Value of Information in Competitive Economies with Incomplete Markets*

by

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Abstract

We study the value of public information in competitive economies with incomplete asset markets. We show that generically the welfare effect of a change in the information available prior to trading can be in any direction: there exist changes in information that make all agents better off, and changes for which all agents are worse off. In contrast, for any change in information, a Pareto improvement is feasible, i.e. attainable by a planner facing the same informational and asset market constraints as agents. In this sense, the response of competitive markets to changes in information is typically not socially optimal.

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1 Introduction

The objective of this paper is to analyze the value of public information in the setup of a competitive exchange economy under uncertainty in which agents trade in asset markets to reallocate risk. It is well-known that when markets are complete, and agents are initially uninformed, the receipt of a public signal prior to trading impairs risk sharing and cannot improve welfare. Indeed, if the true state of the world is revealed before markets open, no mutually beneficial risk sharing trade is possible. This effect has come to be known as the *Hirshleifer effect*, after Hirshleifer (1971) who produced an early example of it. If markets are incomplete, the arrival of information still reduces the possibilities of trading gains from risk sharing, but a second welfare effect arises. Provided some residual uncertainty remains, agents can hedge it more effectively by conditioning their portfolios on the available information. We refer to this as the *Blackwell effect*, after Blackwell (1951) who compared the value of different information structures in single-agent decision problems.

We begin our analysis with an example of the welfare impact of the arrival of information before markets open in which the Hirshleifer and Blackwell effects can be clearly disentangled. We show that, depending on the nature of this information, one or the other effect dominates so that in equilibrium welfare can worsen or improve. In the rest of the paper we extend the analysis to a general two-period exchange economy with a single consumption good and a single round of asset trade wherein we study the welfare effects of arbitrary changes in information, starting from a situation where agents may have some initial information, changes that do not necessarily entail an increase or decrease in the information available to agents prior to trading. While in our general analysis the welfare impact of a change in information cannot be interpreted in terms of the Hirshleifer and Blackwell effects, it is nevertheless possible to provide an analogous interpretation that is helpful in understanding how competitive markets deal with changes in information.

For this purpose it is instructive to compare the change in agents' welfare in equilibrium to welfare changes that can be achieved by a hypothetical planner who faces the same constraints as agents in terms of the available assets and information. We show that for a generic economy a (strict) Pareto improvement can be attained, for any change in information, by tailoring agents' portfolios to the new information. When we compare competitive equilibria associated with different information structures, an additional welfare effect obtains because of adjustments in equilibrium asset prices. We find that the overall effect on welfare may be positive or negative. More precisely, we show that generically, if markets are sufficiently incomplete, there exists a change in information that makes all agents better off as well as a change that makes all agents worse off; indeed, there is a change in information such that any subset of agents is better (or worse) off.

Thus agents' welfare can vary in any direction, depending on the change in information, in line with our findings in the example. Moreover, we can interpret the welfare effect of a change in information as having a positive component due to agents being able to tailor their portfolios to the available information, and a second component that may be positive or negative for any individual agent arising from the adjustment in equilibrium asset prices induced by the change in information. This is in the same spirit as the decomposition of the overall welfare effect in the example into the Blackwell and Hirshleifer effects.[1] Our analysis provides an answer to the question of how competitive markets respond to changes in information, and how equilibrium changes in welfare compare with welfare changes that are attainable. Our results lead to the conclusion that competitive markets typically do not deal with changes in information in a way that is welfare-improving even though it is feasible to do so.

There is an extensive literature on the value of information in a competitive pure exchange setting. A number of papers have followed Hirshleifer's lead in comparing competitive equilibrium allocations before and after the arrival of information. Assuming complete markets, Schlee (2001) derives some conditions under which more information is Pareto worsening.[2] In an economy with incomplete markets, Green (1981) and Hakansson et al. (1982) provide (quite restrictive) conditions under which better information leads to a Pareto improvement, while Milne and Shefrin (1987) present an example showing that welfare can also decrease. In contrast to the various examples in the literature, the example in the present paper allows a clear understanding of the welfare impact of the arrival of information by explicitly decomposing the overall change in welfare into the Hirshleifer and Blackwell effects.

In our general welfare analysis we utilize the analytical apparatus developed by Geanakoplos and Polemarchakis (1986), and later generalized by Citanna et al. (1998), to study the welfare effects of local perturbations in competitive economies with incomplete markets. These methods have been employed in particular by Cass and Citanna (1998) and Elul (1995) to show that generically the welfare effect of the introduction of a new asset can be in any direction. We provide an analogous result on the effect of a change in public information prior to trading.[3] While there are some parallels between these two problems, they are analytically quite distinct. The introduction of a new asset enlarges the set of feasible allocations, while this is not

[1] Unlike the Blackwell effect, which we can properly speak of only for an increase in information, the first (positive) component is present for any change in information. The second component bears a closer relationship to the classical Hirshleifer effect. Indeed, Gottardi and Rahi (2001) argue that the Hirshleifer effect is due to changes in equilibrium prices, induced by a change in information, that alter agents' budget sets. See footnote 13 for further discussion in the context of this example.

[2] For the case of complete markets, an alternative to the welfare analysis of competitive equilibria is provided by Campbell (2004), who compares welfare at allocations associated with different information structures when these allocations have to be feasible and, in addition, satisfy ex-post individual rationality constraints. He shows that any increase of information in the sense of Blackwell (1951) has a negative effect on agents' welfare in the sense that each allocation associated with the better information is ex-ante Pareto dominated by some allocation associated with the worse information.

[3] We are able to exploit differential techniques by employing a smooth parametrization of changes in information, just as Cass and Citanna (1998) and Elul (1995) are able to use these techniques by modeling the introduction of an asset in a way that avoids discontinuities.

in general true for a change in information.[4] Furthermore, the price changes that underpin the welfare effects are different. In our single-good setting, only the current asset prices are affected by a change in information. In contrast, the results of Cass and Citanna (1998) and Elul (1995) also depend on adjustments in the future relative prices of physical commodities that modify the set of transfers of state-contingent consumption achievable through asset trading.[5]

The aforementioned techniques have also been used by Citanna and Villanacci (2000) to study the effect on welfare of the information revealed by prices in an asymmetric information economy with multiple physical goods and nominal assets. In their model, there is a continuum of equilibria which can be parametrized by the future (state-contingent) price level. The authors compare a non-revealing equilibrium to a nearby equilibrium of the same economy at which asset prices reveal some information, and show that generically there is an arbitrary change in welfare when going from one equilibrium to the other, with the utility of any individual agent going up or down. In contrast, we consider an exogenous change in public information, and show that the welfare change is arbitrary when we move from any equilibrium of a generic economy to a nearby equilibrium associated with the new information structure. Moreover, in our model with a single physical good and real assets, the welfare change can be attributed entirely to the change in information under consideration; it arises from agents being able to condition their portfolios on the new information and from the adjustment in equilibrium asset prices resulting from the change in information. In the analysis of Citanna and Villanacci (2000), on the other hand, welfare effects cannot be traced solely to the change in information, which is endogenous, being the information revealed by asset prices. In their welfare results, an important role is played by two additional price effects which do not arise in our model: adjustments in relative spot commodity prices in the future, and changes in real asset payoffs induced by changes in the future price level. The latter effect is unrelated to the change in information.

The paper is organized as follows. We describe the economy in Section 2. In Section 3 we provide an example to illustrate the welfare effects of changes in information. The remainder of the paper is devoted to our formal results, on attainable welfare changes in Section 4, and on equilibrium changes in welfare in Section 5. Proofs are collected in the Appendix.

2 The Economy

There are two periods, 0 and 1, and a single physical consumption good. The economy is populated by $H \geq 2$ agents, with typical agent $h \in H$ (here, and elsewhere,

[4]We elaborate on this point in footnote 12.

[5]In a single-commodity environment, Elul (1999) shows that it is generically possible to introduce an asset in order to make all agents better off (while it can never be the case that all agents are worse off). The argument relies on identifying an asset whose introduction leaves the prices of all existing assets unchanged. Thus price effects play no role in this result.

we use the same symbol for a set and its cardinality). Uncertainty, which is resolved at date 1, is described by S states of the world.

Agent $h \in H$ has endowments $\omega_0^h > 0$ in period 0 and $\omega^h \in \mathbb{R}^S_{++}$ in period 1. He has time-separable expected utility preferences with von Neumann-Morgenstern utility functions $u_0^h : \mathbb{R}_{++} \to \mathbb{R}$ for period 0 consumption and $u^h : \mathbb{R}_{++} \to \mathbb{R}$ for period 1 consumption. We assume that u_0^h and u^h satisfy the standard smooth preference assumptions, i.e. they belong to the set $\mathcal{U} := \{\phi : \mathbb{R}_{++} \to \mathbb{R} \,|\, \phi \in \mathcal{C}^2, \phi' > 0, \phi'' < 0, \lim_{x\to 0} \phi'(x) = \infty\}$.

Asset markets, in which $J \geq 2$ securities are traded, open at date 0. At date 1 assets pay off. The payoff of asset j in state s (in terms of the physical consumption good) is denoted by r_s^j, and the vector of asset payoffs in state s by $r_s \in \mathbb{R}^J$. By default all vectors are column vectors, unless transposed. Thus $r_s^\top = (r_s^1 \dots r_s^J)$. Let R be the $S \times J$ matrix whose s'th row is $r_s^\top$. We assume, without loss of generality, that R has full column rank J. Markets are complete if $J = S$, and incomplete if $J < S$.

Prior to trading, agents observe a public signal correlated with the state of the world s. This signal does not directly affect utility functions, endowments or asset payoffs. We fix a finite set of "signal realizations" Σ, $\#\Sigma \geq 2$, with a typical element of Σ denoted by σ. The uncertainty over fundamentals, parametrized by $s \in S$, and the public signal, parametrized by $\sigma \in \Sigma$, can then be jointly described by a probability measure on $S \times \Sigma$, i.e. by the probabilities $\pi := \{\pi_{s\sigma}\}_{s\in S, \sigma\in\Sigma} \in \mathbb{R}^{S\Sigma}_+$, where $\pi_{s\sigma}$ denotes $\text{Prob}(s, \sigma)$. Let $\pi_{s|\sigma} := \text{Prob}(s|\sigma)$, $\pi_s := \text{Prob}(s)$, and $\pi_\sigma := \text{Prob}(\sigma)$.[6] The uncertainty over fundamentals is thus given by the marginal distribution over S, $\{\pi_s\}_{s\in S}$.

We refer to the vector π as an *information structure*.[7] More precisely, an information structure lies in the set[8]

$$\Pi := \left\{ \pi \in \mathbb{R}^{S\Sigma}_+ \,\middle|\, \pi_s > 0, \forall s \in S;\ \pi_\sigma > 0, \forall \sigma \in \Sigma;\ \sum_{s,\sigma} \pi_{s\sigma} = 1 \right\}.$$

This specification admits a range of possible information structures. It includes information structures that have full support over S for every σ, i.e. $S_\pi(\sigma) := \{s \in S \,|\, \pi_{s\sigma} > 0\} = S$ for all σ. The set of these information structures is the interior of Π, given by $\Pi^0 := \{\pi \in \mathbb{R}^{S\Sigma}_{++} \,|\, \sum_{s\sigma} \pi_{s\sigma} = 1\}$. A subset of Π^0 consists of information structures satisfying the independence condition $\pi_{s\sigma} = \pi_s \pi_\sigma$, for all s, σ, in which case the public signal provides no information about s; we refer to such information structures as uninformative. The set Π also includes information structures

[6]Note that π_s and π_σ are scalars.

[7]This is a slight abuse of terminology since π represents both the uncertainty over fundamentals and the information of agents. Describing information in terms of the joint distribution $\{\pi_{s\sigma}\}_{s\in S,\sigma\in\Sigma}$ is clearly equivalent to using the conditional distributions $\{\pi_{s|\sigma}\}_{s\in S}$ and the marginal distribution $\{\pi_\sigma\}_{\sigma\in\Sigma}$, with $\pi_{s\sigma} = \pi_{s|\sigma}\pi_\sigma$. But working directly with the joint distribution, captured by the single vector π, is notationally more convenient.

[8]The restriction on the marginal distributions over S and Σ is without loss of generality.

for which the support $S_\pi(\sigma)$ is a strict subset of S for some signal realizations. A special case of the latter is one where the signal induces a partition over S.

We take as given the date 0 endowment ω_0^h and preferences u_0^h of all agents $h \in H$, and parametrize economies by agents' date 1 endowments and preferences, and the information structure. Let $\omega := \{\omega^h\}_{h\in H} \in \Omega := \mathbb{R}^{SH}_{++}$, and $u := \{u^h\}_{h\in H} \in \mathcal{U}^H$. An economy is then a tuple $(\omega, u, \pi) \in \mathcal{E} := \Omega \times \mathcal{U}^H \times \Pi$, specifying the period 1 endowments and utility functions of agents and the information structure. We formalize our notion of genericity as follows. The sets Ω and Π are endowed with the usual (Euclidean) topology. The set $\mathcal{U}$ is endowed with the $\mathcal{C}^2$ uniform convergence topology on compact sets, i.e. the sequence u_n^h in $\mathcal{U}$ converges to u^h if and only if $u_n^h, u_n^{h\prime}$ and $u_n^{h\prime\prime}$ converge uniformly to $u^h, u^{h\prime}$ and $u^{h\prime\prime}$ respectively, on any compact subset of $\mathbb{R}_{++}$. The set of economies $\mathcal{E}$ is endowed with the product topology. By "generic subset of Ω," we mean "for an open, dense subset of Ω," and likewise for $\mathcal{U}^H$, Π and $\mathcal{E}$. By "generically" or "for a generic economy" we mean "for an economy in an open, dense subset of $\mathcal{E}$."

