THE SOCIAL VALUE OF ASYMMETRIC INFORMATION REVISITED

XUE-ZHONG HE[1], LEI SHI[2] AND MARCO TOLOTTI[3]

[1]University of Technology Sydney

[2]Macquarie University

[3]Ca' Foscari University of Venice

Date: February 12, 2019.

Acknowledgement: We thank Phil Dybvig, George Jiang, Kai Li, Nan Li, Jun Liu, Mark Loewenstein, Yajun Wang, Russ Wermers, Huanhuan Zheng and Min Zheng and the participants of Computing in Economics and Finance (Milan, 2018), China Finance Review International Conferences (Shanghai, 2018), the 2-nd International Scholars Finance Conference (Chengdu, 2018), the 6-th Asia Quantitative Finance Conference (Guangzhou, 2018), the 10th Workshop on Dynamic Models in Economics and Finance (Urbino, 2018) and seminars at the University of Western Australia (Perth), Central University of Finance and Economics (Beijing), and South China Normal University (Guangzhou) for helpful comments. We also acknowledge the financial support provided by Ca' Foscari University of Venice under the grant *"Interactions in complex economic systems: contagion, innovation and crises"* and the Australian Research Council (ARC) under Discovery Grant (DP130103210).

[1]University of Technology Sydney, Business School, Finance Department, PO Box 123, Broadway, NSW 2007, Australia; `tony.he1@uts.edu.au`

[2]Macquarie University, Faculty of Business and Economics, Department of Applied Finance and Actuarial Studies, 4 Eastern Road, North Ryde, NSW 2109, Australia; `l.shi@mq.edu.au`

[3]Ca' Foscari University of Venice, Department of Management, S.Giobbe - Cannaregio 873, I - 30121 Venice, Italy; `tolotti@unive.it` .

Abstract. In contrary to previous literature, we show in the Grossman-Stiglitz model of noisy rational expectation that the social value of asymmetric information can be improved with more informative prices when being informed is uncertain. Investors always benefit from a privately payoff-relevant information, but they have to pay more to increase the probability of observing the information. In equilibrium, this trade-off can lead to high-risk, high return investments. Consequently the marginal expected utility gain from observing the information is not completely washed out by the cost of information acquisition, which leads to Pareto-optimal equilibrium and improves investors' welfare.

Key words: Social value of asymmetric information, information uncertainty, risk premium, efficiency, welfare.

JEL Classification: D82, G12, G14

1. Introduction

In exchange economics with costly information, it is well recognized that the more information prices convey, the worse off everybody can be. Therefore, public information has no social value; this is the so-called *Hirschleifer effect.*[1] Essentially, reducing the cost of private information makes more investors to be informed. This improves price efficiency on the one hand and leads to low-risk/low return investments on the other hand, which then reduce investor welfare. Is this always the case when information is more complex and uncertain? This paper introduces information uncertainty into a noisy rational expectation equilibrium model to answer this question. We find that, when investors pay to reduce the uncertainty to be informed, improving price efficiency can be socially valuable. Intuitively, the individual investor is always better off with the information. Facing the uncertainty to be informed, to benefit from acquiring information, investors pay more to increase the probability to be informed. This trade-off can lead to high-risk/high-return investments, improving investor welfare. This paper explores when such investors' welfare improvements are likely to arise.

The role of information on financial markets, in particular the effect of public information on price efficiency has been studied extensively. In summary, the literature shows that, the more (public) information is available to investors, the more informative the price is and the thinner is the space to make profits. This hurts investors' perception, thus diminishing exposures and risk sharing. More recently, the debate on social welfare of public information has involved mandatory financial disclosure, questioning the *salvific* role of disclosure itself (see Kurlat and Veldkamp (2015), Gargano, Rossi and Wermers (2017) and Goldstein and Yang (2017)). If, from one hand, information improves price efficiency, its consequences on *social welfare* are still debated. After the pioneering works by Allen (1984) and Laffont

[1]The name follows after Hirshleifer (1978) where the role of information in the framework of technological uncertainty is discussed. Franklin Allen has been the first to acknowledge the implications of this effect for financial markets, especially in the context of exchange economies (see Allen (1984)).

(1985), in a recent contribution, Angeletos and Pavan (2007) have posed again the question about the *efficient use of information* and the *social value of information.*

Security prices reflect available information about future payoffs, while uncertainty about the payoffs in different dimensions can affect security prices differently. Uncertainty about multi-national or complex firms can be very different from the uncertainty of firms from particular industries. Even firms from the same industries can be exposed to multiple dimensions of uncertainty, such as cash flows, technological innovations, and firms idiosyncratic developments differently. Recent empirical evidence shows the revolution in information technology over the last 50 years has increased the price efficiency for the S&P 500 firms but indeed decreased for the average public firms (Bai, Philippon and Savov (2016)). With increasing uncertainty on *multi-dimensional information* in financial markets,[2] it seems *heroic* to assume that investors are certain to be fully informed for a predetermined fixed cost. We, instead, assume that traders pay to increase their probability of being correctly informed. Put differently, we can think about an information market where investors can have access to different sources of information of different quality: the higher is the quality, the higher is the probability of being informed. However, there is a cost/disutility effect: investors pay more to increase the probability to be informed. Such a trade-off between cost and benefit of uncertain information plays a central role in traders' decision making and can have important implications to market efficiency and social welfare.

With information uncertainty, this paper examines such trade-off in an otherwise standard Grossman and Stiglitz (1980) model of noisy rational expectation equilibrium (NREE). The results partially confirm the Hirschleifer effect and its implications; making effort to be informed typically improves market efficiency but at the cost of reducing social welfare. However, in contrast with the majority of the aforementioned literature on the social role of information, we demonstrate that

[2]The role of multi-dimensional and complex information markets has been widely discussed in recent literature. Among the others, Zhang (2006) discusses information uncertainty and its role in shaping prices; Veldkamp (2006) considers different information providers with different prices/quality. In Gorban, Obizhaeva and Wang (2018) authors assume the presence of high and low quality signals and uncertainty about the number of high quality informed agents.

when the information acquisition process is probabilistic and the cost of information is increasing and convex in the cost or effort to be informed, the trade-off can lead to high-risk, high return investments. Consequently the marginal expected utility gain from observing the information is not completely washed out by the cost of information acquisition, which leads to Pareto-optimal equilibrium and improves investors' welfare. Therefore there is possibility of detecting an increase of market quality associated with a beneficial increase in social welfare.

When information is imperfect and costly in a competitive economy, Grossman and Stiglitz (1980) show that information efficiency of a price system depends on the proportion of individuals who are informed. The more individuals choose to be informed, the more efficient the price becomes, the less valuable the information is, and the less incentive individuals choose to be informed. Therefore in equilibrium, *"the number of individuals who are informed is itself an endogenous variable"* and the price becomes more informative when there are more informed traders. Concerning the welfare, informed trading always improves (marginal) welfare in the sense that an individual is always better-off for being informed rather uninformed, however in aggregate more informed trading always reduces the welfare for both informed and uninformed traders. In particular, the social welfare is always higher when traders are all uninformed than when they are all informed. Put differently, we detect a sort of *Prisoner's dilemma* situation.[3] The social welfare would be better off if nobody is informed. However, the single individual is rationally driven to being informed, at least with some positive probability. Therefore, in the equilibrium, the market may end up into a sub-optimal equilibrium (from the welfare viewpoint) typical of a coordination-failure situation.

Regarding the endogenous information production, we endogenize traders' decision on their optimal effort to become informed when facing information uncertainty. We model a continuum of agents playing an *information game* inspired by global games (Morris and Shin (2002)). Differently from classical global games, the

[3]In Kurlat and Veldkamp (2015), a similar situation in which the economy would be better off if nobody is informed, is discussed but they focus on financial market anomalies in information disclosure. For a recent review about the implications of information disclosure in terms of market quality and welfare, we refer the reader to Goldstein and Yang (2017).

strategy of the players is expressed in terms of the *probability of being informed*. With this respect, our model resembles some recent literature on *probabilistic choice models* (Mattsson and Weibull (2002)) and classical results in information theory (Hobson (1969)). In Mattsson and Weibull (2002), an individual optimally makes an effort to deviate from the status-quo (a reference probability) and *change the likelihood* of a finite set of possible scenarios in order to get closer to implementing a more desired outcome. Given that the reward is always higher for informed than uninformed, traders choose their optimal information acquisition strategy to maximize the trade-off between a higher expected reward of being more informed and a higher cost. When individuals set an optimal trade-off between the expected reward and the cost of deviating from the status-quo, Mattsson and Weibull (2002) show that the disutility of the optimal effort is related to the information entropy. We rephrase this game-theoretic setting as a monetary reduction of wealth due to the investment in the information acquisition. Eventually, we model a two-stage optimization scheme based, firstly, on a strategic information game and, secondly, on a classical mean and variance investment decision problem. We characterize a unique Nash equilibrium in the vector of probabilities of traders being informed and a NREE in asset pricing. With the cost of information being increasing and convex, we show that traders' optimal effort depends on their risk aversion and the information structure. The resulting endogenous information equilibrium leads to outcomes that are significantly different from the Grossman-Stiglitz model.

In the Grossman-Stiglitz model, traders can decide to pay a fixed cost to becoming informed for sure. When information is complex and multi-dimensional, being informed becomes *per se* an uncertain process. The more they pay for the information, the more likely they will discover the right signal. As in the Grossman-Stiglitz model, in equilibrium, the proportion of investors to be informed is determined endogenously and the price becomes more informative when there are more informed traders. However, different from the Grossman-Stiglitz model, the cost is not fixed, but contingent on market proportion of investors to be informed themselves. The optimal cost or effort for investors to becoming informed depends on investors' risk aversion and market information structure. Intuitively, more informed trading reduces dividend risk but increases information risk for uninformed traders, which dominates the dividend risk. This effect becomes more significant when price becomes less informative, the supply is less noisy, or traders are less risk averse, generating a hump-shaped risk premium relation to the informed trading. This hump-shaped risk premium in informed trading then provides high return, high risk investments, which improve investors' welfare. Therefore, differently from the previous studies about the market implications of the Hirschleifer effect in exchange economies, we show that there are situations in which market efficiency can be improved without harming social welfare.

The structure of the paper is as follows. We first introduce the model and traders' optimization problem and then characterize the equilibrium in Section 2. In Section 3, we conduct a welfare analysis, together with risk premia and price efficiency. Section 4 extends the analysis to explicitly model trading motives as a possible route to endogenous supply. Section 5 concludes and all the proofs are collected in the Appendix.

2. The Model

We consider the Grossman-Stiglitz model as our baseline model.[4] There is a continuum of homogenous traders, indexed by $i \in (0,1)$, who are price-takers and can invest in two assets; a risk-free asset with a gross rate of return $R > 1$ and a risky asset which pays a random dividend $\tilde{D}$ at the end of the period. As in the Grossman-Stiglitzmodel, the dividend is given by

$$\tilde{D} = d + \tilde{\theta} + \tilde{\epsilon}, \tag{2.1}$$

where $\tilde{\theta} \sim \mathcal{N}(0, v_\theta)$ and $\tilde{\epsilon} \sim \mathcal{N}(0, v_\epsilon)$, respectively, and $d = \mathbb{E}[D]\ (> R)$ is a constant.

Different from the Grossman-Stiglitz model in which $\tilde{\theta}$ is fully observable at a fixed cost c, we assume that $\tilde{\theta}$ can only be observed with a probability of p at an increasing and convex cost of $c(p)$ in general. To have the most intuitive expressions, we present

[4]In the last decades, several generalizations to a dynamic setting of the classical Grossman-Stiglitzmodel have been proposed. Among the others, Wang (1993), Veldkamp (2006), Kyle, Obizhaeva and Wang (2017). For the sake of simplicity, we prefer to stick with the standard static one-period model.

our main results for a quadratic cost function $c(p) = p^2$ for $p \in [0,1]$.[5] That is, p is the probability of becoming fully informed about the private information $\tilde{\theta}$ and a trader observes it after paying the cost $c(p)$. This implies that traders need to pay more in order to increase their probability to be informed. Note that if traders can only choose $p = 0$ or $p = 1$, we are back to the Grossman-Stiglitz model in which traders are either informed (with $p = 1$) or uninformed (with $p = 0$).

The risk-free asset is in zero net supply and the risky asset has a noisy net supply of $\tilde{z} \sim \mathcal{N}(0, v_z)$. Note that the supply shock $\tilde{z}$ can be due to liquidity demand or noise trading.[6] The random variables $\tilde{\theta}$, $\tilde{\epsilon}$ and $\tilde{z}$ are independent of each other.

Traders are risk averse with a CARA utility function, i.e., $u(\tilde{W}_i) = -e^{-\alpha \tilde{W}_i}$, where α is the absolute risk aversion and $\tilde{W}_i$ is trader i's terminal wealth. Let x_i be the number of shares trader i holds and $\tilde{P}$ be the price of the risky asset, then trader i's terminal wealth becomes

$$\tilde{W}_i = x_i(\tilde{D} - R\tilde{P}) + W_{i,0}R, \tag{2.2}$$

where $W_{i,0}$ is trader i's initial wealth. We assume traders' initial wealth is zero. Therefore, their trading motive is purely speculative; they make profit by supplying liquidity to the noise/liquidity traders.

