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Quantum indeterminacy and the double-slit experiment

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1 Introduction

In Calosi and Wilson (2018), we argue that on many interpretations of quantum mechanics (QM), there is quantum mechanical indeterminacy (QMI), and that a determinable-based account of metaphysical indeterminacy (MI), as per Wilson (2013) and (2016), properly accommodates the full range of cases of QMI. Here we argue that this approach is superior to other treatments of QMI on offer, both realistic and deflationary, in providing the basis for an intelligible explanation of the interference patterns in the double-slit experiment. We start with a brief overview of the motivations for QMI and for a determinable-based account of MI (Sect. 2). We then apply a developed 'glutty' implementation of determinable-based QMI to the superposition-based QMI present in the double-slit experiment, and positively

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compare the associated explanation of double-slit interference with that available on a metaphysical supervaluationist account of QMI (Sect. 3). We then present and respond to objections, due to Glick (2017) and Torza (2020), either to QMI (Sect. 4) or to our specific account of QMI (Sect. 5); in these sections we also positively compare our treatment of double-slit interference to that available on Glick's deflationary treatment of QMI. We conclude with some dialectical observations (Sect. 6).

2 A determinable-based approach to QMI

2.1 The appearance of QMI

Taking standard presentations (in textbooks, or in Feynman 1963) at face value, QM violates the classical supposition of 'value definiteness', according to which the properties—a.k.a. 'observables'—of a particle or system always have precise values. Such indefiniteness or indeterminacy is core to what is supposed to be distinctive about quantum phenomena, as per, e.g., the indeterminate location of a particle in a double-slit experiment, the indeterminate life-status of Schrödinger's cat, the failure of a particle to have precise values of both position and momentum at a time, the failure of components of a spin-entangled state to have determinate values of spin, and so on. As has been frequently observed,[1] the indeterminacy in such cases is most plausibly taken to be metaphysical—not merely epistemological, much less semantic.

The motivation for there being QMI in these cases stems from the 'Eigenstate-Eigenvalue Link', which is, as Lewis (2016) puts it, "a fairly standard way of understanding quantum states" (76):

> *Eigenstate-Eigenvalue Link* (EEL): A quantum system S has a definite (determinate) value v for an observable O iff S is in an eigenstate of O having eigenvalue v.

As we observe in our (2018), attention to EEL (or its variants[2]) suggests three different sources of QMI, as involving superposition, incompatible observables, or entanglement:[3]

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[1] See, e.g., Darby (2010), Skow (2010), Bokulich (2014), Lewis (2016), Torza (2020), Calosi and Wilson (2018).

[2] Some suggest that certain interpretations of QM require revision of EEL as a principle connecting the quantum formalism to the having of a definite value of a given observable. For example, Albert and Loewer (1992) suggest that since collapse on the Ghirardi-Rimini-Weber (GRW) interpretation of QM leaves a lingering 'tail' of indeterminacy, EEL should be replaced by the Fuzzy Link (FL):

> (FL): A quantum system has a definite value v for a particular observable O iff the square projection of its state onto an eigenstate of O is greater than $1 - P$, for some (suitably small) P.

See also Lewis's (2016) discussion of the 'Vague Link'. As we note in our (2018), the need for such revision is controversial (see Frigg 2009) and also appears to be pragmatically motivated, in a way leaving seeming QMI intact. We later revisit whether GRW or other interpretations of QM are committed to QMI; at this point we aim simply to present the usual EEL-based motivations for QMI.

[3] Plausibly, superposition is the most general source of QMI, since in cases of incompatible observables O_1 and O_2, some eigenstates of O_1 are superpositions of eigenstates of O_2, and entanglement states are superposition states. Even so, just as the properties of *colour*, *red*, and *blue* are related (with the first being more general than the second and third) yet interestingly distinct, we similarly maintain that these sources of quantum MI are related yet interestingly distinct; see Calosi and Wilson (2018) for discussion.