Consider an economy $(\omega, u, \pi) \in \mathcal{E}$. For signal realization σ, let $p_\sigma \in \mathbb{R}^J$ be the vector of asset prices (date 0 consumption serves as the numeraire), and $y_\sigma^h \in \mathbb{R}^J$ the portfolio of agent h. The consumption of agent h for signal σ is then given by $c_{0\sigma}^h := \omega_0^h - p_\sigma \cdot y_\sigma^h$ at date 0, and $c_{s\sigma}^h := \omega_s^h + r_s \cdot y_\sigma^h$ in state s at date 1. Let $y_\sigma := \{y_\sigma^h\}_{h\in H}$, $y := \{y_\sigma\}_{\sigma\in\Sigma}$, $c_0 := \{c_{0\sigma}^h\}_{h\in H, \sigma\in\Sigma}$, and $p := \{p_\sigma\}_{\sigma\in\Sigma}$. A competitive equilibrium is defined as follows:

Definition 1 *Given an economy $(\omega, u, \pi) \in \mathcal{E}$, a competitive equilibrium consists of a portfolio allocation y, and prices p, satisfying the following two conditions:*

(a) *Agent optimization: $\forall h \in H$ and $\sigma \in \Sigma$, y_σ^h solves*

$$\max_{x\in\mathbb{R}^J} \Big(u_0^h \left[\omega_0^h - p_\sigma \cdot x\right] + \sum_s \pi_{s|\sigma}\, u^h \left[\omega_s^h + r_s \cdot x\right] \Big). \tag{1}$$

(b) *Market clearing: $\forall \sigma \in \Sigma$,*

$$\sum_h y_\sigma^h = 0. \tag{2}$$

We will often refer to an equilibrium (y, p) of the economy (ω, u, π) as a π-equilibrium in order to emphasize the dependence of the equilibrium on the information structure. Since portfolios uniquely determine period 1 consumption, we can write a π-equilibrium allocation as (c_0, y).

We wish to study the welfare consequences of changes in information. Given $\pi \in \Pi$, we consider the set of alternative information structures $\hat{\Pi}(\pi) \subset \Pi$ that represent purely a change in information relative to π without affecting the uncertainty over fundamentals. Formally, $\hat{\Pi}(\pi)$ is the set of vectors $\hat{\pi}$ in Π satisfying the following conditions:

a1. $\sum_{\sigma\in\Sigma} \hat{\pi}_{s\sigma} = \pi_s$, for all $s \in S$; and

a2. $\{\hat{\pi}_{s\sigma}\}_{s\in S}$ is not proportional to $\{\pi_{s\sigma}\}_{s\in S}$, for some $\sigma \in \Sigma$.

Condition **a1** says that $\hat{\pi}$ does not alter the marginal distribution over S implied by π, while condition **a2** says that there is an actual change in information.[9] If $\hat{\pi} \in \hat{\Pi}(\pi)$ we say that $(\pi, \hat{\pi})$ is an *admissible* pair of information structures.

We begin with an example to illustrate the welfare effects of changes in information.

3 An Example

Consider an economy with four equally likely states and two assets. The state space is $S = \{s_1, s_2, s_3, s_4\}$, and the asset payoff matrix is

$$R = \begin{pmatrix} 1+\delta & 0 \\ 0 & -1 \\ -1 & 0 \\ 0 & 1+\delta \end{pmatrix},$$

for some $\delta > 0$. There are two agents. For simplicity we assume that no consumption takes place at date 0 and agents have no endowment in that period. Both agents have the same quadratic utility function, $u(c) = c - \frac{a}{2}c^2$, for period 1 consumption,[10] where the parameter a is positive and chosen to be small enough to ensure that the marginal utility of consumption is positive in every state in equilibrium. Agents' endowments are given by the matrix

$$\begin{pmatrix} \overline{\omega} - \alpha + \beta & \overline{\omega} + \alpha - \beta \\ \overline{\omega} - \alpha - \beta & \overline{\omega} + \alpha + \beta \\ \overline{\omega} + \alpha - \beta & \overline{\omega} - \alpha + \beta \\ \overline{\omega} + \alpha + \beta & \overline{\omega} - \alpha - \beta \end{pmatrix},$$

the (s,h)'th element of which is the endowment in state s of agent h. The parameters α and β are positive, and $\overline{\omega} > \alpha + \beta$. Notice that there is no aggregate uncertainty in this economy. We can think of endowments as being subject to two kinds of shocks, the α-shock and the β-shock, which affect the agents in opposite directions.

The information of agents is described by a signal σ that can take values σ_1 or σ_2 with equal probability; thus $\Sigma = \{\sigma_1, \sigma_2\}$. We consider an initial situation in which agents have no information; the information structure is π, which is uninformative with $\pi_{s|\sigma} = \frac{1}{4}$, for all s, σ. We analyze the welfare effect of two alternative changes in information, to either $\hat{\pi}$ or $\hat{\pi}'$; in both cases the signal conveys information about

[9] As an example of a "change" in information that does not satisfy **a2**, suppose π is uninformative, $\hat{\pi}_{s|\sigma} = \pi_{s|\sigma} = \pi_s$ for all s, σ, and $\hat{\pi}_\sigma \neq \pi_\sigma$ for some σ. Clearly $\hat{\pi}$ is also uninformative.

[10] While for our general results we assume that utility functions lie in the set $\mathcal{U}$ specified in Section 2, for the purpose of the example it is convenient to use quadratic utility which is clearly not a member of this class.

the state of the world s, inducing a partition over S represented by the event trees in Figures 1 and 2 respectively. Thus $\hat{\pi}$ reveals the α-shock, while $\hat{\pi}'$ reveals the β-shock. In both cases, the two states in each cell of the partition are equally likely. Conditional probabilities are shown on each branch.

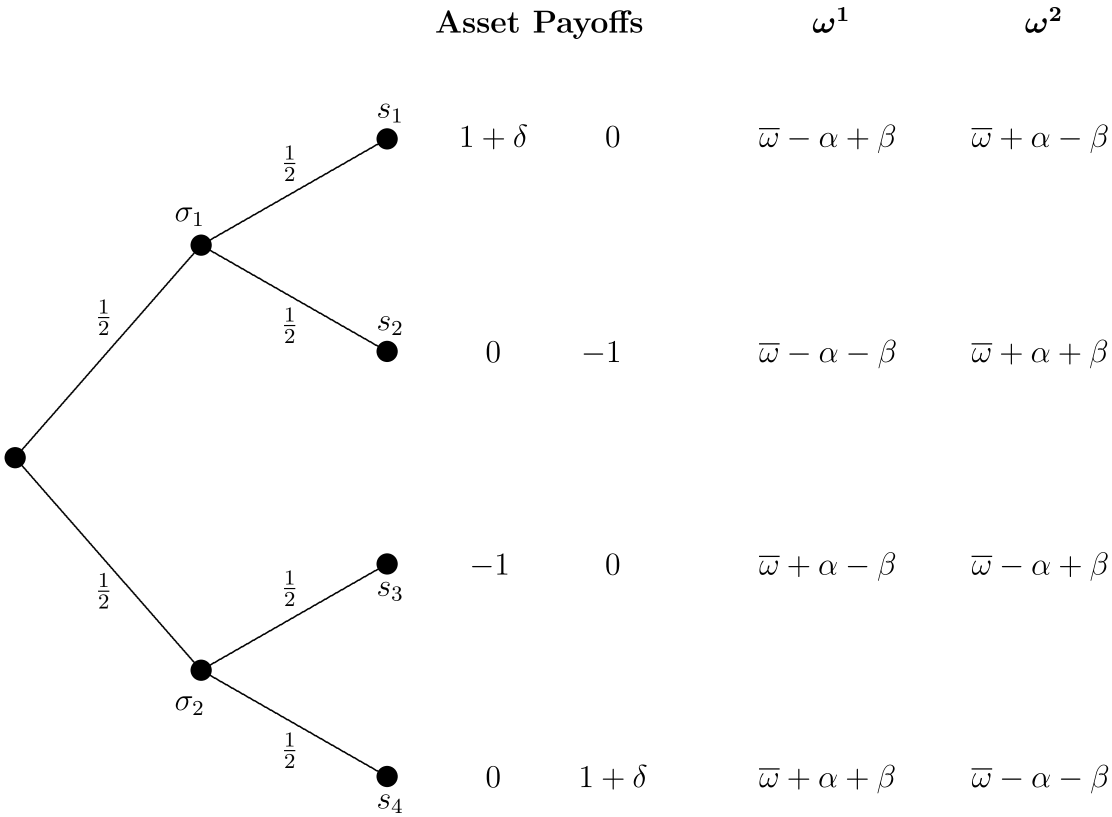

Figure 1: Agents are worse off under $\hat{\pi}$ (Hirshleifer dominates Blackwell)

As a benchmark it is useful to consider the first-best symmetric allocation in which both agents consume their ex-ante (i.e. evaluated prior to the receipt of the public signal) expected endowment $\overline{\omega}$, in all states.[11] The ex-ante expected utility of both agents is given by

$$\begin{aligned}\overline{U} &:= \overline{\omega} - \frac{a}{2}\overline{\omega}^2 \\ &= U_A + \frac{a}{2}(\alpha^2 + \beta^2),\end{aligned}$$

[11]This is the unique competitive equilibrium allocation of this economy if markets are complete and the signal is uninformative.

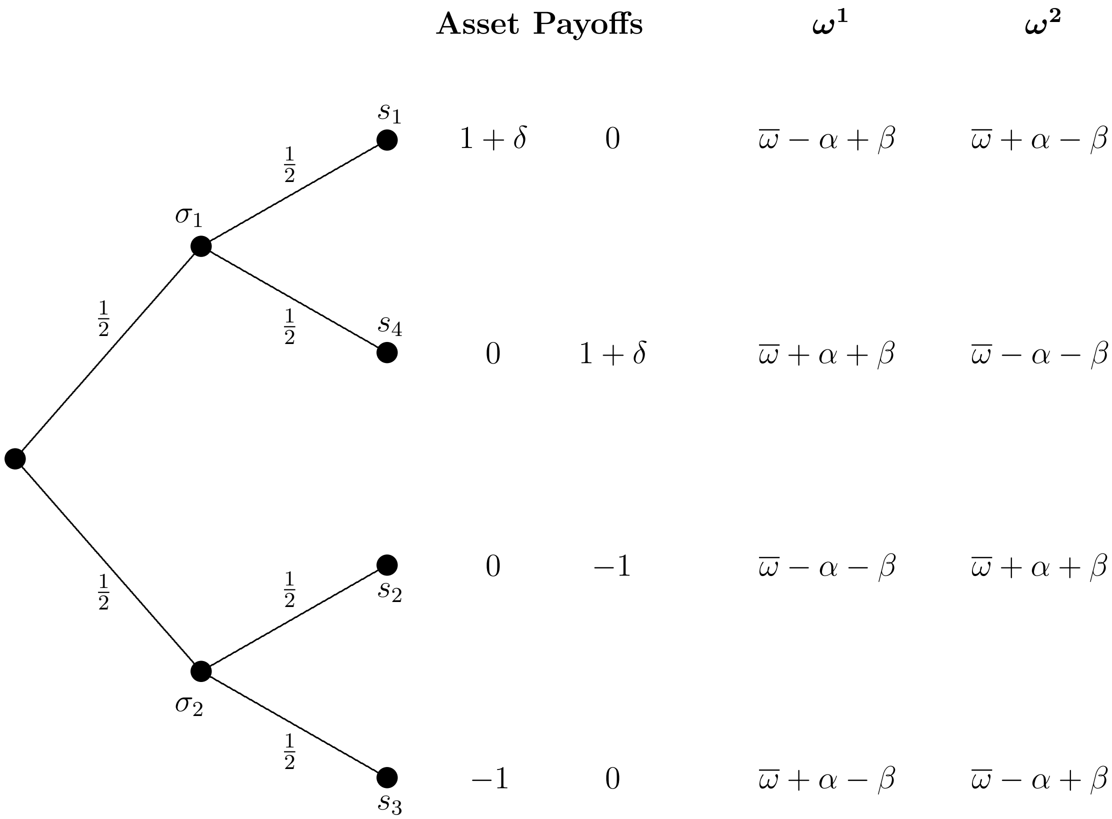

Figure 2: Agents are better off under $\hat{\pi}'$ (Blackwell dominates Hirshleifer)

where U_A is the expected utility of both agents in autarky. Thus the first-best utility gain from sharing the α-shock is $\frac{a}{2}\alpha^2$, while the corresponding gain from sharing the β-shock is $\frac{a}{2}\beta^2$.

