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Abstract

The implementation of Revenue Management (RM) techniques in non profit performing arts organizations presents new challenges compared to other sectors, such as transportion or hospitality industries, in which these techniques are more consolidated. Indeed, performing arts organizations are characterized by a multi-objective function that is not solely limited to revenue. On the one hand, theatres aim to increase revenue from box office as a consequence of the systematic reduction of public funds; on the other hand they pursue the objective to increase its attendance. A common practice by theatres is to incentive the customers to discriminate among themselves according to their reservation price, offering a schedule of different prices corresponding to different seats in the venue. In this context, price and allocation of the theatre seating area are decision variables that allow theatre managers to manage these two conflicting goals pursued. In this paper we introduce a multi-objective optimization model that jointly considers pricing and seat allocation. The framework proposed integrates a choice model estimated by multinomial logit model and the demand forecast, taking into account the impact of heterogeneity among customer categories in both choice and demand. The proposed model is validated with booking data referring to the Royal Danish Theatre during the period 2010-2015.

Keywords Multi-objective optimization; Pricing; Seat allocation; Multinomial logit model; Theatre demand

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1 Introduction

In the seminal article by Baumol and Bowen (1966) the authors claim how theatres will be more and more dependent on subsidies, due to their productivity lag. However, the last decades' tendency shows that public funds allocated to non profit performing arts organizations in Western countries (Marco-Serrano, 2006) are decreasing[1]. This fact has forced theatres to increase other sources of revenue, including box office revenue. In addition, such organizations pursue the aim to increase the attendance, for a couple of reason: first, they feel the mission to spread culture to as broad a segment of the populace as possible (Hansmann, 1981) legitimizing their social value; second, they prefer to avoid empty seats in the venue that can have negative effect on the reputation of the theatre.

In this context, managers of the performing arts organizations can implement Revenue Management (RM) techniques in order to balance between the rate of occupancy and the profitability of theatre. The most common among these techniques is realized through market segmentation based on the price leverage, that leads to different pricing scheme. For istance, price reductions are offered to customers segments, such as students and senior citizens, who are supposed to be less able to pay. Discounts are offered also to those customers - subscribers - who buy in advance a bundle of tickets, assuring a long-term commitment toward the theatre. Due to heterogeneity in price sensibility within the same customer segment, one usual practice by theatre is to use a non-linear tariff system offering a schedule of different prices according to the quality of the product. In this case, different prices are charged according to the seat location in the venue in order to better capture consumers' willingness to pay. Indeed, this mechanism incentives customers to discriminate themselves by choosing the seating area they prefer. So, beside the pricing strategy, also the seat allocation across these fare classes (i.e seating area) represents a decision that may foster an orientation by the theater towards either the maximization of the total attendance or the maximization of revenue. In the first case we expect that theatre would increase the accessibility of the most expensive seating areas for all the customers: to do this, it is convenient to propose a scheme in which the prices of the different seating are closer downward. This mechanism will lead to an increase of the size of the expensive seating area and, in addition, can favorite a customer buy-up behaviour (i.e buying a ticket for a more expensive fare class when the

[1] This framework holds also for our case study: the Royal Danish Theatre. According to the National Danish Statistics (http://www.statbank.dk), the public subsidy to the Royal Danish Theatre decreases from 608 675 Danish crowns in the 2011/2012 season, to 573 900 Danish crowns in the 2014/2015 season.

ticket for the required seating area is not available). In the second case, we expect that theatre would strengthen the self-discrimination operated by customers. Thus, the allocation policy will strongly depend on the type of customer that attends the performance: if the performance attracts an audience group (as young customers) that is supposed to be highly price sensitive, the theatre would enlarge the cheapest seating area in order to prevent a loss in revenue. In the opposite case, the theater would take advantage of the inelastic demand by enlarging the expensive seating area.

With respect to this pricing and allocation strategy, it becomes essential not only the demand forecasting, but also the understanding of the customers' behavior with respect to price discrimination by seating area. Since Talluri and Van Ryzin's (2004) paper, discrete choice models have emerged as a standard approach in the RM literature to incorporate the buy-up and buy-down behavior.

This paper proposes an optimization model that jointly considers the pricing and allocation problem in the performing arts context. To this end, the demand forecasting is integrated with a customer choice model. In order to accomodate for heterogeneity in preference over seating areas, we adopt a multinomial logit model (MNL) using customer's characteristics and performance-production attributes as variables to be interacted with the characteristics of the choice alternatives.

We aim to contribute to the RM and theatre demand literature by demonstrating that discrete choice and optimization models can effectively be employed to assist theatre managers in both setting price and seating allocation. Indeed, theatres, as providers of performing arts services, meet the requirements for the implementation of RM techniques, such as the capacity constraint and the perishability issue (i.e the ticket value deteriorates as time goes on and it is null when the performance starts). However, as already said, subsidized theatres, compared to the dominant sector in RM literature like the transportation and hospitality industries, are organizations which pursue other objectives in addition to the revenue maximization. Therefore, we employ a bi-objective optimization model to study the optimal pricing and seat allocation with conflicting objectives related to revenue and total attendance.

Our model has been implemented to a data set provided by the Royal Danish Theatre which refers to the period 2010-2015. A simulation is conducted considering three performances that differ from each other by characteristics that affect the demand.

To the best of our knowledge, there are no studies in the RM literature that has considered jointly the pricing and allocation problem in a context in which the organization present an objective function that is not limited solely limited to revenue.

In this sense, we believe that this is the added value of this article, providing guidances to theatre managers in adapting the price and size of seating area according to the weight given to each objective to pursue.
The remainder of the paper is organized as follows: Section 2 presents the relevant literature on RM in the performing arts context; Section 3 describes the research framework, whereas Sections 4 and 5 present respectively the demand estimation and the choice model. Section 6 describes the optimization model, whereas Section 7 presents the results of our simulation. Finally Section 8 provides some conclusions.