1. *Superposition*. In general, a superposition of eigenstates of an observable O is not an eigenstate of O. Hence, given EEL, any physical system S in such a superposition will fail to have a determinate value of O.
2. *Incompatible Observables*. Let O_1 and O_2 be incompatible observables. In general, an eigenstate of O_1 will not be an eigenstate of O_2, and vice versa. Hence, given EEL, any physical system S in such an eigenstate of O_1 will fail to have a determinate value of O_2.
3. *Entanglement*. Let S_3 be a system composed of two other systems S_1 and S_2, and let S_3 be in an eigenstate of an observable $O_3 = O_1 + O_2$, where O_i is defined over the Hilbert space of system S_i. In general, an eigenstate of O_3 will not be an eigenstate of either O_1 or O_2. Hence, given EEL, a system S_3 in such an eigenstate of O_3 will be such that the subsystems S_1 and S_2 will fail to have determinate values of O_1 and O_2, respectively.

The appearances of QMI give rise to several questions which have been the topic of recent debate, including: Can the appearances be taken at realistic face value, on at least some interpretations of QM? And if so, which account of MI is best suited to accommodate QMI?

2.2 Determinable-based QMI

In our (2018) we offered answers to these questions. We started by observing that the primary reason for thinking that seeming QMI cannot be taken at metaphysical face value reflects concerns that MI is incoherent (Evans 1983) or unintelligible (Dummett 1975; Lewis 1986), but that in the wake of recent work addressing these concerns, the notion of MI is no longer seen as untenable. Two different approaches to MI have come to the fore. One is a metaphysical supervaluationist approach, along lines proposed by Akiba (2004) and later developed by Barnes (2010) and Barnes and Williams (2011); here metaphysical indeterminacy is located at the ‘meta-level’, as indeterminacy in which of some range of precise (maximally determinate) states of affairs obtains. The other is a determinable-based approach, along lines proposed by Wilson (2013 and 2016); here metaphysical indeterminacy is located at the ‘object-level’, as indeterminacy in a given state of affairs itself. As we argued (expanding on the critiques in Darby 2010 and Skow 2010), a supervaluationist approach does not properly accommodate QMI—not just on the orthodox interpretation which was the focus of Darby’s and Skow’s discussions, but also on certain readings of the Everettian and GRW interpretations. However, we then argued, a determinable-based approach can accommodate all three forms of QMI on any of these interpretations.

As prefigured, we aim here to substantiate and expand on this result, with proper accommodation of the double-slit experiment as a focus. We start by sketching a determinable-based account. Schematically stated, the account is as follows:

> *Determinable-based MI*: What it is for a state of affairs to be MI in a given respect R at a time t is for the state of affairs to constitutively involve an object (more generally, entity) O such that (i) O has a determinable property P at t,

and (ii) for some level *L* of determination of *P*, *O* does not have a unique level-*L* determinate of *P* at *t* (Wilson 2013, 366).

There are two ways for failure of unique determination to occur, associated with 'gappy' and 'glutty' MI, respectively:

1. 'gappy' MI: no determinate of the determinable is instantiated, hence *a fortiori* no unique determinate of the determinable is instantiated.
2. 'glutty' MI: more than one determinate of the determinable is instantiated, such that no determinate is properly taken to be 'the' unique determinate of the determinable. There are moreover two variants on the glutty theme: one where multiple determinates are instantiated, albeit in relativized fashion, and one where multiple determinates are instantiated, each to degree less than one. (We'll expand on these variants down the line.)

As discussed in Wilson (2013) and (2016), *Determinable-based MI* has certain general advantages, including that such an account ...

1. ... reduces MI to a pattern of instantiations of properties of the sort with which we are already familiar, and so (unlike a supervaluationist account), does not take MI to be primitive.
2. ... does not introduce propositional indeterminacy, and so (unlike a supervaluationist account) does not require introducing an indeterminacy operator into one's semantics or logic.
3. ... is thoroughly compatible with classical logic and semantics, and so (unlike a supervaluationist account[4]) requires no revision in these classical theories.

In addition to arguing, in our (2018), that *Determinable-based MI* can accommodate the full range of sources of QMI, we considered, for each source, whether it might be better treated in gappy or rather glutty fashion. We found that in certain cases of superposition QMI, there appeared to be advantages to a glutty treatment. We now expand on a glutty determinable-based approach to double-slit QMI, with an eye to highlighting some specific advantages of this approach.

3 A determinable-based account of the double-slit results

3.1 The double-slit experiment: an overview

We start by reviewing the experimental setup and salient outcomes of the double-slit experiment. As is familiar (see, e.g., Feynman 1963 and Barrett 2001), the setup involves a particle source which emits particles (e.g., electrons or photons) of the same wavelength in the direction of a screen covered with closely spaced particle detectors. Between the particle source and the screen is a barrier containing two slits, each of which may be opened or closed. A given run of the experiment consists in either setting one slit open and one slit closed, or else setting both slits open, and then firing a large number of particles at the screen, one at a time.

