A data envelopment analysis approach to evaluate the performance of mutual funds

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1. Introduction

The data envelopment analysis (DEA) is an optimization based technique that has been proposed by Charnes, Cooper and Rhodes (1978) to measure the relative efficiency of public sector activities and no profit organizations, such as for example educational institutions and health services.

The DEA efficiency measure is computed by solving a fractional linear programming model that can be converted into an equivalent linear programming problem which can be easily solved.

Afterwards, the same methodology has been applied to many profit oriented companies, too. For a review of various applications, see for example Seiford (1996).

The main purpose of this contribution is to use the DEA methodology in order to compute a mutual fund performance index that can take into account many conflicting objectives together with the costs required by the investment. In particular, the traditional performance indexes proposed in the literature do not allow to consider investment costs such as the subscription and redemption costs, while the DEA approach can naturally include many costs among the inputs of the model.

It will be seen that the DEA performance index for mutual funds can be considered as a generalization of many traditional ratios such as Sharpe, Treynor, and reward to half-variance ratios.

Moreover, the results of the DEA technique can be used in order to identify, for each inefficient decision making unit, a corresponding efficient set (called peer group) which represents a "virtual" composite portfolio. This composite portfolio can be seen as a personalized benchmark and characterizes the portfolio style.

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The structure of the paper is as follows. In Section 2 we briefly describe the data envelopment analysis approach and focus in particular on the input-oriented CCR model. In Section 3 we recall some traditional numerical indexes that have been proposed in the literature to evaluate the performance of mutual funds. The DEA performance index of mutual fund investments is proposed in Section 4. In Section 5 the DEA index is tested by measuring the relative efficiency of a set of mutual funds on the Italian financial market. Section 6 describes how to build composite portfolios (peer groups) to be used as benchmarks. Finally, some concluding remarks are presented in Section 7.

2. The data envelopment analysis approach

The data envelopment analysis (DEA) is an optimization based technique that allows to measure the relative performance of decision making units which are characterized by a multiple objectives and/or multiple inputs structure.

Operational units of this kind, for example, tipically include no profit and governmental units such as schools, hospitals, universities. In these units, the presence of a multiple output–multiple input situation makes difficult to identify an evident efficiency indicator such as profit and complicates the search for a satisfactory measure of efficiency.

Charnes, Cooper and Rhodes (1978) proposes a measure of efficiency which is essentially defined as a ratio of weighted outputs to weighted inputs. In a sense, the weighted sums allow to reduce the multiple input–multiple output situation to a single "virtual" input–"virtual" output case; the efficiency measure is then taken as the ratio of the virtual output to the virtual input. Of course, the higher the efficiency ratio is, the more efficient the unit is.

Such a weighted ratio requires a set of weights to be defined and this can be not easy. Charnes, Cooper and Rhodes's idea is to define the efficiency measure by assigning to each unit the most favourable weights. On the one hand, this means that the weights will generally not be the same for the different units. On the other hand, if a unit turns out to be inefficient, compared to the other ones, when the most favourable weights are chosen, we cannot say that this depends on the choice of the weights!

The most favourable weights are chosen as the ones which maximize the efficiency ratio of the unit considered, subject to the constraints that the efficiency ratios of the other units, computed with the same weights, have an upper bound of 1. Therefore, an efficiency measure equal to 1 characterizes the efficient units: at least with the most favourable weights, these units cannot be dominated by the other ones in the set. As a result we obtain a Pareto efficiency measure in which the efficient units lie on the efficient frontier (see Charnes, Cooper, Lewin and Seiford (1994)).

Let us define:

$j = 1, 2, \ldots, n$	decision making units
$r = 1, 2, \ldots, t$	outputs
$i = 1, 2, \ldots, m$	inputs
y_{rj}	amount of output r for unit j

x_{ij}	amount of input i for unit j
u_r	weight given to output r
v_i	weight given to input i

DEA efficiency measure for the decision making unit j_0 ($j_0 = 1, 2, \dots, n$) is computed by solving the following fractional linear programming model

$$\max_{\{v_i, u_r\}} \quad h_0 = \frac{\sum_{r=1}^{t} u_r y_{rj_0}}{\sum_{i=1}^{m} v_i x_{ij_0}} \tag{2.1}$$

subject to

$$\begin{aligned} &\frac{\sum_{r=1}^{t} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1 \qquad && j = 1, \dots, n \\ &u_r \geq \epsilon && r = 1, \dots, t \\ &v_i \geq \epsilon && i = 1, \dots, m \end{aligned} \tag{2.2}$$

where ϵ is a convenient small positive number that prevents the weights be zero.

The above ratio form has an infinite number of optimal solutions: in fact, if $(v_1, \dots, v_m, u_1, \dots, u_t)$ is optimal, then $\beta(v_1, \dots, v_m, u_1, \dots, u_t)$ is also optimal for all $\beta > 0$. One can define an equivalence relation that partitions the set of feasible solutions of problem (2.1)-(2.2) into equivalence classes. Charnes and Cooper (1962) proposes to select a representative solution from each equivalence class. The representative solution that is usually chosen in DEA modelling is that for which $\sum_{i=1}^{m} v_i x_{ij_0} = 1$ in the input-oriented forms and that with $\sum_{r=1}^{t} u_r y_{ij_0} = 1$ in the output-oriented models.

