

A replication of Pindyck's willingness to pay: on the sacrifice needed to obtain results

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Abstract

We present a verification, an extension and a reanalysis of “Uncertain outcomes and climate change policy”, R. Pindyck, *Journal of Environmental Economics and Management*, 2012. As far as verification is concerned, we are able to reproduce the results provided in Pindyck's work in many cases and convincingly confirm the quality of the work. Some discrepancies are present, they are due to rounding or related to specific sets of parametric values and do not change the economic interpretation or significance of the results. The re-estimation of the model with more recent data on climate change made available in 2014 shows that temperature increments are now deemed to be higher in mean but less dispersed. As a consequence, the willingness to pay doesn't vary much with respect to the original paper. We also modify the functional form describing the impact of temperature increase on the growth rate of consumption and obtain much bigger and potentially problematic increments of the willingness to pay.

Finally, the paper demonstrates that the numerical results are sensitive to a variety of technical settings used in the computations and suggests that great care is needed in obtaining estimates and employing results in policy discussions.

Keywords: Replication, environmental policy, climate change, economic impact, willingness to pay.

JEL codes: D81, Q51, O44.

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1 Introduction

This paper presents a verification as well as an extension and a reanalysis of the work “Uncertain outcomes and climate change policy” by R. Pindyck, *Journal of Environmental Economics and Management* (Pindyck, 2012), P12 hereafter. Pindyck incorporates the distribution for the (uncertain) temperature change and the distribution of the (uncertain) impact of this change on the growth of consumption and computes the willingness to pay (WTP), i.e., “the fraction of consumption [...] that society would be willing to sacrifice, now and throughout the future, to ensure that any increase in temperature at a specific horizon H is limited to τ ”. These fractions are typically below 2% and it is stated in P12 that this is consistent with the adoption of a moderate abatement policy.

P12 is a sound paper tackling difficult questions with crystalline thinking and terse prose. The work was cited often (31 times on Scopus and 113 on Google scholar¹) in a relatively short lifespan. Assumptions and methods are clearly spelled out, as indeed proven by the fact that most of the paper could be reproduced with no access to the original code or files. Most of important arguments are crucially based on figures numerically resulting from the model, and data and estimates are based on *IPCC* Fourth Assessment (IPCC, 2007c,a,b). We believe the results in P12 are important and insightful and summarize in a clever way a vast amount of knowledge on climate change. The computations involved in the model are technically demanding and, basically, require to evaluate many 3-dimensional non-trivial integrals (over a long span of time, over an estimated distribution of temperature changes, over an estimated distribution of an impact coefficient). Our replication was often facilitated by the working paper (Pindyck (2009), hereafter P09) which, we deem, was a detailed preparation of the contributions that were later streamlined and distilled in P12. The possibility to read two “versions” of the same work and access, when needed, alternative wording of the same procedures or descriptions is a fortunate circumstance, hence we hope more scholars will routinely publish in the future all the drafts of the papers that ultimately result in a “definitive” publication on a journal. The examination of multiple interrelated stages of development of a scientific research can illuminate technical passages, as well as clarify the logical path linking the original ideas to the final upshot.

Replication is of paramount importance in science and lies at the very heart of what differentiates science from cheap talk and non-scientific arguments. There is an increasing awareness of the need for more replication studies and too many scholars sadly admit that have attempted with faltering or no success to reproduce others’ work (Baker, 2016). Replication, however defined, is in our opinion very important for another reason: boldly put, we believe that replicating a paper is the only way to (fully) understand it. This may be also true for theoretical works (say, reworking all the proofs) but we have no doubt that this is needed to master numerical, empirical or simulative work. The amount of effort needed is often inordinate but, even though the verification is never published as a standalone paper, the rewards are hefty and one of the authors of this article just believes that most, if not everything, of what he knows comes from the hard times spent

¹The number of citations was recorded on October 31, 2017.

in struggling with the details of papers to be replicated.

For additional clarity, we'll no longer use the word "replication" in what follows for reasons that are convincingly exposed in Clemens (2017): the term is used in the literature to refer to distinct procedures and there is no agreed standard on its precise meaning. In this paper, we present a *verification*² as well as an *extension* and a *reanalysis*³ of P12 as defined in Clemens (2017). In the first part relative to verification, we aim at reproducing the results of P12 using the *same specification of the model and the same data*. Neither P12 nor P09 explicitly state the software used for the computations but Robert Pindyck, in a personal communication, made clear that MATLAB was used and provided us with some code. In what follows, we used R, (R Core Team, 2015), a popular and reliable free software platform for statistical and numerical computing and data visualization. While we stress that some scholars, like Anderson et al. (2008), appear to require that a "verification" should use the same software (and perhaps the same hardware), we believe that the use of MATLAB, R or any other professionally trusted software (e.g. Octave, Mathematica) should not alter the substantial results of a research. In other words, if different results are obtained with different pieces of software, the case is indeed worth studying as done in McCullough and Vinod (2003).⁴ Subsequently, we move forward and perform an extension re-estimating everything using more recent data, and a reanalysis showing how the results contained in P12 change if we alter the specification of the model. In particular, we firstly use data from *IPCC* Fifth Assessment (IPCC, 2014) whereas P12 was based on the previous *IPCC* Fourth Assessment, (IPCC, 2007c,a,b). Secondly, we change the specification assuming the change in temperature affects consumption's growth rate convexly, in contrast with P12 where linear dependence is assumed.

We obtained two main results from our verification. The first is that most of P12 can be reproduced quite accurately and even if discrepancies are present in some of our figures, they are small and do not affect the economic meaning or interpretation in any way.

The other outcome of the verification is more general and a bit troublesome: our

²A *verification* has the aim of reproducing the results of the original paper using the *same specification of the model and the same data*. Therefore, a verification should not produce discrepant results unless there are plain errors or fraud in the original work.

³*Reanalysis* and *extension* are robustness checks to the original work, having the aim of exploring the stability of results of the original work using different data and/or alternative model specifications. More specifically, a *reanalysis* uses the same data of the original work but with a different model specification, while an *extension* runs the original model using different data. Hence, if for a verification we expect the results to be the same of the original work, there is no reason to expect the same after an extension or a reanalysis.

⁴From Axtell et al. (1996): "We have identified a few cases in which an older model has been reprogrammed in a new language, sometimes with extensions, by a later author. For example, Michael Prietula has reported reimplementing a model from Cyert and March (1963) and Ray Levitt has reported a reimplementation of Cohen, March and Olsen (1972). However, these procedures are not comparisons of different models that bear on the same phenomena. Rather they are "reimplementations", where a later model is programmed from the outset to reproduce as closely as possible the behaviour of an earlier model. Our interest is in the more general and troublesome case in which two models incorporating distinctive mechanisms bear on the same class of social phenomena, be it voting behaviour, attitude formation, or organizational centralization".

results (and, hence, our ability to verify P12) are sensitive in many cases to choices of parameters used in the computation but otherwise having no deep relationships with the model. For instance, even though some integrals are naturally defined on the real (half) line, integration routines require to set an upper limit for the domain: while this should be intuitively irrelevant, it turns out that it can introduce non obvious and large biases. A more detailed discussion is deferred to Section 6 but our experience emphasizes that it may be difficult to select the “right” parameters leading to the “correct” results, especially if one has not an article, like P12, as a target for fine-tuning.

Regarding the extension, we change the data source and “redo the paper” to give the flavour of how our understanding and policy vary based on two subsequent *IPCC* reports. Essentially, more recent data support a temperature change distribution over next century that is higher on average and more concentrated. The effects on the WTP, thus are opposite as the higher mean would increase our willingness to pay but, at the same time, as extreme events are less likely, smaller risk tends to curb the WTP. The overall effect is a slight increase in the willingness to pay for strong mitigation and a slight decrease in the willingness to pay for moderate mitigation.

We then alter the specification of the model, still keeping the original data to be comparable with P12, and assume that the growth rate of consumption is convexly (as opposed to linearly) affected by the temperature change. This incorporates in the model a more cautious and risk-averse attitude as large (albeit rare) increments in the temperature can have drastic effects. This reanalysis produces (moderately) higher levels of the willingness to pay. The sensitivity of results to the choice of the damage function, about which “we know almost nothing” (Pindyck, 2013), suggests caution in the interpretation of results for policy making and sheds light on the need for further research on the economic implications of climate change.

The paper is organized as follows. Section 2 briefly summarizes the model contained in Pindyck’s work and describes the major conclusions of P12. In Section 3, we present our verification strategy and explain the functioning of the routines we have used in the verification. P12 is a rich paper with plenty of numerical results and robustness tests or discussions. We reproduced a vast body of outcomes, including pictures, key tables and robustness checks of the original paper. Section 4 is devoted to the extension, while Section 5 is devoted to the reanalysis. Section 6 discusses in detail some of the most relevant results of the previous parts. We then conclude with some final remarks and suggestions for future research.

2 The model

This section describes the model presented in P12. It is assumed that the temperature increase T_H at horizon H is distributed as a three-parameter displaced gamma density of the form:

$$T_H \sim f(x) = f(x; r = r_T, \lambda = \lambda_T, \theta = \theta_T) = \frac{\lambda^r}{\Gamma(r)} (x - \theta)^{r-1} e^{-\lambda(x-\theta)}, \quad x \geq \theta,$$

where θ is the displacement parameter and $\Gamma(r) = \int_0^\infty s^{r-1} \exp(-s) ds$ is the Gamma function. If T_H is the increase in temperature after H years, the increase at time t , T_t evolves according to

$$T_t = 2T_H[1 - (1/2)^{t/H}], \quad (1)$$

so that, in particular, $T_t \rightarrow 2T_H$ as $t \rightarrow \infty$.

