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Prospect theory: An application to European option pricing

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Abstract
Empirical studies on quoted options highlight deviations from the theoretical model of Black and Scholes; this is due to different causes, such as assumptions regarding the price dynamics, markets frictions and investors' attitude toward risk. In this contribution, we focus on this latter issue and study how to value European options within the *continuous cumulative prospect theory*. According to prospect theory, individuals do not always take their decisions consistently with the maximization of expected utility. Decision makers have biased probability estimates; they tend to underweight high probabilities and overweight low probabilities. Risk attitude, loss aversion and subjective probabilities are described by two functions: a value function and a weighting function, respectively. In our analysis, we use alternative probability weighting functions. We consider the pricing problem both from the writer's and holder's perspective, obtaining an interval for the prices of call and put options.

Keywords
Behavioral Finance, Cumulative Prospect Theory, European Option Pricing.

JEL Codes
C63, D81, G13.

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1 Introduction

Black and Scholes model is considered as a milestone in the option pricing literature, it is widely applied in financial markets, and has been developed in many directions. Nevertheless, empirically one observes systematic differences between the market prices of financial options and the corresponding theoretical prices; this is due to different causes, such as assumptions regarding the price dynamics, and volatility in particular, the presence of markets frictions, information imperfections, and investors' attitude toward risk. Normally one tries to improve the performance of models considering more complex dynamics for the prices of the underlying assets, but leaving unchanged decision maker's preferences. An alternative approach is to price options considering behavioral aspects of the operators. In this contribution we focus on this issue and study how to evaluate options within the *cumulative prospect theory* developed by Tversky and Kahneman (1992).

According to the prospect theory, individuals do not always take their decisions consistently with the maximization of expected utility. Decision makers are risk averse when considering gains and risk-seeking with respect to losses. They are loss averse; people are much more sensitive to losses than they are to gains of comparable magnitude. People evaluate gambles based on potential gains and losses relative to a *reference point*, rather than in terms of final wealth. Individuals have also biased probability estimates; they tend to underweight high probabilities and overweight low probabilities. Kahneman and Tversky (1979) provide empirical evidence of such behaviors. Risk attitude, loss aversion and subjective probabilities are described by two functions: a value function, which is typically concave for gains and convex for losses, and a weighting function, which models probability perception.

In many financial application, and in particular when dealing with options, prospects involve infinitely many values or a continuum of values; hence, prospect theory cannot be applied directly in its original or cumulative versions. Davis and Satchell (2007) provide results for the cumulative prospect value in order to deal with continuous outcomes.

The literature on behavioral finance (see e.g. Barberis and Thaler, 2003, for a survey) and prospect theory is huge, whereas a few studies in this field focus on financial options. A first contribution which applies prospect theory to options valuation is the work of Shefrin and Statman (1993), who consider covered call options in a one period binomial model. A list of paper on this topic should include: Poteshman and Serbin (2003), Abbink and Rockenbach (2006), Breuer and Perst (2007), and more recently Wolff *et al.* (2010). Following this direction, we apply the cumulative prospect theory in the continuous case in order to evaluate European plain vanilla options, extending the model to the European put option. We consider both the positions of the writer and the holder and obtain an interval estimation for the option price. We also use alternative probability weighting functions, and discuss some numerical examples. Empirical evidence suggests that the prospect theory can yield appreciable results when applied to the pricing of option contracts.

The rest of the paper is organized as follows. Section 2 synthesizes the main features of prospect theory and introduce the value and the weighting functions; Section 3 focuses on the application of continuous cumulative prospect theory to European option pricing; in Section 4 numerical results are provided and discussed.

2 Prospect Theory

Prospect theory (PT), in its formulation proposed by Kahneman and Tversky (1979), is based on the subjective evaluation of *prospects*. Originally PT deals only with a limited set of prospects[1].

With a finite set[2] of potential future states of nature $S = \{s_1, s_2, \dots, s_N\}$, a prospect is a vector

$$(\Delta x_1,\, p_1; \Delta x_2,\, p_2; \dots; \Delta x_N,\, p_N)$$

of pairs $(\Delta x_i, p_i)$, $i = 1, 2, \dots, N$. Assume $\Delta x_i \leq \Delta x_j$ for $i < j$, $i, j = 1, 2, \dots, N$, and $\Delta x_i \leq 0$ $(i = 1, 2, \dots, k)$ and $\Delta x_i > 0$ $(i = k+1, \dots, N)$. Prospects assign to any possible state of nature s_i a subjective probability p_i and an outcome Δx_i. Outcome Δx_i is defined relative to a certain *reference point* x^*; being x_i the absolute outcome, we have $\Delta x_i = x_i - x^*$.

2.1 Cumulative Prospect Theory

Cumulative prospect theory (CPT) developed by Tversky and Kahneman (1992) overcomes some drawbacks of the original prospect theory. Risk attitude, loss aversion and subjective probabilities are described by two functions: a value function, which is typically concave for gains and convex for losses, and a weighting function, which models probability perception.