In this way the fractional problem (2.1)-(2.2) can be converted into an equivalent linear programming problem which can be easily solved. Using the input-oriented form we thus obtain the so called input-oriented CCR (Charnes, Cooper and Rhodes) linear model; its dual problem is also useful, both for computational convenience (as it has usually less constraints than the primal problem) and for its significance:

Input-oriented CCR primal model

$$\max \quad \sum_{r=1}^{t} u_r y_{rj_0}$$

subject to

$$\begin{aligned} &\sum_{i=1}^{m} v_i x_{ij_0} = 1 \\ &\sum_{r=1}^{t} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \\ &\qquad\qquad j = 1, \dots, n \\ &-u_r \leq -\epsilon \qquad r = 1, \dots, t \\ &-v_i \leq -\epsilon \qquad i = 1, \dots, m \end{aligned}$$

Input-oriented CCR dual model

$$\min \quad z_0 - \epsilon \sum_{r=1}^{t} s_r^+ - \epsilon \sum_{i=1}^{m} s_i^-$$

subject to

$$\begin{aligned} &x_{ij_0} z_0 - s_i^- - \sum_{j=1}^{n} x_{ij} \lambda_j = 0 \quad && i = 1, \dots, m \\ &-s_r^+ + \sum_{j=1}^{n} y_{rj} \lambda_j = y_{rj_0} && r = 1, \dots, t \\ &\lambda_j \geq 0 && j = 1, \dots, n \\ &s_i^- \geq 0 && i = 1, \dots, m \\ &s_r^+ \geq 0 && r = 1, \dots, t \\ &z_0 \quad \text{unconstrained.} \end{aligned}$$

The CRR primal problem has $t+m$ variables (the weights u_r and v_i which have to be chosen so as to maximize the efficiency of the targeted unit j_0) and $n+t+m+1$ constraints.

It can be seen (see for example Charnes, Cooper, Lewin and Seiford (1994)) that CCR model gives a piecewise linear production surface which, in economic terms, represents a production frontier: in fact, it gives the maximum output empirically obtainable from a decision making unit given its level of inputs; from another point of view, it gives the minimum amount of input required to achieve the given output levels. The input-oriented models focus on the maximal movement toward the frontier through a reduction of inputs, whereas the output-oriented ones consider the maximal movement via an augmentation of outputs.

DEA model (2.1)-(2.2) is the first, simplest and still most used DEA technique. Nevertheless, in the meantime a number of extensions and variants have been proposed in the literature to better cope with special purposes; for a review see for example Charnes, Cooper, Lewin and Seiford (1994). Though born to evaluate the efficiency of no profit institutions, soon afterwards the DEA technique has been applied to measure the efficiency of any organizational unit; for example it has largely been used to compare the performance of different bank branches.

3. Numerical indexes for measuring mutual funds performance

In order to completely rank a set of investment funds, some numerical indexes have been proposed in the literature that evaluate the fund performance by taking into account both expected return and risk and synthesize them in a unique numerical value. On the other hand, we have to point out that these indicators do allow to compare any couple of portfolios, by suggesting to choose the one with the higher index value, but are based on strong assumptions on the market behaviour and investors' preferences.

Let us consider one of the alternative portfolios whose performance has to be evaluated and let R be the random portfolio return. Let us denote by $E(R)$ the expected return and by $\sigma_R = \sqrt{Var(R)}$ the standard deviation of the return. Moreover, let us define by $E(R) - r$ the expected excess return as the difference between the portfolio expected return and the riskless rate of return.

One of the most used performance indicator is the *reward to variability ratio* proposed by Sharpe (1966), which is defined as the ratio between the expected excess return and the standard deviation

$$I_{Sharpe} = \frac{E(R) - r}{\sigma_R}. \tag{3.1}$$

Sharpe index measures the portfolio performance by means of the expected differential return $E(R) - r$ per unit of risk. We observe that the standard deviation of the returns may be a proper risk measure when the investor holds only one risky asset and the returns probability distribution is symmetric.

To relax, at least partly, the strongest assumptions of Sharpe ratio some variants of the reward to variability ratio index have been proposed.

Ang and Chua (1979) suggests to use two performance indexes having the same meaning of Sharpe ratio but using two different risk indicators which take into account only the (undesirable) negative deviations from the mean or from a threshold value. The *reward to half-variance index*

$$I_{half-var} = \frac{E(R) - r}{HV_R} \tag{3.2}$$

measures the risk using the *half-variance* risk indicator that represents the average of the squared negative deviations from the mean

$$HV_R = E\left(\min\left[R - E(R), 0\right]\right)^2 . \tag{3.3}$$

Let us notice that when the return random variable is symmetric $HV = 2\sigma^2$.

