt, iam satis superque diximus.



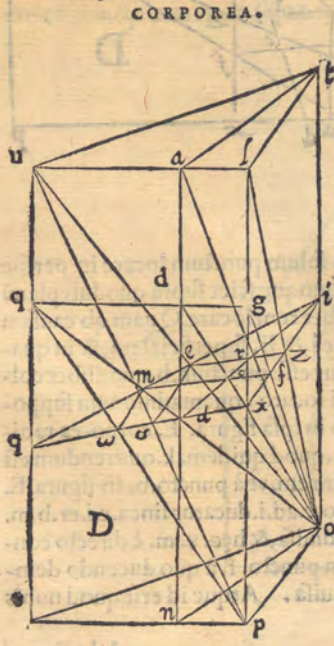
Quamobrem, punctum. x. aut intra, aut extra lineam. q. d. veniat, hunc modum sequentes, in errorem non incidemus, imò efficietur quadrilaterum. q. r. superficiale, simile, & æquale corporeo.

C A P. I I I I.

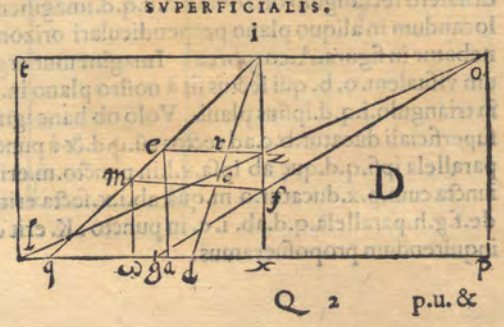
**P**VNCTVM verò. i. (quod verum est punctum perspectiuæ, vt practici dicere solent) quid sit, hac via & ratione sub nostram cognitionem cadit: quòd nihil aliud

aliud est, quàm punctum, varijs sectionibus commune, & huiusmodi punctum, oculi non est, quemadmodum multi Pictores, Sculptores, Architecti, & Perspectivi ignari, ipsum punctum, oculum appellando, falso crediderunt, quasi punctum, i. perspectivæ oculus esset.

In supradictis igitur figuris manifeste elucescit causa diminutionis obiectorum, & altitudinis trianguli æqualis ei, quæ est oculi à plano orizzontali, ut etiam distantie. p. l. p. x. & cuiusvis tandem rei. Sed ut huius effectus scientia magis in univrsum pareretur. Volo duas hic subscriptas figuras. D. corpoream, & D. superficialem à vobis considerari, in quarum corporea, linea. p. l. sit extra duas. u. s. et. a. n. ut in figura. B. locata, ita tamen ut planum trianguli. i. q. d. disjunctum sit à rectangulo superficiali, id est, ut separatim existat à linea. q. d. latere ipsius rectanguli, & sit etiam obliquum, respectu ipsius rectanguli, id est ut communis sectio dicti plani cum superficie a. s. orizzontalis ipsi. u. a. parallela non sit, sed sit obliqua, si tamen idem planum perpendicularare dictæ superficiæ orizzontali. a. s. erit: & dicta communis sectio exprimitur characteribus. q. d. x. nunc in figura corporea habebimus figuram. e. r. c. m. in plano, quod visualem pyramidem secat, medio cuius figuræ. e. r. c. m. oculi positus in. o. rectangulum orizzontale conspiciat. Volentes vero nunc in figura. D. superficiali eam describere, faciem us. p. x. superficialem, æqualem corporeæ, eiq. addemus. x. l. æqualem corporeæ, aut sumemus. p. l. eidem corporeæ æqualem, quam secabimus in puncto. x. eodem planè modo, quo corporea reperitur diuisa; erigemus deinde. p. o. et. x. i. æquales corporeis. Secabimus deinde. x. q. æqualem corporeæ, & ducemus. q. i. et. l. o. vnde habebimus triangulos. o. p. l. et. i. x. q. similes & æquales corporeis ex. 4. primi Eucli. Secabimus deinde. q. x. in puncto. d. eadem ratione, qua secta fuit corporea, & ducemus lineam. d. i. vnde habebimus triangulos. i. d. q. et. i. d. x. similes corporeis, & mediante triangulo. i. q. d. huiusque habebimus situs duorum laterum figure rectanguli degradati, id est situs ipsius. e. m. et. r. c. etiam si adhuc nesciatur in qua parte ipsius. i. q. & ipsius. i. d. esse debeant. Quod si scire volumus secabim. p. l. in puncto. g. similis corporeæ, si in ipsa tamen corporea prius protraxerimus lineam. q. d. latus rectanguli vsque ad. p. l. in puncto. g. Ducetur deinde linea. o. g. superficialis, quæ secabit lineam. i. x. in puncto. f. linea vero. o. l. in puncto. z. punctis sitis in. i. x. superficiali, præcisè ut in corporea, quemadmodum quilibet ex se facile cognoscere potest. Deinde in corporea, in superficie orizzontali ducatur. p. q. et.

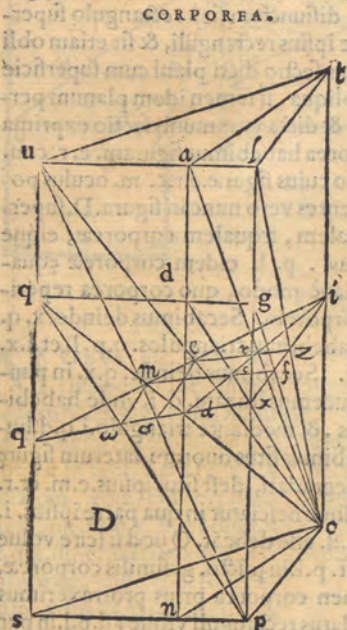


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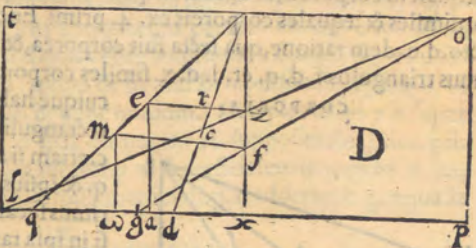
Q 2 p. u. &

p.u.& imaginemur.o.q.in superficie.t.s.vnde trianguli.o.p.q.et.o.p.u.erunt perpen-  
 diculares orizonti ex. 18. lib. 11. et. m. et. z. e. commutes sectiones dictorum duorum  
 triangulorum cum plano trianguli. i. q. x. ipsi quoque plano ex. 19. eiusdem lib. erunt  
 perpendiculares. Nunc autem secetur. q. x. superficialis in punctis. m. et. z. eadem ra-  
 tione; qua corporea secta fuit à duabus. p. q. et. p. u. à quibus punctis. m. et. z. superficia  
 libus ductæ sint duæ. m. et. z. e. perpendiculares vsque ad latus. i. q. in punctis. m. et.  
 e. que situm habebunt in. i. q. superficiali præcisè, vt in corporea, ex. 26. primi, du-  
 cendo deinde in superficiali duas. m. f. et. e. Z. ea æquales erunt corporeis ex. 4. pri-  
 mi; & sic anguli. i. e. z. et. i. m. f. & eę duę lineę. e. z. et. m. f. sectę erunt à linea. i. d. in duobus  
 punctis. r. et. c. vnde. e. r. et. m. c. æquales erunt corporeis ex. 26. primi, sed ita quo-  
 que se habent duę. e. m. et. r. c. si verum est qđ dif-  
 ferentię rerum æqualium sint adinuicem etiam  
 æquales. Hac ratione igitur habebimus figu-  
 ram quadrilateram. m. e. r. c. superficialem om-  
 ninò similem, & æqualem corporea. Istamen  
 modus prolixus est, & arduus, quam ob cau-  
 sam neque ego vnquam cũ vsui accommoda-  
 darem, neque alijs, vt eodem vterentur sua-  
 derem.



CORPORA.  
 bus punctis. r. et. c. vnde. e. r. et. m. c. æquales  
 erunt corporeis ex. 26. primi, sed ita quo-  
 que se habent duę. e. m. et. r. c. si verum est qđ dif-  
 ferentię rerum æqualium sint adinuicem etiam  
 æquales. Hac ratione igitur habebimus figu-  
 ram quadrilateram. m. e. r. c. superficialem om-  
 ninò similem, & æqualem corporea. Istamen  
 modus prolixus est, & arduus, quam ob cau-  
 sam neque ego vnquam cũ vsui accommoda-  
 darem, neque alijs, vt eodem vterentur sua-  
 derem.

SUPERFICIALIS.



C A P. V.

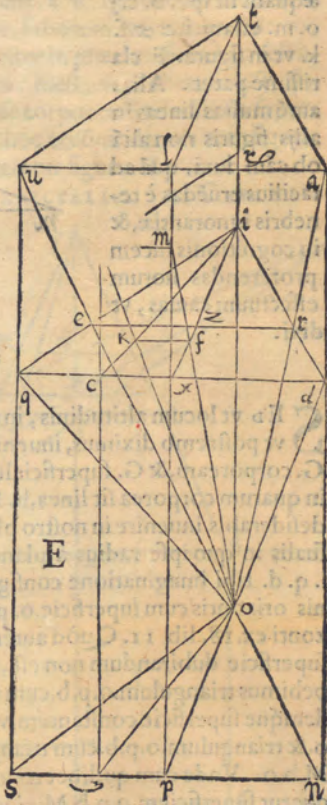
**E**ST igitur sciendum, quòd qui sciuerit vnum solum punctum locare in perspe-  
 ctiva, eo modo quem nunc proponam, facìle quoque sciet supra quoduis planũ  
 (quod tamen sit perpendiculare orizonti) quamlibet rem locare. Quam ob causam  
 imaginemur hic subscriptas duas figuras. E. corporea, & E. superficialem, & in qua-  
 drilatero rectangulo orizontali. a. u. q. d. imaginemur esse punctum. b. quodlibet col-  
 locandum in aliquo plano perpendiculari orizonti locato, quemadmodum suppo-  
 nebatur in figura. A. corporea. Imaginemur ergo in ipsa figura. E. corporea radi-  
 um visuale. o. b. qui sectus sit à nostro plano in. k. quod quidem. k. quarendum est  
 in triangulo. i. q. d. ipsius plani. Volo ob hanc igitur rem, vt à puncto. b. in figura. E.  
 superficiali ducatur. b. c. ad rectos cũ. q. d. & à puncto. c. ad. i. ducatur linea. c. i. et. b. m.  
 parallela ipsi. q. d. que ab ipsa. x. l. in puncto. m. erit diuisa, & hec. x. m. è directo con-  
 iuncta cum. p. x. ducatur. o. m. que ab. i. x. secta erit in puncto. f. à quo ducendo dein-  
 de. f. g. h. parallela. q. d. ab. i. c. in puncto. K. erit diuisa. Atque id erit quod nobis  
 inquirendum proposueramus.

Ad cuius

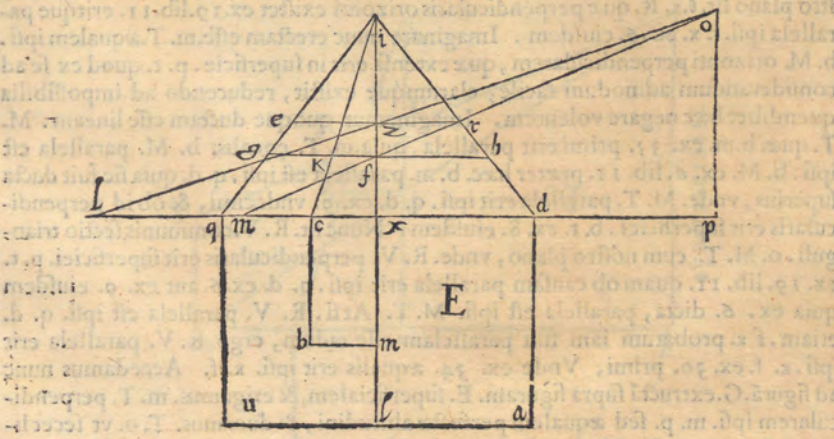
DE PERSPECT.

Ad cuius rei speculationē, imaginatione concipiamus lineam b.c. corpoream, protractam esse usque ad y. lineam s.n. & imaginatione sit comprehensa linea y.o. et b. & parallela eidem, ideo ob rationes iam dictas de figura. A. hæ tres lineæ o.y.i.c. et b. b. simul cum linea o. b. erunt in vna eademque superficie plana, quam characteribus .y. & notemus .et. i. c. eius erit sectio communis cum plano, in quo quæritur punctum, et f. k. ipsius plani cum triangulo o.b.m. erit sectio communis, & parallela ipsi .q. d. ex 6. lib. 11. quia k. f. perpendicularis est superfici. p. t. ex 19. eiusdem cum triangulo o. b. m. eidem superfici. p. t. ex 18. eiusdem perpendicularis existat. Vnde perspicue patet ratio quare protracta sit parallela .b. c. et quare ducta sit .i. c. et coniuncta .x. m. cum .x. p. directe, & quare ducta sit .o. m. et .f. k. Laudo igitur ut semper præsupponatur .p. x. perpendicularis basi ipsius plani & præsupponatur, (ut rem totam vno verbo complectar) superficies .p. t. perpendicularis plano, & orizonti. Quod reliquum est, necessarium non est, nisi ad specularandum. Necessariæ ergo non sunt aliæ lineæ, quam .p. x. p. o. x. i. b. c. et .x. m. è directo coniuncta cum .p. x. (quæ .x. m. coniuncta æqualis

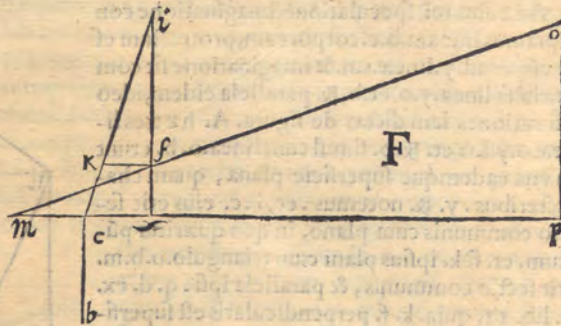
CORPOREA.



SUPERFICIALIS.



æqualis sit ipsi. b. c.)  
o. m. etiam. i. c. et. f.  
k. vt in figurâ. F. cla  
rissimè pater. Alias  
autè multas lineas in  
alijs figuris non aliâ  
ob cãm duxi, quâ ad  
facilius eruẽdas è te  
nebris ignorantia, &  
in cognitionis lucem  
proferendas horum  
effectuum causas, vt  
dixi.

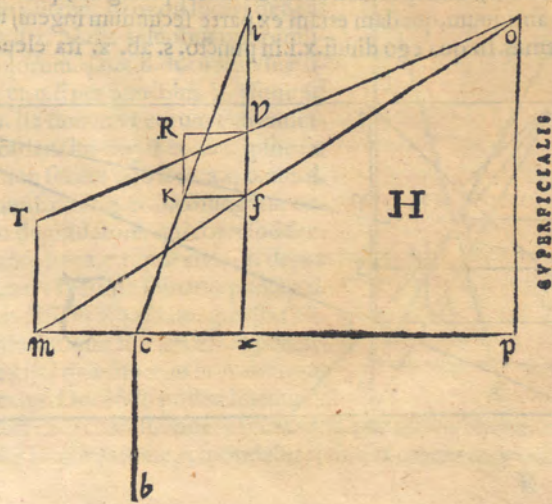
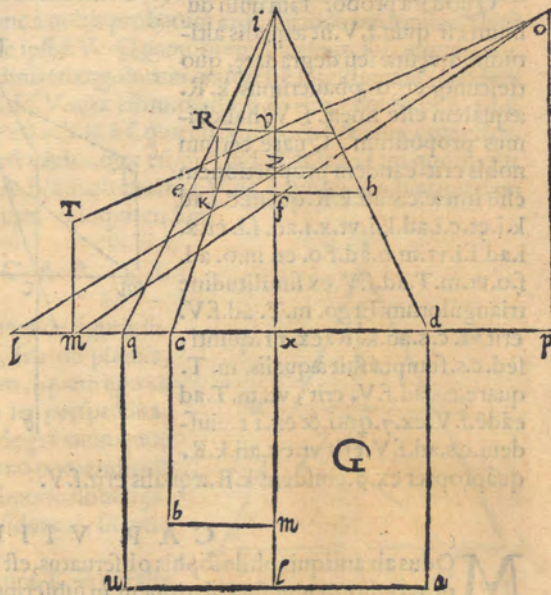
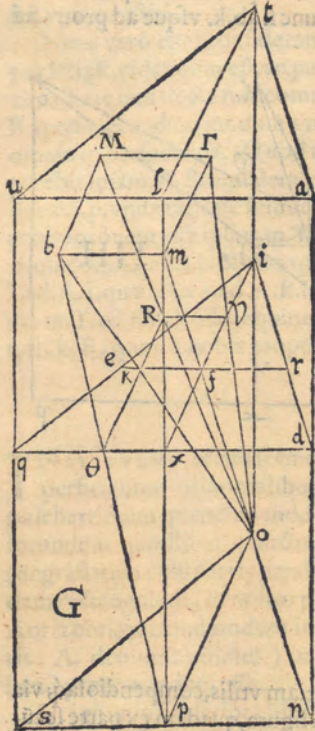


## C A P. V I.

**S**ED vt locum altitudinis, in nostro plano perpendiculari orizonti, & ita locatū, vt postremo diximus, inueniamus; duas hęc subscriptas figuras considerabimus. G. corpoream, & G. superficialem, similes duabus. E. E. proximè præcedentibus, in quarum corporea sit linea, b. M. altitudinis perpendicularis orizonti. Quare si desiderabis inuenire in nostro plano situm puncti. M. idest punctum radij. o. M. visualis in quo ipse radius à plano est diuisus, quod sit. R. quamuis extra triangulū i. q. d. tibi imaginatione confige ductam esse lineam. p. b. quæ erit sectio communis orizontis cum superficie. o. p. b. M. quæ superficies erit perpendicularis ipsi orizonti ex. 18. lib. 11. Quod autem non minus. o. p. quàm. M. b. fit in vna eademque superficie dubitandum non est, quia si imaginabimur ductam esse lineam. p. M. habebimus triangulum. o. p. b. cum triangulo. M. b. p. communibus partibus in vna eademque superficie constantem, vt triangulum quoque. o. p. M. cum triangulo. M. b. o. & triangulum. o. p. b. cum triangulo. o. p. M. & triangulum. M. b. p. cum triangulo. M. b. o. Vnde cum quilibet triangulus in vnica tantum superficie sit ex. 2. lib. 11. sequetur superficiem. o. p. b. M. planam esse, & vnicam, cuius communis sectio cum nostro plano sit. b. k. R. quæ perpendicularis orizonti existet ex. 19. lib. 11. eritque parallela ipsi. i. x. ex. 6. eiusdem. Imaginare nunc erectam esse. m. T. æqualem ipsi. b. M. orizonti perpendicularem, quæ extensa erit in superficie. p. t. quod ex se ad considerandum admodum facilè, clarumque existit, reducendo ad impossibilia quemlibet hęc negare volentem. Imaginemur quoque ductam esse lineam. M. T. quæ. b. m. ex. 33. primi erit parallela, quia. m. T. æqualis. b. M. parallela est ipsi. b. M. ex. 6. lib. 11. præter hęc. b. m. parallela est ipsi. q. d. quia sic fuit ducta superius, vnde. M. T. parallela erit ipsi. q. d. ex. 9. vndecimi, & ob id perpendicularis erit superfici. b. t. ex. 8. eiusdem. Nunc sit. R. V. communis sectio trianguli. o. M. T. cum nostro plano, vnde. R. V. perpendicularis erit superfici. p. t. ex. 19. lib. 11. quam ob causam parallela erit ipsi. q. d. ex. 6. aut ex. 9. eiusdem quia ex. 6. dicta, parallela est ipsi. M. T. At si. R. V. parallela est ipsi. q. d. etiam. f. k. probatum iam fuit parallelam esse eidem, ergo. R. V. parallela erit ipsi. k. f. ex. 30. primi, Vnde ex. 34. æqualis erit ipsi. k. f. Accedamus nunc ad figurâ. G. extractâ supra figuram. E. superficialem, & erigamus. m. T. perpendicularem ipsi. m. p. sed æqualem perfectæ altitudini, & ducamus. T. o. vt fecerit lineam. i. x. in puncto. V. ab ipso ducentes. V. R. parallelam ipsi. q. d. ducendo de  
inde

inde . k . R . parallelam ipsi . i . a . habebimus altitudinem . k . R . quam quærebamus in nostro plano . Quod cum sui natura clarum euadat , laborem ratiocinandi de eo ,  
**CORPOREA .** cui libet vel mediocriter in præclarissima hac scientia erudito relinquo . id est , ut probetur . k . R . superficialē , æqualē esse corporeæ . Sed tollēdo superficialitē linearum , & hoc accommodantes ut in figura .  
**F .** diligenter consideretur figura . H .

SYPERFICIALIS . CAP . VII .

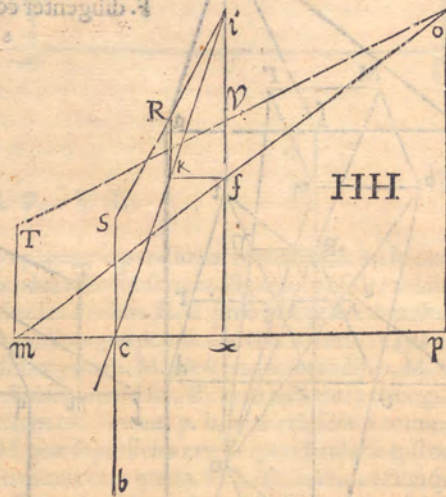


SYPERFICIALIS

C A P. V I I.

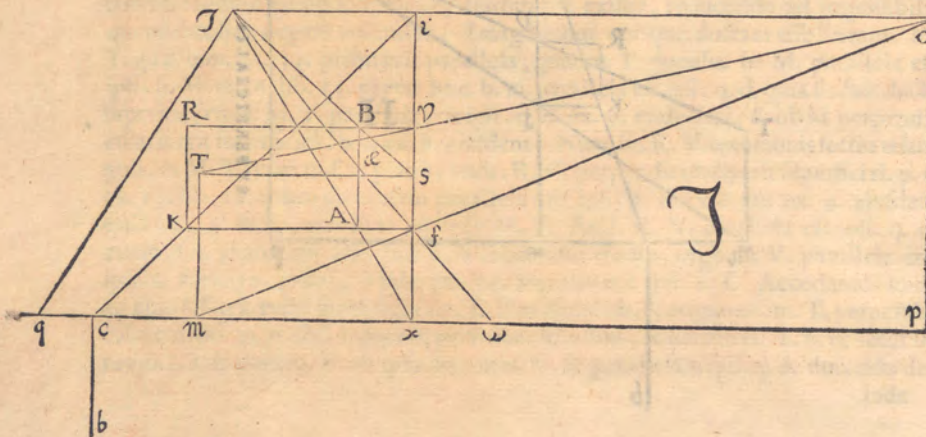
**A**liam tamen inveni viam breuiorem vt in figura. H. H. in qua sit punctus. b. perfecti, & k. degradati plani. Nunc ducatur. b. c. s. ad rectos cum. p. m. indefinite, quæ quidem abscindatur in puncto. s. ita quòd. c. s. æqualis sit altitudini perfectæ, deinde coniungatur recta. s. cum. i. Tunc si ab. k. vsque ad protractã i. s. ducta fuerit. k. R. parallela lineæ. c. s. hæc. R. k. erit altitudo quæ sita seu degradata.

Quod ita probo. Iam nulli dubium est quin. f. V. sit æqualis altitudini quæ sita seu degradate, quotiescunq; ergo pbauerimus. k. R. æqualem esse lineæ. f. V. habebimus propositum. Quare certum nobis erit eandem proportionem esse lineæ. c. s. ad. k. R. quam. c. i. ad. k. i. et. c. i. ad. k. i. vt. x. i. ad. f. i. et. x. i. ad. f. i. vt. m. o. ad. f. o. et. m. o. ad. f. o. vt. m. T. ad. f. V. ex similitudine triangulorum. Ergo. m. T. ad. f. V. erit vt. c. s. ad. k. R. ex. I I. quinti, sed. c. s. sumpta fuit æqualis. m. T. quare. c. s. ad. f. V. erit, vt. m. T. ad. eadẽ. f. V. ex. 7. qnti, & ex. I I. eiusdem. c. s. ad. f. V. erit vt. c. s. ad. k. R. quapropter ex. 9. eiusdem. k. R. æqualis erit. f. V.



C A P. V I I I.

**M**odus ab antiquis philosophis obseruatus, est etiam vtilis, compendiosaq; via progreditur, cuius speculationem, in subscripta figura, quadam ex parte secundum morem antiquum, quadam etiam ex parte secundum ingenij mei vires cõstructa, cognoscemus. In qua ego diuisi. x. i. in puncto. s. ab. x. ita eleuato, quanta est vera





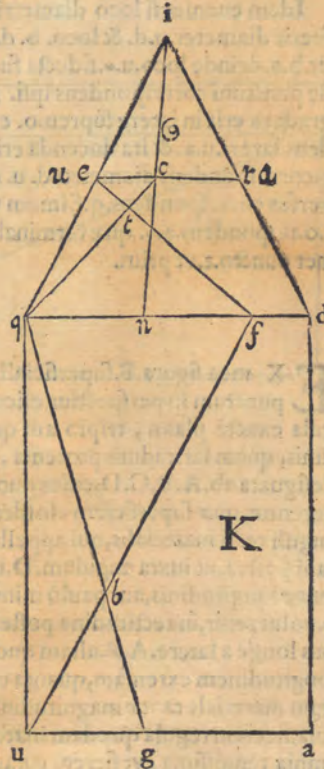
vera altitudo ipsius. M. T. et I. s. duxi supponendo esse. I. punctu perspetiua secundu antiquos, id est angulum supremum trianguli antiquorum a puncto que. k. meo duxi k. f. parallelam ipsi. c. m. p. vsque ad. i. x. in puncto. f. & a puncto a communi ipsi. k. f. et. i. x. vsque ad. I. s. duxi quoque. A. B. parallelam ipsi. i. x. atque hac omnia ex more antiquo praestiti.

Nunc vero eum considerans modum, quem ego de figuris. G. H. antecedentibus praescrpsi, videndum est, an punctum. B. tribus lineis. A. B. I. s. et. R. V. quarum hec vltima a me iam ducta fuit, commune existat, id est vtrum. A. B. aequalis existat ipsi. K. R. quam secundum modum a me adinuentum, reuera scimus esse desideratam altitudinem in perspetiua. Quod tunc a nobis probatum erit, quando rationibus clare patebit ipsam. A. B. aequalem esse ipsi. f. V. Quamobrem ducamus. I. f. vsque ad. o. lineam. c. p. vnde ratione similitudinis triangulorum manifeste intelligemus, eandem proportionem esse ipsius. m. T. ad. f. V. quae est. m. o. ad. f. o. & eius, quae est. m. o. ad. f. o. quae est. o. I. ad. f. I. & eius, quae est. o. I. ad. f. I. quae est. x. I. ad. A. I. & eius, quae est. x. I. ad. A. I. quae est. x. s. ad. A. B. id est vt eius, quae est. m. T. ad. A. B. sed idem quoque erat de. m. T. ad. f. V. Vnde sequitur. A. B. aequalem esse. f. V. ex. 9. quinti Eucli. atque etiam ipsi. k. R. quod a nobis propositum est inquirendum.

CAP. IX.

INSTITVENS etiam sermonem de figuris superficialibus orizontalibus, seu de plantis, pulcherrimum quendam modum, quem ego ad locandum quodlibet punctum in perspetiua, (degradatum cum fuerit parallelogramum quoddam rectangulum, in nostro plano perpendiculari orizonti, quemadmodum in superioribus figuris. A. demonstraui) consideraui, silentio haud praetereundum esse.

Sit igitur in subscripta hic figura. K. in parallelogramo perfecto punctum. b. quod locari debeat in degradato. e. q. d. r. Nunc a duobus quorumlibet quatuor angulorum. q. u. a. d. ducuntur duae lineae occultae. q. g. et. u. f. per punctum. b. vsque ad latera. q. d. et. u. a. ita tamen vt eorum extremitates. g. et. f. intus cadant inter. q. d. et. u. a. ipsorum laterum, id est vt non fecent duo latera. q. u. aut. d. a. Deinde punctum. f. inter. q. et. d. coniungatur occulte cum angulo degradato. e. qui correspondet. u. perfecti, mediante linea. e. f. quae erit. u. f. degradata in nostro plano. Deinde sumatur punctum. n. in linea. q. d. tam distans a. q. quam. g. distat ab. u. ducaturque linea. i. n. quae lineam. e. r. in puncto. c. diuidet, quod ex ijs, quae superius iam diximus ad ipsum. g. referetur. Ducendo postea lineam occultam. q. c. patebit eam correspondere lineae. q. g. quae secans lineam. e. f. in puncto. t. hoc, communi scientiae ratione, respondebit ipsi. b. vt omnes cognoscent.

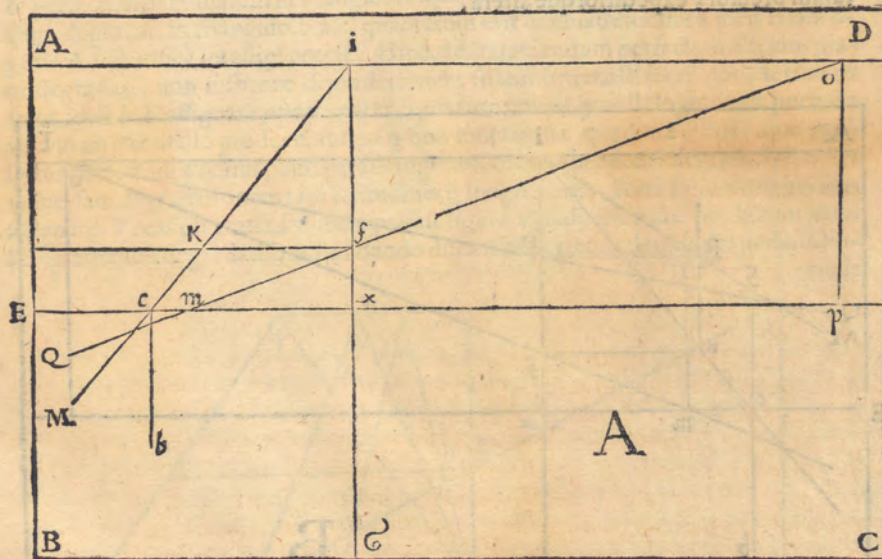


R Sed



DE PERSPECT.

inter. o. i. et. p. E. intercedebat, altitudinem oculi ab horizonte significabat. Signaui etiam lineam. i. G. perpendicularem lineæ. E. p. cui affigi possët non nihil chartæ quotiescunq; volebam in perspectiua aliquid delineare. Quod cum facere desiderabam, ponebam perfectum optimè affixum in quadrangulo. E. G. & in quadrangulo. E. i. aliquid folium papyri affigebam. Ponamus nunc, me voluisse constituere punctū. b. sumebā angulū rectū materialē, seu normā, & eius vnum latus, iuxta latus. B. c. ponebam, atque aliud per punctum. b. transire faciebam, & vbi hoc latus lineā E. p. diuidebat, punctum. c. signabam per quod efficiebatur, vt regula. i. M. transfirer, quiesceretq; aliquantulum aliquo modo in huiusmodi situ, opera deinde circini interuallum. b. c. sumebam, & in. p. E. à puncto. x. versus. E. punctum. m. signabā: per quod faciebam, vt transfirer regula. o. Q. quæ lineam. x. i. in puncto. f. diuidebat. Angulum deinde rectum materialem accipiebam, cuius vnum latus. A. B. ponebam, aliud verò per punctum. f. transibat, quod quidem latus regulam. i. M. in puncto. k. (quod statim super folio papyri signabatur) intersecabat, atque hoc erat punctū, quod quærebam, puncto. b. correspondens. Huiusmodi effectus rationes ab ijs, quæ superius dixi eliciuntur. Atque hæc ad bases rerum, vt in subscripta figura elucescit, spectabant.



CAP. XI.

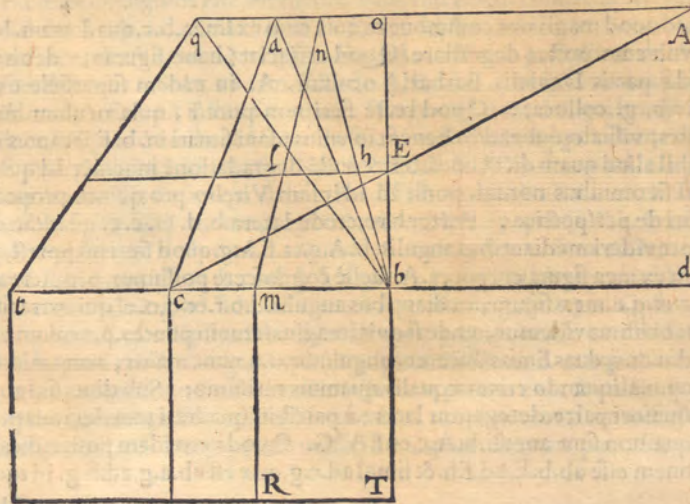
**A**D degradandas deinde altitudines, vsus sum mea figura tam. H. quã èt. H. H. vt mihi sese offerebat occasio. In primis ratione modi figuræ. H. curabã, vt



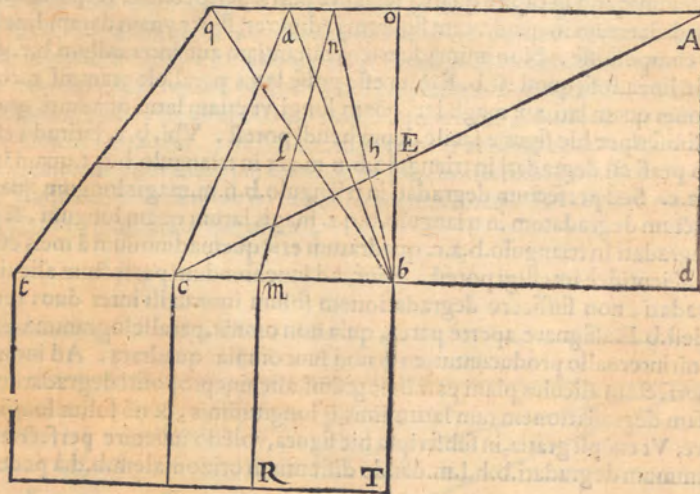
JACOBO SOLDATO MEDIOLANENSI  
 Serenissimi Ducis Sabaudia Architecto peritissimo.

CAP. VII.

**S**UPERIORIBVS diebus non diu postquam de perspectiuis inter nos sermonem habuimus, dum animus totus adhuc in his esset. Illud in mentem venit quod eximius ille vir, & profundissima doctrina, nec vnquam satis laudatus Daniel Barbarus se accepisse profiteretur à Ioanne Cusino Parisiense. Nec parum mirabar peritissimum illum Cusinum, quod in capite quarto secunda partis perspectiua, vt quodpiam planum quadrilaterum in quadratam figuram redigeret, super vnam datam lineam quadratam composuisse. Non animaduertens distantiam aut interuallum. b.c. degradatum in linea. b.f. (quod est. b. E.) ita esse posse latus parallelogrammi rectanguli magis longi quam lati, aut magis lati quam longi, vt etiam latus quadrati, quod beneficio subscriptae hic figurae facile deprehendi potest. Vbi. b. c. latitudo esse potest, tam perfecti degradati in triangulo. b.n.m. aut in triangulo. b.q.t. quam in triangulo. b.a.c. Sed perfectum degradati in triangulo. b.n.m. magis longum quam latum & perfectum degradatum in triangulo. b.q.t. magis latum quam longum, & perfectum degradati in triangulo. b.a.c. quadratum erit quemadmodum à meis etiam figuris. A. scientificè intelligi potest. Hinc, ad inueniendum perfectum alicuius plani degradati, non sufficere degradationem solum interualli inter duos terminos solos, idest. b. E. assignare, aperte patet, quia non omnia parallelogramma perfecta ab vno tm interuallo producuntur, eo qd non sunt omnia quadrata. Ad inquirendum igitur perfectum alicuius plani parallelogrammi, alicuius propositi degradati, oportet vniuersam degradationem tam latitudinis, q̄ longitudinis, & nō solius longitudinis assignare; Vt exēpli gratia, in subscripta hic figura, volēdo inuenire perfectum parallelogrammum degradati. b.h.l.m. dando distantiam horizontalem. b.d. à pede. d. hominis



minis vsque ad plani situm in quo degradatio facta sit : statim altitudo. A. oculi à pe  
 de , quæ tanta semper esse debet quanta est altitudo trianguli. b.n.m. qui clauditur  
 protrahendo. m. l. et. b. h. vsque ad concursum in. n. in lucem prodibit . Oportet  
 deinde erigere lineam. b.o. perpendicularem lineæ. d.b.m. & vsque ad eandem pro  
 ducere lineam. l.h. in puncto. E. et à puncto. A. per. E. vsque ad. c. ipsius. d.b.m. produ  
 ctæ ducere. A. E. c. atque deinde protrahere lineam. o. b. vsque ad. T. ita vt. b. T. equa  
 lis sit ipsi. b. c. & ad ipsam à puncto. m. ducere parallelam. m. R. & à puncto. T. ducere.  
 T. R. parallelam ipsi. b. m. Vnde ex. 34. primi Eucli. m. R. æqualis erit ipsi. b. T. et. R.  
 T. ipsi. m. b. & anguli in rectos euadent, atque hoc parallelogrammum rectangulum  
 erit verum perfectum degradati. b. m. l. h. ob rationes à me circa figurã. A. adductas .



Sed est hic quod magis nos commoueat, quia cum ex linea. b. c. quadratum. b. g. pro  
 duxerit, vult eum postea degradare. Quod vt faciat ( hanc figuram videbis in cap.  
 4. secundæ partis Danielis Barbari ) oculum. A. in eadem superficie extensa,  
 quadrati. b. g. collocat. Quod rectè fieri non potest, quia oculum hoc mo  
 do locantes, visualesque radios beneficio vnus plani situati in. b. f. secantes in ipso  
 plano, nihil aliud quam dictã lineã. b. f. & nullã degradationẽ inueniẽt. Id quod, & si  
 natura sua sit omnibus notum, ponit tũ id ipsum Vitelio pro quinta propositione  
 quarti libri de perspectiua. Præter hæc, credit latera. b. d. et. c. e. quadrati degra  
 dati semper videri mediantibus angulis. b. A. c. et. f. A. g. quod fieri nõ potest, quem  
 admodum ex mea figura corporea. A. facillè cognoscere possumus, propterea quòd  
 latera. d. r. et. q. e. meæ figuræ, mediantibus angulis. d. o. r. et. q. o. e. qui extra superfi  
 ciem. s. a. existunt videntur, vnde si quis imaginaretur in puncto. p. oculum esse, &  
 ab ipso ad. u. et. q. duas lineas duceret, angulus. q. o. u. nunc maior, nunc minor esset  
 angulo. q. p. u. aliquando etiam æqualis, quamuis rarissime; Sub diuersis igitur an  
 gulis, pro maiori parte, deteguntur latera, à partibus quadrati tam degradati, quàm  
 perfecti, quæ non sunt anguli. b. A. c. et. f. A. G. Quod vero idem postea dicat eam  
 proportionem esse ab. b. E. ad. f. h. & simul ad. c. g. quæ est ab. a. g. ad. h. g. id tuo relin  
 quam

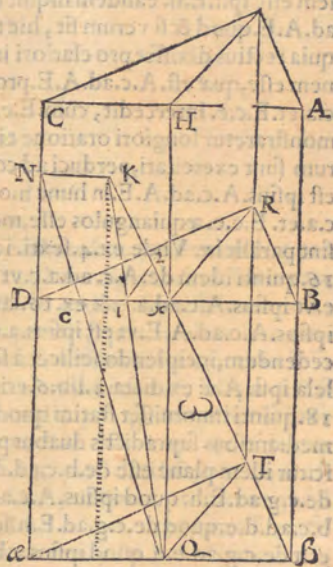
quam iudicio. Tibi quoque considerandum relinquo; cum rationabilis degradatio esse debeat, qua ratione necessarium sit, ut distantia resque, in vna & eadē proportionē cum altitudine oculi ad rem degradatam existant? Cum postea degradauerit quadratū, is scriptor, in figura. d. b. c. e. eum bene & ex perspectiua optimis legibus degradatum fuisse probare nititur; solum probans. d. e. æqualem esse ipsi. E. h. q̄. E. h. secundū ipsum est degradatio lateris. c. g. & cū superius dixerit, se tria quadrati plana degradauisse, quia. b. E. degradat. b. c. et. E. h. degradat. c. g. et. f. h. degradat. f. g. nec quidem de lateribus. b. d. et. c. e. loquitur; quia si. c. g. perfecti, degradatum est in. E. h. et. d. e. rectè protracta existit, cum sit æqualis ipsi. E. h. cum etiam. b. d. et. c. e. rectè protractæ esse debeant: qua de causa ipsi. b. E. et. f. h. quæ, ex ipso, sunt degradationes. b. c. et. f. g. æquales esse non debent? Posset is mihi quidem respondere, q̄ hoc pacto nulla superficies clauderetur. Ergo tria latera. b. c. c. g. et. g. f. nō benè sunt degradata, eiusq̄; pportionalitates male intellectæ nil probant. quia si dictæ proportionalitates, nobis tutò promitterent degradationes, ab eo primum effectas, in linea. b. f. esse bonas, ergo duæ. b. d. et. e. c. falsæ existerent, quarum quælibet maior est. b. E. et. f. h. ex. 18. primi Eucli. Omittamus etiam quod vbi is scribit eam esse rationem, aut comparisonem ab. A. d. ad. b. E. quæ est ab. d. c. ad. b. c. eandemque esse ab. E. h. ad. c. g. quæ est ab. A. E. ad. A. c. nil probet; nec similitudinem triangulorum, nec aliquam propositionem Eucli. citans. In quo excusari non potest, quod non soleat Euclidem, aut alium quemuis autorem citare, cum vel in ipso operis principio capite. 3. primæ partis, Apollonium Pergeū Euclidemq̄; & si etiam præter rem, citet. Deinde quū idem probare vult. d. e. æqualem esse ipsi. E. h. eandem inquit esse proportionem. a. b. ad. a. d. quæ est ipsius. A. c. ad. A. E. quod & si verum sit, hic tamen modus ratiocinandi nullo ordine nititur, quia rectius dixisset pro clariori intelligentia ipsius. a. c. ad. a. e. eandem proportionem esse, quæ est. A. c. ad. A. E. propter similitudinem, quæ inter duos triangulos. A. c. a. et. E. c. e. intercedit, cum. E. e. supponatur parallela ipsi. A. a. quod etiam ut demonstraretur longiori oratione ei opus fuisset si voluisset intellectum eorum, qui parum sunt exercitati, perducere ad cognoscendū idem planè futurum de. a. c. ad. a. e. ut est ipsius. A. c. ad. A. E. in hunc modum, id est probando primum duos triangulos. A. c. a. et. E. c. e. æquiangulos esse, mediante. 29. primi Eucli. cum. A. a. et. E. e. inuicem sint parallelæ. Vnde ex. 4. sexti. idem extitisset de. A. c. ad. E. c. ut. a. c. ad. e. c. et. ex 16. quinti idem de. A. c. ad. a. c. ut ipsius. E. c. ad. e. c. & ex. 19. eiusdem de. A. E. ad. a. e. ut ipsius. A. c. ad. a. c. & ex. 16. iam dicta de. A. E. ad. A. c. ut ipsius. a. c. ad. a. c. id est ipsius. A. c. ad. A. E. ut est ipsius. a. c. ad. a. e. Aut hoc alio modo, qui breuior est procedendum, incipiendo scilicet à secunda sexti Eucli. dicendo q̄ existente. E. e. parallela ipsi. A. a. ex dicta. 2. lib. 6. erit idem de. c. E. ad. E. A. ut de. c. e. ad. e. a. vnde ex. 18. quinti innotuisset statim quod de. c. A. ad. E. A. ut de. c. a. ad. e. a. extitisset. Nunc mediantibus supradictis duabus propositionibus id est. 29. primi, & 4. sexti, cognoscitur idem planè esse de. b. c. ad. d. e. quod ipsius. a. c. ad. a. e. & ex eisdem idem esse de. c. g. ad. E. h. quod ipsius. A. c. ad. A. E. vnde ex. 11. quinti bis repetita idem erit de. b. c. ad. d. e. quod de. c. g. ad. E. h. sed cum ex supposito. c. g. sit æqualis ipsi. c. b. idem erit de. c. g. ad. e. d. quod ipsius. c. b. ad eandem ex. 7. quinti, vnde ex. 11. idem erit de. c. g. ad. E. h. quod eiusdem. c. g. ad. e. d. ex. 9. igitur eiusdem. d. e. æqualis erit ipsi. E. h. atque hic verus est modus ducendi intellectum parum exercitatum in cognitionis campum. quem quidem mihi obseruandum proponerem si onus scribendi susciperem ijs, qui in scientijs parum versati sunt, quos tanquam puerulos manu ducere

cere oportet. Ratio verò ab ipso adducta propter quam. E. representatur oculo altius quam. b. nempe eo quod. A. superftet ipsi. E. nihil valet, quia si inferius esset, idem contingeret, sed hoc euenit eo quod. E. altius est ipso. b. Idem dico de. h. vbi similiter decipitur. Idem etiam in. 7. cap. fallitur in secundo modo, quem ostendit pro secundo quadrato aliquo degradato à parallelogrammo degradato magis longo quàm lato, cum ducat parallelam. l. m. ad. b. c. à puncto. l. intersectionis ipsius. o. c. id, quod non rectè efficitur quemadmodum ex rationibus à me allegatis circa meas figuras. A. A. facile innotescit.

Nono deinde cap. contrario planè ordine, quam oporteret processit, quia cū angulus. 2. trianguli perfecti magis distet à plano super quod degradari debet triangulum, quàm latus. 1. 3. oppositum dicto angulo. 2. & per consequens longè remotior sit ab oculo, ipse in degradato, eū magis propinquum esse facit, è contra cap. 10. rectè fecit contra id, quod capite. 9. tradiderat.

Quod autem deinceps in prima parte. 11. & vltimi capitis asserit est, admittendū. Quod verò in secunda parte ab eo traditur, idest alius quidam modus quem de trasferendis punctis à perfecto in degradato proponit, non est modus vniuersalis; quia si altitudo. T. Q. oculi à plano horizontali, non esset æqualis medietati lateris. B. D. perfecti, interualla. a. b. c. d. e. lateris. B. D. admittenda non essent.

Pro cuius rei intelligentia fit in subscripta hic figura corporea.  $\omega$ . parallelogrammum rectangulum A. B. C. D. in plano horizontali, & linea. Q. H. illud per medium diuidat, quæ sit parallela duobus lateribus. A. B. et. C. D. in cuius quolibet puncto. Q. fit infimus terminus altitudinis oculi, & in. T. ad perpendicularum ipsius. Q. fit verus situs eiusdem, tantum eleuatus à. Q. quanta est medietas ipsius. D. B. fitque figura corporea finita similis meæ. A. vnde. Q. T. æqualis erit ipsi. Q. æ. & planum perpendiculare orizonti, super quod punctum. k. perfecti duci debet sit. R. D. B. sintque ductæ per imaginationem lineæ. T. K. Q. K. et sit. K. N. perpendicularis lateri. C. D. à quo puncto. N. imaginatione sit cõpræhensa linea. N. Q. atque hæ tres lineæ sectæ sint à plano in punctis. c. i. et. 2. quorum punctū. 2. erit quæsitum plani. Imaginemur nunc duos triangulos. K. T. Q. et. N. Q. æ. qui secti erunt à plano. R. B. D. quorum communes sectiones erunt. 1. 2. et. D. c. & quia. N. K. D. i. et. æ. Q. inuicem sunt parallelæ, sequitur eandem proportionem futuram ipsius. Q. K. ad. K. i. quæ est ipsius. æ. N. ad. N. D. imaginatione concipiendo à puncto. K. vsque ad. æ. Q. quandam parallelam ipsi. N. æ. quemadmodum ex te ipso intelligere potes. Sed ratione similitudinis triangulorum ita se res habet de. æ. Q. ad. D. c. vt de. æ. N. ad. N. D. vt quoque de. T. Q. ad. 2. 1. quemadmodum ipsius. Q. K. ad. K. i. vnde ex. 11. quinti, idem erit de. Q. T. ad. 1. 2. quod de. Q. æ. ad. c. D. & ex. 16. eiusdem de. Q. T. ad. Q. æ. quod de. 1. 2. ad. c. D. & existente. æ. Q. ex supposito æquali ipsi.



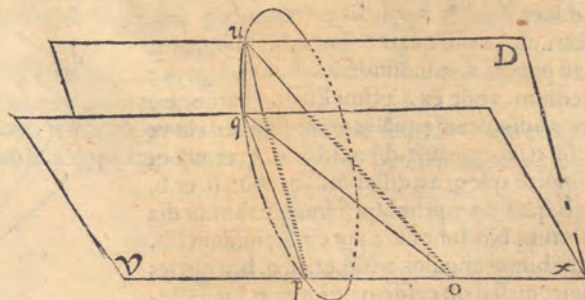


DE PERSPECT.

.1.2. Vnde huiusmodi regula tunc bona redditur, quando T. Q. æqualis est ipsi. ce. Q. idest medietati ipsius. D. B. at verò si æqualis non esset hoc minime sequeretur, vt faciliè patet. Quòd verò. 2. R. z. & sint benè disposita, dubitandum non est, quia punctum. i. meæ hic subscriptæ figuræ, quod coresponder K. eius figuræ adeò distat à medio. R. X. trianguli. R. B. D. vt. 2. cum. 1. 2. dicto medio. R. X. ex. 6. Vndecimi fit parallela. Idem de reliquis dico. quod manifestè cognosci potest, ab eo, quod in superius positis figuris corporeis dixi. Huiusmodi modus ducendi res in perspectivâ, non solum à Gallis, sed à Germanis etiam in vsum reducitur. Sed quia ad hæc vsq; tempora eiusdem perfectionis ratio, quam ego superius proposui, nõdum in lucem emerfit, factum fuit, vt errorũ laqueis irretirentur, fumentes. T. Q. modo maiorem, modo minorem medietate lateris. D. B. Cum hunc igitur modum hic Auctor vniuersalem esse putet, labitur in errorem, cum debuisset longitudinem ipsius. T. Q. debere esse æqualem medietati ipsius. D. B. proferre. Afferit deinde distantiam ipsius. T. Q. à latere. B. D. æqualem esse debere lateri. C. D. quod necessarium non est, quia in quibuslibet distantijs, iusta operatio fieri potest, quemadmodum in subscripta hic figura facile patet, idest, quòd quibuscunq; modis. c. D. æqualis remaneat ipsi. 1. 2. & sic intervalla, quæ p. transuersum aguntur vsq; ad mediũ trianguli. D. R. B. Neque etiam probandus est auctor ille, cum pro oculo, suum. T. loco. Q. à me positi, ponit, cum is locus sit verus situs pedis eius qui respicit, & non oculi. Quòd autè Auctor iste, modo vniuersali intelligat, vt iam diximus, cõsideretur figura tertij modi primi cap. tertix partis, in qua suum oculum ( vt ita dicam ) ponit in. o. altius seu distans à rectitudine lateris. c. d. plus quam fit totum latus. d. b.

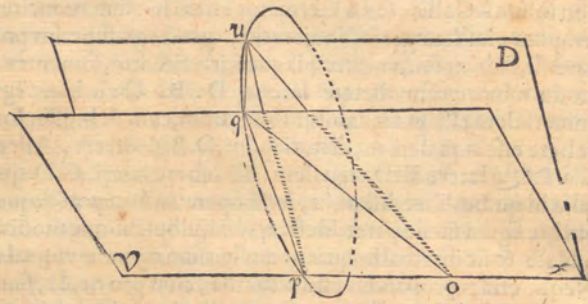
AD EVNDEM IACOBVM.  
CAP. XIII.

**T**VAS accepi literas omnis humanitatis & officij plenas, in quibus requiris causam, quæ me in alijs meis literis impulit ad dicendũ, angulũ. q. o. u. modo maiorem, modo verò minorem futurum angulo. q. p. u. meæ figuræ corporeæ. A. hanc igitur ob causam imagineris in subscripta hinc figura duo triangula. q. o. u. et. q. p. u. quorum. q. p. u. perpendiculariter sit super superficie trianguli. q. o. p. collocatum, præcisè vt in mea figura corporeæ. A. superficies verò trianguli. q. o. p. fit exempligratia. V. M. & trianguli. u. o. p. fit. V. D. quarum cõmunis sectio sit. V. p. o. x. non est enim dubitandum quin triangulum. q. p. u. sit perpendicularare triangulo. q. o. p. cũ hoc ex. 18. lib. 11. Eucli. perpendicularare sit superfici. a. s. in qua reperitur triangulum. q. p. u. & hoc ex linea. o. p. perpendiculari dictæ superfici. a. s. Nunc dico angulũ. q. o. u. modo maiorem, modo minorem esse angulo. q. p. u. Notissimum igitur primum nobis est



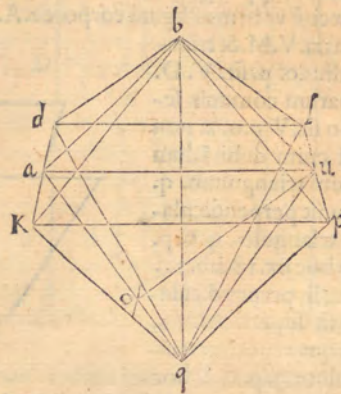
S est

est angulum. p. q. u. obtusum esse; Imaginemur ergo circa triangulum. p. q. u. circumscriptum esse circulum, cuius portio. p. q. u. minor erit medietate eiusdem medij circuli, vt iam ex 30. Eucli. lib. tertij nouisti. nunc imaginemur dictum circulum circum lineam. q. u. loco axis versus. x. moueri, vnde girus eiusdem, per quem transibat linea V. x. remouebitur ab eadem linea non nihil cum motus erit à primo situ vsquequò ad secandam dictam lineam. V. x. in alio quodam puncto inter. p. et. x. redibit; quod quidem punctum si erit inter. o. et. x. angulus. q. o. u. maior erit angulo. q. p. u. Sed si idem punctum erit inter. p. et. o. dictus angulus. q. o. u. minor erit. q. p. u. de qua q. d. e re tu ipse mediante. 20. lib. 3. et. 16. lib. primi certior fieri potes. Valde miror q. hanc Ioannis Cusini dicta ad hæc vsque tempora tanto in prætio sint habita, vt ab excellentibus scriptoribus quasi si proprij eorum ingenij partus essent, de verbo ad verbum vt thesauros, in suis ipforūmet libris rescripta fuerint, quemadmodum iam omnes admonui in megnomonica Orontium, Munsterum, aliosq. permultos fecisse.



C A P. X I I I I.

**E**X ijs, quæ de nonnullis effectibus ducendo in perspectiua tertium corpus regulare, q. octo triangulis æquilateralibus est terminatum, scire desideras, hoc vnū est caput: vnde fiat, aut quomodo probetur quaslibet duas facies oppositas eiusdem corporis octoædri inuicè æquidistantes esse. Quamobrem sit hic subscriptū octoædri, cuius diameter vna sit. b. q. et. b. p. l. vna ex faciebus, cui opponatur facies. q. k. d. quas adinuicè æquidistantes esse contendo sint aliæ duæ facies, quæ inter has ponuntur. b. d. k. et. q. p. l. & à punctis extremis. b. q. diametri. ductæ sint quatuor lineæ. b. a. b. u. q. a. q. u. ad puncta. a. et. u. diuidentia. k. d. et. l. p. per medium, vnde ex 4. primi Eucli. quatuor hæ lineæ adinuicem æquales erunt sumendo eas vt bases triangulorū. a. d. b. u. l. b. a. d. q. et. u. l. q. adinuicè quoq. æquidistant. a. b. ab. u. q. et. b. u. ab. q. a. ex. 27. primi; q. a. si imaginabimur diametrum. b. q. tunc ex. 4. aut ex. 8. eiusdem lib. habebimus angulos. a. b. q. et. u. q. b. æquales inuicem; sed ob easdem rationes. p. l. parallela est ipsi. d. k. vnde ex 15. lib. 11. facies. b. p. l. parallela sit, aut æquidistans ipsi. q. d. k. id est primam propositum.



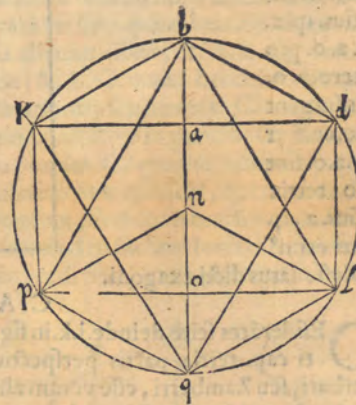
Ad

Ad habendam deinde quantitatem distantiae, aut interualli simul cum situ, in facie. q. d. k. quem latus. p. l. perpendiculariter respicit. Imaginemur à puncto. u. super q. a. cadere lineam perpendicularem. u. o. quae illico reperitur cum triangulum. a. u. q. ex lateribus datis & cognitis constet, quodquidē triangulum, medietas est quadrilateri, seu. rumbi. q. a. b. u. cui vnaquæque dictarum quatuor facierum perpendicularis existit ex. 4. et. 18. lib. 11. & ob id linea. u. o. extensa in superficie dicti quadrilateri, & perpendicularis linea. q. a. perpendicularis erit faciei. q. d. k. & ex. 29. primi, angulus. b. u. o. rectus erit, ut etiā angulus. o. u. l. ex. 2. definitione lib. 11. vnde ex. 4. eiusdem lib. o. u. perpendicularis erit faciei. b. p. l. Habebimus ergo situm in facie. q. d. k. qui respicietur ad angulos rectos à linea. p. l. qui quidem erit in perpendiculari à puncto. o. ad. q. a. ducta.

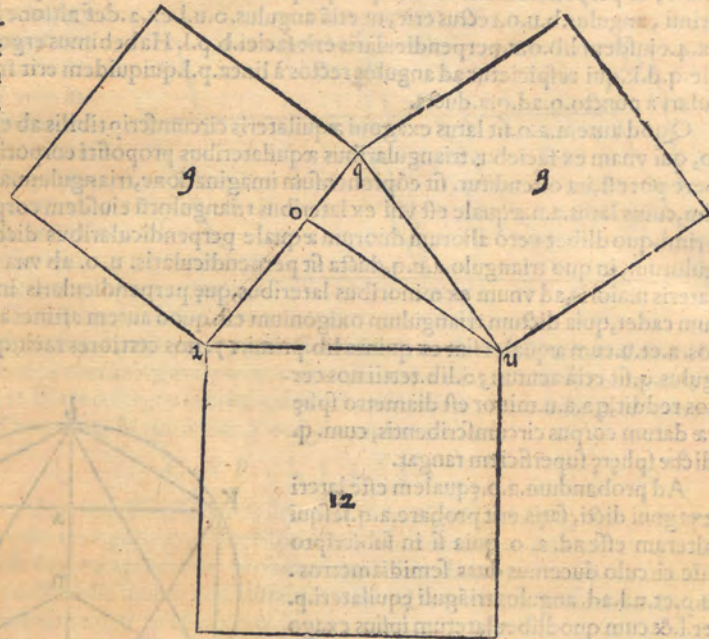
Quòd autem. a. o. sit latus exagoni æquilateris circumscriptibilis ab eodem circulo, qui vnam ex faciebus triangularibus æquilateribus propositi corporis circumscribere potest, ita ostenditur. sit cõprehensum imaginatione, triangulum. a. q. u. separatum, cuius latus. a. u. æquale est vni ex lateribus triangularū eiusdem corporis ex. 33. primi, quo dlibet verò aliorum duorum æquale perpendicularibus dictorum triangularum, in quo triangulo. a. u. q. ducta sit perpendicularis. u. o. ab vna extremitatū lateris maioris, ad vnum ex minoribus lateribus, quæ perpendicularis intra triangulum cadet, quia dictum triangulum oxigonium est. quod autem attinet ad duos angulos. a. et. u. cum æquales sint ex quinta lib. primi, 17. nos certiores facit; quod verò angulus. q. fit etiā acutus: 30. lib. tertii nos certos reddit, q. a. a. u. minor est diametro sphærae datum corpus circumscribentis, cum. q. dictæ sphærae superficiem tangat.

Ad probandum. a. o. equalem esse lateri exagoni dicti, satis erit probare. a. q. sesquialteram esse ad. a. o. quia si in subscripto hic circulo ducemus duas semidiametros. n. p. et. n. l. ad. angulos triaguli æquilateri. p. et. l. & cum quodlibet laterum ipsius exagoni, æquale sit semidiametro circuli ex. 15. lib. 4. habebimus ex. 8. primi, angulum. n. p. l. æqualem angulo. q. p. l. Vnde ex. 4. eiusdem. o. n. equaliserit ipsi. o. q. id est. q. a. sesquialtera erit ad. a. o.

Ad probandum nunc in triangulo. a. q. u. a. q. sesquialteram esse ad. a. o. est quoque sciendum primò omne latus trianguli æquilateri in potentia sesquitertium esse ad perpendicularem eiusdem trianguli, quod vndecima lib. 14. Eucli. breuiter demonstratum est.

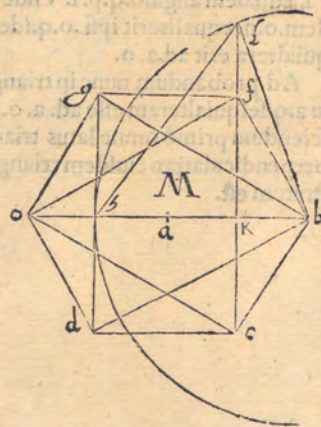


Ponamus nunc quadratum lateris. a. u. esse. 12. clarum erit quodlibet quadratum aliorum duorum laterum. a. q. et. u. q. futurum nouem, ex ijs quæ posteriore loco diximus, & quia quadratum ipsius. q. a. est tantò minus aliorum duorum quadratorum summa, quantum est duplum producti ipsius. q. a. in. a. o. ex. 13. secundi, sed alia duo quadrata simul collecta faciunt. 21. à quo numero subtrahendo quadratum ipsius. a. q. id est nouem, remanebit numerus. 12. pro duplo producti ipsius. q. a. in. a. o. cuius dupli media pars, id est simplex productum ipsius. q. a. in. a. o. erit 6. Sed quia quadratum ipsius. q. a. est nouem, eius radix. q. a. erit. 3. per quã diuidendo. 6. productum ipsius. q. a. in. a. o. pro latere. a. o. confluent duo, cum ergo. a. o. sint duo tertia ipsius. a. q. certi erim<sup>9</sup> a. o. esse latus dicti exagoni.



C A P. X V.

Desiderantes scire deinde. l. k. in figura. M. quarti cap. tertiæ partis perspectiue Danielis Barbari, seu Zamberti, esse veram altitudinẽ corporis octoædri, primũ scire debemus quod existere. b. h. vt etiã. b. l. tripla ad. b. k. vt ex ijs, quæ superius iã diximus, facile percipi potest; ex penultima primi. b. l. in potentia, sesquioctaua erit ad. k. l. ipsa est. k. l. dupla in potẽtia ad. h. k. & ob id ducta cũ esset. h. l. existeret in potentia tripla ad. h. k. & sesquialtera ad. l. k. & sesquitercia ad. l. b. & sic ad. h. b. vnde. l. h. æqualis esset vni ex lateribus triãguli equilateri dicti corporis. Ex rationibus igitur superius hinc positis. l. k. erit altitudo dicta, id est distantia inter duas facies inuicem oppositas, octoædri.



Neq; volo te ignorare aliũ nõ paruũ fuisse errorẽ illius Zamberti: cum eodẽ capite affirmet angulos octoædri rectos esse cũ sint acuti, nã vnusquisq; minor est angulo cubi solido. DE

# DE MECHANICIS.

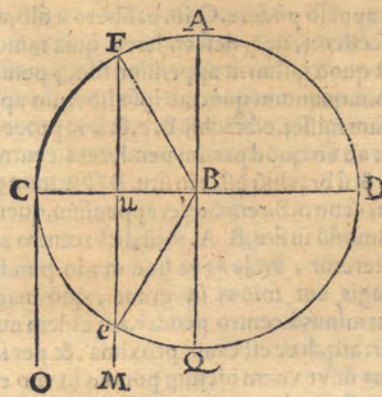


*CRIPSERUNT* multi multa, & quidem scitissimè, de mechanicis, at cum natura vsusq; aliquid semper vel nouum, vel latens in apertum emittere soleant, nec ingenui aut grati sit animi, posteris inuidere, si quid ei contigerit comperuisse prius tenebris inuolutum: cum tam multa ipse ex aliorum diligentia sit consequutus. Paucula quadã futura, vt reor, non ingrata his qui in hisce mechanicis versantur, nusquam ante hac tentata, aut satis exactè explicata in medium proferre volui: quo vel iuuandi desiderium, vel saltem non ociosi ingenioli argumentum aliquod exhiberem, atque vel hoc vno modo me inter humanos vixisse testatum relinquerem.

## De differentia situs brachiorum libræ.

### C A P. I.

**O**MNE pondus positum in extremitate alicuius brachij libræ maiorem, aut minorem grauitatem habet, pro diuersa ratione situs ipsius brachij. sit exempli gratia. B. centrum, aut, quod diuidit brachia alicuius libræ, & A. B. Q. verticalis linea, aut, vt rectius dicam, axis orizontis, & B. C. vnum brachium dictæ libræ, & in C. sit pondus, & C. O. linea inclinationis, seu itineris. C. versus centrum mundi, cum qua. B. C. angulum rectum constituat in puncto. C. Existente igitur in huiusmodi situ brachio. B. C. dico pondus. C. grauius futurum, quam in alio quolibet situ. quia supra centrum. B. omninò non quiescet, quemadmodum in quouis alio situ faceret. Ad quod intelligendum, sit dictum brachium, in situ. B. F. cum eodem pondere in puncto. F. & linea itineris seu inclinationis dicti ponderis sit. F. u. M. per quam lineam dictum pondus progredi non potest, nisi brachium. B. F. breuius redderetur. Vnde clarum erit quòd pondus. F. aliquantulum supra centrum. B. mediante brachio. B. F. nititur. Est quidem verum, quòd pondus. C. nec ipsum etiam per lineam. C. O. proficiscetur, quia iter extremitatis brachij est circularis, & C. O. in vno quodã puncto est contingens. Sit hoc iter. A. C. Q. Oportet nunc præsupponere pondus extremitatis brachij debere tanto magis cetro. B. inniti, quanto magis linea suæ inclinationis (ponamus. F. u. M.) propinqua erit dicto centro. B. quod sequenti cap. probabo, vt exempli gratia, sit. F. super. u. punctum medij ex æquo inter. C. et. B. quapropter. u. B. æqualis erit. u. C. vnde fe-



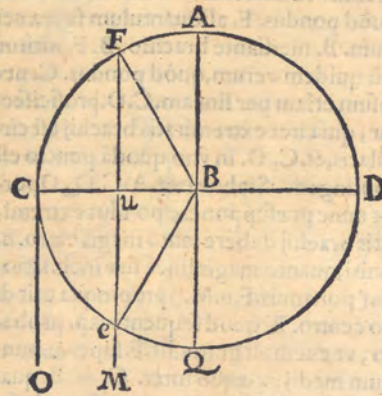
quetur

quetur dictum pondus grauius futurum pro parte. F. C. quam pro ea, quæ est. A. F. & minus supra centrum. B. pro dicta parte. F. C. quam pro parte. A. F. quieturum; & dictum brachium quanto magis horizontale erit à situ. B. F. tantò mitius supra dictum centrum. B. quiescet, & hac ratione grauius quoque erit, & quanto magis vicinum erit ipsi. A. à dicto. F. tantò magis super centrum. B. quoque quiescet, vnde tantò quoque leuius existet. Idem dico de omni situ brachij per girum inferiorem. C. Q. vbi pondus pendeat à centro. B. dictum centrum attrahendo, quemadmodum superius illud impellebat. Hæc verò omnia cap. sequenti melius percipientur.

*De proportione ponderis extremitatis brachij libræ  
in diuerso situ ab orizontali.*

C A P. I I.

**P**R O P O R T I O ponderis in. C. ad idem pondus in F. erit quemadmodum totius brachij. B. C. ad partem. B. u. positam inter centrum & lineam. F. u. M. inclinationis, quam pondus ab extremitate. F. liberum versus mundi centrū conficeret. Quod vt facilius intelligamus imaginemur alterū brachium libræ. B. D. & in extremo. D. locatum aliquod pondus minus pondere. C. vt. B. u. pars. B. C. minor est. B. D. clarè cognoscetur ex. 6. lib. primi de ponderibus Archimedis, quòd si in puncto. u. collocatum erit pondus ipsius. C. libra nihil penitus à situ orizontali dimouebitur. Sed perinde est quòd pondus. F. æquale. C. sit in extremo. F. in situ brachij. B. F. quæ vt sit in puncto. u. in situ ipsius. B. u. orizontali. Ad cuius rei euidentiam imaginemur filū. F. u. perpendicularare, & in cuius extremo. u. pendere pondus, quod erat in. F. vnde clarum erit quòd eundem effectum gignet, ac si fuisset in. F. quod, vt iam diximus remanens affixum puncto. u. brachij. B. u. tantò minus graue est situ ipsius. C. quantò. u. B. minus est ipso. B. C. Idem assero si brachium esset in situ. e. B. quod facile cognoscere poterimus, si imaginemur filum appensum ipsi. u. brachij. B. C. & vsque ad. e. perpendicularare, in quo extremo appensū esset pondus æquale ponderi. C. & liberū ab. e. brachij. B. e. vnde libra orizontalis manebit. Sed si brachium. B. e. consolidatum fuisset in tali situ cum orizontali. B. D. & appeso pōdere. C. in. e. libero à filo, nec ascēderet, neq; descenderet. quia tantum est quod ipsum sit appensum filo, & pendet ab. u. quantum quòd ab ipso liberum appensum fuisset. e. brachij. B. e. & hoc procederet ab eo quòd partim penderet à centro. B. & si brachiū esset in situ. B. Q. totum pōdus centro. B. remaneret appensum, quemadmodū in situ. B. A. totū dicto centro anniteretur. vnde fit vt hoc modo pondus magis aut minus sit graue, quò magis aut minus à centro pendet, aut eidem nititur: atq; hæc est causa proxima, & per se, qua fit vt vnum idemq; pondus in vno eodemq; medio magis aut minus graue exi-



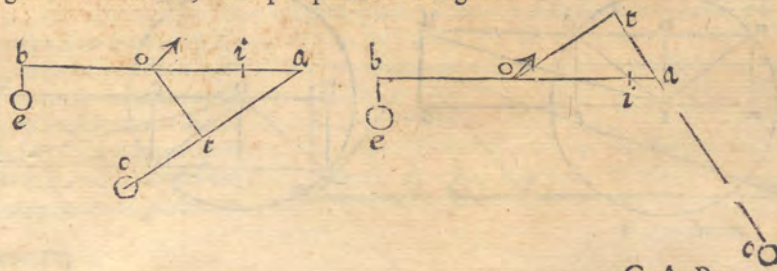
stat.

stat. Et quamvis appellem latus. B. C. horizontale, supponens illud angulum rectum cum. C. O. facere, unde angulus. C. B. Q. fit ut minor sit recto, ob quantitatem vnius anguli æqualis ei, quem duæ. C. O. et. B. Q. in centro regionis elementaris constituunt, hoc tamen nihil refert, cum dictus angulus insensibilis sit magnitudinis. Ab istis autem rationibus elicere possumus, quod si punctus. u. erit ex æquo medius inter centrum. B. & extremum. C. pondus. F. aut. M. pendebit, aut nitetur pro medietate dicto centro. B. & si dictum. u. erit propius. B. quam puncto. C. pendebit ab ipso, aut nitetur ipsi amplius quàm ex medietate, & si magis versus. C. minus quàm ex medietate nitetur.

*Quod quantitas cuiuslibet ponderis, aut virtus mouens respectu alterius quantitatis cognoscatur beneficio perpendicularium ductarum à centro libræ ad lineam inclinationis.*

C A P. I I I.

EX ijs, quæ à nobis hucusque sunt dicta, facile intelligi potest, quæ quantitas. B. u. quæ ferè perpendicularis est à centro. B. ad lineam. F. u. inclinationis, ea est, quæ nos ducit in cognitionem quantitatis virtutis ipsius. F. in huiusmodi situ, constituens videlicet linea. F. u. cum brachio. F. B. angulum acutum. B. F. u. Ut hoc tamen melius intelligamus, imaginemur libram. b. o. a. fixam in centro. o. ad cuius extremitates appensa duo pondera, aut duæ virtutes mouentes. e. et. c. ita tamen quæ linea inclinationis. e. id est. b. e. faciat angulum rectum cum. o. b. in puncto. b. linea verò inclinationis. c. id est. a. c. faciat angulum acutum, aut obtusum cum. o. a. in puncto. a. Imaginemur ergo lineam. o. t. perpendicularem lineæ. c. a. inclinationis, unde. o. t. minor erit. o. a. ex. 18. primi Euclidis. fecerunt deinde imaginatione o. a. in puncto. i. ita ut o. i. æqualis. sit. o. t. & puncto. i. appensum sit pondus æquale ipsi. c. cuius inclinationis linea parallela sit lineæ inclinationis ponderis. e. supponendo tamen pondus aut virtutem. c. ea ratione maiorem esse ea, quæ est. e. quia. b. o. maior est. o. t. absque dubio ex. 6. lib. primi Archi. de ponderibus. b. o. i. non mouebitur situ, sed si loco. o. i. imaginabimur. o. t. consolidatam cum. o. b. & per lineam. t. c. attractam virtute. c. similiter quoque continget ut b. o. t. communi quadam scientia, non moueatur si tu. Est ergo quod proposuimus verum quantitatem alicuius ponderis respectu ad eam, quæ est alterius debere deprehendi à perpendicularibus, quæ à centro libræ ad lineas inclinationis exiliunt. Hinc autem innotescit facillimè, quantum vigoris, & vis pondus, aut virtus. c. ad angulum rectum cum. o. a. minime trahens, amittat. Hinc quoque corollarium quoddam sequetur, quod d. quanto propinquius erit centrum. o. libræ centro regionis elementaris, tanto quo minus erit graue.

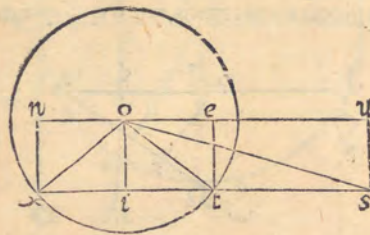
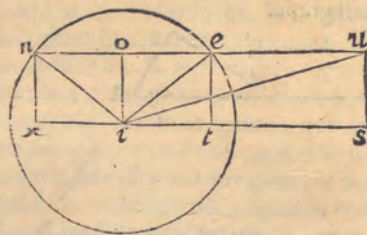


C A P.

*Quemadmodum ex supradictis causis omnes Staterarum & vectium causæ dependeant.*

## C A P. I I I I.

**V**Is brachij longioris alicuius stateræ, aut vectis, maior breuioris, ab ijs, quæ in superioribus capitibus diximus, idest quæ nitatur pendeatue magis aut minus à centro pondus in extremitate brachij maioris positum, oboritur. Quamobrem illud à nobis primò est cognoscendum, stateras, aut vectes, puras mathematicas lineas non esse, sed naturales, hincque existere corpora cum materia coniuncta. Nunc igitur imaginemur. n. s. eam superficiem esse, quæ secundum longitudinem axem stateræ scindit. & supponamus ipsius centrum esse primum in. i. & maius brachium esse .i. u: minus autem. i. n. & lineam verticalem. i. o. quæ tanta sit, quanta est spissitudo, aut crassities ipsius stateræ à superiori latere ad inferius, ad faciliorem intelligentiam, supponendo. n. s. parallelogramam. Positis igitur duobus ponderibus æqualibus in extremitatibus brachiorum, experientia innotescit, quod pondus ad. u. s. appensum, violentiam faciet ponderi appenso ad. n. x. sed nos volumus inuestigare causam huius effectus, quæ à nemine vnquam literarum monumentis, quæ sciam, consignata fuit. Iam diximus stateram, aut vectem materialem esse & n. s. eius superficiem mediam, supponendo. i. esse centrum quo nititur dicta statera aut vectis; Cum hoc ergo ita se habeat, sint. u. s. et. n. x. lineæ inclinationum ponderum, & imaginemur, quod dicta pondera pendeant à punctis. u. et. n. vt reuera pendent, etiam si appensa essent sub. s. et. x. quia punctum. u. & punctum. n. ita coniuncta sunt cum. s. et. x. ut qui vnū trahit alterum quoque trahat. Imaginemur quoque duas lineas. i. u: i. n. et. i. e. quæ i. e. faciunt angulum. o. i. e. æqualem angulo. o. i. n. Hinc clarè nobis patebit, si quis ipsi e. pondus ipsius. u. (quod æquale est ponderi. n.) appenderet, id eandem planè vim haberet, quam pondus ipsius. n. habet, & stateram neque sursum, neque deorsum moueret, quia ambo pondera ad centrum. i. mediantibus lineis. e. i. et. n. i. ex æquo anniterentur, sed dicto pondere posito in. u: linea. u. i. per quam pondus centro annitur, magis horizontalis quam. e. i. fit, & linea. u. s. inclinationis longius distans à centro. i. quam linea. e. t. vnde huiusmodi pondus magis quoque liberum à centro. i. resultat. magisque ponderosum, quam cum erat in. e. ratione eorum, quæ primo & secundo capitibus diximus, & ob hanc causam superat pondus positum in. n. Sed si centrum fuerit. in. o. imaginabimur duas lineas. o. s. et. o. x. & supponemus quod pondera posita sint in. s. et. x. vnde existente magis orizontali linea. o. s. quam erit. o. x. & linea u. s. inclinationis longius distans à centro. o. quam linea. e. t. eius pondus erit quoque



gravius

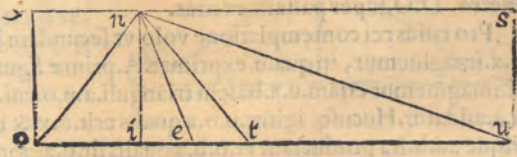


gravius, quia tantò minus pendeat à centro. o. & ratiocinando, vt superius diximus, inueniemus eundem effectum verum esse. In stateris, rectè & propriè appellari potest. x. i. s. aut. n. o. u. horizontalis, sed in omni vectium specie, hoc tantum per quamdam similitudinem dicitur. Idem contemplari licet supponendo centrum in medio inter. o. et. i. quod vnusquisque ex se absque alterius auxilio facile præstare poterit.

*De quibusdam rebus animaduersione dignis.*

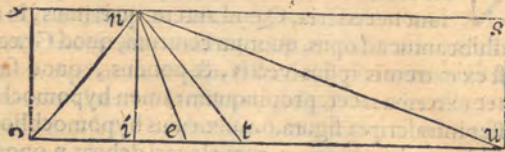
C A P. V.

**N**on omittenda mihi videtur quædam, quæ ad tractationem vectium admodum sunt necessaria. Quod autem quærimus, in eo consistit, quod aliqui vectes adhibeantur ad opus, quorum centrum, quod Græci hypomochliò appellant vnum est ex extremis ipsius vectis, & pondus, quod sursum eleuari debet, inter ipsam et extrema iacet, propinquum tamen hypomochlio, vt exempli gratia, si vectis esset infra scripta figura. o. s. u. x. cuius hypomochlion esset in puncto. o. & pondus in puncto. n. clarum erit, quod cum eleuari debeat. n. oportebit quoque opera manus eleuari. u. Nunc considerandum est quomodo pondus. n. annitatur ad. u. Hanc ob causam imaginabimur rectas lineas. n. o. n. i. n. e. n. t. et. n. u. quarum. n. i. versus mundi centrum sit posita, et. n. t. faciat angulum. i. n. t. æqualem angulo. i. n. o. Nunc ponendo aliquam virtutem in. i. æquali inclinatione ad superius constante, vt. n. ad inferius (remota tamen grauitate materiæ vectis) huiusmodi virtus, totum pondus ipsius. n. communi quadam scientiæ notione sustinebit. & si pondus ipsius. n. esset in. x. è directo super. o. totum pondus super hypomochlio se haberet, & tanta virtus ipsius hypomochlij sufficeret ad resistendum pro sustinendo, quanta est grauitas ipsius ponderis, sed ipsum iterum ponamus in. n. ibi clarum erit, quod si alia virtus à parte inferiori ad superiorem vectis non opponitur, excepto tamen hypomochlio, oportebit virtute cuiusdam partis ponderis. n. (absque consideratione tamen, vt iam dixi, ponderis materiæ vectis) vt vectis à parte. s. u. deprimatur, & dixi vnus cuiusdam partis ponderis. n. quia alia eiusdem ponderis pars annititur ipsi hypomochlio. o. mediante linea. o. n. quæ angulos rectos cum. o. x. non facit. Si autem à puncto. t. opponet sese huiusmodi resistentia, vt vectis non deprimatur, clarum erit communi scientiæ, quod virtus ponderis. n. diuisa erit per medium æqualiter, cuius vna medietas super. o. quiescet, & alia super. t. mediantibus duabus lineis. n. o. et. n. t. Imaginemur nunc resistentiam t. ablatam esse, positamque in. e. clarum quoque erit, quod maior pars ponderis. n. ipsi. e. annitetur beneficio lineæ. n. e. quàm ipsi. o. cum linea. n. i. inclinationis ipsi. e. sit propinquior quam. o. quia omnis resistentia aut in. i. aut in. e. aut in. t. aut in. u. est loco centri, quemadmodum est. o. & alter alterius opera iuuatur. Si verò eadem resistentia posita erit in. u. clarum quoque erit, quod minor pars ponderis. n. annitetur ipsi. u. quàm ipsi. o. cum dicta. n. i. à centro. u. longius quam à centro. o. distet, & proportio partis ponderis. n. in. o. ad proportionem partis ponderis. n. in. u. non erit secundum proportionem angulorum. u. n. i. et. o. n. i. sed secundum proportionem. u. i. ad. i. o. quod clarè comprehendendi potest ab



T huius

huius effectus conuerso, id est, vt quemadmodum nunc supponuntur. o. et. u. esse duo centra quibus sustinet pondus. e. ipsius. n. imaginemur. n. esse quoddam centrum à quo pendeant duo pondera. o. et. u. sic inuicem proportionata, ut sunt. u. i. et. i. o. certe horum ponderum causa statera. o. s. quam vectem appellabamus à nulla parte inclinabitur. Redeuntes nunc ad propositum, dicemus q̄ annitente pondere ipsius. n. minus ad. u. quam ad. o. id est ad. t. minori vi opus erit in. u. quàm in. t. ad attollendum pondus ipsius. n. & sic per consequens quantò longius erit punctum. u. ab. t. tantò minori quoque vi egebit, & consequenter quando vis, aut resistentia in. u. ita proportionata erit illi, quæ est ipsius. o. vt est. o. i. ad. i. u. vectis non mouebitur. Sed quando erit proportio maior, resistentiæ ipsius. u. ad eam, quæ est ipsius. o. ea, quæ est. o. i. ad. i. u. tunc vectis à parte ipsius. u. s. eleuabitur, si vero proportio minor esset quàm. o. i. ad. i. u. tunc vectis ab eadem parte deprimetur.



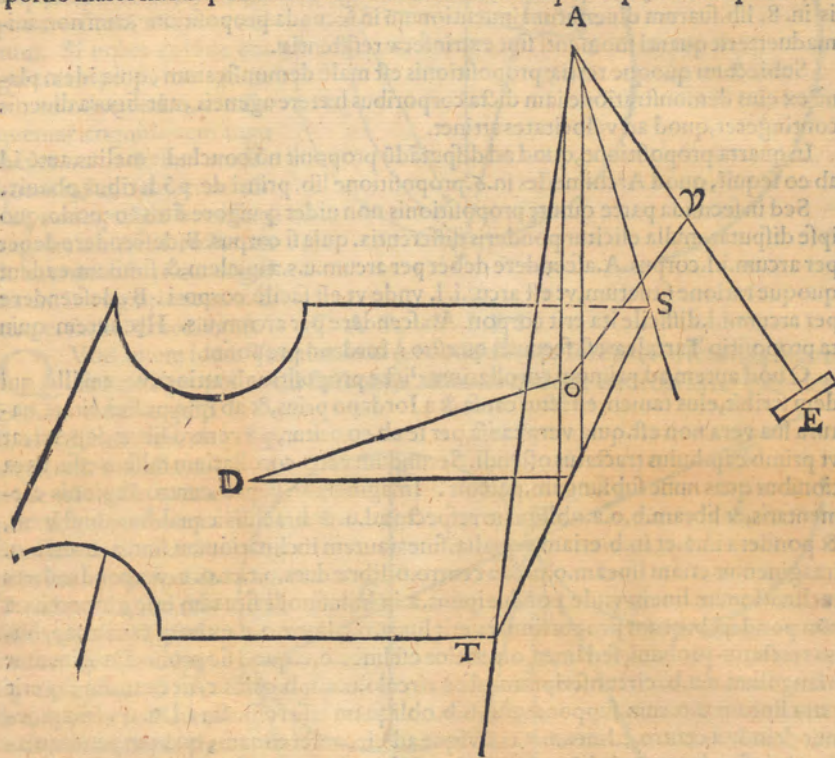
*De ratione cuiusdam vis aduictæ.*

C A P. V I.

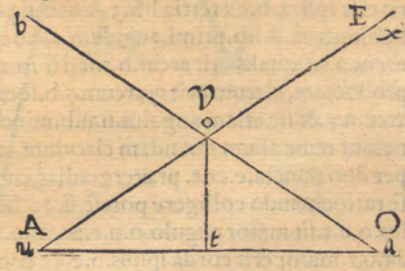
**Q** Vibuscum in locis vtuntur quidã quodã instrumẽto pistorio ad subigendã pastam, vnius tantum hominis vi adhibita, quæ quidem machina cum mihi digna contemplatione esse videatur, eius aliquam rationem proponere volui, pro cuius descriptione imaginemur planum, in quo sedet ille, qui voluit pastam, & in quo ipsa pasta est reposita. T. S. D. & triangulum. T. A. S. immobile perpendiculareque superficiẽ dicti plani, angulo autem. A. coniunctum lignum. A. E. vt semidiametrum mobilem, & æqualem perpendiculari ipsius trianguli, unde. A. loco centri erit et. D. O. sit semidiameter, qui pastam contundit, & ab eius extremo. O. (quod. O. quando. D. O. horizontalis est, in basi dicti trianguli reperitur) veniat lignum. O. V. quod cum. A. V. sit æquale perpendiculari imaginatæ ab angulo. A. basi. T. S. denotatũ tñ ut vulgo dicitur seu flexile in. O. & in. V. vt elleuare atq; deprimere semidiametrum. D. O. possit, et. V. O. sit æqualis. A. V. et. V. medium sit inter. A. et. E. vnde. A. V. cum. O. V. æquales erunt. A. E. sunt deinde duo ligna perpendicularia ab. A. ad basim fixa, & immobilia inter se adeo distantia, vt inter ipsa pertrãseãt. O. V. et. D. O. supra & infra, ne deuiet semidiametrum. D. O. In extremitate deinde ipsius. E. sit lignum quoddam tenue, vt digitus pollex, ad angulos rectos cum. A. E. quod ab aliquo, qui antedictam machinam stet, manibus teneatur, qui quidem homo id ipsum lignum, id est semidiametrum. A. E. à superficie trianguli dicti, ad se trahendo, & deinde versus eundem triangulum impellendo, vim quandam maximam mediante semidiametro. D. O. super pastam excitat.

Pro cuius rei contemplatione volo vt secundam hanc subscriptam figuram. b. a. u. x. imaginemur, in qua. u. exprimat. A. primæ figuræ, & a. denotet. O. & o. V. & x. E. imaginemur etiam. u. a. basem trianguli. a. u. o. cui. o. t. perpendicularis distãt basi. u. a. addatur. Hucusq; igitur. u. o. æqualis erit. o. x. & ipsi. o. a. imaginemur etiam. a. o. vsque ad. b. ita productam vt. o. b. æqualis sit. o. a. ponamus etiam pondus in. a. impellere

Iere versus u. vnde linea eius inclinationis fit semper a. u. supponamus etiam a. o. b. esse librā, aut stateram, aut vectem, & o. eius centrum, vnde vis, aut virtus ipsius a. proportionalis erit ipsi. o. t. respectu virtutis, aut vis imaginata in. b. inclinationis perpendicularis ipsi. b. a. quæ quidem virtus, aut vis in. b. proportionalis erit ipsi. b. o. ex tertio capite huius tractatus; Si ergo fuisset posita in. b. virtus quædam ad angulum rectum, trahens lineam. b. o. tam proportionatam virtuti perpendiculari ipsius a. quam est. o. t. proportionata ipsi. o. b. statera. b. o. a. non moueretur, sed quæuis portio maior in. b. superaret. a. cum autem fuerit. o. x. æqualis ipsi. o. b. idē planē eue-



niet, communi quadam scientia, ponendo virtutem. b. in. x. Quantitas ergo virtutis in. x. quæ superare debet resistentiam in. a. quæ ipsi. u. contra ponitur, debet habere aliquantulum maioris proportionis ad resistentiam, quæ in. a. angulum rectum efficeret cum. a. o. ea, quæ est. o. t. ad. o. x.



T 2 DE

*De quibusdam erroribus Nicolai Tartalea circa pondera  
corporum & eorum motus, quorum aliqui desumpti  
fuerunt à Jordano scriptore quodam antiquo.*

C A P. V I I.

**C**um magis amici veritatis esse debeamus quam cuiusquam hominis, quemadmodum Aristo. scribit, detegam hoc loco quosdam errores Nicolai Tartalea de ponderibus corporum, & velocitatibus motuum localium. Et primum decipitur in 8. lib. suarum diuersarum inuentionum in secunda propositione, cum non animaduertit quanti momenti sint extrinsecæ resistantiæ.

Subiectum quoque tertiæ propositionis est malè demonstratum, quia idem planè ex eius demonstratione iam dicta corporibus hætereogencis, aut figura diuersis contingeret, quod ad velocitates attinet.

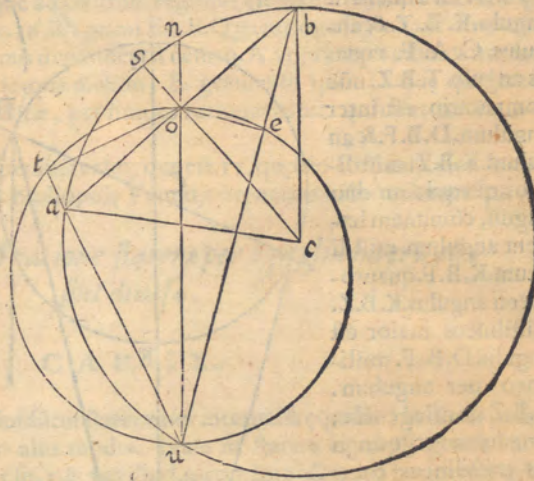
In quarta propositione, quod ad disputadū proponit nō concludit melius, autè id ab eo sequit, quod Archimedes in 6. propositione lib. primi de pōderibus pbauit.

Sed in secunda parte quintę propositionis non uidet q̄ uigore situs eo modo, quo ipse disputat, nulla elicitor ponderis differentia. quia si corpus. B. descendere debet per arcum. i. l. corpus. A. ascendere debet per arcum. u. s. æqualem, & similem. eadem quoque ratione situatum, vt est arcus. i. l. vnde vt est facile corpori. B. descendere per arcum. i. l. difficile ita erit corpori. A. ascendere per arcum. u. s. Hęc autem quinta propositio Tartalea est secunda quæstio à Jordano proposita.

Quòd autem ad primum corollarium dictæ propositionis attinet, verum ille qui dem scribit, eius tamen effectus causa & à Jordano prius, & ab ipso postea citata, natura sua vera non est. quia vera causa per se ab eo oritur, q̄ à centro libræ dependeat vt primo cap. huius tractatus ostendi. Secundum verò corollarium falsum esse, ijs rationibus quas nunc subiungam, patebit. Imaginemur. u. pro centro regionis elementaris, & libram. b. o. a. obliquam respectu ad. u. & brachiis æqualibus constātem, & pondera in. a. et in. b. etiam æqualia. lineæ autem inclinationum sint. a. u. et. b. u. imaginemur etiam lineam. o. u. & à centro. o. libræ duas. o. t. et. o. e. perpendiculares inclinationum lineis; vnde pondus ipsius. a. in huiusmodi situ tam erit proportionatum ponderi. b. quam proportionata erit lineam. o. t. lineam. o. e. ex eo q̄ tertio cap. huius tractatus probauit, sed lineam. o. t. maior est lineam. o. e. quod sic probo. Imaginemur triangulum. u. a. b. circumscriptum esse à circulo. u. a. n. b. cuius. c. sit centrum, q̄ erit extra lineam. u. o. cum supponatur. a. o. b. obliquam esse respectu ad. u. o. Imaginemur deinde à centro. c. lineam. c. o. s. vsque ad circumferentiam, quæ perpendicularis erit ipsi. a. b. ex tertia lib. 3. Eucli. si postea imaginemur duas lineas. c. a. et. c. b. habebimus ex. 8. lib. primi, angulum. a. c. o. æqualem angulo. b. c. o. Vnde ex. 25. lib. 3. arcus. a. s. æqualis erit arcui. b. s. sed si imaginabimur. u. o. ad circumferentiam vsque productam, clarum erit q̄ arcum. s. b. secaret in puncto. n. vnde arcus. n. b. minor erit arcu. n. a. & sic etiam angulus. n. u. b. minor erit angulo. n. u. a. ex ultima lib. 6. Imaginemur nunc alium quendam circulum, cuius. o. u. sit diameter, cuius circumferentia per duo puncta. e. et. t. prætergradiat, cum in ipsis sint anguli recti, quod quilibet ex se ratiocinando colligere potest, si. 30. lib. 3. in mentem reuocauerit. Sed cum angulus. o. u. t. sit maior angulo. o. u. e. arcus. o. t. maior erit arcu. o. e. ex vltima. 6. vnde corda. o. t. maior erit corda ipsius. o. e. ex conuerso. 27. lib. 3. quod est propositum. Pondus igitur ipsius. a. in huiusmodi situ, pondere ipsius. b. grauius erit. Quod è directo ijs repugnat quæ Tartalea in 2. parte quintæ propositionis ediderit, & per consequens 2. corollarij falsitatem ostendit, vt eam quoque, quæ in 6. propositione latet, quia cū

pro-

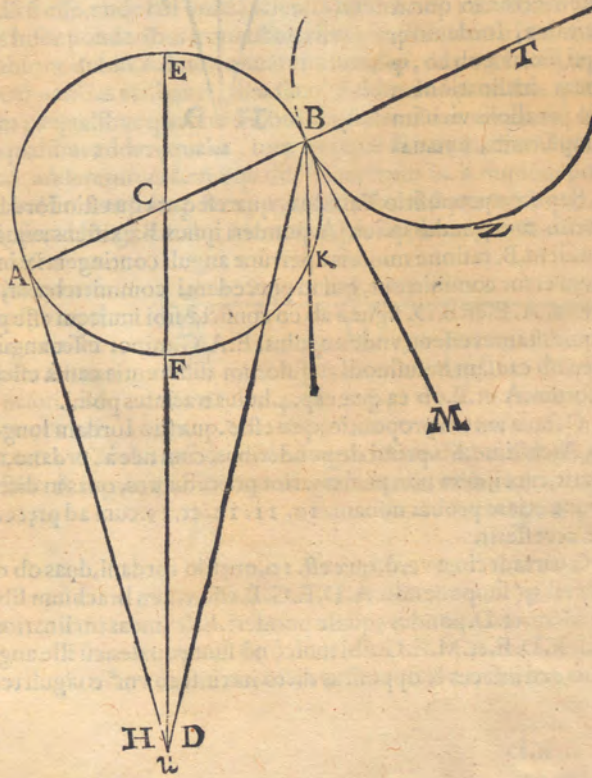
proportio p<sup>o</sup>deris. a. ad pondus ipsius. b. eadem sit cum ea quę est. o. t. ad. o. e. sub cognitione nostram cadere potest, primum cognoscendo angulos obliquitatis librę, id est angulos. b. o. u. et. a. o. u. quia oportet semper supponere situm aliquem notum. Si nobis deinde cognita erit proportio ipsius. o. u. ad. o. b. et. ad. o. a. assequemur cognitionem anguli. b. et. o. a. u. & per consequens ipsius. o. a. t. eius residui, unde postea beneficio angulorum. e. et. t. rectorum & laterum. o. b. et. o. a. cognitorum in cognitionem. o. t. et. o. e. facile deueniemus.



CAP. VIII.

Quod autem idem Tartalea in. 6. propositione, & Jordanus in secunda parte. secunda propositionis scribunt, maximum quoque errorem in se continet.

Dicunt enim angulū h. a. f. differentem ab angulo. d. b. f. alia ratione non esse quàm per angulum contactus duorum circularū, ut in sua figura scribit Tartalea; id quod falsissimum est. Quā ob causam in subscripta figura sit libra. B. A. & eius centrum. C. et. u. centrū regionis elementaris, et. A. u. et. B. u. lineę inclinationū. Imaginemur deinde lineam. B. K. parallelā ipsi. A. u. quę gyrum. B. E. A. in puncto. K. communi scientię precepto scindet, & habebimus angulum. K. B. Z. æqualem angulo. H. A. F. id est. u. A. F. (quia. H. u. et. D. unū sunt) cum ex. 29. libr. primi Euclidis angulus.

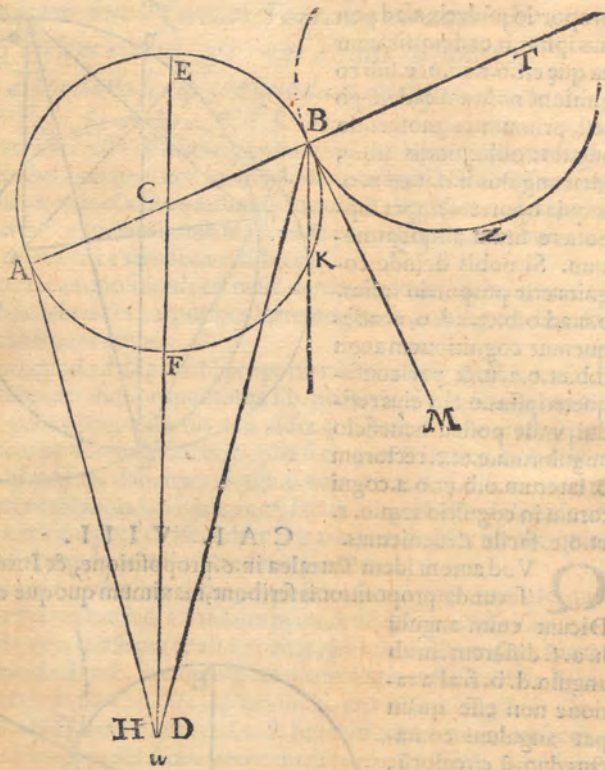


lus u. A. C. æqualis fit angulo. K. B. T. & angulus. C. A. F. æqualis angulo. T. B. Z. nūc comparatio est inter angulum. D. B. F. & angulum. K. B. Z. mistilineos, qui quidem duo anguli, cōmunem habent angulum mistilineum. K. B. F. quapropter si angulus. K. B. Z. mistilineus maior est angulo. D. B. F. mistilineo per angulum. K. B. Z. contingentia, circulorum ergo angulus mistilineus communis. K. B. F. æqualis erit mistilineo, angulo. D. B. F. pars videlicet sui toro. Omnis autem error in quem Tartalea, Iordanusque lapsi fuerunt ab eo, & lineas inclinationum pro parallelis vicissim sumperunt, emanavit.

Septima propositio Tartaleæ, quæ est quinta quæstio Iordani mihi videtur excipienda risu, cum pondus ipsius. A. ponderi ipsius. B. existens æquale, grauius sit pondere eiusdem. B. ratione minoris apertura anguli contingentia in. A. quam in. B. in quo idem error committitur, qui in præcedenti committebatur, cum scilicet ipse putet lineas. A. E. et. B. D. figuræ ab eo confictæ sibi inuicem esse parallelas, quæ etiam si æquidistantes essent (unde angulus. E. A. G. minor esset angulo. D. B. F.) non eam tamen ob causam huiusmodi angulorum differentia causa esset differentia grauitatū ipsorum. A. et. B. ob ea quæ cap. 4. huius tractatus posui.

Octaua autem propositio, quæ est. 6. quæstio Iordani longè melius demonstratur ab Archi. in. 6. lib. primi de ponderibus, cum nec à Iordano, nec à Tartalea probata fuerit, cum ijdem non probauerint præcedentes, quas in dicta. 8. Tartalea citat, qui neque etiam probat nonam. 10. 11. 12. et. 13. cum ad præcedentes probandas minime accesserit.

Quarta decima verò, quæ est. 10. quæstio Iordani, duas ob causas est falsa, quarum vna est, & supponendo. A. D. E. G. B. esse vnum brachium libræ, et. A. punctum cætri eiusdem, et. D. pondus æquale ponderi. E. & lineas inclinationum. D. K. et. E. M.) anguli. K. D. E. et. M. E. G. sibi inuicem non sunt æquales; cū ille angulus sit intrinsecus, hic verò extrinsecus & oppositus dicto intrinsecos vni<sup>9</sup> trianguli terminati à. D. E. à. D. K. et. E.



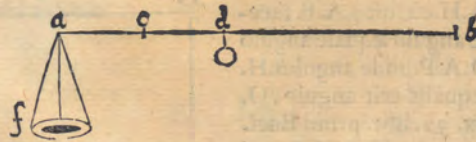
et. E. M. lineis productis vsque ad centrum regionis elementaris, vnde dictus angulus. M. E. G. maior est alio, ex. 16 lib. primi Eucli. Qua ratione fit, vt hanc ob causam E. grauius sit ipso. D. cum minus dependeat à centro. A. vt primo cap. huius tractatus iam dixi. Alia quoque est ratio, qua dictum. E. grauius sit ipso. D. quæ quidem est maior distantia à centro. A. libræ, per similes rationes capit. 4. huius tractatus citatas.

Decimaquinta quoq; nil penitus valet, quæ est. 11. quæstio Iordani, cuius Authoris opusculum opera Traiani Bibliopolæ Venetijs è tenebris in lucem emerfit.

*Quòd summa ratione statera per aequalia interualla sint diuisa.*

C A P. I X.

Magna cum ratione diuiditur statera per interualla equalia, in libras, aut in vncias, aut quoquo alio modo. Nam fit statera exempli gratia. a. b. & punctum, quod eam sustinet sit. c. & vas illud, quod continet id, quod ponderari debet f. Imaginemur nunc quod pondus brachij. c. b. ab una parte, & pondus brachij. c. a. cum eo, quod est dicti vasis. f. ab altera parte, sint causæ, quibus statera. a. b. c. stet horizontalis. cui sic horizontali manenti imaginemur ad punctum. a. adiunctum esse pondus, veluti vnus libræ. & ad punctum. d. tam distanti à. c. ut est. a. ab ipso. c. aliud quoque pondus vnus libræ additum esse, vnde cõi quadã scientia statera, non mouebitur situ. quæ existentibus duobus hisce ponderibus equalibus, altero in. d. & altero in. a. remota cum essent. d. b. et. f. absque dubio. a. d. non mutaret situm, sed. d. b. et. f. in situ, in quo reperiuntur, à centro paribus viribus prædita sunt. Addendo igitur. d. b. ipsi. d. et. f. ipsi. a. summa earum, equalibus quoque viribus constabunt. ex communi sententia, quæ habet si equalibus addas equalia, tota quoque fiet equalia. Si verò ponderi ipsius. a. aliud adderetur eidem equalè, haberemus in. a. duplum pondus ei quod est ipsius. d. sed volentes vt solum cum pondere ipsius. d. statera stet horizontalis, si dictum pondus ipsius. d. longè distabit à centro. c. per duplum ipsius. c. a. id est ipsius. c. d. id quod volumus assequemur, beneficio supradictarum rationum, adiuti opera sextæ lib. primi de ponderibus Archimedis. Et si quis aliud quoque pondus adiungeret ipsi. a. equalè illi priori, ad efficiendum, vt statera semper horizontalis maneret, oporteret, vt pondus ipsius. d. ab. c. longè distaret, ita vt huiusmodi distantia tripla esset primæ, & sic per quosdam quasi gradus interualla redderentur equalia.



Quòd

*Quòd linea circularis non habeat concauum cum conuexo coniunctum, & quod Aristo. circa proportiones motuum aberrauerit.*

## C A P. X.

**A**ristoteles in principio quaestionum Mechanicarum ait lineam, quæ terminat circulum videtur conuexum habere coniunctum cum concauo, quod falsum est: quia huiusmodi linea partes nullas secundum latitudinem habet, ( vt ipse etiam confirmat ) sed est idem conuexum circuli: linea verò quæ terminus est superficiæ ambientis, & amplectentis circulum est eadem concauitas dictæ superficiæ eundem circulum ambientis, quæ nullam conuexitatem habet. & hæc duæ sunt lineæ, quarum vna diuersa est ab alia, neque altera alterius, quod ad conuexum, & ad concauum attinet.

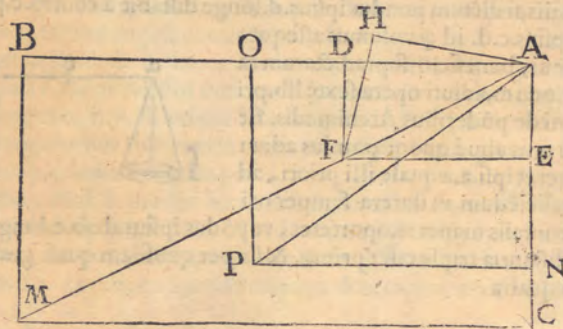
Sed illud, quod Aristoteles scribit de duplici respectu motus vnius puncti secundum vnã datã proportionem, non sufficit, ille enim sic ait.

Sit proportio secundum quam latum fertur, quam habet. A. B. ad. A. C. et. A. qui dem fertur versus. B. A. B. verò subterferatur versus. M. C. latum autem sit. A. quidẽ ad. D. vbi autem est. A. B. versus. E. Quoniam igitur latioris erat proportio, quam. A. B. habet ad. A. C. necesse est &. A. D. ad. A. E. hanc habere rationem. Simile igitur est proportione paruum quadrilaterum maiori. Quamobrem etc.

Cui respondeo, punctum. A. quod mouetur in linea. A. M. ab. A. versus. M. vsque ad. F. non moueri ab aliqua proportione determinata magis quàm ab alia: vnde nõ solum possumus imaginari dictum punctum. A. moueri ab. A. vsque ad. F. eiusdem velocitatis sub alia quadã proportione, sed etiam sub alia, quæ iam datã contraria sit, vt est proportio ipsius. A. C. ad. A. B. imaginãtes moueri. A. versus. C. et. A. C. versus. B. M. de latã. Dico etiam idem. A. moueri vsque ad. F. secundum proportionem ipsius. A. O. ad. A. N. Quamobrem imaginemur à puncto. F. lineam. F. H. cum linea. F. A. efficere angulum æqualem angulo. O.

P. A. & à puncto. A. lineã A. H. cū lineã. A. F. facere angulũ æquale angulo O. A. P. unde angulus. H. æqualis erit angulo. O. ex. 32. libr. primi Eucl. & triangulũ. A. H. F. qui angulum erit triangulo. A. O. P. Quam ob causã eadẽ proportio erit ipsi⁹ A. H. ad. F. H. quæ ẽ ipsius A. O. ad. O. P. punctum igitur. A. vsque ad. F. mouetur secundum proportionem etiam ipsius. A. O. ad. O. P.

Huiusmodi igitur consideratio, ab Aristotele facta, nullius est momenti.



Quòd



*Quod Aristo. in prima mechanicarum questionum eius quod inquirat, ueram causam non attulerit.*

C I A P. I X I.

**Q**uærens Aristoteles unde fiat, ut ex libra, quæ brachia habent alijs longiora, sint exactiores cæteris, ait hoc euenire ratione maioris velocitatis extremorum earundem. Quod verum non est; quia hic effectus nil aliud est, quam clarius proponere ob omniū oculos obliquitatem brachiorum à linea horizontali, & ostendere etiam facilius à dicto horizontali situ exire brachia iam dicta. Quæ quidem per se neque à velocitate, neque à tarditate motus, sed à ratione vectis, & à maiori interuallo inter secundum situm extremorum à primo proficiuntur. Vt exempli gratia, imaginemur magnam libram. A. B. horizontalem, cuius centrum sit .E. et pondus. B. maius sit pondere ipsius. A. unde conceditur, quod ob hanc rationem dicta libra, situm mutabit, qui secundus situs sit in. H. F. Imaginemur etiam parua quædam libram. a. e. b. horizontalem, quæ pondera habeat. a. et. b. æqualia duobus ponderibus alterius libraræ & secundus situs sit in. h. f. ita tamen ut anguli circa .e. æquales sint ijs, qui sunt circa. E. id est. b. e. f. sit equalis. B. E. F. Nunc dico situm .H. F. exactiorē futurum & clariorem situ. h. e. f. ratione interualli. B. F. maioris, interuallo. b. f. quod. B. F. in eadem proportione maior est ipso. b. f. in qua. B. E. maius est. b. e. quod autem interuallum. B. F. breuiori, aut longiori temporis spacio quam. b. f. sit factum, nil planè refert. Ratione vectis deinde, dico quod si supponemus duas libras pares æqualesq; in omni alio respectu, præter quam in brachiorum longitudine, pondus. B. maiorem vim habebit ad deprimentum brachium. E. B. quam pondus. b. quia libra materialis, cum sustineatur ab .E. e. & non à puncto mathematico, sed à linea, aut superficie naturali in materia existente, unde aliqua resistentia ipsi motui brachiorum oritur, & hanc ob causam, supponendo hanc resistentiam æqualem tam in. E. quam in. e. clarum erit ob ea, quæ in cap. 4. huius tractatus ostendi. B. cum minus dependeat ab. E. aut minus quoque eidem. E. annitatur, ponderosum magis futurum, quam. b. & hac de causa mouebit ad partem inferiorem, maiori cum agilitate, brachium. E. B. multo magis etiam illud ipsum deprimet, id est maiorem etiam angulum. B. E. F. quam erit angulus. b. e. f. faciet.



bono

V DE

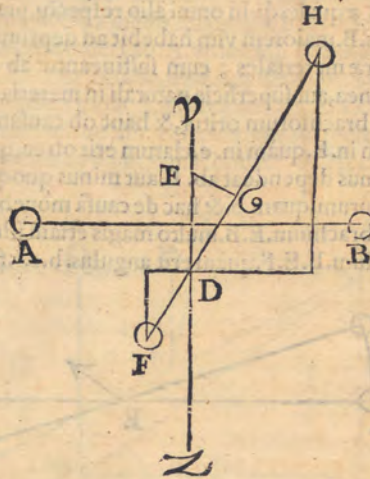
*De vera causa secunda, & tertia quæstionis mechanica  
ab Aristotele non perspecta.*

C A P. X I I.

**A**ristoteles in secunda quæstionum mechanicarum quærens illius rationem sic scribit.

Cur siquidem sursum fuerit spartum quando deorsum lato pondere quispiam id admouet rursus ascendit libra: si autem deorsum constitutum fuerit non ascendit, sed manet? an quia sursum quidem sparto existente plus libræ extra perpendicularum sit (spartum enim est perpendicularum) quare necesse est deorsum ferri id, quod plus est, quare & cætera.

Sed vera causa, vnde fiat, vt si spartum fuerit sursum, & brachium vnum ipsius libræ deprimendo, & idem liberum deinde permittendo, ad situm horizontalem redeat, non solum est maior quantitas ponderis brachiorum quæ iam prætergressa est vltra verticalem lineam, sed etiam est longitudo brachij eleuati, quæ vltra verticalem lineam reperitur, vnde eius extremi pondus redditur grauius in proportione, quam in hoc exemplo proponam, sit. A. B. libra in situ orizonrali, cuius spartum sit. E. super ipsam. & deprimentes brachium ipsius. A. vsque ad. F. eius situs sit in. F. H. vnde medium punctum. G. prætergressum erit lineam verticalem. V. Z. versus. B. quæ. V. Z. fecabit brachium. F. G. in puncto. D. vnde. D. H. longius erit ipso. F. D. Nunc nobis supponendum est id, quod verissimum existit, dictam scilicet libram in situ. F. H. etiã si sustineatur à puncto. E. idem tamen futurum ac si sustentaretur in puncto. D. vnde sequitur, quod pondus appensum ex ipsa. H. ita grauius reddatur, ipso. F. in eadem proportione, quæ maior est. D. H. ipso. D. F. ob rationes quas in primis huius tractatus capitibus posui, vt etiam si. D. H. quod materiale esse supponitur, nullam planè grauitatem haberet, solus tamè excessus vis ponderis in. H. positi, longè maior pondere in F. collocato pro maiori longitudine ipsius D. H. sufficiat. ad præstandum vt libra ad situm horizontalem redeat.



In secunda deinde huius quæstionis parte, in qua scribit libram in situ, in quo posita est, firmam manere, toto cælo aberrat, quia necessariū est, vt omninò cadat, eò usq; quò spartum sursum remaneat: ablato tamen omni impedimento, quod nulla eget probatione, cum natura sua clarissimè pateat.

Causa, deinde, vera tertiæ quæstionis non est ea, quam Aristoteles ponit, sed huiusmodi effectus ab eo, quod capitibus. 4. et. 5. huius tractatus proposui originem habet.

Quod

*Quod Aristotelis ratio in 6. quaestione posita non sit admittenda.*

CAP. XI I I.

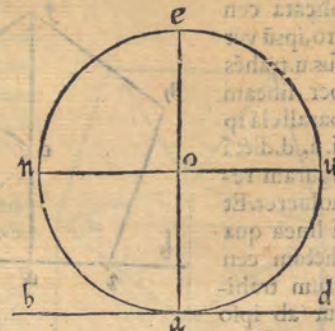
**V**olens Aristoteles rationem proponere, unde fiat, ut naus velocius moueatur cum antennam altiorem quam cum depraessioem habet, id ad veētis rationem refert, quod verum nō est. Huiusmodi enim ratione nauis tardius potius, quam velocius ferri deberet, quia quanto altius est velum, vi venti impulsū, tātō magis proram ipsius nauis in aquam demergit. Sed huiusmodi effectus à maiori potius quantitate venti quam recipit, quam ab alia aliqua causa oritur, quia ventus liberius vehementiusq; in altiore parte, quam in depraessione vagatur & perflat.

*Quod rationes ab Aristotele de octaua quaestione confictae sufficientes non sint.*

CAP. XI I I I.

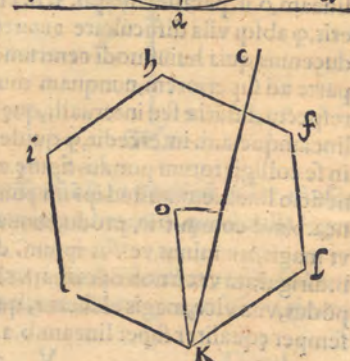
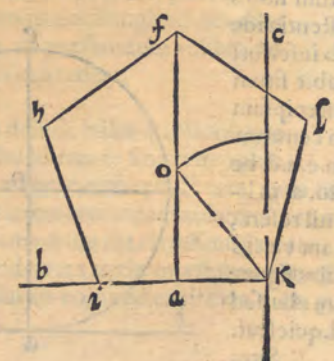
**R**ationes etiam ab Aristotele propositae pro indaganda octauae quaestionis veritate, in qua quaerit unde fiat, ut corpora rotunda figurae, ad voluendū sint faciliora reliquis, quarum reuolutionum corporum tres species assignat, quarū vna est, ut rotarum curruū; altera ut rotarum puteorum, aut trochlearum, quibus hauritur aqua; & tertia, ut paruorum vasorum a figulis fabricatorum, sufficientes nō sunt.

Incipiens autem à prima dico dubium non esse, quin tangente corpore aliquo rotundo aliquod planum mediante solo quodam puncto contingat, quemadmodum probat Theodosius in. 3. lib. primi & Vitellio in. 7. 1. lib. primi, & ducēdo per centrū sphaerae lineam vsque ad punctum contactus, ipsa erit perpendicularis plano contingenti sphaeram dictam, ut probat idē Theodosius in. 4. lib. primi Al hazē in. 2. 5. quarti, & Vitellio in. 7. primi. Verum etiam est omnem inclinationem ponderosam huiusmodi corporis homogenei totam hanc lineam aequaliter omni ex parte circūdare; cuius quidem rei exemplum in carta describere possumus mediante figura circulari hic subscripta. a. n. e. u. contigua linea recta. b. d. in puncto. a. unde. e. o. a. perpendicularis erit ipsi. b. d. ex. 1. 7. lib. 3. Eucli. & tantū ponderis habebimus à parte. a. u. e. quantum ab ipsa. a. n. e. Nunc igitur si imaginabimur ductum esse centrum versus. u. per lineam. o. u. parallelam ipsi. a. d. clarum nobis erit, q; absq; vlla difficultate aut resistantia idē ducentus, quia huiusmodi centrum ab inferiori parte ad superiorem, nunquam mutabit situm respectu distantiæ seu interualli, quæ inter ipsum lineamque. a. d. intercedit, q; quidem centrum in se colligit totum pondus figure. a. n. e. u. & beneficio lineæ. e. o. a. illud ipsum puncto. a. in linea. b. a. d. committit, productum. a. nil refert, ut magis, aut minus versus ipsum. d. aut versus b. dirigat; ita ut cū non oporteat ut huius figure pondus, vna vice, magis eleuetur, quam alia, sed semper aequaliter super lineam. b. a. d. quiescat.



V 2 Sitq;

Sitq; semper diuisum à linea.a.o.e.per medium, sequitur communi quodam conceptu, nullam nobis difficultatem oborituram, dictum centrum ad quam voluerimus partem ducendo, quemadmodum à qualibet alia figura, quæ perfectè rotunda non esset, emergeret; Vt exèpli gratia, si imaginabimur pentagonum.K.i. h.f.l. quiescere sup eandè lineã.a.b. K. ita ut primũ totũ latu. i.K. in linea.b.K. extēdat, ducēdo postea centrum.o. (ponamus.) versus.l.dubium non est, quin oporteat, vt dictum centrum.o. à linea.b.d. eleuetur, ab eademq; magis distet, voluens se per arcũ vnum circuli, q; p suo semidiametro habeat.o.K. quæ maior est ipsa.o.a.ex. 18. li. primi Eucli. vnde si a puncto.K. imaginabimur lineam. K. c. respicientem centrum regionis elementaris, dubium non est, quin si velimus transferre cētrum hoc à priori situ vsq; ad dictam lineam, oporteat addere pondus parti ipsius.l. quæ à linea.K. c. fuit secta, aut aliquid de ipso pondere partis centri detrahere. quod quibusuis modis fiat, arduum certè est ad efficiendum; neque hoc etiam accidit figuræ perfectè rotundæ, cum cētrum q; perfectè in medio ipsius ponderis est, reperitur semper in lineã perpendiculari ipsi plano, in quo animaduertendum est, q; etiam si ipsum planum appellem; pro plano tamen perfectò intelligi nolo, sed pro superficie perfectè spherica circa centrum à corporibus grauib; expetitum; nam ratione magnæ amplitudinis huiusmodi superficie, nullam differentiam notatu dignam à perfectò aliquo plano exigui interualli ad curuitatem eiusdem superficie imaginari poterimus. Sed ut redeamus ad sermone de reuolutione figuræ rotundæ susceptum, clarũ igitur erit quamlibet minimam vim (vt ita dicam) quæ trahat, aut impellat centrum.o. versus.u. huiusmodi figuram reuoluturam, cuius media pars ad trahendum, aut impellendum punctum.e. sufficere; Imaginemur autem q; linea.n.o.u. esset libra quædã in figura perfectè rotunda.a.n.e.u. posita, & vis, quæ trahere centrum deberet, diuisa esset per medium, cuius medietas appensa esset extremitati. u. diametri.n.o.u. clarũ erit, q; absque vlla difficultate reuolueret figuram super lineam.b.a.d. versus. d. quia huius vis, aut pondus nullũ contra pondus haberet vltra centrum.o. uersus. n. q; centrum. o. perpetuo quiescit sup. a. in linea. e. o. a. per medium diuidente semper totum pondus figuræ suppositæ. Tantò facilius ergo tota dicta vis applicata centro, ipsũ versus.u. trahēs per lineam parallēlã ipsi. a. d. dictã figuram reuolueret. Et si linea qua dictum centrum trahitur ab ipso



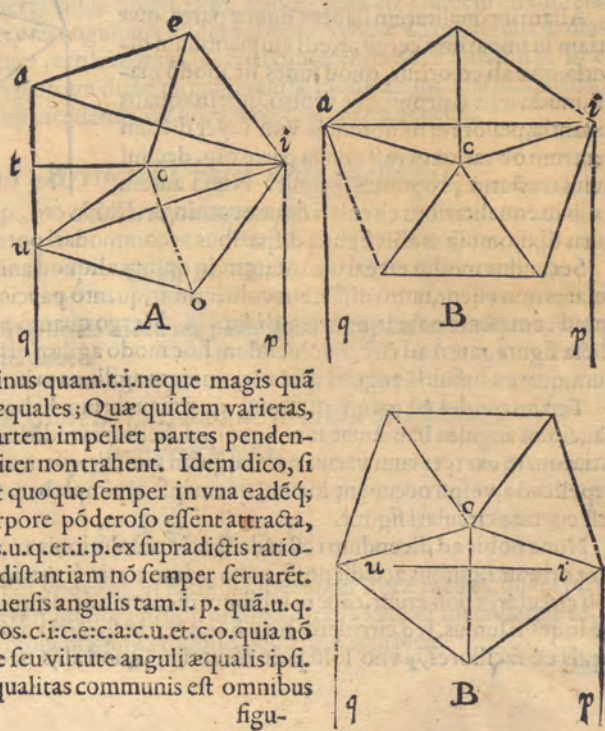
b.a.

b.a.d. non æquedistaret, sed sursum traheret super. u. aut subter, aliquid de sua vi virtuteq; amitteret, & tantò plus, quantò inclinata magis esset versus. a.o.e. & tandem cum esset vnita cum. a.o.e. aut ad superius, aut ad inferius quantalibet ui, etiam si infinita, figuram extra situm primæ lineæ. a.o.e. non moueret, sed si sursum traheret se iungeret eam à linea. b.a.d. non ob id tamen efficeret, ut centrum. o. exiret extra primam lineam. a.o.e.

Secunda verò species, tribus reuolutionum modis, absque axis mutatione constare potest, idest modo, quo reuoluuntur trochleæ mediante fune, & quo reuoluuntur aliquæ rotæ, in quibus aliquod animal incedit; & quo reuoluuntur illæ, quæ in hominis manu circunuoluuntur medio alicuius manubrij inflexi. Hi omnes modi cum circulari figura magis, quàm cum alia quauis, faciliores euadunt. Et primò si priorem modum considerabimus, vt mediante fune quælibet figura, quæ circularis non sit, voluatur, supponamus exemplo debere reuolui pentagonum æquiangulum. a.e.i.o. u. circa centrum. c. mediante fune. q. u. a. e. i. p. necessariò occurrent (in hac figura angulorum, laterumq; disparium) plures inæqualitates, quæ reuolutionem eiusdem figuræ irregularem efficiunt; quarum vna erit, quod duæ partes funis, idest. u. q. et. i. p. non erunt in vna eademq; inter se distantia semper, quod facile intellectu erit, si imaginabimur ductas esse lineas. a. i. u. i. et. i. e. t. si funis duo pondera habebit alterum altero maius, suis extremis appensa, vnde debeat figura virtute ponderis maioris circunuolui: dictæ duæ partes. u. q. et. i. p. eiusdem funis, mudi centrum, dum firmæ manebunt, respicient; sed permittentes pondera libera; maius, efficiens vt circunuoluatur figura; efficiet, vt aliquando vnum ex lateribus, eiusdem figuræ mundi quoq; centrum respiciet, vt in

figura. A. sicq; etiam linea. i. c. t. (pro exemplo) erit mensura distantie funium inter ipsas, & deinde circunuolueno etiam distabunt inter se per lineam. i. a. aut. i. u. vt in figura. B. Innotuit exemplo, & sic etiam aliquando erunt magis distantes, quàm linea t. i. & minus quàm. i.

a: nunquam tamen minus quàm. t. i. neque magis quàm i. a. aut. i. u. quæ sunt æquales; Quæ quidem varietas, in hanc, & in illam partem impellet partes pendentes funis, vnde æqualiter non trahent. Idem dico, si extrema. q. et. p. essent quoque semper in vna eademq; distantia; neque à corpore pôderoso essent attracta, quia aliæ partes ipsius. u. q. et. i. p. ex supradictis rationibus vnam eademq; distantiam nõ semper seruarét. vnde fieret vt cum diuersis angulis tam. i. p. quàm. u. q. traherét semidiametros. c. i. c. e. c. a. c. u. et. c. o. quia nõ semper traherent ope seu virtute anguli æqualis ipsi. c. i. p. Hæc autem inæqualitas communis est omnibus



figu-

figuris rectilineis tam paris, quàm disparis numeri. Sed aliam quandam maiorem inæqualitatem habent hæ figuræ numeri disparis, quæ est, quòd quâdo linea. t. i. tam .u. q. quàm ipsi. i. p. perpendicularis fuerit, idest quâdo. t. i. cum dictis partibus funis angulos rectos constituerit, tunc ratione longitudinis ipsius. c. i. maioris quam .t. c. (quia cum sit. c. i. equalis ipsi. c. a. et. c. a. maior ipsa. c. t. c. i. etiam maior fit ipsa. c. t.) pondus aut vis ipsius. p. superabit eam quæ est ipsius. q. sed

quando. t. erit in opposita parte, et. i. in ea, quæ est ipsius. t. q. eadem ob causam superabit. p. & sic motum faciet irregularem, & nõ uniformem; & ob id etiam perarduum, præterictus, quos infligunt anguli in partem pendentem ascendentem funis, quâdo vnum ex lateribus vnitur cum fune.

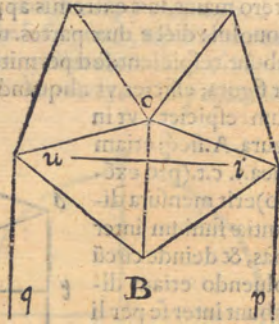
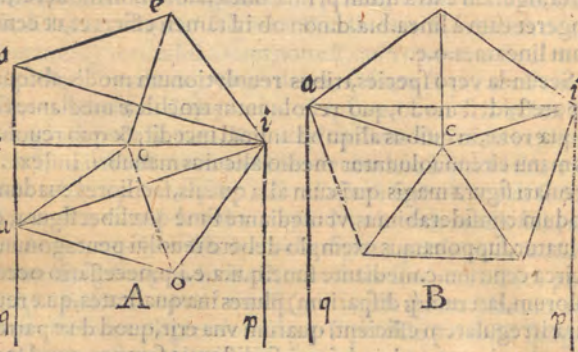
Aliam inæqualitatem habent figuræ pares, quæ etiam in imparibus cernitur, etsi aliquantulum diuersa; quæ ab eo oritur, quod funes sit modo magis, modo minus propinque centro; quæ inæqualis distantia, maiorem minoremq; vim super dictum centrum ob rationes in secunda parte cap. decimi huius tractatus propositas, gignit. Nulla autem ex ijs inæqualitatibus circulari figuræ contingit. Illud verò, quod de pentagonis figuris dixi, omnibus alijs figuris disparibus accommodari potest.

Secundus modus est earum rotarum, in quibus aliquod animal incedit, quæ si circulares non essent, tantò difficilius voluerentur, quantò pauciores angulos haberent, quod cum per se pateat, non demonstro. Si ergo quantò plures angulos habeat dicta figura, tantò ad circunuoluendum hoc modo agilior erit. Circularis igitur figura, quæ ex infinitis angulis efficitur, omnium agilissima erit.

Tertius modus est earum rotarum, quæ manubrium habent, quæ etiam quantò pauciores angulos habebunt, tantò quoq; difficiliore reddentur, tam ratione inimicitia; quam exercet cum vacuo natura, quàm violentiæ, quam anguli aëri faciunt, eum expellendo, vt ipsi occupent locum, quem ipse aër implebat. Quod nullo modo potest enenire circulari figuræ.

Nunc nobis ad dicendum restat de specie reuolutionis rotarum, quæ parallelæ sunt orizzonti, quibus accidit posse volui primo tertioq; modo secundæ speciei, & ob id si circulares non erunt, eadem subibunt incommoda, de quibus in secunda illa specie loquuti sumus, sed circulares rotæ huius tertie speciei ad reuoluendum erunt reliquis eò faciliores, q; vno solû polo nituntur; Quod alijs nequaquam conceditur.

Super



Super hac tertia specie formari potest problema, vnde fiat, vt quiescens huiusmodi rota parallela orizonti super vnum punctum, & quantò fieri potest existens equalis, si eam circunuoluamus maiore qua poterimus vi, & eadem postea dimittentes non perpetuò circunuoluatur.

Hoc quidem, quatuor fit ob causas. quarum prima est, quia huiusmodi motus, eius rotæ non sit naturalis. secunda est, quia etiam si rota super punctum mathematicum quiesceret, oporteret tamen vt superius alterũ haberet polum, qui ipsam orizontalem teneret, qui quidem munimento aliquo corporeo indigeret; vnde fricatio quædam consequeretur, ex qua resistentia prodiret.

Tertia est, quia aer contiguus eam perpetuò astringit, hocq; modo eius motui resistit.

Quarta est, quia quælibet pars corporea, quæ à se mouetur, impetu eidem à qualibet extrinseca virtute mouente impresso, habet naturalem inclinationem ad rectum iter, non autem curuum, vnde si à dicta rota particula aliqua suæ circumferentiæ distingeretur, absque dubio per aliquod temporis spatium pars separata recto itinere ferretur per aerem, vt exemplo à fundis, quibus iaciuntur lapides, sumpto, cognoscere possumus, in quibus, impetus motus impressus naturali quadam propensione rectum iter peragit, cum euibratus lapis, per lineam rectam contiguam giro, quem primo faciebat, in puncto, in quo dimissus fuit, rectum iter instituat, vt rationi consentaneum est.

Eadem, quoque ratione fit, vt quantò maior est aliqua rota, tantò maiorem quoque impetum, & impressionem motus eius circumferentiæ partes recipiant, vnde sepe euenit, vt dum eam sistere volumus, id cū labore & cum difficultate agamus; quia quantò maior est diameter vnus circuli, tantò minus curua est eiusdem circumferentiæ, & tantò propius accedit angulum eiusdem circumferentiæ ad quantitatem duorum angulorum rectorum rectilineorum, idest circumferentiæ ad rectitudinem lineæ rem. Vnde earundem partium dictæ circumferentiæ motus ad inclinationem sibi à natura tributam, quæ est incedendi per lineam rectam, magis accedit.

*Quod Aristotelis ratio nonæ quæstionis  
admittenda non sit.*

C A P. X V.

**V**era ratio nonæ quæstionis à secunda parte decimæ cap. huius tractatus, & non aliunde, accersiri debet.

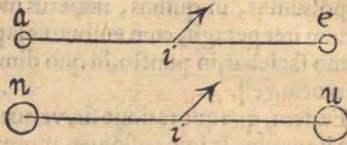
*Quod Aristotelis rationes de decima quæstione  
sint reijciendæ.*

C A P. X V I.