2 Literature review

Opposed to a huge amount of RM research devoted to transportation industry, the issues related to pricing and allocation problems in performing arts organizations have not received a great deal of attention. This disparity is due to different factors: first, RM is not a pervasive practice in most of these organizations. Second, this kind of organizations is characterized by peculiar features, in primis the multi-dimensional nature of their objectives.
The literature of cultural economics has been dealing with the objectives of performing arts institutions. Since most performing arts institutions are nonprofit firms this taps into a more general literature on the objectives of nonprofit firms (e.g. Hansmann (1980) and Steinberg (1986)). Steinberg (1986) suggests that nonprofit firms are either service maximizers or budget maximizers or something in between. However, in the performing arts, the concept of service is not straight forward. Several authors (e.g. Throsby and Wither, 1979, Throsby, 1994, and Hansmann, 1981) have suggested three different measures of output: 1) Quality, 2) Audience size and 3) Budget. Several empirical studies have shown that the performing arts are primarily output maximizers (either quality or quantity), and less budget maximizers (see e.g. Luksetich and Lange, 1995; and Gapinski, 1985). For an overview of the literature see Brooks (2006). To our knowledge no studies have been made, dealing with the optimization decisions in the performing arts, when the repertoire is planned (based on quality decisions), and the theatre wants to make the optimal decision on how to maximize attendance as well as revenue, based on decision on prices and seat categories.
Most of the research related to the demand-management decision in the theatre sector has focused on the price discrimination practice. Hansmann (1981) claims that in the nonprofit performing arts sector, price discrimination is not effective due to the difficulty to identify customers with inelastic demand. Therefore, according to

the author, the only form of discrimination that nonprofit enterprise can apply is by asking for a voluntary donation, in order to extract a part of consumer's surplus. Seaman (1985) raises some doubts about Hansmann's (1981) hypotheses: the author measures the degree of price discrimination (such as: the number of different prices charged and the standard deviation of the prices charged) to a set of non profit performing art organization. He concludes that price discrimination varies significantly across art forms (opera, ballet, theater, symphony concert) and that the organizations that discriminate more are characterized by a high ratio between fixed cost and attendance. Huntington (1993) justifies the adoption of price discrimination by seating area, by referring to the Rosen's (1974) utility model (i.e the hedonic price model), as there are observable differences between different seats. Moreover, the author compares the box office revenue between theaters operating a single price policy and those operating a discrimination pricing policy: he finds that the price range policy is statistically significant and positively correlated with the revenue of the theatre, controlling for seat capacity and the number of performances per year. Rosen and Rosenfield (1997) describe a model in which theater venue has two types of seats: (high and low quality), and the theatre manager knows the distribution of reservation price for both seat category. First, the authors solve the pricing problem, given the quantity of seats for each category. Second, the authors solves the allocation problem, given the optimal pricing policy. Leslie (2004) considers the Broadway show "Seven Guitars" and estimates a structural econometric model of price discrimination based on a individual consumer behavior model, that incorporate all the types of price discrimination (by seating area and social category). The model allows him to perform different experiment using alternative pricing policies. Tereyagoglu *et al.* (2012) use the data from the ticket purchase transaction of the shows of a symphony orchestra in the northeast region of the US, in order to employ a proportional hazard framework to analyze how pricing and discount actions over time affect the timing of customers purchase.

3 Research framework

3.1 The Royal Danish Theater

The Royal Danish Theatre was founded in 1748 and is the Danish national theatre. It has three main Stages in Copenhagen. The Old Stage from 1874, a new Royal Opera House from 2005 and a new Royal Playhouse from 2008. The Opera House and the Playhouse has a main stage and smaller stages for experimental productions. It is one of the few theaters in the world offering both opera, ballet and theatre per-

formances as well as classical concerts. Today The Old Stage is the house, where ballet is performed.
The law of the Royal Danish Theatre states that it is the national theatre for the whole country and the entire population. Besides, it has an obligation to produce a broad repertoire of high artistic quality within ballet, opera and plays. It is obligated to continue the classical traditions as well as developing the performing arts in new and contemporary ways. A special concern is on productions of Danish origin. The Royal Danish Theatre is on the state budget under the Ministry of Culture, and has a number of more specific obligations in agreement with the current Minister of Culture. Included in these obligations are general cultural policy goals, like having special productions for children and youth, and to keep prices to a level that make the theatre accessible for all socio-economic groups.
In 2015 the theatre had a total budget of 705,4 million DKK (94 million Euros), of which 76 percent were public support from the Government. The theatre had 165,8 million DKK (22 million Euros) in own earnings, of which 69 percent (15 million Euros) was from ticket sales, the rest was income from sponsors etc.
Due to its obligations as a national theatre, it has to decide its repertoire based on a number of parameters, namely quality and variety, understood as a fairly large number of different productions from the classical repertoire as well as new productions, developing the performing arts, and Danish as well as international production from the world repertoire. Besides, it has to decide the number of performances of each production during the season and how they are scheduled on weekday and weekends. It will create a loss in earnings if a given production is played less than demanded by the audience as well as if a performance is played more times than demanded by the audiences (empty seats). There are high fixed costs take a new producing on stage (due to rehearsal time, designing the staging etc.), but the costs to prolong a production with extra performances are small, and the marginal costs are lower than the marginal revenue (Bille Hansen, 1991).
Finally, it has to decide its price policy, including price differentiation based on different audience groups (like young, subscribers and senior) as well as seat categories, time of the performance, the type of the performance, the production costs etc.

3.2 Problem description

In this paper we assume that the repertoire decisions are already determined by the theater, both with regard to the variety of productions and the number of performances of each production during a season. With this restriction the theatre has to decide on the price and the allocation of seat categories for the individual performances. It is assumed, that the theatre wants to optimize both attendance and revenue, where the former finds an upper limit in the theatre capacity. Our bi-objective optimization model incorporates the demand forecast and the customers' seat choice model. The latter is estimated with a multinomial logit (MNL) model that predicts the probability to choose a particular seating area as a function of price and performance characteristics. From Baldin and Bille (2016) we know that some audience groups (especially young people) are quite price sensitive, while other groups are very insensitive to price (e.g., subscribers). Therefore, we estimate one demand forecast for each customer category; whereas the choice model accounts for heterogeneity by including choice-invariant variables that accounts also for the customer category.

The methodological procedure in this paper follows the study by Hetrakul and Cirillo (2014) that proposes, in a railway setting, an optimization model in which discrete choice models and demand function are integrated, in order to calculate the price and fraction of the demand to be accepted for each origin-destination pair.

4 Demand forecast

4.1 Sample selection

The demand estimation is based on booking data from the sale system of the Royal Danish Theater for the period 2010/2011 to 2014/2015. The sample is constituted by 401 opera performances which took place during that period. We estimate a demand function for each customer category identified, with which we refer to the price type applied by the theater in the price discrimination process across buyers. Hence, we assume that the market segmentation is solely based on the price leverage. Among the numerous price type existing (including customers with a loyalty card, employees, group sales, disabled...), we consider the three main customer categories that together account for nearly 80% of the total tickets sold: standard ticket buyers (45.9%) who pay the full price for the ticket; young (under 25 years)-student customers (6.1%) for which tickets are discounted by 50%; and subscribers (26.1%). Regarding the subscribers categories, Royal Danish Theatre applies two type of subscription: a fixed subscription, in which the bundle of events included is predetermined by the theatre, and a "choose your own" subscription that allows the customers to choose the productions they want to see. In the first case, a discount of 15% is applied, whereas in the second case the discount drops to 10%. In order to simplify the optimization model, we merge the two types of subscription, considering the average discount of 12.5%. In this category we include also the additional tickets that a subscriber can purchase, besides his subscription. For example, when a subscriber buys a performance ticket of a production that it is not included in the subscription, also this ticket is discounted by 10%.