The *reward to semivariance index*, instead,

$$I_{semivar} = \frac{E(R) - r}{SV_R} \tag{3.4}$$

measures the risk with *semivariance* which is the average of the squared negative deviations from a fixed threshold value h ; this value represents the minimum target for the returns to be considered desirable by the investor

$$SV_R = E\left(\min\left[R - h, 0\right]\right)^2 . \tag{3.5}$$

Another performance measure with a structure that is analogous to Sharpe ratio is the *reward to volatility ratio* proposed by Treynor (1965)

$$I_{Treynor} = \frac{E(R) - r}{\beta} \tag{3.6}$$

where the portfolio risk is measured by the β of the portfolio, i.e. the ratio of the covariance between the portfolio return R and the market portfolio return R_m to the variance $Var(R_m)$ of the market portfolio return

$$\beta = \frac{Cov(R, R_m)}{Var(R_m)}. \tag{3.7}$$

We may observe that using β coefficient as risk indicator entails the assumption that the investor has diversified his investments so that they are equivalent to a quota of the market portfolio. Moreover, we have to point out that Treynor index assumes $\beta > 0$ and is meaningless otherwise.

Jensen (1968) introduces an index that finds inspiration in the volatility estimation of a risky portfolio obtained in the C.A.P.M. framework through the following linear regression

$$E(R) - r = J + \beta(E(R_m) - r). \tag{3.8}$$

Jensen index measures the portfolio performance by means of the intercept J of equation (3.8). In particular, a significantly positive value for the intercept J means that the mutual fund management has obtained positive results that overcome those obtained by the market portfolio.

4. A DEA measure of mutual funds performance

Other techniques that have been introduced to evaluate the performance of mutual funds refer to multi-criteria decision making methods; these approaches recognize the existence of a trade-off between conflicting objectives such as the portfolio expected return and its risk. Among these methods, we cite a PROMETHEE (*preference ranking organization methods for enrichment evaluation*) approach proposed by Cardin, Decima and Pianca (1992) and applied to the Italian market.

On the other hand, we have seen in Section 2 that DEA takes into account a multiple input-multiple output situation by computing a performance measure that is based on the virtual output/input ratio. Let us partition the conflicting objectives of a multi-criteria problem into two groups: an output set which includes the desirable objectives (those to be maximized) and an input set including the undesirable ones (those to be minimized). Then the DEA methodology may be used, in some sense, as a multi-criteria approach in which the weights that allow to aggregate the objectives are not fixed in advance in a subjective manner but are determined as the most favourable weights for each unit (and may be different for the various units). On this subject refer to Joro, Korhonen and Wallenius (1998) which makes a direct comparison between DEA and multiple objective linear programming models.

Our idea is to use the DEA methodology in order to compute a mutual fund performance index that can take into account many conflicting objectives together with the costs required by the investment.

In effect, we can assign the desirable objectives, such as the return, to the output set and both the undesirable objectives and the investment costs to the input set. For example, all the possible risk measures, such as standard deviation or half-variance, may be included among the inputs as undesirable objective to be minimized, while subscription costs and redemption fees are included in the input set as they represent investment costs that have to be taken into account.

By applying a DEA approach with these definitions, we obtain a performance measure that may be seen as a generalization of many of the traditional performance indicators presented in Section 3. In fact, Sharpe, Treynor, half-variance and semi-variance indexes are ratios between the expected excess return (an output) and a risk indicator (an input). The DEA efficiency measure is a ratio of a weighted sum of outputs (the portfolio return and eventually some other desirable objectives) to a weighted sum of inputs (one or more risk measure, the investment costs that the traditional index do not consider, and eventually some other undesirable features).

A first attempt to apply the DEA methodology in order to obtain an efficiency mutual fund indicator that modifies Sharpe index is the DPEI index developed by Murthi, Choi and Desai (1997). The DPEI index considers the mutual fund return as output and the standard deviation and transaction costs as inputs. Nevertheless, among the various transaction costs, they consider also operational expenses, management fees and purchase and sale costs incurred by the management, which are costs that have already been deduced from the net return. On the contrary, we have preferred to take into account subscription costs and redemption fees that directly burden the investors but not the expenses that have already been deduced from the net return of the portfolio.

As we have seen, the DEA approach allows to consider many outputs and many inputs. However, in this contribution we propose some performance measures that take into account only one output, the portfolio expected return $E(R)$, and many inputs. The inputs considered are a risk measure, (eventually) the β as a measure of risk which is relevant when the investor's portfolio is well diversified and one or more subscription and redemption costs.

Let us consider a set of n mutual funds with expected return $E(R_j)$, $j = 1, 2 \ldots, n$, and denote, as before, the levels of the m inputs of fund j by x_{ij}, $i = 1, 2, \ldots, m$. Let us compute the relative efficiency of fund j_0. The DEA performance index of mutual fund investments that we propose, I_{DEA}, is defined as the maximum value of the objective function, computed with respect to the output weight u and the input weights v_i, $i = 1, 2, \ldots, m$, of the following fractional linear problem

$$\max_{\{v_i,u\}} \quad I = \frac{uE(R_{j_0})}{\sum_{i=1}^{m} v_i x_{ij_0}} \tag{4.1}$$

subject to

$$\begin{aligned} &\frac{uE(R_j)}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, && j = 1, \ldots, n \\ &u \geq \epsilon \\ &v_i \geq \epsilon && i = 1, \ldots, m \end{aligned} \tag{4.2}$$