**A**ristotelis rationes, vnde fiat, vt facilius moueantur libræ vacuæ, quàm plene ad propositam disputationem non pertinent; quia semper ineunda est ratio proportionis virtutis mouentis super mobile; quod ipse non fecit.

Sit

Sit exempli gratia libra a. i. e. quæ in vtraque extremitate vnciam vnã solum ponderis obrineat, & sit libra n. i. u. æqualis priori, quæ pro singulâ extremitate vnã ponderis libram habeat. Aristoteles admiratur, quòd addendo ipsi e. mediã ponderis vnciam, brachium i. e. velocius cadat, quàm adijciendo ipsã mediã vnciã ipsi u. brachij i. u. Quod à duabus causis proficiscitur, quarum prior est, magna differentia proportionis vnus libræ ad medietatem vnus vnciæ, ad proportionem vnus vnciæ ad ipsam medietatem, quia si pondus adiectum extremo u. dimidiæ esset libræ, & cum eadem tarditate brachium moueret, optimo iure in admirationem posset Aristoteles duci. Sed hoc fieri non posset, quia ipsum deprimeret cum eadem quasi velocitate, quæ mediã vnciã brachium i. e. Dixi autem quasi, quia non nihil discriminis intercederet, quod proficiscitur à secunda ratione. Et hæc, resistentia est, quæ oritur à sparto, quia quânto maius pondus continet libra, tantò magis pramittit spartum in loco, in quo sustinetur; vnde maior resistentia in circunvolutione eiusdẽ sparti, in loco, in quo quiescit, exoritur, quia ipsum est corpus materiale. Si quis autem vellet, vt brachium i. u. eadem agilitate, quæ i. e. descenderet, oporteret, vt proportio dimidiæ libræ adiectæ ponderi ipsius u. quod est vnus libræ, vim suam haberet, quæ excederet resistentiam sui sparti (medio brachiorum maiorum ijs qui sunt. a. i. e.) ita proportionatam, vt proportionata est vis dimidiæ vnciæ ipsi e. iunctæ, resistentiæ sui sparti. Huiusmodi rationes cum rotis grauioribus leuioribusq; & ijs, quæ à corporibus quibuslibet grauib; impelluntur, accommodatæ fuerint, titubantem intellectum confirmabunt.



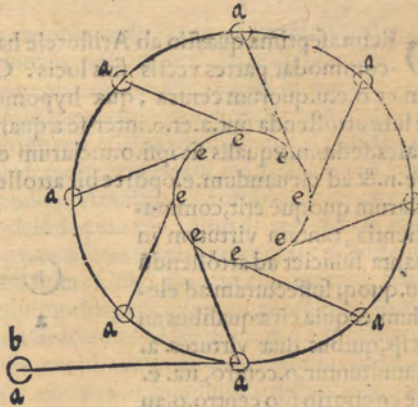
*De vera causa. 1 2. quæstionis mechanica.*

C A P. X V I I.

Vera ratio, cur multò longius corpus aliquod graue impellatur funda, quam manu, inde oritur, quòd circunvoluendo fundam, maior impræssio impetus motus fit in corpore graui, quàm fieret manu, quod corpus liberatum deinde cum fuerit à funda, natura duce, iter suũ à puncto, à quo profiliit, per lineam contiguam giro, quem postremò faciebat, suscipit. Dubitandumq; non est, quin dicta funda maior impetus motus dicto corpori imprimi possit, cũ ex multis circumactibus, maior semper impetus dicto corpori accedat. Manus autem eiusdem corporis motus, dum illud ipsum circunvoluitur (pace Aristotelis dixerim) centrum non est, neque funis est semidiameter. Immo manus quam maximè fieri potest in orbem cietur; qui quidem motus in orbem, vt circumagatur etiam ipsum corpus, cogit, quod quidem corpus, naturali quadam inclinatione, exiguo quodam impetu iam incepto, vellet recta iter peragere, vt in subscripta figura patet, in qua. e. significat manum. a. corpus. a. b. lineam rectam tangentem girum. a. a. a. a. quando corpus liberum remanet. Verum quidem est, impræssum illum impetum, continuò paulatim decrescere vnde statim inclinatio grauitatis eiusdem corporis subingreditur, quæ sese miscens cum impræssione facta per vim, non permittit, vt linea. a. b. longo tempore recta permaneat,



maneat, sed citò fiat curua, cum dictum corpus. a. duabus virtutibus moueatur, quarum vna est, violentia impraſſa, & alia natura, contra opinionem Tartalea, qui negat corpus aliquod motibus violento & naturali ſimul & ſemel moueri poſſe. Neq; eſt ſilètio prætereūduſ hac in re qdā notatu dign⁹ effectus, qui eiufmodi eſt, q̄ quanto magis creſcit impetus in corpore. a. cauſa tuſ ab augmento velocitatis giri ipſius. e. tãtò magis oportet, vt lenſiat ſe trahi manu à dicto corpore a. mediante fune, quia quanto maior impetus motus ipſi. a. eſt impreſſuſ, tantò magis dictum corpus. a. ad rectum iter peragendum inclinatur, vnde vt recta incedat, tantò maiore quoque vi trahit.



*De decimatertia quaſtione.*

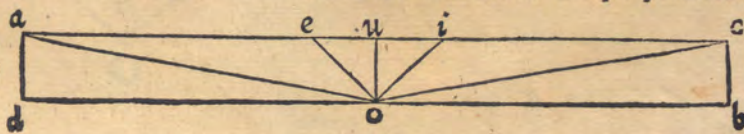
C A P. X V I I I.

**D**ecimatertia quaſtio ad vectem omnino eſt referenda. Imaginari debemus axem cylindrici iugi, hypomochlion eſſe. Quod reſtat, illud ipſum totum dependet à 4. quintoq; cap. huius tractatuſ. Vna tamen differentia inter hanc machinam, vectemq; reperitur, quæ eſt, q̄ iugum aliquam reſiſtentiam pro coniunctione calcata in loco, in quo voluitur, magis quàm hypomochlion vecti efficiat.

*De decimaquarta quaſtione.*

C A P. X I X.

**R**ationes etiam decimaquarta quaſtionis dependent ab ijs, quæ ſunt vectis, vt exempli gratia ſit lignum. a. b. c. d. frangendum in medio, annitendo genibus in punctum. o. clariffimè tunc videbimus, q̄ tenentes manuſ longè à medio, in locis a. et. c. faciliuſ minoriq; cum labore illum frangemus, quàm ſi eaſdem vicinas medio eiufdem ligni in locis. e. et. i. poneremus. Cuiuſ rei rationeſ eadē ſunt cū ijs, quæ primiſ huius tractatuſ capitibuſ propoſita fuerunt. Imaginemur lineas rectaſ ductaſ à puncto. o. ad loca. a. e. i. et. c. hinc manifeſtè perſpiciemus eorum, quæ iam diximus ratione, q̄ loca. e. et. i. mediantibuſ duabuſ lineiſ. e. o. et. i. o. magis annitentur. o. centro, quàm loco. a. et. c. duaruſ linearuſ. a. o. et. c. o. beneficio; vnde vim quoq; maiorem habebūt in. a. et. c. quàm in e. et. i.



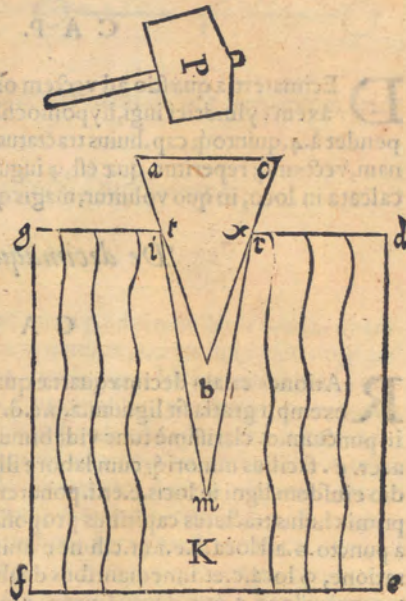
De ueritate. 17. questionis.

C A P. XX.

**D**ecima septima quaestio ab Aristotele haud bene percepta fuit, quia is non accommodat partes vectis suis locis. Quamobrem imaginemur duos vectes. a. o. n. et. o. e. u. quorum centra, quae hypomochlia appellantur sint. o. & pondera, quae sunt attollenda sint. a. et. e. inter se aequalia, & distantiae sint. a. o. et. e. o. sibi inuicem aequales, sed. o. n. aequalis sit ipsi. o. u. clarum erit, quod ad eleuandum. a. oportebit depri- mere. n. & ad eleuandum. e. oportebit attollere. u. Et quia omnia supponuntur aequa- lia, clarum quoque erit, commu- ni scientia, tantam virtutem in n. quanta sufficit ad attollendu- a. in. u. quoque; suffecturam ad ele- uandum. e. quia cum aequalibus an- gulis ijs, quibus duae virtutes. a. et. n. annituntur. o. centro, ita. e. et. u. e. contrario suo centro. o. an- nituntur. & omnes rationes pro- vecte. a. o. n. quarto quintoque; huius tracta- tus capitibus citatae, vecti. o. e. u. vt satis fu- perque; dixi in dicto capit. 5. conuenire pos- sunt.



Nunc sit aliqua pars ligni cindenda se- cundum venulas suas. d. e. f. g. & sit cuneus a. b. c. qui vi mallei. P. vsque ad. t. x. pene- trarit. Hinc clarum erit, quod apertura i. m. r. ligni, postquam infigitur cuneus se- cundum venas, longior erit parte. x. b. t. cu- nei, quae ingressa est. Oportet nunc ima- ginari duos vectes similes supradictae. u. e. o. in hunc modum, vt puncta. i. r. ligni sint loco. u. extremi ipsi<sup>9</sup> vectis, et. t. x. loco vir- tutis applicatae ipsi. u. & resistentia circa punctum. m. loco ponderis. e. vectis. o. e. u. dicti, & pars. K. quasi immediata post. m. versus extremitatem. f. e. ligni, sit loco hy- pomochlij. o. Hinc fiet vt quanto longio- res erunt linea. i. m. K. et. r. m. K. tanto quo- que facilius virtutes. t. x. impellent. i. r.



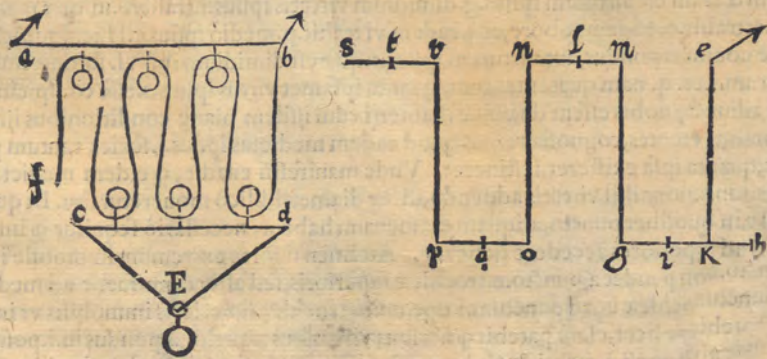
De

DE MECHANICA

De vera & intrinseca causa trochlearum.

CAP. XXI.

Pro intelligenda vera, & intrinseca ratione, unde fiat ut multitudo rotularum in trochleis causa sit, ut exigua vis sursum moueat, aut attollat potera magna. Imaginemur duas hic subscriptas trochleas explicatas transfuersaliter in hunc modum, id est sit paruū tignū. a. b. fixum & parallelū orizonti. cui sint rotulæ appense ab inferiori parte ad superiorem huiusq; e regione opposit<sup>o</sup> sit aliud tignū. c. d. quod moueri possit ab imo ad sumum, super quod totidem sint rotulæ aut radij, cū annexa postea fuerit funis puncto. b. fixo, eam faciendo pertransire per rotulas tam à parte superiore, quam ab inferiore; & appensum deinde cum erit paruū illi tigno. c. d. mobili pondus. E. ducendo postmodum extremum. f. funis transeuntis per rotulas, idem planè fiet quod à trochleis simul unitis fieri solet. Cuius quidem effectus ratio sub nostra cognitionem cadet facilius in huiusmodi figura. Imaginemur separatim stateram; g. h. cuius cētrum sit. K. ita situm, ut brachium. g. k. sit duplum ad brachium. K. h. supponendo igitur in puncto. g. pondus, aut uirtutem mouentem unius libræ, & in h. duarum librarum, absq; dubio hæ duæ uirtutes in huiusmodi distantijs à centro æquales inuicè erūt, ob rationes prioribus capitibus iam allatas, & statera orizontalis manebit. Unde clarum erit, quæuis etiam exigua uirtus adiuncta ipsi. g. mouebit stateram extra orizontalem situm. Nunc si puncto. i. ex æquo medio inter. g. et. K. applicata erit uirtus ipsius. h. non amplius considerato brachio. K. h. inclinante uirtute ipsius. i. eandem partem uersus, in quam inclinabat, quando erat in. h. sed uirtus ipsius. g. inclinet contrario modo, diuersoq; ab eo, quo inclinabat prius; clarum quoq; erit, communi conceptu, & ob ea, quæ cap. 5. huius tractatus sunt dicta. g. h. semper in eodem situ absque motu mansuram, hancq; stateram appellabimus mobilem, & primam. Imaginemur nunc à puncto. e. fixo descendere funem. e. K. quæ fulciat punctum. K. extremum diametri. g. K. quam intelligo pro diametro vnus ex rotulis inferioribus trochleæ; & sit. n. l. m. diameter vnus ex rotulis superioribus alterius paruū tigni defixi à parte inclinationis ipsius. g. & parallela diametro. g. K. cuius diametri centrum fixum sit. l. & sit coniunctum. g. punctum, à fune cum puncto. m. quæ tã perpendicularis sit primo diametro. g. i. K. quàm secundo. n. m. id est ita ut anguli. n. m. g.



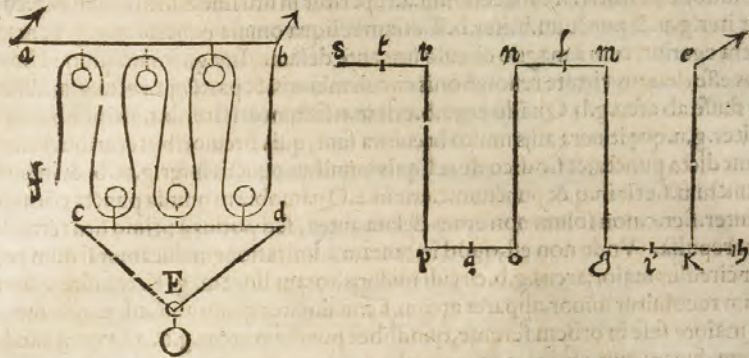
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et. m. g. k. sint recti. Imaginemur quoq; virtutem ipsius. g. applicatam esse extremo. n. cum inclinatione tamen contraria, id est ad inferiorem partem, quæ quidem virtus communi quodam conceptu eandem possidebit vim sustentandi immobilem diametrum. g. i. k. quam habebat, quæ erat in. g. cum inclinatione ad superiorem partem, & sic etiam diameter. n. l. m. non magis ab una, quam ab alia parte declinabit, quia cum quædam virtus in. n. reperiatur æqualis medietati virtutis ipsius. i. quæ virtus ipsius. i. vim habet deprimendi ipsum. g. id est. m. pro dimidia sui ipsius parte, sequitur. n. m. debere immobilem permanere. Nunc si alia diameter rotulæ mobilis erit desumpta, quæ sit. p. q. o. cuius centrum sit. q. in situ parallelo ipsi. n. l. m. & sic collocata, vt coniungendo. o. cum. n. anguli. m. n. o. et. n. o. p. sint recti: si imaginati fuerimus trāslatum esse pondus ipsi<sup>o</sup>. n. in. o. cū eadē inclinatione ad depræssiorem partem, illud ipsum, ac si esset in. n. communi conceptu, sine alicuius diametri mutatione præstabit. Et si centrum. q. fixum esset, & extremo. p. appositum fuisset pondus ipsius. o. cum inclinatione ad superiorem partem, idem etiam planè præstaret, etiam si nullum ullius diametri situm, communi scientia, mutaret, cum extremum. m. deorsum sit ductum à. g. virtute dimidiæ partis ipsius. i. & ab alia huic simili. m. quoque deorsum sit tractum ab. o. : quod quidem. o. deorsum est alteratum, ob inclinationem ad superius à virtute posita in. p. supponendo centrum. q. fixum. Sed si loco centri fixi, imaginabimur in. q. pondus aliquod æquale ipsi. i. quod duplum erit in virtute ad eam, quæ est ipsius. p. & ipsius quoque. g. sequetur etiā eadem immobilitas horum trium diametrorum. Quia cum sit huiusmodi pondus seu virtus in. q. cum inclinatione contraria virtuti in. p. quæ æquipollet dimidiæ parti ipsius. q. & sic ei quæ est ipsius. o. similiter quia. o. tractum est supra ab. n. virtute ipsius. g. quod. m. deorsum tradit, idcirco quanta erit vis quam habebit virtus in. q. ferendi deorsum diametrum. p. o. tanta quoque virtutes ipsorum. p. et. o. æquales, & æqualiter distantes à. q. ipsum ad superiorem partem inclinabunt. Quamobrem nec ascendet, nec descendet, nec locum mutabit. Supponamus nunc quartum diametrum rotulæ. s. r. r. quæ sit secunda rotularum fixarum, parallela ipsi. p. o. & in eo situ, quo coniungendo extrema. r. p. anguli o. p. r. et. p. r. s. sint recti, & imaginemur virtutem ipsius. p. reperiri in. s. cum inclinatione tamen contraria, id est deorsum versus, ex his idē quoque planè sequetur, id est quæ nulla harū quatuor diametrorum mouebitur. quia eundem effectū cū inclinatione deorsum versus efficeret dicta virtus in. s. quem in. p. cum inclinatione sursum versus, et iam dictum est virtutem ipsius. g. dimidium virtutis ipsius. i. trahere. m. quæ mediā te. n. attrahit. o. eodem robore, et. s. eadem vi trahit. p. medio ipsius. r. Hucusque scientificè nouimus pondus, aut virtutem ipsius. s. quæ est dimidium ipsi<sup>o</sup>. i. sustinere vim ipsorum. i. et. q. nam quater tantum, quanta ipsamet virtus ipsius. s. esse conspicitur. Et si adiuncte nobis essent duæ alie diametri cum iisdem planè conditionibus iisdem rationibus vtentes, cognosceremus quod eadem medietas ipsius. i. sexies tantum ponderis, quanta ipsa existeret, festineret. Vnde manifestū euadit, quod eidem medietati ipsius. i. in. s. non nihil virtutis addendo, dictæ diametri, illicò moueretur situ. Et quia rotulæ in quolibet puncto, aliquam diametrum habent, necessariò sequitur quod inferiores ad superiores accedere debeant. Attamen si forte extremum immobile ipsius funis non pendet à puncto. e. trochleæ superioris, sed alligatum fuerit ad mediū inferioris trochleæ ut ad punctum. i. ope unius trochleæ superioris immobilis vt in figura. A. videre licet, clarè patebit quod à tribus virtutibus æqualibus pondus in. i. positū sustinebitur; hoc est à. g. ab. i. & ab. k. quarū vnaqueque tertia pars erit ipsius. i. in contrariam partē, hoc est tertia pars resistentiæ. propterea quod ex æquo inter se distāt.

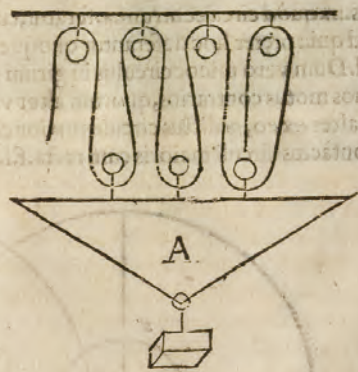
g. i.

alia virtus, ut si  
 ne virtus in sine  
 dicitur tota virtus  
 est. noluit. n. in. p.  
 distat in virtute  
 alia virtus, ut si  
 sed ut alia virtus  
 i. virtus. i. virtus  
 as conet. i. p. o. &  
 c. n. s. virtus. i. p.  
 virtus. i. p. o. &  
 do aliqua virtus  
 & virtus. i. p. o. &  
 virtus. i. p. o. &  
 virtus. i. p. o. &  
 virtus. i. p. o. &

g. i. et. K: Quia propter augebitur virtus per numeros impares, hoc modo; Nam. g. esset tertia pars resistentiæ, quemadmodum prius media erat. Idem infero de. m. n. o. p. r. et. s. Sed cum oporteat pondus. q. tantum esse vt sufficiat resistentiæ in. o. et. p. ipsum sustinere, idcirco ipsum pondus. q. subsequalter erit pōderi in. i. positi. Quapropter. s. quinta pars erit ponderum. i. et. q. Deinde si adhuc. duo diametri vnus inferior, alter verò superior additi fuerint cum pondere æquali. q. ad medium diametri inferioris, tunc pondus. s. erit septima pars trium ponderum. i. q. & tertij additi, ex



supradictis rationibus. Et quia virtus sustinens totale pondus trochleæ inferiori appensum in tot diuiditur partes æquales, quot sunt diametri orbiculorum trochleæ inferioris, quando extremum immobile finis alligatum fuerit trochleæ superiori, vt puta in puncto. e. cum verò alligatum fuerit trochleæ inferiori, virtus primi diametri. g. i. K. trochleæ inferioris semper sesqui altera erit vnicuique aliorum diametrorū; ideo virtus resistentiæ alterius extremi mobilis funis, puta. s. submultiplex erit totalis ponderis, eo modo quo diximus, cuius virtus, seu grauitas diuiditur seu distribuitur diametris inferioris trochleæ vt dictum est.

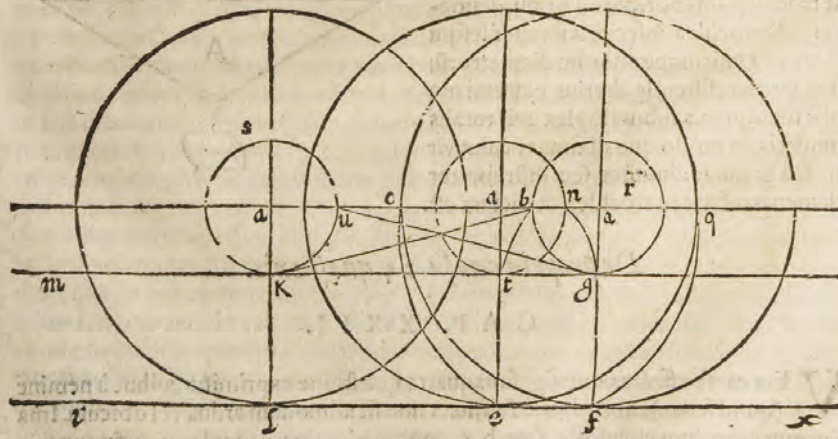


*De propria causa. 24. quæstionis.*

CAP. XXII.

Vera causa effectus, qui vigesimaquarta quæstione exprimitur, adhuc à nemine (quod sciam) animaduersa fuit, licet non sit admodum ardua vel obscura. Imaginemur ergo duos circulos. c. f. et. b. g. concentricos, itaq; simul coniunctos, vt si ipsum vnus feratur in orbem, alius quoque circumagatur, eo modo, quo curruum rotæ voluuntur. Et imaginemur primò super lineam. f. i. reuolui maiorem, & quando idem circulus erit in. l. dictam lineam. f. i. tangere circumferentiam eiusdem in puncto. c.

cto. c. vnde linea. g. m. mediante. K. continget circumferentiam circuli minoris in puncto. b. et. K. g. ex. 34. primi Eucli. æqualis erit ipsi. f. l. quia ex. 17. tertii, anguli. f. et. g. sunt æquales, vnde ex. 28. primi. f. l. et. g. K. sunt parallelæ. & sic erunt. k. l. cum. f. g. ex eadem supradicta. Ratio autem, qua arcus. g. b. transferit lineam. g. K. maiorem ipsa, est, quia dum mouetur, quodlibet punctum ipsius. g. b. virtute reuolutionis ipsius. f. c. omne punctum eiusdem arcus. g. b. vltius versus. K. quam si moueretur virtute reuolutionis ipsius. g. b. super lineam. g. m. deferretur. vt exempli gratia, quando virtute reuolutionis maioris circuli, centrum. a. reperitur in situ lineæ. l. K. punctum. g. confecerit iter. g. u. & punctum. b. iter. b. K. etiam reliqua omnia puncta inter. g. b. magna itinera egerint, cum à magno circulo sint ante delata. Imaginemur quoque hos circulos esse delatos virtute reuolutionis circuli minoris, & partē. g. t. rectē. g. m. dimensionem fuisse ab arcu. g. b. Quādo ergo. b. erit in. t. factum erit iter. b. t. ab ipso. b. et. g. faciet iter. g. n. quæ itinera alijs multò breuiora sunt, quia breuioribus cruribus reuoluta sunt dicta puncta, & sic dico de reliquis omnibus punctis inter. g. et. b. & in hoc casu punctum. f. erit in. q. & punctum. c. erit in. e. Quamobrem omnia puncta cōtingentiæ inter. f. et. c. non solum non erunt delata antea, sed potius à primo situ retrorsum erunt repulsa. Vnde non est, quòd in tantam admirationem ducamur si dum reuoluitur circulus maior, arcus. g. b. circuli minoris, totam lineam. g. K. transire videtur, & dum reuoluitur minor, apparet arcum. f. c. maius iter quam ab. f. ad. e. non facere, cum maiore sese in orbem ferente, quodlibet punctum arcus. g. b. ad vnam eandēq; partem duos motus obtineat. vt exempli gratia punctum. b. non solum mouetur versus. m. quòd circa. centrum. a. feratur, cum ipsum etiam centrum moueatur versus. m. sed quia præter hoc deferantur quoque à circulo maiori versus. m. vsque ad lineam. k. l. Dum verò minor circulus in girum ducitur, habet quodlibet punctum arcus. f. c. duos motus contrarios, quorum alter versus. i. virtute reuolutionis circuli minoris, & alter ex eo, quòd dictus circulus maior circa centrum. a. voluatur, vnde omne punctum contactus circuli maioris cum recta. f. i. retrorsum pellitur versus. x.



## D E M E C H A N .

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*De uera causa. 30. quaestionis.*

## C A P . XXIIII.

**V**era ratio, cur homo dum sedet (non tamen Turcarum more) velit sese in pedes erigere, calcaneos retrahit, vt efficiat angulum acutum, cum femoribus coxis à parte inferiori, & ventrem inclinat, ad constituendum etiam angulum acutum in superiori parte, ea est; vt totius corporis pondus, ex æquo, idest ab oppositis partibus circundet lineam rectam, quæ transit per locum, in quo consistunt pedes versus mundi centrum. idest, ut edatur æquilibrium ponderis ipsius corporis circum lineam illam, quæ sub pedibus inferuit pro sparto. Vnde aperiendo, deinde dictos duos angulos circa dictam lineam, absque vlla difficultate erigitur corpus, & absque periculo in alterutram partem cadendi.

*De ratione. 35. & ultima quaestionis.*

## C A P . XXV.

**V**era ratio, quare, quæ reperiuntur in vorticibus aquarum, semper versus medium ipsarum vertiginum vniuntur, inde promanat, quod media vertiginum semper depressiora sunt. vnde quod dicta corpora ad medium accedant, nihil aliud est, quàm ipsa corpora suo pondere grauitateque descendere, figura enim vorticibus est quasi conica, & concaua cum angulo deorsum, & gyro basis sursum. Atque hæc vera est huius effectus causa, & non ea quam Aristoteles ponit, à quo aliarum omnium quaestionum, quas ego omisi rationes sunt benè propositæ.

DISPV-

# DISPUTATIONES DE QVIBVSDAM PLACITIS ARISTOTELIS.



**A**NTA est certè Aristotelis amplitudo atque authoritas, vt difficillimum ac periculosum sit quidpiam scribere contra quam ipse docuerit, & mihi presertim, cui semper visa est viri illius sapientia admirabilis. Veruntamen studio veritatis impulsus, cuius ipse amore in seipsum si viueret excitaretur, in medium quadã proferre non dubitavi, in quibus me inconcussa mathematica philosophia basis, cui semper infisto, ab eo dissentire coegit.

*Qualiter & ubi Aristoteles de uelocitate motuum naturalium localium aliter tractauerit quam nos sentiamus.*

## C A P. III.

**V**Olens Aristoteles probare vacuum non esse in rerum natura. 8. cap. lib. 4. physicorum ait, idem corpus per varia diuersaq; media, vt per aerẽ, & per aquã si moueretur, proportionem uelocitatis eiusdem corporis per aerem, ei, quã per aquam fit, vnã eandemq; futuram cum ea, quã est subtilitatis aeris ad subtilitatem aquã. In postrema autem parte eiusdem capituli sic scribit: Nam cum ea quẽ maiorem vel ponderis vel leuitatis præstantiam habent, si simili figura sint, spaciũ par, & æquale, maiore celeritate conficere cernamus, ea quã magnitudines inter se habent, proportionẽ: profectò idem etiam per inane fieret. Aliam quoque rationem proponit philosophus. 2. cap. sexti physicorum scribens eademmet proportionẽ, qua tempus diuiditur, magnitudinem etiam diuidi. Sexto autem cap. primi de cœlo scribit, tempora eandem proportionem habere, quam habent è conuerso pondera; vt si media pars vnus ponderis, vnus horã spatio moueretur, vniuersum pondus in media hora moueretur. Secundo cap. lib. 3. de cœlo duobus in locis apertè comonstrat uelocitatem corporis minoris, maiori corpori comparatam, in eadem existere proportionẽ, in qua dicta corpora adinuicem relata existunt. Quinto cap. eiusdem lib. idem affirmat, exemplo ab igne desumpto. Ex alijs etiam plurimis locis cognosci potest, sensisse Aristotelem duo corpora eadem specie, & figura prædita eandem planè proportionem in suorum motuum uelocitatibus, quam in suis magnitudinibus habent, retinere. Alij quoque permulti eandem opinionem retinuerunt, & omniũ postremus Nicolaus Tartalea, secunda propositione vigesimionĩ quãsitũ octauĩ libri, vbi profiteretur se demonstratiuè probare hanc propositionem veram existere; neq; videt quã magna resistantiarum sit differentia, quã tam ex diuersitate figurarum, quã ex magnitudinum varietate exoriri potest; quas quidem diuersitates ne considerat quidem.

Quẽdam



*Quaedam supponenda ut constet cur circa uelocitatem motuum naturalium localium ab Aristotelis placitis recedamus.*

C A P. I I.

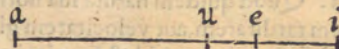
CUM susceperimus prouinciam probandi quod Aristoteles circa motus locales naturales deceptus fuerit, sunt quedam primo verissima & obiecta intellectus per se cognita presupponenda, ac primum qualibet duo corpora, graui, ut leuia, area æquali, similiq; figura, sed ex materia diuersa constantia, eodẽ que modo situm habentia, eandem proportionem uelocitatis inter suos motus locales naturales, ut inter suamet pondera aut leuitates in vno eodemq; medio, seruatura. Quod quidem natura sua notissimum est si considerabimus non aliunde maiorem tarditatem, aut uelocitatem gigni, quàm à .4. causis ( dummodo medium vni for me sit & quietum ) idest à maiori aut minori pondere aut leuitate; à diuersa figura; à situ eiusdem figuræ diuerso, respectu lineæ directionis, quæ recta inter mundi centrum, & circumferentiam extenditur; & ab inæquali magnitudine. Vnde patebit, quòd figuram non variando, nec in qualitate nec in quantitate, neque eiusdem figuræ situm, motum fore proportionatum uirtuti mouenti, quæ erit pondus aut leuitas. Quod autem de qualitatẽ, de quantitate & situ eiusdem figuræ dico, respectu resistentiæ ipsius medij dico: Quia dissimilitudo aut inæqualitas figurarum, aut situs diuersus non parũ alterat dictorum corporum motus, cum figura parua facilius diuidat continuitatem medij, quam magna; ut etiam celerius idem facit acuta, quàm ob tusa; & illa quæ cum angulo, qui antecedit mouebitur uelocius quàm illa, quæ secus. Quotiescunq; igitur duo corpora vnam eandemq; resistentiam ipsorum superficiebus, aut habebunt aut recipient, eorum motus inter se ipsos eodem planè modo proportionati consurgent, quo erunt ipsorum uirtutes mouentes: & è conuerso, quotiescunq; duo corpora vnam eandemq; grauitatem, aut leuitatem, & diuersas resistentias habebunt, eorum motus inter se ipsos eandem proportionẽ sortientur, quã habebunt eorum resistentiæ conuerso modo; quæ quidem resistentiæ inter se ipsas, eandem proportionem quàm ipsarum superficies habebunt, aut in qualitate sola si figuræ, aut in quantitate sola, aut in situ, aut in aliquibus ex dictis rebus, eo tamen modo, qui superius positus fuit, ut scilicet corpus illud quod alteri comparatum, æqualis erit ponderis, aut leuitatis, sed minoris resistentiæ, existet uelocius altero, in eadẽ proportionẽ, cuius superficies resistentiam suscipit minorem ea quæ alterius est corporis, ratione facilioris diuisionis continuitatis aeris, aut aquæ; Vt exempli gratia, si proportio superficiẽ corporis maioris superficiẽ minoris sesquitertia esset, proportio uelocitatis dicti corporis maioris, uelocitati corporis minoris, esset sub sesqui tertia; unde uelocitas minoris corporis, maior esset uelocitate corporis maioris, quẽ admodum quaternarius numerus ternario maior existit.

Aliud quoque supponendum est, uelocitatem scilicet motus naturalis alicuius corporis grauis, in diuersis medijs, proportionatam esse ponderi eiusdem corporis in iisdem medijs; Vt exempli gratia, si pondus rotale alicuius corporis grauis significatum erit ab a. i. quo corpore posito in aliquo me-



Y dio

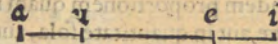
dio minus denso, quàm ipsum sit, (quia in medio se denfiore si poneretur, non graue esset, sed leue, quemadmodum Archimedes ostendit) illud medium subtrahat partem. e. i. vnde pars. a. e. eiusdem ponderis libera maneat; & posito deinde eodem corpore in aliquo alio medio denfiore, minus tamen denso quam ipsum sit corpus, hoc medium subtrahat partem. u. i. dicti ponderis, vnde pars. a. u. eiusdem ponderis remanebit. Dico proportionem velocitatis eiusdem corporis per mediũ minus densum, ad velocitatem eiusdem per medium magis densum futuram vt. a. e. ad. a. u. vt est etiam rationi consonum, magis quàm si dicamus huiusmodi velocitates esse, vt. u. i. ad. e. i. cum velocitates à virtutibus mouentibus solum (cum figura vna, eademq; in qualitate, quantitate situq; erit) proportionentur. Quæ nunc diximus, planè similia sunt ijs, quæ supra scripsimus, quia idem est dicere, proportionem velocitatum, duorum corporum heterogeneorum, sed similibus figura, & magnitudine æqualium, in vno solo medio, æqualem esse proportioni ponderum ipsorum, vt si dicam<sup>9</sup> proportionem velocitatum vnius solum corporis per diuersa media eandem esse cum ea, quæ est ponderũ dicti corporis in iisdem medijs.



*Posse uelocitatem alicuius corporis proportionem contrariam in diuersis medijs habere cum densitate eorum.*

### C A P. I I I.

**P**ossibile est in rerum natura corpus aliquod huiusmodi densitate præditum reperiri, vt velocitas eius motus naturalis per aerem, velocitati per aquam ita proportionata existat, vt est densitas aquæ densitati aeris. Densitas aquæ notetur (exempli gratia) per. u. i. & ea, quæ aeris est per. e. i. & pondus alicuius corporis in aere per e. a. & pondus eiusdem corporis in aqua per. u. a. ita tamen, quod eadem proportio sit. e. a. ad. u. a. vt. u. i. ad. e. i. vnde per vltimam suppositionem præcedētis capituli, proportio velocitatis prædicti corporis per aerem, proportioni eiusdem corporis per aquam erit, vt e. a. ad. u. a. ergo per. i. r. quinti, vt. u. i. ad. e. i.



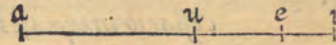
*Oscitanter ab Aristotele nonnihil prolaturum cap. 8. lib. 4. Physicorum.*

### C A P. I I I I.

**E**X supra dictis patet in vniuersum non esse verum quod Aristo. 8. cap. 4. lib. phisicorum scribit, velocitates scilicet motuum alicuius corporis per diuersa media, proportionatas esse densitatibus eorundem mediorum: Quæ circa, - fit proportio. u. i. ad. e. i. vt densitatis aquæ ad aereã densitatem. et. e. a. ad. u. a. vt ponderis alicuius corporis in aere ad pondus eiusdem in aqua, ita tamen vt maior aut minor proportio sit. e. a. ad. u. a. quam. u. i. ad. e. i. vnde existente proportione velocitatis per aere

ad

ad velocitatem per aquam vt. e. a. ad. a. u. non erit ergo vt. u. i. ad. e. i. Ob hanc igitur causam nimis dissentaneum est rationi, opinari proportionem velocitatis omnium corporum grauium per aerem vnam eandemq; esse cum velocitate eorundem per aquam, quemadmodum Aristoteles sentit.



*Exempla dictorum.*

C A P. V.

**P**onamus, exempli gratia, aquam esse in densitate dupla ad aerem, & aliquod graue corpus in aqua duplum ad densitatem ipsius aquae, vnde dictum corpus in densitate ad aerem quadruplum erit; quam ob causam, mediam sui ponderis totalis partem in aqua, & in aere quartam partem, ex. 7. lib. de insidentibus aquae ab Archimede conscripto, amitteret. Moueretur igitur in aqua virtute illius mediae partis ponderis sui, in aere aut virtute triu quartarum; vnde proportio facultatis moueris dicti corporis in aere ad facultatem mouentem eiusdem in aqua sesquialtera erit. hocq; corpus appelletur. A. Sit aliud quoque corpus, quod. B. nomenetur, simile figura, & magnitudine corporea corpori. A. sed densitate, in proportione sesquialtera ad aquam, & densius erit aere in proportione tripla. quamobrem corpus. A. grauius erit corpore. B. in aere in proportione sesquialtera, vnde etiam velocius erit ipso. B. in aere in eadem proportione, sed corpus. B. in aere, duplo maius pondus habebit, quam in aqua, cum in aere remaneant ei duae ponderis tertiae partes, & in aqua vna tantum, ita vt Aristoteli concedam corpus. B. in aere, quam in aqua velocius futurum in eadem proportione, in qua, aqua est densior aere, ex Euclidis vndecima propositione lib. quinti. Sed praeter haec omnia, si corpus. A. esset etiam velocius in aere, quam in aqua, in eadem proportione, sequeretur ex. 16. dicti lib. quinti proportionem velocitatis. A. in aqua ad velocitatem ipsius. B. in aqua etiam sesquialteram esse. Sed cum corpus. A. in densitate ad aquam duplum sit, & corpus. B. sesquialterum ad ipsam aquam, sequetur proportionem ponderis ipsius. A. ad pondus ipsius. B. in aqua esse in proportione dupla; Vnde ex primo supposito capitis secundi proportio velocitatis. A. ad velocitatem. B. in aqua dupla erit, non sesquialtera. Si ergo proportio velocitatis. A. ad eam quae est. B. in aqua dupla est, & ea, quae est. B. in aere, ad eam, quae est ipsius per aquam est etiam dupla (vnde ea quae est. A. per aquam equalis erit ei, quae est. B. per aerem, ex. 9. lib. quinti) & cum ea, quae est. A. sit ei, quae est. B. per aerem sesquialtera, erit ergo ea, quae est. A. per aerem, ei, quae est suimet ipsius per aquam sesquialtera, non autem dupla, ex. 7. eiusdem libr. quinti. Hisce rationibus accedimus ad confirmandam veritatem vltimi suppositi cap. 2. proportionem videlicet velocitatis motus naturalis in diuersis medijs alicuius corporis ponderosi in ipsis medijs esse eandem cum ea, quae est inter pondera dicti corporis in dictis medijs. de ijs tamen medijs intelligendo, quae unctuosae, aut pingua non sunt, ut sunt oleum, lac, aut huiusmodi alia, quae a qualibet minima qualitate frigoris aut caloris alterantur, & impermeabiles fiunt.

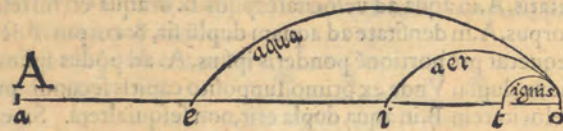
p aerem . A . 6 .		p aquam . A . 4 .
p aerem . B . 4 .		p aquam . B . 2 .
p aquam . B . 2 .		

Y 2 Quod

*Quod proportionēs ponderum eiusdem corporis in diuersis medijs proportionēs eorum mediōrum densitatum non seruant. Unde necessariō inæquales proportionēs uelocitatum produciuntur.*

## C A P. V I.

**O**Mne corpus graue variat proportionem ponderis per diuersa media, unde proportionēs uelocitatum inæquales existunt. Ut exempli gratia, si fuerit corpus. A. cuius pondus totale sit. o. a. quod in aqua diminutum sit ratione partis. e. o. ita ut ei solum relinquatur pondus. a. e. & in aere adempta sit ei pars. i. o. unde solum remaneat pondus. a. i. Supponamus aliud quoque medium in eadem proportione minus densum, quàm aer, quemadmodum aer minus densus est, aqua, in quo, corpus. A. ammittat partem. t. o. ponderis sui, unde ex. 7. lib. de insidentibus aquæ Archimedis, eadem proportio erit. e. o. ad. i. o. quæ est. i. o. ad. t. o. Supponamus quoque eandem proportionem esse. a. i. ad. a. e. est. e. o. ad. i. o. tunc dico non futuram eandem proportionem. t. a. ad. a. i. quæ est. i. o. ad. t. o. Cum sit ergo proportio. a. i. ad. a. e. ut. e. o. ad. i. o. erit disiunctim. e. i. ad. e. a. ut. e. i. ad. i. o. Quare ex. 9. libr. quinti erit. a. e. æqualis. i. o. sed cum ita se habeat. e. o. ad. i. o. ut. i. o. ad. t. o. ita quoque se habebit, ex undecima quinti. a. i. ad. e. a. ut. i. o. ad. t. o. Cum autem (ut uidimus). a. e. æqualis sit ipsi. i. o. non poterit esse proportio. t. a. ad. i. a. ut est. o. i. ad. t. o. quia si hoc esset, esset etiam disiunctim proportio. i. t. ad. i. a. ut est. i. t. ad. t. o. & ex supradicta 9. lib. quinti. a. i. æqualis esset. t. o. Maximum autem inconueniens esset. t. o. minorem. o. i. id est minorem. a. e. æqualem esse. a. i. quæ maior est. a. e. Ostensiuè tamen idem hoc modo probari potest, ut existente. i. o. equali ipsi. a. e. per consequens quoque erit minor ipsa. a. i. cum. a. e. pars sit ipsius a. i. Per eandem tamen rationem. o. t. minor est. o. i. Tanto magis igitur minor erit. t. o. ipsa. i. a. Unde ex. 8. libri quinti maiorem proportionem habebit. i. t. ad. t. o. quam ad. i. a. & ex. 28. eiusdem lib. i. o. ad. t. o. maiorem proportionem habebit, quàm. t. a. ad. i. a. ex. 12. igitur dicti quinti maiorem proportionem habebit. i. a. ad. e. a. quàm. t. a. ad. i. a. ita ergo se habebunt ipsorum uelocitates.



*Corpora grauia aut leuia eiusdem figura et materia sed inæqualis magnitudinis, in suis motibus naturalibus uelocitatis, in eodem medio, proportionem longè diuersam seruatura esse quam Aristoteli uisum fuerit.*

## C A P. V I I.

**E**st mihi nunc probandum quod in uno eodemque medio duo corpora inæqualia, sed simili figura & materia, mouebuntur naturali motu, diuersa tamen ratione ab

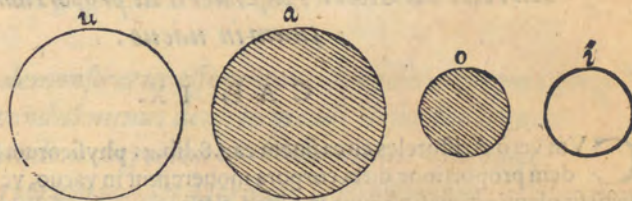
ca,

ea, quàm Aristoteles præscripsit.

Sint igitur corpora. a. et. o. inæqualia, eadē tamen figura & materia prædita, quorum. a. maius sit, & per consequens in eadem quoque proportione grauius ipso. o. in qua est maius, communi omnium sententia.

Scribit ergo Aristoteles proportionem velocitatis corporis. a. ad eam, quæ est corporis. o. (naturaliter se vnoquoque mouente) eandem futuram, quæ est magnitudinis, aut grauitatis corporis. a. ad magnitudinem, aut grauitatem corporis. o. Imaginemur igitur corpus u. eadem magnitudine & figura, qua corpus. a. præditum est, sed eandem grauitatem obtinere, quæ communicata est corpori. o. quod ex quauis materia constet. Hinc ex primo supposito secundi capitis certi erimus proportionem velocitatis corporis. a. si comparatur cum velocitate corporis. u. futuram, vt eā, quæ est ponderis corporis. a. ad pondus ipsius corporis. u. Ex. 9. igitur lib. quinti Euclidi. cogitur fateri Aristoteles velocitatem corporis. o. esse vnā eandemq; in specie, quæ est corporis. u. Quod primo supposito cap. secundi huius lib. planè repugnet. Igitur hæc Aristotelis opinio falsā est. Idem quoque probaretur mediante corpore. i. æquali magnitudine, similiq; figura cum corpore. o. prædito, sed, quod ad quantitatem attinet, æquali corpori. a. vnde ex primo supposito cap. secundi huius libri in eadem pro-

portione veloci<sup>9</sup> esset corpore. o. in qua grauius est. ex. 9. igitur quinti cogitur Aristoteles affirmare tā velox esse corpus a. quā est corpus i. vnde idem planè inconueniens emergit ex secundo supposito cap. secundi huius lib.

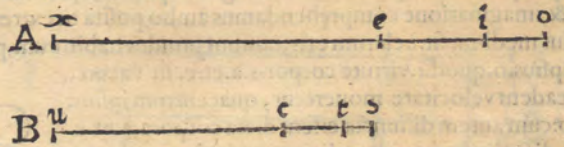


*Quod duo corpora inæqualia eiusdem materia in diuersis medijs eandem velocitatis proportionem retinebunt.*

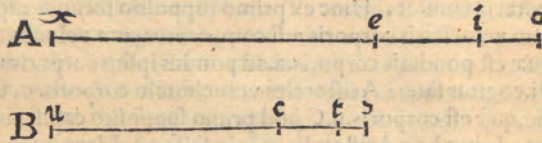
CAP. VIII.

Qualibet duo corpora inæqualia simili tamen figura & eadem materia constantia, naturaliter se per diuersa media mouentia, vnā eandemq; semper proportionem velocitatum seruant.

Sint duo corpora. A. et. B. sibi inuicem inæqualia quorum. A. sit maius, sed simile figura & idem materia, cuius pondus totale sit. x. o. & pondus totale ipsius. B. sit. u. s. Imagine-mur quoque corpus. A. positum in aqua amitte-re partē. o. e. ponderis. o. x.



o. x. et. B. quoque in eodem loco amittere. c. s. et. A. in aëre partem. i. o. et. B. partem. . t. s. Nunc quia corpus aqueum, cui correspondet. e. o. æquale est ipsi. A. & corpus aqueum, cui correspondet. c. s. æquale est ipsi. B. vt est ab Archimede probat ū: communi quadam scientiæ ratione, sequitur eandem proportionem futuram. o. x. ad. e. o. quæ est. u. s. ad. c. s. ob eademq; rationes idem erit de. x. o. ad. i. o. ut. u. s. ad. t. s. & idē etiam erit de. o. x. ad. s. u. vt de. e. o. ad. c. s. vt etiam de. o. i. ad. s. t. Vnde ex. 19. lib. quinti erit de. x. i. ad. u. t. quemadmodum de. x. o. ad. u. s. idem dico de. x. e. ad. u. c. Ex 11. igitur dicti lib. erit. de. x. i. ad. u. t. quemadmodum de. x. e. ad. u. c. ex quibus quidē proportionibus, si subtrahantur proportionēs resistētiarum extrinsecus aduenētium, proportionēs quæ remanebunt, ex tertio communi axiomate ab Eucli. in principio primi lib. posito, ad inuicem erunt æquales, secundum quas eorundem corporum sunt velocitates.



*An rectè Aristoteles diseruerit de proportionibus motuum in uacuo.*

C A P. I X.

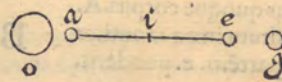
Cum verò Aristoteles circa finem cap. 8. lib. 4. physicorum subiungit quod eadem proportione dicta corpora mouerentur in vacuo, vt in pleno, id pace ei dictū sit planè erroneū est, quia in pleno dictis corporibus subtrahitur proportio resistētiarum extrinsecarum à proportione ponderum, vt velocitatum proportio remaneat, quæ nulla esset, si dictarum resistētiarum proportio, ponderum proportioni æqualis esset, & hanc ob causam diuersam velocitatum proportionem in vacuo haberent ab ea, quæ est in pleno.

*Quòd in uacuo corpora eiusdem materie equali uelocitate mouerentur.*

C A P. X.

Quòd supradicta corpora in vacuo naturaliter pari uelocitate mouerentur, hac ratione assero.

Sint enim duo corpora. o. et. g. omogenea, et. g. sit dimidia pars ipsius. o. sint alia quoque duo corpora. a. et. e. omogenea primis, quorum quodlibet æquale sit ipsi. g. & imaginatione comprehendamus ambo posita in extremitatibus alicuius lineæ, cuius medium sit. i. clarum erit, tantum pondus habiturum, punctum. i. quantum centrū ipsius. o. quod. i. uirtute corporis. a. et. e. in vacuo, eadem uelocitate moueretur, quæ centrū ipsius. & cum autem disuncta essent dicta corpora. a. et. e. à dicta linea, non ideo aliquo modo suam uelocitatem



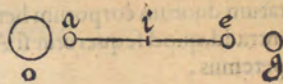
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tem mutarent, quorum quodlibet esset quoque tam velox, quam est. g: igitur. g. tam velox esset quam. o.

*Corpora licet inæqualia eiusdem materiae & figurae, si resistentias habuerint ponderibus proportionales aequaliter movebuntur.*

C A P. X I.

EAdem ratione, quam cap. antecedente præscripsimus, posset ostendi, si duo corpora. o. et. g. suas resistentias, ita ad inuicem proportionatas haberent, ut sunt eorum pondera, in pleno pari velocitate prædita esse, quod in fine capitis noni leuiter attingi, quia punctum. i. tam velox esset, ut centrum ipsius. o. cum à tanto pondere i. motum esset; quanto centrum ipsius. o. atque tan tam resistentiam duo corpora. a. et. e. quæ ipsam o. solum haberet ex hypothefi, dicta tamen corpora. a. et. e. tam separata, quam coniuncta, eandem velocitatem retinerent. g. igitur tam velox esset, quam. o.

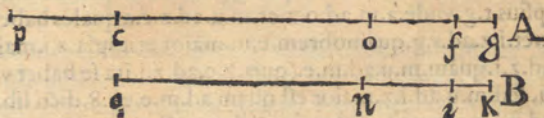


*Maiores hic demonstratur esse proportio ponderis corporis densioris ad pondus minus densi in medijs densioribus, quam sit eorundem corporum in medio minus denso, nec corporum pondera seruare proportionem densitatis mediorum.*

C A P. X I I.

P Roposita nobis cum fuerint duo corpora. A. et. B. area corporea æqualia, quorum. A. densius sit ipso. B. probabo in medio magis denso, maiorem proportionem futuram ponderis ipsius. A. ad pondus. B. quam in medio minus denso.

Sit igitur. p. g. pondus totale ipsius corporis. A. et. q. k. ipsius corporis. B. vnde. p. g. maius erit ipso. q. k. Sit quoque. o. g. pondus, quod medium magis densum subtrahit à pondere. p. g. et. n. k. sit pondus, quod idem medium subtrahit à pondere. q. k. et f. g. sit pondus, quod medium minus densum subtrahit à. p. g. et. i. k. illud, quod idem mediũ subtrahit ab. q. k. vnde. o. g. æquale erit. n. k. et. f. g. ipsi. i. k. quia quod ad areã attinet, corpora supponuntur æqualia, vnde proportio. p. f. ad. q. i. maior erit ea, quæ est. o. f. ad. n. i. communi scientiæ notione, quia si scinderet aliqs. p. f. in puncto. c. ita. vt. c. f. æquale esset ipsi. q. i. proportio. c. f. ad. q. i. esset vt ea, quæ est. o. f. ad. n. i. (hoc est nulla) sed



fed proportio. p. f. ad. q. i. maior effēt ea, quæ est. c. f. ad. q. i. ex. octa ua lib. quinti, vnde ex. 12. eiusdem lib. maior effēt. p. f. ad. q. i. quàm. o. f. ad. n. i. ex. 33. igitur eiusdem, maior erit proportio. p. o. ad. q. n. quàm. p. f. ad. q. i. Sic quoque se habebunt ad inuicem velocitates, quod est propositum. Cum autem proportio. p. o. ad. q. n. maior sit, quàm. p. f. ad. q. i. permutando igitur maior erit proportio. p. o. ad. p. f. quàm. q. n. ad. q. i. aut euerſim maior erit proportio. q. i. ad. q. n. quàm. p. f. ad. p. o. vnde si proportio p. f. ad. p. o. effēt ac ea, quæ est. o. g. ad. f. g. non effēt. q. i. ad. q. n. ut est. o. g. ad. f. g. aut vt. n. k. ad. i. k. quod idem est, de quibus quidem rebus, exemplis propositis quinto capite mētionem feci.

Velocitatibus autem sequentibus pondera, sequitur proportionem velocitatum duorum corporum heterogeneorum eandem non esse per diuersa media, contra id, quod sequeretur si Aristotelis opinionem. 8. cap. lib. 4. physicorum reciperemus.

*Longe aliter veritatem se habere quam Aristoteles doceat in fine libri septimi physicorum.*

## C A P. X I I I.

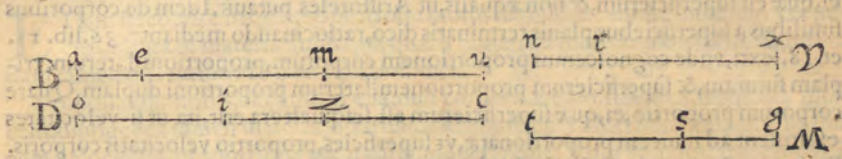
**N**on tam facile est assignare proportionem velocitatum duorum corporum naturalium, quam Aristoteles ultimo cap. lib. 7. physicorum putauit.

Quamobrem sint duo corpora. B. et. D. materia magnitudineq; diuersa, pondere tamen, & figura similia, & proportio resistantiarum, quas recipiunt à medio dū mouentur, sit. ut. o. i. ad. a. e. denotentur deinde velocitates totales absque vlla resistantia ab. a. u. et. o. c. quæ æquales erunt ad inuicem per communem scientiam ex supposito, sint alia deinde duo corpora. V. et. M. eodem modo se habentia ut prima. B. et. D. in eodem medio, sed ex diuersa materia ab ea, quæ est illorum duorum corporum, magnitudine tamen & figura ijsdem similia: significantur quoque eorundem resistantiæ per. r. s. et. n. r. & eorundem velocitates à nulla ex resistantijs diminutæ, per. n. x. et. t. g. vnde. n. r. æqualis erit. a. e. et. r. s. ipsi. o. i. et. n. x. ipsi. t. g. n. x. tamen et. t. g. non erunt æqualia. a. u. et. o. c. Sed exempli gratia, ponamus ea esse minima. Supponamus nunc. e. u. velocitatem esse quæ remanet ipsi. B. cum applicata erit resistantia. a. e. dicto corpori. B. quæ diminutam facit totam. a. u. per. a. e. sitq; i. c. ea, quæ remanet ipsi. o. c. corporis. D. et. r. x. ea, quæ remanet. n. x. corporis. V. et. s. g. ea, quæ est ex. t. g. corporis. M. Vnde communi omnium cōsensu assequemur. e. u. maiorem futuram. r. x. et. i. c. ipsa. s. g. Scindatur deinde. a. m. ad æqualitatem. n. x. et. o. z. ipsius. t. g. vnde. a. m. ad. o. z. et. m. u. ad. z. c. æquales habebimus, ut quoque. e. m. ad. r. x. et. i. z. ad. s. g. quamobrem. e. m. maior erit ipsa. z. i. maior igitur erit proportio. z. c. ad. z. i. quàm. m. u. ad. m. e. (quia. z. c. ad. z. i. ita se habet vt. m. u. ad. i. z. ex. 7. lib. quinti, sed. m. u. ad. i. z. maior est quam ad. m. e. ex. 8. dicti lib. vnde ex. 12. eiusdem. z. c. ad. ad. z. i. maior erit, quàm. m. u. ad. m. e. Ergo ex. 28. maior proportio erit. c. i. ad. z. i. quam



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quam u. ad. m. c. & ex. 27. maior erit proportio. c. i. ad. u. e. quam. z. i. ad. e. m. id est. s. g. ad. r. x. quod Aristoteli in mentem non venerat. Alijs quoque modis idem probari potest, vt si diceret aliquis, maiorem proportionem esse. e. m. ad. m. u. quam. i. z. ad. z. c. (quia. e. m. ad. m. u. eadem est ratio vt ad. z. c. ex. 7. quinti, sed proportio. e. m. ad. z. c. maior est quam. i. z. ad. z. c. ex. 8. eiusdem, ergo ea, quæ est. e. m. ad. m. u. ex. 12. maior erit, quam. i. z. ad. z. c.) vnde componendo, ea quæ est. e. u. ad. m. u. maior erit illa, quæ est. i. c. ad. z. c. & permutâdo, quam ea, quæ est. e. u. ad. i. c. ea, quæ est. m. u. ad. z. c. & ex. 33. quinti, ea, quæ est. e. m. ad. i. z. maior erit ea, quæ est. e. u. ad. i. c.



*Quid sequatur ex supradictis.*

CAP. XIII.

EX præcedenti capite manifestè deprehenditur, in vniuersum Aristotelis opinionem veram non esse in prima parte vltimi capituli. lib. 7. physicorum; quia in eo loco supponens ipse corpus. B. præcedentis capituli esse dimidiam partem ipsius D. quantum ad aream corpoream spectat (sunt tamen pondere ad inuicem æqualia) ait. B. futurum duplo velocius ipso. D. Ego verò præcedenti capite accepi. e. u. pro velocitate residua corporis. B. (subtracta ea tamen parte, quam ei resistentia adimit, quæ erat. e. a.) et. i. c. pro ea, quæ est corporis. D. et. r. x. pro ea, quæ est corporis. V. et. s. g. pro ea, quæ est corporis. M. Dicat nunc Aristoteles, quæ nam harum duarum proportionum dupla erit? quia si earum aliqua talis erit, alia nullo modo esse poterit, vt iam ostendi, etiam si duo corpora. V. et. M. easdem conditiones habeant, quas. B. et. D. Ratio autem, quæ Aristotelem induxerit ad illud credendum, nulla alia esse potuit, quàm quod putarit resistentias proportionatas esse magnitudinibus corporis, id est quemadmodum. B. erat corporaliter dimidia pars ipsius. D. sic etiam haberet medietatem eius resistentiæ, quam habuisset corpus. D. Quod etsi verum esset, non tamen sequeretur necessariò in quibuslibet corporibus futuram velocitatum proportionem eandem, quæ resistentiarum est, vt superiore capite ostendimus.

*Num rectè senserit Philosophus resistentias proportionales esse cum corporibus mobilibus.*

CAP. XV.

QVOD Aristoteles crediderit resistentias proportionatas esse corporibus, errauit. Si superficies iisdem proportionatæ essent, dubium non est, quin resistentiæ quoque ipsæ, iisdem proportionatæ existerent, supponendo eas similes situ, dum eadem corpora mouerentur. Sed eadem proportio non est inter superficies,

cies, & quæ inter corpora reperit: Aristoteles igitur in eo defecit. Quod autem inter superficies non eadem sit proportio, quæ inter corpora extat, si primo ad sphericas mentem verterimus, intelligemus proportionem eam, quæ inter duas sphericas reperitur triplam semper existere ei, quæ est inter ipsarum diametros ex vltima. 12. libr. Euclid. Est autem proportio, quæ est inter superficies sphericas æqualis ei, quæ est ipsorum circulorum maiorum ex. 16. libr. quinti, cum ex. 31. primi de sphaera & cylindro Archimedis, omnis spherica superficies quadrupla, sit maiori circulo ipsius sphaera, sed proportio, quæ est inter dictos circulos, est dupla ei, quæ est inter eorundem diametros ex. 2. libr. 12. Euc. ergo proportio, quæ est inter corpora, sesquialtera erit ei, quæ est superficieum, & non æqualis, ut Aristoteles putauit. Idem de corporibus similibus à superficiebus planis terminatis dico, ratiocinando mediante. 36. libr. 11. et. 18. sexti, vnde cognoscemus proportionem corporum, proportioni laterum, triplam futuram, & superficieum proportionem, laterum proportioni duplam. Quare corporum proportio, ei, quæ superficieum est, sesquialtera erit, ita ut si velocitates extitissent ad inuicem proportionata, vt superficies, proportio velocitatis corporis. B. ei, quæ est corporis. C. fuisset subsesquialtera proportioni corporum, & non æqualis eidem.

*Id ipsum aliter demonstratur.*

C A P. X V I.

**A**lio quoque modo probari potest non esse in vniuersum verum id, quod Aristoteles in prima parte capitis vltimi lib. 7. physicorum ait, sic scribens.

Si. A. quidem sit id quod mouet. B. verò id quod mouetur, et. C. sit longitudo per quam, et. D. tempus in quo est motum, in tempore nimirum æquali, potentia æqualis. A. dimidium ipsius. B. per duplum mouebit ipsius. C. per ipsum autem. C. in dimidio temporis. D. sic enim erit rationis similitudo.

Sit ergo corpus. c. septimi capitis pondere æquali corpori. u. eiusdem capitis, sed area corporea minus ipso. u. pro medietate. Simile tamen figura. Imaginemur nunc tertium aliud corpus omogeneum ipsi. u. quod sit. i. magnitudine & figura simile ipsi o. vnde minor erit ipso. u. pro media parte, & hanc ob causam ipsum. u. erit duplo magis graue, quam ipsum. i. & per consequens ipsum quoque. o. duplo grauius erit quam sit ipsum. i. ex. 7. libr. quinti Euclidis. Ipsum ergo corpus. o. duplo velocius erit, quam ipsum. i. ex primo supposito cap. 2. huius lib. Vnde ex. 9. quinti, velocitas ipsius i. æqualis esset ei, quæ est ipsius u. cum Aristoteles scribat. o. quoque futurum duplo velocius ipso. u. q. cap. 7. huius lib. falsum esse demonstraui.

*De alio Aristot. lapsu.*

C A P. X V I I.

**S**cribit Aristoteles in ultimo cap. lib. 7. physicorum in hunc modum.  
Si duo quædam seorsum per tantum spatium tanto tempore duo seorsum pondera mouent, & composita per longitudinem æqualem, æqualiue in tempore, compositum ex ponderibus vtriusque mouebunt, est enim in eis eadem ratio.

Quod

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Quod in vniuersum nec etiam potest esse verum in pleno, quia cap. 14. iam probauimus, non eandem proportionem esse inter superficies corporum, & ipsa corpora.

*Quomodo dignoscatur proportio velocitatis duorum similium corporum omogeniorum inaequalium.*

CAP. XVIII.

**E**tiam si reperire in qua proportione motus naturaliter moueantur duo corpora, figura & materia similia, inaequalia tamen ad inuicem, non facile sit, ostendam tamen qua ratione id consequi possimus.

Proponantur nobis, exempli gratia, duo corpora. a. et. o. spherica, inaequalia inuicem, omogenea tamen materia, quorum. a. maius sit; si voluerimus inuenire in qua nam velocitatis proportione naturaliter mouerentur. Volo ut inquiratur corpus. i. sphericum, alia tamen & diuersa materia constans, sed pondere equale corpori. o. & superficie tam proportionata superficiei corporis. a. quam est ea, quae est sui ponderis ad pondus ipsius. a. Hoc facto, indagetur, quanam erit proportio inter superficies corporum. i. et. o. quae semper dupla est, vel subdupla ei quae est diametrorum; ut iam cap. 15. dixi, & haec proportio superficierum sphericarum ipsi. o. et. i. subtrahatur ab aequalitate, quod igitur remanebit, erit proportio velocitatum inter duo corpora. o. et. i. id est inter. o. et. a. ut exempli gratia, si proportio superficiei. o. superficiei ipsius. i. sesquitertia esset, sub

trahendo eam ab aequalitate, remaneret proportio subsesquitertia, unde velocitas corporis maioris (quod in presenti loco supponitur esse. o.) ei, quae est corporis minoris, quale est corpus. i. subsesquitertia esset; aut dicamus quod. i. esset velocius ipso. o. in proportione sesquitertia ex secundo supposito secundi capitis huius libri. Sed. i. tam velox est quam ipsum. a. ex 11. cap. ergo proportio velocitatis ipsius. a. sesquitertia erit ei. quae est ipsius. o.

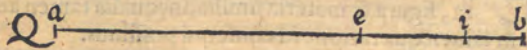


*Quam sit inanis ab Aristotele suscepta demonstratio quod vacuum non detur.*

CAP. XIX.

**E**x ijs, quae superius demonstrauimus facile cognosci potest irritam esse eam rationem, quam Aristoteles. 8. cap. lib. 4. physicorum ad destruendum vacuum, confluxit. Ut igitur idem facilius ostendamus, comprehendamus imaginatione infinita media corporea, quorum vnum altero rarius sit, in qua placuerit nobis ex proportionibus, incipiendo ab uno, imaginemur etiam corpus. Q. densius primo medio, cuius corporis, totalis grauitas sit. a. b. & positum in ipso medio, amittat partem. e. b. ipsius grauitatis, & in secundo medio amittat. i. b. & sic per gradus vnde nobis patebit

dicto corpori. Q. Nunquam remansuram suam totalem grauitatem. a. b. in quolibet ex dictis medijs. Nunc si quærat à me Aristoteles proportionem velocitatis corporis. Q. per vacuum ad velocitatem dicti corporis per plenum, ego ei proponam proportionem ipsius. a. b. ad. a. e. exempli gratia, dicens, q̄ quæadmodum. a. b. maius est ipso. a. e. sic etiam corpus. Q. velocius erit in vacuo, quam in pleno, dicti autem pleni densitatem appellabimus. e. b. Aristoteles dicit nunc, q̄ aliud quoddam medium in eadem proportione subtilius ipso. e. b. desumatur; quemadmodum. a. e. minus est ipso. a. b. fit ergo istud. i. b. in quo Aristoteles credit corpus Q. futurum tam velox ut in vacuo, in quo aberrat, q̄a proportio velocitatis corporis. Q. in medio. i. b. ad velocitatem eiusdem in medio e. b. ita se habebit, ut. i. a. ad e. a. ex ultimo supposito capit. 2. huius libr. quæ minor esset ea, quæ est. a. b. ad. a. e. ex. 8. lib. quinti Eucli.



*Non satis dilucide Aristotelem de loco ratiocinatum fuisse.*

C A P. X X.

**Q**Uæ Aristoteles de loco scribit multas in se continent difficultates. Primum, cap. 4. lib. 4. physicorum ait, omne corpus esse in suo proprio loco, supponendo vnum centrum pro loco grauium, et unam circumferentiam pro loco leuium corporum. Sed quomodo punctum potest esse locus ipsius corporis, cum omni dimensione capacitate q̄; sit denudatum? vnde si centrū locus esset corporum grauium, omnia dicta corpora grauia, extra proprium locum existerent, quia nullum ex iis est, q̄ fit in centro. Adde quod neque hoc cum loci definitione ab ipso posita consentiret cum ipse dicat in eodem cap. locum esse superficiem quandam, & non interuallum, licet huiusmodi definitio falsa appareat primo ex incouenienti falso, quod ipse hinc sequitur dicit, id est, quod si locus interuallum esset, infinita loca existerent, quod reuera nec ob hanc causam inconueniens existit, quia eodem planè modo quo aliquod corpus potest esse infinita corpora, (quod ipse diceret in potentia) sic etiam interuallum aliquod posset esse infinita interualla. Cum autem dicat superficies corporis ambientis esse locum eius corporis, quod continetur, cogitur dicere lineam, quæ circumdat superficiem, superficiem locum esse, & puncta ipsius lineæ, quod reuera absurdum est. Locus corporis est interuallum illud eadem magnitudine & figura, qua corpus ipsum præditum est, quod si non esset, sed esset superficies, quemadmodum Aristoteles voluit, maximum inconueniens sequeretur, scilicet æquales locos capere inæqualia corpora, aut corpora æqualia, locos inæquales occupare, quod scitu facillimum est, cum Theon super Ptolomei Almagestum iam probarit sphericam superficiem maius interuallum corporeum continere, quam aliam quauis superficie dictæ sphericæ æqualem, vnde possent facillè reperiri duo loci, quorum alter millies altero maior esset, capaces tamen corporum æqualium, aut reperiri duo corpora, quorum alterum millies maius esset altero, quæ tamen corpora apta essent ad occupandos locos æquales, quamuis Aristoteles dicat, locum, neque maiorem neque minorem esse debere locato. Sed interualla corporea æqualia à quauis figura terminata, continebunt semper corpora æqualia. Corporeum igitur interuallum est

reuera

reuera locus corpori adæquatus, cum corpus in interuallum superficiale non intret, quamuis interuallum corporeum ingrediatur. Et hoc modo nullū est corpus, quod in mundo aut extra mundum ( dicat autem Aristoteles quicquid voluerit ) locum suum non habeat.

*Utrum bene Aristoteles senserit de infinito.*

C A P. X X I.

**T**Ractans Aristoteles in fine quinti cap. lib. 3. physicorum de infinito ait, impossibile cum sit inuenire locum infinitum, & omne corpus in loco cum sit, impossibile quoque esse in rerum natura aliquod infinitum corpus reperiri. Omittamus quòd cum Aristoteles debuerit beneficio loci destruere infinitum, ordine peruerso de infinito prius, quàm de loco disputationem instituat; sed dicamus ipsum intelligere de infinito corporeo, & cum probauerimus corporis locum esse corporeum in interuallum, non autem superficiem, neque opus sit in definitione interualli mentionem aliquam facere terminorum, vnde ipsum infinitum esse potest, neque aliqua ratione de hac re dubitari potest; hoc modo nullum inconueniens sequeretur, quòd extra cælum reperiri possit corpus aliquod infinitum, quamuis, id ipse nulla euidenti ratione inductus perneget. Sensit quoque, absque eo, quod aliquam rationem proponat, aliquid extra cælum reperiri quemadmodum apparet ex fine cap. 9. lib. primi de cælo, cum etiam ait cap. 8. lib. 8. physicorum, infinitas partes alicuius continui esse solum in potentia, non item in actu, hoc non est illico concedendum, quia si omne totum continuum, & re ipsa existens, in actu est, omnis quoque eius pars erit in actu, quia stultum esset credere, ea quæ actu sunt, ex ijs, quæ potentia existunt, componi. Neque etiam dicendum est continuationem earundem partium efficere, vt potentia sint ipsæ partes, & omni actu priuatæ; Sit exempli gratia linea recta. a. u. continua quæ deinde diuidatur in puncto. e. per æqualia, dubium non est, quin ante diuisionem, medietas. a. e. tam in actu (licet coniuncta cum alia. e. u.) reperiretur, quàm totum. a. u. licet à sensu distincta non esset. Idem affirmo de medietate. a. e. idest de quarta parte totius. a. u. & pariter de octaua, de millesima, & de quauis, ita vt essentia actualis infiniti hoc modo tutò concedi possit, cū ita sit in natura. Sed peius etiam sensit Aristoteles eodem loco capitis quinti lib. 3. physicorum, negando infinitum posse connumerari inter quantitates, dicens vnam aliquam quantitatem intelligi vt cubitum, tricubitum, & cætera; vbi non considerat eadem etiam ratione intelligi posse aliquam quantitatem infinitorum cubitorum, & in quantitatis definitione nullam esse necessitatem terminorum, vt exempli gratia in definitione numeri, non est necessitas alicuius determinati numeri, quia multitudo, non minus infinita, quàm finita, intelligi potest. Vbi postea cap. 8. libr. 4. physicorum ait nullam esse differentiam inter infinitum, & vacuum, reuera nihil absurdius hoc dicere fingereue poterat.

*Exagitur ab Aristotele adducta temporis definitio .*

## C A P. X X I I.

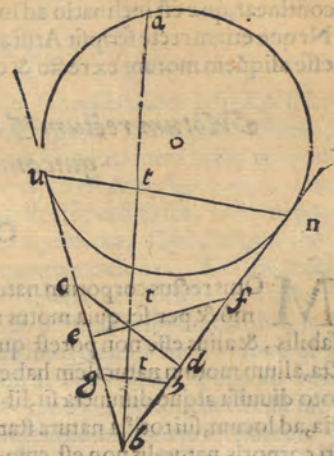
CVM senserit Aristoteles tēpus absque motu esse nō posse, ea tamen ab inuicē separans, volēs definire tēpus ait, ipsū esse mot<sup>o</sup> mensurā numerūq;. Quæ quidem definitio, natura sua non est bona, quia tempus, neque numerus est, neque etiā est mēsurā motus p se, sed tñ p accidēs, quia nihil est, qđ numeret aut mensuret aliud, quod non sit eiūsdē speciei cū illo quod mēsuratur, aut numero circumscribē, vt exēpli gratia, nulla vnquā superficies p se numerabit aut mēsurabit lineā, aut corpus; nec lineā superficiem aliquā, aut corpus: nec corpus lineā aliquā aut superficiem; Sed lineā lineam mēsurabit; superficies superficiem; & corpus corpus; etiā si tam vna ex iis quantitibus quā altera sit continua. Cum verò motus non sit tempus, neque tempus sit motus, sed inter se maximē differant, sequetur ex iis, alterum nullo modo per se esse mēsuram alterius, nisi per accidens. Et si alicui videtur, qđ ad significandam aliquam quantitatem motus, dicere huiusmodi operationem duarum horarum, aut duorum dierum, aut duorum annorum spatio completam esse, sit ponere tantum tempus: animaduertēre debet hoc simpliciter non esse verum, quia horarum, dierum, & annorum interualla, imaginatione concipiūtur vt motus corporum cęlestium, sine quibus, neque anni, neque dies, neque horę existerent, etiā si omnis motus sit (vt ita dicam) locatus in tempore, ut corpus in loco, vnde motus motu, & tempus tempore, non autem aliud ab alio mēsuratur. Tempus ex necessitate (phylosophicē tamen loquendo) res est æterna, motus non item, quia diuersis modis terminari potest & cessare, & interim dum cessabit quiescet corpus, quod primo mouebatur. nihilominus tamen, tempus continuabit cursum suum. Tempus igitur potius locus motus erit dicendum, quā numerus aut mēsurā eius, & tale est, vt consummatum uideatur à continuo quodam fluxu vnus instantis, quemadmodum iam dixi in. 38. capite meę gnomonica; & cum dico ab vno instanti, vnum in specie, & non in numero intelligo, quod à sensibus nostris percipi non potest, neq; etiam notari, quia nouum semper instans nobis occurrit. & si aliquis aliquod exemplū (lar go modo) incompræhensibilitatis ipsius instantis desideraret, imaginetur rotam aliquam albam, in qua sit nigrum aliquod punctum sensibile, aut è contra rotam nigrā imaginetur, in qua sit punctum album, quæ rota velocissimē moueatur; huiusmodi punctum, nullo modo assignari poterit, magis ab una parte quā ab altera; immo se se nobis offeret semper in forma lineæ circularis. possumus aliquo modo etiam sumere exemplum à sono, quia omnis chorda cuiuslibet instrumenti musici, dum sonus editur, tremitt, unde huiusmodi sonus, appellari potest aggregatum aliquod ex innumerabilibus sonis. eodem modo se habet sonus, quem edunt campanę, & omnia instrumenta tam naturalia, quā artificialia, quæ quantò velocius tremūt, tantò acutiorē generant sonum, & quantò tardius, tantò grauiorem. Neque est quòd in admirationem ducamur, quòd sensui unum aliquod continuum appareat id, quod discretorum est multitudo (non putet tamen aliquis me negare continuitatem successiuam ipsius temporis) quod clare cognosci potest à niue, aut à chrystallo, aut à vitro, aut à saccharo in minutissimas partes redactō, quæ continuam aliquam albedinē nobis ad inspiciendum offerunt, quod nihil aliud est, quā innumerabilis quædam multitudo minorum reflexorum. Idē dico de spūto, & qualibet spuma, & quan-

to minutiora sunt corpuscula à quibus vt à speculis reflectitur lumen, tantò magis aggregatū illud albū apparet. Hæc autē exempla cū sint, nec non largo modo sumpta, mirū non erit si claudicare videbūt. Sed ut ad motū, & tēpus reuertamur (quæ sunt cōtinua successiua) Aristoteles in definiendo tempore, non reduxit in mentem, quod scribit decimo metaphysicę et. 4. cap. secundo. libr. de cęlo, omnia videlicet, ab eo, quod minimum est in suo genere, mensurari, & ex seipso in physicorum libris, tempus non est de genere motus; ergo eius ipsius rationum ui, tempus non erit mensura motus, sed motus quidem potest mensurare motum, videlicet velocior minus velocem, & breuior longiorem; & numer⁹ mensurā numero, & tempus tempore in quantum longum est, aut breue, non in quantum velox, aut tardum; Nullum autem inconueniens sequetur sumendo tempus tam proportionale motui, quam locus corpori, quia motus decem miliarium, quæ aliquis vnus horæ spatio conficiat, erit proportionalis corpori denso, & motus vnus miliaris eadem hora peracti, proportionalis erit corpori raro; & quemadmodum corpus densum occupat minus interuallum loci, contra quam fiat in corpore raro: sic etiam motus velox breuiori temporis spatio peragetur, quam tardus.

*Motum rectum esse continuum, uel dissentiente Aristotele.*

C A P. X X I I I.

**A**ristoteles. 8. capi. 8. physicorum ait impossibile esse aliquid per lineā rectā nunc vno modo, nunc altero, id est eundo, & redeundo per dictā lineam in extremis absque quiete moueri. Id quod contrā possibile esse dico. Pro speculatione cuius rei imaginemur circulum. u. a. n. motu continuo circa centrum. o. in quā libet partem, aut dextrā, aut sinistrā ferri; & imaginemur pūctum. b. extra ipsum, ubi magis nobis videbitur, à quo ducantur duæ lineæ recte. b. u. et. b. n. contiguæ ipsi circulo in punctis. u. et. n. Imaginatione quoque inter has duas lineas, alteram quæ sit. u. n. aut. c. d. aut. e. f. aut. g. h. constituamus in qualibet parte, sumemus etiam punctum. a. circumferentiæ dicti circuli, à quo vsque ad. b. lineam. b. a. imaginemur fixā in. b. sed quod remanet mobile, secundum quod mouebitur punctum. a. unde aliquādo hæc linea erit eadem cum. b. u. & aliquando cum. b. n. & aliquando ab. b. u. versus. b. n. proficiscetur, & aliquando ab. b. n. versus. b. u. vt accidit lineæ directionis, & retrogradationis planetarum, unde circulus. u. a. n. erit, vt epiciclus et. b. vt terræ centrum. Clarum nunc erit, quòd quando linea. b. a. eadem erit cum. b. u. aut cum. b. n. non quiescet, quia in instanti reuertetur, quia. b. u. et. b. n. in puncto, dictū circulum rangunt, & dicta. b. a. interfecabit semper aliquam ex dictis u. n. aut. c. d. aut. e. f. aut. g. h. quod intersectionis punctum sit. t. Imaginemur nunc quod secundū punctum. t. aliquid per aliquam ex dictis lineis mouca-



moueat, clarum erit quod tale aliquid, nunquam quiescet, etiam si sit in quouis extremis. Aristotelis igitur opinio, tuta non est.

*Idem uir grauisissimus an bene senserit de motibus corporum  
uolentis & naturalibus.*

C A P. X X I I I.

**A**ristoteles in fine. 8. physicorum sentit corpus per vim motum, & separatum à primo mouente, moueri, aut motum esse per aliquod tempus ab aere, aut ab aqua, quæ ipsum sequitur. quod fieri non potest; quia imo aer, qui in locum desertum à corpore subintrat ad fugandum vacuum, non solum hoc corpus non impellit, sed potius id cohibet à motu, quia aer per vim à corpore ducitur retrò, & diuisus à parte anteriori à dicto corpore, resistit similiter, & quantum dictus aer in dicta parte condensatur, tantum in posteriori rarefit, unde per vim sese rarefaciens non permittit, ut dictum corpus cum ea velocitate fugiat, cum qua auferret, quia omne agens in agendo patitur. Quamobrem cum aer à dicto corpore rapiatur, corpus quoque ipsum ab aere rapitur. Huiusmodi autem rarefactio aeris, naturalis non est, sed uolenta; & hanc ob causam resistit, & ad se trahit, sed non sufferente natura, ut inter unum & aliud ex dictis corporibus reperiatur vacuum; iccirco sunt hæc semper contigua, & mobile corpus aerem deserere cum nequeat, eius velocitas impeditur. Huiusmodi igitur corporis separatum à primo mouente uelocitas oritur à quadam naturali impressione, ex impetuositate recepta à dicto mobili, quæ impressio & impetuositas, in motibus rectis naturalibus continuò crescit, cum perpetuò in se causam mouentis, id est propensionem eundi ad locum ei à natura assignatum habeat. Aristo. 8. cap. primi lib. de cælo, dicere non deberet quod quanto propius accedit corpus ad terminum ad quem, tanto magis sit uolens; sed potius, quod quanto longius distat à termino à quo, tanto uolens existit. quia tanto maior sit semper impressio, quanto magis mouetur naturaliter corpus, & continuò nouum impetum recipit, cum in se motus causam contineat, quæ est inclinatio ad locum suum eundi, extra quem per vim consistit. Neque etiam rectè scripsit Aristo. 9. cap. lib. 8. physicorum et. 2. lib. primi de cælo esse aliquem motum ex recto & circulari mixtum, quod omninò impossibile est.

*Motum rectum & naturalem non esse primo & per se  
quicquid Aristoteli uisum sit.*

C A P. X X V.

**M**otus rectus corporum naturalium sursum, aut deorsum, non est naturalis primò & per se, quia motus naturalis perpetuus est, aut ut melius dicam, incessabilis, & alius esse non potest quam circularis, nullaque pars cum suo toto coniuncta, alium motum naturalem habere potest, quam eum, qui est totius. si autem à suo toto diuisa atque disuncta sit, liberèque uagetur, spontè, & quam breuissima potest uia, ad locum, sui totius à natura statutum proficiscitur. hic motus primò, & per se dicti corporis, naturalis non est, cum à causa naturæ suæ contraria, sit generatus, id est, ab eo,



ab eo quod sit extra suum locum, ubi contra naturam suam reperitur. Vnde huiusmodi motus, partim & non omnino, naturalis est. Is autem proprius est & naturalis motus, qui dicti corporis essentiam conseruat: hoc autem non prestat hic rectus, cum destruat, ergo hic motus primo & per se naturalis non est.

*Omne corpus esse in loco proprio graue, ut Aristoteli placuit, non est admittendum.*

## CAP. XXVI.

Arist. 4. cap. lib. 4. de celo sic scribit.

Suo enim in loco grauitatem habent omnia præter ignem, signum cuius est utrum inflatum plus ponderis, quam vacuum habere, &c.

Quo in loco, manifestè indicat se causam nec grauitatis, nec leuitatis corporum naturalium nosce, quæ est densitas aut raritas corporis grauis, aut leuis, maior densitate, aut raritate mediæ permeabilis, in quo reperitur.

Exemplum quod ipse de utre inflato proponit, debuisset saltem ei oculos ad veritatem, quæ clarissimè fulget, inspiciendum aperire. Verisimum est, utrum inflatum plus ponderis habere quam vacuum, aut quando aer in eo non est per vim inclusus.

Ratio autem huius rei est, quia quando inflatus est, ea quantitas aeris, in eum per vim iniecti, minorem occupat locum, quam si eidem liberè vagari permitteretur, unde violenter, quodam modo, condensata est, & quia corpus densum in minus denso, semper descendit, & minus densum in magis denso ascendit: Hanc ob causam utrum inflatus plenus corpore magis denso, quam est medium quod eum circumdat, descendit, non quia aer in aere, aut aqua in aqua sit grauis.

*Haud admittendam opinionem Principis Peripateticorum de circulo, & sphaera.*

## CAP. XXVII.

Cum Aristoteles senserit circulum esse figurarum superficialium primam, & sphaeram esse primam corporearum propter earum periferias, decipitur. Sunt enim vltimæ, non primæ. Sunt quidem (in quo rectè sentit) perfectæ, licet rationem huius rei non nouerit. Nam centrum cuiuslibet rei, eiusdem rei principium est, & eæ figuræ, quæ ipsum æqualiter circumdant, possunt appellari perfectæ, siue sint superficiales, siue corporeæ, & e contra illæ, quæ contrario modo se habent, imperfectæ. Quod autem perfectum est, licet natura sit primum, est tamen vltimum generatione. Sed quando Aristoteles duas dictas figuras pronuntiauit primas, ut perfectas, prioritate scilicet ea, quæ oritur à perfectione, verum dixit; sed quando de figuris superficialibus loquens, vult circulum esse primum, quia ab vna tantum linea terminetur; nõ minus pro circulo, quam pro oxigonia seu elipsi, aut cucurbitali, aut aliis multis figuris ab vna tantum linea terminatis concludit. Neque etiam hæc ratio perfectionem circuli monstrat, quia aliæ figuræ, à lineis curuis terminatæ, eandem conditionem fortiuntur. Circulus sphaeræque, non ex vno solo angulo recto constant, ut idem Aristoteles putat.

cap. 4. lib. 4. de celo, etiam si triangulus ex duobus angulis rectis confurgat, sed sunt figurę infinitorum angulorum rectorum, & hanc ob causam à me dicuntur vltimæ & perfectę, quia infinito nihil addi potest. Numerus angulorum rectorum circuli, est minor duplo infinito per duo infinita angulorum contingentia, quæ duo infinita minora sunt quouis angulo acuto rectilineo, & numerus angulorum rectorum solidorũ spheræ, minor est quadruplo infinito per 4. infinita angulorum solidorum cõtinentia, quæ 4. infinita, minora sunt quouis angulo solido acuto terminato à tribus planis. Triangulus inter figuras planas superficiales est primus, & circulus vltimus; & pyramis quadrilatera, inter corpora est prima, & spheræ vltima.

*Occultam fuisse grauisimo Stagira causam scintillationis stellarum.*

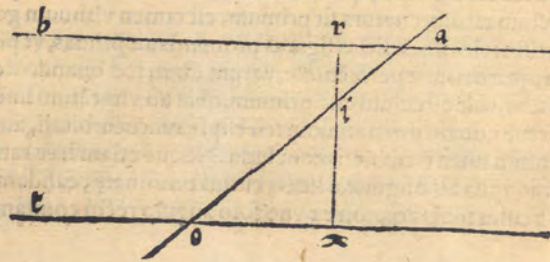
C A P. X X V I I I.

**V**Bi Aristoteles ait scintillationem stellarum fieri ratione aspectus nostri ob, maximam distantiam, maximum errorem committit, vt etiam facit quum putat visionem fieri extramittendo, contra id, quod alio loco, immo contra veritatem ipsam asseruit. Scintillatio ergo stellarum, neque aspectus nostri ratione, neque alicuius mutationis earundem stellarum, sed ab inæqualitate motus corporum diaphanorum mediorum nascitur, quæadmodum clarè cernitur, quòd si inter aliquod obiectum, & nos, aliquis fumus, qui ascendat, intercesserit, videbimus obiectum illud quasi tremere. Hoc autem tantò magis fiet, quantò magis distabit obiectum ab ipso fumo; vnde admirationi locus non erit, si stellas fixas magis scintillare, quam errantes cernamus. Lumen stellæ ad oculum nostrum accedens, perpetuò per diuersas diaphaneitates penetrat, medio continuorum motuum corporum mediorum, vnde continuò eorum lumen variatur, & hoc in lóginquis magis, quàm in propinquis stellis apparet, quemadmodum ab exemplo de fumo allato, & etiam ab aliquibus vitris ex superficie non plana, sed irregulari constantibus, quilibet cognoscere potest.

*Dari continuum infinitum motum super rectam atque finitam lineam.*

C A P. X X I X.

**O**Mnes hætenus fenserunt impossibile esse dari per imaginationem motum continuum & perpetuũ super vnã lineam rectam finitã in quo tñ decipiuntur. Imaginemur iò duas lineas parallelas. a. b. et. t. x. quarũ b. a. sit infinita à qualibet parte, & in ea imaginemur punctum. a. moueri continuò ad quam voluerimus partem,



## DISPUTATIONES.

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& in linea. t. x. imaginemur punctum fixum, quod sit. c. imaginemur etiam inter. c. et a. vnam lineam rectam. c. a. & inter duas parallelas dictas. r. x. fixam, & motus puncti sit ab. b. versus. a. ita ut. c. a. fecet. r. x. in puncto. i. quod interfectionis punctum mouebitur ab. r. versus. x. continuo, in tempore infinito, neque vnquam idem erit cum puncto. x.

*Non esse solis calorem à motu locali ipsius corporis solaris,  
ut Aristoteli placuit.*

## C A P. X X X.

**I**D nullo planè modo est admittendum quod Aristoteles credidit calorem solis à motu locali ipsiusmet corporis solaris, & non à lumine, prouenire, quemadmodum manifestè asserit primo meteororum cap. 3. circa finem sic scribens.

Vt igitur tepor gignatur atque calor, solis latitudo duntaxat, satis est efficere, &c. sed cap. 7. lib. 2. de cœlo sic scribit, Caliditas autem ab ipsis, lumenq; ideo fit, quia aer ab illorum motione fricatur.

Vbi non solum ostendit se opinari, quòd motus corporum cœlestium sit causa calor, sed etiam luminis, paulò autem post dicit, superiorum autem corporum vnum quodque fertur in sphaera, vt ipsa quidem non igniantur. Opinio profecto absurda. Nam cum corpus solare fixum sit in spissitudine sui orbis deferentis, secundum communem opinionem, non mouetur per se, sed accidentaliter, cum scilicet fertur à dicto suo orbe, vnde fieri potest, vt in motu sui orbis, nullum ex orbibus suorum deferentium augis fricet, sed si fricaret, id faceret mediante vno solo puncto, vt cuilibet, aliquantulum in mathematicis versato patet. Quam ob causam, rationi cõsentaneum non esset credere, quòd tantum calor gigneretur. Quod tamen si possibile esset, quid ergo fricatio superficierum orbis sui, cum iis, quæ sunt deferentium augis efficere? Quàdo tamen hoc fieret, vt scilicet à fricatione superficierum procederet calor, nil planè discriminis inter hyemen, & æstatem intercederet, nec inter calorem diei, & noctis, nec inter unam horam, aut alteram; sed secundum Aristotelis rationes, Venus, Mercuriusq; magis calefacere quam sol deberet, cum ita sint veloces vt ipse Sol, & eodem magis propinqua terræ. Verum Aristotelis temporibus, nullum aliud planetam quam solem putabatur philosophi supra Lunam esse. Atque etiam cõtiggeret mense Decembri, quam Iunio, magis inualesceret calor, cum huiusmodi mense sol ad nos propius accedat, quam mense Iunii. per differentiam maiorem diametro regionis elementaris, (nam solaris eccentricitas maior est semidiametro elementaris regionis) non considerans Aristoteles differentiam caloris, quæ nascitur ex eo, quod Sol aut altius supra horizontem excurrat, aut infra eundem feratur; neque eam, quæ à longitudine, aut breuitate diei proficiscitur. Sed quia Aristoteles eodem cap. tertio Meteororum intelligit de motu rpto, id est diurno, siue dicamus vniuersali, hinc sequi deberet, quod Sol maiorem caloris vim mense Martij & Septembris, quam aliis mensibus, profunderet. quia in iisdem temporibus, sol virtute huiusmodi motus velocior existat, quam alio quolibet tempore anni, cum tunc per æquatorum circumuoluatur. Multa quoque alia incommoda sequerentur si Aristotelis rationes admitteremus. Sed clarè uidemus, mediante reflexione aut refractione radiorum solarium, quod vniente sese lumine, unitur quoque, & augetur calor, atque omnis res ad comburendum apta accenditur, & inflammatur. In lumine igitur

tur continetur calor, & non in motu ipsius solis, & ita in lumine sedem habet, ut si sol quiesceret, neque in orbe suo circumageretur, infelicissima esset ea regio, in cuius Zenith ipse reperiretur.

*Vnde caloris solis prodeat incrementum æstate, et hyeme decrementum.*

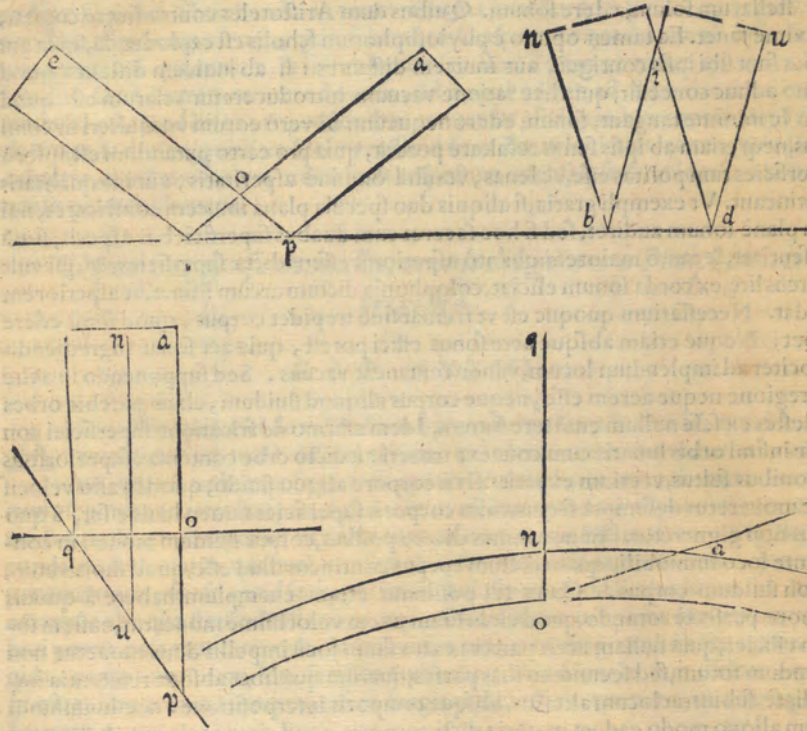
## C A P. X X X I.

**C**Um capite superiore ostenderim calorem solis non aliunde, quam à lumine provenire, ostendam nunc ex ordine, ex quot, quibusq; causis oriatur magna differentia eius caloris æstatis ad hyemem, quarum nonnullæ ab antiquis observatæ fuerunt, aliæ autem à nemine, quod sciam. Sunt autem quinque ad minus eæ causæ, quarum vna est diuturna solis mora, tempore æstatis supra horizontem, quæ causa ab antiquis posita, & citata fuit. Aliam quoque huius rei causam iidem antiqui dicebant esse propinquitatem solis nostro Zenith, sed hæc causa immediata non est, quia ab ea tres causæ immediatæ nascuntur; quarum vna est maior unio radij reflexi cum radio incidenti; secunda maior quantitas luminis in superficie terræ; tertia, minor resistentia vaporum ipsi radio luminoso facta; quarta verò est impressio caloris facta in terra, quæ cum aliis causis coniuncta calorem adauget. quæ quidem causæ nemini adhuc, quod sciam, in mentem venerunt. Quod autem attinet ad maiorem coniunctionem radii reflexi cum incidente, quisque, uel saltem mediocriter in cathoptrice cognitione versatus hoc verum esse cognoscet. Ut hoc tamen innotescat facilius. Imaginemur. q. p. et. b. d. esse duas particulas æquales superficiem ipsius terræ, super quas cadant duo radii luminosi solis. e. q. et. n. d. quorum. e. q. sit ad modum obliquus, et. n. d. quasi perpendicularis, vnde radii reflexi. p. a. et. b. u. ascendant cum angulis æqualibus eis, qui sunt suorum cadentium, cum omnis angulus reflexionis semper æqualis sit angulo suæ incidentiæ, ut cuilibet in cathoptrica, vel mediocriter versato patet. Mixtio autem primorum obliquorum erit. q. o. p. & ea, quæ est minus obliquorum. b. i. d. quorum duorum triangulorum nullus unquam erit, qui dubitare possit. q. o. p. non esse minorem. b. i. d. cum anguli. q. et. p. trianguli. q. o. p. acutiores sint angulis. b. et. d. trianguli. b. i. d. ex supposito. Quod uero attinet ad maiorem quantitatem luminis super terræ superficiem; Imaginemur radium. a. q. cuius respectu etiam imaginemur duos superficiem terræ situs, quorum vnus sit. q. o. cui dictus radius sit perpendicularis, & alter. q. p. cui radius. a. q. ex obliquo incidat. Imaginemur ergo triangulum. q. o. p. cuius angulus. o. rectus est ex supposito, unde. q. o. minor erit. q. p. ex. 18. primi Euclidis. hinc fit, ut super. q. o. cadat vniuersum lumen, quod super. q. p. diffunditur. Sit. q. u. æqualis. q. o. & sit imaginatione protracta. u. n. æquidistans. p. o. a. vnde. q. u. illuminata erit à radio. n. q. minore radio. a. q. ergo minus calida erit superficies. q. u. ipsius terræ, quam. q. o. quia maius lumen in se maiorem calorem includit: quod manifestè apparet in radiorum vnione mediante reflexione, aut refractione. Sed quod attinet ad minorem resistentiam vaporum ad ipsum radium luminosum, etsi primo capite meæ Gnomonicæ leuiter id attigerim, nihilominus tamen, & idem ipsum hoc loco proponam. Denotetur, exempli gratia, superficies terræ ab. o. g. et ea, quæ est vaporum ab. n. a. supponatur etiam sol in situ.

q. qui

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q. qui sit Zenith pūcti. o. & etiā in. p. ipsi orizōti propinquus, aut extra Zenith, cuius duos radios. q. o. et. p. o. Imaginemur, quorum duæ partes. a. o. et. n. o. erunt aliquo modo ab ipsi vaporibus offuscata, sed. o. n. breuior est. o. a. ex. 7. lib. 3. Eucli. minorem ergo resistentiam habebit à vaporibus sol in Zenith, quàm extra eundem commorans, & quantò longius erit idem ab ipso Zenith, tanto maiorem resistentiam à dictis vaporibus inferri ex eadem. 7. lib. 3. Eucli. dicemus.



*Nullum corpus sensus expers à sono offendi, praterquam Aristoteles crediderit.*

CAP. XXXII.

**P**osse sonum corpus aliquod, quod sensu sit destitutum, vt Aristoteles. 9. cap. lib. 2. de cęlo putauit, offendere, est falsum.

Corpus enim non nisi à corpore potest lædi, non ergo à sono, cum sonus corpus non sit. Sed aer, & ignis, cum è contra sint corpora, hoc faciliè præstare possunt implendo aliquem locum velociter atd excludendum vacuum; vnde generatur sonus. Quod hucusque à nemine animaduersum fuisse comperio.

Pyta-

*Pythagoreorum opinionem de sonitu corporum cælestium non  
fuisse ab Aristotele sublatam.*

C A P. X X X I I I.

**S**enserunt Pythagorici orbes cælestes dum circunuoluuntur, non autem corpora stellarum solum, ædere sonum. Quibus dum Aristoteles contradicere cogitat, maximè fauet. Ea tamen opinio è phylosophorum scholis est explodenda, quia aut orbes sunt sibi ipsis contigui, aut inuicem distantes: si ab inuicem distantes (quod nemo adhuc concessit, quia hac ratione vacuum introduceretur) clarum est, quod cum se minime tangant, sonum edere nequeunt: Si verò eorum vnus alteri sit contiguus, neq; etiam ab ipsis sonus resultare poterit, quia pro certo putandum est, ipsorū superficies tam politas esse, ac lenas, vt nihil omnino asperitatis, aut inæqualitatis contineant. Vt exempli gratia, si aliquis duo specula plana inuicem confricaret, nullum planè sonum audiret, sed si hoc faceret cum duabus superficiebus asperis, sonū perferiret, & tanto maiorem; quanto asperiores essent dictæ superficies, & qui vult vt arcus liræ, ex corda sonum eliciat, colophonia dictum arcum illinet, vt asperiozem reddat. Necessarium quoque est vt tremat siue trepidet corpus, quod sonū edere debet; Neque etiam absque aere sonus effici potest, quia aer sonat ingrediendo velociter ad implendum locum, vt non remaneat vacuum. Sed supponendo in ætherea regione neque aerem esse, neque corpus aliquod fluidum, clarè patebit orbes cælestes ex sese nullum emittere sonum. Idem affirmo de fricatione superficiei concavæ infimi orbis lunaris cum conuexa materiæ à dicto orbe contentæ; superioribus rationibus fultus, vt etiam experientia à corpore aliquo fluido, quod in alio velocifimè moueretur desumpta fretus, cuius corporis superficies tamen lenis esset, à quo sonus non gigneretur. Et non minus dicere possum, corpus fluidum moueri in continente loco immobili, quam dictum corpus continens illud esse, quod moueretur, & non fluidum corpus. Cuius rei possumus etiam exemplum habere à quouis corpore perfectè rotundo, quod circa suum axem velocifimè moueatur, nullum sonum efficiet, quia nullam aeris partem extra suum locū impellit dum mouetur non secundum totum, sed secundum suas partes, quarum quælibet absque resistentia immediatè subintrat locum alterius, absque temporis interpositione. nec huiusmodi locum aliquo modo eadem materia dicti corporis, quod circunuoluitur: destitutum dimittat. Sed si Pythagorici de alia quadam harmoniæ specie ab ea, quæ est sonorum, vt à diuersis velocitatibus motuum, aut à diuersis magnitudinibus aut distantiis, aut stellarum influxibus intellexissent, rectè sensissent ex parte, non autem omnino, quia ea harmoniam efficere nequeunt, quæ ad inuicè secundum interualla harmonica proportionata non sunt, vt sunt dupla, sesquialtera, sesquitertia, sesquiquarta, sesquiquinta, supertripartientia quintas, superbipartientia tertias, & quæ ab ijs dependēt id est coniuncta sunt cum duplis; de consonantijs loquendo. de dissonantijs idem dico, quæ harmonicis inseruiunt modulationibus, vt sesquioctauū, sesquinonū, sesquidecimū, sesquiigesimū quartū, sesquioctogesimū, & superbipartientia vigesima quintas. Verū quidem est nonnulla harmonica interualla in aspectibus cōperta fuisse, vt Ptolomeus ostendit, & alii quoque asserunt. inest tamen huic rei non nihil difficultatis. vt exempli gratia, si subtrahamus diatessaron extra diapason, remanet diapente, & si à diapente subtrahamus semiditonum, remanet ditonum (quæ duæ

consonantiæ, eum habent respectum ad inuicem, quem habent diatessaron, & diapente, quia quemadmodum semiditonum, & ditonum simul coniuncta, componunt diapente, sic diatessaron, & diapente simul vnita componunt diapason; & quemadmodum terminus, qui diuidit diapason in diatessaron, & diapente, est mediator harmonicus inter extrema diapason diuisi, sic etiam terminus, qui diuidit diapente in semiditonum, & ditonum, mediator est harmonicus inter extrema ipsius diapente diuisi) subtrahendo deinde à diapason semiditonum remanet exachordum maius, & ab eodem diapason subtrahendo ditonum remanet exachordum minus, quæ quidē nō accidunt aspectuum circulo, quia subtrahendo aspectum quadratum ab opposito, remanet aliud quadratum, & subtrahendo sextilem à trino remanet quoque alius sextilis. Quod autem attinet ad motus, ad magnitudines, ad distantias, & ad influxus, nihil est, quod hisce proportionibus conueniat, sed quia hæc omnia depēdent ab infinita, & diuina puidētia Dei, necessariō fit vt istæ velocitates, eæ magnitudines, distantia, & influxus, talem ordinem, & respectum inter seipsa, & vniuersum habeāt, qualis perfectissimus fit.

*De raro et denso nonnulla, minus diligenter à Peripateticis  
perpensa.*

C A P. XXXIII.

**A**ntiqui Peripatetici de videndo in hyeme animalium halitu. Id, quod in æstate non euenit, malè disputauerunt, quia hoc nascitur à condensatione halitus, quæ ab ambiente frigore fit. quia halitus is ab ore, aut naso animalis exiēs non est purus aer attractus primò, sed mixtus est cum quodam vapore excrementitio, & subtili, quo semper ab ea parte euacuat corpus, qui statim ab aere frigido circumdatur, & densatur, quam ob causam ab ipso ea luminis pars reflectitur, quæ cum penetrare non potest, quod in hypocaustis, huiusmodiq; calidis locis non fit. Idem exemplo ab aqua statim à cisternis, aut profundis puteis in hyeme extracta comprobari potest, quia tunc temporis, huiusmodi aqua, cum magis calida sit, quàm frigida, emittit vaporem, qui facillimè videtur, ob rationem iam dictam, quod in æstate non cernitur in aqua, etsi ea magis calida esset, quàm ea, quæ in hyeme hauritur.

Ratio autem, quàm ab antiperistasi desumptam citarunt iidem ad inquirendum, cur aqua subterranea magis calida, aut minus frigida, hyberno tempore, quàm ea, quæ est supra terram sit, vana est, quia hoc non aliunde fit, quàm ab eo, quod terræ pori à frigoris siccitate sint clausi, vnde vapores & exalationes non tam facillè exire possunt. quam obrem calefiunt subterraneæ partes. Fimum, foenum, frumentum hac in re sunt nobis exemplo, in quibus sepius visum est ignem accendi.

Priore illa quoque ratione de antiperistasi dicta, volunt philosophi maiorem caliditatem hyeme, quàm æstate in animalium stomacho contineri, non animaduertentes siccitatem, frigiditatis partes superficiales corporis, restringēt, sanguinem versus originem suam impellere, qui in eo loco copiosior cum sit, eas partes tunc temporis calefacit magis.

Neque etiam iidem nouerunt causam, vnde fiat, ut in æstate impleto vase vitreo, aut argenteo, aut ex materia non porosa constante, aqua frigida, vas sudet, quod tempo-

tempore hyemis, non nisi in calidis locis euenit, quem sudorem, dicebant ipsi, esse eandem aquam, quæ per porros vasis exiret, quod falsissimum est, quia si per porros aqua frigida exiret multò magis exiret calida, cum subtilior sit, & ad penetrandum aptior. Sed hoc non aliunde oritur, quàm à condensatione aeris vas circumdantis, causata à frigiditate vasis refrigerati ab aqua, quemadmodum tempore hyberno clarè videmus manè superficies interiores vitri fenestrarum sudare, quia extrinsecù frigus refrigerando vitrum, intrinsecum aerem sibi contiguum congelat.

Neque silentio inuoluendum est, nec Aristotelem, neque alium ex suis fautoribus animaduertisse densum, & rarum esse causam ventorum. Rarū autem & densum, mediante calore & frigore fit, & si à partibus, in omogeneis, licet argumētari, de toto deducat consequentiam qui velit, obseruans in calidis aestatis diebus, dum aliqua nubecula ad Solem cooperiendum incedit, ibi statim agitationem aeris sentiri; ea verò nubecula prætergressa cum fuerit, & in ea parte, aer ad pristinam raritatem causatam à calore Solis redierit, quiescit; huiusmodi autem aeris agitatio, à hulla certè exhalatione proficiscitur, sed à motu solum locali, quem dum condensatur, facit. Omne densum natura sua frigidum est; omne rarum calidum, & è conuerso. Et frigida aura, quæ à stabellis causatur, non solum à nouo aere qui nos tangit, sed etiam à denso, quod in agitatione eiusdem aeris fit, nascitur.

Cum autem de raritate & densitate disputationem susceperim, non sine ratione mihi videt illorū opinionē explodēdā esse, qui Lunæ maculas nō aliud esse dixerunt, quàm aliquas partes rariore aliiis eiusdem Lunæ partibus, non obseruantes rarum, & densum, proportionabilia lumini, quod ab huiusmodi corporibus reflectitur, non esse. quia corpus aliquod rarum aliquando aptum erit ad reflectendum maius lumen, quàm corpus minus rarum ut manifestè apparet à nubibus reflecti lumen: quod ab aere non fit. Non defuerunt qui contrarium dixerunt, id est, eas Lunæ partes, densiores esse; neque unquam aliquis fuit qui de diaphano, aut opaco mentionem fecerit, quia melius est credere, eas partes diaphanas, siue perspicuas magis esse, quàm alias, quæ per aliquod spatium, solis radio ingressum permittant, & aliæ partes eū sint opacæ ipsum à superficie reflectant. diuersa tamen ratione à speculo, cum in plenilunio tota ferè Lunæ pars illuminata cernatur, quamuis dictum lumen extensiuè & intensiuè sit minus eo, quod ipsa in nouilunio recipit. Indignum autem mihi videtur ijs responderè, qui dixerunt huiusmodi maculas, terræ umbras existere, cum crassissimæ ignorantia tenebris sint circumfusi, vt etiā fuit Cornelius Agrippa, qui primo de occulta philosophia dicens se nosse modum quendā naturalem à Pythagora inuentum, quo in Luna id totum, quod ipse super speculum scripsisset, videretur. ostendit manifestè se ignorare luminum umbrarumq; naturam. quia nulla vnquam umbra generari potest à corpore, quod aut opacum non sit, aut officio opaci non fungatur, vt nunc dicemus de diaphaneitate aquæ. Neque corpus opacum illuminatum adūbrare potest, nisi opacum illud in linea recta situm obtineat, quæ inter lucidum & illuminatum extenditur. Neque etiam respondebimus ijs, qui sentiunt quotiescunque nulla esset terra, sed totus hic globus esset aqua, toties non futuram eclipsim lunarem, ratione diaphaneitatis aquæ. Quod falsissimū est, quia omne corpus sphericum quantumvis diaphanum sit, dummodo sit densius aere, luminosos radios refrangit, & eos ad inuicem interfecare facit, qui deinde vltra intersectionis punctū disgregantur, ita vt amittant illuminationis actum. Adde q̄ et si huiusmodi corpus aqueum, sphericum non esset, sed cubicum, illud super aliquā ex eius superficiebus ad angulos rectos radius solaris percuteret, non eum tamen penetraret, quia dictus radius perpetuò debilitatur, & eò magis, quo maiorem profunditatem in diaphano eius



eius corporis, quod sit densius aere acquirat, nec totus radius unquam dictum corpus ingreditur, cum ab eius superficie magna pars reflectatur. Resistit ergo huiusmodi corpus lumini, & quanto magis spissum aut profundum existit, tanto validius resistit. Habemus huius rei testes, piscatores unonum, in ijs mundi partibus, quæ paucis ab hinc annis Hispanorum opera nobis innotuerunt, qui affirmant ad maris vsq; fundum lumen Solis non peruenire.

Immediata ratio, cur nebule in ijs locis in quibus cõspiciuntur permaneant, & uic altiores, nunc verò depressores cernantur, non ea est, quam Aristoteles cap. 3. lib. 1. meteororum proponit, sed inde oritur, quòd sint eadem densiores ea parte aeris, quæ ipsis supereminet, & rariore e a, quæ ipsis subjacet. Quòd autem alicuius corporis densitas maior ea, quæ est medijs, in quo reperitur, causa sit, ut ipsum corpus descendat, & maior raritas eiusdem corporis, ea, quæ est quoque medijs, efficiat, ut dictum corpus ascendat, iam Archimedes in lib. de insidentibus aquæ docuit.

Rectissime instituit natura, ut corpora densiora versus loca angustiora, & minora (intelligendo ea loca orbicularis figuræ) quæ ad centrum propius accedunt, & rariora ad ampliora loca, & maius spatium occupantia, sese reciperent. tum quia eadem quantitas materiæ condensatæ, eget minori loco quam rarefacta, cū etiam, quia cū corpus densum non ita ad velocitatem motus localis, ut rarum, idoneum sit, ad eas partes accedat, quæ motibus tardioribus magis sunt aptæ, corpora autè rara ad eas, quæ velocioribus motibus sunt aptiores sese transferant. præterquam quod reuera appareat pro maiori parte, corpus magis densum, minus diaphanum; aut magis opacum futurum, quam rarum, licet sæpissimè videamus contrarium, ut superius inuimus. est tamen naturale proportionatumq; magis opacum densò, & diaphanum raro, quam è contra. Quamobrem summa ratione inducta natura voluit, ut corpora magis opaca, aut minus diaphana, magis vicina centro colligantur, ut spatium, quòd remanet, absque vilo impedimento à radijs solaribus penetrari possit. Tres autem eæ causæ, quas hoc loco posui, propriæ sunt, immediatæ, & per se, ex quibus fit, ut corpora densiora descendant, & rariora ascendant in medijs minus densis, aut minus raris dictorum corporum, quæ à nemine, quod sciam, hucusque propositæ fuerunt.

Qui autem asserunt cucurbitæ, quam apponunt chirurgi, effectum ex eo nasci, quod calidi sit attrahere, valdè aberrant à vero quia hoc, non nisi à raro, & à densò immediatè, à calido & frigido causatis efficitur, quia aer in cucurbita rarefactus à calore & per consequens dilatatus, statim ut à dicto calore deseritur, iterum condensatur & tantò citius, quantò aer ambiens frigidior existet, & quia eadem materia cum condensata fuerit minorem semper occupat locum, restringens igitur sese in cucurbita aer dum condensatur, necessariò fit, ne ulla, scilicet pars vacua remaneat, quod cum alius aer ingredi cucurbitam nequeat aliud corpus ingrediatur. Idem cum amphora in qua nullum aliud, quam aereum sit corpus experiri possumus, si eam ad ignem primò calefactam, deinde cū ore in amplo aliquo cyatho, aut alio vase. vino, aut aqua pleno vbi videbimus huiusmodi liquorem statim sursum ferri, quia dum calefit amphora, rarefit quoque aer qui in ea continetur, & quia rarefcit dilatatur, & quia dilatatur, eget maiore loco; & ideo magna pars eius foras exit; Cum verò ea aeris portio, quæ intus remanserit, iterum condensatur ob defectum caloris, restringitur, minoriq; indiget loco; Quod cum ita se habeat, necessarium est, ne aliquis locus vacuus remaneat, ut aliud quoddam corpus ingrediatur, cum ad ingrediendum aeri non patuerit aditus. quòd si corpus admodum non erit fluxile, aut humidum, ita ut ingredi amphoram possit ita amphoræ hærebit, ut non cito diuelli possit, & eo modo sepe

cū admiratione videm<sup>9</sup> fragile vas vitreū magnū, & graue lapideū corpus eleuare. Sed vt ad densum & ad rarum redeamus, mihi videtur frigidum esse consequentem qualitatem densi, & calidum rari, quia quæuis res dum calefit, rarefit, & quælibet materia dum refrigeratur, simul condensatur. Qua ratione fit, vt terra frigidior fit aqua, & ignis calidior fit aere.

Nec propriè locutus est Aristoteles. 9. & 10. capite primi lib. & secundo secundi methcororum cum dixerit calorē Solis eum esse, qui sursum humores, vaporesq; euehat, quia Sol nil aliud facit, quam calefacere, cuius caloris ratione, ea materia rarefit, & ob rarefactionem leuior facta ascendit, non quia sursum à Sole feratur.

Quæ subsequuntur, cum raro ac denso symbolum habere videntur. cum raro, scilicet calidum, humidum, leue, sublime, diaphanum, lumen, clarum, lux, albū, dies, motus, velox, simplex, disgregatum, molle, lene, acutum, subtile, coctum, spaciosū, dulce, voluptas, audacia, læritia, liberalitas, veritas, industria, amor, misericordia, humanitas, sanitas, vita, & iis similia. Cum denso verò frigidum, siccum, graue, imum, opacum, vmbra, obscurum, tenebræ, nigrum, nox, quies, tardum, mixtum, congregatum, durum, asperum, obstusum, crassum, crudum, angustum, amarum, dolor, timor, melancholia, auaritia, mendacium, inertia, odium, crudelitas, feritas, infirmitas, mors, & ijs similia.

Verum est quod ea ratio, qua Aristoteles ait aerem humidum esse, parui est momenti, quia similiter de igne inferri posset, qui facilius à termino alieno, quā aer, aut aqua terminari potest.

*Motum rectum curuo posse comparari etiam dissentiente  
Aristotele.*

C A P. X X X V.

**S**ed vt ad Aristotelē redeamus, rectè dicere non potest motum rectum ad curuū comparabilem non esse. 4. cap. lib. 7. physicorum, vbi errat quoque dicens reperiri non posse lineam aliquam rectam alicuius circuli circumferentiæ æqualem. quia Archimedes iam probauit in lib. de quadratura circuli, triangulum illum orthogonium, cuius vnum ex lateribus circulantibus angulum rectum æquale esset semipiametro alicuius circuli, & aliud circumferentiæ, æqualem futurum dicto circulo. Illud igitur triangulum orthogonium, quod æquale erit alicui circulo, & habebit ali-quod ex suis lateribus circulantibus angulum rectum æquale semidiametro dicti circuli, aliud quoque latus ipsum angulum rectum circundans, ex necessitate, circumferentiæ dicti circuli habebit æquale. Potest igitur dari vna quædam recta linea equalis circulari contra Aristotelis opinionem, qui non benè reuocauit in mentem, quod scripsit de reatiuis, cum dixit quadraturam circuli posse quidem dari, etsi tūc tps de ea nō haberet scientia. Si igit dicta quadratura dari pōt, potest etiā dari vna recta linea equalis circumferentiæ eiusdē circuli, ob rationes iā dictas. Sed si Arist. dixisset, circularem corporum cælestium motum, comparabilem non esse recto corporum elementarium, verum dixisset, non quia eorum alter circularis, alter verò sit rectus, sed quia cælestis regularis sit, neque modò tardus, modò velox, sed vnam semper & eandem velocitatem retinens, mor<sup>9</sup> aut, qui est corporū elementarium

tarium è contrà se habeat, præter id, q̄ nunquam fuit, neque sit futurus aliquis horū rectorum, qui naturales dicuntur, qui tam velociter moueatur, ut motus cœli, quia si voluerimus considerare motum diurnum, 24. horarum, secundum opinionem communem, reperiemus calculando, Lunam in quadraturis cum Sole, dum inuenitur in æquatore, singulis horarum minutis moueri per 500. milliaria Italica vel circa, & in coniunctionibus, & oppositionibus ipsius Solis, 1000. vel circa, & Solem tempore æquinoctiorū. 18000. & Saturnū circa æquatoris sitū. 260000. & ampli⁹ de stellis aut fixis circa æquatorem positis quibus cogitet, quod reuera difficillimum quibusdam videbitur, quod quidem non occurrit secundum pulcherrimam Aristarchi famij opinionem, diuinitus à Nicolao Copernico expressam, contra quam nil planè valent rationes ab Aristotele; neque etiam à Ptolomeo propositæ. Motu verò proprio, quo libet horæ minuto, Sol mouet per milliaria circa. 48. Luna quādo cōiuncta est, aut opposita reperitur Soli, 36. milliaria, & in quadraturis. 18. Saturnus. 24. Iupiter. 40. Mars. 100. Venus. 26. Mercur. 5. Sed Saturnus motu rapido, vno horæ minuto mouet circa. 260000. milliaria, vt diximus Iupiter circa. 170000. Mars. 75000. Venus. 10000. Mercurius. 2000. corpus autē elementare, & si moueret motu recto hoc mō, & velocius etiam corpore cœlesti, non obseruans tamē uniformitatem, ut dictum cœleste facit, cum eodem nullo modo comparari posset, quia rectus dictus naturalis, suam semper velocitatem adauget, ob continuam impressionem, quam recipit à causa perpetuò coniuncta cum ipso corpore, quæ est propensio illa naturalis eundi breuiori quadam via ad locum suum, ita vt etiam si dictum corpus elementare à motu tardiore ad velociorem, superare posset motū alicuius corporis cœlestis, ij duo motus interfecarent sese in vno solo pūcto, quod diuidi distribuiq; in partes nequiret, id est non nisi in vno solo temporis instanti redderentur æquales, vt ita dicam. Neq; solū loquor de circulari cœlesti cum recto elementari, sed de qualibet alia motuum specie, siue sint ambo recti, siue ambo curui, quando aliquis eorum irregularis erit.

*Minus sufficienter explosam fuisse ab Aristotele opinionem credentium plures mundos existere.*

C A P. XXXVI.

**M**Aior ratio, qua Aristoteles eorum opinionem, qui plures esse mundos dixerunt, refutare nititur, in eo consistit, quod is credat partes terræ, quæ alijs mundis assignarentur, ad huius mundi centrum inclinationem habere, & sic ignem illorum, propensionem habiturum ad circumferentiam huius.

Quæ certè ratio tam debilis est, vt per se cadat, non considerans ipse, quod si essent dicti mundi, eorum quilibet suum proprium centrum, suamq; propriam circumferentiam haberet, terrasq; & ignes haberent inclinationem ad centra circumferentiasq; suorum mundorum, absque eo, q̄ vna terra, alterius centrum appeteret, vt exempli gratia, si doctissimi Aristarchi opinio est vera, rationi quoq; consentaneum erit maximè, vt quod Lunæ contingit, cuilibet etiā ex alijs quinque planetis eueniat, id est, vt quemadmodum Luna suorum epicyclorum ope circū terram voluitur, quasi per circumferentiam alterius cuiusdam epicycli, in quo terra sit instar centri naturalis (id est sit in medio) delati ab orbe annuo circa Solem; Sic etiam Saturnus, Iupiter, Mars, Venus, atque Mercurius, circum aliquod corpus in medio sui epicycli

cli maioris, situm habens, voluantur; quod quidem corpus, & aliquem quoque habeat motum circa suum axem, sit opacum, ijs conditionibus, quæ terræ sunt similes, præditum existat, & in dicto epyciclo sint res similes istis lunaribus.

*An rectè loquutus sit Phylsophus de extensione luminis per uacuum.*

C A P. X X X V I I.

**A**ristoteles secundo lib. de anima sentit quod per vacuum non extenderetur lumen, quod procederet à corpore lucido. Quod verisimile nõ est; quia quæ admodum quanto rarius est aliquod corpus, tanto aptius est ut diaphanum existat; & quanto rarius est dictum corpus, tanto minorem quantitatem materiæ contineat; sic quanto magis diaphanum est, cum ex per exigua materia constet, tanto magis liber transitus luminis patet; Vnde quanto minor quantitas materiæ erit in dicto spatio, tanto nitidius pertransibit lumen. Sequitur ergo, quod ubi nulla esset materia, totum lumen libere transiret. Color cæruleus quem videmus in profunditate aquæ, & aeris, color est aquæ & aeris, qui denotat resistantiam factam ab aere & ab aqua ipsi luminis; Quod quidem lumen ubi corpus aliquod non esset, minime reflecteretur, sed absque ullo impedimento rectè transiret.

*An rectè phylsophia penus Aristoteles senserit de loco impellendo à pyramide.*

C A P. X X X V I I I.

**A**ristoteles 8. cap. lib. 3. de cælo, disputans contra antiquos de elementorum figuris, ait pyramidem implere posse locum corporeum. quod verum non est. Cubus quidem id facit ab 8. enim cubis perfectè impletur locus, sed non item. 12. pyramides, ut Aristoteles sensit (id est sex super aliquam exagonam figuram superficialem & sex sub eadem) id præstant, cum potius maius vacuum remaneat ad quamlibet partium supra, & infra, quam plenum. Rectius Aristoteles egisset, si probasset ratione immobilitatis convenire pyramidem terræ, quam cubum. quamvis, de horum corporum altero, sit stultum hoc credere. decepti tamen fuerunt antiqui, credentes cubum ad motum minus idoneum esse, quam reliqua quatuor corpora regularia (loquor autem habita volubilitatis ratione) quia pyramidale est illud, quod ita se habet, ut multis rationibus probari potest, quarum una hæc nobis sufficiat. Scimus iam ex communi conceptu corpus sphericum esse magis volubile, instabileque, quam alia sint. Illud ergo corpus, cuius figura ad sphericam magis accedet, ad voluendum, & ad mouendum facilius erit quouis alio, quod æqualis sit quantitatis, & sibi omogeneum materia, ut exempli gratia corpus. 20. basium ad voluendum, & ad mouendum promptius erit eo, quod ex. 12. constat, & id, quod est. 12. eo, quod est. 8. & id, quod est. 8. eo, quod est. 6. & id, quod est. 6. ut cubus est, eo, quod est. 4. cuiusmodi est pyramidale. Huc accedit, quod pyramidale corpus aliam conditionem habet, quam cubicum, cum in quavis facie inaltera-

## DISPUTATIONES.

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rabile sit, cubicum autem e contra sit alterabile unde quaque, suaque; quadrata in rhombos mutare possit, iisdem existentibus lateribus.

*Examinatur quam valida sit ratio Aristotelis de  
inalterabilitate Cæli.*

## CAP. XXXIX.

**A**ristoteles textu. 22. primi lib. de Cælo ita inquit.  
Accidit autem, & hoc per sensum sufficienter, quo ad humanam dixisse fidem, & omni præterito tempore secundum traditam inuicem memoriam, nihil videtur transmutatum neque secundum totum vltimum cælum, neque secundum partem ipsius propriam vllam.

Hoc autem in loco Aristo. non considerauit, quod similiter de terra dici posset, quando ipsa ita eminus prospiceretur, imo absque dubio putandum est, quod si terra lucē Solis prædita esset, & aliquis ipsam ab octauo orbe vellet videre, nullo pacto cerne-  
ret, cum sidera illa quæ primæ magnitudinis vocantur, & quæ plusquam centies maiora ipsa terra putantur non nisi vt puncta videantur.

IN

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 I N Q V I N T V M  
 EVCLIDIS LIBRVM.



Quamuis omnia libri quinti Euclid. uerissima sint. Animaduertimus tamen permultos summa cum difficultate eorum demonstrationes percipere. Praecipue ubi quinta, aut septima definitiones eiusdem libri necessariae sunt. Illae enim adeo obscurae uidentur, ut longè facilius admissuri sint haec nostra postulat at tanquam clariora. Atque etiam tanquam intellectui commodiora, quam sit illud quintum idemque ultimum postulatum eiusdem in primo libro positum, de linea duas alias secante. Quandoquidem ijs nostris postulatis admissis, sequentia Theoremata per facillima reddentur.

Horum autem primum est.

QVOD tota composita ex aequali numero partium aequalium, sunt inuicem aequalia.

Vt si quis diceret omnes proportionales quae compositae sunt ex aequali numero aliarum proportionum inuicem aequalium, sunt etiam inuicem aequales, quod Euclides conatur demonstrare in. 2. 2. et. 2. 3. quinti libri.

S E C V N D V M.

QVOD si à totis aequalibus detractae fuerint aequales partes, quae remanent erunt partes inuicem aequales.

Et è conuerso si aequalibus aequalia addas composita erunt inuicem aequalia.

Quod in ipsis proportionibus hoc loco semper intelligendum est.

T E R T I V M.

Quae est Euclidis septima propositio.

QVOD si fuerint plures termini aequales inuicem, ratio seu proportio unius ipsorum ad alium tertium terminum maiorem, minorem uicè, sed eiusdem generis, erit eadem quae cuiusuis alterius termini ad eundem tertium. Et è conuerso, quae fuerit proportio tertij termini ad unum praedictorum aequalium, eadem erit, specie, cum alio eorundem terminorum.

QVAR-

## IN QVINT. LIB. EVCLI.

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## QVARTVM.

*Euclidis uerò nona propositio.*

QVOTIESCUNQVE proportio vnus plurium terminorum collatorum cum aliquo tertio eiusdem generis, eadem fuerit cum ea quæ est cuiusuis alterius dictorum terminorum cum eodem tertio, aut proportio dicti tertij, cum aliquo dictorum, eadem fuerit cum ea quæ ipsius est ad aliquem alium eorundem terminorū, tunc eiusmodi termini, æquales erunt inter se.

## QVINTVM.

*Euclidis uerò octaua propositio.*

QVOTIES plures erunt termini, quorum vnus fuerit maior altero, si comparentur alicui tertio eiusdem generis, proportio maioris ad tertium illum, maior erit ea, quæ est minoris ad prædictum tertium, & proportio illius tertij ad maiorem, minor erit ea, quæ eiusdem tertij ad minorem terminum comparati.

## SEXTVM.

*Euclidis uerò decima propositio.*

QVOTIES proportio vnus, ex pluribus terminis comparatis ad aliquem tertium, maior fuerit proportionem alicuius alterius dictorum cum eodem tertio, primus ille terminus, altero maior erit. Et quoties proportio tertij termini ad vnum quæ ad alterum terminum maior fuerit, eiusmodi terminus altero minor erit.

## SEPTIMVM.

*Euclidis uerò undecima propositio.*

PROPORTIONES, quarum vnaquæque cum aliqua tertia æqualis est, ipsæ quoque inter se sunt æquales. Vt illud, Quæ vni & eidem sunt æqualia, sibi inuicem sunt æqualia.

## OCTAVVM.

*Euclidis uerò duodecima propositio.*

QVOTIESCUNQVE proportio vnus ex pluribus antecedentibus cum suo ex pluribus consequentibus, æqualis fuerit ei cuiusuis alterius dictorum antecedentiū, cum suo plurium consequentium, proportio totius aggregati antecedentium cum toto aggregato consequentium, dictæ primæ proportioni æqualis erit, nempe illius antecedentis ad suum consequens.

## NONVM.

*Euclidis vero tertiadecima propositio.*

QVOTIESCVNQVE aliqua proportio plarium proportionum inuicem æqualium, tertia aliqua proportione, maior aut minor fuerit, qualibet prædictarum æqualium inter se, tertia illa proportione maior aut minor pariter erit.

DECIMVM.

QVOTIESCVNQVE fuerint ex vna parte plures termini (sive coniuncti sive disuncti sint) æquales singuli vni tertio termino; ex altera verò parte totidem fuerint alteri tertio termino æquales, proportio aggregati priorum terminorum ad suum tertium, æqualis erit proportioni aggregati reliquorum terminorum ad suum tertium, & è conuerso, ita se habebit primus tertius terminus ad suos multos terminos, sicut se habet secundus tertius terminus ad suos simul sumptos.

VNDECIMVM.

Aggregatum ex partibus proportionalitatis continuæ, quod inter maximum, & minimum terminum omnium terminorum proportionalium comprehenditur, semper multiplex est ad singulas partiales proportiones, ex quibus ipsum componitur.

DVODECIMVM.

Quæuis proportio quocunque modo diuisa fuerit, ex iis partibus componitur, in quas diuiditur.

*Cum enim hæc propositiones sint ita conspicuæ ipsi intellectui, ut absq; dubio inter obiecta ipsius intellectus connumerari possint, nullus sanæ mentis eas negabit.*

THEOR. I. II. ET III.

Primum, secundum, & tertium theorema quinti Euclidis ab ipso satis exactè demonstratur, studiosus itaque autorem consulat.

THEOREM. IIII.