Let us denote now with Δx_i, for $-m \leq i < 0$ negative outcomes and with Δx_i, for $0 < i \leq n$ positive outcomes, with $\Delta x_i \leq \Delta x_j$ for $i < j$. According to CPT, an investor's subjective value for a prospect is

$$V = \sum_{i=-m}^{n} \pi_i \cdot v(\Delta x_i)\,, \tag{1}$$

[1]The book of Wakker (2010) present a thorough discussion on the argument

[2]Infinitely many outcomes may also be considered. See Schmeidler (1989).

with decision weights π_i and value function $v(\Delta x_i)$, which is based on relative outcomes. The function v is tipically convex in the range of losses and concave in the range of gains.

In the CPT, subjective values $v(\Delta x_i)$ are not multiplied by probabilities p_i. Decision weights π_i are computed on the basis of a probability weighting function w^- for losses and w^+ for gains, considering cumulative probabilities instead of p_i:

$$\pi_i = \begin{cases} w^-(p_{-m}) & i = -m \\ w^-\left(\sum_{j=-m}^{i} p_j\right) - w^-\left(\sum_{j=-m}^{i-1} p_j\right) & i = -m+1, \ldots, -1 \\ w^+\left(\sum_{j=i}^{n} p_j\right) - w^+\left(\sum_{j=i+1}^{n} p_j\right) & i = 0, \ldots, n-1 \\ w^+(p_n) & i = n. \end{cases} \tag{2}$$

2.2 Framing and Mental Accounting

Prospect theory postulates that decision makers evaluate outcomes with respect to deviations from a reference point rather than with respect to net final wealth. The definition of such a reference point is crucial due to the fact that individuals evaluate results through a value function which gives more weight to losses than to gains of comparative magnitude. Individual's framing of decisions around a reference point is of great importance in prospect theory.

People tend to segregate outcomes into separate *mental accounts*, these are then evaluated separately for gains and losses. Thaler (1995) argues that, when combining such accounts to obtain overall result, typically individuals do not simply sum up all monetary outcomes, but use *hedonic frame*, such that the combination of the outcomes appears best possible.

Consider a combination of two sure outcomes Δx and Δy, the hedonic frame can be described as follows (see Thaler, 1999):

$$V = \max\{v(\Delta x + \Delta y), v(\Delta x) + v(\Delta y)\}. \tag{3}$$

Outcomes Δx and Δy are aggregated, and in such a case we have $v(\Delta x + \Delta y)$, or segregated $v(\Delta x) + (\Delta y)$, depending on what yields the highest prospect value.

An extension of the hedonic frame rule is (see also Breuer and Perst, 2007):

$$\begin{aligned} V = &\sum_{i=1}^{N} \pi_i \cdot \max\{v(\Delta x_i + \Delta y), v(\Delta x_i) + v(\Delta y)\} + \\ &+ \left(1 - \sum_{i=1}^{N} \pi_i\right) \cdot \max\{v(0 + \Delta y), v(0) + v(\Delta y)\}, \end{aligned} \tag{4}$$

where Δx_i are possible results with subjective probabilities π_i and Δy is a sure result.

Regarding the valuation of financial options, different aggregation or segregation of the results are possible. One can consider a single option position (*narrow framing*) or a portfolio, a naked or a covered position. It is also possible to segregate results across time: e.g. one can evaluate separately the premium paid for the option and its final payoff.

2.3 Continuous Cumulative Prospect Theory

In order to apply prospect theory in its cumulative version to option valuation, one has to deal with continuous results. Davis and Satchell (2007) provide the continuous cumulative prospect value V:

$$V = \int_{-\infty}^{0} \Psi^{-}[F(x)]\, f(x)\, v^{-}(x)\, dx + \int_{0}^{+\infty} \Psi^{+}[1-F(x)]\, f(x)\, v^{+}(x)\, dx, \qquad (5)$$

where $\Psi = \frac{dw(p)}{dp}$ is the derivative of the weighting function w with respect to the probability variable, F is the cumulative distribution function and f is the probability density function of the outcomes x; v^{-} and v^{+} denote the value function for losses and gains, respectively.

2.4 The value function

Specific parametric forms have been suggested in the literature for the value function; some examples are reported in table 1. A function which is used in many empirical studies is the following value function

$$\begin{array}{ll} v^{-} = -\lambda(-x)^{b} & x < 0 \\ v^{+} = x^{a} & x \geq 0, \end{array} \qquad (6)$$

with positive parameters which control risk attitude ($0 < a \leq 1$ and $0 < b \leq 1$) and loss aversion ($\lambda > 1$). Function (6) has zero as reference point; it satisfies the properties required by Kahneman and Tversky (1979) for a value function: it is concave for positive outcomes and convex for negative outcomes, it is steeper for losses. Figure 1 shows an example of the value function defined by (6). Parameters values equal to one imply risk and loss neutrality.