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[4] See Williamson (1994, Ch. 6), for relevant discussion.

In trials where one of the slits is closed, the observed histogram of particles hitting the screen is the pattern we expect from classical physics (see Fig. 1):[5]

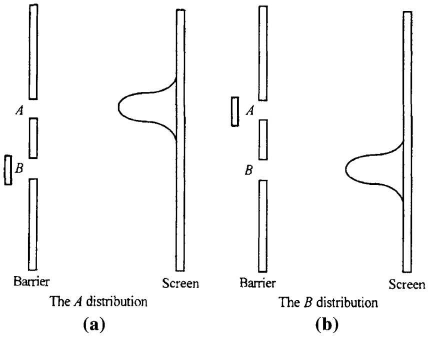

Fig. 1 Single-slit patterns

What if both slits are open? Given that only one particle is fired at a time, and might go through either slit, the pattern expected from classical physics would be an overlay of the two single slit patterns (see Fig. 2):

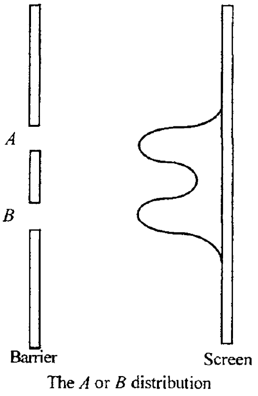

Fig. 2 Classical double-slit pattern

Famously, however, the classically expected pattern is not what is actually observed. Rather, when both slits are open, the histogram exhibits interference (see Fig. 3):

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[5] The following figures are taken from Barrett (2001, 4).

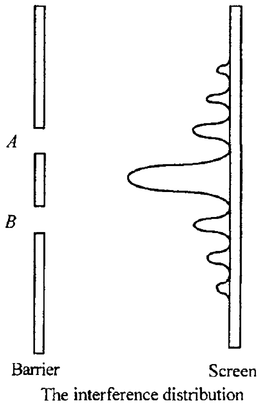

Fig. 3 Quantum double-slit pattern

The question then arises: how is this interference pattern produced, given that only one particle makes the journey from source to detector at a time? As Feynman (1963) remarks just prior to discussing the double-slit experiment, this phenomenon is mysterious:

> We choose to examine a phenomenon which is impossible, *absolutely* impossible, to explain in any classical way, and which has in it the heart of quantum mechanics. In reality, it contains the only mystery. [...] In telling you how it works we will have told you about the basic peculiarities of all quantum mechanics. (1-1)

Given that this experiment encodes the ‘heart’ and the ‘mystery’ of QM, it would be a considerable benefit of a given account of QMI if it were able to provide an intelligible basis for understanding how a sequence of single particles could produce the double-slit interference pattern. As we’ll now argue, a glutty implementation of a determinable-based account of QMI provides an intelligible basis for this pattern, whereas a metaphysical supervaluationist account does not do so.

3.2 The double-slit experiment: a partial explication

We now present what has frequently been offered as an intuitive explication of double-slit interference, which goes some but not all of the way towards rendering the phenomenon intelligible.

To start, the double-slit phenomenon is treated in the quantum formalism as a case of superposition,[6] where a particle traveling from source to detector is represented as being in a state $\frac{1}{\sqrt{2}}(|A\rangle + |B\rangle)$, where $|A\rangle$ represents the state of the

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[6] Such a treatment is forced by the fact that if detectors are placed at each of the slits, the resulting pattern is just that associated with classical physics.

particle's traveling through slit A but not slit B, and $|B\rangle$ represents the state of the particle's traveling through slit B but not slit A.[7]

Now, taking the formalism at face value in this and other cases, superpositions involve additive combinations of instantiated determinate states. Classic descriptions of cases of superposition often reflect this additive or conjunctive understanding, as when Einstein (1939) describes Schrödinger's case in glutty terms: "At a fixed time parts of the Ψ-function correspond to the cat being alive and other parts to the cat being pulverized". Such an understanding suggests a natural and commonly endorsed strategy for explaining the interference in the double-slit experiment. As Dirac (1930) initially put the suggestion:

> So long as the photon is partly in one beam and partly in the other, interference can occur when the two beams are superposed. (8–9)