Quartum vero Theorema Euclidis ego sic demonstraré. fit, verbi gratia, proportio . a. ad. b. quæ est. c. ad. d. sumptis multiplicibus. e. et. f. ad. a. et. c. æqualiter, item multiplicibus. g. et. h. ad. b. et. d. dico proportionem. e. ad. g. esse eandem quæ est. f. ad. h. Habemus enim ex. 10 postulato præmissò, eandem futuram proportionem. e. ad. a. quæ est. f. ad. c. & ita. b. ad. g. quæ est. d. ad. h. ex præsupposito verò cū sic se habeat. a. ad. b. sicut. c. ad. d. erit ex primo postulato eadè proportio. e. ad. g. quæ est. f. ad. h. Nam proportio. e. ad. g. componitur ex eis quæ sunt. e. ad. a. et. a. ad. b.





b. et. b. ad. g. & similiter proportio. f. ad. h. componitur ex eis quæ sunt. f. ad. c. et. c. ad. d. et. d. ad. h.

T H E O R . V . E T V I .

CIRCA 5. et. 6. theorema nihil notandum occurrit.

T H E O R . V I I . V I I I . I X . X . X I . X I I . X I I I .

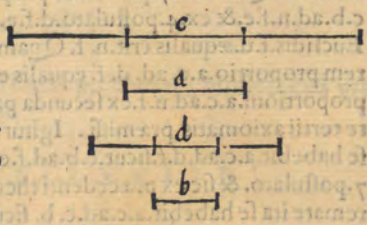
**T**heoremata à 6. in. 13. cum sint de obiectis intelligibilibus, sine vlllo medio, ab intellectu cognitis, inter axiomata à me relata fuerunt. 7. inquam quinti Euclid. fecimus tertium Postulatum, 8. quintum, 9. quartum, 10. sextum, 11. septimum, 12. octauum, 13. nonum.

T H E O R E M . X I I I I .

**Q**uartumdecimum Theorema ex Euclide demonstrabitur, mutatis tantum theorematibus ab interprete notatis, ita vt loco. 7. 8. noni, & decimi citetur tertium. 5. 4. et. 6. postulatum à me propositum.

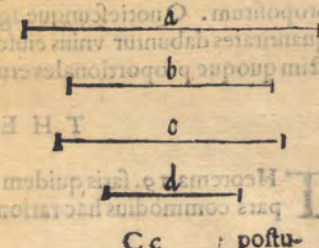
T H E O R . X V .

**Q**uintumdecimum Theorema sic demonstrabo; Sit, exempli gratia, a. terminus antecedens. et. b. consequens, quibus duo multiplices sumantur. c. et. d. Dico eandem proportionem habiturum. c. ad. d. quam. a. ad. b. habet. In primis enim manifestè patet quamlibet partem ipsius. c. habituram eandem proportionem cum qualibet parte. d. quam habet. a. ad. b. quare ex. 7. et. 8. postulato propositum elucefcet.



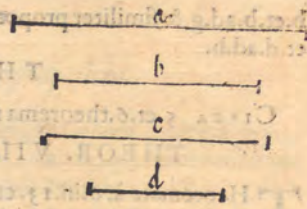
T H E O R E M . X V I .

**S**extumdecimum theorema sic demonstrabitur. Sit, exempli causa, eadem proportio. a. ad. b. quæ est. c. ad. d. Dico qd ita se habebit. a. ad. c. sicut. b. ad. d. Cogitemus itaque alterum istorum terminorum. c. aut. b. medium inter. a. et. d. quare primum intelligamus. b. inter. a. et. d. proportio ipsius. a. ad. d. componetur ex ea quæ est. a. ad. b. & ea quæ est. b. ad. d. ex. 12. postulato. Et ex eodem, illa ipsa proportio. a. ad. d. pariter componetur ex ea quæ est. a. ad. c. & ea quæ est. c. ad. d. sumpto. c. pro medio termino. Ex quo sequitur, aggregatum duarum proportionum, videlicet. a. ad. b. et. b. ad. d. æquale esse aggregato. a. ad. c. et. c. ad. d. ex quibus aggregatis æqualibus si duas proportiones æquales subtraxerimus, eam videlicet quæ est. a. ad. b. & illam quæ est. c. ad. d. supererunt duæ proportiones inter se æquales. erit enim proportio. a. ad. c. æqualis proportioni. b. ad. d. ex prima parte secundi postulati diuisim.



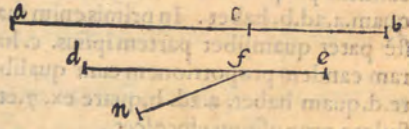
Alia etiam ratione id ipsum demonstrari potest, sumpto. b. pro medio termino inter. a. et. c. et. c. pro termino medio inter. b. et. d. quare proportio. a. ad. c. componetur ex. a. ad. b. et. b. ad. c. illa verò quæ est. b. ad. d. ex. b. ad. c. et. c. ad. d. ex. 12.

postulato. Sed cum proportio a. ad. b. equalis sit  
 proportioni c. ad. d. communis autem b. c. propor-  
 tio. itaque a. ad. c. equalis erit. b. ad. d. ex secunda  
 parte. 2. postulati compositè, & sic habebimus pro-  
 positum, ita quòd quotiescunque dabuntur. 4. qua-  
 titates ex una parte proportionales, illæ ipsæ ex  
 altera proportionales erunt.



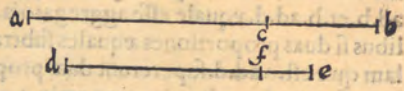
T H E O R. X V I I.

**D**ecimiseptimi theorematìs hæc est demonstratio. Ita se habeat a. c. b. ad. c.  
 b. sicut se habet. d. f. e. ad. f. e. Probo ita se habere a. c. ad. c. b. sicut se habet. d.  
 f. ad. f. e. Cogitemus itaque alterum terminum scilicet. n. f. qui sic se habeat. ad. f. e.  
 sicut se habet. a. c. ad. c. b. Quare ex præcedenti theoremate ita se habebit. a. c. ad. n.  
 f. sicut se habet. c. b. ad. f. e. & ex. 8. postulato ita se habebit. a. c. b. ad. n. f. e. sicut se habet.  
 c. b. ad. f. e. Sed cum ex præsupposito ita se habeat. a. c. b. ad. c. b. sicut se habet.  
 d. f. e. ad. f. e. ideo ex præcedenti theoremate ita se habebit. a. c. b. ad. d. f. e. sicut se ha-  
 bet. c. b. ad. f. e. demonstratum autem est ita se habere. c. b. ad. f. e. sicut se habet. a. c. b.  
 ad. n. f. e. Quare ex. 7. postulato proportio. a. c. b. ad. d. f. e. equalis erit proportioni. a.  
 c. b. ad. n. f. e. & ex. 4. postulato. d. f. e. equalis erit. n. f. e. Itaque ex 3. postulato primi  
 Euclidis. f. d. equalis erit. n. f. Quamob-  
 rem proportio. a. c. ad. d. f. equalis erit  
 proportioni. a. c. ad. n. f. ex secunda par-  
 te tertij axiomatis præmissi. Igitur ita  
 se habebit. a. c. ad. d. f. sicut. c. b. ad. f. e. ex  
 7. postulato. & sic ex præcedenti theo-  
 remate ita se habebit. a. c. ad. c. b. sicut. d. f. ad. f. e. quod erat propositum: Quoties-  
 cunque igitur dabuntur. 4. quantitates coniunctim proportionales, diuisim quoque  
 proportionales erunt.



T H E O R E M. X V I I I.

**T**heorema. 18. hac ratione demonstrari potest. Detur proportio a. c. ad. c. b. si-  
 milis ei quæ est. d. f. ad. f. e. probo ita se habere. a. c. b. ad. c. b. sicut se habet. d. f.  
 e. ad. f. e. In primis notum est ex. 16. theoremate ita se habiturum, a. c. ad. d. f. si-  
 cut. c. b. ad. f. e. Quare ex. 8. postulato ita  
 se habebit. a. c. b. ad. d. f. e. sicut. c. b. ad. f. e.  
 Itaque ex. 16. theoremate ita se habebit.  
 a. c. b. ad. c. b. sicut. d. f. e. ad. f. e. Quod erat  
 propositum. Quotiescunque igitur. 4.  
 quantitates dabuntur vnus cuiusdemq; generis disiunctim proportionales, coniun-  
 ctim quoque proportionales erunt.



T H E O R E M. X I X.

**T**heorema. 19. satis quidem apud Euclidem demonstratur: eius tamen tertia  
 pars commodius hac ratione demonstrari poterit (nempe) quod cum sit pro-  
 portio

IN QVINT. LIB. EVCLI. 203

portio. a. ad. b. quæ est. c. ad. d. probabo ita se habituram proportionem. b. ad. a. sicut se habet. d. ad. c. hoc argumento: si. a. ad. b. ita se habet sicut. c. ad. d. ex. 16. theoremate ita se habebit. a. ad. c. sicut. b. ad. d. Quare sic se habebit b. ad. d. sicut. a. ad. c. Itaque ex eodem. 16. ita se se habebit. b. ad. a. sicut. d. ad. c.

$$\frac{a}{c} = \frac{b}{d}$$

THEOREM. XX.

Quamuis. 20. theorema apud Eucli. perfectè demonstratum fuerit, potest nihilominus & hac via demonstrari. Sic se habeat proportio. a. ad. b. sicut se habet. c. ad. d. & proportio. b. ad. e. sicut. d. ad. f. Dico qd si. a. maius fuerit. e. pariter. c. maius erit. f. & si. a. minus fuerit. e. c. quoq; minus erit f. sin verò æquale, et æquale erit. Nam ex primo postulato certi sumus ita se habere proportionē. a. ad. e. sicut se habet proportio. c. ad. p. Quare ex. 12. theor. ppositū manifestū erit.

$$\frac{a}{c} = \frac{b}{d}$$

$$\frac{b}{d} = \frac{e}{f}$$

THEOREM. XXI.

Vigesimum primum theorema, satis apud Eucli. probatum, nihilominus præscripto nunc modo demonstrari poterit.

THEOREM. XXII. XXIII.

DVO hæc theoremata in primum postulatum collegimus. Sequentia verò cum exactè apud Eucli. demonstrantur non est cur nos in ijs immoremur.

PHYSICA,  
ET MATHEMATICA  
RESPONSA.

JO. BAPTISTAE BENEDICTI PATRITII  
Veneti, Philosophi Mathematici.

Ad Lectorem.



**V** Nil magis virtutis est proprium, quam  
agitari, & incessabili motu prodesse. Ac velu  
ti fulgidum sydus ante oculos spectantiū com  
micare. Ita mihi mathematicis ijsq; maxi  
mè philosophicis speculationibus dedito, sapi  
sime, ut in principium summorum aulis, &  
amplissimis ciuitatibus degenti, ubi multa sem  
per Nobilium mira curiositate, sciendi desiderio, & conferendi cu  
piditate referta, uersantur ingenia, contigit, modo ab his, modo ab  
illis, aut uerbis tentari, aut literis promocari ad differendum, de  
his, in quorum studijs uersamur. Quarum concertationum & re  
sponsonum, quoniam non omnino indigna existimaui, quæ memoria  
comendarentur, partem aliquam apud me conseruaui. Vbi uerò  
per ocium licuit, collegi, relegi, ac tandem de manu mittere decreui.  
Tum ut scientia ipsa quo magis diffundetur, crescat; & quicquid  
ualeo, sine inuidia in communem utilitatem conferam. Tum ut ui  
rorum præstantissimorum, qui me suis interrogationibus excitaue  
runt, quantum in me erit, gratitudinis ergo, nomina reddam im  
mortalia, & eorum exemplo alios, ocio sordidiore abiecto, quod solet  
curialium præcipuè excelsa ingenia corrumpere, ad sciscitandum  
conferendum, & differendum, de rebus serijs, & quæ usui aliquan  
do esse possint, & quandoq; euulgari mereantur, alliciam. Tu inte  
rim nostris laboribus fruiere, & nostram diligentiam boni, & equi  
consule, & Vale.

DE

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IO. BAPTISTAE BENEDICTI

# DE TEMPORVM EMENDATIONE

IO. BAPTISTAE BENEDICTI  
Patritij Veneti, Philosophi  
Mathematici.

*AD SERENISS. EMANVELEM PHILIB.  
Allobrogum & subalpinarum gentium Ducem  
Inuictissimum.*

E P I S T O L A.



IRVM, Quàm lectione epistolæ seu ( vt vocant ) Breuis. S. D. N. Gregorij XIII. Pont. Max. quod ad me nuper tua Celsitudo misit ex Nicea, vt meam de ea re sententiam proferrem, delectatus sim; ex quo, non tantum recta illius mens ac verè sancta cogitatio, sed etiam aperta maximaq; si ad exitum perducatur, imo summè necessaria vniuerso orbi vtilitas percipi potest; qua de re memini cum Celsitudine tua aliquando sermonem habuisse. Vidi præterea cum ipso breui transmissum compendium Domini Aloisij Lili: cuius mihi sententia perplacet, de correctione eius diei, qui 134. quoque anno præter, necessitatem, gignitur. qui sanè dies perpetuæ retrogradationis ingressus Solis in Zodiaci signa, causa fuit. quod ita perspicuè patebit. Cum Numa Pompilius anni cursum correxit emendauitq; , ea sanè mente id videtur præstitisse, vt principium Ianuarij primi mensis anni, præcisè in ipso hyemalis solstitij puncto collocaretur. quod hac tempestate, dictam ob causam adeo retrocessit, vt circa vndecimam diem Decembris esse reperiatur. quod si centesimo trigesimo quarto quoque anno detractus dies vnus fuisset, nihil erroris prorsus accidisset. Atq; dies hic ( vt alias Celsit. tuæ significauit ) inde generatur, quod quarto quoque anno addentes nos ad quarti anni dies. 365. diem horarum. 24. ob errorem annum horarum quinque minorum. 49. secundorum ferè. 16. ( anni æqualis siue medij ) fallimur quarto quoque anno in minutis. 42. secundis propè. 56. amplius quàm par sit minutis scilicet. 10. secundis ferè. 44. singulis annis; qui numerus. 134. multiplicatus, diem penè horarum. 24. constituit; penè inquam, quia minutum vnū deesset tñ modo, & secunda. 44. si decè illa minuta, & 44. se cunda annua, exquisita essent atque perfecta; quæ tamen differentia nullius adeo esset momenti, aut certè perexigui, vt vix exactis. 111086. annis, diem vnum afferret. Itaq; planè necessaria eiusmodi esset emendatio, aptaq; eius ratio à D. Lilio ostenditur, prout etiam Petrus Pitatus Veronensis tradidit, in eo, quem de vera anni quantitate tractatu conscripsit, nempe vt tribus primis centesimis annis, centesimus quisque annus communis sit, quartus subsequens centesimus intercalaris: quod sanè fieri necesse est. Nam

cùm

cùm tribus centesimis cõmunibus, tres quartas diei partes plus æquo detraxerimus, non enim centesimo quoque anno, sed centesimo trigesimo quarto, dictus dies detrahi debet, postquam tres integros dies, qui quadringentis detrahendi erant, trecentorum annorum spacio detraxerimus; sitque 134. penè tertia pars. 400. quarto annorum centenariorum, tres quartæ diei partes recuperabuntur; atque ita in fine quadringentesimorum annorum omnia exactè suo loco restituta erunt. Idcirco dictus iam quadringentesimus annus intercalaris & non communis constituendus erit, non alia de causa, quam ut bissexti ordinem sequamur.

Is verò modus, qui à D. Lilio traditus est, de ratione inueniendi singulis mensibus Nouilunij diem, interdum fallere nos posset vno die; prout Ianuario proximè lapsò accidit; quo ex præscripto modo nouilunij, dies nonus illius mensis esse debuisset, qui fuit tamen dies septimus, sexta decima hora cum dimidia post meridiem. Neque etiam tutum est, via integrorum dierum, nulla habita horarum aut minorum ratione, nec minus ea, quæ à Pitato tradita est, mediorum seu æqualium motu progredi: At censerem potius veros motus sequendos esse ex calculis exactarum tabularum, quales Prutenicas esse iudico; Et cum solius Paschæ causâ laboremus hac in re, pleniluniorum verorum, in multis annos tabulas formarem, quæ æquinoctia veralia sequuntur, cum assignatione diei Paschatis præcisè, prout fecit Pitatus; non via tamen æqualium pleniluniorum sed verorum. Porro quod ad Paschatis celebrationem attinet, rationi consentaneum est, concilij Niceni decretum ea de re feruari, prima scilicet dominica die post primum plenilunium, quod æquinoctium vernale sequitur; hoc tamen animaduerso, si dictum plenilunium primum post æquinoctium contingens, diè dominicum fortiretur; nulla ratione tali die Pascha celebrandum esse; verum subsequenti, ne cum Hæbreis consentiat Ecclesia Christi: quæ fuit causa, ut in decreto concilij Niceni statutum sit, à quartadecima, in vigesimam primam celebrari debere: Quod mihi Petrus Pitatus non animaduertisse videtur, cum ex ei<sup>9</sup> sententiâ in suis tabulis diè Paschate declarata, huiusce anni Pascha celebrandum fuerit. 23. Martij, ipsomet de plenilunij non tantum æqualis, sed veri.

Dies autem Paschatum elapsorum, quos hæcenus examinaui, reperi omnes concordare cum ea regula, quam nonnulli de diè carnis priuui tradiderunt. nempe primum diem martis post nouilunium Februarij, carnis priuui diem esse; non autem cõsactione Patrum concilij Niceni, qua statuerunt à vigesima prima Martij dirigendum esse Paschatis diem, ut porè qui sibi persuaserunt, circa eum diem æquinoctium perpetuò esse debere; prout tunc temporis erat. Non itaq; error accidit, quod Pascha ex huiusmodi suppositione concilij, post vigesimam primam lunæ celebratur, cum seruata regula concilij non fuerit. Prout manifestum est de Paschate anni. 1566. celebrato. 14. Aprilis (quæ fuit. 24. lunæ) quod. 7. dicti mensis celebrandum erat. Tum anno. 1569. 10. Aprilis solenne fuit Pascha, quod tertia eiusdem esse debuerat. Anno deinde 1572. 6. Aprilis, dies fuit Paschatis, quæ. 30. Martij futura erat, anno vero 1575. in tertiam Aprilis Pascha incidit, casurum in. 27. Martij.

Cum igitur (ut ex diplomate ad Celsit. tuam misso patet) S. D. N. mens sit atq; voluntas, ut quisque liberè in medium proferat quid hæc de re sentiat: quædam mihi non omnino præmittenda occurrunt, quæ tantis coeptis non nihil adiumenti forsasse adferre queant.

Atque illud in primis non tantum ut corrigatur Calendarium ob Pascha cæteraque festa mobilia ab illo manantia, ut decreto concilij Niceni fancitum est, scilicet ut ipsum Pascha celebretur prima dominica post primum plenilunium, quod æquinoctium

noctium vernale proxime sequitur; verum etiam quò anni principium emendetur, scilicet ut ad suum verum principium reuocetur annus. nempe ad diem hyemalis solstitij, quæ prima Ianuarij dies esse debet.

Deinde, tot dierum menses constituantur, quot hac nostra tempestate, sol in ipsis Zodiaci signis versatur. Postremò, quædam festa immobilia in alios dies transferantur, celebrenturq; aptis temporibus: quod à. S. D. N. mente dissentire minimè videtur. cum non magis de festis mobilibus quam immobilibus agat, imo etiam planè æquum sit, ut habeatur utrorumque ratio, quò statutis temporibus celebrentur.

Ut autem ad primam Ianuarij diem verum principium anni reuocetur; censerem ex eo anno, quem corrigere voluerimus, non modò dies. 10. esse detrahendos, verum etiam vnum & viginti, illo ipso anno; idq; duplici via; aut partiendo menses, atque ex illis demendo eos dies, qui minus ad rem hanc facere videbuntur, ac tum remaneat annus trecentorum quadraginta quatuor dierum ita ut decem menses sint dierum duorum spatio solito breuiores, alter mensis vno deficiat: aut constituto Decembri dicti anni dierum decem, dies autem ille, qui decimum proxime sequitur, sit & primus Ianuarij, & dies solstitij, ob quam causam existimarem consultiusimam eius modi annum esse milesimum quingentesimum septuagesimum nonum. Quò quam primum. S. D. N. Pontifex max. suis temporibus huius correctionis manifestos effectus experiri & perpendere, atque dispositionem anni non solum principio, sed ceteris partibus suis in vniuersum tam concinnè apteq; respondere, & astrorum moribus, & Ecclesiæ sacrosanctæ sanctionibus, se authore latari possit.

Omnino itaq; iudico detrahendos esse vnum & viginti dies elapsi erroris: non decem tantum, quo hyemalis conuersio ad initium Ianuarij reuocetur; idque ne à communi opinione de ipso anni principio veritas discrepet, quæ principium Ianuarij, anni principium arbitratur. etenim cum credant omnes annu à Ianuario inchoari, veritas autem ipsa sic se habeat, ut nobis septentrionalibus tunc inchoet annus, cum ad nos Sol accedere incipit, aut dies augetur; non conuenit principia eiusmodi separata & discrepantia esse. Et hanc fuisse Numæ Pompilio mentem credibile est, qui ad annum Romuli decem mensium, Ianuarium & Februarium addidit, ut principium Ianuarij principium esset anni: cuius rei argumentum esse potest, quod C. Iulij Cæsaris temporibus (qui multis annis post Numam fuit) atq; vti Pont. Max. corrigendorum festorum curam suscepit hyemale solstitium per aliquot dies retrocesserat; nec mirum tamen esset, si Numæ temporibus, exactè prima Ianuarij die non fuisset hyemale solstitium, adhuc pubescente in Italia Astronomia.

Huiusmodi autem correctio dierum. 21. post. 2300. annos à Numa, quæ sit perpetuo seruitura, media emendatione ea, quæ de tribus centesimis annis communibus, & quarto intercalari, superius proposita fuit, non repudianda ei videatur, qui sciet, qua ratione Numæ Pompilij annus corrigeretur, octauo quoque anno, intercalando annum vltimum medijs diebus. 90. quo prima dies Ianuarij ad verum principium anni, hoc est hyemale solstitium, reduceretur.

Alio item argumento cuique patere potest, priscos Romanos statuisse annum ab hyemali solstitio initium sumere, ut inquit Ouidius primo Fastorum.

Bruma noui prima est, veterisq; nouissima Solis.  
Principium capiunt Phæbus, & annus idem.  
eo quod diem naturalem à medio noctis inchoarent, ab eo puncto scilicet, quo Sol ad nostrum hemispherium accedere incipit.

Tribuebant igitur veteres diei, atque anno principium ab eo puncto, quo Sol ad

ad nos accedit: cum punctum Zodiaci, quod tropicum hyemalem Capricorni nobis producit, respondeat puncto meridiani sub terra, in quo Sol semel in die reperitur: Quod aperte norunt hi, qui sub polo boreali constituti sunt. Atque facile discernere possumus, diem scilicet & annum, quasi sibi ad inuicem medio suarum partium respondere; solstitium inquam hyemale, mediæ nocti, æstiuum meridiei, æquinotium vernale ortui Solis, autumnale occasui. Quam tamen similitudinem, multo quam nos manifestius deprehendunt, hi qui (ut diximus) sub polo boreali versantur.

Quod si quis dubitet hac ratione correcto anno, quo nam pacto ad calculos cœlestes motus medijs tabulis astronomicis hæctenus in lucem æditis redigi possint, id facillimum sanè erit, exempli gratia; aliquis planetæ situm, aut alicuius stellæ fixæ, quo cunque die mensis anni correcti inuenire cupit, detrahat ex huiusmodi tēpore dies. 21. ab Aera Christi, cum residuo supputet stellam, cuius situm scire desiderat; sumpta quacunque tabula, supputatio erit exacta: Cuius ratio cuilibet manifesta erit, qui sciet annum ut potè. 1579. dierum. 344. tantummodo constitutum fuisse. Nam in iisdem locis cœli prima die Ianuarij correcti, erunt stellæ quibus esse solebant. 11. Decembris præcedentis anni ex supputatione tabularum: atque ita deinceps. Alia præterea via idem perfici posset inuentione omnium motuum cœlestium ipso principio anni. 1580. correcti: hoc statuto, ut hi motus radices essent Aera S. D. N. Gregorij xlii. quod si alio tēpore quispiam motus cœlestes ad calculos redigere voluerit, supputabit ab Aera huiusmodi, quæ anno. 1580. principium habuerit: Quæ ut nobis nomen sortiatur, idque merito ex nomine Gregorij. XIII. Pont. Max. appelletur; exemplo antiquarum, quæ ex Principum nominibus sunt appellatæ: ut tanto Pontifici, cum ex alijs multis, tum etiam ex hac non infima re, inter mortales immortale nomen comparetur. Ei verò summæ, quæ ex huiusmodi Aera Gregoriana ex tabulis colligetur, ipsiusmet Aera radices addantur, ut exactus calculus habeatur. Et hæc sit primæ sententiæ nostræ explicatio.

Altera erit numerum dierum mensium anni alia ratione quam nunc se habeat, ordinandum esse: nempe ut Ianuarius, Nouember atque December dies. 29. singuli contineant, Februarius, Martius, & October. 30. Aprilis, Maius, Augustus, & September dies. 31. Iunius, ac Iulius. 32. atque id hac potissimum de causa, ut Sol unum quodque signum calendis mensium ingredi possit. Nam detractis (ut dictum est) diebus. 21. & reuocato ingressu Solis in principium Capricorni ad principium Ianuarij, in quo signo hac nostra tempestate, Sol, dies propè. 29. & quartam vnam versatur: si Ianuarius. 29. dies continebit, exactis hisce diebus, ingredietur Aquarium circa principium Februarii; hæret autem hoc nostro sæculo in Aquario Sol dies propè. 29. cum dimidio; quare si Februarius erit. 30. dierum, elapsis ipsis diebus, Sol ingredietur pisces circa principium Martii: & sic de cæteris.

Quamobrem si generali correctione annus emendandus erit, pulcherrimè accidet, si menses anni cum duodecim partibus cœlestibus, itineris annui Solis, concordauerint; eisque aptè responderint. Quæ ex re, variæ utilitates promanabunt, præsertim Nautis, Agricolis, Medicis, & alijs qui vera principia, & interualla temporum perspecta habebunt: terminos item & interualla incrementi & diminutionis dierum & noctium, & eorundem æqualitatis. Exempli causa, scient omnes principium Ianuarij, esse non modo anni principium, verum etiam hyemis, esse minimam anni diem, & eius noctem maximam; principium incrementi diei, & diminutionis noctis; atque etiam omnia illa, quæ ex huiusmodi conuersione Solis ad nos dependent. pariter scient omnes primam diem Iulij, non tantum æqualiter annum diuidere, sed principium



cipium quoque esse æstatis, maximam diem, noctem minimam totius anni; principium diminutionis diei & incrementi noctis, vnà etiã ea, quæ Solis conuersionem ad australes sequuntur.

Necnon intelliget vnusquisque primam diem Aprilis, primamq; Octobr. æquinoctiorum dies esse; primam autem diem Aprilis, initium veris; Octobris Autumnus; Item Aprilis diem esse eum, quo dies noctis prolixitatem vincere incipit; Octobris, quo nox diei longitudinem superat, & alia huiusmodi, quæ ab æquinoctijs depêdêr.

Si vero quispiam obijciat, modum hunc nostrum & ordinem perpetuum esse non posse, ob motum augis Solis; quod punctum cum fuerit in principio Capricorni, tunc Sol hærebit in signo Sagittarij, 32. diebus, totidem in Capricorno, in Geminis vero 29. totidem in Cancro; ex quo sequetur prioribus cõtrarius effectus; huic ego respondebo, tale quidpiã non euenturum, nisi exactis ab hoc anno annis. 24000. quod si mundus posthac totidem annis, quot fuit antehac, perdurauerit, punctus augis non amplius à situ præsentis, quàm .45. gradibus distabit. Verum demus modum nostrum & regulam in annos ter, aut quater mille subseruire posse, nec amplius, certè hoc toto tempore nullius momenti penè erit, quæ accidere poterit mutatio, tamen si elapsis quatuor millibus annorum Februarius esse debet. 29. dierum. Aprilis & Nouember. 30. Iunius & October. 31. Augustus. 32. in aliis verò mensibus nihil mutandum erit. Ecce quam sit nullius momenti mutatio.

Quæ si Iulij Cæsaris temporibus fuissent animaduersa nunquam omiſſa fuissent, sed scientiæ Astronomicæ nondum (vt ita dicam) confirmata ætas, cum alibi, tunc maxime in Italia, quo minus hæc aut scirentur aut statuerentur impediebat.

Tertia ratio est, vt non solùm festa mobilia, verum et immobilia ad meliorem regulam (vt dictum est) reuocentur, si suis temporibus celebranda erunt. Quorum primùm est Natiuitas Domini, & quæ ab ea pendet; nempe Circuncisio, Epiphania, Purificatio, Annunciatio, & Natiuitas Io. Baptistæ. ita vt dies Natalis Domini celebretur prima die anni, cum Dei filius nasci voluerit circa verum principium anni, quod à solstitio hyemali initium ducit, & in ipso principio diei naturalis ex Romanorùm sententia, media scilicet nocte, tanquam qui summæ lætitiæ principium, post longos & graues filiorum Adæ mæiores, esset allaturus. Nec forsitan Ianuarij nomini, à veteribus Iano bifronti dicati hæc mutatio non conueniret, cum in ipso seruatore, duæ veluti frontes & formæ vnitæ sint, duæ scilicet naturæ diuina & humana. Hac ratione abusus tollitur, natus ex diuersis moribus Tabulariorum, quorum alij monumenta, seu quæ uocant Instrumenta, à die Natiuitatis Domini incõhant, alij à Circuncisione, alij à Calendis Martij, nonnulli à Paschate; quæ varietas innumerabiles lites affert & abusus propè infinitos, ob dubiam & ancipitem scripturam. Indictionum præterea ordini, hic noster modus nihil officiet; celebrato Natali celebrabitur Circuncisio octaua Ianuarij, Epiphania. 13. eiusdem. Purificatio. 11. Februarij quæ erit 40. dies à Natiuitate seruatoris. Prima Aprilis Annunciatio Virginis solennis erit, ipso nempe die æquinoctij, natiuitas Diui Io. Baptistæ celebrabitur Prima Iulij die quæ erit solstitij æstiu, cum illa diminutionem capit. vt rectè Diuus Augustinus illa verba Io. Baptistæ interpretatus fuerit. Illum oportet crescere, me autem minui: in quibus sit tantus Doctõr philosophatur, vt tempus etiam natiuitatis serui & domini præclare notet dicens, natus est seruus cum decreſcunt dies, natus est Dominus cum crescere incipiunt.

Insignes etiam Theologi admonuerunt habendam rationem esse nonnullorum festorum, vt Diui Antonij, diuorum Fabiani & Sebastiani, & aliorum sanctorum,

si forte in octauam Epiphaniæ inciderint: Verum hæc. S. D. N. curæ erunt, ut in aptissima tempora transferantur.

Admonuerunt præterea transferendos esse dies festos Beati Stephani, Ioannis, & Innocentium, vt quemadmodum factum est hæctenus, diem natalis proximè sequantur, ob multorum Doctorum, non recentium modo, sed etiam antiquorum obseruantiam; qui suis omelijs & concionibus multa piè, de mysterijs successionis Fæstorum huiusmodi tradiderunt.

Cuperent etiam præclari Theologi diem Assumptionis Beatæ virginis incidere in primam Septembris, Natiuitatem autem in. 25. vt quemadmodum toto illo mense in signo Virginis sol versabitur, ita Ecclesia Dei in celebrandi tantæ Virginis matris Dei laudibus occupetur.

Atque hæc sunt Serenissime Princeps, quæ longa & attenta cogitatione à me examinata, atque perpensa fuerunt; quæ si tam diligenter & accuratè expendentur ab his, quorum interest, quam mihi apta & rationi consentanea, ac vera penitus, imo (quod me magis afficit) etiam tibi visa fuerunt; non dubito quin placitura sint; & votis summi Pont. aliqua ex parte satisfactura. eò magis quòd te iubente, & cogitata à me, & scripta fuerint. Vale Princeps Serenissime, & qua soles hylaritate cetera nostra, etiam has breues vigilias suscipe & foue. Dat. Augustæ Taurinorum Kal. Aprilis. MDLXXVIII.

**T. Celsitudinis.**

**Deditissimus Mathematicus.**

**Io. Bap. Benedictus.**

**D B**

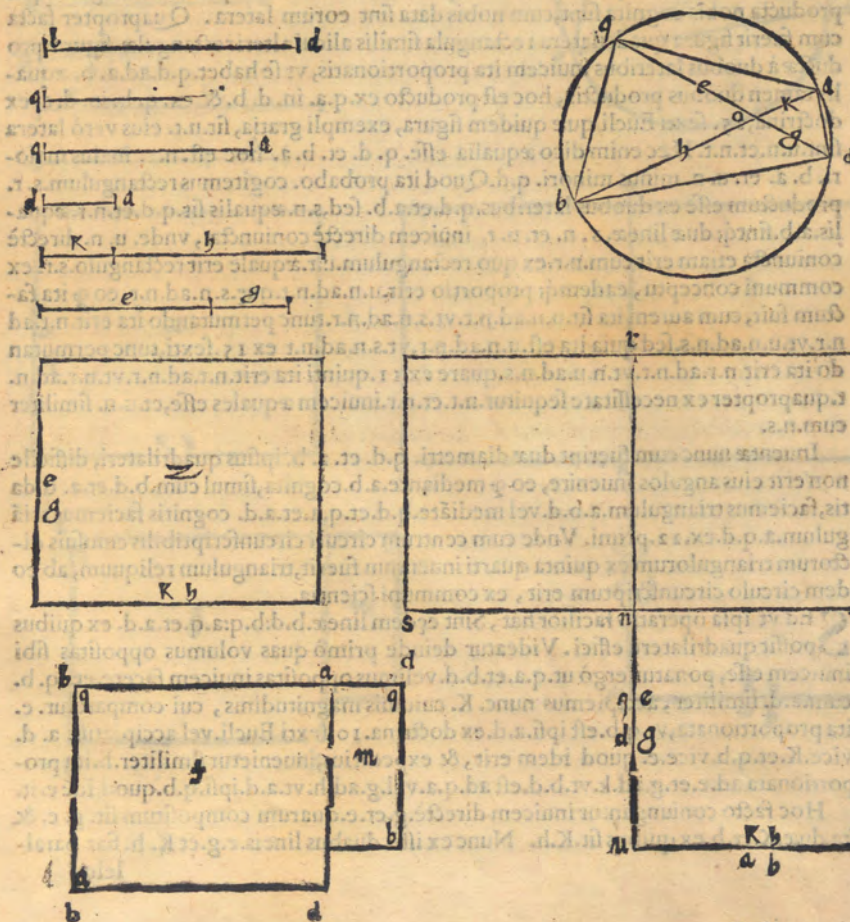
# DE CIRCVLO

## AMBIENTE QVADRILATERVM.

AD SERENISS. CAROLVM EMANVELEM  
Pedemontis Principem.

**P**roblema quod à celsitudine tuà nobis propònitur non solum possibile est, sed facile etiam ad soluendum, hoc est quod circulus talis inueniatur, qui possit circumscribere, seu capere quadrilaterum ex quatuor datis rectis lineis terminatum, vel sic, datis quatuor rectis lineis ex quibus quadrilaterum possit effici, tale efficiatur vt circa ipsum, circulus possit circumscribi.

Sint igitur. 4. lineæ propositæ. b. d. q. b. a. q. et. a. d. ex quib<sup>9</sup> possibile sit quadrilaterum constitui, tale verò constituatur, vt aliquis circulus possit ipsum circumscribere, imaginemur autem hoc factum esse, quod quidem quadrilaterum sit. a. d. q. b. cuius



Dd 3

diametri sint. q. d. et. a. b. quæ se inuicem interfecerint in puncto. o. vnde cum anguli contra se positi circa. o. æquales inuicem sint ex. 15. primi Eucli. & angulus. a. q. d. æqualis angulo. a. b. d. & angulus. q. b. a. æqualis angulo. q. d. a. et. b. q. d. angulo. b. a. d. ex. 20. tertij tunc triangulus. a. o. q. similis erit triangulo. d. o. b. et. q. o. b. similis triangulo. a. o. d. ex definitione. Vnde eadem proportio erit ipsius. q. o. ad. b. o. quæ ipsius q. a. ad. b. d. & ipsius. b. o. ad. o. d. eadem quæ. q. b. ad. a. d. & ipsius. q. o. ad. o. a. eadem quæ. q. b. ad. a. d. proportio igitur. q. o. ad. o. d. cognita nobis erit, vt composita ex ea quæ est. q. o. ad. o. b. ex. o. b. ad. o. d. quæ nobis cognitæ sunt, mediante proportione ipsius. q. a. ad. b. d. & ipsius. q. b. ad. a. d. proportio similiter ipsius. b. o. ad. o. a. nobis cognita erit, vt composita ex proportione ipsius. b. o. ad. o. q. & ipsius. o. q. ad. o. a. cognitæ, mediante proportione ipsius. b. d. ad. q. a. & ipsius. q. b. ad. a. d. cum autē proportio ipsius. q. o. ad. o. b. nobis cognita sit, tunc nobis cognita erit proportio ipsius. q. d. ad. a. b. Nam vt. q. o. ad. o. b. est vt. a. o. ad. o. d. ex similitudine, quare proportio compositi ex primo, & quarto terminorum ad compositum ex. 2. & tertio, cognita erit. sed quod sit ex. q. d. in. a. b. cognitum nobis est, vt æquale duobus productis, hoc est ex. q. a. in. d. b. & ex. q. b. in. d. a. ex secunda primi Almagesti. quæ producta nobis cognita sunt, cum nobis data sint eorum latera. Quapropter facta cum fuerit figura quadrilatera rectangula similis alicui alteri rectangulæ figuræ productæ à duobus lateribus inuicem ita proportionatis, vt se habet. q. d. ad. a. b. æqualis tamen duobus productis, hoc est producto ex. q. a. in. d. b. & ex. q. b. in. d. a. ex doctrina. 25. sexti Eucli. quæ quidem figura, exempli gratia, sit. u. t. eius verò latera sint. u. n. et. n. t. Hæc enim dico æqualia esse. q. d. et. b. a. hoc est. n. t. maius maiori. b. a. et. u. n. minus minori. q. d. Quod ita probabo. cogitemus rectangulum. s. r. productum esse ex duobus lateribus. q. d. et. a. b. sed. s. n. æqualis sit. q. d. et. n. r. æqualis. a. b. sintq; . s. n. et. n. t. inuicem directè coniunctæ, vnde. u. n. directè coniuncta etiam erit cum. n. r. ex quo rectangulum. u. t. æquale erit rectangulo. s. r. ex communi conceptu, eademq; proportio erit. u. n. ad. n. t. quæ. s. n. ad. n. r. eo qd ita factum fuit, cum autem ita sit. u. n. ad. n. t. vt. s. n. ad. n. r. tunc permutando ita erit. n. t. ad. n. r. vt. u. n. ad. n. s. sed quia ita est. u. n. ad. n. r. vt. s. n. ad. n. t. ex 15. sexti, tunc permutando ita erit. n. r. ad. n. t. vt. u. n. ad. n. s. quare ex 11. quinti ita erit. n. t. ad. n. r. vt. n. r. ad. n. t. quapropter ex necessitate sequitur. n. t. et. n. r. inuicem æquales esse, et. u. n. similiter cum. n. s.

Inuentæ nunc cum fuerint duæ diametri. q. d. et. a. b. ipsius quadrilateri, difficile non erit eius angulos inuenire, eo qd mediante. a. b. cognita, simul cum. b. d. et. a. d. datis, faciemus triangulum. a. b. d. vel mediante. q. d. et. q. a. et. a. d. cognitis faciemus triangulum. a. q. d. ex. 22. primi. Vnde cum centrum circuli circumscripibile cuiusuis dictorum triangulorum ex quinta quarti inuentum fuerit, triangulum reliquum, ab eodem circulo circumscriptum erit, ex communi scientia.

**S**ed vt ipsa operatio facilior fiat, Sint eadem lineæ. b. d. b. q. a. q. et. a. d. ex quibus possit quadrilaterū effici. Videatur deinde primò quas volumus oppositas sibi inuicem esse, ponatur ergò ut. q. a. et. b. d. velimus oppositas inuicem facere, et. q. b. cum. a. d. similiter, accipiemus nunc. K. cuiusuis magnitudinis, cui comparatur. e. ita proportionata, vt. q. b. est ipsi. a. d. ex doctrina. 10. sexti Eucli. vel accipiat. a. d. vice. K. et. q. b. vice. e. quod idem erit, & expeditius, inuenietur similiter. h. ita proportionata ad. e. et. g. ad. k. vt. b. d. est ad. q. a. vel. g. ad. h. vt. a. d. ipsi. q. b. quod idē erit.

Hoc facto coniungantur inuicem directè. g. et. e. quarum compositum sit. g. e. & ita duæ. K. et. h. ex quibus sit. K. h. Nunc ex istis duabus lineis. e. g. et. K. h. fiat parallelo-

lelo

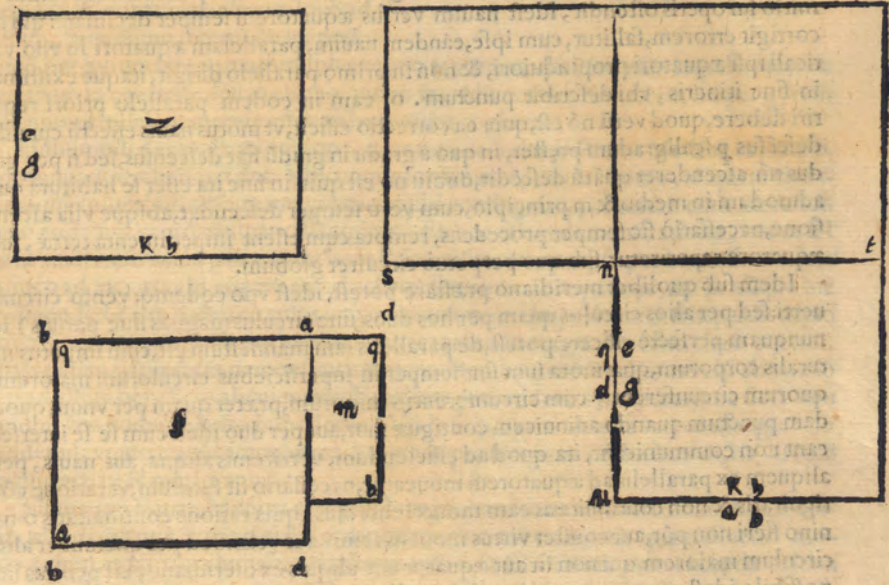
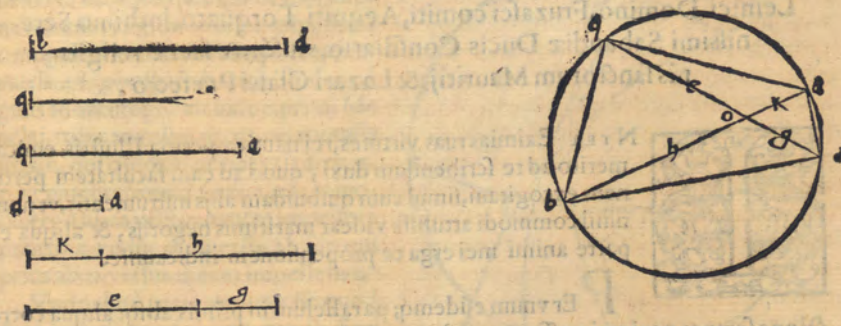
EPISTOLAE

lelogrammum. Z. deinde fiant alia duo parallelogramma rectangula quorum vnum sit ex. q. a. in. b. d. reliquum vero sit ex. q. b. in. a. d. quæ quidem sint. f. m.

Quo facto designetur rectangulum. u. t. ex. 25. sexti, quod æquale sit duobus rectangulis. f. et. m. simile tamen. Z. cuius rectanguli vnum latus correspondet. e. g. reliquum vero. K. h. in proportione, sed in æqualitate, vnum correspondet. q. d. reliquum vero. a. b. diametris ipsius quadrilateri.

Accipiat nunc latus illud quod correspondet. K. h. hoc est ipsi. a. b. maius scilicet, & simul cum. b. d. et. a. d. formetur triangulum. a. b. d. ex. 22. primi Eucli. circa quod circumscribatur circulus ex. 5. quarti. & inuentum erit quod querebamus.

PER



214

PER EVNDEM PARALLELVM  
absque correctione semper nauigari  
non posse.

*Vbi notantur Petri Nonij lapsus in correctione erroris nauis.  
Et alij Petri Medina errores.*

ILLVSTRISSIMO ANDREAE PROVANAЕ  
Leinici Dominò, Fruzasci comiti, Aequiti Torquato, in thimo Sere-  
nissimi Sabaudia Ducis Consiliario, eiusq; & sacrae religio-  
nis sanctorum Mauritij, & Lazari Claisi Praefecto.



INTER Eximias tuas virtutes, rei nauticae peritia Illustris emicat merito ad te scribendum duxi, quod ad eam facultatem perti-  
nens excogitavi, simul cum quibusdam alijs instrumentis, vt non-  
nihil commodi attulisse videar maritimis negotijs, & aliqua ex  
parte animi mei erga te propensionem indicauisse.

PER vnum eundemq; parallelum in primis absq; aliqua corre-  
ctione semper nauigari posse, omnino nego. Nam, verum est id quod Petrus Nonius in  
initio sui operis ostendit, id est nauim versus aequatorem semper declinare: qui cum  
corrigit errorem, fallitur, cum ipse, eandem nauim, parallelam aequatori in vno ver-  
ticali ipsi aequatori propinquiori, & non in primo parallelo dirigit, itaque existimat  
in fine itineris, vbi describit punctum. o. eam in eodem parallelo priori repe-  
riri debere, quod verum non est, quia ea correctio efficit, vt motus nauis effectum cuiusdam  
descensus per scaligradum praestet, in quo a gradu in gradum fiat descensus, sed si gra-  
dus tum ascenderet quam descendit, dubium non est quin in fine ita esset se habiturae que  
admodum in medio & in principio, cum vero semper descendat, absque vlla ascen-  
sione, necessario sic semper procedens, remota cum essent impedimenta terrae, sub  
aequatore reperiretur, sub quo perpetuo circuiret globum.

Idem sub quolibet meridiano praestare potest, id est vno eodemq; vento circun-  
uerti: sed per alios circulos quam per hos duos (siue circulus magnus siue parvus) id  
nunquam perfecte efficere potest, de parallelis iam manifestum est, cum impetus na-  
turalis corporum, quae mota sunt sint semper in superficiebus circulorum maiorum,  
quorum circumferentiae cum circumferentijs minorum, praeter quam per vnum quod-  
dam punctum quando adiuicem contiguae sunt, aut per duo id est cum se se interse-  
cant non communicant, ita quod ad efficiendum, vt triremis aliqua, aut nauis, per  
aliquem ex parallelis ad aequatorem moueatur, necessario sit futurum, vt ratione con-  
tigitatis & non continuitatis eam moueri curemus. quia ratione continuitatis om-  
nino fieri non potest, aut constet virtus mouens remis, aut velis. Sed per quemlibet alium  
circulum maiorem, qui non sit aut aequator aut aliquis ex meridianis, est penitus im-  
possibile, id est vt vnus venti vi nauis impellatur. Quod vt clare pateat, sit orizon.  
a. c. b. d. & aequator. c. q. r. d. & vnus meridianorum sit. a. r. n. t. b. in quo. n. sit Zenit sub  
quo primum nauis reperiat et. r. sit polus septentrionalis. Ponamus etiam quod  
azimur.



Superius posita meae demonstrationis ope, deuenimus in cognitionem magnitudinis arcus. n. q. cognoscimus etiam angulum. n. q. t. vnde nobis manifestum esset quo vento oporteret iter facere. cum à puncto. q. nauis aliqua discessura esset, in eodem azimuth proposito. Idem etiam dico de puncto. u. cum cogniti essent arcus. n. u. et. n. r. vt supponitur, simul cum angulo. r. n. u. vnde cognitus esset nobis angulus. n. u. r. ex 11. primi lib. Copernici, ex quo ventus nobis cognitus foret.

Modus autem quem idem Medina cap. 9. lib. tertij ad cognoscendam distantiam vnus meridiani ab alio præscribit, in genere est falsus, etiam si is ab antiquis eum de sumat, qui, hic non viderunt quam magna inter meridianos differentia sit interuallorum eorum quæ sunt vicina polis & eorum quæ sunt circa æquatorem.

Falsus est etiam modus ab eo traditus ad cognoscendos gradus longitudinis per medium itineris cogniti, in quouis parallelo extra æquatorem facti, & hoc cap. 14. libri tertij eiusdem, & primo cap. lib. 4. continetur, vbi. 17. leucas cum dimidia cuilibet gradui tam paralleli quam meridiani assignat.

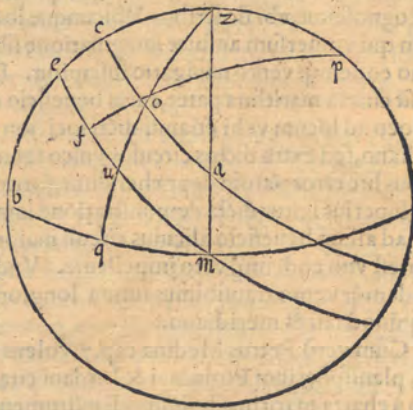
Falsum est etiam quod ab eo asseritur, Solem, cum reperiretur in æquatore, circa eos qui sub ipso æquatore habitant, vnus diei noctisque spatium per omnes ventos circunvolui. quia illis æquator idem est cum verticali, qui duos tantum rhombos producit, id est orientis, & occidentis: hic verò error, in secundo cap. lib. 6. habetur.

Falsum est etiam quod profert Solem ijs qui habitant spheram obliquam, quilibet hora tertia, regulariter ab vno rhumbo ad alium ex præcipuis id est ab vno azimuth ad alium progredi, quemadmodum eadem cap. 2. lib. 6. et. 7. cap. septimi libri, scribit. Huius autem rei falsitas ita facile depræhendetur, ponamus hemispherium orientale, verbi gratia, cuius meridianus sit. p. z. b. æquator aut. e. m. vnus verò parallelorum septentrionalium sit. c. a. in quo Solem existere ponamus, orizon autem sit. b. m. zenit vero. z. polus arcticus. p. sit postea azimuth. z. q. à meridiano distans per gradus. 45. qui quidem azimuth in hoc hemispherio erit rhombus illius venti, quem vulgo Itali Sirocum dicunt, et. z. m. sit azimuth verticalis qui in hoc hemispherio erit rōbus venti orientalis, ita quæ secundum Medinam à rhumbo. z. m. ad. z. q. Sol absoluet spatium temporis trium horarum, & aliud æquale temporis spatium absoluet à rhombo. z. q. ad. z. b. ex ipso Medina, vnde arcus. a. o. paralleli esset graduum. 45. & item arcus. o. c.

Ponamus nunc Solem reperiri in æquatore, vbi per ipsum Medinam arcus. u. m. similiter esset graduum. 45. & sic. u. e. protracto ergo arcu. p. o. f. palam erit arcum. f. e. fore graduum. 45. sed cum arcus. e. u. sit graduum. 45. ex supposito ipsius Medinæ, sequeretur arcum. e. f. æqualem esse arcui. e. u. pars igitur æqualis erit suo toto.

Id etiam quod Petrus Nonius pagina 124. et. 125. lib. de arte nauigandi contra nautas de distantijs Solis à meridiano scribit, hanc opinionem Petri à Medina & eorum qui idem ei persuaferunt falsam esse demonstrat.

Fal-





Falsum est etiam id quod cap. 3. lib. 6. pronuntiat, ita dicens.  
 Quod cum verum esset à parte oriëntali insularum quæ azore dicuntur, pyxidem  
 versus eum ventum qui vulgò Græcus dicitur, & ab occidentali versus eum qui Ma-  
 gister dicitur, vergere, huius rei nulla est ratio.

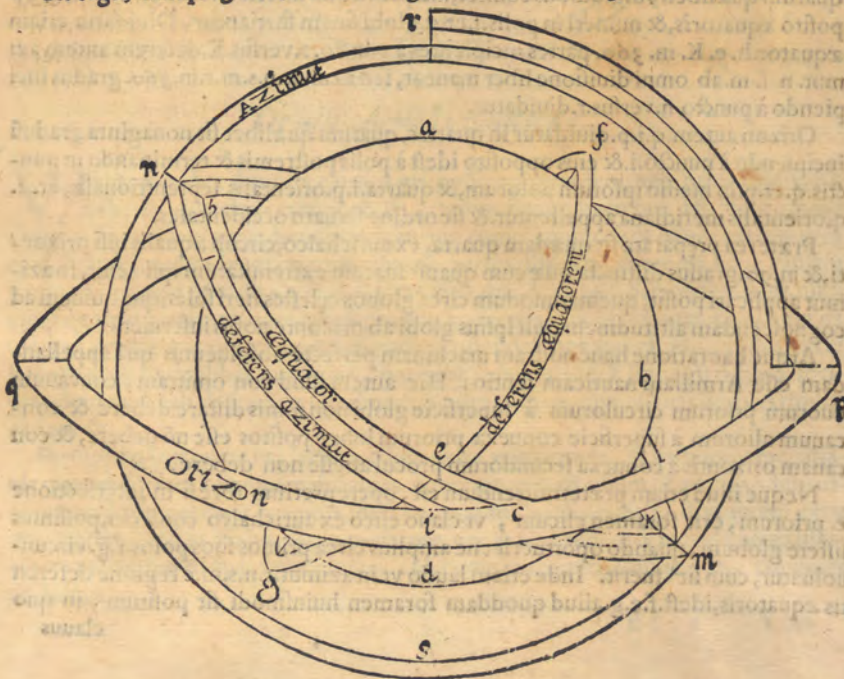
Ego enim huiusmodi rationem reperiri posse contendo, quæ talis est, quia pars  
 rosæ (ut vocant) à magnete tacta, ad aliquod punctum, aut situm globi terre, in eo-  
 dem meridiano insularum, quæ Azore dicuntur, ultra situm poli arctici in terra diri-  
 geretur, ita ut situs dicti poli in terra esset in dicto meridiano, inter locum qui ab in-  
 dice rosæ aut pyxidis respiceretur, & dictas insulas, id quod superius scripto meridia-  
 no facile cognosci potest, sumendo pro insulis situm. e. in meridiano et. z. pro polo,  
 et. p. pro loco qui à pyxide sit visus, imaginando deinde pyxidem in. f. magis orienta-  
 li quam est. e. clarum est lineam quæ respicit (ponamus) f. p. versus Græcum & ab alia  
 parte versus Magistrum declinare.

*De Armilla Nautica.*

A D E V N D E M.

Cum sæpe viderim quam in magnis æquoribus nos fallant, atq; decipiant mari-  
 timæ, seu nauigatorie chartæ, quemadmodum aliquoties inter nos sermonem  
 habuimus: in id totus incubui ut aliquam machinam excogitarem, quæ difficilis non  
 esset, efficeretq; ut naus super aquæ globum, beneficio circulorum maiorum, quam  
 optimè posset, id est breuissimo itinere ab uno loco ad alium ferretur. Id q; mihi ex  
 animi voto successurum putavi, beneficio quinq; circulorum circundantium aliquè  
 globum terrestrè & maritimum, quales ij sunt qui in inferiori Germania à Gerardo  
 Mercatore struuntur, qui vno pede cum dimidio diametri constet, id est sesquipede.

Sit ergo, exempli gratia, huiusmodi globus. a. b. d. circa quem duo circuli, aut cir-



culares lineæ ex aurichalco applicentur inuicem coniuncti per medium ad angulos rectos, quorum prior. f. e. g. in se globi polos mediantibus extremitatibus axis mundi contineat, qui quidem poli à punctis suarum interfectionum per quartã ex æquo in punctis. f. et. g. ita distent, vt globus circa eosdem, in situ longitudinis mundi volui possit. Huiusmodi autem circulus, æquatoris deferens appelletur.

Secundus autem circulus sit. h. e. K. cum primo ad angulos rectos in puncto. e. & in suo opposito connexus, & is appellabitur æquator, & poli. f. g. primi poli dicentur.

Circa huiusmodi duos circulos, alios etiã duos existere vellẽ simul cõiunctos medio ad angulos rectos. In quibus quidẽ interfectionis punctis sint duo poli, qui hos duos circulos cum secundo priorum idest cum æquatore in duobus punctis inuicem oppositis connectant; quæ æquatoris puncta à punctis interfectionis eiusdem cum suo deferente, ratione vna quarta distent, quorum duorum circulorum primus sit. n. i. m. quem deferentem azimuth appellabimus; secundus. r. n. s. m. azimuth locorum nominabimus. eorundem interfectionis recta, puncta sint. n. et. m. à quibus duo poli ex aurichalco confecti similes primis. n. h. et. m. K. vsque ad puncta. h. et. K. æquatoris perueniant, qui spissitudinem æquatoris distantem à puncto. e. vna quarta penetrẽt, ita vt æquator circum circa. n. h. et. m. K. in situ latitudinis mundi verti queat. Ee hos, secundos polos nominabimus.

Alius deinde circulus. q. i. p. duos posteriores circulos ambiat, cum deferente tamen azimuth mediantibus duobus polis in puncto. i. & in suo opposito ex æquo distantibus à secundis polis vnus quartæ spatio iungatur. Ita vt dictum deferens azimuth circa hos tertios polos volui possit, atque hunc circulum. q. i. p. orientem vniuersalem vocabimus. Hic vero orizon super quatuor quartas circuli, aut super quatuor paruis columnis, ut fieri solet innixis suæ basi, ita ponatur, vt moueri non possit.

Primus autem circulus. f. e. g. deferens æquatoris in. 4. partes æquales diuidatur, quarum quælibet. 90. gradibus constat, incipiendo ab interfectionibus. e. & eius opposito æquatoris, & numeri in polis. f. et. g. globi finem sortiantur. Diuidatur etiam æquator. h. e. K. in. 360. partes incipientes à puncto. e. versus. K. deferens autem azimuth. n. i. m. ab omni diuisione liber maneat, sed azimuth. n. s. m. r. in. 360. gradus incipiendo à puncto. n. versus. r. diuidatur.

Orizon autem. q. i. p. diuidatur in quartas, quarum quælibet sit nonaginta graduũ incipiendo à puncto. i. & eius opposito idest à polis postremis & terminando in punctis. q. et. p. in medio ipsorum polorum, & quarta. i. p. orientalis septentrionalis, et. i. q. orientalis meridiana appellentur. & sic ordine seruato occidentales.

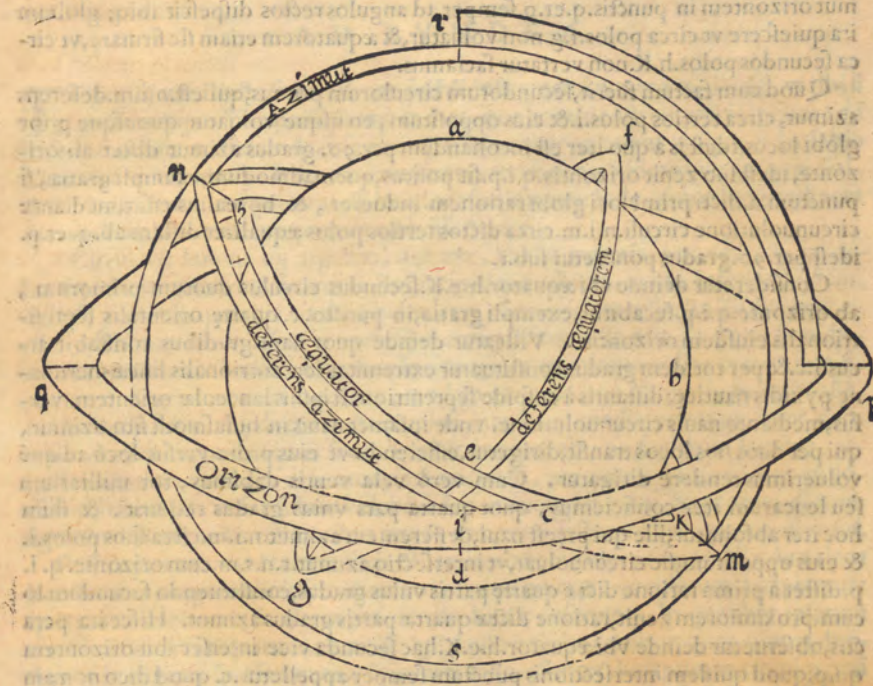
Præterea præparata sit quædam quarta, ex aurichalco, circuli æqualis ipsi orizonti, & in. 90. gradus distincta quæ cum quauis suarum extremitatum ipsi zenit, in azimuth applicari possit, quemadmodum circa globos cælestes fieri solet; quæ quidem ad cognoscendam altitudinem poli ipsius globi ab orizonte nobis inseruiet.

Atque hac ratione hanc nostram machinam perfectè absoluemus quã appellandam esse Armillam nauticam sentio. Hic autem illud non omittam, concauum duorum priorum circulorum à superficie globi non nimis distare debere & concauum aliorum à superficie conuexa priorum longe positos esse nõ debere, & concauum orizontis à conuexa secundorum procul abesse non debere.

Neque illud etiam prætermittendum est, operepretium fore si in interfectione e. priorum, erit foramen elicum, vt clauo elico ex aurichalco confecto, possimus fissere globum, quando oportuerit, ne amplius circa primos suos polos. f. g. circumuoluatur, cum sit<sup>9</sup> fuerit. Inde etiam laudo vt in azimuth. r. n. s. m. è regione deferentis æquatoris, idest. f. e. g. aliud quoddam foramen huiusmodi sit positum, in quo

clauus

claus clicus vsque ad circulum.f.c.g.perueniens, æquatorum sistere possit, ne circa secundos polos.h.et.K.amplius moueatur quum noluerimus eum mutare situm.



*De usu Armille nautica.*

**V**T autem nostra Armilla nautica uti possimus pyxidem nos prius oportebit habere, diuersam tamen ab ijs, quibus nautæ hæcenus vsi fuere: nolo enim ut tā crassa minerua beneficio vëtorum communium circa hanc rem nos geramus, sed ratione graduum orizontis in.360.partes distincti, atque ob hanc causam sentio, ut ima pars pyxidis penitus detecta videatur, & in.360.partes diuidatur, nilq; aliud quam quandam lanceolam supra eius acum esse volo, quæ dum mouebitur nauis, per gradus quamlibet orizontis partem ostendet; hos autem.360.gradus, ita se habere volo, ut quælibet quarta.90.contineat, supputatioq; à linea meridiana incipiat, & in verticali desinat, ut huiusmodi diuisio cum ea, quæ est orizontis Armillæ eadem sit.

Præsupponantur nunc in globo duo loci extra æquatorum, & in diuersis meridianis quomodolibet adinuicem distantes, à quorum vno ad alium sit nauigandum itinere quo ad fieri poterit breuiori, idest per gyrum circuli maioris, dixi autem extra æquatorum, idest ut ambo, nec in æquatore, nec in uno eodemq; meridiano existant, quia ut aliàs dixi in huiusmodi locis, vnico tantum vento comite, iter conficere possumus.

Ec 2 Volo

Volo primum vt mediante circūuolutione globi circa primos polos. f. g. & æquatoris circa secundos. h. k. hoc est per longitudinem, & latitudinem, hi duo loci in globo propofiti sub azimuth. r. n. s. m. fecundorum circularum fitū fortiantur, qui azimuth orientem in punctis. q. et. p. semper ad angulos rectos dispefcit ibiq; globum ita quiescere vt circa polos. f. g. non uoluatur, & æquatorem etiam sic firmare, vt circa secundos polos. h. k. non uertatur faciamus.

Quod cum factum fuerit, fecundorum circularum primus, qui est. n. i. m. deferens azimuth, circa tertios polos. i. & eius oppositum, eo usque uoluatur quousque prior globi locus, id est is à quo iter est incohandum per. 90. gradus azimuth distet ab orizonte, id est sub zenit orientis. q. i. p. sit positus, quemadmodum, exempli gratia, si punctum. a. dicti primi loci globi rationem indueret, & borealius esset, mediante circūuolutione circuli. n. i. m. circa dictos tertios polos æqualiter distans ab. q. et. p. id est per. 90. gradus poneretur sub. r.

Consideretur deinde ubi æquator. h. e. k. secundus circulus duorum primorum, ab orizonte. q. i. p. fecabitur, exempli gratia, in puncto. c. quartæ orientalis septentrionalis eiusdem orizontis. Videatur deinde quot nam gradibus constabit arcus. i. c. & per totidem gradus constituatur extremitas septentrionalis lineæ meridianæ pyxidis nauticæ, distantis à cuspide septentrionali ipsius lanceolæ orientem uersus, mediante nauis circūuolutione. unde ipsamet nauis in huiusmodi situ azimuth, qui per duos hos locos transit, dirigitur, efficiendo vt eius prora uersus locū ad quē uoluerimus tendere dirigatur. Cum uerò uela uentis dabimus, tot milliarium seu leucarum iter conficiemus, quot quarta pars unius gradus requirit. & dum hoc iter absoluitur, ille qui præest nauis, deferentem azimuth. n. i. m. circa suos polos. i. & eius oppositum, sic circūuoluat, vt interfectio azimuth. r. n. s. m. cum orizonte. q. i. p. distet à prima ratione dictæ quartæ partis unius gradus, constituendo secundum locum, proximiorē zenit, ratione dictæ quartæ partis gradus azimuth. Hisce ita peractis, obseruetur deinde ubi æquator. h. e. k. hac secunda uice interfecabit orizontem q. i. p. quod quidem interfectionis punctum semper appelletur. c. quod dico non amplius in eadem distantia mansurum, ut prius à puncto. i. sed aut longius distabit, aut propius accedet, vt in præfenti exemplo. quemadmodum ex se manifestum est, cū poli globi, id est æquatoris sint extra azimuth, vt præsupponitur, quia loci sunt in diuersis meridianis.

Pro huiusmodi autem distantia ratione denuo dirigatur nauis prout æquator. h. e. k. in orizonte. q. i. p. nobis ostendet, atque hoc modo omnium iter quasi breuissimum fiet. dico autem, quasi, quia omnibus modis necessariò conficitur iter contortum & in formam serpigineæ lineæ. Applicantes deinde per uices extremitatem quartæ appositæ (de qua superius mentionem fecimus) ipsi zenit. r. efficientes ut per fitum poli globi pertranscat, deueniemus in cognitionem altitudinis eiusdem ab orizonte, & per consequens quantum itineris per latitudinem eiusdem globi peregerit. mediante deinde interfectione orizontis. q. i. p. cum æquatore, cognoscemus quantum itineris per longitudinem eiusdem globi, in ipso æquatore fuerit peractum,

*Instrumentum ad ortum, & occasum Luna cognoscendum  
qualibet anni die.*

A D E V N D E M .

**E**Cce tibi vir Illustris. modū conficiēdi instrumenti nuper à me inuēti, vt tibi si gnificauī, quo scire possis fermè in dies, qua hora (de astronomicis loquor) ad determinatum parallelum & absque multa supputatione, etiam absque Astrolabio Luna oriatur occidatq; . In quo instrumento poteris etiam videre quo in signo Sol, & sæpius itidem Luna permeat, & huiusce aspectus cum Sole, atque longitudinem diei noctisq; toto anni tempore exactè discernere.

Circularis lamina ex argento, aut ære, aliaue materia paranda est, in cuius superficie ambarum facierum Zodiacus delineabitur, modo inferius depicto, deinde pro anno quinque circuli sibi inuicem cōcentrici, at respectu Zodiaci excentrici cēlabuntur in ea, adeo vt vtriusque centri distantia sit pro. 32. parte semidiametri concavitatis Zodiaci è regione locis augis, temporis qui nostra ætate circa finem secundi gradus cancri inuenitur, eandem viam, in hoc, sequuti, quam Stoflerus in dorso Astrolabij docet. At nomina mensium media ponantur inter duos maiores circulos, postea inter secundum, & tertium ab vna facierum laminæ, arcus semidiurni, ab altera vero arcus seminocturni, per quinos quosque dies collocentur, ita exactè, vt hic subter videbis. adeo vt numeri dierum & ipsorum dierum signa sint in interuallis vicinioribus centro communi dictorum quinque circularum.

Posteaquam ab vna & altera facierū laminæ hæc insculpta fuerint, aliæ duæ circulares laminæ, magnitudinis semidiametri minimi quinque circularum accipiantur: quarum vna pro ortu, & altera pro occasu Lunæ deferuiet. In qualibet ipsarum constituētur circuli quatuor, eo modo qui paulo inferius cernitur, quos omnes diuidemus in triginta spacia æqualia: & in interuallo quod inter duos primos circulos positum est, triginta dies annotabim⁹ qui ipsos Lunæ triginta dies præscribēt, vt in figura.

Postmodum in lamina quæ ortus Lunæ indicabit, ac duorum maiorum circularū interuallo è regione numeri .1. videlicet primi diei, ponemus horas. 12. & minuta. 48. ex aduerso diei secundi ho. 13. et min. 36. ex opposito tertij ho. 14. min. 24. & sic successiuè augendo per min. 48. & indicem è diuerso diei. 30. statuēdo, qui coitus Lunæ cum Sole significabit: atque lineas aspectuum, vt inferius videre est facilè inueniemus.

Alteram in lamina quæ occasum Lunæ indicabit, postquam distincta fuerit, vt altera. 30. dies ac ceteræ lineæ, eo modo quo in superiori collocabuntur, at numeri interualli maioris, aliter disponentur, vt potè ex aduerso diei primi solum. 48. minuta describi debent, è directo secundi diei ponenda erit hora vna cum minutis. 36. & è regione tertij inscribentur. 2. horæ, & min. 24. & sic ex ordine per. 48. minuta augendo.

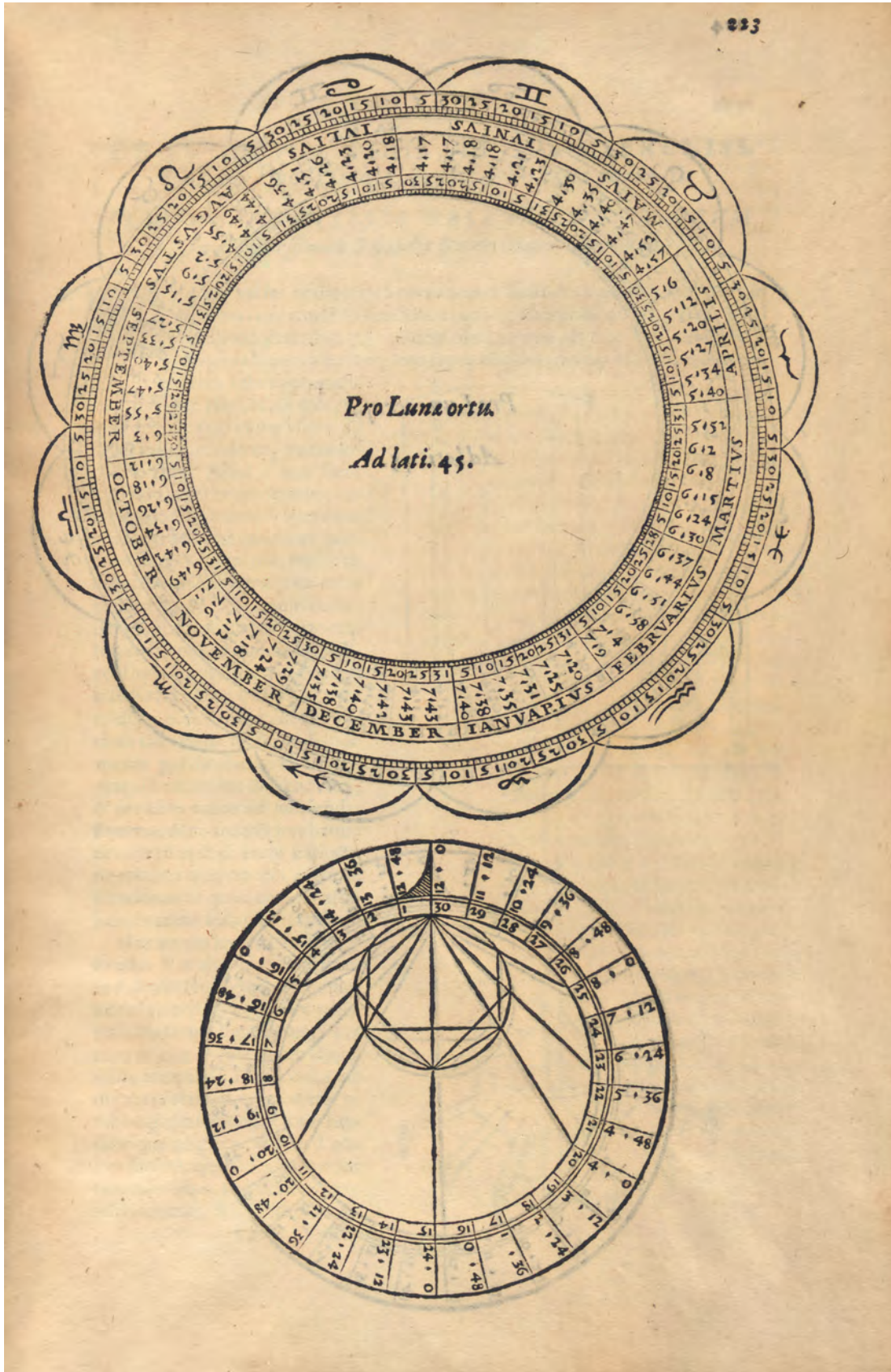
Nunc lamina ortus Lunæ, cum anno arcuum seminocturnorum, & illa occasus cum anno arcuum semidiurnorum concētrari debet, & ita nostrum instrumentum perfectum erit & absolutum.

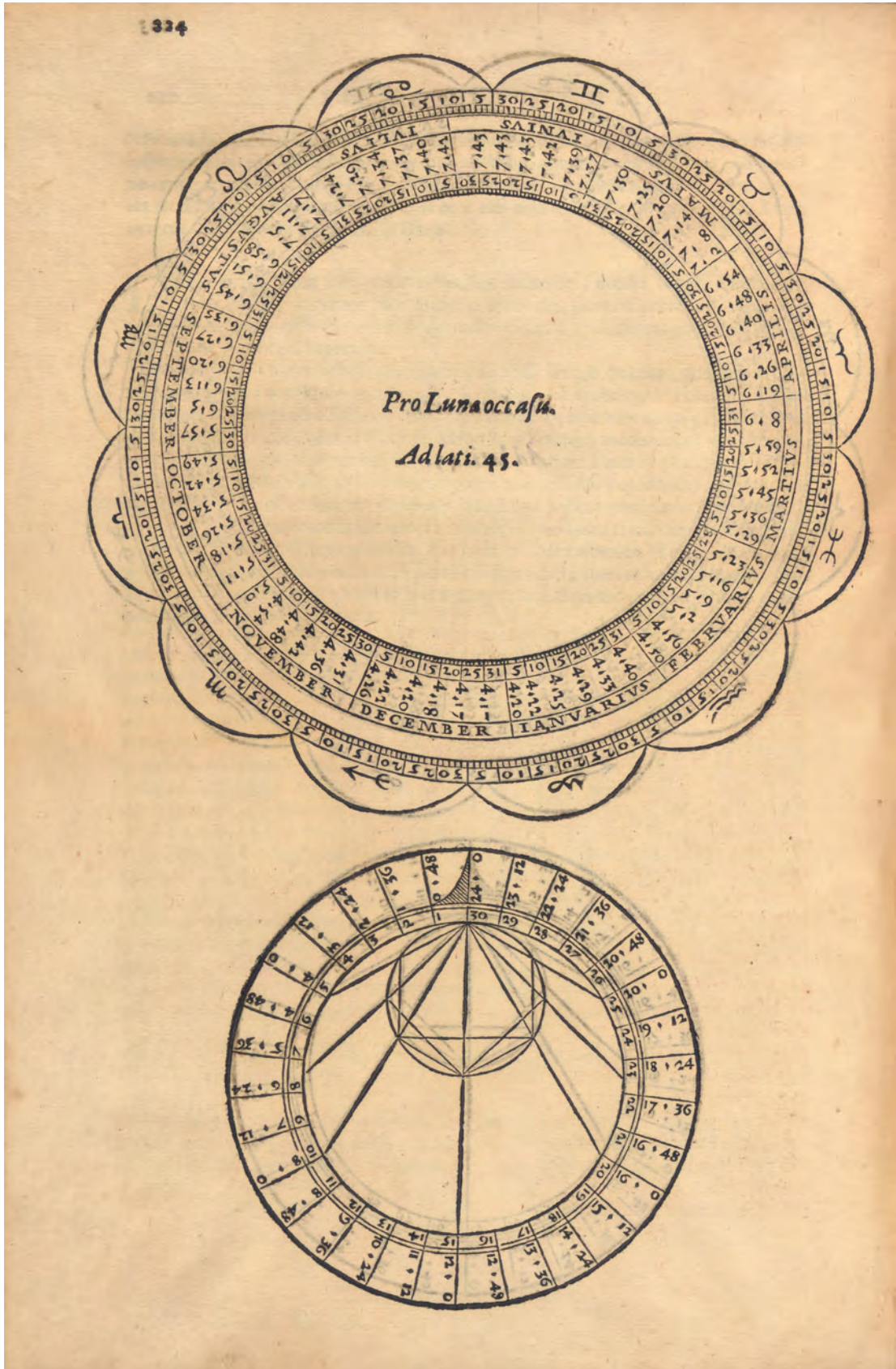
Quoties igitur voluerimus medio instrumento dignoscere fermè in tali horizonte qua hora Luna oriatur, ita necesse erit volubilem rotam ortus flectere, ut index veniat è regione diei mensis in quo talis operatio fit & tali rota firma manente perspicere,

cre ex aduerso diei Lunæ, numerum horarum & minorum in maiori interuallo ipsius rotæ notatorum, qui cum arcu seminocturno anni, quo cum in ipsa rectitudine centri conueniet colligetur, & summa quæ ex tali supputatione proueniet apertas faciet horas astronomicas, quibus ferè etiam non exactè in die proposito Luna oriatur. Idipsum fiet pro occasu Lunæ.

**D**IEM atatis Lunæ iam totus orbis scit inuenire, media supputatione numeri Epactæ currentis cum numero mensium, sumpto principio à Martio, adiunctis diebus mensis currentis, & detracto numero. 30. à summa prædicta, si ab ipsa dictus numerus. 30. superatur.

Sed ne aliquis putet sufficere tantummodo additionem quatuor quintarum horæ qualibet die. à nouilunio inchoando, sciendum est huiusmodi recessum Lunæ (quamuis non ita exactæ fiat) non computandū esse ab horizonte aliquo, sed à recto, seu à meridiano quod idem est, quemadmodum unicuique mediocriter erudito patere potest. At propositum nobis non est scire qua hora Luna in meridiano reperitur, sed in nostro obliquo horizonte, in parte orientali seu occidentali, propterea igitur addendus est, ei summæ temporis, qua Luna distat à meridiano, arcus semidiurnus, vel seminocturnus illius loci Zodiaci, in quo Luna reperitur illa die in proposito parallelo, ut sciatur proximè, qua hora (ex astronomicis) Luna erit in horizonte orientali, vel occidentali dicti paralleli. supra dicta enim additio quatuor quintarum horæ tantummodo, sufficiens erit temporibus æquinoctij, sed aliis anni temporibus falli ratione iam dicta.







DE LVCERNA SPIRITALI QVAM SERENISS.

Sabaudia Duce. D. meo collendis. anno. 1570. construxi.

CLARISS. FRANCISCO BARBARO VENETORVM

apud Serenissimum Sabaudia Ducem Oratori Illustrissimo.

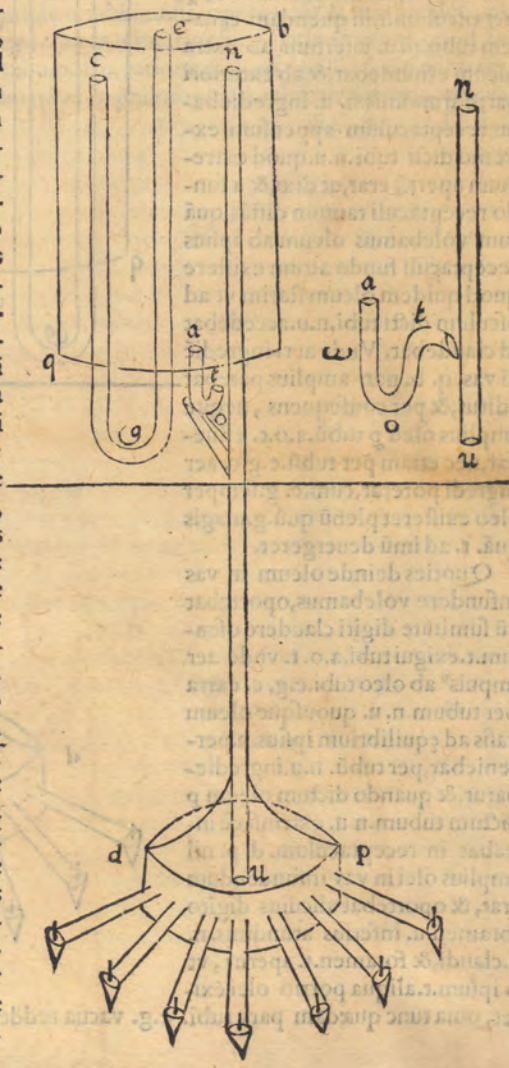


ERON varias ac diuersas hydraulicas, & spiritales machinas proposuit, inter quas tamen nullam similem, ei qua ego Serenissimo Sabaudia Duce construxi, describit, que quidem fuit Lucerna, & erat huiusmodi, vt a magno aliquo vase oleo pleno supra alicuius triclinij tabulatum posito,

subtilis quidam tubus perpendiculariter per tabulatum exiret, & in dictum triclinium vsque ad medium descenderet, ita tamen vt hic solus tubus, non item vas oleo plenum cerneretur, cuius quide tubi inferiori extremitati iunctum esset quoddam paruum receptaculum olei, simile cooperulo alicuius pyxidis, e cuius ambitu prope basim multi diuersi quae tubi aequales & orizontales, cuiusuis longitudinis profilerent, quorum quilibet in extremitate sua, exiguam quandam pyramidem, appesa haberet, in qua ellchnu esset cu mixo. oleu deinde medio ppendicularis tubi ad receptaculu extrinsece descendeat, & per alios tubos ad nutriendas flammam dum arderet ferebatur: at vero cu eadem erant extinctae ne minima quidem olei gutta descendeat: id quod eos qui astabant in admirationem trahebat.

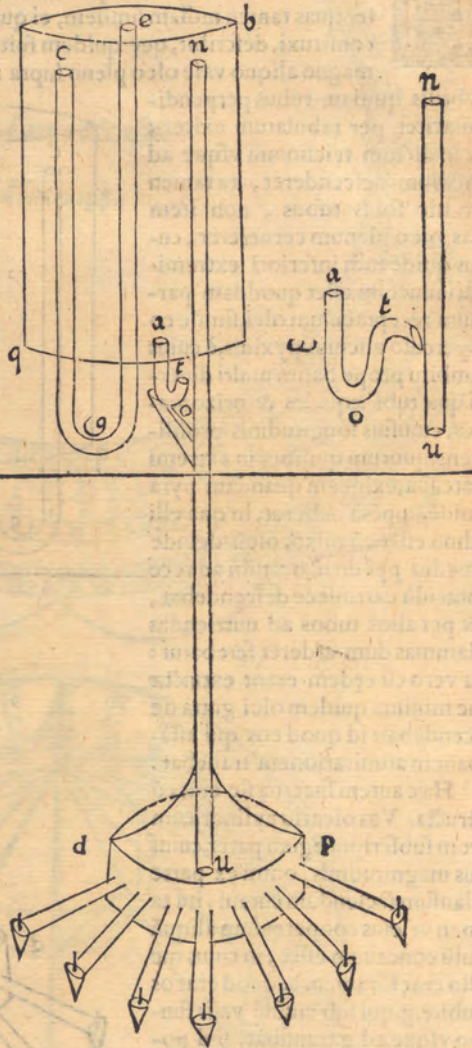
Hac autem lucerna sic erat constructa. Vas oleariu cylindricum vt in subscripta figura paret, cuiusuis magnitudinis, omni ex parte clausum faciendum curau, ita tamen vt eius cooperculum aliquantulu concauum esset, in cuius medio erat foramen. e. quod erat os tubi. e. g. qui sub eiusdem vasis fundo vsque ad. g. transibat, sed postea sursum, quasi vsq; ad cooperculum in situ. c. ab inferiori parte reflectebatur, & ibi terminaba.

Ff Vnde



Vnde oleum quod in vas infundebatur per foramen. e. dictum vas postea ingrediebatur per foramen. c. Habebat deinde tubum. n. u. rectum, qui à situ. n. propinquo co operculo ad libellam extremi. c. incipiebat, & per fundum contignationemq; vsque ad centrum supradicti receptaculi (circa quod tuborū ope appensę erant ellychniorum pyramides) transibat, atque huiusmodi tubi. n. u. extremitates tam superius quam inferius erant apertę, & hic tubus acris erat. Pręterea aliud quoddam foramen in vasis fundo feceram, cui paruum tubū. a. o. t. reflexum, ita tamen, vt. o. alius esset quam. g. aptauerā, atq; p hunc reflexum tubum. a. o. t. oleum vasis exhibat, q per osculum. t. in quendam canalem tubo. n. u. insertum, ab extra oleum effundebat, & ab exteriori parte arundinis. n. u. ingrediebatur receptaculum appensum extremo dicti tubi. n. u. quod extremum apertū erat, ut dixi, & à fundo receptaculi tantum distās, quātum volebamus oleum ab ipsius receptaculi fundo altum existere quod quidem oleum statim vt ad osculum dicti tubi. n. u. accedebat id claudebat. Vnde aeri ingrediendi vas. q. b. non amplius patebat aditus, & per consequens, neque amplius oleū p tubū. a. o. t. effluebat, nec etiam per tubū. e. g. c. aer ingredi poterat, cum. c. g. semper oleo existeret plenū quū. g. magis quā. t. ad imū deurgeret.

Quoties deinde oleum in vas infundere volebamus, oportebat cū sumitate digiti claudere osculum. t. exigui tubi. a. o. t. vnde aer impuls⁹ ab oleo tubi. e. g. c. extra per tubum n. u. quousque oleum vasis ad equilibrium ipsius. n. u. perueniebat, per tubū. n. u. ingrediebatur. & quando dictum oleum p dictum tubum. n. u. extrinsecē intrabat in receptaculum. d. p. nil amplius olei in vas infundendum erat, & oportebat alicuius digito foramen. u. inferius arundinis. n. u. claudi, & foramen. t. aperiri, vt p ipsum. t. aliqua portio olei exiret, quia tunc quædam pars tubi. e. g. vacua reddebatur, & cum per. t. nil amplius olei



olei egrediebatur, aperiebatur. u. & per ipsum. t. denuo tantum olei exire permitte-  
 bamus, quantum in réceptaculo ad claudendum foramen. u. idoneum existeret. Ra-  
 tio verò, quæ me mouit, ut punctum. g. inferius ipsius. t. constituam, est, quia cū clau-  
 sum erit. u. per dictum. t. oleum non amplius egredietur, quia pondus olei in tubo. c.  
 g. maius euadet oleo quod vsque ad. t. progredieretur, tubum autem. e. g. c. reflexum  
 facio, ne cogamur claudere foramen. e. quia hoc difficile præstaretur, tubum etiam  
 a. o. t. sursum versus reflexum constitui, ut aerem ab ingressu per foramen. t. arceré,  
 quia huiusmodi aer nunquam descendit si corpus magis densum non descendat.

Verum est, quod melius erit, ut maiores difficultates euitemus, statuere dictum tubum  
 a. o. t. ita curuum ut est. s. qui cum suo extremo inferiori ipsi. n. u. sit contiguus ita ta-  
 men ut dictum extremum inferius non sit inferius quam. o. quia totum oleum exiret.

Volui etiam ut superior extremitas. n. tubi. n. u. sit in aere vasis & non in oleo, ne  
 per eam oleum exeat, quia cum extremitas. u. inferior sit. g. totum oleum quod su-  
 peraret osculum. n. per dictum tubum. n. u. ratione maioris ponderis egrederetur,  
 quæadmodum cuilibet, vel mediocriter in philosophicis rebus versato innotescet.

DE

*[The following text is extremely faint and largely illegible due to fading and bleed-through from the reverse side of the page. It appears to be a continuation of the philosophical or scientific discourse.]*

## DEFENSIO EPHEMERIDVM.

AD ILLVST. D. BERNARDVM

Trotium.



EDITA erant scripta quædam, quorum titulus ANIMADVER-  
SIONES IN EPHEMERIDAS. & breuis alia disputatio de er-  
roribus calculorum Astronomicorum. ac demû Thefes quæ-  
dam typis data. 11. Augusti. 1581. quæ omnia cum ad manus  
meas peruenissent, non potui, non eis animum ad mouere, cû  
ibi de his studijs ageres, in quibus partem non exiguam an-  
norum meorum consumpsi. nec tamen scribere aliquid sta-  
tueram; tum quod existimarem viros Astronomiæ peritos  
facile, quanti facienda essent ea quæ edita erant iudicatu-  
ros, alijs verò haud gratam futuram harum rerum tractationem. Tum quod si ingenue meam sententiam profer-  
re voluisssem, non poteram, sine maxima authoris molestia ferè omnia reprobare.  
Quandoquidem solet vnusquisque indignationem concipere ex his, quæ suæ opi-  
nioni repugnant, id omne malevolentia potius quam veritatis studio tribuens. Qui  
nimo cum nec deessent qui dicerent in me ipsum directa ea tela fuisse, nullam fidem  
eis adhibendam duxi, nec enim qui in ephemeridas inuehitur, me arguere potest,  
qui nullas ephemeridas scripsi, nec tabulas composui. Nec si author quidpiam ex  
his quæ à nobis edita fuissent impugnare voluisset, ægrè ferre debuisssem, modo à ve-  
ritate nusquam deuiasset. Liberum enim est cuique scribere quodlibet. nec Ari-  
stotelem afficit iniuria, quicumque illi fidem suam non accommodat, & si valdè ini-  
quus sit, quisquis maiorum opiniones veras, & ab omnibus merito comprobatas nõ  
admittit. Hinc mihi satis oïbus fecisse videbar, cum his qui de scriptis illis me inter-  
rogauerat respondisssem, ea non satis firmis esse innixa fundamētis, & quod ad talia  
tractanda opus fuisset exercitatore iudicio. Verumtamen cum tu vnus maxime om-  
nium desideres tibi clarius, quæ nam de his mea sit sententia explicari, non tam tuis  
precibus deuictus quam mea ipsius cupiditate de te benemerendi impulsus, non su-  
stineo diutius animum tuum hesitantem relinquere. Atque vt tibi adeo honesta cu-  
pienti morè geram paucis hisce scriptis incultis quidem, vt ab homine omni pro-  
fus facundia destituto exaratis, sed ex quibus nihilominus facile, atque perspicue, vt  
spero, conceptum animi nostri percipere possis, si tamen eam præstantis ingenij tui  
aciem adhibueris, qua soles intima quæque scientiarum penetrare, nostræ opinio-  
nis summam perstrinxi, quæ ad te mittere decreui. & quamuis ipsa res de qua agitur,  
quæ exactiorem desiderat expositionem, prolixior me esse cogit quam voluisssem.  
multa tamen me obmississe intelliges, non admodum necessaria his quibus  
Astrologiæ noti sunt termini, vt tuarum occupationum rationem me etiam habere  
intelligeres, atque vt summam oblectationem cõcipiam animo si me tibi aliqua ex  
parte satisfecisse intellexero ita humanitati tuæ gratiam habebo, quæ mihi occasio  
nè p̄buit, imò verò me impulit, ad ea proferenda: quæ grata esse possint tui similibus,  
id est p̄claro & cãdido ingenio præditis, atq; ad euellendã ex eorũ animis falsam opi-  
nionem

nionem, si quam fortasse ex illorū scriptorum lectione cōceperunt circa ea, de quibus nunc sum acturus.

Quemadmodū igitur ab hoc autore ter scriptum fuit de cōtradictionibus, siue erroribus Ephemeridum, & earum calculos sequentium, & de ratione qua cognosci potest situs & locus alicuius superioris planetæ, diuersus ab eo, qui ab ipsis Ephemeridibus assignatus est, ita disputationem hanc meam diuidam in tres partes, quo scilicet minus confusè, & magis distinctè à me scribatur, p̄supponendo, vt animaduertere potes, huius scriptoris intentionem, aliam non fuisse, quā ostēdere, quod scriptores Ephemeridum diuersimode eiusdem temporis locum planetæ assignauerunt, & quod cum faciant modo nimium velociter currere, modo nimium in vno signo morari, vt (exempli gratia) Martem interdum faciunt morari sex, aut septem mensibus in vno signo. Idq; postea in causa esse ait, vt Astrologi iudicarij fallātur, & simul careant certis fundamentis rationum quibus futura iudicent, & prædicant. Primum ergo videndum est, quam rectè hic vsus sit arte, & scientia, vt aliorum opiniones, & scripta redarguere possit. Deinde videbimus quomodo verum sit, & possibile id quod ab Astrologis hæcenus creditum, atque traditum est, & qua ratione possint fieri veri calculi à peritis regularum scientiæ.

In primo igitur tractatu inscripto Animaduerſiones, præsupponit Author professoris huius scientiæ nescire inuenire vera loca planetarum, quia vtuntur Ephemeridibus, in quibus eorum loca non rectè sunt notata. Quod secundum ipsum oritur, ex errore calculatorum, seu computistarum, potius quam ex varietate tabularum, à quibus Ephemerides sumptæ sunt, hoc tamen verum non est, Ephemeridas, scilicet, ita inter se differre, ratione errorum computistarum tantummodo, sed potius ratione ipsarum tabularum, & si interdum contingere possit error aliquorum minorum, nec non graduum, non propterea Ephemerides ita spernendæ sunt. In multis enim calculis, tales errores excusabiles sunt, cum ab innumerabilibus propè accidentibus oriri possint, præsertim in calculis prutenicis.

Videatur deinde vbi is profert quinquagesimum enuntiatum centiloquij Ptolemæi, satis mendose. Ptolemæus enim ibi sic ait.

Non obliuiscaris esse centum viginti coniunctiones, quæ sunt in stellis erraticis, in illis enim est maior scientia eorum quæ sunt in hunc mūdum suscipiendi incrementum, & decrementum.

Nam, neque eo in loco, neque alibi, Ptolemæus quidquam eius dicere voluit quod ab hoc profertur.

Pergatur postea in pag. 2. & videbitur hunc existimare absurdum quod Saturni, & Iouis coniunctio vera anni. 1563. potuerit esse in Leone signo igneæ triplicitatis cum eorum coniunctio vera anni. 1544. fuerit in Scorpione, signo triplicitatis aqueæ, & cum coitus eorum anni. 1583. futurus sit in Piscibus, signo pariter triplicitatis aqueæ. Ita enim ait.

Nam postquam duæ stellæ coierint, non prius sub alio alterius triplicitatis signo inter se sunt conuenturæ, quam per omnia signa quæ eiusdem ternarij cum primo extiterint prius coniungantur. Ita sentit Ptolemæus, cæteri q; non aspernandi nominis Astronomi.

Et tamē Ptolemæus nunquam quidquā huius rei attigit, & quamuis Albumasar & Alchibitius de eo loquātur, is tamen eos non intellexit, cum illi ibi nō agant de

pe-

periodis apparentibus, aut veris, sed de mediocribus aut æqualibus, & quidem rectè dicunt, quia lineæ eorum mediorum motuum non coeunt in aliquo signo alterius triplicitatis, prius quam pertransierint omnia signa illius, in qua inceperunt. Itaque nullum inconueniens sequitur, si in veris cõiunctionibus non reperitur hæc regula. Fieri enim potest, ut lineæ mediorum motuum coniungantur in vno signo, corpora verò eorum planetarum coeant in alio, cum rarò eueniat, ut linea medij motus, eadem sit cum linea veri.

Nunc quidem tamè non affirmauerim, nec negauerim eorum coniunctionem anni. 1563. fuisse potius in Cancro, quam in Leone. Sed tantum dicam vanū esse credere id euenisse propter similem naturam, aut qualitatem signorum. Hunc enim respectum non habent illi planetæ in veris suis coniunctionibus. Exempli autem causâ ponamus, quod rectè supputatæ fuerint coniunctiones annorum. 1484. 1504. & 1524. quod attinet ad differentiam duodecatemoris, scilicet prima in. 24. gradu Scorpij, secunda in. 20. Cancrj, tertia in. 10. Piscium. Cum secunda anticipauerit trigonum perfectum cum prima, gradibus. 4. & tertia anticipauerit trigonum perfectum cū secunda gradibus. 10. si forte prima ut facta fuit in. 24. gradu Scorpij facta fuisset in. 2. gradu eiusdem, planū est quod secūda facta fuisset in. 28. gradu geminorum & tertia in. 18. Aquarij, quæ signa sunt diuersa triplicitatis ab illa Cancrj. Insuper si coniunctio anni. 1544. quæ fuit in. 28. gradu Scorpij fuerit recta correspondēs præcedenti, anni. 1524. per gra. 18. sine dubio si coniunctio anni. 1524. facta fuisset in. 18. gradu Aquarij, illa anni. 1544. fuisset in. 6. Scorpij signi alterius triplicitatis quàm sint Gemini. Præterea, ut anno. 1544. cõiunctio facta est in. 28. gra. Scorpij, & 1563 in. 29. Cancrj, ponendo eas esse rectas, quod attinet ad superandum trigonum vno gradu, si anno. 1544. facta fuisset in. 30. Scorpij, anno. 1563. proculdubio facta fuisset in primo gradu Leonis. Et suppositis ijs interuallis, quæ supersunt, aut desunt perfectis trigonis, si coniunctio anni. 1524. fuisset in. 20. gradu Piscium, anno. 1544. fuisset in. 8. Sagittarij. Quæ quidem omnia aduersantur opinioni huius scriptoris. Quod autem opinatur coniunctionem anni. 1583. fore in Ariete, sic dicens pagina secunda.

Non erit ab re si & eandem Saturni, & Iouis coniunctionem in primo igneæ triplicitatis signo, quod est Aries futuram afferamus anno. 1583. si ab accidentibus nobis licet, ut ab omnibus passim conceditur, planetarum loca discernere.

In eo fallitur, nã neque Saturnus, neque Iupiter, errât à vero per. 9. nec. 8. gra. ac neque quidè in quibusvis Ephemeridibus aut tabulis. Itaque videbit eiusmodi cõiunctione contra sententiã suã fieri in Piscibus, nõ aut in Ariete. (ut postea res ipsa nos docuit sub mense Aprili, post quæ scripta hæc epistolã, vulgariq; sermone trãsmisã, sed antequã in latinū trãslata, & huic volumini inserta cū alijs Typographo cõmitteretur.)

Vbi postea meminit magnæ periodi annorum. 960. non tantum ei cogitandum erat hæc fuisse opinionem antiquorū, ut videri pòt apud Albumasarè & Alchibiciū, sed etiam perpendendum an esset vera, priusquã ei adhereret. Hic enim fuit vnus ex erroribus illius ætatis, quæ nondum penetrauerat intima huius scientiæ. Sunt tamen illi antiqui excusatione aliqua digni. Ponebant enim vigesimo quoque anno præcisè fieri mediam coniunctionem Saturni cū Ioue, & in quolibet signo eiusdem triplicitatis cõiungi quater. Itaque in qualibet triplicitate dicebant eos coire duodecies.

Quod

Quod secundum primum suppositum finiebatur spacio annorum. 240. qui numerus fit. ex. 20. duodecies multiplicatis. Et quia triplicitates sunt. 4. ideo credebant in spacio annorum. 960. qui numerus fit ex. 240. quater multiplicato, perfici. 48. conjunctiones, priusquam redirent ad se coniungendos in eodem loco, ubi prius iuncti fuissent. Primum autem suppositum, quod vigesimo quoque anno iugerentur, colligebant sic ratiocinantes. Si Saturnus annis. 30. peragit suum cursum per omnia signa Zodiaci, Iupiter autem peragit eum annis. 12. Saturnus ambulauerit. 4. signa, et. 4. quintas partes signi, siue gra. 24. dum Iupiter peragit integrum ambitum idest annis. 12. Itaque desunt ei anni. 8. ad perueniendum ad. 20. quibus. 8. annis Saturnus pambulat signa tria & quintam partem unius signi. i. gradus. 6. qui iuncti dictis signis. 4. & gra. 24. faciunt signa. 8. quae Iupiter item percurrit in annis. 8. atque ita in annis. 20. Iupiter percurrit. 20. signa antequam perueniat ad Saturnum, cum Saturnus eodem tempore perfecerit cursum signorum. 8. Eandem conclusionem etiam fortasse collegerant ex dictis suppositis, dicentes, si Saturnus annis. 30. ambulat. 12. signa proculdubio annis. 20. ambulat. 8. signa, quo tempore Iupiter perambulat. 20. ad rationem. 12. signorum in annis. 12.

Verum hoc suppositum non est bonum, quoniam, si ita esset, conjunctiones horum duorum planetarum nunquam exirent ex vna triplicitate, & non modo. 960. quoque anno, sed etiam sexagesimo rursus coniungerentur in eodem puncto. nec conjunctiones eorum (semper autem intelligo de medijs) unquam egrederentur ex illis tribus signis Zodiaci.

Sed periodus aequalis Saturni, est dierum circiter. 10740. atque ita minor an. 30. atque etiam. 29. cum dimidio. periodus autem aequalis Iouis, est circiter. 4328. ut ego eam comperio, quidquid alij dicant, utque planius alias ostendam. Itaque haec periodus Iouis, etiam minor est ann. 12. praetermittendo in supputatione tantum Saturni quam Iouis quasdam minutias horarum & earum partium, quae hac in re pro nihilo haberi possunt. Atque his duabus periodis eccentricorum duorum planetarum possumus cognoscere interuallum quod erit inter vtramque mediam conjunctionem, hoc modo agendo, & ratiocinando.

Si Saturnus diebus. 10740. circuit gradus. 360. diebus. 4328. qui sunt periodus Iouis, conficiet gradus. 145. & min. 4. idest min. 8704. & eadem regula inueniemus quod Saturnus. 30. quibusque diebus, conficiet min. 60. & secunda. 20. Iupiter autem singulis. 30. diebus, conficiet min. 149. & secunda. 43. unde subtrahendo minuta Saturni a minutis Iouis, supererunt min. 89. cum secun. 23. Itaque Iupiter. 30. quibusque diebus velocitate cursus, superabit Saturnum minutis. 89. cum secundis. 23. Atque dicendo, si minuta. 89. cum secundis. 23. dant nobis dies. 30. supradicta, minuta. 8704. dabant nobis dies. 2921. quibus iunctis cum diebus. 4328. periodus Iouis, efficiuntur dies 7249. idest anni Aegyptij. 19. cum diebus. 314. & haec erit aequa periodus temporis inter vtramque conjunctionem horum duorum Altiorum planetarum. Ut autem planius ostendatur hanc operationem rectam esse (nam demonstrationem speculatiuam huius operationis in. 113. Theoremate nostrae Arithmetice cuique videre licet) fieri potest his alijs calculis.

Si Saturnus diebus. 10740. transit per gra. 360. in spacio dierum. 2921. transibit per gradus. 97. min. 54. quibus iunctis cum gra. 145. min. 4. supra notatis, efficiuntur gra. 242. min. 58. Deinde, si Iupiter spacio dierum. 4328. transit per gra. 360. igitur spacio. 2921. per eandem regulam inueniemus eum transire gradus. 242. mi. 58. qui numerus par est illi Saturni. Cum ergo Iupiter confecerit vnum ambitum post con-

iunctionem cum Saturno, vt rursus affequatur Saturnum, transeundum ei erit p gra. 242. min. 58. iter confectum à Saturno toto tempore annorum. 19. & dierum. 314. ad rationem graduum. 360. diebus. 10740. (possimus etiã dicere gra. 243. quia præ terminimus quasdam exiguas particulas periodorum perfectorum cuiusque planetę in superioribus supputationibus.) Illos verò gradus. 243. Iupiter conficiet diebus. 2921. ad rationem graduum. 360. diebus. 4328. Atq; ita vt diximus, ab vna coniunctiõne ad aliam intererunt anni. 19. Aegyptij cum diebus. 314. vel circa.

Nunc autem vt videatur an tabulę Alfonsi conueniant cum hoc nro calculo, cõsiderabimus, q̄ Era (vt vocant) dicti temporis annorum. 19. cum diebus. 314. est duarum tertiarum sexagenarum, secundę nullius, & 53. primarũ siue dierum. Et per hęc Eram colligendo motum mediocrem, tum Saturni, tum Iouis, omisis radicibus, & incipiẽdo ab Ariete, comperiemus q̄ vtriusq; planetę lineę eiusmodi motus transibunt per min. 56. tertij gradus Sagittarij, idest coniunctę erunt.

In fine postea secundę periodi, cuius era erit. 4. tertiarum, secundę. 1. et. 47. primarum sexagenarum, locus mediocris vtriusq; erit in min. 56. gra. sexti Leonis. In fine verò tertię periodi, cuius era erit. 6. tertiarum. 2. secundarum, et. 41. primę, locus eorum mediocris inuenietur in. 56. minuto gradus. 9. Arietis. Atq; ita deinceps in fine cuiusq; periodi, locus eorũ mediocris coniunctim semper distabit à loco mediocri præcedentis coniunctiõnis gradibus. 117. idest in trigono antecedenti, minus gra. 3. Vnde apparet has coniunctiõnes procedere in contrariam partem respectu ordinis signorũ Zodiaci, sed respectu ordinis graduum signorũ, semper progrediuntur ordine per ternos gradus nunquam retrogradientes. Hinc sequitur, vt non duodecies in omni triplicitate coniungantur hi duo planetę, vt antiqui putauerunt, sed decies tantum. & ad summum ter in singulo signo, spatio annorum. 198. & dierum. 220. aut circiter, non autem. 240. nec. 242. Atque decem vices comprehendunt gra. 27. & vltima vice inueniuntur in signo sequenti alterius triplicitatis. Exempli gratia, ponamus q̄ prima vice cõiungantur in gra. 2. Arietis, secunda coniunctiõ erit in. 5. Sagittarij, tertia. in. 8. Leonis, quarta in. 11. Arietis, quinta in. 14. Sagit. 6. in. 17. Leonis, sexta in. 20. Arietis, octaua in. 23. Sagittarij, nona in. 26. Leonis, decima in. 29. Arietis, et vndecima erit in gra. 2. Capricorni signi sequētis triplicitatis. Decem igitur interualla singula annorum. 19. & dierum. 314. faciunt annos. 198. & dies. 220. Immo per tabulas Alfonsi, eiusmodi periodus non modo non reperitur annorũ. 242 nec. 240. vt antiqui credidere, sed tribus diebus minor annis. 198. & diebus. 220. idest per dictas tabulas inuenitur esse annorum. 198. & dierum. 217. tantum, qui numerus multiplicatus per. 4. triplicitates, efficiet periodum maiorem, quę erit annorum 794. & dierum. 138. quo tempore dicti planetę redeunt ad eundem locum vbi primum se coniunxere.

Vt exempli gratia, locus mediocris Saturni & Iouis in fine annorum. 198. dierum 217. reperitur in gradu. 30. Sagittarij. Si quæsiuerimus hunc locum per aggregatũ annorum. 794. & dierum. 138. cum annis. 198. & diebus. 217. quorum summa est. 992. & dies. 355. inuenietur locus mediocris ipsorum planetarũ in dicto vltimo gradu Sagittarij. Sed si quæsiuerimus eorum locum mediocrem per aggregatum annorum. 198. & dierum. 217. cum annis. 960. quod erit summa annorum. 1158. & dierũ 217. reperiemus Iouem in gradu. 18. Sagittarij & Saturnum in. 16. Leonis distantibus inter se duabus eorum lineis motuum mediocrium gra. circiter. 122. Atq; Iupiter præcedet, & oportebit q̄ coniunctiõ eorum mediocris fuerit multis annis ante omittendo (vt dixi) radices, quia satis est inuenire interuallum inter lineas eorum mediocrium motuum.

Debe-



Debebat igitur author animaduersionum non quasi cæcus cæcos sequi, sed prius laborare, vt certior fieret, an interuallum annorum. 960. Verum esset.

Sed peius est, q̄ idem author paulo inferius citat coniunctiones horum duorum planetarum anni. 1493. et. 1512. quas nescio vnde sumpserit.

Nam, et si inter hos annos est interuallum annorum. 19. tamen tantum abest, vt couerint dictis annis, vt Saturnus anno. 1493. ante finem Augusti fuerit in. 28. gradu Aquarij, Iupiter verò in. 28. Leonis ex diametro oppositi. Et anno. 1512. per totum mensem Iunium & Augustum, Saturnus fuerit in Libra, Iupiter verò in Ariete, itaque inter se similiter oppositi, & si perfecta oppositio non fuit postea nisi ad finem Iunij ann. 1513. & locus Montereij ab eo citatus, vbi ait eum ponere coniunctionem anni. 1484. in gra. 23. min. 4. Scorpij, est mendosus. Nam ipse Montereij ponit dictam coniunctionem in mi. 42. gra. 24. non autem in min. 4. ipsius gradus. Sed hic error nullius est momenti, fortasse qui impræssorum incuria irrepsit.

Pergatur postea obsecro ad paginam. 3. ipsarum Animaduersionum, vbi hic conatur ostendere calculatores non obseruasse verum modum, sic dicens.

Anno. 1484. Nouembris. 25. Saturno locum constituit Montereij in grad. 23. min. 4. Scorpij. Anno postmodum subsequenti qui est. 1485. eundem in min. 7. Sagittarij collocat. 21. Februarij die. Inter q̄; tempora duo intersunt menses dies. 26. At cum ex motus sui natura Saturnus hoc temporis spacio gradus. 4. non debeat transcendere, sit tamen inter vtrunq; tempus differentia graduum. 7. minorum. 3. quæ ratione sui motus requirunt menses. 6. vt eos perficiat, constat plusquam tribus mensibus fallere nos Saturnum.

Hic videre licet quam veram viam hic securus sit ad aperiendos errores Ephemeridum, & miseri Montereij, qui Saturnum claudum facit tantum itineris conficere tribus mensibus, quãtũ vix confecisset mensibus. sex. Sed fortasse ratiocinat hoc modo.

Si motus naturalis Saturni facit vt circumeat totum cælum annis. 30. igitur mensibus. 30. conficiet duodecimam partem circuitus, cum menses. 30. sint duodecima pars annorum. 30. & quia duodecima pars circuitus cæli intelligitur constare ex. 30. gradibus, igitur quilibet mensis postulabit gradum vnum. Ideo illi. 6. aut. 7. gradus postulant tempus, amplius mensium sex.

Atque eiusmodi mira ratiocinatio potest in. 2. exemplo eius, inscripto.

De eodem ex eodem

Vbi miratur, q̄ Montereij faciat Saturnum ambulare gra. 9. min. 10. in mensibus. 7. & diebus. 6. Ad quod iter Saturno seni opus esset saltem mensibus. 9. eius iudicio.

Sed si hoc miratur, quid dicturus fuisset, si animaduertisset, quod idem calculator Montereij facit Saturnum ambulare immo volare gra. 9. min. 48. non in 7. sed in 2. mensibus cum dimidio, videlicet à. 10. die Iunij vsque ad. 26. Augusti eiusdẽ anni. 1504.

Quid si etiam animaduertisset q̄ à. 10. die Iunij supradicti vsque ad. 16. Ianuarij anni sequentis, faciunt Saturnum, sursum, deorsum cursitare amplius gra. 17. mi. 54. Immo si animaduertisset, quod anno. 1524. Stoflerinus ab initio anni, vsque ad medium Maium, id est mensibus. 4. cum dimidio, facit Saturnum ambulare gra. 15. Profectò ob has velocitates, eius iudicio, tam absurdas, obstupuisse.

Vbi autem in tergo eiusdem pagine ait, quod gradibus. 13. min. 42. respondent menses. 19. errauit in calculo, nam ex eiusmodi tempore secundum eius regulam effice-

ficerentur singuli ambitus Saturni ad rationem annorum amplius. 40.

Videamus nunc ubi agit de Ioue, & reperiemus quod in primo exemplo circa annum .1484. reprehendit Monteregium, quia facit Iouem ambulare gradus. 14. cum min. 6. in mensibus. 2. diebus. 4. ad quod iter, ut ipse ait, opus esset saltem mensibus. 11. atque ita secundum ipsum, Ioui opus esset anno vno pro singulo medio signo. Vbi bonus hic vir pariter cæcutit.

Idem in secundo exemplo sumpto à Stoflerino ait, quod Ioui ad cursum vnus gradus, & min. 5. opus est diebus. 30. non autem mensibus. 7. & diebus. 28. ubi ostendit, se paruum discrimen facere inter Iouem, & Saturnum.

Miratur postea quod Stoflerinus faciat laborare generosum Iouem ferè mensibus sex in vno gradu. Sed multo magis, ut puto, miratus esset, si vidisset, quod idem Stoflerus in eodem anno facit, quod Iupiter die. 4. Ianuarij sit in eodem puncto, in quo postea reperitur die vltima Augusti. At fortasse dici posset, quod Iupiter propter prædantiam, & bonitatem suam factus est Rex omnium Deorum, ut ait Homerus, & ideo expulit e sede Saturnum, & ascendit in altiori cælo. Vnde euenit ut factus fuerit lentior in cursu, Saturnus autem velocior. Aut iam tot annos esse natum Iouem, ut iure credi possit eum iam factum esse senem, & pariter tardior in se mouedo. aut tunc temporis illum detentum fuisse in sibi dilecta Arcadia cum Calisto. Aut fortasse erat in alta specula intentus audiendo ingenti certamini Timoclis & Damidis, vnde pendebat exitium aut gloria familiæ suæ, nam alioquin Stoflerus non depræhendisset eum tam otiosum & morantem. Sed iam relinquamus Saturnum & Iouem, & ad Martem veniamus.

Ferox & inquietus Mars, qui semper bella & ignes spirare solet, etiam, & ipse ab Astrologis factus est piger, & languidus, ut velint eum nonnunquam commorari in vno signo sex aut septem mensibus, quod nullo pacto placet auctori Animaduersionum, cum pag. 4. ita scribat.

Quod citra notam, ab omnibus creditur posse obseruari, quamuis à nobis non accipitur.

Itaque ei videtur impossibile. Quia Mars peragit suum circuitum minus. 2. annis. Sed audacior fuisse videtur, qui voluerit arguere tot egregios viros antiquos, & recentiores, qui vti diligentes rerum cælestium obseruatores, ipsis oculis certi facti sunt tam de his effectibus Martis, quam aliorum, vnde coacti sunt fingere tantam magnitudinem eius epicycli, cum ipse nunquam obseruauerit motum, nec huius nec alterius planetæ, sed tantum viderit eius moram in Ephemeride scriptam. Si enim saltem diceret, se aliquo tempore obseruasse iter Martis, & comperuisse aliorum opinionem falsam, attulisset aliquem colorem sententiæ suæ. Sed si obseruasset, non scripisset postea contra, ut puto. Res enim ita se habet, quod Mars in omni circuitu sui epicycli transiens per inferiorem partem ipsius epicycli, semper commoratur multis mensibus in vno duodecatemorio Zodiaci, scilicet. 6. et. 7. mensibus, atque etiam amplius, quod quidem ego sæpe obseruavi, præsertim anno. 1565. et. 1566. hoc ordine. Primum inspiciens Ephemeridas stadij, reperi quod Mars secundum eum egrediebatur retrogradationem circa diem. 12. Ianuarij anni 1566. in. 16. grad. Geminorum. Et similiter quod anno. 1565. die vltima Augusti Mars futurus erat in eodem supradicto loco, priusquam retrogradi inciperet. Postea inueni, quod post retrogradationem die. 11. Aprilis. 1566. Idem Mars futurus erat in gra. 16. Cancrî, itaque in his. 30. gradibus à. 16. Geminorum ad. 16. Cancrî consumebatur spatium mensium. 7. & dierum. 11.

Quo

Quo supputato, sumpsi instrumenta, & ad experimentum me paravi, & vltima nocte mensis Augusti anni. 1565. reperi Martem esse in dicto gradu geminorum vt scribebat Stadius. Deinde singulis ebdomadibus obseruans retrogradationem; vidi circa finem Octobris quod retrogradi incipiebat, & ea retrogradatio perseverauit vsque ad medium mensem Ianuarium, aut circiter, anni. 1566. obseruavi postea etiam situm eiusdem planetae die. 11. Aprilis sequentis eumq; inueni in gradu. 16. Cancri, vt eum posuerat Stadius. Atque ita experimentum meum conuenit cum calculo Stadij, comperiq; eum non errasse: Et sic quisque binis quibusq; annis poterit certior fieri de veritate. Si autem delectationis causa id experiiri volueris, expectato primam retrogradationem Martis, cuius initium secundum Stadium futurum est circa diem. 20. Nouembris anni. 1582. & finis circa diem. 10. Februar. 1583. circa grad. 9. Cancri, & animaduerte quando Mars erit circa dictum gra. 9. Cancri prius quam retrogradi incipiat, quod erit circa diem. 19. Septem. 1582. Deinde aspice quum erit in grad. 9. Leonis, quod erit circa diem. 7. Mai. 1583. & videbis q; ipse Mars in his gra. 30. morabitur p menses. 7. & dies. 18. atq; vt eius rei periculum facias, obserua noctem praecedentem diei. 19. Septem. 1582. locum longitudinis eius stellae, & idem postea obserua nocte praecedente diei. 7. Mai, aut nocte sequenti. 1583. & inter duos hosce terminos obserua aliqua alia nocte statum eius. Manifestoq; videbis Martem consumere totum dictum tempus in hoc duodecemorio. Et quicumque aliquid intelligit in hac facultate quamuis non viderit Ptolomei Almagestum, minori labore posset per calculos scientificos colligere veritatem, suppositis tamen terminis scriptis in theoreticis planetarum. Qui enim vidit Almagestum vel reuolutiones orbium coelestium Nicolai Copernici, non potest de hoc vlllo pacto dubitare. Sed qui nondum tantopere progressus est, saltē capiat huius rei notitiam vniuersalem, hoc modo. Supponat primum eccentricitatem deferentis epicycli Martis, esse. 6. partium taliū, quales sunt sexagesimae semidiametri ipsius deferentis, & semidiametrum epicycli esse, partium supradictarū. 39. cum dimidia, & quod argumenta vera, in temporibus primarum stationum (cum epicyclus est in auge, aut in eius opposito, aut in lūgitudinib. mediocribus) iā ab antiquis rectē supputata sint, sicuti sunt. Et praesupponat motum diurnum centri epicycli. min. 31. cum dimidio, quamuis reuera sit min. 31. & secundorum. 27. aut circiter, nunc quidē praetermittens, quod vnus habeat respectum ad augem mediam epicycli, & alter ad centrum aequantis. Atque his praesuppositis fingat (exempli gratia) quod centrum epicycli sit in quouis longitudinum mediarum, & Mars in prima maxima aequatione argumenti, scilicet in prima linea, quae attingens epicyclum, à centro mundi pergat ad circumferentiam Zodiaci, quae erit illa linea cōtangentia à qua proficiscēs Mars perget ad lineam primae stationis, vt postea retrogradiatur, veluti si in infraposita figura maiori, cētrū mūdi esset. o. & vnus arcus eccentrici esset. a. b. c. d. & vna ex lineis mediocribus longitudinum esset. o. c. f. & centrum epicycli. c. qui notabitur per a. f. e. g. & lineae contingentes epicyclum in punctis. i. et. t. sint notatae. o. i. et. o. t. & linea primae stationis. o. n. b. & linea secundae. o. u. d. si igitur Mars esset in puncto. i. angulus. i. o. e. maximae aequationis argumenti esset gra. 40. minut. 55. quāuis talis maxima aequatio argumenti in longitudinibus mediocribus Alfonsi ponatur esse gra. 41. minut. 10. quod euenit quia calculatores ipsarum tabularum interuallum. o. c. quod in eo situ epicycli interponitur inter centrum mundi, & centrum dicti epicycli, acceperunt partium sexaginta praecisē, nihili facientes minuta illa. 18. aut circiter, quae verē sunt praeter dictas partes. 60. quandoquidem euenit vt dictum interuallum in

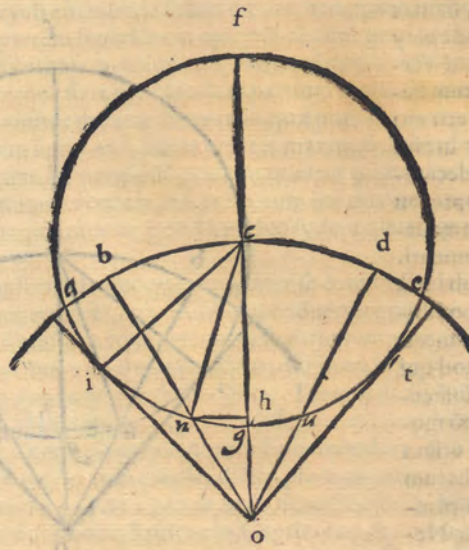
tali situ epicycli sit basis vnus trianguli orthogonij, cuius vnum ex illis duobus lateribus est semidiameter eccentrici partium. 60. præcisè, aliud est interuallum eccentricitatis partium. 6. eiusmodi. Angulus ergo. i. o. c. vt dixi, erit partium. 40. minu. 55. qui angulus continuò variatur secundum situm epicycli. & cum centrum eius est in auge eccentrici. est minimus quã esse possit. estq; tantum grad. 36. minu. 46. & in opposito ipsius augis est grad. 47. minu. 1. maximus quam alibi vnquam sit, & si c. continuò variatur, secundum situm, quem habet epicyclus in eccentrico. Qui quidem angulus inuenitur per doctrinam. 27. et. 28. libri primi Monteregij de triangulis. Nam triangulus. c. i. o. est semper reſt angulus in puncto. i. & latus. c. i. respectu semidiametri est datum. Quod. c. i. erit veluti partium. 39. cum dimidia, et dictum interuallum. o. c. veluti partium. 60. minu. 18. & quia datur nobis etiam eccentricitas veluti partium. 60. talium, & cum. c. o. sit linea veri motus epicycli, & latus similiter vnus trianguli, cuius duo latera sunt supradicta, scilicet semidiameter eccentrici, & eccentricitas, inter se comprehendentes angulum datum. Nam semper præsupponitur datus locus centri ipsius epicycli, cum ipse est extra auge aut oppositum eius quia in auge linea. o. c. constat ex semidiametro eccentrici & interualli eccentricitatis. & in eius opposito, ipsa linea. o. c. est minor dicto semidiametro eccentrici per interuallum dictæ eccentricitatis. Vnde etiam possumus extra auge, vel oppositum eius cognoscere. o. c. tanquam latus dicti trianguli duorum laterum cū angulo cognitorum. Idq; per. 49. propositionem libri primi eiusdè Monteregij cum scilicet dictus angulus nõ fuerit reſt. Nam si fuerit reſt videbitur per. 27. et. 28. supra citatas.

Cum igitur habeamus angulum. c. o. i. gra. 40. mi. 55. angulus. o. c. i. tanquam reliquus ex reſto, erit grad. 49. mi. 5. cui respondet arcus. i. g. epicycli confectus à Marte in diebus circiter. 105. ad rationem min. 28. aut circiter in singulos dies, prætermittis nunc quidem minutijs cum exigui momenti sit error. 15. aut. 20. dierum ad verificationem longæ morè Martis in vno duodecatesimario, atque per hoc tempus centrum epicycli conficit gradus. 55. minu. 7. aut circiter, ad rationem minorum. 31. cū dimidio in singulos dies. qui numerus graduum. 55. minu. 7. differt à numero graduũ. 40. minu. 55. maximæ æquationis argumenti gradibus. 14. mi. 12. nec refert quod gra. 55. minu. 7. habeant respectum ad centrum æquantis, magis quam ad centrum mūdi, quia differentia non est tanta, vt possit inducere errorem mensium. Hinc sequitur quod in fine dictorum dierum. 105. Mars erit in linea. o. c. veri motus epicycli, sed gradibus. 14. minu. 12. vltcrius quam in primo loco, in quo erat in Zodiaco, & erit in medio suæ retrogradationis. Sed quoniam Mars manifestè retrogradi non incipit in puncto. i. contingentia, imo ab illo puncto vsque ad terminum primæ stationis lineæ. o. n. interponitur arcus. i. n. epicycli, qui est graduum. 32. minu. 14. Idq; cognoscitur subtrahendo arcum. f. i. n. graduum. 163. mi. 9. qui est inter auge, & primam stationem, à gradibus. 180. (qui arcus. f. i. n. erit verum argumentum, quod similiter variatur secundum situm epicycli, etsi eiusmodi varietas, nobis nõ est magni momenti, vnde possumus præsupponere, quod. c. centrum epicycli non alteret interuallũ. c. o. à centro mūdi, cū non possit intercedere, error mēsiũ reliquum verò. g. n. graduum. 16. minu. 51. subtrahendo ex arcu. g. i. graduum. 49. minuti. 5. vnde reliquus nobis erit arcus. n. i. graduum. 32. minu. 14. in eiusmodi tamen situ mediocrium longitudinum. Nunc hic arcus epicycli graduum. 32. mi. 14. fit à stella Martis diebus. 69. ad rationem supradictam, omittendo quod ipsa stella habeat respectum ad auge mediocrem epicycli, & quod dicta aux mediocris mutet distantiam à vera propter motum epicycli, quod nunc quidem parui refert, in quibus diebus. 69. cen-

trum

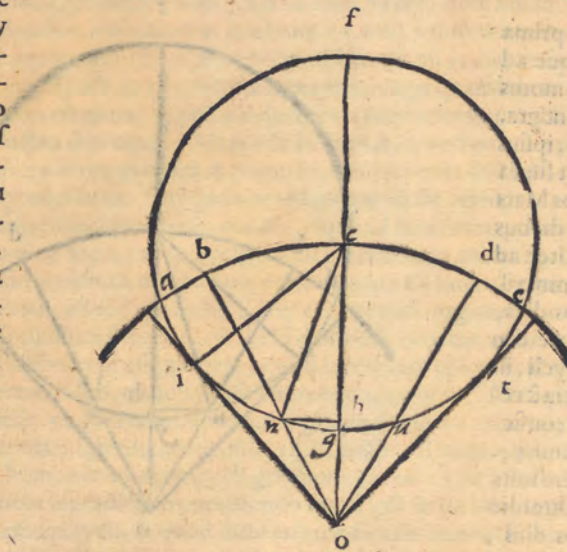
trum epicycli conficit gra. 36. min. 13. ad rationem supradictam. Restat nunc nobis inuenire angulum. b. o. c. in centro mundi inter duas lineas, b. o. et. c. o. quarum prior est primae stationis, altera est veri motus epicycli, quod facile intelligemus per dictam. 49. lib. 1. Monteregij, cum duo latera. n. c. et. c. o. & angulus. n. c. o. sint nobis nota. Hoc autem fiet fingendo lineam. n. h. perpendicularerem ad. o. c. quam tanquam sinus anguli. n. c. h. erit partium. 28986. talium qualium. n. c. esset partium. 100000. & c. h. tanquam sinus anguli. c. n. h. restantis ex vno recto, erit partium. 95706. dicendo postea si. n. e. tanquam sinus totalis partium. 100000. dat nobis. n. h. partium. 28986 quid dabit nobis diameter. n. c. tanquam partium. 39. mi. 30. inueniemus. n. h. venire nobis ex partibus 11. mi. 27. & idem faciendo de. c. h. inueniemus quod veniet nobis partium. 37. mi. 48. quibus subtractis ex tota. c. o. quae est partium. 60. mi. 18. reliqua erit nobis. h. o. partium. 22. min. 30. capiendo postea radicem quadratam summae quadrati. n. h. cum quadrato. h. o. veniet nobis. n. o. partium. 25. min. 12. talium qualis. n. h. est partium. 11. min. 27. si igitur ad. o. n. tanquam partium. 25. min. 12. respondet. n. h. partium. 11. minuti. 27. linea. n. h. ad. o. n. tanquam partium. 100000. respondet part. 45436. tanquam sinus anguli. n. o. h. qui angulus erit gra. 27. minut. 1. subtracto postea hoc angulo ab angulo. c. o. i. graduum. 40. minut. 55. remanebit angulus. n. o. i. graduum. 13. minut. 54. inter lineam contingentiam, & lineam primae stationis in eiusmodi situ. Et ideo Mars accesserit ad lineam. o. c. veri motus epicycli. Sed quia linea. o. i. contingentiam, propter motum centri epicycli, in dictis diebus. 69. confecerit gradus. 36. minut. 13. (praesupposita semper eadem distantia. o. c. quamuis nonnulla sit differentia, quam nunc praetermittimus) & Mars in dicto tempore retrogressus fuerit per dictum angulum gra. 13. mi. 54. quibus deductis, ex. 36. & min. 13. restabunt gra. 22. min. 19. itaque in diebus. 69. Mars promotus fuerit a primo situ gra. 22. min. 19. aut circiter, prius quam retrogradatio eius incipiat esse appa-

rens.  
Nunc a prima statione vsque ad lineam veri motus epicycli sunt gra. 16. min. 51. ipsius epicycli, ut supra vidim<sup>9</sup> quos Mars transit in diebus 36. aut circiter ad rationem min. 28. in singulos dies, quo tempore centrum epicycli, in tali distantia a centro mundi conficeret gra. 18. mi. 54. ad rationem min. 31. cum dimidio in singulos dies, quibus deductis ex gra. 27. min. 1. anguli



guli. c. o. n. remanebunt gra. 8. min. 7. pro numero dimidiæ retrogradationis quum Mars erit in linea. o. c. veri motus epicycli. Quibus gradibus. 8. min. 7. subtractis à grad. 22. minu. 19. per quos Mars progressus erat, supererunt grad. 14. minut. 12. quibus ipse in media retrogradatione existens in linea. o. c. veri motus epicycli promotus erit à principio primi situs. Quod cum eò concordat quod supra diximus circa hos gradus. 14. min. 12. ultra primum situm in spatio dierum. 105. vt supra, ad tantum enim ascendunt. 69. et. 36. nunc fingendo q̄ Mars pergat in suo motu compositio qui constat ex his duobus circulis, vi eccentrici, & epicycli (quanquam vt dixi omitimus illam summam subtilitatem seu scrupulositatem còtinuæ inæqualitatis distantia centri epicycli à centro mundi, & præterimus etiam irregularitatē eius circa centrum mundi, propter regularitatem eius circa centrum æquantis, atque etiam missum facimus motum epicycli recti à sua media auge) fingendo inquam, quod Mars dicto motu suo pergat, vsque ad punctum secundæ stationis, præteribunt alij dies. 36. vt prius, quibus iunctis cum. 105. fiet summa dierum. 141. & Mars retrogressus erit per alios grad. 8. minut. 7. quibus subtractis à gradibus. 14. min. 12. per quos progressus erat supererunt. gra. 6. min. 5. quibus ipse Mars in fine suæ retrogradationis promotus erit à primo loco vnde moueri cœpit. Iner hanc igitur secundam stationem lineæ. o. u. d. & lineam. o. t. secundæ contingentia, Mars diebus. 69. vt prius, confecerit gradus. 32. min. 14. sui epicycli, & eodem tempore linea contingens. o. t. ambulauerit gra. 36. min. 13. vt prius, à quo itinere subtracto angulo secundo. d. o. t. graduum. 13. min. 54. supererunt gra. 22. min. 19. vt prius, quos Mars ambulauerit directè & apparenter, quibus additis ad gra. 6. minut. 5. quibus Mars progressus erat à principio motus, fient gra. 28. min. circiter. 24. quibus processerit à priori loco in diebus. 177. idest in. 141. et. 36. qui sunt ferè menses. 6. Itaque Mars partim sursum partim, deorsum ambulans detentus erit mēsisibus 6. in grad. 28. Zodiaci, atq; si finxerimus quod epicyclus moueatur versus oppositum augis, longior erit mora planetæ in eiusmodi duodecenario, propter augmentum æquationis argumenti. Itaque probata à nobis est possibilitas huius moræ Martis. Quod quidem mihi sufficere videtur nõ modo tibi, sed etiam cuius, qui harum scientiarum principia teneat. Neque enim nunc docere volo eos qui

in



in ijs sunt consumati, nec curam mihi suscipere erudiendi imperitos. Satis igitur sit ostendisse, quod qui scripsit Martem commorari posse tam multos menses in vno signo, non impossibilem rem tradidit. Immo per observationes huius veritatis milles factas, Astrologi fecere supradictas suppositiones necessarias ad reducendum in suas causas, & ad regulam, eiusmodi verissimos effectus.