2.1. **Information Uncertainty and Trading.** There are three dates, $t = 0, 1, 2$. At date $t = 0$, each trader-i chooses strategically p_i and pays the cost $\mu c(p_i)$, where μ is a scalar measuring the sensitivity to the cost. We refer to this stage as the *information game*. Next, at date $t = 1$, a Boolean random variable $\tilde{\omega}_i$ is drawn independently for each trader i with $\mathbb{P}(\tilde{\omega}_i = 1) = p_i$ and $\mathbb{P}(\tilde{\omega}_i = 0) = 1 - p_i$. If $\tilde{\omega}_i = 1$, the trader observes $\tilde{\theta}$ and becomes informed (type I). If $\tilde{\omega}_i = 0$, the trader does not observe $\tilde{\theta}$ and stays uninformed (type U). Then, the value of $\tilde{\theta}$ is realized and each trader, depending on his type, chooses his optimal demand $x_i^*(P)$ in the risky asset, where P is the price of the risky asset. Finally, at date $t = 2$, with the

[5] We have also considered other cost functions that are increasing and convex in p. In general, we can also assume that $p \in [p_0, 1)$ and $c(p_0) = 0$, where $p_0 \in [0, 1)$ is a reference or status quo probability of becoming informed. For simplicity, we assume $p_0 = 0$ in this paper.

[6] In Section 4, we model the behaviour of liquidity/noise traders explicitly using endowment shocks. For now we simply take the noisy supply as given.

noise demand $\tilde{z}$, the market equilibrium price $\tilde{P}$ is determined by market clearing, each trader gets their allocation of shares according to their optimal demand and the equilibrium price. Then, the dividend $\tilde{D}$ is paid and consumption occurs.

2.2. **Information Acquisition and Trading Decisions.** Concerning the trading decision, since dividend payoff is normally distributed (and the information cost c does not depend on the investment strategy), the standard solution for trader i's optimal holding of the risky asset is given by

$$x_i^* = \frac{\mathbb{E}[\tilde{D} - R\tilde{P}|\mathcal{F}_i]}{\alpha \mathbb{V}ar[\tilde{D} - R\tilde{P}|\mathcal{F}_i]}, \tag{2.3}$$

where $\mathcal{F}_i$ is the information set for trader i.

Regarding the information acquisition, by taking into account the associated cost, trader i's objective at date $t = 0$ is to choose his probability p_i of being informed to maximize

$$\mathcal{U}(p_i; \lambda) = [p_i V_I(\lambda) + (1 - p_i) V_U(\lambda)]\, e^{\alpha\mu\, c(p_i)}, \tag{2.4}$$

where

$$\lambda = \int_0^1 \omega_i \, di = \int_0^1 p_i di$$

is the *market fraction of informed traders*, which we will use as a state variable, and

$$\begin{array}{ll} V_I(\lambda) = \max_{x_i} \mathbb{E}\left\{\mathbb{E}\left[u\left(x_i(\tilde{D} - P)\right)\Big|\mathcal{F}_I\right]\right\}, & \mathcal{F}_I = \{\theta, P\}, \\ V_U(\lambda) = \max_{x_i} \mathbb{E}\left\{\mathbb{E}\left[u\left(x_i(\tilde{D} - P)\right)\Big|\mathcal{F}_U\right]\right\}, & \mathcal{F}_U = \{P\}, \end{array}$$

are the expected utilities of the informed and uninformed, respectively. Note that $V_I(\lambda)$ and $V_U(\lambda)$ depend on λ since the equilibrium price P itself depends on λ. Therefore, when needed, we will denote the price as P_λ.[7] Also, we assume traders take λ as given, or more precisely, each trader conjectures the average choice of p_i by all other traders before giving his best response in a non-cooperative strategic game.[8] Two technical lemmas are now stated; the first provides the solution to traders' optimal portfolio and information acquisition decisions given the market fraction λ. This result is based on a first order condition argument. The second

[7]More precisely, in equilibrium, $P_\lambda = h_\lambda(\tilde{\theta}, \tilde{z})$ is a random variable, where h_λ is a deterministic function depending on λ.

[8]Being λ a function of $p = (p_i)_{i \in [0,1]}$, the problem of finding an optimal vector p^* results in a non-cooperative strategic game.

lemma provides a concavity (second order) condition ensuring that the optimization problem is well-defined. For convenience, we denote by

$$\gamma(\lambda) = 1 - \frac{V_I(\lambda)}{V_U(\lambda)} \tag{2.5}$$

the relative benefit of informed to uninformed. We also introduce the following notations, $v_x \equiv \mathbb{V}ar[\tilde{x}]$, $\sigma_{x,y} \equiv \mathbb{C}ov[\tilde{x}, \tilde{y}]$, $\beta_{x,y} \equiv \sigma_{x,y}/v_x$ and $\rho_{x,y} \equiv \sigma_{x,y}/\sqrt{v_x v_y}$, for any two normally distributed random variables $\tilde{x}$ and $\tilde{y}$. Following the NREE literature (Admati (1985), Admati and Pfleiderer (1987)), we postulate a linear price

$$\tilde{P} = \frac{1}{R}(d + b_\theta \tilde{\theta} - b_z \tilde{z}), \tag{2.6}$$

where b_θ and b_z are two positive coefficients to be determined in equilibrium.

Lemma 2.1. *Assume traders' expected utility is concave in p_i, i.e., $\mathcal{U}''(p_i;\lambda)(= \partial^2\mathcal{U}(p_i;\lambda)/\partial p_i^2) < 0$ and the equilibrium price P has the form of (2.6). Then*

(i) *trader i's optimal demand in the risky asset is given by*

$$x_i^*(P) = \begin{cases} \frac{d+\theta-RP}{\alpha v_\epsilon}, & \omega_i = 1; \\ \frac{d-RP}{\alpha v_U}, & \omega_i = 0, \end{cases} \tag{2.7}$$

where, conditional on his type $k \in \{I, U\}$,

$$v_U \equiv \frac{v_\epsilon + v_{\theta|P}}{1 - \beta_{P,\theta}}, \qquad v_{\theta|P} \equiv (1 - \rho^2_{\theta,P})v_\theta, \qquad \beta_{P,\theta} = \frac{\sigma_{\theta,P}}{v_P};$$

(ii) *the expected utilities of trader i, conditional on his type $k \in \{I, U\}$, is given by*

$$V_k(\lambda) = -\frac{1}{\sqrt{1 + \xi_k(\lambda)}}, \qquad \xi_k(\lambda) \equiv \frac{v_{\chi_k}}{v_{D|\mathcal{F}_k}}, \tag{2.8}$$

where

$$\xi_I(\lambda) = \frac{(1 - b_\theta^2)^2 v_\theta + b_z^2 v_z}{v_\epsilon}, \qquad \xi_U(\lambda) = \frac{(1 - \beta_{\theta,P})(b_\theta^2 v_\theta + b_z^2 v_z)}{v_U},$$

and $\chi_k \equiv \mathbb{E}[\tilde{D} - R\tilde{P}|\mathcal{F}_k]$ is the conditional risk premium;

(iii) *trader i's optimal choice of probability to be informed is given by*

$$p_i^* = g^{-1}\left(\frac{1}{\alpha\mu}\frac{\gamma(\lambda)}{1 - \lambda\gamma(\lambda)}\right), \tag{2.9}$$

where $g(p_i) \equiv c'(p_i)$.

When trader's optimization problem is well-defined, (2.7) gives the optimal demand, (2.8) defines the value function to be informed and uninformed, while (2.9) provides the optimal probability choice.

Lemma 2.2. *The function* $\mathcal{U}(p_i;\lambda)$ *as defined in* (2.4) *is concave in* p_i *if and only if*

$$\frac{\gamma(\lambda)}{g(\lambda)}\left[\frac{2g(\lambda)}{1-p_i\gamma(\lambda)}-\frac{g(p_i)}{1-\lambda\gamma(\lambda)}\right]\leq\frac{g'(p_i)}{g(p_i)},\ \textit{for all}\ \ p_i\in[0,1]. \tag{2.10}$$

A sufficient condition for $\mathcal{U}''(p_i;\lambda)<0,\ \lambda\in[0,1]$, *is given by*

$$\gamma(\lambda)<\frac{1}{2}\min_{p_i\in[0,1]}\left(\frac{1}{\frac{g(p_i)}{g'(p_i)}+\frac{p_i}{2}}\right); \tag{2.11}$$

moreover, if $c(p_i)=p_i^2$, (2.11) *reduces to*

$$\gamma(\lambda)<\frac{1}{3}. \tag{2.12}$$

Some remarks are needed. First, note that, as soon as one of the concavity conditions in Lemma 2.2 is met, the optimal probability p_i^* in (2.9) is the same for all agents. In this sense, similarly to the Grossman-Stiglitz model, the cost $c(p_i^*)$ paid for acquiring the signal is the same for all agents, although endogenously computed. Having said that, our traders' optimization problem differs significantly from that of the Grossman-Stiglitz model, which has the following solution,

$$p_i^*=\begin{cases}0, & \gamma(\lambda)<1-e^{-\alpha c};\\ 1, & \gamma(\lambda)>1-e^{-\alpha c},\end{cases} \tag{2.13}$$

where c is a fixed cost. Note that $\gamma(\lambda)$ measures the relative benefit of informed to uninformed. Therefore, Grossman-Stiglitz model's information equilibrium requires $\gamma(\lambda)=1-e^{-\alpha c}$ or equivalently $V_I(\lambda)e^{\alpha c}=V_U(\lambda)$. In other words, the expected utility of the informed after the cost exactly matches the expected utility of the uninformed. As a results, traders are indifferent between becoming informed or staying uninformed. In our model, due to the information uncertainty, the cost is associated with the average expected utility of being informed (with probability p_i) and uninformed (with probability $1-p_i$), characterized in (2.4).

As said, the optimization scheme of information acquisition and portfolio choice for trader i can be separated in two stages and solved *backwards*. At date 1, trader

i decides his portfolio choice x_i^* *given his type*, i.e., the realization of ω_i. This stage amounts to determining V_I and V_U. At date 0, agents play the information game: by averaging on the likelihood of being informed and forming an expectation about other traders' actions, traders strategically set optimal strategies, $p^* = (p_i^*)_{i\in(0,1)}$. Finally, to close the model, the equilibrium price P of the risky asset is determined by the market clearing condition as in the standard noisy rational expectation equilibrium.

2.3. **Information and Asset Market Equilibria.** Before characterizing the information and asset market equilibria, we first introduce the following definition.

Definition 2.1. We say that probability $p^* = (p_i^*)_{i\in(0,1)}$, market fraction of informed λ^*, and price P^* of the risky asset are in equilibrium if

(i) $p^* = (p_i^*)_{i\in(0,1)}$ is a Nash equilibrium, meaning that for every $i \in (0,1)$,

$$\mathcal{U}(p_i^*;\lambda) \geq \mathcal{U}(p_i;\lambda) \qquad \text{for all } p_i \in [0,1];$$

(ii) the following *consistency equation* is satisfied[9]

$$\lambda^* = \mathbb{E}\left[\int_0^1 \omega_i^*\, di\right] = \int_0^1 p_i^*\, di, \tag{2.14}$$

here ω_i^* is the random variable associated to the optimal probability p_i^*;

(iii) the price $P^* = P_{\lambda^*}$ satisfies market clearing condition

$$\int_0^1 x_i^*(P)\, di = \tilde{z}, \tag{2.15}$$

where $x^*(P) = (x_i^*(P))_{i\in(0,1)}$ is the optimal investment strategy profile.

We now characterize the equilibrium in the following proposition.

Proposition 2.3. *Assume* (2.10) *holds and denote by*

$$n = \frac{v_\theta}{v_\epsilon}, \qquad \xi_0 = \alpha^2 v_z v_D, \qquad \xi_1 = \alpha^2 v_z v_\epsilon,$$

the informativeness of the private signal, and the expected trading profit when no traders are informed and when all traders are informed, respectively. Then

[9] At the equilibrium, expectations realize so that the fraction of informed, λ, exactly matches the value expected by the traders when using the revealed vector of probabilities p^*.

(i) *the equilibrium market fraction of informed traders,* λ^**, is determined by* $\lambda^* = \lambda$ *satisfying*

$$\lambda = g^{-1}\left(\frac{1}{\alpha\mu}\frac{\gamma(\lambda)}{1-\lambda\gamma(\lambda)}\right); \tag{2.16}$$

(ii) *the linear equilibrium price of the risky asset is given by*

$$\tilde{P} = \frac{1}{R}(d + b_\theta\tilde{\theta} - b_z\tilde{z}), \tag{2.17}$$

where

$$b_\theta = \frac{\lambda\bar{v}}{v_\epsilon}, \qquad b_z = \alpha\bar{v}, \tag{2.18}$$

and

$$\frac{1}{\bar{v}} = \frac{\lambda}{v_\epsilon} + \frac{1-\lambda}{v_U}, \qquad v_U = v_D + \frac{1}{\alpha}\left(\frac{v_\theta}{v_z}\right)\left(\frac{\lambda}{\alpha v_\epsilon}\right) = v_D\left(1 + \frac{n\lambda}{\alpha\xi_0}\right). \tag{2.19}$$

As argued before, under the mild concavity conditions of Lemma 2.2, equilibrium probabilities p_i^*, $i \in [0,1]$, all collapse to the same value p^* solving (2.9); moreover, by virtue of (2.14), we have $\lambda^* = p^*$.[10] Note that, in the Grossman-Stiglitz model, although the asset market equilibrium is identical to ours, the information equilibrium differs significantly. Instead of (2.16), Grossman-Stiglitz model requires $\gamma(\lambda) = 1 - e^{-ac}$, which means that the expected utility of the informed traders, after having paid the cost c, is exactly the same as that of the uninformed traders (who do not pay any cost). Differently in our model, every trader optimally chooses to pay a cost equal to $\mu c(p_i^*) = \mu c(\lambda^*)$ in equilibrium, and $p_i^* = \lambda^*$ becomes the (optimal) probability of observing the signal $\tilde{\theta}$. As we will see in Section 4, this difference from the Grossman-Stiglitz model leads to very different welfare implications.