2.5 The weighting function

The weighting function models risk attitude towards probabilities. Empirical evidence suggests a particular shape for probability weighting: small probabilities

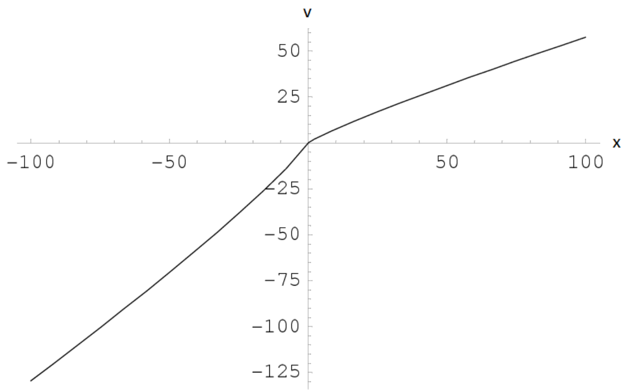

Figure 1: Value function (6) with parameters $\lambda = 2.25$ and $a = b = 0.88$

Table 1: Alternative value functions

function	formula
Linear	$v(x) = x$
Logarithmic	$v(x) = \ln(a+x)$
Power	$v(x) = x^a$
Quadratic	$v(x) = ax - x^2$
Exponential	$v(x) = 1 - e^{-ax}$
Bell	$v(x) = bx - e^{-ax}$
HARA	$v(x) = -(b+x)^a$

are overweighted whereas individuals tend to underestimate large probabilities. This turns out in a typical inverse S-shape weighting function: the function is concave (probabilistic risk seeking) in the interval $(0, p^*)$ and convex (probabilistic risk aversion) in the interval $(p^*, 1)$, for a certain value of p^*. In many empirical studies, the intersection between the weighting function and the linear weighting function $w(p) = p$ (elevation) is for p^* in the interval $(0.3, 0.4)$. Different parametric forms for the weighting function with the above mentioned features have been proposed[3]; some examples are reported in table 2.

In this contribution, we first consider the functional form suggested by Tversky and Kahneman (1992);

$$w^-(p) = \frac{p^{\gamma^-}}{\left(p^{\gamma^-} + (1-p)^{\gamma^-}\right)^{1/\gamma^-}} \qquad w^+(p) = \frac{p^{\gamma^+}}{\left(p^{\gamma^+} + (1-p)^{\gamma^+}\right)^{1/\gamma^+}}, \tag{7}$$

[3]See Diecidue *et al.* (2009) for a discussion and references therein.

Table 2: Alternative weighting functions.

function	formula
Linear	$w(p) = p$
Power	$w(p) = p^\gamma$
Lattimore *et al.*	$w(p) = \frac{\delta p^\gamma}{\delta p^\gamma + (1-p)^\gamma}$
Tversky-Kahneman	$w(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{(1/\gamma)}}$
Wu-Gonzales	$w(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^\delta}$
Prelec-I	$w(p) = e^{-(-\ln p)^\gamma}$
Prelec-II	$w(p) = e^{-\delta(-\ln p)^\gamma}$

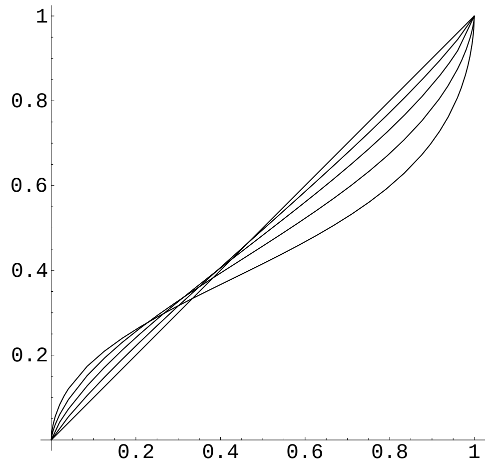

Figure 2: Weighting function (7) for different values of the parameter γ. As γ approaches the value 1, the w tends to the linear function

where γ^- and γ^+ are positive constants (with some constraint in order to have an increasing function). Note that $w(0) = 0$ and $w(1) = 1$. Hence in equation (5) we have:

$$\Psi = \frac{dw(p)}{dp} = \gamma p^{\gamma-1}\left[p^\gamma + (1-p)^\gamma\right]^{-1/\gamma} - \\ - p^\gamma\left[p^{\gamma-1} - (1-p)^{\gamma-1}\right]\left[p^\gamma + (1-p)^\gamma\right]^{-(\gamma+1)/\gamma}. \tag{8}$$

The parameter γ captures the the degree of sensitivity to changes in probabilities from impossibility (zero probability) to certainty; the lower the parameter, the higher is the curvature of the function.

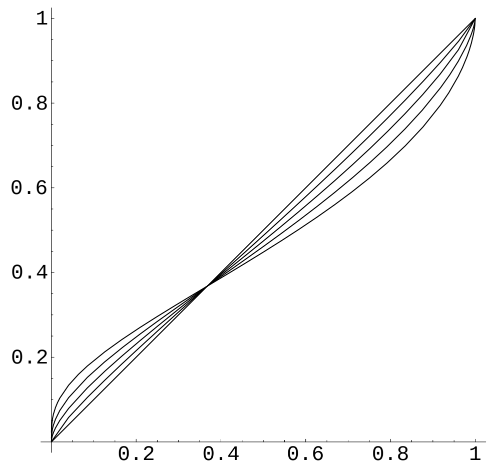

Figure 3: Prelec's weighting function (9) for different values of the parameter γ