Under the assumed incomplete asset structure, agents are not able to insure fully. It is straightforward to check that there is a π-equilibrium in which the two assets have the same (nonzero) price. Agent 1 buys an amount $\frac{2+\delta}{1+(1+\delta)^2}\,\alpha$ of asset 1 and sells an equal amount of asset 2. Agent 2 trades the same amounts in the opposite direction. The ex-ante expected utility of both agents is given by

$$U_\pi := \overline{U} - \frac{a}{2}\,\Delta\,\alpha^2 - \frac{a}{2}\,\beta^2,$$

where

$$\Delta := \frac{\delta^2}{2[1+(1+\delta)^2]}.$$

By going long in one asset and short in the other, agents can transfer consumption between the events $\{s_1, s_2\}$ and $\{s_3, s_4\}$. Thus they are able to partially insure the α-shock. The proportion of the potential welfare gain from insuring this shock, $\frac{a}{2}\alpha^2$, that is not realized in equilibrium is given by Δ. In other words, $\frac{a}{2}\Delta\alpha^2$ is the magnitude of the unrealized welfare gain from trading the α-shock. Notice that $\Delta \in (0, \frac{1}{2})$ and is strictly increasing in δ. As δ tends to zero, risk sharing relative to the α-shock approaches the first-best, i.e. full insurance. On the other hand, agents are not able to insure the β-shock at all, since that would require them to trade the assets in one direction in the event $\{s_1, s_2\}$ and the opposite direction in the event $\{s_3, s_4\}$. Since the equilibrium relative price of the assets is 1, it is not optimal to insure the β-shock in one of these events at the cost of exacerbating it in the other.

Given the information structure $\hat{\pi}$, markets are complete conditional on σ.[12] There is a unique equilibrium in which the consumption of each agent, conditional on σ, is riskless. The consumption of agent 1 is $\overline{\omega} - \alpha$ and $\overline{\omega} + \alpha$ in σ_1 and σ_2 respectively, while for agent 2 it is the other way around. The ex-ante expected utility of both agents is given by

$$U_{\hat{\pi}} := \overline{U} - \frac{a}{2}\,\alpha^2.$$

Since $\hat{\pi}$ reveals the α-shock, this risk cannot be shared in equilibrium; hence the utility loss $\frac{a}{2}\alpha^2$ relative to the first-best. On the other hand, agents can trade the assets in opposite directions in the events $\{s_1, s_2\}$ and $\{s_3, s_4\}$, which allows them to fully insure the β-shock. The equilibrium welfare gain from changing the information structure from π to $\hat{\pi}$ is

$$U_{\hat{\pi}} - U_{\pi} = -\frac{a}{2}(1-\Delta)\,\alpha^2 + \frac{a}{2}\,\beta^2.$$

The Hirshleifer effect is captured by the term $\frac{a}{2}(1-\Delta)\alpha^2$, which is equal to the welfare loss, relative to the π-equilibrium, arising from agents' inability under $\hat{\pi}$ to share the α-shock.[13] The second term $\frac{a}{2}\beta^2$ is the magnitude of the Blackwell effect, the welfare gain from being able to trade the β-shock under $\hat{\pi}$, which was not possible under π. The Hirshleifer effect is increasing in α and decreasing in δ, while the Blackwell

[12] In this example, the release of information leads to an increase in the asset span (conditional on σ), and is thus analogous to the introduction of new assets. This is not the case in general, however. For example, for information structures in Π^0 (which is a generic subset of the set of all information structures Π), i.e. those for which the conditional distribution over S has full support for every realization of the signal, the asset span conditional on the signal is the same for every realization and equal to the asset span with no information.

[13] As mentioned in footnote 1, the Hirshleifer effect can be interpreted in terms of changes in equilibrium prices induced by the change in information. In the present example, agents face two budget constraints, one for each value of σ. Under π, these constraints are identical, and hence reduce to a single budget constraint which allows agents to transfer wealth between the events $\{s_1, s_2\}$ and $\{s_3, s_4\}$. The change in information from π to $\hat{\pi}$ causes equilibrium prices to diverge in the two events, resulting in two distinct budget constraints. Agents cannot transfer wealth between these events without violating one of the budget constraints. In Hirshleifer's original two-state example, each of the two events consists of a single state; thus in each event the budget set faced by an agent is trivial, containing only the agent's endowment in that event.

effect is increasing in β. For sufficiently large α and/or sufficiently small δ and β, the Hirshleifer effect dominates the Blackwell effect, and both agents are worse off.

Next consider the information structure $\hat{\pi}'$. As with $\hat{\pi}$, markets are complete conditional on σ. Again there is a unique equilibrium, with riskless consumption for both agents conditional on σ. Ex-ante expected utility is given by

$$U_{\hat{\pi}'} := \overline{U} - \frac{a}{2}\,\beta^2.$$

Since $\hat{\pi}'$ reveals the β-shock, this risk cannot be traded in equilibrium; hence the utility loss $\frac{a}{2}\beta^2$ compared to the first-best. On the other hand, the α-shock can be smoothed out completely. The equilibrium welfare gain from changing the information structure from π to $\hat{\pi}'$ is

$$U_{\hat{\pi}'} - U_{\pi} = \frac{a}{2}\,\Delta\,\alpha^2.$$

It consists of only one term, which is equal to the unrealized welfare gain from trading the α-shock under π. This is the Blackwell effect. The Hirshleifer effect is zero in this case. While agents are unable to insure the β-shock under $\hat{\pi}'$, due to this shock being revealed prior to trading, it was not insurable even under the uninformative information structure π, because of the structure of asset payoffs. Clearly both agents are better off under $\hat{\pi}'$ for all values of α, β and δ.

Let us assume that the parameters α, β and δ are such that the Hirshleifer effect dominates the Blackwell effect under $\hat{\pi}$. Comparing competitive equilibrium allocations before and after the release of information, we see that agents are ex-ante worse off under $\hat{\pi}$, and ex-ante better off under $\hat{\pi}'$, relative to the π-equilibrium. We now consider what a hypothetical planner is able to achieve, when he is restricted by the same feasibility condition as competitive markets, i.e. $\sum_h y^h_\sigma = 0$ for all σ, and has access to the same information. We show that, for both $\hat{\pi}$ and $\hat{\pi}'$, the planner can attain an allocation that is Pareto improving ex-post (i.e. conditional on both values of σ), and hence also ex-ante.[14]

Under $\hat{\pi}$, the planner can improve ex-post upon the π-equilibrium by assigning to each agent the sum of his $\hat{\pi}$-equilibrium and π-equilibrium portfolios, for both values of σ. He is thus able to exploit the Blackwell effect (in this case, insuring the β-shock) just as competitive markets can under $\hat{\pi}$, while also insuring the α-shock to the extent that competitive markets can under π. Under $\hat{\pi}'$, the competitive equilibrium is itself an ex-post (not just ex-ante) Pareto improvement relative to the π-equilibrium. Clearly the $\hat{\pi}'$-equilibrium allocation is feasible for the planner as well (though he cannot improve upon it since it is ex-post efficient).

[14]The first-best allocation in which both agents have a riskless consumption of $\overline{\omega}$ is a feasible allocation under both $\hat{\pi}$ and $\hat{\pi}'$, and is an ex-ante Pareto improvement relative to the π-equilibrium. However, it is not in general an ex-post Pareto improvement. The ex-post expected utility of both agents at this allocation is $\overline{U}$ for both values of σ. One can verify that, under $\hat{\pi}$, the expected utility of agent 1 conditional on σ_2 is higher than $\overline{U}$, for sufficiently small β. Similarly, under $\hat{\pi}'$, the expected utility of agent 1 conditional on σ_1 is higher than $\overline{U}$, for δ sufficiently small relative to β.

To summarize, when markets are incomplete, the release of information expands the set of achievable utility levels via the Blackwell effect. The planner can take advantage of this and so can competitive markets. At the same time, the release of information can impair risk sharing due to the Hirshleifer effect, by precluding the sharing of risks that have already been resolved. The planner is not subject to this effect since the initial equilibrium allocation remains feasible even after the arrival of information. The example suggests that while equilibrium welfare effects can go in either direction, depending on the nature of the information being released, the planner can achieve an ex-post (hence also ex-ante) Pareto improvement for any increase in information.

In the remainder of the paper we extend the analysis to a general economy wherein we consider the effects of arbitrary changes in information. In the case of non-ordered information structures, welfare changes cannot be interpreted in terms of the Blackwell and Hirshleifer effects. Nevertheless, there is a sense in which the findings of the example remain valid. In Theorem 1 we establish that, for any change in information, an ex-post welfare improvement can generically be attained by tailoring agents' portfolios to the new information structure. An additional welfare effect arises in equilibrium because of the adjustment in asset prices induced by the change in information; we show in Theorem 2 that this may have a positive or negative effect on welfare. More precisely we show that, for a generic economy, there is a change in information that makes all agents (ex-ante) better off as well as a change that makes all agents worse off. Identifying the set of information changes that lead to a welfare improvement (or worsening) is not possible, however, at the level of generality of this result; clearly such a characterization can be obtained only under strong restrictions on the environment as in the example. We combine Theorems 1 and 2 in Corollary 1 to argue that the conclusions drawn from the example, summarized at the end of the previous paragraph, have generic validity, i.e. competitive markets typically do not respond to changes in information in a way that is welfare-improving even though it is feasible to do so.

4 Attainable Changes in Welfare

In this section we consider allocations attainable by a hypothetical planner who is subject to the same asset market and informational constraints as agents. We say that an allocation $(\hat{c}_0, \hat{y}) := \left\{\hat{c}^h_{0\sigma}, \hat{y}^h_\sigma\right\}_{h\in H,\sigma\in\Sigma}$ is *feasible* if $\sum_h \hat{c}^h_{0\sigma} = \sum_h \omega^h_0$ and $\sum_h \hat{y}^h_\sigma = 0$, for all $\sigma \in \Sigma$.

Note that a feasible allocation is defined as an allocation contingent on σ, and is therefore trivially measurable with respect to the information available to agents. Moreover, since all signals have the same set of possible realizations Σ, allocations that are measurable with respect to $\hat{\pi}$ are the same as those that are measurable with respect to π. Thus a change in information does not affect the set of feasible allocations. However, it does affect the set of attainable utility levels: the value to an agent of making his consumption conditional on σ depends on the information conveyed by it. For example, if π is uninformative and $\hat{\pi}$ induces a partition over S, under π nothing is gained by (nontrivially) conditioning on σ, but under $\hat{\pi}$ this is typically not true for values of σ associated with distinct cells of the partition. Even if $\hat{\pi}$ does not induce a partition, it is in general beneficial to condition on σ as long as the conditional distribution over S varies with σ.

Given a π-equilibrium, if no welfare improvement is feasible under the new information structure $\hat{\pi}$, i.e. when agents evaluate consumption using the beliefs determined by $\hat{\pi}$, we say that the π-equilibrium is $\hat{\pi}$-efficient. As in the example in the previous section, this gives rise to two welfare notions according to whether agents evaluate allocations ex-ante or ex-post, i.e. before or after the observation of the signal. Formally:

Definition 2 *A π-equilibrium (y, p) is <u>ex-ante $\hat{\pi}$-efficient</u> if there does not exist a feasible allocation $(\hat{c}_0, \hat{y})$ that ex-ante Pareto dominates the π-equilibrium allocation, given the information structure $\hat{\pi}$, i.e.*

$$\sum_{s,\sigma} \hat{\pi}_{s\sigma}\Big(u_0^h\left[\hat{c}_{0\sigma}^h\right] + u^h\left[\omega_s^h + r_s \cdot \hat{y}_\sigma^h\right] - u_0^h\left[\omega_0^h - p_\sigma \cdot y_\sigma^h\right] - u^h\left[\omega_s^h + r_s \cdot y_\sigma^h\right]\Big) \geq 0,$$

for all $h \in H$, where at least one of these inequalities is strict.

Definition 3 *A π-equilibrium (y, p) is <u>ex-post $\hat{\pi}$-efficient</u> if there does not exist a feasible allocation $(\hat{c}_0, \hat{y})$ that ex-post Pareto dominates the π-equilibrium allocation, given the information structure $\hat{\pi}$, i.e.*

$$\sum_{s} \hat{\pi}_{s\sigma}\Big(u_0^h\left[\hat{c}_{0\sigma}^h\right] + u^h\left[\omega_s^h + r_s \cdot \hat{y}_\sigma^h\right] - u_0^h\left[\omega_0^h - p_\sigma \cdot y_\sigma^h\right] - u^h\left[\omega_s^h + r_s \cdot y_\sigma^h\right]\Big) \geq 0,$$

for all $h \in H$ and $\sigma \in \Sigma$, where at least one of these inequalities is strict.