Non oportebat autem scriptorem harum animadversionum tantopere eiusmodi mora commoueri, sed cogitare quod fortasse calculi facti fuerunt eo tempore quo miser Mars à Vulcano rete vincitus erat. Vnde cum non ita celeriter se expedire posset iter eius segnius peractum fuit. Aut quod quum vulneratus fuit in bello Troiano, vis eius & agilitas per aliquantulum temporis imminuta fuit. Atque si hic etiam intellexisset cum aliquando fuisse in potestate Othi, & Ephialtis vincitum & carceri inclusum menses tredecim, dum ab Eribea solutus fuit, ut tu, antiquos sequens, eleganter scribis in illis tuis pulcherimis dialogis. non existimasset, credo, tam absurdum quod alius eum detinisset sex aut septem mensibus, sed operam dedisset ut te intelligeret quid sibi vellet tam longa captiuitas.

Sed ut ad rem redeamus. Idem pag. 4. ait, quod verus motus Martis distat à medio circiter dies. 8. supponens medium motum esse dierum. 683. quod etiam falsum est. Sed utcunque sit, fallitur. Solet enim periodus veri motus Martis esse dierum circiter. 708. modo paulo plus, modo paulo minus, & interdum potest etiam esse multo breuior, sicuti erit à die. 3. Decembris anni. 1593. vsq; ad initium Iunij. 1595. Tunc enim erit tantum dierum. 545. & non quidem sine ratione, nam dicto initio Decembris Mars paulo ante cæperit esse directus, cum centrum epicycli erit circa medium Tauri, & eius stella in principio Arietis & initio Iunij. 1595. Mars parum distabit ab initio retrogradationis, regressus tamen ad initium ipsius Arietis, & centrum epicycli erit circa medium Aquarij, in cuius signi medio, hac ætate reperitur oppositum augis, & in quo situ, æquationes argumèti sunt, quam maxime esse possint, quum centrum epicycli circuiuerit solum circiter tres quartas totius ambitus, & Mars circuiuerit per partem superiorem epicycli circiter gradus. 252. Hoc autem dico, ut ostendam possibilitatem huius eius extraordinariæ velocitatis. Nam quicumque voluerit poterit certior fieri, per calculum partium motus Martis.

Vbi autem postea idem author miratur interualla, quæ ponuntur inter coniunctiones Iouis, & Martis in eodem signo, eaquæ vocat errores maximos, ostendit se non rectè considerasse motus eorum. Et præcipuè primum miratur quod inter annum. 1528. et. 1553. Iupiter & Mars nunquam coeant in Leone, cum hæc duæ coniunctiones inter se distent ann. 25. afferens pro ratione, quod hæc duo sidera, altero quoque anno coniunguntur, sic dicens.

» Qui sciet has duas stellas secundo quoque anno inter se coniungendas, mirabitur  
» quomodo non poterunt enumeratores, huiusmodi animaduertere errores.

Et præter hanc rationem fortasse et considerauit, quod in dicto temporis interuallo Iupiter sæpe fuit in Leone, ut ann. 1540. et. 1541. Mars autem in eo sæpe fuit. Vnde impossibile euidentur eos non conuenisse in dicto signo. Idemquæ dici potest de alijs coniunctionibus eorundem planetarum, atque has differentias temporum inter dictas coniunctiones ipse tribuit erroribus calculorum Ephemeridum, non autem tabularum, ut supra dixit, sed nescio quare vellet dictos planetas coire in Leone, si quum Iupiter in eo erat anno. 1540. et. 1541. & in eo deambulabat, Mars interea erat modo in Libra, modo in Scorpione, Sagittario, Capricorno, & alijs signis vsq; ad Cancrum, in quo cum repertus fuit anno. 1541. cogitans congruere cum Ioue in Leone,

ne, comperit eum inde aufugisse. Idq; fortasse, Iupiter data opera fecit, vt huiusmodi Astrologos in admirationem induceret.

Idem dico de alijs coniunctionibus horum duorum.

Quod postea ait, eos secundo quoque anno coniungi, animaduertendum est, quæ (vt iã dixi) duæ sunt species coniunctionum, quarum vna est linearum eorum mediocrium motuum, altera corporum eorum, saltem in longitudine, cum ambo inueniuntur in eodem circulo, qui transit per polos ecclipticæ, nam eos inueniri in eadẽ linea recta trãscũte per centrum mundi, rarissimum est. Atque coniunctio supra dictarum linearum vocatur mediã, & inter Iouem & Martem fieri solet spatio dierũ 816. cum dimidio, aut circiter. Altera dicitur vera, siue apprensã, & irregularissima, quæ quidem non seruat tempus determinatum. Quare quamuis altero quoque anno coniungantur, & Iupiter duodenis annis transeat per totum Zodiacum, non ideo necesse est, vt in spatio 24. annorum coniungantur in singulis signis, nunquam in eo deficientes, vt ipse credit loquens de veris coniunctionibus apparentibus, eo quod sint irregularissima, vt dixi.

Atque si quis velit inuenire periodum coniunctionum mediocrium horum duorum planetarum, ita faciendum erit. Sumat periodum motus mediocri Iouis, quæ est dierum. 4328. & Martis, quæ est dierum. 687. in quo tempore Martis, Iupiter ambulat gra. 57. min. 8. & diebus. 30. conficit. grad. 2. minut. 29. & secun. 23. ad rationem gra. 360. in diebus. 4328. Mars verò ad rationem graduũ. 360. in diebus. 687. singulis. 30. diebus conficit. gra. 15. mi. 43. secũ. 14. vnde differentia inter eos est graduum. 13. mi. 15. secũ. 51. per quam diuidendo productum graduum. 57. min. 8. in dies. 30. obueniẽt dies. 129. & duæ tertiæ. quibus addendo periodum Martis fient. 816. cum dimidio, aut circiter. Atque hæc est periodus infallibilis mediocrium coniunctionum Iouis cum Marte.

Nunc venientes ad tabulas Animaduersionum, videbimus hæc mirabilia eius, in quo consistant & vbi sint tam multi insignes errores.

Primum igitur neminem latet quod calculus Saturni, à Leouitio editus, difert à calculo Stadij circiter gra. 2. aut. 3. cum Leouitius faciat eum progredi per tantum in teruallum, modo plus, modo minus, & similiter Iouem. sed longe minori differentia, & sepe gra. 1. minus, atque in alijs planetis differunt, modo plus, modo minus. Huic igitur mirũ videtur, quod vnus ex his calculatoribus detineat Saturnum pluribus mensibus in vno signo, & alter in alio, non animaduertens dictam differentiam esse eius rei causam. Miratur item, quod vnus ex is faciat Saturnum morari paucis mensibus in vno signo, alter vero eum ibi detineat integris annis. Vt exempli gratia, versus finem suæ tabulæ Saturni, dicit quod Leouitius eum carceri includit in geminis annis. 2. mense vno, & diebus. 9. Stadius vero clementior eum liberat intra menses. 3. & dies. 14. Sed hic non cogitat, quod Stadius facit eum ingredi in geminos anno. 1559. diẽ. 10. Iunii, & ambulare directum vsque ad diẽ. 6. Septembris, eiusdem anni gra. 6. min. 34. eumq; postea retrogradum inde exire die. 22. Decemb. eiusdem anni, cum ingreditur in Taurum, vbi partim retrogradus, & partim directus manet vsque ad diẽ. 20. Februa. 1560. rediens postea in geminos, in quibus manet vsque ad diẽ Iunii. 1561. & inde ingreditur in Cancrum, ambulatq; directus. gra. 4. min. 59. vsque ad diẽ. 4. Octob. Vnde retrogradus rursus intrat in Geminos die. 28. Decemb. eiusdem anni, atque ibi partim retrogradus partim directus manet vsque ad diẽ. 12. Apr. 1562. itaque in pluribus vicibus facit eum morari in Geminis dies circiter. 816. idest circiter menses. 27. sumpsit autem hic scriptor breuissimam



uiffimam moram caufa comparationis cum calculo Leouitij, vt faceret differentiam apparere maiorem. Tamen in quouis dictorum temporum nunquam inuenietur Leouitius differre à Stadio plus gradibus tribus integris. Idem fecit in multis alijs locis dictorum virorum eos conferens tum in Saturno tum in Ioue, & Marte, putās magnum esse errorem, q̄ planeta non perambulet totum signum, in quod est ingressus vel directus vel totum retrogradus. Atque hæc opinio similis est superiori de coniunctionibus veris Saturni, & Iouis, vbi dicit quod nunquam coniunguntur in vno signo alterius triplicitatis, nisi perferat coniunctionem in omnibus signis primæ triplicitatis. Verum vt superfedeam vltius disputare, mihi videtur, quod hæc tunc dixi, posse tibi satisfacere, quod attinet ad sciendam sententiam meam super dictis Animaduersionibus latinè scriptis. Hoc tamen non prætermittam, q̄ hic non animaduertit, nepe q̄ differentia locorum planetarū quæ sūt inter ephemeridas Leouitij & Stadij, euenere, quia vnus supputat cū radicibus, & fundamentis Alfonso alter verò Reinoldi ex Copernico recentius obseruatis, ita idem euenire poterit futuris temporibus, si supputati fuerint dicti motus, & loci cum recentioribus obseruationibus cum impossibile sit tam subtiliter, tanque perfectè supputare loca & motus eorum, vt longo interuallo temporis non comperiantur in eis aliqua differentia, cuius rei remedium est semper sequi recentiores obseruationes & tabulas.

Atque vt tibi satisfaciam etiam circa alia scripta vulgari lingua edita mensibus. 4 post latina, etsi intelligere potes, qualia possint esse alia eius scripta, ex ijs quæ supra dicta sunt, atque etiam ex eo, quod dicit se misisse multa exempla suarum Animaduersionum in varias terras, illis qui profitentur has scientias, aut earum studiosi sunt, nec quenquam inuenisse qui ad tā laudabilem prouinciam motus sit, nec vidisse, q̄ aliquis responderit eius rationibus, laudabilem prouinciam, autem puto, q̄ intelligat correctionem ephemeridum, verens, ne culpa calculatorum, qui eas sumpserunt e tabulis, tam differentes sint, vt quibusdā locis cap. 1. Videtur, & præcipue vbi sic ait.

» Perche essendo impossibile alli studiosi di dette scientia di non seruirsi delle  
 » ephemeridi, maggiormente a quelli che non fanno seruirsi delle tauole, e cono-  
 » scendo d'incorrere in errori senza hauerui altro rimedio, farebbono forzati di ab-  
 » bandonare i studij loro.

Quamquam circa finem dicti capituli redeat in meliorem viam & aduersetur sibi-  
 » bipsi vbi sic ait.

» Che poi essi possessori della scienza, &c.

Etiam aperiam tibi, quæ mea sit de ijs sententia.

Hic igitur in scriptis Italicis, vt morderet aliquem ex ijs, qui eius superiora scripta non laudauerant, occasionem capit aperiendi aliquos illius errores, per editionem collationis quorundam calculorum a se collectorum illius, atque etiam aliorum, cuius calculi sunt in secunda, & septima figura. Sed prius quam veniamus ad defensionem harum duarum figurarum vide obsecro quam alienum ei videatur, quod alij dixerint differentiam ephemeridum non esse magni momenti, non afferens respectum vllum, qui enim dixerunt eiusmodi differentiam non esse magni momenti id dixerunt habito respectu ad signum in quo est planeta, vt (exempli gratia) quamuis in ponendo loco Saturni Leouitius interdum differat à Stadio gradibus. 3. quum vterque cum ponat in eodem signo, tunc id nullius momenti est, & sic in coniunctionibus aut alijs aspectibus duo, aut. 3. gradus non faciunt alterationem sensibilem, cum virtus coniunctionum, & aspectuum insit, & duret per multos gradus ante aut post ipsum punctum. Nec quicquam tamè est qui dubitet, quin prætare scire subtiliter ipsum punctum. Nec vnquam fuit aliquis qui negauerit re-

ferre vt anni directionum correfpondeant gradibus æquatoris. Et præterea in ephemeridibus videntur certè motus & afpectus luminarium, quamuis in fit differètia minorum. Nam non differunt gradibus, præter fitum parum diftantem à vero omnium planetarum, quorum cognitio in cœlo, quamuis circa eorum locum error effer gra. 10. tamen in hoc prodeffet, & tempus afpectus eorum, etfi non diei præcifè, quia influentia eiuſmodi afpectuum, præterquam Lunæ durat multis diebus, & non vno tantum. præterquam quod ipfæ ephemerides oftendūt nobis tempus ecclipſiū, in quo certè non differunt nec diebus nec multis horis, & itidem multa alia.

Non ſunt igitur contemnendæ ephemerides, nec habendæ pro re nullius pretij, vt hic ait.

Quod attinet ad illa alia, quæ hic vocat errores ephemeridum, tam de apparenti coniunctione Saturni cum Ioue in ſignis alterius triplicitatis prius quam peregerit præcedentem, quam de faciendo currere Saturnū, & de retinendo Ioue, de detinendo Marte. 6. aut. 7. menſibus in vno ſigno, de Marte, & Ioue non coeuntibus ſingulis. 24. annis in quolibet ſigno, & eius generis alia, minime verum eſt quod ſint errores, quamuis huic præbuerint occaſionem toties errandi.

Comparatio poſtea inter eius calculos ſumptos partim ex tabulis Iunctini, & partim ex ephemeridibus Stadij tanquam calculis Copernici, & calculos figurarum ſuper eis poſitarum ſupputatarum à diuerſis per ephemeridas Alfonſinas, etiam propoſita ab eo eſt ad oftendendum magnam & monſtruoſam differentiam, vt ait cap. 2. vbi miratur, quod cum ex communi ſententia calculi Copernici meliõres ſint, calculatores dictarum figurarum potius eos ſumpferint à tabulis Alfonſi, quam Copernici. Quæ admiratio quam aliena ſit, conſiderandum permittam cuius intelligenti harum facultatum, cum ſæpe accidere poſſit. vt cum aliquis velit ſcire ſolum vniuerſalia alicuius geneſis, ſiue natiuitatis, cum non inueniantur ephemerides Copernici, ſed tantum Alfonſi, calculator vtatur tantum ephemeridibus, quas inuenit, tū cauſa vitandi tædij calculi tabularum, qui magni laboris eſt, præcipuè in tabulis Prutenicis Reinoldi. tum quia ſuperflua ei eſt ſumma ſubtilitas, cum non curet laborare circa directiones vt factū eſt pro ſecūda figura ab hoc propoſita, quæ erat anni. 1551 quo non inueniebatur ephemerides Copernicæ, quæ non editæ ſunt antè annum 1554. præterquam quod ille nobilis vir pro quo ſupputata fuit dicta ſecunda natiuitas dubitabat de anno, vt hic ſimiliter ſcit. quare potuiſſet perdi tempus, & labor, ſi ſupputata fuiſſet per tabulas Reinoldi, nam Iunctini tabulæ nondum editæ fuerant. Calculus poſtea ſeptimæ figuræ, qui erat reuolutio dictæ ſecundæ natiuitatis, duab<sup>9</sup> de cauſis non factus eſt per tabulas prutenicas, primum, quia eius anni. 1580. non inueniebantur amplius ephemerides Copernicæ. Ephemerides enim Stadij incipiètes ab ann. 1554. deſinūt ann. 1576. & cõtinuatæ poſtea quæ perueniunt vſque ad annum. 1600. non peruenere ad manus calculatoris ante hunc annum. 1581. Altera ratio eſt, quia in reuolutio nibus, quoniam in eis non ſunt directiones, non ponuntur à doctis, ne minuta quidem. quare non ſolum non curant eas ſupputare per tabulas, ſed nec exquisitè quidem per ephemeridas. Calculi poſtea ab hoc ſumpti ex tabulis Iunctini, & poſiti ſub dicta ſecunda figura, adeò rectè facti ſunt, vt cum ſecundum ipſas tabulas oporteat Saturnum eſſe circa. 32. minutum gradus. 23. Aquarij, ipſe eum ſcribat in gra. 11. mi. 3. dicti ſigni. Iupiter ſimiliter qui ſecundum dictas tabulas inuenitur circa finem gradus. 5. Cancri, ab eo ponitur in min. 28. gra. 19. eiuſdem. ex quibus planetis Saturnus in figura poſitus eſt in min. 27. grad. 23. Arietis, Iupiter autem in min. 3. gra. 6. Cancri, Vnde ſecundum verum, inter calculum Alfonſi, &

fi & Iunctini in Saturno non erat differentia plus quam minu. 5. & in Ioue min. 4. sed secundum calculum huius in Saturno fuisset differentia gra. 11. minu. 54. & in Ioue grad. 13. minu. 35. Atque hæ sunt quidem differentia magnæ, & monstruosæ vt ipse eas vocat, vt etiam est illa Veneris, & Mercurij inter tertiam figuram, & eius calculum sumptum, non quidem à tabulis laboriosis, sed à simplicibus ephemeridibus Stadij, quæ differentia nõ est quidem paucorum graduum, cum sit tertiæ partis coeli in quolibet dictorum planetarum. Huiusmodiq; monstra certè non sunt orta à tabulis siue ephemeridibus diuersis, sed sunt partus huius authoris.

Pergens postea assidue bonus hic vir hominibus dare specimen doctrinæ suæ aperiendo (vt conatur) aliorum errores, proponit duas differentias inter primam figuram, & suum calculum suppositum Saturni, & Iouis. Primum de Saturno ait, qd cum differentia sit gra. 1. min. 30. ostendit in directione, qd accidens sit euenturum anno vno, & mensibus sex ante, aut post, quasi eiusmodi differentia esset partium æquatoris, sicuti est partium Zodiaci. Idem dico de differentia Iouis. Quod quidem, manifestum est inditium scientiæ suæ, & quantum ea intelligat de quibus loquitur.

Quod postea attinet ad differentiam inter Copernicum & Alphonsum, circa Solè, nullus est harum scientiarum peritus, qui id nesciat, & similiter de differentia situs cæli in reuolutionibus annuis.

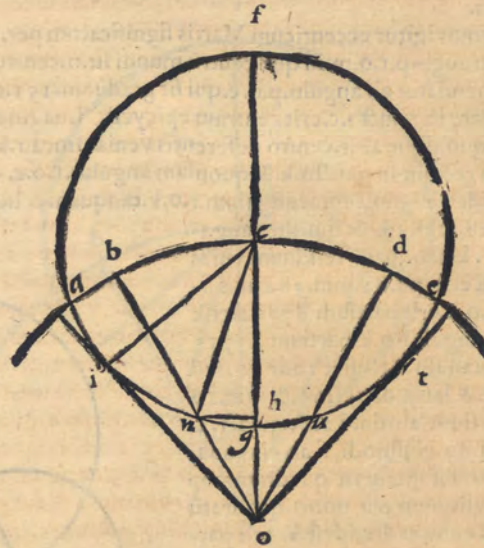
Quod vero ait septimam figuram malè supputatâ fuisse, si non est maximus certè non est minimus monstruosorum eius errorum. Vbi itidem videri potest, quam alienus hic sit ab hac scientia. Nam si saltem curasset sibi ab aliquo supputandum locum Solis per tabulas Alphonfi in instanti minorum. 36. pomeridianorum, certior factus esset quod in illo puncto Sol inueniebatur in minu. 54. grad. 11. Geminorum, id est præterierat gra. 10. cum min. 54. vel si curasset sibi inueniendum tempus, per dictas tabulas cum grad. 10. min. 54. Geminorum vt faciendum est, sequendo tamè Alphonsum, & non per calculum Solis positum in ephemeridibus, vt parum periti facere solent, vidisset qd inuenta essent min. 36. pomeridiana. Leuis tamen occasio huic fuit suspicandi eiusmodi tempus esse falsum, quod viderit in illa figura Solè positum esse cum gra. 11. & non cum gra. 10. min. 54. non animaduertens ita notatum fuisse Solem, vt omnes alios planetas, scilicet sine minutis, quum, vt dixi, in reuolutionibus non adhibeatur tanta scrupulositas.

Quod deinde ait, in illa figura Solem positum esse in decima domo, & non in. 9. id relinquam iudicio eorum qui sciunt numerare domos, saltè posuisset autoritate sua Solem in dicta decima diuersè ab exemplo ei dato ab amico, vt ostenderet se dicere verum, vt in secunda figura discrepat ab ipso exemplo in collocando Leone, Virgine, & Libra, & Scorpio, quos malè locauit, & si alii bene se habent.

Atque quod hæcenus à me dictum est, satis sit ad intelligendum quale sit reliquæ dictæ eius disputationis. Si enim velim pergere notare omnia eius errorum loca, esset mihi inanis labor, & tibi nimia molestia. Et quamuis non defuerint præstantissimi viri, qui visis eius scriptis familiariter eum monuere, & tu ipse, vt audiui, cum instrumèto theoreticè in manib; ei ostèderis quo mò Mars possit morari amplius sex mensibus in vno signo, & præterea cum iam ab initio Taurinum aduenit, mecum communicauerit illa sua prima scripta, egoq; eum monuerim, quod in varijs reb; fallebatur, dissuasèrimq; ne ea imprimenda curaret, quia nullum honorem inde referret, eum hortans, vt potius alijs rebus operam daret, atque ei dixerim quod ad animaduersiones differentiarum, ephemeridum attinet, quod id iam oēs animaduertent. Mibi respondit se decreuisse illa edere, vt postea fecit, & tot admonitionibus non

æq̄ uescens, die. 11. Augusti edidit chartam illam impressam inuitans ad disputationem quotquot adhererent contrariæ sententiæ, volens sustinere Martē non posse commorari in vno signo amplius duobus mensibus, supponens partem principiorum ab omnibus admittorū, & in fine paginæ exponens modum, quo utitur ad probationem suæ intentionis. Puto autem quod secum ratiocinabatur de Marte, ut fecit de Saturno in scripto latino, hoc modo. Si Mars in duobus annis ambulat per omnia. 12. signa, necesse est igitur, ut in mensibus duobus ambulet per vnum signum, cum menses. 2. sint duodecima pars annorum duorum. Sed ibi statim in ipso initio committit errorem graduum ferē. 7. dicens, quod mediū motus Martis inueniebatur signorum. 4. & gra. 17. cum eo tempore dictus mediū motus non esset reuera plus quam sign. 4. grad. 10. mi. 36. verum hoc ad ea, quæ sequuntur exigui est momenti. Is postea particulatim colligit mediū motum Martis ad diem. 29. Mai anni. 1514. quem ait esse signorum. 9. gra. 27. minu. 53. & tamen reuera erat tantū signorum. 9. gra. 21. mi. 29. sed missum faciamus etiam hunc errorem tanquā à primo pendentem. Cum deinde ibidem ponit centrum epicycli, similiter errat, nam centrum epicycli nunquam poni debet ubi est linea mediū motus, nisi sit in auge, aut in opposito augis eccentrici, quia debebat collocare ipsum centrum tāto post lineā mediū motus, quanta erat æquatio centri, quia mediū centrum Martis tunc erat minus signis sex, & aux eccentrici eius erat in sexto minuto grad. 16. Leonis. Tamen hoc etiam leue est. Præsupponamus igitur quod centrum epicycli esset in grad. 28. Capricorni, ut ipse credidit, id est gradibus. 7. vltius quam erat reuera. Ait postea se comperisse Martem ambulasse signa. 4. & grad. 22. eius epicycli, sed non explicat an intelligat de argumento medio, an de vero, quod vocatur æquatū, nam si intelligatur de medio, hoc esse non potest, cum mediū esset signorum. 4. gra. 24. mi. 35. sed si intelligatur de vero, ut iure credendum est (alioquin etiam errasset) certe falsum est. Nā, verum, erat signorum. 4. grad. 29. minu. 39. Itaque Mars non distabat à linea veri motus epicycli amplius gradibus. 30. & minu. 21. ipsius epicycli, & æquatio argumenti secundo correctā erat gra. 44. minu. 2. à quo subtracta æquatio ne centri, quæ erat gr. 5. minu. 4. (cum centrum epicycli deberet tanto spacio esse post lineam mediū motus quantum supra dixi) supererant gra. 38. minu. 58. addendi gradibus, & minu. mediū motus, qui cum reuera essent grad. 21. & minu. 29. Capricorni, perueniebant ad minu. 27. grad. 1. Piscium. Sed præsupposito secundū ipsum, quod mediū motus esset grad. 28. Capricorni, & quod Mars esset non solū ubi hic ait, sed etiam in prima linea contingentia epicycli, id est in prima linea maximæ æquationis argumenti, & præsupposito etiam quod dicta æquatio esset æqualis illi, quam haberet ad mediū Aquarij scilicet grad. 47. quum centrum epicycli est in opposito augis, manifestum est, quod eiusmodi linea contingentia non tranfieret vltra grad. 15. Piscium, & tamen hic ait, quod linea veri motus Martis vadit ad grad. 16. Arietis. vnde oporteret, quod æquatio argumenti esset plus quam grad. 78. Quod si verum esset, & o. c. etiam esset partium. 54. secundum distantiam proximiorē centro mundi, semidiameter epicycli esset eiusmodi partium. 52. minut. 49. & quum Mars esset in. g. id est in opposito veræ augis epicycli, dum centrum epicycli esset in eiusmodi distantia à terra, distantia. o. g. id est à terra ad Martem non esset plus, quam vna sola pars ex dictis, cum minut. 11. cum partes. 52. minut. 49. ad. 54. sint ut sinus anguli gra. 78. qui est partium. 97814. ad sinum totalem partium 100000. Nam iam supra dixi, quod triangulus. o. c. i. est rectangulus. Hinc sequetur

retur, quod in interuallo. o. g. vni<sup>9</sup> partis, & min. 11. respectu. o. c. partium. 54. Collocaretur semidiameter terrę cum spissitudine aeris, ignis, cęlorum Lunę, Mercurij, Veneris, & Solis, præterquam quod vt inter Solem, & terram sunt circa 605. diametri ipsius terrę, inter terram, & Martem cum esset in auge sui epicycli, & epicyclus in auge eccentrici, inuenirentur circa. 60000. diametri eiusdem terrę,



& tamen ea distantia siue interuallum non potest continere. 5000. diametri terrę. Et quod plus est, hic tam vastum facit hunc suum epicyclum, vt ambiente Marte per inferiorem eius partem, necesse ei esset manere in vno duodecatemorio multo plus quam. 7. aut. 8. mens. vnde hic multo magis miraretur quam prius. Hinc cernere licet quam rectè facti sint hi eius calculi.

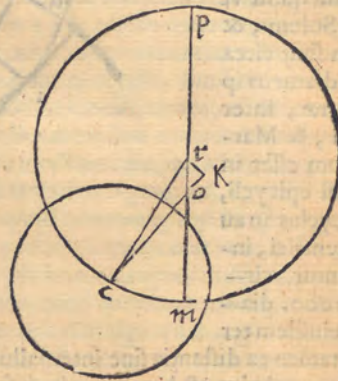
Vt autem etiam hinc aliqua vtilitas capiatur (prætermisissis inconuenientibus vna cum falsis suppositis huius) Videamus ordine scientifico vbi poterat esse verus locus Martis, aut vero proximus, die. 29. Mai anni. 1514. quem hic exempli causa sumit. Idq; tam ad defensionem tabularum Alfonsi, quam ephemeridum ex eis collectarum. quę quidem exactę sunt, vt quisque peritus facile videre poterit, non autem calculatę à tam stupidis hominibus, vt à vero aberrant etiam gradibus. 46. vt hic ait se depræhendisse.

Primum igitur supponemus eosdem illos terminos, quos ipse nec debet, nec potest negare, præter ea quę supra supposita sunt, nempe quod semidiameter epicycli sit partium. 39. minu. 30. & eccentricitas partium. 6. talium qualium est semidiameter deferentis diuisus in. 60. & quod dicto tempore aux eccentrici Martis esset circa minutum. 5. grad. 16. Leonis, scilicet graduum. 135. min. 5. & quod linea motus medioeris esset circa minu. 30. gradus. 22. Capricorni, & quod verum centrū Martis esset grad. 151. minut. 20. & quod argumentum verum esset grad. 149. minu. 39. atq; ita ostendam, neque tabulas, neque ephemerides errare, ne quidem vno gradu, ac ne quidem multis minutis, non modò tam monstruosa differentia, vt ipse ait.

Quare primum nobis scientificè inueniendum est, quanta esset distantia. o. c. præ-

præcise idest interuallum inter centrum mundi, & centrum epicycli Martis in huiusmodi situ.

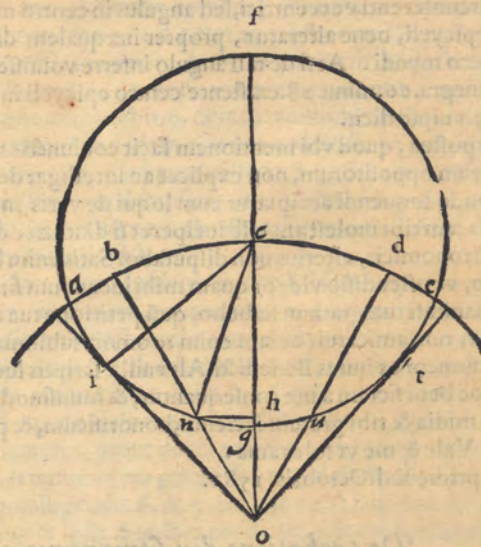
Fingemus igitur eccentricum Martis significatum per. p. c. m. cuius centrum sit. r. & lineam augis. p. r. o. m. in qua centrū mundi ut. o. centrum autem verum epicycli, comprehendatur ab angulo. p. o. c. qui sit graduum. 151. min. 30. secundum suppositum. Quare in puncto. c. erit centrum epicycli. Imaginemur ergo. c. o. productam à parte. o. quousque ab. r. centro deferentis veniat linea. r. k. perpendiculariter, faciens angulum rectum in puncto. k. & quoniam angulus. r. o. c. datur nobis graduum. 151. min. 30. ideo cognoscemus angulum. r. o. k. tanquam reliquum ex duobus rectis, qui erit gra. 28. min. 30. & similiter angulum. o. r. k. tanquam residuum vnius recti, qui erit gra. 61. min. 30. cuius sinus idest. o. k. erit partium 87881. et. k. r. vt sinus anguli. r. o. k. partium. 47715 talium qualium. o. r. esset 100000. sed vt. o. r. est. 6. latus. o. k. erit. 5. & min. 16 et. r. k. partium. 2. min. 52. & quia. r. c. est partiū. 60. eiusmodi, si ab eius quadrato subtractum fuerit quadratum ipsius. r. k. reliquum erit nobis quadratū ipsius. k. c. cuius radix, idest. k. erit partium. 59. min. 56. à qua. c. k. subtrahendo postea. k. o. partium. 5. minu. 16. remanebit. o. c. partium. 54. min. 40. pro distantia quæsitā.



Fingamus postea epicyclum. f. n. g. in quo argumentum verum graduum. 149. minu. 39. sit arcus. f. n. vbi Mars inueniatur in. n. per quem punctum transeat linea. o. n. veri motus Martis. Deinde inueniamus angulum. c. o. n. æquationis argumenti, modo iam dicto, idest ducendo sinum. n. h. arcus. n. g. qui arcus tanquam reliquus argumenti veri, iam præsuppositi, ex dimidio circulo, erit graduum. 30. minu. 21. & n. h. eius sinus partium. 50528. sinus similiter anguli. n. c. h. et. c. h. tanquam sinus anguli. c. n. h. restantis ex uno recto grad. 59. minu. 39. erit partium. 85295. taliū qualium. c. n. sinus totus esset partium. 100000. sed vt partium. 39. & min. 30. sinus. c. h. erit partium. 34. min. 5. et. n. h. partium. 19. mi. 57. reliquum postea. h. o. ex. o. c. partium. 20. min. 35. quia iam supra inuenimus. o. c. esse partium eiusmodi. 54. minu. 40. vnde. o. n. vt radix quadrata summæ duorum. n. h. et. h. o. erit partium. 28. minu. 41. talium qualium. n. h. inuenta fuit partium. 19. min. 57. quæ. n. h. erit postea partium, 69552. talium qualium. n. o. partium. 100000. & sumpta dicta. n. h. vt sinus dictarum partium, dabit nobis angulum. n. o. h. quæsitum gra. 44. min. 4. qui per tabulas Alfonso inuentus est gra. 44. min. 2. par huic, vt dici potest. Qui angulus gra. 44. minu. 4. collectus cum angulo veri centri iam supposito graduum. 151. minu. 20. & cum angulo augis eccentrici Martis, similiter suppositæ grad. 135. min. 5. dabit nobis summam veræ distantie Martis à principio Arietis grad. 330. min. 29. quod aliud non significat, nisi quod Mars inuenietur in minu. 29. primi gradus Piscium. Et Stoflerus in suis ephemeridibus ponit eum in. 22. minuto dicti primi gradus, cuius diffe-

rentia à tabulis  
est minut. 5. tan  
tū, & à meo cal  
culo min. 7. vide  
licet minima.

Nunc autem  
nolui sumere ip  
sum angulum æ  
quationis à tabu  
lis propter duas  
rationes, primū  
quia ne hic qui  
dem repræhen  
sor in hoc voluit  
credere dictis ta  
bulis. Sed id vo  
luit videre pro  
prijs oculis i sua  
theorica Martis.  
Vbi iuenit quod  
linea. o. n. tranfit  
per gra. 16. Arie  
tis. Secunda ra  
tio est, vt videat



tur quod dictæ tabulæ rectè supputatæ sunt, super dictis suppositis.

Sed vt videat quantus sit medius motus Martis die. 29. Mai colligit frustatim, q̄  
elegantè colligere poterat, vna opera in columellis ipsius medij motus eiusmodi  
stellæ per eram eiusdem temporis, quæ erat. 2. primarum sexagenarum. 33. secunda  
rum. 32. tertiarum, et. 52. quartarum.

Primum deinde suppositum quod scribit, scilicet, quod diameter epicycli sum  
ptus in longitudine media sit signorum. 2. & grad. 19. vti superfluum est, ita etiam fal  
sum, nam eiusmodi diameter in dicto loco, non occupat ad centrum mundi plus quã  
gra. 66. min. 28. id est signa. 2. gra. 6. min. 28. quia proportio. o. c. ad semidiametrum  
epicycli in eiusmodi loco est ut partium. 60. minu. 18. ad partes. 39. min. 30. quæ duæ  
lineæ intellectu, vt latera vnus trianguli rectanguli, habebunt pro basi aliam lineam  
partium similibus. 72. mi. 5. Quæ intellecta vt sinus totus dabit semidiametrum epi  
cycli partium. 54798. tanquam sinum subiectum angulo gra. 33. min. 14. pro medie  
tate illius, quod quatitur.

Nec prætermittenda mihi videtur ratio, qua credere possumus, hunc cogi  
tasse, quod diameter epicycli compleat spatium duorum signorum cum gradibus.  
19. quæ quidem ratio alia esse non potest, nisi quod cum iste inuenerit, in comētarijs  
Theoricarum, semidiametrum huiusmodi epicycli esse partium. 39. min. 30. talium  
qualium sunt. 60. illæ quæ sunt semidiametri huius eccentrici, dictas igitur partes. 39  
mi. 30. hic putauit esse gradus Zodiaci, & propterea dixit diametrum huiusmodi epi  
ycli esse signorū duorū, & gra. 19. qui numerus. 79. duplus est numero. 39. cū dimidio  
hoc autem dixit accidere in longitudinibus medijs, quia si hic intellexisset de pro  
portione horum duorum diametrorum, quæ est ut. 120. ad. 79. non specificasset lo  
cum

cum epicycli, cum ipsa proportio nullo modo alteratur existente epicyclo vbi volueris ipsius circumferentiæ eccentrici, sed angulus in centro mundi, cui subiacet dictus diameter epicycli, bene alteratur, propter inæqualem distantiam centri epicycli ab ipso centro mundi. At si de tali angulo inferre voluisset, iam probavi ipsum continere solum gra. 66. minu. 28. existente centro epicycli in longitudinibus medijs & non gra. 79. vt ipse dicit.

Omitto postea, quod vbi mentionem facit coniunctionum Solis cum Marte augium & earum oppositorum, non explicat an intelligat de veris an de medijs. Nam si ex eius modo loquendi accipiat eum loqui de veris multum erraret.

Sed quia iam tibi molestum esse inciperet si diutius te detinere in his contentionibus astronomicis, vterius non disputabo. Satis enim hæcenus explicavi sententiam meam, vt ostendisse videor quam mihi iucundum sit tibi moré gerere. In quo etiam humanitati tuæ gratiam habebó, quæ petitione tua occasionem mihi de deris efficiendi, vt tum amici tui (amant enim te omnia sublimia ingenia) tum alij, si quæ falsam opinionem ex huius Benedicti Altræuillæ scriptis sumpserint, eam relinquant, & per te hoc beneficium à me consequantur, & huiusmodi occasionem, & iuandi hominum studia & tibi gratum faciendi, honorificum, & per gratum mihi fuisse intelligant. Vale & me vt soles ama.

Taurini pridie Kal. Octobris. 1581.

*De probatione diuisionis numerorum.*

A D E V N D E M.

**I**Nter alia quæ à me scire cupis, vir doctissime, hoc vnum est, vt ex literis tuis accepi, vnde sit vt prisca nostri probatione numeri nouenarij potius quam septenarij vbi fuerint, & qua ratione non idem proueniat ex probatione numerorum octonarij, senarij, vel quinarj, aut cuiuslibet alterius. Vnde pariter oriatur quod in partitionis probatione necessum sit probationum euentus multiplicare cum probatione diuisoris, ac eam quæ est producti postea cum probatione fractionis in summam colligere, &c. Ad hæc in primis respondeo, cum aliquoties accidere possit tales probationes nos fallere posse, id quod si in tali summa similis numerus, ut puta septem, aut nouem, plus vel minus æquo iustouè positus fuerit, attamen per raro euenire potest, vt quis per nouenarium potius quam per septenarium decipiatur. Exempli gratia, ponamus summam esse. 100. quam numerus nouenarius vndecies solum ingreditur, at septenarius quatuordecies, vnde quis septi<sup>9</sup> ex septenario, hac ratione, quam ex nouenario numero se posse errare facile depræhendet, et si ex probatione nouenarij magis quam septenarij, vt practici scribunt, duabus de causis errare possimus. Alia tamen ratio mihi suppetit, ob quam credibile est ipsos potius nouenario adiutos fuisse, quam septenario, quæ est ob sui cum velocitatem tum facilitatē, neque enim in septenario est adeo facilis. Nā quamuis, tam vna quam altera aliud non sit, quam numerorum ordines diuidere (si de summis primo loquamur) aut è summa superfluum ordinum colligere, & videre an idemmet superfluum ex eadem summa emanet; attamen cum modus, qui in hoc adhiberi potest in nouenario quæ in septenario velocior sit, & ob id probationem nouenarij seligunt potius quam septenarij.

Verum



Verum nolo te in ea, quæ falsa est, opinione consistere, non idem, & cum octonario, senario, vel quinario, aut quouis alio numero posse efficere, cum eadem met ratio, quæ in septenario, aut nouenario, et in cæteris perhibeatur. Ponamus exemplū hos tres ordinum numeros velle supputare, quorum primus sit. 679. secundus. 846. & tertius. 935. & illorum summā. 2460. nunc maiorem numerum primi ordinis ab octonario mensi, proijciendo, remanebit. 7. deinde maiorem numerum demendo à secundo ordine, residuum erit. 6. ac si idem in tertio ordine fecerimus, erit nobis reliquum. 7. Demum tria hæc residua in vnum collecta. 20. efficiunt, à quibus si numerum maiorem ab octonario mensum dempseris, supererunt. 4. & totidem à summa. 2460. remanebunt, reiecto maiori numero ab octonario menso. Atque idem medio quouis alio numero, euenire potest.

Cuius ratio tam per se clara atque euidentis est, quod si summam trium reliquorū, quæ est. 20. à summa. 2460. subduxeris, remanebunt. 2440. pro summa trium numerorum dictorum ordinum ab octonario mensorum, cui numero addito. 16. pro maiori numero summæ reliquorū, qui ab octonario mensus sit, supererunt. 4. At si per senariū experimētū feceris, remanebit. 0. & sic de reliquis per ordinem procededo.

Verum posses sciscitari, quare velocius, excessus ordinum, potius per nouenariū, quam per cæteros numeros, prout docet practici, inueniri queat, videlicet aggregando prius duas figuras numerorum primæ summæ, deinde alias duas. Exemplum sit primus ordo. 679. colligendo. 6. et. 7. faciunt 13. & cum hæc summa sit duarum figurarum, supputantur & ipsæ, è quibus prodeunt. 4. & consimilis erit probatio numeri. 67. facta per. 9. quod idem est, ac si quis diuidat. 67. per. 9. ex quo reliqui erunt semper. 4.

At quo ratio huiusce rei perspicuè dignosci possit, in primis sciendum est, cuique ex se cognitum, atque exploratum esse, denarium numerum vnitate nouenarium superare, & ex hoc sequitur, sex denarios continere in se sex nouenarios, & sex vnitates.

At sex vnitates, vna cum. 7. faciunt. 13. & quia in. 13. est denarius, igitur in illo erit vnitas supra. 9. Quæ vnitas addita ternario, præbet nobis superfluum, per quod. 67. superat. 54. iunctum cum. 9. scilicet summam. 63.

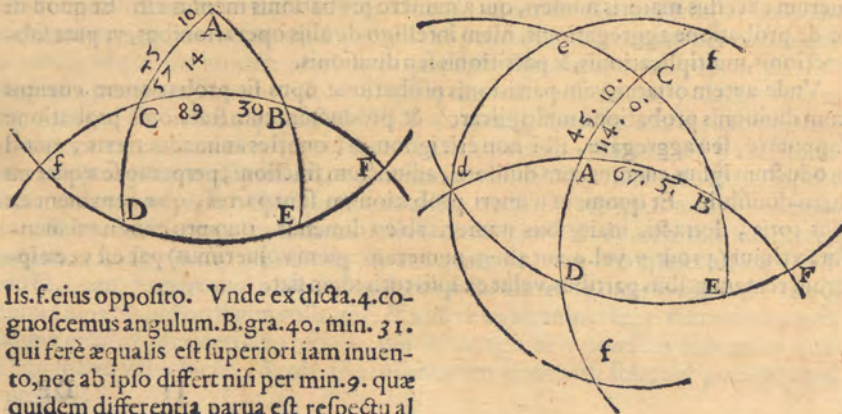
Idem dici non potest de octonario, septenario, vel senario, & de reliquis, quoniam numerus denariorum, in cæteris minoribus nouenario non præbet illico numerum excessus maioris numeri, qui à numero probationis mensus est. Et quod dico de probatione aggregationis, idem intelligo de alijs operationibus, vt puta subtractionis, multiplicationis, & partitionis seu diuisionis.

Vnde autem oriatur, vt in partitionis probatione opus sit probationem euentus cum diuisionis probatione multiplicare, & productum cum fractionis probatione supputare, seu aggregare, tibi non erit ignotum, quoties animaduertes, quod productum ipsius euentus cum diuisione, adiunctum fractioni, perpetuo se æquat numero diuisibili. Et quoniam numeri probationum sunt partes, quæ remanent ex ipsis totis, detractis maioribus numeris ab eo dimensis, quo pro communi mensura utimur (prout. 7. vel. 9. aut alium numerum, quem voluerimus) par est vt ex ipsarum remanentibus partibus, velut ex ipsis totis idem fiat.

*De falacia operationis triangulorum sphericorum.*

## A D E V N D E M.

QUod diebus præteris tibi significavi, idem nunc confirmo, scilicet sphericorum triangulorum operationem sæpe nos fallere, ut exempli gratia, si propositus nobis fuisset triangulus. A. B. C. cuius angulus. A. nobis datus esset graduum. 114. mi. 0. & eius latus. A. B. graduum. 67. min. 5. & latus. A. C. graduum. 45. mi. 10. si reliquos angulos cum tertio latere etiam cognoscere voluerimus, ex methodo. 11 primi Copernici propositum obtinebimus. unde latus. B. C. esset graduum. 89. min. 30. angulus vero. C. graduum. 57. min. 14. angulus autem. B. grad. 48. min. 38. Quare ultimus hic angulus. B. falsus esset, eo quod operatio parvorum triangulorum in causa est, quotiescunque eorum latera tam brevia sint, ut non excedant unum gradum, quare ipsorum angulorum veram quantitatem non tribuunt, propterea igitur cum voluerimus veram quantitatem ipsius anguli. B. oportet postquam inuenerimus angulum. C. mediante arcu. D. E. supponere alium polum in. B. deinde producere. B. A. vsque ad. d. et. B. C. vsque ad. e. imaginando. B. d. et. B. e. duas quartas esse magnorum circularum, extendendo postea. d. e. vsque ad intersectionem cum. A. C. & eundem ordinem prosequendo, tunc. e. d. nobis ostendet angulum. B. esse gra. 40. mi. 22 quæ erit eius vera quantitas. Cuius quidem rei experientiam possumus etiam facere, hoc modo, esto, exempli gratia, quod nobis datus sit angulus. C. graduum. 57. min. 14. cum latere. A. C. gra. 45. min. 10. & latus. B. C. gra. 89. min. 30. Tunc si ordinem. 11. dicti lib. sequemur, obtinebimus intentum, hoc modo scilicet supponendo in. A. polum, & non in. B. ducendo etiam. A. B. et. A. C. sed. A. B. vsq; ad gra. 90. ducendo postea. D. E. ita quod ab omni parte concurrat cum latere. B. C. producto, unde tam. f. C. B. F. quam. f. D. E. F. erunt semicirculi magnorum circularum. quare. C. D. nobis cognitus erit gra. 44. min. 50. & sic etiam angulus. D. C. f. gra. 57. min. 14. ex 4. dicti lib. postea habebimus. F. I. gra. 60. min. 54. & angulum. f. gra. 53. mi. 24. aggregatum postea. f. C. cum. C. B. habebimus. f. B. gra. 150. min. 24. qui si a semicirculo de prius fuerit, nobis remanebit. B. F. gra. 29. mi. 36. cum angulo. F. cognito cum sit æqua-



lis. f. eius opposito. Unde ex dicta. 4. cognoscemus angulum. B. gra. 40. min. 31. qui ferè æqualis est superiori iam inuento, nec ab ipso differt nisi per min. 9. quæ quidem differentia parua est respectu al-  
terius

terius differentia quam supra inuenerimus.

Superius enim dixi non esse ponendum polum in. B. eo quod. B. C. fit gra. 89. mi. 30. vnde nobis prodijisset triangulus. f. C. D. trium valde paruorum laterum, quorum latus. C. D. esset gra. 0. mi. 30. & latus. f. l. gra. 0. mi. 55. & latus. F. D. gra. 0. mi. 47. vnde angulus. f. gra. 32. min. 40. falsus esset, qui quidē postea nobis daret. D. E. gra. 45. minu. 16. falsum similiter.

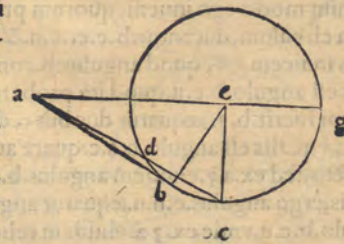
*De passione circuli hactenus incognita.*

A D E V N D E M.

**D**Vbitandum quidem nō est quin passiones circuli innumerabiles penē sint, quae quidem omnes ferē casu inueniuntur, vt mihi nunc accidit, quam tibi mitto, hāc autem est, quod quadratum lineae. a. g. in figura hic subscripta semper æquale est ei productō, quod fit ex. a. e. in diametro circuli. g. c. b. simul sumpto cum quadrato inscripibili in dicto circulo, & simul cum quadrato lineae. a. b. contingētis ipsum circulum, supponendo. a. g. per centrum ipsius circuli transire.

Pro cuius demonstratione à centro. e. duco semidiametrum. e. c. perpendicularē ipsi. g. a. & à puncto. c. ad. a. duco. c. a. quae secabit circumferentiam ipsius circuli in pūcto. d. eo, quod angulus. c. acutus est. Nunc ex. 35. tertij, productum. c. a. in. a. d. æquale est quadrato. a. b. productum autem. a. c. in. d. c. æquale est quadrato inscripibili in circulo. g. c. b. ex. 130. primi Vitellionis, i qua propositione ipse Vitellio supplet pro eo, quod in quinta propositione libri de lineis spirabilibus Archimedis desideratur, sed quadratum. a. c. æquale est ijs duobus productis. per. 2. secundi Eucli. ergo quadratum. a. c. æquale erit quadrato inscripibili in circulo. d. c. g. & quadrato. a. b. sed quadratum lineae. a. c. æquale est duobus quadratis, hoc est lineae. a. e. & lineae. e. c. ex pitagorica, quare ex communi conceptu duo quadrata lineae. a. e. & lineae. e. c. hoc est lineae. e. g. quod idem est, æqualia erunt duobus iam dictis, hoc est inscripibili, & ei, quod fit ex. a. b. sed quadratum lineae. a. g. æquale est quadrato lineae. a. e. & quadrato quod fit ex. e. g. & duplo illius quod fit ex. a. e. in. e. g. hoc est productō. a. e. in diametrum. Quare quadratum lineae. a. g. æquale est quadrato circumscripibili, & quadrato lineae. a. b. & productō lineae. a. e. in diametrum circuli. d. c. g.

Breviori etiam methodo demonstrare possumus quadrata lineae. a. e. et. e. g. æqualia esse quadrato circumscripibili, & quadrato lineae. a. b. ducendo lineam. e. b. quae æqualis est lineae. a. g. tali methodo, hoc est, considerando, quod quadratum inscripibile semper duplum est quadrato semidiametri, vel medietati circumscripibili, quod quidem nihil aliud est, nisi æquale esse ijs duobus quadratis, hoc est lineae. e. b. & lineae. e. g. sed quadratum lineae. a. e. æquale est ijs duobus quadratis, hoc est lineae. a. b. & lineae. b. e. vnde quadratum lineae. a. e. cum quadrato lineae. e. g. æquale est quadrato circumscripibili, simul collecto cum quadrato lineae. a. b.



*Demonstrationes quarundam propositionum de quibus agit  
Cardanus capite primo libro. 16. de  
subtilitate.*

A D E V N D E M.

**E**A quæ Cardanus in primo cap. lib. 16. de subtilitate ita scribit, quod si diametros producat extra quantumlibet, alia verò diametro in centro secetur ad rectos, ex huius sine &c. quæ quidem secundum illum est vndecima proprietas circuli, quoniam te id non intelligere scribis, idemq; dicis etiam de duodecima, & similiter de tribus illis passionibus, quas ipsæ communes facit circulo, defectio, seu ellipsi, & hyperboli, tibi breuiter respondebo.

Circa vndecimam proprietatem circuli verum dicit. Imaginemur circulum. p. d. q. à duabus diametris, inuicem ad angulos rectos coniunctis, diuisum. p. d. et. d. g. diuidatur enim quarta. q. d. per quot partes æquales volueris, mediantibus punctis. b. a. o. ducanturq; ab iisdem punctis tot perpendiculares diametro. d. g. quæ sint. b. m. a. n. et. o. s. quæ quidem erunt parallelæ diametro. q. p. coniungatur deinde extremitas. d. diametri. d. g. cum primo puncto. b. & protrahatur. d. b. vsque ad concursum cum diametro. p. q. protracto in puncto. h. Nunc dico. q. h. quæ adiacet diametro. q. p. æqualem esse omnibus dictis perpendicularibus, quapropter coniungantur puncta. m. a. n. o. et. s. q. & producantur vsque ad adiacentem diametro. q. p. in punctis. c. et. e. vnde habebimus angulos. b. a. o. q. inuicem æquales ex. 26. tertij, cum verò. o. s. a. n. et. b. m. parallelæ sint ipsi. p. h. tunc anguli. b. h. c. a. c. et. o. e. q. æquales erunt angulis. d. b. m. m. a. n. et. n. o. s. ex. 29. primi: quare anguli. h. c. e. q. erunt inuicem æquales, vnde ex. 28. eiusdem. b. h. m. c. n. e. et. s. q. erunt inuicem parallelæ, & ex. 34. e. q. æqualis erit. o. s. et. e. c. æqualis. n. a. et. m. b. æqualis. c. h. verum est igitur propositum.

Duodecima vero proprietas est, ut si fuerit circulus. a. b. e. q. cuius duo diametri ad rectos coniuncti sint. a. e. et. q. b. & diameter. a. e. protractus indeterminate ad partem e. tunc si ab extremo. b. diametri. q. b. ducta fuerit. b. n. u. extra circulum, seu. b. u. n. intra circulum, vt in subiecta figura patet, ita vt secta sit à circumferentia circuli in puncto. n. vel à diametro in puncto. u. semper id quod sit ex. u. b. in. b. n. æquale erit quadrato inscriptibili in dicto circulo, hoc autem diuersimodè cognosci potest, tribus enim modis ego inueni, quorum primus ita se habet. Nam si punctus. u. fuerit extra circulum, ducantur. b. e. et. e. n. & habebimus duos triangulos. b. n. e. et. b. e. u. similes inuicem, eo, quod angulus. b. communis ambobus existit, & angulus. b. n. e. æqualis est angulo. b. e. u. quod ita probatur, nam angulus. b. n. e. cum angulo. b. a. e. (ducta cum fuerit. b. a.) æquatur duobus rectis ex. 21. tertij, sed ex quinta primi angulus. b. e. a. æqualis est angulo. b. a. e. quare angulus. b. n. e. cum angulo. b. e. a. æquatur duobus rectis, sed ex. 13. eiusdem angulus. b. n. e. cum angulo etiam. e. n. u. æquatur duobus rectis, ergo angulus. e. n. u. æquatur angulo. b. e. a. quare angulus. b. n. e. æquatur etiã angulo. b. e. u. vnde ex. 32. eiusdem reliquus angulus. b. u. e. æqualis erit reliquo angulo. b. e. n. latera igitur erunt proportionalia ex. 4. sexti, vnde ita se habebit. u. b. ad. b. e. vt. b. e. ad. b. n. ex. 16. sexti igitur verum erit propositum.

Sed si punctus. u. intra circulum fuerit, triangulus. b. e. n. similis erit triangulo. b. u. e. nam angulus. b. ambobus communis erit. Angulus vero. b. n. e. æqualis est angulo. b. e. u. ex. 26. tertij, quare ex. 32. primi reliquus angulus. b. e. n. æqualis erit reliquo angulo

angulo. b.u.e. vnde ex. 4. sexti eadem proportio erit ipsius. b.n. ad. b.e. quæ. b. e. ad b.u. quare ex. 16. eiusdem patebit propositum.

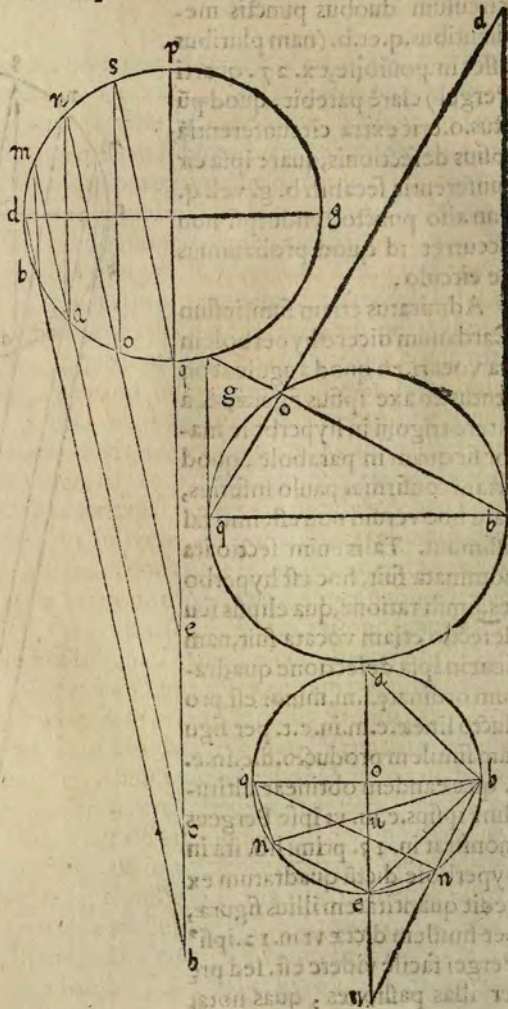
Secundus autem modus ita se habet, ducta. q.n. habebimus duo triangula orthogonia similia inuicem. b.q.n. et. b.u.o. eo quod angulus. b. communis ambobus existit, quare ex. 4. sexti ita se habebit. u.b. ad. b.o. vt. q.b. ad. b.n. vnde ex. 15. eiusdem quod fit ex. u.b. in. b.n. æquale erit ei, quod fit ex. q.b. in. b.o. Sed ex. 16. eiusdem, qd fit ex. q.b. in. b.o. æquatur quadrato, b.e. quia. b.e. media proportionalis est inter diametrum & semidiametrum eiusdem circuli. ex. 4. eiusdem, quare quod fit ex. u. b. in b.n. æquale erit quadrato ipsius. b.e.

Tertius modus adiungitur, & est quod cum quadratum. u.b. existente. u. extra circulum æquale sit ei, quod fit ex. u.b. in. b.n. simul sumpto cum eo, qd fit ex. u.b. in. u.n. ex secunda secundi, & idem quadratum. u.b. æquale duobus quadratis. u.o. et. o. b. ex penultima primi, ideo duo dicta producta æqualia erunt dictis duobus quadratis. o. u. scilicet et. o. b. sed quadratum

o. u. æquatur ei, quod fit ex. a. u. in. e. u. & ei quod fit. ex. o. e. in se ipsam ex. 6. secundi, quare duo iã dicta producta æqualia erunt duobus dictis quadratis, o. b. scilicet. et. o. e. & ei quod fit ex. a. u. in. u. e. sed quod fit ex. b. u. in. u. n. æquale est ei quod fit ex. a. u. in. u. e. ex. 35. 3. relinquit ergo vt id qd fit ex. u. b. in. b.n. æquale sit duobus quadratis. o. b. et. o. e. quare & quadrato ipsius. b. e. ex Pitagorica.

Si autem punctū. u. fuisset intra circulum idem eueniret. Nam quadrato. b. e. æquatur duo quadrata. o. b. et. o. e. sed vice quadrati. o. e. dicemus quadratū. o. u. cum eo quod fit ex. a. u. in. u. e. ex. 5. secundi, id est quadratum. o. u. cū eo quod fit ex. b. u. in. u. n. ex. 34. tertij, vnde quadratum b. e. æquale erit quadrato. o. b. & quadrato. o. u. id est quadrato b. u. ex Pitagorica simul cū producto. b. u. in. u. n. id est producto n. b. in. b. u. quod æquale est quadrato. b. u. cum producto. b. u. in. u. n. ex. 3. secundi.

Circa tres passiones communes postea circulo hyperboli, & defectiōni notandum est primā patere ex. 36: primi Pergei, secundum



cundam verò ex. 37. et. 38. eiusdem, propterea quod in. 37. probat mediante maiori diametro ipsius hyperbolis & defectiois. In. 38. autem mediante minori diametro ordinatè ad maiorem.

Tertia autem passio, non nisi circulo convenit; pace ipsius Cardani dictum sit.

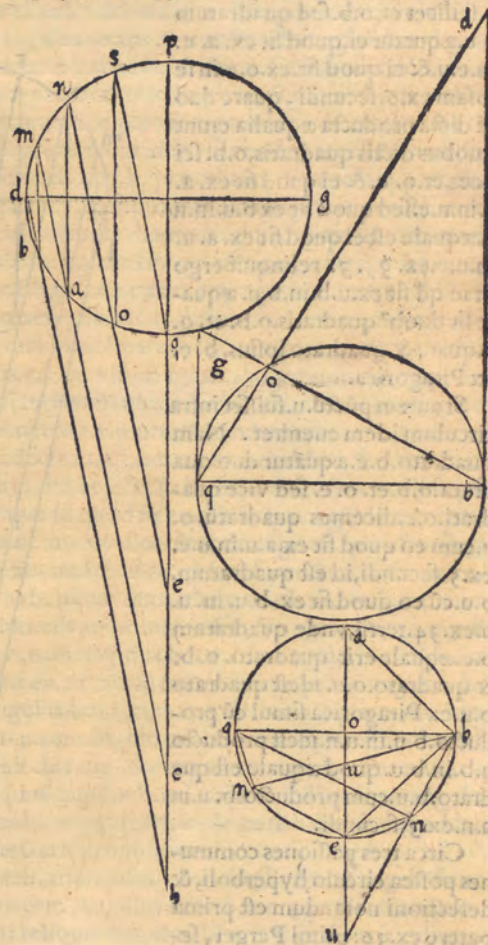
Quapropter sit circulus. q. o. b. cuius diameter sit. q. b. contingentes vero ab extremitate diametri sint. d. b. et. q. g. per punctum autem. o. quoduis, ipsius circūferentiæ, transeant. b. o. g. et. q. o. d. tunc dico productum. q. o. in. q. d. vel. b. o. in. b. g. equale esse quadrato. q. b. quod ita probo.

Nam angulus. q. b. d. seu. b. q. g. rectus est ex. 17. tertij Eucli. et. b. o. q. similiter rectus ex. 30. ipsius lib. angulus verò. b. q. d. seu. q. b. g. communis est. quare. b. q. media proportionalis erit inter dictas lineas. q. d. et. q. o. & inter. b. g. et. b. o. Vnde sequetur propositum ex. 16. 6. Eucli.

Sed si circa diametrum. q. b. mente fingamus aliquam elipsim, quæ tangat ipsum circulum duobus punctis mediantibus. q. et. b. (nam pluribus esset impossibile, ex. 27. quarti Pergei) clarè patebit, quod punctus. o. erit extra circūferentiã ipsius defectiois, quare ipsa circūferentiã secabit. b. g. vel. q. d. in alio puncto, vnde ipsi non occurret id quod probauimus de circulo.

Admiratus etiam sum, ipsum Cardanum dicere hyperbolem ita vocari, eo quod angulus contentus ab axe ipsius figuræ, & à latere trigoni in hyperbole maior sit quam in parabole, quod etiam confirmat paulo inferius, nam hoc verum non est, imo falsissimum. Talis enim sectio ita nominata fuit, hoc est hyperboles, simili ratione, qua elipsis seu defectio etiam vocata fuit, nam sicut in ipsa defectioe quadratum ordinatæ. l. m. minor est producto lineæ. c. m. in. e. t. per figuram similem producto. d. e. in. e. t. quæ eandem obrineat altitudinē ipsius. e. m. vt ipse Pergeus monstrat in. 13. primi lib. ita in hyperbole dictū quadratum excedit quantitatem illius figuræ, per similem dictæ vt in. 12. ipsi Pergei facile videre est. sed præter illas passiones, quas notat

Car-



Cardanus in supradicto capite, multæ aliæ sunt, cum corollarium primæ tertij Eucli. sit passio propria ipsius circuli, & idem dico de propositione. 3. 4. 7. 8. 9. 11. 12. 13. 14. 15. 17. 18. 19. 20. 30. 31. ipsius tertij lib. nec non de. 8. 9. et. 10. tertijdecimi, & de prima. 3. 4. 5. 6. et. 7. quartidecimi eiusdem. Idem infero de ea quod scripsi Mario Nizzolio, Francisco Vimercato, Francisco Contareno, Angelo Agrimenfori, & de alijs nonnullis à me excogitatis.

DE FINE CORPORVM COELESTIVM,  
& eorum motu.

*Illustri viro, Philiberto Pingonio Sabaudo Cusiacenſium  
Baroni.*

**C**VM antea meo nomine Sebastianus nôſter omnia ferè tibi retuliſſet, inter alia, quæ relinquebantur tibi dicèda, hoc vnum erat, quod ſi abſque lumine ſuperiori, in quem finem facta fuerint corpora cœleſtia ſcire deſideras, & humanam rationem ſequi volueris, putandum tibi non erit ea ſolum effecta eſſe, vt tam vile corpus, vt eſt terra aquis irrigata, animalia, & plantas regant, cum ea corpora ſint diuina, in numero incomprehenſibilia, maximis magnitudinibus, & motibus velociffimis, prædita, id etiam minus putabunt hij, qui opinionem Ariſtarchi Samij, & Nicolai Copernici ſequuntur, quorum ratione fieri non poteſt, vt credant, eius, quod ex vniuerſo reliquum eſt, alium finem non habere, quam regimen huius centri epicycli Lunarum, vt illorum more loquar. Quæ enim turpe eſſet ſi centra aliorum epicyclorum planetarum tali regimine priuarentur, id quod nullo modo cum ratione conſentit, ſi tam vera eſt ea opinio, quemadmodum rationabiliorè eam exiſtimât. Neq; quidquam valet opinio Ariſtotelis, qui corpora cœleſtia, ab ortu, & interitu libera eſſe ſentit. dicens ſuperioribus ſæculis, à noſtris antiquis nullam vnquam animaduerſam fuiſſe alterationem in cœlo, cum non videat ſi quis eſſet in cœlo, neq; etiam obſeruari poſſet alterationes quæ in terra, & circa terram fiunt, quæ in partibus, & non in toto ſpectantur: vnde etiam fieri poteſt, vt in cœlo ſint particulares alterationes, quæ à nobis tamen, qui ab illis longè diſtamus, non comprehendantur, terra, mareq; (quamuis minimum reſpectu ipsius terræ) ratione totius ita ſe ſemper habuerūt quæ admodum ſeſe habere corpora cœleſtia videmus, ſed alteratio, ratione tantum aliquarum minimarum partium quaſi inſenſibilem, ſi cum toto comparentur ſit. Quis enim ſcit, vt iam tibi dixi, quin, quemadmodum Luna circa terram voluitur, ipſaq; terra ſit veluti centrum epicycli maioris eiusdem, vt Ariſtarchus Samius, & Nicolaus Copernicus cenſuerunt; ſic etiam Saturnus, Iupiter, Mars, Venus, atq; Mercurius circa alia huiusmodi corpora, huic terræ ſimilia, in orbem agantur, quaſi ſpecula, lumen Solis ſuo centro ex reflexione, deferentia (ſuppoſita dico vera illorum opinione) Nollè tamen tibi è mente excidere, vt aliàs te monui, q; ſi communis opinio vera eſt, neceſſario fatendū ſit corpus ſolare, dū in æquatore reperitur motu diurno quolibet horæ minuto, magis quàm decè & ſeptè mille miliaria pagere, ideſt paulo minus quam. 18000. miliaria, Saturnum verò cum ſimiliter eſt in æquatore, eodem tēporis ſpatio, quaſi tercentamille miliaria Italica conficere, & ſic per gradus alia corpora velociora alijs moueri, quæ quidem omnia, cū ſimplici gyro terræ circa ſuum

axem

axem (vt dicunt) tolluntur, quod sufficit ad recipiendum lumen, & influentias illorum corporum. Et ita, veluti princeps corporum vniuersi, intra vnum annum circa eam vertitur. Ita etiam sufficeret, vt ipsa terra circa dictum diuinum corpus solare, interfecando axem diurnum cum axe annuali (cum ab eo lumen, calorem, & influentiam suscipere debeat) circunuolueret. Rationes autem a Prole- meo in contrarium adductæ apud ipsos, nullæ sunt, quia quælibet pars (vt inquit) retinet naturam totius, præterquam q̄ aer, & aqua, quæ ipsam terram circundât, plañe eundem naturalem impetum motus obtineant, qui tanto lentior est, quanto longius distat ær ab ipsa terra, secundum etiam talem opinionem, nulla necessitas foret, vt locus fixarum terminaretur aliquibus superficiebus, conuexa scilicet, & de- uexa.

*De Luce, Lumine, & Colore, De obiectu oculi, De lumine Luna, & Rubedine nubium.*

A D E V N D E M.

**Q**uod proximè quærebam, an sit lux aliqua, quæ à corpore lucido non proueniat, mihi facillè ad considerandum videtur. hic enim oportet, vt nos ad id quod perpetuò videmus referamus, existimo autem te velle dicere lumen, non lucem, quia propriè lux, qualitas ea visibilis appellatur, quæ est in corpore lucido, à quo quidem corpore lumen effunditur; lumen verò, ea qualitas esse dicitur, quæ extra ipsum corpus reperitur, à luce, quæ in dicto corpore manet emanans. vnde patet, nullam lucem absque corpore subiecto esse posse, id quod cum fieri quæret, idè de quolibet alio accidente dici posset, id est quod ex se, & absque aliquo subiecto subsisteret.

Lumen deinde à luce proficisci patet, q̄ penetrat diaphanum, neque aliquo modo suum actum ostendit, nisi, aut per incidentiam, aut ratione opaci, ex reflexione, cuius superficiei colorem induit. Atque hæc est causa, vt inter crepusculum matutinum, aut vespertinum, nox etiam si sit serena, adeo obscura nobis appareat, quamuis totum vniuersum diaphanum, extra conum vmbra, quæ ex terra prouenit sit vndiq; radijs luminosis Solis collustratum; qui quidem radij, non nisi à suamet reflexione à Luna, & ab alijs stellis (vt corporibus opacis, quæ resistunt lumini, ne vltius penetrare possit, vnde retrò redit) comprehenduntur.

Ais etiam propria visus obiecta plura esse, nominans pro vno, colorem, & lucem pro alio. Ego autem respondeo, obiectum oculi esse vnicum tantum, id est lumen. Quod ad lucem spectat, iam tibi dixi, eam esse quandam qualitatem in corpore lucido, & non extra ipsum positam, à quo quidem corpore, cum non exeat, oculi obiectum esse nequit, sed lumen quidem ab ipsa luce productum. Color etiam, qui est in corpore colorato, obiectum oculi esse non potest, cum dictum corpus non deferat, sed lumen quidem ab eodem corpore reflexum, & huiusmodi corporis colore tinctum: vnde tam lumen incidens, quam reflexum colore est semper imbutum.

Illud quidem coloratum est qualitate lucis corporis lucidi, a ut medij, per quod transit, sed hoc colore corporis, a quo reflectitur.

Neque etiam te ignorare volo, lumina reflexa colorata, non reflecti à puris propriisq; superficiebus corporum coloratorum, eo qd̄ pauca corpora tam opaca reperiuntur, ut immediatè lumen à superficie propriè reflectât, sed lumen penetrat ali-

quan-



quantulum dicta corpora, & ita illorum colore afficitur, vbi verò non penetrat, non coloratur colore corporis illius.

Sed vt ad propositum redeamus, dico lumen tantum esse visus obiectum, quod si colore est imbutum, aut tale est ratione coloris lucis, quæ eum mittit, aut ratione medij per quem transit, aut ratione corporis, vnde reflectitur. etsi superficies corporis vnde lumen reflectitur esset omnino priuata colore, sub aspectum non caderet, vt etiam cum huiusmodi superficies læuigata, & polita est secundum continuitatē suarum partium, videlicet, vt speculi radio tamen non profundante, & ideo perfectissimi morum quorundam speculorum superficies non cernuntur, sed lumen tantum reflexum, colore alicuius alterius superficies, aut à luce, corporis lucidi, aut à medio per quem transit, conspicitur. Ego verò non assero colorem non esse quid diuersum à lumine, sed imaginis lumen esse veluti animam, aut substantiam & colorem corporis formam accidentalem, cum nullum lumen à sensu visus percipi possit, quod aliquo modo colore non sit imbutum: & eundem respectum quem sonus ad auditum, lumen ad oculum habet, quia vt sonus secundum eam velocitatem, quæ à motione aeris, aut aquæ, ex collisione aliorum corporum producitur ad euitandum vacuum, acutus, vel grauis sentitur, ita lumen originem ducens à corpore lucido per medium diaphanum aeris, aut aquæ, aut alterius huiusmodi corporis ad oculum transit colorem lucis, aut medij per quod transit, aut vnde reflectitur induit.

Quod verò Luna nullum ex se habeat lumen, sufficiens inditium est nos ipsam tantò magis obscuram videre, quantò magis in cono umbræ terræ immergitur, & si eo tempore ipsam videmus rubeo colore affectam, hoc enim accidit, quia radij solares vnde quaque refranguntur à vaporibus ipsam terram circumstantibus, quæ quidem refractione fit versus axem conij umbræ terræ, & propterea umbra dicti conij non est æqualiter obscura, seu tenebrosa, circa vero axem ipsius conij, magis quam circa eius circumferentiã, obscura vñ, & quia corpus lunare tale est, vt facillimè recipiat quaecumque lumen, quod etiam manifestè videtur dum ipsa Luna reperitur secundum longitudinem inter Solem, & Venerem, quod pars Lunæ lumine Solis destituta, à lumine Veneris aliquantulum illustratur, quod ego sæpè vidi, & multis ostendi. Propterea dum ipsa Luna in cono umbræ terræ reperitur adhuc videtur. Rubedo etiam illa nubium post Solis occasum, vel ante ortum, aliunde non prouenit, nisi à qualitate vaporum, per quos solares radij transeunt, à quibus vaporibus, tali colore ipsi radij afficiuntur, eomet modo quo radius, cuiusuis corporis lucidit, trãsiens per vitrum, seu aliud diaphanum coloratum.

## DE ICTV BOMBARDÆ SEC V N D V M

diuersas eleuationes. Et de quibusdam erroribus Nicolai Tartaleæ, circa idem.

*Illustri D. Iosepho Cambiano ex Russia Dominis, equiti  
srenuo, & tormentis bellicis Serenissimi Ducis  
Sabaudia Prefecto.*



Cogitavi quædam dum ocio frui licuit per absentiam Ducis Serenissimi, quæ ad te scribere placuit, vt si probaueris in lucem quandoque proferre non dubitem, si despexeris, ocius supprimam, sunt autem huiusmodi.

Vnde fiat vt tormentum bellicum vehementi<sup>9</sup> feriat ictu superius delato quam horizontali, vt Tartalea scribit, quæsito secundo libr. primi quæsitorum, à nemine adhuc (quod sciam) traditum est.

Rationes verò Tartaleæ nullius sunt momenti, quia si validæ essent, sequeretur vt inclinata bombardæ, adeo vt angulus sub horizonte factus æqualis esset ei, qui supra horizontem est, ictum bombardæ in vtroque huiusmodi situ eundem esse futurum. & si aliqua differentia oriretur ratione grauitatis pilæ ab ipsa bombardæ emissæ, hoc fieret, vt scilicet velocior esset in motu inclinato quam in eleuato cum pondus, motui adeo non opponatur. Id quod non ita se habet, vera enim causa vnde fiat, vt bombardæ eleuata vehementius feriat, quàm ea quæ est minus alta, eadem est ferè, in genere, cum ea, quæ aliquod corpus materia magis densa, sed simile & æquale alteri corpori materiæ minus densæ velocius mouetur ab vna eademque, aut æquali vi compulsam. Est eadem etiam in specie ei, quæ maiorem effectum producit puluis, qui in locis subterraneis ponitur quum vasis optimè colligatis ferro includitur. Est etiam similis ei, quæ longius impellitur pila, quæ ludimus, ab aliquo instrumento ligneo, quando percutitur contra, quam cum secundum suum motum projicitur. Id quod inde fit, quia virtus mouens maiori vi, & intensiori huiusmodi corpus percutit, quia corpus quod moueri debet, quanto magis resistit virtuti mouenti (certum tamen terminum præscribendo) in exiguo eo temporis spatio, tanto maiorem virtutem colligit, quæ ipsum deinde tanto cum impetu mouet, & tanto magis impellens concomitatur, vt maiorem effectum efficiat, quam si ad mouendum sese facile reddidisset. Atque hoc supradictis ictibus eleuatis accidit, quia grauitas pilæ, ea est quæ resistens virtuti mouenti, dat ei commoditatem colligendi dictam virtutem, multo magis quam esset ea, quæ ad depressiorem eleuationem eam impelleret. Et quia huiusmodi multiplicatio virtutis, nullam proportionem cum pondere pilæ gerit, volo inferre quod dum colligitur tanta virtus, colligitur multo plus eo, quod ad impellendam dictam pilam sufficeret, ratione magnæ velocitatis augmenti, quia quanto plus temporis ei conceditur ad commutandam puluerem in ignem, tanto maior quantitas ignis progignitur, vnde fit, vt tanto maiori loco indigeat, quam obrem tanto magis impellit, sed vt dixi, tanta cum velocitate adaugeat, vt huiusmodi virtus longè superet resistentiâ pòderis pilæ, & sic est causa, vt effectus, quod experiètia innouit escit producat. Sed ea ratio, quæ sese idè author in tertio quæsito ad aliquod impossibile, circa iter ipsius pilæ Legatum Hispanum redu-

reducere putat, nullo fundamento nititur, quia non est semper dicendum, quod quāto velocior sit quædam pila, tanto rectius moueatur, quia ei dici posset, vsque ad certum quendam terminum velocitatis, per tantum spatij eam aptam esse, vt recta perfectè moueatur, sed si velocius iret, non tamen futurum, vt per idem spatium rectius moueretur, sed quod per longius spatium recta motum perageret, & sic nihil haberet quod replicaret, præter quam quod ipse supponit id quod in 18. quæsito negat, in quo ait pilam uicinam orificio, non adeo uelocem esse, quam cum aliquantulum ab eodem est remota, ratione resistentiæ sui cyllindri aerei. Sed quod pila, recta eat quanto altior, aut depressior bōbarda erit, iō fit, quia linea inclinationis naturalis cum linea inclinationis uiolentiæ angulum rectum non facit, unde quanto longius distat à recto huiusmodi angul<sup>o</sup>, siue sit acutus siue obtusus, tanto minorem uim habet, eodem planè ferè modo quem tertio capite mei tractatus de rebus mechanicis descripsi. Quia in iētibus eleuatis, iter inclinationis uiolentiæ ipsius pilæ uersus terminum ad quem, incipiendo à loco ipsius pilæ cum itinere inclinationis naturalis, angulum obtusum, & in iētibus inclinatis acutum constituit. Neque etiam hic prætermittam notatu dignum errorem, quem Tartalea eodem loco committit, cū putet indifferenter aliquod corpus impellere, aut percutere maiori cū impetu cum est in itinere recto. Quia sequeretur quod aliquod corpus graue perpendiculariter sursum uersus proiectam, in qualibet parte sui itineris, semper fortius percuteret, quam in qualibet parte itineris alterius cuiusuis eleuationis obliquæ, quod quā fit falsum, tibi considerandum relinquo.

Est etiam falsa ea ratio, quam in quarto quæsito idem adducit, quia aer in motu non tantum durat, quantum ipse putat, imò huiusmodi uiolenta agitatio, citò cessat & citius etiam, quam si extra aliquam bombardam cum tanta uiolentiā impullisset faccum plumis plenum.

Ratio etiam quam in 18. quæsito de eo, quod pila pertranseat illud corpus cyllindricum aereum adducit, est planè uana, quia statim aer, qui prius in bōbarda erat inclusus, extra ipsam erūpit, cedit, à pilaq; diuiditur, vt si nunquam eam figuram induisset, neque aer ambiens ei resistit. Sed quod velocior sit in certa quadam distantia, quam in principio erat, si hoc uerū esset, ab alia causa dependeret, quæ partim si milis esset ei, quæ efficit, vt corpora in motibus naturalibus, cum longius distant à termino unde naturaliter sese mouerunt, sint uelociora, quia per aliquod spatium huiusmodi corpus moueretur quemadmodum motu naturali cietur.

Ratio autem eius quare pila, aut globus bombardæ sibi et ab eodem in septimo quæsito nil ualet, quia hoc fit cum pila aliquam paruam concauitatem habet.

In 27. autem quæsito ait, quod retrahendo signum, iētus altius tenderet, quod potest etiam esse falsum, cum hoc non sit necessarium, quia pila dum descendit, fortasse tangeret scopum.

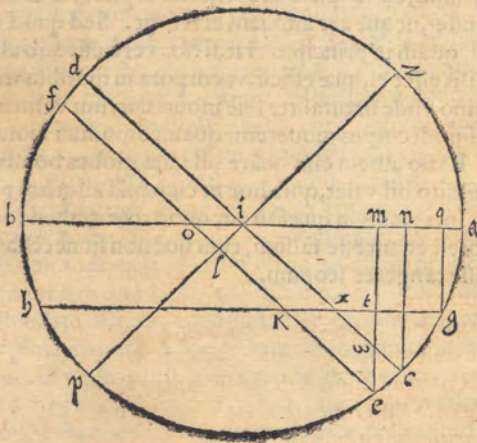
*De erroribus Ioannis Stadij.*

## A D E V N D E M.

**F**iguram quam ponit Ioannes Stadius pag. 147. in lib. suarum tabularum Prutenicarum, à Nicolao Copernico sumpsit pag. 64. à tergo in libr. reuolutionum cœlestium, sed ipse Stadius eam non intellexit, omitto, quod mutauerit characteres ipsius figure, vt illa sua videatur, quod nihil refert, alterat etiam demonstrationē, sed ipsum putare. i. K. perpendicularem à centro circuli semper dependere, est intolerabilis error; nec vnquam verificatur hoc, nisi quando punctum. K. interfectionis diametrorum parallelorum, forte reperitur in axe mundi. Reliqua verò suæ demonstrationis, si non intelligis, minimè miror, eo quod ipsemet Stadius seipsum confundit. Veram autem demonstratio nem huiusmodi figuræ in dicto libr. Copernici clarè videbis. Quod verò diuersè cogitavi nunc accipito.

Cum nobis cognita sit maxima eclipticæ declinatio, vt puta. a. c. si latitudo etiã stellæ nobis data fuerit, vt puta. c. e. cognitus nobis erit totalis arcus. a. e. & eius sinus. e. m. & quia notus etiam nobis est sinus arcus. a. c. hoc est. c. n. & corda. e. f. medio eius arcus. e. p. f. minoris media circumferentia, per duplum latitudinis datæ, vnde. e. l. eius dimidium nobis cognitum erit, vel vt sinus arcus. e. p. cognitus etiam nobis est sinus. q. g. declinationis. a. g. datæ, cui æqualis est. m. t. ex. 34. primi Euclid. vnde. i. e. t. nobis cognita remanet, cum verò duo trianguli. i. c. n. et. t. e. K. æquianguli sint, propter duas parallelas. e. m. et. n. c. ex. 28. primi, & propter duas. a. b. et. g. h. & propter duas. c. d. et. e. f. eo quod ex communi scientia anguli. c. et. e. sunt æquales, cum ex. 29. dicti lib. vnusquisq; æqualis sit angulo. m. s. i. ita etiam infero de angulis. e. K. t. et. c. i. n. quorū vnusquisque æqualis est angulo. a. x. t. & sic de alijs dico, eo quod vnusquisque eorū æqualis est angulo. m. vnde cum cognitum nobis sit latus. n. c. et. c. i. et. t. e. notū etiam nobis erit. e. K. ex. 19. septimi, eo q̄ ex. 4. sexti sunt inuicem proportionalia, detrahendo postea. e. K. ab. e. l. cognito, vel è contra, hoc ab illo, nobis innotescet. K. l. sinus longitudinis stellæ.

Valde etiam miror id, quod dictus Stadius pag. 9. illius libr. scribit, hoc est, Solem maiorem esse Luna, solum. 1644. vicibus, propterea q̄ cum affirmet Solem maiorem esse terra (vt etiam in Almagesto videre est) 166. vicibus cum tribus quartis, terram vero maiorem Luna. 39. vicibus cum quarta parte, tunc Solem oporteret maiorem esse Luna. 6545. vicibus, & non. 1644.



De

*De cognitione latitudinum stellarum.*

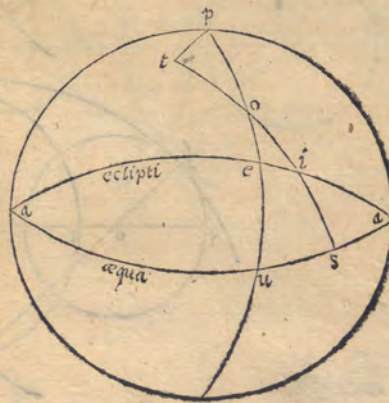
## A D E V N D E M.

**A**D cognoscendam latitudinem stellæ, eiusq; declinationem, Monteregus in 10. propositione. 8. li. Almagesti methodū satis docuit, sed si alia aliqua methodo hoc idem cognoscere voluerimus, oportebit nos prius altitudinem poli cognoscere, deinde altitudinem meridianam ipsius stellæ, nec non horam, quādo ipsa stella in meridiano supra terram reperitur, qua hora mediante, illicò cognoscemus punctum eclipticæ à meridiano intersecto, eo tempore, quo stella cœlum mediat supra terram. Et quia ex cognita altitudine poli, illico cognoscitur altitudo æquatoris, cuius altitudinis differentia ab altitudine stellæ est declinatio ipsius stellæ, habebimus ideo eius declinationem cognitam; qua mediante ad cognoscendū etiam latitudinem ita faciemus.

Sit exempli gratia. p. o. u. meridianus. u. a. verò æquator. e. a. autem ecliptica, & o. centrum astri. u. o. verò eius declinatio ab æquatore, et. e. a. arcus eclipticæ inter æquatorem, & meridianum, hoc est minor quarta, et. a. u. ascensio recta ipsius arcus, et. u. e. sit declinatio puncti. e. eclipticæ ab æquatore, residuū vero declinationis stellæ sit. o. e. quæ oīa nobis cognita erunt, sitq; t. polus eclipticus, à quo per. o. vsque ad eclipticam transeat quarta. t. i. in qua querendus erit arcus. o. i. hoc modo.

Primum arcus. o. u. e. u. e. o. a. e. et. a. u. nobis cogniti sunt, cum angulo. a. declinationis eclipticæ, & cum angulo. u. recto, vnde ex. 4. primi Copernici, cognoscemus angulum. a. e. u. collateralem, & eius. o. e. i. quare in triangulo. o. e. i. cognoscemus angulū e. et deinde. i. vt rectū, & latus. o. e. ergò ex eadē. 4. cognoscemus arcū. o. i. quæsitum, & similiter arcum. e. i. qui coniunctus vel dēptus ab. a. e. tribuet nobis longitudinem stellæ, sed quia huiusmodi operatio in paruis triangulis valde fallit. Ideo tibi suadeo alia methodo, hoc facere, hoc est inuenire angulum. o. trianguli. t. p. o. cuius duolatera. t. p. et. p. o. cognita nobis sunt, cum angulo. p. Nam. o. p. est complementum declinationis stellæ, et. p. t. est arcus coluri solstitiorum inter duos polos, & angulus. p. residuum ex recto. t. p. a. duorum colurum dempto angulo. a. p. u. cognito ascensionis recte, vnde angulus. u. o. s. vt contrapositus cognitus remanet. angulus verò. u. rectus est, & arcus. o. u. cognitus, quare cognitus nobis erit arcus. u. s. & angulus. u. s. o. vnde arcus. a. s. nobis cognitus remanebit cū angulo. a. s. i. residuo ex duobus rectis. Et quia etiam angulus. s. a. i. cognitus est, cum sit angulus maximę declinationis Zodiaci ab æquatore. Ideo in triangulo. a. s. i. cuius duo anguli. a. et. s. cum latere. a. s. dantur, facile inueniemus arcum. s. i. cū arcu. a. i. sed a. i. erit longitudinis stellæ dempto postea. s. i. ex. s. o. iam inuento habebimus arcum. i. o. latitudinis ipsius stellæ.

Hæc autem tibi scribo non vt ipsis vtaris, sed potius vt tibi morem gerā, cum breuissima methodus sit illa, quā Monteregus scripsit i. 10. p. propositione. 8. li. in Almagest.



*Qualiter circulus designari possit alios duos circulos  
propositos includens.*

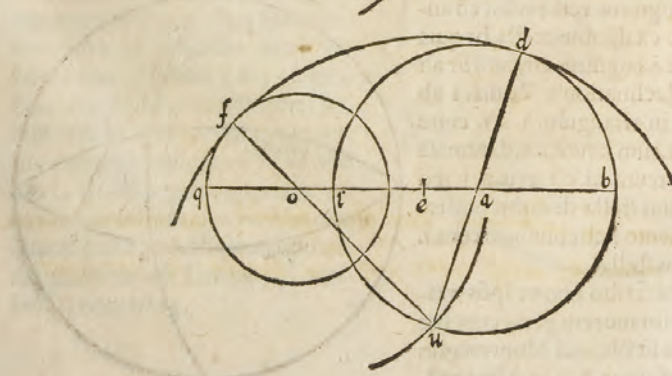
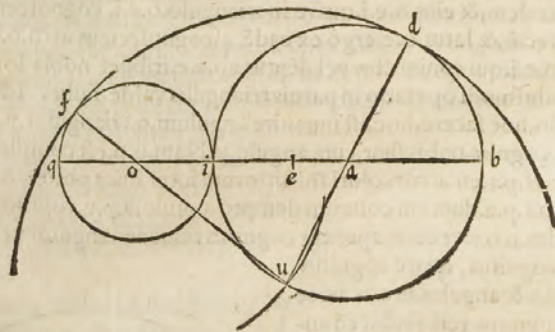
CLARISS. PETRO PIZZAMANO.

**S**uperioribus diebus per tuas literas à me quaesivisti, ut modum tibi scribere vellem, quo circulus designari possit circumscribens alios duos propositos circulos.

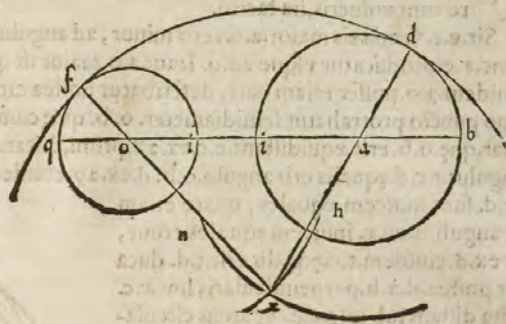
Qua in re ut tibi satisfaciam quod maximè cupio ita rem accipe.

Propositi circuli sint, aut inter se contigui, aut interfecantes vel separati. Esto primū contiguos esse, qui sint. d. b. et. f. q. quorū. d. b. maior sit et. f. q. minor, eorū vero centra sint. a et. o. punctū autem cōtingentię sit. i. Nūc, per a. b. a. o. q. per cētra eorum ab vna circumferentia ad aliam, quę quidem linea transibit per punctum. i. ex 11, tertij Eucli. deinde à diametro maiori abscindatur. i. e. ad æqualitatem minoris semidiametri, quo factō sumatur distantia inter. e. et. b. circino mediante factōq; centro. o. scindatur, alio circini pede, circumferentia maioris circuli in puncto. u. à quo si mente concipiemus duas lineas. u. a. d. et. u. o. f. transeuntes per eorum centra. a. et. o. vsque ad circumferentias in punctis. d. et. f. ipse erūt inuicem æquales, eo quod. e. i. supra fuit æqualis. o. f. et. o. u. æqualis. e. b. quare. u. f. æqualis erit. b. i. sed. u. d. etiā æqualis. b. i. ergo. u. d. æqualis erit. u. f. & circulus, cuius u. d. vel. u. f. erit semidiameter, contiguus erit ipsis propositis circulis ex conuerso. 11. iam dictæ. Idem dico pro circulis se inuicem secantibus.

Sed

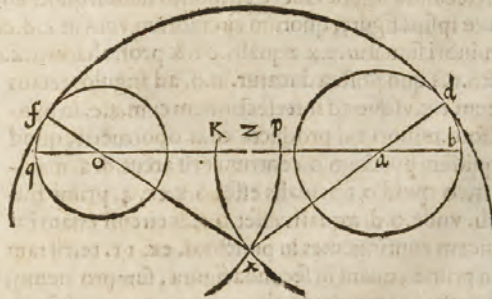


Sed si circuli propositi seiuncti fuerint, sumatur. b. i. diameter maioris, qui fiat semidiameter vnus circuli circa centrum. o. & hic circulus vocetur. h. x. coniungatur deinde semidiameter. o. i. minoris circuli cum semidiametro. a. i. circuli maioris, & ex huiusmodi composita linea, fiat vnus semidiameter. a. x. circuli. x. n. concentrici cum maiori, & à puncto. x. intersectionis horum circularum (posito quod se inuicem interfecerint) ducantur per eorum centra. x. a. et. x. o. vsque ad ipsorum circumferencias in punctis. d. et. f. due lineæ, vnde habebimus. x. d. æqualem. x. f. eo quod tam in x. d. quam in. x. f. reperiuntur diametri, & semidiametri amborum circularum, facto denique centro. x. vnus circuli, cuius semidiameter æqualis sit vni earum. x. d. vel. x. f. solum erit problema, dicta ratione.



Si verò distantia duorum propositorum circularum tanta fuerit, quod secundi circuli nequeant se inuicem tangere, vel secare, tunc alia via incidendum erit, quæ talis est & generalis. Diuidatur tota. q. b. per æqualia in puncto. z. circa quod signetur duo puncta ab ipso equidistantia. k. et. p. distantia vero. a. k. facta sit semidiameter esse vnus circuli. k. x. circa centrum. a. distantia autem. o. p. semidiameter alterius circuli. p. x. circa centrum. o. qui quidem circuli se inuicem secant in puncto. x. à quo cum ductæ fuerint. x. a. d. et. x. o. f. per centra dictorum circularum, ipse erunt inuicem æquales, eo quod cum. b. k. æqualis sit. q. p. igitur. x. d. et. q. p. erunt inuicem æquales, sed. f. x. æqualis est q. p. quare. x. f. æqualis erit. x. d. tunc si. x. centrum fuerit vnus circuli, cuius semidiameter sit vna dictarum, problema solutum erit.

Talis etiam solutio commoda erit ad inueniendum dictum circulum cuiusvis magnitudinis, dato tamen eius diameter, maior sit. b. z. cum in nostra potestate sit accipere puncta. k. et. p. proxima vel remota ab ipso. z. ad libitum. Vnde absque vlla diuisione ipsius. q. b. per medium, satis erit signare puncta. k. et. p. duabus distantijs mediantibus. b. k. et. q. p. inuicem æqualibus, & etiam propositis.



*In soluis casib, diuisa qz bisariam, quæ quædam punctis fiat centrum, circulus descriptus qz tangens semper datos circ: in punctis qz stringet.*

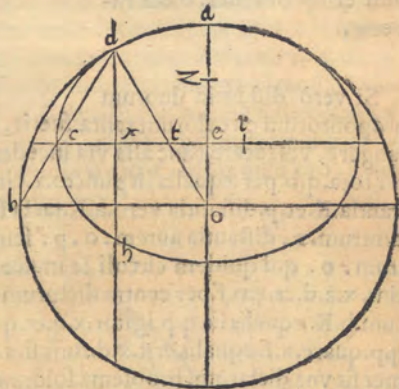
Figuram

*Figuram superficialem ellipsi similem , ex datis axibus circino mediante delineari posse.*

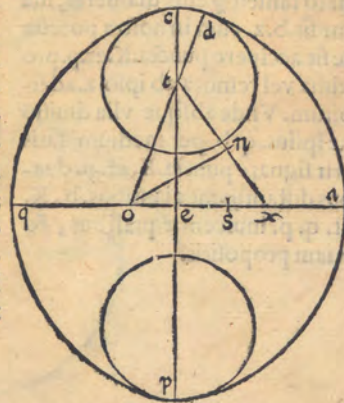
A D E V N D E M.

**F**iguram superficialem ellipsi similem, ex datis axibus, circino mediante delineare cum volueris, ita facito.

Sit. e. c. femiaxis maior. a. e. verò minor, ad angulum rectum inuicem coniuncti, tunc. a. e. producaturs vsque ad. o. Itaq; a. o. maior sit quam distantia inter. o. et. c. que quidem. a. o. posset etiam dari, describatur postea circulus. a. d. b. circa centrum. o. à quo puncto protrahatur semidiameter. o. b. quæ cum. a. o. angulum rectum constituat, que. o. b. erit æquidistans. e. c. ex. 28. primi, ducatur postea. b. c. d. et. o. t. d. vnde angulus. t. c. d. equalis erit angulo. o. b. d. ex. 29. eiusdem. ex quinta autem anguli. b. et. d. sunt inuicem æquales, quare etiam & anguli. d. et. c. inuicem æquales erunt, & ex. 6. eiusdem. t. c. equalis erit. t. d. ducatur postea. d. x. h. perpendicularis lineæ. c. e. ita distans sub ipfa. c. e. vt arcus circularis circa. t. delineatus ex semidiametro. t. d. aptus sit eam secare, sumpto postea. r. tam distante ab. e. vt. t. reperitur ab ipso e. et. z. ab. e. vt. o. ab eodem, ducendo postea duos alios arcus magnitudinis priorum circa centra. r. et. z. habebimus propositum.



Sed cum quis voluerit prius arcus minorum circularum delineare circa maiorem a xem, fiant cuiusvis magnitudinis, vt in secunda figura videre est, posito tamen quod eorum diameter, minor sit minore axe ipsius figure, quorum circularum vnus sit. c. d. circa. t. eius centrum, deinde in axe minori sumatur. a. x. æqualis. c. t. & protrahatur. t. x. que per equalia diuidatur in puncto. n. à quo postea ducatur. n. o. ad angulos rectos cum. t. x. vsque ad intersectionem cum. a. e. in puncto. o. minori axi producta cum oportuerit, quod quidem punctum. o. centrum erit arcus. d. a. maioris, eo quod. o. t. æqualis esset. o. x. ex. 4. primi Eucli. vnde. o. d. æqualis esset. o. a. & circuli etiam inuicem contingentes in puncto. d. ex. 11. tertij tam in prima, quam in secunda figura, sumpto deniq; puncto. s. tam remoto ab. e. quam. o. reperitur ab eodem, ipsum, centrum erit alterius arcus oppositi, possemus etiam absq; diuisione ipsius, t. x. constituere angulum. x. t. o. æquale angulo. t. x. o. vnde ex 6. primi haberemus. o. t. æqualem. o. x.



De



*De inuentione axis propositæ portionis datæ sphaeræ.*

A D E V N D E M.

**V**T axem propositæ alicuius datæ sphaeræ inuenire possis ita tibi operandum est vt gratia exempli. Propositæ nobis est sphaera. c. i. e. t. diametri cognitæ. proposita etiam est nobis eius portio. n. e. u. axis. e. a. cognitæ minoris semidiametro, data etiam nobis est proportio alterius portionis minoris hemisphaerio. i. e. t. ad portionem. n. e. u. quæritur nunc quantus sit axis. e. x. secundæ portionis hoc est desideramus cognoscere proportionem. e. x. ad. e. a. vel ad diametrum ipsius sphaeræ.

Cuius gratia reperitur primò proportio circûferentiæ maioris circuli ipsius sphaeræ ad eius diametrum, quæ ferè est vt. 22. ad. 7. ex Archimede.

Quo facto, inueniatur quantitas superficialis huiusmodi maioris circuli, quæ semper æqualis est producto quod fit ex semidiametro in dimidium circûferentiæ ipsius circuli, ex eodem Archimede. Et sic cognoscemus quartam partem superficiæ sphaericæ sphaeræ propositæ ex. 31. primi lib. de sphaera, & cyllindro Archimedis.

Deinde sumatur tertia pars producti, quod fit ex semidiametro in superficiem maioris circuli, & habebimus conum, cuius basis erit circulus maior, altitudo verò semidiameter propositæ sphaeræ ex. 9. duodecimi Eucli.

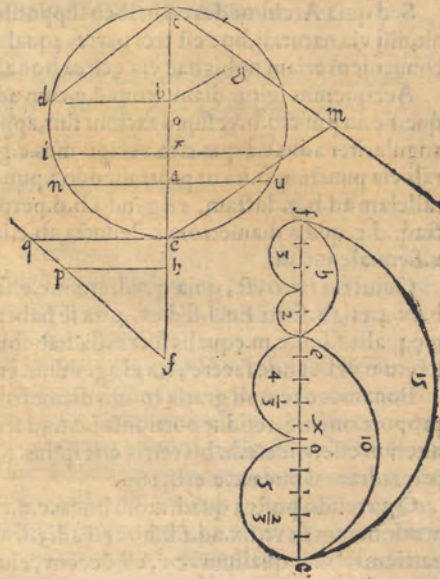
Quadruplum postea huiusmodi coni, erit quantitas soliditatis, seu corporeitas totius sphaeræ ex. 32. dicti lib. Archimedis.

Imagemur postea in sphaerica portione. n. e. u. lineâ. e. u. à sùmitate ad extremitatē basis, cuius. e. u. quantitatem cognoscemus, hoc modo scilicet, sumendo radicē quadratam producti. c. e. in. e. a. eo quod quadratum. e. u. æquale est quadrato a. u. & quadrato. a. e. ex penultima primi Eucli. hoc est producto quod fit ex. c. a. in. a. e. ex. 34. tertij eiusdē, & quadrato. a. e. hoc est producto, quod fit ex. c. e. in. e. a. ex. 3. secundi eiusdem.

Inuenta postea. e. u. ponamus eam vnus circuli semidiametrum esse, cuius superficialis quantitas etiam inueniatur, vt supra dictum est, quæ quidē æqualis erit superficiæ portionis n. e. u. ex. 40. primi li. Archimedis de sphaera, & cyllindro.

Hæc autem quantitas vltimo inuenta multiplicetur cum tertia parte semidiametri datæ sphaeræ, & habebimus soliditatem vnus coni æqualis aggregato soliditatis portionis. n. e. u. simul sumptæ, cū soliditate vnus coni, cuius axis fit. a. o. residuū semidiametri nostræ sphaeræ dempta. a. e. ba

L I      fis



sis verò eadem quæ est portionis, cuius diameter est. n. u. ex. 9. 12. Eucli. & ex. 42. id est vltima primi Archimedis de sphaera, & cylindro.

Nunc autem ex hoc aggregato iam vltimo dicto detrahatur conus, cuius. o. a. est axis et. n. u. diameter basis, qui quidem conus nobis cognitus est, cum. a. n. semidiameter eius basis, nobis cognita sit ex. 34. 3. Eucli. & sic quantitas eius basis, & ita tertia pars. a. o. eius axis, quæ multiplicata cum dicta basi, cuius. n. u. est diameter, producit dictum conum, qui quidem conus, vt diximus, demptus cum fuerit ex dicto aggregato, relinquet nobis soliditatem portionis. n. e. u. vnde cognoscemus proportionem istius portionis ad totam sphaeram propositam.

Sed cum nobis proposita sit proportio portionis. n. e. u. ad portionem. i. e. t. cognoscemus etiam soliditatem huius secundæ portionis. i. e. t. & similiter proportionem huius ad totam sphaeram, & ad residuum etiã ipsius sphaeræ hoc est portio. i. c. t.

Protrahatur nunc diameter. c. e. à parte. e. vsq; quo. e. f. æqualis sit. c. o. semidiametro sphaeræ, quæ quidem. f. e. diuidatur in puncto. h. ita vt proportio. f. h. ad. h. e. æqualis sit portio. i. c. t. ad portionem. i. e. t. quod quidem hoc modo efficitur. applicabimus lineam. f. q. (indeterminatam) cum. f. e. ad quemuis angulum in puncto. f. in qua accipiemus duas lineas. f. p. et. p. q. inuicem ita relatas, vt se habent in proportione duæ iam dictæ portiones, hoc est, vt. i. c. t. portio ad portionem. i. e. t. ducento postea. q. e. et. p. h. parallelam ad ipsam. q. e. diuisam habebimus. f. e. in eadem proportione vt dictum est ex. 2. sexti, & 11 quinti Euclidis, vnde. c. e. f. et. f. h. nobis cognite erunt.

Oportebit nos nunc cognoscere quantitatem. c. x. hoc modo, videlicet, queramus quadratum, cuius. c. x. eius sit radix, cui quadratum lineæ. c. e. cognitum, ita sit proportionatum, vt est linea. x. f. ad lineam. f. h. quæ nobis cognita est, quod rectè factum erit ex eo, quod scripsit Archimedes in. 4. secundi de sphaera, & cylindro.

Sed quia Archimedes eo in loco supponit id, quod nec ipse, nec alius adhuc inuenit, nisi via naturali, hoc est tres partes æquales ex proportione data effici, non erit in conueniens etiam nobis hac via, circa hoc aliquid dicere.

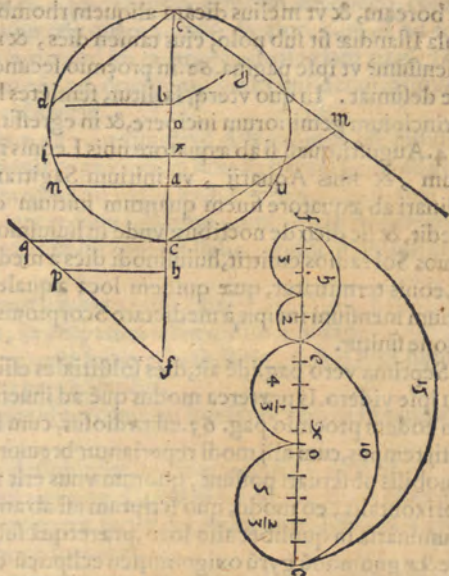
Accipiemus igitur diametrum. c. e. cum addita. e. f. eius semidiametro, diuidemus quæ. f. e. in puncto. h. vt supra factum fuit, applicabimus postea. c. m. indeterminatam angulariter ad. c. e. à qua. c. m. accipiemus. c. g. æqualem. f. h. queremus deinde naturali via punctum. b. ita ut protrahendo à puncto. e. (altero extremo diametri) e. m. parallelam ad. b. g. ductam, erigendo. b. d. perpendiculararem ad. c. e. in puncto. b. protrahatq; d. c. quæ à diametro. e. c. deducta ab. c. incohando vsque ad. x. relinquat nobis. x. f. æqualem. c. m.

Cuius rei ratio est, quia quadratum. c. e. se habet ad quadratum. c. d. vt. c. e. ad. c. b. ex. 4. et. 18. sexti Eucl. sed ex. 4. ita se habet. m. c. ad. c. g. vt. e. c. ad. b. c. & cum sit. c. g. æqualis. f. h. si. c. m. æqualis fuerit. f. x. habebimus propositum. Quod si quis per discretum vellet hoc facere, ita ei agendum erit.

Ponamus exempli gratia totum diametrum. c. e. propositæ sphaeræ esse ut decem, proportionemq; residuæ portionis. i. c. t. ad secundam. i. e. t. hoc est. f. h. ad. h. e. sesquialteram esse, vnde. e. h. bis tertia erit ipsius. f. h. totaq; linea. c. f. erit. 15. et. f. h. erit. 3. & quadratum lineæ. c. e. erit. 100.

Querendo postea quadratum lineæ. c. x. cui quadratum. c. e. hoc est. 100. ita proportionatum sit vt. f. x. ad. f. h. hoc est ad. 3. si autem cogitauerimus. c. x. esse nouem partium talium qualium. c. e. est decem, eius quadratum erit. 81. et. x. f. erit. 6. partium talium qualium. c. f. est. 15. dicendo postea si. 100. dat. 81. (ex regula de tribus)

x. f. hoc est. 6. dabit. 4. integra cum. 86. centesimis, sed nos vellemus nobis prouenire tria, eo q̄ ita est. f. h. qua propter descendere nos oportebit à nouem ad. 8. & ab. 8. ad. 7. & à. 7. ad. 6. tunc inueniemus. c. x. oportere esse circiter quinque cum duabus tertijs, operādo postea ex regula de tribus, si dixerimus quando. 100. nobis dat. 32. cum nona parte integri, tunc nouem cum tertia parte integri dabit. 2. cū. 296. de. 300. hoc est. 2. cum circa. 49. quinquagesimis, quæ quidem quantitas, cum propinquissima sit lineæ. f. h. trium integrorum dicemus. c. x. esse quinque integrorum cum duabus tertijs partibus vnus integri, et. e. x. residuum, hoc est axem quæsitum portionis. i. e. t. esse circa. 4. integra cum tertia parte vnus integri.



DE ERRORIBVS THOMAE PORCACHII & Benedicti Bordonij in eorum insularijs.

*Excellentissimo D. Io. Baptista Famello Ciui Decurioniq̄ Taurinensi Philosopho, Medico, & in Accademia eius Ciuitatis Medicina Practica Ordinario, Primarioq̄, professori celeberrimo.*

**D**ij perdant tuas adeo molestas, & assiduas curas, quæ te nimis à suauioribus studijs distrahunt, & à nobis longius abducunt. Nam, ut tibi quiete, ita mihi ingentem adimunt voluptatem. Sed ne in aliquo erga te deficere videar, quæ tibi olim promisi, nunc mitto.

Negari quidem non potest, quin fuerit laboriosum opus Porcachij, & Benedicti Bordonij, hoc est insularium, qui rectè etiam fecissent, cum loqui eos oportebat de terminis spheræ ratione situs locorum, si seipfos alicuius excellentis Cosmographi consilio submisissent. Considera quæso, quomodo admitti possit, id quod ait Porcachius initio sui operis, id est Islandiam sub Polo arctico iacere, inter austrum, & boream: omittamus etiam quod idem in Progmio lib. secundi, vbi ait Biarmiam, (& non Islandiam) esse sub dicto polo arctico: in eodemq̄ principio repetit ipsam Islandiam inter austrum, & boream per centum leucas Germanicas extendi, deinde versus occidentem, ea duo stupenda miracula conspici. Vide quæso, quomodo incolæ sub aliquo ex polis, habere possint occidentem, orientem, magistrum, austrum,

& boream, & vt melius dicam aliquem rhombum. Sed quomodo fieri potest, vt insula Islandiæ sit sub polo, eius tamen dies, & nox maior non sit longior spatio triū mensium? vt ipse pagina. 62. in proëmio secundi lib. affirmat, quamuis hoc à Bordo ne desumat. In quo vterq; fallitur, sentiētes huiusmodi diem ab ingressu Solis, in principium geminorum incipere, & in egressu à Leone terminari, id est à. 12. Maij ad 14. Augusti, quasi si ab æquatore finis Leonis ita declinaret, vt principium geminorum, & finis Aquarij, vt initium Sagittarij, nam ratio postulat, tantum declinari ab æquatore finem quantum initium diei, vbi maximus dies. 24. horas excedit, & sic dico de noctibus: vnde in huiusmodi regione, vbi per tres menses continuos Sol radios emittit, huiusmodi dies à medietate Tauri incipit, & in medietate Leonis terminatur, quæ quidem loca æqualem declinationem habent, & sic nox trium mensium incipit à medietate Scorpionis, & in medietate Aquarij, eadem ratione finitur.

Septima verò pag. idē ait, dies solstitiales esse circa. 24. Iunij. Q̄, an tūc esset verū, tu ipse videto. Is præterea modus quē ad inueniēdū orientē, & occidētē præscribit in eodem proëmio pag. 63. est rædiosus, cum semper expectare nos cogat æquinotij tempus, cum alij modi reperiantur breuiores, qui in qualibet reuolutione primi mobilis obseruari possunt, quorum vnus erit mediante inuentione lineæ meridiane horizontalis, eo modo, quo scriptum est ab antiquis mediante Sole, aut Luna, quæ luminaria in quolibet alio loco, præterquā sub polo efficiunt, vt extremitas vmbre rectæ gnomonū gyrū oxigoniū, seu eclipticū ducat, id est in ijs locis, quorū zenit est inter polum, & circulum arcticum, quemadmodum facit, vt alijs, existentibus ipsis luminariibus extra æquatorem, & circulos arcticos gyrum hyperbolicum reddant. Sed id quod eidem Porcachio impossibile esse apud eos, qui habitant sub polo videtur, id est vt multis rationibus, vt ipse dicit, fieri non possit, ut fiat immediata quædam, & subita mutatio à continuo die ad continuam noctem absque eo quod ijs, saltem semel concessa sint dies, & nox terminata duodecim horarum, est magis ad mirandum impossibile, quod imaginari possimus, nam necessarium esset, ut orizon-habitatorum sub polo secaret æquatorem contra id, quod superius admiserat, id est orizotē Biarmiæ, esse eūdē cū circulo æquinotiali. Vide etiam quid is ab antiquis colligat, loquens de iis, quæ in insula Taprobana ad finem pag. 186. admirabilia sunt, scribens eiusdem insule habitatoribus, Lunam super terram non apparere ab octauo usque ad decimum sextum diem: præterquam, quod etiam scribit, in eadem insula, tramuntanam non uideri, quod falsum est, quia hæc à polo arctico circiter quatuor gradibus distat nostris temporibus. unde ab ijs qui sunt sub æquatore, cum ea supra orizontem est, conspici potest, cum iisdem singulis diebus oriatur, & occidat. Idem etiam pro re admirabili scribit, uideri Canopum, qui à polo antarctico plus quam quadraginta gradibus distat.

*De erroribus Lucilli Philalthei.*

A D E V N D E M.

**Q**uod Lucillus Philalthus tam eximius Mathematicus sit, ut ipsum Antonius Berga facit, ego quidem non uideo. In suis enim commentariis de Cælo, dicit primum, Pyramidem, quæ inter corpora regularia primum locum tenet,  
fex

sex basibus constare, pag. 15. 583. 632. et. 647. Omitto errorem ab eodem commissum in fine pag. 39. ubi oleum grauius esse quam aquam. fatetur, cum id ad res mathematicas non spectet: Omitto etiam quod idem neget astrologiam pag. 74. 79. & quod etiam dicat pag. 89. Deum esse ad orientem, non considerans aliquibus populis nostrum orientem esse occidentem.

Quod idem ait pag. 241. Astrologiam esse antiquiorem Astronomia est falsissimum, quia iudicialia semper præsupponit cognitionem situs stellarum, quæ ab Astronomia petitur. Mouebit tibi risum quod ait pag. 307. his verbis.

Verum propriè media dicitur illa, quæ rectam spheram omninò habet, quæ eundem polum orientis & mundi obtinet, quæ orientem habet diuidentem spheram æquè secundum angulos rectè.

Paulo inferius continuans sermonem de sphaera recta, ait.

Et nisi tumor terræ, & gibum esset, ijs perpetuus esset dies sine nocte.

Linea verò. 56. ait habitatores spheræ rectè habere. 4. solstitia, sese ipsum huius rei planè ignarum prodens. 310. autem pag. sic scribit.

Quoniam repercutiuntur radij, & per idem centrum transeunt, ob id stupam appositam centro radius accendit.

Quem quidem errorem ab Euclide desumit, et. 15. linea pag. 636. repetit.

Si vis ridere, legito. 16. primas lineas. 357. pag. Quod idem deinde dicat circa finem. 396. pag. lucem esse substantiam corporis lucidi & corpoream, subijciam tuo iudicio, vt etiam quod ait. 397. pag. his verbis vtens.

Idcirco animalia illa, quæ nocte vagantur perpolita, dum volant, aerem terunt nocturnum, & fulgent.

Et pag. 398.

Multitudo radiorum non admodum facit ad excitandum calorem si solum incidat sine repercussu, nec recta incidere iuuerit.

Quod falsum est cum radius incidens longè magis quam reflexus calefaciat. In fine autem. 405. sic scribit.

Sol in ortu & in occasu longius apparet, iccirco reuolui creditur. Hinc etiam in abside stare putatur, & in opposito absidis, vnde solstitia vocant, sed nobis in Cancro, antipodibus verò in Capricorno tum Sol abesse longius apparet vtrisque.

An hoc quid peius dici potest? Circa vero. 40. lineam pag. 459. sic scribit.

Si enim alij planetæ, & stellæ fixæ reciperent à Sole lumen, dum accederent ad Solem, vel recederent, aut contra, Sol ad eas appropinquaret, & abscederet, easdem lucis vicissitudinis subiret, quas Luna.

Hoc autem nondum deprehensum est, quin etiam Mercurius, Venus, suo interposito, Solem occultarent nobis, vt Luna.

Paulo inferius sic ait. Rursus æquè Saturnus, Iupiter, Mars, subire deliquium, more Lunæ, aut saltem obiectu terræ inter Solem & ipsos, quia tum ob interpositam terram non possent haurire lumen à Sole.

Hæc verò omnia, talia sunt, qualia ab ijs qui incipiunt intelligere spheram non proferrentur. Omittamus, quod ait deinde.

Accedit quod si astra lumen à Sole acciperent eiusdem caloris essent. Itaque omnia sicarent, & nulla essent frigidæ constitutionis contra Astrologos.

Quia hac ratione, Luna, quæ negari non potest, quin ab ipso Sole lumè accipiat, eiusdem caloris esset cum eodem Sole. Sunt ea etiam ridenda, quæ idem ait pag. 460. lineis. 18. 19. 23. 26. 27. 29. quasi ea lux infinita (vt ita dicam) magni Solis, non

in

in alium finem sit effecta quam ad illuminandam superficiem huius excrementi ipsius vniversi ad vtilitatem hominum, imò, vt rectius dicam, animalium. vide etiam pag. 632. et. 633. vbi Aristotelem de implendo loco non intellexit, cum citet spheram, loco pyramidis, & inter. 46. et. 47. lineas dicat quadratum esse quid multiplex, cum sit vnicum tantum in specie, quia species est quadrilateri, & quadranguli, sed vbi in. 6. linea pag. 633. ait.

Item hexagonus.

Magnum errorem committit, vt etiam cum. 12. linea. 636. pag. scribens. Pyramidis, siue planum, siue solidum, habet acutissimum, & in. 2. libr. de anima pag. 215. dicat de die posse videri stellas in speculo posito in vase aqua pleno, quod reuera est valde absurdum. Alios eiusdem errores tibi non patefacio, quia iam nihil amplius orij mihi est, sed eos tu ipse perspicere, & cognoscere facile poteris, & multo plures quidem, quam putas.

*Cur maius lumen extenuet minus.*

PIRRO DE ARZONIS.

**E**X tuis literis intellexi id, quod etiam sine ijs exploratum mihi erat. Sed concedo tantum esse dicere vbi est maius lumen, minus non discerni, quantum inter diu stellas non videri: immo est etiam magis vniuersale, quia idem multis aliis luminibus, præter ea quæ sunt stellarum, ea ratione contingit, quia ingrediente per pupilam, tam lumine maiori, quam minori, reflexum ipsius maioris in oculo, in situ minore, efficit, vt ipsum minus confundatur, & distingui nequeat, quemadmodum aperte cognosci potest in aliquo cubiculo, cuius parietes dealbati sint, in quo, vnicum tantum sit exiguum foramen, per quod aliqua lumina reflexa ab obiectis extrinsecis intra ipsum cubiculum ingredi possint, vnde imagines obiectorum in parietibus conspiciuntur, sed si per idem foramen ingrederetur etiam primarius radius Solis, reflexus huiusmodi radij efficeret, vt dictæ imagines, magis aut minus euanescerent, prout dictus reflexus radij solaris, maiori, minorive vi polleret.

Ad hoc tamen propositum, nolo tibi silentio inuolui mirabilem quendam effectum eiusmodi rei. Hoc est vt fiat foramen illud rotundum, magnitudinis tamen vnus specilli, quod foramen obturetur mediante vno illorum specillorum, quæ profenibus (non breuis visionis) conficiuntur, hoc est quorum ambæ superficies conuexæ sunt, non autem concauæ. Deinde opponatur folium album papiri, adeo distans à foramine, vt extrinseca obiecta in eo appareant. Quæ quidem obiecta si à Sole illustrata fuerint, tam clara, & distincta videbuntur, vt nihil pulchrius delectabiliusq; videri poterit, inuersa tamen. Sed si ea directa videre voluerimus, hoc optime faciemus, mediante reflexione alicuius speculi plani.

*Cur hyems valde frigida sequatur æstatem in qua calor vigerit.*

NOBILISSIMO, NECNON INGENIOSISSIMO  
Gabrieli Buschæ, Mediolanensi.

**Q**UOD dixi hyemem valde frigidam sequi æstatē, in qua calor vigerit, inde nascitur, quia calor terræ, aquæ, & æris, non est naturalis horum corporum, vt est frigus, cum calor à Sole procedat, qui ea calefacit suo lumine, vnde quod æstate Sol præter modum calefaciat terrā, ideo cōtingit, quod minora impedimēta contraria sortiatur, & cum eandem postea deserit, ad aliam partem æquatoris transmigrās terra ad suam qualitatem reddit, maiori cum impetu, eo modo, quo res in motibus localibus naturalibus, qui etiā terminos sibi præfixos, & constitutos excedunt, hinc etiā hyeme fit glacies, ex calefacta prius aqua, quæ durior postea est atque frigidior alia. Aestas etiā quæ sequitur hyemem valde frigidam, non erit admodum calida, quia Sol inueniens contrarium naturale valde potens, non tam facile illud pellere potest, vnde etiā si in Geminis, Cancro, & Leone, moram trahat, non sufficit tamen ut magnum calorem imprimere possit. Vnde sequitur duas æstates quarum una sequatur aliam, in eodem loco, uehementi calore præditas esse non posse, quemadmodum nec duas hyemes excessiuo frigore, remotis tamen accidentibus uentorum, pluuiarum, & niuium.

QVOD MALE SENSERIT NICOLAUS TARTALEA  
circa attractionem machinæ tormentalis.

A D E V N D E M.

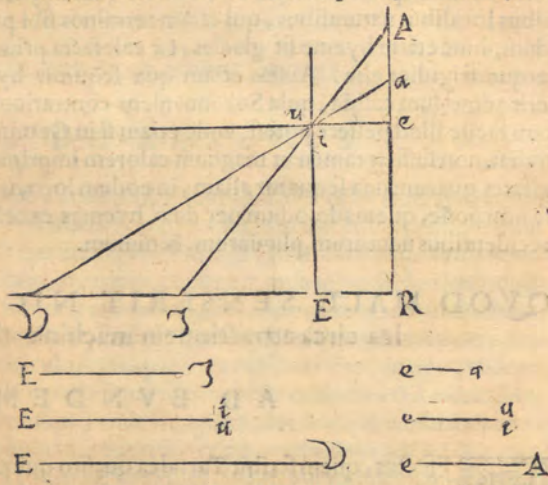
**E**ffectus, quem scribit Tartalea quæsito quinto primi lib. necnon quæsito 21. et. 24. maxima cum ratione esse uidetur, non tamen ea quam ipse in quinto profert, quia uerum non est, vt quanto aliquid fit calidius, tãto uehementius attrahat, eo quod si etiā huiusmodi res, in eodem calore, in quo semel reperitur, firma maneret; neque attraheret, neque aliquid impelleret. Nam dum aliquod corpus calefit, dilatatur, & per consequens circumcirca undiq; trudit; & partes uasis debiliores cedunt. dum uerò dictum corpus refrigeratur, restringitur, & dum in unum cogitur, si reperiat in uase, quod aer, aqua, aut aliud aliquod corpus ingredi nequeat, dictum uas à quo circumdatur frangit, ne aliqua pars loci uacua remaneat, sed si aliquod corpus ingredi potest, illud ipsum ad se attrahit, quemadmodum uidere licet in cucurbitulis. Vnde sequitur eam propositionem, qua dicitur, calidi est attrahere, ueram non esse, quia si hoc fieret, quanto aliquid calidus efficeretur, tanto magis attraheret, & e contra, cum tamen planè contrarium appareat, cum quanto magis aliquid calefit, tanto uehementius impellat, & quanto magis frigeat, tanto plus attrahat. Quapropter uerius dicemus, frigidum esse attrahere, calidum uerò expellere, quamuis per accidens. Ex quo sequitur, ut quanto calidior facta fuerit materia aliqua, aliquo loco determinata, redeundo postea ad suam priorem frigiditatem, tanto minori loco indigeat, similiter etiā è conuerso accidit, ut quanto frigidior reperitur talis materia, tanto maiori loco, postea

stea egeat ipsa ualde calefacta. Quod Tartalea in quinto quæsto non animaduert erat.

*Solutiones aliqua, circa alimetriam.*

A D E V N D E M.

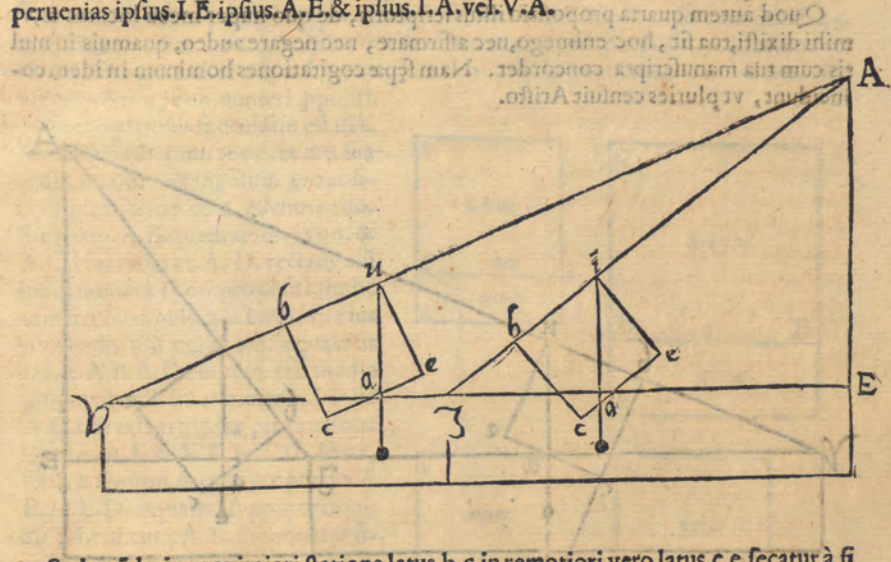
**T** Vas literas accepi, tuasq; dubitationes consideravi, quas quidem non inutiles inueni, quo uerò ad primam, dico te oportere illud Theorema speculari ordine huiusmodi methodi, uidelicet quod quotiescunq; habuerimus angulū aliquē cuiusuis amplitudinis, puta. A. R. V. cuius duo latera. R. A. et. R. V. indeterminata intelligantur, si ab aliquo puncto inter ipsas posito, puta. u. quod etiam uocetur. i. duæ fuerint. 4. lineæ ipsi dictis lateribus, hac scilicet cōditiōe, qđ duæ ex dictis. 4. sint parallele ipsi laterib<sup>9</sup>, puta u. e. et. u. E. reliquæ uero duæ seccent ipsa latera, ut V. u. a. et. I. u. A. Dico nunc proportionem. e. A. ad. e. a. eamdem esse, quæ. E. V. ad. E. I. Nam scimus proportionem E. i. ad. E. i. eandem esse quæ e. i. ad. e. A. ex similitudine triangulorū, similiter proportionē. E. u. ad. E. V. eadē quæ e. a. ad. e. u. quare aggregata ex istis erunt inuicem equalia, uel si maus ex e qua proportionalitate, quod idem est, ita se habebit. E. I. ad. E. V. ut. e. a. ad. e. A.



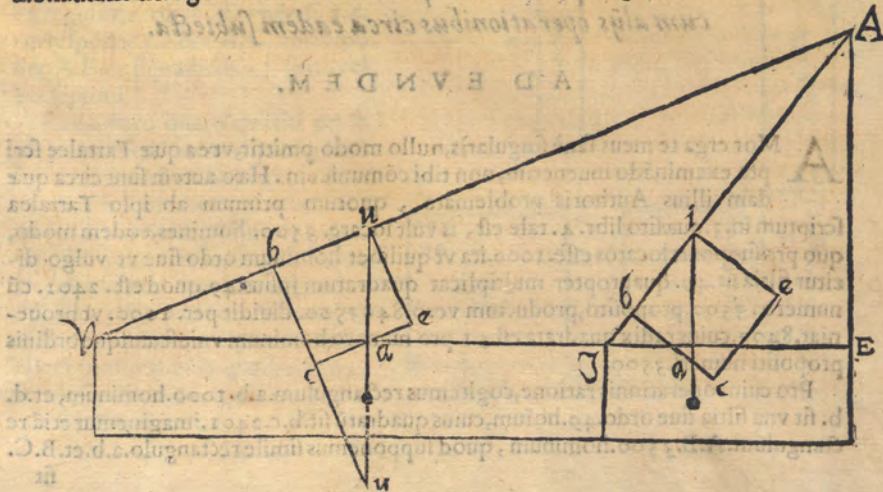
Supposito nunc plano horizontali. V. E. Altitudineq; inaccessiblei. A. E. Duæ uerò stationes oculorum sint. V. et. I. lineæ autem uisuales sint. V. A. et. I. A. Et quadratum geometricum sit. b. e. Supponatur nunc pro prima dubitatione, quod in ambabus stationibus filum perpendiculare seccet latus. e. c. non autem. b. c. (nam quando in ambabus stationibus filum seccat latus. b. c. nullum tibi dubium oritur, imo manifestè patent partes lateris. b. c. terminatas à. b. & à filo proportionales esse. V. E. & I. E. sumpto. E. pro. b. et. I. V. pro punctis seccatis à filo, ex euidēti similitudine triangulorum quadrati cum triangulis. A. E. V. et. A. E. I.) Sed cum in presenti casu reperiatur triangulum. u. e. a. minus, in statione remotiori, simile triangulo maiori. V. E. A. & triangulum maius. i. e. a. proximioris stationis, simile triangulo minori. I. E. A. (quod in alio iam dicto, casu non accidit, ut unum triangulorum, minus scilicet, simile sit uno triangulorum, maiori scilicet & è conuerso) Non omnino absque ratione dubitas quo pacto fieri possit ut. a. e. remotioris stationis ad. a. e. propinquioris ita se habeat quemadmodum. I. E. ad. E. V. Quapropter si præcedentem figuram diligenter-



genter inspexeris, omnis tua dubitatio euanesct. in qua figura apertè vi debis correspondentiam talium triangulorum inter se, nec magis, nec minus quam in infra- scripta hic figura cernere licet, quamuis in hac, triangula quadrati, separata sint ab imaginarijs. A.E.V. et. A.E.I. in supradicta vero coniuncta, & inuicem communican- tia in puncto. u. i. quod quidem nihil refert. Dempta igitur. a. e. minori ex. a. e. ma- iori, reliquum ita se habebit ad. a. e. minorem, vt, V. I. ad. I. E. quod nunc tibi clarè patebit. Vnde ex te poteris ordinem operationis profèqui, vt in cognitionem peruenias ipsius. I. E. ipsius. A. E. & ipsius. I. A. vcl. V. A.