More specifically, the suggestion is that, insofar as a particle is associated with a wave function expressing the probability of its being found in a certain location upon measurement, then if such a particle could somehow go through both slits at the same time, then it or its wave-function could appropriately interfere with each other, in the way Barrett (2001) describes:

> These two wave-packets spread out and interfere with each other in the region between the barrier and the screen and then the composite wave hits the screen. (5–6)

As Lewis (2016) emphasizes, what is being proposed here is 'self-interference':

> Note that since the interference pattern is present even though the electrons pass through the apparatus singly, each electron interferes with *itself*, not with other electrons. (5)

Supposing that self-interference makes sense, then the natural explanation of double-slit interference follows, along lines described by Maudlin (2019):

> [T]he interference manifests itself just as it does for water waves. The two parts of the wavefunction at $t = 0$, the parts in front of the two slits, have equal magnitude and phase, because the plane wave that hit the barrier had equal magnitude and phase in those locations. At any point on the screen where the difference in the distances to the two slits is a multiple of the wave length, the two superposing waves arrive with the same phase, and the resulting wavefunction has twice the amplitude of each. But if the difference of the distances is a half wavelength (or 3/2, or 5/2, etc.), then the two superposing wavefunctions have opposite phase and equal amplitude. Added together at that point they cancel out [...] leaving the wavefunction with zero amplitude. By Born's rule, a flash has no chance to occur there. The alternating regions of

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[7] This is a simplification. The initial wavefunction of each particle is an eigenstate of momentum; hence it has the form of a plane-wave $|\psi\rangle = e^{ipx/\hbar}$, corresponding to a much more complicated superposition of position states. The simplification is frequently adopted in the literature (see, e.g., Barrett 2001), and for present purposes is harmless.

> high probability and zero probability yield the interference fringes as many flashes accumulate. (52–53)

Now, an explanation of double-slit interference as involving self-interference of a single particle (its wave function) with itself goes part but not all of the way towards making sense of the double-slit results. For it remains to explain how a single particle and its associated wave function can go through both slits, or be "sensitive" to both slits, given that the particle is, empirically speaking, an individual. As Feynman (1963) puts it, in setting up the puzzle posed by the double-slit experiment:

> [L]et us try to analyze the [double-slit interference pattern] to see whether we can understand the behavior of electrons. The first thing we would say is that since they come in lumps, each lump, which we may as well call an electron, has come either through hole 1 or through hole 2. [Let us call this] 'Proposition A'. (1-5)

Given that an electron can be in two different places—i.e., go through both slits—at the same time, then we can make sense of the two superposing wave functions (as Maudlin puts it) interfering in ways conforming to the results of the double-slit experiment. But how can an electron be in two different places at the same time? How can an electron go through both slits, given that it is (for all experiment tells us, and notwithstanding its association with a probabilistic wave function) an individual: a 'lump', not a wave?

As Feynman notes, there have been efforts to explain how interference might come about "in terms of individual electrons going around in complicated ways through the holes" but "None of them has succeeded". Feynman takes these failures to support taking a quietist attitude towards apparent failures of Proposition A. But if one wants to explain double-slit interference in terms of self-interference, more needs to be said. For the explanatory strategy suggested by the remarks of Dirac, Barrett, Lewis, and Maudlin is one that clearly rejects Proposition A, and the rejection of Proposition A is not clearly consistent. How, after all, might one thing—a 'lump', not a wave—be in two different places at the same time, in such a way that it (or its associated wave function) can "interfere with itself"?

3.3 A glutty determinable-based explanation of double-slit interference

As we'll now argue, *Determinable-based MI* provides an intelligible basis for making sense of how a particle (its wave function) might be located in two places at the same time, such that it (its wave function) can interfere with itself.[8]

Recall that cases of superposition are paradigmatic of seeming QMI: in such cases, the system is not in an eigenstate of the observable in question, and so, by lights of EEL, has an indeterminate value of that observable or associated property. Recall also that according to *Determinable-based MI*, MI reflects that a given

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[8] As will become clear, our proposal is not that self-interference involves 'multilocation' of the sort recently defended, e.g., in Eagle (2016).

determinable property instance is not uniquely determined, and that the failure of unique determination may reflect either that no determinates are instantiated (as per 'gappy' MI), or that too many determinates are instantiated (as per 'glutty' MI). Here, a glutty implementation of *Determinable-based MI* seems most apt, in accommodating superposition QMI in a way that moreover fills in the intuitive explication of double-slit interference.