As an alternative, we also consider the Prelec's (1998) weighting function

$$w(p) = \exp\left[-(-\ln p)^{\gamma}\right] \qquad p \in (0,1), \tag{9}$$

for which one easily obtains

$$\Psi(p) = \frac{\gamma}{p}(-\ln p)^{\gamma-1} \exp\left[-(-\ln p)^{\gamma}\right]. \tag{10}$$

More generally, Prelec suggests the two parameter function

$$w(p) = e^{-\delta(-\ln p)^{\gamma}}. \tag{11}$$

The weighting function w may be one of the main causes of the options' mispricing through its effect to the prospect value (see Shiller, 1999). Figures 2 and 3 show some examples of weighting functions defined by (7) and (9) for different values of the parameters. As the parameters tend to the value 1, the weight tends to the objective probability and the function w approaches the 45^{o} line. One can assume different parameters for probabilities when the outcome is in the domain of gains or losses. In order to use a more parsimonious model, one assumes $\gamma^{+} = \gamma^{-}$.

An interesting parametric function is the *switch-power weighting function* proposed by Diecidue *et al.* (2009), which consists in a power function for probabilities below a certain value $\bar{p}$ and a dual power function for probabilities above $\bar{p}$;

$$w(p) = \begin{cases} cp^{a} & p \leq \bar{p} \\ 1 - d(1-p)^{b} & p > \bar{p}. \end{cases} \tag{12}$$

Such a function has five parameters: a, b, c, d, and $\bar{p}$, which reduce to three by assuming continuity of $w(p)$ at $\bar{p}$ and differentiability. All the parameters are positive, due to continuity and monotonicity. For $a,b \leq 1$ the function w has an inverse S-shaped form, while for $a,b \geq 1$ the weighting function in convex for $p < \bar{p}$ and concave for $p > \bar{p}$. As $\bar{p}$ tends to 0 one obtains a power function, whereas as $\bar{p}$ approaches 1 the function w reduces to a dual power function. Both parameter a and b govern the curvature of w when $a \neq b$. In such a case $\bar{p}$ may not lie on the 45^{o} line, hence it has not the meaning of dividing the region of over- and underweighting of the probability.

3 European options valuation

Let S_t be the price at time t, with $t \in [0,T]$, of the underlying asset of a European option with maturity T; in a Black-Scholes setting, the underlying price dynamics is driven by a geometric Brownian motion. As a result, for the probability density function (pdf) in (5) we have

$$f(S_T) = \frac{1}{S_T \sigma \sqrt{2\pi T}} \exp\left(\frac{-\left[\ln(S_T/S_0)-(\mu-\sigma^2/2)T\right]^2}{2\sigma^2 T} \right), \tag{13}$$

where μ and $\sigma > 0$ are constants. The cumulative distribution function (cdf) of S_T is

$$F(S_T) = \Phi\left(\frac{\ln(S_T/S_0)-(\mu-\sigma^2/2)T}{\sigma\sqrt{T}} \right), \tag{14}$$

where $\Phi(\cdot)$ is the cumulative distribution function of a standard Gaussian random variable.

Wolff *et al.* (2010) provide the prospect value of writing L call options and compare their results for the call option with those obtained applying Heston (1993) stochastic volatility model.

Let c be the option premium with strike price X. At time $t = 0$, the option's writer receives c and can invest the premium at the risk-free rate r, obtaining Lce^{rT}. At maturity, he has to pay the amount $L(S_T - X)$ if the option expires in-the-money. In what follows, we will consider a single contract, $L = 1$.

Considering zero as a reference point, the prospect value of the writer's position in the *time segregated* case is

$$V_s = v^+\left(ce^{rT}\right) + \int_X^{+\infty} \Psi^-\left(1-F(S_T)\right) f(S_T) v^-\left((X-S_T)\right) dS_T, \tag{15}$$

with $f(S_T)$ and $F(S_T)$ being the pdf and the cdf defined in (13) and (14), respectively, of the future underlying price S_T, and v is defined as in (6).

In equilibrium, we equate V_s at zero and solve for the price c:

$$c_s = e^{-rT} \left(\lambda \int_X^{+\infty} \Psi^- \left(1 - F(S_T)\right) f(S_T) \left(S_T - X\right)^b dS_T \right)^{1/a}, \tag{16}$$

which requires numerical approximation of the integral.

When considering the *time aggregated* prospect value, one obtains:

$$\begin{aligned} V_a &= w^+ \left(F(X)\right) v^+ \left(c\, e^{rT}\right) + \\ &+ \int_X^{X+c\exp(rT)} \Psi^+ \left(F(S_T)\right) f(S_T) v^+ \left(c \exp(rT) - (S_T - X)\right) dS_T + \\ &+ \int_{X+c\exp(rT)}^{+\infty} \Psi^- \left(1 - F(S_T)\right) f(S_T) v^- \left(c \exp(rT) - (S_T - X)\right) dS_T. \end{aligned} \tag{17}$$

In this latter case, the equilibrium option price has to be determined numerically.