Notice that both the allocations (c_0, y) and $(\hat{c}_0, \hat{y})$ are evaluated at the same odds, given by $\hat{\pi}$.[15]

It is well-known (Diamond (1967)) that competitive equilibria of a two-period, single-good economy with incomplete markets are (ex-post) constrained Pareto efficient. Thus, in our terminology, π-equilibria are ex-post π-efficient. This condition can be characterized in terms of the equality of agents' marginal rates of substitution between assets and period 0 consumption, for every σ, i.e.

$$\frac{\sum_s \pi_{s\sigma}\, u^{h\prime}\left[\omega_s^h + r_s \cdot y_\sigma^h\right] r_s^j}{u_0^{h\prime}\left[\omega_0^h - p_\sigma \cdot y_\sigma^h\right]} = \frac{\sum_s \pi_{s\sigma}\, u^{\check{h}\prime}\left[\omega_s^{\check{h}} + r_s \cdot y_\sigma^{\check{h}}\right] r_s^j}{u_0^{\check{h}\prime}\left[\omega_0^{\check{h}} - p_\sigma \cdot y_\sigma^{\check{h}}\right]}, \quad \forall h, \check{h} \in H;\ j \in J;\ \sigma \in \Sigma. \tag{3}$$

[15] It would not be sensible to evaluate (c_0, y) at π, and $(\hat{c}_0, \hat{y})$ at $\hat{\pi}$. Doing so can lead to the possibility of a "Pareto improvement" with no change in the allocation (unless the allocation under consideration is σ-invariant for any given s).

This means that, if there is no change in information, an ex-post welfare-improving reallocation of portfolios cannot be found. In this sense, competitive markets make efficient use of the available information. In contrast, we show that if markets are incomplete π-equilibria are generically ex-post $\hat{\pi}$-inefficient, for every $\hat{\pi} \in \hat{\Pi}(\pi)$. In other words, while a competitive equilibrium makes efficient use of the available information, a Pareto improvement can typically be achieved for *any change* in information.

We begin by verifying that incompleteness is necessary for this result. When markets are complete (and the initial information structure has full support), at a competitive equilibrium there are no further gains from trade, whatever the change in agents' information is. Hence no portfolio reallocation can be found that allows an improvement in agents' welfare.

Lemma 1 *Suppose markets are complete and $(\pi, \hat{\pi})$ is an admissible pair of information structures with $\pi \in \Pi^0$. Then every π-equilibrium is ex-post $\hat{\pi}$-efficient.*

Proof :
If markets are complete and $\pi \in \Pi^0$, condition (3) for the π-efficiency of a π-equilibrium (y, p) is equivalent to the equality of agents' marginal rates of intertemporal substitution for every σ:

$$\frac{u^{h\prime}\big[\omega_s^h + r_s \cdot y_\sigma^h\big]}{u_0^{h\prime}\big[\omega_0^h - p_\sigma \cdot y_\sigma^h\big]} = \frac{u^{\check{h}\prime}\big[\omega_s^{\check{h}} + r_s \cdot y_\sigma^{\check{h}}\big]}{u_0^{\check{h}\prime}\big[\omega_0^{\check{h}} - p_\sigma \cdot y_\sigma^{\check{h}}\big]}, \qquad \forall h, \check{h} \in H;\ s \in S_\pi(\sigma);\ \sigma \in \Sigma. \tag{4}$$

Since $S_{\hat{\pi}}(\sigma) \subset S_\pi(\sigma) = S$ for all σ, condition (4), and hence also (3), must in fact hold when π is replaced by $\hat{\pi}$. Therefore, the π-equilibrium (y, p) is ex-post $\hat{\pi}$-efficient, and this is true for all $\hat{\pi} \in \hat{\Pi}(\pi)$. □

When markets are incomplete, condition (3) still holds at a π-equilibrium, but does not imply (4). In fact for a generic economy competitive equilibria are Pareto inefficient, so that (4) does not hold (this is a standard result; see, for example, Magill and Quinzii (1996)). We now show that, in addition, a π-equilibrium is generically ex-post $\hat{\pi}$-inefficient, i.e. condition (3) is violated when π is replaced by $\hat{\pi}$, indeed for any $\hat{\pi} \in \hat{\Pi}(\pi)$. Intuitively, when we change π to $\hat{\pi}$, (3) continues to hold only under restrictive conditions on agents' marginal utilities for state-contingent consumption. These conditions are not satisfied for a generic economy. We prove the result under the assumption that the asset payoff matrix R is in general position, i.e. every $J \times J$ submatrix of R is nonsingular.[16]

Theorem 1 *Suppose markets are incomplete and R is in general position. Suppose further that $(\pi, \hat{\pi})$ is an admissible pair of information structures such that at least one element of the pair $(\pi, \hat{\pi})$ lies in Π^0. Then there is a generic subset of Ω such that every π-equilibrium is ex-post $\hat{\pi}$-inefficient.*

[16]General position is a generic property of the set of $S \times J$ matrices, for $S \geq J$.

Theorem 1 allows for arbitrary changes in information,[17] including those that entail an increase or decrease in the support of the information structure.[18] In particular, it implies that generically an increase in information generates additional gains from trade, so that agents would trade again if asset markets were to reopen after the increase in information. This idea is developed more fully in Gottardi and Rahi (2012), which provides several extensions of Theorem 1 in the specific context of retrading after the arrival of information. However, Theorem 1 suffices for our present purpose, which is to contrast welfare changes in equilibrium to those that can be achieved by a planner.

5 Equilibrium Changes in Welfare

We have shown that, for any π-equilibrium, and for any change in information from π to $\hat{\pi}$, generically there is a feasible allocation that is ex-post Pareto improving. While this is no longer true if we restrict attention to feasible allocations that are also $\hat{\pi}$-equilibrium allocations, we show in this section that generically there *exists an information structure* $\breve{\pi}$ and a corresponding $\breve{\pi}$-equilibrium such that all agents are better off ex-ante. But there also exists a $\breve{\pi}'$ and a corresponding $\breve{\pi}'$-equilibrium such that all agents are worse off. Indeed, the effect of a change in information on equilibrium welfare can be in any direction. Unlike the welfare changes considered in the previous section, the change in equilibrium welfare is attributable not only to the effect of the change in information on the available gains from trade, but also to the effect of this change on equilibrium asset prices. The proof of this result exploits the general framework laid out in Citanna et al. (1998), and exposited more fully in Villanacci et al. (2002), Chapter 15, to analyze the effect on equilibrium welfare of perturbing a parameter of the economy.

Given a π-equilibrium (y, p), and a $\hat{\pi}$-equilibrium $(\hat{y}, \hat{p})$, let $W(\hat{y}, \hat{p}, \hat{\pi}; y, p)$ be the vector of differences in the ex-ante expected utilities of agents between the two equilibria. This vector is given by

$$\left\{\sum_{s,\sigma} \hat{\pi}_{s\sigma}\Big(u_0^h\big[\omega_0^h - \hat{p}_\sigma \cdot \hat{y}_\sigma^h\big] + u^h\big[\omega_s^h + r_s \cdot \hat{y}_\sigma^h\big] - u_0^h\big[\omega_0^h - p_\sigma \cdot y_\sigma^h\big] - u^h\big[\omega_s^h + r_s \cdot y_\sigma^h\big]\Big)\right\}_{h\in H}$$

We consider local changes in $(\hat{y}, \hat{p}, \hat{\pi})$ in a neighborhood of (y, p, π). An ex-ante Pareto improvement is attained in equilibrium if $W(\hat{y}, \hat{p}, \hat{\pi}; y, p)$ is nonnegative and nonzero. We show in fact that for a generic economy, for any vector in $\mathbb{R}^H$, there exists a $\hat{\pi}$ and a corresponding equilibrium $(\hat{y}, \hat{p})$ such that the change in equilibrium welfare given by $W(\hat{y}, \hat{p}, \hat{\pi}; y, p)$ is equal to the specified vector. In other words, it is possible

[17] The assumption that either π or $\hat{\pi}$ has full support is just a matter of convenience. The result holds even if neither has full support, provided **a2** and the market incompleteness condition apply for the relevant subset of S.

[18] Notice that an ex-post Pareto improvement can typically be attained even if $\hat{\pi}$ resolves all the uncertainty in the economy, since there is still room for intertemporal reallocation of consumption.

to choose $\hat{\pi}$ in order to generate a local change in welfare in any direction. Our result requires that markets be sufficiently incomplete, and that the heterogeneity of agents not be too large.

Theorem 2 *Suppose $S \geq 2JH$, $J \geq H-1$, and R is in general position. Then, for a generic subset of $\mathcal{E}$, for any π-equilibrium (y,p), there exists a local change in information $d\hat{\pi}$ satisfying $(\pi + d\hat{\pi}) \in \hat{\Pi}(\pi)$, and a corresponding local change in the equilibrium $(d\hat{y}, d\hat{p})$, such that $W(y + d\hat{y}, p + d\hat{p}, \pi + d\hat{\pi}; y, p)$ is any desired nonzero vector in $\mathbb{R}^H$.*

We can now summarize the sense in which Theorems 1 and 2 generalize the example in Section 3.

Corollary 1 *Suppose $S \geq 2JH$, $J \geq H-1$, and R is in general position. Then, for any (u, π) in a generic subset of $\mathcal{U}^H \times \Pi$, there is a generic subset of Ω such that for any π-equilibrium, there exist $\hat{\pi}$ and $\hat{\pi}'$ in $\hat{\Pi}(\pi)$ for which the following statements hold:*

i. There is a $\hat{\pi}$-equilibrium at which all agents are worse off ex-ante.

ii. There is a $\hat{\pi}'$-equilibrium at which all agents are better off ex-ante.

iii. The π-equilibrium is ex-post $\hat{\pi}$-inefficient as well as ex-post $\hat{\pi}'$-inefficient.

The proof, which is in the Appendix, is not an immediate consequence of Theorems 1 and 2. For a given economy (ω, u, π) in the generic subset for which Theorem 2 applies, statements (i) and (ii) of the corollary hold. However, the generic subset of endowments for which statement (iii) holds, given by Theorem 1, may not contain ω. An additional argument is required to establish the result.

Corollary 1 tells us, in particular, that while there is (almost) always a change in information that makes all agents worse off in equilibrium, for the same change in information there is a feasible allocation at which all agents are better off (even ex-post). Thus, while competitive markets make efficient use of the available information (in the sense of being constrained efficient for a given information structure), they typically do not deal with changes in information in a welfare-enhancing manner even though it is feasible to do so.

A Appendix

In our analysis we use the following shorthand notation for matrices. Given an index set $\mathcal{N}$ with typical element n, and a collection $\{z_n\}_{n\in\mathcal{N}}$ of vectors or matrices, we denote by $\operatorname{diag}_{n\in\mathcal{N}}[z_n]$ the (block) diagonal matrix with typical entry z_n, where n varies across all elements of $\mathcal{N}$. For a given vector or matrix z, $\operatorname{diag}_{n\in\mathcal{N}}[z]$ is then the diagonal matrix with the term z repeated $\#\mathcal{N}$ times. In similar fashion, we write $[\ldots z_n \ldots_{n\in\mathcal{N}}]$ to denote the row block with typical element z_n, and analogously for column blocks. We drop reference to the index set if it is obvious from the context: for example $\operatorname{diag}_{h\in H}$ is shortened to diag_h, and $[\ldots z_s \ldots_{s\in S, s\neq s_1}]$ to $[\ldots z_s \ldots_{s\neq s_1}]$. We use the same symbol 0 for the zero scalar and the zero matrix; in the latter case we occasionally indicate the dimension in order to clarify the argument. We denote by I_N the $N\times N$ identity matrix. A "$*$" stands for any term whose value is immaterial to the analysis. We will sometimes need to order the set S (and similarly the sets Σ and H) as $\{s_1, s_2, \ldots\}$, s_1 being the first state, and so on.

We begin by providing a characterization of competitive equilibria. The first-order conditions for the utility-maximization program (1) are:

$$\sum_s \pi_{s\sigma}\left(u^{h\prime}\left[\omega_s^h + r_s\cdot y_\sigma^h\right] r_s - u_0^{h\prime}\left[\omega_0^h - p_\sigma\cdot y_\sigma^h\right] p_\sigma\right) = 0, \qquad \forall h\in H;\ \sigma\in\Sigma. \quad (5)$$

The tuple (y,p) is a competitive equilibrium if and only if it satisfies equations (2) and (5). Let $\pi^\sigma := \{\pi_{s\sigma}\}_{s\in S}$, and define

$$f_\sigma(y_\sigma, p_\sigma, \omega, u, \pi^\sigma) := \sum_s \pi_{s\sigma}\left(u^{h\prime}\left[\omega_s^h + r_s\cdot y_\sigma^h\right] r_s - u_0^{h\prime}\left[\omega_0^h - p_\sigma\cdot y_\sigma^h\right] p_\sigma\right), \quad \forall h\in H,$$

$$g_\sigma(y_\sigma) := \sum_h y_\sigma^h.$$

Then the equations that characterize a competitive equilibrium, (2) and (5), can be written as

$$F_\sigma(y_\sigma, p_\sigma, \omega, u, \pi^\sigma) := \begin{pmatrix} f_\sigma(y_\sigma, p_\sigma, \omega, u, \pi^\sigma) \\ g_\sigma(y_\sigma) \end{pmatrix} = 0, \qquad \forall\sigma\in\Sigma, \quad (6)$$

or more compactly as

$$F(y, p, \omega, u, \pi) := \begin{pmatrix} f(y, p, \omega, u, \pi) \\ g(y) \end{pmatrix} = 0, \quad (7)$$

where $f := \{f_\sigma\}_{\sigma\in\Sigma}$, and $g := \{g_\sigma\}_{\sigma\in\Sigma}$. Here the endogenous variables are (y,p), while (ω, u, π) are parameters.[19]

[19] For ease of notation, we will often drop parameters that remain fixed and play no role in the argument at hand. For example, in the proof of Theorem 2, we write $F(\hat{y}, \hat{p}, \hat{\pi})$ instead of $F(\hat{y}, \hat{p}, \omega, u, \hat{\pi})$.