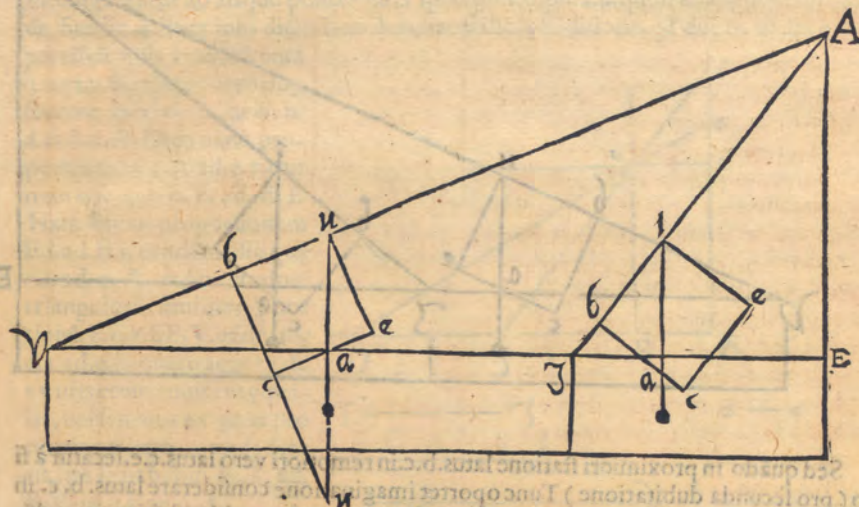
Sed quādo in proximiori statione latus. b. c. in remotiori vero latus. c. e. secatur à fi- lo ( pro secunda dubitatione ) Tunc oportet imaginatione considerare latus. b. c. in remotiori statione distentum esse vsque ad filum in puncto. n. vbi videbis triangulū. u. b. n. simile triangulo. A. E. V. ita vt. i. b. a. simile suo. A. E. I. reperitur, vbi tam in vno



Mm quam

quam in altero. i. b. et u. b. correspondebit ipsi. A. E. et. b. n. ipsi. E. V. et. b. a. ipsi. E. I. quapropter iubeo, vt quæras quantum sit latus. b. n. ex regula de tribus, dicens si. a. e. tribuit mihi. e. u. quid mihi dabit. u. b. eo quod. a. e. u. simile est. u. b. n. reperto autem latere. b. n. ex quo dempto. b. a. breuioris distantie, residuum respondebit ipsi. I. V. vt scis, vnde prosequendo operationem tibi cognitam, obtinebis intentum, hoc est cognosces reliqua interualla. Nihil enim miror demonstrationem Tartaleæ circa huiusmodi operationem te minime satisfecisse.

Quod autem quarta propositio illius scriptoris, de quo nuper mecum locutus es, vt mihi dixisti, tua sit, hoc enim ego, nec affirmare, nec negare audeo, quamuis in multis cum tua scriptura concordet. Nam sepe cogitationes hominum in idem coincidunt, vt pluries censuit Aristo.



*Demonstrationes quorundam problematum Nicolai Tartaleæ cum alijs operationibus circa eadem subiecta.*

A D E V N D E M.

**A** Mor erga te meus sanè singularis, nullo modo pmittit, vt ea quæ Tartaleæ scripta examinãdo inuenerim, non tibi cõmunicem. Hæc autem sunt circa quædam illius Authoris problemata, quorum primum ab ipso Tartalea scriptum in. 3. quæsito libr. 4. tale est, is vult locare. 3500. homines, eodem modo, quo præsupponit locatos esse. 1000. ita vt quilibet hominum ordo siue vt vulgo dicitur filia sit. 49. quapropter multiplicat quadratum ipsius. 49. quod est. 2401. cù numero. 3500. proposito, productum verò. 8403500. diuidit per. 1000. vt proueniat. 8403. cuius radix quadrata est. 91. pro numero hominum vniuscuiusque ordinis propositi numeri. 3500.

Pro cuius operationis ratione, cogitemus rectangulum. a. b. 1000. hominum, et. d. b. sit vna filia siue ordo. 49. hoium, cuius quadratũ sit. b. c. 2401. imaginemur etiã rectangulum. A. B. 3500. hominum, quod supponemus simile rectangulo. a. b. et. B. C. fit

fit quadratum ipsius. D. B. Nunc supponendo. A. B. simile. a. b. clarum erit ex diffinitione similium figurarum, quod eadem proportio erit. A. D. ad. D. B. quæ. a. d. ad. d. b. hoc est. A. D. ad. D. C. ut. a. d. ad. d. c. hoc est. A. B. ad. B. c. ut. a. b. ad. b. c. ex prima sexti, vel. 18. seu. 19. septimi, tunc cum dixerimus si. a. b. ita responderet ad. b. c. ergo. A. B. correspondet etiam ita ad. B. C. quare ex regula de tribus rectè fit multiplicando. A. B. per. b. c. productum verò diuidendo per. a. b. ex. 15. sexti vel. 20. septimi, cuius prouentus radix quadrata erit quod quærebatur.

Sed aliter idem posse fieri specularus sum, hoc est multiplicando numerum. 49. ordinis. 1000. hominum cū radice quadrata numeri. 3500. propositi, productum verò diuidere per radicem quadratam ipsius. 1000. unde prouentus. 91. erit numerus vnus ordinis. 3500. numeri ppositi.

Cuius opationis speculario est ista.

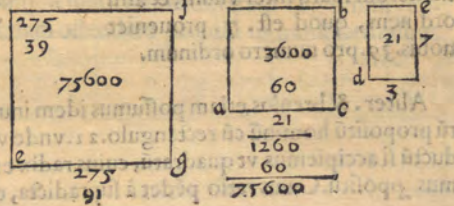
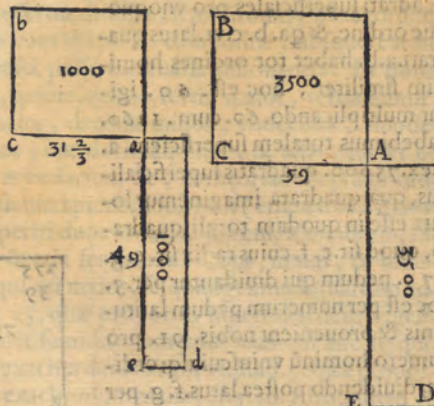
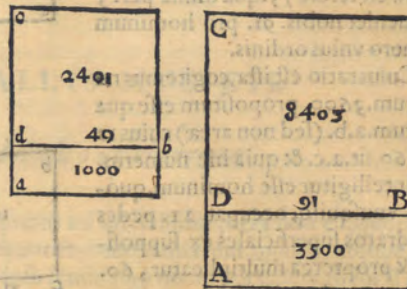
Sit. a. b. quadratum. 1000. et. a. e. sua radix et. a. d. rectangulum propositum ipsius. 1000. et. a. c. vnus ordo.

Sit etiam. A. B. quadratum. 3500. & A. C. eius radix et. A. D. rectangulū ipsius numeri. 3500. propositi, simile tamen rectangulo. a. d. et. A. E. eius vnus ordo Cū enim. a. b. æquale sit a. d. et. A. B. A. D. tūc. a. c. erit media proportionalis inter. a. e. et. e. d. & sic A. C. erit etiam media proportio

lis inter. A. E. et. E. D. per. 16. sexti, seu. 20. septimi, & quia proportio. A. E. ad. E. D. æqualis est proportioni. a. e. ad. e. d. cum. A. D. supponatur simile. a. d. ergo proportio. A. E. ad. A. C. æqualis erit proportioni. a. e. ad. a. c. quæ medietates sunt totorū æqualium, rectè igitur fiet si procedamus ex regula de tribus, dicendo si. a. c. correspondet. a. e. tūc. A. C. correspondet. A. E. ex supradictis. 15. sexti. vel. 20. septimi.

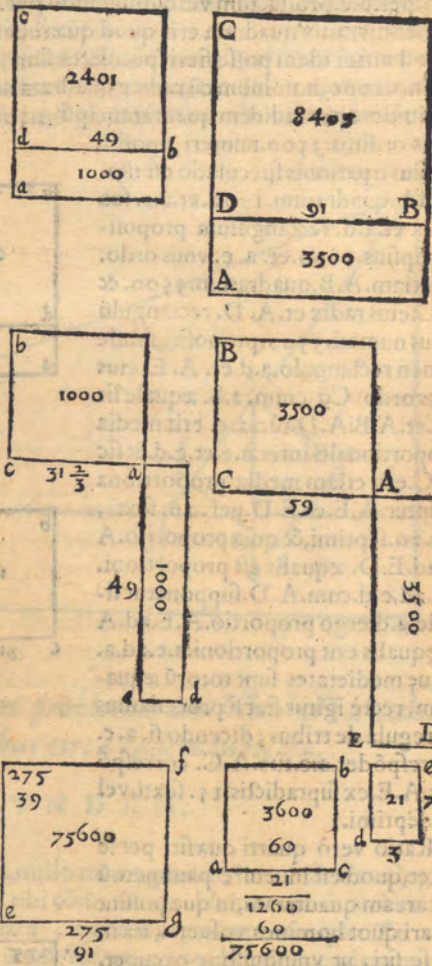
Ratio verò quarti quæsti per se patet, quod est inuenire pauimentū seu aream quadratam, in qua possint locari quot homines volueris, ita inter se siti, ut vnusquisque occupet. 7. pedes ipsius areæ in longitudinem et. 3. per latitudinem à lateribus.

Seu ex proposito hominum numero inuenire numerum ipsorum locabilem in aliqua area quadrata, ita, ut vnusquisque occupet. 21. pedes quadratos ipsius area.



Sed aliter idem fieri posse inueni, hoc est multiplicādo radicem quadratam propositi numeri hominum per. 21. & productum item multiplicando per eandem radicem, & huiusmodi producti radicem diuidendo per. 3. vnde prouentus esset numerus hominum vnius ordinis. Exēpli gratia proponantur. 3600. homines, multiplicabimus huiusmodi numeri radicem quadratam hoc est. 60. per. 21. hoc est per productum quod fit ex. 7. cū 3. & resultabit nobis. 1260. quod si multiplicabitur, per. 60. hoc est per eandem radicem, resultabit nobis. 75600. cuius producti radix quadrata est ferē. 275. qua diuisa per. 3 proueniet nobis. 91. pro hominum numero vnius ordinis.

Cuius ratio est ista, cogitemus numerum. 3600. propositum esse quadratum. a. b. (sed non areæ) cuius radix. 60. sit. a. c. & quia hic numerus. 60. intelligitur esse hominum, quorum vnusquisq; occupat. 21. pedes quadratos superficiales ex supposito, & propterea multiplicatur, 60. cum. 21. vnde nobis veniat. 1260. quadrati superficiales pro vnoquoque ordine, & q. a. b. c. vt. latus quadrati. a. b. habet tot ordines hominum similiter, hoc est. 60. igitur multiplicando. 60. cum. 1260. habebimus totalem superficiem. a. b. ex. 75600. quadratis superficialibus, quæ quadrata imaginemur locata esse in quodam totali quadrato, quod sit. e. f. cuius radix sit. e. g. 275. pedum qui diuidantur per. 3. hoc est per numerum pedum latitudinis & proueniet nobis. 91. pro numero hominū vniuscuiusq; ordinis, diuidendo postea latus. f. g. per numerum spatij inter vnum, & aliū ordinem, quod est. 7. proueniet nobis. 39. pro numero ordinum.



Aliter. & breuius etiam possumus idem inuenire, hoc est multiplicando numerū propositū hominū cū rectangulo. 21. vnde veniet nobis productū. 75600 quod productū si accipiemus vt quadratū, cuius radix erit. 275. quæ diuidatur p. 3. habebimus propositū. Cuius ratio pēdet à supradicta, eo qđ loco multiplicādi. a. c. (hoc est. 60.) per

60.) per. 21. deinde productū etiam multiplicare per. b. c. (hoc est. 60.) breuius erit multiplicare totum numerum. 3600. per. 21. cetera verò facere, vt diximus.

Sed vnaquæq; istarum operationum, aliquid imperfectionis patitur, eo quod cū aliquis cuperet quadratum perfectum superficiale habere, absq; aliquo defectu, vel excessu, aliquid aliud adhuc facere oporteret, hoc est, inuentum cum fuerit quadratum. e. f. cum suis radicibus. e. g. et. g. f. pedum. 275. vnaquæque, vt in dicto exemplo factum est, oportebit numerū quærere minorem ipso. 275. sed proximiorē mensurabilem ab. 3. & ab. 7. quod facillè fiet si diuiserimus. 275. per. 21. detrahendo fracta diuisionis ab ipso. 275. quæ quidem fracta in hoc exemplo sunt. 2. vnde remanebit. 273. pro numero laterum quadrati superficialis, in quo possent locari. 3549. homines, eo ordine quo supra dictum est, quorum scilicet vnusquisque obtineat. 21. pedes superficiales.

### DE INTERVALLIS MUSICIS.

*Cypriano Rorè Musico celeberrimo.*



**P**INIO Hæctoris Eufonij Cypriane mi dilectissime, vera non est, quod aliquis rectè possit intelligere rationes consonantiarum musicæ, absque cognitione illarum mediante ipso sensu, imo nemo pot calere theoriâ musicæ, nisi aliquo mō versatus sit in praxi. Quō enim cognosci poterūt quid nam sint diapason, diapente, diatesseron, ditonus, semiditonus, hexachordum maius, aut minus, & consonantiæ ex ijs cum diapason compositæ, absque earum praxi? vnde sequetur neq; etiam cognosci posse interualla dissonantia. Et purus practicus non intelliget quid sit octaua, quinta, quarta, tertia maior, tertia minor, sexta maior, sexta minor, decima maior, decima minor, vndecima, duodecima, decimatertia maior, aut minor, aut decimaquinta, & aliæ, ita vt ad comparandam perfectionem musicæ necessarium sit, & theoriâ & praxim addiscere. Cum præterea Ludouicus Folianus apertè monstrarit (etiam si id a diatonico sintono Ptolomei desumpserit) reperiri duos tonos, maiorem, & minorem, id est sesquioctauum, & sesquionum, & tria semitonia, maius, minus, & minimum, id est sesquiquintumdecimum, qui est maius, sesquiugesimumquartum id est minimum, & mediocre, vt. 27. ad. 25. quæ proportio superbipartiens vigesimaquintas appellatur, & cum cognouerit semiditonus consonantem esse sesquiquintum, ditonus sesquiquartum, & hexachordum minus, vt. 8. ad. 5. quæ proportio dicitur supertripartiens quintas, & hexachordum maius, vt. 5. ad. 3. hæc autem vocatur superbipartiens tertias; omnium simplicium consonantiarum cognitioni, extremam imposuit manum. Et quia tibi etiam ostendere promisi in modulationibus

hæc



hæc omnia interualla seruari, ideo ad te mitto septem hic subscripta exemplâ, in quorum primo, & secundo, inter diesim, et. b. in superiori, agnosces interuallum minimi semitonij, & si ibi sit diesis, tanquam terminus ad quem, et. b. tanquam terminus à quo: quod autem inter diesim et. b. sit semitonium minimum, faciliè agnosces si subtraxeris decimâ minorè à maiori, quâ facit superius cū inferiori, idest cū bassu.

Qua quidem modulatione tu etiam vsus es in cantilena illa, quæ Galica lingua incipit. Hellas comment. Eadem, ego quoque in meis cantilenis latino sermone compositis, quæ Moreta vocantur aliquando vsus sum.

Sed in tertio exemplo inuenies semitonium maius, necessariò genitum in superiori, si sextam maiorem cum bassu efficere volueris, quia tenor, à ditono cum superiori ad diapentem, & ad vnisonum cum bassu procedit, vbi quiescit, progrediendo postea bassus ad semiditonum cum tenore, tunc si à proportione huius septimæ, quæ est vt. 9. ad. 5. hoc est superquadrupartiens quintas demptum fuerit hexachordum maius, seu sexta maior, quæ est vt. 5. ad. 3. remanebit proportio. 37. ad. 25. quæ maior est quam. 32. ad. 30.

In quarto exêplo habebis semitonium minus in superiori, quod quidem remanet ex subtractione ditoni cõsonãtis ab diatessaron cõprehensa à superiori cum tenore.

In quinto exemplo videbis tonum minorem, & tonum maiorem successiue vnum post alium in tenore, detrahendo primo semiditonũ à diatessaron, quod superius facit cum tenore, vel detrahendo diapente ab hexachordo maiori, quod facit tenor cum bassu, vnde remanet tonus minor sesquiquonius, detrahendo postea diatessaron à diapente, quod superius facit cum tenore, remanebit tonus maior sesquioctauus.

In sexto exemplo deinde videbis tenorem ascendere per duos tonos minores successiue vnum post alium in tenore, si depseris semiditonũ à diatessaron cū superiori.

In. 7. exêplo demum videbis superiorè ascendere per duos tonos maiores successiue vnũ post aliũ, si dempseris diatessaron à diapente, quod facit tenor cū superiori.

De

The musical score consists of seven numbered measures, each containing musical notation on four staves. The notation includes various note values (minims, crotchets, quavers) and rests, illustrating the interval relationships described in the text. The staves are arranged in two pairs, with the top pair using a treble clef and the bottom pair using a bass clef. The notes are connected by stems, and some are beamed together. The overall structure is a sequence of seven measures, each demonstrating a specific interval relationship between notes on different staves.

## EPISTOLAE.

279

*De eodem subiecto.*

## A D E V N D E M.

**Q**Uod aliàs tibi dixi, verum est, quod necessarium nullo modo fit, vt modulando, desinat cantilena in eodem tono (quod Græci phthongum appellant) à quo incepit. immo necessariò semper ferè, altius, aut depræsius terminatur, per differentiam alicuius interualli æqualis, vel multiplicis ipsi commati sesquioctuagesimæ, quod quidem comma, quamuis cantabile non sit, insensibiliter tamen generatur, & toties ab aliqua parte ipsius cantilenæ possit dictū comma generari, versus acutum, vel graue, quod in fine ipsius cantilenæ, vocis phthongus reperitur distans à primo per interuallum alicuius toni sesquinoni, seu sesquioctauipius, minúsue, vt in subscripto exemplo clarè videre potes in prima figura, vbi superius à.g. primæ cellulæ ad.g. secundæ, interest vnum cõma, eo quod progrediens superius in prima cellula ipsius cantilenæ à quarta ad quintam cum tenore, ascendit per tonum sesquioctauum, à prima cellula deinde ad secundam, tenor ascendit similiter per tonum sesquioctauum cum transeat à quinta ad quartam, quod facit cum superiori, in secunda cellula postea, cum superius descendat à maiori sexta ad quintam, quod facit cum bassu, seu à quarta ad tertiam minorem, quod facit cum tenore, tunc descendit per tonum sesquinonum, ita quod non reuertitur ad eundè phthongum, vbi prius erat in prima cellula, sed reperitur per vnū cõma altius, qđ quidè cõma est differentia inter tonū sesquioctauū & sesquinonū, vt alias tibi demonstrauit.

Progrediendo igitur hoc modo, videbis quod cum tenor à secunda cellula ad tertiam transeat à tertia minori ad quartam, quod facit cum superiori, descendit per tonum sesquinonum, vnde in tertia cellula altius remanet quam in prima per vnū comma, in qua tertia cellula, cum iterum transeat superius à quarta ad quintam, & facit cum tenore, eleuatur per tonum sesquioctauum, prosequendo deinde tali ordine, videbis in quarta cellula cantilenam auctam per duo commata, in sexta, aut cellula per tria commata, in octaua verò per .4. commata, vnde hac methodo, si cantilena prolixior debito esset, vel si talia interualla frequentiora reperirentur, possit cantilena à principio ad finem differre per .9. commata, & plus etiam, quæ quidem inter-



interualla superant tonum sesquionum, & si essent. 10. commata superarent tonum sesquioctauum, eo quod aggregatum ex.9. commatibus continetur sub istis duobus terminis hoc est. 150094635296999121, et. 134217728000000000. quæ quidem proportio maior est proportione sesquinona, summa verò. 10. commatum continetur sub. 12157665459056928801. et. 10737418240000000000. quæ proportio maior est tono sesquioctauo, quod autem dico de ascensu cantilenæ, idem assero de eiusdem descensu, & hoc non tantum per interuallum illius commatis, quod est differentia toni maioris à minori, sed etiam per illud quod est differentia semitonij maioris à minori, vt in secundo exemplo hic subscripto videre est in descensu cantilenæ per comma & comma, vt differentia inter semitonia maiora & minora, vbi in prima cellula discedens bassus à quinta cum superiori, & ab vnifono cum tenore descendens ad tertiam minorem cum ipso tenore, facit cum superiori septimã maiorem, quæ est vt. 9. ad. 5. superquadripartiens quintas scilicet, à qua discedens postea superius, vt faciat cum bassu sextam maiorem, descendit per semitonium maius, à qua sexta maiori descendens bassus, & ascendens per quartam, efficit cum dicto superiori tertiam maiorem, à qua discedens superius, vt efficiat quartam cum ipso bassu (qui quidem bassus transit in tenorem) ascendit per semitonium minus, differens à semitono maiori per vnum comma, vnde cantilena remanet depressa per vnum comma. cum deinde idem faciat inter tertiam, & quartam cellulam, per aliud comma descendit, & sic toties facere posset, vt postremo valde deprimatur cantilena à primo phthongo.



Quod autem hic supradictum est, circa instrumenta artificialia non accidit, quia propter organa, & clauicimbula concordantur certo quodam ordine, ita vt omnes consonantia, excepta diapason, seu octaua, sint imperfectæ, hoc est, aut diminutæ, aut superantes à iusto, vt exempli gratia, omnes quintæ sunt diminutæ, quartæ verò sunt excessiue, quod quidem fit, vt tertiæ, & sextæ, non multum auribus dissonent, eo quod si quintæ omnes, & quartæ, perfectæ essent, tunc omnes sextæ, & tertiæ intollerabiles essent, & à perfectis differrent per vnum comma, quod manifestum nobis erit hoc modo, accipiamus tres diapentes, seu quintas, consequenter successiuas vnã post aliam, hoc est tres proportiones sesquialteras, quarum aggregatum erit vt. 27. ad. 8. quæ proportio, dicitur tripla supertripartiens octauas, & quæ à practicis appel-



appellaretur tertiadecima maior, vt exempli gratia, effct Gamaut cum secundo elami, tunc talis tertiadecima valde odiosa esset sensui auditus, à qua, si dempra fuerit diapason, seu octava, remaneret quoddam hexachordum maius, seu sexta maior, auribus valde inimica, sub proportione. 13. ad 8. sed hæc proportio differret à proportione superbipartientetertias perfecti hexachordi maioris, hoc est sextæ maioris consonantis, per proportionem sesquioctuagefimam, hoc est per vnum comma, quod quidem est etiam differentia aggregati trium sesquialterarum, à tertiadecima maiori consonanti, hoc est excessus proportionis triplæ supertripartientis octauas, supra triplam sesquiterciam, quæ est summa ipsius duplæ cum superbipartientetertias.

A tali summa igitur trium sesquialterarum efficitur tertiadecima maior dissonans excedens consonantem per vnum comma (cuius proportio est. 81. ad. 80.) quæ consonans continetur in proportione. 10. ad. 3. vt supra dixi.

Hæc igitur est vera ratio, propter quam debemus comma distribuere in organis & clauicymbalis, cum ab aggregato trium quintarum producat talis excessus supra perfectam, seu consonantem tertiamdecimam maiorem, quod quidem aggregatum, cum demptum fuerit à quintadecima, relinquet nobis tertiam minorem dissonantem, & mancam, per eundem excessum à consonanti. quæ quidem tertia minor dissonans subtracta à diapente seu quinta perfecta, relinquet nobis tertiam maiorem dissonantem, quæ consonantem excedit per eundem excessum commatis, & hæc demum tertia maior dissonans, dempra ex diapason, seu octava, relinquet nobis hexachordum minus, hoc est sextam minorem dissonantem, & mutilam à consonanti per eundem excessum commatis. De huiusmodi verò commatis distributione doctissimè scripsit Excellentissimus Zarlinus in secunda parte Institutionum Harmonicarum.

Sed quia sensus auditus non potest exactè cognoscere debitam quantitatem excessus, vel defectus, intendendo vel remittendo chordas instrumentorum, ideo hanc viam sequutus sum.

Sit exempli gratia, hic subscriptus ordo lignorum tangentium seu pinarum incipiens ab. G. desinens ad. g. ita quod inter ipsos terminos sit ea consonantia quæ vocatur vigesima secunda, quæ primus. b. inter. D. E. quod est nigrum ipsius Elami grauissimum, quod grosso modo facio consonans cum. G. grauissimo per sextam minorem, deinde cū ipso primo. b. ipsius elami concordo suum octauum & quintumdecimum, quo perfectius possum, deinde accipio. b. molle secundum ipsius. b. fabmi quod concordo cum. b. primo ipsius Elami per quintam imperfectam, deinde cum hoc. b. secundo ipsius bfabmi concordo secundum. f. per quintam similiter imperfectam, cum quo. f. postea concordo tertium. c. per similem quintam, quem tertium. c. postea conféro cū secundo. b. ipsius elami, ita quod inter se consonent per sextam maiorem tolerabilem, & si sic inuenio, tunc nihil muto has tres chordas hoc.

G A B b C \* D b E F \* g \* a b b c \* d b e f \* g \* a b b c \* d b e f \* g \*

1 1 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 3 3 4.

Namque est

est. b. secundum ipsius bfabmi, f. secundum, et. c. tertium, sed si dictum tertium. e. valde dissonans esset cum. b. secundo ipsius elami, tunc ipsum. c. intendo, aut remitto, quousque aliquo modo sit consonans per sextam maiorem aliquantulum excessiuam cum. b. secundo ipsius elami, cum quo postea. c. consonare aliquantulum facio. f. secundum per quintam defectiuam, & cum hoc demum. b. secundum ipsius bfabmi, quo facto concordo secundum. c. cum tertio per octauam, cum quo secundo. c. postea concordo tertium. g. per talem quintam, quod ipsum tertium. g. cum secundo. b. ipsius bfabmi consonet tolerabiliter per sextam maiorem aliquantulum excessiuam, deinde cum isto tertio. g. concordo tertium. d. per talem quintam, ita quod ipsum. 3. d. concordet tolerabiliter cum. 2. f. per sextam maiorem excessiuam, postea cum hoc. 3. d. concordo. 2. d. per octauam perfecte, cum quo. 2. d. postea concordo. 3. a. per quintam, ut in alijs factum est, ita ut cum. 2. c. consonet talis sexta maior, ut supra dictum est, cum quo. 3. a. postea concordo. 3. e. per quintam, ut dictum est, ita quod cum. 3. g. faciat sextam maiorem ut supra, postea cum hoc. e. concordo. 2. e. per octauam, cum quo concordo. b. quadrum tertium per quintam, ut dictum est, ita quod cum. 2. d. faciat sextam maiorem similem alijs superius dictis, cum quo. b. quadrato tertio concordo tertium nigrum ipsius. f. per quintam, ita quod cum. 3. a. faciat sextam maiorem, ut supra, deinde cum hoc concordo. 2. f. nigrum per octauam, cum quo, per quintam concordo 3. c. nigrum ita quod cum. 2. e. faciat sextam dictam, demum cum hoc concordo. 4. g. nigrum per quintam, ita quod faciat cum. 3. b. quadrato sextam dictam, & sic ad ultimam quintam peruenio, supra quod. g. nigrum nulla quinta amplius reperitur; postea cum istis chordis concordo per octauas omnes alias ab acutis ad graues.

G A B b C \* D b E F \* g \* a b b c \* d b e f \* g \* a b b c \* d b e f \* g \*  
 I I I I I I I I I I 2 2 2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 3 3 4 4.

Valde etiam admiratione dignum est, quod perfectiores quæque consonantia, ita in harmonica diuisione sibi inuicem conueniant, ut diapason cum diapente, cum diapason diapente, cum ditono, cum hexachordo maiori cum bisdiapason, cum decima septima maiori. Nam in ipsa diapason, harmonicè locatur diapente in parte grauiori, & diatessaron in acutiori. In diapente verò harmonicè locantur ditonus maior in parte grauiori, & tonus minor in acutiori. In ditono harmonicè locantur tonus maior in parte grauiori, & tonus minor in acutiori. In hexachordo maiori, harmonicè locantur diatessaron in parte grauiori, & ditonus in acutiori. In diapason diapente, harmonicè locantur diapason in parte grauiori, & diapente in acutiori. In bisdiapason, harmonicè locantur decima maior in parte grauiori & hexachordum minus in acutiori. In decima septima maiori, harmonicè locantur diapason diapente in parte grauiori, & hexachordum maius in parte acutiori. Ita quod tonus sesquioctauus in ditono, proportionalis est ipsi ditono in diapente. Tonus verò sesquiquinonius in ipso ditono, proportionalis est triemitonio, vel sesquitonio seu semiditono (quod idem est) in diapente. Ditonus autem in diapente, proportionalis est ipsi diapente in diapason. Sesquitonius verò in diapente, proportionalis est diatessaron in diapason. Et sic de singulis. Ita quod tonus sesquioctauus in ditono, ditonus in diapente, diatessaron in hexachordo maiori, diapente in diapason, diapason in diapason diapente, decima maior in bisdiapason, diapason diapente in decima septima

septima maiori, omnia sibi inuicem sunt proportionalia, idem etiam dico de reliquis partibus, cum relatæ fuerint ad sua tota.

Nec alienum mihi videtur à proposito instituto, speculari modum generationis ipsarum simplicium consonantiarū; qui quidem modus fit ex quadam æquatione percussionum, seu æquali concursu vndarum aeris, vel conterminatione earum.

Nam, nulli dubium est, quin vnifonus sit prima principalis audituq; amicissima, nec non magis propria consonantia; & si intelligatur, vt punctus in linea, vel vnitas in numero, quam immediate sequitur diapason, ei simillima, post hanc verò diapente, cæteræq;. Videamus igitur ordinem concursus percussionum terminorum, seu vndarum aeris, vnde sonus generatur.

Concipiatur igitur mente monochordus, hoc est chorda distenta, quæ cum diuisa fuerit in duas æquales partes à ponticulo, tunc vnaquæq; pars eundem sonum proferet, & ambæ formabunt vnifonum, quia eodem tempore, tot percussiones in aere faciet vna partium illius chordæ, quot & altera: ita vt vndæ aeris simul eant, & æqualiter concurrant, absque ulla interfectione, vel fractione illarum inuicem.

Sed cum ponticulus ita diuiserit chordam, vt relicta sit eius tertia pars ab vno latere, ab alio vero, duæ tertie, tunc maior pars, dupla erit minori, & sonabūt ipsam diapason consonantiā, percussiones verò terminorum ipsius, tali proportione se inuicem habebunt, ut in qualibet secunda percussione minoris portionis ipsius chordæ, maior percutiet, seu concurret cum minori, eodem temporis instanti, cum nemo sit qui nesciat, quod quo longior est chorda, etiam tardius moueatur, quare cum longior dupla sit breuiori, & eiusdem intensiois tam vna quam altera, tunc eo tempore, quo longior vnum interuallum tremoris perfecerit, breuior duo interualla conficiet.

Cum autem ponticulus ita diuiserit chordam, ut ab uno latere relinquatur duæ quintæ partes, ab alio verò tres quintæ, ex quibus partibus generatur consonantia diapente; tunc clarè patet, quod eadem proportione tardius erit vnum interuallum tremoris maioris portionis, vno interuallo tremoris minoris portionis, quam maior portio habet ad minorem; hoc est tempus maioris interualli ad tempus minoris erit sesquialterū; quare non cōuenient simul, nisi perfectis tribus interuallis minoris portionis, & duobus maioris; ita quod eadem proportio erit numeri interuallorum minoris portionis ad interualla maioris, quæ longitudinis maioris portionis ad longitudinem minoris; vnde productum numeri portionis minoris ipsius chordæ in numerum interuallorum motus ipsius portionis, æquale erit producto numeri portionis maioris in numerum interuallorum ipsius maioris portionis; quæ quidem producta ita se habebunt, vt in diapason, sit binarius numerus; in diapente verò senarius; in diatessaron duodenarius, in hexachordo maiori quindenarius; in ditono vicenarius, in semiditono tricenarius, demum in hexachordo minori quadragenarius: qui quidem numeri non absque mirabili analogia conueniunt inuicem.

Voluptas autem, quam auditui afferunt consonantiæ sit, quia leniuntur sensus, quemadmodum cōtra, dolor qui à dissonantijs oritur, ab asperitate nascitur, id quod facile videre poteris cum conchordantur organorum fistula.

## DE IVSTITIA COMMVTATIVA.

*Francisco Ferrario Ancisa Iurisconsulto senatoriq; apud  
subalpinos grauisimo.*



**S**AEPIVS inter nos dum oportunitas vicinarum ædium, & amoris mutui vis, ad familiaria trahunt colloquia ego de meis mathematicis, tu de tuis legibus, in quibus tractandis magnum tibi nomen comparasti loquuti sumus. Cum vero nonnunquam de mirabili iustitiæ commutatiuæ instituto non ingratus incidisset sermo, dixi modum, quo formam suam à proportionalitate arithmetica dissociata, & non a coniuncta desumat, à nemine literis proditum esse, libet autem nunc per otium latius explicare. dixi enim à dissociata, & non coniuncta proportionalitate, quia in coniuncta, seu continua nullo pacto fieri potest talis commutatio, cum semper quatuor terminos ad minus transeat, vt nunc videbimus.

Exempli gratia, Petrus ex suis bonis tribuat Ioanni aliquid valoris quinquaginta aureorum.

Vnde priusquam Ioannes aliquid ex suis bonis retribuat Petro, bona ipsius Petri diminuta erunt per quinquaginta aureos, bona verò ipsius Ioannis, aucta totidem aureis.

Ecce nunc quo pacto constituti sunt. 4. termini in proportionalitate arithmetica, per quos fit talis permutatio, sed nondum æquata, nisi fiat æqualis retributio à Ioanne ad Petrum, vt videbimus.

Cogitentur itaque. 4. termini arithmetice proportionales. C. A. B. D. Ita quod. A. mediante significentur bona Ioannis. B. vero Petri, prius quam Petrus aliquid ex bonis suis tribuat Ioanni. Tunc Petrus secat partem vnã ex. B. eamq; dat ipsi Ioanni, vnde ipsi Petro remanet. D. Ioanni autem. C. quatuor igitur termini constituti sunt. B. D. C. A. quorum. B. primus. A. quartus. C. vero tertius. D. aut secundus, sed B. et. A. sunt in sua naturali mediocritate absque defectu vel excessu sui ipsius. Non ita tamen se habet. C. et. D. quia. D. deficit. C. autem excedit à sua priori quantitate. Nihilominus isti. 4. termini constituti sunt in ipsa arithmetica proportionalitate, nam eadem quantitate qua. D. diminuta est à. B. eadem. C. aucta est supra. A.

Sed quia. B. et. A. tantummodo iusti sunt termini. C. verò et. D. iniusti, vt ad suam priorem æqualitatem reuertantur, oportebit ex. C. secare aliquam partem æqualis valoris ei, qua. C. superat. A. vel qua. D. minor est. B. & ipsam partem addere ipsi. D. vt bona Petri reuertantur ad priorem suam quantitatem ipsius. B. & bona Ioannis remaneant æqualia. A. vt prius.

Quare necessarium non est, vt talis proportionalitas sit coniuncta ( vt inquit Eufratius seu Michael Ephesius, super quinto capite libr. quinti Ethicorum ) tribus terminis contenta, imò oportet ut ipsa dissociata sit, ut diximus, vbi non est necesse quod. A. æqualis sit. B. aliquo modo.



DE

DE MOTV MOLAE, ET TROCHI, DE AMPVL-  
lis aquæ, de claritate aeris, & Lunæ noctu fulgentis, de æter-  
nitate temporis, & infinito spacio extra  
Cælum, Cœliq; figura.

*Illust. Ioanni Paulo Capra Nouariensi Sabaudia Ducis hospicij  
Magistro, viro ingenij præstantia, & morum candore,  
non minus quam familia nobilitate conspicuo.*



I vera esset animorum illa transmigratio quam sibi Italicæ sapientiæ Pater Pythagoras effinxerat, tuam, meamq; existimarem animam canis, quandoque venatici fuisse.

Quæris à me literis tuis, an motus circularis alicuius molæ molendinæ, si super aliquo d punctum, quasi mathematicū, quiesceret, posset esse perpetuus, cum aliquando esset mota, supponendo etiam eandem esse perfectè rotundam, & leuigatam. Respondeo huiusmodi motum nullo modo futurum perpetuum, nec etiam multum duraturum, quia præterquam quod ab aere qui ei circumcirca aliquā resistantiam facit stringitur, est etiam resistantia partium illius corporis moti, quæ eum motæ sunt, naturā, impetum habent efficiendi iter directum, vnde cum simul iunctæ sint, & earum vna continuata cum alia. dum circulariter mouentur pariuntur violentiam, & in huiusmodi motu per vim vnitæ manent, quia quanto magis mouentur, tanto magis in ijs crescit naturalis inclinatio recta eundi, vnde tanto magis contra suammet naturam voluntur, ita vt secundum naturam quiescant, quia cum eis proprium sit, quando sunt motæ, eundi recta, quanto violentius voluntur, tanto magis vna resistit alteri, & quasi retrò reuocat eam, quam antea reperitur habere.

Ab eiusmodi inclinatione rectitudinis motus partium alicuius corporis rotundi fit, vt per aliquod temporis spaciū, trochus cum magna violentia seipsum circumagens, omnino rectus quiescat super illam cuspidem ferri quam habet, non inclinans se versus mundi centrum, magis ad vnā partē, quam ad aliam, cum quælibet suarum partium in huiusmodi motu non inclinet omnino versus mūdi centrum, sed multo magis per transuersum ad angulos rectos cum linea directionis, aut verticali, aut orizontis axe, ita vt necessariò huiusmodi corpus rectum stare debeat. Et quod dico ipsas partes non omnino inclinare versus mundi centrum, id ea ratione dico, quia non absolute sunt unquam priuatæ huiusmodi inclinatione, quæ efficit vt ipsum corpus eo puncto nitatur. Verum tamen est, quod quanto magis est velox, tanto minus premit ipsum punctum, imò ipsum corpus tātò magis leue remanet. Id quod aperte patet sumēdo exēplū pilæ alicuius arcus, aut alicuius alterius instrumētī, seu machinæ missilis, quæ pila quanto est velocior, in motu violento, tanto maiorem propulsionem habet rectius eundi, vnde versus mundi centrum tanto minus inclinatur, & hanc ob causam leuior redditur. Sed si clarius, hanc veritatem videre cupis, cogita illud corpus, Trochum scilicet, dum velocissime circūducitur secari, seu diuidi in multas partes, vnde uidebis illas omnes, non illico uersus mundi centrum descen-

descendere, sed recta horizontaliter, vt ita dicam, moueri. Id quod à nemine adhuc (quod sciam) in trocho est obseruatum. Ab huiusmodi motu trochi, aut huius generis corporis, clarè perspicitur, quàm errent peripatetici circa motum uolentum alicuius corporis, qui existimant aerem qui subintrat ad occupandum locum à corpore relictum, ipsum corpus impellere, cum ab hoc, magis effectus contrarius nascatur.

Quod deinde ampullæ iungantur in aqua, non fit ratione sympathiæ, de qua loquitur Fracastorus, nam per accidens iunguntur, quia cum alia ad aliam accedit, quælibet earum tentat ascendere ab ea parte, à qua inuicem hærent, quemadmodum efficiunt iuxta labrum uasis, ea enim superficies aquæ vicina circumferentiæ uasis aliquantulum ascendit in uasè, qui non est omnino plenus.

Ad id deinde quod de claritate noctium scribis, miror cur non videas, quod quàm to magis obscura nox apparet, non dico ratione nubium, sed distantia Solis sub horizonte ab eodem horizonte, tanto magis claram, & luminosam sese nobis ostendit Luna in quintadecima, quia cum Sol est in Sagittario, & Capricorno, Luna est in Geminis, & in Cancro, unde in media nocte, eius radius per valde exiguam quantitatem vaporum transit, quia tunc ipsa est valde propinqua axi horizontis, & præterea in huiusmodi tempore anni & noctis, aer est magis purgatus, quàm in qualibet alia temporis parte, quia hieme Sol non potest excitare multos vapores, & ij, qui at tolluntur, nocte à frigore statim congelati ratione grauitatis decidunt, unde remanet aer multo clarior, qua ratione apparent stellæ minutæ, & Cœlum ipsdem magis ornatum, quàm in quolibet alio anni tempore.

Dicere deinde, quemadmodum hic mundus est ætatis septem, aut octomillium annorum, ita nunc potuisset esse (si Deus uoluisset) ætatis quinquagintamillium; ergo erat tempus; ita se habet, ac si diceremus, quemadmodum hic mundus est tantæ magnitudinis, ita etiam quinquagies maior esse potuisset, ergo est spatium, aut interuallum corporeum, quod eum capere potuisset.

Illud, nihil, Aristotelis extra Cœlum, nullo modo nobis inferuit pro eiusdem Cœli spherica rotunditate, cum cuiusque alterius ex infinitis figuris Cœlum ipsum esse possit, secundum suam superficiem conuexam. Nam Cœlum ea ratione sphericum non est, quod magis sit capax, quia ei innumerabiles alias figuras adeo magnas poterat concedere causa diuina: sed sphericum est effectum, ne partem aliquam haberet sui termini superfluum, quia nullum corpus à breuiori termino quam à spherico terminari potest.

*De reuolutione rotæ putealis & alijs  
problematibus.*

A D E V N D E M.

**F**Vnis cui appensa est situla, longè facilius axi inuoluitur, si ipsi axi affixa sit rota, atque item commodius eò fiet, quo amplior rota erit, & axis exilior.

Commodissimè autem, si ipsa rotæ extrema circumferentia, ex materia minori, & densiori, ac proinde grauiori constabit. Cuius rei ratio multiplex est. Nempe quia omne corpus graue, aut sui natura, aut vi motum, in se recipit impressionem

nem & impetum motus, ita vt separatim à virtute mouente per aliquod temporis spatium ex seipso moueatur. nam si secundum naturam motu cieatur, suam velocitatem semper augebit, cum in eo, impetus & impressio semper augeantur, quia coniunctam habet perpetuò virtutem mouentem. Vnde manu mouendo rotam, ab eaq; eam remouendo rota statim non quiescet, sed per aliquod temporis spatium circunuertetur.

Secunda causa est, quia quoduis graue corpus, aut per naturam, aut per vim motum, rectitudinem itineris naturaliter appetat, quod clarè cognoscere possumus, proijciendo lapides funda, & circuducentes brachium, nam funes tanto maius pondus acquirunt, & manum tanto magis onerant, quanto velocius voluitur funda, & incitatur motus, quod ab appetitu naturali infito ei corpori per lineã rectam progrediendi procedit. Vnde fit, vt pondus circunferentiã ipsius rotã, tanto facilius circunuoluatur, & ex seipso tanto longiori tempore moueatur, quanto longius distat à centro, cum eius iter tanto minus sit curuum. Hanc igitur ob causam, rota, quanto maior erit, eiusq; pondus tanto magis vicinum circunferentiã, tanto magis durabit impetus motus assumptus.

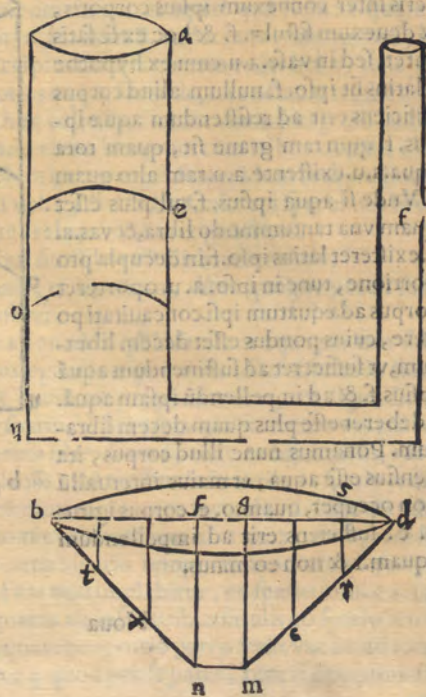
Tertia causa est, quod funis dum circunuoluitur, vicinius axi mathematico reuolutionis, quam corpus graue circunferentiã rotã, ratione vectis, cum rota est in motu, eius impetus non obrinet resistentiam æqualem à contrario pondere aquã in situ la positã.

*De machina, qua aquam impellit & subleuat.*

A D E V N D E M.

Vnde fit vt in fonte mandauerim, vas seu mortarium in quod ingreditur instrumentum, quod aquam impellit, diametrum suã concauitatis, habere non oportere maiorem diametro fistulã, per quam debet ascende re aqua, ratio est, quia si maius esset, necessarium esset aliquod instrumentum quo aqua impelleretur multo grauius toto corpore aqueo, quod aptum esset implere aliquam fistulam adeo altam, vt est fons, quã tamen esset adeo lata vt est mortarium.

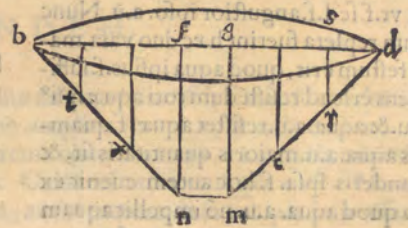
Sit exempli gratia, tota fistula, seu hirundo, per quam ascendit aqua. f. mortarium verò sit. a. u. quod tam altũ fit vt. f. sed. f. angustior ipso. a. u. Nunc cum repleta fuerint hæc duo vasa, manifestum erit, quod aqua ipsius. f. sufficiens erit ad resistẽdum toti aquã ipsi<sup>o</sup> a. u. & aqua. a. u. resistet aquã. f. quamuis aqua. a. u. maioris quantitatis sit, & ponderis ipsa. f. hoc autem euenit ex eo quod aqua. a. u. nõ impellit aquam f. toto



f. toto suo pondere, propterea quod pondus diuiditur proportionaliter supra basim vasis.

Sit exempli gra vas aliquod. b. d. n. m. conicæ figuræ, seu truncus conici concaui aqua plenus, cuius orificij diameter sit. b. d. & multiplex diametro. m. n. infimæ basis. cogitemus etiam. b. d. diuisum in tot partes, quarum vnaquæq; æqualis sit, m. n. imaginemurq; tot lineas perpendiculares descendere versus mundi centrum ad puncta. r. c. m. et. t. x. m. vt in subscripta hic figura videre est, per quas cogitemus tot superficies curuas conicasq; inter quas, mente concipienda est aqua, quæ pondere suo quiescet supra maiorem superficiem illa, quæ æque distans esset mundi centro, seu quam supra basim. m. n. vt exempli gratia consideretur aqua inter. g. m. et. s. r. cuius pondus distribuitur secundum latitudinem. m. r. quæ maior est. g. s. cogitemus igitur. m. c. æqualem esse. g. s. manifestum erit, quod. m. c. non sustinebit totum pondus aquæ, quæ inter. g. m. et. s. r. reperitur, eo quod omnis pars aquæ ad perpendiculum inclinatur versus mundi centrum, quapropter fundus seu basis. m. n. non sustinet aliud pondus quàm aquæ. f. m. sed si quis hoc in dubium reuocaret dicens, quod aqua circumscribens situm corporis aquei. f. m. impellit lateraliter dictum corpus aqueum, respondendum est, quod ex æquo huius corporis. f. m. aqua impellit etiam aquam circumsstantem eo, quod sunt corpora homogenea, cum in corporibus homogeneis æquales partes habeant æquales vires.

Sed redeundo ad vasa. a. u. et. f. dico quod sicut aqua. f. sufficit ad resistendū aquæ a. u. ita quodlibet aliud pondus æquale. f. cuiusvis materiæ, in fistula. f. positum, sufficiens erit, dummodo illud corpus ita sit adæquatū concauitati fistulæ. f. quod non permittat transitum aliquem aquæ vel aeris inter convexum ipsius corporis, & deuezum fistulæ. f. & hoc ex se satis patet, sed in vase. a. u. cum ex hypothesi latius sit ipso. f. nullum aliud corpus sufficiens erit ad resistendum aquæ ipsius. f. quin tam graue sit, quam tota aqua. a. u. existente. a. u. tam alto quam f. Vnde si aqua ipsius. f. nil plus esset quam vna tantummodo libra, & vas. a. u. existeret latius ipso. f. in decupla proportione, tunc in ipso. a. u. oporteret corpus adæquatū ipsi concauitati ponere, cuius pondus esset decem librarum, vt sufficeret ad sustinendum aquam ipsius. f. & ad impellendū ipsam aquam. f. deberet esse plus quam decem librarum. Ponamus nunc illud corpus, ita densius esse aqua, vt maius interuallū non occupet, quam. o. e. corpus igitur o. e. sufficiens erit ad impellendum aquam. f. & non eo minus.



Noua



NOVA SOLVTIO PROBLEMATIS DE VASE  
pleno liquoris.

*Nicolao Caluxio Serenissimi Ducis Sabaudia à secretis.*



QVOD à me postulas est problema ab alijs iam scriptum, sed illud tibi alio medio solvam.

Proponitur vas plenum liquore aliquo, puta aqua, quod tres habeat fistulas ad basin, quarum vnaquæque possit euacuare ipsum vas, inæquales tamen, ita quod prima tam lata sit, ut spatio vnus horæ possit ipsum euacuare totum; secunda vero spatio duarum horarum, tertia autem spatio trium horarum. Tunc quæritur quanto tempore omnes tres fistulæ simul apertæ euacuabunt ipsum vas. Ad hoc volo ut quæraturo primo quanta pars aquæ vnaquæque fistula euacuabit in aliquo dato tempore, quod facile est, ut puta, prima fistula, spatio dimidiæ horæ euacuabit dimidium vas, eo quod spatio integræ horæ potest totum euacuare, secunda fistula, eodem temporis spatio, euacuabit quartam partem ipsius vasis, tertia verò fistula, eodemmet spatio temporis dimidiæ horæ, euacuabit sextam partem ipsius vasis, quæ omnia fracta simul collecta faciunt vndecim duodecimas partes totius vasis, vnde manifestum erit, quod omnes fistulæ pariter apertæ, spatio dimidiæ horæ euacuabunt vndecim duodecimas partes totius aquæ, sed nos cupimus scire, quanto tempore, totum vas euacuabitur, apertis omnibus fistulis, quapropter dicemus ita; Si vndecim duodecimæ partes consumunt minuta. 30. temporis, quantum consumunt omnes partes aquæ quæ sunt. 12. quare ex regula de tribus prouenient nobis minuta. 32. cum. 8. vndecimis vnus minuti, hoc est cum. 43. secundis horæ ferè, vel si accipiemus tres quartas vnus horæ, tunc prima fistula emittet tres quartas partes totius aquæ, secunda, tres octauas eiusdem aquæ, tertia verò, quarta pars, tunc omnia, hæc collecta, faciunt vnum integrum cum tribus octauis. Si dixerimus igitur quando vnum integrum cum tribus octauis absument. 45. minuta temporis, ergo illud solum integrum absument idem ut supra hoc est min. 32. cum. 8. vndecimis vnus minuti vel. 43. secundis. Cuius rei speculatio tam coniuncta est operationi, quod vna cognita, reliqua statim cognoscitur.

Idem eueniet de implendo vase tribus similibus fistulis mediantibus.

Secundum quæsitum ab alijs traditum, tuum etiam, aliter quoque potest solui, propterea non præmittam tibi satisfacere.

Problema itaque tale est, ut sit vas aliquod in quod infunditur aqua per tres fistulas, sed dum infunditur aqua, eadem egreditur per duas alias fistulas in fundo vasis positas, sed tres superiores sint inuicem proportionatæ, ut supradictum est, primaque inferiorum talis sit, ut spatio. 4. horarum possit totum vas euacuare, secunda autem possit spatio. 6. horarum idem facere, vnde ex supradictis, vas implebitur à tribus fistulis superioribus, clausis existentibus inferioribus, spatio temporis minorum. 32. cum. 8. vndecimis hoc est min. 32. cum. 43. secundis, deinde per duas fistulas inferiores possit euacuari spatio temporis horarum. 2. et mi. 24. ex supradictis.

Supponamus igitur omnes fistulas operari spatio temporis minorum. 32. cum secundis. 43. tunc manifestum est quod vas non implebitur, eo spatio min. 32. cum secundis. 43. sed tanta aqua deficiet, quanta ab inferioribus fistulis eo spatio temporis min. 32. secun. 43. potest euacuari, quare proportio partis vasis vacuæ, ad totum vas, erit ut min. 33. ferè ad horas. 2. min. 24. quod per se patet, tunc si demptum fue-



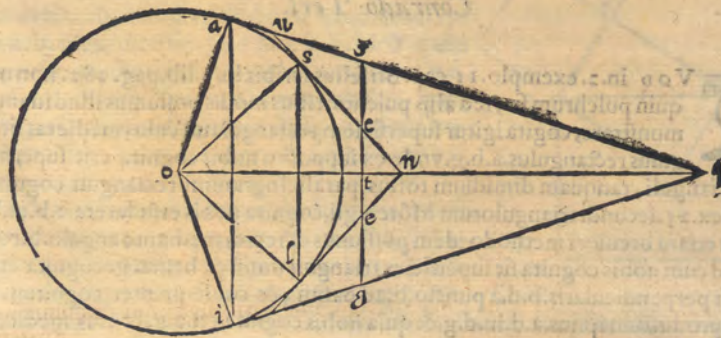
*Quod incendium, ex reflexione radiorum solarium, non fiat in centro speculi sphaerici, & aliquid contra Cardanum, & de motu radiorum solarium.*

A D E V N D E M.

**I** Terum tibi dico, quod radij illi solares, qui à diuersis punctis ipsius solaris corporis veniunt, transeunt per centrum speculi sphaerici concâui, quamuis à superficie speculi ad centrum ipsum reflectantur, vt aliàs tibi dixi, nihilominus nullo modo possunt aliquod obiectum incendere duabus ex causis, quarum vna est, quia cum Sol valde remotus sit à nobis, val de etiam acutus generatur angulus coni radiorum in centro speculi, vnde à parua superficie ipsius speculi reflectuntur, quare paucissimi radij sunt qui reflectantur in ipso centro, & propterea non sufficiunt ad combustionem alicuius obiecti. Alia verò causa est, quod quamuis multi, & sufficientes radij fuissent ad còburendù velociter quoduis obiectum. impossibile tamen omnino esset, vt aliquod obiectum comburerent, propterea quod cum radij incidentes debeant per centrum transire, obiectum combustibile, vt opacum, obstaret ipsis radijs, ne vltius transirent, vnde nulla fieret reflexio, sed etiam si disti radij in centro reflexi, sufficerent ad combustionem, incidentes hoc magis efficerent. & ita absque vlllo speculo, omnia & in quolibet loco comburerentur, quod manifestè falsum est. Desine igitur mihi citare Lucillum Philalteam, qui in philosophia mathematica fuit omnium imperitissimus. Verum speculum vstorium illud est quod ab Alhazem Deinde à Vitellione describitur.

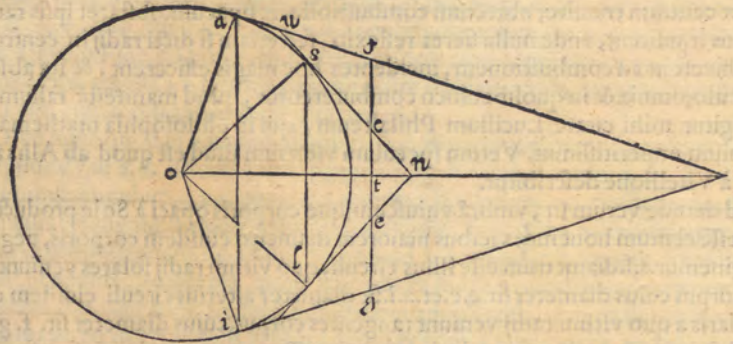
Quod deinde verum sit, vmbra vniuscuiusque corporis opaci à Sole productam semper esse centum nouemq; vicibus maiorem diametro eiusdem corporis, nego.

Imagemur. s. l. diametrum esse illius circuli, quo vltimi radij solares veniunt tangentes corpus cuius diameter sit. c. e. et. a. i. sit diameter alterius circuli eiusdem corporis solaris à quo vltimi radij veniunt tangentes corpus, cuius diameter sit. f. g. in eadem distantia, & eodem situ prioris corporis. Tunc conus vmbrae ipsius. f. g. sit. f. g. q. & ipsius. c. e. sit. c. n. e. centrum autem solare sit. o. conorum verò axes sint. t. n. q. tunc ex supposito. q. f. a. n. c. s. n. e. l. et. q. g. i. erunt omnes contigui corpori solari, vnde ex 17. tertij Eucli. anguli. o. a. q. et. o. s. n. erunt recti. protracta deinde cum fu erit a. s. habebimus angulos. u. a. s. et. u. s. a. minores duobus rectis. Quare. n. s. concurret



cum a. q. in puncto. u. Nunc verò si umbra. t. q. tanto maior est. f. g. quanto. 109. est vno et. r. n. etiam tãto maior. c. e. ergò eadem proportio erit. q. t. ad. t. f. quæ. n. t. ad. t. c. sed cum angulus. t. communis sit ambobus triangulis. q. t. f. et. n. t. c. sequitur ex. 6. sexti dictos triangulos æquiangulos esse. Vnde si anguli. t. n. c. et. t. q. f. æquales inuicem sunt, ergo. q. f. æquidistans erit. n. c. quod est impossibile, quia nunc demonstrauimus ipsas concurrere in puncto. u. Quare non est eadem proportio. q. t. ad. t. f. quæ n. t. ad. t. c. decipitur ergo Cardanus in. 4. lib. de subtilitate.

Circa illud deinde quod à me quæris, hoc est, quæ sit causa, quod nos videmus radium solarem tardissimè moueri, cum alias tibi dixerim ipsum qualibet hora circa terram quindecim gradus perficere, respondeo, quod radius ille quem videmus, eximpli gratia, in aliquo cubiculo, nunquam est idem numero, sed quia ipsi radij nullo modo differunt inter se, nisi in numero, propterea putamus eundem semper esse, cum semper alius, atque alius sit, quorum vnusquisque (de illis loquor, qui ad hanc terræ globum perueniunt) circa terram reuoluitur spatio. 24. horarum, & cum quilibet circulus diuidatur in. 360. gradus, quorum vigesimaquarta pars est. 15. verum est igitur, quod tibi iam dixeram.



### OPERATIONES DIVERSAE AB ALIIS

Michaelis Stifelij.

*Conrado Terl.*



**V**OD in. 2. exemplo. 11. cap. Stifelius scribit in. 3. lib. pag. 282. non nego quin pulchrum sit, sed alijs pulchrioribus modis possumus illud idem demonstrare; cogita igitur superficiem rectangulam, cuius medietas sit triângulus rectangulus. a. b. g. vnde ex supposito nobis cognita erit superficies ipsius trianguli, tanquam dimidium totius parallelogrammi rectanguli cogniti.

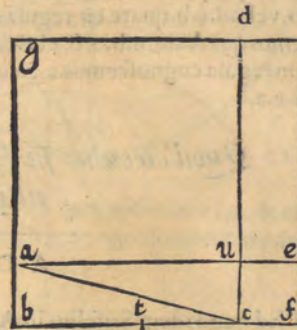
Quare ex. 25. secundi triangulorum Móteregij, cognita nobis erunt latera. a. b. et. b. g. Alia etiam breuiori methodo idem possumus effi cere, mediante angulo. b. recto, eo quod cum nobis cognita sit superficies trianguli simul cù basi. a. g. cognita etiam nobis sit perpendicularis. b. d. à puncto. b. ad basim, & consequenter cognitum nobis erit productum ipsius. a. d. in. d. g. & quia nobis cognita est. a. g. & eius medietas, ideo

ideo vnaquæque eius pars. a. d. et. d. g. similiter nobis cognita erit ex quinta secundi Eucl. vnde ex penultima primi habebimus propositum.

Possimus item circulum mente concipere cuius. a. g. sit diameter, & ab eius centro. c. protracta cum fuerit. e. b. quæ nobis cognita erit, vt medietas ipsius. a. g. de cuius potentia, dempta cū fuerit potentia ipsi<sup>9</sup> b. o. remanebit nobis potentia ipsius. d. e. & ita eius longitudo, quæ addita medietati. e. g. & detracta à dimidio. e. d. erunt nobis cognitæ. a. d. et. d. g. vnde. b. g. et. b. d. remanebunt nobis cognitæ ex dicta penultima primi Eucl. huiusmodi figuram videbis in dicto. 25. problemate. 2. li. Montisregij.

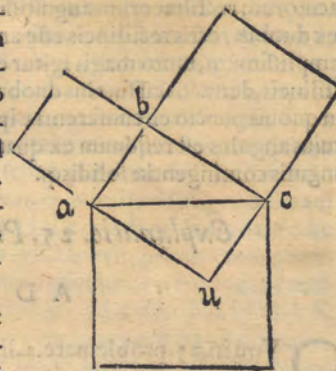
Aliter etiam possumus hoc idem efficere.

Sit rectangulus hic subscriptus. a. b. c. u. superficiei cognitæ simul cum diametro. a. c. extendatur imaginatione. b. c. vsque ad. f. ita quod. c. f. æqualis sit. c. u. intelliganturq; quadrata. g. f. g. u. et. u. f. vnde sūma quadratorū. g. u. u. f. cognita nobis erit ex penultima primi. nam. a. c. data nobis fuit, quare summā. g. u. u. b. et. u. f. cognoscemus, cui sūmæ addito suplemento. d. e. æquali. u. b. dabit nobis cognitū quadratum. g. f. totale, quare cognoscetur eius radix. b. f. cognita igitur. b. f. cum producto. b. u. illico ex. 5. secundi cognoscetur. b. c. et. c. f. forte cognita. b. f. diuisa p æqualia in puncto. r. & per inæqualia in pūcto. c. Nam quadratū ipsius. t. f. cognitum, equatur rectangulo. b. u. cū quadrato ipsius. t. c. depto igitur rectangulo, b. u. ex quadrato ipsius. t. f. relinquetur quadratum ipsi<sup>9</sup>. t. c. cognitum & eius radix. t. c. qua addita ipsi medietati. b. t. & depta ex medietate. f. t. relinquetur propositum.



Similiter de tertio exemplo eiusdem Stifelij infero.

Sit rectangulus. a. b. c. u. cuius diametri. a. c. quantitas, simul cum proportione laterum. b. c. et. b. a. nobis data sit. cum autem scire voluerimus eius superficiem. b. u. clarum est, quod cum nobis data sit proportio. b. c. ad. b. a. illico cognoscemus etiā proportionem quadrati ipsius. b. c. ad quadratum ipsius. b. a. cum dupla sit ei quæ. b. c. ad. b. a. ita etiam & aggregati dictorum quadratorum ad quadratum ipsius. b. a. hoc est nota erit nobis proportio quadrati ipsius. a. c. diagonalis ad quadratum ipsius. a. b. idem dico de quadrato. b. c. idest quod proportio quadrati ipsius. a. c. ad quadratum. b. c. cognita nobis erit, sed. a. c. data nobis fuit, quare cognoscemus etiam omnia dicta quadrata eorumq; radices. a. b. et. b. c. quare & superficiem rectanguli quaesitam.



Quartum exemplum etiam faciliori via potest solui, propterea, quod cum nobis cognita sit basis trianguli cum summa reliquorum laterum, & cū angulo opposito basi ipsius reliqua cognita nobis emergunt ex. 15. problemate secundi lib. de Triangulis ipsius Montisregij.

Vcl

Vel si tibi placet, accipe hanc aliam methodum à me excogitatum.

Duplicetur triangulū. a. b. c. orthogoniū, & fiat rectangulū. b. u. vt in mea figura secundi exempli hic vides. produca turq; b. c. quousque. c. f. æqualis sit. c. u. vnde. b. f. cognita nobis erit ex hypothesi, quare cognoscemus etiam quadratum. g. f. à quo demptū cum fuerit aggregatū quadratorum. g. u. et. u. f. nobis cognitū (nam quadrata. g. u. et. u. f. æqualia sunt quadrato ipsius. a. c. diagonalis datæ) remanebit aggregatum supplementorū cognitum, quare eius medietas cognoscetur idest. b. u. vnde ex 5. secundi Eucli. vt superius diximus cognoscetur etiam. b. c. et. c. f. distinctæ.

Idem assero de exemplo Gemmæ Frisij à Stifelio citato in Appendice regulæ falsi.

Sit gratia exempli rectangulum hic subscriptum. a. b. datæ superficiæ data etiam nobis sit proportio. a. e. ad. e. b. laterum producentium, cogitemusq; a. e. producta vsque ad. o. ita vt. e. o. æqualis sit ipsi. e. b. imagine mus etiā perfectum esse quadratum. b. o. vnde ex prima sexti seu. 18. vel. 19. septimi vel. 15. quinti eadem proportio erit ipsius. a. b. ad. b. o. vt. a. e. ad e. o. vel ad. e. b. quare ex regula de tribus, cognoscemus quadratum. b. o. & eius radicē. e. o. & ex eadem regula cognoscemus. a. e. cum cognita nobis sit. e. o. simul cum proportione. e. o. ad. e. a.



*Quod circulus sit figura infinitorum angulorum hoc est  
ultima polygoniarum.*

A D E V N D E M.

Sed quod idem Stifelius in Appendice secundi libri dicat circulum esse figuram polygoniam, non est ita mirandum, nam & alij multi doctissimi viri hanc veritatem cognouerunt, de Leone Baptista Alberto nihil dicam, cum ipse fateatur hoc accepisse à philosophis, vt etiam refert Arist. de sphaera tertio de caelo. considera quæso in circulo, quod cum angulus contingentiae sit angulus, quamuis omnium acutorum rectilineorum angustissimus, vnde ex communi ratione sequitur reliquum ex duobus rectis rectilineis esse angulum, & si omnium obtusorum rectilineorum sit amplissimum, tanto magis igitur erit angulus, id quod remanet ex duobus rectis rectilineis, detractis cū fuerint duobus angulis contingentiae, qui quidem angulus erit in quouis puncto circumferentiae ipsius circuli, idem intelligendum est de sphaera, cuius angulus est residuum ex quatuor rectis solidis, detractis cum fuerint quatuor angulis contingentiae solidisq;.

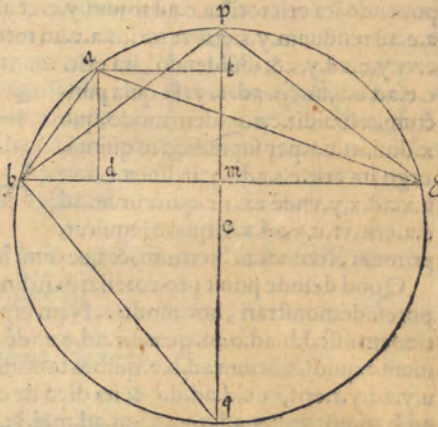
*Explanatio. 25. Problematis lib. 2. Monteregij.*

A D E V N D E M.

Quod in. 25. problemate. 2. lib. de triangulis Monteregium non intelligas, mirum non est, eo quod quandoque bonus dormitat Homerus. Puto enim illud problema ab ipso Monteregio non fuisse visitatum. Sed ne me aliquo modo culpes, accipe hanc aliā methodū à me aliter etiā excogitatā in eadem ipsius figura.

Propo-

Propositū fit nobis triangulum. a. b. g. cuius basis data sit cum area, seu perpendiculari. a. d. cum angulo etiam. a. ad cognoscendum autem. a. b. et. b. g. cogitemus circumlum. a. b. q. g. circumscribere ipsum triangulum cuius diameter. p. q. ad rectos fecet basim. b. g. in puncto. m. cogitemus etiam. b. p. et. p. g. unde ex. 20. tertij Euclid. angulus. b. p. g. æqualis erit angulo. a. & angulus. m. p. b. erit eius dimidium, quod ex te ipso cognosces, & angul<sup>o</sup>. p. b. m. similiter cognoscetur, quare ex. 29. primi eiusdem Montere gij cognoscemus. p. m. et. p. b. (nam. b. m. datum fuit, vt dimidium totius basis. b. g.) ducta postea. b. q. ex eadē. 29. cognoscemus. p. q. cum. p. b. iam cognita fuerit, à qua. p. q. (diametro) depra p. m. remanebit. q. m. cognita, cū qua iuncta cum fuerit. m. t. æquali. a. d. perpendiculari, dabitur. q. t. et. t. p. inter quas. a. t. media proportionalis locatur, quare cognoscemus. a. t. quæ sinus est arcus. a. p. unde cognitus erit arcus a. p. sed arcus. p. g. cognitus est median te angulo. p. b. g. cognito, qui quidem arcus. p. g. si coniunctus fuerit cum arcu. p. a. cognoscemus compositum. a. g. & eius chorda similiter (hoc est secundū latus) qua cognita, illico cognoscemus chordam a. b. hoc est tertium latus trianguli propositi.



*Quadam notanda in Federicum Comandinum.*

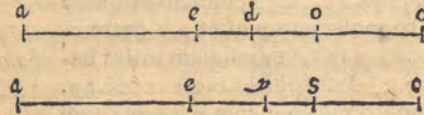
A D E V N D E M.

**P**Vtabas enim me ioco dixisse Federicum Comandinum non omnino irreprehensibilem esse, vide igitur, quod scribit in quinto lemmate in decima propositionem libr. 2. de insidentibus aquæ Archimedis, volens demonstrare eandem esse proportionem. l. b. ad. b. m. quæ. c. e. ad. e. a. vbi est aliquo modo prolixum, mediante linea. c. p. cum suis partibus, citans etiam antecedens lemma extra propositum, eo quod nec in antecedente lemmate, nec in alio, ipse vnquam probauerit proportionem. c. d. ad. d. q. esse, vt. l. b. ad. b. m. sed ne putes me falli, tibi demonstrabo non esse necessarium ducere lineam. c. m. p. vel. q. p. eo quod cū per quintam lib. de quadratura parabolæ Archimedis, ita sit. c. d. ad. d. e. vt. l. b. ad. b. m. existente a. c. dupla ipsi. d. e. et. e. c. dupla ipsi. g. c. et. l. d. dupla ipsi. l. b. erit, primo componendo. c. e. ad. e. d. vt. l. d. ad. d. m. & per æqualitatem proportionum, ita erit. e. g. ad. e. d. vt. b. d. ad. d. m. & per. 19. quinti Eucli. ita erit. e. g. idest. g. c. ad. g. d. vt. b. d. idest. l. b. ad. b. m. sed. c. g. ad. g. d. est vt. c. e. ad. e. a. ratio est, quia componendo ita est. c. d. ad. d. g. vt. c. a. ad. a. e. & hoc est, quia permutando, ita est. a. c. ad. d. c. vt. a. e. ad. d. g. & hoc verum est ex. 19. quinti eo quod totius. a. c. ad. totum. d. c. est vt. abscissi. e. c. ad. abscissum. g. c. vt. supradixi.

Sed

Sed etiam alio vniuersaliori modo potes probare, quod ita sit.  $u.x.ad.x.y.vt.c.e.$  ad.  $e.a.$  cogitando in linea.  $c.a.$  punctum quoddam quod vocabimus similiter.  $y.$  in tali situ locatum, quod diuidat.  $c.a.$  eadem proportione qua.  $y.$  diuidit.  $u.s.$  vnde cum  $e.s.$  diuisa eodem modo etiam sit à puncto.  $s.$  ex supradicta quinta lib. de quadratura parabolæ, erit igitur proportio.  $a.y.$  ad.  $y.c.$  vt.  $e.s.$  ad.  $s.c.$  per.  $11.$  quinti Eucli. & componendo ita erit toti<sup>9</sup>.  $a.c.$  ad totum.  $y.c.$  vt abscissi.  $s.c.$  ad abscissum.  $s.c.$  quare residui  $a.e.$  ad residuum.  $y.s.$  erit vt totius.  $a.c.$  ad totum.  $y.c.$  & permutando, ita erit.  $a.c.$  ad.  $a.e.$  vt.  $y.c.$  ad.  $y.s.$  & diuidendo, ita erit.

$c.e.$  ad.  $e.a.$  ut.  $c.s.$  ad.  $s.y.$  & quia punctum.  $s.$  diuidit.  $c.a.$  eodem modo, quo  $x.$  diuidit.  $u.s.$  per supradictam quintã, ergo ita erit.  $c.s.$  ad.  $s.y.$  in linea.  $c.a.$  vt  $u.x.$  ad.  $x.y.$  vnde ex.  $11.$  quinti.  $c.e.$  ad  $e.a.$  erit, vt.  $u.x.$  ad.  $x.y.$  quare sequitur, primum, secundum, tertium, & quartum lemma superflua esse.



Quod deinde ponit pro corellario in fine.  $6.$  lemmatis, aliter quam per.  $6.$  lemma potest demonstrari, hoc modo. Nam superius demonstraui eandem proportionem esse.  $l.b.$  ad.  $b.m.$  quæ.  $c.e.$  ad.  $e.a.$  idẽ dico de proportione.  $u.x.$  ad.  $x.y.$  & omnium æquidistantium ad.  $h.e.$  quibus rationibus mediantibus eodem modo scies, quod  $u.y.$  ad.  $y.r.$  erit, vt.  $c.d.$  ad.  $d.c.$  & ita dico de omnibus æquidistantibus. ad.  $h.e.$  vnde.  $l.b.$  ad.  $b.m.$  erit vt.  $u.x.$  ad.  $x.y.$  et.  $l.m.$  ad.  $m.d.$  vt.  $u.y.$  ad.  $y.r.$  per.  $11.$  quinti, sed cum sit.  $l.b.$  ad.  $b.m.$  vt.  $u.x.$  ad.  $x.y.$  componendo erit.  $l.m.$  ad.  $b.m.$  vt.  $u.x.$  ad.  $x.y.$  & euerfim.  $b.m.$  ad.  $m.b.$  erit, vt.  $x.y.$  ad.  $y.u.$  & per æquam proportionalitatem erit.  $b.m.$  ad.  $m.d.$  vt.  $x.y.$  ad.  $y.r.$  quod est propositum.

Non video etiam, quare ipse ducat lineam.  $s.r.$  cum in ipso contextu nihil faciat de dicta.  $s.r.$

Comentum postea contextus.  $P.$  pulchrius esset, si diceret, quod cum ita sit totius,  $l.a.$  ad totum.  $a.d.$  sic se habebit abscissum.  $a.i.$  ad abscissum.  $a.z.$  eo quod ita est, vt scies, hoc est in proportione dupla, ergo residui.  $i.l.$  ad residuum.  $d.z.$  erit vt totius.  $a.l.$  ad totum.  $a.d.$  hoc est in proportione dupla.

### De Visu.

#### A D E V N D E M.

**R**atio vnde fiat, vt videamus distinctè omnes colores, cum in qualibet aeris parte, quo lumina reflexa possunt peruenire mixta sint, & non distincta, oritur à paruitate ipsius pupillæ oculorum, & à magna expansione virtutis visiuæ in superficie concaua orbis continentis humores diaphanos oculorum per ramusculos nerui optici remotè ab ipsa pupilla. & quamuis radii luminosi frangantur ab vnoquoque humore diuersimodè, hoc nihilominus maximè iuuat ad distinctionem radiorum, sed & si directè procederent, idem ferè eueniret, non tamen suis locis, cogita exempli gratia lineam.  $a.u.$  vt communis sectio cuiusdam plani secantis spheram oculi, per centrum ipsius, & pupillæ, et.  $o.$  punctum sit proximum centro ipsius pupillæ, sed interius aliquantulum, extra autè oculū, sint varij colores, vt.  $c.n.t.$  in dicto plano.

Iam nulli dubium est quod lumina quæ producuntur ab.  $c.n.t.$  ad.  $o.$  in ipso.  $o.$  mixta,



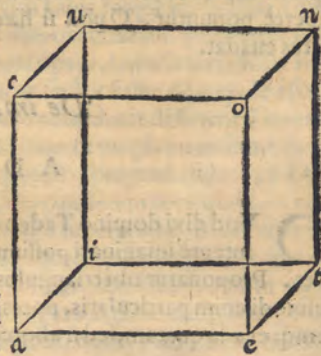
xta, & non distincta, procedendo igitur ulterius ipsi radij citra o. tunc difgregantur, & separantur abinuicem, & cum perueniunt ad lineam. a.u.e. sentiuntur distincti alij ab alijs. Cuius quidem rei, exemplum manifestum accipere possumus a quouis cubiculo ex omni parte clauso, quod transitum nullum permittat radijs luminosis, nisi per aliquod paruum foramen, in quo foramine, & extra ipsum cubiculum, omnes radij mixti erunt, sed in obiecto pariete ipsius cubiculi videbuntur distincti, vnde sequitur, quod quo remotius erit obiectum. c.n.t.ab.o. tanto acutior erit angulus. c.o.t. & suus contrapositus similiter, & per consequens linea. e.u.a. breuior erit, & punctum. o. propinquius etiam erit ipsi lineae. a.u.e. quae omnia efficiunt, ut nobis obiectum. c.t. paruum, & minus distinctum, seu magis confusum appareat.



DE APPARENTI DISTANTIA PARTIVM  
hæmisphærij.

Anselmo Fucaro.

GRATA mihi tuæ literæ fuerunt, quibus ostendis non paruum desiderium sciendi vnde fiat, quod cum dies illucescit, & est serena pars Coeli, circa axem orizontis demissior appareat, quam aliæ partes, cum ab alijs (quod sciã) satis expressum non fuerit, sed quia de eo a me aliquid scire desideras dicam quod mihi vñ. Scias non solum multitudinem obiectorum oppositorum efficere, ut aliqua res alia longius distare videatur, ut alij putarunt, sed etiam diuersitates colorum, quamobrem cum decipiamur, credentes Coelum esse præditum colore cæruleo, cum is color, aeri, non Coelo conueniat, & videntes huiusmodi colorem circa axem orizontis magis densum, quam versus ipsum orizontem, ratione exiguae reflexionis, a pauca quantitate vaporum inter nostrum situm, & reflexionis locum, iudicamus Coelum proximior esse circa dictum axem, quam sint aliæ partes; præterquam, quod is color, qui videtur terminare, aut impedire radium visualem (aduertas tamen me hac in re platonice non esse) eo semper propinquior esse videtur, qui ei locum dat, & hanc ob causam videntes nos densitatem cærulei circa axem orizontis, & cernentes amplitudinem gyri aliarum partium, adducimur, ut putemus eam partem vicinior esse. Neque illud etiam omittam hoc etiam fieri ratione imaginationis, vnde etiam multis contrarium euenire potest, id est ut eis magis profundum videatur Cælum, circa axem orizontis, quam vicinum gyro eiusdem orizontis, iudicantibus eam partem longinquo rem esse, quae sese magis obscuram oculo demonstrat, & eam propinquiorē quae sese clariorē ostendit, ut ei et contingere potest, qui subscriptam figuram cubicam non quidem ductam secundum ordinem opticum, sed ita, ut omnia latera opposita inuicem sint parallela, prospiciet, id est. a.i.ad.e.t.et.c.u.ad.o.n.et.a.i.ad.c.u.et.e.t.ad.o.n. vnde sequitur, ut aliquando quadratum. a.o. videbitur citra, et. i.n. ultra dictum cubum aliquando verò e conuerso.



Pp De

## DE PHILOSOPHIA MATHEMATICA.

*Dominico Pisano.*

**S**I omnia vno colore constarent, & corporum vmbra à luminibus non distinguerentur, neque diuersitas situs, lumina, quæ veniunt ad oculum non alteraret; perinde esset, ac si essemus cæci. Miror quod cum in Aristotele sit versatus, in tuis tamen scriptis philosophum à Mathematico separes, quasi mathematicus non sit adeò philosophus, vt est naturalis, & metaphysicus, cum multo magis quam ij philosophus sit appellandus, si ad veritatem suarum conclusionum respiciamus. Verum quid est, te in huiusmodi errore solū non versari; sed grauius est, quod cum vos videatis etiam res morales sub philosophiæ appellatione cadere, non animaduertatis diuinas scientias mathematicas etiam philosophiæ nomine ornandas esse. Quod si eiusdem nomen penitus considerare velimus, inueniemus aperte, mathematico magis illud ipsum quàm cuiilibet alio conuenire, cum nullus ex alijs tam certo sciat id quod affirmat quam mathematicus, neque aliquis sit, qui in cognitionis, & scientiæ cupiditatem magis ducatur, vt aperte patet, cum nec etiam ipsi sensui det locum, neque aliquid præsupponat, quod non sit ita verum & intellectui notum, vt nulla quæuis potentia, illud esse falsum ostendere queat. Sed quia Græci, qui ad placitum nomina rebus imposuerunt, voluerunt etiam, non solum mathematica, sed etiam naturalia, metaphysica, & moralia, sub communi philosophiæ nomine contineri. Vt aut tibi satisfaciam autoritate Aristotelis, quem tanopere colis, primum considera, nunquam eum de philosopho mentionem facere quin prius aperiat de quo philosopho loquatur, atque hoc semper præstat, exceptis quibusdam locis, vt cap. 2. lib. 4. Metaphysicorū, vbi de philosopho in genere loquens, ait, proprium esse philosophi, vt res omnes speculetur atque hoc in principio quia ti textus asserit, cum in quarto iam ostenderit mathematicum esse philosophum: omitto quod in 2. textu secundi physicorum idem affirmet, æquum esse appellare philosophiam scientiam veritatis, & finem speculatiuæ existere veritatem. An non idem in primo cap. 6. metaphysicæ philosophiam speculatiuam, mathematicis physicis & supernaturalibus rebus contineri? An non idem paulo inferius scribit physicam primam futuram, si aliæ substantiæ quam naturales non reperirentur? considera deinde quid dicat in fine tertij cap. lib. 11. quo loco nil clarius esse potest, lege etiam quæ 6. cap. eiusdem libri ab eodem adducuntur, & quæ in 8. cap. 12. libri textu. 44. aperte ponuntur. Quod si hæc tibi non sufficiunt, vereor ne tuus morbus desperatus euadat.

*De imaginatione specierum.*

A D E V N D E M.

**Q**UOD dixi domino Tadeo est, quod aliquas particularium species, perfecte & integrè imaginari possumus, alias non item, id tibi melius exemplo innotescet. Proponatur tibi triangulus æquilaterus datæ magnitudinis, datiq; coloris, huiusmodi enim particularis, potes imaginatione tibi fingere integram speciem, totalemq; eadæquatam, sed si aliquam speciem aliquando vniuersaliorem imaginatio

ne

ne concipere velles , quemadmodum vnus trianguli æquilateri , tali magnitudine , sed non præfinito colore constantis , hoc minime præstare posses . quia nullam rem visibilem priuatam colore imaginari possumus . nec etiam potes imaginari speciem ali cuius trianguli æquilateri , indeterminatæ magnitudinis , & indefiniti coloris , quæ cuiilibet particulari cuiusuis magnitudinis , & coloris postea applicari queat . Species deinde alicuius trianguli æquicruri , aut vnus trianguli laterum inæqualium , aut trian guli in genere , aut tandem figuræ , considerato tu ipse , an possit sub imaginationem cadere . Possumus quidem huiusmodi speciem ( ratione mediante ) intelligere , vn de quamlibet speciem rei particularis visibilis , compositæ , ex figura , magnitudine , & colore , perfectè imaginari possumus , & huiusmodi conceptus erit specialissima species , quia in infinito suorum indiuiduorum , nunquam fiet , vt aliquod eorum , ali quo modo ab alijs differre possit ; admonens te , nil reperiri , quod differat , aut in se partem aliquam habeat , quod aliquid aliud non obtineat , quin dicta differentia sit specifica , eius tamen solum partis quæ differt ab alia duorum indiuiduorum , vnus , eiusdemq; speciei . quia si est in magnitudine nulla planè magnitudo reperitur , quæ sua specie non sit dotata , quod si non esset , inter res omnes nulla æqualitas eluceret : & si in figura , & colore , idem affirmo , aliter nulla res similis esset alteri , neque aliqua similitudo reperiretur . Idem de quolibet alio obiecto sensibili dico . Ratio autem eorum omnium quæ dixi est , quia imaginatiua nihil aliud intellectui ostendere potest , quam id quod recipit à sensu , & cum sensus , alio modo moueri non possit quam supradictò , hanc ob causam verum est , quæquid scripsi . Vnde triangulum æquilaterum datæ magnitudinis , erit genus triangulorum æquilaterū eiusdè datæ magnitudinis , sed diuersorū colorū , erit etiā species trianguli æquilateri indeterminatæ magnitudinis , & hic deinde erit species trianguli , & hic postea species figuræ . Idem de alijs omnibus rebus per gradus dico , quæ sicut à sen su , ita etiam ab imaginatione longè recedunt , adeo vt has species specialissimas tan tum , id est eas solum , quas hic superius descripsi , integrè capere possit : at verò gene ra , quanto vniuersaliora sunt , ab eadem imaginatione , tanto longius distant .

*De maculis Luna, & eius lumine.*

A D E V N D E M .

**M**Aculæ Lunæ , nihil aliud sunt , quàm partes ipsius Lunæ magis perspicuæ , à qui bus , lumen non reflexum , sed penetrans , nobis occultatur ; quemadmodū via lactea , nihil aliud est , quam pars octauæ orbis magis opaca , à qua lumen Solis reflexum , sese nobis ostendit . Quod autem Maurolicus scribit folio . 64 . cap . de astro rum fulsionibus , circa Lunā , est falsum ; primo , quia non considerat differentiā inten sionis luminum inter Venerem , & Lunam , cum lumē illius sit magis intensum , quam Lunæ , quia quilibet qui sano sit oculo , facile potest comprehendere , si Lu na esset , vbi est Venus , aut Venus vbi reperitur Luna ( quibus in locis eiusdem ma gnitudinis nobis apparent ) ipsa Luna à Venere longè superaretur , & excedere tur splendore , & lumine , ita vt si etiam verum esset , quod per tres gradus inter ualli sese nobis proderet sexagesima pars luminis ( quod in quadraturis nec in villo alio situ verum euadit , respectu ad Solem , id est vt tres gradus differentiæ situs , con stituant sexagesimam partem differentiæ luminis respectu nostri ) non ideo tamen

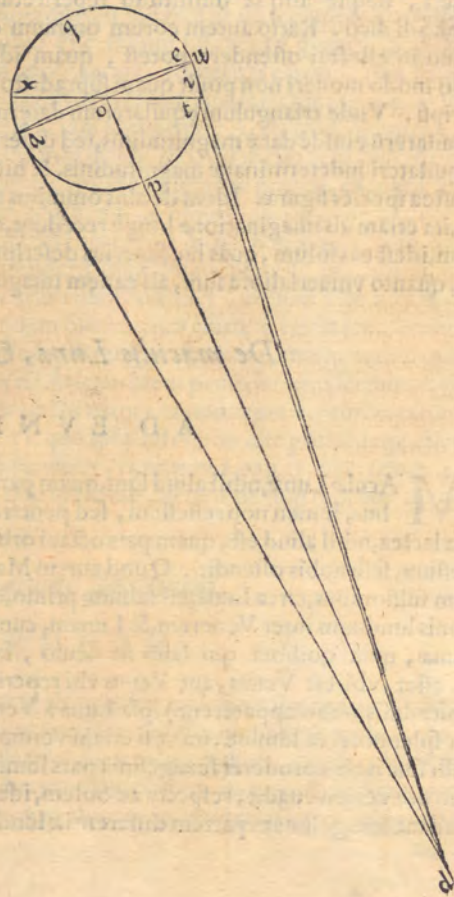
dictum Jumen conspiceretur, quia non sufficit extensio luminis, cum eiusdem intensio sit etiam necessaria. Sed id quoque tibi dico, quod etiam si dicta sexagesima pars totius luminis lunaris, eadem intensione splendoris, & luminis Veneris, in tali distantia trium graduum à Sole prædita esset, non eam tamè videremus, ratione obliquitatis curvæ, & sphericæ superficiei Lunæ, respectu nostri, in huiusmodi situ: id quod tibi ita demonstratum volo.

Pars superficialis lunaris globi, quæ nos respicit sit. a. p. u. quam accipere possumus pro medietate ipsius superficiei totalis, eo quod respectu nostri visus, insensibiliter, ab ipsa medietate differat, pars autem à Sole visa sit. u. q. a. cogitemus etiam circum. a. p. u. q. vnum esse ex maioribus ipsius globi, cuius superficies trāseat per oculum vidētis, vnde pars eius. a. p. u. diuidet vmbra per æqualia, reliqua verò pars. a. q. u. diuidet per æqualia lumen ipsius Lunæ à Sole receptum, ita quod pars illuminata, erit medietas. u. q. a. excessus verò, cum nostro visui incompræhensibilis sit, pro nihilo reputetur, cuius causa est, maxima illa distantia, quæ inter Solem, & Lunam reperitur, quamuis Sol maior sit Luna multis millibus vicium, eo quod tunc inter Solem, & Lunam reperiuntur plus quam. 570. diametri terræ.

Supponamus nunc Lunam remotam esse à loco ipsius cōiunctionis cum Sole per 3. gradus. vnde quæadmodum prius lumen erat in gyro. a. q. u. nunc reperitur in gyro. x. q. t. ita quod. t. u. erit sexagesima pars ipsius. a. p. u. quæ à vero sensibiliter non discedit.

Imaginentur nunc duæ rectæ lineæ ductæ ab oculo. d. ad puncta. t. et. u. verum tamen est quod linea. d. u. secabit arcum. t. u. sed ita propinqua puncto. u. quod erit ei ferè contingens, vnde absque sensibili errore possumus arcum. t. u. intelligere inter duas lineas. d. t. et. d. u. quapropter tale lumen compræhendetur, ferè, sub angulo. t. d. u. quem quidem angulum oportet nos videre, cuius magnitudinis existat, respectu totalis anguli a. d. u. protracta cum fuerit. d. a.

Producatur primo. d. t. vsque ad diametrum in puncto. i. deinde per puncta. a. et. u. ducatur arcus. a. e. u. circa. d. cætrum, ad quem ducatur linea. d. t. i. in puncto. e. sed quia, cum diameter. a. u. tam brevis sit respectu distantia à terra, tempore interlunij, vnde minor ceteris parte ipsius distantia existit, sequit nos posse absque sensibili errore cogitare, a puncto. d. ad quoduis punctum ipsius diametri omnes lineas ad angulos rectos cum ipso diametro, & insensibilis inæqualitatis



litatis à linea. d. o. Accipiemus igitur. t. i. pro sinu arcus. t. n. qui est graduum. 3. hoc est sexagesima pars semicirculi graduum. 180. quapropter. t. i. erit partium. 5233. tallium qualium. o. u. est. 100000. cuius. t. i. quadratum demptum cum fuerit à quadrato semidiametri. o. t. relinquet nobis quadratum ipsius. o. i. quæ quidem. o. i. vt radix quadrata, erit partium. 99862. tallium qualium semidiameter est. 100000. vnde. i. u. residuum diametri, remanebit partium. 138. Vel sic, cum cognitus sit nobis arcus. t. u. illicò cognoscemus sinum arcus. p. t. complementū vnius quartæ, qui sinus æqualis erit ferè arcui. o. i. partium. 99862. vnde. i. u. erit, vt dictum est, partium. 138. quæ quidem. i. u. æqualis est ferè sinui arcus. u. e. & ita etiam. u. e. quare si diuisa fuerit tota. a. u. partiū. 200000. per. 138. proueniet nobis. 1449. & sic angulus. t. d. u. erit vna partium. 1449. anguli. a. d. u. Consideremus igitur quomodo fieri potest, vt oculo comprehendatur hæc tam parua particula luminis lunaris.

## SOLUTIONES ALIQUAE.

*Paulo Aemilio Raifetaim.*

**P**ost eas literas quas proximè ad te dedi, Franciscus Monardus mihi retulit tuas nonnullas dubitationes circa nostrum Theorema Arithmeticum. 116. quarum prima est, quod si numerus. a. cogitat<sup>9</sup>, esset æqualis. 4. tunc ipse non esset multiplex ipsi. 4. de quo tamen nullam mentionem feci. Idem etiam inquis, si. a. fuisset. 5. 6. 7. nec non. 1. 2. et. 3. Cui respondi, quod quauis nullam fecerim mentionem de æqualitate ipsius. a. cum. 4. nihil tamen refert, propterea quod quando ita fuisset, nihilominus easdem conditiones subiret, quemadmodū si fuisset duplus, triplus, aut quadruplus. eò quod à genere multiplici, æqualitas, formam diuersam non induat. Quare idem eueniet si. a. fuerit. 4. 5. 6. 7. vt si esset. 8. 9. 10. et. 11. & sic de cæteris, excepto quod in proprijs multiplicibus, vel in superantibus ipsis multiplicibus. a. mensuratur ab ipso. 4. plus quam semel. Quod autem dicis. de. 1. 2. et. 3. nihil est, quia, vt in secunda summa, hoc est in tertio termino maximo, reliquus tertius terminus, id est. 9. non comprehendetur, ita nobis indicabit primum numerum sumptum minorem esse quaternario. Quæ omnia, ex ipsa nostra theoria ibidem expressa manifestantur. Quid autem circa hoc Frater Lucas dicat, nescio, quia ipsius opus ad manus meas nunquam peruenit, satis enim mihi fuit, in Tartalea hanc praxim vidisse, ratio vero nullibi à me reperta fuit. Tartalea enim multos citat auctores, quorum scripta ego nunquam vidi, vt Leonardi Pisani, Prodocimi, Petri Borghi, Fratris Lucæ, Ioannis Sfortunati, cæterorumq; similium.

ELI-

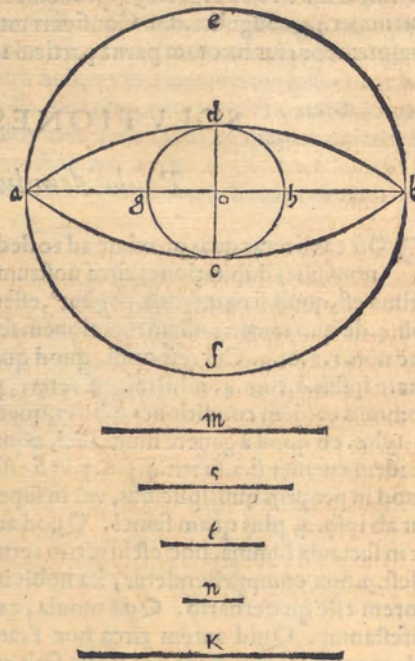
ELIPSIM PROPOSITAM QUALITER  
quadrare valeamus.

*Illustri Viro Francisco Mendozza.*



Q UOD antea tuo nomine fecerat Marcus Antonius amicus noster sufficiebat. Sed quia, quæ nunc à me petis, talia sunt, vt sine tripartita æqualiter aliqua data proportione non possit aliquis exactè intentum perficere, nihilominus, supposita diæta diuisione, reliqua facilia erūt. Primū enim est. Propositam Ellipsim quadrare.

Sit igit̃ Ellipsis proposita. a. b. d. c. cuius axes sint. a. b. et. d. c. dati, seu reperi ex 47. secūdi Pergei, sintq; duo circuli. a. e. b. f. et. g. d. h. c. circa easdem diametros, tūc proportio. a. b. ad. d. c. dimidiū erit proportionis circulorum ex. 2. 12. Euclid. sed proportio. a. b. ad. d. c. æqualis est proportioni maioris circuli ad Ellipsim. ex. 5. Archimedis in lib. de conoidibus, quapropter proportio Ellipsis ad minorem circulum altera medietas erit totius proportionis circulorum, hoc est maioris ad minorem, quare Ellipsis media proportionalis erit inter eos circulos. Nunc verò cum ex Archimede repertæ fuerint duæ figuræ rectilineæ æquales duobus circulis iam dictis, & inter has, reperta fuerit alia media proportionalis propositum obtinebimus.



*Spharoidem propositam cubare.*

A D E V N D E M.

P ROPosita sphæroides erit, aut prolata, aut oblonga, sit prius prolata, sitq; a. b. diameter circuli, qui eam per æqualia secat, circa quam. a. b. vt circa axem intelligatur sphæroides oblonga, cuius spissitudo sit. d. c. axis prolatae, cogitemus nūc duas sphæras. a. e. b. f. et. g. d. h. c. circa dictos axes. Vnde quatuor corpora habebimus, hoc est duas sphæras, & duas sphæroides, quas probabo continuas proportionales inuicem esse.

Consideremus igitur duos conos rectos, quorum. a. b. diameter sit eorum basium, altitudo autem maioris, æqualis sit semidiametro majori, hoc est medietati. a. b. altitudo

titudo verò minoris, æqualis sit semidiametro minori, hoc est medietati. d. c. vnde habebimus proportionem coni maioris ad conum minorem, eadem quæ est diametri maioris ad diametrum minorem, quod ex. 2. parte. 11. duodecimi Eucli. nec non ex. 9. eiusdem manifestum est, sed conus minor, est quarta pars spheroidis prolatae ex. 29. Archimedis in lib. de conoidalibus, & conus maior, est etiam quarta pars spheræ, ex. 32. primi lib. de spherâ, & cyllindro, quare ex communi scientia, eadẽ proportio erit spheræ maioris ad spheroidem prolatam, quæ. a. b. ad. d. c. sed proportio. a. b. ad. d. c. est tertia pars proportionis maioris spheræ ad minorẽ. Consideremus nũc alios duos conos rectos, vnius & eiusdẽ basis, cuius diameter sit. d. c. sed altitudo maioris, æqualis sit semidiametro spheræ maioris, altitudo verò minoris, sit æqualis semidiametro minoris spheræ, vnde ex dictis rationibus habebimus proportionẽ maioris coni ad minorẽ, vt quæ est. o. b. ad. o. d. hoc est vt. a. b. ad. d. c. & ex dictis propositionibus ita se habebit spheroides oblonga ad spheram minorem vt. a. b. ad. d. c. hoc est tertia pars proportionis spheræ maioris ad minorem. Quare proportio spheroidis prolatae ad oblongam, erit reliqua tertia pars proportionis maioris spheræ ad minorẽ. Quapropter hæc quatuor corpora continua proportionalia inuicem erunt.

Nunc verò quarenda est inter. a. b. & suas duas tertias partes vna media proportionalis, quæ sit. K. & ex Archimede, inuentum sit quadratum æquale circulo, cuius sit. K. diameter. Vnde proportio circuli (cuius. a. b. est diameter) ad circulum cuius. K. est diameter, sesquialtera erit ex. 2. 12. Eucli.

Ducatur deinde quadratum lineæ. K. in lineam. a. b. & proueniet nobis corpus quoddam, quod æquale erit spheræ maiori, ex corellario. 32. primi de spherâ & cyllindro, cuius corporis, latus cubus sit. m.

Idem facere oportebit mediante. d. c. minoris spheræ, cuius corporis cubica radix sit. n.

Nunc verò inter. m. et. n. inueniantur duæ mediæ proportionales. s. t. & ex. s. producatür cubus, qui æqualis erit spheroidi prolatae propositi, cubus vero. t. æqualis erit spheroidi oblonge, cuius axis esset. a. b.

Si autem spheroides oblonga nobis proposita fuisset, eodem methodo solueretur problema.

*Quadratum circulis mediantibus designare.*

A D E V N D E M.

**M**odus autem conficiendi quadratum ex circulis supra datam lineam, vt Dominum Gasparem docui, facillimus est.

Sit enim lineæ. b. a. 46. propositionis primi Euclidis, positoq; pede immobili circuli in puncto. a. secundum quantitatem lineæ. a. b. propositæ fiat circulus, similiter circa punctum. b. alius circulus eiusdem magnitudinis, erecta deinde sola. a. c. perpendiculari ipsi. a. b. ex puncto. a. ipsa secabitur à circumferentia circuli. cuius centrum est. a. in puncto. c. vnde. a. c. æqualis erit. a. b. posito demum pede immobili ipsius circuli in puncto. c. secundum longitudinem ipsius. c. a. fiat alius circulus, qui æqualis erit reliquis duobus circulis cum eorum semidiametri æquales sint, & hic vltimo factus secabit circulum, cuius centrũ est. b. in puncto. d. à quo cum ductæ fuerint. d. c. et. d. b. recte

rectè habebimus quod volumus. nam omnia latera sunt inuicem equalia ex conditionibus circuli, angulus autem. a. rectus effectus fuit, tunc si imaginatione cogitata fuerit diameter. b. c. ex. 8. primi, concludemus angulum. d. esse rectum deinde ex. 5 et. 32. eiusdem concludemus etiam reliquos angulos rectos esse.

Circa verò id quod mihi scripsisti de igne perpetuo putans nugas esse, quod Romæ inuentæ fuerint lucernæ ardentes in sepulchris antiquorum. Ego quidem minime puto eas nugas esse, propterea quod tales lucernas non vnus tantum aut duo viderint, sed multi homines fide dignissimi. Præterea cum ais id nulla ratione posse fieri. Respondeo quod maxima ratione possibile esse puto, quam quidem rationem ita esse oportet, quod primum lucerna sit perfectè circumclusa, vt materia in ea constituta nullo modo exire possit, deinde quod materia inflamabilis talis sit, vt excrementum fuliginosum ex flamma transmissum, tangendo superficiem deuiam ipsius lucernæ, aptum sit in pristinum humorem congelari, siue transformari, vnde materia prima per tres formas perpetuò transibit, hoc est per humorem, siue oleum tale, vt diximus, per ignem, seu flammam, & per vaporem, seu exhalationem fuliginosam aptam condensari, atque in priorem humorem illicò reuerti.

### DE DIVISIONE TRIANGVLI SECVNDVM propositam proportionem.

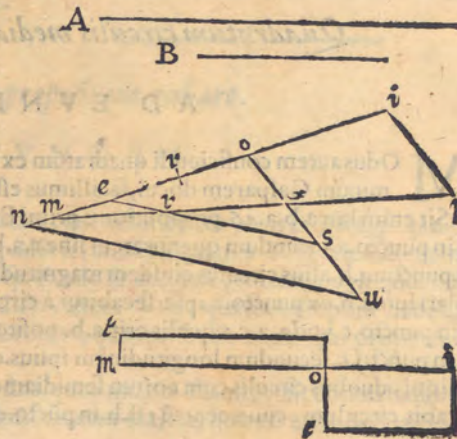
*Michaeli Angelo Muciasco.*



V O B mihi proponis, tale est, vt scilicet tibi modum scribam diuidendi triangulum propositum secundum datam proportionem à linea transeunte per punctum notatum extra triangulum.

Triangulū igit à te mihi propositum sit. n. o. u. considero primū quod si quis ipsum diuiderit in duas partes mediante. e. s. parallela ad. n. u. ea proportione, quam mihi proponis. deinde inuenerit in dicta. e. s. punctum. r. per quod transiens alia linea à puncto. p. proposito, ita quod efficiat duo triangula. m. r. e. et. r. s. x. inuicem equalia, problema solutum erit. eo quod triangulum. m. o. x. æquale esset triangulo. e. o. s. & quadrilaterum residuum. m. n. u. x. etiam equalis esset quadrilatero. e. n. u. s.

Sed dum punctum. r. uenarer, alia via mihi in mentem venit, cognoui igitur quod quum propositum expeditum fuisset, hoc est, qd si à puncto p. protracta esset linea. p. n. quæ triangulum. n. o. u. in duas partes inuicem ita proportionatas diuisisset, vt se haberet. A. et. B. ita se haberet productū n. o. in. o. u. ad productum. m. o. in. o. x. vt trianguli. n. o. u. ad triangulum m. o. x. quod quidem non est difficile speculari, ex methodo. 24. sexti, eo quod





eo quod tam proportio producti. n. o. in. o. u. ad productum. m. o. in. o. x. quam proportio trianguli. n. o. u. ad triangulum. m. o. x. componitur ex proportione. u. o. ad. o. x. & ex proportione. n. o. ad. m. o. unde proportio dictorum productorum nobis cognita erit, eo quod cum nobis cognita sit proportio. A. ad. B. ut data; cognita etiam nobis erit coniuncta, hoc est. A. B. ad. B. & propterea ea quae trianguli. n. o. u. ad triangulum. m. o. x. & similiter productorum. Quasiui postea modum inveniendi duas dictas lineas. m. o. et. o. x. & cognoui quod si producta fuerit. p. i. æquidistans lineæ. o. x. producendoq; .o. n. quousque cum. p. i. se interfecarent in puncto. i. inueniendopostea lineam quandam, quæ ducta cum. p. i. efficeret rectangulum æquale rectangulo cognito quod ex. m. o. in. o. x. potest fieri, quod cognitum dico, eo quod nobis cognita est proportio data, & rectangulum etiam. n. o. in. o. u. deinde secando ab. o. n. partem æqualem lineæ iam inuentæ, quæ sit. o. t. Inueniendo postea, ex. 28. sexti lineam. o. m. cuius productum in. m. t. æquale sit producto. t. o. in. o. i. unde ex. 15. eiusdem proportio. o. i. ad. m. o. eadem esset, quæ. m. t. ad. o. t. & componendo, ita se haberet. m. i. ad. m. o. ut. m. o. ad. o. t. sed ex. 4. sexti, ita esset. p. i. ad. o. x. ut. m. i. ad. m. o. quare ex. 11. quinti, ita esset. p. i. ad. o. x. ut. m. o. ad. o. t. unde ex. 15. sexti productum. o. x. in. m. o. æquale esset producto. p. i. in. o. t. & sic haberemus intentum.

Sed si punctum. m. caderet in punctum. n. idem esset, si verò punctum. m. transfiret n. oporteret nos facere hoc in latere. n. u. ipsum quærendo in linea. n. u. ducendo primum lineam. p. i. æquidistantē. u. x. & producendo. u. n. ad partem. u. prosequendo, quæ superius iam dictum est.

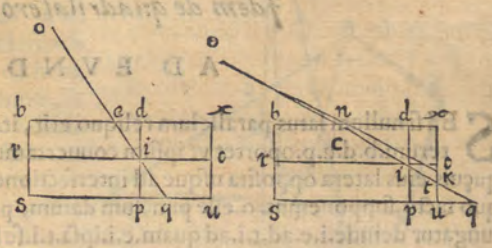
*Idem facere de parallelogrammo.*

A D E V N D E M.

Datum parallelogrammum in duas partes diuidere, secundum aliquam datam proportionem à linea transeunte per punctum propositum.

Sit exempli gratia, datum parallelogrammum. b. u. datum verò punctum. o. extra figuram, proportio autem ea sit, quæ. A. ad. B. ut supra. Nunc diuidatur primò rectangulum datum per æqualia, mediante linea. r. c. parallela ambobus lateribus. b. x. et. s. u. quæ quidem linea diuidatur in puncto. i. ita quod eadem proportio sit. r. i. ad. i. c. ut. A. ad. B. protrahatur deinde à puncto. o. linea. o. i. q. quæ secabit ambo duos latera. b. x. vel. s. u. intra terminos eorum, vel tantum. b. x. reliquum verò extra terminos. s. u.

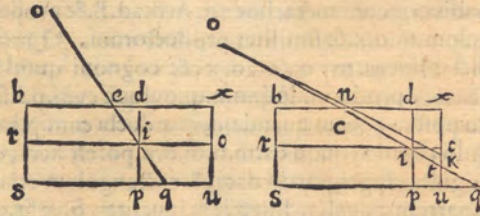
Nunc autem si intra dictos terminos transibit, ut in prima figura videre potes, problema solutum erit, eo quod si à puncto. i. protracta fuerit. p. d. parallela ad. u. x. habebimus ex prima sexti eandem proportionem. s. d. ad. p. x. ut. r. i. ad. i. c. hoc est ut. A. ad. B. sed triangulus i. e. d. æqualis est triangulo. i. q. p. ut tibi facile patebit, unde quadrilaterum. e. q. u. x. æquale erit quadrilatero. d. u. ex communi



Q. q. scien-

scientia. Quare ex 9. quinti, ita erit. s. d. ad dictum. d. u. ut ad quadrilaterum. e. q. u. hoc est vt. A. ad. B. ex. 11. eiusdem.

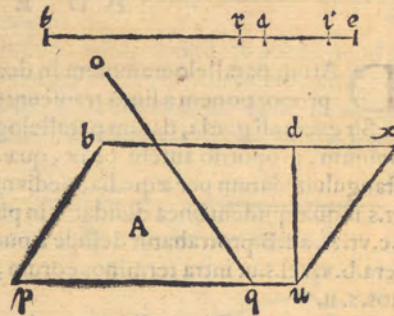
Sed si punctum. q. fuerit extra ut in . 2. figura videre est . tunc manifestum erit , q̄ triangulus. e. x. t. maior erit parallelogrammo. d. u. per triangulum. q. t. u. cum triangulo. q. i. p. æqualis triangulo. d. i. e. excedat quadrilaterum. i. t. u. p. per triangulum dictū. q. t. u. quapropter cum diuisus fuerit triangulus. e. x. t. mediante linea. o. n. K. ita q̄ quadrilaterū. e. n. K. t. fit æquale triangulo. q. t. u. ex doctrina præcedenti , habebimus propositum.



*Idem de frusto trianguli.*

A D E V N D E M.

Sed si quadrilaterum dictum esset frustum alicuius triāguli ut in figura. A. hic subscripta videre est, supposita, b. d. parallela ad. u. p. ita faciendum esset , ducendo scilicet parallelam. u. x. ad. b. p. quæ producatur vsque ad concursum cum. b. d. in puncto. x. sitq; proportio data inter. t. a. et. a. e. quas duas lineas cogitemus inuicem directè coniunctas , tunc diuidatur tota. t. e. in puncto. i. ita vt. t. i. ad. i. e. fit vt quadrilaterum. p. d. ad trigonum. u. d. x. deinde diuidatur t. i. in puncto. r. tali modo vt. t. r. ad. r. i. se habeat vt. t. a. ad. a. e. quo factō ex doctrina præcedenti diuidatur totum parallelogrammum. p. x. mediante linea. o. q. secundum quod se habet. t. r. ad. r. e. Atque ita solum erit problema , vt ex tē ipso ratiōnarifacile p̄otes.



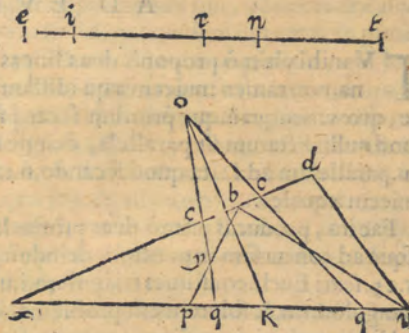
*Idem de quadrilatero in genere.*

A D E V N D E M.

Sed si nullum latus parallelum reliquo erit , ita faciendum erit. si sit tale quadrilaterum. b. d. u. p. oportet vt ipsum conuertamus in triangulum, producendo duo quouis eius latera opposita usque ad intersectionem ut pote. u. p. et. d. b. in puncto. x. quo tacto, supponemus. o. esse punctum datum, proportio verò data sit. t. r. ad. r. i. ad iungatur deinde. i. e. ad. t. i. ad quam. e. i. ipsa. t. i. se habeat vt quadrilaterum. b. d. u. p. se

se habet ad triangulum. b. p. x. ducatur postea. o. q. quæ diuidat totale triangulum. d. u. x. in duas partes inuicem ita proportionatas, ut se habent. t. r. et. r. e. quæ quidem partes sint. c. d. u. q. et. c. q. x. ut in primo problemate tibi monstraui, & habebis propositum, dato quod punctum. c. sit inter

b. et. d.  
Sed si forte linea. o. q. secabit. b. x. hoc est si punctum. c. esset inter. b. et. x. manifestum est, quod. c. q. secaret. b. p. in puncto. y. vnde in tali casu, alio modo operandum esset, hoc est ducendo. b. u. quæ diuideret quadrilaterum in duo triangula, & ut se haberet triangulum. b. d. u. ad triangulum. b. p. u. vellem vt ita secaretur t. i. in puncto. n. vt ita se haberet. t. n. ad. n. i. ut dictum est de istis duobus triangulis, deinde prout se habet. n. r. ad. r. i. ita secaret triangulum. b. p. u. mediante linea. o. K. ex doctrina primi problematis, & ita haberes propositum.

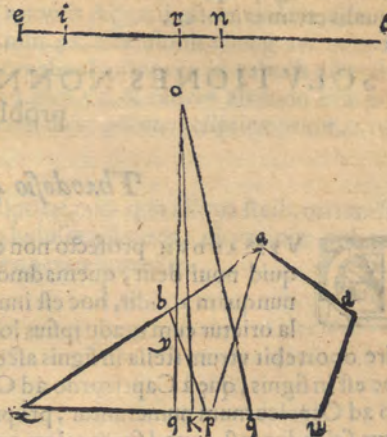


*Idem de Pentagono, Exagono, & de reliquis.*

A D E V N D E M.

**P**entagonum, seu hexagonum, vel alias quasuis multilateras figuras propositas ita diuidere, vt dictum est de trilateris, & quadrilateris.

Sit exempli gratia pentagonus. a. d. u. p. b. quem secare volumus mediante linea. o. q. in duas partes inuicem se habentes, vt se habent. t. r. et. r. i. oportet igitur ut ipsum pentagonum reducas ad quadrilaterum. x. a. d. u. quod diuidatur secundum præcedentem doctrinam, vt se habet. t. r. ad. r. e. vnde si punctum. q. incidit inter. p. et. u. tunc habebis propositum, si verò incidet inter. p. et. x. clarum erit quod linea. o. q. secabit latus. p. b. trianguli. b. x. p. in puncto. y. quapropter duces lineam. a. p. vt claudat triangulum. a. b. p. diuidaturq; t. i. in puncto. n. ita vt. t. n. ad. n. i. se habeat, vt quadrilaterum. a. d. u. p. ad triagulum. a. b. p. deinde huc triangulum. a. b. p. diuidas mediante linea. o. K. vt. n. r. ad. r. i. ex doctrina primi problematis & habebis propositum. Idem dico de hexagono, reducendo ipsum ad pentagonum, & item de eptagono, ipsum reducendo ad exagonum, & idem infero de infinito ipsarum superficialium figurarum rectilinearum.



*De duobus triangulis aequalibus inter lineas  
inuicem inclinatas.*

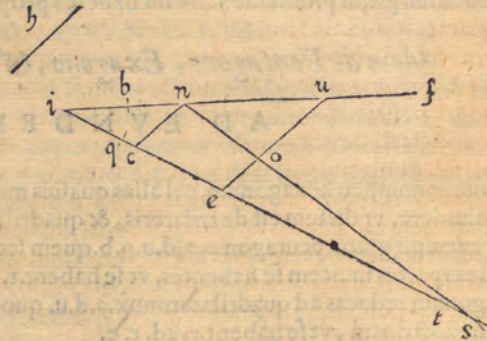
A D E V N D E M.

**T**V mihi vltimò proponis duas lineas rectas. b.f. et. q.s. in eadem superficie plana, non tamen inuicem æquidistantes, proponis etiam. n.t. in eadem superficie, quæ vnquamque priorum secat, proponis etiam lineam. h. tali conditione, quod nulli distantum sit parallela, deinde scire cupis qua arte aliquis possit ducere. e.u. parallelam ad. h. ita quod secando. n.t. constituat duos triangulos. n.o.u. et. t.o.e. inuicem æquales.

Fac ita, producas primò duas primas lineas à parte, in qua inuicem inclinantur, vsque ad concursum in puncto. i. deinde à puncto. n. duces. n.c. parallelam ad. h. postea ex. 25. sexti Eucli. constituas triángulum. i. u. e. simile triángulo. i. c. n. æquale tamen triángulo. i. t. n. & solutum erit problema.

Vel sic, inuenies. i. e. mediam proportionalem inter. i. c. & i. t. duces postea. e. u. parallelam lineæ. h. vel. c. n. quod idem erit ex. 30. primi Eucli. & solutum erit problema.

Nam ex. 17. sexti eadem proportio erit triánguli. i. c. n. ad triángulum. i. e. u. ut. i. c. ad. i. t. Quare ut triánguli. i. c. n. ad triángulum. i. t. n. ex prima sexti, et. 11. quinti. Vnde ex. 9. eiusdem. i. e. u. æqualis erit. i. t. n. Quapropter. o. n. u. æqualis etiam erit. o. e. t.



SOLUTIONES NONNULLAE QVORVNDAM  
problematum.

*Theodosio à Raifestaim.*

**D**VBITANDVM profecto non est, quin quotidie hominibus studiosis aliquid noui desit, quemadmodum, quod tibi nunc occurrit, mihi nunquam accidit, hoc est inuenire orientem, cui aliqua propofita stella oriatur cum gradu ipsius longitudinis. pro cuius rei operatione te prius scire oportebit vtrum stella in signis ascendentibus, vel descendentibus reperitur, hoc est in signis, quæ à Capricorno ad Cancrum procedunt, vel in illis, quæ à Cancro ad Capricornum numerantur, propterea quod si in signis ascendentibus inuenitur, sciendum est, quod supra talem orientem polus mundi australis attollitur, sed si in signis descendentibus reperitur, tunc polus borealis eleuatur supra dictum orientem

horizontem, vt exempli gratia, canicula quæ à Græcis Prochyō vocatur, reperitur in 24. minuto vigefimi gradus Cancrī, quapropter polus borealis eleuatur ſupra horizontem, cui ipſa oritur cum eodem gradu, & minuto eclipticæ illius ſigni. ſed quia volumus etiam ſcire veram quantitatem arcus eleuationis huiusmodi poli, propterea accipiemus in tabula generali Monteregij numerum qui vocatur radix aſcenſionum, è regione numeri longitudinis ipſius ſtellæ, qui quidem numerus in præſenti exemplo erit gra. 107. cum minutis. 53. qui eſt cuiuſdā arcus æquatoris, qui incipit in principio Arietis, & in circulo longitudinis deſinit, hoc eſt ab horizonte quaſiſito, ita quod talis numerus erit aſcenſio obliqua huiusmodi puncti eclipticæ illi horizonti, qua aſcenſione mediante, ſimul cum gradu, & minuto longitudinis in tabulis aſcenſionum obliquarum, inueniemus gradum, & minutum altitudinis pollaris, quod quærebatur, eodem ordine ac methodo, quo vtimur ad inueniendum in tabulis poſitionum, polum circuli poſitionis alicuius aſtri, mediante declinatione & diſtantia à meridiano cuiusdem aſtri, vt ſcis. Vnde in præſenti exemplo eleuatio poli borealis ſupra talem horizontem erit gra. 7. cum minutis. 45.

Sed ſi ſtella fuerit in medietate aſcendente, tunc certi erimus polum australem ſuper dictum horizontem attolli, nam idem eſt quaerere altitudinem vnius polorum mundi à tali horizonte, quod diſtantiam dicti poli à circulo ſecundum quem longitudo terminatur, qui etiam longitudinis dicitur, eo quod tunc temporis talis circulus vnus & idem eſt cum horizonte. Sumatur ergo exempli gratia ſtella, quæ in ore piſcis australis eſt, quæ, pro nunc, ſit in gradu. 20. cum minutis. 14. Aquarij longitudinis, & in gradu. 23. cum nullo minuto meridianæ longitudinis. Tunc certi erimus horizontem, cui dicta ſtella oritur cum eiufmodi puncto eclipticæ, depreſſum eſſe à parte australi ſub illoq; polo, ſed quia propoſitum eſt ſcire etiam quantitatem huiusmodi depræſſionis, reperiemus in tabula generali gradum, & minutum æquatoris, correſpondentem tali puncto longitudinis à circulo longitudinis terminato, qui quidem numerus in præſenti exemplo erit gra. 317. cum minutis. 46. & hic numerus, vt diximus eſt aſcen. obli. ad dictum horizontem, vbi polus australis attollitur, & deſcenſio obliqua, vbi polus borealis eleuatur. Quapropter ſi à. 317. gradibus cum minutis 46. demptus fuerit dimidiuſ circulus gra. 180. remanebunt gra. 137. cum minutis. 46 & punctus oppoſitus gradibus. 20. cum. 14. minutis Aquarij eſt in eodem numero Leonis, & mediantibus iſtis gradibus. 137. min. 46. aſcenſionis, cum grad. 20. min. 14. Leonis inueniemus eleuationem poli borealis ab horizonte in tabulis aſcenſionum obliquarum Monteregij, hoc eſt gra. 17. min. 53. & eadem altitudo erit poli australis ſupra horizontem à quo Fomahant cum dicto puncto eclipticæ oritur, in qua longitudine dicta ſtella reperitur.

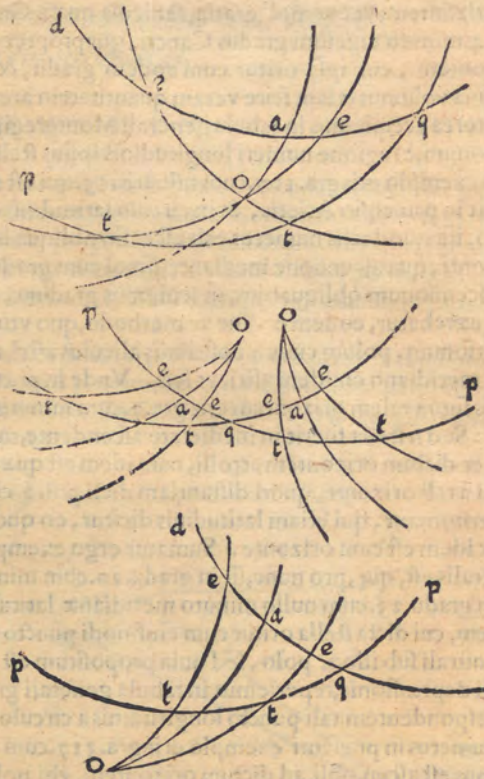
Sed ſi propoſitus nobis fuerit punctus eclipticæ, cum quo aliqua ſtella oritura ſit, & oporteat inuenire vbi, hoc eſt horizontem huiusmodi ortus, eleuationem poli arctici, ſeu antarctici ſupra talem horizontem, ita operandum eſſet.

Sit ſup

Sit exempli gratia stella. *o*. ecliptica verò. *d*. *q*. æquator autem. *p*. *q*. punctus verò eclipticæ, cum quo stella oritura sit. *e*. orizon de mû. *o*. *e*. vbi stella oriri possit cû puncto. *e*. Nam cum stella proponitur, datur etiam eius longitudo, nec non latitudo, quare arcus. *a*. *q*. & arcus. *a*. *o*. nobis cognitus erit, cum supponatur arcus. *a*. *o*. esse circuli latitudinis, et. *a*. *o*. latitudo ipsius stellæ, & angulus *a*. rectus erit, & quia punctum. *e*. datur, ergo arcus. *a*. *o*. & arcus. *a*. *e*. simul cû angulo. *a*. recto cogniti sunt, vnde ex. 11. primi lib. copernici, angulus. *a*. *e*. *o*. cognoscetur, & angulus. *q*. *e*. *o*. similiter, vt residuum ex duobus rectis quo. *e* mediante cum angulo. *q*. declinationis ab æquatore, mediante *q*. latere. *q*. *e*. cognito, cognitus quoque nobis erit angulus. *e*. *t*. *q*. ex 12. eiusdem. qui quidem angulus erit altitudinis æquatoris ab orizonte quæsito, qui demptus à 90. gradibus, dabit altitudinem poli ab orizonte quæsito.

Inuenire postea gradum eclipticæ, cum quo stella data oriatur ad orizontem propositum, nullius est difficultatis.

Ponamus exempli gratia, aliquem scire velle gradum eclipticæ, cum quo canicula oritur ad orizontem, cuius polus boreus eleuatur per gradus. 44. quæ canicula supponatur habere gradus. 19. cum min. 24. Cancræ longitudinis, & gra. 16. min. 10. latitudinis meridianæ, quærere primum oportet eius declinationem ex doctrina. 2. problematis tabularum directionum Monteregij, quæ erit graduum. 6. cum minutis. 5. septentrionalis, deinde inuenire eius ascensionem rectam ex doctrina. 4. problema tis eiusdem Monteregij, quæ erit gra. 108. mi. 42. deinde mediante declinatione iam inuenta in tabulis differentiarum ascensionum sub polo. 44. accipiemus differentiam ascensionum, qua differt recta ab obliqua, quæ in præsentem exemplo erit gra. 5. min. 5. quæ dempta ab ascensione recta stellæ, vt præsens exemplum exigit, relinquet nobis ascensionem obliquam stellæ propositæ ad polum. gra. 44. quæ erit gra. 102. minu. 47. qua mediante, in tabulis ascensionum obliquarum poli. 44. habebimus gradum & minutum eclipticæ cum quo stella oritur. quod in casu nostro erit gra. 1. min. 8. Leonis, sed si tecum non fuerint tabulæ dictæ, potes eleganter omnia hæc perficere via triangulorum sphericorum.



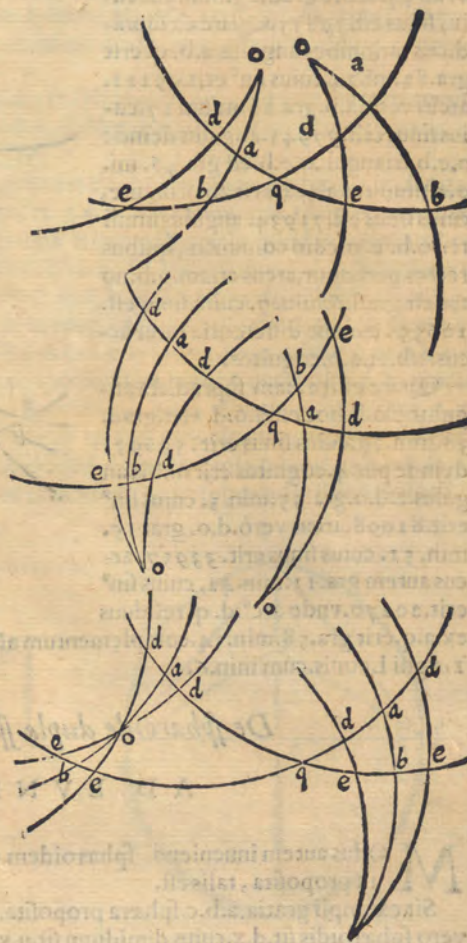
Via triangulorum idem facere.

Sit ex epli gratia. q. b. æquator, ecliptica verò. q. a. propositus aut orizon sit. o. c. d. & stella data sit. o. in orientali parte orizontis, circulus verò. a. o. ille sit, qui transiēs per polos eclipticæ & per centrum stellæ terminat longitudinem ipsius stellæ, & in ipso sit eius latitudo. Nunc propositum sit inuenire arcum. d. q. eo quod illic sciemus punctum. d. qua propter oportet nos prius cognoscere arcum. d. a. qui demptus, vel additus arcui. a. q. prius cognito ex supposito (nam data nobis est longitudo, & latitudo stellæ) dabit nobis. d. q.

Cum igitur voluerimus arcum. d. a. cognoscere, ita faciemus. nam. q. a. cognitus nobis est ex supposito vt dictum est. angulus quoque. a. q. b. qui declinationis eclipticæ ab æquatore est, angulus deinde. a. (trianguli. a. b. q.) rectus est, ergo ex. 4. primi copernici cognitus nobis erit arcus. a. b. nec non angulus. a. b. q. vnde angulus. o. b. c. residuus ex duobus rectis in duobus primis hic subscriptis figuris nobis itidem cognitus erit, etiam & arcus. b. o. residuus siue compositus ex arcu. a. o. cognito ex supposito cū sit arcus latitudinis ab ecliptica. Tunc in triangulo. o. b. e. cognoscimus latus. o. b. & angulum. o. b. e. nec non angulum. b. e. o. qui est altitudinis æquatoris ab orizonte, quare ex. 12. dicti lib. cognitus nobis erit angulus. b. o. c. Consideremus deinde triangulum. a. o. d. cuius angulus. a. rectus est, & angulus. a. o. d. cū latere. a. o. etiam cognitus, vnde ex supradicta. 4. nobis cognitus erit arcus. a. d. & consequenter cognoscemus arcum. d. q. eius residuum, seu compositum, quem quærebamus.

Sed si hac via inuenire desideras, cui orizonti proposita stella oriatur cum eodem eclipticæ puncto. a. longitudinis, hoc aliud nihil esset, quam cognoscere amplitudinem anguli. a. b. q. eo quod talis orizon, idem circulus esset. a. b. o. vnde cum quis sciret vnum illorum angulorum, quem æquator efficit cum orizonte, reliqua illic ei innotescunt, sed dictus angulus. b. iam diximus quomodo cognoscatur.

Pona-



Ponamus nos scire velle p̄ctum  
eclipticę, cum quo Procyon oritur  
polo. 44. o. dato, quod stella in gra.  
19. cum min. 24. Cancrī, reperiatur  
distans ab ecliptica per gra. 16. min.  
10. meridiem versus. vnde arc<sup>o</sup>. a. q.  
erit gra. 70. min. 36. eiusq; sinus par-  
tium. 94321. talium qualium totalis  
est. 100000. arcus verò. a. o. gra. 16.  
minut. 10. sinus erit 27845. angu-  
lus autem. a. q. e. declinationis zodia-  
ci ab equatore grad. 23. min. 30. cu-  
ius sinus est. 39875. Quare ex supra-  
dictis rationibus angulus. a. b. q. erit  
gra. 82. mi. 24. cuius sin<sup>o</sup> erit. 99122.  
arcus vero. a. b. gra. 22. minu. 17. cu-  
ius sinus erit. 37945. angulus deinde  
o. e. b. trianguli. o. c. b. est gra. 46. mi.  
o. altitudinis æquatoris ab horizonte,  
cuius sinus est. 71934. angulus simili-  
ter. o. b. e. medio coniuncti, quibus  
rectus perficitur, arcus etiam. o. b. no-  
tus est grad. 6. min. 7. cuius sinus est.  
10655. cum sit differentia inter ar-  
cus. a. b. et. a. o. cognitos.

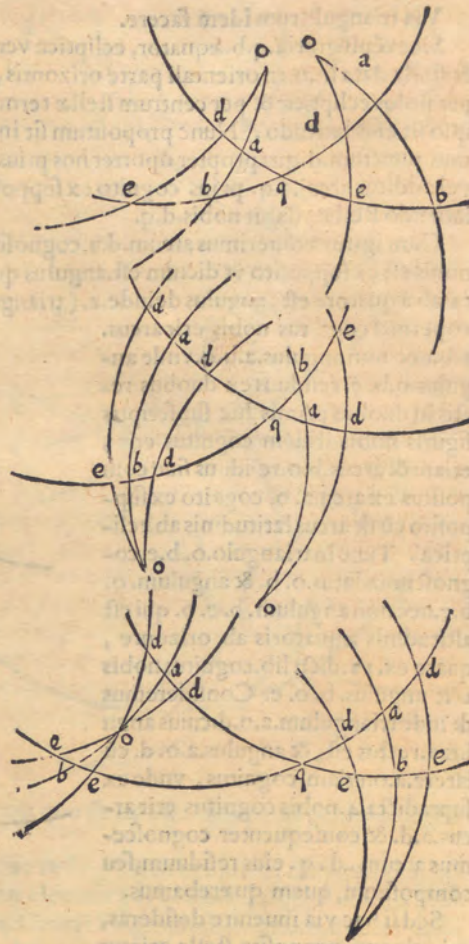
Quare ex. 12. iam supradicta angu-  
lus. e. o. b. hoc est. a. o. d. erit. grad.  
36. min. 39. cuius sinus erit. 59693.  
deinde per. 4. cognitus erit nobis an-  
gulus. a. d. o. gra. 55. min. 5. cuius sin<sup>o</sup>  
erit. 81998. arcus verò. d. o. gra. 19.  
min. 51. cuius sinus erit. 33957. ar-  
cus autem gra. 11. min. 42. cuius sin<sup>o</sup>  
erit. 20270. vnde arc<sup>o</sup>. d. q. residuus  
ex. a. q. erit gra. 58. min. 54. complementum aut quartæ erit gra. 31. mi. 6. hoc est gra.  
1. signi Leonis. cum min. 6.

*De spheroidē duplæ spheræ propositæ.*

A D E V N D E M.

**M**odus autem inueniendi spheroidem ex dato axe, quod duplum sit spheræ  
proposita, talis est.

Sit exempli gratia. a. b. c. spheræ proposita. cuius semidiameter sit. o. c. semiaxis  
vero spheroidis sit. d. x. cuius dimidium sit. u. x. tunc ex doctrina. 9. sexti Euclid. inue-  
niatur. g. h. media proportionalis inter. u. x. et. c. o. deinde sicut se habet. u. x. ad. g. h.  
facie-





faciemus, quod diameter. a. b. dictæ sphaeræ ita se habeat ad. e. f. ex. 10. sexti, quæ e. f. erit reliqua axis quæ sita. Vnde constituta cum fuerit ellipsis. d. f. t. e. ex dictis axibus, deinde circumuertendo ellipsim circa maiorem axem, constituemus sphaeroidem oblongam, si autem circumuertemus ipsam circa minorem axim constituemus sphaeroidem prolatam.

