Matrix-State Particle Filter for Wishart Stochastic Volatility Processes

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Abstract: This work deals with multivariate stochastic volatility models, which account for a time-varying variance-covariance structure of the observable variables. We focus on a special class of models recently proposed in the literature and assume that the covariance matrix is a latent variable which follows an autoregressive Wishart process. We review two alternative stochastic representations of the Wishart process and propose Markov-Switching Wishart processes to capture different regimes in the volatility level. We apply a full Bayesian inference approach, which relies upon Sequential Monte Carlo (SMC) for matrix-valued distributions and allows us to sequentially estimate both the parameters and the latent variables.

Keywords: Multivariate Stochastic Volatility; Matrix-State Particle Filters; Sequential Monte Carlo; Wishart Processes, Markov Switching.

1. Introduction

Many financial time series share some common features, also known as stylized facts. The following features: time-varying volatility, clustering in volatility and excess of kurtosis are well described by univariate stochastic volatility models. See the seminal works of Taylor (1986, 1994) and Jacquier *et al.* (1994).
In order to capture dependencies and spill-over effects between the volatility of different variables, the univariate models have been successfully extended to the multivariate case by Harvey *et al.* (1994), Aguilar and West (2000) and Chib *et al.* (2006). We also refer the reader to Asai and McAleer (2006a) and Asai *et al.* (2006) for an updated review on *multivariate stochastic volatility* (MSV) models.
Earlier MSV models assume a constant correlation between the observable variables. This assumption is quite unrealistic for many economic and financial series (see for example Engle (2002) and Pelletier (2006)). Thus in the last few years there has been an increasing attention to a new class of stochastic volatility models, which account for time-varying and stochastic correlation structure.
In this work we deal with the MSV models proposed by Philipov and Glickman (2006) and Gourieroux *et al.* (2004). They use two different stochastic representations of a

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Wishart autoregressive process for the stochastic volatility. Their models allow for time-varying variances and covariances, which determine implicitly a time-varying stochastic correlation structure between asset returns. For an updated discussion on the different ways of introducing the time-varying correlation into MSV models we refer to Asai and McAleer Asai and McAleer (2006b, 2005).

The first contribution of this work is to propose some extensions of the Wishart MSV model. In particular following So *et al.* (1998) we are interested in capturing different regimes in the volatility behavior. To this aim we assume that the parameters of the Wishart autoregressive process are driven by a Markov-Switching process. We study the effect of the Markov-switching process on the volatility and on the correlation structure of the observable, which results implicitly determined by the volatilities and the covariances.

In this work we propose a on-line filtering approach for the Wishart-type MSV models. The on-line estimation of the latent Wishart process is very useful in a applied context, but represents a challenging inference problem. The inference task is difficult not only due to the nonlinear and non-Gaussian dynamics, but also due to the fact that the posterior distribution has not closed form and that the hidden states are matrices, possibly of high dimension. Note that the Kalman Filter was originally defined for vector-valued state (Kalman (1960) and Kalman and Bucy (1960)), but in many applications (such as finance and engineering) the states of a stochastic system are naturally defined by a matrix. In our work the time-varying volatility structure of set of time series are naturally defined by a nonsingular positive-definite matrix. We propose a general probabilistic state space representation of the MSV model and solve the inference problem by using nonlinear filtering techniques. In this work, we bring into action sequential Monte Carlo techniques (Berzuini *et al.* (1997), Arulampalam *et al.* (2001), Doucet *et al.* (2001)) and apply them to the Wishart-type MSV models.

The proposed regularised particle filters belong to the family of simulation based methods. These include the simulated maximum likelihood method proposed by Danielson and Richard (1993), and Danielsson (1994), MCMC method proposed by Jacquier *et al.* (1994) and improved by Kim and Chib (1998), maximum likelihood Monte Carlo (Sandmann and Koopman (1998)), the simulation method using important sampling and antithetic variables proposed by Durbin and Koopman (2000), efficient method of moments (EMM) by Gallant and Tauchen (1996).

The last contribution of the work is to propose a full Bayesian approach to the joint estimation of the parameter and the latent variable. We augment the state space of the system by considering the parameter as latent variable. The original state-space model can be represented in a vector form and the filtering problem can be applied to the vectorized form. The vectorized version of the system is useful because it allows us to evaluate many quantities of interest, such as general measures of filtering and prediction abilities of the model. However the vectorized version is not suitable for dealing with constrains on the hidden states. In our work and following Bar-Itzhack *et al.* (2006) and Choukroun (2003) we do not vectorize the system and follow an alternative route, which make advantage of the probabilistic structure of the model. Note that our general state-space representation includes as special case the linear and Gaussian matrix-valued state space models studied in Bar-Itzhack *et al.* (2006) and Choukroun (2003).

In that case the Matrix Kalman Filter proposed by the authors is optimal solution of the filtering problem when compared to our nonlinear filtering approach. Note however that our state-space...