As done in many studies of climate change the effect of temperature increase is linked to the Gross Domestic Product (GDP) through the loss function $L(T) = \exp(-\beta T^2)$. The GDP (or consumption) at H is then $L(T_H)GDP_H$, where GDP_H is the “would have been” GDP at $t = H$ with no warming. Clearly, the loss affects the level of GDP but it is argued in P12 that a model incorporating effects on the growth rate of GDP is more appropriate. Hence, assuming that in the absence of warming the GDP would grow at constant rate g_0 and that T_t decreases the instantaneous growth rate to

$$g_t = g_0 - \gamma T_t, \quad (2)$$

we obtain the path of the growth rate as

$$g_t = g_0 - 2\gamma T_H[1 - (1/2)^{t/H}].$$

Hence, consumption (or GDP) $C_t = C_0 \exp(\int_0^t g(s) ds)$ (with warming), can be computed as

$$C_0 \exp\left(-\frac{2\gamma H T_H}{\ln(1/2)} + (g_0 - 2\gamma T_H)t + \frac{2\gamma H T_H}{\ln(1/2)}(1/2)^{t/H}\right), \quad (3)$$

for any t . Normalizing consumption C_0 at 1 and equating final consumption (3) at H with what would be obtained using a loss function on levels, one gets

$$\exp\left(-\frac{2\gamma H T_H}{\ln(1/2)} + (g_0 - 2\gamma T_H)H + \frac{2\gamma H T_H}{\ln(1/2)}(1/2)^{H/H}\right) = L(T_H) \exp(g_0 H) = \exp(g_0 H - \beta T_H^2),$$

and, subsequently, the relationship between β and γ :

$$\gamma = 1.79 \frac{\beta T_H}{H}. \quad (4)$$

Typically, integrated assessment models in the literature provide estimates of β , which can be converted into values for γ that are finally used to fit a displaced gamma density $f_\gamma(y) = f_\gamma(y; r_\gamma, \lambda_\gamma, \theta_\gamma)$, for the random variable γ appearing in (2).

We define the social utility function

$$U(C_t) = \frac{C_t^{1-\eta}}{1-\eta},$$

where η is the index of relative risk aversion of the society. It is convenient in what follows to set $u(C_t) = C_t^{1-\eta}$ so that $U(C_t) = \frac{1}{1-\eta} u(C_t)$.

The willingness to pay $w^*(\tau)$ is the “fraction of consumption – now and thorough the future – society would sacrifice to ensure that an increase in temperature at a specific

horizon H is limited to an amount τ ", see p. 292 in P12. Pindyck's paper does not deal with the practically significant problem that $w^*(\tau)$ may not be enough to keep T below τ but assumes that the society is willing to sacrifice up to a fraction $w^*(\tau)$ of consumption to truncate the distribution $f(T)$, so that $T \leq \tau$. More formally, if no action is taken social welfare would be

$$\begin{aligned} W_2 &= \iiint U(\tilde{C}_t) e^{-\delta t} f(x) f_\gamma(y) dt dx dy \\ &= \frac{1}{1-\eta} \iiint u(\tilde{C}_t) e^{-\delta t} f(x) f_\gamma(y) dt dx dy \\ &= \frac{1}{1-\eta} G_\infty, \end{aligned} \tag{5}$$

where the tilde emphasizes the random nature of the quantity, $0 \leq t \leq \infty^5$, the uncertain temperature increase x spans the interval $\theta_T \leq x \leq \infty$ and the impact coefficient γ is in $\theta_\gamma \leq \gamma \leq \infty$.

If society sacrifices a fraction $w^*(\tau)$ of consumption, we have two effects in the computation of social welfare: firstly, only the remaining part of consumption, $C'_t = (1 - w(\tau))C_t$, is used as an argument of the utility function; and, secondly, integration with respect to the variable x will be bounded to τ and be taken with respect to a truncated and renormalized density $f_\tau(x)$, where $f_\tau(x) = \mathbf{1}_{x \leq \tau} f(x) / F(\tau)$ and the normalizing constant is

$$F(\tau) = \int_{\theta_T}^{\tau} f(x) dx.$$

Hence, given the upper threshold τ for temperature increase, social welfare (under sacrifice) is

$$\begin{aligned} W_1(\tau) &= \iiint U(\tilde{C}'_t) e^{-\delta t} f_\tau(x) f_\gamma(y) dt dx dy \\ &= \frac{(1 - w(\tau))^{1-\eta}}{1-\eta} \iiint u(\tilde{C}'_t) e^{-\delta t} f_\tau(x) f_\gamma(y) dt dx dy \\ &= \frac{(1 - w(\tau))^{1-\eta}}{1-\eta} G_\tau, \end{aligned} \tag{6}$$

where the integration domains are $0 \leq t \leq \infty$, $\theta_T \leq x \leq \tau$ and $\theta_\gamma \leq \gamma \leq \infty$, respectively, for the variables t, T and γ .

The willingness to pay $w^*(\tau)$ is then the solution of the equation $W_2 = W_1(\tau)$. Using (5) and (6) WTP can be written as

$$w^*(\tau) = 1 - \left[\frac{G_\infty}{G_\tau} \right]^{\frac{1}{1-\eta}}. \tag{7}$$

⁵ H is the forecasting horizon, but damages are evaluated also beyond that period

Observe that ultimately the WTP can be readily computed once the two 3-dimensional integrals G_∞ and G_τ are evaluated. It turns out that these computations are far from trivial in a variety of parameters' constellations and require considerable care to be performed. Indeed, in Section 5.2 of P12 a simple case is examined in which no uncertainty is assumed on T and γ , which are replaced by a known T_H and by the mean $\bar{\gamma}$ of density f_γ (denoted as g in P12), see Figure 4 in P12. Technically speaking, this makes the previous integrals 1-dimensional and, more importantly, results can be contrasted with more general situations where uncertainty plays a role, spreading the set of feasible outcomes in ways depicted by the estimated densities for T and γ .

Our presentation of the model differs from the one in P12 as we emphasize the fact that relevant quantities are obtained taking 3-dimensional integrals whereas slightly more abstract mean operators are used to describe the very same objects in Pindyck's work. Incidentally, we hope that two equivalent descriptions may benefit different readers or clarify, if needed, both notations and their precise meanings.

Coming to the main concrete claims of P12, we believe it's fair to say that the author interprets his own results as an indication that “moderate abatement policies” should be pursued in the face of the large uncertainty surrounding the amount of future temperature increase and its unknown impact. This broad conclusion is stated in the abstract, in the introduction and elsewhere in the paper. In the concluding remarks, the argument takes an analogous flavour: asking “whether a stringent [abatement] policy is needed now”, Pindyck says results “are consistent with beginning slowly”. A similar lesson, we believe, can be drawn from the simplified example contained in Section 5.2 of P12: if the temperature increase is *known* to be $T_H = 6^\circ\text{C}$ in $H = 100$ years under business as usual and *known* economic impact, then the willingness to pay to have no warming, $w^*(0)$, is *still only 2.2%* (italics are ours).

These considerations spurred us to assess the robustness of the model, using more recent data (Section 4) or changing part of the model specification the model (Section 5), in order to check how much can be retained of the gist of the original paper in different setups. However, in Section 3, we begin with a verification of most of the results contained in P12.

3 Verification

3.1 R

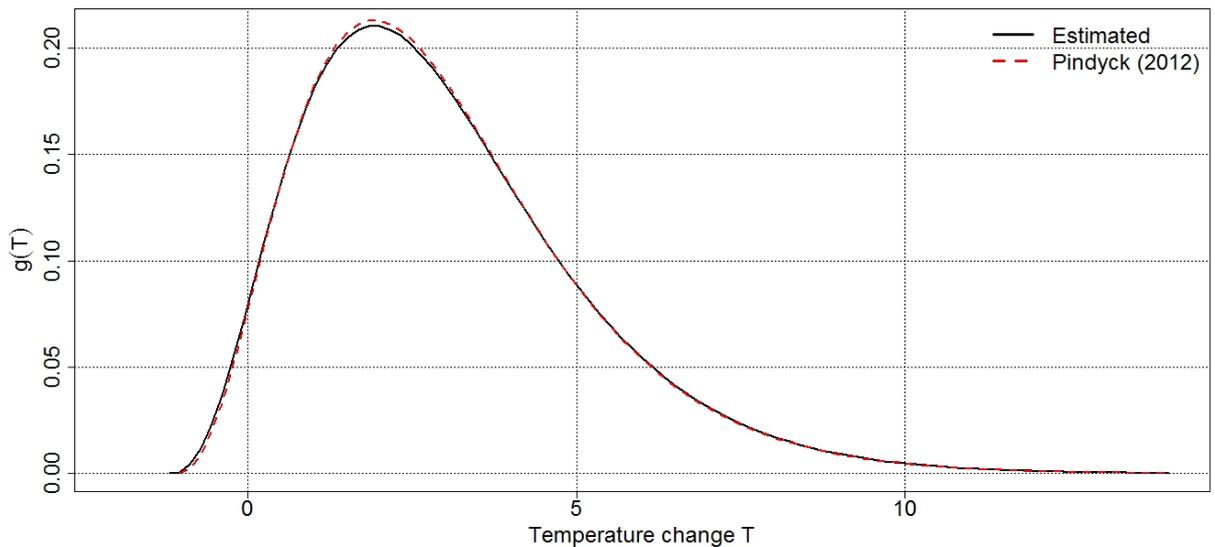
All the computations of this paper are obtained using the R platform, R Core Team (2015). R is a free software environment for statistical computing and graphics. Among the alternatives, we chose R since it is free, it is easy scriptable and, among the many existing packages, there is one (`cubature`, Johnson and Narasimhan (2013)) specifically developed to evaluate multiple-dimension integrals.⁶

⁶The code is available upon request

3.2 Estimation of the displaced gamma density

P12 assumes a displaced gamma density for both T_H and γ . Almost all the integrals for the calculation of the WTP involve these two random variables. Therefore the results of the paper heavily depend on this preliminary estimation. Like in P12, we fit the parameters of a displaced gamma density for the random variable T_H with $\mathbb{E}(T_H) = 3^\circ\text{C}$, $\mathbb{P}(T_H \leq 7^\circ\text{C}) = 5\%$ and $\mathbb{P}(T_H \leq 10^\circ\text{C}) = 1\%$. We obtain $r_T = 3.9$, $\lambda_T = 0.92$ and $\theta_T = -1.22$ that can be compared with the values reported in P12: 3.8, 0.92 and -1.13 , respectively. Figure 1 displays the two distributions which, despite the slightly different value of the parameters, appear to be almost indistinguishable. We decide to number our figures the same way they were numbered in P12: hence, to ease the comparisons for the readers, Figure X in this paper always corresponds to Figure X in P12 (of course, this may also be a bit perplexing as, say, there is no Figure 2 in this work and we jump from Figure 1 to 3). Along the same lines, we will retain the original numbering found in P12 in Sections 4 and 5, adding a literal suffix to the proper numeral.

Figure 1: Distribution of temperature change T_H .