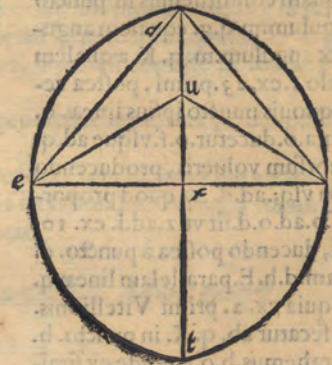
Quod autem talis operatio rationalis sit, nulli dubium erit, quotiescunque cognoscet conum rectum. e. u. f. æqualem esse cono recto. a. c. b. ex. 2. parte. 12. duodecimi Euclid. & quod cum conus. e. d. f. duplus sit cono. e. u. f. ex lemmate collecto ab 11. duodecimi, conus. e. d. f. duplus existit etiam cono. a. c. b. ex. 7. quinti. Cum deinde ex. 32. primi lib. de sphaera, & cylindro sphaera. a. c. b. q. quadrupla sit cono. a. c. b. ipsa consequenter dupla erit cono. e. d. f. sed ex. 29. primi de conoidalibus, dimidium sphaeroidis. e. d. f. t. hoc est. e. d. f. dupla est cono. e. d. f. Quare talis medietas æqualis est sphaeræ propositæ, tota que sphaeroides dupla erit sphaeræ datæ. Quod autem dico de proportione dupla, idem infero de qualibet alia, sumendo. u. x. ita proportionatam ad. d. x. vt proponitur.

Sphaeram autem inuenire quæ dimidia sit sphaeroidis propositæ nullius erit negotij, quotiescunque inuentus fuerit modus diuidendi vnâ datam proportionem in tres æquales partes.

Sit proposita sphaeroides. e. f. d. t. cuius axes ex consequentia dantur. e. f. et. d. t. que quidem sphaeroides sit primo oblonga, et. u. x. sit dimidium axis maioris. imagineatur etiam conus. e. u. f. vt supra. Imagineatur etiam factum esse, quod proponitur, hoc est, vt sphaera. a. b. c. q. sit dimidium ipsius sphaeroidis, vnde conus. a. c. b. æqualis erit cono. e. u. x. vt supra demonstratum est, & sit. g. h. media proportionalis inter. u. x. et. o. c. Iam visum superius fuit, quod eadem proportio erat ipsius. u. x. ad. g. h. quæ. a. b. ad. e. f. quare eadem quæ. o. b. ad. e. x. sed. u. x. et. e. x. dantur. inter quas. g. h. et. o. b. vel. o. c. (nam. o. c. æqualis est. o. b.) mediæ proportionales sunt, eo quod cum. g. h. media proportionalis sit inter. u. x. et. o. c. & proportio. o. b. ad. e. x. æqualis sit ei, quæ. u. x. ad. g. h. hoc est ei quæ. g. h. ad. o. c. vel. ad. o. b. quare quotiescunque inuentæ fuerint. g. h. et. o. c. vel. o. b. mediæ proportionales inter. d. x. et. x. c. ipsa. o. c. vel. o. b. erit semidiameter sphaeræ quæ sitæ. eodem modo faciendum erit si sphaeroides fuerit prolata.



g  
h



R r Modus



ad.o.d.hoc est vt.z.ad.l.hoc est vt.c.ad.y.quare triangulū. p.q.o.ita erit proportio  
natū triangulo.o.q.b.vt.c.ad.y.constituo deinde ex. 25. sexti duo triangula simi-  
lia duobus.p.q.o.et.o.q.b.æqualiaq̄.c.et.y.que sint.a.i.u.et.n.t.x.fecetur postea. q.  
g.in puncto.a.ita, quod.q.æ.æqualis sit.i.a.duco postea.a.e.æquidistantem.ad.p.b.  
& sic habebimus duo triangula. q.x.æ.et.q.x.e.vt quærebantur,quamuis duo trian-  
gula.a.i.u.et.t.n.x.easdem habeant conditiones.

## DE IMPERFECTA SOLVTIONE PROBLE-

mat̄is Nicolai Tartaleæ ad Cardanum. De animad-  
uerfione in Ptolomeum. De incendio carbo-  
num à vento

*Clarissimo Dominico Morefmo.*



**S**ic propositam tibi quæstionem te diu agitauisse, nec tamen solutio-  
nem assequi potuisse, aduerte igitur ipsam falsam, idest impossibilem  
esse, quemadmodum etiam decimumoctauum quæsitum propositum à  
Cardano Tartaleæ, ab ipso Tartalea solutum minimè fuit. Quiquidem  
Tartalea vult circulum describi circa triangulum per quintam libri quarti Euclidis,  
vt in fine ferè quintæ partis suarum mentularum affirmat, neque videt in quinta  
quarti Euclidem vti vndecima primi, & in vndecima primi, quarta aut octaua eius-  
dem, quas, ipse Euclides ostensuè non demonstrauit. Quapropter oportebat Tar-  
taleam demonstrasse omnes propositiones ad hoc necessarias ostensuè vsq; ad pri-  
mas indemonstrabiles, quia ad demonstrandam scientificè aliquã propositionem,  
aut à propositione in propositionem vsque ad prima principia vniuersalia ( vt ali-  
quando ego feci ) est retrogradandum, aut ab ipsis principijs incipiendum successi-  
uè eousque progrediendo donec ad propositionem quam demonstrare volumus  
perueniamus.

Quod ad Ptolomeum in geographia attinet, dico eum mihi non satisfacere, cum  
sumit portionem arcus circuli maioris inter vnã ciuitatem, & aliam, ea ratione  
quam describit. Quod si vsus fuisset modo Menelai, ab ipso met. deinde in suū Al-  
magestum vsurpato, aut Monteregij triangulorum sphericorum, quem Copernicus  
adhibuit ( qui tamen modus, tempore Ptolomei, nondum fortasse in lucem vene-  
rat ) bene egisset.

Quod deinde ad suum illud instrumentum geometricum attinet, est imperfectū,  
vt ostendi domino Pandulfo.

Motum autem aeris, aut mauius ventum, accendere ignem, non solum ratione an-  
tiperistasis, quam affers euenit, sed etiam quia à carbonibus accensis totam excre-  
mentitiam materiam, quæ eos circumdat, auferat.

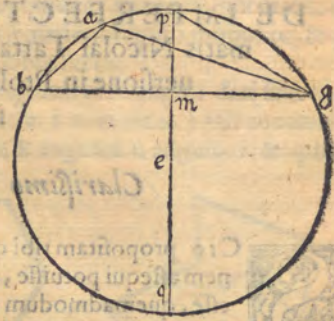
*Alia dilucidatio propositionis. 25. lib. 2. Monteregij.*

A D E V N D E M.

**S**cribis te non intelligere. 25. propositionem lib. 2. Monteregij. cum nec scias  
reperire diametrum circuli circumscripibilem circa propositum triangu-  
lum,

R r 2 lum,

lum, cuius data sit b asis tantummodo simul cum angulo, qui ipsi basi opponitur. Imagineris igitur triangulum datum esse obtusiangulum. a. b. g. cuius basi. b. g. sit nobis data simul cum angulo. a. ei opposito, obtusoque; Considera etiam circulum. a. b. g. q. ipsum triangulum circumscribentem, cuius diameter. q. e. p. transeat per. m. punctum medium ipsius. b. g. tunc protractis imaginatione. e. g. et. g. p. certum erimus angulos. circa. m. rectos esse ex. 3. tertij Eucli. angulumque. q. e. g. duplum esse angulo. q. p. g. ex. 19. eiusdem, unde aequalem angulo. a. qui etiam duplus est angulo. q. p. g. quapropter proportio arcus. q. g. ad arcum g. p. tibi cognita erit; & proportio etiam chordae. p. g. ad sinum. m. g. arcus. g. p. & quia. m. g. ut dimidium ipsius. b. g. tibi data est, cognosces etiam. p. g. ut. m. g. & sic tertium latus. m. p. trianguli orthogonij. p. m. g. & q. a. ex. 34. tertij quod fit ex. p. m. in. m. q. est aequale ei quod fit ex. b. m. in. m. g. ideo cum diuisum fuerit productum. b. m. in. m. g. per. p. m. proueniet. m. q. quapropter habebis totum. q. p.

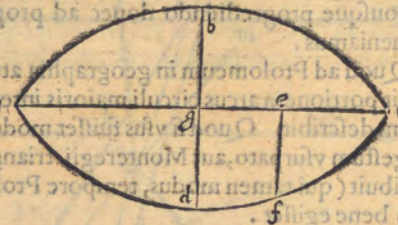


Idem efficies, si eum angulus. a. acutus fuisset.

*Modus inueniendi puncta elliptica via Pergei.*

**A D I E V N D E M.**

**M**odus inueniendi puncta elliptica, via. 21. primi lib. Pergei ex datis axis, ut ubi alias significati, talis est. Sit exempli gratia maior axis propositus. a. c. minor autem. b. d. cum ergo volueris inuenire punctum circumferentiae correspondentem puncto. e. maioris axis, inueniemus primo latus tetragonicum producti. a. g. in. g. c. quod fit. h. latusque tetragonicum producti. a. e. in. e. c. quod fit. i. deinde inueniemus lineam. K. tertiam in continua proportionalitate cum. h. et. i. unde. i. erit media proportionalis inter. h. et. K. & ut. h. proportionalis erit ad. K. inueniemus. e. f. cui. g. d. medietas secundi axis ita se habeat, quae postea iuncta axi maiori, ad angulos rectos in puncto. e. dabit situm puncti. f. quaesiti ex dicta. 21. primi lib. Pergei, sed talis modus prolixus est.

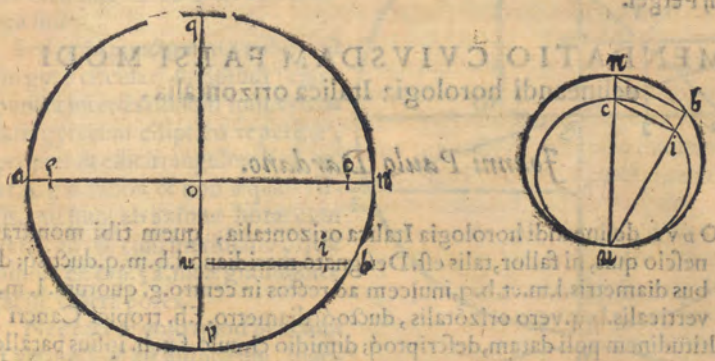


**A D I E V N D E M.**

Accipio

Accipe igitur hunc alium. Sit propositus maior axis. q. p. minor vero. e. c. ad angulos rectos se inuicem fecantes in puncto. o. describatur circulus. q. n. p. a. cuius diameter sit axis maior, in quo accipiat punctum, quod volueris, vt puta. u. a quo protrahatur u. b. parallela ad. o. c. n. designetur postea separatim circulus. u. b. n. cuius diameter aequalis sit semidiametro prioris circuli, ita etiam fiat circulus. u. i. c. contingens circulum. u. b. n. in puncto. u. cuius diameter sit. u. c. aequalis dimidio axi minori. accipiat deinde in circulo maximo longitudo. u. b. quae collocetur in circulo mediocri a puncto. u. quae secabitur a minimo circulo in puncto. i. cum itaque longitudo. u. i. mensurata fuerit in. u. b. maximi circuli a puncto. u. habebimus propositum.

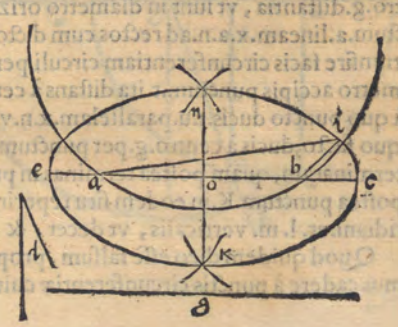
Cuius rei ratio est, quia. u. b. mediocri circuli diuiditur a gyro minimi in puncto i. eadem proportione, qua diuisa est. u. n. in puncto. c. quod manifestum est ex similitudine triangulorum. u. b. n. et. u. i. c. imaginatae cum fuerint duae. b. n. et. i. c. sed ita esse oportet parallelas maximi circuli, quotiescunque circumferentia ipsius ellipsis transitura sit per. c. vt in. 5. 1. cap. meae gnomonicae ostensum fuit.



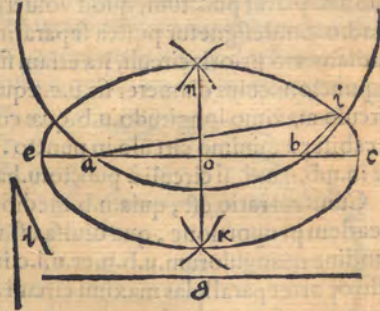
*Modus designandi angulum, certo modo conditionatum.*

A D E V N D E M.

N Vllius reuera difficultatis mihi videtur esse, quotiescunque nobis proposita fuerint duo puncta. a. et. b. simul cum angulo. d. necno linea. g. ducere duas lineas a dictis punctis terminatas, quae constituat angulum aequalem dato, & ipsae directe coniunctae constituant lineam aequalem datae. Nam ducatur linea indefinita per puncta proposita, cuius lineae pars illa, quae intercepta fuerit inter dicta puncta, diuidatur per aequalia in puncto. o. etiam & linea data, quarum medietates accipio in linea indefinita protracta a puncto. o. medio



dio, vt vna earum sit. o. c. reliqua verò sit. o. e. deinde aperiatur circinus quantum. o. c. positoq; vno pede in. b. designetur cum altero duo arcus. n. K. posito iterum vno pede in. a. designentur alij duo arcus interfecantes primos in punctis. n. K. Deinde à puncto. n. ad. K. ducetur linea. n. K. quæ per punctum. o. transibit, quam. n. K. mente concipio, vt axis minor vni<sup>9</sup> ellipsis, cuius. e. c. sit axis maior, quibus axibus mediantibus designetur ellipsis. n. c. K. e. considero deinde. a. b. vt chordã vnius circuli, seu portionis circularis, quæ capax sit vnius anguli æqualis angulo. d. proposito, ex. 32. tertij Euclid. cuius circumferentia, circumferentiam ipsius ellipsis secabit in duob<sup>9</sup> punctis quorum vnũ sit. i. à quo protractæ cum fuerint duæ lineæ. a. i. et. i. b. habebis propositum, cum. a. i. iuncta cum. i. b. æquetur. e. c. ex. 52. tertij Pergei.



EMENDATIO CUIVSDAM FALSI MODI  
delineandi horologia Italica orizontalia.

*Joanni Paulo Dardano.*

**M**ODVS delineandi horologia Italica orizontalia, quem tibi monstrauit nescio quis, ni fallor, talis est. Designato meridiano. l. b. m. q. ductisq; duobus diametris. l. m. et. b. q. inuicem ad rectos in centro. g. quorum. l. m. sit verticalis. b. q. vero orizontalis, ductoq; diametro. f. h. tropici Cancræ secundum altitudinem poli datam, descriptoq; dimidio circulo. f. z. h. ipsius paralleli, inuentoq; puncto. z. horæ proposiæ, & ab eo ducta per perpendiculari. z. r. ad. f. h. & à puncto. r. ducta. r. o. y. parallela ad diametrum. q. b. orizontalem, ducis postea. f. o. et. r. t. vsque ad orizontalem. q. b. parallelas ad diametrum. l. m. verticalem. Determinato postea gnomone. g. s. in orizontis axe, ducta q; umbrarum linea. s. K. parallela orizontali, ducta q; y. g. K. ad terminandam. s. K. delineas deinde separatim circulum q. x. b. n. magnitudinis prioris, qui quidem circulus significet orizontem ipsam, in quo ductis diametris. q. g. b. et. l. g. m. accipis in diametro. q. g. b. puncta. a. et. o. ita à centro. g. distantia, vt sunt in diametro orizontali prioris circuli, ducis postea per punctum. a. lineam. x. a. n. ad rectos cum dicto diametro, deinde per tria puncta. n. o. x. transire facis circumferentiam circuli per quintã quarti Euclidis, postea in dicto diametro accipis punctum. t. ita distans à centro, & ex eadem parte, vt in priori circulo, à quo puncto ducis. r. u. parallelam. x. n. vsque ad circumferentiam. x. n. in puncto. u. quo factò, ducis à centro. g. per punctum. u. ipsius circularis circumferentiæ. g. u. indeterminatam, quam postea terminas in puncto. K. ita quod. g. K. æqualis sit. s. K. Ducas postea punctum. K. in eodem situ reperiri, respectu duorum diametrorum. q. b. meridiani. et. l. m. verticalis, vt decet, & oportet punctum horæ proposiæ existere.

Quod quidem dico esse falsum, propterea quod perpendiculares quas cogitamus cadere à punctis circumferentiæ cuiusuis paralleli supra quemuis orizontem obliquum

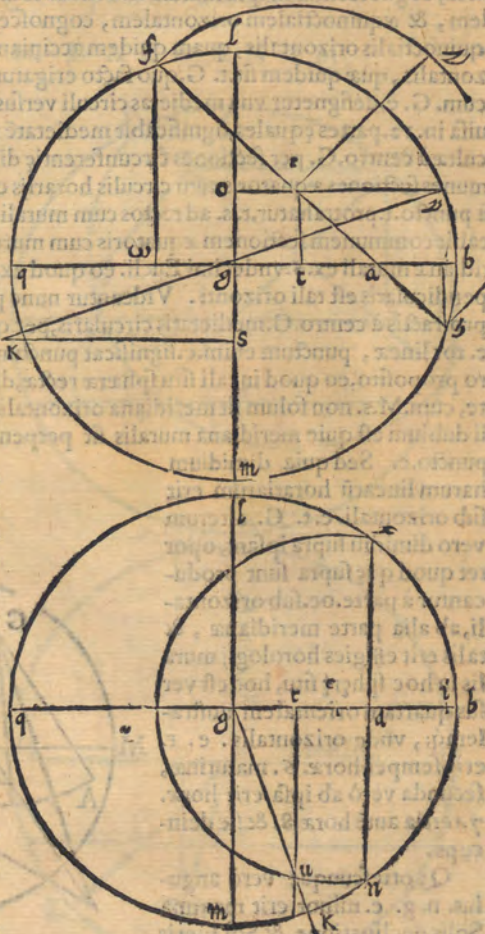
liquum secantem æquatorem, omnes cadunt in gyro elliptico, oxygonio, seu deflectionali, & non circulari. Vnde per supradicta tria puncta. n. x. oporteret transferre talem circumferentiam, & non circularē, quæ circumferentia esset vnus ellipsis, cuius minor axis in diametro. b. q. esset. ab. s. vsque ad. i. terminum fini h. i. arcus. h. b. in analemate, maior verò axis esset magnitudinis. f. h. diametri paralleli, quæ trāsisset per punctum. c. medium inter. s. et. i. quæ quidem circumferentia tota esset intra circulum. q. n. b. x. cōtiguaq; gyro. q. n. b. x. in punctis. n. x.

Si ergò circumferentia. n. x. esset elliptica tunc punctum. u. in horizonte illud esset vbi caderet sinus altitudinis horæ, et. t. u. æqualis esset. r. z. communi sectioni paralleli cum almicantrat ex. 34. primi Euclid. et. u. g. æqualis esset. o. y. communi sectioni almicantrat cum meridiano, vel cum azimut illius horæ ex. 4. primi, cum. g. t. æqualis sit ipsi. o. r. et. t. u. ipsi. r. z. & angulus. t. trianguli. g. t. u. rectus, quem admodum. r. qui comprehenditur ab. z. r. et. r. o. vnde anguli. K. g. m. et. K. g. b. rectè se haberent, distantia verò inter. K. et. g. iã rectè sumpta fuit.

Sed quia punctum. u. vt plurimū (in gyro circulari sumptum) extra puncta intersectionum ipsius circularis gyri cum elliptico reperitur, propterea efficit angulos. K. g. m. et. K. g. b. falsos, & non æquales illis, qui sunt ab azimut horæ cum verticali, & cum meridiano, quæ omnia ex cap. 5 2. meæ gnomonicæ facillè videre potes.

Nec tacere volo quod punctum u. verum, hoc est ellipticum, inueniri posset ea via quam scripsi in eodem. 5 2. cap. qua mediante docui demum inuenire punctum. s. r. orientis, quamuis in presenti ca. fu. s. r. perpendicularis esset supra minorem axem ipsius ellipsis, quāuis supra maiorem axem, quod tamen minimè mutat ordinem, imò rationes eedem sunt, tam in vna, quam in alia operatione, sed vt illicò idipsū habeas, fac vt. t. u. æqualis. sit. r. z. & tunc punctum. K. erit quæsītū, quod ego in. 5 2. cap. meæ gnomonicæ, ijs verbis significauī.

» Itaq; mediis binis triangulis ijs,  
» medioq; azimut Solis pariter horologia fabricari poterunt.



De





maiori numero declinationis quam fuerit angulus.n.g.e.tunc talis paries illuminabitur ab ipso Sole à mane vsque ad vesperam .

Huius quidem rei speculatio, vnicuique manifesta erit, qui rationes.46.cap.nostræ gnomonicae prius intellexerit,vbi manifestè apparet proportionem semidiametri horologij ( si ita eam appellare licet ) ad semidiametrum æquatoris horarij semper esse,vt.e.t.ad.t.g.hoc est proportio maioris inæqualitatis. nolo etiã prætermittere.quin te admoncam,vt nullo pacto confidas in longioribus vmbri, eo quod valde nos decipiant, cum semper iusto breuiores sint.

*Declaratio quorundam verborum nostra Gnomonica.*

*Defensioq; nostra contra Christophorum Clauium.*

AD EVNDEM DARDANVM.

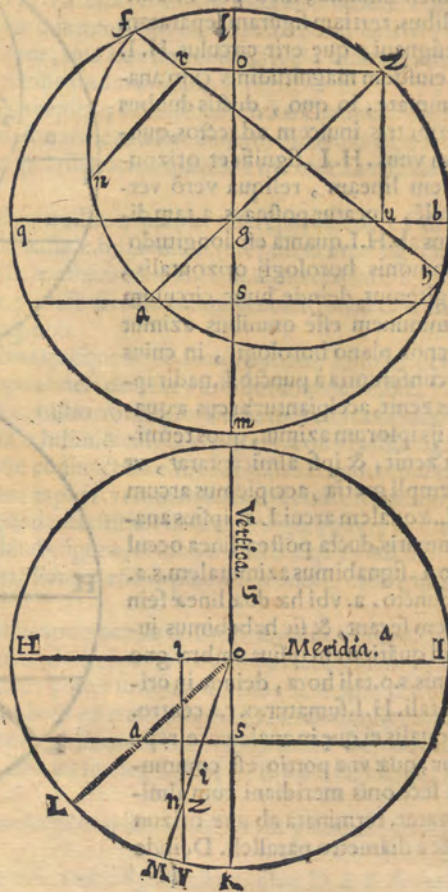
**T**Vas demum accepi literas, quibus mihi significas te totum. 52. caput meæ gnomonicae intellexisse, præter illa verba, quæ etiam superioribus diebus ad te scripsi, hoc est.

Itaq; medijs binis triangulis ijs, medioq; azimuth Solis, pariter horologia fabricari poterunt.

Quapropter ne aliquid tibi desit, scire debes, me nihil aliud, eo in loco inferre voluisse, quàm quod punctum horæ propositæ in plano horologij horizontali reperiri potest, ope longitudinis vmbrae gnomonis, & eius declinationis à verticali linea, seu à meridiana horizontali, iam in ipso horologij plano ductis.

Exempli gratia, sit analemma. l.q.m.b.in quo.l.m.fit verticalis. q.b.verò horizontalis. f.n.h. autem sit semicirculus, cuiusvis paralleli æquinoctiali, cuius diameter sit.f.h. et.n.fit Solis locus in ipso parallelo: n.r.autem sit rectus sinus arcus.f.n:et.r.o.z.sectio communis ipsius almicantrat cum meridiano,et.s.a. cõmunis sectio azimuth Solis cum plano horologij,et.s.g.gnomon, et. z.g.a.radius Solis.z.u.verò sinus altitudinis ipsius Solis, vbi videre potes duo triangula dicta esse. z. u.g.

Ss &c

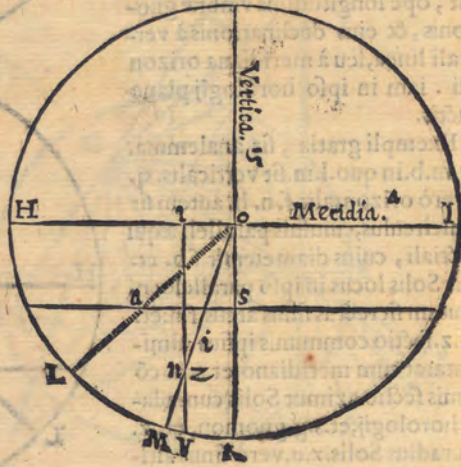
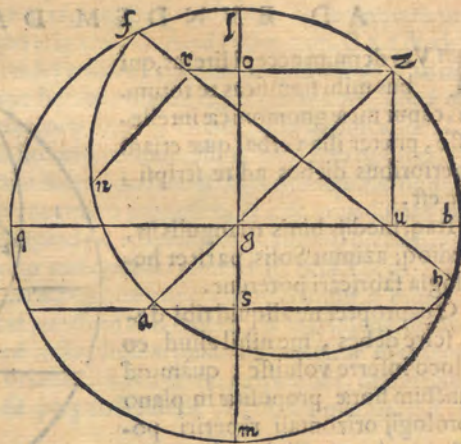


et. g. s. a. quibus mediantibus cognoscitur longitudo vmbrae gnomonis hoc est. s. a.

Cum autem dico, medioq; azimuth Solis, nihil aliud significare volo, nisi angulum, quem terminat linea azimuthalis horologii, hoc est umbra gnomonis cum linea meridiana, seu cum verticali in ipso plano horologii. qui quidem anguli, æquales sunt ijs, qui in triangulo constituto ex. n. r. ex. r. o. & ex. o. z. reperiuntur, cuius quidem trianguli, angulus puncti. r. rectus est, angulus verò terminatus ab. n. r. et. o. z. ille est quem constituit azimuth cum verticali, vel ipsi æqualis, ut coalternus, reliquus verò in puncto. o. ille est quem azimuth facit cum meridiano, vel ipsi æqualis ut coalternus.

Vnde quotiescunque volueris in aliquo plano, orizzonti parallelo, lineas horarias ducere, iudico optimum fore si separatim designatæ fuerint hæ tres figuræ, hoc est analemma meridianum, vel azimuthale, ut ita dicam, deinde parallelus inferuiés pro tropicis, ut ego feci cap. 5 r. meæ gnomonica, quæ duæ figuræ, sufficietes erunt

pro omnibus horologijs, tam horizontalibus quam muralibus, non tamen omnino, ideo pro horizontalibus, tertiam figuram separatam designavi, quæ erit circulus. H. I. K. eiusdem magnitudinis cum analemmate, in quo, ductis duobus diametris inuicem ad rectos, quorum vnus. H. I. significet horizontalem lineam, reliqua verò verticalis, ducatur postea. s. a. tam distans ab. H. I. quanta est longitudo gnomonis horologii horizontalis, cogitemus, deinde hunc circulum communem esse omnibus azimuth necnon plano horologii, in cuius circumferentia à puncto. k. nadir ipsius zenit, accipiantur arcus æquales ijs ipsorum azimuth, quos terminat zenit, & ipsi almicantrat, ut exempli gratia, accipiemus arcum k. L. æqualem arcui. L. z. ipsius analemmatis, ducta postea linea occulta. o. L. signabimus azimuthalem. s. a. in puncto. a. vbi hæ duæ lineæ se inuicem secant, & sic habebimus iustam quæritatem ipsius vmbrae gnomonis. s. o. tali hora, deinde in horizontali. H. I. sumatur. o. r. à centro. o. æqualis ei quæ in analemmate reperitur, quæ vna portio est communis sectionis meridiani cum almicantrat, terminata ab axe horizontalis, & à diametro paralleli. Deinde du-



ducat. r. V. ad rectos cum. H. I. vsque ad circumferentiam, in qua accipiatur. r. n. æqualis ei quæ est in parallelo, ducatur postea. o. n. M. & habebimus triangulum, o. r. n. similem æqualemq; triangulo iam supradicto. Vnde angulus. H. o. M. ei æqualis erit, quem azimuth facit cum meridiano, & angulus. M. o. k. ei æqualis, quem azimuth constituit cum verticali, ita quod si talis circulus. H. k. I. esset planum horologij horizontalis, supposito. o. pro pede gnomonis, secundo postea. o. M. in puncto. i. ita vt. o. i. æqualis esset. s. a. dato quod. o. M. ducta sit ad partem sibi conuenientem, respectu. o. k. ipsa pro verticali supposita, quod tibi relinquo, cum hoc facillimum sit, tunc punctum. i. esset quod quærebamus. Quod verò de vno puncto dico, idem de omnibus infero.

Vbi verò mihi significas Christophorum Clauium, me duobus in locis meæ gnomonica redarguere, iam vidi. Circa primum locum igitur, qui est in pagin. 161. ita inquit.

Non enim defunt, qui vel omninò negent, inter quos est Ioannes Baptista Benedictus in sua gnomonica cap. 70. et. 71. vbi alia, & multo longiore ratione conatur arcus signorum describere, vel certe dubitent, hoc modo rectè posse describi arcus signorum, cum rationem non videant, qua hæc nostra descriptio quam quidem omnes scriptores sine vlla demonstratione tradunt nitatur.

Abique dubio raptim transcurrit illa capita. 70. 71. Reuerendus Clavius alioquin non scripisset, quod ego alia & multo longiore ratione conatus sim arcus signorum describere &c. præsertim cum eadem prorsus ratio, quæ ibi à me tradita est, illa sit, quam ipse suis scriptis inferuit.

Meus igitur modus in dictis capitibus traditus, minime discrepat ab eo, sed ab illorum modo, quorum opinio est interualla. e. h. u. u. n. n. m. et. m. d. meæ figuræ in pagi. 75. posita, æqualia esse interuallis. e. h. u. u. n. n. m. et. m. d. præcedentis figuræ, qui etiam supponunt. t. e. meæ figuræ. 75. esse directè coniuncta cum linea. e. h. u. n. m. d. & propterea versus finem. 73. pag. dixi.

Aduertat autem quam diligentissime quisque ne se decipi patiat à subscripta figura semicirculi. Q. æ. m. cum reliquis lineis ductis, ex antiquorum more, &c.

Eo quod non defuerunt aliqui, ex verustioribus (quorum scripta ad meas manus peruenerunt) qui sumentes interualla. e. h. u. u. &c. figuræ. pag. 75. æqualia illis figuræ pag. 74. putauerunt lineam. t. e. directè coniunctam esse cum. e. h. &c. quod quidem maximi erroris causa erat, & propterea cap. 71. verum modum ostendi, seruando illam eandem suppositionem, hoc est quod interstitia. e. h. u. u. &c. figuræ pag. 75. æqualia sint interstitijs. e. h. u. u. &c. præcedentis figuræ, & ideò in dicto cap. 71. dixi.

Supposito deinde. f. e. b. lineam meridianam esse in plano horizontali, ceteræ lineæ horariæ erunt prædictæ.

Stantibus igitur his suppositis, vt habeantur omnia scientificè, volui, vt intelligeretur pyramis quadrilatera, eo modo quo dixi, cap. 71. vbi clarè patet eandem pyramidem esse, quam Pater Clavius (tacitè) posuit in figura horologij, vt ipse docuit propositione secunda, lib. secundi, cuius basis est triangulum. H. I. F. suæ figuræ (exempli gratia pro quinta hora post meridiana) Alterum verò triangulum à me cogitatum, terminatum ab. t. e. e. d. et. ab. t. d. eleuata in mea figura, est in sua triangulum. D. I. F. & propterea dixi.

Nam. t. e. et. e. d. vtræq; in plano horologii non sunt, quamuis in plano æquatoris tres sint, &c.

Angulus verò. e. quem dico rectum esse, in sua figura est angulus. D. I. F. & mea

t. d. imaginata, est sua. D. F. Tertium deinde triangulum, quod in mea figura terminatur ab. t. d. ab. f. d. & ab. f. t. in sua est triangulum. D. F. H. vnde mea. f. t. respondet suę. H. D. & mea. f. d. suę. H. F. & mea. t. d. suę. D. F. Quartum autem triangulum f. t. e. in mea figura, respondet suo. H. D. I. & meum punctum. t. suo. D. Nunc triangulum rectangulum, quod dico separatim constituere, est illud tertium dictum correspondens suo. D. F. H. vt ipse facit in sequenti figura, quod ipse vocat. D. C. H. & me<sup>o</sup> radius. t. x. in sua figura, ille est qui terminatur ab. D. & ab initio Tauri, & Virginis.

Et quamuis ego non scripserim talem figuram, vt ipse fecit, nihilominus ipsam verbis descripsi comet modo, & propterea dixi.

Quam diuisionē, si in triangulo seorsum descripto inuenire voluerimus, res erit inuentu facillima, cum rectum angulum. f. t. d. (respondentem suo. H. D. C.) prædicti trianguli tertij ea ratione diuiserimus, &c.

Quapropter Reuerendus Clavius non animaduertit meam rationem aliam non esse, nec puncto longiorem sua, cum eademmet ipsa sit.

Citavi etiam Munsterum cap. 30. eo quod in ea impressione, quam tunc præ manibus habui, vidi in ea figura, quam ipse vocat fundamentum horologiorum, literam c. positam esse loco. f. et. f. loco. c. quod causæ fuit, vt omnia mendosa viderentur, recentiores autem impressiones correctæ sunt.

Rursus alio in loco mihi accidit vt reprehenderim Alexandrum Piccolomineum in libris de sphaera, qui quidem dicebat eas figuras superficiales, quæ paucioribus angulis circumscriberentur, capaciores esse alijs, dummodo earum periphæriæ essent æquales.

Nunc autem correctæ sunt eo in loco impressiones, & qui non viderit primas, putabit me immeritò ipsum reprehendere.

Idem etiam dico de eo capite ipsius Piccolominei, in iisdem libris, vbi tractat de modo, quo vsi sunt antiqui ad diuidendum zodiacum in. 12. signa, quod erat circa finem quarti libri.

Nunc verò, in recentioribus impressionibus, illud caput positum non est. Impressiones autem illæ, vbi talia dixit, duæ fuerunt, quarum prima erat anni. 1540. secunda verò. 1552. Venetijs apud Andream Puteum.

Alius verò locus ipsius Reuerendi Clauij, contra meas reprehensiones, est circa finem pag. 298. & circa. 299. vbi ita scribit.

Ex his liquido constat, non rectè à Ioan. Baptista Benedicto in sua gnomonica capit. 49. reprehendi hanc rationem describendi horologij declinantis, qua omnes ferè alij scriptores vtuntur, quoniam, vt ex demonstratione à nobis allata constat, rectè per eam lineæ horarię in plano, quod à verticali declinat ducuntur. Modus autem quem eo loco præscribit differentem ab eo, quem nos tradidimus certus etiam est, sed nulla ratione nostro contrarius, quia nos constituimus. D. E. F. angulum declinationis plani à verticali circulo propriè dicto, ipse autem loco huius anguli affigit angulum declinationis eiusdem plani à Meridiano circulo, vnde mirum non est modum ipsius à nostro discrepare. Quod si cõstitueremus. D. E. F. angulum declinationis plani à Meridiano, ut ipse (quemadmodum forsitan ab alijs putauit fieri) & in reliqua descriptione progredieremur, vt tradidimus, proculdubio horologij declinans perperam describeretur, vt rectè docet.

Optimè scripsisset Reuerendus Clavius, si verum fuisset, quod antiqui sumerent declinationem superius dictā à verticali propriè dicto, & non à meridiano. Sed ego dico, authores à me citatos. capit. 49. meę gnomonicę sumere dictam declinationem

nem plani à meridiano, & non à dicto verticali.

Con sidera primum in Munistero cap. 16. suae horologiographiae, ubi clarè docet accipere angulum comprehensum inter meridianum, & planum propositum, ubi etiam ponit quandam figuram aedificij cum pariete super quo designatum est quoddam horologium, & ubi se manifestè declarat, ita dicens.

Nam ipsarum partium complementum. propositum indicabit angulum, quantum videlicet fuerit arcus eiusdem circuli. d. e. f. g. à puncto. g. vsque ad productam lineam meridianam interceptus, qui vnà cum ipso. f. g. quadrantem integrare videtur, vt in sequenti figura: quoniam arcus. f. g. est sexaginta partium, qualium. e. f. quadrās nonaginta, vnde concluditur reliquam partem hoc est, datum inclinationis angulū, fore partium triginta similiū.

Orontius verò cap. 13. iisdē vtitur verbis, cum figura simili ad reliqua autem ipsius B. Clauij, videnda nondum mihi ortum fuit. quod si dabitur, tibi libenter dicam quid sentiam.

### DE MODO DUCENDI LINEAS HORARIAS super cyllindro immobili.

*Hieronymo Ferrerio artium & Medicinae Doctori peritissimo.*

**D**ESIGNARE horarias lineas super cyllindro immobili, ad orientemque perpendiculariter erecto difficile tibi non erit, (quod à me postulasti) si modum. 53. cap. meae gnomonicae obseruaueris, accipiendo tamen pro linea horizontali in tabula non aliquam rectam lineam, sed circularem, similemque circumferentiae ipsius cyllindri, dico autem similem, eo quod si gnomon. o. x. supra tabulam signatus, & perpendicularis ipsi horizontali circulari. b. i. x. esset dimidia, vel tertia vel quarta pars gnomonis cyllindro infixi, oporteret, vt semidiameter circuli. b. i. x. etiam esset medietas, vel tertia, aut quarta pars semidiametri cyllindri, vt omnes arcus huiusmodi circuli inter ipsos azimuth intercepti similes sint arcibus cyllindri, quod à te ipso facile videre scientificè poteris. reliqua nihil mutanda erunt ab eo, quod scripsi circa figuram. 53. cap. vt dixi. Vnde inuenta cum fuerit distantia horizontalis puncti. b. à pede gnomonis. x. nec non quantitas azimuthalis muralis b. t. quae semper ab horizontali perpendiculariter descendit, illico punctam. t. horae propositae in cyllindro inuenietur.



Nunc

iboniuuH

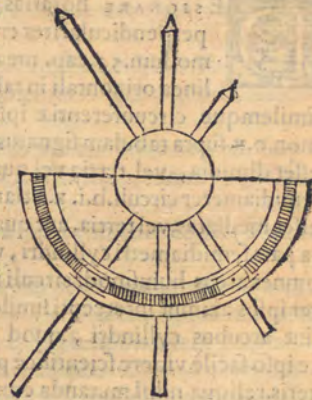
Nunc verò cum duo puncta alicuius horarię lineę inuenta fuerint, quę à Solis situ in diuersis parallelis efficiuntur, si voluerimus ipsam lineam horariã ducere, sciendum primò est ipsam lineam horariam esse communem sectionem circuli horarij, illius horę cum superficie cyllindrica, & propterea ellipticam, vt ostendit Serenus in. 19. primi lib. quod etiam elicere possumus ab eo, quod Archimedes in. 10. propositione libr. de conoidalibus, scribit. Quapropter oportet nos instrumentum prius componere, modo circini, sed trium crurum, quę omnia in eadem plana superficie sint, ea tamen arte factum, vt quodlibet illorum possimus prolongare, necnon contrahere, ut cum duo extrema firmata fuerint, media possit circunduci circa centrum, seu punctum commune illarum intersecctionum simulq; possit produci, necnon abbreviari vel augeri, & diminui, vt mediante sua extremitate inferiori possimus delineare gyrum ellipticum horarium, dum cętrum ipsorum crurum adhæreat extremitati gnomonis, reliquę vero extremitates ipsorum crurum sint supra puncta inuenta ipsius horę. oportet etiam vt hoc instrumentum à tergo ipsorum crurum habeat in superiori parte superficiem quandam semicircularẽ, quę fit vice vnus partis illius superficiẽ, in qua supponuntur omnia crura instrumenti, & hoc quantum fieri potest, quod quidem fieri debet, ne crus medium, hoc est mobile, exeat à tali superficie, seu declinet ab ea, quę semper supponitur in situ circuli horarij talis horę. oportet etiam, vt iuxta circumferentiam dimidij circuli sint duo gyri eiusdem materię inter se parum distantes, ita ut crura possint moueri, intra hos gyros, & dimidium circulum, & quod inter hos gyros locatę sint duę cochleę, seu duo helices, vt quando voluerimus, possimus firmare ipsa crura extrema, dum eorum extremitates fuerint supra puncta inuenta illius horę, deinde in dorso istius instrumenti, circa centrum coniunctionis, rectẽ factum erit si aliqua concauitas fuerit, in qua, extremitas gnomonis possit locari, dum ducere voluerimus aliquam horariam lineam.

Tale instrumentum excogitauì ad fugiendum tedium inueniendi dictam ellipticam ex punctis

Nunc autem sciendum est, quod vnus tantummodo gnomon sufficiens non erit pro tota die æstiuã, neque duo, nisi valde breues fuerint respectu semidiametri cyllindri, & in situ medio quartarum meridionalium nostro orizzonti, quorum autem longitudo ita inuenienda esset.

Sit exempli gratia circulus. a. b. e. u. cyllindri orizzontis vice, diuisusq; à duobus diametris. d. e. et. e. f. quarum. c. f. fit pro meridiana: d. e. autem pro verticali, fitq; e. punctus orientalis: d. verò occidētalıs. f. autem meridionalis. et. c. septentrionalis, computeturq; maxima Solis amplitudo ab. f. versus. e. quę terminetur ab. q. ita quod arcus. f. q. minor sit quã graduum. 45. aliter impossibile esset duobus tantummodo gnomonibus mediantibus tota die æstiuã horas videre.

Quo factò ducatur ab. q. q. p. contingens circulum & à centro circuli. o. per punctum. u. medium quartę ducatur. o. u. i. vsque ad contingentem. q. p. vnde. u. i. longitudo erit vniuscuiusque gnomonis, qui gnomones infixi erunt in medio dictarum quartarum.



Huiusmodi



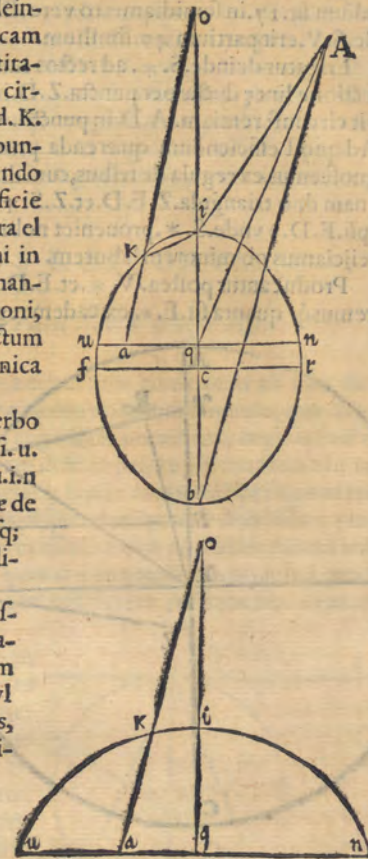




rem, quæ ita reperientur, efficiemus primo angulum conij, qui fit. i. A. b. quem diuidemus per æqualia mediante. A. q. constituendo. A. i. huius anguli æqualem. A. i. superficie conicæ et. A. q. diuidentem, æqualem parti. A. q. axis conij, ducendo postea ab. i. per. q. lineam vnã quousque concurrat. A. b. in puncto. b. habebimus. i. b. pro maiori axi ipsi ellipsis, quod per se clarum est, cuius medietas fit. i. c. sed. i. q. ipsius. i. b. æqualis est ipsi. q. i. ipsius conij, ex quarta primi Euclij. et. q. b. ipsius. i. b. æqualis alteri parti inuisibili. Reliquum est, vt reperiamus minorem axem, quem vocabimus. f. r. ducatur ergo primum. q. a. u. n. ad f. c. os cum. i. b. æqualisq; ei quæ est conij, & diuisa similiter in. a. quæ. u. n. ipsius conij nobis cognita est ex lateribus. A. u. et. A. n. & ex angulo conij, et. a. q. æqualis est. e. p. ex. 34. primi. Nunc certum erimus ex. 21. primi Pergei, quod eadem proportio erit quadrati. u. q. ad quadratum ipsius. f. c. quæ producti ipsius. i. q. in. q. b. ad productum ipsius. i. c. in. c. b. & cum cognita nobis sint hæc tria producta hoc est. i. q. in. q. b. et. i. c. in. c. b. et. u. q. in seipsa, cognoscemus etiã quartum ipsius. f. c. & sic. f. c. eiusq; duplum. f. r. cogniti nobis itaque cum sint hi duo axes. i. b. et. f. r. formabimus ellipsim. Deinde producemus axim. b. i. à part. e. i. quousque. i. o. æqualis sit ei quæ extra conum est, deinde ducemus. o. a. quæ circumferentiam ellipticam secabit in puncto. K. vnde habebimus quantitatem ipsius. o. K. et. K. i. rectam. inde mediante circino si acceperimus rectam distantiam ab. i. ad. K. in ellipsi, deinde firmando pedem circini in puncto. i. in superficie conica, & cum alio signando lineam vnã curuam ad partem. K. in superficie conica, sumendo postea interuallum. o. K. extra ellipsim, deinde firmando vnum pedem circini in extremitate gnomonis, cum alio postea signando aliam lineam curuam in superficie ipsius conij, quæ primam fecit in puncto. K. hoc erit punctum quæsitum horæ propositæ in superficie conica proposita.

Sed si talis sectio fuerit parabole, vel hyperbole, tunc mediante suo diametro. i. q. cum basi. u. q. n. cognita, designabimus ipsam sectionem. u. i. n. ope mei instrumenti in calce meæ gnomonicæ descripti, deinde diuisa. u. q. in. a. p. ducta q; q. i. vsq; ad. o. ducta q; o. a. habebimus punctum. K. Reliqua faciendæ sunt, vt dictum est de ellipsi.

Inuenta modo cum fuerint duo puncta eiusdem horæ propositæ, ducemus ab vno ad aliud, lineam horariam, mediante circino trium crurum, quem tibi scripsi nudius tertius pro cylindro, quæ quidẽ linea erit portio gyri ellipsis, seu hyperbolæ, vel parabolæ, vt à te ipso cogitare potes.



Tt Quæ-

QVAEDAM NOTATA DIGNA IN  
Ptolomeum.

*Bartolomeo Christino Serenissimi Sabaudia Ducis apparitore.*

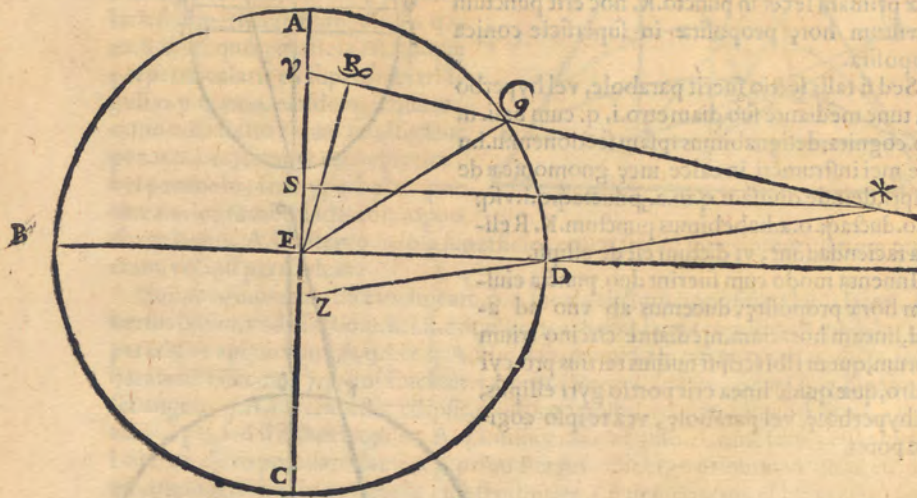


X tuis literis cognoui quo erga me animo esses, qualiq; voluntate, sed ne tua pulcherrima studia aliquo modo imperfecta relinquant, vel ego tibi deesse videar, dum Problemata geographica Magni Ptolomei consideras, aduerte, quod si putares in figura. 6. cap. libr. 7. geographiæ eiusdem (vt multi credunt) lineam. V. \* . secare circumferentiam. A. D. in puncto. G. ita vt punctus. G. sit tropici æstiu, idest arcum. D. G. esse graduum. 24. cum illis incideres in maximum errorem. Quapropter considera quæ nunc tibi scribo.

Sit circulus. A. B. C. D. huius centrum. E. supponaturq; semidiameter. E. D. esse partium. 120. quarum. E. Z. in alio semidiametro. C. E. ei orthogonaliter coniuncto, talium sit. 17. in semidiametro vero. E. A. accipiatur. E. S. talium. 24. et. E. V. 64. unde. S. V. erit partium 40. similitum.

Erigatur deinde. S. \* . ad rectos cum. E. A. in puncto. S. quæ terminetur ab intersectione lineæ ductæ per puncta. Z. D. in puncto. \* . ducatur demum. V. \* . quæ secabit circumferentiam. A. D. in puncto. G. Quærat nunc quantitas ipsius. G. D. Ad quod efficiendum quærenda primum est quantitas ipsius. S. \* . quam illico cognoscemus ex regula de tribus, cum dixerimus, si. 17. dat nobis. 120. quid dabit. 41. (nam duo triangula. Z. E. D. et. Z. S. \* . sunt inuicem similia, cum. S. \* . parallela sit ipsi. E. D. ) unde. S. \* . proueniet nobis ex similibus partibus. 289. cum fracto, quod rejiciamus ob minorem laborem.

Producantur postea. V. \* . et. E. D. vsque ad eorum concursum in puncto. o. quæremusq; quanta sit. E. o. ex eadem regula, cum dixerimus, si. 40. dat nobis. 289. quid



dabit

dabit. 64. (nam duo triangula. V.S. \* .et. V.E. .sunt inuicem similia eadem ratione) vnde. E. .veniet nobis ex talibus partibus. 462.

Coniungatur nunc quadratum ipsius. E. V. quod est. 4096. cum quadrato ipsius. E. . quod est. 213444. & habebimus quadratum ipsius. V. . talium partiū. 217540.

Dicemus postea si. 217540. dat nobis. 4096. quid dabit quadratum ipsius. V. . vt sinus totus quod est. 1000000000. vnde veniet pro quadrato ipsius. V. E. talium partium, superficialium scilicet. 18827211. cuius radix erit. 13721. & erit sinus anguli. V. . E. qui erit grad. 7. min. 53. vnde angulus. . V. E. erit grad. 82. min. 7. eius vero sinus erit partium. 99054.

Nunc autem quia angulus. E. V. . est acutus, imaginemur. E. R. ductam esse ad rectos ipsi. V. . sitq; etiam ducta ipsa. E. G. Vnde habebimus angulum. R. E. V. graduum. 7. min. 53. eius vero sinus. R. V. partium. 13721. (propter similitudinem triangulorum. E. R. V. et. . E. V. ) talium scilicet, qualium. E. V. fuerit. 100000. Sed qualium. E. V. est. 64. talium erit. 8. cum tribus quartis, cuius. R. V. quadratum erit partium. 76. cum dimidio similia sed superficialium, quo quidem quadrato dempto ex quadrato ipsius. 64. quod est. 4096. remanebit quadratum ipsius. E. R. partium. 2871. quo etiam quadrato. E. R. dempto ex quadrato. E. G. partium. 14400. remanebit quadratum ipsius. R. G. partium. 11529. cuius radix. R. G. erit partium. 107. taliū qualium. E. G. est. 120. sed qualium. E. G. erit. 100000. talium. R. G. erit partium. 89166. quæ vt sinus anguli. R. E. G. habebit pro ipso angulo, gra. 63. min. 5. qui collecti cum gra. 7. min. 53. anguli. V. E. R. dabunt totum angulum. A. E. G. grad. 70. min. 58. cuius complementum ex grad. 90. erit. G. D. graduum. 19. min. 2. & non. 24. vt omnes ferè putant.

DE REFLEXIONIBVS RADIORVM.

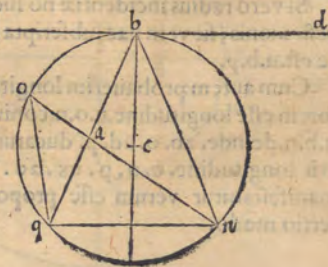
Excellentissimo Philosopho Francisco Vimercato.



VONTIAM non videbatur quiescere animus tuus, cum paucis ab hinc diebus tibi siscitanti respondissem, nec tamen rationem omnium, quæ dixeram exactè explicare per temporis angustiam potuissem, cogitavi ad te per hanc occasionem scribens, & iam dicta repetere, & omnium tibi rationem subiungere, & vt mihi plenius satisfaciam, & tibi commodè perlegenti facilius sit veritatem intueri. Scripsisti enim in tuis disputationibus, vir doctissime, quod omnis res visa per speculū quodcūque, sub breuissimis lineis cōprehendatur à visu.

Propositio hæc non est vniuersaliter vera (quamuis etiam ab alijs omnibus pro tali posita sit) cum in speculis concauis non semper verificetur, vt nunc tibi demonstrabo.

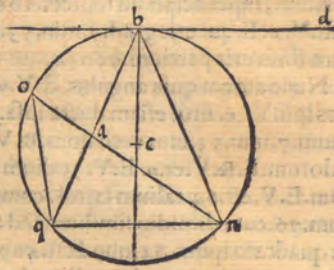
Esto quod linea recta. b. d. tangat circulum b. o. q. n. qui sit communis sectionis superficiei reflexionis, & sphericæ alicuius speculi sphericæ concaui, & punctum contingentiæ sit. b. à quo exeant duæ lineæ. b. q. et. b. n. efficientes duos angulos inuicem æquales circa perpendicularem. b. c. res autem visa primò sit in ipsa circumferentia huiusmodi circuli in puncto. n. oculus vero in puncto. q. ipsius circumferentiæ. Dico nunc duas



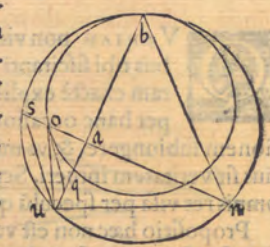
Tt 2 12 lineas

lineas, b. q. et. b. n. simul sumptas longiores esse omnibus alijs lineis exeuntibus ab ip-  
 sis punctis. q. n. quæ in aliquo puncto dictæ circumferentiæ simul concurrant.

Sint igitur aliæ duæ. q. o. et. n. o. quas probare volo simul sumptas, esse minores dua-  
 bus simul sumptis. q. b. et. n. b. Nam ex. 20. tertij Eucli. cognoscimus angulos. q. b. n.  
 et. q. o. n. inuicem æquales esse, & similiter angulos. b. n. o. et. b. q. o. deinde ex. 15. pri-  
 mi eiusdem habemus angulos contra se positos,  
 circa. a. esse etiam inuicem æquales. Vnde ex. 4.  
 sexti, habebimus proportionem. a. b. ad. a.  
 o. eandem esse, quæ. a. n. ad. a. q. & sic. b.  
 n. ad. o. q. Quare ita erit. a. b. n. ad. a. o. q. vt. a. n.  
 ad. a. q. sed cum. a. n. maior sit. q. a. ex. 18. primi,  
 eo quod angulus. b. q. n. (qui æqualis est angulo.  
 b. n. q. ex. 5. eiusdem) maior est angulo. a. n. q.  
 qui pars est ipsius. b. n. q. ergo latera simul sum-  
 pta. a. b. n. maiora erunt lateribus. a. o. q. sed ex.  
 20. primi. a. b. n. etiã maior erit. a. n. vnde ex. 25.  
 quinti. q. a. b. n. maior erit. n. a. o. q. quare sequi-  
 tur verum esse propositum.



Sed si oculus esset in. u. quemadmodum in subscripta hic secunda figura videre est,  
 res autem visibilis in. n. ambo extra dictum circulum, esto etiam primum. b. u. æqua-  
 lis. b. n. probabo similiter. u. b. n. maiores esse. u. o. n. Nam angulus. o. maior est angulo.  
 b. eo quod si circulum. u. b. n. cogitemus circumscribere triangulum. u. b. n. ducen-  
 do vsque ad suam circumferentiam. o. n. in puncto. s. deinde ducendo. u. s. habebimus  
 ex. 20. tertij angulum. u. s. n. æquale angulo. u. b. n. sed cū angulus. u. o. n. exterior trian-  
 guli. u. o. s. existat, ipse maior erit angulo. s. ex. 16. primi. duco postea. o. q. parallelam  
 ad. u. s. quæ secabit. a. u. in puncto. q. & habebimus angulum. a. o. q. æqualem angulo.  
 n. s. u. ex. 29. eiusdem, hoc est angulo. n. b. u. sed ex su-  
 pradictis rationibus, lineæ. q. b. n. simul sumptæ maio-  
 rem efficient longitudinem, quam. q. o. n. Nunc cum  
 ipsi. q. b. addita fuerit. u. q. & vice. q. o. sumpta fuerit ali-  
 qua linea minor ipsa. u. q. o. eo amplius. u. q. b. n. maior  
 erit, quod quidem hoc modo faciendum. Acci-  
 piatur. o. u. vt comes. o. n. quæ minor est ambabus. o.  
 q. et. q. u. ex. 20. primi, ita enim habebimus propositū.  
 sed breviori modo hoc ipsum videbis ex præcedenti,  
 & ex. 21. primi Euclid. Nam ex præcedenti. u. b. n. longior est  
 ipsa. u. s. n. ex. 21. autem primi. u. s. n. longior est  
 ipsa. u. o. n. ergo verum est propositum.



Si verò radius incidentiæ nõ fuerit æqualis radio  
 reflexionis, sit vt in hac subscripta tertia figura vide  
 re est. u. b. p.

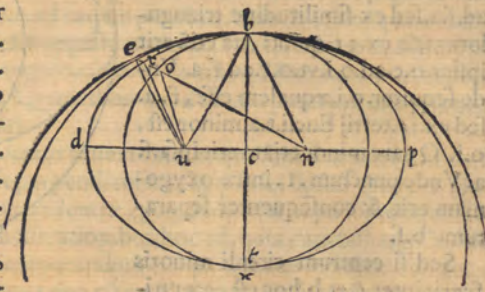
Cum autem probauerim longitudinem. u. b. n. ma-  
 iorem esse longitudine. u. o. n. coniungatur. n. p. cum  
 u. b. n. deinde. ab. o. ad. p. ducatur. o. p. quæ minor  
 erit longitudine. o. n. p. ex. 20. primi, & illicò  
 manifestabitur verum esse propositum, etiam hoc  
 tertio modo.



Si

Si autē res visibilis oculusq; ambo fuerint intra circulum, tūc possibile esset quod lōgītudo. u. b. n. modo maior, modo minor, modo verò æqualis esset ipsa. u. o. n. nūc. Quod etiam affirmo de. u. b. p. similiter etiam eueniet si vnus terminorum. u. vel. n. fuerit intra circumferentiam, reliquus verò extra ipsam.

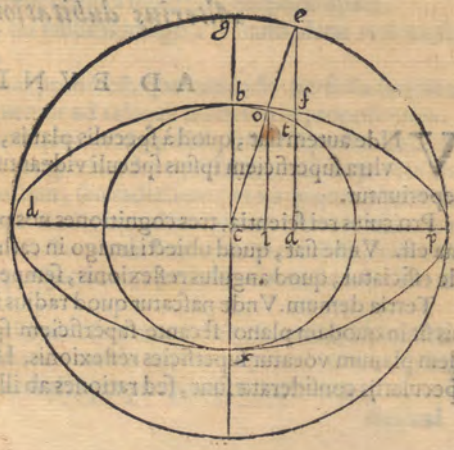
Consideremus nunc hic infra scriptam. 4. figuram vbi. d. b. p. sit circumferentia oxygonia seu ellipticā (quod idem est) cuius maior axis sit. d. p. in quo, duo termini. u. n. sint centra eius generationis: b. x. verò sit minor axis. Imaginemur etiam circulum. b. o. x. cuius semidiameter sit. c. b. non maior medietate minoris axis, ne circumferentia huiusmodi circuli fecet circumferentiam oxygoniam. Cogitemus etiam circulum. b. e. cuius semidiameter, minor non sit minori axe. b. x. ipsius oxygoniæ, ne se inuicem fecent huiusmodi circumferentiæ, sint etiam ambo eorum centra in linea. b. x. minoris axis, & punctum. b. sit commune vnicuique earum periphæriarum, vnde minor circulus, totus intra, maior autem, totus extra ipsam figurā oxygoniam erit, Nunc ad partem. o. r. e. vbi non communicant inuicem ipsæ circumferentiæ ducantur. n. o. r. e. u. o. u. r. et. u. e. & per. b. et. r. cogitetur transire alium circulum, cuius centrum in axe. b. x. sit. t. omnesq; isti circuli imaginentur trium diuersorum sphericorum speculorum, vnde pro generatione ipsi oxygoniæ, seu ex. 52. tertij Pergei, habebis longitudinem. u. r. n. æqualem esse longitudini. u. b. n. & ei, quæ est. u. o. n. (vt minor ipsa. u. r. n. ex. 21. primi Euclidis) minor ipsa. u. b. n. & longitudinem. u. e. n. (vt maior ipsa. u. r. n. ex eadem. 21. primi Eucli.) maior ipsa. u. b. n. Sed si quis vellet hoc demonstrare ope circuli, vni<sup>9</sup> tātūmodo speculi, multiplicādo ipsas oxygonias quæ admodum de ipsis circulis fecimus, obtineret similiter propositum.



*Solutio dubitationis.*  
AD E V N D E M.

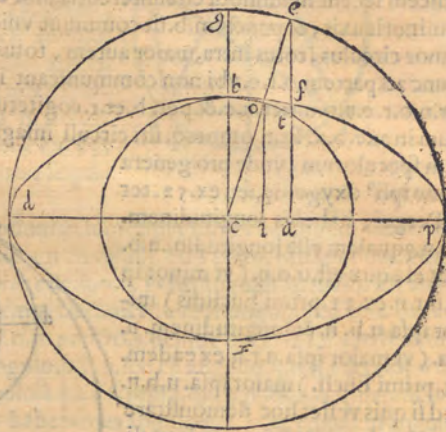
**R**ationalis est dubitatio tua, vtrum (cū circulus minor hoc est. b. o. habeat suum centrum in minori axe inter centrum oxygoniæ, et. b. existente. b. extremo axis minoris, communeq; ambobus circumferentijs circuli scilicet & oxygoniæ) dictus circulus minor, plura puncta communia habeat cum ipsis circumferentijs.

Cui dubitationi respōdeo quod quotiescunque centrum alicuius circuli fuerit idem cum. c. centro oxygoniæ, vel inter. c. et. b. in intervallo scilicet minoris axis, existente. b. sua extremitate communi ambabus cir-



circunferentijs, ipsas circunferentias inuicem contiguas esse oportebit in puncto. b. tantummodo.

Esto primum quod centrum. c. commune existat, vt dictum est. sit etiam centrum vnus circuli, cuius diameter sit idē cū maiori axe. d. p. & in gyro oxygoniæ accipiat-ur punctum. f. proximum. b. quantum fieri poterit, tunc protrahatur. f. a. e. parallela ipsi. g. c. vsque ad gyrum maioris circuli in puncto. e. quæ cum. d. p. rectos efficiet angulos. ex. 29. primi Eucli. secabitq; gyrum circuli. b. o. minoris in puncto. t. quod di- co esse intra oxygoniam, separatumq; ab. f. Quapropter duco. c. e. quæ secabit cir- cunferentiam circuli minoris in puncto. o. à quo puncto duco etiam. o. i. parallelam ad e. a. Deinde confidero, quod ex ra- tionibus ab Archimede adductis in quinta propositione libri de conoi- dalibus, & spheroidibus, eadem proportio erit ipsi<sup>9</sup>. g. c. ad. b. c. quæ ipsius. e. a. ad. f. a. vnde permutando ita erit ipsius. g. c. ad. e. a. vel. b. c. ad f. a. hoc est ipsius. e. c. ad. e. a. vt. o. c. ad. f. a. sed ex similitudine triangu- lorum, & ex. 11. quinti, ita etiã erit ipsius. o. c. ad. o. i. vt. o. c. ad. f. a. Vn- de sequitur. o. i. æqualem esse. f. a. sed ex. 14. tertij Eucli. t. a. minor est. o. i. Quare minor etiam erit ipsa. f. a. Vnde punctum. t. intra oxygo- niam erit, & consequenter separa- tum. ab. f.



Sed si centrum circuli minoris fuerit inter. c. et. b. hoc est eccentri- cum ipsius oxygoniæ, ipse tanget concentricum in puncto. b. tantummodo, vt in. 3. Euclidis libro probatur. Vnde tanto magis distans erit punctum. t. à puncto. f. quod erit propositum.

### *Alterius dubitationis solutio.*

#### A D E V N D E M.

Vnde autem fiat, quod à speculis planis, obiectorum imagines, ita distantes ultra superficiem ipsius speculi videantur, vt obiecta citra ipsam superficiem reperiuntur.

Pro cuius rei scientia, tres cognitiones nos primum habere oportet, quarum pri- ma est. Vnde fiat, quod obiecti imago in catheto incidentiæ videatur. Secūda. vnde efficiatur, quod angulus reflexionis, semper æqualis sit angulo incidentiæ.

Tertia demum. Vnde nascatur quod radius incidentiæ simul cum radio reflexio- nis sit in quodam plano secante superficiem speculi semper ad rectos, quod qui- dem planum vocatur superficies reflexionis. Huiusmodi tres passionis, ab omnibus specularijs consideratæ sunt, sed rationes ab illis traditæ, mihi non satisfaciunt.

Nam

Nam circa æqualitatem angulorum reflexionis & incidentiæ, iam tibi probaui illud non vniuersaliter euenire à breuitate aggregati radiorum incidentiæ reflexionisq; . Sed hoc nascitur potius ab eo, quod cum radius incidentiæ non possit superficiem corporis opaci penetrare, reflectit, vt citra ipsam cū angulo æquali ei, quem faceret cum eadem superficie vltra ipsam si tranfuiisset.

Exempli gratia sit .a. obiectum .b. autē oculus in figura .A. et .c. e. superficies ipsius speculi .d. verò sit punctum ipsius superficiæ, à quo ad oculum reflectitur imago ipsius .a. Nunc si radius .a. d. incidentiæ, recta incederet sub .c. e. efficeret angulum .e. d. h. æqualem angulo .c. d. a. eius contrapósito, sed quia impeditur ipsæ radius ab opacitate ipsius speculi .c. e. ne vltius incedat, propterea reflectitur ab ipsa superficie speculi, constituens cum ipsa angulum .e. d. b. æqualem angulo .e. d. h. sed quia angulus .c. d. a. est etiã æqualis ipsi angulo .e. d. h. propterea angulus .e. d. b. æqualis existit angulo .c. d. a; per accidens igitur sequitur .a. d. et .d. b. simul sumptas, breuiorem facere longitudinem omni alia, quæ ab ipsa superficie .c. e. ad eadem puncta .a. b. ducta esset, quare naturæ intentio est efficere angulum .e. d. b. æqualem angulo .c. d. h. vnde ex accidenti potest sequitur, ipsum æqualem esse angulo .c. d. a. & deinde quæ lineæ .a. d. et .d. b. constituent longitudinem breuiorem. Quare illud quod omnes putabant esse primum & per se, vltimum est, & ex accidenti.



Quare vero superficies, quæ vocatur reflexionis, in qua sunt duæ lineæ, hoc est incidentiæ, reflexionisq;, semper sit perpendicularis superficiæ ipsius speculi: Hæc est ratio, quia cum quilibet radius incidentiæ, perpendicularis ipsi superficiæ speculi, in seipso reflectit, ex iisdem dictis rationibus, hoc est, quia cum tali angulo vult reflecti, cum quali transiret, ita etiam putandum est, quod radius incidens obliquus, cum in seipsum non possit redire, quia non est perpendicularis superficiæ speculi, reflectitur tamen per planum erectum ipsi superficiæ speculi, vt in eo, cui magis resistit superficies corporis opaci, quàm alicui alij plano ipsius infiniti inclinorum planorum, ab vtraque parte ipsius plani perpendicularis, quod vnum etiam tantummodo est, & in quo, radius maiorem vim obtinet reflectendi, seu in eo, in quo radius ipse cum maiori resistentia repercutitur à superficie corporis opaci.

Postremo sciendū vnde oriatur, & rei visibilis imago, à speculo plano reflexa, semper in catheto incidentiæ videatur.

Pro cuius rei ratione cognoscendum primò est, quo modo sit perfecta simplexq; visio, & non reflexa, deinde prosequemur ad reliqua huius tertiæ propositionis.

Animaduertendum igitur est, quod quotiescunq; obiectum aliquod visibile aspiciamus, nos nunquam perfectè illud comprehendere possumus, nisi in puncto concursus, seu intersectionis axium visualium, seu radialium (vt ita loquar) quæ intersectione, nos efficimus ope reuolutionis oculorum adinuicè, hoc est voluendo vnum versus alium, ita vt in situ ipsius obiecti, seinuicem fecerit axes iam dicti, tunc enim vtroque oculo mediante, exacte rem perspicimus, ceteris .8. circumstantijs non obstantibus.

Vnde stantibus oculis in tali situ, altero respectu alterius, si eorum alter rectus, seu velatus fuerit, tunc alio tantummodo oculo mediante, videbimus obiectum, in ea distantia, exactius, quam in quauis alia propinquiori, & remotiori.

Animal

Animal igitur, secundum distantiam obiecti, oculum accommodat ad recipiendum quam exactissimè speciem ipsius obiecti, & hoc voluendo ambos oculos, vnum versus alium, ita quod intersecctio axium fit in situ seu loco dicti obiecti, nam tunc videant ambo vel aliquis eorum solus, in tali distantia exactè obiectum videbit.

Vnde sequitur obiectum visibile, compræhensibile non esse ab vno tantummodo oculo in quolibet situ axis ipsius oculi, sed in eo, vbi alius axis intersecatur à dicto. Quæ quidem intersecctio potest fieri propinqua, vel remota à visu, ad certos tamen terminos vsque.

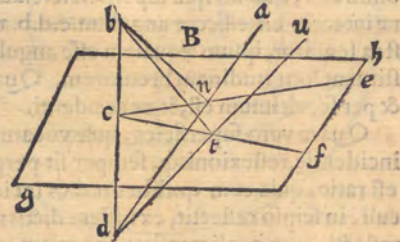
De huiusmodi axium visualium intersecctione scribit Alhazem in. 2. et. 15. propositione tertij lib. Vitellio verò in. 32. et. 45. eiusdem.

Quod igitur dico, verum est, idest, quod si vno tantummodo oculo aspiciemus obiectum aliquod, ipsum nunquam perfectè prospicietur, nisi cum oculus ita situs fuerit, vt eius axis cum axe alterius in loco obiecti se inuicem secent, quamuis alter oculus nihil videat, cū aut duobus oculis in tali situ cõstitutis obiectum videmus, vnum tantummodo nobis cernere videbimur, & si extra talem punctum intersecctonis ipsum obiectum positum fuerit, tunc duo talia obiecta nobis apparebunt, sed huiusmodi rei causam alias tibi manifestabo.

His igitur cognitis, ponamus aliquam speculi superficiem esse. g. h. in figura. B. obiectum autem visibile. b. oculos vero. a. et. u. punctum autem. n. in superficie speculi, à quo imago ipsius. b. reflectit ad. a. & punctum. t. à quo reflectitur ad. u. et. c. e. sit cõmunis sectio superficiæ reflexionis radiorum. b. n. a. et. c. f. sit cõmunis sectio superficiæ reflexionis radiorum. b. t. u. quarum vnaquæq; superficies reflexionis, erecta est ad superficiem speculi. g. h. vt supra diximus. Nunc ex. 19. vndecimi Eucl. sequitur communem sectionem harum duarum superficiærum. (b. c. d. scilicet) ad rectos etiam esse supra superficiem speculi. g. h. tum qua. b. c. quælibet linearum. a. n. vel. u. t. reflexarum (productæ cum fuerint) se inuicem intersecabunt eo quod duo anguli. d. c. n. et. d. n. c. simul collecti minores sunt duobus rectis, & ita. d. c. t. cum. d. t. c. cum anguli. a. n. e. et. u. t. f. reflexi, ipfis contrapõsiti, æquales sint angulis. b. n. c. et. b. t. c. incidentiæ, quorum vnusquisq; ex. 32. primi, minor est recto.

Dico etiam quod in eodem puncto huiusmodi catheti. b. c. d. in quo intersecabuntur à linea. a. n. in eodem secabuntur à linea. u. t. & quod punctum dicti concursus, tantum depressum erit sub superficiem speculi. g. h. quantum. b. supra ipsam reperietur. Nam anguli. b. n. c. et. d. n. c. sunt inuicem æquales, anguliq; b. c. n. et. d. c. n. recti. c. n. verò commune, vt trianguli. d. c. n. æquale erit lateri communi. b. c. vt trianguli. b. c. n. Idem etiam dico de latere. d. c. vt ipsius trianguli. d. c. t. quod æquatur lateri. b. c. vt trianguli. b. c. t. Vnde cum. b. c. vnum, & idem sit: d. c. igitur etiam erit, & ipsum vnū & idem, quod erit propositum.

Nunc autem cum hi duo radij se inuicem secent in puncto. d. ergo in ipso puncto. d. videbimur nobis videre imaginem obiecti. b: cū ope duorū istorū radiorū. n. a. et. c. u. ita inuicem istorū, videamur nobis imaginem prospicere. Vnde si in tali casu, vnus





oculorum clauderetur, nihilominus cum reliquo obiectum vidissemus in eodē ipso loco. d. & non in alio ex superius dictis rationibus.

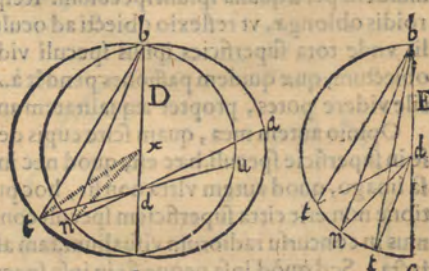
Et si stantibus ijs terminis volueremus pupillam oculi. u. versus aliam. a. ad aspiciendum punctum. n. in superficie. g. h. ipsius speculi, hoc est si fecerimus quod axes visuales seinuicem secarent in ipso puncto. n. tunc videremur nobis videre duas imagines ipsius obiecti. b. intra speculum, eo quod obiectum, propter hoc non cessaret reflectere ad oculos ab ipsis punctis. n. et. t. quapropter recipiendo radium. t. u. in situ axis oculi. u. & radium. n. a. in situ axis oculi. a. hi axes ex necessitate (vt probauimus) seinuicem secant in puncto. d. vnde vnā tantummodo imaginem ipsius obiecti nobis apparebit.

Ex his igitur omnibus potes facillē videre omnem imaginem, cuiusuis obiecti, reflexam à speculo, reperiri in ipso catheto incidentiæ, cum ipse semper sit communis sectio duarum superficialium reflexionis, in quo catheto concurrunt ipsæ axes visuales.

Ex ijsdem etiam dictis rationibus facile comprehendere poteris, vnde fiat, vt videamus imaginem reflexam à speculis sphericis concauis citra ipsorū superficiem, & non vltra. Quod nunquā euenit, nisi quando punctū. d. intersectionis ipsorū radiatorū visualium (quod alio in loco non fit, nisi in catheto incidentiæ hoc est in communi sectione duarum superficialium reflexionis. Dato quod obiectum non sit in vna eademque superficie, in qua reperi fuerint axes visuales, hoc est dato, q. ambo axes visuales non sint in vna eademque superficie reflexionis) reperitur citra & non vltra superficiem ipsius speculi.

Ad cuius rei euidentiā non præmittā dicere, quod cum debeant semper superficies reflexionum perpendiculares esse, vel ad rectos secare superficiem ipsius speculi, ipsarum communes sectiones cum superficie speculi sphericis, semper erunt circūferentiæ magnorū circulorum illius spheræ, cuius portio est speculum propositum, vt etiam Vitellio affirmat in prima sexti libri. Vnde vnusquisque cathetus incidentiæ transibit per centrum speculi, cum ipse sit communis sectio duarum superficialium reflexionis, quare in ipso catheto erit punctum intersectionis ipsorum axium visualium ex necessitate, vt videbimus, si vnā tantummodo imaginē obiecti nobis videremur videre.

Exempli gratiā, sint duæ superficies reflexionis speculi sphericis concaui. b. n. c. a. et. b. t. c. u. obiectumque sit. b. oculi autem sint. a. u. punctum verò superficiē speculi, à quo obiectum emittit reflexionem suę imaginis ad oculum. a. sit. n. punctum autem à quo eandem reflectit oculo. u. sit. t. communis autem sectio harum duarum superficialium sit. b. c. sed. x. centrū sit speculi, radius verò incidentiæ superficiali. b. n. e. erit. b. n. cuius reflexus sit. n. a. radij autem alterius superficiali erunt. b. t. et. t. u. Imaginemur nunc duos semi-diametros. x. n. et. x. t. quæ angulos. b. n. a. et. b. t. u. per æqualia diuidant ex supposito.



Nunc ijs suppositis, si vnā tantummodo obiecti imaginem videbimus,

Vu clarum

clarum erit ex rationibus supradictis nos ipsam videre in cōmuni concursu ipforum axium visualium, qui axes cum reperiantur vnà cum ipsis radijs reflexis. n. a. et. t. u. ex necessitate se inuicem secabūt in catheto. b. c. cum extendantur in ipsis superficiebus reflexionum, quæ superficies nihil aliud commune inuicem habent, quam cathetum dictum. b. c. sit igitur in puncto. d.

Ex his dictis alia oritur necessitas, hoc est, quod quotiescunque vnā tantummodo imaginem obiecti. b. videmus, dato quod duæ superficies reflexionis sint, & non vna tantum, tunc angulos. n. et. t. semper inuicem æquales esse oportebit. Vnde arcus. n. c. et. t. c. ex necessitate inuicem æquales erunt.

Scimus enim ex. 3. sexti Euclid. quod eadem proportio erit ipsius. b. n. ad. n. d. quæ ipsius. b. x. ad. x. d. & ipsius. b. t. ad. t. d. similiter, quare ipsius. b. n. ad. n. d. erit vt ipsius. b. t. ad. t. d. Vnde sequitur. b. n. æqualem esse ipsi. b. t. et. n. d. ipsi. t. d. vt à medio circulo. E. potes videre, quomuis etiam. b. non esset extremum diametri, sed vbiunque volueris in ipso diametro, vel etiā protracta, eo quod punctum. n. & punctum. t. in eodem semicirculo, vel in æqualibus semicirculis, non possēt aliter in ipsa circumferentia locari, eādem seruando proportionem. b. n. ad. n. d. vt. b. t. ad. t. d. propterea quod in omni alio situ existente puncto. t. ipsa. b. t. esset aut maior aut minor ipsa. b. n. et. t. d. aut minor, aut maior ipsa. t. d. ex. 7. & 14. tertij Eucl. vnde aut maior, aut minor proportio esset ipsius. b. t. ad. t. d. quam ipsius. b. n. ad. n. d. & non eadem.

Nunc è conuerso si. b. n. et. b. t. sunt sibi inuicem æquales, & sic. n. d. cum. t. d. sequitur ex. 8. primi Eucl. angulos. n. et. t. inuicem æquales esse.

Ab iisdem speculationibus potes etiam videre vnde accidat quod partes superiores alicuius obiecti reflexæ à tali speculo concauo videntur nobis inferiores esse, & inferiores appareant superiores, & dextræ sinistra, & sinistra dextræ. quod autem hucusque demonstraui de speculis planis, & sphericis concauis, ratiocinare tu iisdem medijs circa spherica conuexa, vbi clarè videbis puncta huiusmodi speculi conuexi, à quibus reflectitur imago obiecti ad ambos oculos, semper oportere æquidistantia esse à puncto communi ipsius superficie speculi, & catheto incidentiæ, dum vnā tantummodo imaginem ipsius obiecti videmus, & à diuersis superficiebus reflexionum.

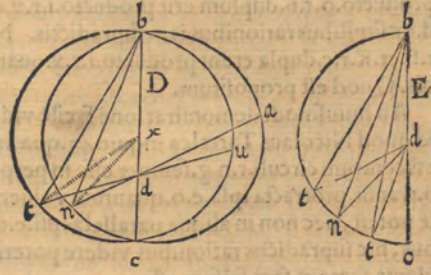
Nolo etiam prætermittere, quod nunc mihi succurrit, hoc est quod posset aliquis duos situs inuenire, vnum pro oculo, alterum verò pro obiecto, respectu alicuius speculi concaui, spheroidis prolata, vt reflexio ipsius obiecti videretur, vt linea diuidens per æqualia ipsam speculum. Respectu verò alicuius speculi concaui spheroidis oblongæ, vt reflexio obiecti ad oculum veniret à tota superficie ipsius speculi, vnde tota superficies ipsius speculi videretur colorata illo colore cuius esset obiectum, quæ quidem passiones pendēt à. 48. tertij lib. ipsius Pergei, vt ex te ipso facile videre potes, propter æqualitatem angulorum reflexionis, & incidentiæ.

Opinio autem mea, quam scire cupis de imagine obiecti reflexa, quam putas esse in superficie speculi, hæc est, quod nec in superficie, nec ultra, nec citra eam est ipsa imago, quod autem ultra non sit, hoc puto nulli dubium esse. eadem etiam ratione non erit citra superficiem speculi concaui, quamuis ipsam nos comprehendamus in concursu radiorum visualium, tam ab vno speculo quam ab alio reflexione facta. Sed quod ipsa neque sit in ipsa speculi superficie, manifestum erit ex hoc, quod duo spectantes in eodem speculo, duas diuersas imagines vident, tres, aut tres, quatuor, & sic deinceps, vnde tot essent imagines supra superficiem speculi, quot obiecta, & tamen ita non est, nec plus est in vno loco ipsa imago, quam in alio,

nisi

nisi in obiecto ipso, lumen enim ab ipso obiecto reflexum, seipsum diffundit vndique, & radij ipsius luminis reflexi, vt plurimum se inuicem secant. Vnde in ipso aere sunt omnes misti. Quapropter natura sagacissima pupillam oculi animalibus tam paruam construxit ad superficiem tam amplae sphaerae ipsius oculi, vt distincta viderentur omnia obiecta.

Nolo etiam tibi tacere, quod quotiescunq; oculorum pupillae positae fuerint inter cathetum incidentiae, & superficiem speculi sphaerici concaui, vt puta in lineis d.t.et.t.n. in figura D. tunc nullo pacto possemus videre vniam imaginem obiecti, sed duas nec non confuses, propterea qd nullo pacto radij t.d. et n.t. reflexi poterint ambo vniri cu ambobus axibus visualibus, eo quod axes visuales nunquam possunt inuicem intersecari post visum, sed semper ante ipsum, vnde nec inuicem paralleli possunt esse.

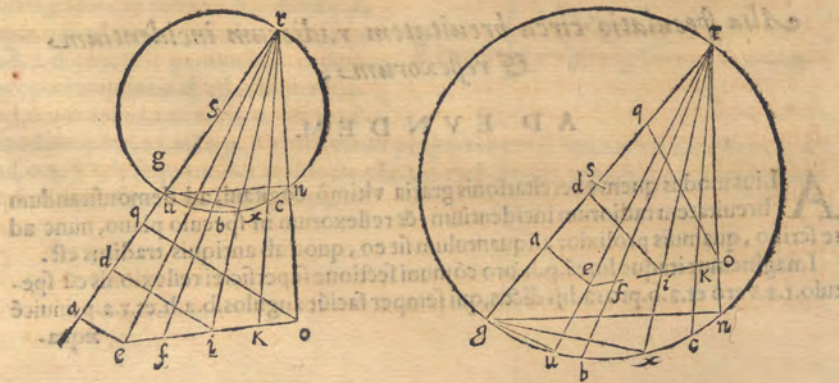


Dico etiam, quod si obiectum incidit in eadem superficie, in qua duo axes visuales, vel radij reflexi reperiuntur, hoc est in vna eademq; superficie reflexionis, tunc locus imaginis non erit in catheto incidentiae, eo quod interfectio axium visualium non erit in ipso catheto sed extra, in qua interfectione fit visio vnus tantummodo imaginis, quod antiqui non animaduertent. Hoc autem dico de speculo sphaerico concauo.

*Speculatio cuiusdam propositionis arithmeticae.*

A D E V N D E M.

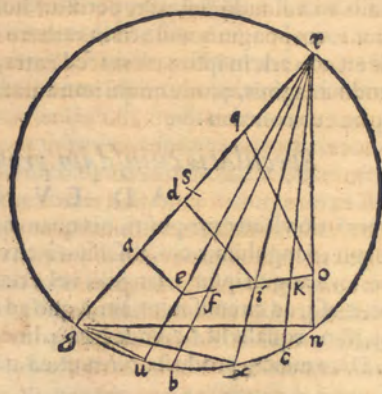
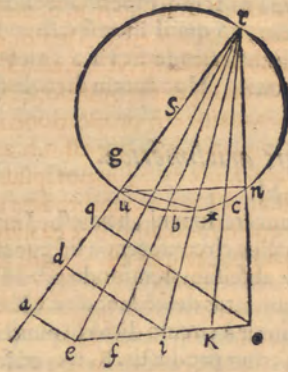
Speculatio vltimae propositionis quam numerorum via inueni, haec est. Imaginemur triangulum r.e.o. abscisum a circulo, in cuius circumferentia sit punctum r. superioris anguli ipsius trianguli, vel etiam non sit abscisum dummodo protrahatur linea vsq; ad circumferentiam, a quo ad oppositum latus descendant duae r.K.et.r.f. ita qd K.o. aequalis sit. f.e. vnde haec 4. linea secabuntur a circulo dicto in punctis. n.c.b.u. Dico nunc qd producta. o.r.n. et. e.r.u. aequalia erunt productis. K.r.c. et. f.r.b.



Vu 2 quapro-

quapropter cogitemus. r. a. indeterminatam transire per centrum. s. ipsius circuli, similiter etiam. r. i. ad punctum medium lateris. e. o. deinde à tribus punctis, e. i. o. imaginemur tres perpendiculares ad. r. a. hoc est. e. a. i. d. et. o. q. & ubi circulus fecat. r. a. sit punctum. g. protractis deinde. g. n. g. x. et. g. u. habebimus triangulum. a. e. r. similem triangulo. g. u. r. unde clarum erit productum. g. r. a. æquale esse producto. e. r. u. productumq; .g. r. q. æquale esse producto. o. r. n. nam trianguli. g. r. n. et. o. r. q. sunt inuicem similes, sed productum. g. r. a. simul cum producto. g. r. q. duplum est producto. g. r. d. ex prima sexti, eo quod. a. r. q. dupla est. d. r. & ideo productum. e. r. u. simul cū producto. o. r. n. duplum erit producto. i. r. x. quod quidem æquale est producto. g. r. d. ex similibus rationibus iam supradictis. Nunc ex similibus rationibus producta. f. r. b. et. K. r. c. dupla erunt producto. i. r. x. quare prima producta æqualia erunt secundis. Quod est propositum.

Ab huiusmodi demonstratione facile videre poteris non esse generaliter verum, id quod Nicolaus Tartalea inquit. 43. quæsito vltimæ partis suorum tractatum, hoc est centrum circuli. r. n. g. semper esse in perpendiculari, quæ à puncto. r. ad lineam. e. o. transit, protracta ipsa. e. o. quantum volueris, imò in quacunque alia linea ipsum esse potest, nec non in aliqua parallela ipsi. e. o. quemadmodum ex te ipso, mediantibus, hic supradictis rationibus videre poteris, unde ex necessitate sequitur illud problema semper ferè falsum esse.



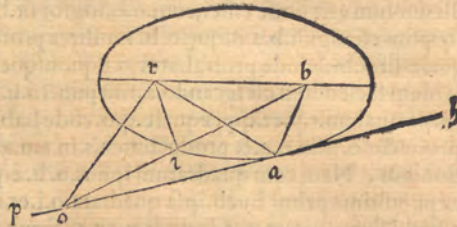
*Alia speculatio circa breuitatem radiorum incidentium & reflexorum.*

A D E V N D E M.

**A**lius modus quem exercitationis gratia vltimò cogitavi, ad demonstrandum breuitatem radiorum incidentium, & reflexorum in speculo plano, nunc ad te scribo, quamuis prolixior aliquantulum sit eo, quod ab antiquis traditus est. Imaginemur itaque lineã. p. h. pro cõmuni sectione superficiẽ reflexionis cū speculo. r. a. verò et. a. b. pro radijs dictis, qui semper faciunt angulos. b. a. h. et. r. a. p. inuicẽ æqua-



Alia etiam via possumus idem concludere. Imaginemur maiorem axem alicuius ellipsis transire per duo puncta. r. et. b. supponendo ipsa puncta, ea esse, quae ita axem diuidunt, ut singula producta sectionum sint, ut inquit Pergeus. imaginemur, etiam. p. h. contiguam esse ipsi ellipsi in puncto. a. unde si protractae fuerint duae. r. a. et. b. a. habebimus ex. 48. tertij ipsius Pergei angulos. b. a. h. et. r. a. p. inuicem aequales. Ducendo postea ad quoduis punctum ipsius p. h. duas. b. o. et. r. o. certi erimus, quod secabuntur a gyro oxygono, quarum vna secta sit in puncto. i. ducta postea. i. r. clarum erit ex. 52. dicti, quod longitudo. b. i. r. aequalis erit longitudini. b. a. r. & minor ipsa. b. o. r. ex. 21. primi Euclid.



*De errore Euclidis circa speculum vstorium.*

A D E V N D E M.

**V**erum speculum vstorium, illud non est, quod ab Euclide traditum fuit, & quod tu etiam putas. Nam Euclides errat, cum credat radios reflexos a superficie sphaerica concaua se inuicem in centro speculi interfecare. Nam cum omnes lineae rectae a centro, & circumferentia alicuius sphaerae terminatae, sint eidem circumferentiae perpendiculares, sequeretur ex necessitate radios incidentiae etiam perpendiculares eidem superfici ei esse, cum anguli incidentiae semper aequales sint angulis reflexionis, unde etiam ex necessitate sequeretur punctum corporis lucidi, a quo radij luminosi exeunt, in centro speculi reperiri. quod quidem falsissimum est.

Alia etiam via possum hanc ostendere impossibilitatem, & tibi probabo, quod in nullo aliquo puncto possunt inuicem conuenire ipsi radij reflexi omnes.

Sit igitur. l. a. c. cois sectio superfici ei reflexionis cum speculo, cuius centrum sit. o. punctum vero lucidum sit. g. protrahaturq; . g. o. a. Nunc autem primum dico, quod radij reflexi a punctis diuersarum distantiaru ab. a. non coincident inuicem in aliquo puncto lineae. g. o. a. sint ergo duo puncta. u. et. r. diuersarum distantiaru ab. a. a quibus veniant duo radij incidentiae. g. r. et. g. u. radius vero reflexus ab. r. sit. r. e. protrahatur u. e. quam dico esse non posse radium reflexum ab. u. quotiescunque eius incidens descendat ab. g. Protrahantur ergo duae lineae. o. r. et. o. u. unde cum dixerit aliquis u. e. reflexu esse ipsius. g. u. igitur anguli. g. u. o. et. o. u. e. erunt inuicem aequales, & sic etiam erunt duo. g. r. o. et. o. r. e. unde ex tertia sexti & 11. quinti Eucli. proportio . g. u. ad. u. e. aequalis esset ei, quae. g. r. ad. r. e. quod quidem impossibile esse demonstrabo, eo quod cum. g. u. maior sit. g. r. ex. 8. tertij, erit ex. 8. quinti proportio ipsius. g. u. ad. r. e. maior proportione ipsius. g. r. ad. r. e. sed ex. 7. tertij. u. e. minor est. r. e. erit igitur ex dicta. 8. quinti maior proportio ipsi. g. u. ad. u. e. quam. g. u. ad. r. e. unde eo ma

gis

gis erit maior proportio ipsius.g.u.ad.u. e. quam ipsius.g.r.ad.r.e. ergo non æqualis, quapropter impossibile est.u.e. esse radium reflexum incidentis radij.g.u. Vnde sequitur concursum radiorum reflexorum à speculo sphaerico concauo non esse in vno, & eodem puncto ipsius catheti incidentiæ, quando à situ non æquidistanti ab ipso catheto reflectuntur, ex hac speculatione etiã videre licet, verum esse id quod in.3.Epistola tibi scripsi nempe, quod quotiescunque axes visuales, vel radij reflexi, in vna eademq; superficie reflexionis fuerint, tunc imago obiecti nullo modo videbitur in catheto incidentiæ, in speculo sphaerico concauo.



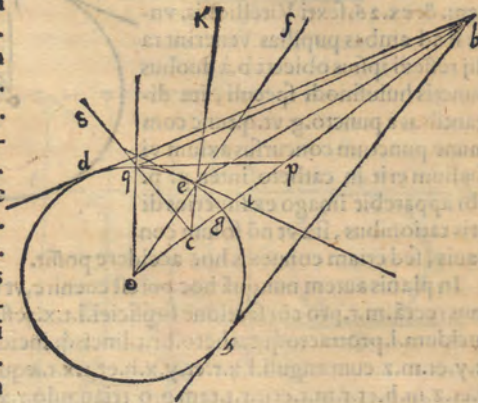
*Alterius dubitationis solutio.*

ADEVNDEM.

**N**on absque ratione dubitas, vtrum etiam in sphaericis speculis conuexis idem accidat, hoc est, an radij reflexi à punctis inæqualis distantiz à catheto incidentiz conueniant inuicem in eodem catheto.

Ad quod respondeo, non concurrere in dicto catheto, sed extra ipsum, & similiter extra ipsum videbitur imago.

Pro cuius rei ratione, imaginemur superficiem reflexionis alicuius speculi sphaerici conuexi. b. d. h. g. cuius communis sectio cum superficie sphaerica sit linea circularis. d. e. h. et. o. eius ceterum, à quo protrahatur. g. b. indeterminata, et. o. g. sit semidiameter circuli. d. g. h. et. o. c. sit plus medietate ipsius. o. g. accipiaturq; linea. e. c. minor ipsa. o. c. sed maior ipsa. c. g. quod difficile non erit, locando immobilem pedem circini in puncto. c. aperiendo ipsum aliquantulum plus quam. c. g. sed minus quam. c. o. signando circumferentiam. d. e. h. in puncto. e. quod ex. 7. tertij possibile est, protrahatur postea. o. e. f. indeterminatè. Faciemus deinde angulum. f. e. b. æqualem angulo. o. e. c. protracta postea cum fuerit. c. e. K. indeterminatè, habebim<sup>9</sup> duos angulos. b. e. f. et. f. e. K. æquales inuicem mediante. r. s. primi, ita qd si radius incidens veniet à puncto. b. ad. e. reflexus erit. e. K. qui quidem reflexus



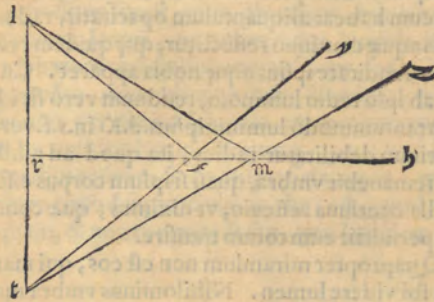




puncti. l. clarum igitur nunc habes, quod in sphaerico concauo, seu conuexo, non omnes radij reflexi conueniunt in vno, eodemq; puncto catheti incidentiae, quemadmodum in planis accidit, in quibus semper vnum, & idem punctum est ipsis commune in ipso incidentiae catheto.

Non praetermittam etiam hunc alium breuiorem modum speculandi aequalitate depressionis imaginis sub speculo plano, ei qua supra reperitur ipsius obiecti, in catheto incidentiae, quemadmodum nunc

ultimò diximus, hoc est quod cum imago obiecti. l. reflexa à puncto. x. reperiatur in linea. y. x. t. & imago eiusdem obiecti reflexa à puncto. m. reperiatur in linea. z. m. t. & istae duae lineae se inuicem secent in puncto. t. ipsius catheti, existente. r. t. aequali. r. l. vt nunc vidimus, ergo semper imago reflexa à speculo plano, nobis apparebit in ipso catheto, tam vltra speculum, quam citra ipsum, repta fuerit ipsū obiectū quod nec Alhazem, nec Vitellio, nec alius aliquis (quod sciam) adhuc scientificè demonstrauit. exempla enim vel experientia non faciunt scire. Credo etiam te non dubitare quin duae lineae. y. x. et. z. m. inuicem concurrant, cum anguli. t. x. m. et. t. m. x. minores sint duobus rectis cum aequales sint angulis. l. x. m. et. l. m. x.



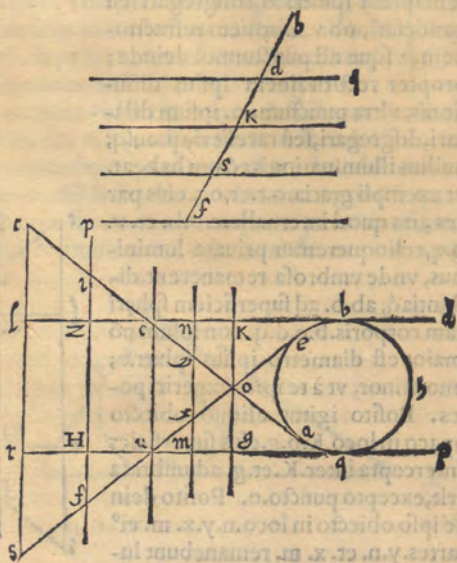
*De rotunditate vmbrae terrae in eclipsibus Lunaribus.*

A D E V N D E M.

Rotunditas vmbrae in eclipsibus lunaribus oritur tã à rotunditate maris, quã terrae, & si terra esset etiã cuiusuis alterius figurae, quã sphaerice, dummodo aqua impleret locū sphaericitatis à terra derelictū, nihilominus vmbra esset rotunda, quae quidem ab aqua produceretur, quãuis Alexander Piccolhomineus aliter sentiat in libro de magnitudine terrae, & aquae.

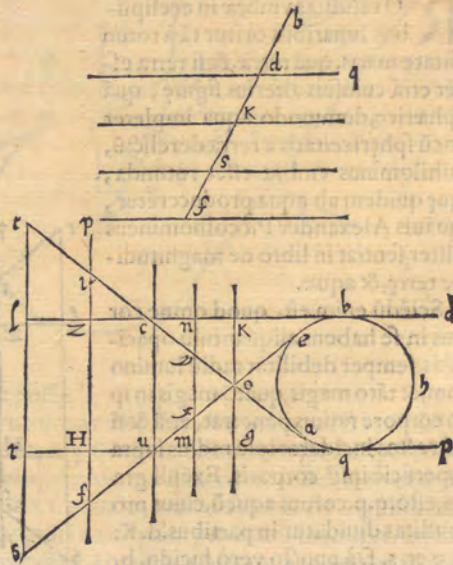
Sciendū enim est, quod omne corpus in se habens aliquantulū opacitatis, semper debilitat radiū luminis, & tãto magis, quãto magis in ipso corpore radius penetrat, etiã & si ad rectos incideret ipse radius supra superficiē ipsius corporis. Exēpli gratia, esto. q. p. corpus aqueū, cuius profunditas diuidatur in partibus. d. K: K. s. et. s. f. à puncto verò lucido. b.

X x descen-



descendat radius. b. d. K. s. f. ad libitū hoc est rectè vel obliquè, cuius pars. b. d. in ipso aere existat. Nunc manifestum erit partem. b. d. ipsius radij clariorem seu minus im-peditā esse quam. d. K. quod ex eo etiam cognoscere possumus quia. b. d. reflectitur à puncto. d. superficiè corporis aquei, quapropter minus luminosa remanebit pars. d. K. cum non tota claritas. b. d. descendat in corpore aqueo, sed vna eius pars reflectatur, reliqua verò tantummodò descendat, deinde pars. K. s. ex necessitate debilior erit ipsa. d. K. eo quod succedit post ipsam. d. K. propter hoc etiam, quia cum corpus aqueum habeat aliquantulum opacitatis, radius. d. K. ab omni puncto ipsius spissitudinis aquæ continuo reflectitur, quæ quidem reflexio est illud lumen cæruleum, quod in profunditate ipsius aquæ nobis apparet. Cum igitur reflexio ipsa semper detrahat ab ipso radio luminoso, residuum verò sit id quod penetrat, ideo. K. s. erit vna pars tantummodò luminis ipsius. d. K. in. s. f. verò aliqua pars luminis ipsius. K. s. & sic continuo debilitatur radius, ita quod ad nihilum vsque deuenit, & ultra tale corpus remanebit umbra, quasi si ipsum corpus esset perfectè opacum, cuius rei causa, est illa continua reflexio, vt diximus, quæ continuo adimit aliquid ex ipso radio, nec permittit eum totum transire.

Quapropter mirandum non est eos, qui margaritas quærunt in fundo maris nul-lum ibi videre lumen. Nihilominus umbra maris, quam dico nos posse videre in superficie corporis lunaris, ab alia etiam ratione prouenire posset. Imaginemur enim aggregatum terre, marisq; esse tantummodò aqueum, quod quidem esset perfectè sphericum ratione centri grauitatis, supponamusq; ipsū esse valde diaphanum, ita quod radij solares ipsum penetrassent. Tunc dico quod in superficie corporis luna-ris produceret umbram. Pro cuius intelligentia cogitemus subscriptam hic figuram b. h. q. a. e. esse spheram aliquam crystallinam, & ad partem. b. h. q. sit radius lumino-sus solaris qui ipsam illuminet, cuius radij extremitates sint. d. b. l. et. p. q. r. supponen-do. d. l. et. p. r. terminos esse vnus plani secantis ipsum radium per axem, tunc vide-bis ipsum radium. b. p. q. d. transeū-tem ipsam spheram, congregari seu condensari, ob vniformem refractionem, vsque ad punctum. o. deinde; propter rectitudinem ipsius diffu-sionis, vltra punctum. o. ipsum dila-tari, disgregari, seu rarefieri, quousq; nullius illuminationis actum habeat. vt exempli gratia. o. t. et. o. s. eius par-tes, ita quod interualla. e. o. b. et. u. o. q. relinquerentur priuata lumini-bus, vnde umbrosa remanent. di-stantiaq; ab. o. ad superficiem spheri-cam corporis. b. e. d. q. non solum nō maior est diametro ipsius spheræ; imo minor, vt à te ipso experiri po-tes. Posito igitur aliquo obiecto opaco in loco. K. o. g. eius superficies intercepta inter. K. et. g. adumbrata erit, excepto puncto. o. Posito dein de ipso obiecto in loco. n. y. x. m. ei⁹ partes. y. n. et. x. m. remanebunt lumine



mine destituta interuallumq; tantummodò inter. y. x. illuminatum erit, sed si in loco. c. u. positum fuerit, tunc totum. c. u. illuminatum erit, sed debili modo propter detractionem factam à reflexione in superficie corporis sphaerici, vt supra diximus.

Posito deinde obiecto in loco. i. z. H. f. tunc partes. z. i. et. H. f. rectos Solis radios habebunt cum aliquibus refractis, sed. z. H. paucissimum habebit lumen, propter disgregationem radiorum. Posito postea ipso obiecto in loco. t. l. r. s. tanto minus lumen habebit pars. l. r. propter dictam disgregatione, seu dissipatione radiorum, & sic successiue quanto remotius positum fuerit ipsum obiectum, tanto minus illuminabitur. vnde ita remotum poterit locari, ut nullus actus luminis in eo videatur, de radijs scilicet, qui per sphaeram chrySTALLINAM transibunt, sed videbitur umbra ipsius sphaere in obiecto proposito, cum nullum actum illuminationis in eo loco obiecti habeant radij transeuntes per dictam sphaeram. quapropter partes. t. l. et. r. s. illuminatae erunt à Sole, et. l. r. omnino lumine destituta.

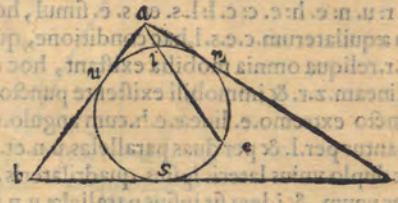
Quòd vero tolerabilior sit oculis radius reflexus Solis à superficie aquae, quàm à superficie alicuius speculi, oritur ab eo, quod supra diximus, hoc est, quod magna pars ipsius luminis penetrat in aquam, & non totum reflectit, quod quidem non accidit speculis opacis.

DE LONGITVDINE DVORVM LATERVM  
cuiusvis trianguli supra tertium.

Hieronymo Fenarolo.

**Q**UOD quolibet duo latera continentia rectum angulum cuiusvis trianguli orthogonij, longiora sint tertio latere, per diametrum circuli in eo inscripti, ab alijs iam demonstratum fuit. Sed quòd quolibet duo latera cuiusvis trianguli longiora sint tertio per latus tetragonicum, quadrupli producti cuiusvis lineae descendens ab angulo contento à dictis duobus lateribus ad oppositam partem circuli inscripti, in partem extrinsecam ipsius lineae, nullus (quod sciam) vnquam scripsit, vel animaduertit.

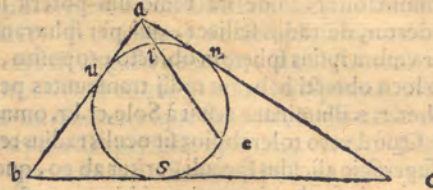
Sit exempli gratia triangulus. a. b. c. quem volueris, in quo describarur circulus. u. s. n. & puncta contingentiae sint eadem. u. s. n. à puncto vero. a. descendat linea. a. i. e. quae terminetur à circumferentia in puncto. e. ipsius circumferentiae, vbi volueris. Dico nunc latera. a. b. et. a. c. longiora esse latere. b. c. per latus tetragonicum quadrupli producti ipsius. a. e. in. a. i. Nam certi sumus ex vltima parte penultimae tertij Euclij. n. c. et. s. c. aequales inuicem esse, & similiter. b. s. et. b. u. vnde ex communi conceptu dicta latera maiora erunt ipso. b. c. per. a. u. et. a. n. quae duae partes sunt inuicem aequales dicta ratione, & quadratum lineae aequalis aggregato earum, esset quadruplum quadrato cuiusvis earum ex. 4. secundi, sed ex penultima tertij, productum. a. e. in. a. i. aequale est quadrato ipsius. a. u. vel ipsius. a. n.



X x 2 Ve-

Verum est igitur quod. a. b. cum. a. c. longiores sint ipsa. b. c. per latus tetrago-  
nicum quadrupli eius quod fit. ex. a. e. in. a. i. quod fuit propositum.

Illud etiam non est spernendum, quod quotiescunque data fuerint omnia latera  
alicuius trianguli, illico possumus cognoscere puncta. u. n. s. contingentiae circuli in  
scripti, ope viciniae partis penultima tertij, eo quod ex illa iam scimus, quod de-  
trahendo. b. c. ex aggregato aliorum duorum laterum, remanebit. u. a. et. a. n. qua-  
rum vnaqueque nota erit, cum illarum quaelibet, medietas sit residui cogniti, detra-  
hendo postea vnam illarum ab altero  
duorum laterum. a. b. vel. a. c. rema-  
nebit. u. b. vel. c. n. equalis. b. s. vel. c.  
s. vnde similiter nobis innotescet  
punctum. s. cum duobus punctis. u.  
et. n. à quibus duobus punctis, si  
duae perpendiculares ad talia latera  
ductae fuerint, vbi hae perpendicu-  
lares seinuicem secabunt, ibi cen-  
trum circuli inscriptibilis erit in trian-  
gulo proposito.



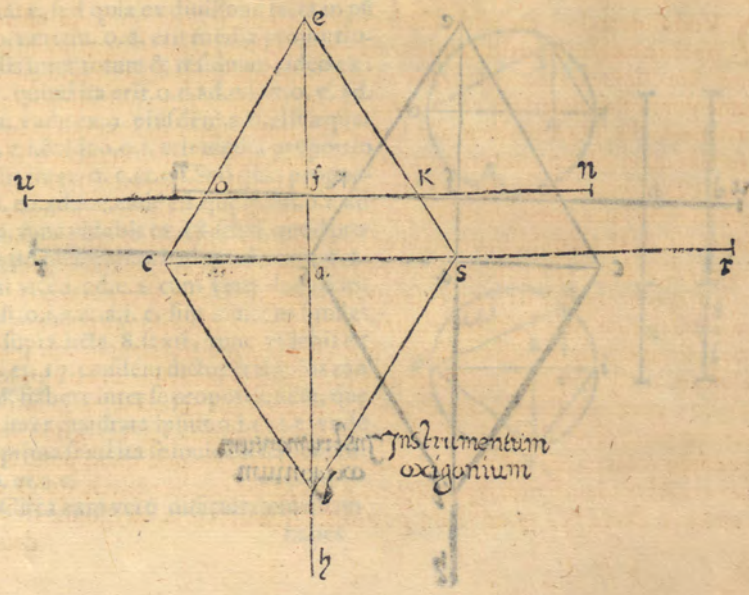
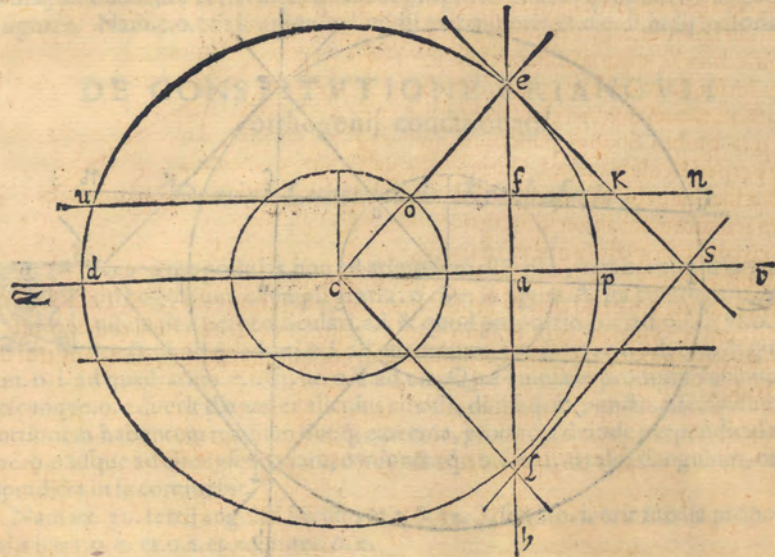
Inter alia, quae tibi dixi de Iride, quod memoria non tenes, nihil aliud est nisi  
quod cum Iris videtur, non eodem loco ab omnibus videtur, quia reflexio est, &  
vt reflexio luminis à speculo non omnibus ab eodem puncto fit, ita etiam tibi dixi  
de Iride.

### *De Instrumento oxygonio, seu elliptico.*

A D E V N D E M.

**Q**uod aliquando à me audiisti falsum non est, scilicet possibile esse ( vt  
speculatus sum ) particulare instrumentum fabricari ad designandum oxy-  
goniam, seu ellipticam sectionem, quae à Pergeo defectio appellatur, quod quidem  
instrumentum valde diuersum est ab alijs, quae alias inueni, pro ipsis conicis sectio-  
nibus delineandis. Occasionem aut huiusmodi instrumenti inueniendi mihi pra-  
buit secunda dubij solutio quam feci ann. 1568. grauius philosopho Francisco Vimer-  
cato, nam cum viderim in ea figura. f. a. semper aequalē esse. o. i. suae parallelae scilicet,  
vnde cum recta linea fuerit protracta per. o. et. f. ipsa foret semper equidistans. d. p. ex  
33. primi. Eucli. Venit mihi in mentem modus construendi hoc subscriptum instru-  
mentum, tali ordine, videlicet, coniungendo septem hic subnotatas lineas materia-  
les. z. r. u. n. e. h. e. c. l. l. s. et. s. e. simul, hoc modo, scilicet fabricando quadrila-  
terum aequilaterum. c. e. s. l. hac conditione, quod immobili existente puncto. c. in li-  
nea. z. r. reliqua omnia mobilia existant, hoc est quod punctum. s. moueatur per di-  
ctam lineam. z. r. & immobili existente puncto. e. vt extremum lineae. e. h. hoc est  
coniuncto extremo. e. lineae. e. h. cum angulo. c. e. s. reliqua puncta lineae ipsius. e. h.  
moueantur per. l. & per duas parallelas. u. n. et. z. r. longitudo vero. e. h. sit compo-  
sita ex duplo vnus lateris ipsius quadrilateris. Oportet deinde quod punctum. f.  
semper vnum, & idem sit ipsius parallelae. u. n. moueatur tamen per. e. h. quod qui-  
dem punctum illud erit, quod vnam portionē circumferentiae oxygoniae sectionis  
desi-

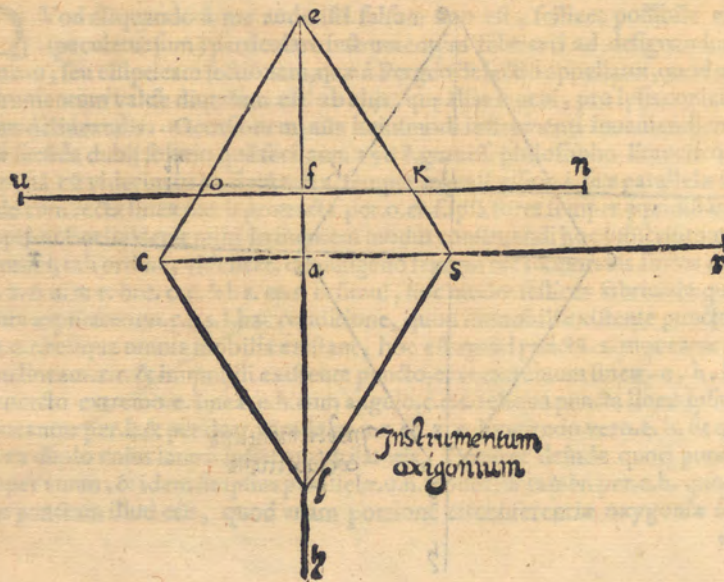
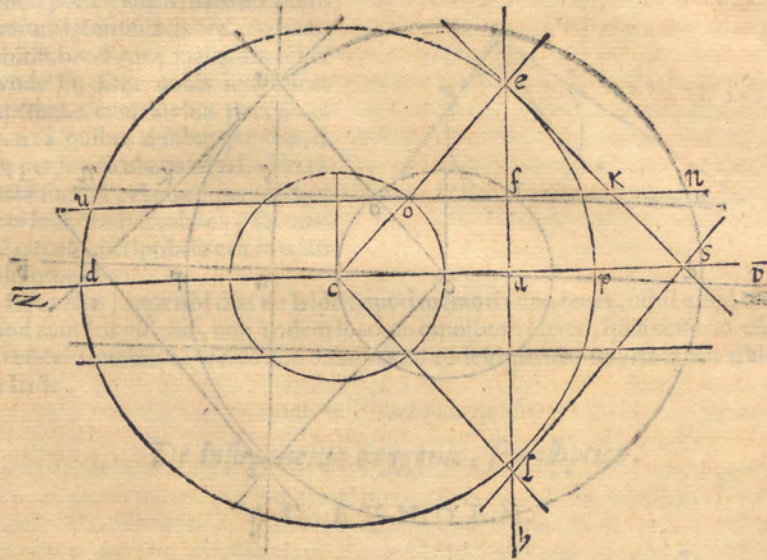
designabit, puncta verò . o . et . K . vt puncta laterum . c . e . et . s . e . æquãdistantia à punctis . c . et . s . eadẽ semper sint , ita tamen vt puncta lineæ . u . n . semper diuersa existãt , & quodlibet ipsius quadrilateri latus , æquale sit medietati maioris axis ipsius oxygoniæ sectionis delineandæ , et . c . o . seu . s . K . ( quod idem est ) sit æqualis medietati axis minoris dictæ sectionis , et . z . r . æqualis duplo . e . h . vnde , quando puncta . e . et . l . coniuncta simul erunt , similiter coniuncta simul erunt . c . e . et . e . s . cum . c . l . et . l . s .  
 Quapropter



Instrumentum oxygonium

Quapropter puncta. e. l. f. et. p. extremum axis maioris, in eodem met loco erunt, hoc est in aliquo extremorum maioris axis, & cum punctus. s. coniunctus fuerit cum centro. c. punctus. f. parallelus. u. r. in extremo axis minoris erit, & in eodem loco erit cum. o. & cum. K. In extremitatibus vero lineæ. z. r. necesse est, ut sint duo pūcta ferri a, ad firmandum ipsam. z. r. super subiectam lineam significantem maiorem axem propositę sectionis.

Volo



Volo etiam quod ad partem. c. l. s. quadrilateri constituta sit alia parallela ad. z. r. & in æquali distantia ab ipsa quemadmodum. u. n. distat ad eademmet. z. r. ad eandem operationem faciendam. Vnde in vno tantummodo itinere puncti. s. ab. r. vsq; ad. c. designabimus quartam partem sectionis, conuerso postea instrumento, hoc est posito puncto. r. vbi prius erat. z. et. z. vbi erat. r. aliam delineabimus quartam, & sic ad oppositam partem ipsius. z. r. faciendum erit. Hoc instrumentum possumus etiam ita construere, vt puncta. o. et. K. possint collocari in lateribus. c. e. et. e. s. vbi nobis magis libuerit, ita vt licebit in qualibet proportione axiũ proposita, oxygoniam designare. Nam. c. o. erit longitudo dimidij axis minoris, et. c. e. dimidij maioris.

DE CONSTITVTIONE TRIANGVLI orthogonij conditionari.

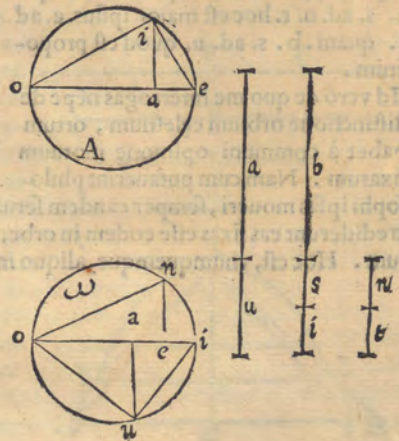
Domino Ludouico de Rocchaforte.



Vobis à me postulas, non est admodum difficile, cupis enim triangulum orthogonium, exempli gratia. o. i. e. in figura. A. ita constituere, vt diuisum sit à perpendiculari. a. i. & quod proportio. o. e. ad. o. i. sit vt. o. i. ad i. e. & quod quadrati. o. i. ad quadratum. o. a. sit vt. e. i. ad. e. a. & quadratum. o. i. ad quadratum. e. i. sit. ut. o. a. ad. e. a. Quæ omnia in promptu veniunt, quotiescunque. o. e. fuerit diameter alicuius circuli, diuisa q; in puncto. a. secundum proportionem habentem medium duosq; extrema, protracta deinde perpendiculari. a. i. ad. o. e. usque ad circumferentiam, coniunctaq; o. i. et. i. e. tale triangulum, omnia supradicta in se continebit.

Nam ex. 30. tertij angulus. i. rectus erit, & ex. 8. sexti. o. i. erit media proportionalis inter. o. e. et. o. a. et. e. i. inter. o. e. et. a. e. sed quia ex diuisione facta in puncto. a. etiam. o. a. erit media proportionalis inter totum & residuum, ideo ex. 11. quinti ita erit. o. e. ad. e. i. vt. o. e. ad. o. a. vnde ex. 9. eiusdem. a. o. erit æqualis. e. i. & ideo. o. i. erit media proportionalis inter. o. e. et. e. i. Sed quia proportio. e. i. ad. a. e. eadẽ est, quæ ipsius. o. e. ad. o. a. tunc videbis ex. 18. sexti, quod proportio quadrati. o. i. ad quadratum. o. a. erit vt. e. i. ad. e. a. cum vero duo trianguli. o. i. a. et. a. i. e. sint inuicem similes ex supradicta. 8. sexti, tunc videbis ex 18. et. 17. eiusdem dictos triangulos eandem habere inter se proportionem, quæ est inrer quadrata ipsius. o. i. et. i. e. vnde ex prima sexti ita se inuicem habebunt. a. o. et. a. e.

Circa eam verò difficultatem quam habes



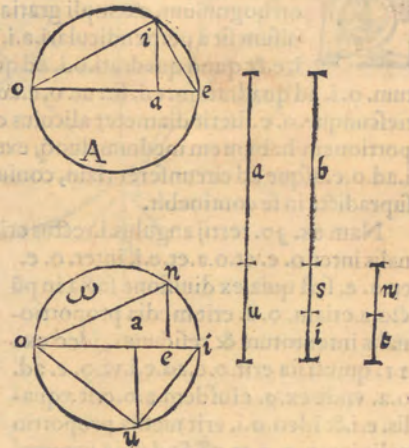
habes in circulo.  $\omega$ . vbi fateris te non videre qua ratione eadem proportio fit quadrati.  $o. u.$  ad quadratum.  $o. n.$  vt lineæ.  $o. a.$  ad lineam.  $o. e.$  partes diametri.  $o. i.$  ipfius circuli, terminata à perpendicularibus.  $u. a.$  et.  $n. e.$

Hoc necessario contingit, propterea quod cum fuerint protractæ.  $u. i.$  et.  $n. i.$  tunc habebimus ad partem.  $o. u. i.$  triangulum.  $o. u. i.$  diuifum in duo triangula fimilia ipfi totali triangulo. Idem etiam dico ad partem.  $o. n. i.$  vnde ex tali fimilitudine habebimus.  $o. u.$  mediam proportionalem inter.  $o. i.$  et.  $o. a.$  et sic.  $o. n.$  erit media proportionalis inter.  $o. i.$  et.  $o. e.$  quare ex. 16. sexti, quadratum.  $o. u.$  æquale erit producto ipfius  $o. i.$  in.  $o. a.$  & quadratum.  $o. n.$  æquale producto.  $o. i.$  in.  $o. e.$  sed ex prima eiusdem, eadem proportio est ipfius.  $o. a.$  ad.  $o. e.$  quæ producti ipfius.  $o. i.$  in.  $o. a.$  ad productum.  $o. i.$  in.  $o. e.$  quare, ex cõmuni conceptu, ita erit quadrati.  $o. u.$  ad quadratum.  $o. n.$  Et hæc est alia circuli paffio.

Reliqua verò difficultas quam te habere fcribis, est, quare cum duæ lineæ  $a. u.$  et.  $b. s. i.$  fint inuicem æquales, diuifæ verò non æquali modo, fed tali, quod.  $a.$  maior fit quam.  $u.$  et.  $b. s.$  maior quam.  $i.$  quomodo potest fieri, quod si.  $u.$  maior fuerit.  $i.$  proportio.  $a. ad. i.$  maior fit quam ipfius.  $b. s. ad. u.$

Hoc etiam ex necessitate cuenit, eo quod si accepta fuerit.  $t. n.$  æqualis.  $u.$  ab ipfaq; abfcifa fuerit.  $t.$  æqualis.  $i.$  & ab.  $b. s.$  abfcifa.  $s.$  æqualis.  $n.$  habebimus.  $a.$  et  $b.$  inuicem æquales, vnde habebis maiorem proportionem ipfius.  $b. ad. t.$  quàm  $s. ad. n.$  quod cum clarum per fe fit, tibi relinquo. fed ex. 27. quinti, proportio  $b. ad. s.$  maior erit quam.  $t. ad. n.$  & ex 28. eiusdẽ pportio.  $b. s. ad. s.$  maior erit, quam.  $t. n. ad. n.$  & ex. 27. maior proportio erit ipfius.  $b. s. ad. n. t.$  quam.  $s. ad. n.$  ergo ex. 33. maior erit ipfius.  $b. ad. t.$  quàm  $b. s. ad. n. t.$  hoc est maior ipfius.  $a. ad. i.$  quam.  $b. s. ad. u.$  quod est propofitum.

Id verò de quo me interrogas nẽpe de diftinctione orbium cęlestium, ortum habet à communi opinione motuum fixarum. Nam cum putauerint philofophi ipfas moueri, femper eandem feruãdo inuicem diftantiam, non sine ratione crediderunt eas fixas eße eodem in orbe, idem etiam poſtea de planetis opinauerunt. Hoc est, vnumquemque, aliquo in orbe, fixo exiſtere.



DE



DE MODO DIVIDENDI PARABOLAM  
propositam secundum datam proportionem.

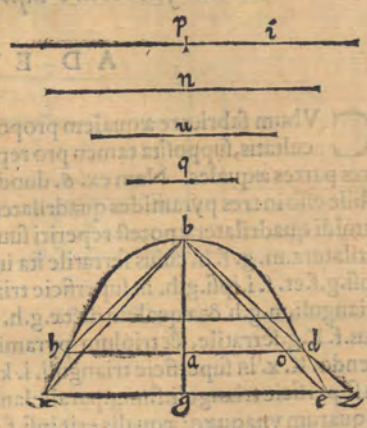
*Pamphilo Gotbfrid.*



VOD à me quæris, est quidem possibile, non tamen adhuc inuentum, quoniam nemo ad huc usque diem diuisit vnã datã proportionem in tres æquales partes, sed si hoc pro facto concesseris, nunc tibi morem geram. Nam proponis mihi parabolẽ. x. b. e. cum proportione. p. ad. q. cupisq; scire modum diuidendi ipsã parabolẽ vna mediante lineã parallela ipsi basi, ita vt eandem habeat proportionem tota parabola ad partem abscissã, quã est inter. p. et. q. Ad quod faciendum, supponendum primò datã proportionem inter. p. et. q. diuisã esse in tres partes æquales, duabus lineis medianibus. n. et. u. quã mediã proportionales vocabuntur inter. p. et. q. deinde à quouis puncto circumferentiẽ ipsius figurẽ ducatur parallela basi. x. e. postea verò per puncta media harum duarum æquidistantiũ protrahatur. g. b. quã diameter erit sectionis, ex 28. secundi Pergei, diuidatur deinde hæc diameter in puncto. a. ita quod eadem proportio sit ipsius b. g. ad. b. a. quã ipsius. p. ad. u. quod tibi facile erit, secando à lineã. p. partem. i. æqualem ipsi. u. tali modo postea diuidendo. b. g. ex. 12. sexti, ducatur a puncto. a. ipsã. d. h. parallelam ipsi. x. e. & habebitur propositum.

Pro cuius rei ratione, scies primum quod. h. d. diuisa erit à diametro. b. g. per æqualia ex. 7. primi Pergei, vel si cogitabimus aliquã lineã tangentem ipsã parabolam in puncto. b. tunc ex quinta secundi ipsius Pergei habebimus ipsã esse parallelam. e. x. & ex. 30. primi Euclidi. erit similiter æquidistans. d. h. vnde ex. 46. primi eiusdem Pergei. h. a. æqualis erit. d. a. Protrahatur deinde. e. b. d. b. x. b. et. h. b. vnde ex. 17 lib. de quadratura parabolæ Archimedis, habebimus eandem proportionem superficiẽ totalis parabolæ. x. b. e. ad trigonum. x. b. e. quã portionis. h. b. d. ad suum trigonũ, eo quod tã vnã quã alia erit sesquitercia, ei? etiã medietates sic se habebũt.

Vnde permutando, proportio medietatis totalis parabolæ ad medietatem partialem ipsius, æqualis erit proportioni trianguli g. b. e. ad triangulum. a. b. d. sed ex. 20. primi Pergei, eadem est proportio quadrati ipsius. g. e. ad quadratum ipsius. a. d. quã. b. g. ad. b. a. hoc est, vt. g. e. ad. a. o. ex similitudine triangulorum, & quia. b. g. ad. b. a. est sicut. p. ad. u. ita igitur erit quadrati ipsius. g. e. ad quadratum ipsius. a. d. quare. g. e. ad. a. d. erit ut. p. ad. n. ex. 18. sexti Euclid. sed cum ex. 24. eiusdem proportio trianguli. b. g. e. ad triangulum. b. a. d. composita sit ex proportione. g. e. ad. a. d. et. ex. g. b. ad. b. a. hoc est. g. e. ad. a. o. & quia proportio. g. e. ad. a. o. æqualis est ei quã. p. ad. u. ex. 11. quinti Euclid. & proportio. g. e. ad. a. d. æqualis est ei quã. p. ad. n. hoc est vt. u. ad. q. ergo proportio trianguli. b. g. e. ad triangulum. b. a. d. composita erit ex ea quã. p. ad. u. & ex ea quã. u. ad. q. æqualis ergo erit ei, quã p. ad. q. & ita medietates parabolarum, & eorum dupla.



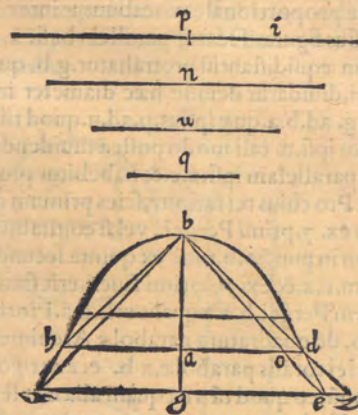
Yy Cor-

COROLLARIUM.

Proportio maioris portionis ad minorem semper erit sesquialtera proportioni ipsius. b.g.ad.a.b. eo quod cum sit proportio totalis portionis ad partialem vt trianguli. b.g.e.ad.b.a.d. & hæc sesquialtera proportioni ipsius. g.e.ad. a. o. hoc est vt ipsius. b.g.ad. b.a. ideo proportio ipsarum portionum erit similiter sesquialtera proportioni diametrorum.

Deinde si protractæ fuerint. b.d.et. g.e. quousque conueniant in puncto. z. habe bis inter. g.z.et.a.o. duas. g.e.et.a.d. medias proportionales in proportionalitate continua, eo quod cum (ex ijs quæ supra diximus.) a.d. media proportionalis sit inter. g.e.et.a.o. & proportio. g.z.ad.g.e. vt ipsius. a.d.ad.a.o. eo quod ipsius. g.z. ad. a. d. & ipsius. g.e.ad.a.o. est vt ipsius. b. g. ad. b.a. ex similitudine triangulorum, ideo dictæ pportiones erunt inuicé æquales. Vnde permutatim ita erit ipsius. g. z.ad.g. e. vt ipsius. a.d.ad.a.o. & ut ipsius. g.e.ad.a.d.

Amplius etiam dico, quod proportio parabolæ totalis ad partialem, eadem est, quæ cubi ipsius. g.e.ad cubum ipsius. a.d. & ex cõsequenti, vt cuborum earundem basium, eo quod cum sit, ex. 36. vndecimi Euclid. proportio cubi ipsius. g.e.ad cubum ipsius. a.d. tripla ei quæ ipsius. g.e.ad.a.d. ideo æqualis erit ei quæ trianguli. b.g.e.ad triangulum. b. a.d. cum proportio horum duorum triangulorum composita sit (vt supra vidimus) ex ea quæ. g.e.ad.a.o. & ex ea quæ. g.e.ad.a. d. & hæc mediætas illius, sed trianguli ita se inuicem habent, vt parabolæ, quare ipsæ parabolæ se inuicem habebunt, vt cubi ipsarum basium.



*Cubum fabricare æqualem pyramidi propositæ.*

A D E V N D E M.

Cubum fabricare æqualem propositæ pyramidi quadrilateræ, nullius erit difficultatis, supposita tamen pro reperta diuisione cuiusuis datæ proportionis in tres partes æquales. Nam ex. 6. duodecimi Eucli. patet omne corpus ferratile diuisibile esse in tres pyramides quadrilateras æquales, scimus etiam quod cuilibet pyramidi quadrilateræ potest reperiri suum ferratile. Sit igitur proposita pyramis quadrilatera. m. g. f. h. cuius ferratile ita inueniemus, ducendo primum. h. i. parallelam ipsi. g. f. et. f. i. ipsi. g. h. in superficie trianguli. f. g. h. et. m. K. ipsi. g. h. in superficie trianguli. m. g. h. & æqualem dictæ. g. h. ducetur postea. K. h. et. K. i. & habebimus corpus. f. K. g. ferratile, & triplum pyramidi propositæ. Nunc duplicemus ipsum, ducendo. K. x. in superficie trianguli. i. k. h. parallelam, æqualemq; ipsi. i. h. et. m. y. in superficie trianguli. f. m. g. parallelam, æqualemq; ipsi. f. g. ducatur postea. g. y. et. h. x. quarum vnaquæq; æqualis erit ipsi. f. m. vnde habebimus corpus. f. x. parallelepipedum, & sexcuplum ipsi pyramidi propositæ.