2.4. **Existence and Uniqueness of Nash Equilibrium.** We now examine the existence and uniqueness of the Nash equilibrium with respect to parameter μ, which measures the cost sensitivity. Intuitively, $\lambda \to 0$ in equilibrium as $\mu \to \infty$; $\lambda = 1$ when μ is small enough; otherwise $\lambda \in (0,1)$. This is demonstrated as following.

[10]We stress the fact that, in principle, there could be multiple equilibira in λ for the fixed point argument (2.16) even if the optimization problem is well-defined in p^*. In the following we provide sufficient conditions for uniqueness to keep this paper more focused on welfare analysis and market implications. We leave this intriguing discussion on multiple equilibira for future research.

Proposition 2.4. *Assume* (2.10) *holds and* $c(p) = p^2$. *Then*

(i) $\lambda^* = 0$ *as* $\mu \to \infty$;

(ii) $\lambda^* = 1$ *when* $\mu \leq \bar{\mu} := \frac{1}{2\alpha}\frac{\gamma_1}{1-\gamma_1}$, *where* $\gamma_1 \equiv \gamma(1) = 1 - \sqrt{\frac{n+\xi_1}{n+\xi_0}}$;

(iii) $\lambda^* \in (0,1)$ *when* $\mu > \bar{\mu}$; *furthermore the Nash equilibrium is unique under condition* (2.12).

Moreover, the equilibrium price P^*, *satisfying* (2.17), *is characterized by parameters* b_θ *and* b_z *defined in* (2.18) *and* (2.19), *evaluated at the equilibrium* λ^*.

Proposition 2.4 provides a necessary and sufficient condition (on the parameter μ) for thee existence of a non-trivial Nash equilibrium $0 < \lambda^* < 1$ and a sufficient condition (on the relative benefit γ) for the uniqueness. In general, the equilibrium fraction of informed traders is expected to increase as traders become less sensitive to the cost function. Put differently, we expect λ^* to be decreasing in μ. However, it turns out that monotonicity is not guaranteed.

Proposition 2.5. *The equilibrium* $\lambda = \lambda(\mu)$ *is decreasing in* μ *if and only if*

$$\frac{\Gamma'(\lambda)}{\Gamma(\lambda)} \leq \frac{g'(\lambda)}{g(\lambda)}, \qquad \Gamma(\lambda) = \frac{\gamma(\lambda)}{1-\lambda\gamma(\lambda)}; \tag{2.20}$$

or equivalently

$$\frac{\gamma^2(\lambda)+\gamma'(\lambda)}{1-\lambda\gamma(\lambda)} \leq \frac{g'(\lambda)}{g(\lambda)}. \tag{2.21}$$

In particular, for $c(p) = p^2$, *condition* (2.21) *becomes*

$$\lambda[\gamma^2(\lambda)+\gamma(\lambda)+\gamma'(\lambda)] \leq 1. \tag{2.22}$$

In particular, at $\lambda = 0$, *condition* (2.22) *is always satisfied; while at* $\lambda = 1$, *condition* (2.22) *becomes*

$$\sqrt{\frac{\xi_1+n}{\xi_0+n}}\left[1+\frac{\xi_1+n}{\xi_0+n}\right] \leq 3\frac{\xi_1+n}{\xi_0+n} + \frac{n^2\xi_1}{[\xi_0+n]^2}. \tag{2.23}$$

Proposition 2.5 provides conditions for the equilibrium $\lambda = \lambda(\mu)$ to be decreasing in μ or, put differently, it provides a less restrictive condition for the uniqueness of the Nash equilibrium λ^*. Note that, since $\lambda < 1$ and $\gamma'(\lambda) < 0$, condition (2.22) is always satisfied as soon as (2.12) is satisfied. This leads to the following theorem in which we summarize the main results of this section.

Theorem 2.6. *Consider the optimization problem* (2.4) *with* $c(p) = p^2$*. Suppose that* $\mu < \bar{\mu}$ *and* $\gamma(\lambda) < 1/3$*. Then, there exists a unique equilibrium* (P^*, λ^*) *such that (i)* $\lambda^* \in (0,1)$*, solves* (2.16) *and is decreasing in* μ*; and (ii)* P^* *is given by* (2.17)*.*

3. Welfare Analysis

In this section, we analyze traders' welfare and examine market conditions in which traders' welfare increases when prices become more informative in equilibrium.

3.1. The Welfare in the Grossman-Stiglitz Model.

To better understand the welfare effect of the information uncertainty, we first review the welfare results in the Grossman-Stiglitz model. The following lemma is helpful for our analysis.[11]

Lemma 3.1. *The expected utilities of informed and uninformed traders are given by, respectively,*

$$V_I(\lambda) = f(\lambda) V_U(\lambda) = -\frac{1}{\sqrt{B(\lambda)}} \qquad \text{and} \qquad V_U(\lambda) = -\frac{1}{f(\lambda)\sqrt{B(\lambda)}}, \tag{3.1}$$

where

$$f(\lambda) = \sqrt{\frac{\xi_1 + n\lambda^2}{\xi_0 + n\lambda^2}}, \qquad B(\lambda) = 1 + \frac{B_1(\lambda)}{B_2(\lambda)},$$

$$B_1(\lambda) = \xi_1 \left[\xi_1 n (1-\lambda)^2 + (\xi_0 + n\lambda)^2\right], \qquad B_2(\lambda) = \left[(1+n\lambda)\xi_1 + n\lambda^2\right]^2.$$

In addition, $V_I(\lambda) > V_U(\lambda)$*,* $V_I'(\lambda) < 0$ *and* $V_U'(\lambda) < 0$ *for* $\lambda \in [0,1]$*.*

In the Grossman-Stiglitz model, with a fixed information cost of c, the social welfare of market participants is defined by $\mathcal{U}^*_{GS}(\lambda) = \lambda V_I(\lambda) e^{\alpha c} + (1-\lambda) V_U(\lambda)$. Based on Lemma 3.1, it is straightforward to show that

$$\mathcal{U}^*_{GS}(0) = -\frac{1}{\sqrt{1+\xi_0}} \qquad \text{and} \qquad \mathcal{U}^*_{GS}(1) = -\frac{1}{\sqrt{1+\xi_1}} e^{\alpha c}. \tag{3.2}$$

Note that $\xi_0 = \xi_1(1+n)$ and hence $\mathcal{U}^*_{GS}(0) > \mathcal{U}^*_{GS}(1)$. Therefore, in terms of welfare, traders are better off under the *no-information* equilibrium than under the *full-information* one (even if the cost of acquiring information is zero), which demonstrates the well-documented Prisoner's dilemma situation in welfare.

[11] The results in Lemma 3.1 are based on Grossman and Stiglitz (1980) and Allen (1984), but presented more explicitly for convenience of the analysis.

As a textbook example, consider the case where we just have two players. By fixing all the relevant parameters equal to one ($v_\epsilon = v_\theta = v_z = \alpha = 1$), we obtain $V_U(0.5) < V_I(1) < V_U(0) < V_I(0.5)$, where $\lambda = 0.5$ accounts for the situation where the two players choose a different action. Eventually, the normal-form game has a payoff matrix as in Table 1. The resulting Prisoner's dilemma illustrates a situation in which traders fail to coordinate towards the best outcome (represented by $V_U(0)$) and come up with a socially less preferable Nash equilibrium.

	I	U
I	-0.70; -0.70	-0.56; -0.76
U	-0.76; -0.56	-0.57; -0.57

TABLE 1. Two-player payoff matrix.

Furthermore, from Lemma 3.1, $V_I(\lambda) - V_U(\lambda) > 0$ for $\lambda \in (0,1)$; hence being informed is always beneficial. The expected utilities of both informed and uninformed traders are decreasing in λ, however the difference $V_I(\lambda) - V_U(\lambda)$, can either increase or decrease in λ. In equilibrium, $V_I(\lambda)e^{\alpha c} = V_U(\lambda)$, hence $\mathcal{U}^*_{GS}(\lambda) = V_U(\lambda)$. Therefore, as the cost of acquiring information is reduced, more traders become informed, which improves price efficiency[12] but reduces welfare. Hence, we obtain the following remark:[13]

Remark 3.2. In the Grossman-Stiglitz model, only the *no-information* equilibrium, with $\lambda = 0$, is *Pareto-optimal* with respect to the social welfare. In fact, $\mathcal{U}^*_{GS}(\lambda) < \mathcal{U}^*_{GS}(0)$ for $\lambda \in (0,1)$.

3.2. **The Welfare Improvement.** To examine the welfare effect of information uncertainty in equilibrium, we fist have the following equilibrium condition,

$$\alpha\mu g(\lambda)\bar{V}(\lambda) = -[V_I(\lambda) - V_U(\lambda)],$$

[12]It is well known that, in the Grossman-Stiglitz model as in our model, price informativeness measured by $\rho^2_{\theta,P} = 1/(1+m)$, where $m = \xi_1/(n\lambda^2)$, is an increasing function of λ.

[13]Note that a similar result also holds for some related recent contributions on information markets. See, for example, Kurlat and Veldkamp (2015) and Veldkamp (2006).

where

$$\bar{V}(\lambda) \equiv \lambda V_I(\lambda) + (1-\lambda)V_U(\lambda).$$

From Lemma 3.1, we then have

$$\alpha\mu g(\lambda) = -\frac{V_I(\lambda) - V_U(\lambda)}{\bar{V}(\lambda)} = \frac{\gamma(\lambda)}{1-\lambda\gamma(\lambda)},$$

where γ has been defined in (2.5) (and hence $\gamma(\lambda) = 1 - f(\lambda)$ from Lemma 3.1). Following (2.4), we now introduce traders' welfare in equilibrium.

Definition 3.1. In equilibrium, the overall welfare of the (speculative) traders is measured by

$$\mathcal{U}^*(\lambda) \equiv \mathcal{U}(\lambda;\lambda) = \bar{V}(\lambda)e^{\Phi(\lambda)}, \tag{3.3}$$

where

$$\Phi(\lambda) \equiv \frac{c(\lambda)}{g(\lambda)}\frac{\gamma(\lambda)}{1-\lambda\gamma(\lambda)} = \frac{1}{2}\frac{\lambda\gamma(\lambda)}{1-\lambda\gamma(\lambda)}$$

represents the cost function in equilibrium.[14]

Based on Definition 3.1, it is straightforward to show that

$$\mathcal{U}^*(0) = -\frac{1}{\sqrt{1+\xi_0}} \qquad \text{and} \qquad \mathcal{U}^*(1) = -\frac{1}{\sqrt{1+\xi_1}}e^{\Phi(1)}. \tag{3.4}$$

Therefore as in the Grossman-Stiglitzmodel, in terms of welfare, traders are better off under the *no-information* than under the *full-information* equilibria even if the cost of acquiring information is zero.

To better understand this dilemma, note that as λ increases, price becomes more sensitive to the signal, i.e, b_θ increases. When $\lambda = 1$, we obtain that $d + \tilde{\theta} - R\tilde{P} = \alpha v_\epsilon \tilde{z}$. Therefore, the informed traders are only compensated by the risk premium since the information they receive have been fully reflected by the equilibrium price, i.e., $b_\theta = 1$. On the other hand, when $\lambda = 0$, which would be the outcome when information is extremely costly, the price is uninformative since $b_\theta = 0$. Therefore, traders are compensated by the risk premium, i.e, $d - R\tilde{P} = \alpha(v_\epsilon + v_z)\tilde{z}$, which is however larger than in the case of $\lambda = 1$, because traders perceive a larger dividend risk, thus a larger price discount is required.

[14]This condition is satisfied exactly when (2.16) is met. With a slight abuse of notation, we write λ in place of λ^*.

As in the Grossman-Stiglitz model, prices become more efficient as traders become less sensitive to the cost (to reduce the information uncertainty). However, differently from the Grossman-Stiglitz model, we show that, in contrast to Remark 3.2, traders' welfare can be improved as the prices become more efficient. In particular, we provide necessary and sufficient conditions for the welfare to be increasing in the fraction of informed traders, i.e., $(\mathcal{U}^*)'(\lambda) \geq 0$.

First, it follows from Lemma 3.1 that $\bar{V}(\lambda)$ and hence $\mathcal{U}^*(\lambda)$ can be written as

$$\bar{V}(\lambda) = [1 - \lambda\gamma(\lambda)]V_U(\lambda), \qquad \mathcal{U}^*(\lambda) = [1 - \lambda\gamma(\lambda)]V_U(\lambda)e^{\Phi(\lambda)}. \tag{3.5}$$

Note that $0 < \gamma(\lambda) < 1$ and hence $0 < 1 - \lambda\gamma(\lambda) < 1$ for $\lambda \in (0, 1]$. Relatively to the expected utility to be uninformed, (3.5) shows two opposite effects of the information uncertainty on traders' welfare: a positive effect on the information benefit,[15] defined as $(1 - \lambda\gamma(\lambda))^{-1}$, and a negative effect on the cost component $e^{\Phi(\lambda)}$ (to reduce the information uncertainty). With the cost function $c(p) = p^2$, we have $\Phi(\lambda) = \big[(1 - \lambda\gamma(\lambda))^{-1} - 1\big]/2$; therefore the negative cost effect increases in the information benefit. The welfare improves when the positive effect overwhelms the negative one.