The core idea is that in the double-slit experiment, the associated QMI reflects that, on any given pass of the experiment, the emitted particle has the determinable property *having traveled from the source to the detector* (which property is itself a determinate of *position* or of *being spatiotemporally located*), but does not have a unique determinate of that determinable, due to too many of the determinates of the determinable, associated in particular with the states $|A\rangle$ and $|B\rangle$, being instantiated, in glutty fashion. (Note that the determinable property here and in other cases of superposition QMI is not to be identified with the property of being in the superposition at issue, notwithstanding that the determinates of the relevant determinable are associated with components of the superposition state. We will revisit this issue down the line.) Here as in other cases where multiple determinates of a determinable are concurrently instantiated, there is no non-arbitrary way to identify one of the determinates as 'the' determinate of the instantiated determinable. In particular, the particle does not have one and only one of the determinate properties associated with the states $|A\rangle$ and $|B\rangle$. Rather, there is a sense in which the particle has both properties, and correspondingly travels through both slits, without inducing the metaphysical correlate of a contradiction.

Indeed, on a glutty implementation, there are two ways in which the consistent concurrent having of the determinate properties might occur.

The first variant draws on what Wilson (2013) calls 'multiple relativized determination', as illustrated by (an available understanding of) the case of an iridescent feather. The colour of such feathers—e.g., the throat feathers ('gorgets') of certain hummingbirds—shifts from red to blue, depending on the angle of viewing. The scientific understanding of this phenomenon suggests that the determinate color of an iridescent feather is relative to perspective.[9] Depending on the operative account of colour, the perspective at issue might advert to a conscious observer, or rather to a spatial ray. What is important for our purposes is that there are available understandings of the relativization at issue which are both reasonable and compatible with several specific accounts of color, according to which an

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[9] As Johnsgard (1997) puts it: "The highly iridescent feathers of the hummingbird gorgets are among the most specialized of all bird feathers [...]. The colors do not directly depend on selective pigment absorption and reflection, as do brown and blacks produced by the melanin pigments of non-iridescent feathers. Rather, they depend on interference coloration, such as that resulting from the colors seen in an oil film or soap-bubble [...]. Put simply, red wavelengths are longer than those at the violet end of the spectrum and generally require films that are thicker or have higher refractive indices than those able to refract bluish or violet light. Thus, the optimum refractive index for red feathers is about 1.85; for blue feathers it is about 1.5 [...]. When an optical film is viewed from about, it reflects longer wavelengths than when viewed from angles progressively farther away from the perpendicular. Thus, a gorget may appear ruby red when seen with a beam of light coming from directly behind the eye, but as the angle is changed the gorget color will shift from red to blue and finally to black, as the angle of incidence increases (121–26).

iridescent feather has the determinable *colour*, but no unique determinate of that determinable; rather, the feather has a determinate colour (*red*, *blue*) only relative to a given perspective.[10] More specifically, the suggestion in Wilson (2013) is that one can reasonably maintain the following about what we might call ‘the feather case’:

1. The form of colour determination in the feather case indicates that determination may be a relativized phenomenon: which determinate determines a given determinable at a time may depend on specific circumstances.
2. The feather case indicates that at least sometimes, multiple such circumstances may hold at the same time *t*: one person can look at the feather at *t* and see one colour (one spatial ray can be associated with one colour); another person can look at the feather at *t* and see a different colour (another spatial ray can be associated with a different colour). That is, we can take the case to involve multiple relativized determination.
3. An iridescent feather has only a single instance of the determinable property *colour* at a given time *t*. It would, in particular, be redundant to take the feather to possess multiple instances of this determinable property at *t*.
4. In the feather case, the various relativized determinates are on a par; hence it would be arbitrary and inappropriate to attribute one rather than another of the determinate instances to the feather, as being the ‘unique’ determinate of the determinable property *colour* which the feather possesses at t.
5. So, in the feather case, the feather has the determinable property *colour* at a time, and it does not have a unique determinate of this determinable at that time.

The feather case illustrates one way the conditions in *Determinable-based MI* may be satisfied, in glutty fashion. In particular, while such multiple relativized determination prevents attributing a unique determinate to the shade to the feather, there remains a sense in which the feather can, in relativized fashion, consistently be both red and blue at a time.[11]

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[10] See Wilson (2013) for further discussion of this interpretation and its compatibility with several specific accounts of colour.