For the purpose of model simplicity, there are some remarkable categories that, given their low number of attendees per performance, are not considered. For example, tickets for senior customers, which are entitled to a discount of 50%, represents only 2.5% of the tickets sold just because this discount is made available only for some performances decided by theatre management. Indeed, as many senior customers are subscribers, it does not result convenient to offer this discount for all the performances. We exclude also the young/student subscribers, which accounts for 0.74% of the total theater market: their discount is 65% for a fixed subscription and 60% for a "choose your own" subscription.

4.2 Demand estimation

Following the literature, we adopt a double-log specification, which is the most popular functional form adopted in estimating theatre-attendance demand (Seaman, 2006). For each category j, the following demand function is estimated:

$$ln(D_j) = \alpha_j + \beta_j ln(p_j) + \gamma_j' z + \epsilon_j \tag{1}$$

so as:

$$D_j = exp(\alpha_j + \beta_j ln(p_j) + \gamma_j' z + \epsilon_j) \tag{2}$$

where, for given a performance, D_j is the number of tickets sold to category j, p_j is the average price of ticket deflated by CPI[2] charged to category j: in particular, we take the average price of the different seat categories offered by the theatres. z is a vector of performance and production characteristics, while ϵ_j is an error term.

Concerning the performances scheduling, we include three dummy variables to take into account the weekly seasonal effect: WKDAY denotes performances run during weekdays (from Monday morning to Friday morning); WKEND indicates performances run during Friday and Saturday evening or during the evening before a public holiday. Finally SUNDAY denotes performances that take place on Sunday or in a public holiday day. This latter group of performances are "matinee" as no evening performances take place on Sunday. Except on Sundays, in the other days of week performances can take place either on monday-afternoon or during the evening. We denote with EVE performances that take place during the evening.

[2] CPI data are collected by *Statistics Denmark*: http://www.dst.dk/en

In order to capture the seasonality effect, we construct month dummies variables for each month of the year, except for July and August when the theatre is closed. In addition, following Corning and Levi (2002) we include also REMAIN and TOTPERF denoting respectively the number of remaining and total perfomances of a given production. We find also a significant interaction between these two variables: indeed, this interaction term allows to weight the amount of remaining performance: for instance, the effect of the second to last performance changes when the total number of performances are twenty or, for example, five.

We also control for the production characteristics: to capture the popularity of an opera show, we introduce the variable POP measured as the number of times the production is performed worldwide during the same year it has been performed in the Royal Danish Theatre [3]. However, it should be considered that some Danish production (e.g *Maskarade*, *Livlægens besøg*) are popular in Denmark but not worldwide. To control for this aspect, we include the dummy DANISH, denoting Danish productions. Moreover, the dummy variable NEWDKT controls for productions that take place for the first time at Royal Danish Theater.

We also control for the year the production has been created by introducing three dummies: 1920-2015, 1850-1919, BEFORE 1850.

As our analysis is based on performances running in 5 years, we include a time trend t variable. Finally, considering that the total capacity of the theatre can change due to production requirements, we add the variable CAPACITY indicating the number of the available seats for the show.

In estimating the demand function for subscribers, we add a new variable SUBYEAR as the log number of subscribers in the current season. Indeed subscriptions are sold in advance and the number of subscribers is known to the theatre before the season starts. However, for customers who buy a fixed subscription, it is unknown their distribution among performances, given a production.

Table 1 provides a descriptive statistics of the data.

We estimate (1) by OLS with robust standard-error. Although more sophisticated models are available for a forecast analysis (Ainslie *et al.*, 2005), not necessarily such techniques provide a significant improvement (Andrews *et al.*, 2008; Eliashberg *et al.*, 2009).

We have also checked for multicollinearity issues that does not seem to arise.

Table 2 shows the estimation results of the demand function for all the categories considered.

[3]We collect these data through "Operabase", a website designed to collect statistics about operatic activity worldwide: http://operabase.com

Variable	**Mean**	**SD**	**Min**	**Max**
Price (standard ticket)	456.06	74.93	208.96	661.13
Standard tickets sold	562.34	252.11	62	1117
Young tickets sold	73.46	63.65	0	576
Subscribers ticket sold	310.33	151.37	0	716
REMAIN	7.49	5.38	1	30
TOTPERF	14	6.07	6	30
CAPACITY	1482.89	45.51	1297	1529
POP	213.17	186.00	1	507
SUNDAY	0.174	0.380	0	1
WKEND	0.257	0.437	0	1
WKDAY	0.568	0.496	0	1
EVE	0.733	0.443	0	1
JANUARY	0.157	0.364	0	1
FEBRUARY	0.117	0.322	0	1
MARCH	0.149	0.357	0	1
APRIL	0.115	0.319	0	1
MAY	0.147	0.355	0	1
JUNE	0.047	0.2127	0	1
SEPTEMBER	0.027	0.163	0	1
OCTOBER	0.085	0.279	0	1
NOVEMBER	0.125	0.331	0	1
DECEMBER	0.030	0.171	0	1
1920-2015	0.160	0.366	0	1
1850-1919	0.486	0.500	0	1
BEFORE 1850	0.354	0.479	0	1
DANISH	0.027	0.163	0	1
NEW DKT	0.651	0.477	0	1
t	3.06	1.295	1	5

401 observations

Table 1: Descriptive statistics of OLS variables

Results of the demand estimation reveal that price elasticity differs across customer category. Young customers is the audience group most price sensitive: a 1 % increase in ticket price results in approximately 1.84 % decline in demand. Standard ticket buyers are less price sensitive as the price elasticity is less than unity: a 1 % increase in ticket price results in approximately 0.49 % decline in demand. Consistent with previous results in literature (Felton, 1994; Baldin and Bille, 2016), subscribers are the least price sensitive: for our sample the price coefficient is even not statistically significative. This result is not surprisingly as literature has shown in some cases even a positive price elasticity in the demand for performing arts, configuring the theatrical experience as a Veblen good (Laamanen, 2013).
The results for the single ticket buyers show a strong explanatory power ($R^2 = 0.75$) and almost all variables are statistically significant. In particular, Table 2 shows that, for this type of customers, the demand is higher for Friday/Saturday evening performances. The number of times a title is rerun (TOTPERF), which is sup-