Proposition 3.3. *In equilibrium,*

(i) *traders are better off than an (fictitious) uninformed trader, who does not pay any cost to acquire private information, i.e.* $\mathcal{U}^*(\lambda) \geq V_U(\lambda)$ *for* $\lambda \in (0, 1)$ *if and only if*

$$e^{\Phi(\lambda)} \leq \frac{1}{1 - \lambda\gamma(\lambda)}; \tag{3.6}$$

(ii) *traders' welfare is better than the welfare in the* no-information *equilibrium* $\lambda = 0$ *(where all traders make no effort to reduce the information uncertainty), i.e.,* $\mathcal{U}^*(\lambda) \geq V_U(0)$, *if and only if*

$$\frac{V_U(\lambda)}{V_U(0)}e^{\Phi(\lambda)} \leq \frac{1}{1 - \lambda\gamma(\lambda)}; \tag{3.7}$$

[15]Since $V_U(\lambda) < 0$, the benefit is actually reflected by the reciprocal of $1 - \lambda\gamma(\lambda)$.

(iii) *traders' welfare is improving in the fraction of informed traders, i.e.,* $(\mathcal{U}^*)'(\lambda) \geq 0$*, if and only if*

$$\frac{V_U'(\lambda)}{V_U(\lambda)} \leq \frac{1}{2}\frac{[1-2\lambda\gamma(\lambda)][\gamma+\lambda\gamma'(\lambda)]}{[1-\lambda\gamma(\lambda)]^2}; \tag{3.8}$$

In particular, at $\lambda = 0$*,* $(\mathcal{U}^*)'(0) \geq 0$ *if and only if*

$$\frac{V_U'(0)}{V_U(0)} = \frac{n\xi_0}{1+\xi_0} \leq \frac{1}{2}\gamma(0) = \frac{1}{2}\left(1 - \frac{1}{\sqrt{1+n}}\right); \tag{3.9}$$

Proposition 3.3 (i) and (ii) show that traders' welfare are better off than that of the Grossman-Stiglitzmodel when the positive effect of information benefit dominates the cost effect for all $\lambda \in [0,1]$; in particular, than the welfare in the *no-information* equilibrium $\lambda = 0$ when the dominance becomes stronger (note that $V_U(\lambda)/V_U(0) \geq 1$). The reason being that they can strategically choose the probability $p_i^* = \lambda$ to observe $\tilde{\theta}$ so that the welfare gain is not completely washed out by the cost of information.

More importantly, Proposition 3.3 (iii) shows that price efficiency can actually improve traders' welfare if the reduction in the expected utility of the uninformed traders due to an increase in λ is below a certain threshold, which depends on the welfare difference between the informed and uninformed, i.e., $\gamma(\lambda)$, and its derivative. Let $\bar{\gamma}(\lambda) := \frac{1}{1-\lambda\gamma(\lambda)}$ represent the information benefit effect. Condition (3.8) can be written as $\frac{V_U'(\lambda)}{V_U(\lambda)} \leq \bar{\gamma}'(\lambda)\left[\frac{1}{\bar{\gamma}(\lambda)} - \frac{1}{2}\right]$, or equivalently,

$$\frac{V_U'(\lambda)}{V_U(\lambda)} + \frac{1}{2}\Phi'(\lambda) \leq \frac{\bar{\gamma}'(\lambda)}{\bar{\gamma}(\lambda)},$$

meaning that, in equilibrium *lambda*, the marginal welfare cost due to an increment in the fraction of informed traders is less than the marginal welfare gain due to an increment in the probability of becoming informed.[16] In particular, the *no-information* equilibrium is not necessarily the best-case scenario (as in the Grossman-Stiglitz model). More specifically, if condition (3.9) is satisfied, there exist a $\hat{\lambda} > 0$ such that $\mathcal{U}^*(\hat{\lambda}) > \mathcal{U}^*(0) = V_U(0)$ for $\lambda \in [0, \hat{\lambda})$.

The intuition is as follows. There are two opposing effects on the welfare when the fraction of informed traders increases and price becomes more informative. The

[16] In equilibrium the fraction of informed is the same as the probability of becoming informed, though the interpretation of λ is different.

first is the (negative) *Hirshleifer effect*: revealing private information reduces payoff uncertainty, which distorts risk-sharing between the informed and uninformed traders. Put differently, they jointly make less trading profit from the noise demand $\tilde{z}$. In the limiting case as $v_\epsilon \to 0$, their trading profit approaches zero. The second (positive) effect comes from the increased probability to observe the private signal $\tilde{\theta}$, thus being able to place demands $x_i^*(P)$ that are more positively correlated with the payoff $\tilde{D}$. Essentially, (3.8) shows the condition under which the second effect dominates the first, which results in a welfare improvement. At $\lambda = 0$, this dominance occurs when the squared Sharpe ratio $\xi_0 = \alpha^2 v_z v_D$ is relatively small, while the information advantage of the informed over the uninformed traders, $\gamma(0)$, is relatively large.

The above intuition is illustrated by Panels (A) and (C) in Fig. 3.1. Panel (C) plots the equilibrium welfare functions for four values of the informativeness of the private signal n. From which, we have the following two observations. First, the initial welfare improvement from the no-information equilibrium increases as the informativeness n increases, leading to a more hump-shaped welfare function and a more significant welfare improvement for lower equilibrium λ. Second, there is a strictly positive Pareto-optimal equilibrium λ^* on the welfare improvement so that $\mathcal{U}^*(\lambda^*) \geq \mathcal{U}^*(\lambda)$ for all $\lambda \in [0, 1]$. In addition, the Pareto-optimal equilibrium λ^* decreases in n, meaning that the Pareto-optimal equilibrium can only be achieved at a lower fraction of informed traders when the information become more informative. Panel (A) plots the regions $\Omega(\lambda)$ for the welfare improvement with the given equilibrium λ and $(\mathcal{U}^*)'(\lambda) = 0$ on the boundaries. It shows that the parameter region in (n, v_D) shrinks in λ, satisfying $\Omega(\lambda_1) \subset \Omega(\lambda_2)$ for $\lambda_1 > \lambda_2$. Clearly, when $(\mathcal{U}^*)'(0) > 0$, we have $(\mathcal{U}^*)(\lambda) \geq (\mathcal{U}^*)(0)$ for $\lambda \in [0, \bar{\lambda})$ with some $\bar{\lambda} > \lambda^*$. Panels in Fig. 3.2 demonstrate that the region for $(\mathcal{U}^*)(\lambda) \geq (\mathcal{U}^*)(0)$ is larger than the region for $(\mathcal{U}^*)'(0) > 0$ for different values of λ. Therefore, different from the Grossman-Stiglitz model, traders' welfare is better than that in the no-information equilibrium for $0 < \lambda\bar{\lambda}$. More importantly, for $0 \leq \lambda \leq \lambda^*$, traders' welfare can be improved as the prices become more efficient, resulting a positive relation between price efficiency and social welfare.

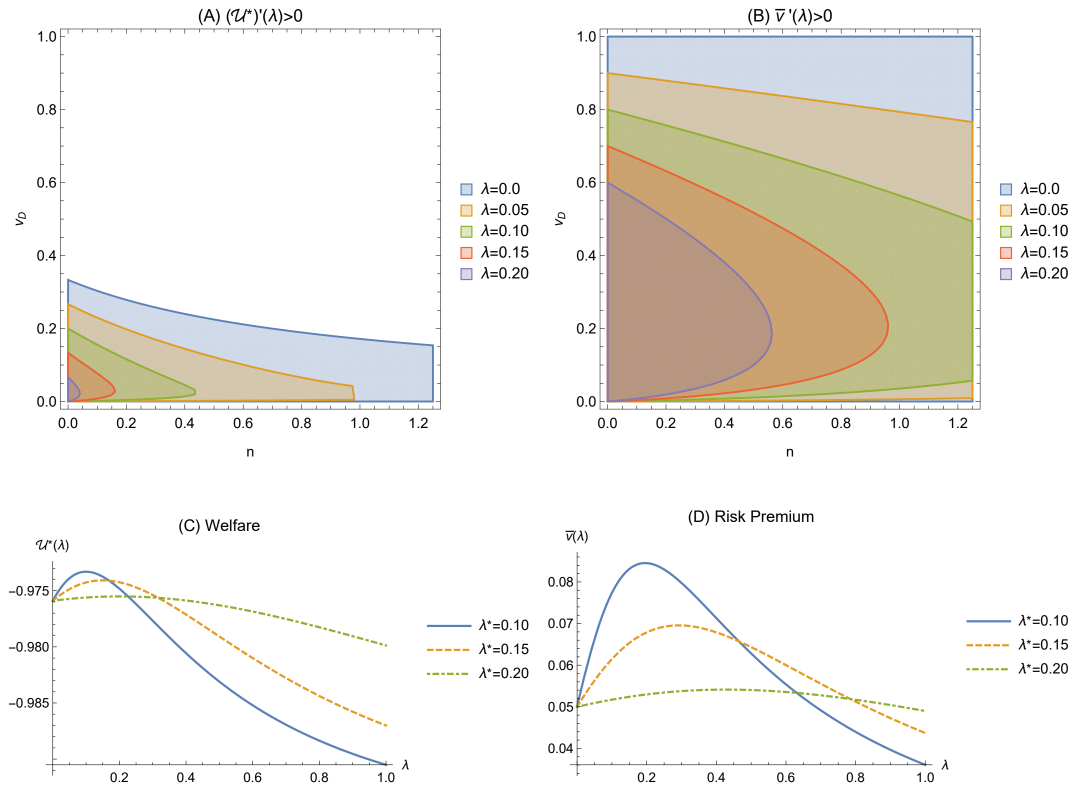

FIGURE 3.1. Panels A and B show that regions marked by $\{n, v_D\}$ in which $(\mathcal{U}^*)'(\lambda) > 0$ and $\bar{v}'(\lambda) > 0$, respectively, for $\lambda \in \{0, 0.05, 0.10, 0.15, 0.20\}$. Panels C and D show $\mathcal{U}^*(\lambda)$ and $\bar{v}(\lambda)$ for $0 \leq \lambda \leq 1$, where $v_D = 0.05$ and n is chosen such that $(\mathcal{U}^*)'(\lambda^*) = 0$ with $\lambda^* \in \{0.10, 0.15, 0.20\}$ and $n \in \{0.386, 0.144, 0.020\}$. We set $\alpha = 1$ and $v_z = 1$.

To better understand the underlying mechanism, we next try to relate traders' welfare improvement to the behaviour of risk premium as a function of the state variable.

3.3. **Relationship between Welfare Improvement and Risk Premium.** In this subsection, we try to draw a connection between the condition for the improvement of traders' welfare and the expected liquidity cost of the noise demand, $\mathbb{E}\left[-\tilde{z}\tilde{P}\right] = (\alpha\bar{v})v_z$, which is the product of the risk premium $\alpha\bar{v}$ and the variance of the noise demand. It can be seen from Equation (2.18) in Proposition 2.3 that $\bar{v}$ is not necessarily decreasing in the fraction of the informed traders λ. Similarly,

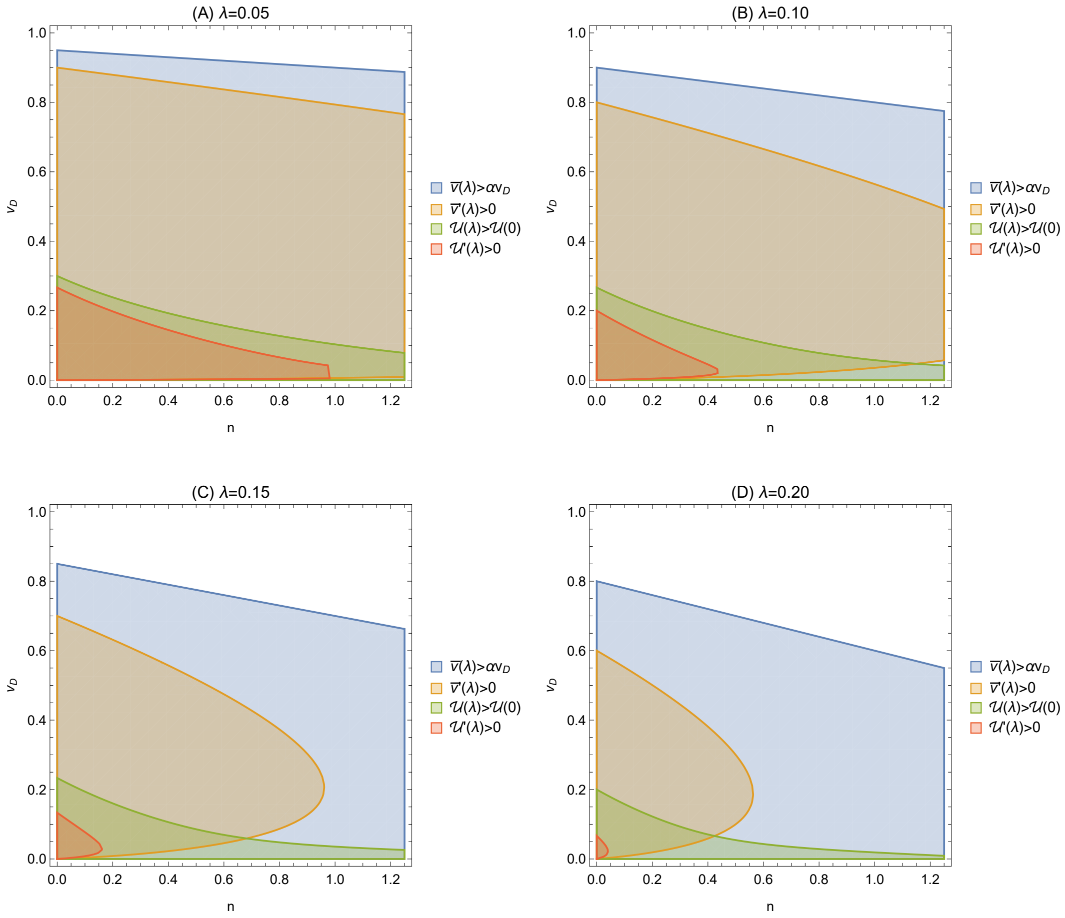

FIGURE 3.2. Regions characterized by $\{v_D, n\}$ for $\lambda \in \{0.05, 0.10, 0.15, 0.20\}$, where $(\mathcal{U}^*)'(\lambda) > 0$, $(\mathcal{U}^*(\lambda) > \mathcal{U}^*(0)$, $\bar{v}'(\lambda) > 0$ and $\bar{v} > \alpha v_D$. We set $\alpha = 1$ and $v_z = 1$.

the total risk faced by the uninformed traders can be larger than the unconditional variance of the dividend payoff, i.e., $v_U \geq v_D$.