In the case of a put option one cannot use put-call parity arguments; the prospect value of the writer's position in the time segregated case is

$$V_s = v^+ \left(p\, e^{rT}\right) + \int_0^X \Psi^- \left(F(S_T)\right) f(S_T) v^- \left((S_T - X)\right) dS_T\,, \tag{18}$$

and one obtains

$$p_s = e^{-rT} \left(\lambda \int_0^X \Psi^- \left(F(S_T)\right) f(S_T) \left(X - S_T\right)^b dS_T \right)^{1/a}. \tag{19}$$

In the time aggregated case the put option value is implicitly defined by the following equation:

$$\begin{aligned} V_a = & \int_0^{X - p\, e^{rT}} \Psi^- \left[F(S_T)\right] f(S_T)\, v^- \left[p\, e^{rT} - (X - S_T)\right] dS_T + \\ & + \int_{X - p\, e^{rT}}^X \Psi^+ \left[1 - F(S_T)\right] f(S_T)\, v^+ \left[p\, e^{rT} - (X - S_T)\right] dS_T + \\ & + w^+ \left[1 - F(X)\right] v^+ \left[p\, e^{rT}\right]. \end{aligned} \tag{20}$$

3.1 Option valuation from holder's perspective

When one considers the problem from the holder's viewpoint, the prospect values both in the time segregated and aggregated cases changes. Holding zero as reference point, the prospect value of the holder's position for a call option in the time segregated case is

$$V_s^h = v^- \left(-c\, e^{rT}\right) + \int_X^{+\infty} \Psi^+ \left(1 - F(S_T)\right) f(S_T) v^+ \left((S_T - X)\right) dS_T, \tag{21}$$

with $f(S_T)$ and $F(S_T)$ being the pdf and the cdf defined in (13) and (14) of the future underlying price S_T, and v is defined as in (6).

We equate V_s^h at zero and solve for the price c, obtaining:

$$c_s^h = e^{-rT} \left(\frac{1}{\lambda} \int_X^{+\infty} \Psi^+ \left(1 - F(S_T)\right) f(S_T) \left(S_T - X\right)^a dS_T \right)^{1/b} . \tag{22}$$

In the time aggregated case, the prospect value has the following integral representation:

$$\begin{aligned} V_a^h = {} & w^- \left(F(X)\right) v^- \left(-c\, e^{rT}\right) + \\ & + \int_X^{X+c\exp(rT)} \Psi^- \left(F(S_T)\right) f(S_T) v^- \left((S_T - X) - c\exp(rT)\right) dS_T + \\ & + \int_{X+c\exp(rT)}^{+\infty} \Psi^+ \left(1 - F(S_T)\right) f(S_T) v^+ \left((S_T - X) - c\exp(rT)\right) dS_T . \end{aligned} \tag{23}$$

In order to obtain the call option price in equilibrium, one has to solve numerically the problem.

In an analogous way one can derive the put option prospect values for the holder's position.

4 Numerical Results

We have calculated the options prices both in the time segregated and aggregated case. Tables 3 and 4 report the results for the European calls and puts, respectively, for different strikes and values of the parameters. Consider a single contract ($L = 1$). When we set $\mu = r$, $a = b = 1$, $\lambda = 1$, and $\gamma = 1$, we obtain the same results as in the Black-Scholes model (BS prices are reported in the second column). We compare BS premia with prices obtained considering the parameters used in Tversky and Kahneman (1992). Segregated values combined with TK sentiment yield too high option prices. We then used the *moderate sentiment* parameters as in Wolff *et al.* (2010) and compare the prices obtained considering different weighting functions: in particular, we applied (7) and Prelec's function (9); results are reported in the last two columns of the tables.

Table 5 reports the results for the call option evaluated by the holder[4]. The prices are below the writer's results. In all cases, segregated prospect values combined with TK sentiment provide too low option prices to be used in practice.

If one considers the pricing problem both from the writer's and holder's perspective, it is possible to obtain an interval for the prices of call and put options.

[4]Similar results are obtained for the put option.

In table 6 we report some results for the call option: Balck-Scholes price always lies in the interval bounded by the holder's price from below and the writer's price from above. The range of such an interval depends on the value of the parameters which govern investor's sentiment (attitude toward risk and loss aversion and probability bias). More moderate sentiment implies smaller estimate intervals.

As already observed, the weighting function w may be one of the main causes of the mispricing. Option prices are sensitive to the choice of the parameters values. In order to analyze the sensitivity of the option values to prospect theory parameters, we performed some numerical experiments. We isolate the effect of the probability bias, considering a linear value function, hence assuming risk neutrality over gains and losses (with parameters $a = b = 1$ and $\lambda = 1$), and letting varying the parameter γ of the weighting function.

In the examples we used the TK weighting function, distinguishing the cases with sensitivity to probability risk both for positive and negative outcomes, assuming $\gamma^+ = \gamma^-$ in order to have a more parsimonious model (see figure 4), then holding one of the parameter equal to 1 as in the results shown in figure 5 ($\gamma^+ = 1$) and figure 6 ($\gamma^- = 1$). We computed the option prices for different values of the volatility on the underlying asset price (the other option parameters are $S_0 = 100$, $X = 100$, $\mu = r = 0.01$, $T = 1$), both in the time segregated and aggregated cases; note that the option values in the segregated prospect for the writer do not depend on the parameter γ^+.