The structure of the work is as follows. Section 2. proposes some new Wishart MSV models, those parameters are governed by a Markov-switching process. Section 3. deals with the inference problems and proposes a full Bayesian inference approach, which relies upon Sequential Monte Carlo algorithms. We suggest to take advantage of the matrix structure of the states to reduce the computational complexity of the nonlinear filtering procedure. Section 4. exhibits a simulation analysis of the proposed inference approach.

2. Wishart Stochastic Volatilty Processes

Let $\mathcal{M}_{+}^{k} \subset \mathbb{R}^{k\times k}$ denote the set of all the real-valued symmetric and positive-definite matrices of dimension k. Let $\{\boldsymbol{y}_t\}_{t\geq 0}$, with $\boldsymbol{y}_t \in \mathbb{R}^k$, represent the stochastic process of log-returns and $\{\Sigma_t\}_{t\geq 0}$, with $\Sigma_t \in \mathcal{M}_{+}^{k}$, the matrix-valued stochastic process of the variance-covariances.
We define the following multivariate model with Wishart stochastic volatility

$$\boldsymbol{y}_t \sim \mathcal{N}_k(\mathbf{0}_k, \Sigma_t) \tag{1}$$
$$\Sigma_t^{-1} \sim \mathcal{W}_k\left(\nu_t, \Xi_t\right) \tag{2}$$

for $t = 1, \ldots, T$, where $\mathcal{N}_k(\mathbf{0}_k, \Sigma_t)$ is a k-variate normal distribution with null mean vector. $\mathcal{W}_k(\nu_t, \Xi_t)$ denotes a Wishart distribution of dimension k, with possibly time-varying and stochastic degrees of freedom parameter ν_t and scale-matrix Ξ_t. The Wishart distribution accounts for the positive definiteness of the variance-covariance matrix and the time-varying parameters can be governed by other latent or exogenous variables.
In the following examples we present some alternative stochastic representations of the Wishart stochastic volatility process defined above. In the first example we follow Philipov and Glickman (2006), in the second one Gourieroux *et al.* (2004). The remaining examples extend the basic Wishart MSV models by introducing a Markov-switching process, which accounts for sudden changes in the volatility level. The proposed models can be considered a multivariate extension of the So *et al.* (1998) univariate stochastic volatility process with Markov-switching.

Example 1 - Autoregressive Wishart Process (ARW(1))
In order to introduce a first-order autoregressive dynamic in the stochastic volatility we set

$$\Sigma_t^{-1} \sim \mathcal{W}_k\left(\nu, \frac{1}{\nu} A^{1/2}(\Sigma_{t-1}^{-1})^d (A^{1/2})'\right) \tag{3}$$

where $A \in \mathcal{M}_{+}^{k}$, $A^{1/2}$ is its Cholesky decomposition and d is a scale factor. The matrix power is defined by $A^n = A^{n-1}\cdot A$, $n \geq 1$, $A^0 = I_k$, with I_k the identity matrix. The Wishart process is stationary for $d \in (-1, +1)$, as discussed in Philipov and Glickman (2006). In our study we consider diagonal matrix $A = \text{diag}\{(a_1, \ldots, a_k)\}$ and scale factor $d \in (0, 1)$, in order to avoid an inverse relation between two consecutive realisations of the covariance process.
Fig. 1 shows observations and latent volatilities simulated from a stochastic volatility model of dimension $k = 5$. Fig. 2 exhibits the time evolution of the stochastic correlation structure, which is implicit in the the simulated covariance process. In the simulation we use the following parameter setting: $d = 0.3$, $\nu = 19$ and $A^{-1} = 0.0125\,\text{diag}\{\iota'\}$ where $\iota = (1, \ldots, 1)' \in \mathbb{R}^5$.

$\square$

Figure 1: Simulated stochastic volatility model. Right column: simulated observable process $\boldsymbol{y}_t$. Left column: simulated latent volatilities $(\sigma_{11t}, \ldots, \sigma_{55t})$, where σ_{ijt} denotes the (i, j)-th element of $\Sigma_t^{1/2}$. In the simulation study we assume $d = 0.3$, $\nu = 19$, $A^{-1} = 0.0125\, I_5$, where I_k denotes the identity matrix of dimension k.