Using the same device discussed in Section 2, the fractional problem (4.1)-(4.2) can be converted into an equivalent input-oriented linear model. The resulting primal and dual problems are as follows

Mutual funds primal model

$$\max \quad uE(R_{j_0})$$

subject to

$$\begin{aligned} &\textstyle\sum_{i=1}^{m} v_i x_{ij_0} = 1 \\ &uE(R_j) - \textstyle\sum_{i=1}^{m} v_i x_{ij} \leq 0 \\ &\qquad\qquad j = 1, \ldots, n \\ &-u \leq -\epsilon \\ &-v_i \leq -\epsilon \qquad i = 1, \ldots, m \end{aligned}$$

Mutual funds dual model

$$\min \quad z_0 - \epsilon s^+ - \epsilon \textstyle\sum_{i=1}^{m} s_i^-$$

subject to

$$\begin{aligned} &x_{ij_0} z_0 - s_i^- - \textstyle\sum_{j=1}^{n} x_{ij}\lambda_j = 0 \quad i = 1, \ldots, m \\ &-s^+ + \textstyle\sum_{j=1}^{n} E(R_j)\lambda_j = E(R_{j_0}) \\ &\lambda_j \geq 0 \qquad j = 1, \ldots, n \\ &s_i^- \geq 0 \qquad i = 1, \ldots, m \\ &s^+ \geq 0 \\ &z_0 \quad \text{unconstrained.} \end{aligned}$$

By letting $\nu_i = v_i/u$, $i = 1, 2, \ldots, m$, problem (4.1)-(4.2) can equivalently be written in the following (output-oriented) reduced form

Reduced primal model

$$\min \quad \sum_{i=1}^{m} \nu_i \frac{x_{ij_0}}{E(R_{j_0})}$$

subject to

$$\sum_{i=1}^{m} \nu_i x_{ij} \geq E(R_j) \qquad j = 1, \ldots, n$$

$$\nu_i \geq \varepsilon \qquad i = 1, \ldots, m$$

Reduced dual model

$$\max \quad \sum_{j=1}^{n} \lambda_j E(R_j) + \varepsilon \sum_{i=1}^{m} s_i^-$$

subject to

$$\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = \frac{x_{ij_0}}{E(R_{j_0})} \qquad i = 1, \ldots, m$$

$$\lambda_j \geq 0 \qquad j = 1, \ldots, n$$

$$s_i^- \geq 0 \qquad i = 1, \ldots, m$$

where ε is a suitable small positive number.

5. An empirical analysis

We have tested the DEA performance index of mutual fund investments proposed in Section 4 on the Italian financial market. We have considered the weekly logarithmic returns of 47 mutual funds, for which homogeneous information are available, and of the Milan stock exchange Mibtel index. Moreover, we have also considered the instantaneous rate of return of the 12 months B.O.T. measured on a weekly base. The last two assets have been included as they can be considered, in some way and in a DEA context, as “natural benchmarks”. The data regard the Monday net prices in the period 1/1/1997 to 31/12/1998 (104 weeks).

The mutual funds have been chosen from different classes (using the Assogestioni classing valid in the period considered: Az denotes a stocks fund, Bi a balanced fund and Ob a bonds one), with different total capital and from different management companies, as follows

Total capital (thousand millions of lire)	Az2	Az3	Az4	Bi1	Bi2	Ob3	Ob5	Total
(0, 500]	7	2	2		3	5	2	21
(500, 1000]	2	1	1	1		3	1	9
(1000, 1500]		3		2	1			6
(1500, 2000]		1	1					2
(2000, 5200]	1	1	1	2		1	3	9
Total number	10	8	5	5	4	9	6	47

Az2 = International stocks funds, Az3 = Italian stocks funds, Az4 = European stocks funds, Bi1 = Italian balanced funds, Bi2 = International balanced funds, Ob3 = International bonds funds, Ob5 = Italian bonds funds.

As noted in the previous section, we consider as unique output the portfolio expected return. It is worth noting that we prefer to use the expected return as output instead of the excess return, as would be suggested by a generalization of the Sharpe index, in order to limit the presence of negative values among the outputs. Just to allow the comparison with the riskless rate of return, we have included a B.O.T. among the funds to be compared.

Among the inputs, we have considered a risk measure; this has been chosen either as the portfolio standard deviation σ_R or as the square root of the half-variance HV_R. Moreover, in some analysis we have also included the β coefficient

Table 1. Comparison between various traditional indexes (Sharpe, Treynor, Jensen and reward to half-variance) and the DEA performance indexes with 3 and 7 inputs.