Intuitively, a larger proportion of informed traders reduces payoff uncertainty, however it also increases adverse selection risk for the uninformed traders so that they trade less aggressively knowing that price contains private information. Therefore, the aggregate risk can increase in λ if the adverse selection risk plays the most prominent role. The next corollary pins down the conditions for this to occur and also its connection with welfare improvement.

Corollary 3.4. *On the aggregate risk $\bar{v}$ (per unit of supply, that is $\frac{1}{z}\frac{\partial \mathbb{E}[\tilde{D} - R\tilde{P}|\tilde{z} = z]}{\partial \lambda}$):*

(i) *when $\xi_0 > 1$, it decreases in λ;*

(ii) *when $\xi_0 \leq 1$, it increases in λ if and only if*

$$\lambda \leq \bar{\lambda} = \frac{1}{n}\left[\sqrt{\xi_1(n+\xi_0)} - \xi_0\right];$$

(iii) *when the aggregate risk $\bar{v}$ is decreasing in λ, traders' welfare $\mathcal{U}^*(\lambda)$ is also decreasing at $\lambda = 0$, i.e., $(\mathcal{U}^*)'(0) < 0$.*

Corollary 3.4 shows that, close to the *no-information* equilibrium $\lambda = 0$, more informed trading improves both risk premium and traders' welfare only if the expected trading profit $\xi_0 \leq 1$. Intuitively, when the expected trading profit is large, the welfare of the uninformed traders decrease at a faster rate, as shown by Equation (3.9), although this can be offset by a relatively small n (signal informativeness), a small n also reduces $\gamma(0)$ (welfare gain by observing the private signal). Therefore, an initial welfare improvement from the no-information equilibrium is always accompanied by an increase in risk premium and expected liquidity cost. Note that the converse is not true, i.e., an increase in risk premium is not a sufficient condition for welfare improvement at the *no-information* equilibrium.

Figures 3.1 and 3.2 illustrate the above results numerically. Panel (D) in Fig. 3.1 plots the equilibrium risk premium $\bar{v}(\lambda)$ for the four values of the informativeness n, showing the exact same pattern as the welfare to the informativeness of the information. Together with Panel (C) in Fig. 3.1, it shows a positive connection between social welfare and the risk premium. Panel (B) in Fig. 3.1 plots the regions for the risk premium improvement for the given equilibrium λ with $\bar{v}'(\lambda) = 0$ on the boundaries. It shows that the parameter region for the risk premium improvement shrinks in λ. In addition, comparing Panels (A) and (B) in 3.1, for the given λ, the parameter region for the welfare improvement is always a subset of the parameter region for the risk premium improvement. This implies a positive connection from social welfare to the riks premium. This illustrates the risk premium channel to the welfare improvement, but not otherwise, as shown in Corollary 3.4. This mechanism channel becomes even more clearly in Figures 3.2, showing that, for given equilibrium λ, the parameter region for $\bar{v} > \alpha v_D$ is the largest, followed by the regions for $\bar{v}'(\lambda) > 0$, $(\mathcal{U}^*)(\lambda) > (\mathcal{U}^*)(0)$, and $(\mathcal{U}^*)'(\lambda) > 0$ as subsequent subsets. Therefore, the improvement in social welfare comes from high return, high risk investments, which however not sufficient for better social welfare.

4. Modelling Trading Motives Explicitly

In this section, rather assuming noise in supply, we follow Bond and García (2017) explicitly to motivate trading using *endowment shocks*.

Each trader i receives an endowment, $e_i\tilde{D}$, at the end of the trading period. Thus, trader i's terminal wealth is given by[17]

$$\tilde{W}_i = (x_i + e_i)(\tilde{D} - \tilde{P}) + e_i\tilde{P}. \tag{4.1}$$

We assume that e_i is known to trader i, but not known to other traders. Moreover $e_i = \tilde{z} + \tilde{u}_i$, where $\tilde{z} \sim \mathcal{N}(0, v_z)$ is an aggregate endowment shock and $\tilde{u}_i \sim \mathcal{N}(0, v_u)$ is an idiosyncratic shock, thus $v_e \equiv \mathbb{V}ar[\tilde{e}_i] = v_z + v_u$.

4.1. **Optimization problem.** As before, each trader i's objective is to determine the optimal probability p_i^* of observing the private signal θ, in order to maximize his expected utility of terminal wealth,

$$\mathcal{U}(p_i; \lambda, e_i) \equiv [p_i V_I(\lambda, e_i) + (1 - p_i)V_U(\lambda, e_i)]\, e^{\alpha\mu\, c(p_i)}, \tag{4.2}$$

where

$$\begin{aligned} V_I(\lambda, e_i) &= \max_{x_i} \mathbb{E}\left\{\mathbb{E}\left[u(\tilde{W}_i)|\theta, P, e_i\right]|e_i\right\}, \\ V_U(\lambda, e_i) &= \max_{x_i} \mathbb{E}\left\{\mathbb{E}\left[u(\tilde{W}_i)|P, e_i\right]|e_i\right\} \end{aligned}$$

are trader i's expected utility depending on whether or not he observes the private signal θ. Note that apart from θ, trader i also has a another private signal, which is his own endowment shock e_i. Intuitively, e_i helps trader i to forecast the aggregate endowment shock $\tilde{z}$, which is negatively correlated with the equilibrium price $\tilde{P}$. For example, after observing the same price, a trader who receives a positive endowment shock will infer a larger value for θ than a trader who receives a negative endowment shock.

[17]For simplicity, we assume the the payoff of the risk-free asset, $R = 1$.

Conditional on his information set, trader i's optimal portfolio is given by

$$x_i^*(\mathcal{F}_i) = \frac{\mathbb{E}\left[\tilde{D} - \tilde{P}|\mathcal{F}_i\right]}{\alpha \mathbb{V}ar\left[\tilde{D} - \tilde{P}|\mathcal{F}_i\right]} - e_i, \tag{4.3}$$

where the information set $\mathcal{F}_i = \{\theta, P, e_i\}$ if trader i is informed and $\mathcal{F}_i = \{P, e_i\}$ if he is uninformed. As before, we conjecture a linear equilibrium price,

$$\tilde{P} = d + b_\theta \tilde{\theta} - b_z \tilde{z}. \tag{4.4}$$

Next, we characterize the solution to traders' optimization problem. The optimal demand for the uninformed and informed traders are given by

$$x_i^*(P, e_i) = \frac{(1-\kappa)(d-P) - \kappa \beta_{e,P} e_i}{\alpha v_U} - e_i, \tag{4.5}$$

where

$$\kappa = \frac{\sigma_{\theta,P}}{v_P - \frac{\sigma_{e,P}^2}{v_e}}, \qquad \text{and} \qquad v_U = (v_\epsilon + v_\theta)\left(1 - \frac{\rho_{P,D}^2}{1-\rho_{e,P}^2}\right) = v_D - \kappa\sigma_{\theta,P},$$

and

$$x_i^*(\theta, P, e_i) = \frac{d + \theta - P}{\alpha v_\epsilon} - e_i, \tag{4.6}$$

respectively.

Next, we compute expected utilities for the informed and uninformed traders, i.e., $V_I(\lambda, e_i)$ and $V_U(\lambda, e_i)$. Next, we work out . First, trader i's welfare given his information set is given by

$$\mathbb{E}\left[u(\tilde{W}_i)|\mathcal{F}_i\right] = -\exp\left\{-\alpha e_i P - \frac{1}{2}\frac{\chi_i^2}{v_i}\right\}, \tag{4.7}$$

where $\chi_i \equiv \mathbb{E}\left[\tilde{D} - \tilde{P}|\mathcal{F}_i\right]$ and $v_i \equiv \mathbb{V}ar\left[\tilde{D} - \tilde{P}|\mathcal{F}_i\right]$. Since, conditional on the investor i's endowment shock e_i, the price P and expected excess return χ_i follow a bivariate normal distribution, we can obtain the following expression for trader i's welfare given his own endowment shock.

Proposition 4.1.

$$\mathbb{E}\left\{\mathbb{E}\left[u(\tilde{W}_i)|\mathcal{F}_i\right]|e_i\right\} = -\exp\left\{-A_0 e_i + \frac{1}{2}A_1 e_i^2\right\}\left(\frac{\nu}{v_i}\right)^{-1/2}, \tag{4.8}$$

where

$$A_0 = \alpha\, d, \qquad A_1 = \frac{\alpha^2(v_{P|e}(v_\epsilon + v_\theta) - \sigma^2_{\theta,P}) - \beta^2_{e,P} - 2\alpha(v_\epsilon + v_\theta - \sigma_{\theta,P})\beta_{e,P}}{\nu}$$

and

$$\nu \equiv v_{\chi_i} + v_i = v_{P|e} + (v_\epsilon + v_\theta) - 2\sigma_{\theta,P}, \qquad v_{P|e} \equiv \mathbb{V}ar\left[\tilde{P}|e_i\right] = v_P - \frac{\sigma^2_{e,P}}{v_z + v_u}.$$

From Proposition 4.1, the expected utility gain of becoming informed is independent of his endowment shock e_i, i.e.,

$$\gamma(\lambda) \equiv \frac{V_I(e_i;\lambda) - V_U(e_i;\lambda)}{-V_U(e_i;\lambda)} = 1 - \sqrt{\frac{v_\epsilon}{v_D - \kappa\sigma_{\theta,P}}}. \tag{4.9}$$

Therefore, the solution to trader i's optimization problem in (4.2) is given by (2.9) just as in the baseline model. Also, the concavity condition, $\mathcal{U}''(p_i;\lambda,e_i) < 0$, is satisfied if (2.12) is true, where $\gamma(\lambda)$ is given by (4.9).

4.2. **Equilibrium.** Since we assume the risky asset is in zero net supply, market clearing requires

$$\int_0^1 [\lambda x_i^*(\theta, P, e_i) + (1-\lambda)x_i^*(P, e_i)]\, di = 0, \tag{4.10}$$

where λ is the fraction of informed traders. In the next proposition, we determine the coefficient b_θ and b_z in equilibrium.

Proposition 4.2. *For given $\lambda \in (0,1)$, let $\Psi \equiv v_z/(v_z + v_u)$, there exists a linear equilibrium price of the risky asset,*

$$\tilde{P} = d + b_\theta\tilde{\theta} - b_z\tilde{z}, \tag{4.11}$$

where $x \equiv \frac{b_\theta}{b_z}$ solves

$$x = \frac{1}{\alpha v_\epsilon}\left(\lambda + \frac{1-\lambda}{\Psi^{-1} + x^{-2}(v_\epsilon^{-1} + v_\theta^{-1})v_u}\right), \tag{4.12}$$

$$b_\theta = \frac{1}{1 + x^{-2}\frac{v_\theta^{-1} + \lambda v_\epsilon^{-1}}{v_u^{-1} + v_z^{-1}}} + \frac{\lambda}{\frac{v_\epsilon + \lambda v_\theta}{v_\epsilon + v_\theta} + x^2\frac{v_u^{-1} + v_z^{-1}}{v_\epsilon^{-1} + v_\theta^{-1}}}, \tag{4.13}$$

and λ is the solution of

$$\lambda = g^{-1}\left(\frac{1}{\alpha\mu}\frac{\gamma(\lambda)}{1 - \lambda\gamma(\lambda)}\right), \tag{4.14}$$

where $\gamma(\lambda)$ is given by (4.9), assuming the sufficient condition for concavity in (2.12) is satisfied for the equilibrium λ.

4.3. **Welfare.** The welfare of trader i, given his endowment shock, e_i can be measured by

$$\mathcal{U}^*(\lambda; e_i) \equiv \mathcal{U}(\lambda; \lambda, e_i), \qquad \alpha\mu = \frac{1}{2}\frac{\gamma(\lambda)}{\lambda\left(1-\lambda\gamma(\lambda)\right)}, \tag{4.15}$$

since every trader optimally choose the same probability $p_i^* = \lambda$ in the Nash equilibrium.

Next, we consider two special cases where $\lambda = 0$ and $\lambda = 1$. Note that for $\lambda = 0$, the equilibrium price becomes $\tilde{P} = d - \alpha(v_\theta + v_\epsilon)\tilde{z}$ and trader i's optimal portfolio is $x_i^*(P, e_i) = \frac{d-P}{\alpha(v_\theta+v_\epsilon)} - e_i$. On the other hand, when $\lambda = 1$, the equilibrium price and trader i's optimal portfolio are given by $\tilde{P} = d + \tilde{\theta} - \alpha v_\epsilon \tilde{z}$ and $x_i^*(\theta, P, e_i) = \frac{d+\theta-P}{\alpha v_\epsilon} - e_i$. The following proposition characterizes traders' overall welfare.