Variable	**Single tickets**	**Young**	**Subscribers**
Intercept	2.3538**	6.7917****	−0.1729
	(1.017)	*(1.986)*	*(3.1428)*
Log price	−0.4904***	−1.8440****	−0.1315
	(0.1811)	*(0.3994)*	*(0.7346)*
SUNDAY	0.22741****	−0.0459	0.2652
	(0.0654)	*(0.1321)*	*(0.1980)*
WKEND	0.4620****	0.0083	−0.0338
	(0.0357)	*(0.0644)*	*(0.0926)*
EVE	−0.1205**	−0.0220	0.1626
	(0.0596)	*(0.1106)*	*(0.1838)*
REMAIN	−0.0575****	−0.0483***	−0.0122
	(0.0089)	*(0.0173)*	*(0.0249)*
TOTPERF	0.0365****	−0.0081	−0.0472***
	(0.0049)	*(0.0097)*	*(0.0154)*
REMAIN x TOTPERF	0.0020****	0.0018**	0.0013
	(0.0004)	*(0.0007*	*(0.0009)*
JANUARY	0.2743*	−0.0458	1.0828**
	(0.1603)	*(0.2493)*	*(0.5359)*
FEBRUARY	0.3370**	0.1374	1.1611**
	(0.1590)	*(0.2415)*	*(0.5034)*
MARCH	0.3383**	−0.0316	1.2604**
	(0.1568)	*(0.2350)*	*(0.4980)*
APRIL	0.4957***	−0.0492	1.2728**
	(0.1580)	*(0.2384)*	*(0.5195)*
MAY	0.5726****	−0.1028	1.3395***
	(0.1542)	*(0.2318)*	*(0.5044)*
JUNE	0.5192***	−0.2148	1.1711**
	(0.1631)	*(0.2764)*	*(0.4956)*
SEPTEMBER	−0.2083	−0.9979***	1.5058***
	(0.1949)	*(0.3619)*	*(0.5473)*
OCTOBER	0.0237	−0.3690	1.3119**
	(0.1597)	*(0.2449)*	*(0.5255)*
NOVEMBER	0.0575	−0.2394	1.0168**
	(0.1554)	*(0.2315)*	*(0.5030)*
POP	0.0007****	0.0022****	−0.0013****
	(0.0001)	*(0.0002)*	*(0.0002)*
1850-1919	0.6935****	0.1236	0.6683***
	(0.0758)	*(0.1903)*	*(0.2374)*
BEFORE 1850	0.6385****	0.1108	0.8165****
	(0.0743)	*(0.1851)*	*(0.2292)*
DANISH	−0.1132	0.8734****	−0.5989***
	(0.0858)	*(0.1247)*	*(0.1997)*
NEWDKT	−0.0648*	0.1970***	−0.0524
	(0.0376)	*(0.0771)*	*(0.0774)*
CAPACITY	0.0037****	0.0045****	0.0008
	(0.0004)	*(0.0009)*	*(0.0010)*
t	−0.0101	0.0606**	0.1007**
	(0.0164)	*(0.0303)*	*(0.0508)*
SUBYEAR			0.5192****
			(0.1348)
R-square	0.7512	0.4213	0.3830
Model F-value	51.64****	13.22****	5.02****
No. of observations	401	401	401

Robust st.error listed under coefficients
$^{****}p < 0.001$ $^{***}p < 0.01$ $^{**}p < 0.05$ $^{*}p < 0.10$

Table 2: Estimation results of demand functions

posed to be an indicator of the total expected demand for that production, has a positive impact on the demand for a single performance. Moreover, given the same production, each performance has a 5.75 % higher demand than the previous, holding fixed the number of times a performance is rerun. This is probably due to a word-of-mouth effect (Laamanen, 2013). Furthermore, we can deduce that single ticket buyers prefer traditional and less risky productions than the more experimental ones: indeed the productions that take place for the first time at Royal Danish Theatre have a negative impact on demand; whereas popularity score has a positive impact, as well as those production composed before 1919.

Results for young customers and subscribers have a lower explanatory power (R^2 is respectively 0.42 and 0.38). For the former, there is a positive word-of-mouth and time trend effect. Furthermore, the Danish productions have a strong positive effect on demand, as well as the popularity of the production worldwide; but also the productions that take place for the first time at Royal Danish Theater seem to be appealing to young customers.

Concerning subscribers, we note a significant month-seasonality and time trend effect. Contrary to single ticket buyers, subscribers seem to appreciate less conventional productions, as the coefficient associated with the popularity score is negative. On the other side, productions composed before 1850 seem to be preferred by this audience group.

Table 3 compares the actual attendance with the values predicted by the demand functions. The prediction capability of the model is measured with different indicator, such as root mean squared error, mean absolute error, average error and Pearson correlation between predicted and actual. In addition we perform the out of sample validation. We consider 74 performances run during season 2015/2016 that are not included in our sample. The demand functions for such performances are estimated using the coefficients obtained for our initial sample, and their final estimations are compared with the actual attendance.

Whereas the average errors is decidedly higher for the out of sample performances than the sample performances; the other measures are similar among the two groups of performances.

	Root mean squared errors	Mean absolute errors	Pearson correlation	Average errors
2010/2011-2014/2015				
Single tickets	148.33	114.55	0.78	12.76
Young	52.09	28.36	0.68	9.92
Subscribers	139.22	106.44	0.56	26.20
2015/2016				
Single tickets	155.31	133.06	0.83	-86.11
Young	53.52	30.69	0.56	12.11
Subscribers	111.02	100.84	0.57	-33.58

Table 3: Predictive performance of the demand functions

5 Customer choice model

5.1 Sample selection

The choice model concerns the price discrimination across seating area. The theatre policy has been refined in the last years. In 2010 the *OperaHouse* offered 5 different price zones, 6 price zones in 2011 and 8 seating area from 2012 onwards (Figure 1).

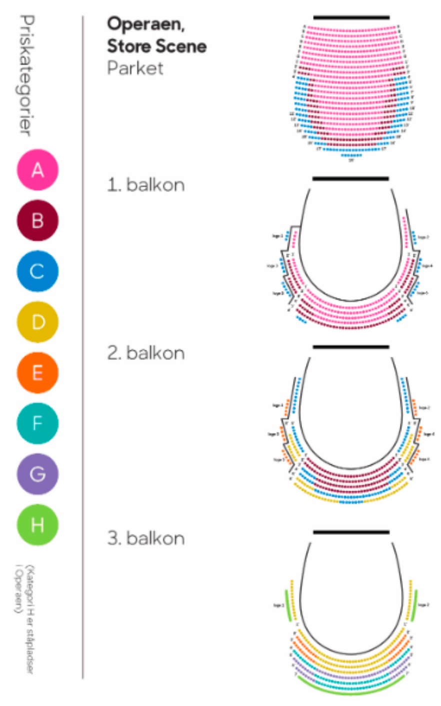

Figure 1: Price zones at the Opera House

The subdivision is not physically evident: for example, zone "price A" includes both

stall seats and first balcony seats; whereas zones “price B” comprehend stall seats as well as first and second balcony seats, and so forth. This allows the theatre manager to be quite flexible in the subdivision of the venue.
Since the number of price zones changed through the period under examination, we aggregated productions with more than five price zones into five seat categories. The procedure adopted follows Baldin and Bille (2016), to which we refer for details.
For logistic reason, it has not been possible to collect data for the choice model estimation for the whole sample considered in the demand function. Our sample consists in 103322 bookings which involve 11 opera productions and 122 performances.