Suppose $\pi \in \Pi^0$. Consider the equilibrium system (6) for a given value of σ. These are $JH + J$ equations, equal to the number of unknowns (y, p). The Jacobian of this equation system can be written as follows:

$$D_{y_\sigma, p_\sigma, \omega} F_\sigma = \begin{pmatrix} D_{y_\sigma, p_\sigma} f_\sigma & D_\omega f_\sigma \\ D_{y_\sigma, p_\sigma} g_\sigma & 0 \end{pmatrix},$$

with

$$D_\omega f_\sigma = \text{diag}_h \left[\ldots \pi_{s\sigma}\, u^{h\prime\prime}\left[\omega_s^h + r_s \cdot y_\sigma^h\right] r_s \ldots_s \right]$$

and

$$D_{y_\sigma, p_\sigma} g_\sigma = \left(\ldots I_J \ldots_h \quad 0 \right).$$

The matrix $D_\omega f_\sigma$ has full row rank since R has full column rank and the term $\pi_{s\sigma} u^{h\prime\prime}[\omega_s^h + r_s \cdot y_\sigma^h]$ is nonzero, for all h, s, σ. Clearly, $D_{y_\sigma, p_\sigma} g_\sigma$ has full row rank as well (indeed, $D_{y_\sigma} g_\sigma$ has full row rank). By a standard argument, for a generic subset of endowments $\Omega_{R\sigma}$, the number of equilibria conditional on σ (zeros of F_σ) is finite, and positive. Hence, for endowments in the generic subset $\Omega_R := \cap_{\sigma \in \Sigma} \Omega_{R\sigma}$, the number of equilibria (zeros of F) is finite as well.

Since we exploit the transversality theorem repeatedly in the proofs below, it is useful to summarize the argument at the outset. Consider a function $\Psi : \mathbb{R}^n \times \mathcal{E} \to \mathbb{R}^m$, $m > n$. For $e \in \mathcal{E}$, let Ψ_e be the function $\Psi(\cdot, e)$. The argument involves identifying such a function Ψ, such that the desired result can be formulated as $\Psi_e^{-1}(0) = \varnothing$, for every e in a generic subset of $\mathcal{E}$. We show that the Jacobian $D_{x,e}\Psi$ has full row rank at all zeros (x, e) of Ψ, i.e. Ψ is transverse to zero. By the transversality theorem, there is then a dense subset $\hat{\mathcal{E}}$ of $\mathcal{E}$ such that, for each $e \in \hat{\mathcal{E}}$, $\Psi_e : \mathbb{R}^n \to \mathbb{R}^m$ is transverse to zero. It follows that $\Psi_e^{-1}(0) = \varnothing$. In other words, the equation system $\Psi_e(x) = 0$ has no solution since the number of (locally) independent equations m exceeds the number of unknowns n. A standard argument (see, for example, Citanna et al. (1998)) establishes that the set $\hat{\mathcal{E}}$ is open, and hence a generic subset of $\mathcal{E}$.

In order to economize on notation, we use the shorthand $u_{0\sigma}^{h\prime} := u_0^{h\prime}\left[\omega_0^h - p_\sigma \cdot y_\sigma^h\right]$, $u_{s\sigma}^{h\prime} := u^{h\prime}\left[\omega_s^h + r_s \cdot y_\sigma^h\right]$, and similarly for the second derivatives, $u_{0\sigma}^{h\prime\prime}$ and $u_{s\sigma}^{h\prime\prime}$.

Proof of Theorem 1:
A π-equilibrium (y, p) is ex-post $\hat{\pi}$-efficient if and only if the marginal rates of substitution between assets and period 0 consumption, evaluated at $\hat{\pi}$, are equal across agents, for every σ, i.e. condition (3) holds with π replaced by $\hat{\pi}$. Letting

$$\mu_\sigma^{h\check{h}}(y_\sigma, p_\sigma) := \frac{u_{0\sigma}^{h\prime}}{u_{0\sigma}^{\check{h}\prime}},$$

we can rewrite this condition as follows:

$$\sum_s \hat{\pi}_{s\sigma} \left(u_{s\sigma}^{h\prime} - \mu_\sigma^{h\check{h}}\, u_{s\sigma}^{\check{h}\prime} \right) r_s^j = 0, \qquad \forall h, \check{h} \in H;\ j \in J;\ \sigma \in \Sigma. \tag{8}$$

Fix a σ for which $\{\hat{\pi}_{s\sigma}\}_{s\in S}$ is not proportional to $\{\pi_{s\sigma}\}_{s\in S}$. We will show that a Pareto improvement can be achieved conditional on every σ with this property.

We assume that $\pi \in \Pi^0$ in the following argument, outlining in the last paragraph of the proof how it extends to the case where π does not have full support. Let

$$R^* := \begin{pmatrix} \operatorname{diag}_s[\hat{\pi}_{s\sigma}]R & \operatorname{diag}_s[\pi_{s\sigma}]R \end{pmatrix}.$$

We claim that $\operatorname{rank}(R^*) \geq J+1$. If $S_{\hat{\pi}}(\sigma) = S$, this is immediate from the following result in Geanakoplos and Mas-Colell (1989) (Step 4, page 31):

Fact *Suppose markets are incomplete, and R is in general position. Consider nonzero scalars θ_s, θ'_s, $s \in S$, such that $\{\theta_s\}_{s\in S}$ is not proportional to $\{\theta'_s\}_{s\in S}$. Then, $\operatorname{diag}_s[\theta_s]R$ and $\operatorname{diag}_s[\theta'_s]R$ do not have the same column span.*

On the other hand, if $S_{\hat{\pi}}(\sigma) \subsetneq S$, R^* is row-equivalent to

$$\hat{R}^* := \left(\begin{array}{c|c} \operatorname{diag}_{s\in S_{\hat{\pi}}(\sigma)}[\hat{\pi}_{s\sigma}]\, R_1 & \operatorname{diag}_{s\in S_{\hat{\pi}}(\sigma)}[\pi_{s\sigma}]\, R_1 \\ \hline 0 & \operatorname{diag}_{s\notin S_{\hat{\pi}}(\sigma)}[\pi_{s\sigma}]\, R_2 \end{array}\right),$$

where R_1 consists of the rows of R corresponding to the states in $S_{\hat{\pi}}(\sigma)$, while R_2 consists of the remaining rows. If we delete the rows of $\hat{R}^*$ corresponding to the redundant rows of its upper left block, and also delete the rows corresponding to the redundant rows of its lower right block, we are left with a block-triangular matrix whose diagonal blocks have full row rank, and hence whose rank is equal to the sum of the ranks of the diagonal blocks. It follows that for the full matrix $\hat{R}^*$, $\operatorname{rank}(\hat{R}^*) \geq \operatorname{rank}(R_1) + \operatorname{rank}(R_2)$, which is at least $J+1$ due to the general position of R. Consequently, $\operatorname{rank}(R^*) = \operatorname{rank}(\hat{R}^*) \geq J+1$.

Let $r^j \in \mathbb{R}^S$ be the payoff of asset j. Since $\operatorname{rank}(R^*) \geq J+1$, we can pick j such that $\operatorname{diag}_s[\hat{\pi}_{s\sigma}]\, r^j$ lies outside the column span of $\operatorname{diag}_s[\pi_{s\sigma}]R$, and hence the matrix

$$\begin{pmatrix} \ldots \hat{\pi}_{s\sigma}\, r^j_s \ldots_s \\ \ldots \pi_{s\sigma}\, r_s \ldots_s \end{pmatrix} \tag{9}$$

has full row rank $J+1$.

We will show that, for a generic subset of Ω, there is no solution to the equation system

$$\Psi_1(y_\sigma, p_\sigma, \omega; \pi^\sigma, \hat{\pi}^\sigma) := \begin{pmatrix} F_\sigma(y_\sigma, p_\sigma, \omega; \pi^\sigma) \\ \sum_s \hat{\pi}_{s\sigma} \left(u^{h_1\prime}_{s\sigma} - \mu^{h_1h_2}_\sigma u^{h_2\prime}_{s\sigma}\right) r^j_s \end{pmatrix} = 0,$$

i.e. the ex-post $\hat{\pi}$-efficiency condition (8) is violated. The Jacobian of Ψ_1 is

$$D_{y_\sigma, p_\sigma, \omega}\Psi_1 = \left(\begin{array}{c|c} * & \operatorname{diag}_h\left[\ldots \pi_{s\sigma}\, u^{h\prime\prime}_{s\sigma} r_s \ldots_s\right] \\ \hline D_{y_\sigma, p_\sigma} g_\sigma & 0 \\ \hline * & \left[\ldots \hat{\pi}_{s\sigma}\, u^{h_1\prime\prime}_{s\sigma} r^j_s \ldots_s\right] \quad * \end{array}\right),$$

which is row-equivalent to

$$\left(\begin{array}{c|c|c} * & \ldots\hat{\pi}_{s\sigma}\, u^{h_1\prime\prime}_{s\sigma} r^j_s \ldots_s & * \\ \hline * & \ldots\pi_{s\sigma}\, u^{h_1\prime\prime}_{s\sigma} r_s \ldots_s & 0 \\ \hline * & 0 & \mathrm{diag}_{h\neq h_1}\left[\ldots\pi_{s\sigma}\, u^{h\prime\prime}_{s\sigma} r_s \ldots_s\right] \\ \hline D_{y_\sigma,p_\sigma} g_\sigma & 0 & 0 \end{array}\right).$$

This matrix is in turn column-equivalent to

$$\left(\begin{array}{c|c|c} * & * & \begin{pmatrix} \ldots\hat{\pi}_{s\sigma}\, r^j_s \ldots_s \\ \ldots\pi_{s\sigma}\, r_s \ldots_s \end{pmatrix} \\ \hline * & \mathrm{diag}_{h\neq h_1}\left[\ldots\pi_{s\sigma}\, u^{h\prime\prime}_{s\sigma} r_s \ldots_s\right] & 0 \\ \hline D_{y_\sigma,p_\sigma} g_\sigma & 0 & 0 \end{array}\right). \tag{10}$$

This is a block-triangular matrix. The top right block is the same as (9) which has full row rank. The other two diagonal blocks of (10) have full row rank as well, by the properties of π-equilibria established at the beginning of the Appendix. Hence the matrix (10) has full row rank.

We have shown that the Jacobian $D_{y_\sigma,p_\sigma,\omega}\Psi_1$ has full row rank, at every zero of Ψ_1. Thus Ψ_1 is transverse to zero, and $\Psi_{1\omega}^{-1}(0) = \varnothing$, for every ω in a generic subset of Ω.

In this proof we assumed that $\pi \in \Pi^0$. If not, then $\hat{\pi} \in \Pi^0$, due to the assumption in the theorem that at least one of these information structures has full support. Then $\mathrm{rank}(R^*) \geq J+1$ by the same argument as above, swapping the roles of π and $\hat{\pi}$. Dropping any assets that are redundant with respect to the states in $S_\pi(\sigma)$, let $\breve{y}_\sigma$ and $\breve{p}_\sigma$ be the portfolios and prices of the remaining assets. The above argument shows that, for a generic subset of Ω, there is no solution to the equation system $\Psi_1(\breve{y}_\sigma, \breve{p}_\sigma, \omega; \pi^\sigma, \hat{\pi}^\sigma) = 0$. In particular, $D_{\breve{y}_\sigma,\breve{p}_\sigma,\omega} F_\sigma$ has full row rank, and consequently so does $D_{\breve{y}_\sigma,\breve{p}_\sigma,\omega}\Psi_1$. This establishes the result. $\square$

Proof of Theorem 2:
We fix an economy $e = (\omega, u, \pi) \in \Omega \times \mathcal{U}^H \times \Pi^0$, and consider a π-equilibrium (y, p). Thus $F(y, p, \pi) = 0$. We consider $(\hat{y}, \hat{p}, \hat{\pi})$ in a neighborhood of (y, p, π) such that $(\hat{y}, \hat{p})$ is a $\hat{\pi}$-equilibrium, i.e. $F(\hat{y}, \hat{p}, \hat{\pi}) = 0$.