[11] To be sure, as is discussed in Wilson (2013), there are also understandings of the feather case which do not involve multiple relativized determination. For example (as a referee noted), one might accommodate the relativization in the feather case by taking the determinate colours as well as their associated determinables to be dyadic relations between objects and perspectives. This understanding strikes us as an unparsimonious and metaphysically inapropos way of treating what appears to be a singly instanced, unrelativized instantiation of the determinable *colour*, but again, for our purposes what is important is that an understanding in terms of multiple relativized determination makes sense. Here it may also be worth observing that properly metaphysical accommodation of the relativity at issue does not require that the perspectives (more generally, circumstances) be ‘built in’ to the properties at issue. On the contrary, in recent literature on perspectival facts this approach is commonly rejected; for example, Lipman (2016) argues that one should not account for what he calls “perspectival variance” by “saying that the apparent properties or relations merely turn out to have higher adicity—that these cases simply reveal a hidden argument place” (44), and Berenstain (2020) characterizes a perspectival fact as a fact expressed by a proposition whose truth value depends on the perspective of a particular observer, where the locus of relativization is the truth value of the proposition as opposed to a purportedly relational fact (or constitutive property). Nor is there any reason to think that non-relational conceptions of perspectival variance are not property metaphysical; see Evans (2020) for discussion as applied to the case of colour, in particular.

Similarly, we suggest, it is reasonable and compatible with the quantum facts that the case of the particle in the double-slit experiment involves multiple relativized determination: while superposition prevents attributing a unique trajectory to the particle, there remains a sense in which the particle can, in relativized fashion, consistently travel through both slits at a time. In the case of an iridescent feather, the circumstances to which the determinate colours are relativized are perspectives, subjective (observers) or objective (spatial rays). What circumstances are the determinates in the double-slit case relativized to? For the most part, we leave this to physicists to determine; however, we offer one negative and one speculative positive consideration. First, the basis of relativization should not be understood in terms of potential measurement outcomes, since we are here taking seriously that the particle (its wave function) is interfering with itself, in ways requiring the location determinates associated with states $|A\rangle$ and $|B\rangle$ to be instantiated prior to measurement. Second, and more positively, we speculate that the determinates may be relativized to certain possible trajectories for the particle, such that the particle has the property associated with the state $|A\rangle$ relative to all possible trajectories from the source to the detector passing through slit A but not slit B, and similarly for $|B\rangle$.[12] In any case, the notion of multiple relativized determination provides an intelligible basis for self-interference to occur.[13]

The second variant on the ‘glutty’ theme, representing another strategy for rendering self-interference intelligible, draws on a view according to which instantiation can come in degrees. Such a view accommodates satisfaction of the conditions in *Determinable-based MI*, since it plausibly suffices for a determinable to not be uniquely determined that none of its determinates are instantiated to degree 1.[14] In application to QMI, the quantum formalism itself provides a natural means of

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[12] Such an approach is reminiscent of the path integral formulation of QM (see Feynman and Hibbs 1965). As a referee observed, insofar as position and momentum are incompatible observables, quantum particles cannot (Bohmian mechanics aside) have perfectly well-defined trajectories. Correspondingly, our talk of possible particle trajectories should be understood as shorthand for one or other of the following three understandings. First, such talk may advert to possible classical trajectories—that is, trajectories that would be available if the particles were to behave classically. Second, such talk may advert to sequences of different spatial positions and regions that quantum particles can occupy without having definite position and momentum. Third, such talk may advert to approximations of classical trajectories. One way of developing this last strategy is along lines of Wallace’s (2008) remark that “If a system happens to be in a quasi-classical state $|\mathbf{q}(t), \mathbf{p}(t)\rangle \otimes |\psi(t)\rangle$ [...] then its evolution will accurately track the phase-space point $(\mathbf{q}(t), \mathbf{p}(t))$” (47); here particle trajectories can be taken to be represented by the quasi-classical evolution of the phase-space point $(\mathbf{q}(t), \mathbf{p}(t))$.