5.2 MNL estimation

After estimating the demand for each performance, in this section we propose a choice model for the seating area decision. To this aim, we adopt a multinomial logit (MNL) approach. Hence, we assume that each customer chooses the seat that maximizes his utility. The independent variables that enter in the model as choice's attributes are: *price* and a dummy variable for each seat category. These variables aim to capture the tradeoff behavior between cheap seats with low visibility and/or acoustics and more expensive high quality seats. Moreover, in order to address heterogeneity, we allow the price sensitivity and the marginal utility of the seating areas to vary across customer categories. The price coefficient interacts also with variables related to the performance characteristics.
The utility of a customer that buys a ticket which refers to the seating area s, for the performance i, can be formulated as:

$$U_{sj} = V_{sj} + \epsilon_{sj} \tag{3}$$

with

$$V_{sj} = p_{sj} \cdot (\beta_1 + \beta_2 \cdot young + \beta_3 \cdot sub + \gamma' z) + seat_s \cdot (\delta_1 + \delta_2 \cdot young + \delta_3 \cdot sub) \tag{4}$$

where *young* and *sub* are dummy variables denoting whether the customer is respectively a young customer or a subscriber. This implies that single ticket buyers are treated as the base category. z is a vector of performance and production characteristics. In our estimation, such characteristics are represented by the dummy variables SUNDAY and WEEKEND, already defined in the demand function. Moreover, we used the number of times the production is performed worldwide during the same year, in order to define three dummy variables that denote the degree of popularity of the production: $LowPopularity$ (for productions run less than 50 times worldwide) treated as base variable; $MediumPopularity$ (for productions run between 50 and 150 times worldwide) and $HighPopularity$ (for productions run more than 150 times worldwide). Finally $seat_s$ is a dummy variable denoting whether the seat belongs to area s or not. $Seat1$ is used as baseline in order to guarantee identification of the model.

Assuming that the error components in (2) are independent and identically distributed according to a Gumbel distribution, the probability of a customer belonging to category j purchasing a ticket of seating area s (among the 5 seating areas) is given by:

$$Pr(s \mid j) = \frac{exp[V_{sj}]}{\sum_{t=1}^{5} exp[V_{tj}]} \tag{5}$$

Estimation results for the MNL model are displayed in Table 4.

	Coefficient	*t-stat*
Price	**-0.00129******	−11.88
Price-Young	−0.0109****	-26.32
Price-Subscribers	0.00074****	3.87
Price-Popularity Medium	0.00005	−1.03
Price-Popularity high	0.00036****	−1.93
Price-Wkend	0.00004	0.91
Price-Sunday	0.00023****	5.55
Seat 2	**0.782******	28.93
Seat 2 - Young	0.427****	7.09
Seat 2 - Subscriber	1.51	22.84
Seat 3	**1.37******	34.83
Seat 3 - Young	0.474****	5.44
Seat 3 - Subscriber	1.56****	18.78
Seat 4	**1.87******	36.04
Seat 4 - Young	1.12****	9.81
Seat 4 - Subscriber	1.73****	16.92
Seat 5	**1.95******	29.37
Seat 5 - Young	1.56****	11.36
Seat 5 - Subscriber	1.86****	14.58
No. of observations		103322
ρ^2		0.102
Adjusted ρ^2		0.102
Null log-likelihood		- 166290.344
Final log-likelihood		- 149291.813

$^{****}p < .001$

Table 4: Estimation of multinomial logit model

As expected, young customers are highly price sensitive, followed by standard-ticket

buyers and subscribers. In addition, the price coefficient increases significantly when we consider popular productions as well as, surprisingly, performances that take place on Sunday.
With regard to the seat quality, the coefficients reflect an expected pattern: keeping the price fixed, an increase of the quality of the seat leads to a greater utility. This pattern holds for all the customer categories considered. Contrary to Baldin and Bille (2016), we can not compare the marginal utility of the seat categories across customers categories because each category has its own price coefficient. However, in terms of willingness to pay (WTP), i.e the ratio between the coefficient of the attribute and the price coefficient, it results that this value is greater for subscribers, followed by standard-ticket buyers and young customers.

6 Optimization model

The optimization model we propose considers the two objectives of the theatre, i.e., to maximize revenue and attendance, in a constrained bi-objective maximization framework. It incorporates both the demand function and the customers' seat choices described in the previous section. The decision variables are the prices p_{sj}, for each seating area s and each customer category j. As these prices affect the demand and the customers' seat choice (as described by (formula...-.)), the optimal prices determine the optimal splitting into fare classes of the seats in the theatre.
The expected revenue and attendance can be written as, respectively,

$$Revenue = \sum_{j=1}^{3} D_j(p_j) \cdot \left[\sum_{s=1}^{5} Pr(s \mid j) \cdot p_{sj} \right] \tag{6}$$

and

$$Attendance = \sum_{j=1}^{3} D_j(p_j) \cdot \left[\sum_{s=1}^{5} Pr(s \mid j) \right], \tag{7}$$

where D_j is the number of tickets sold to category j, defined by the estimated demand function (2); p_j is the average price for a customer belonging to category j; $Pr(s \mid j)$ is the probability of buying a ticket of seating area s, given the customer category j, for the considered performance, as defined by (5). The maximum number of seats that can be sold is bounded by the capacity of the theatre C:

$$\sum_{j=1}^{3} D_j(p_j) \cdot \left[\sum_{s=1}^{5} Pr(s \mid j) \right] \leq C \ . \tag{8}$$

Moreover, we have to consider a set of constraints that are required by the theatre policy:

$$p_{(s-1)j} < p_{sj} < p_{(s+1)j}, \quad \text{for each } j \text{ and } s \tag{9}$$

and

$$p_{s(j-1)} < p_{sj} < p_{s(j+1)}, \quad \text{for each } j \text{ and } s. \tag{10}$$

As seen in Section 4.1, both the ticket price for a young customer and for a subscribers are obtained discounting the standard ticket price, given a seating area s.
However, we allow for a more flexible relationship:

$$0.4 \cdot p_{standardticket} < p_{young} < 0.6 \cdot p_{standardticket}, \tag{11}$$

$$0.7 \cdot p_{standardticket} < p_{subscriber} < 0.9 \cdot p_{standardticket}. \tag{12}$$