Let

$$\Phi(\hat{y}, \hat{p}, \hat{\pi}; y, p, \pi) := \begin{pmatrix} W(\hat{y}, \hat{p}, \hat{\pi}; y, p) \\ F(\hat{y}, \hat{p}, \hat{\pi}) \\ \left\{\sum_\sigma \hat{\pi}_{s\sigma} - \pi_s\right\}_{s\in S} \end{pmatrix}.$$

Notice that $\Phi(y, p, \pi; y, p, \pi) = 0$. As we vary $(\hat{y}, \hat{p}, \hat{\pi})$ in a neighborhood of (y, p, π) we move locally along the equilibrium manifold. The theorem is established by showing that the Jacobian of Φ with respect to $(\hat{y}, \hat{p}, \hat{\pi})$ has full row rank at $(\hat{y}, \hat{p}, \hat{\pi}) =$

(y, p, π), i.e. $D_{\hat{y},\hat{p},\hat{\pi}}\Phi(y, p, \pi; y, p, \pi)$ has full row rank. This full rank property implies that there is a local change $(d\hat{y}, d\hat{p}, d\hat{\pi})$ such that $d\Phi(y, p, \pi; y, p, \pi)$ is any desired vector. In particular, $(d\hat{y}, d\hat{p}, d\hat{\pi})$ can be chosen so that $dW(y, p, \pi; y, p) := W(y + d\hat{y}, p + d\hat{p}, \pi + d\hat{\pi}; y, p)$ is any desired nonzero vector in $\mathbb{R}^H$, $dF(y, p, \pi) := F(y + d\hat{y}, p + d\hat{p}, \pi + d\hat{\pi}) = 0$, and $\sum_\sigma d\hat{\pi}_{s\sigma} = 0$ for all $s \in S$. The condition $dF = 0$ ensures that $(y + d\hat{y}, p + d\hat{p})$ is a $(\pi + d\hat{\pi})$-equilibrium. The condition $\sum_\sigma d\hat{\pi}_{s\sigma} = 0$ implies that $(\pi + d\hat{\pi})$ satisfies condition **a1** in the definition of the set $\hat{\Pi}(\pi)$. Condition **a2** in this definition must also be satisfied, for otherwise there is no change in the equilibrium allocation and consequently no change in welfare. Therefore, $(\pi + d\hat{\pi}) \in \hat{\Pi}(\pi)$.

The matrix $D_{\hat{y},\hat{p},\hat{\pi}}\Phi(y, p, \pi; y, p, \pi)$ is given by

$$\left(\begin{array}{c|c|c}
0 & \begin{matrix} \vdots \\ \ldots -\pi_\sigma u^{h\prime}_{0\sigma} y^{h\top}_\sigma \ldots_\sigma \\ \vdots_h \end{matrix} & 0 \\
\hline
* & * & \operatorname{diag}_\sigma \begin{pmatrix} \vdots \\ \ldots \left(u^{h\prime}_{s\sigma} r_s - u^{h\prime}_{0\sigma} p_\sigma\right) \ldots_s \\ \vdots_h \end{pmatrix} \\
\hline
\operatorname{diag}_\sigma[\ldots I_J \ldots_h] & 0 & 0 \\
\hline
0 & 0 & \ldots I_S \ldots_\sigma
\end{array}\right).$$

The top left block is zero because of the first-order condition (5). This matrix is row/column-equivalent to

$$\left(\begin{array}{c|c|c}
\begin{matrix} \vdots \\ \ldots\ u^{h\prime}_{0\sigma} y^{h\top}_\sigma \ldots_\sigma \\ \vdots_h \end{matrix} & 0 & 0 \\
\hline
0 & \operatorname{diag}_\sigma[\ldots I_J \ldots_h] & 0 \\
\hline
* & * & Q(y)
\end{array}\right), \tag{11}$$

where

$$Q(y) := \left(\begin{array}{c}
\operatorname{diag}_\sigma \begin{pmatrix} \vdots \\ \ldots\ \left(u^{h\prime}_{s\sigma} r_s - u^{h\prime}_{0\sigma} p_\sigma\right)\ \ldots_s \\ \vdots_h \end{pmatrix} \\
\hline
\ldots\ I_S\ \ldots_\sigma
\end{array}\right). \tag{12}$$

Later in the Appendix we show that, for every competitive equilibrium of a generic economy, under the dimensionality restrictions in the statement of the theorem, the portfolios and marginal utilities of agents satisfy the following conditions:[20]

C1. The vectors $\{y^h_\sigma\}_{h\in H,\, h\neq \breve{h}}$ are linearly independent, for all $h, \breve{h} \in H$ and $\sigma \in \Sigma$.

C2. The vectors $(\ldots u^{h_1\prime}_{0\sigma} \ldots_\sigma)$ and $(\ldots u^{h_2\prime}_{0\sigma} \ldots_\sigma)$ are not collinear.[21]

C3. $Q(y)$ has full row rank.

Each of these conditions holds for a different generic subset of $\mathcal{E}$ (see Lemmas A.1, A.3 and A.5). In the remainder of the proof, we assume that the economy e considered in the foregoing analysis is in the intersection of these three generic subsets. Thus e is a generic economy for which conditions **C1**, **C2** and **C3** hold. We will prove that, for such an economy, (11) has full row rank, and hence so does the Jacobian $D_{\hat{y},\hat{p},\hat{\pi}}\Phi(y,p,\pi;y,p,\pi)$.[22]

Since (11) is a block-triangular matrix, it suffices to show full row rank of the three diagonal blocks. The middle block clearly has full row rank. The bottom right block $Q(y)$ has full row rank due to **C3**.

It remains to show that the top left block of (11) has full row rank. The last $H-1$ rows of this block are linearly independent by condition **C1**. We claim that the first row is not in the span of the last $H-1$ rows. Suppose not. Then the first row $[\ldots u^{h_1\prime}_{0\sigma} y^{h_1\top}_\sigma \ldots_\sigma]$ can be written as a unique linear combination of the remaining rows $\{[\ldots u^{h\prime}_{0\sigma} y^{h\top}_\sigma \ldots_\sigma]\}_{h\neq h_1}$, i.e. there exist unique coefficients $\{\gamma^h\}_{h\neq h_1}$ such that

$$y^{h_1}_\sigma = \sum_{h\neq h_1} \gamma^h \frac{u^{h\prime}_{0\sigma}}{u^{h_1\prime}_{0\sigma}} y^h_\sigma, \qquad \forall \sigma \in \Sigma.$$

By the market-clearing condition, this linear combination must in fact be $y^{h_1}_\sigma = -\sum_{h\neq h_1} y^h_\sigma$, for all σ. It follows that the ratio $u^{h_1\prime}_{0\sigma}/u^{h_2\prime}_{0\sigma}$ is σ-invariant, violating condition **C2**. $\square$

In the remainder of the Appendix we establish conditions **C1**, **C2** and **C3**, that were used in the proof of Theorem 2, for a generic subset of $\mathcal{E}$, and provide a proof of Corollary 1.

Lemma A.1 *Suppose $J \geq H-1$ and $\pi \in \Pi^0$. Then, for a generic subset Ω_Y of Ω, condition* **C1** *is satisfied at any π-equilibrium.*

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[20] The upper bound on the number of agents relative to the number of assets, $J \geq H-1$, is clearly necessary for condition **C1**, which in turn is used in the proofs of conditions **C2** and **C3**. In addition, **C2** and **C3** rely on a property that agents' marginal utilities for assets vary sufficiently with respect to π, across both s and σ (see Lemma A.2); a lower bound on the number of states, $S \geq J(H+1)+1$, is needed to establish this property. Finally, the proof of condition **C3** requires a tighter lower bound on S, given by $S \geq 2JH$.

[21] The choice of agents h_1 and h_2 is just a matter of convenience.

[22] Notice that, in order to establish the full rank property of $D\Phi$, we do not apply a transversality argument to the map Φ, i.e. we do not perturb parameters of Φ. Instead, we use transversality to show the generic validity of conditions **C1**, **C2** and **C3**.

Proof :
Without loss of generality we can take $\breve{h} = h_1$. Consider the equation system

$$\Psi_2(y_\sigma, p_\sigma, \psi_2, \omega) := \begin{pmatrix} F_\sigma(y_\sigma, p_\sigma, \omega) \\ [\dots y_\sigma^h \dots_{h \neq h_1}]\, \psi_2 \\ \psi_2^\top \psi_2 - 1 \end{pmatrix} = 0,$$

where $\psi_2 \in \mathbb{R}^{H-1}$, with typical element ψ_{2h}, $h \in H, h \neq h_1$. Thus we have appended $J+1$ equations and $H-1$ unknowns to the equilibrium system $F_\sigma = 0$. Under the dimensionality condition $J \geq H-1$, the number of equations exceeds the number of unknowns, for given ω. We will show that, for a generic subset of endowments, this system has no solution, and hence the vectors $\{y_\sigma^h\}_{h \neq h_1}$ are linearly independent at any equilibrium.

The Jacobian of Ψ_2 is

$$D_{y_\sigma, p_\sigma, \psi_2, \omega} \Psi_2 = \left(\begin{array}{c|c|c|c} D_{y_\sigma} f_\sigma & D_{p_\sigma} f_\sigma & 0 & D_\omega f_\sigma \\ \hline \dots I_J \dots_h & 0 & 0 & 0 \\ \hline 0 \quad [\dots \psi_{2h} I_J \dots_{h \neq h_1}] & 0 & * & 0 \\ \hline 0 & 0 & 2\psi_2^\top & 0 \end{array}\right),$$

which is column-equivalent to

$$\left(\begin{array}{c|cc|c} 0 & & * & D_\omega f_\sigma \\ \hline * & \left(\begin{array}{c|c} I_J & \dots I_J \dots_{h \neq h_1} \\ \hline 0 & \dots \psi_{2h} I_J \dots_{h \neq h_1} \end{array}\right) & 0 & 0 \\ \hline 2\psi_2^\top & & 0 & 0 \end{array}\right).$$

This is a block-triangular matrix. At any zero of Ψ_2, $\psi_2 \neq 0$ (since $\psi_2^\top \psi_2 = 1$), and hence the bottom left and middle blocks have full row rank. Furthermore, $D_\omega f_\sigma$ also has full row rank. It follows that the Jacobian $D_{y_\sigma, p_\sigma, \psi_2, \omega} \Psi_2$ has full row rank, at every zero of Ψ_2. Thus Ψ_2 is transverse to zero, and $\Psi_{2\omega}^{-1}(0) = \varnothing$ for all $\omega \in \Omega_{Y\sigma}$, a generic subset of Ω. The set Ω_Y in the statement of the lemma is given by $\Omega_Y := \cap_{\sigma \in \Sigma} \Omega_{Y\sigma}$.
$\square$

In order to establish conditions **C2** and **C3**, we will need to independently perturb the marginal utilities of agents for different signal realizations σ. This cannot be achieved via endowment perturbations, since endowments do not vary with respect to σ. Instead, we perturb the information structure π. More precisely, at any $\pi \in \Pi^0$, we consider derivatives with respect to the probabilities over S for two signal realizations σ_1 and σ_2, i.e. with respect to $\pi^1 := \{\pi_{s\sigma_1}\}_{s \in S}$ and $\pi^2 := \{\pi_{s\sigma_2}\}_{s \in S}$, along directions that leave π_{σ_1} and π_{σ_2} unchanged. In particular this ensures that the perturbed probabilities remain in Π^0.

We denote by $\overline{\pi}_{\sigma_i}$ the value of π_{σ_i} that remains fixed in such a perturbation (with derivatives being evaluated at π, this value is equal to $\sum_s \pi_{s\sigma_i}$). As a first step we show in Lemma A.2 below that, for a generic subset of endowments, the Jacobian of

$$Z(y_{\sigma_1}, p_{\sigma_1}, y_{\sigma_2}, p_{\sigma_2}, \pi^1, \pi^2, \omega) := \begin{pmatrix} f_{\sigma_1}(y_{\sigma_1}, p_{\sigma_1}, \pi^1, \omega) \\ \sum_s \pi_{s\sigma_1} - \overline{\pi}_{\sigma_1} \\ f_{\sigma_2}(y_{\sigma_2}, p_{\sigma_2}, \pi^2, \omega) \\ \sum_s \pi_{s\sigma_2} - \overline{\pi}_{\sigma_2} \end{pmatrix}$$

with respect to π^1 and π^2 has full row rank at any equilibrium. Lemma A.2 is invoked in all the lemmas that follow.[23]

Lemma A.2 *Suppose $S \geq J(H+1)+1$, and R is in general position. Let $\pi \in \Pi^0$ be an information structure with $\sum_s \pi_{s\sigma_i} = \overline{\pi}_{\sigma_i}$, $i = 1, 2$. Then, for a generic subset Ω_Z of Ω, at any π-equilibrium, $D_{\pi^1,\pi^2} Z$ has full row rank.*

Proof :
For $\sigma = \sigma_1, \sigma_2$, we have

$$f_\sigma = \sum_s \pi_{s\sigma}\, u^{h\prime}\left[\omega_s^h + r_s \cdot y_\sigma^h\right] r_s - \overline{\pi}_\sigma\, u_0^{h\prime}\left[\omega_0^h - p_\sigma \cdot y_\sigma^h\right] p_\sigma, \qquad h \in H.$$

Therefore,

$$D_{\pi^1,\pi^2} Z = \operatorname{diag}_{\sigma \in \{\sigma_1,\sigma_2\}} \left(\begin{array}{c} \vdots \\ \ldots \;\; u_{s\sigma}^{h\prime} r_s \;\; \ldots_s \\ \vdots_h \\ \hline \ldots \;\; 1 \;\; \ldots_s \end{array} \right).$$

Let

$$\Gamma_\sigma := \begin{pmatrix} \vdots \\ \ldots \;\; u_{s\sigma}^{h\prime} r_s \;\; \ldots_{s \leq JH+1} \\ \vdots_h \end{pmatrix}$$

and $\mathbf{1}^\top := (1\ 1 \ldots 1)_{1\times(JH+1)}$. We claim that, for a generic subset of Ω, at a π-equilibrium, the square matrix

$$\breve{\Gamma}_\sigma := \left(\begin{array}{c} \Gamma_\sigma \\ \hline \mathbf{1}^\top \end{array} \right)$$

has full rank, i.e. for $\psi_3 \in \mathbb{R}^{JH+1}$, there is no solution to

[23] The assumptions in the statement of Lemma A.2 are implied by those in the statements of Lemmas A.3, A.4 and A.5.