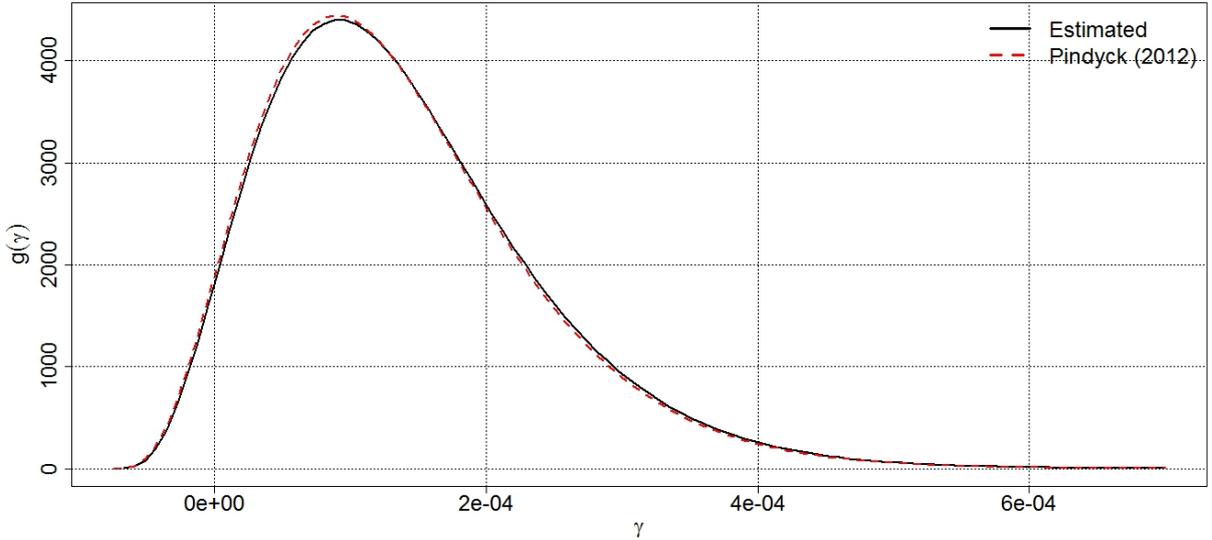


The distribution of the damage coefficient γ is calibrated in order to fit a displaced gamma density such that $\mathbb{E}(\gamma) = 0.0001363$, $\mathbb{P}(\gamma \leq 0.0000450) = 0.17$ and $\mathbb{P}(\gamma \leq 0.0002295) = 0.83$.⁷

⁷The procedure used to calculate these moments is not explicitly stated in P12, but can be inferred from a footnote of P09. These moments of the γ distribution are implied by the corresponding moments of β . IPCC (2007a) reports that, for a 4°C warming level, the expected production loss in levels $Loss_{T_H}$ is 3%, with a 66% confidence interval being 1-5%. We can obtain the values of β that are coherent with these projections using the equation $\exp(-\beta T_H^2) = 1 - Loss_{T_H}$. These estimates imply that $\bar{\beta} = 0.00190$, $\beta_{0.17} = 0.000628$ and $\beta_{0.83} = 0.00321$. The moments for γ are then obtained from (4)

We obtain $r_\gamma = 4.43$, $\lambda_\gamma = 20939$ and $\theta_\gamma = -7.28 \cdot 10^{-5}$. Again, our estimates are quite close to the values $r_\gamma = 4.50$, $\lambda_\gamma = 21431$ and $\theta_\gamma = -7.46 \cdot 10^{-5}$ reported in P12, and the plots corresponding to the two densities are basically undistinguishable (see Figure 3). The numerical approximation of the parameters for γ has been less trivial than it was needed for T_H , due to the different order of magnitudes of the three parameters; for details we refer the reader to Section 6.

Figure 3: Distribution of loss function parameter γ .



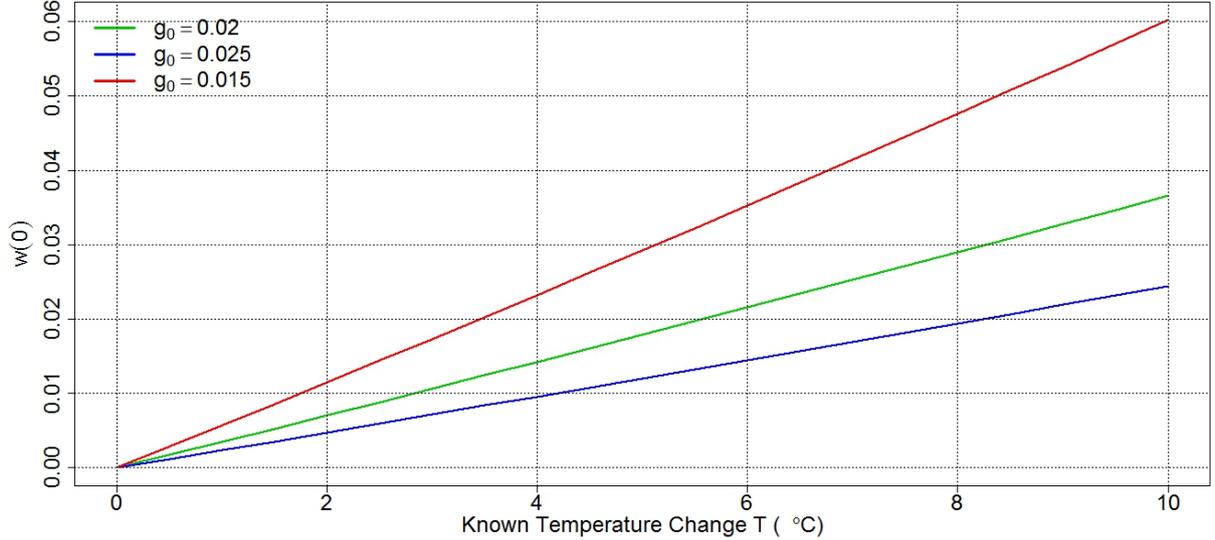
3.3 Estimation of Willingness to Pay

The computation of several WTPs with different values taken by key parameters is clearly one of the most important features of P12. The WTP is graphically displayed in Figures 4-7 as well as tabulated in Tables 1 and 2. In what follows, we reproduce the original Figures 4, 5 and 6 and recompute Table 1.

First, some benchmark WTPs are computed in a setup with no uncertainty on future temperature increase and economic impact, and Figure 4 depicts the WTP $w^*(0)$ to keep warming at zero as a function of a known T_H , assuming a fixed value for $\gamma = \bar{\gamma} = \mathbb{E}(\gamma) = 0.0001363$, and showing three possible scenarios for the growth rate $g_0 = 0.015$, 0.02 or 0.025 (the remaining parameters are the index of relative risk aversion $\eta = 2$ and discount rate $\delta = 0$). This is, therefore, a scenario with no uncertainty where the integrals needed to evaluate the WTP are 1-dimensional. It is, of course, formally impossible to test whether two figures are the same, but an eyeball test of the twin figures in P12 and in this paper (and lots of zooming!) show that they are essentially displaying the very same quantities. Perhaps more concretely, to exemplify the meaning of Figure 4 it is reported in P12 that

when $T = 6$ and $g_0 = 0.02$ then $w^*(0)$ is about 0.022 or 2.2%. For comparison, our own computations produce 0.02156.

Figure 4: $w^*(0)$, known temperature change T_H , $\eta = 2$, $g_0 = 0.015, 0.020, 0.025$, and $\delta = 0$.



Second, WTPs are displayed in Figure 5, allowing for uncertainty. In the picture, the functions $w^*(\tau)$ are depicted in four scenarios combining different risk aversion η and baseline growth rate g_0 . In this setup, full 3-dimensional integrals are involved and care is needed to set apparently irrelevant (technical) parameters, as detailed in Section 6. It is again hard to discern any differences in the two versions of Figure 5 of this paper and of Pindyck's one.

Third, we focus on Figure 6 where the dependence of $w^*(3)$, namely the WTP to limit the increase in temperature to 3°C, is plotted as a function of risk aversion η (under two different discount rate δ). This picture is interesting as it turns out that its replication is difficult, in particular, if η approaches 1 or when $\eta = 4$. In the first case, we have an evident singularity in the definition of $U(\cdot)$ and can resort to the fact that, in this situation, the utility function is – up to constant – a logarithmic function. It is less clear why high values of η prove to be relatively ill-posed for the integration routine `cubature`. While additional details are deferred to Section 6, we observe that our Figure 6 is extremely similar to the one found in P12.

We finally move to Table 1, which lists 19 pairs of WTPs and allows for a more rigorous comparison of numeric figures obtained tilting the reference values of some parameters. In particular, the two WTPs $w^*(0)$ and $w^*(3)$ are tabulated and, unless otherwise indicated, $\delta = 0$, $\eta = 2$, $g_0 = 0.02$, $\mathbb{E}(T) = 3^\circ C$, $\mathbb{E}(\gamma) = 0.0001363$ and social utility is computed on a span of time of 5 centuries ($t_{max} = 500$).

Generally speaking, our estimates of the WTP (in the second and fourth columns) match very well the ones in P12 (placed side by side in the third and fifth columns). For

Figure 5: $w^*(\tau)$, T_H and γ uncertain, $\eta = 1.5, 2$, $g_0 = 0.015, 0.020, 0.025$, and $\delta = 0$.