Funds	Classes	Sharpe	Treynor	Jensen	Rew.HV	DEA 3in.	DEA 7in.
Stocks funds							
Arca 27	Az2	0.092	0.006	0.000	7.601	0.155	0.155
Azimut Borse Int.	Az2	0.096	0.006	0.003	5.875	0.125	0.128
Centrale Global	Az2	0.130	0.008	0.003	9.219	0.107	0.121
Epta-International	Az2	0.094	0.005	0.002	5.752	0.157	0.164
Fideuram Azione	Az2	0.105	0.007	0.003	6.281	0.094	0.113
Fondicri Int.	Az2	0.117	0.008	0.003	9.195	0.115	0.134
Genercomit Int.	Az2	0.102	0.007	0.002	7.108	0.123	0.147
Investire Int.	Az2	0.117	0.007	0.003	8.233	0.158	0.158
Prime Global	Az2	0.093	0.006	0.002	5.726	0.146	0.154
Sanpaolo H. Intern.	Az2	0.103	0.006	0.003	6.762	0.104	0.113
Centrale Italia	Az3	0.207	0.009	0.007	10.780	0.181	0.199
Epta Azioni Italia	Az3	0.177	0.008	0.006	8.833	0.203	0.210
Fondicri Sel. Italia	Az3	0.179	0.008	0.006	9.258	0.190	0.219
Genercomit Azioni Italia	Az3	0.190	0.008	0.006	10.654	0.238	0.239
Gesticredit Borsit	Az3	0.177	0.008	0.006	9.507	0.188	0.210
Imi Italy	Az3	0.184	0.008	0.007	8.836	0.162	0.182
Investire Azion.	Az3	0.184	0.008	0.006	9.522	0.231	0.232
Oasi Azionario Italia	Az3	0.172	0.007	0.006	8.844	0.275	0.275
Azimut Europa	Az4	0.124	0.006	0.004	6.630	0.152	0.155
Gesticredit Euro Az.	Az4	0.135	0.007	0.004	8.429	0.157	0.169
Imi Europe	Az4	0.144	0.008	0.004	7.882	0.122	0.139
Investire Europa	Az4	0.114	0.006	0.003	6.937	0.215	0.216
Sanpaolo H. Europe	Az4	0.148	0.008	0.004	10.751	0.122	0.132
Balanced funds							
Arca BB	Bi1	0.194	0.009	0.003	22.584	0.295	0.295
Azimut Bil.	Bi1	0.177	0.008	0.003	18.746	0.203	0.209
Eptacapital	Bi1	0.181	0.008	0.003	18.299	0.201	0.214
Genercomit	Bi1	0.199	0.009	0.004	19.917	0.179	0.223
Investire Bil.	Bi1	0.199	0.009	0.003	20.577	0.289	0.290
Arca TE	Bi2	0.115	0.009	0.002	15.707	0.207	0.207
Fideuram Performance	Bi2	0.128	0.006	0.003	10.704	0.198	0.198
Fondo Centrale	Bi2	0.119	0.007	0.002	12.281	0.079	0.094
Genercomit Espansione	Bi2	0.060	0.004	0.001	6.495	0.120	0.151
Bonds funds							
Arca Bond	Ob3	0.136	0.048	0.001	45.558	0.311	0.311
Azimut Rend. Int.	Ob3	0.043	0.022	0.000	9.067	0.095	0.099
Epta 92	Ob3	0.094	0.024	0.001	31.317	0.129	0.147
Genercomit Obb. Estere	Ob3	0.075	0.015	0.000	31.428	0.334	0.339
Imi Bond	Ob3	0.175	0.058	0.001	69.177	0.128	0.172
Investire Bond	Ob3	0.161	0.034	0.001	54.800	0.238	0.241
Oasi Bond Risk	Ob3	0.154	0.077	0.001	43.410	0.254	0.254
Primebond	Ob3	0.094	0.042	0.001	29.108	0.182	0.226
Sanpaolo H. Bonds	Ob3	0.073	0.027	0.000	26.161	0.098	0.116
Bpb Tiepolo	Ob5	0.159	0.031	0.000	288.082	1.000	1.000
Centrale Tasso Fisso	Ob5	0.233	0.039	0.001	123.803	0.215	0.269
Eptabond	Ob5	0.218	0.034	0.001	155.131	0.283	0.371
Fideuram Security	Ob5	-0.198	-0.026	0.000	-649.469	1.000	1.000
Oasi Btp Risk	Ob5	0.303	0.068	0.002	118.703	0.589	0.589
Prime Reddito Italia	Ob5	0.151	0.024	0.000	86.910	0.180	0.267
Mibtel		0.170	0.007	0.007	7.133	0.214	0.215
B.O.T.						1.000	1.000

Table 2. Results obtained with the DEA performance indexes for the different classes of stocks funds and for different inputs: with standard deviation or half-variance as a risk indicator, with a subscription and a redemption cost (3 inputs), with β as additional input (3+1), with all the subscription and redemption costs (7 inputs), with β as additional input (7+1).