Finally figure 7 compares the results for two volatility values; we make different hypothesis about probability weighting when considering gains or losses. All the prices in figures 4 to 7 converge decreasing to the Black and Scholes values as γ approaches to 1 (and when $a = b = 1$ and $\lambda = 1$). It is also interesting to note that, when considering the writer's payoff, the major effect on the option value is explained by the parameter γ^-, as is also intuitive (the segregated prospect value does depend only on the parameter γ^-).

5 Concluding remarks

Prospect theory has recently begun to attract attention in the literature on financial options valuation; when applied to option pricing in its continuous cumulative version, it seems a promising alternative to other models proposed in the literature, for its potential to explain option mispricing with respect to Black and Scholes model. In particular, we focus on the probability bias and compared the results obtained with two alternative weighting functions.

It is worth noting that hypothesis on the segregation of the results are also important: in particular, results obtained in the time aggregated case with moderate sentiment do not deviate too far from BS prices.

As a final comment, note that option prices are sensitive to the choice of the values of the parameters. Calibrating model parameters to market data and studying market sentiment is an important issue which requires further investigation.

References

[1] Abbink K., Rockenbach B. (2006) Option pricing by students and professional traders: A behavioral investigation, *Managerial and Decision Economics* **27**, 497–510

[2] Barberis N., Thaler R. (2003) A survey of behavioral finance. In: Constantinides G.M., Harris M., Stulz R., (eds) *Handbook of Economics and Finance*, Elsevier Science, 1052–1121

[3] Black F., Scholes M. (1973) The pricing of options and corporate liabilities, *Journal of Political Economy* **81**, 637–654

[4] Breuer W., Perst A. (2007) Retail banking and behavioral financial engineering: The case of structured products, *Journal of Banking and Finance* **31**, 827–844

[5] Davies G.B., Satchell S.E. (2007) The behavioural components of risk aversion, *Journal of Mathematical Psychology* **51**, 1–13

[6] Diecidue E., Schmidt U., Zank H. (2009) Parametric weighting functions, *Journal of Economics Theory*, **144**, 1102–1118

[7] Heston S.L. (1993) A closed-form solution for options with stochastic volatility with application to bond and currency options, *The Review of Financial Studies*, **6**, 327–343

[8] Kahneman D., Tversky A. (1979) Prospect theory: An analysis of decision under risk, *Econometrica* **47**, 263–292

[9] Lattimore P.K., Baker J.R., Witte A.D. (1992) The influence of probability on risky choice: A parametric examination, *Journal of Economic Behavior and Organization* **17**, 377–400

[10] Poteshman A.M., Serbin V. (2003) Clearly irrational financial market behavior: Evidence from the early exercise of exchange traded stock options, *Journal of Finance* **58**, 37–70

[11] Prelec D. (1998) The probability weighting function, *Econometrica* **66**, 497–527

[12] Schmeidler D. (1989) Subjective probability and expected utility without additivity, *Econometrica*, **57**, 571–587

[13] Shefrin H., Statman M. (1993) Behavioral aspects of the design and marketing of financial products, *Financial Management* **22**, 123–134

[14] Shiller R.J. (1999) Human behavior and the efficiency of the financial system. In: Taylor J.B., Woodford M. (eds) *Handbook of Macroeconomics* Vol. 1C, Elsevier Amsterdam, 1305–1340

[15] Thaler R.H. (1985) Mental accounting and consumer choice, *Marketing Science*, **4**, 199–214

[16] Thaler R.H. (1999) Mental accounting matters, *Journal of Behavioral Decision Making*, **12**, 183–206

[17] Tversky A., Kahneman D. (1992) Advances in prospect theory: cumulative representation of the uncertainty, *Journal of risk and uncertainty* **5**, 297–323

[18] Wakker P.P. (2010) *Prospect Theory: For Risk and Ambiguity*, Cambridge University Press, Cambridge

[19] Wolff C., Versluis C., Lehnert T. (2010) A cumulative prospect theory approach to option pricing. LSF Research Working Paper Series 09–03, Luxembourg School of Finance

Table 3: Call option prices in the Black-Scholes model and in the segregated and aggregated prospects. The parameters are: $S_0 = 100$, $\mu = 0.05$, $r = 0.05$, $\sigma = 0.2$, $T = 1$, and $L = 1$. In the BS model: $a = b = 1$, $\lambda = 1$, and $\gamma = 1$. The Tversky-Kahneman parameters are: $a = b = 0.88$, $\lambda = 2.25$, $\gamma^+ = 0.61$, and $\gamma^- = 0.69$. The moderate sentiment parameters are: $a = b = 0.988$, $\lambda = 1.125$, $\gamma^+ = 0.961$, and $\gamma^- = 0.969$.