$$\Psi_3(y_\sigma, p_\sigma, \psi_3, \omega) := \begin{pmatrix} F_\sigma(y_\sigma, p_\sigma, \omega) \\ \Gamma_\sigma(y_\sigma, p_\sigma, \omega)\, \psi_3 \\ \mathbf{1}^\top \psi_3 \\ \psi_3^\top \psi_3 - 1 \end{pmatrix} = 0.$$

The Jacobian, $D_{y_\sigma, p_\sigma, \psi_3, \omega} \Psi_3$, is row-equivalent to

$$\left(\begin{array}{c|c|c} * & * & D_\omega \begin{pmatrix} f_\sigma \\ \Gamma_\sigma\, \psi_3 \end{pmatrix} \\ \hline 0 & \begin{array}{c} \mathbf{1}^\top \\ \psi_3^\top \end{array} & 0 \\ \hline D_{y_\sigma, p_\sigma} g_\sigma & 0 & 0 \end{array} \right). \tag{13}$$

We wish to show that this matrix has full row rank at any zero of Ψ_3. As we have seen already, $D_{y_\sigma, p_\sigma} g_\sigma$ has full row rank. Also, ψ_3 is orthogonal to $\mathbf{1}$ and nonzero (since $\psi_3^\top \psi_3 = 1$). Hence, due to the block-triangular structure of (13), it suffices to show that the upper right block, given by

$$D_\omega \begin{pmatrix} f_\sigma \\ \Gamma_\sigma\, \psi_3 \end{pmatrix} = \left(\begin{array}{c} \mathrm{diag}_h \left[\ldots\ \pi_{s\sigma}\, u_{s\sigma}^{h''}\, r_s\ \ldots_s \right] \\ \hline \mathrm{diag}_h \left[\left(\ldots \psi_{3s}\, u_{s\sigma}^{h''}\, r_s \ldots_{s \le JH+1} \right) \quad (0 \ldots 0) \right] \end{array} \right),$$

has full row rank. This matrix is row-equivalent to a block-diagonal matrix, with blocks indexed by h. The h'th block is

$$\left(\begin{array}{c|c} \ldots \pi_{s\sigma}\, u_{s\sigma}^{h''}\, r_s \ldots_{s \le JH+1} & \ldots \pi_{s\sigma}\, u_{s\sigma}^{h''}\, r_s \ldots_{s > JH+1} \\ \hline \ldots \psi_{3s}\, u_{s\sigma}^{h''}\, r_s \ldots_{s \le JH+1} & 0 \end{array} \right). \tag{14}$$

This matrix is block-triangular as well, and its upper right block has full row rank since it has at least J columns and R is in general position. It remains to show that the lower left block of (14) has full row rank. Let $\bar{S}$ be the subset of states for which $\psi_{3s} \neq 0$. This is a nonempty subset at any zero of Ψ_3.[24] Then we have $\sum_{s \in \bar{S}} \psi_{3s} u_{s\sigma}^{h'} r_s = 0$. Since R is in general position, and $u_{s\sigma}^{h'}$ is nonzero for all s, we must have $\#\bar{S} \geq J + 1$. Full row rank of the lower left block of (14) now follows from the general position of R.

We have shown that the Jacobian $D_{y_\sigma, p_\sigma, \psi_3, \omega} \Psi_3$ has full row rank, at every zero of Ψ_3. Thus Ψ_3 is transverse to zero, and $\Psi_{3\omega}^{-1}(0) = \varnothing$ for all $\omega \in \Omega_{Z\sigma}$, a generic subset of Ω. This establishes that $\breve{\Gamma}_\sigma$ has full row rank. It follows that $D_{\pi^1, \pi^2} Z$ has full row rank for the generic subset of endowments $\Omega_Z := \Omega_{Z\sigma_1} \cap \Omega_{Z\sigma_2}$. $\square$

Lemma A.3 *Suppose $S \geq J(H+1) + 1$, $J \geq H - 1$, and R is in general position. Then, for a generic subset of $\Omega \times \Pi$, condition* **C2** *is satisfied at any equilibrium.*

[24] $\bar{S}$ may depend on the zero of Ψ_3 we are considering, but this does not affect our argument.

Proof :
Let $\pi \in \Pi^0$. We restrict endowments to lie in the generic subset $\Omega_Y \cap \Omega_Z$ of Ω for which condition **C1** holds and $D_{\pi^1,\pi^2} Z$ has full row rank (see Lemmas A.1 and A.2), and show that, for a generic subset of Π^0, and hence of Π, there is no solution to

$$\Psi_4(y_{\sigma_1}, p_{\sigma_1}, y_{\sigma_2}, p_{\sigma_2}, \pi^1, \pi^2) := \begin{pmatrix} F_{\sigma_1}(y_{\sigma_1}, p_{\sigma_1}, \pi^1) \\ F_{\sigma_2}(y_{\sigma_2}, p_{\sigma_2}, \pi^2) \\ \sum_s \pi_{s\sigma_1} - \overline{\pi}_{\sigma_1} \\ \sum_s \pi_{s\sigma_2} - \overline{\pi}_{\sigma_2} \\ u^{h_1\prime}_{0\sigma_1} u^{h_2\prime}_{0\sigma_2} - u^{h_1\prime}_{0\sigma_2} u^{h_2\prime}_{0\sigma_1} \end{pmatrix} = 0.$$

The Jacobian, $D_{y_{\sigma_1}, p_{\sigma_1}, y_{\sigma_2}, p_{\sigma_2}, \pi^1, \pi^2} \Psi_4$, is row/column-equivalent to

$$\left(\begin{array}{ccc|c} & * & & D_{\pi^1,\pi^2} Z \\ \hline D_{y_{\sigma_1}} g_{\sigma_1} & 0 & 0 & \\ 0 & D_{y_{\sigma_2}} g_{\sigma_2} & 0 & 0 \\ * & * & D_{p_{\sigma_1}, p_{\sigma_2}} \left[u^{h_1\prime}_{0\sigma_1} u^{h_2\prime}_{0\sigma_2} - u^{h_1\prime}_{0\sigma_2} u^{h_2\prime}_{0\sigma_1} \right] & \end{array}\right). \tag{15}$$

Furthermore,

$$D_{p_{\sigma_1}} \left[u^{h_1\prime}_{0\sigma_1} u^{h_2\prime}_{0\sigma_2} - u^{h_1\prime}_{0\sigma_2} u^{h_2\prime}_{0\sigma_1} \right] = u^{h_1\prime}_{0\sigma_2} u^{h_2\prime\prime}_{0\sigma_1} y^{h_2}_{\sigma_1} - u^{h_1\prime\prime}_{0\sigma_1} u^{h_2\prime}_{0\sigma_2} y^{h_1}_{\sigma_1},$$

which is nonzero since $y^{h_1}_{\sigma}$ is not collinear with $y^{h_2}_{\sigma}$ by condition **C1**. Since $D_{y_\sigma} g_\sigma$ has full row rank for all σ, it follows that the lower left block of (15) has full row rank. Since $D_{\pi^1,\pi^2} Z$ also has full row rank, the Jacobian $D_{y_{\sigma_1}, p_{\sigma_1}, y_{\sigma_2}, p_{\sigma_2}, \pi^1, \pi^2} \Psi_4$ has full row rank. Thus Ψ_4 is transverse to zero, and $\Psi^{-1}_{4\pi}(0) = \varnothing$ for all π in a generic subset of Π^0. $\square$

In order to establish condition **C3**, we will employ finite-dimensional perturbations of agents' period 1 utility functions. This procedure requires the following result.

Lemma A.4 *Suppose $S \geq J(H+1)+1$, and R is in general position. Then, for a generic subset $\Omega_C \times \Pi_C$ of $\Omega \times \Pi$, at any equilibrium, $c^h_{\hat{s}\hat{\sigma}} \neq c^h_{\check{s}\check{\sigma}}$ for all $(\hat{s}, \hat{\sigma}) \neq (\check{s}, \check{\sigma})$, and for all $h \in H$.*

Proof :
Consider first the case where $\hat{s} \neq \check{s}$ and $\hat{\sigma} = \check{\sigma}$. Without loss of generality, we can take $\hat{s} = s_1$ and $\check{s} = s_2$, and prove the result for the first agent, h_1, for a given σ. We will show that, for a generic subset of Ω, there is no solution to

$$\Psi_5(y_\sigma, p_\sigma, \omega) := \begin{pmatrix} F_\sigma(y_\sigma, p_\sigma, \omega) \\ c^{h_1}_{s_1\sigma} - c^{h_1}_{s_2\sigma} \end{pmatrix} = 0.$$

The Jacobian, $D_{y_\sigma,p_\sigma,\omega}\Psi_5$, is row-equivalent to

$$\left(\begin{array}{c|c} * & D_\omega\begin{pmatrix} f_\sigma \\ c^{h_1}_{s_1\sigma}-c^{h_1}_{s_2\sigma}\end{pmatrix} \\ \hline D_{y_\sigma,p_\sigma}g_\sigma & 0 \end{array}\right),$$

and

$$D_\omega\begin{pmatrix} f_\sigma \\ c^{h_1}_{s_1\sigma}-c^{h_1}_{s_2\sigma}\end{pmatrix} = \left(\begin{array}{c} \mathrm{diag}_h[\ldots\ \pi_{s\sigma}\, u^{h\prime\prime}_{s\sigma}\, r_s\ \ldots_s] \\ \hline 1\quad -1\quad 0_{1\times(S-2)}\ \big|\ 0 \end{array}\right). \tag{16}$$

Note that the zero block in the lower right of (16) corresponds to all agents other than h_1. The matrix (16) is row-equivalent to

$$\left(\begin{array}{ccc|c} \pi_{s_1\sigma}\, u^{h_1\prime\prime}_{s_1\sigma} r_{s_1} & \pi_{s_2\sigma}\, u^{h_1\prime\prime}_{s_2\sigma} r_{s_2} & [\ldots\pi_{s\sigma}\, u^{h_1\prime\prime}_{s\sigma} r_s\ldots_{s>2}] & \\ & & & 0 \\ 1 & -1 & 0 & \\ \hline & 0 & & \mathrm{diag}_{h\neq h_1}[\ldots\pi_{s\sigma}\, u^{h\prime\prime}_{s\sigma}\, r_s\ldots_s] \end{array}\right). \tag{17}$$

The lower right submatrix of (17) has full row rank since R has full column rank. The $(1,3)$ block of the upper left submatrix of (17) also has full row rank due to the general position of R (note that the assumed dimensionality condition implies that $S \geq J+2$). Consequently, the whole matrix (17) has full row rank, and hence so does (16). Furthermore, due to the full row rank of $D_{y_\sigma,p_\sigma}g_\sigma$, the Jacobian $D_{y_\sigma,p_\sigma,\omega}\Psi_5$ has full row rank as well. Thus Ψ_5 is transverse to 0, and $\Psi_{5\omega}^{-1}(0)=\varnothing$ for all $\omega\in\Omega_{1\sigma}$, a generic subset of Ω. Let $\Omega_1 := \cap_{\sigma\in\Sigma}\Omega_{1\sigma}$.

Now consider the case where $\hat\sigma \neq \check\sigma$, while $\hat s$ may or may not be equal to $\check s$. Without loss of generality, we can take $\hat\sigma=\sigma_1$, $\check\sigma=\sigma_2$, and $\hat s = s_1$, and prove the result for the first agent, h_1. Let $\pi\in\Pi^0$. We restrict endowments to the generic subset Ω_Z for which $D_{\pi^1,\pi^2}Z$ has full row rank (see Lemma A.2), and show that, for a generic subset of Π^0, and hence of Π, there is no solution to

$$\Psi_6(y_{\sigma_1},p_{\sigma_1},y_{\sigma_2},p_{\sigma_2},\pi^1,\pi^2) := \begin{pmatrix} F_{\sigma_1}(y_{\sigma_1},p_{\sigma_1},\pi^1) \\ F_{\sigma_2}(y_{\sigma_2},p_{\sigma_2},\pi^2) \\ \sum_s \pi_{s\sigma_1}-\overline{\pi}_{\sigma_1} \\ \sum_s \pi_{s\sigma_2}-\overline{\pi}_{\sigma_2} \\ c^{h_1}_{s_1\sigma_1}-c^{h_1}_{\check s\sigma_2}\end{pmatrix} = 0.$$

The Jacobian, $D_{y_{\sigma_1},p_{\sigma_1},y_{\sigma_2},p_{\sigma_2},\pi_1,\pi_2}\Psi_6$, is row-equivalent to

$$\left(\begin{array}{c|c} * & D_{\pi^1,\pi^2}Z \\ \hline \left(\begin{array}{c|c} D_{y_{\sigma_1},p_{\sigma_1}}\begin{pmatrix} c^{h_1}_{s_1\sigma_1} \\ g_{\sigma_1}\end{pmatrix} & * \\ \hline 0 & D_{y_{\sigma_2},p_{\sigma_2}}g_{\sigma_2}\end{array}\right) & 0 \end{array}\right).$$

Since $D_{\pi^1,\pi^2}Z$ has full row rank, it suffices to show that the lower left block of the above matrix has full row rank. This block is itself block-triangular, and $D_{y_{\sigma_2},p_{\sigma_2}}g_{\sigma_2}$ has full row rank. The other diagonal term, given by

$$D_{y_{\sigma_1},p_{\sigma_1}}\begin{pmatrix} c^{h_1}_{s_1\sigma_1} \\ g_{\sigma_1}\end{pmatrix} = \left(\begin{array}{c|c|c} r_{s_1}^\top & 0_{1\times J(H-1)} & 0 \\ \hline I_J & \ldots I_J \ldots_{h\neq h_1} & 0 \end{array}\right),$$

also has full row rank since r_{s_1} is nonzero by the general position of R.