[13] As with the feather case, the suggestion here is that one *can* intelligibily understand the phenomenon of self-interference as involving multiple relativized determination, not that one *must* do so (indeed, we will shortly consider an alternative understanding). Again, what is important for our purposes is that one is not forced to endorse a metaphysical accommodation of the seeming self-interference in terms which do not involve metaphysical indeterminacy of one or other glutty variety, as on, e.g., a quantum variation on the relational proposal discussed in footnote 11. Here again, recent literature on perspectival facts (including Lipman 2016; Berenstain 2020; Evans 2020) is relevant, especially since much of this literature either focuses on or is intended to apply to cases of seeming perspectivalism in quantum mechanics.

[14] The degree-theoretic approach here is different from that in Smith and Rosen (2004); in particular, we reject three claims that Smith and Rosen accept, including that all fundamental properties are maximally precise, that MI involves an object’s being an ‘intermediate instance’ of a precise property, and that

extracting the degree of instantiation of a particular determinate (i.e., eigenvalue) from the square moduli of the coefficient of the corresponding eigenvector in a given quantum state, as per the following degree-theoretic variation on EEL:

> DEEL: A quantum system instantiates $O = x$ to a degree y iff $\sqrt{y}$ is the coefficient of the x's eigenvector in the quantum state of S.

(Note that instantiation to degree 0 here corresponds to not instantiating the determinate in question.) By way of illustration, consider a system S and observable O with eigenvectors $|\psi\rangle$ and $|\varphi\rangle$, having eigenvalues 1 and -1, respectively. Given that the state of S is $|\omega\rangle$, first write the state of S using the eigenvectors of O as a basis, along the following lines: $|\omega\rangle = c_1|\psi\rangle + c_2|\varphi\rangle$. Then extract the degree of instantiation of the different eigenvalues from the coefficients of the respective eigenvectors. Here, S instantiates $O = 1$ to degree $|c_1|^2$, and S instantiates $O = -1$ to degree $|c_2|^2$.

A degree-theoretic version of a glutty determinable-based implementation can provide an intelligible basis for self-interference, given (as again seems plausible) that it suffices for a feature to contribute to causing a given effect that the property is instantiated to non-zero degree. One might wonder if effects produced by properties instantiated to degree less than 1 would be different from effects produced by those properties when instantiated to degree 1, such that, e.g., were *being a force of 5 Newtons* to be instantiated to degree .5, that would be equivalent to the instantiation to degree 1 of the feature *being a force of 2.5 Newtons*. Such an understanding would be incorrect, however: the *same* property is instantiated, not some weaker counterpart of it. Moreover, on various accounts of causation, properties instantiated to a degree less than 1 could produce effects indistinguishable from those that would be produced if the properties were instantiated to degree 1—if, say, causes are probability-raisers. Correspondingly, the interference effects associated with a single particle concurrently instantiating properties associated with states $|A\rangle$ and $|B\rangle$ to degree less than 1 would be indistinguishable from those that would be produced were two particles to concurrently instantiate the properties associated with these states to degree 1—again providing an intelligible basis for self-interference.

Two further points, applicable to either variant of a glutty implementation, are worth noting.

First, cases of QMI are typically associated with properties whose interdependence prevents their being mutually instantiated. In the case at hand, this interdependence and associated mutual exclusion can be seen as the familiar

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Footnote 14 continued
'fuzzy logic' is the correct logic of MI. In particular, and unlike degree-theoretic approaches which depart from classical logic in allowing 'degrees of truth', our approach is that (as applied, e.g., to the quantum cases at hand) sentences of form 'system s has value v of observable O' are incomplete, and hence not truth-evaluable. Rather, it is sentences of form 'system s has value v of observable O to degree n' that are truth-evaluable, in line with both classical semantics and classical logic. This approach is in line with the more general supposition of determinable-based metaphysical indeterminacy as not inducing any propositional indeterminacy. See Wilson (2016) and Calosi and Wilson (in progress) for further details.

variety associated with determinates of a single determinable: just as nothing can be both red and blue all over, *simpliciter*, nor can a single particle go through slit A but not slit B, and slit B but not slit A, *simpliciter*. At best the determinate properties must be had by the particle in relativized or degree-theoretic fashion.[15]

Second, the determinable in the double-slit case, as in other cases of superposition QMI, is not to be understood as the property of *being in such-and-such superposition*. For a crucial feature of determinables is that they continue to be instantiated when (uniquely) determined (see Wilson 2017); but when a superposition is resolved into a unique determinate value (e.g., upon measurement), the associated superposition property does not continue to be instantiated. We registered this line of thought in our (2018), but there continues to be some unclarity about how to understand the determinables (and determinates) at issue in applications of *Determinable-based MI* to QMI; we now say a bit more about this for the case of the double-slit experiment, and expand further on our view in §5.