Inue-

Inueniatur nunc quadratum. u. n. æquale sextæ parti superficiæ. f. i. g. h. quod per se facile erit, deinde accipiatur altitudo corporis. f. x. ducendo vnam perpendiculara rem à puncto. m. ad basim. f. g. h. quæ sit. n. e. qua mediante, cum quadrato. u. n. fabri cetur solidum parallelepipedum. u. e. quod erit æquale dictæ pyramidi ex. 33. vnde cimi Euclid.

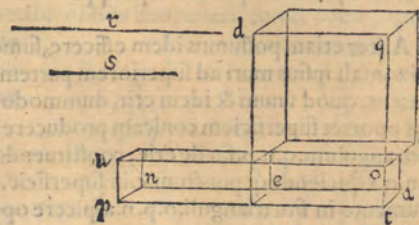
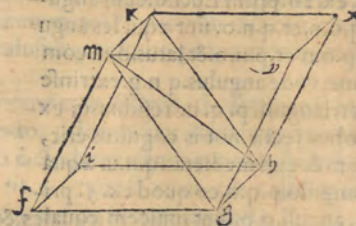
Repertæ nunc sint duæ mediæ proportionales. r. s. inter. n. e. et. n. p. quarum. s. sit proximior ipsi. u. p. ex qua. s. si constitutus fuerit cubus, habebimus propositum.

Pro cuius rei ratione, cogitemus corpus. u. e. productum esse vsque ad. a. o. per longitudinem. s. latus dicti cubi, qui quidem cubus sit. d. b. vnde proportio corporis. u. e. ad corpus. e. o. erit, vt superficiæ. p. e. ad superficiem. t. e. ex. 33. undecimi, ipsa verò superficies sibi inuicem erunt vt. n. e. ad. e. a. ex prima sexti, quare proportio corporis. u. e. ad corpus. e. o. dupla erit proportioni ipsius. s. ad. n. p. sed cum ex. 33. vndecimi, proportio cubi. d. b. ad corpus. e. o. sit vt quadratū. q. b. ad quadratum. o. a. & cum proportio. q. b. ad. o. a. dupla sit ei quæ. q. o. ad. o. t. ex. 18. sexti, erit igitur proportio cubi. d. b. ad corpus. e. o. dupla ei quæ. q. o. ad. o. t. hoc est ei quæ. s. ad. n. p. sed ita erat corporis. u. e. ad corpus. e. o. quare ex. 9. quinti, cubus. d. b. æqualis erit corpori. u. e. hoc est pyramidi proposita.

Sed si oportebit cubum maiorem vel minorem ipsa pyramide reperire, in qua proportione tibi placuerit, tunc opus erit aliud quadratum inuenire, quod in ea proportione se habeat ad quadratum. u. n. quam volueris, quo mediante simul cum altitudine pyramidis consequemur propositum.

Aduertendum tamen quod fabricare ipsum corpus ferratile. k. f. h. & solidum. f. x. necessarium non est, nisi pro demonstratione. idemq; dico de alijs solidis, nam pro simplici operatione huiusmodi problematis, absque aliqua re necessaria ad speculandum, ita faciendum erit.

Data pyramide. m. f. g. h. accipe ei<sup>9</sup> altitudinem à puncto. m. vsque ad superficiem basis. f. g. h. quæ sit. n. e. accipe deinde latus tetragonum quadrati. u. n. æqualis tertiæ partis ipsius basis. f. g. h. quod latus sit. n. p. inter quod, et. n. e. inuenta cum fuerint duæ lineæ mediæ proportionales. s. et. r. quarū. s. proximior sit. n. p. quæ quidē. s. erit latus cubi quaesiti.



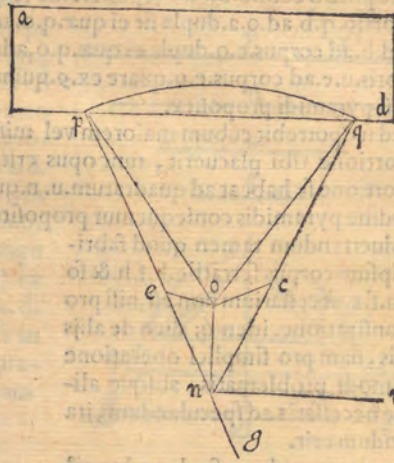
Yy 2 Duplex

*Duplex modus parallelam horizontalem alicui muro proposito una tantummodo statione ducendi.*

A D E V N D E M.

**D**Vcere parallelam horizontalem alicui muro recto proposito una tantummodo statione, non solum possibile est sed etiam facile.

Sit exempli gratia murus rectus. a. d. situs verò. o. n. Si cupimus ducere. n. u. parallelam dicto muro, accipiatur quadratum geometricum, seu scala altimetra vel aliquod simile instrumentum, quo mediante à situ. o. videbimus punctum. q. quod volueris ipsius muri, dexteram versus, inferius tamen, ipso. o. vnde formatum habebimus triangulum. n. o. q. Quo facto ad partem sinistram cum eodem angulo. n. o. q. oportebit nos inuenire punctum aliquod. p. in dicta superficie muri, & tunc habebimus angulum. n. o. p. æqualem angulo. n. o. q. vnde angulus. q. n. p. nobis cognitus erit, duoq; latera. n. q. et. n. p. erunt inuicem æqualia, ex. 26. primi Euclid. cum anguli. q. o. n. et. q. n. o. sint æquales angulis. p. o. n. et. p. n. o. & latus. o. n. commune, vnde angulus. q. n. g. extrinsecus trianguli. p. q. n. residuusq; ex duobus rectis nobis cognitus erit, etiam & eius medietas. q. n. u. æqualis angulo. p. q. n. eo quod ex. 5. primi, anguli. q. p. sunt inuicem æquales, & ex. 32. eiusdem, æquales sunt extrinsecò. q. n. g. & ex 27. n. u. erit parallela ipsi. q. p.

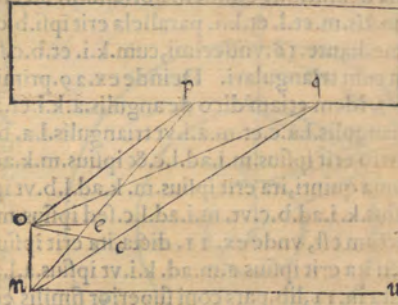


Aliter etiam possumus idem efficere, sumendo duo illa puncta in suprema linea horizontali ipsius muri ad superiorem partem aspiciendo, quemadmodum ad inferiorem, quod vnum & idem erit, dummodò non aspiciamus horizontaliter, eo quod nos oportet superficiem conicam producere, linea visuali mediante. cognoscere autem angulum. q. n. p. facile erit, constituendo primò instrumentum in situ trianguli. o. n. q. aspiciendoq; punctum. c. in superficie. n. q. o. & sic in alia parte, existente instrumento in situ trianguli. o. p. n. aspiciere oportet punctum. e. proximum puncto. n. vbi possit metiri angulum. c. n. e.

Sed si situs puncti. n. talis esset, vt ab eo non posset aliquis murum videre ad rectos angulos, aspiceremus punctum. q. sub horizontali ab oculis nostris, in horizontali tamen puncti. n. ita quod angulus. o. n. q. rectus existat, quo facto obseruando angulum. n. o. q. eo mediante, medianteque. n. o. cum angulo. o. n. q. cognoscemus quantitatem distantie. n. q. idem etiam faciendum est cum alio puncto. p. quod volueris, & mediantibus duobus punctis inuicem proximis. c. e. cognoscatur angulus

gulus. p. n. q. vnde ex methodo. 56. primi triangulorum Monteregij, cognoscemus reliqua. trianguli. q. p. n. Constitucendo postea angulum. q. n. u. aequalem angulo. n. q. p. propositum habebimus.

Si etiam puncta. q. p. lineæ. q. p. horizontali in eodem plano non existerent cum puncto. n. nihil referret, dummodo in pavimento notetur puncta. c. e. proxima. n. in iisdem superficiebus triangulorum. n. o. p. et. n. o. q. vnde. n. c. et. n. e. erunt communes sectiones dictarum superficierum cum superficie pavimenti supra quam fit statio.



CONI RECTI DIVISIO A PLANO  
parallelo basi secundum datam proportionem.

Raphaeli de Auria.



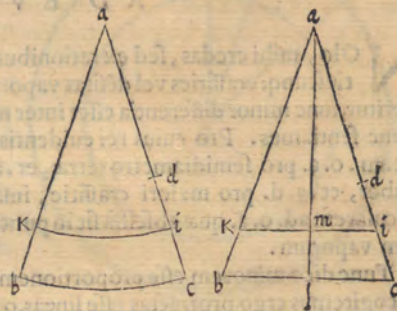
VOTIESCVNQVE volueris conum rectum diuidere à plano parallelo basi secundum vnam datam proportionem, nullius tibi erit difficultatis, concessa tamé pro inuenta diuisione cuiusuis propositæ proportionis per tres æquales partes.

Sit exempli gratia conus rectus. a. b. c. secandus vt dictum est, accipiatur latus ipsius, quod sit. a. c. ipsumq; diuidatur in puncto. d. secundum illam proportionem quam desideras, hoc est ipsius. a. c. ad. a. d. quo facto, inter totum. a. c. et. a. d. inueniantur duæ lineæ proportionales, quarum maior sit. a. i. tunc si conus. a. b. c. sectus fuerit à plano per punctum. i. parallelo basi, habebimus quod querebamus.

Cuius rei ratio, primò est, quia quotiescunque conus aliquis sectus fuerit ab aliquo plano parallelo basi ipsius, pars superior similis semper erit totali cono, quod ita probo, cogitemus conum sectum esse à plano per axem. a. l. vnde ex. 3. primi Pergei, talis sectio triangularis erit, quæ sit. a. b. c. et. b. c. diameter erit basis.

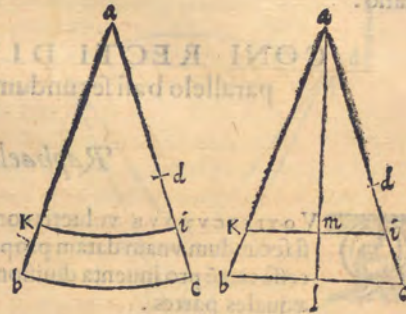
Imagemur deinde. K. i. communem esse sectionem huiusmodi trianguli cum plano parallelo ipsi basi, tunc tale planum, circularè erit ex. 4. primi ipsius Pergei. K. i. verò, eius diameter erit, et. a. m. su<sup>o</sup> axis.

Cum verò. a. l. sit perpendicularis ipsi basi conii totalis, eo quod rectus supponitur, ideo eadem. a. m. l. erit perpendicularis etiam ipsi secundo plano circulari, ex conuersa. 14. vndecimi Euclid. vnde ex secunda



fecunda definitione eiusdem libr. a. m. l. efficiet angulos rectos cum duabus. b. c. et. K. i. in punctis. m. et. l. et. k. i. parallela erit ipsi. b. c. ex. 28. primi, quod etiam potest concludi mediante. 16. vndecimi, cum. k. i. et. b. c. sint communes sectiones duorum planorum cum triangulari. Deinde ex. 29. primi anguli. a. i. m. et. a. c. l. erunt inuicem æquales, idem etiam dico de angulis. a. k. i. et. a. b. c. anguli postea ad. a. communes sunt triangulis. l. a. c. et. m. a. i. vt triangulis. l. a. b. et. m. a. k. Vnde ex. 4. sexti, eadem proportio erit ipsius. m. i. ad. l. c. & ipsius. m. k. ad. l. b. vt ipsius. a. m. ad. a. l. Quare ex vndecima quinti, ita erit ipsius. m. k. ad. l. b. vt ipsius. m. i. ad. l. c. & ex. 13. eiusdem, ita erit ipsius. k. i. ad. b. c. vt. m. i. ad. l. c. sed ipsius. m. i. ad. l. c. est vt ipsius. a. m. ad. a. l. quod iam dictum est, vnde ex. 11. dicta, ita erit ipsius. k. i. ad. b. c. vt ipsius. a. m. ad. a. l. & ex. 16. dicti ita erit ipsius. a. m. ad. k. i. vt ipsius. a. l. ad. b. c. Quare ex definitione ab Euclidi. posita in. 11. lib. pars conii superior similis erit cono totali.

Deinde sciendum est illud quod Euclid. scribit in. 10. duodecimi lib. hoc est,  $\frac{a}{b} = \frac{a}{c} = \frac{a}{d}$  proportio duarum pyramidum inuicem similibus, triplicata est ei diametrorum suarum basium, hoc est, quod proportio. b. c. ad. k. i. tertia pars erit proportionis totius pyramidis. a. b. c. partiali pyramidi. a. k. i. sed ita est ipsius. a. c. ad. a. i. vt ipsius. b. c. ad. k. i. ex. 4. sexti cum trianguli. a. b. c. et. a. k. i. sint æquianguli, quod ex ijs, que superius diximus facile comprehenditur. Quare proportio. a. c. ad. a. i. tertia pars erit proportionis totius conii. a. b. c. ad eius partem abscissam. a. k. i. sed eadem proportio ipsius. a. c. ad. a. i. erat etiam tertia pars proportionis ipsius. a. c. ad. a. d. Quare ex communi conceptu, proportio totius pyramidis, ad partem abscissam, æqualis erit proportioni ipsius. a. c. ad. a. d.



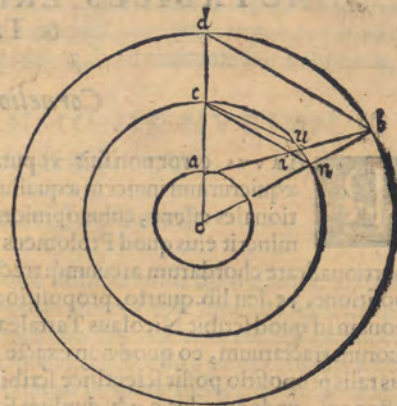
*De differentia caloris Solis propter vaporum  
altitudinem.*

A D E V N D E M.

**N**Olo, mihi credas, sed ex rationibus, quas tibi scribo considera, quod quotiescunq; crassities vel dẽfitas vaporũ, seu altitudo, maior esset ea, que nunc reperitur, tunc minor differentia esset inter maiorem minoremq; calorem Solis, quam nunc sentiamus. Pro cuius rei euidencia, imaginemur in hac subscripta figura, lineam. o. a. pro semidiametro terræ, et. a. c. pro crassitie vaporum, vt nunc se habet, et. a. d. pro maiori crassitie, imaginemurque lineam. a. b. quasi perpendiculararem ad. o. a. quæ abscissa sit in puncto u. à circumferentia. c. u. inferiori priorum vaporum.

Tunc dico minorem esse proportionem ipsius. a. b. ad. a. d. quam ipsius. a. u. ad. a. c. cogitemus ergo protractas esse lineas. o. b. d. b. c. u. et. c. n. quæ. c. n. secabit. a. u. in puncto

puncto. i. ex communi conceptu, & parallela erit ipsi. d. b. ex secunda parte secundæ sexti, vnde ex prima parte eiusdem, ita erit ipsius. b. i. ad. i. a. vt. d. c. ad. c. a. & coniunctim ita erit ipsius. b. a. ad. a. i. vt ipsius. d. a. ad. a. c. & permutatim ipsius. a. b. ad. a. d. erit, vt. a. i. ad. a. c. sed cum. a. u. maior sit ipsa. a. i. vt omne totum maius est sua parte. maior proportio erit ipsius. a. u. ad. a. c. quam ipsius. a. i. ad. a. c. hoc est quam ipsius. a. b. ad. a. d. Verum igitur est propositum.

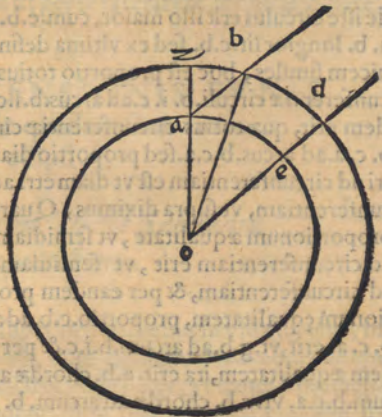


*De differentia caloris Solis respectu altitudinis ipsius.*

AD EVD E M.

**Q**uod à me postulas deinde, ita se habet. Inquis enim, quod cum differentia inter maiorem, minoremq; calorem, oriatur etiam ex differentia maioris quantitatis vaporum ad minorem, per quam quantitatem vaporum transit lumen Solis (vt alias etiam tibi dixi) velles nunc scire quantitatem ipsius differentie, quæ inter duas Solis datas altitudines supra horizontem reperitur.

Quapropter imaginemur circulum. a. c. pro magno terræ, et. z. b. d. pro magno vaporum, supponatur etiam quod angulus. z. o. d. vel. z. a. b. qui sunt inuicem ferè æquales, sit angulus distantie Solis à zenit, z. a. verò sit spissitudo vaporum, et. a. b. radius transiens per vapores dictos. nunc queratur proportio, quæ est inter. a. b. et. a. z. qua inuenta, angulo. z. a. b. mediante, quæremus eandem mediante angulo. z. a. b. maiore priori, vel ipso minore, vnde cognoscemus differentiam duarum. a. b. quæ quidem inæquales inuicem erunt, eo quod supponatur. a. z. immutabilis, & hoc ita faciemus. Imaginabimur. o. b. quæ claudat triangulum. a. b. o. & quia. a. z. cognita est quam Alhazem docet inuenire, cognoscimus etiã o. a. vt semidiametrum terræ, vnde. o. b. et. o. a. duo latera trianguli. a. o. b. cognita erunt simul cum angulo. o. a. b. residuo duorum rectorum, eo quod reliquus. z. a. b. datus est. Quare. a. b. cognita erit respectu. o. a. et. o. b. et. a. z. quæ est eorum differentia. Nunc si idem faciemus cum alia. a. b. sub diuerso angulo, habebimus propositum.



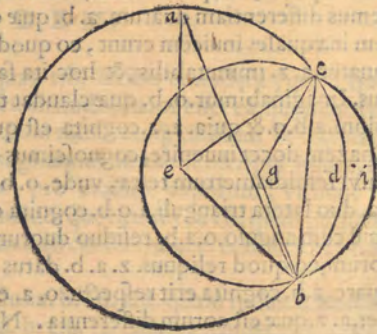
NOTA-

NOTABILES ERRORES ORONTII  
& Tartalea.

*Cornelio Bitonto.*

**P**ARVVS error non fuit, vt putabat Orontius, quod anguli triangulorum æquicrurium inuicem æqualium, basibus oppositi, iisdem basibus proportionales essent, cuius opinionis causa fuit quod nunquam viderit vel meminisset eius quod Ptolomeus scripsit lib. primo Almagesti, vbi de disproportionalityte chordarum arcuumq; tractat, vel quod scribit Vitellio lib. primo propositione. 35. seu lib. quarto, propositione. 27. quod idem est. Sed nec ego tibi proponam id quod scribit Nicolaus Tartalea diuisioni. 28. quinti capitis quartæ partis suorum tractatum, eo quod non exactè scientificè scripserit, nec vniuersaliter, quâuis talis propositio possit scientificè scribi, accipiendo. b. c. in eius figura, pro latere octagoni, vnde angulus. a. e. b. duplum foret angulo. b. e. c. collocato postea. b. c. in arcu. a. b. punctum. c. medium fuisset dicti arcus, et. e. c. diuideret. a. b. per æqualia, ex quinta primi, nec non ad rectos ex. 3. tertij, vnde ex. 18. primi, clare vidiffemus non esse proportionem. a. b. ad. b. c. vt anguli ad angulum. Sed vniuersaliori modo possumus hoc speculari. Nam manifestè scimus, eandem esse proportionem circumferentiæ ad diametrum in omnibus circulis tam maioribus, quam minoribus. Sine igitur duo anguli. a. e. b. et. e. c. b. cuiusuis amplitudinis, quorum latera. e. a. e. b. et. e. c. sint inuicem æqualia, protrahatur. b. a. et. b. c. Tunc dico maiorem proportionem esse anguli. a. e. b. ad angulum. b. e. c. quam. a. b. ad. c. b. ducatur enim. b. g. ita qd faciat angulum. g. b. c. æqualem angulo. e. b. a. protrahatur postea. c. g. quæ idem faciat in puncto. c. vnde. g. b. et. g. c. æquales inuicem erunt ex. 6. primi, & quia angulus. a. æqualis est angulo. e. b. a. ex quinta eiusdem, ideo ex. 32. dicti, et. 4. sexti, horum duorum triangulorum latera, erunt inuicem proportionalia. Constituto deinde. g. centro, & secundum semidiametrum. g. b. vel. g. c. quod idem est, descripto circulo. b. i. c. necnon circulo. b. c. a. circa centrum. e. opè semidiametri. e. b. et. e. a. vnde iste circulus erit illo maior, cum. c. b. maior sit. g. b. ex. 14. quinti. cum ex. 14. tertij a. b. longior sit. c. b. sed ex vltima definitione tertij, arcus. b. i. c. et. b. c. a. erunt inuicem similes, hoc est proportio totius circumferentiæ circuli. b. i. c. ad arcum. b. i. c. eadem erit, quæ totius circumferentiæ circuli b. c. a. ad arcum. b. c. a. sed proportio diametri ad circumferentiam est vt diametri ad circumferentiam, vt supra diximus; Quare ex proportionum æqualitate, vt semidiametri ad circumferentiam erit, vt semidiametri ad circumferentiam, & per eandem proportionum æqualitatem, proportio. c. b. ad arcum. b. c. a. erit, vt. g. b. ad arcum. b. i. c. & per eandem æqualitatem, ita erit. a. b. chordæ ad arcum. b. c. a. vt. c. b. chordæ ad arcum. b. i. c. & permutando, ita erit chordæ. a. b. ad chordam. c. b. vt arcus. b. c. a. ad arcum. b. i. c. sed arcus. b. i. c. maior est arcu. b. d. c. ex commu

ni





ni scientia. Quare maior proportio erit acus. b. c. a. ad arcum. b. d. c. quam ad arcum b. i. c. ex. 8. quinti. Vnde ex vltima sexti et. 12. quinti, proportio anguli. a. e. b. ad angulum. c. e. b. maior erit quam chorda, siue basis. a. b. ad chordam siue basim. c. b.

DE CAUSA SVSPENSIONIS NVBIVM  
in aere contra Antonium Bergam.

*Clarissimo Francisco Venerio.*

**E**GO enim non tantum miror ea quæ mihi scripsisti de opinione Ortenij quantum quod Antonius Berga putat nubes à Sole suspensas teneri, id plañe falsum est, vera causa huiusmodi effectus, alia nulla est, nisi earunde raritas hoc est, cum rariores sint ipso aere subiecto, propterea supra ipsū natant & stant sub eo qui rarior ipsis est, eo quod corpora rariora posita in medio non tam raro, ascendunt, & densiora in medio minus denso descendunt. Nam si Sol ipsas nubes suspensas in aere teneret, hoc interdum tantummodo fieret, sed non est, cur non descendunt vsque ad terram, & in eodem loco semper manent? Sciendum igitur est nubes ascendere in altum quousque inueniant aerem eiusdem raritatis cuius ipsæ sunt. Raritas enim & densitas non sunt res visibiles nisi per accidens, quemadmodum etiam leuitas, & grauitas, opacitas verò & diaphaneitas magis comprehenduntur, opacitas enim ex reflexione radiorum luminosorum, diaphaneitas verò comprehenditur ex penetratione ipsorum radiorum, opacitas autem nubis non est densitas, cum valde diuersa sit densitas ab opacitate, sicut raritas ab diaphaneitate, vt aliàs dixi. Et quando dicit, quod Sol calefaciendo aerem ipsam nubem ambientem, rarefaciat eum magis quam ipsam nubem respondeo, hoc verum non esse, propterea quod radius Solis non multum calefacit ea corpora, quæ ipsi per mittunt liberum transitum. vnde corpora quanto magis diaphana sunt tanto minus ab ipso radio luminoso calefiunt, sed ea quæ magis opaca sunt, magis etiam calefiunt & per consequens magis rarefiunt, cum calidi sit per se rarefacere, & non attrahere, vt ipse & ferè omnes alij putant.

DE RATIONE EXTENSIONIS FVNIS  
cuiusdam libramenti, & de quadam simboleitate circuli cum ellipsi.

*Angelo Ferrario Serenissimi Ducis Sabaudia  
Agrimensori expertissimo.*

**I**BI in mentem veniet, quod cum superioribus diebus in villa lucenti, in qua degebat Serenissimus Dux noster, dum viridarium ad æquilibrium reducebas, effemus, à te quæsiui an scires vnde fieret, vt stante libramento ad angulos rectos supra suum pedem, funis quæ extrema eiusdem libramenti cum pede in formam trianguli æquicruris coniungit, magis distentus existeret, quam cum dictum libramentum cum pede obliquum remanet, ita vt huius-

Z z modi

modi funis cum libramento triangulum scalenum constitueret.

Exempli gratia, ponamus lineam. d. b. c. esse libramentum. et. b. e. u. eius pedem, funem autem, qui aliquando cum libramento facit triangulum ifocellum, & aliquando scalenum, esse. d. e. c. esto etiam quod in figura. A. dictus triangulus. d. e. c. fit ifocellus, & in figura. B. scalenus. Tunc quaesivi à te an scires rationem, quare funis. d. e. c. in figura. A. esset distensus, & in figura. B. laxus quemadmodum videbamus. cum mihi responderis, nescio quid, quod nunc memoria nō teneo, sed quia pollicitus sum me tibi eam afferre, propterea nunc ad te mitto. Scias ergo huiusmodi rationem nihil aliud esse nisi quod in figura. A. duæ lineæ. c. e. et. d. e. simul è directo iunctæ longiores sint illis, quæ reperiuntur in figura. B. sed quia funis tam in figura. B. quam in figura. A. vnus, & idem est, ideo in figura. B. laxatus est, & non in tensus, ut in figura. A. Sed vt huiusmodi veritatis certam notitiam habeas, infra scriptum circulum mente concipe. f. e. i. cuius semidiameter, æqualis sit. b. e. & diameter sit. f. i. in quo imaginare esse tuum libramentum. d. b. c. & figuras. A. et. B. & probabo lineas. d. e. c. figuræ. A. longiores esse lineis. d. e. c. figuræ. B.

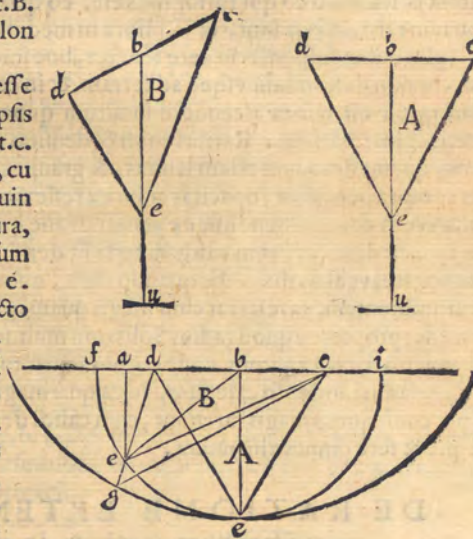
Imaginemur igitur lineam. b. e. esse dimidium minoris axis alicuius ellipsis cuius quidem figuræ ponamus. d. et. c. centra ipsius circumscriptionis esse, cuius circumferentia, nulli dubium est, quin extra propositum circulum transitura, & in vno tantummodo puncto ipsum circulum tactura sit, qui existat. e. figuræ. A. separatim tamen à puncto e. figuræ. B. Tunc si protracta fuerit linea. d. e. figuræ. B. vsque ad girum ellipticum in puncto. g. à quo ad punctum. c. ducta etiam sit linea g. c. tunc manifestum erit duas lineas d. e. et. e. c. figuræ. A. simul iunctas, æquales esse duabus. d. g. et. g. c. simul positis, vt etiam ex. 52. tertij

Pergei facile videre est, sed ex. 21. primi Euclid. iam certò scimus. d. g. c. longiores esse. d. e. c. figuræ. B. ergo. d. e. c. figuræ. A. longiores sunt. d. e. c. figuræ. B. quod est propositum.

Quod etiam mihi nunc circa hoc succurrit, tibi libenter significo, hoc est, quod sicut in ellipsi duæ lineæ. d. e. c. figuræ. A. simul iunctæ, sunt semper æquales duabus lineis. d. g. g. c. in longitudine, ita in circulo duæ. d. e. c. figuræ. A. æquales sunt in potentia duabus. d. e. c. figuræ. B.

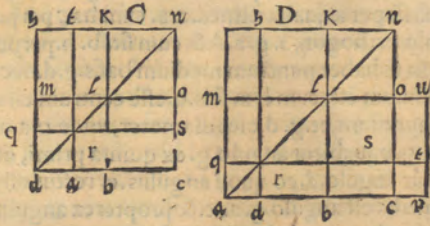
Manifestum enim primum est ex penultima primi in figura. A. quadratum. c. c. æquale esse duobus quadratis scilicet. e. b. et. b. c. & quadratum. c. d. æquale duobus. e. b. et. b. d. Quare quadrata. e. c. et. e. d. æqualia sunt quadratis. e. b. figuræ. A. et. e. b. figuræ. B. et. b. c. et. b. d. hoc est duplo quadrati. e. a. (ducta cum fuerit. e. a. perpendicularis ad. c. b. d. a.) duplo quadrati. a. b. ex penultima primi, & duplo quadrati. b. c. Sed quadrata. d. c. et. e. c. figuræ. B. æqualia sunt duplo quadrati. a. c. & quadrato a. d.

&



& quadrato. a. c. ex eadē. Nunc videndum est virū duplū quadrati. a. e. cū duplo quadrati. b. a. cū duplo quadrati. b. c. sit æquale duplo quadrati. a. e. cū quadrato. a. d. & cum quadrato. a. c. Sed quia tam ex vna parte quā ex alia habemus duplum quadrati. a. e. Videndum igitur erit vtrum duplum quadrati. a. b. simul cum duplo quadrati. b. c. æquale sit quadrato. a. c. cum quadrato. a. d. sed hoc manifestum est. ex. 10. secundi Euclidis, dato quod punctū. a. sit inter. f. et. d. sed si fuerit inter. d. et. b. hoc manifestum erit ex. 9. secundi dicti, nihilominus accipe hunc alium modum.

Sit hic subscriptum quadratum. D. ex. a. c. in seipsā producta, cuius diameter sit a. n. protrahanturq; parallelę. d. h. b. K. l. m. o. et. r. q. s. eiq; addatur. c. p. ad. a. e. æqualis tamen. d. a. sitq; protracta. p. u. vsque ad. m. o. u. vnde habebimus. a. n. pro totali quadrato, et. p. s. pro partiali, & æquali quadrato lineę. a. d. Videndum nunc est, vtrū hæc duo quadrata æqualia sint duobus quadratis lineę. a. b. & duobus lineę. b. c. Nā duo quadrata lineę. b. c. sint. K. o. et. h. l. videndum nunc est utrum residuum æquale sit duobus quadratis lineę. a. b. quorum vnum sit. m. b. alterum verò. l. p. quod superat. l. c. et. s. p. figurę. D. per supplementum. o. t. cui æquale est parallelogrammum. h. m. figurę. D. sed si punctus. a. positus fuerit inter. d. et. b. constituto quadrato. d. u. cū omnibus parallelis, vt in figura. C. videre licet, in qua figura videbimus quadrata. r. n. et. d. r. æquari duplo quadratorum. l. n. et. r. l. nam in quadrato. r. n. ipsā duo quadrata. l. n. et. r. l. capiuntur, reliquum est igitur vt videamus an duo supplementa. l. t. et. l. s. cum quadrato. d. r. sint æqualia dictis quadratis. l. n. et. r. l. sed quadratum. d. l. æquatur quadrato. l. n. videndum igitur est, an duo supplementa. l. t. et. l. s. cum quadrato. d. r. sint æqualia duobus quadratis. d. l. et. r. l. sed quadratum. d. l. æquatur quadrato. d. r. & supplemento. l. t. mediante. q. l. & supplemento. r. b. supplementum verò. l. s. superat supplementum. r. b. per quantitatem æquale quadrato. r. l. quare duo supplementa. l. t. et. l. s. cum quadrato. d. r. æquantur quadrato. d. l. cū quadrato. l. r. verum igitur est duas. d. e. e. c. figurę. A. æquales esse in potentia duabus d. e. e. c. figurę. D. quæ quidem affectio circuli, à nemine fuit adhuc (quod sciam) detecta.



Zz 2 DE

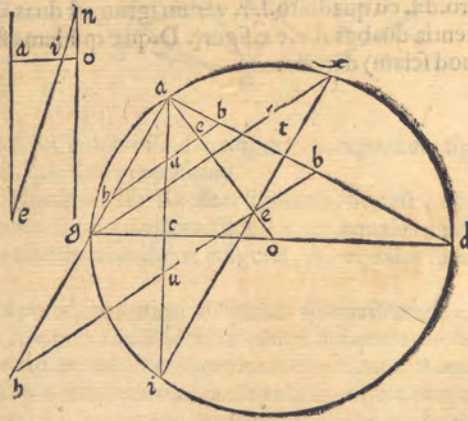
DE AVGMENTO PONDERIS CORPORIS  
ad stateram appensi, & quadam alia demonstratione,  
& quibusdam erroribus Tartaleæ.

*Mutio Groto.*



I ea quæ à me audiisti non credis, considera quæso libram seu stateram o. a. cuius centrum non longitudinis sed ponderum sit. i. quæ statera, vt horizontaliter consistat, oportebit pondus extremitatis. o. ita se habere ad pondus extremitatis. a. ut. a. i. se habet ad. o. i. quod te scire puto, imaginemur nunc duas lineas. a. e. et. o. n. parallelas infinitasq; & à puncto. n. immobili, & fixo extra stateram, transeat per. i. linea. n. i. e. Cogitemus etiam punctum. e. inter sectionis ipsius. n. i. e. cum. a. e. progredi vniformiter continuoq; ab. a. per lineam. a. e. vnde punctum. i. interfectionis ipsius. n. i. e. cum. a. i. o. semper vicinius fiet puncto. o. nec vnquam cum illo vnus erit, quamuis moueatur tempore infinito. Nunc autem dico, quod cum stateram. o. i. a. oporteat semper horizontalem esse virtute ponderis, o. oportebit pondus. o. in infinitum etiam augeri, quotiescunq; pondus. a. nunquam diminui voluerimus vel econtra hoc in infinitum diminui, si illud nunquam augeri voluerimus.

Sed re vera non putabam te indigere aliqua demonstratione, quod linea. b. h. diuisa sit per æqualia à linea. c. a. cum hæc perpendicularis sit ab. a. ad basim. g. d. in triângulo orthogonio. g. a. d. & cum sit. b. h. perpendicularis ad. a. o. ex supposito quæ. a. o. in se habet punctum medium basis. g. d. nec nõ illud anguli recti. a. quod per se clarissimum est, cum iam scis. o. esse centrum circuli circundantis triangulum. g. a. d. orthogonium, et. g. d. eius diameter, vnde. o. a. æquabitur ipsi. o. g. quapropter angulus o. a. g. æquabitur angulo. g. ex quinta primi, deinde ex. 32. eiusdem, angulus. h. æquabitur angulo. d. eo quod angulus. e. rectus est, quemadmodum et. a. sed angulus. d. æqualis est angulo. g. a. c. & propterea angulus. h. erit etiam æqualis angulo. h. a. u. vnde. h. u. æqualis erit ipsi. u. a. ex. 6. primi, cum postea angulus. o. a. d. æqualis sit angulo. d. ex quinta primi erit angulus. a. b. e. æqualis angulo. g. ex. 32. dicta, eo quod e. rectus est, & ex eadem æqualis erit angulo. d. a. c. vnde. u. b. erit æqualis ipsi. u. a. ex. 6. dicti, & ideo æqualis erit ipsi. u. h. Reliqua vero illius propositionis credo ex te omnia posse intelligere, excepto, qd vt tibi significavi si à puncto. i. communi ipsi. a. e. u. & circumferentiæ, ducta fuerit. i. x. ad punctum. x. commune vni parallelæ à puncto. g. ipsi h. b. & circumferentiæ, quod dicta. i. x. ad rectos erit ipsi. a. b. d. eo quod cum angulus. a. g. x. æqualis sit



fit angulo. a. h. b. propter æquidistantiam dictam, æqualis etiam erit angulo. d. & arcus. a. x. æqualis arcui. a. g. vnde angulus. a. i. x. æqualis erit. d. sed angulus. i. a. d. communis est triangulis. c. a. d. et. i. a. t. quare angulus. a. t. i. rectus erit, vt. c. hoc est. i. x. perpendicularis erit ipsi. a. d.

Sed vbi tibi scripsi circa finem illius epistolæ, Tartaleam errasse in quinta propositione primi lib. suæ nouæ scientiæ, non sine ratione illud scripsi. Nam, inquit ipse, nullum corpus æquè graue potest in aliquo temporis spatio moueri motu naturali, violentoq; simul mistis. Vbi decipitur, eo quod non animaduertit incrementum velocitatis vnus motus, simul esse cum decremento velocitatis alterius, eodemq; tempore, vt manifestè patet in itinere corporis, ab ipso pro exemplo assumpto, hoc est quod velocitas motus in spatio. c. d. crescit vt naturalis, & decrescit vt violenta. nã crescit orizontem versus & decrescit in remotione à linea. a. b. sed si à puncto. c. ad punctum. d. motus esset purè violentus, vt putat Tartalea, corpus illud minimè descenderet, eo quod uirtus mouens, in. a. posita, nullo pacto potest talem effectum efficere, vnde ab ipsa natura prouenit descensio illius corporis propter grauitatē, quã dictum corpus habet in tali medio, aeris scilicet, & non ex violentia aliqua. Sed si dixisset ipse, illum motum esse purum naturalem, hoc esset falsum, eo quod purus naturalis motus alicuius corporis non impediti, extra locum suum, fit per lineam rectam, & non per curuam, vt videre est inter. c. et. d.

In vltima propositione deinde eiusdem lib. quæ. 6. est decipitur similiter, & hæc deceptio oritur ab ignoracione quintæ, & à putando motum naturalem non esse causam ipsius descensus per spatium. c. d. Sed quia tibi significauit expeditiorem viam reperiri ad cognoscendā proportionem inter. a. h. et. a. e. in vltima propositione secundi lib. ipsius Tartaleæ, ipsam nunc tibi scribo. Nã iam scis angulum. h. l. i. diuisum esse per æqualia ab. P. l. & quod. a. h. et. h. p. æquales inuicem sunt ex. 6. primi Euclidi. vnde. p. i. et. a. h. æquales erunt inuicem similiter, sed ex. 3. sexti ita est ipsius. a. l. ad. l. i. vt ipsius. a. p. ad. p. i. & coniunctim ita erit. a. l. i. ad. l. i. vt. a. i. ad. p. i. sed. a. l. cognita est ex eius quadrato, et. l. i. etiam, cum æqualis sit ipsi. a. i. vnde ex regula de tribus notam habebimus. p. i. respectu. a. i. & ita respectu. a. e. si hypotheses ipsius Tartaleæ veræ sunt.

*Alia demonstratio impossibilitatis diuidendi per æqualia  
proportionem superparticularem in  
discretis.*

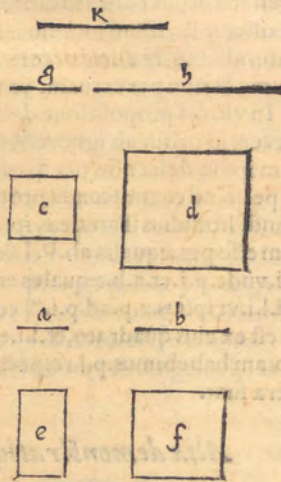
A D E V N D E M.

**Q**uod à me postulas, hoc est scientiam impossibilitatis diuidendi per æqualia proportionem superparticularem in numeris latis à Campano in. 8. octani potes habere, Iacobus Faber Stapulensis etiam idem tractat in libello suæ musicæ demonstratæ. Sed si etiam alia via idem desideras, quamuis longiori, nihilominus vniuersaliori, considera duos numeros. g. et. h. inuicem relatos secundum proportionem superparticularem, quam volueris. Tunc dico impossibile esse, vt per æqualia diuidatur, quod si dixeris possibile esse, fit per te. K. medius numerus  
propor-

proportionalis inter.g.et.h.quare.g.et.h.non erunt minimi in ea proportione, quia vnitas diuisibilis esset si.g.h.minimi fuissent, quod non conceditur, sint igitur minimi in dicta proportione.a.et.b.quorum differentia erit vnitas, vt scis, sitq;.c.quadratum ipsius.g.et.d.quadratum ipsius.K.tunc clarum erit ex.11.octau, quod proportio ipsius.c.ad.d.eadem erit qua.g.ad.h.hoc est vt ipsius.a.ad.b.vnde si vnus terminorum.a.vel.b.esset quadratus, reliquus etiam quadratus esset ex.22.octau, & ex 16.eiusdem,inter.a.et.b.reperiretur aliquis medius numerus proportionalis, quod fieri non potest ex hypothesi, cum inter.a.et.b.nullus sit numerus, quia differunt inter se per vnitatem tantummodo. Nunc autem cum nullus numerorum.a.vel.b.quadratus sit, ponatur quod.f.quadratus sit ipsius.b.et.c.fit productum ipsius.a.in.b.vnde ex.18.septimi, proportio ipsius.e.ad.f.erit vt.ipsius.a.ad.b.hoc est vt ipsius.c.ad.d.quapropter.e.erit quadratus ex.22.octau, cuius latus tetragonicum esset mediū proportionale inter.a.et.b.ex.20.septimi, quod est impossibile, vt iam dixi, cum.a.et.b.sint inuicem consequentes, vnus post alium immediatè.

Superius enim dixi hunc modum esse vniuersalem, hoc est quod hac methodo possumus in cognitionem venire, quod non solum in duas æquales partes diuidi non possit, sed nec in tres, nec quatuor nec quot volueris. Primum enim quod non in tres diuidatur à te ipso cognoscēs ope cuborū vice quadratorū, ope vero censuū cēsūū, vel qui cognouerit eam proportionē esse indiuisibilem per æqualia, illicō etiam cognoscet indiuisibilem esse per quatuor partes, ope verò primorum relatorum, cognoscet non esse diuisibilem per quinque partes, & sic de ceteris, sed mediantibus ijs quas scripsi de istis dignitatibus in libro Theoremātū arithmeticoꝝ.

Id autem quod Illustrissimus Daniel Barbarus scribit in quinta parte suæ perspectiuæ, si supra aliquo immobili, atque magno pariete facere volueris, te oportebit hoc ex reflexione radij solaris à speculo plano perficere.



## DE INVENTIONE DIAMETRI circuli circumscribentis triangulum.

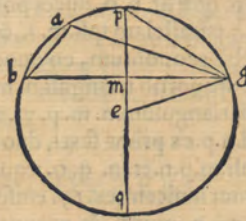
*Franchino Triuultio.*



**V**OD mihi nunc proponis est triangulum, cuius basis cum angulo sibi opposito dantur. Vellesq; diametrum circuli apti eum triangulum circumscribere inuenire in diserto.

Sit igitur triangulum.a.b.g. cuius basis.b.g.simul cum angulo.a.ei opposito data sit in numeris. Imaginetur ergo circulus circumscribens ipsum triangulum.b.p.g.q.cuius diameter sit.q.p.perpendicularis eius basi. b.g.vnde. b.g.diuisa erit per æqualia ab ipso diametro in puncto.m. per tertiam tertij, protrahatur etiam  
c.g.

e.g. vnde angulus.g.e.q. æqualis erit angulo.b.a.g. portionis, cum duplus sit angulo q.p.g. medietati anguli ipsius portionis ex. 19. tertij, ita quod angulus.q.e.g. nobis cognitus erit, & similiter arcus.g.q. & consequenter arcus.p.g.refiduum medij circuli, & sic.m.g. eius sinus re-ctus, & etiam chorda.p.g.vt dupla sinus dimidij arcus.p.g.& sic.p.m.eius sinus versus, vel vt tertium latus trian-guli orthogonij.p.g.m.vnde nobis cognita erit propor-tio ipsius.b.g. (quæ dupla est ipsi.m.g.)ad.m.p. & quia productum.p.m.in.m.q.æquale est ei, quod fit ex.b.m.in.m.g. ex. 34. tertij, quapropter nobis cognita erit pars q.m. quæ cum.p.m.complet totum diametrum.q.p. vn-de nobis cognita erit proportio ipsius.b.g.ad.q.p. qua mediante cognoscemus diametrum secundum partes il-las quibus propoftra fuerit. b.g.

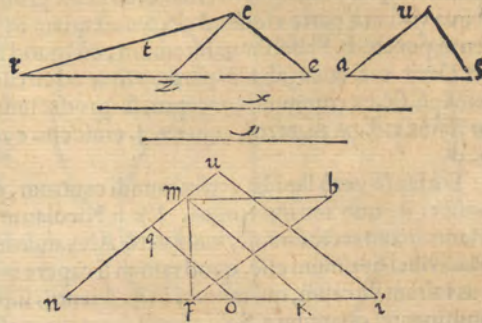


Hoc autem problema non in numeris sed in continuo ab Euclid. ponitur in. 32. tertij.

*De inuentione alterius trianguli conditionati.*

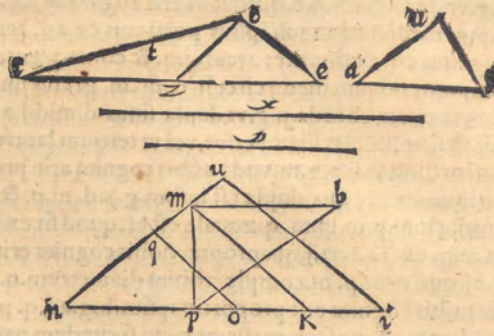
A D E V N D E M.

Q Votiescunq; etiam inuenire voluerimus triangulum aliquem, puta . n. q. o. æqualem triangulo.t. (exempli gratia) propofito, qui habeat angulum.n. æ-qualem angulo.a. dato, latera vero continentia ipsum angulum.n. sint inuicem pro-portionata vt.x.et.y. ita faciemus, accipiemus lineam.n.m. cuius volueris magnitu-dinis, supra quam constituemus triangulum.m.n.p.æqualem triangulo.t. hac metho-do, hoc est prolongando latus.r.z.trianguli.t. quod fit.r.e. ita vt duplum fit ipsi. r. z. du-cendo postea.c.c. habebimus ex. 38. primi triangulum . t. esse dimidium totius trianguli.r. c. e. designabimus deinde ex. 44. dicti superficiem.p. n. m. b. parallelo-grammam æqualemq; triangu-lo. r. c. e. habentem angulum.n. æqualem angulo. a. ducatur postea.p. m. & habebimus trian-gulum. m.n.p.æqualem.t. cum angulo.n.æquali angulo.a. pro-ducatur postea.n.p. ita vt.n. K. se habeat.ad.n. m. quemadmo-dum.x. ad.y. quod erit facillimum producendo. n. m. et. n. K. indeterminate si oportuerit, deinde eas ad æqualitatem le-cardo ipsis. x. et. y. efficiendo exempli gratia quod. n. i. fit æqualis ipsi.x.et.n.u. ipsi. y. du-cendo postea.u. i. deinde à puncto.m. ducendo.m. K. æquidistanter. u. i. ex. 31. primi. & sic habebimus ex. 4. sexti proportionem.x. ad. y. esse inter. n. K. et. n. m. inue-



n. K.  
m.in

m. inuenies postea ex. 9. eiusdem lineam aliquam mediam proportionalem inter n. K. et n. p. quæ sit. n. o. duces postea o. q. parallelam ipsi. m. K. & habebis propositum, eo quod cū sit proportio trianguli. n. m. K. ad triangulum. n. m. p. vt. n. K. ad n. p. ex prima sexti, duo triânguli. m. p. n. et. n. q. o. æquales erunt inuicem, ex. 17. eiusdem & ex. 9. quinti, & proportio. o. n. ad. n. q. erit, vt. x. ad. y. ex. 11. dicti, cum ex. 4. sexti sit vt. n. k. ad. n. m.



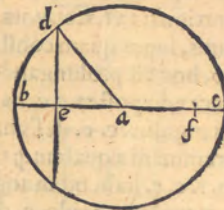
*De producto conditionato.*

A D E V N D E M.

**P**roponis deinde mihi duas rectas lineas, vni quarum, vis vt aliam quandam directè coniungam, ita quod productum huius aggregati in lineam adiunctam æquale sit quadrato alterius.

Vt exempli gratia si fuerint duæ lineæ. e. d. et. e. f. oporteretq; nos ad lineam. e. f. aliam lineam puta. f. c. vel. e. b. iūgere, ita longam, vt productum totius compositi. e. c. vel. f. b. in. f. c. vel. e. b. esset æquale quadrato ipsius. e. d.

Hoc enim nullius esset difficultatis, eo quod quotiescūque. e. d. coniuncta erit cum. e. f. ad rectos, diuisaq; per medium à puncto. a. à quo ducta. a. d. deinde secundum semidiametrum. a. d. designato circulo. b. d. c. & protracta. e. f. à qua volueris parte vsque ad circumferentiam in pūcto. c. seu in puncto. b. habebimus intentum, eò quod si producta fuerit. e. f. etiam ab alia parte, vsque ad circumferentiam, habebimus. b. c. æqualem ipsi. f. c. ex communi conceptu, & productum. e. c. in. e. b. æqualem quadrato ipsius. e. d. ex. 34. tertij, cum ex. 3. eiusdem. e. d. medietas sit chordæ arcus dupli b. d.



De lapsu verò lapidis versus mundi centrum, dum ipsum attingere, ac præterire posset, de quo me interrogas. Dico Nicolauum Tartaleam, nec non Franciscum Maurolicum rectè sensisse, malè verò Alexandrum Piccolhomineum, & exemplum Maurolici optimum esse, quod tamen si capere non potes, crede saltem authoritati bus talium virorum, qui tantum in ijs scientijs superant ipsum Alexandrum Piccolhomineum, quantum à Sole cætera superantur astra.

Lapis igitur ille transfret centrum, redderetq; cum diminutione tamen motus impressi, eo fermè modo vt scribunt iudiciosissimi illi viri, donec post multas reductiones sursum, deorsumq; quiesceret circa centrum mundi. Lucidioris tamen intelligentiæ





etiam collectus cum angulo. o. a. c. grad. 15. hi tres anguli efficient angulum. d. a. c. dictum grad. 108. min. 22.

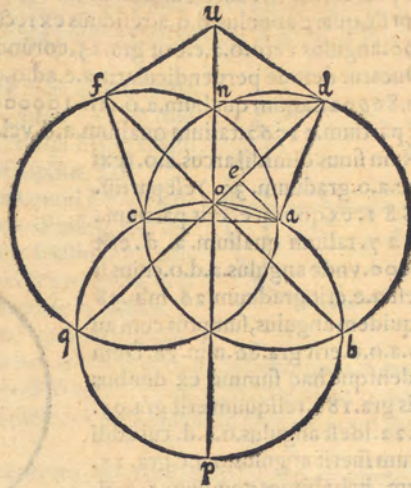
*Examinatio anguli. u.*

Ducatur. d. n. quam quidem. d. n. cognoscemus vt sinus anguli. d. o. n. gra. 45. nam angulus ei contrapositus. q. o. p. est dimidium recti, quare. d. n. erit partium. 70710. talium qualium. d. o. fuerit. 100000. sed. d. o. est partium. 115270. qualium. a. d. est. 100000. nam. e. d. vt sinus anguli. e. a. d. gra. 63. min. 22. est partium. 89389. o. e. vero est partium. 50000. talium qualium. a. o. est. 100000. vt sinus anguli. e. a. o. gra. 30. sed vt. a. o. est partium. 51762. hoc est vt. a. d. est. 100000. ipsa. o. e. erit partium. 25881 quæ iuncta cum fuerit cum. e. d. efficiet. d. o. partium. 115270. vt dictum est, quapropter cum. d. n. sit partium. 70710. talium qualium. d. o. fuerit. 100000. ipsa. d. n. erit partium. 81507. talium qualium. d. o. erit. 115270. id est qualium. d. a. vel. d. u. erit 100000. quæ quidem. d. n. est sinus anguli. d. u. n. graduum scilicet. 54. 36. cuius duplum erit gra. 109. mi. 12. debebat tamen esse. 108. m. o.

*Examinatio anguli. d.*

Accipe angulum. a. d. o. gra. 26. min. 38. vt supra, cui applica angulum. o. d. n. gra. 45. min. 0. simul cum angulo. u. d. n. residuo ex recto graduum. 35. minu. 24. & conficies angulum. a. d. u. grad. 107. minu. 2. & habebis propositum, quem tamen oportebat esse gra. 108. min. 0.

Quod autem omnia rectè supputata sint, ex summa omnium angulorum patere potest. nam collectis omnibus quinque angulis. a. c. d. f. u. simul, hoc est grad. 108. minu. 22. cum gra. 107. min. 2. cum grad. 12. efficient grad. 540. min. 0. summa æqualis sex angulis rectis.



ALIA

ALIA DEMONSTRATIO NONÆ, ET DECIMÆ  
secundi Euclidis.

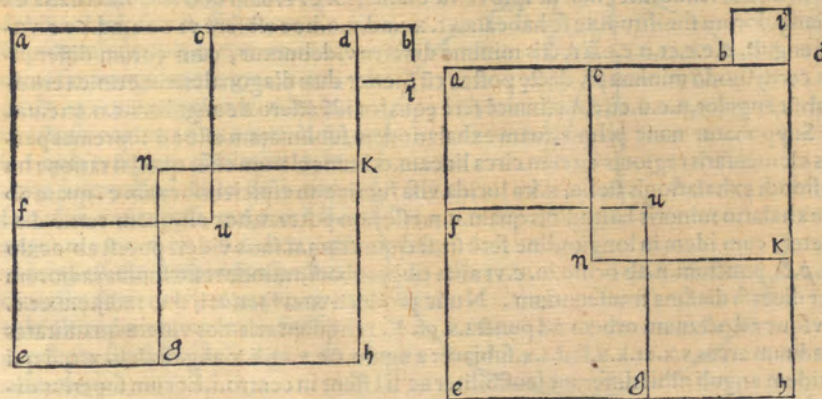
*Petro Catena.*



VAMVIS nona ac decima secundi Euclid. aliter à Comandino & Maurolico demonstratæ fuerint, nihilominus mihi etiam visum est non nihil meo moræ in eas tibi scribere, vt sensibilibus quoque cognoscas illas veras esse.

Esto linea. a. b. pro nona propositione, diuisa per æqualia in. c. per inæqualia verò in. d. quadratum autem. a. d. sit. d. e. quadratum verò. d. b. sit. d. i. quadratum. a. c. sit. c. f. & quadratum. c. d. sit. c. K. clarum enim erit. K. h. æqualem existere ipsi. a. c. secetur igitur. e. h. in. g. ita vt. h. g. equalis existat ipsi. K. h. vnde. g. e. æqualis erit c. d. perficiatur etiam quadratum. h. n. vnde in totali quadrato. a. h. habebis duplū quadrati partis. c. d. nempe. c. K. et. f. g. & quadratum. a. u. cum gnomone. u. g. h. k. cui deficit quadratum æquale. d. i. quadrato, vt sint etiam duo quadrata partis. a. c.

In decima autē propositione, quadratū totalis lineæ. a. d. sit. d. e. & lineæ. b. d. sit. b. i. et c. d. sit. d. n. et. a. c. sit. c. f. et. f. e. sit. e. u. vnde. n. u. æquale erit quadrato. b. i. vnde in quadrato totali. a. h. videbis duo quadrata æqualia. f. c. et. g. k. partis. a. c. & quadratum. c. K. cum gnomone. n. f. e. g. cui addito quadrato. b. i. habebis duplum quadrati partis. c. d.



DE STELLA CASSIOPEIÆ.

*Annibali Raymundo Astrologo Peritissimo.*



OSTQVAM tua doctissima scripta perlegi, consideraui, quod si à multitudi-  
tudi- ne exhalationum in regione elementari acciderit anno. 1572. &  
1573. vt to- tos sex menses ab omnibus per vniuersum terrarum orbem  
vifa fuerit stella illa, quæ est in angulo septentrionali quadrilateri Cassio-  
peiæ

Aaa 2 peiæ

peia tam lucida, vt ipso lucifero videretur rutilantiôr atq; cæterarum ( absque vlla aspectus diuersitate ) magis scintillans. Qui fieri poterat, vt stellæ quæ ab illa partem distant, alioqui multo maiores, non etiam illa clariore apparuerint? sed si aliquis diceret eam exhalationem non ita fortasse dilatari, vt inter nos, & aliam aliquam stellam interponeretur. Tunc ego responderem necessario sequi debere talem exhalationem, tantam latitudinem occupare, quod aliquibus populis aliam aliquam stellam circunvicinâ hac ipsa de qua mentionem fecimus redderet lucidiorem. Sed cum hoc perspectum fuerit nulli, sequebatur lucem illam ab ipsis exhalationibus elementaribus haud posse oriri: quod nobis scintillatio illa maxima permagno fuit inditio, si phas est credere, nâ quo magis aliquid cœleste corpus scintillat, eo longius à nobis distare.

Verum quoniam efflagitasti à me vt aliquid circa huiusce rei speculationem tibi scribam, idcirco tibi morem gerere volens paucis subiungam.

Confidera primo hanc subscriptam primam figuram, in qua. c. a. e. signatur pro Globo terrestri cuius. i. centrum sit et. u. o. n. pro conuexo ignis, sed. K. x. s. pro orbe octauo. x. autem pro stella iam superius dicta, quæ semper fuit, est, & erit, quamuis cæteris tribus nunc obscurior sit. Accipiantur deinde duo loca in superficie terræ, quæ sint. c. et. e. diametraliter inuicem opposita, ita quod circa eorum horizontes possibile sit stellam. x. videre, radijs ipsius stellæ mediantibus. x. n. e. et. x. u. c. quorû partes. n. c. et. u. e. ita breues sint, respectu eorum totorû, vt vix sexcentesima pars sit vna quæq; illarum, nec non. c. e. ita breuis respectu semidiametri octauæ spheræ, quod vix sit vna ex partibus decemmillibus, vt scis, sequitur quod recta terminata ab. u. et. n. minor sensibiliter non sit ipso terræ diametro. c. e. cum duo hæc interualla ex triangulorum similitudine se habeant vt. x. i. ad. x. o. hoc est ferè vt. 602. ad. 601. vnde anguli. n. e. c. et. u. c. e. à rectis minime differre videbuntur, cum eorum differentia certo modo minima sit. ductæ postea cū fuerint duæ diagonales. e. u. et. n. c. terminabunt angulos. n. e. u. et. e. n. c. inuicè ferè æquales, idè assero de angulis. u. c. n. et. e. u. c.

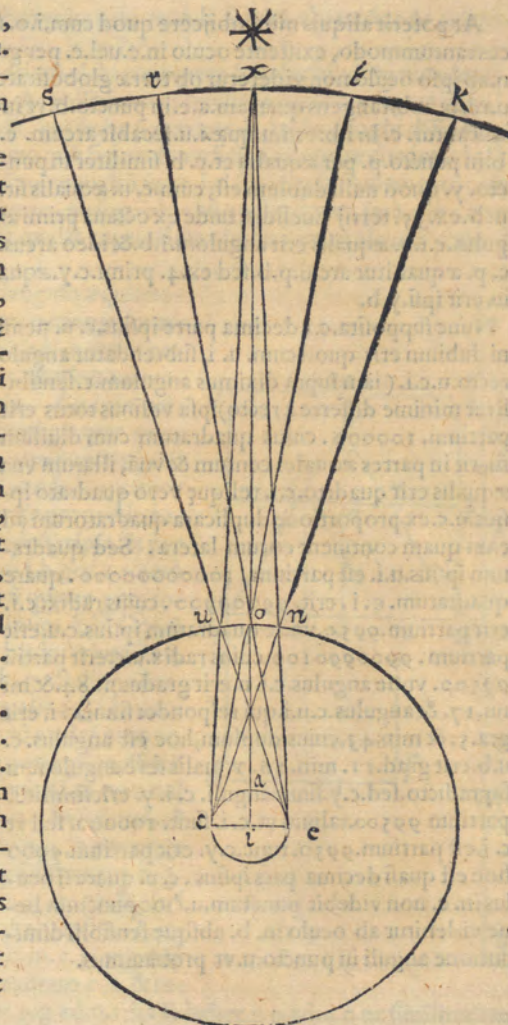
Supponatur nunc primò tuam exhalationem sublimatam esse ad supremas partes elementaris regionis circum circa lineam. o. i. tunc clarum esset quod si ratione huiusmodi exhalationis stella. x. ita lucida visa fuerit tam aspicientibus ab. e. quam ab c. exhalatio minoris latitudinis quam. u. n. esse non poterat, hoc est, quam terræ diameter, cum idem in longitudine ferè sit, sed punctum. u. satis videri potest ab oculo in. e. & punctum. n. ab oculo in. c. vt alias tibi probaui, ratione refractionis radiorum per diuersa diaphana transeuntium. Nunc producti cum fuerint ij duo radij. e. u. et. c. n. vsque ad octauum orbem ad puncta. s. et. K. reliquum erit nos videre quantitates graduum arcus. s. x. et. k. x. sed. s. x. subiacet a ngulo. s. e. x. et. k. x. angulo. k. c. x. qui quidem anguli nihil differunt sensibiliter ac si essent in centro. i. Et cum superius dixerimus angulos. s. e. x. et. k. c. x. sensibiliter minime differre ab angulis. c. n. e. et. e. u. c. si cognouerimus quantitatem istorum, cognita etiam nobis erit quantitas illorum.

Cum igitur semidiameter elementaris regionis maior sit semidiametro terræ, vt 33. ad vnum, & cogitata. c. n. vt dicta semidiameter, quia sensibiliter ab ea minime differt, nunc si supponatur dicta. n. c. vt basis triânguli orthogonij esse partiu. 100000 & dixerimus si. c. n. vt partium. 33. præbet nobis. c. e. duarum partium, quid nobis præstabit eadem. c. n. vt partium. 100000. vnde proueniet nobis. c. e. vt partiu. 6060. cuius angulus. c. n. e. erit graduum. 3. & min. 29. ita etiam erit angulus. k. c. x. cuius ar-

cus

cus.s.x. eorundem graduum erit,  
 & minorum. idem dico de arcu.  
 x.k. Sed circa dictam stellam omnes  
 aliae non distant huiusmodi in  
 teruallo.

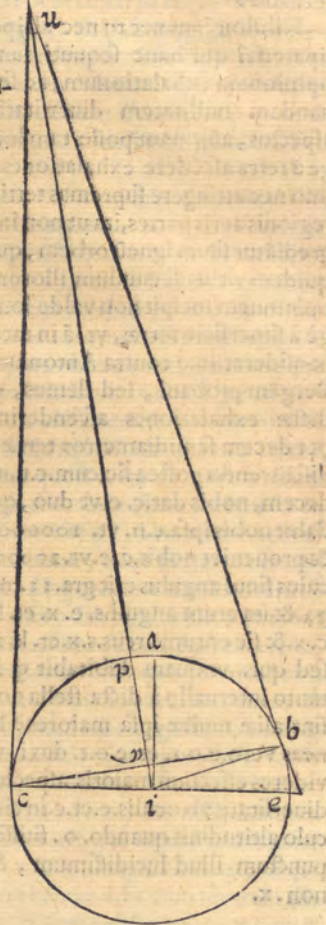
Nihilominus nec tu nec alij pe-  
 ripatetici qui hanc sequuti sunt  
 opinionem exhalationum, ad ser-  
 uandam nullitatem diuersitatis  
 aspectus, affirmant posse tam lon-  
 ge à terra ascēdere exhalationes,  
 imo nec attingere supremas tertie  
 regionis aeris partes, ita ut non in-  
 grediatur suum igneū orbem, qui  
 quidem orbis secundum illorum  
 opinionem incipit non valde lon-  
 ge à superficie terrae, vt iā in mea  
 consideratione contra Antonium  
 Bergam probaui, sed demus, qd  
 dictae exhalationes ascenderint  
 per decem semidiametros terrae,  
 discurrendo postea sic, cum.c.n. ut  
 decem, nobis dat.c. e. vt duo, qd  
 dabit nobis ipsa.c.n. vt. 100000.  
 & proueniet nobis.c. e. vt. 20000.  
 cuius sinus angulus erit gra. 11.mi.  
 32. & ita erunt anguli.s. e. x. et. k.  
 c. x. & sic eorum arcus.s.x.et. k. x.  
 sed quis vnquam dubitabit qd in  
 tanto interuallo à dicta stella non  
 sint aliae multae ipsa maiores? li-  
 neas vero.e.o.r. et. c.o.t. duxi, vt  
 videres effectum maioris aspectus  
 diuersitatis ab oculis.e.et.c. in cir-  
 culo altitudinis quando. o. fuisset  
 punctum illud lucidissimum, &  
 non. x.



Ac

At poterit aliquis mihi obijcere quod cum i.o. fuisset longior i.e. per decem vias tantummodo, existente oculo in e.ucl.c. per gradus 90. ab a. tunc punctus u. vel n. ab ipso oculo non videretur ob terræ globositatem. Imaginemur igitur à puncto u. recta u.b. tangens quartam a.c. in puncto b. vt in secunda figura videre est, in qua ducantur c. b. i. b. et i. u. quæ i. u. secabit arcum. c. b. in puncto p. per æqualia et c. b. similiter in puncto y. quod nulli dubium est, cum c. u. æqualis sit u. b. ex 35. tertij Euclidis, unde ex octava primi angulus c. i. u. æqualis erit angulo u. i. b. & ideo arcus c. p. æquabitur arcui p. b. sed ex 4. primi c. y. æqualis erit ipsi y. b.

Nunc supposita c. i. decima parte ipsius c. u. nemi ni dubium erit quod cum u. i. subtendatur angulo recto u. c. i. ( iam supra diximus angulum c. sensibilibiter minime differre à recto) ipsa vt sinus totus erit partium 100000. cuius quadratum cum diuisum fuerit in partes æquales centum & vnâ, illarum vna æqualis erit quadrato c. i. reliquæ vero quadrato ipsius u. c. ex proportione duplicata quadratorum ad eam quam continent eorum latera. Sed quadratum ipsius u. i. est partium 10000000000. quare quadratum c. i. erit 99009900. cuius radix c. i. erit partium 9950. vnde quadratum ipsius c. u. erit partium 9900990100. cuius radix u. c. erit partium 99500. vnde angulus c. i. u. erit graduum 84. & minu. 17. & angulus c. u. i. qui respondet sinui c. i. erit gra. 5. & minu. 43. cuius duplum, hoc est angulus c. u. b. erit grad. 11. minu. 26. æqualis ferè angulo iam supradicto. sed c. y. sinus anguli c. i. y. erit similiter partium 99500. talium vt c. i. sunt 100000. sed vt c. i. est partium 9950. tunc c. y. erit partium 9900 hoc est quasi decima pars ipsius c. u. quare si oculus in e. non videbit punctum u. hoc punctum bene videbitur ab oculo in b. absque sensibili diminutione anguli in puncto u. vt probauimus.



DE MAGNITVDINIBVS FIGVRARVM  
isoperimetricarum.

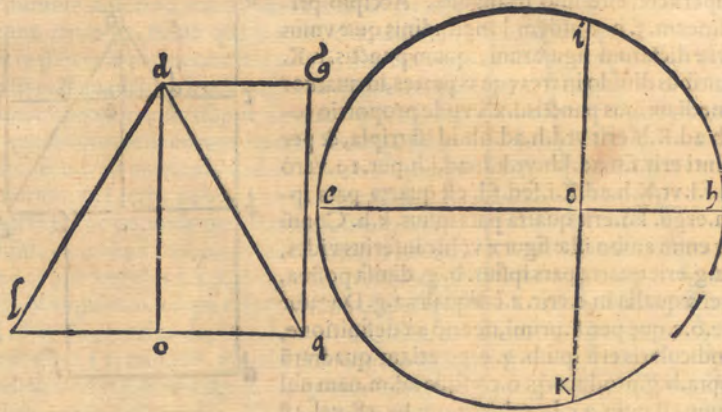
*Domino Ioanni Maria Agatio.*



VAMVIS à Theone supra Ptolomei Almagestum sufficienter traditum sit de magnitudinibus figurarum isoperimetricarum, nihilominus vt tibi morem geram, ea nunc scribo, quæ mihi in mentem venerunt contra Alexandrum Piccolhomineum, antequam aliquid ipsius Theonis vidissem  
Alexan-



Poffumus etiam probare quod periferia quadrati æqualis triangulo æquilatelo minor fit periferia ipsius trianguli æquilateri. Cogita triangulum æquilaterum hic subscriptum. d.l.q. cuius basis. l.q. diuifa fit per æqualia à perpendiculari. d. o. descri- ptūq; fit rectangulum. o.g. quod æquale erit triangulo. d.l.q. sed periferia trianguli maior est periferia rectanguli, nam. l.q. æqualis est. o.g. cum. d.g. sed. q. d. maior est. o. d. ex. 18. primi, vnde. l. d. maior etiam. q.g. cum ex. 34. dicti latera opposita ipsius re- ctanguli sint inuicem æqualia, accipiamus postea. e.c. æqualem. o.d. et. c. h. indire- ctum æqualem. o.g. circa quem diametrum. e.h. intelligatur circulus. e.i.h. k. et. à pun- ctō. c. dirigatur perpendicularis. k.i. ad. e.h. vnde ex. 3. tertij. c.i. æqualis erit. c.k. & ex 34. quod fit ex. c.i. in. c.k. hoc est quadratum ipsius. c.i. æquale erit ei quod fit. ex. e.c. in. c.h. hoc est rectangulo. g.o. hoc est triangulo. d.l.q. sed. e. h. est dimidium perife- riæ ipsius rectanguli. g.o. quæ minor est di midio periferiæ trianguli. d.l.q. vt vidimus et. i. k. est dimidium periferiæ quadrati ipsius. i.c. & minor etiam ipsa. c.h. ex. 14. tertij quare verum est propositum.



Sed quando periferiæ sunt inuicem æquales, poffumus etiam breuiter videre id quod supradiximus, hoc est, quod quadratum, maius fit triangulo æquilatelo. Nam cum. b.g. sesquitertia fit ad. b. a. ergo. b. g. erit vt. 4. et. b. a. ut. 3. vnde. b. q. erit vt. 16 et. b. l. vt. 9. et. c. q. vt. 8. quare. b. l. maius erit ipso rectangulo. c. q. sed. c. q. maius est tri- angulo. b. o. g. cum. q. g. quæ æqualis est. o. g. maior fit. o. c. ex. 18. vel penultima primi, nam si. q. g. æqualis effet. o. c. tunc. c. q. æqualis effet triangulo. b. o. g. ex. 41. primi.

Alia etiam via maiores nostri vsi sunt quæ generalis est vt in Theone supra Al- magestum videre est, medijs perpendicularibus à centris ad latera figurarum, sed quia differētia longitudinum ipsarum perpendicularium alio medio inueniri potest, eo quo ipsi vsi sunt, prætermittere nolo quin tibi scribam.

Ego enim ita discurro.

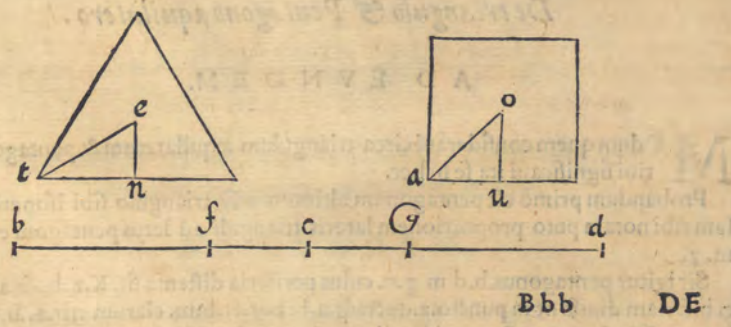
Sint duæ figuræ isoperimetræ æquilateræ & æquiangulæ, puta primò trian- gulum & quadratum quorum centra sint. e. et. o. à quibus centris ad latera sint per- pendiculares. e.n. et. o.u. vnde. n. et. u. diuident latera per æqualia vt scis, ducantur postea. e.t. et. o.a. ad angulos dictorum laterum, vnde habebimus angulum. o.a. u. di- midiu recti, et. c.t.n. tertia pars vnus recti, vt ex te ipso videre potes, quare angulus a. sesqui-



a. sesquialter erit angulo. t. quod vt clarius videas cogita lineam. b. d. cuius medietas sit. c. d. tertia verò pars illius sit. g. d. tunc dico. c. d. sesquialteram esse ipsi. g. d. sit enim f. d. duplum ipsius. g. d. quare. f. d. erunt duæ tertiæ totius lineæ. b. d. & quia eadem proportio est totius. b. d. ad. c. d. quæ. f. d. ad. g. d. ergo permutando eadem erit totius. b. d. ad. f. d. quæ. c. d. ad. g. d. Sed. b. d. ad. f. d. sesquialtera est, verum igitur erit quod angulus. a. sesquialter sit ipsi. t. deinde. t. n. est sesquitercia ipsi. a. u. vt superius vidimus. in eorum duplis. scimus etiam. n. e. esse dimidium ipsius. t. e. eo quod cum. e. t. n. sit tertia pars vnus recti, angulus. t. e. n. erit duo tertia vnus recti, vnde. e. n. erit latus. exagoni æquilateris inscripibilis circulo cuius diameter sit. e. t. quare. e. t. dupla erit ipsi. e. n. in longitudine, sed quadrupla in potentia: t. n. vero tripla in potentia ipsi. n. e. ex penultima primi, quæ omnia etiam ex. 8. tertijdecimi. Eucli. elicere potes, sed t. n. erat sesquitercia ipsi. a. u. in longitudine, hoc est ipsi. o. u. nam. o. u. æqualis est ipsi a. u. quare. n. t. erit minus quam dupla in potentia ipsi. o. u. hoc est, vt. 16. ad. 9. ergo maior proportio erit ipsius. t. n. in potentia ad. n. e. quam ad. o. u. quare etiam in longitudine, maior proportio erit ipsius. t. n. ad. n. e. quam ad. o. u. vnde. o. u. longior erit ipsa. n. e. quod est propositum.

Sed si. o. a. u. esset pentagonus æquilaterus & æquiangulus, similiter probabo perpendicularem. o. u. longiorem esse. n. e. ipsius trianguli æquilateri, dummodo sint isoperimetre. Sit enim. a. u. dimidium lateris pentagoni ex supposito, cuius centrum sit o. tunc proportio. t. n. ad. a. u. erit superbipartientertias, vt ex ordine iam hic supradicto à te facillimè elicere potes, hoc est, vt. 5. ad. 3. et. a. u. minor erit. o. u. eo quod angulus. o. minor erit angulo. a. nam angulus. o. erit quinta pars duorum rectorum, hoc est duarum quintarum vnus recti, vnde angulus. a. residuum vnus recti erit tres quintæ vnus recti, quare angulus. a. maior erit angulo. o. & consequenter latus. o. u. maius latere. a. u. sed. t. n. minor est quam tripla in potentia ad. a. u. eo quod erit vt. 25. ad. 9. cum in longitudine sit vt. 5. ad. 3. sed dicta. t. n. tripla est in potentia ad. e. n. quare. a. u. maior erit ipsa. e. n. sed. o. u. maior est ipsa. a. u. vt diximus, igitur multo magis. o. u. maior est ipsa. a. u. vt dixim⁹ & cōsequēter multo magis. o. u. maior erit ipsa. n. e.

Quotiescunque enim cognoscimus proportionem anguli. o. ad angulum. a. quod quidem facillimum est, nec non proportionem. t. n. ad. a. u. quod, etiam illico cognoscitur, tunc ex scientia cordarum & arcuum omnia etiam facillimè innueniuntur. Verum circa triangulū æquilaterum, & pentagonum, alium modū inueni, sed aliquantulum prolixiorē.

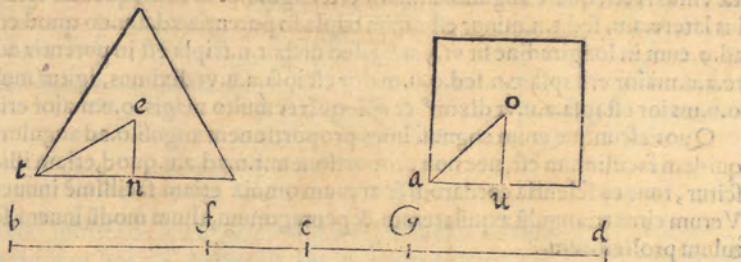


*De incommensurabilitate, in longitudine perpendicularis trianguli æquilateri cum eiusdem latere.*

A D E V N D E M.

**I**D quod à me postulas est omnino impossibile, velles enim duos numeros inuenire inter se ita se habentes, vt se habent perpendicularis in triangulo æquilatero cum vno eius laterum, quod vero hoc fieri non possit, considera in figura præcedenti triangulum æquilaterum. d. l. q. cuius perpendicularis sit. d. o. quæ diuidit. l. q. per æqualia in. o. vnde ex. 4. secundi Euclidis, quadratum. l. q. ( idest. d. q. ) quadruplum erit quadrato. o. q. & ex penultima primi æquale quadratis. d. o. et. o. q. quare erit sesquitergium quadrato ipsius. d. o. & ita quadratum. d. o. erit triplum quadrato ipsius. o. q. hæc autem proportioniones non sunt vt numeri quadrati ad numerum quadratum quod si ita fuissent, sequeretur ternarium numerum esse quadratum ex. 22. octaui. Cum igitur non sint vt numeri quadrati ad numerum quadratum, sequitur ex septima decimi. d. o. esse incommensurabilem ipsi. l. q. seu. d. q. in longitudine.

Vel dicamus ita, proportio quadrati ipsius. l. q. ad quadratum ipsius. o. d. est in genere superparticulari; cum sit sesquitergia, vnde quadratum ipsius. d. o. numeris dari non potest, eo quod si dabilis fuisset, sequeretur, quod inter quadratum ipsius. l. q. & ipsius. d. o. esset aliquis numerus medius proportionalis ex. 16. octaui, vnde ex octaua eiusdem vnitas diuisibilis esset, quod fieri non potest.



*De triangulo & Pentagono æquilatero.*

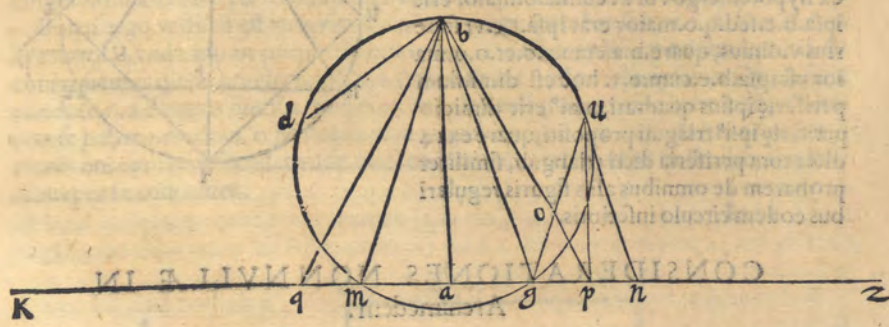
A D E V N D E M.

**M**odum quem consideraui circa triangulum æquilaterum & pentagonum, vt tibi significaui ita se habet.

Probandum primò est pentagonum altiore esse triangulo sibi isoperimetro. Iam tibi notam puto proportionem lateris trianguli ad latus pentagoni esse vt. 5. ad. 3.

Sit igitur pentagonus. b. d. m. g. v. cuius periferia distenta sit. K. z. basis autem. m. g. bifariam diuisa sit in puncto. a. ductaq; a. b: b. g. et. b. m. clarum erit. a. b. perdicularem esse ad. m. g. ex. 8. primi Eucli. cum. b. m. et. b. g. (bases triangulorum. b. d. m. et. b. u.

et. b. i. g. ) sint inuicem æquales ex. 4. eiusdem. *Proq. quoniam ab inuicem circoson*  
 Accipiat deinde vel intelligatur. g. p. æqualis duabus tertijs ipsius. a. g. ducatur  
 que. b. p. quam probabo maiorem esse duplo ipsius. a. p. vnde maior erit latere ipsius  
 trigoni æquilateris, cuius dimidium est. a. p. scimus enim ipsum latus se habere ad. m.  
 g. vt quinque ad. 3. ita etiam. a. p. ad. a. g. vt diximus. *ita b. p. d. latus trigoni*  
 Cum autē angulus. a. b. g. sit quarta pars anguli. b. g. a. ex. 10. quarti & quinta pars  
 vnus recti ex. 32. primi, dictus angulus erit graduum. 18. et. a. g. erit partium. 30902.  
 et. a. b. partium. 95015. et. a. p. 51503. vnde ex penultima primi latus. b. p. erit par-  
 tium. 108075. duplum vero ipsius. a. p. erit. 103006. latus igitur dicti trigoni, quod  
 ab. p. erigitur, secabit perpendicularē. a. b. sub. b. hoc est inter. b. et. a. ex penultima  
 primi. Finitur enim triangulus æquicrurus. b. q. p. quem probaui maiorem esse æ-  
 quilatero isoperimetro pentagono proposito, ducatur q. u. p. ducatur etiam. u. n. pā-  
 rallela ipsi. b. g. quæ concludet triangulum. g. u. n. similem triangulo. m. b. g. eo quod  
 cum angulus. m. b. g. æqualis sit angulo. b. g. u. ex. 16. tertij, per. 27. primi. m. b. et. g. u.  
 erunt inuicem æquidistātes, vnde angulus. b. m. g. æqualis erit angulo. u. g. n. et. ex. 29.  
 angulus. g. u. n. æqualis erit angulo. u. g. b. quare etiam angulo. g. b. m. & angulus. u. n.  
 g. angulo. b. g. m. ex. 31. eiusdem, vnde ex. 4. sexti proportio. g. n. ad. g. m. erit. vt. g. u.  
 ad. m. b. sed cum. g. u. maior sit dimidio ipsius. b. g. ex. 20. primi, hoc est maior dimi-  
 dio ipsius. b. m. ergo. g. n. etiam maior erit ipsa. g. a. quapropter maior erit ipsa. g. p.  
 cum. g. p. minor sit ipsa. g. a. ex hypothesi, ducta deinde cum fuerit. b. n. habebimus  
 triangulum. b. n. g. æqualē triangulo. b. u. g. & maiore triangulo. b. p. g. ex prima sexti  
 vel quia totum maius est sua parte. Triangulus igitur. b. u. g. maior est triangu-  
 lo. b. p. g. quare triangulus. b. u. o. maior erit triangulo. g. o. p. ex communi concepu,  
 idem infero ab alia parte dictarum figurarum. Quare pentagonus. b. d. m. g. u. maior  
 erit triangulo. b. q. p. quem probauimus maiorem esse triangulo æquilatero sibi iso-  
 perimetro.



*Comparatio periferiarum quadrati & trianguli æquilateri  
 circumscriptorum ab eodem circulo.*

AD EUNDEM.

**Q**uod autem periferia quadrati in eodem circulo inscripti, in quo sit triangu-  
 lus æquilaterus, longior sit periferia ipsius trianguli æquilateri, absque vilo  
 Bbb 2 negotio

negotio cordarum & arcuum possumus geometricè demonstrare quod valde desideras.

Quapropter sit circulus.  $b.a.e.q.$  in quo sit triangulū æquilāterum.  $b.e.n.$  & quadratum.  $b.a.q.u.$  cuius periferiam probabo longiorem esse periferia trianguli. Sit enim diameter circuli.  $b.q.$  qui etiam erit diameter quadrati; ut à te scire potes. Sit etiam punctū.  $b.$  commune tam anguli quadrati quam trianguli. vnde sequitur quod dictus diameter secabit latus.  $n.e.$  trianguli ad rectos & per æqualia in  $t.$  Nam cum arcus.  $b.e.$  æqualis sit arcui.  $b.n.$  ex. 27. tertij; remanet ut arcus.  $q.e.$  æqualis sit arcui.  $q.h.$  vnde angulus.  $q.b.e.$  æqualis erit angulo.  $q.b.n.$  ex. 26. eiusdem. quare ex. 4. primi anguli ad  $t.$  erunt recti, et.  $n.t.$  æqualis erit ipsi.  $t.e.$  ut diximus.

Deinde.  $b.e.$  et.  $q.a.$  se inuicem secant in puncto.  $o.$  ut ex se clarum patet; ducatur postea.  $q.e.$  vnde habebimus angulum.  $b.e.q.$  rectum ex. 30. tertij; quare ex. 18. primi.  $q.o.$  longior erit ipsa.  $q.e.$  et.  $q.e.$  longior erit ipsa.  $e.t.$  quare.  $q.o.$  longior erit ipsa.  $t.e.$

Ut probemus postea.  $b.a.o.$  longiorem esse ipsa.  $b.e.$  & producatui.  $b.a.$  ita quod.  $a.p.$  æqualis sit ipsi.  $a.o.$  ducaturq;  $o.p.$  et.  $a.e.$  cum autem etiam dicta. 30. tertij angulus.  $b.a.o.$  rectus sit; erit angulus.  $o.a.p.$  similiter rectus ex. 13. primi, vnde ex. 5. et. 32. eiusdem angulus.  $a.p.o.$  erit dimidium recti, & similiter, ex. 11. eiusdem; angulus.  $b.q.a.$  est dimidium recti quare angulus.  $a.p.o.$  æqualis erit angulo.  $a.q.b.$  sed angulus.  $a.e.b.$  æqualis est angulo.  $a.q.b.$  ex. 20. tertij, ergo angulus.  $b.p.o.$  æqualis erit angulo.  $b.e.a.$  angulus vero  $a.b.e.$  communis est ambobus triangulis.  $a.b.e.$  et.  $o.b.p.$  quare ex. 32. primi anguli.

$b.a.e.$  et.  $b.o.p.$  reliqui ex duobus rectis æquales inuicem erunt. Quare ex quarta sexti, et. 18. quinti proportio.  $b.o.ad.b.p.$  erit, ut  $b.a.ad.b.e.$  sed ex. 18. primi.  $b.o.$  maior est ipsa.  $b.a.$  quare ex. 14. quinti.  $b.p.$  maior erit ipsa.  $b.e.$  sed.  $b.p.$  æquatur ipsis.  $b.a.$  cum.  $a.o.$  ex hypotefi, ergo.  $b.a.$  cum.  $a.o.$  maior erit ipsa.  $b.e.$  sed.  $q.o.$  maior erat ipsa.  $t.e.$  ut superius vidimus, quare.  $b.a.$  cum.  $a.o.$  et.  $o.q.$  maior est ipsa.  $b.e.$  cum.  $e.t.$  hoc est dimidium periferie ipsius quadrati, maior erit dimidio periferie ipsius trianguli propositi, quare ex. 14. dicta tota periferia dicti trianguli, similiter probarem de omnibus alijs figuris regularibus eodem circulo inscriptis.



CONSIDERATIONES NONNVLLE IN  
Archimedem.

Doctissimo atque Reuerendo Domino Vincentio  
Mercato.

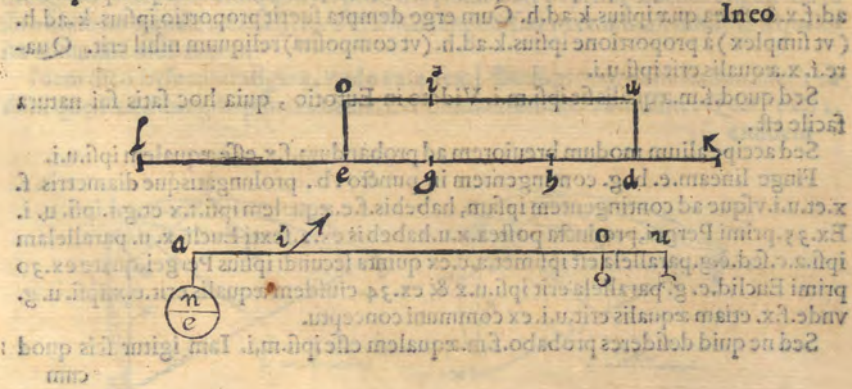
**Q**UOD tibi aliàs dixi verum est. intellectum scilicet non omnino quiescere circa illas duas Archimedidis propositiones, quæ in translatione Tartaleæ sunt sub numeris. 4. et. 5. & in impressione Basileæ sub numeris. 6. et. 7. vbi tractat

tractat de centrīs libræ, seu statera: Aspice igitur in 4. supradicta, quod cum appen-  
 sa fuerint omnes illæ partes ponderum, partibus longitudinis ipsius. l. k. in qua volo  
 ut à punctis. e. et. d. imagineris duas lineas. e. o. et. d. u. inuicem æquales, & ferè per-  
 pendiculares ipsi. l. k. hoc est respicientes mundi centrum; imagineris etiam. o. u.  
 quæ sit parallela ipsi. l. k. quæ diuisa sit in puncto. i. supra. g. Hinc nulli dubium erit;  
 cum. g. fuerit centrum totius ponderis appensi ipsi. l. k. quod. i. similiter erit centrum  
 cum directe locatum sit supra. g. hoc est in eadem directionis linea; quod quidem  
 non indiget aliqua demonstratione; cum per se satis pateat. Vnde ex communi  
 conceptu. o. erit centrum ponderis appensi ipsi. l. h. et. u. erit centrum ponderis ap-  
 pensi ipsi. h. k. Scimus igit. i. esse cætrum duorum; hoc est ipsius. l. h. & ipsius. h. k. con-  
 tinuatorum per totam. l. k. Nunc ergo si consideremus. l. k. diuisam esse; hoc est di-  
 uisam in puncto. h. inuenietur nihilominus. i. centrum esse dictorum ponderum;  
 & quod tantum est, ipsam esse continuam; quantum diuisam in dicto puncto. h. neque  
 ex hoc; punctum. i. erit magis vel minus centrum duorum ponderum. l. h. et. h. k. quo-  
 rum unum pendet totum ab. o. aliud verò totum ab. u. & hoc modo in longitudine;  
 o. u. diuisa ut dictum est, habebimus propositum.

Reliquam propositionem tibi relinquo. Illa verò propositio, quam tibi dixi Archimedes tacuisse in huiusmodi materia  
 est, quod si duo pondera æquilibrant ab extremis alicuius statera; in certis præfixis  
 distantijs à centro. Tunc dico si eorum vno manente alterum moueatur remotius  
 ab ipso centro quod illud descendet, & si vicinius ipsi centro appensum fuerit ascen-  
 det. Hæc enim propositio quotidie omnibus in locis videtur, ipsam verò puto Ar-  
 chimedes prætermisisse ob facilitatem, cum ab antedicta ferè dependeat.

Sit exempli gratia statera. a. u. cuius centrum sit. i. & pondera. u. a. appensa, seu  
 uicem habeant ut. i. u. et. i. a. se inuicem habent. Nunc dico quod si pondus ipsius. u.  
 positum fuerit vicinius centro ut puta in. o. in moto existente pondere. a. quod bra-  
 chium. i. o. u. ascendet, & è conuerso, si remotius positum fuerit; descendet.

Ponatur ergo ut dictum est in. o. vicinius cætro, quapropter brachium. i. o. breuius erit  
 brachio. i. u. unde minor proportio erit ipsius. i. o. ad. i. a. quam. i. u. ad eundem. a. i. &  
 consequenter quam ponderis ipsius. a. (quod sit. n. e.) ad pondus ipsius. u. Quare si ex  
 pondere. n. e. dempta fuerit. e. pars eius, ita quod reliqua pars. n. e. habeat ad pondus  
 o. ut se habet. i. o. ad. i. a. tunc statera non mouebitur; addita verò parte. e. ex com-  
 muni conceptu, a. descendet unde. o. ascenderet conuersum verò ex similibus ratio-  
 nibus per te concludes.



In eo quod à me petis, mittendo te ad Eutotium, tibi non satisfacerem, cum Eutotius citet sextum librum Pergei, quem nunquam vidimus, supponatq; ea, quæ nec ipse nec alius vnquam quod scimus probauit.

Desideras enim demonstrationem illius quod Archimedes dicit inter primam, & secundam propositionem secundi libri, vbi tractat de centrīs grauium, propterea quod illud supponit pro manifesto.

Sit enim figura hic subscripta, ferè similis parabolæ positæ in. 2. propositione dicti libri, vt in impressione Basileensi habetur, sintq; diuisæ duæ. a. b. et. b. c. per æqualia à punctis. x. et. u. protractisq; f. x. et. u. i. ad. b. d. quæ inuicem etiam erunt parallelæ ex. 30. primi Eucli. vnde ipsæ etiam, diametri erunt ipsarum portionum: vt ex eo colligere est, quod in. 49. primi lib. Pergei probatur. Imaginando postea ad puncta. b. f. et. i. tres contingentes, manifestum erit punctum. b. illud esse quod terminat altitudinem huiusmodi portionis, et. f. et. i. terminantia altitudines partialium, ex. 5. secundi ipsius Pergei, eo quod dictæ contingentes parallelæ erunt ipsis basibus, vnde trianguli inscripti, easdem habebunt altitudines, quas portiones ipsæ, quod erit ex mente Archimedis. Et sic deinceps poteris multiplicare angulos figuræ rectilineæ in parabola, quæ designata erit vt desiderat Archimedes, qui quidem dicit, quod protractæ cum fuerint aliæ deinceps post. f. i. ipsæ inuicem equidistantes erūt, diuisæque per æqualia ab. d. b. quod quæuis verū sit, tñ ab Eutotio non satis demonstratū est, cum supponat. a. f. b. æqualem esse ipsi. b. sic probare volens eius diametros æquales esse absque aliqua citata ratione, quæ quidem ratio esset conuersum. 4. propositionis libri de conoidalibus. Sed oporteret nos etiā videre. 6. librum ipsius Pergei, & propterea tibi non satisfacerem.

Esto igitur, ut inuenta sit linea. K. cuius productum in. u. i. æquale sit quadrato ipsius. u. c. inuenta etiam sit linea. h. cuius productum cum. f. x. æquale sit quadrato ipsius. a. x. vnde ex conuerso. 49. primi ipsius Pergei, proportio ipsius. K. ad. b. c. erit vt ipsius. b. c. ad. b. d. & ipsius. h. ad. a. b. vt ipsius. a. b. ad. b. d. Erit igitur ex. 16. sexti Eucli quadratum. b. c. æquale producto ipsius. K. in. b. d. & quadratum. a. b. æquale producto ipsius. h. in. b. d. & ex prima sexti, ita erit ipsius. K. ad. h. vt producti quod sit ex. K. i. in. b. d. ad productum ipsius. h. in. b. d. hoc est vt quadrati ipsius. b. c. ad quadratum ipsius. b. a. ex. 16. et. 11. quinti, hoc est vt quadrati ipsius. u. c. ad quadratum ipsius. a. x. hoc est ut productum ipsius. k. in. u. i. ad productum ipsius. h. in. x. f. Nunc si ipsius. k. ad. h. est vt producti ipsius. K. in. u. i. ad productum ipsius. h. in. x. f. ergo ex. 24. sexti, & communi conceptu, proportio ipsius. k. ad. h. composita erit ex ea quæ ipsius. u. i. ad. f. x. & ex ea quæ ipsius. k. ad. h. Cum ergo dempta fuerit proportio ipsius. k. ad. h. (vt simplex) à proportione ipsius. k. ad. h. (vt composita) reliquum nihil erit. Quare. f. x. æqualis erit ipsi. u. i.

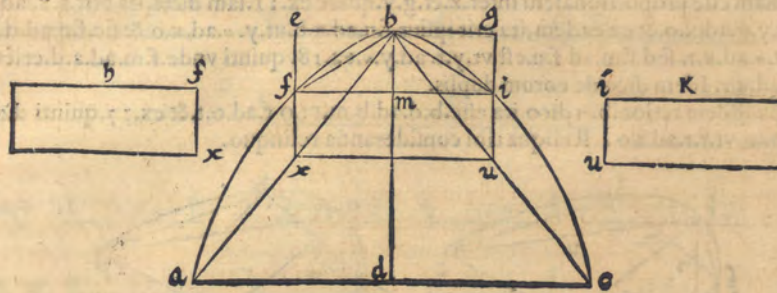
Sed quod. f. m. æqualis sit ipsi. m. i. Videto in Eutotio, quia hoc satis sui natura facile est.

Sed accipe alium modum breuiorem ad probandum. f. x. esse æqualem ipsi. u. i.

Finge lineam. e. b. g. contingentem in puncto. b. prolongatisque diametris. f. x. et. u. i. vsque ad contingentem ipsam, habebis. f. e. æqualem ipsi. f. x. et. g. i. ipsi. u. i. Ex. 35. primi Pergei, producta postea. x. u. habebis ex. 2. sexti Eucli, x. u. parallelam ipsi. a. c. sed. e. g. parallela est ipsi. m. e. a. c. ex quinta secundi ipsius Pergei, quare ex. 30. primi Euclid. e. g. parallela erit ipsi. u. x. & ex. 34. eiusdem æqualis erit. e. x. ipsi. u. g. vnde. f. x. etiam æqualis erit. u. i. ex communi conceptu.

Sed ne quid desideres probabo. f. m. æqualem esse ipsi. m. i. Iam igitur scis quod cum

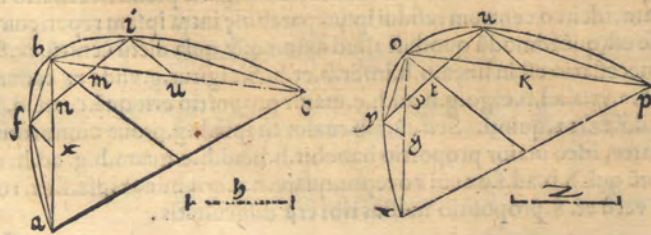
cum sit. f.x. æqualis ipsi. u.i. vt tibi probaui, & inuicem parallelæ ideo. f. i. parallela erit ipsi. x.u. ex. 33. primi Euclidis. Vnde ex. 30. eiusdem, parallela erit etiam ipsi. a. c. sed cum. x.u. diuisa sit ab. d.b. per æqualia, eo quod diuidit. a.c. eodem modo, que ipsi parallela est ex. 2. sexti. Reliqua tibi considerata relinquo. cum verò ambæ. f. x. et. u. i. parallelæ sint ipsi. b. d. sequitur quod cum ex. 34. primi vnaquæq;. f. m. et. m. i. æqualis sit medietati ipsius. x. u. erunt inuicem æquales.



Minime dubitabam tibi non satisfacere Eutocium in. 3. propositione secundi lib. de centrīs Grauium Archimedis, cum citet. 6. librum de elementis conicis, adde quod si aliud in ipso. 6. libro ab eo citato non esset magis ad propositum, quàm ea quæ ab ipso citata sunt, nihilominus adhuc irresolutus maneres.

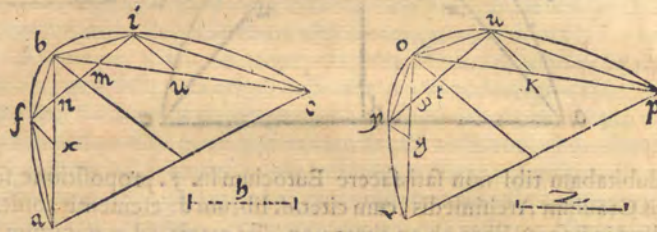
Considera igitur eandem ipsam figuram præcedentem; pro alia verò parabola si mihi dictæ, accipe secundam figuram ipsius tertiæ dictæ propositionis. Deinde imaginabis duo latera. o. x. et. o. p. diuisa esse per æqualia in punctis. g. et. K. protractisq; diametris. g. y. et. K. u. quæ, vt in præcedenti probaui, sunt inuicem æquales, scire debes quod similes parabolæ inuicem aliæ non possunt esse, nisi eæ quæ diametros proportionales suis basibus habeant, similiterq; positæ, hoc est, vt proportio ipsius b. d. ad. a. c. sit eadem quæ ipsius. o. r. ad. x. p. & quod anguli ad. r. sint æquales angulis circa. d. Notentur ergo primum puncta communia ipsius. o. g. cum. y. t. & ipsius. b. x. cum. f. m. characteribus. o. et. n. Nunc igitur scimus. f. m. æqualem esse. m. i. tota m. q;. f. i. parallelam esse ipsi. a. c. Idem dico de. y. t. u. trianguli q;. x. f. n. et. g. y. o. esse similes triangulis. n. m. b. et. o. t. o. quod ita probatur, nam ex. 15. primi Euclid. anguli ad. n. sunt inuicem æquales, ex. 29. verò eiusdem anguli. f. x. n. et. n. b. m. similiter æquales ita etiam. n. f. x. et. n. m. b.

Idem dico in secunda figura, vnde ex. 4. sexti Eucli. proportio. n. f. ad. m. n. erit eadem quæ. f. x. ad. b. m. & ipsius. n. f. ad. x. f. vt. n. m. ad. m. b. ex. 16. quinti. Quare ex. 11. eiusdem



eiufdem erit vt. a.d.ad.d.b. Idem etiam dico in fecunda parabola, fed ipfius. x.o.ad o.r. eft vt.a.b. ad. b. d. ex. 6. fepti Eucli. vnde ex. 11. quinti. n.f.ad.f.x. erit vt. o. y. ad.y.g. Sed in precedenti iam tibi dixi.a.b. mediam proportionalem effe inter. h. et.b.d. Sit nunc.z. pro fecunda parabola, ita ut.h. eft pro prima, vnde.o.x. erit media proportionalis inter.z.et.o.r.& ex. 11. quinti ita erit.h.ad.a.b.vt.z.ad.x.o.& ex. 22. h.ad.a.x.ut.z.ad.x.g.& quia ex. 16. fepti. a.x. media proportionalis eft inter.h.et.f. x. cum fupponatur productum.h.in.f.x. æquale effe quadrato. a. x. Idem dico. x. g. mediam effe proportionalem inter.z.et.g.y. quare ex. 11. iam dicta, ita erit.a.x. ad.f. x.vt.y.g.ad.x.o.& ex eadem, ita erit ipfius.f.n.ad.a.b.ut.y.o.ad.x.o.& fic.f.n.ad.d.a. vt.y.o.ad.x.r. fed.f.m.ad.f.n. eft vt.y.r. ad.y.o. ex. 18. quinti vnde.f.m.ad.a.d. erit vt.y.t.ad.x.r. Idem dico de eorum duplis.

Ex ijsdem rationibus dico ita effe.b.d.ad.b.m.vt.o.r.ad.o.t.& ex. 17. quinti. d.m. ad.b.m.vt.r.t.ad.t.o. Reliqua tibi confideranda relinquo.



In reliquis verò propofitionibus illius lib. nullo pacto poteris dubitare: Verum ne in. 4. aliquid tibi noui exurgat, te fcire volo corollarium. 20. in libr. de quadratura parabolæ docere possibile effe infcriptionem rectilineæ, ea tamen conditione quæ dicit Archimedes.

In quinta poftea animaduertendum eft, quod prima pars, probat tantummodo de centro trianguli, et. 2. pars probat de centro pentagoni, à te ipfo deinde potes probare de centro nonanguli: & fic de cæteris: eo quod cum probatum fuerit de centro figuræ in medio locatæ fi conftitutæ poftea fuerint fimiles figuræ in portionibus lateralibus habebitur propofitum in infinitum.

Idem intelligendum eft in. 3. propofitione quamuis exemplum vltterius non extendatur quam ad pentagonos.

Sexta verò propofitio tibi facilis erit, quæ nihilominus pot demoftrari hoc modo fcilicet. Sint. 4. quætitates. a. b. c. d. ipfius Archimedis fupponedo. a. pro figura rectilinea infcripta in parabola, et. b. pro refiduo ipfius parabolæ et. c. pro triangulo. a. b. c. in medio ipfius parabolæ et. d. pro triangulo. r. Nunc cum. a. maior fit. c. prout totum maior eft fua parte, ideo ex. 8. quinti maior proportio habebit. a. ad. b. quam. c. ad. b. Cum autem. b. minor fit. d. ex fupposito, ideo ex eadem dicta, maior proportio habebit. a. ad. b. quam. c. ad. d. cum verò centrum cuiufuis figuræ plenæ neceffariò fit intra ipfam figuram, idcirco centrum refidui ipfius parabolæ intra ipfam reperietur. quo d ita clarè p fe eft, quæ admodum quoduis aliud axioma, & quia dictum centrū, ex. 8. primi de centrīs, neceffariò eft in linea. b. h. inter. b. et. h. Sit igitur. g. vnde ex eadem. 8. ita erit. g. h. ad. h. e. vt. a. ad. b. ergo. g. h. ad. h. e. maior proportio erit quæ. c. ad. d. hoc eft quam. b. h. ad. f. ex. 12. quinti. Sed cum. h. b. maior fit ipfa. h. g. prout omne totum maior eft fua parte, ideo maior proportio habebit. h. b. ad. h. e. quam. h. g. ad. h. e. vnde multo maiorè quæ. h. b. ad. f. ex cõi cõceptu, quare. h. e. erit minor ipfa. f. ex. 10. quinti.

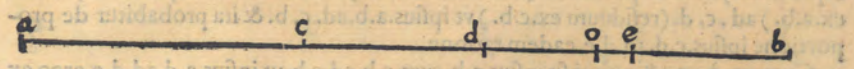
Septima verò et. 8. propofitio nullius tibi erit difficultatis.

Quam-



Quamvis Eutotius scribat super duas ultimas lib. secundi de centrīs grauiū, nihil miror ipsum tibi non satisfacere. Accipe igitur quod ego nunc tibi mitto.

Archimedes eo in loco primū supponit in penultima dicti libri quatuor lineas proportionales. a. b: c. b: d. b: et. e. b: supponit etiam quod proportio quæ est ipsius. e. b. ad. e. a. eadē sit quæ ipsius. f. g. ad tres quintas ipsius. a. d. & quod proportio compositi dupli ipsius. a. b. cum quadruplo ipsius. b. c. cum sexcuplo ipsius. b. d. cum triplo



**V**  $\frac{a \cdot 5}{b} = \frac{b \cdot 10}{c} = \frac{c \cdot 10}{d} = \frac{d \cdot 5}{e}$

**AB**  $\frac{a \cdot 12}{b} = \frac{b \cdot 4}{c} = \frac{c \cdot 6}{d} = \frac{d \cdot 3}{e}$

**A**  $\frac{a \cdot 12}{b} = \frac{b \cdot 4}{c} = \frac{c \cdot 4}{d} = \frac{d \cdot 2}{e}$

**B**  $\frac{b \cdot 2}{c} = \frac{c \cdot 1}{d} = \frac{d \cdot 1}{e}$

**D**  $\frac{a \cdot 12}{b} = \frac{b \cdot 3}{c} = \frac{c \cdot 1}{d} = \frac{d \cdot 1}{e}$

**E**  $\frac{b \cdot 1}{c} = \frac{c \cdot 3}{d} = \frac{d \cdot 2}{e}$

**H**  $\frac{a \cdot 11}{c} = \frac{c \cdot 3}{d} = \frac{d \cdot 2}{e}$

**HA**  $\frac{a \cdot 3}{b} = \frac{b \cdot 6}{c} = \frac{c \cdot 3}{d} = \frac{d \cdot 1}{e}$

**M**  $\frac{b \cdot 11}{c} = \frac{c \cdot 1}{d} = \frac{d \cdot 1}{e}$

**N**  $\frac{a \cdot 11}{b} = \frac{b \cdot 12}{c} = \frac{c \cdot 11}{d} = \frac{d \cdot 1}{e}$

**Q**  $\frac{3}{5} = \frac{b \cdot 2}{c} = \frac{c \cdot 4}{d} = \frac{d \cdot 12}{e}$

**A**  $\frac{b \cdot 2}{c} = \frac{c \cdot 4}{d} = \frac{d \cdot 4}{e} = \frac{e \cdot 12}{a}$

**HA**  $\frac{b \cdot 3}{c} = \frac{c \cdot 6}{d} = \frac{d \cdot 3}{e} = \frac{e \cdot 12}{a}$

Ccc      ipsius

ipsius. b. e. ad compositum quintupli ipsius. a. b. cum decuplo ipsius. c. b. cum decuplo ipsius. b. d. cum quintuplo ipsius. b. e. eadem sit quæ ipsius. g. h. ad. a. d. & vult probare. f. h. esse duas quintas ipsius. a. b.

Cum autem dicit proportionem ipsius. a. c. ad. c. d. & ipsius. c. d. ad. d. e. esse vt ipsius a. b. ad. b. c. & cetera, verum dicit ex. 19. quinti Eucli. eo quod cum ex hypothesi sit ipsius. a. b. totalis ad. c. b. totalem vt ipsius. c. b. partialis (sumpta vt pars abscisa ab. a. b. pro nunc) ad. d. b. partialem (abscisam ab. c. b.) erit ex. 19. dicta ipsius. a. c. (residui ex. a. b.) ad. c. d. (residuum ex. c. b.) vt ipsius. a. b. ad. c. b. & ita probabitur de proportionem ipsius. c. d. ad. d. e. eadem ratione.

Cum verò ex. 18. quinti sit ipsius. a. b. cum. c. b. ad. c. b. vt ipsius. a. d. ad. d. e. ergo ex 22. eiusdem, ita erit ipsius. a. b. cum. c. b. ad. d. b. vt. a. d. ad. d. e. & ex iisdem rationibus eadem proportio erit ipsius. c. b. cum. d. b. ad. b. e. vt. a. d. ad. d. e. quod inquit Archi. Verum etiam erit (ex. 13. quinti) cum dicit eandem proportionem esse ipsius. a. d. ad. d. e. quæ dupli primi antecedentis cum simplo secundi consequentis ad duplum primi consequentis cum simplo secundi consequentis, hoc est dupli ipsius. a. b. c. cū simplo. c. b. d. ad duplum ipsius. d. b. cum simplo. e. b. hoc est dupli. a. b. cum triplo ipsius. b. c. cum simplo. d. b. ad duplum ipsius. d. b. cum simplo. e. b. Nunc duplum. a. b. cum triplo. b. e. cum simplo. b. d. signatum sit caractere. D. suum verò consequens, hoc est duplum. d. b. cū simplo. e. b. significetur à caractere. B. hinc proportio ipsius a. d. ad. d. e. erit vt. D. ad. B.

Inquit nunc Archimedes, si quis sumeret aliquod maius antecedens æquale scilicet duplo ipsius. a. b. cum quadruplo ipsius. b. e. cum quadruplo ipsius. b. d. cum duplo ipsius. b. e. compararetq; illud cum cōsequente. B. clarum esset ex. 8. quinti quod tale antecedens maiorem proportionem haberet ad. B. quam ad. D. hoc est maiorem quàm ipsius. a. d. ad. d. e. ex. 12. quinti.

Nunc si sumpta fuerit aliqua linea, puta. d. o. cui. a. d. dicta habeat proportionem maiorem, clarum erit ex secunda parte decimæ quinti quod. d. o. minor erit ipsa. d. e. Corrige igitur impressionem Basileæ locando characterem. o. inter. d. e. e. eo quod ibi positum non fuit.

Volo nunc quod dictum maius antecedens æquale scilicet duplo ipsius. a. b. cum quadruplo ipsius. b. c. cum quadruplo ipsius. b. d. cum duplo ipsius. b. c. significetur à caractere. A. Hinc habebimus proportionem ipsius a. d. ad. d. o. vt. A. ad. B.

Ex. 18. quinti postea habebimus. A. B. ad. B. vt. a. o. ad. d. o. & proportionalitate euerfa in. 19. dicti ita erit. A. B. ad. A. vt. a. o. ad. a. d. Sed hoc vltimum antecedens in se continet id quod Archimedes scribit, hoc est duplum ipsius. a. b. quadruplum ipsius b. c. sexcuplum ipsius. b. d. & triplum ipsius. b. e. Consequens verò. A. continet duplum ipsius. a. b. quadruplum ipsius. b. c. quadruplum ipsius. b. d. & duplum ipsius. b. e.

Ex supposito deinde ipsius Archimedis & ex conuersa proportionalitate in. 19. dicta, verum est id quod dicit Archimedes, videlicet quod eadem proportio est ipsius. a. d. ad. g. h. quod quintupli ipsius. a. b. cum quintuplo ipsius. b. c. cum decuplo ipsius. b. c. cum decuplo ipsius. b. d. (quod quidem antecedens significetur per. V.) ad duplum ipsius. a. b. cum quadruplo ipsius. b. c. cum sexcuplo ipsius. b. d. cum triplo ipsius. b. e. hoc est ad. A. B.

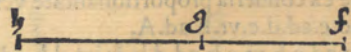
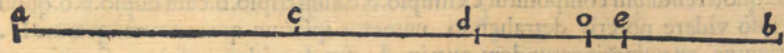
Erit igitur. V. ad. A. B. vt ipsius. a. d. ad. g. h. sed superius vbi signatum est. T. iam probatum fuit ita esse. A. B. ad. A. vt ipsius. a. o. ad. a. d. Ergo ex. 23. quinti Archimedes verum scribit, hoc est quod ita erit ipsius. V. ad. A. vt ipsius. a. o. ad. g. h.

Clarum per se etiam est, id quod Archimedes dicit hoc est quod. V. ad. A. est vt quinque

quinque ad duo, cum quodlibet ingredientium in composito. V. ad quodlibet ingredientium in composito. A. fit ut quinque ad duo. Quare ex. 13. quinti verum dicit. Vnde. a. o. ad. g. h. erit ut quinque ad duo ex. 11. eiusdem ut inquit Archimedes.

Corrige impressionem ubi scriptum est, rursus quoniam. o. a. quia oportet dicere Rursus quoniam. o. d.

Archimedes igitur verum dicit, quod ipse. o. d. ad. d. a. est ut ipse. B. ad. A. ex



V  $\frac{a}{5} = \frac{b}{10} = \frac{c}{10} = \frac{d}{5} = \frac{e}{5}$

AB  $\frac{a}{12} = \frac{b}{4} = \frac{c}{6} = \frac{d}{3} = \frac{e}{3}$

A  $\frac{a}{12} = \frac{b}{4} = \frac{c}{4} = \frac{d}{2} = \frac{e}{2}$

B  $\frac{b}{2} = \frac{c}{1} = \frac{d}{1} = \frac{e}{1}$

D  $\frac{a}{12} = \frac{b}{3} = \frac{c}{1} = \frac{d}{1}$

E  $\frac{b}{1} = \frac{c}{3} = \frac{d}{2} = \frac{e}{2}$

H  $\frac{a}{11} = \frac{c}{3} = \frac{d}{2} = \frac{e}{2}$

HA  $\frac{a}{3} = \frac{b}{6} = \frac{c}{3} = \frac{d}{3}$

M  $\frac{b}{11} = \frac{d}{1} = \frac{e}{1}$

N  $\frac{a}{11} = \frac{b}{2} = \frac{c}{1} = \frac{d}{1}$

Q  $\frac{3}{5} = \frac{b}{2} = \frac{c}{4} = \frac{d}{12} = \frac{a}{12}$

A  $\frac{b}{2} = \frac{c}{4} = \frac{d}{4} = \frac{a}{12} = \frac{e}{12}$

HA  $\frac{b}{3} = \frac{c}{6} = \frac{d}{3} = \frac{a}{3}$

conuerſa proportionalitate in. 19. quinti, cum. a. d. ad. d. o. iam probatum fuit (vbi B.) ita eſſe ut. A. ad. B.

Sed in principio huius ſpeculationis probatum iam fuit ita eſſe ipſius. d. a. ad. d. e. vt ipſius. D. ad. B. vbi notatum eſt. M. quare ex. 23. quinti, Archimedes verum dicit, quod. d. o. ad. d. e. erit vt. D. ad. A.

Sed cum. d. o. ad. d. e. ſe habeat ut. D. ad. A. erit ex conuerſa proportionalitate iam dicta. d. e. ad. d. o. vt. A. ad. D. per euerſam vero erit. d. e. ad. a. o. vt. A. ad ſuum reſiduum. quod reſiduum componitur ex ſimplo. b. c. cum triplo. b. cum duplo. b. o. quod à te ipſo videre poteris detrahendo numeros ipſarum quantitatum quæ in. D. reperiuntur, ex numeris earundem, quæ in. A. quod quidem reſiduum ſignificetur à charactere. E. Vnde ex conuerſa proportionalitate verum dicit Archimedes. hoc eſt quod ita ſe habeat. o. e. ad. d. e. vt. E. ad. A.

Cum autem ſit. a. b. ad. c. b. vt. c. b. ad. d. b. & ita. d. b. ad. e. b. ex ſuppoſito, ideo ex 17. quinti verum dicit Archimedes. hoc eſt quod ita erit ipſius. d. e. ad. e. b. vt. a. c. ad. c. b. & vt. c. d. ad. d. b. & ex. 13. cuiusdem eadem proportio erit tripli ipſius. c. d. ad triplum ipſius. d. b. quæ dupli ipſius. d. e. ad duplum ipſius. e. b. vt inquit Archimedes.

Ex qua. 13. compoſitum ex. a. c. cum triplo ipſius. c. d. cum duplo ipſius. d. e. eandem proportionem habebit ad compoſitum ipſius. e. b. cum triplo ipſius. d. b. cum duplo ipſius. e. b. quam ipſius. d. e. ad. e. b. Sed horum compoſitorum primum ſignificetur per. H. ſecundum vero ſignificatum fuit per. E. vnde. H. ad. E. ſe habeat vt. d. e. ad. e. b. ſed. E. ad. A. iam dictum eſt eſſe vt. o. e. ad. d. e. vbi ſignatum eſt. a. quare ex. 23. quinti eadem proportio erit ipſius. o. e. ad. e. b. quæ. H. ad. A. vt ipſe inquit.

X Ex. 18. poſtea cuiusdem ita erit. o. b. ad. e. b. vt. H. A. ad. A.

Norandum etiam eſt quod ſi collectæ fuerint omnes partes compoſiti. H. A. hoc eſt duplum. a. b. cum duplo. b. e. cum quadruplo. b. c. cum quadruplo. b. d. cum ſimplo a. c. cum triplo. c. d. cum duplo. d. e. habebitur triplum. a. b. triplum. b. d. & ſexcuplum b. c. vt ipſe dixit. Quod autem hoc verum ſit, cum diſtinctæ fuerint omnes partes, vt in ſubſcriptis his lineis videre eſt, videbis quod ſi ex. H. detracta fuerit ſimplex. a. c. quæ quidem poſtea iuncta vni ex partibus quadrupli. b. c. ipſius. A. reſultabit nobis vna integra. a. b. Vnde habebimus triplum ipſius. a. b. & in. A. remanebit triplum ipſius. c. b. Deinde ſi ex. H. auferatur triplum ipſius. c. d. & ipſum addatur tribus partibus quadrupli. b. d. ipſius. A. habebimus tres vices. b. c. quæ ſi iungantur tribus, quæ remanebant in. A. vt dixi, habebimus ſexcuplum ipſius. b. c. & in. A. remanebit ſimplum. b. d. cum duplo ipſius. b. e. Vnde ſi ex. H. demptum fuerit duplum ipſius. d. e. quod quidem iungatur cum duplo ipſius. b. e. habebimus duplum ipſius. b. d. quod coniunctum cum ſimplo. b. d. quod in. A. relictum fuerat, habebimus triplum ipſius d. b. Verum igitur eſt quod inquit Archimedes, hoc eſt, quod. H. A. eſt triplum ipſius. a. b. ſexcuplum ipſius. b. c. & triplum ipſius. b. d.

Verum etiam dicit ex eo (vt ſupra probatum eſt) quod. a. c. d. e. et. d. e. ſe habeant in continua proportionalitate, quare ex conuerſa proportionalitate erunt ſibi inuicem continuæ proportionales.

Nunc autem cum. a. c. d. e. ſint continuæ proportionales in ea proportione in qua ſunt. a. b. c. b. d. b. et. e. b. vt in principio diximus, erit ex. 22. quinti. a. c. ad. d. e. vt. a. b. ad. d. b. & ſic etiam. c. b. ad. e. b. Vnde ex. 24. cuiusdem. a. d. ad. d. e. erit vt. a. b. cum. b. e. ad. d. b. & vt. c. b. cum. b. d. ad. e. b. & ex. 13. dicti vt. a. b. cum. b. e. bis ſumpto, & cum. b. d. ad. e. b. Quare ex conuerſa proportionalitate, vt ſe habeat. e. d. ad. d. a. ita ſe habeat. e. b. cū. d. b. ad. d. b. cū. b. c. duplicato & cū. b. a. vt inquit Archimedes. Nunc antecedens vocetur. M. hoc eſt. b. e. cum. d. b. conſequens vero, hoc eſt

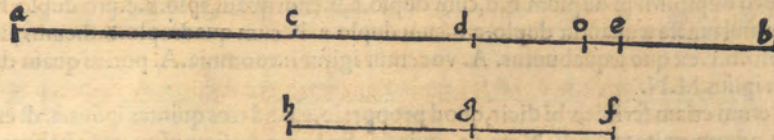
EPISTOLAE.

est. d. b. cum duplo. b. c. cum simplio. b. a. vocetur. N.

Animadvertendum tamen est quod impressio mendosa est ubi dicit.

vnaquæque. c. b: b. d. & cætera,  
 propterea quod dicendum est ita  
 vnaquæq; e. b: b. d.

Nunc ex. 18. quinti, quemadmodum se habet. a. e. ad. d. a. ita se habebit. M. N. ad. N.



$\Psi$	$\underline{a . 5 . b}$	$\underline{b . 10 . c}$	$\underline{b . 10 . d}$	$\underline{b . 5 . e}$
AB	$\underline{a . 2 . b}$	$\underline{b . 4 . c}$	$\underline{b . 6 . d}$	$\underline{b . 3 . e}$
A	$\underline{a . 2 . b}$	$\underline{b . 4 . c}$	$\underline{b . 4 . d}$	$\underline{b . 2 . e}$
B			$\underline{b . 2 . d}$	$\underline{b . 1 . e}$
D	$\underline{a . 2 . b}$	$\underline{c . 3 . c}$	$\underline{d . 1 . d}$	
E		$\underline{b . 1 . c}$	$\underline{b . 3 . d}$	$\underline{b . 2 . e}$
H	$\underline{a . 1 . c}$	$\underline{c . 3 . d}$	$\underline{d . 2 . e}$	
HA	$\underline{a . 3 . b}$	$\underline{b . 6 . c}$	$\underline{b . 3 . d}$	
M			$\underline{b . 1 . d}$	$\underline{h . 1 . e}$
N	$\underline{a . 1 . h}$	$\underline{b . 2 . c}$	$\underline{b . 1 . d}$	
Q	$\frac{3}{5}$	$\underline{b . 2 . d}$	$\underline{b . 4 . c}$	$\underline{b . 2 . a}$
A	$\underline{b . 2 . e}$	$\underline{b . 4 . d}$	$\underline{b . 4 . c}$	$\underline{b . 2 . a}$
HA		$\underline{b . 3 . d}$	$\underline{b . 6 . c}$	$\underline{b . 3 . a}$

Vbi

Vbi autem scriptum est

ad vtrunque simul, b. d; d. a. cum dupla. b. c.  
dicendum est ita,  
ad vtranque simul. b. d. b. a. cum dupla. b. c.

Inquit deinde Archi. quod sicut se habet. e. a. ad. d. a. ita se habebit duplum. M. N. ad duplum. N. Quod quidem verum est ex. 13. quinti, huiusmodi verò antecedens & consequens, Archi. manifestat ex suis partibus, sumendo duplum. e. b. cum duplo b. d. pro duplo. M. & duplum. b. d. cum duplo. a. b. cum quadruplo. b. c. pro duplo. N. que simul iuncta æquantur duplo. e. b. cum duplo. a. b. cum quadruplo. b. d. quadruplo. b. c. ex quo æquabuntur. A. vocentur igitur hæc omnia. A. potius quàm duplum ipsius. M. N.

Verum etiam scribit, vbi dicit, quod proportio. e. a. ad tres quintas ipsius. a. d. erit vt. A. ad tres quintas dupli. N. ex. 22. quinti. Sed cum ex supposito ita se habeat. f. g. ad tres quintas ipsius. a. d. quemadmodum. b. e. ad. e. a. erit ex. 16. quinti verum q̄ dicit Archimed. hoc est, ita se habere. b. e. ad. f. g. vt. e. a. ad tres quintas ipsius. a. d.

Et per. 11. eiusdem verum etiam erit quod sicut se habet. e. b. ad. f. g. ita se habebit. A. ad tres quintas dupli. N. quod quidem duplum. N. significetur per. Q.

Sed superius iam demonstratum fuit (vbi. X.) quod. o. b. ad. b. e. ita se habebat vt H. A. ad. A. & nūc demum probatum fuit ita esse. A. ad tres quintas ipsius. Q. vt. e. b. ad. f. g. Quare ex. 22. quinti ita erit. H. A. ad tres quintas ipsius. Q. vt. o. b. ad. f. g. vt idem inquit.

Sed. H. A. ad. Q. (vt ex suis partibus videre est) ita se habet vt tres ad duo ex. 13. quinti, vt inquit Archimedes.

Ipsē etiam dicit proportionem. H. A. ad tres quintas ipsius. Q. esse vt quinque ad duo. Pro cuius rei euidētia imaginemur tam. H. A. quam. Q. diuisa per quinque partes æquales, vnde ex. 16. quinti habebimus quamlibet quintam partē ipsius. Q. æqualē esse duabus tertijs vniuscuiusque quintæ partis. H. A. vnde tres quintæ ipsius Q. erunt, ex communi conceptu, sex tertiæ vnius quintæ ipsius. H. A. hoc est duæ quintæ ipsius. H. A. Quare. o. b. ita se habebit ad. f. g. vt quinque ad duo ex communi cōceptu, cum. o. b. ad. f. g. probatum fuerit se habere vt. H. A. ad tres quintas ipsius Q. (vbi. Y.) sed iam probatum fuit (vbi. . ) quod. o. a. ad. h. g. erat etiam vt quinque ad duo, hoc est quod. f. h. erit duæ quintæ ipsius. a. b. Quod est propositum.

In vltima

$\overline{a \quad c \quad d \quad o \quad e \quad b}$

$\overline{b \quad g \quad f}$

$\gamma \quad \overline{a \cdot 5. \quad b \quad b \cdot 10. \quad c \quad b \cdot 10. \quad d \quad b \cdot 5. \quad e}$

**AB**  $\overline{a \cdot 12. \quad b \quad b \cdot 4. \quad c \quad b \cdot 6. \quad d \quad b \cdot 3. \quad e}$

**A**  $\overline{a \cdot 12. \quad b \quad b \cdot 4. \quad c \quad b \cdot 4. \quad d \quad b \cdot 2. \quad e}$

**B**  $\overline{b \cdot 12. \quad d \quad b \cdot 1. \quad e}$

**D**  $\overline{a \cdot 12. \quad b \quad o \cdot 3. \quad c \quad b \cdot 11. \quad d}$

**E**  $\overline{b \cdot 1. \quad c \quad b \cdot 3. \quad d \quad b \cdot 2. \quad e}$

**H**  $\overline{a \cdot 11. \quad c \quad c \cdot 3. \quad d \quad d \cdot 2. \quad e}$

**HA**  $\overline{a \cdot 3. \quad b \quad b \cdot 6. \quad c \quad b \cdot 3. \quad d}$

**M**  $\overline{b \cdot 11. \quad d \quad h \cdot 1. \quad e}$

**N**  $\overline{a \cdot 11. \quad h \quad b \cdot 12. \quad c \quad b \cdot 11. \quad d}$

**Q**  $\frac{3}{5} \quad \overline{b \cdot 2. \quad d \quad b \cdot 4. \quad c \quad b \cdot 12. \quad a}$

**A**  $\overline{b \cdot 2. \quad e \quad b \cdot 4. \quad d \quad b \cdot 4. \quad c \quad b \cdot 12. \quad a}$

**HA**  $\overline{p \cdot 3. \quad d \quad b \cdot 6. \quad c \quad b \cdot 3. \quad a}$

In

In vltima verò propofitione fecundi lib. de ponderibus Archi. hoc modo intelligendus est, vt si diceret,

Sit paraboles. a. cuius basis fit. a. c. fitq; d. e. recta parallela dictæ basi. a. c. diameterq; b. f.

Inquit deinde quod linea contingens in. b. parallela erit ipsi. a. c. et. e. d. quod probabimus hoc modo.

Cum. b. f. diameter sit et. a. c. basis, clarum erit ex definitione quod. b. f. diuidet. a. c. per æqualia in. g. Vnde ex. 7. vel etiam ex. 46. primi Pergei. d. e. diuisa erit per æqualia à diametro. b. f. Quare verum dicit ex quinta fecundi ipsius Pergei hoc est quod dicta contingens in puncto. b. parallela erit ambobus. a. c. et. e. d.

Inquit postea quod diuisa cum fuerit pars dianetri quæ inter. d. e. et. a. c. posita est (hoc est. g. f.) per quinque partes æquales, quarum partium media sit. h. k. diuisa etiam imaginatione sit in puncto. i. ita quod proportio ipsius. h. i. ad. i. k. eadem sit quæ inter duo solida quorum vnum (illud scilicet à quo relatio incipit, hoc est antecedens) **R** pro sua basi teneat quadratum ipsius. a. f. cuius etiam solidi altitudo composita sit ex duplo ipsius. d. g. cum simplo. a. f. Aliud verò solidum habeat pro sua basi quadratum ipsius. d. g. eius verò altitudo composita sit ex duplo ipsius. a. f. cum simplo. d. g.

Inquit nunc Archi. quod cum ita factum fuerit, ostendit punctum. i. centrum esse portionis abscissæ à tota sectione, quod frustum nominat signatū characteribus. a. d. e. c.

Sit igitur nunc. m. n. inquit, æqualis diametro. b. f. et. n. o. æqualis. b. g. fitq; x. n. media proportionalis inter. n. m. et. n. o. et. t. n. in continua proportionalitate post. o. n. hoc est quod ea proportio quæ est ipsius. o. n. ad. n. t. eadem sit ipsius. x. n. ad. n. o. Hinc habebimus. 4. lineas in continua proportionalitate sibi inuicem coniunctas. m. n. x. n. o. n. et. t. n.

**A** Vult etiam quod à linea. i. b. incipiens ab. i. versus. g. alia linea abscissa sit, cui linea, ita proportionata sit. f. h. vt. t. m. est ad. t. n. quæ quidem linea signata sit. i. r.

Dicit postea quod diameter. b. f. erit fortasse axis vel aliqua reliquarum diameterum, quod quidem in. 46. primi Pergei videre est, cum omnes dianetri sint inuicem paralleli ipsi axi.

Cum postea dicit, quod. a. f. et. d. g. sunt intentæ ductæque, ibi vult idem inferre, quod Pergeus vocat ordinatè, vt ex. 11. et. 49. primi ipsius Pergei videre licet, vnde ex. 20. eiusdem proportio. b. f. ad. b. g. erit vt quadrati. a. f. ad quadratum ipsius. d. g. vt ipse dicit.

**e** Sed ita erit quadrati. m. n. ad quadratū. x. n. ex. 18. sexti Eucli. Quare ex. 11. quinti quadratum ipsius. m. n. ad quadratum ipsius. n. x. eandem habebit proportionem, quam quadratum ipsius. a. f. ad quadratum ipsius. d. g. Vnde ex. 18. & ex communi scientia, eadem proportio erit ipsius. m. n. ad. n. x. quæ ipsius. a. f. ad. d. g. vt inquit Arch.

Quapropter proportio cubi ipsius. m. n. ad cubum ipsius. n. x. erit vt cubi ipsius. a. f. ad cubum ipsius. d. g. vt etiam dicit ex communi scientia, nec non ex. 36. vndecimi.