Stocks funds	St.dev. 3	3+1	7	7+1	Half-var. 3	3+1	7	7+1
Az2								
Arca 27	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Azimut Borse Int.	0.383	0.383	0.414	0.410	0.377	0.377	0.407	0.407
Centrale Global	0.213	0.213	0.290	0.289	0.211	0.211	0.287	0.287
EptaInternational	0.428	0.428	0.505	0.500	0.422	0.422	0.495	0.495
Fideuram Azione	0.178	0.178	0.283	0.280	0.176	0.176	0.278	0.278
Fondicri Int.	0.260	0.260	0.415	0.413	0.257	0.257	0.408	0.408
Genercomit Int.	0.250	0.250	0.413	0.412	0.249	0.249	0.411	0.411
Investire Intern.	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Prime Global	0.427	0.427	0.507	0.504	0.423	0.423	0.502	0.502
Sanpaolo H. Intern.	0.228	0.228	0.282	0.280	0.225	0.225	0.277	0.277
Mibtel index	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
B.O.T.	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Az3								
Centrale Italia	0.353	0.353	0.442	0.442	0.351	0.351	0.441	0.441
Epta Azioni Italia	0.495	0.495	0.549	0.549	0.490	0.490	0.543	0.543
Fondicri Sel. Italia	0.373	0.373	0.534	0.534	0.372	0.372	0.531	0.531
Genercomit Azioni Italia	0.874	0.874	0.874	0.874	0.870	0.870	0.870	0.870
Gesticredit Borsit	0.392	0.392	0.520	0.520	0.392	0.392	0.519	0.519
Imi Italy	0.287	0.287	0.370	0.370	0.286	0.286	0.369	0.369
Investire Azion.	0.847	0.847	0.847	0.847	0.843	0.843	0.843	0.843
Oasi Azionario Italia	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Mibtel index	0.667	0.667	0.681	0.681	0.657	0.657	0.671	0.671
B.O.T.	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Az4								
Azimut Europa	0.830	0.830	0.948	0.555	0.830	0.830	0.948	0.947
Gesticredit Euro Az.	0.565	0.565	0.816	0.538	0.565	0.565	0.816	0.815
Imi Europe	0.263	0.263	0.395	0.313	0.263	0.263	0.395	0.395
Investire Europa	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Sanpaolo H. Europe	0.343	0.343	0.457	0.337	0.343	0.343	0.457	0.457
Mibtel index	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
B.O.T.	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

as an additional measure of risk which is relevant when the investor's portfolio is well diversified; the market portfolio has been taken as the Mibtel index. In addition, we have considered the per cent subscription costs per different amounts of initial investment (10, 50 and 100 millions of Italian lire) and the per cent redemption costs per year of disinvestment (after 1, 2 or 3 years).

Table 1 presents the overall results of the analysis carried out on all the 47 mutual funds and the two benchmarks. Columns 3 to 6 report the value of Sharpe, Treynor, Jensen and reward to half-variance indexes; the last two columns report the value of the DEA performance index I_{DEA} when the inputs include standard deviation, a subscription cost and a redemption cost (3 inputs) and when the inputs include standard deviation and all the subscription and redemption costs (7 inputs).

It has to be noticed that, contrary to the traditional indexes, the value of which don't change when the set of mutual funds to be compared is modified, the DEA performance index does change its value according to the funds included in the reference set. Therefore, the comparisons presented in Table 1, which are made among all the different selected funds, may be considered of scarse significance from a financial point of view. Nevertheless, the results presented in Table 1 will permit to see, through a comparison with Table 3, how the ranking of the funds varies with the comparison set.

Moreover, from this table, too, one of the most powerful features of DEA performance measure becomes evident, i.e. the possibility to take into account also the investment costs that the other traditional criteria have to neglect. This explains the different ranking of the mutual funds obtained with DEA. Of course, including three levels of subscription and redemption costs (linked to different amounts and durations of the investments) we emphasize the role of costs in the choice of the more efficient fund.

In effect, some experiments carried out, the results of which are not presented here for the sake of brevity, indicate that completely omitting either the subscription costs or the redemption ones the resulting fund ranking does change, while including more than one level for the same type of cost does not substantially affect the ranking.

Table 2 compares in details the results obtained with the DEA procedure using different risk measures, namely standard deviations of the returns versus the square root of the half-variance, and including or omitting the β coefficient, both with one and all levels of subscription and redemption costs. The comparisons reported in table 2 regard the stocks funds and are made separately for each class.

We observe that the two risk measures used give nearly the same results: this means either that for the data under consideration these two risk measures are coherent or that the returns are approximately symmetric. Moreover, neither the inclusion of the β coefficient does substantially modify the results and this could mean that the β, too, though considering the covariance with the market portfolio, does not bring new information to the funds risk.

Table 3 shows the results obtained with the DEA method by comparing the stocks, balanced and bonds mutual funds separately, using standard deviation as a risk measure. In effect, it is interesting to see if the ranking of the funds in a class changes when we enlarge the set of alternative funds that are considered to other classes, too. By comparing the results of table 1 and 3, we note that the (relative) ranking inside each class is substantially preserved. What changes is the absolute value of the performance index, and the fact that when we reduce the set of alternatives the funds with the highest efficiency measure becomes relatively efficient ($I_{DEA} = 1$) even if in the largest set it is not the most efficient one.

Table 3. Results obtained with the DEA performance indexes for the different classes of funds. In column 3 the inputs include standard deviation, a subscription and a redemption cost (3 inputs) while in column 4 standard deviation and all the subscription and redemption costs are included (7 inputs).