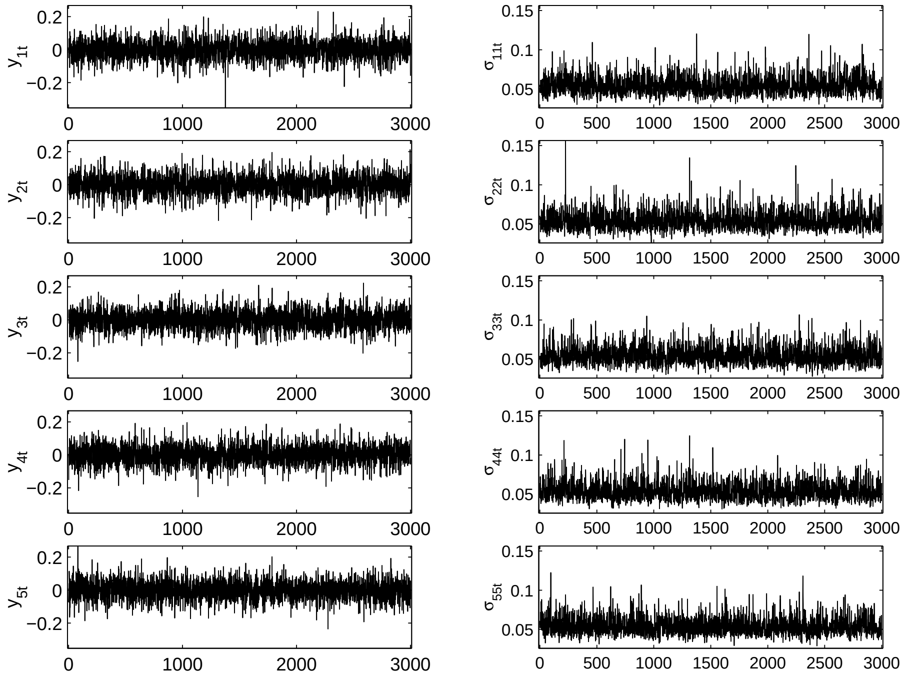
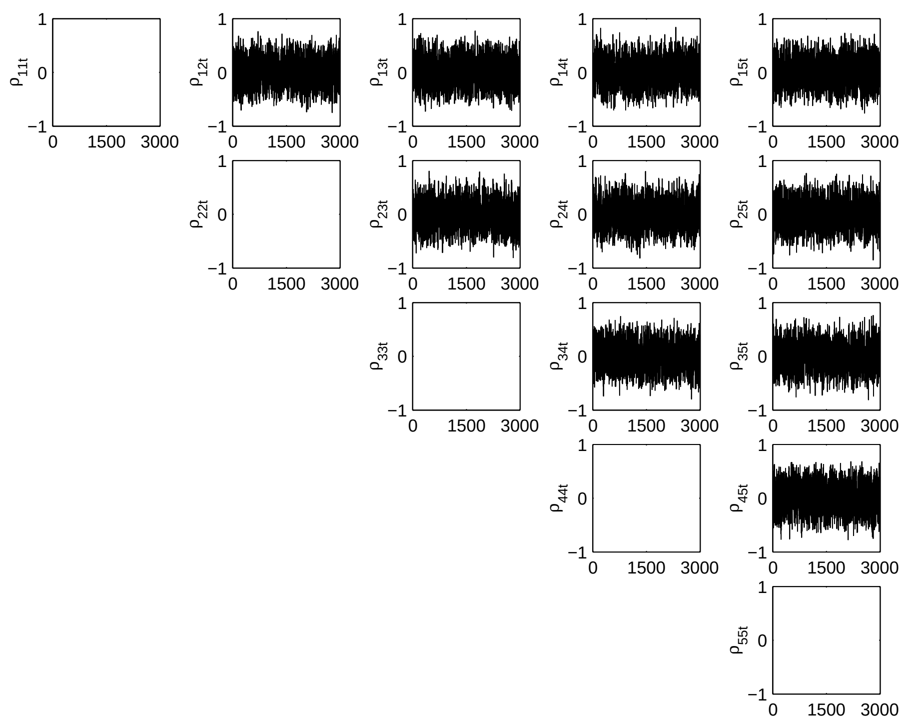


Figure 2: Simulated stochastic correlations. Matrix of graphs representing the time evolution of the components $\rho_{ijt} = \sigma_{ijt}/(\sigma_{iit}\sigma_{jjt})$, with $j \geq i$ of the correlation matrix, where σ_{ijt} denotes the (i, j)-th element of $\Sigma_t^{1/2}$. The time-varying stochastic correlation structure is implicit in the simulated Wishart covariance process.



Example 2 - Wishart Autoregressive Process (WAR(1))
Let $\boldsymbol{x}_{mt} \in \mathbb{R}^k$, with $m = 1, \ldots, \nu$ and $\nu \in \mathbb{N}_0$, be a sequence of first-order vector autoregressive processes: $\boldsymbol{x}_t \sim \mathcal{N}_k(A\boldsymbol{x}_{t-1}, \Sigma)$, where $A \in \mathcal{M}^k$. Then a k-variate Wishart process of the first order is defined as $\Sigma_t = \sum_{m=1}^{M} \boldsymbol{x}_{mt}\boldsymbol{x}'_{mt}$ and its transition density is non-central Wishart, $\Sigma_t \sim \mathcal{W}_k(\nu, \Sigma, A)$. This definition and some generalisations can be found in Gourieroux *et al.* (2004). Note that the model in Philipov and Glickman (2006) is defined also for non-integer values of ν.