Finally, we have the constraint that defines the relation between p_{sj} and p_j

$$p_j = \frac{1}{5} \sum_{s=1}^{5} p_{sj}. \tag{13}$$

7 Optimization results

The bi-objective optimization model we solved consists in maximizing the two objectives, *Revenue* and *Attendance*, under the above defined constraints: the solution of such a problem is the set of Pareto optimal points, the so-called Pareto frontier of the problem. We observe that we are facing a nonlinear bi-objective problem, due to the exponential term both in the demand function and in the formulation of the probability in the multinomial logit model. As usual in multi-objective optimization, in particular in the non-linear case, it is convenient to look for some points of the Pareto frontier; those points should be interesting from the point of view of the decision maker, in our case the direction of the Theatre.
We solved the problem by means of the Synchronous Approach adopted by Miettinen and Mäkelä (2006). Their model, called NIMBUS (Nondifferentiable Interactive Multiobjective BUndle-based optimization System), allows to deal with nondifferentiable and nonconvex multiobjective optimisation problems. The approach is based on the interaction between the decision maker and the solution algorithm and is realized via the Internet based system WWW-NIMBUS (https://wwwnimbus.it.jyu.fi). The single steps of the solution approach consist in the solution of single objective

(sub)problems via classical subgradients methods (see, e.g., Clarke, 1983). Successive single optimisation subproblems are then solved under the guidance of the decision maker: each successive solution is a Pareto optimal solution of the multi-objective problem. At each iteration the decision maker can indicate the preferred way to navigate the set of Pareto optimal solutions, choosing the objectives which value should be improved and, at the same time, which objectives should pay the cost of such improvement. This way the most appropriate solutions from the decision maker point of view are selected from the Pareto optimal solutions set. The software is free for the academic community and is operated directly on an Internet site, requiring neither the download of any software nor huge computing capabilities of the client computer.

As case studies we consider three performances that differ each other by characteristics that affect the demand, in order to verify how different levels of theater occupancy require different pricing and allocation policies, in particular considering the peak-load pricing issue (i.e differentiating prices charged depending on peak and off-peak periods). For purpose of better comparison between the actual pricing and allocation policies and those resulting from the optimization model, we have choosen three performances that show a fitted value of the demand very close to the real demand. The first performance is a high demand performance, namely a Saturday evening performance of *La Tosca* that fills up to 91.05% capacity . The second performance analyzed is a low demand performance, *Djævlene fra Loudun*, run in a weekday: this performance fills less than half of the total capacity (41.98%). The third performance is a medium-popular production, namely *Rusalka*, with 67.86% of the total capacity filled.

We are therefore able to compare the price[4] and seat allocation results of the bi-objective optimization model with the actual results and those resulting from two single objective optimization models: one that maximizes only the Revenue and one that maximizes only the total Attendance (see Tables 5-7).

Some remarks about the implementation of the models: first, for a realistic comparison with the actual data we have considered three performances whose fitted value of the demand is very close to the actual value. Second, for the purpose of realism we have imposed a lower and an upper bound to the 15 decision variables, equal respectively to the half and twice value of the actual price. Third, we subtract from the value of the capacity C the number of tickets sold to other categories not considered, which is considered as already known by the theatre manager.

Table 5 considers the results obtained for a Saturday evening performance of *La*

[4]Price are expressed in Danish crown (DKK):$1 DKK \approx 0.13$€

Tosca. It is a high-demand event almost (but not completely) sold-out.

Seat	Actual		Bi-objective		Revenue max.		Attendance max.	
	Price	no. of seats	Price	no. of seats	Price	no. of seats	Price	no. of seats
Seat1-standard	160	46	300	59	320	67	176	60
Seat2-standard	345	79	341	123	690	105	399	108
Seat3-standard	525	144	625	173	1050	137	408	193
Seat4-standard	720	366	963	211	1440	160	536	285
Seat5-standard	895	258	1360	160	1710	136	565	301
Seat1-young	80	13	134	11	160	16	74	18
Seat2-young	173	23	170	24	299	10	172	19
Seat3-young	263	14	252	17	420	5	226	19
Seat4-young	360	14	386	11	576	2	294	27
Seat5-young	448	8	544	3	684	1	311	37
Seat1-subscribers	140	7	226	1	224	1	153	1
Seat2-subscribers	302	13	293	10	604	9	319	10
Seat3-subscribers	459	24	540	17	918	17	349	18
Seat4-subscribers	630	18	842	33	1260	31	428	35
Seat5-subscribers	783	42	1215	38	1539	37	497	43
Total	682 118	1069	691 167	891	826 714	733	529 468	1174
% improve (Revenue and attendance)			+1.33	-16.65	+21.20	-31.43	-22.38	+9.83

Table 5: Revenue and attendance comparison. Case study: *La Tosca*

The bi-objective optimization model solutions shown in Table 5 (as well as all the other solutions shown in Table 6 and 7) represent one of the points of the Pareto frontier. Hence there are other possible solutions. Figure 2 shows some alternative solutions of the bi-objective model. The solution proposed in Table 5 leads to an increment in revenue of 1.33% and a decrease of the total attendance of 16.65%.

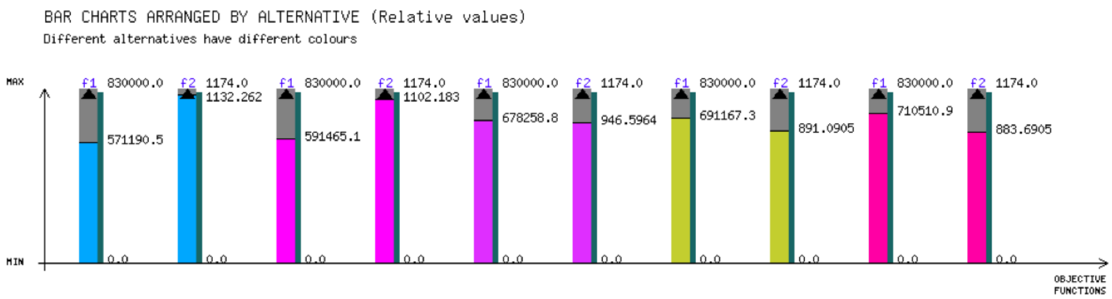

Figure 2: Some alternative optimal values of the bi-objective model

From Figure 2 it is evident the existence of a trade-off among the two objectives: an increase in revenue is associated with a lower value of the attendance and viceversa. It is interesting to observe how price and seat allocation change according to the orientation of the theatre manager towards the two objectives. From Table 5 we can deduce that when the objective is the maximization of the revenue, the theatre exploits the inelasticity that characterizes subscribers and standard tickets by increasing the price to the upper bound. As young customers are price sensitive, their price increases until the loss of young customers is not balanced by a higher revenue per seat. In the attendance maximization perspective, since the performance almost reaches the capacity constraint, the objective is achieved by lowering only the prices of the most expensive seat category.