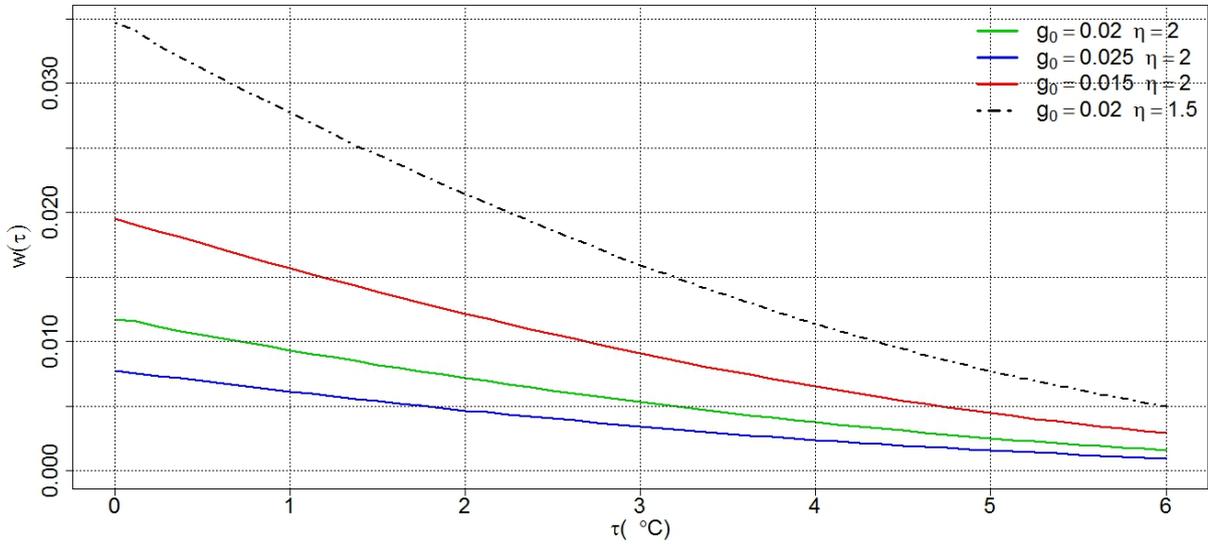


Figure 6: $w^*(3)$ versus η . $g_0 = 0.020$ and $\delta = 0, 0.01$

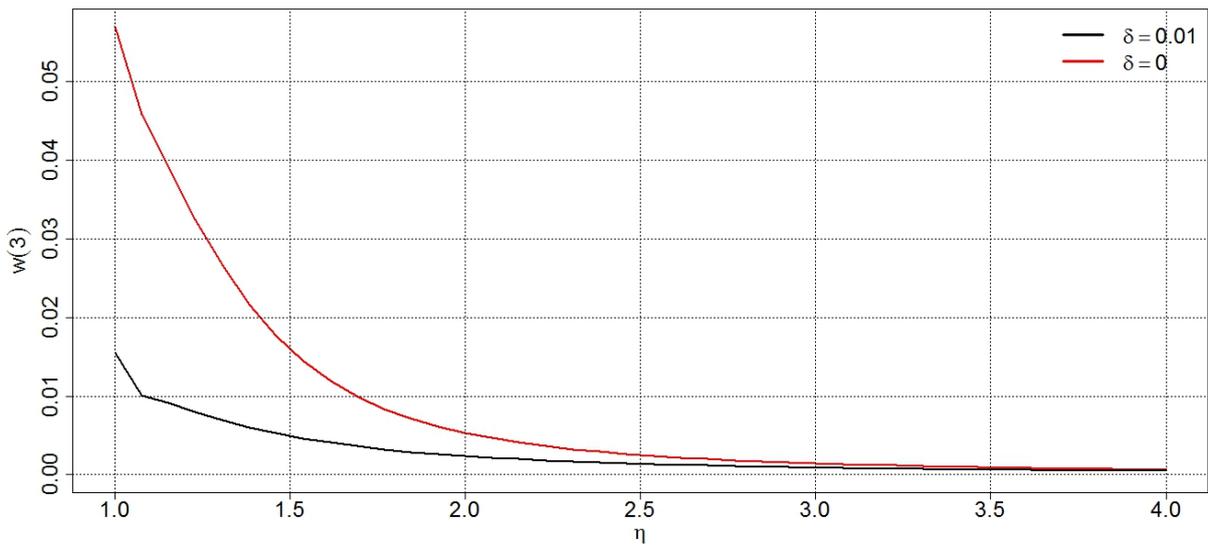


Table 1: WTPs with alternative parameter values.

Cases	$w^*(0)$	P12 $w^*(0)$	$w^*(3)$	P12 $w^*(3)$
1 Base case	0.0118	0.0113	0.0053	0.0059
2 $t_{max} = 300$	0.0112	0.0110	0.0050	0.0056
3 $t_{max} = 1000$	0.0118	0.0113	0.0053	0.0059
4 $g_0 = 0.010$	0.0369	0.0372	0.0179	0.0190
5 $g_0 = 0.005$	0.0761	0.0775	0.0384	0.0407
6 $g_0 = 0$	0.1432	0.1463	0.0750	0.0791
7 $\eta = 4$	0.0015	0.0014	0.0007	0.0008
8 $\eta = 4, g_0 = 0.010$	0.0060	0.1844	0.0029	0.1820
9 $\varepsilon(T_H) = 5^\circ C$	0.0187	0.0189	0.0103	0.0105
10 $\varepsilon(T_H) = 5^\circ C, g_0 = 0.010$	0.0596	0.0599	0.0350	0.0350
11 $\varepsilon(T_H) = 5^\circ C, g_0 = 0.005$	0.1223	0.1232	0.0746	0.0749
12 $\varepsilon(\gamma) = 0.0002726$	0.0240	0.0243	0.0112	0.0116
13 $\varepsilon(\gamma) = 0.0002726, g_0 = 0.015$	0.0402	0.0401	0.0194	0.0198
14 $\varepsilon(T_H) = 5^\circ C, \varepsilon(\gamma) = 0.0002726$	0.0384	0.0373	0.0218	0.0211
15 $g_0 = 0, \delta = 0.01$	0.0369	0.0372	0.0179	0.0190
16 $g_0 = 0, \delta = 0.02$	0.0118	0.0074	0.0053	0.0039
17 $g_0 = 0.005, \delta = 0.01$	0.0196	0.0195	0.0091	0.0098
18 $\eta = 4, g_0 = 0.005, \delta = 0.01$	0.0089	0.0315	0.0045	0.0178
19 $\varepsilon(T_H) = 5^\circ C, g_0 = 0.010, \delta = 0.01$	0.0187	0.0599	0.0103	0.0350

Note: unless otherwise indicated $\delta = 0, \eta = 2, g_0 = 0.020, \varepsilon(T_H) = 3^\circ C, \varepsilon(\gamma) = 0.0001363, t_{max} = 500$ years.

instance, in the first row relative to the baseline case the values of $w^*(0)$ and $w^*(3)$ differ by about $5 \cdot 10^{-4}$. In many cases, we have similar gaps that are insignificant from the practical economic point of view but give the flavour of the “numeric noise” that affect (accurate) estimates obtained by different authors, with distinct software and code. This (small) noise can be attributed to slightly different computational methods being used in different packages, or to dissimilar settings of an abundance of default parameters that are used in standard routines for numerical computations. To give an example: there may be different defaults for stopping criteria; or finer/coarser grids are used when the user is not providing optional specifications.

However, some noteworthy discrepancies can be spotted in rows 8, 16, 18 and 19. Observing preliminarily that two such cases are related to the position $\eta = 4$, in row 8 the WTPs computed in P12 setting $g_0 = 0.01$ are 30 or 60-fold larger than ours. The previous row contains the same WTPs when the growth rate is 0.02 and, in agreement with intuition, halving the growth rate of the economy inflates the WTP to reduce the expected wounds inflicted by climate change to a frail economic growth. While, say, according to our computations, $w^*(0)$ moves from 0.0015 in row 7 to 0.0060 in row 8, a four-fold increase, in P12 we have a spectacular jump from 0.0014 to 0.1844. The same occurrence is visible for $w^*(3)$.

In row 18, again with $\eta = 4$, our $w^*(0)$ and $w^*(3)$ are quite smaller than the WTPs in P12 (differences exceed 2 and 1 percentage point, respectively).

Finally, the last row of Table 1 portrays a large difference in both WTPs. We feel that, nonetheless, there may be a simple material typing error in P12 as the entries in Pindyck’s Table are *exactly* the same as in row 10, whereas we expected the same figure of row 9. This is due to the fact that, as it is possible to notice by looking at (8) of P12,⁸ with $\eta = 2$ an increase in δ compensates for a decrease in g_0 of the same absolute value, implying that a scenario with $\varepsilon(T_H) = 5^\circ C$ and $g_0 = 0.02$ is actually the same as a scenario with $\varepsilon(T_H) = 5^\circ C$ and $g_0 = 0.01$ and $\delta = 0.01$. For the same reason, the entries in rows 1 and 16 in Table 1 should be the same but this does not happen in P12.

All in all, with some exceptions possibly related to low growth rates and extreme values for η , our estimates are often close to the ones obtained by Pindyck (some reasons for what is happening in cases 8 and 18 are discussed in Section 6).

⁸ Equations (7), (8), (9) of P12 report that in the simple case where T_H and γ are known,

$$W = \frac{1}{1 - \eta} \int_0^{+\infty} e^{\omega - \rho t - \omega(1/2)^{t/H}} dt$$

where:

$$\begin{aligned} \rho &= (\eta - 1)(g_0 - 2\gamma T_H) + \delta, \\ \omega &= 2(\eta - 1)\gamma H T_H / \ln(1/2). \end{aligned}$$

When $\eta = 2$, the value of ρ depends on the sum $g_0 + \delta$. Hence, decreases in the growth rate can be offset by equal increases in the discount rate leaving ρ unaffected.

4 Extension: 2014 data

This section provides an extension of the original paper where we change the data used to estimate densities for the temperature increase. As in every extension, the fact that we change the data makes clear that the results shown here cannot be expected to resemble the ones in P12, but should be used to appraise how the original results are affected by the availability of new data. The Fifth *IPCC* Assessment Report released in 2014 (IPCC, 2014) contains new data that can be used to estimate fresh densities for the uncertain quantities used in P12. In particular, IPCC (2014) describes four possible GHG (GreenHouse Gases) possible scenarios, called Representative Concentration Pathways (RCP), which are named after a possible range of radiative forcing values in the year 2100 relative to pre-industrial values. The one without any GHG emissions⁹ mitigation effort beyond current legislation, which can be considered as the baseline path for the present analysis, is RCP8.5¹⁰: the alternative scenarios, with increasing level of mitigation and, therefore, decreasing level of emissions, are RCP6.0, RCP4.5 and RCP2.6.

Under scenario RCP8.5 the forecast is of a $3.7^{\circ}C$ temperature increase from 1986-2005 to 2081-2100, with a “*likely*” range of $2.6^{\circ}C$ to $4.8^{\circ}C$. In Figure 1a, we plot the “old” density for T_H and the “new” density interpreting the term “likely” as a 66% or a 90% confidence interval (in P12 the term “likely” is associated to 66%). Using for the estimation the information $\mathbb{E}(T_H) = 3.7^{\circ}C$, $\mathbb{P}(T_H \leq 2.6^{\circ}C) = 17\%$ and $\mathbb{P}(T_H \leq 4.8^{\circ}C) = 83\%$, the results for the parameters of the gamma displaced density $f_{2014}(x)$ are: $r_{2014} = 7.82$, $\lambda_{2014} = 2.38$ and $\theta_{2014} = 0.42$. Figure 1a shows that the distributions computed with more recent data shift to the right and are more concentrated around the mean of $3.7^{\circ}C$ (and, in particular, the right tail is clearly much thinner than in P12 in either of the two versions obtained from 2014 data).