		TK sentiment		moderate sentiment		moderate sentiment Prelec's w function	
X	c_{BS}	c_s	c_a	c_s	c_a	c_s	c_a
70	33.5401	79.7250	45.6392	37.8210	34.5707	37.7386	34.6245
80	24.5888	59.1367	37.0964	27.7655	25.6391	27.6832	25.7038
90	16.6994	42.2814	29.3001	18.9376	17.7360	18.8640	17.7956
100	10.4506	29.2077	22.3341	11.9501	11.3814	11.9028	11.4247
110	6.0401	19.4366	16.2796	6.9907	6.7702	6.9824	6.8049
120	3.2475	12.3977	11.2361	3.8140	3.7446	3.8394	3.7830
130	1.6395	7.5530	7.2823	1.9564	1.9387	1.9982	1.9819

Table 4: Put option prices in the Black-Scholes model and in the segregated and aggregated prospects. The parameters are: $S_0 = 100$, $\mu = 0.05$, $r = 0.05$, $\sigma = 0.2$, $T = 1$, and $L = 1$. In the BS model: $a = b = 1$, $\lambda = 1$, and $\gamma = 1$. The Tversky-Kahneman parameters are: $a = b = 0.88$, $\lambda = 2.25$, $\gamma^+ = 0.61$, and $\gamma^- = 0.69$. The moderate sentiment parameters are: $a = b = 0.988$, $\lambda = 1.125$, $\gamma^+ = 0.961$, and $\gamma^- = 0.969$.

		TK sentiment		moderate sentiment		moderate sentiment Prelec's w function	
X	p_{BS}	p_s	p_a	p_s	p_a	p_s	p_a
70	0.1262	0.8496	0.8747	0.1555	0.1553	0.1670	0.1666
80	0.6872	3.1572	3.0876	0.8204	0.8145	0.8379	0.8321
90	2.3101	7.9546	7.0728	2.6922	2.6362	2.6932	2.6485
100	5.5735	15.6905	12.5269	6.3848	6.1385	6.3488	6.1483
110	10.6753	26.4833	19.0565	12.0983	11.4250	12.0262	11.4463
120	17.3950	40.2732	26.3766	19.5999	18.2390	19.5062	18.2748
130	25.2994	56.8373	34.2999	28.4329	26.1692	28.3307	26.2097

Table 5: Call option prices in the Black-Scholes model and in the segregated and aggregated prospects from the holder's viewpoint. The parameters are: $S_0 = 100$, $\mu = 0.05$, $r = 0.05$, $\sigma = 0.2$, $T = 1$, and $L = 1$. In the BS model: $a = b = 1$, $\lambda = 1$, and $\gamma = 1$. The Tversky-Kahneman parameters are: $a = b = 0.88$, $\lambda = 2.25$, $\gamma^+ = 0.61$, and $\gamma^- = 0.69$. The moderate sentiment parameters are: $a = b = 0.988$, $\lambda = 1.125$, $\gamma^+ = 0.961$, and $\gamma^- = 0.969$.

		TK sentiment		moderate sentiment		moderate sentiment Prelec's w function	
X	c_{BS}	c_s	c_a	c_s	c_a	c_s	c_a
70	33.5401	12.1241	26.6775	29.8193	32.7509	29.7427	32.7826
80	24.5888	9.0924	18.9273	21.9088	23.8714	21.8318	23.9137
90	16.6994	6.6805	12.7900	14.9708	16.1214	14.9014	16.1617
100	10.4506	4.8156	8.3010	9.4774	10.0519	9.4332	10.0828
110	6.0401	3.3871	5.2167	5.5693	5.8103	5.5628	5.8402
120	3.2475	2.3074	3.1864	3.0552	3.1403	3.0816	3.1787
130	1.6395	1.5131	1.8997	1.5767	1.6024	1.6193	1.6473

Table 6: Call option prices (Black-Scholes formula) and lower (holder's position) and upper (writer's position) estimates in the CPT. The parameters are: $S_0 = 100$, $\mu = r = 0.1$, $\sigma = 0.2$, $T = 1$. In the BS model: $a = b = 1$, $\lambda = 1$, and $\gamma = 1$. The Tversky-Kahneman parameters are: $a = b = 0.88$, $\lambda = 2.25$, $\gamma^+ = 0.61$, and $\gamma^- = 0.69$. The moderate sentiment parameters are: $a = b = 0.988$, $\lambda = 1.125$, $\gamma^+ = 0.961$, and $\gamma^- = 0.969$.

		TK sentiment		moderate sentiment	
X	Black Scholes	segregate prospect	aggregate prospect	segregate prospect	aggregate prospect
80	27.99	10.20–66.76	21.78–40.35	24.91–31.58	27.24–29.04
90	19.99	7.67–49.20	15.27–32.62	17.86–22.61	19.34–21.04
100	13.27	5.66–35.14	10.30–25.62	11.95–15.10	12.78–14.27
110	8.18	4.11–24.30	6.72–19.40	7.47–9.40	7.87–9.03
120	4.71	2.90–16.22	4.27–14.06	4.38–5.48	4.54–5.34