□

Example 3 - Markov-Switching Autoregressive Wishart (MS-ARW(1))
We introduce a Markov-switching process in the dynamics of the ARW(1)

$$\Sigma_{t+1}^{-1} \sim \mathcal{W}_k\left(\nu_{s_{t+1}}, \frac{1}{\nu_{s_{t+1}}} A_{s_{t+1}}^{1/2} (\Sigma_t^{-1})^{d_{s_{t+1}}} (A_{s_{t+1}}^{1/2})'\right) \quad (4)$$

$$s_{t+1} \sim \mathbb{P}(s_{t+1} = j | s_t = i) = p_{ij} \quad \text{with} \quad i, j \in \{1, \ldots, E\} \quad (5)$$

where $A_i \in \mathcal{M}_+^k$, $\forall i \in \{1, \ldots, E\}$. The Markov-switching process accounts for sudden changes of regimes in the volatility behavior. When the parameters $\nu_{s_{t+1}} = \nu$ and $d_{s_{t+1}} = d$, i.e. they are constant over regimes, then the MS-ARW(1) accounts for different regimes in the volatility levels. Fig. 3 and 4 give a simulated example of MS-ARW(1) with $A_{s_{t+1}}$ diagonal matrix, which excludes spill-over effects. The regimes-switching in the matrix $A_{s_{t+1}}$ influences the scale of the observable producing different levels of volatility (see Fig. 3). Note that the correlation matrix (Fig. 4) is note affected by the Markov-switching process.

□

Example 4 - Markov-Switching Wishart Autoregressive (MS-WAR(1))
Let $\boldsymbol{x}_{mt} \in \mathbb{R}^k$, with $m = 1, \ldots, \nu$ and $\nu \in \mathbb{N}_0$, be a sequence of Markov-switching first-order vector autoregressive processes: $\boldsymbol{x}_t \sim \mathcal{N}_k\left(A_{s_t}\boldsymbol{x}_{t-1}, \Sigma_{s_{t+1}}\right)$, where $A_i \in \mathcal{M}^k$ $\forall i \in \{1, \ldots, E\}$. Then $\Sigma_t = \sum_{m=1}^{M} \boldsymbol{x}_{mt}\boldsymbol{x}'_{mt}$ is a MS-WAR(1).

□

3. Matrix-State Particle Filter

The estimation of the latent-variable model presented in Section 2. configures a problem of discrete-time nonlinear filtering, with unknown parameters. In our model neither the Kalman-Bucy nor the Hamilton-Kitagawa filters can be used, thus alternative procedures should be considered. We bring into action a simulation based nonlinear-filtering procedure called Particle Filter, which is in the general class of Sequential Monte Carlo methods.
We follow a full Bayesian estimation approach and propose the joint sequential estimation of the parameters and states of the matrix-valued dynamic model. The Bayesian approach is general enough to account for nonlinear models and for prior information on the parameters. Another feature of this framework is to allow the use of simulation methods in the inference process.

Figure 3: Simulated MS volatility model. Left column: simulated observable process $\boldsymbol{y}_t$ (*black line*) and hidden volatility regimes s_t (*grey line*). Right column: simulated latent volatilities $(\sigma_{11t}, \ldots, \sigma_{55t})$, where σ_{ijt} denotes the (i, j)-th element of $\Sigma_t^{1/2}$. In the simulation study we assume $d = 0.3$, $\nu = 19$, $A_1^{-1} = 0.1\, I_5$, $A_1^{-1} = 0.04\, I_5$ and $A_1^{-1} = 0.0125\, I_5$, where I_k denotes the identity matrix of dimension k.