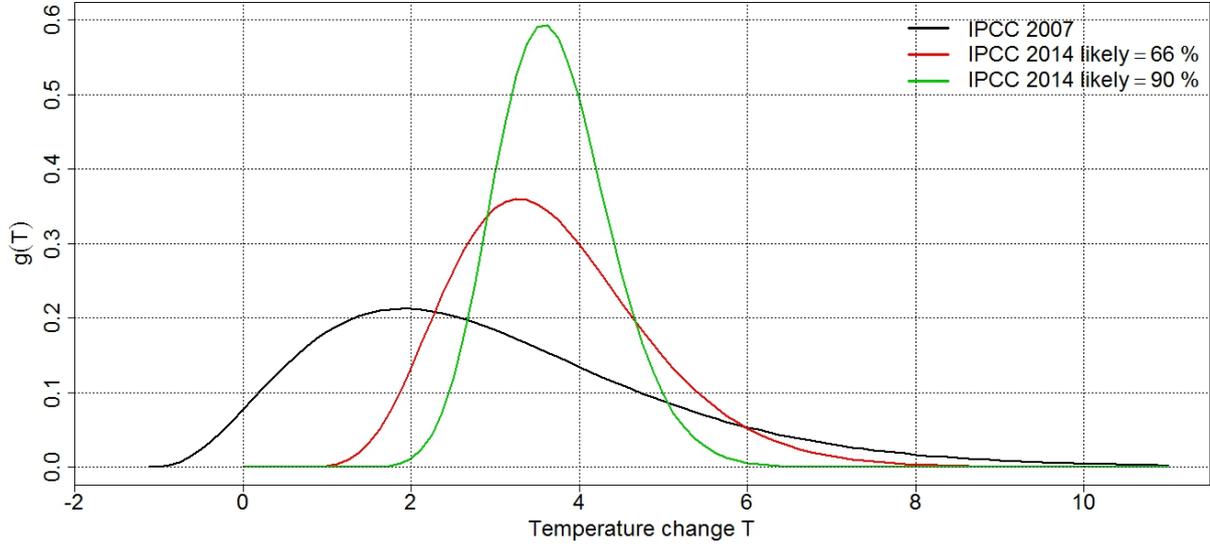
Observe that with the new parameters, being θ_{2014} greater than zero, it is impossible to keep the temperature increment at 0 and, hence, $w^*(0)$ cannot be estimated. In Table 1a we report the WTPs for the same cases listed in Table 1, replacing $w^*(0)$ with $w^*(1)$, under the two interpretations of “likely”. In the third (sixth) column, we display $w^*(1)$ ($w^*(3)$) for the various cases based on IPCC07 and in the fourth and fifth (seventh and eighth) columns the values obtained from 2014 data.

Looking at $w^*(1)$, a scenario related to a stricter abatement policy, WTPs most often grow moving from 2007 to 2014 assessments, meaning that the newer *IPCC* report implies an upward revision of the WTP to curb warming to $1^{\circ}C$. The differences in the WTPs are anyway modest and generally are around 0.2-0.3% or smaller. Scenarios which assume $\varepsilon(T_H) = 5^{\circ}C$ imply a lower WTP under 2014 assessment. This is related to the lower standard deviation underlying the 2014 projections: keeping the same expected value, a lower standard deviation implies a lower WTPs, as explained in P12. In a few other instances, such as the ones where $\eta = 4$ (cases 7, 8, 19), the new values are equal or smaller

⁹Please note that an high GHG emission scenario does not necessarily imply an high "pollution" scenario: pollution depends on the emissions of other substances, like for example SO_2 .

¹⁰Representative Concentration Pathways 8.5, therefore, assumes an increase of $8.5 \text{ W}/m^2$ radiative forcings with respect to preindustrial levels

Figure 1a: Distribution of temperature change T_H , 2014 data.



than the ones in P12. The inspection of the columns 4 and 5 also shows that the WTPs are virtually the same regardless of the chosen interpretation of “likely”.

The examination of the columns relative to $w^*(3)$, instead, reveals that more recent data “suggest” a lower WTP for a moderate abatement policy (namely, limiting the temperature increment to $3^\circ C$), the only exception being case 16 with an high discount rate ($\delta = 0.02$). With respect to the WTP estimated according to 2007 data, differences range from 0.1 to 1.8%. If “likely” is associated to 90% confidence intervals then, typically, WTPs are further reduced by an amount ranging between 0.1 and 0.7%.

Figures 5a and 6a are the counterparts (with more recent data) of Figures 5 and 6 in P12. A careful inspection confirms the previous findings and comments but, perhaps more importantly, may suggest that the inclusion of fresh 2014 data appear to have not changed the gist of the conclusions and lessons in P12. It is true that temperature cannot be kept at the present level, and that strict (moderate) abatement policies are slightly more (less) worthwhile, but changes are perhaps minor in size in many circumstances of practical importance. This was somehow to be expected after the marginal analysis in P12 already pointed out that a hike in the mean temperature would have been offset by a reduction in the standard deviation.

Figure 5a: $w^*(\tau)$, T_H and γ uncertain, $\eta = 1.5, 2$, $g_0 = 0.015, 0.020, 0.025$, and $\delta = 0$, 2014 data. Observe that WTP cannot be plotted when $\tau = 0$ as $\theta_{2014} = 0.42 > 0$.