Corollary 4.3. *The welfare of trader i is characterized by Equation* (4.8)*, where*

$$A_1 = \frac{\alpha^2(v_\epsilon + v_\theta)\left(2v_z/v_e - (v_z/v_e)^2 + \alpha^2(v_\epsilon + v_\theta)v_{z|e}\right)}{1 + \alpha^2 v_{z|e}(v_\epsilon + v_\theta)}, \qquad \frac{\nu}{v_i} = 1 + \alpha^2(v_\epsilon + v_\theta)v_{z|e} \tag{4.16}$$

when $\lambda = 0$ *with* $\mu \to \infty$*, and*

$$A_1 = \frac{\alpha^2 v_\epsilon\left(2v_z/v_e - (v_z/v_e)^2 + \alpha^2 v_\epsilon v_{z|e}\right)}{1 + \alpha^2 v_{z|e} v_\epsilon} + \alpha^2 v_\theta, \qquad \frac{\nu}{v_i} = 1 + \alpha^2 v_\epsilon v_{z|e} \tag{4.17}$$

when $\lambda = 1$ *with* $v_{z|e} \equiv \mathbb{V}ar\left[\tilde{z}|e_i\right] = (v_z^{-1} + v_u^{-1})^{-1}$ *and* $\alpha\mu = \frac{1}{2}\frac{\gamma(1)}{1-\gamma(1)}$.

Moreover, traders are always better off in the no-information equilibrium than in the full-information equilibrium, i.e.,

$$\frac{\mathcal{U}^*(0; e_i)}{\mathcal{U}^*(1; e_i)} = \exp\left\{-\frac{1}{2}\frac{\gamma(1)}{1-\gamma(1)} - \frac{1}{2}\alpha^2\left(\frac{v_u}{v_e}e_i\right)^2 v_\theta\right\}\sqrt{\frac{1 + \alpha^2 v_\epsilon v_{z|e}}{1 + \alpha^2(v_\epsilon + v_\theta)v_{z|e}}} \leq 1. \tag{4.18}$$

Corollary 4.3 shows that, in terms of welfare, the no-information equilibrium (no traders observe $\tilde{\theta}$) dominates the full-information equilibrium (all traders observe $\tilde{\theta}$), since welfare improves for every trader when λ switches from zero to one, regardless of the realization of endowment shocks. In other words, the full-information equilibrium is *not* Pareto efficient, in the sense that there exists an equilibrium with a different λ such that no traders are worse off and at least one trader's welfare improves. Therefore, an important question is whether the no-information equilibrium also dominates any other equilibrium with $\lambda \in (0, 1)$. If so, one may conclude that (at least in this particular model setting), the social value of asymmetric information is strictly negative, and traders can be made better off if no one observes $\tilde{\theta}$, e.g. by increasing the cost parameter μ. However, in the following we show (numerically) that this is *not* the case.

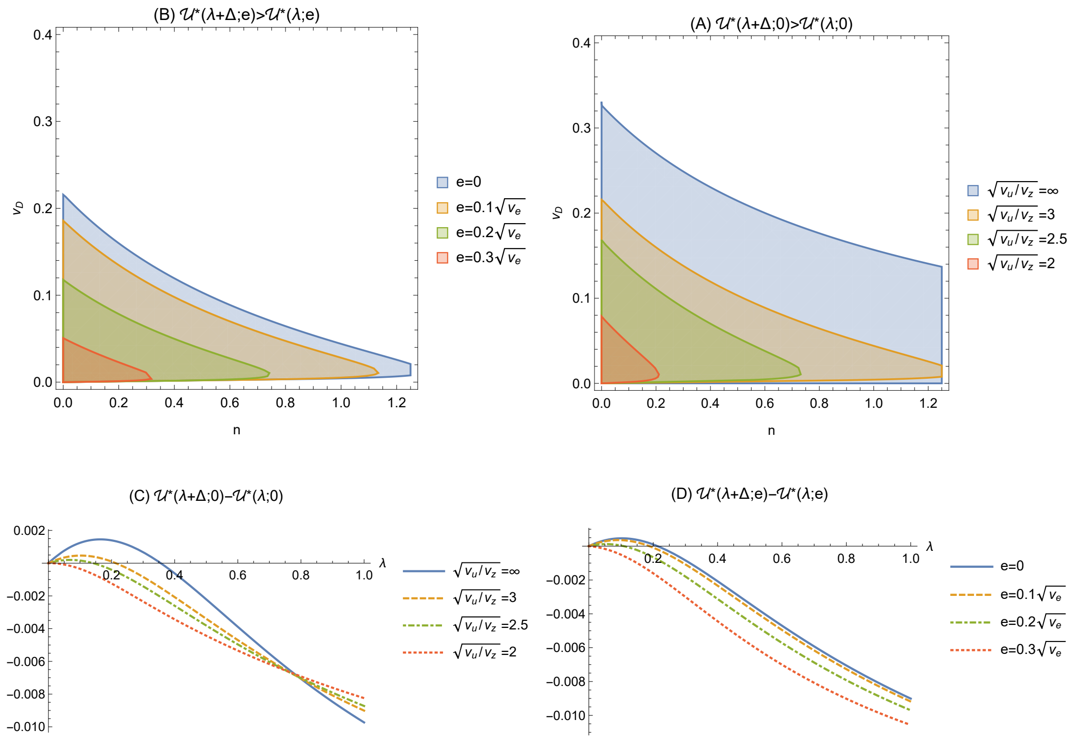

FIGURE 4.1. Panels A and B show that regions marked by $\{n, v_D\}$ in which $\mathcal{U}^*(\Delta; e) > \mathcal{U}^*(0; e)$, where $e = 0$ and $\sqrt{v_u/v_z} \in \{\infty, 3, 2.5, 2\}$ in Panels A, and $e/\sqrt{v_e} \in \{0, 0.1, 0.2, 0.3\}$ and $\sqrt{v_u/v_z} = 3$ in Panels B. Panels C and D show the welfare improvement $\mathcal{U}^*(\Delta; e) - \mathcal{U}^*(0; e)$, where $n = 0.1$. Also, we set $\alpha = 1$ and $v_z = 1$.

In Figure 4.1, it is clear that the welfare improvement region shrinks in v_u and e. Also the λ^* for which $\mathcal{U}'(\lambda^*; e) = 0$ increases with v_u but decreases with e. Therefore, there exists λ close to zero such that the welfare improves for traders with small endowment shocks, i.e., e relatively close to zero. Therefore, equilibria with asymmetric information can be Pareto efficient, since it is not dominated by the no-information equilibrium.

5. Conclusion

In this paper, we model traders' strategic choice of the probability to observe a costly private signal that helps to reduce uncertainty of the future payoff in an otherwise standard noisy rational expectation model of Grossman and Stiglitz (1980). We find that, in contrary to prior literature, paying a cost to increase the probability of becoming informed can be beneficial to speculative traders who are supplying liquidity to either noise traders (with exogenously given liquidity needs) or hedgers who trade to insure against future endowment risk. Due to high-risk, high return investments, the marginal expected utility gain from observing the information is not completely washed out by the cost of information acquisition. Therefore, with information uncertainty, the social value of asymmetric information is not strictly negative as suggested by previous literature. Consequently, an increase of market quality can be associated with a beneficial increase in social welfare.

Appendix A. Proofs

Proof of Lemma 2.1: Since traders' terminal wealth $\tilde{W}_i = x_i(\tilde{D} - \tilde{P})$ is normally distributed, given his type, trader i's optimal demand is given by

$$x_i^*(P) = \frac{\mathbb{E}\left[\tilde{D} - R\tilde{P}|\mathcal{F}_i\right]}{\alpha \mathbb{V}ar\left[\tilde{D} - \tilde{P}|\mathcal{F}_i\right]}. \tag{A.1}$$

For the informed trader who observes θ and P,

$$\mathbb{E}\left[\tilde{D} - R\tilde{P}|\theta, P\right] = d + \theta - RP, \qquad \mathbb{V}ar\left[\tilde{D} - \tilde{P}|\theta, P\right] = v_\epsilon. \tag{A.2}$$

On the other hand, for the uninformed trader who only observes P,

$$\mathbb{E}\left[\tilde{D} - R\tilde{P}|P\right] = (1 - \beta_{P,\theta})(d - RP), \qquad \mathbb{V}ar\left[\tilde{D} - \tilde{P}|P\right] = v_\theta(1 - \rho_{\theta,P}^2). \tag{A.3}$$

Substituting (A.2) and (A.3) into (A.1) leads to (2.7).

Next, we compute trader i's expected utility given their information set $\mathcal{F}_i$, which yields

$$\begin{aligned}\mathbb{E}\left[u(\tilde{W}_i)|\mathcal{F}_i\right] &= -\exp\left\{-\alpha\left(\mathbb{E}\left[\tilde{W}_i|\mathcal{F}_i\right] - \frac{1}{2}\alpha\mathbb{V}ar\left[\tilde{W}_i|\mathcal{F}_i\right]\right)\right\} \\ &= -\exp\left\{-\alpha\left(x_i^*\underbrace{\mathbb{E}\left[\tilde{D}-R\tilde{P}|\mathcal{F}_i\right]}_{\chi_i} - \frac{1}{2}\alpha(x_i^*)^2\underbrace{\mathbb{V}ar\left[\tilde{D}-R\tilde{P}|\mathcal{F}_i\right]}_{v_{D|\mathcal{F}_i}}\right)\right\} \\ &= -\exp\left\{-\frac{1}{2}\frac{\chi_i^2}{v_{D|\mathcal{F}_i}}\right\}. \end{aligned} \tag{A.4}$$

For the informed, $v_{D|\mathcal{F}_I} = v_\epsilon$ and $v_{\chi_I} = \mathbb{V}ar\left[(1-b_\theta)\theta - b_z z\right] = (1-b_\theta)^2 v_\theta + b_z^2 v_z$, whereas for the uninformed trader, $v_{D|\mathcal{F}_U} = v_{D|P} = v_\epsilon + (1-\rho_{\theta,P}^2)v_\theta = v_\epsilon + (1-\beta_{P,\theta}b_\theta)v_\theta$ and $v_{\chi_U} = \mathbb{V}ar[(1-\beta_{P,\theta})(d-RP)] = \mathbb{V}ar[(1-\beta_{P,\theta})(-b_\theta\theta + b_z z)] = (1-\beta_{P,\theta})^2(b_\theta^2 v_\theta + b_z^2 v_z)$.

Next, since the conditional expectation $\chi_i = \mathbb{E}\left[\tilde{D}-R\tilde{P}|\mathcal{F}_i\right]$ itself is a normally distributed random variable for both informed and uninformed traders, we can use following standard result to compute trader i's unconditional expected utility.

Lemma A.1. *Let $X \in \mathbb{R}^n$ be a normally distributed random vector with mean μ and variance-covariance matrix Σ. Let $b \in \mathbb{R}^n$ be a given vector, and $A \in \mathbb{R}^{n\times n}$ a symmetric matrix. If $I - 2\Sigma A$ is positive definite, then $\mathbb{E}\left[\exp\{b^\top X + X^\top AX\}\right]$ is well defined, and given by*

$$\begin{aligned}\mathbb{E}\left[\exp\{b^\top X + X^\top AX\}\right] =& |I-2\Sigma A|^{-1/2}\exp\{b^\top\mu + \mu^\top\Sigma\mu \\ &+ \frac{1}{2}(b+2A\mu)^\top(I-2\Sigma A)^{-1}\Sigma(b+2A\mu)\}.\end{aligned}$$

Applying Lemma A.1 to the conditional expected utility in (A.4) with $X = \chi_i$, $A = -\frac{1}{2}(v_{D|\mathcal{F}_i})^{-1}$, $\Sigma = v_{\chi_i}$, $b = 0$, $\mu = 0$ leads to the desired result and the expressions for $\xi_I(\lambda)$ and $\xi_U(\lambda)$ in (2.8).

Thus, assuming the concavity condition $\mathcal{U}''(p_i,\lambda)$ is satisfied, trader i's optimal choice of p_i is determined by the first order condition,

$$\alpha\mu g(p_i^*) = -\frac{V_I(\lambda)-V_U(\lambda)}{\lambda V_I(\lambda) + (1-\lambda)V_U(\lambda)} = \frac{\gamma(\lambda)}{1-\lambda\gamma(\lambda)},$$

which leads to (2.9).