In relation to the allocation policy, we notice that when the theater is "attendance maximizer" customers are more likely to shift to a higher seat quality (buy-up behaviour) as a consequence of a generalized price reduction. Viceversa, if the theater is "revenue maximizer" customers are more likely to buy a ticket for a cheap seat because they are not willing to pay more. This behaviour is evident when we refer to price sensitive customers. On the contrary, price insensitive customers are not influenced by the theatre policy in their choice of the seating area, which is confirmed in Figures 3 and 4 - respectively for young customers and subscriptions - showing how the probability of buying a ticket of certain seating area changes according to the theatre policy. Thus, the optimal pricing and allocation policy depends on the type of customers the theatre is expected to accomodate.

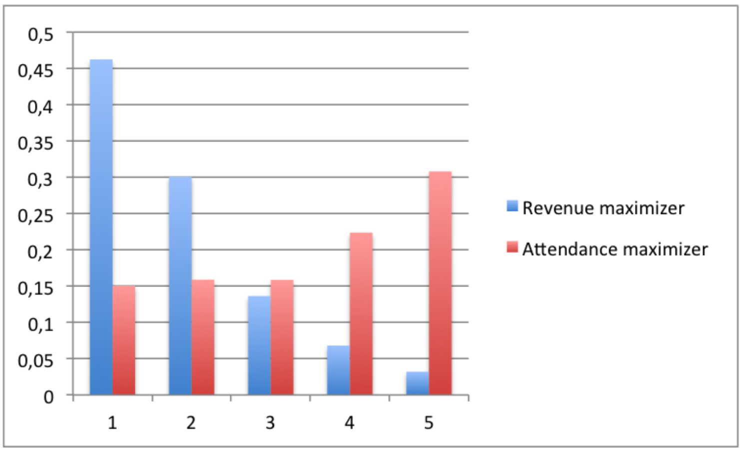

Figure 3: Young customers' choice probabilities

Table 6 considers the results obtained for a weekday performance of *Djævlene fra Loudun*. It is a low-demand event in which the theater is usually occupied approximately only a little bit more than a third of its capacity. One solution obtained solving the bi-objective model allows an increase in attendance of 13.26% and a decrease in revenue of 12.25%. Compared to the previous case, here the theater is forced to reduce prices to the lower bound when it aims to maximize attendance.

Concerning the allocation policy, the pattern previously described is even more evident as the price coefficient of the MNL model decreases when a no popular event is considered. Hence, if the theatre is attendance (revenue) maximizer, it is suggested to increase(decrease) the size of the most expensive (cheapest) seating area, especially when it is expected to attract a price-insensitive audience group (as yung

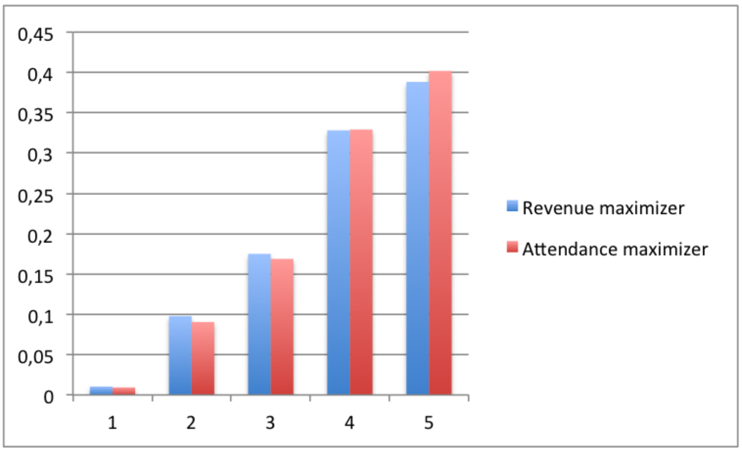

Figure 4: Subscribers' choice probabilities

Seat	Actual		Bi-objective		Revenue max.		Attendance max.	
	Price	no. of seats	Price	no. of seats	Price	no. of seats	Price	no. of seats
Seat1-standard	160	41	110	12	320	8	80	13
Seat2-standard	295	23	216	24	590	20	148	26
Seat3-standard	425	29	366	36	850	26	213	44
Seat4-standard	545	50	505	50	1090	31	307	66
Seat5-standard	695	42	560	50	1352	24	348	69
Seat1-young	80	16	47	9	160	6	40	9
Seat2-young	148	11	91	17	277	4	74	21
Seat3-young	213	1	148	16	340	4	107	26
Seat4-young	273	9	202	27	436	4	137	58
Seat5-young	348	3	226	33	541	2	174	62
Seat1-subscribers	140	3	79	2	224	3	70	2
Seat2-subscribers	258	11	162	23	516	23	129	22
Seat3-subscribers	372	13	287	41	744	38	186	41
Seat4-subscribers	551	86	443	74	981	67	276	79
Seat5-subscribers	608	107	503	88	1216	72	304	97
Total	209 268	445	183 420	504	302 384	331	144 313	637
% improve (Revenue and attendance)			-12.35	+13.26	+44.50	-25.61	-31.04	+43.15

Table 6: Revenue and attendance comparison. Case study: *Djævlene fra Loudun*

customers).

Table 7 considers the results obtained for a Sunday performance of *Rusalka*. This is an intermediate case compared to the previous two. This case is interesting as the bi-optimization model provides a solution that dominates the current value of the objectives. Indeed, the solution proposed allows an increase in revenue of 1.90% and, at the same time, an increase in attendance of 1.83%.