Inquit postea quod proportio totius sectionis. a. b. c. ad portionem. d. b. e. eadem est quæ cubi ipsius. a. f. ad cubum ipsius. d. g. quod verum est, vt aliàs tibi monstravi in diuisione parabolæ secundum aliquam propositam proportionem.

**g** Quando autem dicit quod proportio cubi ipsius. m. n. ad cubum ipsius. n. x. eadem est quæ ipsius. m. n. ad. n. t. verum dicit ex. 36. vndecimi. Vnde ex. 11. quinti ita se habebit totalis sectio. a. b. c. ad portionem. d. b. e. vt. m. n. ad. n. t. & ex. 17. eiusdem ita erit ipsius. m. t. ad. t. n. vt frusti. a. d. e. c. ad sectionem. d. b. e. quemadmodum ipse dicit. Sed quia superius, vbi. A. ipsa. f. h. (quæ est tres quintæ ipsius. f. g.) ad. i. r. ita relata fuit

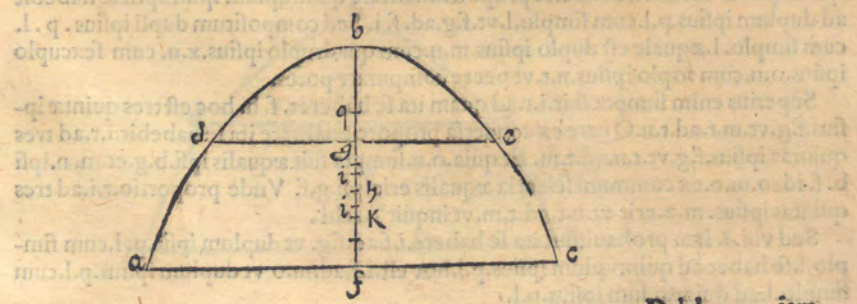


ta fuit vt. m. t. ad. t. n. idcirco ex. 11. quinti ita erit ipsius frusti. a. e. ad sectionem. d. b. e. vt tres quintę ipsius. f. g. ad. i. r.

Inquit deinde quod proportio corporis iam supradicti, quod pro sua basi habeat quadratum ipsius. a. f. altitudinem verò compositam ex duplo ipsius. d. g. cum simplo a. f. ad cubum ipsius. a. f. eadem erit quę dupli ipsius. d. g. cum simplo. a. f. ad. a. f. Quod quidem verum est ex. 33. vndecimi & ex prima sexti.

Sed superius (vbi. a.) iam probauimus eandem proportionem esse inter. m. n. & n. x. quę inter. a. f. et. d. g. ideo ex conuerfa proportionalitate ita erit ipsius. x. n. ad. n. m. vt ipsius. d. g. ad. a. f. sed dupli. x. n. ad simplum. x. n. est vt dupli. d. g. ad. d. g. Quare ex. 22. quinti dupli. x. n. ad. m. n. erit vt dupli. d. g. ad. a. f. & ex. 18. eiusdem ita erit dupli. x. n. cum simplo. m. n. ad. m. n. vt dupli. d. g. cum simplo. a. f. ad. a. f. Quare solidi

Solidum	maius	$\frac{u}{p}$	$\frac{n \cdot 2 \cdot x}{m \cdot 1 \cdot n}$
Cubus	maior	$\frac{u}{m \cdot n}$	
Cubus	maior	$\frac{s}{n \cdot t}$	
Solidum	minus	$\frac{z}{f}$	$\frac{n \cdot 2 \cdot o}{n \cdot 1 \cdot t}$
$\frac{p}{1}$	p. f.	$\frac{m \cdot 1 \cdot n}{n \cdot 2 \cdot x}$	$\frac{n \cdot 2 \cdot o}{n \cdot 1 \cdot t}$
$\frac{s}{1}$	p. f.	$\frac{m \cdot 5 \cdot n}{n \cdot 10 \cdot x}$	$\frac{n \cdot 10 \cdot o}{n \cdot 5 \cdot t}$
$\frac{2}{1}$	p. f.	$\frac{m \cdot 2 \cdot n}{n \cdot 4 \cdot x}$	$\frac{n \cdot 4 \cdot o}{n \cdot 2 \cdot t}$
$\frac{2}{1}$	p. f.	$\frac{m \cdot 2 \cdot n}{n \cdot 4 \cdot x}$	$\frac{n \cdot 6 \cdot o}{n \cdot 3 \cdot t}$
		$\frac{m}{x}$	$\frac{o}{t}$



Ddd iam

iam dicti ad cubum ipsius. a. f. ex. 11. quinti erit vt dupli. x. n. cū simplo. m. n. ad. m. n.  
Superius autem vbi.  $\beta$ . demonstratum fuit ita esse ipsius. m. n. ad. n. t. vt cubi. m. n.  
ad cubum. x. n. & inter.  $\alpha$ . et.  $\beta$ . probatum fuit ita esse cubi. a. f. ad cubum. d. g. vt  
cubi. m. n. ad cubum. x. n. Vnde ex. 11. quinti. m. n. ad. n. t. erit vt cubi. a. f. ad cubum  
d. g.

Dicit postea quod eadem proportio erit inter cubum. d. g. & corpus illud quod  
pro basi habeat quadratum ipsius. d. g. altitudinem verò vt dictum est, quæ est inter  
d. g. & compositum ex duplo. a. f. cum simplo. d. g. quod compositum est altitudo di  
cta, & verū dicit ex ratione superius allegata pro reliquo corpore & cubo ipsius. a. f.  
Quare etiam quemadmodum. t. n. se habet ad duplum ipsius. o. n. cum simplo. t. n.  
ex iisdem rationibus supradictis, vbi loquuti sumus de. x. n. cum. m. n.

Disponantur nūc omnia tali ordine, ita vt. u. primum sit corpus quod pro sua ba  
si habeat quadratum ipsius. a. f. & c.

Et. y. sit cubus ipsius. a. f. et. s. sit cubus ipsius. d. g. et. z. sit corpus quod basim ha  
bet quadratum ipsius. d. g. altitudinem verò vt supradictum est, et. p. sit compositum  
dupli. n. x. cum simplo. m. n. et. l. sit compositum dupli ipsius. n. o. cum simplo. t. n.  
Sed. u. locata sit è regione. p. et. y. è regione. m. n. et. s. è regione. n. t. et. z. è regione. l.  
& habebimus proportionem ipsius. u. ad. y. vt. y. ad. m. n. & ipsius. y. ad. s. vt. m. n. ad.  
n. t. quod superius iam demonstratum fuit, vbi.  $\beta$ . et. s. ad. z. ita se habebit vt. n. t. ad.  
l. vt vltimò probatum fuit. Quare ex. 22. quinti ita se habebit. u. ad. z. vt. p. ad. l.  
quemadmodum dicit Archi.

Et quia vt se habet. u. ad. z. ita facta fuit. h. i. ad. i. K. vbi. R. ideo ex. 11. quinti vt se  
habet. h. i. ad. i. K. ita se habebit. p. ad. l. vt ipse dicit: Et ex. 18. quinti ita erit. h. K.  
ad. K. i. vt. p. l. ad. l. & ex communi conceptu. g. f. se habebit ad. h. K. vt quintuplum  
ipsius. p. l. ad. p. l. & ex. 22. eiusdem ita se habebit. f. g. ad. i. k. vt quintuplum ipsius. p.  
l. ad. l. quintuplum autem ipsius. p. l. compositum est ex quintuplo ipsius. n. m. cum  
decuplo ipsius. n. x. cum quintuplo ipsius. n. t. cum decuplo ipsius. n. o. vt à te facile  
computare potes.

Verum etiam erit ex communi scientia quod. g. f. ad. f. k. est ut quintuplum ipsius  
p. l. ad duplum ipsius. p. l. eo quod superius suppositum fuit. h. K. esse quintā mediam,  
vnde. k. f. relinquebatur pro duabus quintis inferioribus, duplum autem. p. l. com  
positum est ex duplo ipsius. m. n. cum duplo ipsius. n. t. cum quadruplo ipsius. n. x. &  
cum quadruplo ipsius. x. o.

Ex conuersa proportionalitate deinde ita se habet. i. K. ad. i. k. ad. f. g. vt. l. ad. quin  
tuplum ipsius. p. l. et. k. f. ad. f. g. vt duplum ipsius. p. l. ad quintuplum ipsius. p. l. Vnde  
ex. 24. quinti. i. f. se habebit ad. f. g. vt duplū ipsius. p. l. cum simplo. l. ad quintuplum  
ipsius. p. l. Deinde ex conuersa proportionalitate quintuplum ipsius. p. l. se habebit  
ad duplum ipsius. p. l. cum simplo. l. vt. f. g. ad. f. i. Sed compositum dupli ipsius. p. l.  
cum simplo. l. æquale est duplo ipsius. m. n. cum quadruplo ipsius. x. n. cum sexcuplo  
ipsius. o. n. cum triplo ipsius. n. t. vt per te computare potes.

Superius enim sumpta fuit. i. r. ad quam ita se haberet. f. h. hoc est tres quintæ ip  
sius. f. g. vt. m. t. ad. t. n. Quare ex conuersa proportionalitate ita se habebit. i. r. ad tres  
quintas ipsius. f. g. vt. t. n. ad. t. m. Et quia. o. n. sumpta fuit æqualis ipsi. b. g. et. m. n. ipsi  
b. f. ideo. m. o. ex communi scientia æqualis erit ipsi. g. f. Vnde proportio. r. i. ad tres  
quintas ipsius. m. o. erit vt. n. t. ad. t. m. vt inquit Archi.

Sed vbi.  $\theta$ . iam probauimus ita se habere. i. f. ad. f. g. vt duplum ipsi<sup>o</sup>. p. l. cum sim  
plo. l. se habet ad quintuplum ipsius. p. l. hoc est. i. f. ad. m. o. vt duplum ipsius. p. l. cum  
simplo. l. ad quintuplum ipsius. p. l.

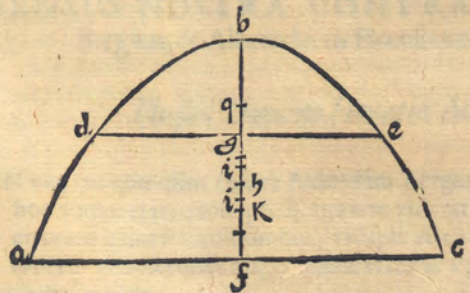
Habemus

Habemus igitur nunc omnēs illas conditiones quas Archimedes in præcedenti propositione supponit. Vnde ex rationibus ibi allegatis sequitur. f. r. esse duas quintas ipsius. m. n. hoc est ipsius. f. b. Quapropter punctum. r. centrum erit ponderis totius sectionis parabolæ ex. 8. secundi lib. de ponderibus eiusdem Archimedis.

Inquit nunc Archimedes, quod existente. q. centro ponderis ipsius parabolæ. d. b. e. partialis, centrum frusti erit in linea recta. q. r. f. ita remotum à centro. r. quod proportio. q. r. ad partem illam ipsius. r. f. quæ reperitur inter centrum. r. & centrum huius frusti æqualis est proportioni totius parabolæ ad partialem. Quod quidem verum est ex. 8. primi libri eiusdem.

Inquit etiam punctum. i. illud esse, eo quod cum probatum sit. f. r. duas quintas esse ipsius. f. b. ideo. b. r. tres quintas erit ipsius. b. f. ut ipse dicit.

<b>Solidum</b>	maius	$\frac{u}{p}$	$\frac{n \cdot 2 \cdot x}{m \cdot 1 \cdot n}$
<b>Cubus</b>	maior	$\frac{y}{m}$	$\frac{n}{n}$
<b>Cubus</b>	maior	$\frac{s}{n}$	$\frac{t}{n}$
<b>Solidum</b>	minus	$\frac{z}{f}$	$\frac{n \cdot 2 \cdot 0}{n \cdot 1 \cdot t}$
$\frac{p}{1}$	p. f.	$\frac{m \cdot 1 \cdot n}{n \cdot 2 \cdot x}$	$\frac{n \cdot 2 \cdot 0}{n \cdot 1 \cdot t}$
$\frac{s}{1}$	p. f.	$\frac{m \cdot 5 \cdot n}{n \cdot 10 \cdot x}$	$\frac{n \cdot 10 \cdot 0}{n \cdot 5 \cdot t}$
$\frac{2}{1}$	p. f.	$\frac{m \cdot 2 \cdot n}{n \cdot 4 \cdot x}$	$\frac{n \cdot 4 \cdot 0}{n \cdot 2 \cdot t}$
$\frac{2}{1}$	p. f.	$\frac{m \cdot 2 \cdot n}{n \cdot 4 \cdot x}$	$\frac{n \cdot 6 \cdot 0}{n \cdot 3 \cdot t}$
		$\frac{m}{x}$	$\frac{0}{t}$



Ddd 2 Sed

Sed, q. b. similiter tres quinta est ipsius. d. b. ex. 8. prædicta. Quare, q. r. tres quinta erit ipsius. f. g. ex. 19. quinti.

Dicamus igitur hoc modo cum. f. b. totum ad totum. b. r. ita se habeat vt abscissum. b. g. ad abscissum. q. b. ex. 7. et. 8. dicti primi libri eiusdem ideo residuum. f. g. ex. f. b. ad residuum. r. q. ex. r. b. erit vt totum. f. b. ad. totum. r. b. ex. 19. quinti Euclidi.

Sed iam sub. & probauimus ita se habere frustum. a. d. e. c. ad parabolam. d. b. e. vt m. t. ad. r. n. sed vt. m. t. ad. t. n. ita assumpra fuit (vbi. A.) i. r. ad quam sic se haberet. f. h. hoc est tres quinta ipsius. f. g. hoc est. q. r. quare ex. 11. quinti proportio frusti. a. d. e. c. ad parabolam partialem erit vt. q. r. ad. r. i. Existente igitur. r. centro totius parabola et. q. centro partialis, ergo. i. centrum erit frusti propositi.

Sed si nullo solido intercedente, voluerimus centrum. i. frusti. a. e. citius inuenire, inueniemus primò centrum. r. totius figurae ex. 8. secundi eiusdem constituendo. b. r. tres quintas totius axis. b. f. & centrum. q. parabola. d. b. e. partialis similiter.

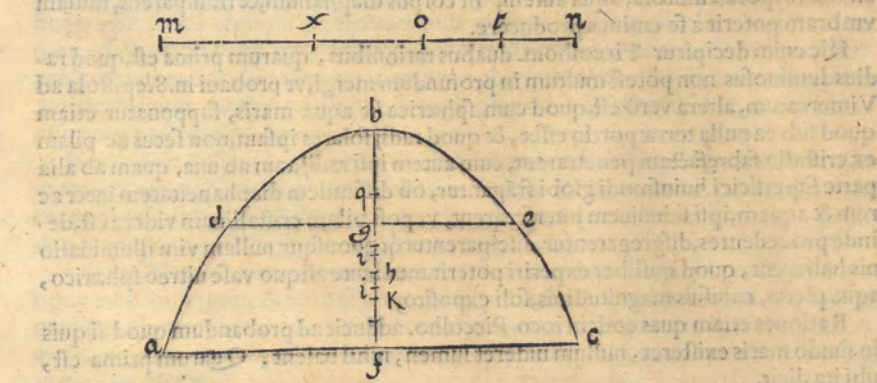
Nunc igitur manifestum est nobis, eandem proportionem fore ipsius. q. r. ad. r. i. quæ frusti. a. e. ad portionem. d. b. e. ex. 8. dicta. Vnde ex coniuncta proportionalitate ita se habebit. q. i. ad. i. r. vt. a. b. c. ad. d. b. e. sed vt. a. b. c. ad. d. b. e. ita se habet. m. n. ad. n. t. eo quod vnaquæque harum duarum proportionum sesquialtera est proportioni. f. b. ad. b. g. eo quod. f. b. ad. b. g. ita se habet. vt. m. n. ad. o. n. quare m. n. ad. t. n. ita se habebit vt. g. i. ad. r. i. vnde disiunctim. m. t. ad. t. n. ita se habebit vt q. r. ad. r. i. Lungatur igitur. r. i. quæ quidem. r. i. ita se habeat ad. r. q. vt. t. n. ad. t. m. vt habeatur centrum frusti.

DE-



b2 s LLG

Solidum	maius	$n$	$m$	$n$	$n$
Cubus	maior	$n$	$m$	$n$	$n$
Cubus	maior	$s$	$n$	$f$	$n$
Solidum	minus	$n$	$f$	$n$	$n$
$\frac{3}{1}$ p. f.		$m \cdot 1 \cdot n$	$n \cdot 2 \cdot x$	$n \cdot 2 \cdot 0$	$n \cdot 1 \cdot f$
$\frac{2}{1}$ p. f.		$m \cdot 2 \cdot n$	$n \cdot 4 \cdot x$	$n \cdot 4 \cdot 0$	$n \cdot 2 \cdot f$
$\frac{2}{1}$ p. f.		$m \cdot 2 \cdot n$	$n \cdot 4 \cdot x$	$n \cdot 6 \cdot 0$	$n \cdot 5 \cdot f$



DEFENSIO NOSTRA CONTRA ANTONIVM

Bergam, & Alexandrum Piccolhomineum.

*Illustri Domino Horatio Muto.*



NOTA ea quae olim contra Antonium Bergam, sermone Italico scripsi, hoc vnum erat, quod ipse Berga non viderat quendam notatu dignum errorem ipsius Piccolhominei, vbi ipse Alexander arguit quendam auctorem in tractatu de magnitudine terra & aquae pag. 37. linea. 26. ita di-

cens, & erit maior aqua.

Quo

Quo in loco clare videtur ipsum putare eandem proportionem inter diametros, quæ inter sphaeras ipsas esse, nec amplius recordari eius quod scripserat pag. 24.

Piccolhom. igitur ibi supponens centrum. D. esse magnitudinis aquæ, & intra sphaeram terrestrem, putat omnino causam esse ut terra superet aquam magnitudine, quasi quod si punctum. D. ut centrum sphaeræ aquæ, vnum idemq; esset cum puncto. E. extremo diametri ipsius terræ, sphaera. A. G. H. sphaeræ. A. B. E. dupla esse deberet, quod quidem nullo pacto fieri potest, quamuis etiam proportio. A. H. ad diametrum A. E. superbipartienteseptimas existeret, quæ minor esset quam sesquitercia, ita quod quando etiam. D. E. maior medietate ipsius. D. H. fuisset, nihilominus tamen terra minor esset aqua, eo quod proportio dupla minor est, quam tripla ad proportionem superbipartienteseptimas, & maior quàm tripla ad proportionem sesquiquartam. Vnde si Piccolhom. supposuisset proportionem ipsius. D. H. ad. C. E. esse sesquiquartam, rectè profectò dixisset, sed dicere quod ubicunque existat punctum. D. intra sphaeram terrestrem, sequitur ipsam esse maiorem aqua, verum non est.

Scripsi etiam quod Piccolho. decipiebatur vbi loquitur de diaphaneitate aquæ pag. 40. ita dicens.

Et cum rationabiliter aliquis existimare non potest, quod umbra quæ facit oriri e eclipses Lunæ, producta sit à terra, & ab aqua simul, ut ab vno corpore aggregato ex ijs duobus elementis, & ad vnam communem sphaeræcitatem reductis, propterea quod cum umbra produci debeat à corporibus opacis, quorum opacitas efficit illa corpora umbrosa, aqua autem, sit corpus diaphanum, & transparent, nullam umbram poterit à se eminus producere.

Hic enim decipitur Piccolhom. duabus rationibus, quarum prima est, quod radius luminosus non potest multum in profundum mergi, ut probaui in. 8. epistola ad Vimeratum, altera verò est, quod cum sphaerica sit aqua maris, supponatur etiam quod sub ea nulla terræ portio esset, & quod radij solares ipsam, non secus ac pilam ex cristallo fabricatam penetrarent, cum autem ipsi radij, tam ab una, quam ab alia parte superficie huiusmodi globi frãgantur, ob dissimilem diaphaneitatem inter aerem & aquam, ipsi seinuicem interfecarent, ut post pilam cristallinam videre est, deinde procedentes, disgregarentur, disciparenturq; quousque nullam vim illuminationis haberent, quod quilibet experiri poterit mediante aliquo vase vitreo sphaerico, aqua pleno, cuiusuis magnitudinis, soli exposito.

Rationes etiam quas eodem loco Piccolho. adducit ad probandum quod si quis in fundo maris existeret, nullum uideret lumen, nihil ualent. Quarum prima est, ubi ita dicit.

Ille qui se in aquam mergit, cum maiorem lucem, quæ supra aquam est, relinquat, iudicat pro magno temporis spatio locum illum obscurum, quemadmodum accidit quando per multum temporis spatium fixis oculis in corpore Solis intuiti sumus, ab eodem postea eosdem amouentes, omnia obscura nobis videntur.

Ipsè autem non considerat quod talis obscuritas quæ sequitur visionem maioris luminis, parum durat, immo cito euanescit, sed in aqua nunquam reuertimur ad videndum, neque vestigium aliquod luminis ibi videtur, in fundo maris dico, quemadmodum nobis nuntiauerunt hi qui margaritas expiscantur in imis partibus ingentium æquorum indicorum.

Secunda uerò ratio ipsius Piccolhom. est ubi ita dicit.

Altera causa quod nobis obscurus appareat locus sub aqua, esse potest obstaculum quod aquæ habent ab opacitate terræ sub eorum fundo, etenim sicut cristallum quamuis

quamuis perspicuū siue transparēs sit, nihilominus propter obstaculum plumbi sub ipso positi, efficit ut radij visuales repercussi reuertantur. ita etiam quamuis aqua sit corpus transparens, nihilominus propter obstaculum terræ opacæ, quæ subsidet in fundo maris efficeret potest obscuras partes illas sub aqua, illis hominibus qui in ipsa aqua merguntur.

In hac secunda ratione decipitur Piccolhom. Primum quia si usque ad imam partem maris, Solis radius ferri posset, ille qui ibi esset, attollens oculos sursum Solem cerneret, deinde aspiciendo ipsum fundum Maris, videret illum, ratione reflexionis luminis ab ipso fundo, & ex eadem ratione speculi ab ipso adducta, quæ contra ipsum est.

Decipitur etiam cum dicat radij visuales à speculo seu plumbo repercuti, eo quod non radij visuales sunt hi qui reflectuntur, sed sunt radij luminosi primarij, seu secundarij qui non ab oculis exeunt sed à corpore lucido.

Scripsi etiam quod si verum esset proportionalitatem continuam quātitatum elementorum ex proportione decupla constare, ignem pro maximo, terram verò pro minimo terminorum sumentes, totum aggregatum ex terra, aqua, aere, & igne, ita esset maius terra, quemadmodum mille centum & undecim ad unum, unde semidia meter regionis elementaris esset quasi aut paulo maior decuplo solum semidiametro terræ, unde inter conuexum ignis, & concuum minimi, seu inferioris orbis lunaris, relinqueretur quidam orbis vacuum spissitudinis vnus interualli plus quam viginti terræ semidiametrorum, quod spatium vacuum orbiculariter, maius existeret ipsa totali regione elementari plus quam trigiesies millies, immo si semidia meter dicti primi orbis lunaris maior esset terrestri ut trigintanouem ad unum, dicti orbis vacuum maior esset elementari regione plus quam. 58208. ad unum, proportionalitatem igitur continuam quæ ex decupla proportionalitate resultat in elementis esse putare est maximus error.

Subdit deinde Berga, hoc voluisse Platonem necessario requiri, ut extrema elementa, nempe ignis & terra cum duobus medijs aere, & aqua coniungerentur, cum in corporibus solidis (quasi Bergæ sint quædam corpora quæ solida non extent) possit dari medium æquale in geometrica proportione.

Sed ubi Plato ad sermonem de numero elementorum se confert, postquam ratione creationis ignis, & terræ se proposuisse putat, ut idem de alijs duobus corporibus medijs probet, comparatione proportionalitatis continuæ geometricæ in tribus terminis, ratione rerum superficialium primò, deinde in quatuor, ratione corporearum vitur, ita dicens.

Vinculorum verò id est aptissimum atque pulcherrimum quod ex se, & ex ijs quæ astringunt, quam maximè unum efficit, &c.

Quo in loco Plato inferre vult de proportionalitate geometrica trium terminorum, in qua ijdem ita se habent, ut medius, primi, vltimiq; vice fungatur, ita ut vtriusque ipsorum extremorum particeps fiat, cum productum quod à medio termino in seipso progignitur idem sit ei quod ab extremis fuit, unde medius, potentia idem est quod productum ab extremis.

Subdit deinde Plato dicens.

Quando enim in tribus numeris, aut molibus, aut viribus, medium ita se habet ad postremum ut primum ad medium, vicissimq; ut postremum cum medio, ita medium cum primo congruit, tunc quod medium est, & primum fit & postremum, postremum quoque, & primum & media fiunt.

Hic

Hiç animaduertendū est omnes interpretes falli, qui hoc loco Platonem de omnibus proportionalitatibus continuis quæ ternario numero ( alia enim Arithmetica, alia geometrica, alia harmonica dicitur ) continentur, intelligendum esse censent, quia de numeris, magnitudinibus, viribusq; , aut ut dici solet, virtutibus mentionem fecerit. Plato enim nihil aliud inferre voluit, quam eandem passionem ( ut ipse recitat ) inter medium extremaq; vnus proportionalitatis continuæ geometricæ, tam in quantitate, quam in qualitate resultaturā, cum tres termini eiusdem essent speciei, & quia quantitas in duas principes primariasq; partes, idest in continuam, & discretam diuiditur, hanc ob causam Plato hoc præcipuè significat numerorum magnitudinisq; vocabulis vtens, quibus vniuersum quantitatis genus complectitur.

Cum verò ait vires, vniuersum qualitatis genus inferre vult. Quia proportio & proportionalitas tam continua quam discreta, non solum inter terminos quanti, sed inter eos etiam qui quali attribuuntur elucet.

Sed quod eo loco de harmonica proportionalitate quæ cū geometrica magis simbola est quam cum Arithmetica Plato minime intelligat, ex eiusdem verbis cum ita scribit manifestè patet.

» Quando enim medium ita se habet ad postremum ut primum ad medium, vicifimq; ut postremum cum medio ita medium cum primo congruit.

Id enim in harmonica proportionalitate non cernitur in qua primus terminus ad postremum, & non ad medium, ita se habet geometricè ut differentia inter primum & medium ad differentiam inter medium & ultimum.

Quod si clarum est ipsum de harmonica proportionalitate nullo modo intelligere, quanto minus de Arithmetica, quæ cum geometrica nihil habet commune.

Cum verò Plato ait.

» Tunc quod medium est & primum fit & postremum, postremum quoque, & primum media fiunt, &c.

Nihil aliud ostendere vult, quam similitudinem quæ inter huiusmodi medium & extrema intercedit, cum ipsum medium ad postremum, quem primus ad seipsum, eundem respectum habeat, in quo est similis primo, & contra ad primum terminū, eundem respectum, quem postremum ad seipsum habet, unde hac ratione ultimum representat, uolens Plato inferre de conuenientia quæ inter media elementa, & extrema intercedit, ut aquæ inter aerem, & terram, cum aqua, ratione suæ frigiditatis, terræ, ratione uero suæ humiditatis aeri similis euadat. Aer uero qui inter ignem, aquamq; ponitur quod ad caliditatem attinet cum igne, quod uero ad humiditatem spectat cum aqua communicet.

Sed quia Plato multis in rebus doctrinam Pythagoricam sequutus est, Pythagorici autem omnia numeris metiebantur, & de omni re secundum numerorum rationem differebant, uidentq; Plato quod inter duos numeros superficiales, inuicemq; similes existentes, unum tantum numerum medium in proportionalitate continua geometrica cadere potest, ideo subiungit.

» Quod si uniuersū corpus latitudinem habere debuisset, nullam uerò profunditatem, unum sanè, tum ad seipsum, tum ad extrema uinciendā interiectum medium sufficisset.

Sequitur postea sic.

» Sed cum soliditatem mundus requireret, solida uerò non uno, sed duobus semper modis copulentur, inter ignem, & terram, Deus, Aerem, Aquamque locauit, &c.

Volens



Volens inferre, quod quemadmodum inter duos numeros solidos, & inuicem similes, vn<sup>o</sup> tantū medius proportionalis intercedere nō potest, sed duo necessariō requiruntur ( vt ex ijs quæ Euclid. 8. lib. 16. 17. 18. et. 19. propositione proponit videntur) ita dictæ ratione inter igneum, terreumq; corpus duo corpora interiecta essēt, non ratione proportionalitatis continuæ in quantitate eorūdem corporum, sed propter similitudinem connexionis, cum productum ex duobus medijs proportionalibus æquale sit productō ab extremis, & idem respectus, quem primum ipsorum quatuor ad secundum habet, secundi ad tertium extet, vnde secundum primo simile euadit, & contra, respectus qui est quarti ad tertium, sit etiam tertij ad secundum, vnde ipsum tertium, ratione vltimi subit, & eius imaginem induit, & hanc ob causam sic scribit Plato.

» Propterea ex huiusmodi rebus numero quaternario conclusis, mundi corpus conflatum est, ea connexum comparatione qua dixi. Ex quo seipsum amicitia concordie complectitur, &c.

Vbi Platonem, elementa maiora, minorāue in proportionalitate continua, nec geometrica, nec alterius cuiusuis generis esse noluisse, clarè perspicitur, sed huiusmodi similitudine, in eo quod media elementa cum extremis conueniunt est vsus, quæ quidem conuenientia, nullibi maior, quam in proportionalitate continua geometrica reperitur. Sed etiam si Plato de huiusmodi corporea elementorum magnitudine seipsum intelligi voluisset, si semidiameter regionis elementaris ex æquo vt. 39 ad vnum, respectu semidiametri terræ fuisset, aqua, ipsam terram, magis quam triagesies, & octies, non solum decies, & aer quoque eandem magis quam. 1500. & ignis magis quam. 5500. partibus magnitudine superaret.

Substantia vero rerum quas scripseram circa finem illius considerationis talis fuit.

Nunc autem tempus esse videtur, vt ego etiam, ne tantum destruxisse, sed etiam construxisse videar aliquid pro veritate disseram.

Non est igitur dubium, solidæ doctrinæ viris, quin præstantissimus Piccolo. secutus sit tutam viam ad explorandum, quod terra maior sit quam aqua, metiendo vtriusque horum corporum superficiem detectam. Omittamus autem compensationem illam curuitatis, & concauitatis vallium, & montium, &c. quam ipse Piccolo. propè finem sexti cap. velle dare fluminibus, stagnis, fontibus, & eiusmodi aquis. eo enim in loco labitur Piccolo. vbi non considerat, quod eiusmodi obliquis superficiebus non respondent anguli solidi centri sphaeræ, qui respiciunt eorum basim ad rectos angulos. Sed postquam Piccolo. comperit superficiem terræ detectam, esse maiorem apparente superficie sphaerica aquæ, proculdubio poterat concludere terram esse maiorem aqua, sicuti fecit, etiā si aqua profunda esset pyramidaliter vsq; ad mundi centrum, idest. 3500. miliaria, supponendo tantum esse huius globi semidiametrum.

Verum quia posset aliquis dubitare circa diligentiam Piccolo. in hiscæ duabus superficiebus dimetiendis, visum est mihi non alienum sequi aliam viam pro hac veritate probanda, supponendo verum esse, quod non vnus solus metitus fuerit, sed multi, idest supponendo verū esse quod maris profunditas mensurari possit, & præterea, quod non modo ipsius maris maxima profunditas non perueniat ad quingentos passus, sicuti refert Piccolo. in fine sui tractatus, & mihi asseruerunt Hispani multi, & Lusitani præstantissimi nautæ, tum Venetijs, tum Parmæ, in Aula Serenissimæ quondam Principis, inter quos, Venetijs fuit Illustris Rodericus Guzmanus, Dominus Franciscus Lopes, Dominus Garzias de Scullia, multiq; alij. Parmæ autem varij

E e e quos

quos omnes recensere molestum esset. Sed etiam supponendo quod maxima pelagi profunditas sit, non modo. 500. passuum, sed etiam. 500. millium passuum, ut dixi, & quod mare sit huius profunditatis, non vno in loco tantum, aut multis, sed quod supra totam etiam faciem terræ; mare tantę profunditatis ipsam terram vndique operiret, id est, quod vbiunque nunc terra detecta est, esset aqua, spissitudinis. 500. millium passuum. Atque ut planius intelligar supponendo quod sicuti totus huius globi semidiameter est milliariū. 3500. Terrestris partis semidiameter esset tñ. 3000. & reliquum semidiametri, id est quingenta milliaria esset crassitudo site profunditas orbis aquei, in quo nihil necesse esset laborare in dimetiendis fontibus, fluminibus, lacubus, stagnis, paludibus, & huiusmodi particulis nullius momenti apud peritos, nec curare subterraneas aquas cauernarum, aut aliorum terræ cauorum, seu terrę porositatum, quę omnia sunt circa ipsius terrę superficiem. Quia verisimile non est naturam eiusmodi caua siue spongiositates produxisse demissius libramenti maris. Supponendo igitur ea quę nunc dicta sunt, terra tamen esset ferè duplo maior aqua, hoc est, vt. 12. ad. 7. Quod quidem, cuius mathematica philosophiæ mediocriter perito, supputatū facillimum est. Cum proportio diametrorum, seu semidiametrorum, tertia pars existat proportionis eorundem sphaerarum. Sed vt parum periti minore labore supputare possint.

Primum sciendum est, quod supponendo diametrum globi, ex terra, & aqua compositi, esse. 3500. milliariū, & semidiameter purę terrestris partis esse. 3000. tantum, eiusmodi proportio erit vt. 7. ad. 6. quia communis maior numerator horum duum semidiametrorum erit. 500. qui in maiorem ingreditur septies, in minorem autem sexies. Et eiusmodi proportio superparticularis, vocatur sesquisepta, cuius triplum erit vt. 57. cum sexta parte ad. 36. & idem erit inter dictum globum compositum, & partem terrestrem simplicem. Quare subtrahendo puram, seu simplicem partem terrestrem, ex composito, reliqua pars erit, vt. 21. cum sexta, pro quantitate aquei orbis, ad quam, terrestris quantitas. 36. erit ferè in eadē proportionē, quę. 12. ad. 7.

Nunc fortasse alienum non erit viderè quanto ferè maior esset terra, quam tota aqua, non dico autē solum de parte illa maximę eius profunditatis, quę nusquam ad quingentos passus peruenit, sed de ficto illo orbe aqueo, profunditatis. 500. passuum, qui totum terrestrem orbem circundaret, & tegetet, supponendo quod per quingentos passus profunditatis, quidquid est terra, esset aqua, id est supposito quod ex totius orbis compositi semidiametro existente. 3500. milliariū, purę terrę semidiameter esset milliariū. 3499. cum dimidio. Supponendo igitur, vt supradixi. Comperietur quod terra esset maior aqua amplius quam. 2333. vicibus. Sed quia partes terrę detectę rumpunt eiusmodi fictum orbem aqueum, quę quidem partes, sunt ampliores superficię aquę, vt obseruauit Piccolo. atque alij præstātes viri, ideo sequetur, vt terra sit maior aqua amplius. 4666. vicibus imo amplius quinquies millicuplo. Si autem quis diceret, in quantitate aquę computari etiam illam, quę gignatur ex vaporibus, qui globum hunc compositum circundant: respondeo quod non modò ei concedo computari eiusmodi aquam, sed supponendo etiam quod totus locus à vaporibus occupatus, qui attolluntur. 52. milliaria supra superficiem huius globi, vt iam supradictum est, totus esset aqueus, & amplius, supponendo quod orbis hic aqueus esset spissitudinis, siue altitudinis quingentorum milliariū supra totum ipsum globum compositum. Tamen terra esset maior ipsa aqua ferè duplo; qua dere, quisque eiusmodi supputationum peritus certior fieri poterit. Vnde itidem

dē m affirmare possemus, terram non solum maiorem esse aqua, sed aqua & præterea aere, si aer non tam altè pertingit, quam multi alij præter Piccolo. sentiunt, qui dicunt inde euenire quod aerea humiditas non tam altè ascendere potest, quoniam humiditas ipsa grauitatem secum affert, præterquam quod nubium situs ostendit supra eas materiam esse rariorem quam sint ipsæ nubes, infra vero densiorem. Corpora enim eouſque ascendunt donec inueniunt constitutionem mediam formæ æqualis (vt ita dicam) suis. Quare materia illa quæ improprie ignis vocatur (non enim est ignis) incipit carere humiditate (qua mediante aer definitur) circa quinquagesimum secundum milliarium supra superficiem terræ, vt iam supradixi à Vitellione demonstratum fuisse. Aristo. autem affert rationē quare nubes altius nō transcēdāt. Vnde apparet tertiam aeris regionem improprie aerem appellari, si humiditate caret, vt ait Arist. qua mediante aer definitur, immo potius retinet ignis naturam, vt etiam asserunt interpretes Aristotelis in primum Meteororum. Qui Aristo. in locis supra citatis itidem ostendit se etiam huius modi esse opinionis.

Quod autem attinet ad probandum quod superficies terræ detecta sit altior quam superficies detecta aquæ, id tam clarum est sua sponte philosophis, qui sciunt quid sit altum, quidue demissum, quod superfluum esset quidquid super hoc dicerem præterquam, quod constat ex demonstratione ab Aristo. facta textu 31. li. 2. de cælo, in quo agit de corporibus in aqua positis, vnde eiusmodi veritas planissime aperitur. Omittimus etiam quod præstantes Moderni omnes, eam pro manifestissima ponūt, sicuti apud omnes sãni iudicij homines reuera existimatur.

Hæc enim sunt quæ in fine illius considerationis scripseram.

Anno autem præterito editus in lucem fuit tractatus quidam Pulcherrimus, ab Excellentissimo, nec non Doctissimo viro Augustino Michele, Patritio Veneto, ad corroborandam opinionem antiquorum, vbi tot authoritates, totq; rationes adducit, vt nil amplius dici possit. At ego sensum, rationemq; & non authoritatem aliquã sequutus sum: cum verò dico sensum, de sensu illorum intelligo, qui profunditatem maris metiti sunt, vt non mihi solum, sed, & Piccolo. & alijs permultis retulerunt; de ratione vero à me adducta, aliorum sit iudicium.

Sed iste mirabilis & Excellentissimus vir, verba mea non accepit in eo sensu, vt ego scripsi, ita vt omnino alienas consequentias sibi confingat, quemadmodū pag. 3. sui tractatus inquit, me non concedere naturam produxisse in magna quantitate, atque immensa, id totum, quod bonum, & necessarium est. Hanc enim consequentiam ipse colligit ex eo, quod ego pag. 19. meæ considerationis contra Antonium Bergam scripseram, quod videntur multa corpora alijs nobiliora, nihilominus minora, eo quod quantitas non sequitur nobilitatem, neque ab ea pendet, ita vt res illa quæ nobilior est, necessarium sit vt etiam maior existat. Sed Excellentissimus ista vir scribit ita me dixisse.

Multa immo infinita corpora sunt nobilia, & necessaria, nihilominus sunt paruę molis.

Vide igitur quantum hoc distat ab illo.

Præterea cap. 12. aliam consequentiam facit, quam ego non tam amplam facio. Ipse enim me inferre vult in alijs terrę partibus cauernas non reperiri, eo quod Montes sint cauernosi. Aspice quæso. pag. 29. meæ considerationis, & clarè videbis me nullo modo negare illas concauitates seu porrositates terræ extra montana loca, circa superficiem terræ, vsque ad æquilibrium, orbiculariter, infimæ profunditatis maris.

Sed putare inferius has porrositates reperiri, cum nulla ratio nobis persuasibilis

Ecc 2 adhuc

adhuc ab aliquo prodita sit, idoneum nullo pacto esset. Rationes autem ab ipso Excellentissimo Augustino adductas circa huiusmodi rem, alij dijudicent, de autoritatibus verò, nihil dicam, quia ab illis petendæ sunt, qui profitentur tales facultates, quorum vnus tantummodo autoritas præualere deberet, contra omnes alias eorū qui nunquam attigerunt summis labris orificia harum scientiarū. Vt si exempli gratia non solum autoritas illorum virorum, quos ipse recensuit, sufficiens esset vt puta Pioccolo. Naibodæ, Bordini, Clauij, reliquorumq; fautorum verè opinionis, sed Francisci Maurolici tantummodo, qui in primo Dialogo suæ cosmographiæ ita inquit.

„ Existimo autem totum terræ corpus rigidum esse saxum, nam si arena esset, aut  
 „ gleba fragilis, ita humorem imbiberet, vt cum eo quasi confunderetur; huc ac-  
 „ cedit, quod si mineræ, ac rupes, quæ sunt grauissimæ partes in ipsa plerunque super-  
 „ ficie comperiuntur, multo magis apud centrum esse debent. Videtur ita ratio exi-  
 „ gere, vt grauiora centro quoque sint propinquiora.

Hæc igitur sola autoritas, instar reliquarum omnium sufficere posset. Verum de autoritatibus minime curandum est, vbi sensus, ratioq; vera illis opponuntur.

Quod autem numerus animalium aquatiliū maior existat numero terrestrium, satis respondimus pag. 41. nostræ considerationis.

„ Sed in cap. 14. Excellentissimas Augustinus ita inquit ( vt etiam superius dixerat )  
 „ quod certiozem cognitionem homo non habet illa, quæ à sensu prouenit. Et quod  
 „ nemo est qui aspiciat terram, & aquam, quod hanc maiorem illa non iudicet, & nõ  
 „ existimet.

Quod autem certiozem cognitionem homo non habeat illa, quæ à sensu prouenit, concedendum non censeo. Nam omnis cognitio mathematica ( cum primum gradum certitudinis obtineat ) ab ipso sensu fieret, quod omnino alienum est à veritate. Sensus enim nunquam vidit incommensurabilitates magnitudinum, vel incoincidentias linearum non tangentium cum curuitate hyperbolica, aut angulum contingentia aliquem, nec ( vt vno verbo dicam ) aliquam conclusionem mathematicam, quam volueris. Neque per sensum est scire, inquit Aristoteles. Cognitio igitur sensitiua, certior non est illa, quæ per habitum scientificum acquiritur.

Ad reliqua verò, supponamus nos tunc fuisse in Arca Noe, cū aquæ cooperiebant omnia cacumina montium, vbi nullum terræ vestigium videbatur, quare proculdubio aquam iudicarem, atque existimarem maiorem terram, dū nulla alia re vteremur nisi sensu absque alio discursu intellectuali, ut reliqua illa animalia irrationalia, quæ nobiscum erant in dicta arca. Nõ sufficit igitur superficiem aquæ tantummodo aspicerem, quia neque tunc temporis, aqua erat maior terra, etiam si non solum tot cubitis attolleretur supra cacumina montium, sed quingenta milliaria, vt supradiximus.

Ratio autem illa, ex infinitis, ab ipso, eo in loco adducta, talis est.

„ Aqua est eccentrica ad terram, & pro cetro habet centrum grauitatis terræ, aqua  
 „ igitur maioris est amplitudinis ipsa terra.

„ Hanc etiam consequentiam alijs relinquo Philosophis dijudicandam.

Subsequitur postea dicens.

„ Præterea proprius locus terræ, est superficies aquæ, igitur terram oportet ab  
 „ aqua tegi.

Ad hoc etiam aliquis posset quærere, quis nam erit locus illius partis terræ detectæ ab aqua? nulli dubium erit quin superficies aeris, & non aquæ existet.

Nunc

Nunc autem si locus terræ est sub aqua, ergo locus aquæ proprius est sub aere, & non sub terra, unde non erit rationabile putare maiorem copiam aquarum existere in cauernis subterraneis, quam supra superficiem terræ. Adde quod locus illarum aquarum non esset superficies aeris, sed terræ, unde non minus locus aquæ esset terra, quam locus terræ, aqua. Sed missa faciamus hæc.

Cap. verò. 20. ita inquit.

Materia elementorum aequalis est. Ergo aqua maior est terra.

<sup>32</sup> Hæc enim consequentia verissima esset. Sed nullus vnquam Philosophus (vt Philosophus dico) concedet totam materiam elementarem, in quatuor æquales partes esse diuisam.

Cap. verò. 21. inquit me dixisse non suffecturam paucam spissitudinem. Eo enim in loco pag. 26. mei tractatus contradicens ipsi Bergæ, dixi, quod secundum ipsum Bergam non sufficeret pauca spissitudo.

Similiter etiam dixi, quod secundum ipsum, quanto remotius diffunditur lumen fortasse tantò magis illuminat. Putans ipse Berga quod in propinquo debilius existeret dictum lumen. Et propter ea dixi, quod apud ipsum fortasse nihil valet illa propositio, quæ dicit. Agens in propinquo, fortius agit quam in remoto.

Cap. autem. 22. vbi Excellentissimus Augustinus inquit, vnum tantummodo elementum non sufficere ad generationem mistorum. Hoc enim concedo, sed hoc nihil ad me spectat, eo quod meum responsum ad Bergam, erat circa transitum luminis, & non circa generationem elementorum.

Cap. demum. 23. pag. 20. linea. 10. vbi scribit me dixisse, iudicare, oportebat scribere, dubitare.

Puto tamen hoc vocabulum esse errorem Thytopographi, quamuis in correctione illud non inuenerim, quia vt ego multoties expertus sum, difficillimum omnes Thytopographi errores corrigere, neque (vt fertur) Argi oculi sufficerent.

Hactenus enim in mei defensionem hæc subiungere volui.

Ad defensionem autem Piccolo. aliorumq; virorum meæ opinionis, nec non de proportione duplicata profunditatis maris ad suam amplitudinem, ex consequentia pyramidalis: alijsq; similibus rationibus, prodeant alij. Huiusmodi tamen Doctissimi viri ingenium, memoriam, nec non doctrinam valde admiror, atque obseruo.

## DE METHODO PRODUCTIONIS FRACTORVM qua vtuntur Pedemontani Agrimenfores.

*Anselmo Rosenburg Agrimensori Casareo.*



METHODVS quàm mihi scribis in Prouincia tua maximè in vsu esse, nimis longa atque proluxa est, Pedemontani verò Agrimenfores in productione fractorum, valde breui methodo vti solent, quam libenter tibi scribo, eo maxime, vt videas quam rationabiliter operentur.

Scire igitur primum te oportet illos, maximam eorum communem mensuram vocare Trabucum, cuius sextam partem vocant Pedem, duodecimam verò pedis, Vnciam, duodecimam autè vnciæ punctū, duodecimam demum puncti; Attomum.

Quotiescunque igitur multiplicant trabucum, per trabucum nulli dubium est quin producant trabucum superficialem scilicet. Similiter

Similiter multiplicando pedes, vncias, puncta, & attoma per trabucum, produ-  
cunt pedes, vncias, puncta, & attoma superficialia rectangula oblonga, quorum lon-  
gitudō est ipsius trabuchi, latitudo vero lineæ dictarum specierum.

Dum vero multiplicant pedem per pedem, nulli dubium est quin producant pe-  
dem quadratum, sed apud ipsos non vocatur quadratum, quamuis reuera ita sit, sed  
illud vocant duas vncias, quæ quidem sunt rectangula oblonga iam hic supradicta,  
quarum vniuscuiusque longitudo sit vnus trabuchi, latitudo vero vnus duodecimæ  
partis ipsius pedis linearis.

Productum autem pedis per vnciam, vocant duo puncta, quæ etiam sunt duo re-  
ctangula oblonga, vt supra.

Productum deinde vnciæ per vnciam, vocant duos attomos, qui etiã sunt duo re-  
ctangula oblonga, vt dictum est, quæ omnia scientificè videbimus.

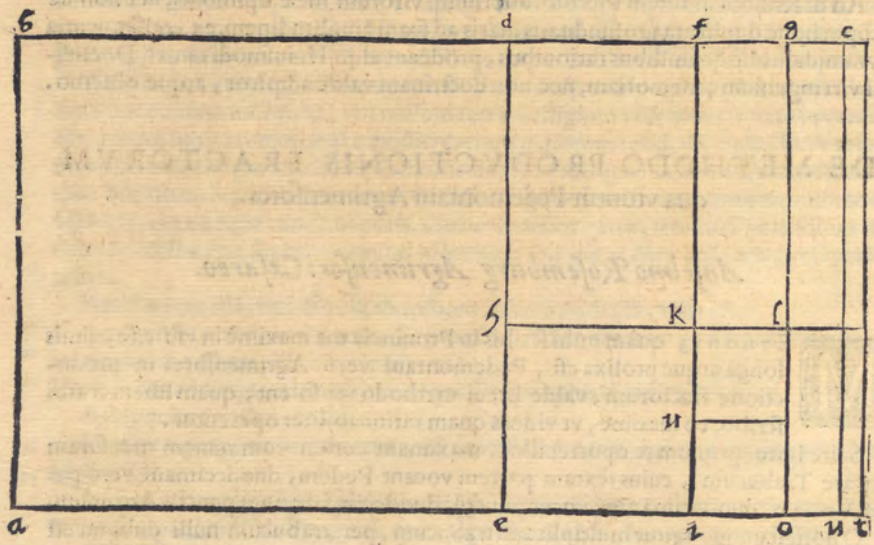
Pro cuius rei cognitione, sit, exempli gratia. a. e. vnus Trabuchus linearis. e. i. ve-  
ro vnus pes. i. o. autem vna vncia, o. u. postea vnum punctum, et. u. t. vnus At-  
tomus.

Vnde. e. i. erit sexta pars ipsius. a. e. et. i. o. duodecima ipsius. e. i. et. o. u. duodecima  
ipsius. i. o. et. u. t. duodecima ipsius. o. u. Sit etiam. a. b. æqualis. a. e. lineæ & sic. e. d. i.  
f. o. g. o. n. & c. terminenturq; parallelogramma. b. e. d. i. f. o. g. u. et. c. t. vnde. b. e. erit  
trabuchum quadratum, et. d. i. pes rectangulus oblongus vt supra, et. f. o. vncia rectan-  
gula oblonga, et. g. u. punctum rectangulum oblongum, et. c. t. attomus rectangu-  
lus oblongus.

De producto igitur trabuchi per trabuchū, nulli dubium est quin sit quadratum.  
a. d. vt superius diximus.

Productum autem trabuchi cum pede erit. d. i. sexta pars ipsius. a. d. cum. e. i. sit sex-  
ta ipsius. a. e. ex prima sexti vel. 18. aut. 19. septimi, siue etiam ex. 15. quinti Eucli.

Productum autem pedis cum pede erit. e. K. quadratum, quod probandum est  
duplum



duplum esse rectangulo. f.o. Nā. K. i. sexta pars est ipsius. f. i. ex supposito, et. i. o. duodecima ipsius. e. i. proportio igitur. e. i. ad. o. i. dupla est proportioni ipsius. f. i. ad. K. i. quare. K. e. duplo maius est ipsius. f. o. eo quod si. i. o. vel. f. g. (quod idem est) duplo maius esset ipso latere presenti. o. i. vel. f. g. tunc. f. o. æquale esset ipsi. K. e. ex. 15. sexti vel. 20. septimi quod quidem. f. o. duplo maius esset ipso presenti. f. o. Rectè igitur inquirunt dicentes productum pedis cum pede esse duas vncias, vel si maius, ita dicas. e. K. sexta pars est ipsius. d. i. ex iam dictis propositionibus. f. o. autem est duodecima ipsius. d. i. ex ipsdem, cum ex supposito. i. o. duodecima sit ipsius. e. i. quare. e. K. duplū erit ipsius. f. o. ex communi notione.

Productum verò pedis cum vncia. sit. K. o. quod probabimus ex ipsdem rationibus duplum esse ipsius. g. u. puncti rectanguli oblongi. Nam. l. o. sexta pars similiter est ipsius. g. o. et. o. u. duodecima ipsius. o. i. quare proportio. i. o. ad. o. n. dupla est proportioni. g. o. ad. o. l. sequitur ergo ex prædictis rationibus. k. o. duplum esse ipsius. g. u. vel sic, ut in præcedenti, cum. K. o. sit sexta pars ipsius. f. o. ex dictis propositionibus. g. u. verò duodecima eiusdem. f. o. ex ipsdem, nam. o. u. duodecima est ipsius. o. i. ergo K. o. duplo maius est ipso. g. u.

Ex ipsdemmet rationibus productum. l. u. pedis cum puncto duplum est ipsius. c. t. atomi rectanguli oblongi.

Probandum nunc relinquitur productum. o. n. vnciæ cum vncia, quod est quadratum, duplum esse ipsius. c. t. atomi rectanguli oblongi. Nam. i. n. est pars vna ex. 72. ipsius. c. u. et. u. t. pars vna ex. 144. ipsius. o. i. ex supposito, quare proportio. i. o. ad. u. t. dupla est proportioni ipsius. c. u. ad. n. i. ex dictis igitur rationibus. o. n. duplo maius est ipso. c. t. Vel si placet dicas. n. o. est vna pars ex. 72. ipsius. f. o. ex supradictis, eo quod. n. i. ita se habet ad. f. i. ut vnitas ad. 72. sed ex ipsdem rationibus. c. t. pars vna ex. 144. est ipsius. f. o. eo quod ita se habet. u. t. ad. o. i. quare. o. n. duplo maius erit ipso. c. t.

Propositum sit nobis nunc, exercitij gratia, quærere superficiem alicuius rectanguli, cuius vnum latus sit trabuchorū. 3. pedum. 2. & vnciarum. 3. aliud vero latus sit trabuchorum. 2. pedum. 3. vnciarum vero. 2.

Huiusmodi autem methodo mediante, multiplicando primum latus dictū. 3. 2. 3. per numerum trabuchorum secundi lateris. 2. scilicet producentur nobis primò trabucha superficialia. 6. pedes. 4. & vnciæ. 6. omnia rectagula, ut dictum est. Multiplicando deinde idem primum latus. 3. 2. 3. per pedes. 3. secundi lateris. Ex trabuchis. 3. primi lateris cum. 3. pedibus secundi, producentur. 9. pedes rectanguli, hoc est vnus trabuchus cum tribus pedibus rectangulis. Ex pedibus autem huius. 2. cum ipsdem alterius lateris. 3. producentur. 12. vnciæ rectangulæ idest vnus pes rectangulus. Ex ipsdem pedibus. 3. secundi lateris, cum. 3. vncijs primi lateris producentur.

Trabucha.	pedes.	vnciæ.	
3.	2.	3.	
2.	3.	2.	
6.	4.	6.	
1.	3.	1.	6.
	1.	6.	8.
			1.
8.	3.	2.	3.

18. puncta rectangula, hoc est vna vncia cum. 6. punctis rectangulis. Deinde ex multiplicatione vnciarum. 2. secundi lateris, cum. 3. trabuchis primi lateris, producentur. 6. vnciæ. Ex multiplicatione postea dictarum. 2. vnciarum secundi lateris cum. 2. pedibus primi, producentur. 8. puncta.

Demum ex iisdem. 2. vncijs secundi lateris cum. 3. primi, producentur. 12. attomi, idest vnum punctum. Quæ omnia collecta facient trabucha. 8. pedes. 3. vncias. 2. & attomi. 3. omnes rectanguli oblongi. Pulcherrima profecto operatio.

Trabucha.	pedes.	vnciæ.	
3.	2.	3.	
3.	3.	2.	
6.	4.	6.	
1.	3.	1.	6.
	1.	6.	8.
			1.
8.	3.	2.	3.

Videamus nunc exercitij causa, vt dixi, quomodo conueniat calculus iste cum calculo ordinario communi?

Nam quotiescunque dicta latera, fracta fuerint in vncias, primum latus erit vnciarum. 243. secundum autem. 182. productum vero vnus in alterum erit vnciarum quadratarum. 44226. quod quidem productum cum diuisum fuerit per. 5184. vncias quadratas vnus trabuchi quadrati, prouentus erit. 8. trabuchorum, reliquus verò numerus, siue fractus, erit vnciarum quadratarum. 2754. qui cum diuisus fuerit per numerum. 144. vnciarum vnus pedis quadrati, prouenient pedes. 19. quadrati cum vncijs. 18. superabundantibus, dicti autem pedes. 19. significant tres pedes rectangulos oblongos cum vno pede quadrato, hoc est cum duabus vncijs rectangulis oblongis, vt supra.

Videndum nunc est, vtrum illæ. 18. vnciæ æquipolleant tribus punctis rectangulis oblongis: sed hoc manifestè videre est, ex hoc, quia quilibet vncia rectangula oblonga componitur ex. 72. quadratis, punctum autem rectangulum oblongum, cum sit duodecima pars ipsius vnciæ rectangulæ oblongæ, ipsum componetur ex. 6. vncijs quadratis. 18. igitur vncijs quadratis, triplum erit ipsius puncti rectanguli dicti. Vnde clarè patet, quod, quotiescunque voluerimus scire proportionem ipsarum vnciarum quadratarum superabundantium, ad punctum rectangulum oblongum, si dixerimus ex regula de tribus, si. 72. (vncia rectangula oblonga) dat. 18. quid dabunt. 12? puncta rectangula oblonga, quarum vnaquæque est duodecima pars ipsius vnciæ rectangulæ oblongæ, in præsentem autem casu prouenient. 3. pro quarto termino quaesito, & habebimus profpitum.



## SOLVTIO CVIVSDAM QVÆSITI.

*Magnifico Ludouico Fauz, zoni amico carissimo.*

**V**I quæfiti folutio quam nescio quis te docuit, valde diuerfa est à vera. quæfitem enim tale fuit.

Reperiuntur quatuor focij, Ludouicus, Hieronymus, Francifcus, & Laurentius quorum primus, Ludouicus fcilicet, pofuit aureos. 6000. Hieronymus verò aureos. 5000. Francifcus autem. 2000. & Laurentius. 1000. quorum fuma faciebat aureos. 14000. interim tamen de tali fuma Ludouicus recepit aureos 2000. Hieronymus verò. 1000. Francifcus autem. 900. & Laurentius. 800. quapropter in fuma refidua Ludouicus non habebat nifi aureos. 4000. Hieronymus etiã 4000. Francifcus. 1100. & Laurentius. 200. quorum fuma erat. 9300. Nunc autem ifti focij cupiunt augere hanc fummam per aureos. 20000. tali tamen conditione quod quilibet tantum tribuat vt in totali fuma, tantam partem unus habeat, quantam alter.

Hoc autem problema tam facile est, & cum fuo theoremate ita coniunctum, quod miror amicum noftrum illud illico non vidiffe.

Accipe igitur illos aureos. 20000. & eos collige cum fuma. 9300. vnde habebis aureos. 29300. pro fuma totali, cuius quarta pars erit. 7325. quã vnufquifq; poftea habebit in dicta fuma. Sed ut reperias quantitatem aureorum quam quilibet prius debet contribuere, vt poftea habeat aureos. 7325. in dicta focietate. Iubeo, vt Ludouicus demat illos aureos. 4000. quos demum habebat, ex. 7325. reliquum autem erit. 3325. qui quidem numerus erit aureorum nunc contribuendorum ipfius Ludouici. Demptis fimiliter aureis. 4000. ex dictis. 7325. remanebunt. 3325. pro contributione ipfius Hieronymi. Deinde fi ex. 7325. extracti fuerint aurei. 1100. relinquent. 6225. pro contributione Francifci. Demptis demum. 200. ex. 7325. refidui erunt. 7125. pro contributione Laurentij, & fic quilibet habebit æqualem portionem in totali fuma.

*Speculatio cuiusdam Methodi reductionis numifmatum  
vnius fpeciei in aliam.*

A D E V N D E M.

**M**irum tibi videtur quo pacto verum fit, quod fuma medietatis cuiufuis numeri illorum numifmatum, quæ hic vocantur Blanci, cum fexta parte eiusdem medietatis, femper fit numerus florenorum huius prouinciæ. Vt exempli gratia, quotiefcunq; reducere voluerimus. 48. Blancos in Florenos, fi medietati ipfius. 48. hoc est. 24. adiecta fuerit fexta pars ipfius medietatis, quæ est. 4. tunc habebimus. 28. & ita dicemus quod. 48. Blanci conftituunt Florenos. 28. quod quidem verum est.

Huiusmodi autem rei fpeculatio ita fe habet. Nam vnufquifque Blancus diuiditur in. 7. æquales partes, quarum. 12. conftituunt vnum Florenum, horum verò numifmatum communis menfura, vocatur Groffus, vt fcis, ex quo fequitur, quod fi

28. Floreni æquantur Blancis. 48. tot Grossi erunt in. 28. Florenis quot in. 48. Blancis. Fingamus igitur, mente, nostram figuram. 79. Theorematis Arithmetici. x. u. o. e. n. supponendo ambo producta. u. x. et. n. e. inuicem equalia existere, & vnum quodque esse grossorum. 336. fit etiam. o. x. vnus Florenus. 12. grossorum. o. n. verò Blancus. 7. eorundem grossorum. o. e. autem Blancorum. 48. Nunc certi erimus ex. 15. sexti vel. 20. septimi Euclidis eandem fore proportionem. o. u. ad. o. e. quæ. o. n. ad. o. x. sed. o. n. est summa medietatis ipsius. o. x. cum sexta parte dictæ medietatis, ita igitur erit. o. u. ipsius. o. e. hoc est summa medietatis. o. e. cū sexta parte medietatis eiusdem, quæ summa in præsentī exemplo erit. 28.

Hac enim speculatione mediante, poteris methodum inuenire conuertendi Florenos in Blancos. Vt si nobis propositi fuerint Floreni. 28. Voluerimusque inuenire quot Blancos faciant, supposita mensura communi, iam supradicta. Nam duplicabimus numerum Florenorum, à quo duplo detrahemus septimam partem, reliquū verò erit numerus quæsitus.

Huiusmodi autem rei ratio est, quia, cum in supradicta figura, proportio. o. e. ad. o. u. equalis existat ei, quæ. o. x. ad. o. n. atque etiam. o. x. fit minor duplo ipsius. o. n. per septimam partem ipsius dupli. o. n. minor erit. o. e. duplo ipsius. o. u. per septimam partem eiusdem dupli ipsius. o. u.

Idem affirmo de quavis conuersione aliorum numismatum, quorum semper. o. x. maior sit. o. n. verò minor. Vt si. o. x. æquivaleret. 7. et. o. n. valeret. 4. et. o. e. valeret 42. quæ quidem. o. e. mensuraretur ab. o. n.

Si cuperemus scire quot. o. x. sint in. o. n. Primo dicemus in. o. n. reperiri summam medietatis sex septimorum ipsius. o. x. collectæ cum vna septima parte ipsius. o. x. seu (vt ita dicam) cum tertia ipsius medietatis. Vnde dempta septima parte ipsius. 42. quæ est. 6. collecta; cum medietate residui, quæ est. 18. habebimus. 24. res, quarum vnaquæque æqualis erit ipsi. o. x.

Sed si quis cupiat reperire. o. e. dato. o. u. duplicet. o. u. à quo demat quartam partem ipsius. o. u. & habebit propositum. Nam ita se habere oportet. o. e. ad. o. u. quemadmodum. o. x. ad. o. n.

### *De lucro mercantili.*

#### A D E V N D E M.

**Q**UOD demum scire à me desideras, est, quod cum vendideris libram vnā mercis pro. 4. solidis, & lucratus fueris. 2. cum quarta parte vnus pro singulis decem libris, scire velles quantum lucri facturus esses in libris decē dando singulam libram pro. 6. solidis.

Nulli dubium est quin decima pars de. 2. cum quarta vnus sit lucrum libræ vnus. Quæ decima pars sunt nouē quadragesimæ partes, & hæc subducta à solidis. 4. reliqui erunt solidi. 3. cum. 31. quadragesimis partibus pro sorte vnus libræ. Quæ fors subtracta à solidis. 6. remanebunt sol. 2. cum. 9. quadragesimis lucri pro libra, quod multiplicatum per. 10. proueniunt sol. 22. cum quarta parte vnus, & tantum ascenderet lucrum, quod fieri posset in libris decem si quamlibet, sol. 3. cum. 31. quadragesimis nobis constaret.

Vel sic multiplicemus sortem vnus libræ per. 10. productum erit. 37. cum tribus quartis

quartis, iterum multiplicemus per. 10. sortem cum lucro vnus librę quod est. 4. productum erit. 40. differens à primo sol. 2. cum quarta parte, multiplicemus pariter per. 10. precium. 6. solidorum proueniens erit. 60. à quo deducendo productum sortis librarum. 10. quod erat sol. 37. cum tribus quartis supererunt sol. 22. cum quarta parte, vt supra.

DE DIGNITATIBVS PLANETARVM.

*Adriano Panetio.*

**Q**UOD eam distinctionem orbium, quę iam inualuit, non teneas, sed putes totum esse quoddam continuum excipiens corpora stellarum, nouum nõ est, nam nonnulli solidę doctrinę Philosophi idem censuerunt. Sed quod attinet ad dignitates planetarum in signis zodiaci, scias huiusmodi ordinem me comprehendere esse desumptum ab ordine antiquo orbium ipsorũ planetarum, qui quidem ordo erat, vt statim post Lunam succederet Sol, post Solem Mercurius, tum Venus deinde Mars, postea Iupiter, & tandem Saturnus per eosdemq; orbis, retro redibant, atque hoc cognoscitur constituendo Cancrum domicilium Lunę, Leonem, Solis, Virginem, Mercurij, Libram, Veneris, Scorpionem, Martis, Sagittarium, Iouis, Capricornum, Saturni, Incipientes deinde ab Aquario, qui ad nos propius accedit eundemq; tribuentes Saturno, Pisces, Ioui, Arietem, Marti, Taurum, Veneri, & Gemellos, Mercurio, septem Planetas cum duodecim signis zodiaci concordēs reddebant.

Quod deinde Aristoteles in libris de sensu & ijs quę sensibus percipiuntur, dicit pupillam oculi esse nigram, non ita se habet, nam idem est, ac si quis diceret nigrũ esse illud medium, quod permittet transitum lumini per suam diaphaneitatem, nullo lumen à seipso reflectens, & etiam ac si quis diceret nigrum esse aerem alicuius cubiculi vnde quaque clausi tenebrofi.

Quod etiam idem Aristoteles volens adducere causam, cur oculus magis materiam aquę, quam aeris participet, dicens id ea ratione fieri, quod aqua magis quam aer seruari possit, eodem libro scribit, est reuera admirandum. Ibi enim clarę demonstrat se planę ignorare, & constructionem oculi, & causam diuersitatis eorum humorum tam in substantia, quam in figura, quę non aliunde dependet quam quod diuersam refractionem radiorum luminosorum producat, qui per pupillam ingrediuntur, vt ad proprios sibiq; destinatos locos dirigantur radij, vt à virtute visiva perfectius sentiantur.

*De ratione Frigiditatis locorum umbrosorum.*

A D E V N D E M.

**V**Era ratio vnde fiat, vt quanto magis sentitur calor in locis expositis Soli, tanto minus sentiatur in umbra, vbi Solis radius non reflectitur, est quia cum refractus est aer à vehemēti calore radij solaris, seipsum colligit, & condensatur in locis, à quibus à calore, ratione rarefactionis, non expellitur, & quia naturaliter calor sequitur rarum, rarum calorem, & frigidum densũ, & densũ frigidum, vt vnici que sanę mentis patet, hanc ob causam sequitur rem ita se habere vt diximus. Possimus etiam absque dubio credere huiusmodi ratione fieri, vt frigus matutini temporis, in crepusculo maius esse eo, quod noctu viguit. Nam materia consistens in cono umbrę terrę, semper densior est ea, quę extra reperitur, imo noua materia continuo condensatur, propter motum umbrę, quę semper corpori solari opponitur. hęc

Fff 2 autem

autem noua condensatio dico semper fit in crepusculo matutino, hoc est in parte con-  
ni à Sole pulsa, in parte vero contrari a ipsius con hoc est in parte crepusculi ve-  
spertini, contrarium accidit, quia potius aliquantulum in hac parte materia con ra-  
rificatur, quia extrinseca condensatur, in parte vero matutina extrinseca rarificatur;  
& propterea intrinseca condensatur.

QVOD RECTE ARIST. SENSERIT COELVM  
casu non esse productum.

*Hieronymo Condrumerio.*

**F**erunt Aristippū tempestate maris ad incognita littora delatum, cum in are-  
na vidisset quasdam figuras geometricas delineatas exultantē lætitia dixisse: Hæc  
sunt hominum vestigia. Nam consonum rationi non erat, vt huiusmodi figuræ casu  
essent impressæ: neque etiam credendum est ingentem hanc machinam tanto or-  
dine constantem fortuito esse productam, cum nulla quantumuis minima eiusdem  
particula, dummodo nitatur ordine, aliquo modo casu effecta fuerit; cum casus ni-  
hil producat, quod regulam & ordinem seruet. Non est igitur producta casu admi-  
randa correspondentia, quæ est obiectorum cum potentijs, luminis cum oculo, so-  
ni cum auditu, saporis cum gustatu, odoris cum odoratu, qualitatum tangibilium cū  
tactu. Si diligenter deinde cuiuslibet rei naturalis operationem considerabimus,  
eas tanta arte constructas videbimus, vt cogamur fateri aliquam prudentissimam,  
& sagacissimam mentem eas formasse, si ergo quælibet mundi pars tanta cum ratione  
& ordine est constructa: quomodo fieri poterit, vt de toto ipso mundo id in dubium  
vocemus, certissimeq; non credamus diuinissimam aliquam mētem esse à qua exqui-  
sissima huius vniuersi harmonia, quæ ex tot tantisq; partibus, maximo ordine ni-  
tentibus conficitur, non dependeat?

V A R I A R E S P O N S A.

*Nicolao Petreio.*

**A**D ea quæ mihi scribis dico, quod excrementa quæ ex corpore sano prodeunt  
in sua ipsorū qualitate sensibili ita se habēt ad facultatē illius partis eiusdem  
corporis, ut eā non lædant, quæadmodū efficeret sputū, si esset amarū, aut quod ex  
cernitur naso fetidū esset. Imagineris igitur quæadmodū possit esse verū id quod idē  
amic⁹ noster ait. Præterea si aliquid tibi in oculū inciderit, an nescis quomodo statim  
affatim affluat humor, vt id foras pellat, vel abducatur? (mirabile opus naturæ.) Dic  
etiā eidem non absque mysterio naturam in tot miserijs senectutem possuisse, cum  
sæpissimè senex mori desideret, ut huius vitæ calamitatibus liberetur, vnde fit, vt  
cum eius aduentum sentiat, minus affligatur. Dicitō etiam eidem, naturam non  
fuisse tam sollicitam de quibusdam partibus quemadmodum est de toto, vnde ma-  
gis rotunda, & polita poterat esse superficies terræ, quam nunc est, quia natura ma-  
gis respicit totum, quam partes, & magis maiores, quam minores.

Dum tuas legerem, me continere non potui quin riferim, id quod scribis te inter-  
rogasse eum Philosophum naturalem, vnde fit, vt ventus sit frigidus, eumq; tibi re-  
spondisse, quod à remotissimis partibus veniat, genereturq; à vaporibus terræ frigi-  
dis. (cum ipsa sit frigida.) Cæterum miror quod ab eo non quæsieris, vnde oriatur  
frigiditas, quæ percipitur ab agitatione aeris, qui quidem à vaporibus terræ non  
proflit, nec à remotissimis partibus ad nos accedit. Sed quia de eadem re me in-

terro-

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terrogas, scito naturā coniunxisse frigiditatem cū densitate, & caliditatem cū raritate, ut sup. diximus, ita ut cum aliquod corpus densat, frigidū reddat, & dum rarefit maiorem caliditatem acquirat, & sic e contra fit, ut quanto magis aliquod corpus refrigeratur, tanto densius reddatur, & quanto calidius fit tanto rarius efficiatur. Quoties igitur agitabitur aer, aut aliud corpus, quod ratione suā subtilitatis, velociter condensari, & rarefieri possit, eius partes densiores semper erunt frigidae, & hanc ob rem quilibet ventus, qui per calida loca non transeat, natura sua frigidus, calidus autem per accidens erit. Hinc fit ut vasa vitrea, & terrea tam in vehemēti frigore, quam in magno aestu frangantur, quia horum vnum fit, ne aliquis locus vacuus remaneat, & aliud ob loci necessitatem, sed hoc non sequeretur, si in materia, qua huiusmodi vas constat, aliqua aeris portio non contineretur.

DE LVMINE LVNÆ, DE FINE LVMINIS,  
de fine motus corporum cęlestium, de albedine,  
de sphaera.

*Clarissimo Antonio Nauaiero.*



**L**V MEN Lunæ etiam si sit lumen reflexum Solis ab ipsa Luna, ab ea tamen non ita reflectitur, ut à superficie polita speculi, cū eius luminis tantā quantitatem super ipsum corpus lunare videamus, & eo modo terminatā quo conspicimus. per se lumen, causa oculi est effectum, per accidens autem puta quod vis. Terra deinde nunquam lunari lumine (quāuis solaris reflexio existat) omnino destituta est, dico etiam, neque in ipsis eclipsibus solaribus vel lunaribus, in solaribus enim cum Sol tot millia vices maior sit Luna, Luna verò minor terra, sequitur, ut terra non omnino priuata remaneat lumine Lunæ, in eclipsibus verò lunaribus Luna semper videtur, gratia luminis solaris, quamuis refracti. Motus corporum cęlestium fit ratione situs, & varietatis virtutis stellæ in diuersis locis, hæc autem varietas absque diuerso situ eiusdem stellæ, nec diuersus hic situs absque motu fieri posset, ita ut motus stellarum sit ratione diuersitatis situum ipsarum, ergo motus, & diuersitas situum, fit, ob diuersam influentiam. Quæ autem de albedine fratri tuo dixeram, erant, quod inter oēs colores albedo, certo quodam modo, maiorē similitudinem habet cum lumine. Primò quia magis coniungitur cum lumine. Secundò quia magis afficit sensum. Tertio quia absque resistantia magis recipit qualitatem aliorum colorum, quam alij colores. Quartò quia maximus est omnium colorum. Quintò quia simplicior est reliquis. Sextò quia disgregat visum. Septimò quia qualitas quæ in niue alba esse videtur, nihil aliud est quam multitudo quædam luminum reflexorum, & non albedo, similis ei, quæ est lactis, aut panni, quæ quidem septima causa efficit, ut ipsam albedinem, magis quam alium quemuis colorem cum ipso lumine compararem, cum nihil sit, quod esse suum trāsmutans, aut apparenter, aut essentialiter, illud ipsum prius non transmutet in formam sibi propinquiorē, ut manifestè patet. Est etiam huius rei octaua ratio, magni ponderis, quia scilicet nullus sit color, qui magis resistat lumini, aut in quem lumen minorem impressionem faciat, quam albedo. Vnde sequitur, obiecta alba, minus esse combustibilia quam alia, cum quælibet res in suum contrarium quam in sibi

sibi simile valentius agat, vt rectè vidit Aristoteles cum dixit, omne contrarium à suo contrario patinatum est.

Inter corpora, multum simplicitatis retinet sphaera.

Circa quod, præter rationes adductas ab Aristotele in libris de Cælo, possumus etiam ratiocinari à facilitate motus vndiq; ab eo quod violentiæ non resistat, ab eo, quod apta nataq; sit quiescere supra quoduis punctum suæ superficiæ, ab eo quod ab aliqua superficie alterius corporis sese tangi non permittat, quæ curvitate concava non adæquetur, nisi medio vnius puncti. Verum est, quod licet hæc vltima ratio nō sit propria sphaeræ, est tamen causa simplicitatis in eo, in quo reperitur, sed propriæ passionis sphaeræ sunt supradictæ, præter quam quod alia eiusdem sphaeræ est propriissima, quæ est distantia eius termini ab vno tantummodo puncto idest ab eiusdem centro, & etiam posse diuidere corpus aliquod medium, cum æquali resistentia circa punctum, quod prius in motu reperitur.

Æqualitas autem rerum, est etiam valde similis simplicitati, & vnitati.

*Comparatio visus, & auditus.*

A D E V N D E M.

**Q**uod ad visum & auditum attinet, magis necessarium esse visum, & nobilio-rem quam auditum existimo, primò quia si quis visu orbatus esset, contra frigus, & calorem, contra famem, & sitim nil providere posset, neque aliud quicquam hoc vocabulum providere significat, neque absque periculo vitæ ab vno loco ad alium ferri posset, neque aliquid arte facere.

Sed si quis destitutus esset facultate audiendi, supradictas tamen operationes præstare posset, neque modo careret, quo animi sui sensa absque beneficio soni, sed ope figurarum & characterum alteri aperiret: neque etiam munere speculandi scientias (excepta musica) destitueretur. Ad scientiam comparandam, longè magis necessarius est visus, quam auditus præterquam, quod visus maiorem numerum obiectorum, & differentiarum rerum percipit, & inter reliquos sensus velocissimè imò in instanti operatur, magis remotè quam alij, & exactius sentit, minusq; quam reliqui afficitur, præterquam quod semper agit, dummodò non dormiat animal. Præterea sese magis patefacit, & prodit anima per oculos, quam per aliud, cuiuslibet sensus, instrumentum. Oculo magis quam alia corporis parte, hominis natura cognoscitur: & si aliquid speculari volumus, quod sine imaginatiua fieri non potest, statim imaginamur nos videre huiusmodi rem, ac si oculo fuisset comprehensa, & ab imagine quæ est vnum ex obiectis oculi, imaginatiua nuncupatur. Beneficio oculorum omnes ferè scientiæ sunt adinuentæ. Auditus nil aliud quam sonum capit, auditus nunquam detulit intellectui figuram, corpus superficiem, aut lineam, materiam, formam, locum, dimensionem, plenum inane, nec innumera alia accidentia, quæ ab oculo comprehenduntur. Quæ verò visui, & auditui sunt communia, sunt etiam tactui communia, vt numerus, motus, maius, & minus, sunt tamen alia qua oculo & tactui communia, quæ auditus non potest capere, vt durum, mollè, acutum, obtusum, asperum, lenè, planum, curuum, concauum, conuexum, magnum, paruum, & supradicta, idest figura corpus & cætera, vt etiam rectum, obliquum, & similia.

Aristo-

Aristoteles circa finem primi capitis libri de sensu ait mediante visu, magis quàm quolibet alio sensu, nos percipere sensibilia communia. Vbi eundem per se, & non per accidens magis necessarium esse quam auditum, tam in ijs quæ ad victum, quam in ijs quæ ad scientiam pertinent esse asserit, quia auditus intellectui confert per accidens. Vide etiam quod idem scribit primo metaphysicorum. Et si ad aliquid perfectè cognoscendum, oculo sese nobis offerrent ea omnia obiecta, quorum species in imaginatiua formamus, ipsa imaginatiua non egeremus. Sed quia hoc fieri non potest, hunc thesaurũ imaginatiuè, seu memoriæ ad conseruandam imaginem omnium obiectorum sensibilem nobis dedit natura, vt ope discursus intellectus circa dictas imagines, rerum veritatem venari possimus. Sed vt ad propositum redeamus, beneficio oculi animal liberum est, cum sine ipso locum mutare nequeat, vt sit tutum. tenebræ, priuatioq; visus sunt ferè vnum, & idem. Neque vllus est sensus, qui sit magis similis intellectui quam visus: neque alij sensus habent obiecta viciissim communia, quæ non sint etiam oculo communia, sed inter oculum, & quem libet alium ex sensibus, inuenientur quidem obiecta communia, quæ cum alijs non communicabunt, vt inter oculum & tactum, figura, acutum, obtusum, & similia, quæ alijs sensibus non percipiuntur. Mediante visu, & auditu etiam, comprehenditur variæ distantia, situsq; obiectorum, nec non proportiones, & alia quæ ab alijs sensibus non comprehenduntur. Multa obiecta deinde sunt subiecta gustatui, quæ alijs accidentibus prædita sunt, vnde cum fuerint semel degustata, talia, qualia sunt ab oculo percipiuntur, quod nullus ex alijs sensibus præstabit. Idem de obiectis odoratus dico. Sensuum nullus est qui maiorem similitudinem gerat cum vigilia & cum vita, quam visus, neque aliquid est, quod magis repræsentet imaginem somni, & mortis, quàm cecitas.

Qui sibi oculos eruit vt melius specularetur maxima stulticia prius obcecatus fuit quia soni magis impediunt speculationem quàm lumina, imò qui commodè vult contemplari, quantum plus potest nititur longius esse ab omni strepitu, magis quàm à locis luminosis, & animal magis lætatur lumine quam sonio: & ad speculationem nos magis inuitat harmonia luminum variorum colorum & figurarum, quàm harmonia sonorum, præterquam quod instrumentum visus totius corporis partium est pulcherrima, & in eminentiori loco locata, si de instrumentis sensuum loquamur, & veluti finestræ animæ. Et si Aristoteles dicat oculos & aures in vno eodemq; orbe existere, volens inferre quod in eodem æquilibrio sint æqualiter alta non ita se habet, quia (si de homine loquamur) oculus est altior aure. Beneficio huius sensus, eorum quæ absunt, & longo iam tempore sunt mortui, animi sensa, & conceptus intelligimus, neque alia ratione rerum omnium memoria seruari potest. Si cabala unquam vera fuit, nulla alia ratione est deleta, quam quia alicuius signi visibilis medio conseruata non fuerit, & quæcunque non scribantur, id est oculo non cõmendantur parum durant cito obliuioni tradunt. In maiori semper pretio fuit pictura quàm musica: obiectis visibilibus magis quam ijs quæ sub auditu cadunt, affectus animi, atq; alia quælibet res naturalis exprimi possunt. Aegyptij volentes significare Deum, oculi medio id præstabant.

Oculus, respectu aliorum instrumentorum sensuum, est quasi epicyclos animæ, neque defuit qui crederet oculum esse principem animi partem.

Oculus à Sole, & à Luna ita dependet, vt qui tempore defectus cuiuslibet luminaris nascitur, statim cæcus euadat, neque aliqua est corporis pars in qua magis appareat

pareat differentia vitæ à morte; quam in oculo. Aristoteles ad finem cap. 15. lib. primi posteriorum ait, clarum esse quod si aliquis sensus deficiat, futurum ut aliqua quoque scientia desit. Considera, quot scientijs careret homo, si visu orbaretur.

Et in tertio de anima ait, eum qui non sentit, nihil intelligere posse; id quod inde confirmat, quia nihil fit in intellectu, quod prius non fuerit in sensu. Plato in timæo ait, oculos nobis attulisse rerum optimarum notitiam, & si oculus non fuisset nihil eorum, quæ ad cælum spectant inueniri potuisse, & cognitione diei ac noctis ab oculis ortum duxisse, ut reuolutiones mensium, & annorum metiri, & tempus agnoscere, & inuestigare ordinem naturæ vniuersalis possemus; quibus philosophiâ nobis comparauimus, ut alia multa omittam, quæ ibi à Platone dicuntur. Addam hic & aliam specialem differentiam inter auditum & visum, quæ est, ut obiectum visus sit permanens, & obiectum auditus transitorium siue successiuum aut, ut alio modo idem dicamus, obiectum visus participes sit æternitatis, illud autem quod est auditus non item, nam auditus temporis subiectus est, visus autem minimè. Vel si dicamus operationem auditus absque tempore fieri non posse cum sit motio, operatio verò visus, nullo indiget tempore, cum ipsa sit momentanea, & propterea instantanea. Nam momentum non est motus, nec instans tempus.

### QVARE HYEME VIDEATUR HALITVS

animalium non autem æstate, & de vento.

*Pancratio Mellano.*

**V**ND E fiat ut hyeme halitum nostrum videamus, & non æstate, ratio est ab eiusdem halitus congelatione, quæ ab extrinseco frigore fit. Prius enim scire debes aerem attractum in pulmone, foras deinde erumpere cum alio vapore aliquantulum crassiore humido, & excrementitio expulso à natura, quæ continuo nostrum corpus euaporare facit, vnde sequitur dum aer foras à pulmone pellitur, maiorem semper materiæ portionem, ea quæ intus attracta est exire: vnde statim ut dicta materia foras expulsa, frigidum aerem offendit, cum constet ex partibus crassis, & obnoxiiis congelationi, condensatur in formam vaporis, ad differentiam aeris ambientis qui in se eas partes crassas non habet, à quibus quidam partibus condensatis, & redditus opacis reflectitur lumen, atque hanc ob causam æstate hoc non fit, quia calor vim condensandi non habet.

Ventus nihil aliud est quam quidam aeris motus, cum condensatur, ob defectum caloris, neque (pace Aristotelis dicam) est exhalatio sicca. Exemplum à Vitruuio allatum nil planè valet, quantum spectat ad venti naturam, cuius rationem à me requiris. Exemplum etiam ventilabri quo tempore æstate utimur negligendum penitus non est, quia eius beneficio, non solum arcemus à nobis aerem ambientem calidum, sed alium etiam aerem circa nos condensamus: & quia ordo naturæ est huiusmodi quod quemadmodum calor sequitur raritatem corporum, sic etiam frigus eorundem densitatem sequatur. Quod si vis ut exemplo illustrem, diligenter obseruato tempore æstatis cum aliqua nubes nobis Solem adimit, vbi aer qui in eius  
vmbra



umbra reperitur, tantum quantum defectus caloris radij solaris fert, qui per vim, dictum aerem rarefactum conseruabat, statim dictum aerem condensari cognosces: & quia ea condensatio homogenea non est, ob diuersas rationes; hanc ob causam percipimus eam aeris impulsione, & inæqualiter, dum verò eadem umbra discedit, ventus, ferè, statim cessat, & saepe ante quam dicta umbra discedat; cuius rei causa est longa mora quam trahit umbra, ita vt prius absoluatur reditus aeris ad formam, quæ ei conuenit in huiusmodi umbra, quam faciet nubes dum Sol deregitur.

Vera non sunt ea, quæ tibi Arnoldus dixit, vt mihi tuis literis significasti. Nam ego ita dixi, videlicet, quod quotiescunque aliquis aspexerit aliquod punctum in superficie speculi, tunc imaginem ipsius post dictam superficiem videbit duplicatam, si verò aspexerit imaginem intra speculum, tunc illud punctum videbit duplicatum, huiusmodi autem rei ratio pendet ab hijs quæ ad Franciscum Vimercatum scripsi, quæ si memoria tenes, nullum tibi dubium remanebit. Nam ea tibi omnia ostendi.

Dum verò dicis omnem proportionem rationalem diuidi posse duobus numeris mediantibus in tres æquas partes, mihi ad memoriam reuocas id quod quidam Vitruuij commentator asserit super primum cap. noni lib. eiusdem Authoris, ita dicens.

» Benè esse potest vt diagonalis ( quadrati scilicet ) numerorum via reperiat, sed  
» fortasse intercedent fracta.

Miror te non memoria tenere quid sint numeri rationales quidue surdi, neque consideras, non solum non esse diuisibilem in tres æquas partes omnem proportionem rationabilem, sed neque in duas, vt sunt superparticulares proportiones, nec non alia innumeræ, sed cum talia scribis te nimis parum versatum in istis rebus ostendis.

Id verò quod tibi dicere volebam nudius tertius de Mercurio erat, quod nullo pacto confidendum est calculis qui fiunt de cursu Mercurij, eo quod eius situs nullo modo obseruabilis est, nam ipse nunquam nec vbiuis locorum orbis terrarum visibilis est altior. 1. gradibus supra horizontem, sed neque confidendum esset si etiã ipsum videremus altum. 20. gradibus, propterea quod magna refractione radiorum infra hos gradus nos valde fallit, quæ quidem refractione, nec vbiq; nec omni tempore vniformis est, propter difformem seu inæqualem crassiciem vaporum quæ continuo mutatur. Imo multoties cum videre putabimus supra horizontem, existente ipso sub horizonte.

*Quod Ouidius transcurrit à motu diurno, ad motum annuum  
præter rem.*

A D E V N D E M.

**T**Vus etiam Ouidius cespitauit, cum pro itinere vnus diei efficiat, vt Phaeton à patre edoctus sit etiam de itinere annuali.

Nam, quod Phaeton petat pro cursu vnus diei, clarè patet ex diuersis locis, & primo vbi ita scribit Ouidius.

» Currus petit ille paternos.

» Inq; diem alipedum ius & moderamen equorum.

Deinde vbi Pater ita loquitur.

» Ardua prima via est, & qua vix manè recentes.

Ggg

Enitun-

„ Enituntur equi medio est altissima celo.  
 „ Vnde mare, & terras ipsi mihi sæpe videre.  
 „ Fit timor & pauida trepidat formidine pectus.  
 „ Vltima prona via est & eget moderamine certo.  
 „ Etiam vbi dicit.  
 „ Dumq; ea magnanimus Phaëron miratur, opusq;  
 „ Perspicit, ecce vigil nitido patefecit ab ortu.  
 „ Purpureas aurora fores, & plena rosarum.  
 „ Atria, diffugiunt stellæ, quarum agmina cogit.  
 „ Lucifer, & cœli statione nouissimus exit.  
 „ Nec non vbi ita inquit.  
 „ Et si (modo credimus) vnum  
 „ Issè diem sine Sole ferunt, incendia lumen  
 „ Præbebant.  
 „ Quod autem à Patre instruatur etiam de cursu annuali,  
 „ videbitur vbi ita dicit.  
 „ Nitore in aduersum, nec me, qui cætera vincit.  
 „ Impetus, & rapido contrarius euehor orbi.  
 „ Et vbi ita loquitur.  
 „ Forsitan & lucos illic, vrbesq; Deorum.  
 „ Concipias animo delubraq; ditia donis  
 „ Esse per infidias iter est, formasq; ferarum.  
 „ Vtq; viam teneas, nulloq; errore traharis.  
 „ Per tamen aduersi, gradieris cornua Tauri.  
 „ Aemoniosq; arcus, violentiq; ora Leonis.  
 „ Sæuaq; circuitu curuantem brachia longo.  
 „ Scorpion atque aliter curuantem brachia cancerum.  
 „ Nec tibi quadrupes animosus ignibus illis.  
 „ Quos in pectore habent quos ore & naribus efflant, &c.  
 „ Sed lucidius etiam hoc videre est inferius vbi ita loquitur.  
 „ Nec tibi directos placeat via quinque per arcus.  
 „ Sectus in obliquum est lato curuamine limes.  
 „ Zonarumq; trium contentus sine, polumq;  
 „ Effugit australem iunctamq; aquilonibus arcton.  
 „ Hac sit iter, manifesta rotæ vestigia cernes.  
 „ Et vbi etiam dicit.  
 „ Ne te dexterior tortum declinet ad anguem.  
 „ Ne ve sinisterior pressam rota ducat ad aram.

*De supputatione quinque corporum regularium.*

*De aliquibus etiam eorum sympathijs.*

A D E V N D E M.

**I**D quod à me desideras, ab alijs etiam factum est, sed ne me putes laborem euitare, non prætermittam aliquid tibi scribere, earum rerum quæ ab Euclide, colle-

gi, methodo etiam qua utebar dum in istis rebus me aliquo modo exercebam.

Quotiescunque igitur scire volueris quantitatem corpulentiae cuiusq; quinq; corporum regularium ab vna eademq; sphaera terminatorum seu circumscribibilium curabis primum, cognoscere quantitatem lateris cuiusq; eorum, talium partium, quantum semidiameter dictae sphaerae sit. 100000. ex tabulis sinuum Nicolai Copernici. Propone igitur tibi ante oculos figuram semicircularem vltimae propositionis. 13. lib. Eucli. & inuenies. c. d. tertiam partem semidiametri. d. b. esse partium. 33333. æqualem sinui arcus. f. e. graduum. 19. mi. 28. qui quidem arcus deptus cū fuerit à tota quarta. b. f. remanebit arcus. e. b. gra. 70. mi. 32. cuius corda erit latus exaedri, quod latus ita cognosces, sumendo scilicet sinum medietatis. b. e. hoc est sinum gra. 35. mi. 16. qui erit partium. 57738. cuius duplum erit partium. 115476. pro latere cubi.

Dempto postea quadrato lateris exaedri, & quadrato totius diametri. a. b. residui radix quadrata, erit. a. e. latus Tetraedri. Vel si duplicaueris sinum dimidij arcus. a. e. qui quidem arcus, componitur ex quarta. a. f. & ex arcu. f. e. iam inuento, siue, vt residuus totius dimidij circuli, dempto. b. e. iam supra inuento, habebimus idem latus. a. e. partium. 163294.

Pro latere verò Octaedri accipere potes radicem quadratam dupli quadrati ipsius. d. b. & habebis. f. b. latus quaesitum. Vel si malis accipe duplum sinus medietatis arcus. b. f. quod duplum erit. f. b. partium. 14142.

Pro latere verò Duodecaedri, diuide latus Exaedri ex methodo. 11. secundi Eucli. cuius maior pars erit latus quaesitum, partium. 71368.

Sed pro latere Icofaedri, te primum oportebit inuenire quantitatem anguli g. d. a. hoc est ipsius arcus. b. n. qui tali angulo subiacet, quod cum pluribus modis inueniri possit, nihilominus, hunc seruabis, inuenies primò quantitatem. d. g. quæ est radix quadrata summæ duorum quadratorum hoc est. d. a. et. a. g. quæ. a. g. æqualis est diametro. a. b. vt scis, dices postea, si. d. g. correspondet ipsi. g. a. cui correspondet. d. h. semidiametro sphaerae tibi veniet. h. k. sinus arcus. a. h. hoc est. b. n. graduum. 63. min. 26. cuius medietas gra. 31. mi. 43. pro sinu suo habet partes. 52571. cuius sinus duplum est partium. 105142. pro latere Icofaedri.

Incipiendo nunc à Tetraedro, scire debes, quod pars. a. c. totius diametri. a. b. æqualis est axi ipsius Tetraedri, quæ quidem. a. c. vt subsesquialtera ipsius. a. b. erit partium. 13333.

Quæres postea quantitatem superficiale[m] vnius faciei ipsius Tetraedri, hac methodo, inueniendo primum radicem quadratam trium quartarum quadrati ipsius. a. e. lateris Tetraedri, eo quod latus hoc, sesquitertium in potentia est ipsi perpendiculari terminatæ ab vno angulorum trianguli æquilateris & à latere ei opposito ex. 11. tertijdecimi ipsius Eucli. quæ quidem perpendicularis, erit partium. 141416. & hæc multiplicata cum medietate lateris trianguli, hoc est cum. 81647. tibi dabit superficiem quaesitam, hoc est basim Tetraedri partium superficialium. 11546192152. Hæc demum basim multiplicando cum tertia parte axis Tetraedri habebis corpulentiam totius Tetraedri, quæ erit. 513158964003488.

Neque tibi hoc loco occultare volo quandam meam animaduersionem, quæ est, quod diameter seu perpendicularis (supradicta) faciei ipsius Tetraedri, semper æqualis est lateri ipsius Octaedri circumscribibilis ab eadem sphaera, hoc est ipsi. b. f. quapropter quotiescunque ipsam perpendicularem habere voluerimus accipiendo b. f. habebimus intentum. Et quod hoc verum sit possumus ita demonstrare.

Primum, notum nobis est, ipsam perpendicularem, triplam esse eius parti, quæ

à centro circuli, ipsum triangulum circumscribentis, terminatur, & à basi, vt in tertio proposito decimaseptimæ quartidecimi Eucli. probatur, ex quo sequitur proportionem huiusmodi perpendicularis ad axem Tetraedri, hoc est ad a. c. sesquioctauam esse in potentia, ex penultima primi Eucli. Sed cum d. c. tertia pars sit ipsius d. a. vt etiam ex 2. proposito, seu corollario decimaseptimæ. 14. lib. discurre licet, cum ex dicto corollario d. c. sit sexta pars ipsius a. b. Quare d. c. quarta pars erit ipsius a. c. vn de a. c. sesquitertia erit ipsi a. d. in longitudine, ideoq; quadratum ipsius a. d. ad quadratum ipsius a. c. erit vt. 9. ad. 16. & ita duplum quadrati ipsius a. d. hoc est quadratum ipsius b. f. ad quadratum ipsius a. c. erit, vt. 18. ad. 16. hoc est sesquioctauum, ergo b. f. æqualis erit dictæ perpendiculari, ex. 9. quinti.

Cubus postea ipsius b. c. erit partium. 1539838575570176.

Pro Octaedro deinde, accipies productum diametri in semidiametrum, quod productum, æquale erit quadrato diuidenti per æqualia Octaedron, hoc igitur productum, multiplicando per. 100000. semidiametrum sphaeræ, tibi dabit columnam quadrilateram cuius tertia pars, erit partium. 6666666666666666. cuius duplum erit ipsum Octaedron partium. 1333333333333.

Pro Icosaedro autem, oportet prius quantitatem perpendicularis inuenire, quæ perpendicularis, per æqualia diuidit basim ipsius Icosaedri, quæ vt radix quadrata trium quartarum quadrati lateris ipsius basis, erit partium. 91055. talium, qualium dictum latus erit partium. 105142. cuius medietas est. 52571. quæ medietas si multiplicata fuerit cum dicta perpendiculari, dabit totam basim superficialem, hoc est superficiem vnus trianguli æquilateris partium superficialium. 4786852405. quo facto, accipe quadratum duarum tertiarum ipsius, hic supra dictæ perpendicularis, ipsumq; dente ex quadrato semidiametri sphaeræ, hoc est, ex quadrato ipsi. 100000 radix postea quadrata residui, erit partium. 79468. & hæc erit perpendicularis à centro sphaeræ ad vnam basim ipsius Icosaedri, quam volueris, quam perpendicularem si multiplicaueris cum quantitate superficiali, hic superius reperta, vnus basis, consequeris columnam trilateram partium. 380401586920540. cuius tertia pars, erit partium. 126800528973513. pro vna ex. 20. Pyramidibus ipsum corpus componentibus. Breuius tamen hoc efficiens, si multiplicaueris basim dictam, cum tertia parte ipsius perpendicularis, hanc postea pyramidem multiplicando per. 20. habebis totam corpulentiam ipsius Icosaedri partium. 2536010579470260.

Pro Duodecaedro demum, accipe sinum gra. 36. qui grad<sup>o</sup> sunt pro dimidio quintæ partis totius gyri circularis, quæ quidē sinus, erit partium. 58778. cuius quadratum si desieris ex quadrato ipsi. 100000. semidiametri circuli circumscribentis aliquē pentagonum æquilaterum, & æquiangulum, tunc radix residui, erit perpendicularis ducta à centro dicti circuli ad medium vnus lateris ipsius pentagoni, quæ perpendicularis, erit partium. 80902. talium qualium medietas lateris dicti fuerit. 58778. Nunc verò dicendo si. 58778. dat. 80902. quid nobis dabit. 35684? medietas lateris ipsius Duodecaedri, vnde dabit. 49116. pro perpendiculari, à centro ipsius pentagoni, ad latus ipsius Duodecaedri, quæ multiplicata cum medietate supradicta ipsius lateris, hoc est cum. 35684. producet vnum ex quinque triangulis componentibus vnus pentagonum, seu vnus basim ipsius Duodecaedri, quod quidem triangulum, erit partium. 1752655344. superficialium, quas si per quinque multiplicaueris habebis vnus basim pentagonam dicti corporis partium. 8763276720. Dicendum postea est, si ad. 80901. conuenit semidiameter circularis partium. 100000. quid conueniet partibus. 49116. dabit. 60711. pro tali semidiametro circulari, cuius quadratum,

tum, si dempseris ex quadrato ipsius. 100000. semidiametro sphaerae, tunc radix quadrata residui, erit perpendicularis à centro sphaerae ad centrum pentagoni partium. 79461. cuius tercia pars, si multiplicata fuerit cum pentagono supra reperto dicti corporis producet vnam ex. 12. pyramidibus componentibus dictum Duodecaedron, quae pyramis, demum, multiplicata per. 12. dabit totam corpulentiam ipsius Duodecaedri partium. 2785354925791680.

Nunc verò si experiri voluerimus vtrum isti calculi duorum corporum maiorum sint rectè supputati, dicem⁹ si ad corp⁹. 12. basiū, quod est partiū. 2785354925791680 conuenit numerus partium. 2536010579470260. ipsius Icosaedri, quid conueniet lateri cubi partium. 115476. & inueniemus conuenire latus ipsius Icosaedri partium 105138. eo quod probatum sit in. 10. propositione. 14. li. Eucl. eandem proportionē esse corpulentiae ipsius Duodecaedri ad corpulentiam ipsius Icosaedri, quae lateris cubi ad latus Icosaedri.

Haec autem corpora, ita sibi inuicem, & cum eorum sphaera harmonicè conueniūt quemadmodum antiqui philosophi inuenerunt, vt mirandū non sit, ipsos credidisse omnia quae natura constant, aliquo pacto ex istis corporibus fieri. Considera quae so quomodo conueniant inuicem Tetraedron, Octaedron, & Icosaedron, cum uniuscuiusque bases sint triangulares aequilaterae intelligendo semper haec corpora ab eadem sphaera circumscribilia.

Octaedron, cum Tetraedro etiam in hoc conuenit, quod latus Octaedri aequale sit ei perpendiculari, quae diuidit basim Tetraedri per aequalia, vt supra demonstrauimus.

Harmonicis etiam interua illis haec duo corpora inuicem concordantur, cum basis Tetraedri ad basim Octaedri seruet proportionem sesquiterciam, consonantiae diatessaron. Et proportio omnium superficierum siue basium Octaedri simul sumptarum, ad omnes bases ipsius Tetraedri simul sumptas sit sesquialtera, consonantiae diapentis. Neque omittendum est, quod proportio Octaedri ad triplum Tetraedri sit, vt latus Octaedri ad latus Tetraedri.

Proportio verò lateris Octaedri, ad axem Tetraedri, potentia est sesquioctaua, vt supra vidimus interuallum scilicet harmonicum toni maioris.

Harmonia verò Tetraedri, & Exaedri cū eorum sphaera, talis est, vt proportio diametri sphaerae, potentia, tripla sit lateri Exaedri, & sesquialtera lateri Tetraedri, ex quo sequitur latus Tetraedri potentia duplum existere lateri Exaedri. Interuallum enim triplum in harmonicis, componitur ex diapason, & diapente, & sonat speciem diapentis. Duplum verò est diapason, sesquialterum autem est diapente, quae consonantiae perfectissimae sunt.

Proportio verò diametri sphaerae, potentia dupla est lateri Octaedri, consonantiae diapason. Ex quo sequitur proportionem lateris Tetraedri ad latus Octaedri, potentia, sesquiterciam esse, hoc est consonantiae diatessaron, & proportionem lateris Octaedri ad latus Exaedri, potentia, sesquialteram esse, ita quod quatuor istae potentiae, id est diametri sphaerae, lateris Tetraedri, lateris Octaedri, & lateris Exaedri constituunt harmoniam ferè perfectissimam, ijs terminis comprehensam. 6. 4. 3. 2. (dixi ferè, quia ditonus supra terminum. 3. vel semiditonus sub termino. 2. hoc loco non reperitur, cuius quidem terminus esset. 2. cum duabus quintis.)

Adde quod diameter sphaerae triplus est longitudine ad perpendicularē ductam à centro sphaerae ad basim Octaedri, quae proportio, vt supra dictum est, dicitur diapason diapente, practici verò eam vocant duodecimam.

Diameter verò sphaerae sesquialter est longitudine axi Tetraedri, consonantiae diapentis. Axis autem Tetraedri sesquitercius est longitudinis semidiametro sphaerae consonantiae diatessaron. Ita quod isti tres termini, qui sunt, diameter sphaerae, axis Tetraedri, & semidiameter sphaerae constituunt etiam valde perfectam harmoniam huiusmodi numeris contentam. 6. 4. 3. corpulentia verò Exaedri ad corpulentiam Tetraedri tripla est, consonantiae iam supradictae diapasondiapente. Si verò de vnisono aliquid videre desideras, considera aequalitatem dupli quadrati diametri ipsius sphaerae, cum omnibus basibus Exaedri, vel potentia diametri sphaerae cum duabus potentijs simul sumptis, quarum vna est lateris Tetraedri, reliqua verò lateris Exaedri, vel aequalitatem numerorum laterum Tetraedri, cum basibus Exaedri. Nec mihi videretur silentio inuoluendum esse, antequam ulterius progrediar notabilem sympatiam inter triangulum aequilaterum, & Tetraedron (quauis triangulum corpus non sit) non solum ob inalterabilitatem harum duarum figurarum. (nam omnes aliae alterabiles esse possunt, iisdem lateribus existentibus, cum ex quadrato rombus, vel ex pentagono equiangulo, pentagonum non aequiangulum &c. efficiatur) sed quod quemadmodum latus trianguli aequilateri sesquitercium potentia est perpendiculari ipsum per aequalia diuidenti, ita latus Tetraedri, sesquialterum est potentia axi ipsius Tetraedri, vnde cum dempta fuerit illa proportio sesquitercia, ex hac sesquialtera relinquetur nobis proportio sesquioctaua, inter perpendicularem trianguli, & axem Tetraedri (quod etiam supra demonstrauimus.) Transeamus nunc haec, nec omitamus tamen sympatias quasdam inter Exaedron, Octaedron, & Tetraedron, hoc est quod eadem proportio sit inter corpulentias Exaedri, & Octaedri, quae inter eorum superficies, nec non, vt latus Exaedri ad semidiametrum sphaerae. Proportio verò basis Exaedri ad basim Tetraedri, vt latus Tetraedri ad perpendicularem diuidentem per aequalia eius basim.

Hactenus satis dictum sit de Tetraedro, Exaedro, & Octaedro cum sphaera. Dicendum nunc censeo aliquid de reliquis duobus mirabilibus corporibus, quamuis ferè omnia haec ab antiquis philosophis inuenta sint, quorum primum est, quod tam basis Duodecaedri, quam Icosaedri, ab vno eodemque circulo circumscripibiles sunt, verum, talis passio accidit etiam basibus Exaedri & Octaedri. Praeterea quemadmodum in Duodecaedro, quilibet angulus solidus terminatur tribus angulis pentagonorum aequiangulorum ita in Icosaedro, quilibet angulus solidus viceversa terminatur quinque angulis triangulorum aequiangulorum. Et tam vnum, quam alterum horum corporum, triginta lateribus continetur. Et tot solidos angulos triangulares, habet Duodecaedron, quot bases triangulares continet Icosaedron.

Et Icosaedron, tot solidos angulos pentagonos, quot bases pentagonas habet Duodecaedron. Et tam vnum quam alterum habet. 60. angulos superficiales. Eademque proportio est omnium basium simul sumptarum Duodecaedri ad omnes bases simul sumptas ipsius Icosaedri, quae corpulentiae ipsius Duodecaedri ad corpulentiam Icosaedri (quamuis haec passio accidat Exaedro cum Octaedro, vt supra diximus) quae quidem proportio, eadem etiam est, quae lateris Exaedri ad latus Icosaedri, vt supra iam dictum fuit.

NOVA

NOVA INVENTIO COMPONENTI ASTROLABIA  
cum Horologijs artificialibus.

*Jacobo Mayeto Ingeniosissimo Horologiorum Serenissimi  
Sabaudia Ducis Artifici.*



NONNUMQUAM consideraui mirabilem pulchritudinem, simul cum utilitate coniunctam, illorum horologiorum, quæ in Germania construuntur cum mobili Rete, seu Aranea Astrolabij super Tabulam regionis, in quibus cõtinuo vident oriri, occidereq; cælestia signa, cælum mediare supra orizontem, necnon sub eo, & ut vno verbo dicam, continuo erecta videtur tota cæli figura. Sed quia talia horologia omnia eorum limbum distinctum habent in .24. horas, quapropter diametrum limbi, minorem duobus palmis, seu semipede esse non oportet ne interstitia horarum iustò breuiora seu angustiora efficiantur, etiam ne intervalla dentium rotæ indicis nimis angusta sint. Sed quia talis magnitudo ut plurimum incommoda existit. Ideo non inutile fore cogitaui, si modus aliquis inuentus fuerit, ut ea omnia efficiantur in limbo diuiso tantummodo in .12. horas æquales, ipsumq; inueni, qui quidem erit, efficiendo ut Tabula (in qua designantur cælestes domus, cum almicantharum, atque azimuth) Reti subiectæ, mobilis sit, tardior tamen ipso Rete cum indice, pro duplo temporis, hoc est, quod eo tempore, quo Aranea cum indice circumuoluetur spacio .12. horarum vno gyro perfecto, ipsa Tabula efficiat tantummodo sex interstitia horarum. Id est dum Tabula dicta efficit vnã integrã reuolutionem, Aranea, seu Zodiacus cum indice, duas efficiat reuolutiones. Ita quod Aranea cum indice perficiet vnã reuolutionem spacio temporis, .12. horarum, Tabula verò perficiet eam spacio temporis, .24. horarum. Vnde sequetur quod Aranea seu Zodiacus cum indice, spacio .24. horarum perfectè circumuoluetur supra Tabulam, & ita huiusmodi horologia, in hoc nihil differrent ab illis supradictis. Ut autem facias dictam tabulam tardio rem duplo temporis Araneæ cum indice, quamuis diuersis modis hoc fieri possit, præstantiorem tamen iudico, si cum Rota indicis, aliã Rotam concentricã coniunxeris, ita tamen, ut vnaqueq; liberè possit volui, similiter si cum ea horologii particula (quæ circumagit Rotam indicis, quæ Italicè ROCHETTO. Germanicè verò TRIBUS vocatur, Latinè aut ipsam vocabo, COLINUM, qui sub rota fusi reperitur) coniunxeris alium colinum quem, secundum vocabo, concentricum verò cum primo, cum eoq; consolidato, numerum verò dentium, tam Rotæ adiunctæ quam secundæ colini, varijs modis poteris inuenire, quorum primus erit, ut numerus dentium secundæ Rotæ duplus existat numero dentium primæ, efficiendo secundum colinum eiusdem numeri dentium quo primum, sed quia intervalla dentium huiusmodi Rotæ, nimis angusta fortasse resultabunt, propterea alios etiam modos inueni, quorum vnus erit (dum numerus dentium primi colini par fuerit) efficiendo secundam Rotam eiusdem numeri dentium cuius est prima. secundum vero colinum, medietatis numeri dentium cuius erit primus. Attamen si primus colinus esset .4. dentium, secundum oporteret esse duorum dentium, unde motus secundæ Rotæ non esset ita continuus. Quapropter alium etiam modum excogitaui, hoc est, cupiendo ut secundus colinus, ex tribus dentibus existat, si primus ex .4. reperitus fuerit, oportebit prius ex regula de tribus, numerum quendam inuenire quo inuento ipsum duplicare, & hunc duplicatum numerum conueniet secundam Rotam habere, ut ipsa possit ab illo colino triu dentium circumuolui in duplo temporis, quo prima à suo colino quatuor dentium.

tium. Exempli gratia, si prima Rota constaret ex. 36. dentibus, dicendum esset, si 4. conuenit cum. 36. cum quibus conuenient. 3. & inueniemus. 27. cum quo numero dicta secunda Rota circunuolueretur eodem tempore à suo colino trium dentium, quo prima à suo quatuor dentium, quare duplicando. 27. haberemus. 54. pro numero dentium dictæ secundæ Rotæ, vt duplo temporis circunuoluator quo prima. Sed si primus colinus constaret ex. 6. dentibus, existente sua Rota ex. 36. vellemusq; q̄ secundus existeret ex. 4. tunc suam Rotam oporteret habere dentes. 48. ex dicta regula. Si autem primus colinus constaret ex numero impari, nihil referret, dummodo huiusmodi numerus impar, seu par, existeret pars propria numeri dentium, vel ipsius dupli primæ Rotæ, hoc est, esset pars aliquota numeri dentium ipsius primæ Rotæ vel ipsius dupli. In ijs verò horologiis in quibus duplum numeri dentium dictæ primæ Rotæ non erit multiplex numero dentium primi colini, hoc fieri non poterit. Ratio enim tam clarè, tibi consideranti, patebit, vt nullis verbis indigeat cum semper numerus dentium secundæ Rotæ multiplex esse debeat numero dentium secundi colini. Idem autem non dico de prima Rota cum suo colino, hoc est; vt numerus primæ multiplex sit numero sui colini, nam hoc necessarium non est. Ponamus exempli gratia primum colinum constare sex dentibus, suam vero Rotam dentibus. 21. cuius quidem numeri, 6. non est pars aliquota, sed dupli ipsius. 21. ipse. 6. est pars aliquota. Nunc verò si voluerimus numerum dentium secundæ Rotæ inuenire, cuius colinus ex quinque dentibus existat (supposito primo ex. 6. constare) tunc ex regula de tribus, diuiso producto, quod fit ex. 21. in. 5. per. 6. exhibit. 17. cum dimidio, cuius duplum esset. 35. qui multiplex est ipsi quinque. Reperto igitur numero secundæ Rotæ, cum numero ipsius colini, oportet nunc scire modum compositionis, seu coniunctionis harum rerum, hoc est duorum colinorum concentricorū (sed de ijs satis iam superius dictum fuit) duarum Rotarum concentricarum cum Tabula, cum Zodiaco, & cum indice, seu Ostensore, cuius quidem Ostensoris medietas tantummodo nobis sufficiet. Sciendum igitur nunc est quod cum primus colinus reuoluat totam primam Rotam, spacio temporis. 12. horarum, oportet vt eius axis, seu arbor voluat ostensorem, Zodiacumq; eodem temporis spacio, & quia Rota hæc inalterabilis est, propter eius coniunctionem cum suo colino, & nos oporteat indicem Zodiacumq;, quotidie ferè, dirigere, suisq; locis collocare, ideo nos oportet, indicem, Zodiacum, & primam Rotam, ita cum axe, seu arbore coniungere, vt possimus dicta omnia efficere. Pars igitur Arboris, seu axis dicti, quæ ingredi debet in primam Rotam, sit rotunda, & contigua ipsi Rotæ, non autem continua, vel cum Rota consolidata. Pars verò quæ per foramen Zodiaci, seu Araneæ transibit, sit quadrata vsque ad summitatē ipsius axis (tali spissitudine, vt in clauis ipsius horologii ingredi possit) & ita foramen ipsius Araneæ, quadratum sit, Ostensor autem circa axem, compositus sit tali ordine, vt circa paruum circulum volui possit, qui paruus circulus habeat quadratum foramen, per quod transeat axis, qui axis aliquantulum emineat supra ostensorem. Sub Aranea vero vel Zodiaco, locata erit Tabula, vt nunc dicemus, sed sciendum est prius, quod inter Tabulam, & suam secundam Rotam, aliam laminam immobilem interpositam esse oportet, quæ circulare foramen habeat, per quod quædam breuis fistula transeat circundans axem & coniungens Tabulam cum sua Rota, cuius quidem fistulæ superficies concaua, rotunda sit, superficies verò extrinseca, non rota, nisi ea pars, quæ secundam Rotam ingreditur, vt in rotundo foramine ipsius Rotæ, dicta fistula volui possit, pars vero extrinseca quæ Tabulam ingredi debet, sit quadrata. Tabula vero quatuor paruissima foramina habeat in extremitati-

bus



bus linearum, meridianæ, & verticalis, vt acu mediante volui possit, prout oportebit.

Perfectum igitur cum fuerit opus hoc, te oportet scire modum ipso vtendi. Quapropter quotiescunque volueris, aspice Solis locum in Zodiaco, Ephemeridibus meridiantibus, idem dico de vnoquoque reliquorum planetarum. Inuento postea Solis loco in nostro Zodiaco horologij, manu mediante, volue ostensorem, ita, vt linea fiducia transeat per gradum Solis, deinde, clauis ipsius horologij mediante, volue indicem, ita cum Zodiaco coniunctum, vt linea fiducia, punctum, seu partem horæ ostendat in limbo horologii, quæ quidem hora notanda est si fuerit ex ijs quæ incipiunt à meridie vsque ad mediam noctem, vel à media nocte vsque ad meridiem, tunc acu supradicta mediante, posita in aliquo illorum quatuor foraminum, circunuoluenda est Tabula, ita, vt extremitas lineæ meridianæ supra horizontem, ex quo incidat inter duodecimam horam, & lineam fiducia, computum incipiendo à duodecima hora, si vero dicta indicis hora fuerit ex ijs quæ incipiunt à media nocte & definunt postea in meridie, oportebit, acu mediante, circunuoluere Tabulam, quousque punctum extremum meridianæ sub terra, medio loco existat inter duodecimam horam, & horam ostensam à linea fiducia. Quo facto continuo videbis erectam cæli figuram. & quia vidisti loca planetarum in Ephemeridibus, videbis etiam eorum loca accidentalibus in domibus scilicet accidentalibus, si aliquas fixarum in Aranea desiderabis, accipere poteris Ocu.  $\gamma$ , cor.  $\Omega$ , spi.  $\mu$ , Liram, Aquilam, & Arcturum, dum locus fuerit capax. Nec te moueat, quod oportebit lineam fiducia supra gra. Solis quotidie collocare, quod nihil refert. Nam oportet etiam quotidie cordam fuso circunuoluere.

DE DEMONSTRATIONIBVS PROPOSITIONVM  
Mathematicarum, nec non de Astrologia Iudiciaria.

*Illustriss. D. Volfardo Aifestain.*

**N**ihil mihi gratius & iucundius afferri potuit tuis literis, quibus te cupidum ostendis sciendi rationem, quare ego non vna methodo ad omnes propositiones demonstrandas vsus sim, hoc est, quare non omnia ea Eucl. Theoremata citem in vnaquaque propositione, quæ ad eam demonstrandam faciunt, quemadmodum in mea Gnomonica vidisti me aliquando omisisse. Respondeo quod mathematicæ demonstrationes, hominibus Euclidis Elementa possidentibus, non indigent aliqua citatione numerorum Theorematum ipsius Euclidis, & si aliquando vsus sum aliqua citatione eorundem, hoc feci propter consuetudinem nostri temporis, vel etiam ad faciliorem intelligentiam illorum, quibus scribebam. Sed omnia quamuis minima citare, vt faciunt nonnulli, mihi, nimis laboriosum, superfluumque videtur, presertim ijs (vt dixi) qui memoria tenent prima Elementa. Hæc igitur est vna ratio. Alia, quia multoties, ita coniuncta est speculatio cum ipsa conclusio, vt mihi sæpius visum sit superfluum, aliquid de ipsa theoria scribere. In iis enim, quæ dum puer eram scripsi, videbis scrupulosam illam methodum, sed postea, non nisi in arduis propositionibus me nihil essenziale prætermittere.

Circa vero id de quo me interrogas, scilicet, vtrum putem omnia vera esse, ea quæ scripta reperiuntur in libris Astrologiæ iudiciariæ. Respondeo quod non, imo puto

puto plurima falsa esse. Nam illa multitudo partium, vt pars vite, pars Hylech, pars futurorum, & relique omnium domorum celestium, salua parte fortunę, sunt mere nuge. Idem dico de faciebus, siue decanis, de terminis, & de gradibus ipsis, vt puta azemenis, puteis, vacuis, fumosis, & de reliquis. De Domibus vero, Exaltationibus, nec non triplicitatibus, experientia confirmat ea vera esse. Idē affirmo de Domibus accidentalibus, rationalibus tamen, non autē de Domibus Campani, & Gazuli. Obseruationes etiam complexionum seu influentiarum ipsorum Planetarum recte factę sunt, que etiam a coloribus ipsorum Planetarum ferē iudicari possunt. Coniunctiones aspectusq; ipsorum inuicem, similiter mirabilia faciunt, & ex maiori parte, ea, que de istis scribuntur vera sunt. Reuolutiones annue similiter, cum Domino anni. Dominum verō orbis Diuiforemq; non approbo, nam hic pender a termino, ille verō ab hora. Nouenarias autem Dodecathemoria, Alfridarias, & multa iis similia omnia nego. Antiscia, vera sunt, idest influunt, malos tamen effectus, alia plus alia verō minus, prout aliqua eorum sunt tetragona, alia verō trigona, alia magna, alia parua, magna sunt, vt Arietis cum Virgine, & Librę cum Piscibus, parua verō, debiliaq; Geminorum cum Cancro, & Sagittarij cū Capricorno. Sed diffusius hęc oīa videbis in meo illo particulari tractatu, de quo tibi aliās dixi, in quo multa videbis, que omnia ab experientia, ex multis a me obseruatis, comprobata sunt, quem quidem tractatum cum quibusdam alijs meis speculationibus in lucem prode re cupio, si fieri poterit, antequam ad directionem mei Horoscopi cum corpore Martis Angetę perueniam, que quidem directio circa annum millesimum quingentesimum nonagesimum secundum eueniet.

F I N I S.

**ERRATA CORRIGITO IUXTA INFRASCRIPITAM TABVLAM**  
 Reliquos vero errores, qui orthographiam respiciunt, benignus lector corrigat, quifciat multos errores in  
 editione irrepsisse, quod non paucos dies morbo fuerim detentus dum praesens opus excuderetur.

Page	Line	Errata	Correcta	Page	Line	Errata	Correcta
3	29	aequalis	aqualis	158	26	versa	versam
8	35	maius	maior	158	26	fit	sint
9	15	in unitate superficialis, erit ac	in unitate superficialis erit, ac	162	22	ciudenda	sciudenda
11	1	proueniens	prouenientem	163	7	oppositus	oppositum
11	8	fiturum	fiturus	164	24	tanta	tanta
11	31	illa nihil aliud sunt	illud nihil aliud est	164	37	adiuncta nobis essent dua alia	adiuncti nobis essent duo alii
11	38	diuidemus	diuidamus	164	39	festineret	festineret
12	1	tertia, sunt	tertie sunt	164	40	dicta	dicti
12	2	productum	productus	164	41	aliquam	aliquem
19	30	proueniens	prouenientem	165	3	sufficiat	sufficiant
19	30	productum equale	productus aequalis	165	4	subsequi alter	subsequi alterum
21	27	eadem	eadem	186	32	suat	suam
24	18	est	esse	187	29	propinqua	propinqui
26	39	est	esse	187	30	philosophi supra	philosophi, proximè supra
41	24	qua	quae	148	31	contingeret mense	contingeret ut mense
41	31	distingueda	distinguedo	187	34	regionis non considerans	regionis in quo non considerant
59	5	subsequens	subsequens	190	11	lenas	lenas
61	3	hac via tenenda	hanc via tenere	190	23	lenis	lenis
61	3	fuit	fuerit	191	10	aliud quadratum	quod
61	45	numerus quæstum	numerus quæstus	191	15	qualis	quis
62	30	quantum est dimidia occupata	quantum est dimidia occupata	193	17	cum	sum
64	1	speculari	considerari	193	36	fit	facit
64	15	tot; sunt termini	tot; esse terminos	193	40	pleno ubi	pleno posuerimus, ubi
64	18	hoc est numerum	hoc est per numerum	195	10	valent	valent apud ipsos
64	19	primo qui minus est	primo qui minus est	196	19	philosophia	philosophia
66	18	minimum	minimus	196	20	pellendo	plendo
67	13	maximus terminum addendus	maximus terminus addendus	198	18	sunt etiam	esse etiam
72	1	num:rum	numerus	204	12	principium	principium
72	8	singulos itinere	singulos in itinere	204	18	indigna	indignas
74	7	itinerarium	itinerantium	204	21	diffundetur, crescat	diffundatur, & crescat
78	21	morium	modum	205	16	aliquando ser	aliquando me ser
78	24	nouentarius	nouentarius	205	19	anno prater, necessitate, gignitur	anno, prater necessitate gignitur
81	10	iuncta	iuncta	206	21	plenilunium, quod	plenilunium fieri, quod
88	35	harmonica	harmonica	207	26	inchoet annus	inchoet annus
91	6	Quare argumetado permutando	Quare permutando	209	9	solis, quod punctum	solis, cuius punctum
95	27	vitiun	viciun	210	10	celebrandis	celebrandis
97	14	summa	summam	212	33	Inuentia	Inuenti
97	30	illud vero quod con	illud vero con	212	33	dua	duo
98	22	desiderarem	desiderarem	214	9	clasi	clasis
103	23	dispositis facit	dispositis, tantum facit	214	10	Inter Eximias	Quia inter Eximias
104	39	dispositis, facit	dispositis, tantum facit	214	26	reperiretur	reperiretur
107	21	excedit	excedit	214	34	necessario sit futurum, re	necessario sit futurum, re
110	41	habuerit eius cerebrum	habuerit cerebrum	215	37	quod si velimus	si velimus
113	46	sufficeret	sufficeret	215	39	potest no eodemq;	no eodemq;
116	8	quarum secundam	quorum secundum	215	45	fit circulus	fit vere circulus
116	8	prima, tertiam	primi, tertium	217	2	Quod cum verum	Quod si verum
116	9	secunda	secunda	217	4	nulla est ratio	nulla est ratio
130	23	ducenda	ducendus	221	31	inuenimus	designabimus
133	10	lineam qua	lineam, qua	221	31	fallat	fallat
133	11	dretam	dratum	222	20	si qua	siq;
134	16	inueniet	inuenimus	225	25	cluium	cluium
137	22	distans	distans	226	14	oleum effundebat	effundebatur
137	45	Notissimum igitur primum	Notissimum primum	227	5	enadit	enadet
139	15	est	fit	227	36	supradicta, minuta	supradicta, minuta
139	28	duas	duos	231	36	progreuntur	progreuntur
141	5	comperuisse	comperisse	232	21	alioi caelo	alioi caelum
142	19	qua	ac	239	12	dum	cum
143	17	linea	lineam	241	41	fferens	cogitans
144	23	patebit, si quis	patebit, quod si quis	243	25	diametri	diametros
145	18	constante	constantem	248	42	fit, & ob id	fit, ob id
146	20	pastam	massam	249	42	alium numerum	alio numero
146	22	pastam	massam	250	13	um voluerimus	inuenimus
148	13	ponit no concludit melius aut	proponit, no concludit, melius aut	251	1	inuenimus	inuenimus
14	41	pratergradiatur	pratergradiatur	251	6	tros	ter
152	6	videtur	videri	252	6	alia vero diametro	alioi vero diametro
153	22	quia libra	quia cum libra	252	8	te id non	te non
153	23			252	12	duobus	duobus
153	23	materiales, cum sustineantur	materiales sustineantur	255	16	in numero	in numero
153	24	existente, unde aliqua	existente, aliqua	255	25	cum non videat si	cum non videat quod si
154	13	fiturum	effecturam	255	36	suo centro	suis centris
155	25	carta	charta	255	39	nulle miliaria	nulle miliariorum
156	23	sufficere	sufficeret	255	41	recenta mille	tercentes mille

Page	Line	Errata	Correcta	Page	Line	Errata	Correcta
257	5	quem	quod	345	19	ipsum reperitum	ipsum, et reperitum
257	11	quem	quod	347	16	reflectit	reflectitur
257	18	quod	nam	352	28	opinaverunt	opinari sunt
258	15	ictum	ictus	353	8	parabolem	parabolas
258	15	eundem esse futurum	idem est et	353	15	hac	hic
258	27	debet, quanto	debet, in huiusmodi situ, quanto	353	18	parallelas	parallela
258	41	quod	qui	356	15	muri, &	muri, in orientali, q. p. et mediate, cuius axis ad orientem erectus sit. o. n. cognoscere
262	17	erunt	supponuntur	356	33	mediate, cognoscere	duplus
263	22	a. K. facta sit	a. K. sit	360	13	duplum	iste circulus
263	22	semidiameter esse minus	semidiameter minus	360	28	unde iste circulus	qui
265	4	cognita	cogniti	361	36	que	diameter
272		in figura ubi est. P.	ponatur. R.	367	25	prolongando	producenda
272	18	E. i. ad. E. i.	E. I. ad. E. i.	369	2	equalem	equale
273	3	licet	liceret	370	4	suas	suam
274	7	te	tibi	370	30	summa	summaria
275	10	diuidere	diuidendo	372	1	caterarum	cateris
276	47	accipimus	accipiamus	372	36	qui qui	qui
278	8	Moteta	Moteta	373	11	hac	hac
280	25	diminuta, quarta	diminuta seu defectiva, quarta	382	41	prolongatis	productis
281	36	grauissimum	grauissimo	383	3	qua	qui
281		in fine pag. vnamquamque Xifram accommoda sub in quoque alphabeti characterem ad priores Xifras prius addendo. x. Idem dico in pag. 282. & inter. D. et E. pone. b.		383	13	ginabis	ginaberis
285	24	antea	ante	383	20	triangulis	triangulig
289	11	quanta pars	quantam partem			in prima parabola obliqua extremitatem inferiorem diametri signabis caractere. d. in secunda vero caractere. r.	
289	25	quanta pars	quantam partem	384			maior proportionem
289	42	est	erit	384	37	proportio habebit	proportionem habebit
290	10	satilis	satilis	385	5	c. b.	c. b.
290	19	equalis	equale	386	22	b. e.	b. c.
290	43	circumscriptibilis	circumscriptibilis	386	29	positum	positus
291	1	od	Quod	386	31	b. c.	b. e.
291	33	detractis	detracti	386	33	& proportionalitate	& ex proportionalitate
291	34	anguli contingente solidisq;	anguli contingente solidisq;	386	34	antecedens in	antecedens. A. B. in
291	24	chorda	chordam	386	40	quod	qua
295	32	lixum	lixum	388	2	B.)	B.)
297	1	distincta, procedendo	distincta sunt, procedendo	388	8	b. cum duplo. b. o.	b. d. cum duplo. b. e.
299	32	reflexum	reflexum	388	45	b. e.	b. c.
299	34	flexum	flexum	388	45	b. e.	b. c.
302	14	eisdem	eisdem	388	46	ad. d. b. cum. e. b.	ad. d. b. cum. e. b.
304	3	ta	tus	390	25	diuisa	diuisas
305	21	Idem facere	Idem possumus facere	392	3	sit paraboles. a.	sit paraboles. a. b. c.
312	37	duple	duplo	392	27	aliq. reliquorum	aliqui reliquorum
313	6	quotiescunque	qui	393		in parabola ubi sub g. est. i. pone r.	
315	23	retrogradandum	retrogradandum	395		& supra solidum minus dicatur cubus minor (leantur duo vero linee sine characterib. de	
316	22	tibi	tibi	394	18	v. y. ad. m. n.	ut p. ad. m. n.
324	41	si constitueremus	si nos constitueremus	394	33	x. o.	n. o.
325	12	simili ad	simili. Ad	394	34	i. k. ad. i. k. ad. f. g.	i. k. ad. f. g.
326	9	longave	ducere	395	11	quintas	quinta
329	5	ipfi ellipsis	ipfi ellipsis	396	1	d. b.	d. b.
333	15	cogitemus	cogitemus	397		Vbi rubrica dicit. Defensio nostra dicitur, contra Anto. Bergam & contra Antonium Bergam, & Alexandrum Piccol.	contra Anto. Bergam & Alex. Piccol atq; defensio nostra contra Excell. August. Michielem
333	17	unde pro genera	unde ex genera	399	32	tionem	tionem
333	20	equalem esse longitudini	equalem longitudini	400	39	num tantum numerum medium	num tantum numerus medius
333	21	es	eam	402	19	dianetrum	semidiameterum
333	21	minor	minorem	403	7	suas	sua
333	23	nor	norem	403	9	iam supra	eo tractatum
333	24	maior	maiozem	403	17	quidquid	quidquam
333	25	maior	maiozem	404	37	miliaribus	miliaribus
333	37	communisq;	communisq;	406	19	trabuchum quadratum	trabuchus quadratus
335	26	reflectitur	reflectitur	410	38	Que fors	qua fors
335	47	& remotiori	vel remotiori	410	41	quamlibet	qualibet
336	19	speculi superficiem	speculi planam superficiem	411	27	dicens	dicat
336	22	reflectit	reflectitur	411	28	eodem libro	et eodem libro
337	8	nam tantummodo imaginem	nam tantummodo imago	412	24	non dependeat?	dependeat?
337	12	ipse	ipfi	413	2	ut supra	ut alias
337	42	quis	qui	413	22	milia rices	millibus riciam
338	14	protracta	protracto	415	32	fenestre	fenestre
338	26	ratiomare	epi. a	416	15	operatio	operationem
340	23	Alius modus	Alium modum				
341	9	equale	equalem				
343	23	videbitur	videri				
344	5	hoc est	hoc est quod				
344	1	punctus	punctum				
345	6	es	cum altitudine				

F I N I S.