Proof of Proposition 2.3: We substitute the linear equilibrium price $P = \frac{1}{R}[d + b_\theta \tilde{\theta} - b_z \tilde{z}]$ into traders' optimal demand functions in (2.7), from which we obtain

$$x_I^*(\theta, P) = \frac{d + \theta - RP}{\alpha v_\epsilon} \qquad \text{and} \qquad x_U^*(P) = \frac{\left(1 - \frac{b_\theta v_\theta}{b_\theta^2 v_\theta + b_z^2 v_z}\right)(d - RP)}{\alpha\left(v_\epsilon + \frac{b_z^2 v_z}{b_\theta^2 v_\theta + b_z^2 v_z} v_\theta\right)}. \tag{A.5}$$

Then, by applying the market clearing condition,

$$\lambda x_I^*(\theta, P) + (1 - \lambda) x_U^*(P) = \tilde{z},$$

we obtain the following equilibrium price,

$$\begin{aligned}\tilde{P} &= \frac{1}{R}\left[\frac{\frac{\lambda}{v_\epsilon}(d + \tilde{\theta}) + \frac{(1-\lambda)\left(1 - \frac{b_\theta v_\theta}{b_\theta^2 v_\theta + b_z^2 v_z}\right)}{v_\epsilon + \frac{b_z^2 v_z}{b_\theta^2 v_\theta + b_z^2 v_z} v_\theta} d - \alpha \tilde{z}}{\frac{\lambda}{v_\epsilon} + \frac{(1-\lambda)\left(1 - \frac{b_\theta v_\theta}{b_\theta^2 v_\theta + b_z^2 v_z}\right)}{v_\epsilon + \frac{b_z^2 v_z}{b_\theta^2 v_\theta + b_z^2 v_z} v_\theta}}\right] \\ &= \frac{1}{R}\left(d + \frac{\lambda \bar{v}}{v_\epsilon}\tilde{\theta} - \alpha \bar{v} \tilde{z}\right),\end{aligned} \tag{A.6}$$

where

$$\frac{1}{\bar{v}} = \frac{\lambda}{v_\epsilon} + \frac{(1 - \lambda)\left(1 - \frac{b_\theta v_\theta}{b_\theta^2 v_\theta + b_z^2 v_z}\right)}{v_\epsilon + \frac{b_z^2 v_z}{b_\theta^2 v_\theta + b_z^2 v_z} v_\theta}.$$

Thus, by matching coefficient to the conjectured equilibrium price, we obtain

$$b_\theta = \frac{\lambda \bar{v}}{v_\epsilon} \qquad \text{and} \qquad b_z = \alpha \bar{v}.$$

Since $b_\theta = \lambda b_z / (\alpha v_\epsilon)$, we obtain an explicit solution for $\bar{v}$ by solving

$$\frac{\lambda}{v_\epsilon} + \frac{(1 - \lambda)\left(1 - \frac{(\lambda b_z/\alpha)\, v_\theta / v_\epsilon}{(\lambda b_z/\alpha)^2 v_\theta / v_\epsilon^2 + b_z^2 v_z}\right)}{v_\epsilon + \frac{b_z^2 v_z}{(\lambda b_z/\alpha)^2 v_\theta / v_\epsilon^2 + b_z^2 v_z} v_\theta} = \frac{b_z}{\alpha}$$

for b_z and substituting the solution back into the expression for $1/\bar{v}$.

On the optimization problem in (2.4), let $\bar{V}(\lambda) \equiv \lambda V_I(\lambda) + (1 - \lambda) V_U(\lambda)$ and $\bar{V}(p; \lambda) \equiv pV_I(\lambda) + (1 - p) V_U(\lambda)$, we have[18]

$$\begin{aligned}\mathcal{U}'(p; \lambda) &= e^{\alpha\mu c(p)}[\alpha\mu g(p) \bar{V}(p, \lambda) + [V_I(\lambda) - V_U(\lambda)]] \\ \mathcal{U}''(p; \lambda) &= \alpha\mu g(p) e^{\alpha\mu c(p)}\left[\left(\alpha\mu g(p) + \frac{g'(p)}{g(p)}\right)\bar{V}(p, \lambda) + 2[V_I(\lambda) - V_U(\lambda)]\right].\end{aligned}$$

Therefore the necessary and sufficient condition for $\mathcal{U}''(p; \lambda) \leq 0$ is

$$\left(\alpha\mu g(p) + \frac{g'(p)}{g(p)}\right)\bar{V}(p, \lambda) + 2[V_I(\lambda) - V_U(\lambda)] \leq 0. \tag{A.7}$$

[18] We drop the subscript i for the remainder of the proof, in order to ease the notation.

Note that in equilibrium,

$$\alpha\mu = -\frac{1}{g(\lambda)}\frac{V_I(\lambda) - V_U(\lambda)}{\bar{V}(\lambda)} = \frac{1}{g(\lambda)}\frac{\gamma(\lambda)}{1-\lambda\gamma(\lambda)}.$$

Also, note that

$$\bar{V}(p,\lambda) = [1 - p\gamma(\lambda)]V_U(\lambda), \qquad V_I(\lambda) - V_U(\lambda) = -\gamma(\lambda)V_U(\lambda).$$

Therefore in equilibrium, (A.7) becomes

$$\left[\frac{\gamma(\lambda)}{1-\lambda\gamma(\lambda)}\frac{g(p)}{g(\lambda)} + \frac{g'(p)}{g(p)}\right](1 - p\gamma(\lambda)) - 2\gamma(\lambda) \geq 0. \tag{A.8}$$

We have from (A.8) that

$$-\frac{\gamma(\lambda)}{1-\lambda\gamma(\lambda)}\frac{g(p)}{g(\lambda)} + \frac{2\gamma(\lambda)}{1-p\gamma(\lambda)} \leq \frac{g'(p)}{g(p)}, \tag{A.9}$$

which leads to condition (2.10).

Moreover, for a sufficient condition, note that (A.7) can be written as

$$V_I(\lambda) - V_U(\lambda) \leq -\frac{1}{2}\left[\frac{g(p)}{g(\lambda)}\frac{V_I(\lambda) - V_U(\lambda)}{-\bar{V}(\lambda)} + \frac{g'(p)}{g(p)}\right]\bar{V}(p,\lambda), \tag{A.10}$$

which can be written as

$$\underbrace{\left[1 - \frac{1}{2}\frac{\bar{V}(p,\lambda)}{\bar{V}(\lambda)}\frac{g(p)}{g(\lambda)}\right]}_{\leq 1}[V_I(\lambda) - V_U(\lambda)] \leq -\frac{1}{2}\frac{g'(p)}{g(p)}\bar{V}(p,\lambda). \tag{A.11}$$

Therefore, a sufficient condition for $\mathcal{U}''(p;\lambda) \leq 0$ is given by

$$V_I(\lambda) - V_U(\lambda) \leq -\frac{1}{2}\frac{g'(p)}{g(p)}\bar{V}(p,\lambda), \tag{A.12}$$

which is equivalent to

$$\left[1 + \frac{1}{2}\frac{g'(p)}{g(p)}p\right]\underbrace{\frac{V_I(\lambda) - V_U(\lambda)}{-V_U(\lambda)}}_{\gamma(\lambda)} \leq \frac{1}{2}\frac{g'(p)}{g(p)} \tag{A.13}$$

that simplifies to the condition (2.12).

Next, if the sufficient condition for $\mathcal{U}''(p;\lambda) \leq 0$ is satisfied, the Nash equilibrium for the choice of probability p_i to observe the private signal θ must be symmetric, since traders are homogeneous, i.e., $p_i^* = \lambda$ for all $i \in (0,1)$, from which we obtain the equilibrium λ in (2.16).

Proof of Proposition 2.4: Note that $\gamma(\lambda) \in (0,1)$. With $c(p) = p^2$, from the equilibrium condition $2\alpha\mu\lambda = \gamma(\lambda)/[1-\gamma(\lambda)]$, it is easy to see that $\lambda \to 0$ as $\mu \to \infty$. For $\lambda = 1$, we have $\mu = \frac{1}{2\alpha}\frac{\gamma(1)}{1-\gamma(1)}$.

It remains to discuss the case $\mu > \bar{\mu}$. To this aim, note that, in case of $c(p) = p^2$, the fixed point (2.16) is equivalent to

$$\lambda^2 - \frac{1}{\gamma(\lambda)}\lambda + \frac{1}{2\alpha\mu} = 0. \tag{A.14}$$

By defining

$$F^1(\lambda) = \frac{1}{2\gamma(\lambda)} - \frac{1}{2\gamma(\lambda)}\sqrt{1 - \frac{2\gamma^2(\lambda)}{\alpha\mu}}; \quad F^2(\lambda) = \frac{1}{2\gamma(\lambda)} + \frac{1}{2\gamma(\lambda)}\sqrt{1 - \frac{2\gamma^2(\lambda)}{\alpha\mu}},$$

(A.14) can be rewritten as

$$[\lambda - F^1(\lambda)][\lambda - F^2(\lambda)] = 0.$$

Assuming $\mu \geq 2\gamma^2(\lambda)/\alpha$ (otherwise the fixed point has no solution and $\lambda^* = 1$), F^1 and F^2 are well-defined. It is not difficult to show that $0 < F^1(\lambda) \leq F^2(\lambda)$. Therefore, since $F^1(0) > 0$, one solution to (A.14) exists if and only if $F^1(1) < 1$. This condition is exactly $\mu > \bar{\mu}$.

Finally, concerning uniqueness, note that $dF^1(\lambda)/d\lambda < 0$. Indeed,

$$\frac{dF^1(\lambda)}{d\lambda} = -\frac{\gamma'(\lambda)}{2\gamma^2(\lambda)}\left(1 - \sqrt{1 - \frac{2\gamma^2(\lambda)}{\alpha\mu}}\right) + \frac{\gamma'(\lambda)}{\alpha\mu\sqrt{1 - \frac{2\gamma^2(\lambda)}{\alpha\mu}}} = \frac{\gamma'(\lambda)}{\gamma(\lambda)}\frac{F^1(\lambda)}{\sqrt{1 - \frac{2\gamma^2(\lambda)}{\alpha\mu}}} < 0.$$

Negativity is due to the fact that $\gamma'(\lambda) < 0$, $\gamma(\lambda) > 0$, and $F^1(\lambda) > 0$. By monotonicity, $\lambda = F^1(\lambda)$ provides at most one solution. Therefore, if a second solution $\tilde{\lambda}$ to the fixed point exists, it must solve $\tilde{\lambda} = F^2(\tilde{\lambda})$. By definition $F^2(\lambda) > \frac{1}{2\gamma(\lambda)}$; therefore, as soon as $\gamma(\lambda^*) < 1/3$, we would have $\tilde{\lambda} = F^2(\tilde{\lambda}) > 3/2$, which is unfeasible. This proves that the solution to the fixed point is unique as soon as the sufficient condition for concavity, $\gamma(\lambda) < 1/3$, is satisfied.

Proof of Proposition 2.5: In equilibrium,

$$\alpha\mu g(\lambda) = -\frac{V_I(\lambda) - V_U(\lambda)}{\bar{V}(\lambda)} = \frac{\gamma(\lambda)}{1 - \lambda\gamma(\lambda)} = \Gamma(\lambda).$$

For $\lambda = \lambda(\mu)$, taking the derivative w.r.t. μ, we have

$$\alpha g(\lambda) = -\lambda'(\mu)[\Gamma'(\lambda) - \frac{g'(\lambda)}{g(\lambda)}\Gamma(\lambda)].$$

Therefore $\lambda'(\mu) \leq 0$ if and only if (2.20) holds.

Applying $c(p) = p^2$ to condition (2.20) leads to condition (2.21). Clearly, (2.21) holds for $\lambda = 0$. For $\lambda = 1$, condition (2.21) becomes

$$\gamma_1^2 + \gamma_1 + \gamma_1' \leq 1.$$

Since $\gamma(\lambda) = 1 - f(\lambda)$, this is equivalent to

$$1 + f_1^2 \leq 3f_1 + f_1'.$$

Using the fact that $f(\lambda) = \sqrt{\frac{\xi_1 + n\lambda^2}{\xi_0 + n\lambda^2}}$, we obtain condition (2.23).

Proof of Proposition 3.3: In equilibrium, the welfare function is given by

$$\mathcal{U}^*(\lambda) = \bar{V}(\lambda) \exp(\Phi(\lambda)),$$

where

$$\bar{V}(\lambda) = \lambda V_I(\lambda) + (1-\lambda)V_U(\lambda) = (1 - \lambda\gamma)V_U(\lambda), \qquad \Phi(\lambda) = \frac{c(\lambda)}{g(\lambda)} \frac{\gamma(\lambda)}{1 - \lambda\gamma(\lambda)}.$$

Hence

$$\mathcal{U}^{*\prime}(\lambda) = e^{\Phi(\lambda)}(-\bar{V}(\lambda)) \left[\frac{\bar{V}'(\lambda)}{-\bar{V}(\lambda)} - \Phi'(\lambda) \right].$$

and $\mathcal{U}^{*\prime}(\lambda) \geq 0$ if and only if

$$\frac{\bar{V}'(\lambda)}{-\bar{V}(\lambda)} \geq \Phi'(\lambda),$$

Note that

$$\frac{\bar{V}'(\lambda)}{-\bar{V}(\lambda)} = \frac{V_U'(\lambda)}{-V_U(\lambda)} + \frac{\gamma(\lambda) + \lambda\gamma'(\lambda)}{1 - \lambda\gamma(\lambda)}$$

and

$$\Phi'(\lambda) = \frac{\gamma(\lambda)}{1 - \lambda\gamma(\lambda)} \left[1 - \frac{c(\lambda)g'(\lambda)}{g^2(\lambda)} + \frac{c(\lambda)}{g(\lambda)} \frac{\gamma(\lambda)}{1 - \lambda\gamma(\lambda)} \right] + \frac{c(\lambda)}{g(\lambda)} \frac{\gamma'(\lambda)}{(1 - \lambda\gamma(\lambda))^2}.$$

Therefore $\mathcal{U}'(\lambda) \geq 0$ if and only if

$$\frac{V_U'(\lambda)}{V_U(\lambda)} \leq \frac{\gamma(\lambda)}{1 - \lambda\gamma(\lambda)} \frac{c(\lambda)}{g(\lambda)} \left[\frac{g'(\lambda)}{g(\lambda)} - \frac{\gamma(\lambda)}{1 - \lambda\gamma(\lambda)} \right] + \left(\frac{\lambda}{1 - \lambda\gamma(\lambda)} - \frac{c(\lambda)}{g(\lambda)} \frac{1}{(1 - \lambda\gamma(\lambda))^2} \right) \gamma'(\lambda).$$

Since $c(p) = p^2$, it follows that

$$S(\lambda) \equiv \frac{1}{2} \left[1 - \frac{\lambda\gamma(\lambda)}{1 - \lambda\gamma(\lambda)} \right] + \frac{\lambda}{2} \left[2 - \frac{1}{1 - \lambda\gamma(\lambda)} \right] \frac{\gamma'(\lambda)}{\gamma(\lambda)} = \frac{1}{2} \frac{1 - 2\lambda\gamma(\lambda)}{1 - \lambda\gamma(\lambda)} \left[1 + \lambda \frac{\gamma'(\lambda)}{\gamma(\lambda)} \right],$$

which leads to (3.8). At $\lambda = 0$,

$$\frac{V_U'(0)}{V_U(0)} = \frac{n\xi_0}{1 + \xi_0}.$$

Applying this to condition (3.8) at $\lambda = 0$ leads to condition (3.9).