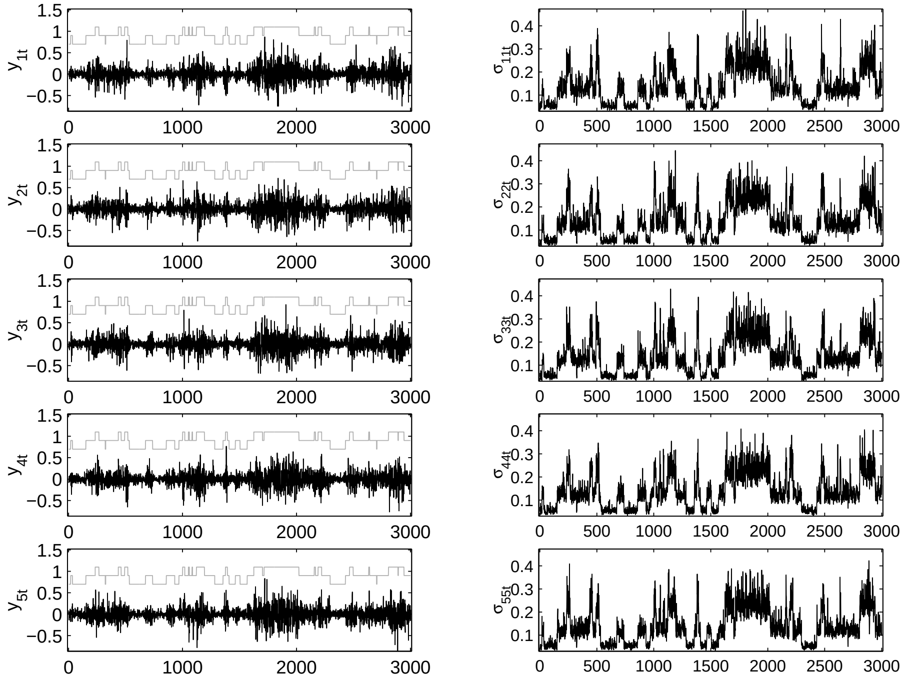
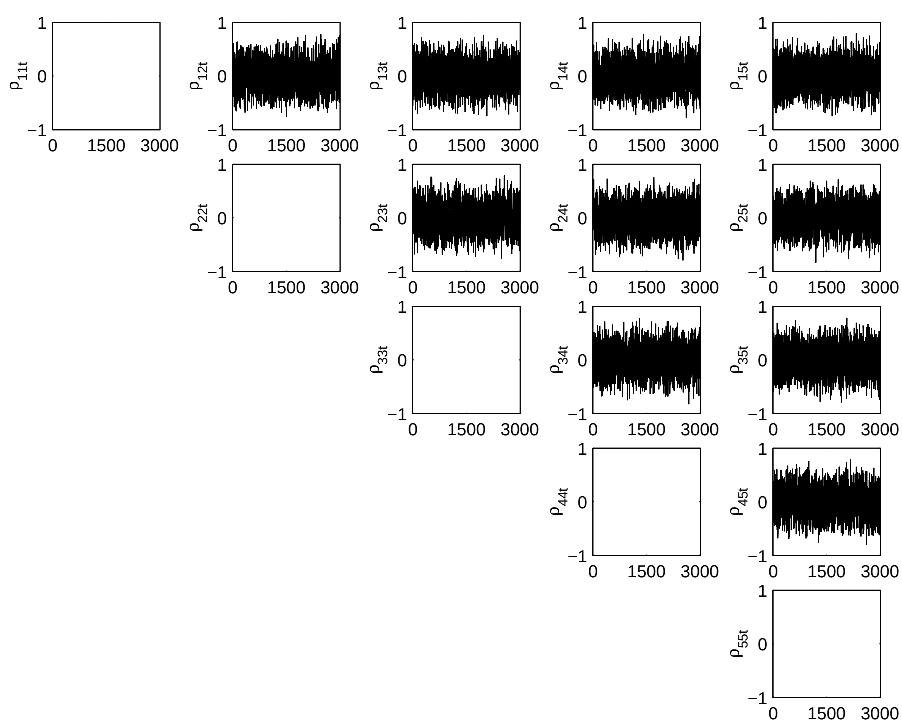


Figure 4: Simulated stochastic correlations. Matrix of graphs representing the time evolution of the components ρ_{ijt} of the correlation matrix. The correlation structure is implicit in the simulated Wishart process and is not affected by the switching dynamics.



3.1 Parameters and Latent Variables Estimation

Following Harrison and West (1997) in this contribution we propose a probabilistic representation of a stochastic dynamic model with matrix-valued states and observations. Let $\mathcal{M}_y \subset \mathbb{R}^{n_y \times m_y}$, $\mathcal{M}_x \subset \mathbb{R}^{n_x \times m_x}$ and $\mathcal{M}_\xi$ be the observation, state and parameter spaces respectively. Let $\mathbf{Y}_t \in \mathcal{M}_y$ denote the matrix of observations, $\mathbf{X}_t \in \mathcal{M}_x$ the matrix-valued hidden state and $\boldsymbol{\xi} \in \mathcal{M}_\xi$ the parameter. A matrix-valued dynamic Bayesian model can be defined as

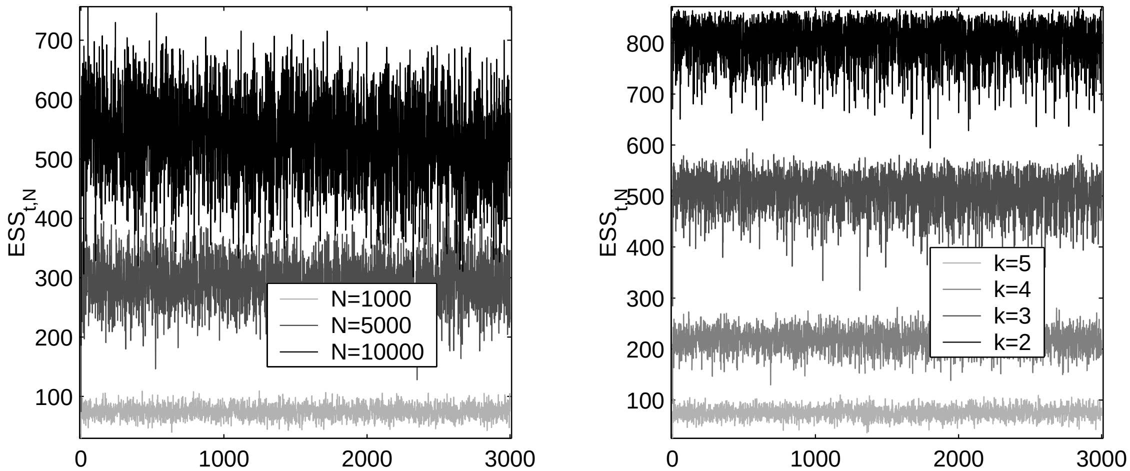
$$\begin{aligned} \boldsymbol{Y}_t &\sim p(\boldsymbol{Y}_t|\boldsymbol{X}_t, \boldsymbol{\xi}) && \text{(measurement density)} && (6)\\ \boldsymbol{X}_t &\sim p(\boldsymbol{X}_t|\boldsymbol{X}_{t-1}, \boldsymbol{Y}_{1:t-1}, \boldsymbol{\xi}) && \text{(transition density)} && (7)\\ \boldsymbol{X}_0 &\sim p(\boldsymbol{X}_0|\boldsymbol{\xi}) && \text{(initial density)} && (8)\\ \boldsymbol{\xi} &\sim p(\boldsymbol{\xi}) && \text{(prior density)} && (9) \end{aligned}$$