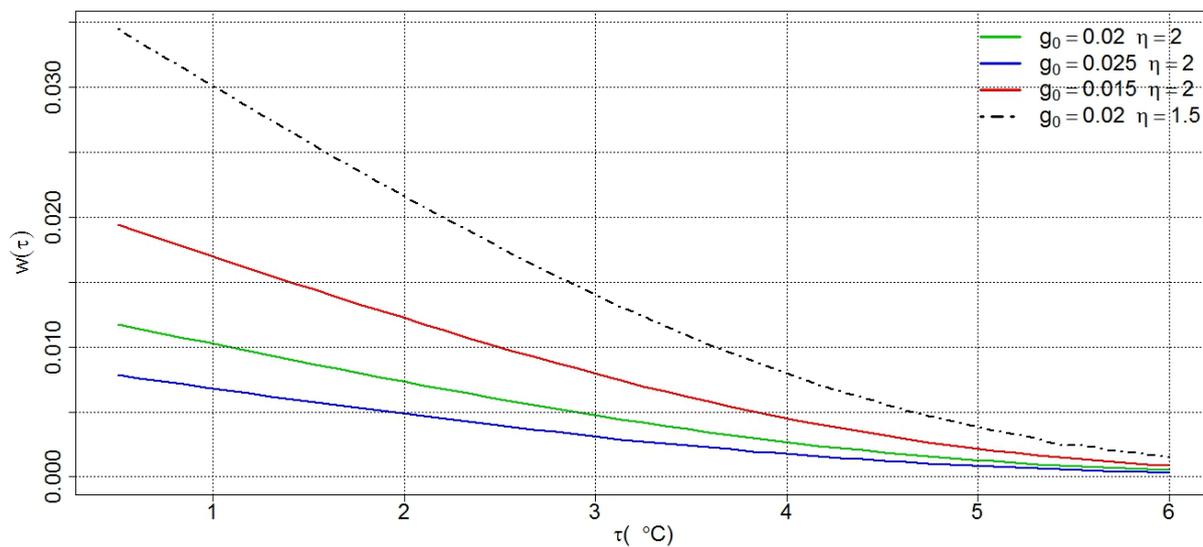


Figure 6a: $w^*(3)$ versus η . $g_0 = 0.020$ and $\delta = 0, 0.01$, 2014 data

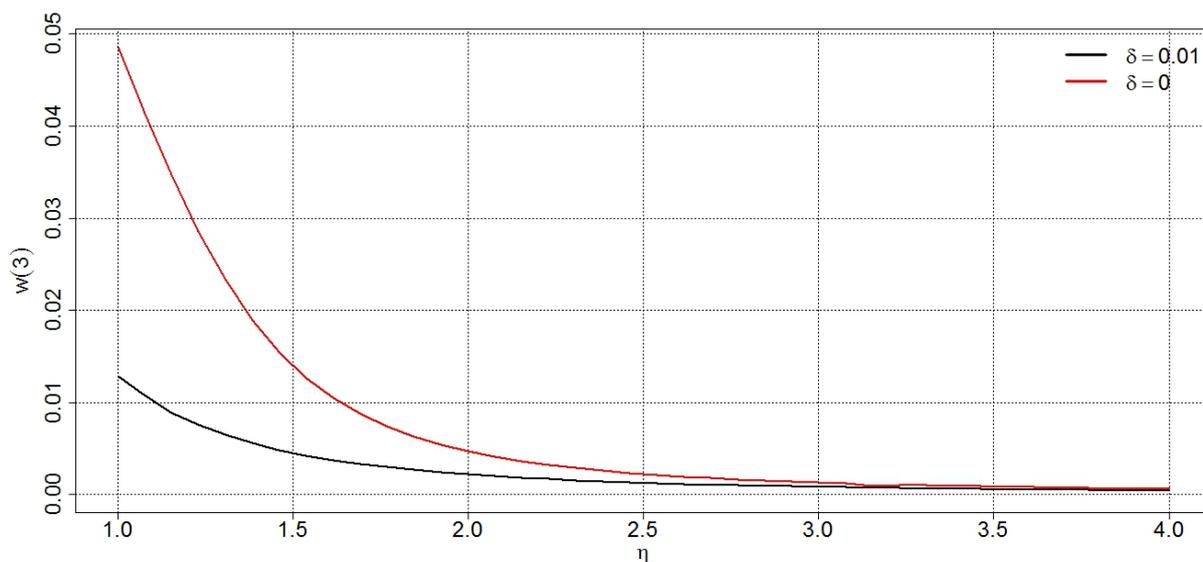


Table 1a: WTPs with alternative parameter values, IPCC (2014) data

Cases		$w^*(1)$ 07	$w^*(1)$ 14 66%	$w^*(1)$ 14 90%	$w^*(3)$ 07	$w^*(3)$ 14 66%	$w^*(3)$ 14 90%
1	Base case	0.0094	0.0102	0.0101	0.0053	0.0048	0.0037
2	$t_{max} = 300$	0.0089	0.0097	0.0096	0.0050	0.0045	0.0035
3	$t_{max} = 1000$	0.0094	0.0103	0.0101	0.0053	0.0048	0.0037
4	$g_0 = 0.010$	0.0300	0.0320	0.0312	0.0179	0.0154	0.0119
5	$g_0 = 0.005$	0.0625	0.0656	0.0639	0.0384	0.0322	0.0250
6	$g_0 = 0$	0.1191	0.1227	0.1190	0.0750	0.0616	0.0476
7	$\eta = 4$	0.0012	0.0013	0.0013	0.0007	0.0006	0.0005
8	$\eta = 4, g_0 = 0.010$	0.0048	0.0052	0.0051	0.0029	0.0025	0.0019
9	$\varepsilon(T_H) = 5^\circ C$	0.0158	0.0150	0.0149	0.0103	0.0087	0.0081
10	$\varepsilon(T_H) = 5^\circ C, g_0 = 0.010$	0.0512	0.0475	0.0469	0.0350	0.0284	0.0262
11	$\varepsilon(T_H) = 5^\circ C, g_0 = 0.005$	0.1064	0.0974	0.0959	0.0746	0.0598	0.0550
12	$\varepsilon(\gamma) = 0.0002726$	0.0193	0.0208	0.0204	0.0112	0.0097	0.0076
13	$\varepsilon(\gamma) = 0.0002726, g_0 = 0.015$	0.0326	0.0347	0.0340	0.0194	0.0166	0.0129
14	$\varepsilon(T_H) = 5^\circ C, \varepsilon(\gamma) = 0.0002726$	0.0327	0.0307	0.0304	0.0218	0.0180	0.0166
15	$g_0 = 0, \delta = 0.01$	0.0300	0.0320	0.0312	0.0179	0.0154	0.0119
16	$g_0 = 0, \delta = 0.02$	0.0094	0.0102	0.0101	0.0053	0.0048	0.0037
17	$g_0 = 0.005, \delta = 0.01$	0.0157	0.0169	0.0166	0.0091	0.0080	0.0062
18	$\eta = 4, g_0 = 0.005, \delta = 0.01$	0.0073	0.0075	0.0073	0.0045	0.0036	0.0028
19	$\varepsilon(T_H) = 5^\circ C, g_0 = 0.010, \delta = 0.01$	0.0158	0.0150	0.0149	0.0103	0.0087	0.0081

Note: unless otherwise indicated, for 2014 estimates $\delta = 0, \eta = 2, g_0 = 0.020, \varepsilon(T_H) = 3.7^\circ C, \varepsilon(\gamma) = 0.0001363, t_{max} = 500$ years. For 2007 estimates $\varepsilon(T_H) = 3^\circ C$ instead of $\varepsilon(T_H) = 3.7^\circ C$. "07" ["14"] represents estimates based on the IPCC 2007 [2014].

5 Reanalysis: Convex damage function

"When assessing climate sensitivity, we at least have scientific results to rely on, and can argue coherently about the probability distribution that is most consistent with those results. When it comes to the damage function, however, we know almost nothing".

Pindyck (2013)

This section provides a reanalysis of the original paper where we change the functional form of (2) defining how much a temperature increase would affect the growth rate g_0 ¹¹. The linear way the increment in temperature affects the economic growth rate is the most speculative part of the analysis, since there is not enough empirical or theoretical support for any specific damage function, and arbitrary choices of the damage function (Weitzman, 2010) or, more generally, on the structure of the model (Pindyck, 2013, 2017) may have non-trivial effects on related policy considerations. Indeed, as far as we would like to understand the consequences of unprecedented warming, we may be tempted to assume

¹¹As in every reanalysis, the fact that we change part of the model makes clear that the results shown here cannot be expected to resemble the ones in P12, but should be used to appraise how the original results are affected by modifications of key parts of the assumptions.

(say, for precautionary reasons) convex damages. Hence, we reanalyse the willingness to pay assuming a convex relationship, in place of the linear specification in (2), between the growth rate g_t of GDP and the level of warming. If we suppose that

$$g_t = g_0 - \gamma' T_t^\alpha, \quad (8)$$

where the parameter $\alpha \geq 1$ can shape different degrees of convex impact and γ' is a constant coefficient. As $T_t = 2T_H[1 - (1/2)^{t/H}]$, we obtain:

$$g_t = g_0 - \gamma' \left\{ T_H \left[1 - \left(\frac{1}{2} \right)^{\frac{t}{H}} \right] \right\}^\alpha$$

As for the linear case, the value of γ' is obtained equating the consumption at horizon H along the path of the growth rate determined by (8) with what will be obtained using a loss function on levels. Given that the path of the growth rate is different with respect to the baseline case exposed in P12, γ' has to be estimated again. We have:

$$C_H = C_0 \exp \left(\int_0^H g_0 - \gamma' (2T_H)^\alpha \left[1 - \left(\frac{1}{2} \right)^{\frac{t}{H}} \right]^\alpha dt \right) \quad (9)$$

$$= 1 \cdot \exp \left(g_0 H - \gamma' (2T_H)^\alpha \int_0^H \left[1 - \left(\frac{1}{2} \right)^{\frac{t}{H}} \right]^\alpha dt \right) \quad (10)$$

Therefore, equating the result obtained through the effect on the growth rate to the one on the level, we get:

$$\exp \left(g_0 H - \gamma' (2T_H)^\alpha \int_0^H \left[1 - \left(\frac{1}{2} \right)^{\frac{t}{H}} \right]^\alpha dt \right) = \exp(g_0 H - \beta T_H^2)$$

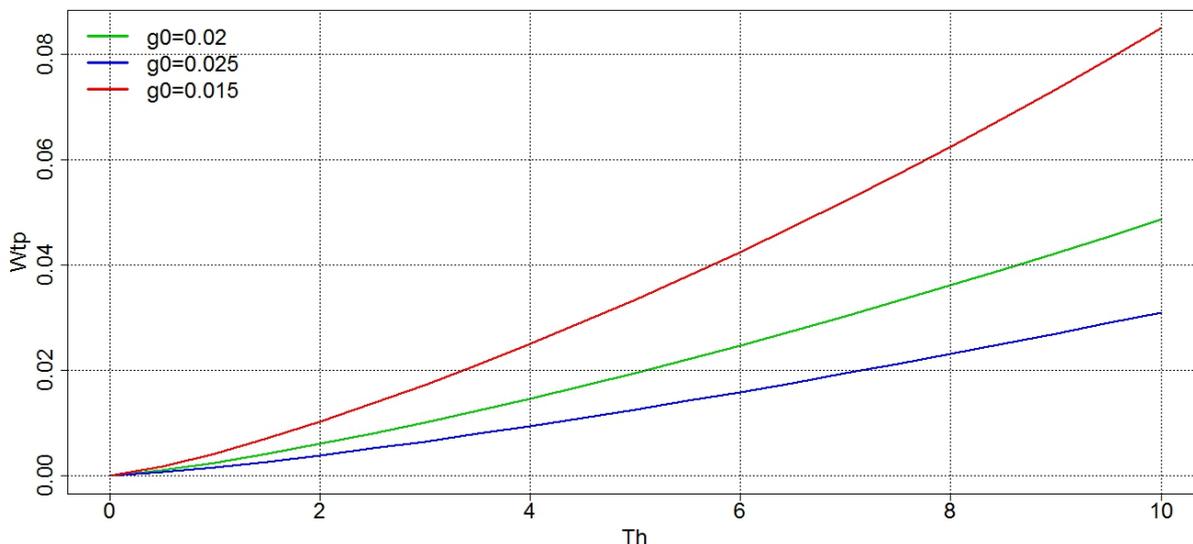
And, finally:

$$\gamma' = \frac{\beta T_H^{2-\alpha}}{2^\alpha \int_0^H \left[1 - \left(\frac{1}{2} \right)^{\frac{t}{H}} \right]^\alpha dt} \quad (11)$$

Note from (11) that γ' is a decreasing function of α . The lower value of γ' under convex damages, with respect to the γ estimated for linear damages, is necessary to have, after 100 periods, the same consumption loss implied by the loss function in levels $\exp(-\beta T_H^2)$. This, in turn, implies that when $\alpha > 1$ we would initially have smaller losses for low increases of T_H to move to greater losses for large increments of the temperature.

To provide some insight, we reanalyse the WTP assuming $\alpha = 1.25$, which implies a modest increase in convexity with respect to the baseline linear case. We obtain $\mathbb{E}(\gamma') = 0.0001068$ which, as just said, is lower than the standard case $\mathbb{E}(\gamma) = 0.0001363$. Figure 4b depicts the situation in which there is no uncertainty on T_H or γ' (and, hence, it should be compared with Figures 4 in this paper or in P12). As expected, the lower γ' produces lower

Figure 4b: $w^*(0)$, known temperature change T_H , $\eta = 2$, $g_0 = 0.015, 0.020, 0.025$, and $\delta = 0$, convex damage function $g_t = g_0 - \gamma' T_t^{1.25}$



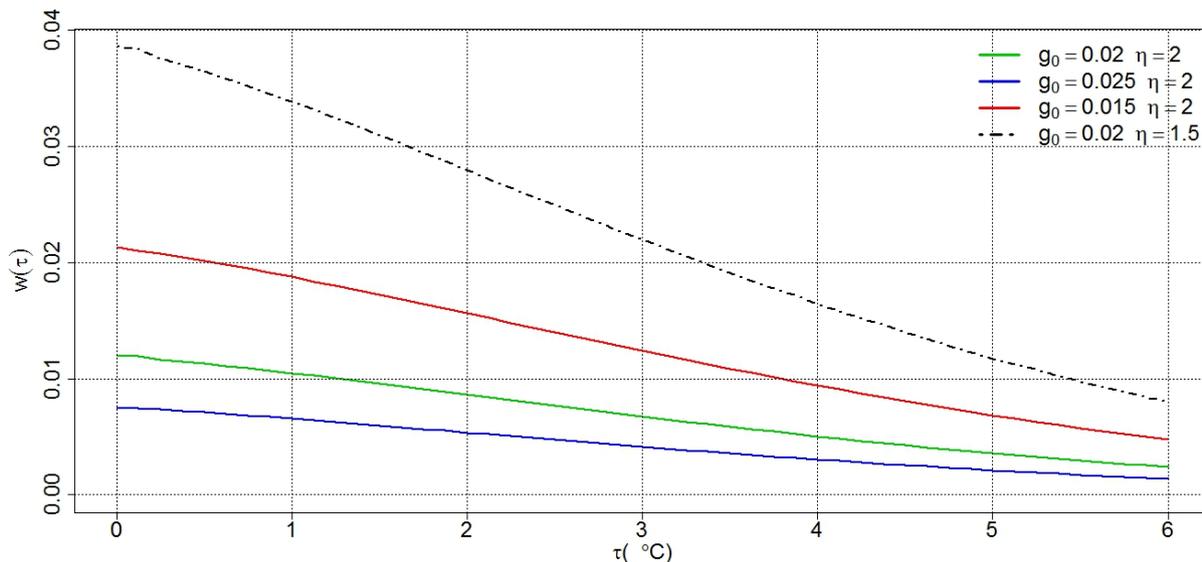
WTPs for low or moderate levels of warming (below 3-4°C), but eventually convex damage takes place for higher warming levels (when T_H is about 8-10°C) inflating the WTP.