Funds	Classes	3 inputs	7 inputs
Stocks funds			
Arca 27	Az2	0.562	0.562
Azimut Borse Int.	Az2	0.295	0.313
Centrale Global	Az2	0.187	0.242
EptaInternational	Az2	0.343	0.389
Fideuram Azione	Az2	0.158	0.233
Fondicri Int.	Az2	0.220	0.320
Genercomit Int.	Az2	0.218	0.329
Investire Intern.	Az2	0.580	0.580
Prime Global	Az2	0.335	0.382
Sanpaolo H. Intern.	Az2	0.195	0.232
Centrale Italia	Az3	0.353	0.442
Epta Azioni Italia	Az3	0.495	0.549
Fondicri Sel. Italia	Az3	0.373	0.534
Genercomit Azioni Italia	Az3	0.874	0.874
Gesticredit Borsit	Az3	0.392	0.520
Imi Italy	Az3	0.287	0.370
Investire Azion.	Az3	0.847	0.847
Oasi Azionario Italia	Az3	1.000	1.000
Azimut Europa	Az4	0.372	0.393
Gesticredit Euro Az.	Az4	0.330	0.398
Imi Europe	Az4	0.199	0.262
Investire Europa	Az4	0.790	0.790
Sanpaolo H. Europe	Az4	0.229	0.272
Indice Mibtel		0.667	0.681
B.O.T.		1.000	1.000
Balanced funds			
Arca BB	Bi1	1.000	1.000
Azimut Bil.	Bi1	0.401	0.431
Eptacapital	Bi1	0.390	0.449
Genercomit	Bi1	0.287	0.445
Investire Bil.	Bi1	0.996	0.996
Arca TE	Bi2	0.702	0.702
Fideuram Performance	Bi2	0.683	0.683
Fondo Centrale	Bi2	0.123	0.169
Genercomit Espansione	Bi2	0.189	0.296
Indice Mibtel		0.631	0.644
B.O.T.		1.000	1.000
Bonds funds			
Arca Bond	Ob3	0.311	0.311
Azimut Rend. Int.	Ob3	0.095	0.099
Epta 92	Ob3	0.129	0.147
Genercomit Obb. Estere	Ob3	0.334	0.339
Imi Bond	Ob3	0.128	0.172
Investire Bond	Ob3	0.238	0.241
Oasi Bond Risk	Ob3	0.254	0.254
Primebond	Ob3	0.182	0.226
Sanpaolo H. Bonds	Ob3	0.098	0.116
Bpb Tiepolo	Ob5	1.000	1.000
Centrale Tasso Fisso	Ob5	0.215	0.269
Eptabond	Ob5	0.283	0.371
Fideuram Security	Ob5	1.000	1.000
Oasi Btp Risk	Ob5	0.589	0.589
Prime Reddito Italia	Ob5	0.180	0.267
B.O.T.		1.000	1.000

6. Peer groups as benchmark portfolios

The measures of the relative efficiency of the decision making units represent only one kind of information resulting from the DEA methodology. The strength of the DEA approach consists in its ability not only to verify if a decision making unit is efficient, relative to the other units, but also to suggest to the inefficient units a "virtual unit" that they could imitate in order to improve their efficiency.

In fact, for each inefficient unit the solution of the input-oriented CCR dual model presented in Section 3 permits to identify a set of corresponding efficient units, called *peer units*, which are efficient with the inefficient unit's weights. The peer units are associated with the (strictly) positive basic multipliers λ_i, that is the non null dual variables.

Therefore, for each inefficient unit j_0 it is possible to build a composite unit with output

$$\sum_{j=1}^{n} \lambda_j y_{rj} \qquad r = 1, \ldots, t \tag{6.1}$$

and input

$$\sum_{j=1}^{n} \lambda_j x_{ij} \qquad i = 1, \ldots, m \tag{6.2}$$

that outperforms unit j_0 and lies on the efficient frontier.

From a financial point of view, this composite unit can be considered as a benchmark for the inefficient fund j_0. Fund j_0 could improve its performance by trying to imitate the behaviour of the efficient composite unit, i.e. of its benchmark portfolio. This (efficient) benchmark portfolio has an input/output orientation or style which is similar to that of the (inefficient) fund j_0.

Therefore, the benchmark composite unit can be useful in studying the style of the portfolio management, and the importance of analyzing the management style of an asset portfolio is now well recognized in finance (see Sharpe (1992)).

The different mutual funds (units) considered in our analysis have as output the (weekly) expected return; thus, they are scaled in terms of the same amount of invested capital. For this reason we have computed normalized multipliers

$$\overline{\lambda}_j = \frac{\lambda_j}{\sum_{k=1}^{n} \lambda_k} \tag{6.3}$$

which indicate the relative composition of the benchmark composite portfolio.

Table 4 reports the peer groups and the relative composition of the benchmark composite portfolios for the different classes of stocks funds in the model with 7 inputs (standard deviation and all the subscription and redemption costs). We may observe that the efficient funds have no need to define a composite benchmark portfolio while they often enter in the benchmark portfolios for the other funds.