with $t = 1, \dots, T$.
The MSV models presented in our work are special cases of this general state space model, where $\boldsymbol{\xi} = ((a_{11}, \dots, a_{kk}), d, \nu)' \in \mathbb{R}^{k+2}$, $\boldsymbol{Y}_t = \boldsymbol{y}_t$, with $\boldsymbol{y}_t \in \mathbb{R}^k$ and $\boldsymbol{X}_t = \Sigma_t$, with $\Sigma_t \in \mathcal{M}_+^k$. The first advantage of the proposed probabilistic representation is that it allows us to naturally introduce nonlinear filtering methods techniques based on simulation methods. Another advantage is that it includes as spacial cases many well know dynamic models. It includes also the matrix-valued linear gaussian model, recently studied in Choukroun (2003) and Bar-Itzhack *et al.* (2006).
In the stochastic filtering literature the matrix-valued state and observations are usually stacked into vectors and the filtering procedure is then applied to the vectorized system. The vectorized representation could be used to define the optimal filter. Note however that working with the vectorized system representation could lead to some difficulties. When the original model is nonlinear and non-Gaussian the analytical form of the vectorized-state transition density is not straightforward and the probabilistic constraints, such as the positive definiteness of the states matrix, on the original model could not be easily imposed in the vectorized system.
In this work we do not vectorize the system and present instead the optimal filtering problem for the original dynamic Bayesian model. In order to estimate the parameter we introduce a general definition of state: $\boldsymbol{Z}_t = (\boldsymbol{X}_t, \boldsymbol{\xi}_t)$ and of augmented state space: $\mathcal{M}_z = \mathcal{M}_x \times \mathcal{M}_\xi$. The state and observation one-step-ahead *prediction densities* and the augmented-state *filtering* and the *smoothing densities* are

$$\begin{aligned} p(\boldsymbol{Z}_{t+1}|\boldsymbol{Y}_{1:t}) &= \int_{\mathcal{M}_z} p(\boldsymbol{X}_{t+1}|\boldsymbol{X}_t, \boldsymbol{Y}_{1:t}, \boldsymbol{\xi}_{t+1}) \delta_{\boldsymbol{\xi}_t}(\boldsymbol{\xi}_{t+1}) p(\boldsymbol{Z}_t|\boldsymbol{Y}_{1:t}) d\boldsymbol{Z}_t && (10)\\ p(\boldsymbol{Y}_{t+1}|\boldsymbol{Y}_{1:t}) &= \int_{\mathcal{M}_z} p(\boldsymbol{Y}_{t+1}|\boldsymbol{X}_{t+1}, \boldsymbol{\xi}_{t+1}) p(\boldsymbol{Z}_{t+1}|\boldsymbol{Y}_{1:t}) d\boldsymbol{Z}_{t+1} && (11)\\ p(\boldsymbol{Z}_{t+1}|\boldsymbol{Y}_{1:t+1}) &= \frac{p(\boldsymbol{Y}_{t+1}|\boldsymbol{Z}_{t+1}) p(\boldsymbol{Z}_{t+1}|\boldsymbol{Y}_{1:t})}{p(\boldsymbol{Y}_{t+1}|\boldsymbol{Y}_{1:t})} && (12) \end{aligned}$$

In this work we extend the *Regularised Auxiliary Particle Filter* (R-APF) due to Liu and West (2001) to matrix-valued state space models and apply it to the proposed Wishart MSV models. Let $\{\boldsymbol{Z}_t^i, w_t^i\}_{i=1}^N$ be a weighted random sample, which is approximating the filtering density at time t. At the time step $t+1$, as a new observation $\boldsymbol{Y}_{t+1}$ arrives, we

Figure 5: The absolute Effective Sample Size at each iteration, estimated over 10 independent runs of the R-APF on different simulated datasets. We apply the R-APF to samples of $3,000$ observations simulated from Eq. (1)-(2). Left-chart: ESS for $k=5$, varying the number of particles: $N=1,000$ (*light grey*), $N=5,000$ (*dark grey*) and $10,000$ (*black*). Right-chart: ESS for $N=1,000$ varying the dimension of the state-matrix: from $k=2$ to $k=5$. The darker the line color, the lower the value of k.