8 Conclusions

This paper has proposed a model that simultaneously optimizes the pricing and seating-allocation policy of a theater. In particular, we present a bi-objective optimization model that integrates the demand forecast and a choice model, where the

Seat	Actual		Bi-objective		Revenue max.		Attendance max.	
	Price	no. of seats	Price	no. of seats	Price	no. of seats	Price	no. of seats
Seat1-standard	160	43	172	35	320	40	84	39
Seat2-standard	345	70	292	68	692	59	204	75
Seat3-standard	525	72	619	88	1052	74	315	120
Seat4-standard	720	150	740	129	1440	83	408	181
Seat5-standard	895	109	782	134	1743	66	483	182
Seat1-young	80	6	79	8	160	7	45	9
Seat2-young	173	18	127	15	301	4	92	17
Seat3-young	263	7	249	6	423	2	183	11
Seat4-young	360	8	296	12	580	1	184	35
Seat5-young	448	2	420	4	702	0	224	36
Seat1-subscribers	140	19	149	5	224	5	71	5
Seat2-subscribers	302	27	259	44	622	43	151	45
Seat3-subscribers	459	35	542	76	920	74	266	82
Seat4-subscribers	630	163	639	146	1260	134	322	160
Seat5-subscribers	783	175	703	177	1568	152	397	194
Total	550 190	929	560 664	946	874 000	745	394 026	1191
% improve (Revenue and attendance)			+1.90	+1.83	+58.85	-19.91	-28.38	+28.20

Table 7: Revenue and attendance comparison. Case study: *Rusalka*

customer chooses one among different seating areas which differ each other in price and quality. The multi-objective nature of our model reflects the multi-dimensional nature of nonprofit performing arts organizations. In our case, the objectives we assume to be maximized are revenue and attendance. The approach adopted allows also to take into account heterogeneity in price sensitivity and choice behaviour across different customer categories. The proposed model is applied to booking data provided by the Royal Danish Theater referring to the period 2010-2015. More precisely we consider three different performances in order to explore the potentialities of the model.

From a management perspective, the model can provide to theatre managers insightful policy implications in terms of demand-management decision. The results obtained confirm the existence of a trade-off between the two theater objectives. When the theater is revenue maximizer, prices charged to price insensitive customers are raised, and the cheapest seating area is enlarged to prevent a loss of revenue. Viceversa, when the theater is audience maximizer, prices are set at lower level, in particular the ones associated to the most expensive seating area. As a consequence, it is recommended to increase the number of seats allocated to the most expensive area, in order to encourage a shift of consumer choices to higher quality seats. The allocation policy just described is particularly effective when a performance is expected to attract customers with an elastic demand, since these customers are more sensitive to price changes in their seat choice; and also when the performance will probably not attract a large audience. Moreover, in one case the bi-objective model provides a solution that gives an improvement in both revenue

and attendance from the current situation.
Overall, our examples clarify that both price and capacity allocation are leverages with which a theatre can calibrate its objectives, even when revenue is not considered as the main goal to pursue.

References

- Ainslie, A., Dreze, X., Zufryden, F. (2005). Modeling movie lifecycles and market share. *Marketing Science*, 24, 508-517.

- Andrews, R. L., Currim, I. S., Leeflang, P., Lim, J. (2008). Estimating the Scan Pro model of store sales: HB, FM or just OLS? *International Journal of Research in Marketing*, 25, 22-33.

- Baldin, A., Bille, T. (2016). Modelling preference heterogeneity for theater tickets: a discrete choice modelling approach on Royal Danish Theater booking data., *Paper presented at the 19th International Conference on CUltural Economics (ACEI 2016), Valladolid (Spain).*

- Baumol, W. J., Bowen, W.G. (1966). *Performing Arts: The Economics Dilemma*, Twentieth Century Fund, New York.

- Bille Hansen, T. (1991). *Det Kgl. Teater - et kulturøkonomisk studie*, AKF Forlaget, Copenhagen.

- Brooks, A.C. (2006). Nonprofit firms in the performing arts, *Handbook of the Economics of Art and Culture*, 1, 473-506.

- Clarke, F.H. (1983). *Optimization and nonsmooth analysis*, Wiley, New York.

- Corning, J., Levy, A. (2002). Demand for live theater with market segmentation and seasonality. *Journal of Cultural Economics*, 26, 217-235.

- Eliashberg, J., Hegie, Q.,Ho, J., Huisman, D., Miller, S. J., Swami, S., Weinberg, C.B., Wierenga, B. (2009). Demand-driven scheduling of movies in a multiplex. *International Journal of Research in Marketing*, 26, 75-88.

- Felton, M.V. (1994). Evidence of the existence of the cost disease in the performing arts. *Journal of Cultural Economics*, 18, 301-312.

- Gapinski, J.H. (1985). Do the nonprofit performing arts optimize? The moral form Shakespeare, *Quarterly Review of Economics and Business*, 25, 27-37.

- Hansmann, H. (1980): The role of nonprofit enterprise, *Yale Law Journal*,89, 835-898

- Hansmann, H. (1981). Nonprofit enterprise in the performing arts.*The Bell Journal of Economics*, 341-361.

- Hetrakul, P., Cirillo, C. (2014). A latent class choice based model system for railway optimal pricing and seat allocation. *Transportation Research Part E: Logistics and Transportation Review*, 61, 68-83.

- Huntington, P. A. (1993). Ticket pricing policy and box office revenue *Journal of Cultural Economics*, 17, 71-87.

- Laamanen, J.P. (2013). Estimating demand for opera using sales system data: the case of Finnish National Opera. *Journal of Cultural Economics* 37, 417-432.

- Leslie, P. (2004). Price discrimination in Broadway theater. *RAND Journal of Economics*, 520-541.

- Luksetich, W.A., Lange, W.A. (1995). A simultaneous model of nonprofit symphony, *Journal of Cultural Economcs*, 19, 49-68

- Marco-Serrano, F. (2006). Monitoring managerial efficiency in the performing arts: A regional theatres network perspective. *Annals of Operations Research*, 145, 167-181.

- Miettinen, K., Mäkelä, M.M. (2006). Synchronous Approach in Interactive Multiobjective Optimization. *European Journal of Operational Research*,170, 909-922

- Rosen, S. (1974). Hedonic prices and implicit markets: product differentiation in pure competition. *The journal of political economy* 82, 34-55.

- Rosen, S., Rosenfield, A. M. (1997). Ticket Pricing 1 *The Journal of Law and Economics*. 40, 351-376.

- Seaman, B. A. (1985). Price discrimination in the arts. *Managerial Economics for the Arts. Association for Cultural Economics, University of Akron*, 47-60.

- Seaman, B. A. (2006). Empirical studies of demand for the performing arts. *Handbook of the economics of art and culture*, 1, 415-472.

- Steinberg, R. (1986). The revealed objective functions of nonprofit firms, *Rand Journal of Economics*,17, 508-526.

- Talluri, K., Van Ryzin, G. (2004). Revenue management under a general discrete choice model of consumer behavior. *Management Science*, 50(1), 15-33.

- Tereyagoglu, N., Fader, P., Veeraraghavan, S. (2012). Filling seats at a theater: Estimating the impact of posted prices and dynamic discounts *Working paper, University of Pennsylvania, Philadelphia.*

- Throsby, D. (1994). The production and consumption of the arts: A view of cultural economics, *Journal of Economics Literature*, 32, 1-29

- Throsby, D., Withers, G.A. (1979). *The Economics of the Performing Arts*, St. Martin's Press, New York.