Once uncertainty is introduced again, WTPs are depicted in Figures 5b, 6b and displayed in Table 1b. While in Figure 5b, which shows WTPs as a function of τ , the modifications due to $\alpha = 1.25$ appear to be minor, Figure 6b draws the attention of the significant effect at least for low values of the parameter η : when $\eta = 1$, the WTP are very close to 2 and 8% when the discount rate δ is 0 or 0.01, respectively. The same numbers in Figure 6 are about 1.5 and 5.5%. Due to the decreasing trend of the WTP as a function of η , differences fade for medium to large values of η .

The first three rows of Table 1b show that little changes, if any, are observed in the baseline case or varying t_{max} while keeping fixed the values of the other parameters. More interestingly, it also discloses that when $\alpha = 1.25$ large effects are caused by the reduction of the growth rate g_0 . Just to provide an example, in row 6 WTPs for $w^*(0)$ and $w^*(3)$ jump to 0.1935 and 0.1316, with increments about 5 percentage points with respect to the standard case where $\alpha = 1$. Even more spectacular hikes are visible in rows 8 and 18, which feature a combination of low g_0 and high η . The explosion of some WTPs seen in the third and fourth columns of the table can be related to the peculiar effects on the utility generated by low growth rates, $\alpha > 1$ and high values for η . Indeed, as $g_t = g_0 - \gamma' T_t^\alpha$, consumption can decrease to infinitesimal levels for combinations of parameters that make the growth rate negative. Consequently, the utility of nearly null consumption can attain very low and negative values, effectively approaching $-\infty$ at fast speed for large values of η . The examination of the fifth and sixth columns shows that setting $\alpha = 1.25$ generally produces relatively small effect on $w^*(3)$ which, we recall, may corresponds to a situation

in which moderate abatement is sought for.

Figure 5b: $w^*(\tau)$, T_H and γ uncertain, $\eta = 1.5, 2$, $g_0 = 0.015, 0.020, 0.025$, and $\delta = 0$, damage function $g_t = g_0 - \gamma'T_t^{1.25}$



This analysis, further, suggests that the use of large α (say, a quadratic damage function would be obtained when $\alpha = 2$) is likely to results in problematic estimates of WTPs close to 1 (i.e., 100%) for some choice of other parameters of the model. Some reflections on the technical difficulties and practical implications of the computation of the WTP in this and other cases form the bulk of next section.

6 Discussion

This section is devoted to the analysis of three important issues faced in the replication or reanalysis of P12. Firstly, we describe why care is needed in the estimation of the coefficients of the gamma displaced densities estimated for T_H and γ . Secondly, we highlight how the statement “*with the other moments of the distribution unchanged*” has to be implemented. Finally, we investigate the effect of seemingly irrelevant and technical positions related to the upper extremes of integration, as $+\infty$ cannot materially be used in (most of) numerical routines routinely available. The next three subsections explore one issue at a time.

6.1 Estimation of the densities

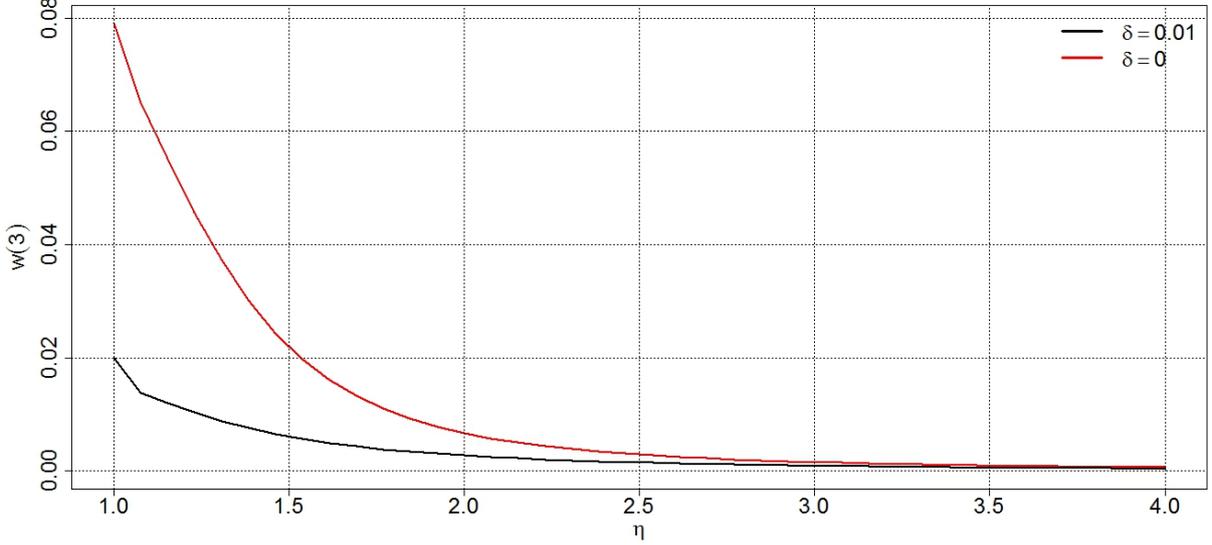
P12 assumes that the two most important uncertain quantities of the model are distributed as displaced gamma distributions, which offer the bonus of a remarkable analytical

Table 1b: WTPs with alternative parameter values, damage function $g_t = g_0 - \gamma' T_t^{1.25}$

Cases	$w^*(0)$		$w^*(3)$	
	$\alpha = 1.25$	$\alpha = 1$	$\alpha = 1.25$	$\alpha = 1$
1 Base case	0.0120	0.0118	0.0067	0.0053
2 $t_{max} = 300$	0.0112	0.0112	0.0062	0.0050
3 $t_{max} = 1000$	0.0135	0.0118	0.0083	0.0053
4 $g_0 = 0.010$	0.0436	0.0369	0.0267	0.0179
5 $g_0 = 0.005$	0.0977	0.0761	0.0633	0.0384
6 $g_0 = 0$	0.1935	0.1432	0.1316	0.0750
7 $\eta = 4$	0.0012	0.0015	0.0007	0.0007
8 $\eta = 4, g_0 = 0.010$	0.7230	0.0060	0.7223	0.0029
9 $\varepsilon(T_H) = 5^\circ C$	0.0214	0.0187	0.0141	0.0103
10 $\varepsilon(T_H) = 5^\circ C, g_0 = 0.010$	0.0801	0.0596	0.0570	0.0350
11 $\varepsilon(T_H) = 5^\circ C, g_0 = 0.005$	0.1776	0.1223	0.1328	0.0746
12 $\varepsilon(\gamma) = 0.0002136$	0.0250	0.0240	0.0147	0.0112
13 $\varepsilon(\gamma) = 0.0002136, g_0 = 0.015$	0.0453	0.0402	0.0276	0.0194
14 $\varepsilon(T_H) = 5^\circ C, \varepsilon(\gamma) = 0.0002136$	0.0450	0.0384	0.0305	0.0218
15 $g_0 = 0, \delta = 0.01$	0.0436	0.0369	0.0267	0.0179
16 $g_0 = 0, \delta = 0.02$	0.0120	0.0118	0.0067	0.0053
17 $g_0 = 0.005, \delta = 0.01$	0.0213	0.0196	0.0124	0.0091
18 $\eta = 4, g_0 = 0.005, \delta = 0.01$	0.8683	0.0089	0.8678	0.0045
19 $\varepsilon(T_H) = 5^\circ C, g_0 = 0.010, \delta = 0.01$	0.0214	0.0187	0.0141	0.0103

Note: unless otherwise indicated, $\delta = 0, \eta = 2, g_0 = 0.020, \varepsilon(T_H) = 3.7^\circ C, \varepsilon(\gamma) = 0.0001068, t_{max} = 500$ years.

Figure 6b: $w^*(3)$ versus η . $g_0 = 0.020$ and $\delta = 0, 0.01$, damage function $g_t = g_0 - \gamma' T_t^{1.25}$



tractability. However, most of what we say would hold, with simple and obvious changes, for any distribution. Essentially, as $f(x|r, \lambda, \theta)$ depends on three unknown parameters to be estimated, three equations should suffice for the estimation and in Section 3.2 we have minimized, say for T_H , the sum of squared deviations from the given “moments”, which were in turn extracted from the literature. As the mean of a displaced gamma is known and equal to $r/\lambda + \theta$ the sum of squared errors is:

$$\left(\frac{r}{\lambda} + \theta - 3\right)^2 + \left(\int_{\theta}^7 f(x|r, \lambda, \theta) dx - 0.95\right)^2 + \left(\int_{\theta}^{10} f(x|r, \lambda, \theta) dx - 0.99\right)^2,$$

where the conditions $E(T_H) = 3^\circ C$, $Pr(T_H \leq 7^\circ C) = 0.95$ and $Pr(T_H \leq 10^\circ C) = 0.99$ can be easily seen. The previous function of r, λ, θ can obviously be optimized but accurate results were obtained only after the selection in the R command `optim` of the numerical method “BFGS” rather than the default choice. Even more crucially, most optimization packages, including the one we have used, implicitly assume by default that the sensitivity of the target function with respect to changes in the variables (or parameters, in our case) are of the same magnitude.