can approximate the prediction and filtering densities in (10) and (12) as follows

$$p_N(\boldsymbol{Z}_{t+1}|\boldsymbol{Y}_{1:t}) = \sum_{i=1}^{N} p(\boldsymbol{X}_{t+1}|\boldsymbol{X}_t^i, \boldsymbol{Y}_{1:t}, \boldsymbol{\xi}_{t+1}) K_h(\boldsymbol{\xi}_{t+1} - \boldsymbol{\xi}_t^i) w_t^i \tag{13}$$

$$p_N(\boldsymbol{Z}_{t+1}|\boldsymbol{Y}_{1:t+1}) \propto \sum_{i=1}^{N} p(\boldsymbol{Y}_{t+1}|\boldsymbol{Z}_{t+1}) p(\boldsymbol{X}_{t+1}|\boldsymbol{X}_t^i, \boldsymbol{Y}_{1:t}, \boldsymbol{\xi}_t^i) K_h(\boldsymbol{\xi}_{t+1} - \boldsymbol{\xi}_t^i) w_t^i \tag{14}$$

A simple way to obtain a weighted random sample from the approximated filtering density at time $t+1$ is to apply importance sampling to the density given in Eq. (14). We propagate each particle of the set through the importance density $q(\boldsymbol{Z}_{t+1}|\boldsymbol{Z}_t^i, \boldsymbol{Y}_{1:t+1}) = p(\boldsymbol{X}_{t+1}|\boldsymbol{X}_t^i, \boldsymbol{Y}_{1:t}, \boldsymbol{\xi}_{t+1}) K_h(\boldsymbol{\xi}_{t+1} - \boldsymbol{\xi}_t^i)$, then particle weights w_{t+1} update as follows

$$\begin{aligned} \tilde{w}_{t+1}^i &\propto \frac{p(\boldsymbol{Y}_{t+1}|\boldsymbol{Z}_{t+1}) K_h(\boldsymbol{\xi}_{t+1} - \boldsymbol{\xi}_t^i) p(\boldsymbol{Z}_{t+1}|\boldsymbol{Z}_t^i, \boldsymbol{Y}_{1:t}, \boldsymbol{\xi}_{t+1}) w_t^i}{q(\boldsymbol{Z}_{t+1}|\boldsymbol{Z}_t^i, \boldsymbol{Y}_{1:t+1})} \\ &\propto w_t^i \, p(\boldsymbol{Y}_{t+1}|\boldsymbol{Z}_{t+1}^i). \end{aligned} \tag{15}$$

4. Simulation Results

We apply the regularised SMC algorithm for matrix-valued state to the MS-ARW(1) given in Example 3. We initialize the algorithm by running a Gibbs sampler on 100 observations. We omit here the details about the choice of the priors, the full conditionals used in the Gibbs sampler. The parameter h in regularisation step of the SMC algorithm has been chosen following usual optimal criteria. In this work we present the simulation results about the efficiency of the algorithm for this model.

We measure the numerical efficiency of the algorithm by evaluating at each iteration the *Effective Sample Size* (ESS), that is defined as

$$\mathrm{ESS}_t = \left(\sum_{i=1}^{N} \left(\frac{w_t^i}{\sum_{j=1}^{N} w_t^j} \right)^2 \right)^{-1} \tag{16}$$

and varies between 1 (all but one particle weights are null) and N (equal weights). A related criterion is the coefficient of variation (see Liu and Chen 1998). For the MSV model in Example 3 we evaluate the ESS varying the number of particles ($N = 1,000$, $5,000$, $10,000$) and the dimension of the state-matrix ($k = 2,3,4,5$). For given N and k, at each time step, the mean ESS has been estimated over 10 independent runs of the R-APF on different simulated datasets. Fig. 5 shows the results.
As one could expect, increasing N improves the absolute efficiency of the filtering procedure (left-chart), although the relative efficiency (ESS_t/N) is decreasing. As showed in the right-chart the efficiency is also affected by the dimension of the latent variable space. The higher the state-matrix dimension is, the lower the numerical efficiency is.

5. Conclusion

In this paper we propose new Wishart stochastic volatility models. The parameter of the Wishart autoregressive process is governed by a first-order Markov chain. We propose a Sequential Monte Carlo approach for the joint estimation of the parameter and the hidden state-matrix. By means of simulation experiments, we discuss the efficiency of the method and the dimensionality problem.

References

Aguilar O. and West M. (2000) Bayesian Dynamic Factor Models and Portfolio Allocation, *Journal of Business and Economic Statistics*, 18, 338–357.

Arulampalam S., Maskell S., Gordon N. and Clapp T. (2001) A Tutorial on Particle Filters for On-line Nonlinear/Non-Gaussian Bayesian Tracking.