Moreover, from this table we may point out a feature of the DEA approach that has to be carefully considered when choosing the inputs. In fact, the efficient units depend on the inputs that are chosen. In particular, the inclusion of inputs of minor importance should be avoided as they could make a fund become efficient on the ground of minor aspects.

Table 4. Peer groups and relative composition of the benchmark portfolios for the different classes of stocks funds in the model with 7 inputs (standard deviation and all the subscription and redemption costs).

Stocks funds	I_{DEA}		Non null standardized multipliers	
Az2				
1. Arca 27	1.000	(efficient)	$\overline{\lambda}_1$ =1.000	
2. Azimut Borse Int.	0.414		$\overline{\lambda}_1$ =0.171	$\overline{\lambda}_{12}$ =0.829
3. Centrale Global	0.290		$\overline{\lambda}_1$ =0.092	$\overline{\lambda}_{12}$ =0.908
4. EptaInternational	0.505		$\overline{\lambda}_1$ =0.153	$\overline{\lambda}_{12}$ =0.847
5. Fideuram Azione	0.283		$\overline{\lambda}_1$ =0.100	$\overline{\lambda}_{12}$ =0.900
6. Fondicri Int.	0.415		$\overline{\lambda}_1$ =0.155	$\overline{\lambda}_{12}$ =0.845
7. Genercomit Int.	0.413		$\overline{\lambda}_1$ =0.127	$\overline{\lambda}_{12}$ =0.873
8. Investire Intern.	1.000		$\overline{\lambda}_8$ =1.000	
9. Prime Global	0.507		$\overline{\lambda}_1$ =0.177	$\overline{\lambda}_{12}$ =0.823
10. Sanpaolo H. Intern.	0.282		$\overline{\lambda}_1$ =0.101	$\overline{\lambda}_{12}$ =0.899
11. Mibtel index	1.000	(efficient)	$\overline{\lambda}_{11}$ =1.000	
12. B.O.T.	1.000	(efficient)	$\overline{\lambda}_{12}$ =1.000	
Az3				
1. Centrale Italia	0.442		$\overline{\lambda}_8$ =0.086	$\overline{\lambda}_{10}$ =0.914
2. Epta Azioni Italia	0.549		$\overline{\lambda}_8$ =0.147	$\overline{\lambda}_{10}$ =0.853
3. Fondicri Sel. Italia	0.534		$\overline{\lambda}_8$ =0.166	$\overline{\lambda}_{10}$ =0.884
4. Genercomit Azioni Italia	0.874		$\overline{\lambda}_8$ =1.000	
5. Gesticredit Borsit	0.520		$\overline{\lambda}_8$ =0.122	$\overline{\lambda}_{10}$ =0.878
6. Imi Italy	0.370		$\overline{\lambda}_8$ =0.065	$\overline{\lambda}_{10}$ =0.935
7. Investire Azion.	0.847		$\overline{\lambda}_8$ =1.000	
8. Oasi Azionario Italia	1.000	(efficient)	$\overline{\lambda}_8$ =1.000	
9. Mibtel index	0.681		$\overline{\lambda}_8$ =0.333	$\overline{\lambda}_{10}$ =0.667
10. B.O.T.	1.000	(efficient)	$\overline{\lambda}_{10}$ =1.000	
Az4				
1. Azimut Europa	0.948		$\overline{\lambda}_7$ =1.000	
2. Gesticredit Euro Az.	0.816		$\overline{\lambda}_7$ =1.000	
3. Imi Europe	0.395		$\overline{\lambda}_7$ =1.000	
4. Investire Europa	1.000	(efficient)	$\overline{\lambda}_4$ =1.000	
5. Sanpaolo H. Europe	0.457		$\overline{\lambda}_7$ =1.000	
6. Mibtel index	1.000	(efficient)	$\overline{\lambda}_6$ =1.000	
7. B.O.T.	1.000	(efficient)	$\overline{\lambda}_7$ =1.000	

By including the subscriptions and redemption costs we get efficiency results that emphasize the lowest investment costs. For example, in the Az2 fund Arca 27 is efficient though neither its expected return is not the highest nor its standard deviation is the lowest one and the reason is probably due to the fact that it does not have any subscription and redemption cost. What's more, the B.O.T. (which has low investment costs) is always efficient and included in the peer groups of the other mutual funds.

7. Concluding remarks

In this paper we propose to use the DEA methodology in order to evaluate the relative efficiency of mutual funds. The DEA performance index for mutual funds represents a generalization of many traditional numerical indexes and permits to take into account many conflicting objectives, as well as the investment costs.

Moreover, the results of the DEA technique allow to identify, for each inefficient fund, a corresponding efficient set which represents a "virtual" composite portfolio. Such a peer group can be seen as a personalized benchmark and characterizes the portfolio style.

Some results obtained by testing the DEA performance index on the Italian financial market indicate the importance of the subscription and redemption costs in determining the fund ranking.

The results suggest that the DEA methodology for evaluating the mutual fund performance may complement the traditional indexes. The DEA approach indeed provides some additional information that may be useful for a careful comparative analysis of the mutual fund performance.

The DEA index we have proposed in this contribution considers many inputs but only one output, the portfolio expected return. A natural extension let to future research may take into account also a multiple output structure.

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