While this is roughly true in the estimation of the density for T_H , it is definitely not the case for the parameters of the density of impact γ , which differ by several orders of magnitude and are reported in P12 to be $r = 4.5$, $\lambda = 21341$ and $\theta = -7.46 \cdot 10^{-5}$. Typically, in similar cases the default choice of the numerical method may fail to be the correct one, possibly resulting in inaccurate results. To tackle this “scaling” problem (Nash, 2014) the user can provide optional information to the algorithm, basically giving the correct magnitudes as an (additional) input. Specifically, in our code we use the option `parscale` to feed the optimizer with the proper parametric scales.

To summarize, it should be kept in mind that in this case other than the defaults computational methods and scaling coefficients were provided to the optimizer in R. Obviously, care and some experience are needed to customize some details prior to minimization and additional scrutiny and checks of the adequacy of the results, with analytical or graphical methods, are advised. The task is hugely facilitated when replicating an existing paper that can be inspected and appear to be much harder if no hint can be guessed, say on starting points or sizes, from other sources.

6.2 On sensitivity analysis

Among the several sensitivity analysis reported in Table 1, some concern an increase in the mean μ of T_H or γ or in both of them. To facilitate the reader in “verifying our verification”, we briefly outline how to obtain the same results of P12.

The statement “*with the other moments of the distribution unchanged*” means that the value of θ must not be altered, and that the values of the parameters r and λ have to be chosen in such a way to leave σ^2 unchanged, get the desired μ and let the moments beyond the second to vary. Exploiting the properties of the displaced gamma distribution, it can be shown that there is a closed-form solution for r and λ as a function of μ , σ^2 and θ :

$$r = \frac{(\mu - \theta)^2}{\sigma^2}$$

and

$$\lambda = \frac{(\mu - \theta)}{\sigma^2}.$$

We adopted this procedure and got estimates for cases 9–14 in Table 1 virtually identical to the ones contained in P12.

6.3 On the upper limits of integration

The quantification of the WTPs entirely relies, as we have seen in previous sections, on the computation of two (multi-dimensional) integrals in (7). In principle, these integrals are to be computed over intervals reaching $+\infty$. However, for practical reason, the support of T_H and γ is truncated and the upper limits in the computations are instead taken to be large numbers, which we denote by T_{max} and γ_{max} . We recall that the same computational shortcut is explicitly mentioned in P12 when, say, the utility of consumption is integrated up to $t_{max} = 500$ (or 1000 in the robustness check of row 3 in Table 1). All the computation in this work used $T_{max} = 15$ °C and $\gamma_{max} = 0.0007$. Intuitively, this is justified by the fact that truncating the distributions should not have a large effect provided that T_{max} and γ_{max} are “big enough”.¹²

Table 2 shows the WTP $w^*(0)$ for the baseline combination of parameters (corresponding to row 1 of Table 1), as a function of the upper limits of integration for T_H and γ . Our

¹²Limiting the upper extremes to 15 and 0.0007, respectively, we are excluding from the analysis an area amounting to probability $2.3 \cdot 10^{-8}$

reference value $w^*(0) = 0.0118$, obtained when $\gamma_{max} = 0.0007$ and $T_{max} = 15$, is singled out and boldfaced in the table. It is clear that replacing $+\infty$ in the integrals with smaller values has little consequences and, unless really too small T_{max} or γ_{max} are chosen, results appear remarkably robust, *for the given set of parametric values* used in the table.

Table 2: Case 1 (baseline) of Table 1. Sensitivity analysis of $w^*(0)$ with several combinations of γ_{max} ($\times 10^{-4}$) (horizontal axis) and T_{max} (vertical axis)

	5	7	9	11	13	15	17	19
12	0.0101	0.0102	0.0102	0.0102	0.0102	0.0102	0.0102	0.0102
13	0.0110	0.0111	0.0111	0.0111	0.0111	0.0111	0.0111	0.0111
14	0.0114	0.0115	0.0116	0.0116	0.0116	0.0115	0.0116	0.0116
15	0.0116	0.0118	0.0118	0.0118	0.0118	0.0118	0.0118	0.0118
16	0.0117	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0118
17	0.0118	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119
18	0.0118	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119
19	0.0118	0.0119	0.0119	0.0119	0.0120	0.0119	0.0119	0.0119
20	0.0118	0.0119	0.0119	0.0120	0.0119	0.0120	0.0119	0.0119
21	0.0118	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119
22	0.0118	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119
23	0.0118	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0120
24	0.0118	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119
25	0.0118	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119
26	0.0118	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119
27	0.0118	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119
28	0.0118	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0121
29	0.0118	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119

Note: $\delta = 0.02$, $\eta = 2$, $g_0 = 0$, $\varepsilon(T_H) = 3^\circ C$, $\varepsilon(\gamma) = 0.0001363$, $t_{max} = 500$ years.

Quite a different behaviour is documented in Table 3, which shows $w^*(0)$ when η increases to 4 and the growth rate g_0 decreases to 0.01 (see the parameters of row 8 of Table 1). Even a cursory look at the table reveals that the WTP dramatically depends on T_{max} and γ_{max} , despite the intuitive belief that they should only play a technical role in the computations, bounding the integration domain. It is quite clear that $w^*(0) = 0.0060$ which, as argued in previous sections, is notably different from the figure shown in P12, is not a robust estimate and its wild fluctuations cast serious doubts on any related policy suggestions. Generally speaking, the increase of T_{max} , as well as γ_{max} appear to fuel the WTP till 100%. This value is reached because there are combinations of T and γ in the integration domain for which consumption “grows” at a negative rate and rapidly approaches infinitesimal levels, due to the small g_0 and to the term $-\gamma T$ in (2), generating (very) negative utility. The effect is enhanced by the relatively large value taken by η , and we have already observed that several WTP are hard to compute when $\eta = 4$, the largest value examined in P12.

The previous discussion demonstrates that reliable computations of WTP may be hard

Table 3: Case 8 ($\eta = 4$, $g_0 = 0.01$) of Table 1. Sensitivity analysis of $w^*(0)$ with several combinations of γ_{max} ($\times 10^{-4}$) (horizontal axis) and T_{max} (vertical axis)

	5	7	9	11	13	15	17	19
12	0.0054	0.0055	0.0055	0.0055	0.0056	0.0058	0.0064	0.0079
13	0.0057	0.0058	0.0058	0.0059	0.0064	0.0083	0.0162	0.0469
14	0.0058	0.0059	0.006	0.0065	0.0093	0.0268	0.1167	0.3613
15	0.0059	0.0060	0.0062	0.0081	0.0251	0.1490	0.4792	0.7474
16	0.0059	0.0061	0.0066	0.0140	0.1033	0.4687	0.7757	0.9098
17	0.0060	0.0061	0.0076	0.0362	0.3279	0.7372	0.9083	0.9682
18	0.0060	0.0062	0.0099	0.1097	0.5974	0.8762	0.9628	0.9888
19	0.0060	0.0063	0.0161	0.2778	0.7785	0.9422	0.9850	0.9961
20	0.0060	0.0065	0.0319	0.4960	0.8805	0.9731	0.9940	0.9986
21	0.0060	0.0068	0.0701	0.6762	0.9359	0.9875	0.9976	0.9995
22	0.0060	0.0073	0.1502	0.7977	0.9656	0.9942	0.9990	0.9998
23	0.0060	0.0082	0.2808	0.8747	0.9816	0.9973	0.9996	0.9999
24	0.0060	0.0098	0.4369	0.9226	0.9902	0.9988	0.9998	1
25	0.0060	0.0126	0.5816	0.9523	0.9948	0.9994	0.9999	1
26	0.0060	0.0175	0.6970	0.9706	0.9972	0.9997	1	1
27	0.0060	0.0261	0.7803	0.9819	0.9985	0.9999	1	1
28	0.0060	0.0408	0.8453	0.9889	0.9992	0.9999	1	1
29	0.0060	0.0653	0.8899	0.9931	0.9996	1	1	1

Note: $\delta = 0.02$, $\eta = 2$, $g_0 = 0$, $\varepsilon(T_H) = 3^\circ C$, $\varepsilon(\gamma) = 0.0001363$, $t_{max} = 500$ years.

under some circumstances or, if you wish, that the model appears to be fragile, being the results too sensitive to “internal” inputs of the numerical software. Clearly, changes in the specification of the utility function may remove some forms of ill-posedness. Additional stability, of course, would be obtained tolerating positive values for δ but we are well aware that the proper level of the intertemporal discount rate is at the heart of a (moral) debate among scholars and economists (Dasgupta, 2008; Pindyck, 2013)¹³.

7 Conclusions

In this paper we present a verification, an extension and a reanalysis of the incisive paper Pindyck (2012). Retracing the path followed by the author has, to a large extent, allowed to verify the accuracy of the estimates of the willingness to pay in order to limit the temperature increase below some threshold τ . This was possible with no access to the original code under a variety of parametric instantiations and critical discrepancies from the results of P12 are present but uncommon, possibly due to one material typing error and likely to be related to a few specific combinations of values taken by the parameters η , related to the risk aversion of the society, intertemporal discount rate δ and growth rate of consumption g_0 .

Our extension and our reanalysis corroborate the main message of Pindyck (2012): we have shown that using more recent data from *IPCC* 2014 does not change the value of the statement that the willingness to pay, given what we know and its sheer uncertainty, is consistent with a moderate abatement policy.

As we know little about the damage function that relates the temperature increase to the decrement in the growth rate of consumption, our reanalysis also investigates how convex damages affect the results, implicitly assuming that rare (and catastrophic) events are greatly valued in the computation of the utility function. While in standard cases this is not changing much the results, in other circumstances the willingness to pay increases in outstanding ways, hinting at some fragility of the model and suggesting caution in the interpretation of policy decisions that may be driven by the model.

We believe that another important outcome of this work was the demonstration that some results critically depend on the values of technical parameters of the numeric algorithms at work in the evaluation, such as the upper extremes of integration. The fragility of this version of the model exemplifies why an oftentimes excruciating effort is needed to verify the results obtained by other scholars, even in this cases where abundant information was available in an extended and detailed working paper written by Pindyck on the very same model.

While we hope that our sacrifice makes a contribution to the ongoing discussion on the usefulness of verifying and reanalysing scientific works, we are well aware that even more sacrifice is surely needed to understand and reduce adverse effects of climate change.

¹³According to Dasgupta (2008) and Pindyck (2013), the different calibration of the discount rate and other crucial parameters is at the very heart of the ten-fold difference between Nordhaus (1994, 2014) and Stern (2007, 2008) in their estimated Social Costs of Carbon.

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