It follows that the Jacobian $D_{y_{\sigma_1},p_{\sigma_1},y_{\sigma_2},p_{\sigma_2},\pi^1,\pi^2}\Psi_6$ has full row rank, at every zero of Ψ_6. Thus Ψ_6 is transverse to zero, and $\Psi_{6\pi}^{-1}(0) = \varnothing$ for all $\pi \in \Pi_C$, a generic subset of Π^0. The set Ω_C in the statement of the lemma is given by $\Omega_C := \Omega_1 \cap \Omega_Z$. $\square$

The space of utility functions $\mathcal{U}$ is infinite-dimensional. For the genericity arguments that we use in order to establish condition **C3**, it suffices to consider a finite-dimensional submanifold of $\mathcal{U}$. This submanifold consists of linear perturbations of the von Neumann-Morgenstern utility index u^h of each agent h in the neighborhood of consumption in each state (s,σ), for a given equilibrium. This is a standard construction (see, for example, Citanna et al. (1998)).[25]

Consider an economy $(\check{\omega},\check{u},\check{\pi}) \in \Omega_C \times \mathcal{U}^H \times \Pi_C$, and a corresponding equilibrium with period 1 consumption allocation $\{\check{c}^h_{s\sigma}\}_{h\in H,s\in S,\sigma\in\Sigma}$. By Lemma A.4, the consumption level of agent h, $\check{c}^h_{s\sigma}$, is distinct across (s,σ), for all h. Therefore, we can find open intervals $\check{B}^h_{s\sigma}, B^h_{s\sigma}$ such that $\check{c}^h_{s\sigma} \in \check{B}^h_{s\sigma} \subsetneq B^h_{s\sigma} \subset \mathbb{R}_{++}$, where the intervals $B^h_{s\sigma}$ are disjoint across (s,σ). Define $\mathcal{C}^2$ functions $\rho^h_{s\sigma} : \mathbb{R}_{++} \to [0,1]$ such that $\rho^h_{s\sigma} = 1$ on $\check{B}^h_{s\sigma}$ and $\rho^h_{s\sigma} = 0$ on the complement of $B^h_{s\sigma}$.[26] Now consider the class of utility functions u^h parametrized by $\nu^h \in \mathbb{R}^{S\Sigma}$:

$$u^h(c,\nu^h) := \check{u}^h(c) + \sum_{s,\sigma} \rho^h_{s\sigma}(c)\,\nu^h_{s\sigma}(c - \check{c}^h_{s\sigma}).$$

It can be verified that, for ν^h sufficiently small in norm, $u^h \in \mathcal{U}$. We have

$$u^{h\prime}(c,\nu^h) = \check{u}^{h\prime}(c) + \sum_{s,\sigma} \rho^{h\prime}_{s\sigma}(c)\,\nu^h_{s\sigma}(c - \check{c}^h_{s\sigma}) + \sum_{s,\sigma} \rho^h_{s\sigma}(c)\,\nu^h_{s\sigma},$$

[25] Unlike Citanna et al. (1998), we perturb the gradient of the utility functions instead of their Hessian. Also, we have state-independent separable utility so additional care has to be exercised in perturbing utilities in different states.

[26] The existence of such a "bump" function is well-known. See Guillemin and Pollack (1974), chapter 1.

so that

$$D_{\nu^h_{s\sigma}} u^{h\prime}(\check{c}^h_{s\sigma}, \nu^h) = 1. \tag{18}$$

Let $\nu_\sigma := \{\nu^h_{s\sigma}\}_{h \in H, s \in S}$, and $\nu := \{\nu_\sigma\}_{\sigma \in \Sigma}$. In order to show that condition **C3** holds, we will perturb agents' period 1 utility functions via perturbations of ν.

Lemma A.5 *Suppose $S \geq 2JH$, $J \geq H - 1$, and R is in general position. Then, for a generic subset of $\mathcal{E}$, condition* **C3** *is satisfied at any equilibrium.*

Proof :
Let Λ_σ be the $JH \times S$ matrix defined by

$$\Lambda_\sigma := \begin{pmatrix} & \vdots & \\ \ldots & \left(u^{h\prime}_{s\sigma} r_s - u^{h\prime}_{0\sigma} p_\sigma\right) & \ldots_s \\ & \vdots_h & \end{pmatrix}.$$

We first establish that

$$\Lambda := \begin{pmatrix} \Lambda_{\check{\sigma}} \\ \Lambda_{\hat{\sigma}} \end{pmatrix}$$

has full row rank, for all $\check{\sigma} \neq \hat{\sigma}$. Without loss of generality we can take $\check{\sigma} = \sigma_1$ and $\hat{\sigma} = \sigma_2$. Let $\hat{\Lambda}$ be the (square) submatrix of Λ consisting of the first $2JH$ columns of Λ. Let $\psi_7 \in \mathbb{R}^{2JH}$. We will show that, for a generic subset of $\mathcal{U}^H \times \Pi$, there is no solution to

$$\Psi_7(y_{\sigma_1}, p_{\sigma_1}, y_{\sigma_2}, p_{\sigma_2}, \psi_7, \nu_{\sigma_1}, \nu_{\sigma_2}, \pi^1, \pi^2) := \begin{pmatrix} F_{\sigma_1}(y_{\sigma_1}, p_{\sigma_1}, \nu_{\sigma_1}, \pi^1) \\ F_{\sigma_2}(y_{\sigma_2}, p_{\sigma_2}, \nu_{\sigma_2}, \pi^2) \\ \sum_s \pi_{s\sigma_1} - \overline{\pi}_{\sigma_1} \\ \sum_s \pi_{s\sigma_2} - \overline{\pi}_{\sigma_2} \\ \hat{\Lambda}(y_{\sigma_1}, p_{\sigma_1}, y_{\sigma_2}, p_{\sigma_2}, \nu_{\sigma_1}, \nu_{\sigma_2})\psi_7 \\ \psi_7^\top \psi_7 - 1 \end{pmatrix} = 0.$$

We restrict ourselves to the generic subset of endowments and information structures $(\Omega_R \cap \Omega_C) \times \Pi_C$ for which Lemma A.4 applies, so that we can parametrize agents' period 1 utility functions by the vector ν, and for which the number of equilibria is finite. Recall that Ω_C is a subset of Ω_Z, for which $D_{\pi^1,\pi^2} Z$ has full row rank (see Lemma A.2).

The Jacobian, $D_{y_{\sigma_1}, p_{\sigma_1}, y_{\sigma_2}, p_{\sigma_2}, \psi_7, \nu_{\sigma_1}, \nu_{\sigma_2}, \pi^1, \pi^2} \Psi_7$, is row-equivalent to

$$\left(\begin{array}{ccc|c|c} & * & & * & D_{\pi^1,\pi^2} Z \\ \hline & * & & D_{\nu_{\sigma_1}, \nu_{\sigma_2}}(\hat{\Lambda}\psi_7) & 0 \\ \hline D_{y_{\sigma_1}, p_{\sigma_1}} g_{\sigma_1} & 0 & 0 & & \\ 0 & D_{y_{\sigma_2}, p_{\sigma_2}} g_{\sigma_2} & 0 & 0 & 0 \\ 0 & 0 & \psi_7^\top & & \end{array}\right). \tag{19}$$

We wish to show that this matrix has full row rank at any zero of Ψ_7. Since $D_{\pi^1,\pi^2}Z$, $D_{y_{\sigma_1},p_{\sigma_1}}g_{\sigma_1}$ and $D_{y_{\sigma_2},p_{\sigma_2}}g_{\sigma_2}$ have full row rank, and $\psi_7 \neq 0$, it suffices to show that the middle block of (19) has full row rank. Using (18), we see that this block is block-diagonal with respect to $h \in H$ and $\sigma \in \{\sigma_1, \sigma_2\}$, with typical diagonal term given by $[\ldots \psi_{7s} r_s \ldots_{s \leq 2JH}]$. This diagonal term has full row rank by the same argument that we used for the bottom left block of (14). It follows that the Jacobian $D_{y_{\sigma_1},p_{\sigma_1},y_{\sigma_2},p_{\sigma_2},\psi_7,\nu_{\sigma_1},\nu_{\sigma_2},\pi^1,\pi^2}\Psi_7$ has full row rank, at every zero of Ψ_7. Thus Ψ_7 is transverse to zero, and $\Psi_{7u\pi}^{-1}(0) = \varnothing$ for all (u, π) in a generic subset of $\mathcal{U}^H \times \Pi_C$.

Now that we have established that Λ has full row rank, consider the matrix Q, defined in (12). We have

$$Q = \left(\begin{array}{c} \mathrm{diag}_\sigma[\Lambda_\sigma] \\ \hline \ldots \ I_S \ \ldots_\sigma \end{array} \right).$$

The upper submatrix of Q has full row rank due to the full row rank of Λ, and clearly the lower submatrix of Q has full row rank as well. If Q does not have full row rank, there exist vectors $a_\sigma \in \mathbb{R}^{JH}$, $\sigma \in \Sigma$, and $b \in \mathbb{R}^S$, not all of which are zero, such that $a_\sigma^\top \Lambda_\sigma + b = 0$, for all σ. Moreover, since Λ_σ has full row rank, $b \neq 0$. It follows that the row spaces of $\{\Lambda_\sigma\}_{\sigma \in \Sigma}$ have a nontrivial intersection. But this contradicts the full row rank property of Λ.

The utility perturbations in this proof apply only to the particular equilibrium under consideration. However, we can repeat the same construction for each equilibrium, and take the intersection of the generic subsets for which the Jacobian of Ψ_7 has full row rank. This intersection is itself a generic subset since the number of equilibria is finite (recall that endowments are restricted to Ω_R). $\square$

Proof of Corollary 1:
Let $\mathcal{E}^*$ be the generic subset of economies for which Theorem 2 applies. An inspection of the proofs of Lemmas A.1–A.5 reveals that $\mathcal{E}^*$ can be written as $\Omega^* \times \Theta$, where Ω^* is a generic subset of Ω, and Θ is a generic subset of $\mathcal{U}^H \times \Pi$ (in particular, we do not perturb endowments jointly with either utilities or probabilities in any of the transversality arguments in these proofs). Moreover, Ω^* is a subset of Ω_R, the set of regular economies (this condition was imposed in the proof of Lemma A.5).

Fix a (u, π) in Θ, and consider an ω^* in Ω^*. Since the economy (ω^*, u, π) is regular, it has a finite number of equilibria, which we index by $i = 1, \ldots, n$. By Theorem 2, for the i'th π-equilibrium, there exist $\hat{\pi}_i$ and $\hat{\pi}'_i$ in $\hat{\Pi}(\pi)$, such that $W \ll 0$ at a $\hat{\pi}_i$-equilibrium and $W \gg 0$ at a $\hat{\pi}'_i$-equilibrium (the information structures $\hat{\pi}_i$ and $\hat{\pi}'_i$ are in a neighborhood of π, and the corresponding equilibria are in a neighborhood of the i'th π-equilibrium). Using the regularity of (ω^*, u, π) once again, there exists $\epsilon_i > 0$, such that the preceding statement applies for all $\omega \in B_{\epsilon_i}(\omega^*) := \{\omega \in \Omega^* \ s.t. \ |\omega - \omega^*| < \epsilon_i\}$, for the same choice of $\hat{\pi}_i$ and $\hat{\pi}'_i$.

By Theorem 1, there is a generic subset $\hat{\Omega}_i$ of Ω such that every π-equilibrium is ex-post $\hat{\pi}_i$-inefficient, and a generic subset $\hat{\Omega}'_i$ of Ω such that every π-equilibrium is ex-post $\hat{\pi}'_i$-inefficient. Therefore, statements (i)–(iii) of the corollary apply for all

$\omega \in O_{\omega^*} := \cap_{i=1}^n [B_{\epsilon_i}(\omega^*) \cap \hat{\Omega}_i \cap \hat{\Omega}'_i]$. Indeed, they apply for all $\omega \in \bar{\Omega} := \cup_{\omega^* \in \Omega^*} O_{\omega^*}$. We claim that $\bar{\Omega}$ is a generic subset of Ω. Let $\epsilon := \min_i \epsilon_i$. Then, clearly, the set O_{ω^*} is an open, dense subset of $B_\epsilon(\omega^*)$. The set $\bar{\Omega}$, being the union of these open subsets of Ω^*, is itself an open subset of Ω^*. Moreover, $\bar{\Omega}$ is dense in Ω^*, since for all $\omega^* \in \Omega^*$, there is a corresponding set O_{ω^*}, the closure of which contains ω^*. Thus $\bar{\Omega}$ is a generic subset of Ω^*, and hence also of Ω. $\square$

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