Proof of Corollary 3.4: The risk premium per unit of supply is given by $\frac{1}{z}\mathbb{E}[\tilde{D}-R\tilde{P}|z] = \alpha\bar{v}$. From (2.19), we have

$$\frac{v_\epsilon}{\bar{v}} = \lambda + \frac{1-\lambda}{1+n+\frac{n}{\xi_1}\lambda}.$$

Then

$$\frac{\partial(v_\epsilon/\bar{v})}{\partial\lambda} = \frac{[1+n+\frac{n}{\xi_1}\lambda]^2 - [1+n+\frac{n}{\xi_1}]}{[1+n+\frac{n}{\xi_1}\lambda]^2}. \tag{A.15}$$

(i) When $\xi_0 > 1$, $\frac{\partial(1/\bar{v})}{\partial\lambda} > 0$ always holds, meaning that the risk premium $\bar{v}$ decreases in λ.

(ii) When $\xi_0 \leq 1$, we have $\frac{\partial(1/\bar{v})}{\partial\lambda} \leq 0$ if and only if

$$1+n+\frac{n}{\xi_1}\lambda \leq \sqrt{(1+n)+\frac{n}{\xi_1}},$$

which leads to the result.

(iii) When $\lambda = 0$, aggregate risk $\bar{v}$ decreases in λ, i.e., $\bar{v}(0) < 0$, if and only if $\xi_0 > 1$. Note that if $\xi_0 > 1$, the necessary and sufficient condition for welfare improvement in (3.9) becomes

$$n \leq \gamma(0) = 1 - \sqrt{\frac{1}{1+n}} \Rightarrow 1-n \geq \sqrt{\frac{1}{1+n}}, \tag{A.16}$$

which requires $n < 1$. However, when $n < 1$, (A.16) becomes

$$(1-n)^2(1+n) \geq 1, \tag{A.17}$$

which never holds since

$$(1-n)^2(1+n) - 1 = n[n(n-1)-1] < 0. \tag{A.18}$$

Proof of Proposition 4.1: In the following proof, we drop the trader specific subscript, as we will always be referring to trader i. First, note that each trader's expected utility conditional on his information set is given by

$$V_K(e;\lambda) \equiv \mathbb{E}\left\{\mathbb{E}\left[u(\tilde{W})|\mathcal{F}\right]|e\right\} = \mathbb{E}\left[-\exp\left\{-\alpha eP - \frac{1}{2}\frac{\chi^2}{v}\right\}|e\right], \qquad K \in \{I,U\}, \tag{A.19}$$

where $\chi \equiv \mathbb{E}[\tilde{D}-\tilde{P}|\mathcal{F}]$ and $v \equiv \mathbb{V}ar[\tilde{D}-\tilde{P}|\mathcal{F}]$, respectively.

First, since (given the endowment shock e) χ and P follow a bivariate normal distribution with mean vector and covariance matrix given by

$$\mu = \begin{pmatrix} \mu_{\chi|e} \\ \mu_{P|e} \end{pmatrix} \qquad \text{and} \qquad \Sigma = \begin{pmatrix} v_{\chi|e} & \sigma_{(\chi,P)|e} \\ \sigma_{(\chi,P)|e} & v_{P|e} \end{pmatrix}, \tag{A.20}$$

where $\mu_{\chi|e} \equiv \mathbb{E}[\chi|e]$, $\mu_{P|e} \equiv \mathbb{E}[P|e]$, $v_{\chi|e} \equiv \mathbb{V}ar[\chi|e]$, $v_{P|e} \equiv \mathbb{V}ar[P|e]$ and $\sigma_{(\chi,P)|e} \equiv \mathbb{C}ov[\chi, P|e]$. Thus, using Lemma A.1 we can establish the following result,

$$V_K(e;\lambda) = -\exp\left\{-\frac{\mu^2_{\chi|e} + \alpha e\left[2\nu\mu_{P|e} + 2\mu_{\chi|e}\sigma_{(\chi,P)|e} + \alpha e\left(\sigma^2_{(\chi,P)|e} - \nu v_{P|e}\right)\right]}{2\nu}\right\}\sqrt{\frac{v}{\nu}}, \tag{A.21}$$

where $\nu = v + v_{\chi|e}$.

If the trader is informed ($K = I$), i.e., $\mathcal{F} = \{\theta, P, e\}$, since $\chi = d + \theta - P$ and $v = v_\epsilon$, we obtain that

$$\begin{aligned} &\mu_{\chi|e} = -\beta_{e,P}e, \;\; \mu_{P|e} = d + \beta_{e,P}e, \\ &v_{\chi|e} = v_\theta + v_{P|e} - 2\sigma_{\theta,P}, \;\; \sigma_{(\chi,P)|e} = \sigma_{\theta,P} - v_{P|e}. \end{aligned} \tag{A.22}$$

Substituting (A.22) into (A.21) leads to the expected utility of an informed trader in (4.8) with $v = v_\epsilon$.

On the other hand, if the trader is uninformed ($K = U$), i.e., $\mathcal{F} = \{P, e\}$, since $\chi = (1-\kappa)(d-P) - \kappa\beta_{e,P}e$ and $v = v_D - \kappa\sigma_{\theta,P}$, $\kappa = \sigma_{\theta,P}/v_{P|e}$, we obtain that

$$\begin{aligned} &\mu_{\chi|e} = -\beta_{e,P}e, \;\; \mu_{P|e} = d + \beta_{e,P}e, \\ &v_{\chi|e} = (1-\kappa)^2 v_{P|e}, \;\; \sigma_{(\chi,P)|e} = -(1-\kappa)v_{P|e}. \end{aligned} \tag{A.23}$$

Substituting (A.23) into (A.21) leads to the expected utility of an uninformed trader in (4.8) with $v = v_D - \kappa\sigma_{\theta,P}$.

Proof of Proposition 4.2: Substituting the optimal demands $x^*(P, e)$ and $x^*(\theta, P, e)$ in (4.5) and (4.6) into the market clearing condition (4.10) leads the following,

$$\frac{(d-P)}{\alpha\bar{v}} + \frac{\lambda}{\alpha v_\epsilon}\tilde{\theta} = \left[1 + (1-\lambda)\frac{\kappa\beta_{e,P}}{\alpha v_U}\right]\tilde{z} \tag{A.24}$$

where

$$\frac{1}{\bar{v}} \equiv \frac{\lambda}{v_\epsilon} + \frac{1-\lambda}{\tilde{v}_U}, \qquad \tilde{v}_U = \frac{v_U}{1-\kappa}.$$

Thus, the equilibrium price can be written as

$$P = d + \underbrace{\frac{\lambda\bar{v}}{v_\epsilon}}_{b_\theta}\tilde{\theta} - \underbrace{\alpha\bar{v}\left[1 + (1-\lambda)\frac{\kappa\beta_{e,P}}{\alpha v_U}\right]}_{b_z}\tilde{z}. \tag{A.25}$$

Therefore, we obtain

$$x \equiv \frac{b_\theta}{b_z} = \frac{1}{\alpha v_\epsilon}\frac{\lambda}{1 + (1-\lambda)\frac{\kappa\beta_{e,P}}{\alpha v_U}}, \tag{A.26}$$

which can be written as

$$x = \frac{1}{\alpha v_\epsilon}\left[\lambda - (1-\lambda)\left(\kappa\beta_{e,P}\frac{v_\epsilon}{v_U}\right)x\right]. \tag{A.27}$$

Since $v_U = v_D - \kappa\sigma_{\theta,P}$, $\kappa = \sigma_{\theta,P}/v_{P|e}$ and $\beta_{e,P} = \sigma_{e,P}/v_e$, also

$$\begin{aligned} &\sigma_{e,P} = -b_z v_z, \qquad \sigma_{\theta,P} = b_\theta v_\theta, \\ &v_{P|e} = b_\theta^2 v_\theta + b_z^2 v_{z|e}, \qquad v_{z|e} = (v_z^{-1} + v_u^{-1})^{-1}, \end{aligned} \tag{A.28}$$

we can obtain that

$$-\left(\kappa\beta_{e,P}\frac{v_\epsilon}{v_U}\right)x = \frac{v_z v_\epsilon v_\theta x^2}{v_u v_z v_D + v_e v_\epsilon v_\theta x^2} = \frac{1}{v_e/v_z + x^{-2}v_u(v_\theta^{-1} + v_\epsilon^{-1})}. \tag{A.29}$$

Substituting (A.29) back into (A.27) leads to (4.12).

Next, given x, we substitute (A.28) into the expression for b_θ and obtain that

$$b_\theta = \frac{\lambda\bar{v}}{v_\epsilon} = \frac{\lambda b_z\left(v_u v_z v_D + v_e v_\epsilon v_\theta x^2\right)}{b_z x^2 v_z v_\epsilon v_\theta - v_e v_\epsilon v_\theta x(1-\lambda) + b_z v_u(v_\epsilon v_\theta x^2 + v_z v_D\lambda)}. \tag{A.30}$$

Since $b_z = b_\theta/x$, (A.30) can be simplified to

$$b_\theta = \frac{v_e v_\epsilon v_\theta x^2 + v_u v_z v_D\lambda}{v_e v_\epsilon v_\theta x^2 + v_u v_z(v_\epsilon + v_\theta\lambda)}, \tag{A.31}$$

which leads to the expression in (4.13).

Proof of Corollary 4.3: For $\lambda = 0$, since the equilibrium price $\tilde{P} = d - \alpha v_D\tilde{z}$, we have

$$\begin{aligned} &v_{P|e} = \alpha^2 v_D^2 v_{z|e}, \qquad \sigma_{\theta,P} = 0, \\ &\beta_{e,P} = -\alpha v_D\frac{v_z}{v_e}, \qquad \nu = v_D(1 + \alpha^2 v_D v_{z|e}). \end{aligned} \tag{A.32}$$

Substituting (A.32) into (4.8) leads to (4.16).

On the other hand, for $\lambda = 1$, since the equilibrium price $\tilde{P} = d + \tilde{\theta} - \alpha v_\epsilon\tilde{z}$, we have

$$\begin{aligned} &v_{P|e} = v_\theta + \alpha^2 v_\epsilon^2 v_{z|e}, \qquad \sigma_{\theta,P} = v_\theta, \\ &\beta_{e,P} = -\alpha v_\epsilon\frac{v_z}{v_e}, \qquad \nu = v_\epsilon(1 + \alpha^2 v_\epsilon v_{z|e}). \end{aligned} \tag{A.33}$$

Substituting (A.33) into (4.8) leads to (4.17).

References

Admati, A. (1985), 'A noisy rational expectations equilibrium for multi-asset securities markets', *Econometrica* **53**, 629–658.

Admati, A. and Pfleiderer, P. (1987), 'Viable allocations of information in financial markets', *Journal of Economic Theory* **43**, 76–115.

Allen, F. (1984), *The social value of asymmetric information*, Rodney L. White Center for Financial Research, The Wharton School, University of Pennyslvania.

Angeletos, G.-M. and Pavan, A. (2007), 'Efficient use of information and social value of information', *Econometrica* **75**(4), 1103–1142.

Bai, J., Philippon, T. and Savov, A. (2016), 'Have financial markets become more informative?', *Journal of Financial Economics* **122**, 625–654.

Gargano, A., Rossi, A. G. and Wermers, R. (2017), 'The freedom of information act and the race toward information acquisition', *The Review of Financial Studies* **30**(6), 2179–2228.

Goldstein, I. and Yang, L. (2017), 'Information disclosure in financial markets', *Annual Review of Financial Economics* **9**, 101–125.

Gorban, S., Obizhaeva, A. A. and Wang, Y. (2018), 'Trading in crowded markets'.

Grossman, S. J. and Stiglitz, J. E. (1980), 'On the impossibility of informationally efficiency markets', *American Economic Review* **70**(3), 393–408.

Hirshleifer, J. (1978), The private and social value of information and the reward to inventive activity, *in* 'Uncertainty in Economics', Elsevier, pp. 541–556.

Hobson, A. (1969), 'A new theorem of information theory', *Journal of Statistical Physics* **1**(3), 383–391.

Kurlat, P. and Veldkamp, L. (2015), 'Should we regulate financial information?', *Journal of Economic Theory* **158**, 697–720.

Kyle, A. S., Obizhaeva, A. A. and Wang, Y. (2017), 'Smooth trading with overconfidence and market power', *The Review of Economic Studies* **85**(1), 611–662.

Laffont, J.-J. (1985), 'On the welfare analysis of rational expectations equilibria with asymmetric information', *Econometrica: Journal of the Econometric Society* pp. 1–29.

Mattsson, L.-G. and Weibull, J. W. (2002), 'Probabilistic choice and procedurally bounded rationality', *Games and Economic Behavior* **41**, 61–78.

Morris, S. and Shin, H. S. (2002), 'Social value of public information', *American Economic Review* **92**(5), 1521–1534.

Veldkamp, L. L. (2006), 'Media frenzies in markets for financial information', *American Economic Review* **96**(3), 577–601.

Wang, J. (1993), 'A model of intertemporal asset prices under asymmetric information', *The Review of Economic Studies* **60**(2), 249–282.

Zhang, X. F. (2006), 'Information uncertainty and stock returns', *The Journal of Finance* **61**(1), 105–137.