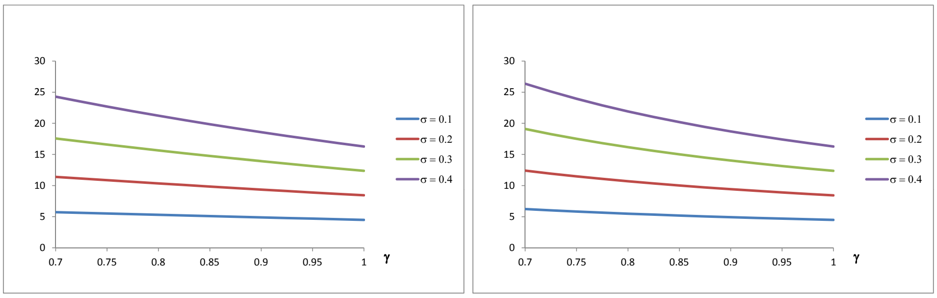

Figure 4: Sensitivity of the call option prices (writer's position) to probability weighting in the CPT. The parameters are: $S_0 = 100$, $X = 100$, $\mu = r = 0.01$, for different values of the volatility $\sigma = 0.1, 0.2, 0.3, 0.4$, $T = 1$; with linear value function ($a = b = 1$, $\lambda = 1$), TK weighting function with parameters varying in the interval $[0.7, 1.0]$ and $\gamma^+ = \gamma^-$, in the segregated (left figure) and aggregate (right figure) cases

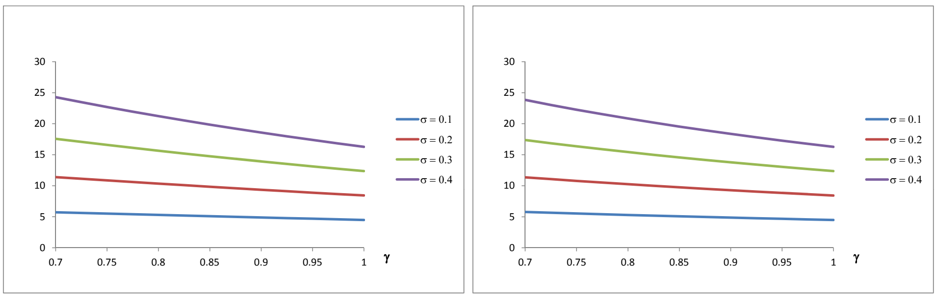

Figure 5: Sensitivity of the call option prices (writer's position) to probability weighting in the CPT. The parameters are: $S_0 = 100$, $X = 100$, $\mu = r = 0.01$, for different values of the volatility $\sigma = 0.1, 0.2, 0.3, 0.4$, $T = 1$; with linear value function ($a = b = 1$, $\lambda = 1$), TK weighting function with parameter γ^- varying in the interval $[0.7, 1.0]$ and $\gamma^+ = 1$, in the segregated (left figure) and aggregate (right figure) cases

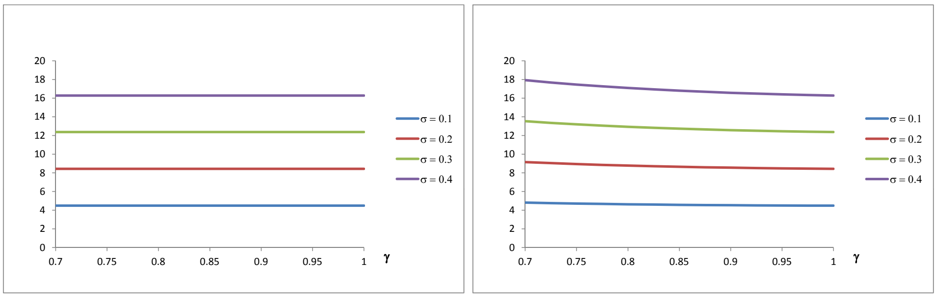

Figure 6: Sensitivity of the call option prices (writer's position) to probability weighting in the CPT. The parameters are: $S_0 = 100$, $X = 100$, $\mu = r = 0.01$, for different values of the volatility $\sigma = 0.1, 0.2, 0.3, 0.4$, $T = 1$; with linear value function ($a = b = 1$, $\lambda = 1$), TK weighting function with parameter γ^+ varying in the interval $[0.7, 1.0]$ and $\gamma^- = 1$, in the segregated (left figure) and aggregate (right figure) cases

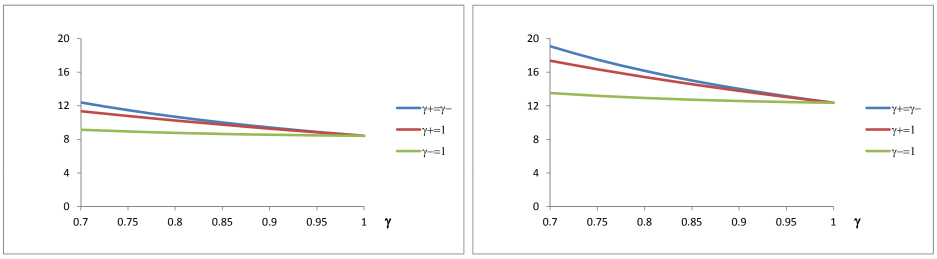

Figure 7: Sensitivity of the call option prices to probability weighting in the CPT (writer's position in the time aggregated case). The parameters are: $S_0 = 100$, $X = 100$, $\mu = r = 0.01$, with $\sigma = 0.2$ (left figure) and $\sigma = 0.3$ (right figure); with linear value function ($a = b = 1$, $\lambda = 1$), TK weighting function with parameters γ^+ and γ^- varying in the interval $[0.7, 1.0]$