Asai M. and McAleer M. (2005) Dynamic Correlations in Symmetric Multivariate SV Models, in: *MODSIM 2005 International Congress on Modelling and Simulation. Modelling and Simulation Society of Australia and New Zealand*, Zerger A. and Argent R., eds.

Asai M. and McAleer M. (2006a) Multivariate Stochastic Volatility Models: Bayesian Estimation and Model Comparison, *Econometric Reviews*, 25, 361–384.

Asai M. and McAleer M. (2006b) The Structure of Dynamic Correlations in Multivariate Stochastic Volatility Models, Econometric Reviews, forthcoming.

Asai M., McAleer M. and Yu J. (2006) Multivariate Stochastic Volatility: A Review, *Econometric Reviews*, 25, 145–175.

Bar-Itzhack I.Y., Choukroun D., Oshman Y. and Weiss H. (2006) Kalman filtering for matrix estimation, Technical Report N. 1266, University of California.

Berzuini C., Best N.G., Gilks W.R. and Larizza C. (1997) Dynamic conditional independence models and Markov chain Monte Carlo methods, *Journal of the American Statistical Association*, 92, 440, 1403–1441.

Chib S., Nardari F. and Shephard N. (2006) Analysis of High-Dimensional Multivariate Stochastic Volatility Models, *Journal of Econometrics*, 134, 341–371.

Choukroun D. (2003) Novel methods for attitude determination using vector observations, PhD Thesis in Engeneering, Israel Institute of Technology, Haifa.

Danielson J. and Richard J.F. (1993) Accelerated Gaussian importance sampler with application to dynamic latent variable models, *Journal of Applied Econometrics*, 8, 153173.

Danielsson J. (1994) Stochastic volatility in asset prices: Estimation with simulated maximum likelihood, *Journal of Econometrics*, 64, 375400.

Doucet A., Freitas J.G. and Gordon J. (2001) *Sequential Monte Carlo Methods in Practice*, Springer Verlag, New York.

Durbin J. and Koopman S.J. (2000) Time series analysis of non-Gaussian observations based on state space models from both classical and bayesian perspectives (with discussion), *Journal of the Royal Statistical Society Series B*, 62, 356.

Engle R. (2002) Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models, *Journal of Business and Economic Statistics*, 20, 3, 339–350.

Gallant A. and Tauchen G. (1996) Which moments to match?, *Econometric Theory*, 12, 657–681.

Gourieroux C., Jasiak J. and Sufana R. (2004) The Wishart Autoregressive Process of Multivariate Stochastic Volatility, Working Paper, CREST.

Harrison J. and West M. (1997) *Bayesian Forecasting and Dynamic Models*, *2nd Ed.*, Springer Verlag, New York.

Harvey A., Ruiz E. and Shephard N. (1994) Multivariate Stochastic Volatiliy Models, *Review of Economic Studies*, 61, 247–264.

Jacquier E., Polson N.G. and Rossi P. (1994) Bayesian Analysis of Stochastic Volatility Models, *Journal of Business and Economic Statistics*, 12, 281–300.

Kalman R.E. (1960) A new approach to linear filtering and prediction problems, *Journal of Basic Engineering*, *Transaction ASME*, *Series D*, 82, 35–45.

Kalman R.E. and Bucy R.S. (1960) New results in linear filtering and prediction problems, *Transaction of the ASME-Journal of Basic Engineering, Series D*, 83, 95–108.

Kim S. S.N. and Chib S. (1998) Stochastic volatility: Likelihood inference and comparison with ARCH models, *Review of Economic Studies*, 65, 361–393.

Liu J. and Chen R. (1998) Sequential Monte Carlo methods for dynamical system, *Journal of the American Statistical Association*, 93, 1032–1044.

Liu J. and West M. (2001) Combined Parameter and State Estimation in Simulation Based Filtering, in: *Sequential Monte Carlo Methods in Practice*, Doucet A. F.J. and J. G., eds., Springer-Verlag, New York.

Musso C., Oudjane N. and LeGland F. (2001) Improving Regularised Particle Filters, in: *Sequential Monte Carlo in Practice*, Doucet A., Freitas J. and Gordon J., eds., Springer Verlag, New York.

Pelletier D. (2006) Regime switching for dynamic correlations, *Journal of Econometrics*, 131, 445–473.

Philipov A. and Glickman M.E. (2006) Multivariate Stochatic Volatility via Wishart Processes, *Journal of Business and Economics Statistics*, 24, 313–328.

Sandmann G. and Koopman S.J. (1998) Maximum likelihood estimation of stochastic volatility models, *Journal of Econometrics*, 63, 289–306.

So M.K.P., Lam K. and Li W.K. (1998) A stochastic volatility model with Markov switch-

ing, *Journal of Business and Economic Statistics*, 18, 429–446.
Taylor S.J. (1986) *Modelling Financial Time Series*, Chichester, Wiley.
Taylor S.J. (1994) Modelling Stochastic Volatility, *Mathematical Finance*, 4, 183–204.