Which determinable and determinates are at issue in the double-slit experiment? As previously, in the first instance the determinable property at issue in this case is plausibly taken to be something like *having traveled from the source to the detector*, having as determinates *having traveled from the source to the detector via slit A but not slit B* and *having traveled from the source to the detector via slit B but not slit A*. This determinable property will, we assume, correspond to a relevant Hermitian operator, with the maximally specific determinates of the determinable, associated with specific trajectories through slit A and slit B, respectively, corresponding to the eigenvalues of the operator.

To be sure, the determinable is not a 'standard' quantum observable. But note that for any experimental question appropriately asked about a given system—e.g., did the particle travel from the source to the detector via slit A?—there corresponds a subspace of the Hilbert space for that system. We can thus construct a projection operator corresponding to the question, which projects onto that subspace. That operator will be an Hermitian operator that, as we will discuss in §5, is reasonably seen as representing a determinable property. Morever, and in any case, we could take the relevant determinable to simply be *position*, having as determinates the occupation of the relevant spatiotemporal regions.[16]

Summing up: An appeal to self-interference provides a natural and commonly endorsed partial basis for explaining double-slit interference—partial, however, since it remains to render intelligible how a particle can be located at both slits at the same time, such that the associated components of its wave-function can interfere. The partial explanation can be filled in and the supposition of self-interference rendered intelligible by appeal to one or other variant of a glutty implementation of

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[15] By way of contrast, in the literature on multilocation, multiple exact location or position is had *simpliciter*.

[16] This suggestion can be made more precise by building on work by Wightman (1962) and developed by Pashby (2016), according to which any region of space r_i can be associated with a projection operator $\hat{P}_{r_i}$.

a determinable-based treatment of superposition QMI.[17] Correspondingly, a glutty implementation of *Determinable-based MI* does double duty, explaining not just the QMI present in this case as constituted (on each run of the experiment) by a particle's having a determinable trajectory but no unique determinate of that determinable, but also how it might intelligibly be that a single particle (its wave function) can engage in self-interference, reflecting that the failure of unique determination is due to each of the relevant determinates (associated with the states $|A\rangle$ and $|B\rangle$) being instantiated, in either relativized or degree-theoretic fashion.[18]

3.4 The double-slit experiment: metaphysical supervaluationism

We turn next to considering whether a metaphysical supervaluationist account of superposition QMI has resources enabling it to provide a comparatively explanatory account of double-slit self-interference. On this approach, MI involves its being indeterminate which state of affairs, of some range of determinate/precise states of affairs—the precisifications—obtains. As Barnes (2010) puts it:

> It's perfectly determinate that everything is precise, but [...] it's indeterminate which precise way things are. (622)

And as Barnes and Williams (2011) put it:

> When p is metaphysically indeterminate, there are two possible (exhaustive, exclusive) states of affairs—the state of affairs that p and the state of affairs that $\neg p$—and it is simply unsettled which in fact obtains. (114–115)

Darby (2010) points out that one might initially see a suggestive parallel between the terms in the superposition and the idea of precisifications:

> One of the terms in the superposition [...] is a term where the cat is alive, the other is not; that is reminiscent of multiple ways of drawing the extension of 'alive', on some of which 'the cat is alive' comes out true, on some, false. (235)

On this approach, then, a superposition is a state whose precisifications are given by the terms of the superposition. Superposition QMI is then taken to reflect its being indeterminate which term (or associated property) of the superposition obtains.

How does such an approach fare as an account of the double-slit experiment? Not well. For the supervaluationist, indeterminacy is unsettledness about which one of a range of maximally precise states of affairs obtains. On this view, it is determinate that only *one* such state of affairs obtains, notwithstanding that it is indeterminate *which* one obtains. Hence in the case of the double-slit experiment, the supervaluationist takes the superposition QMI at issue to reflect its being indeterminate which one of the states $|A\rangle$ or $|B\rangle$ obtains. On this account, there is

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[17] In our (2018), we note certain problems with a gappy implementation of *Determinable-based MI* as applied to cases of superposition-based QMI. We direct the interested reader to that discussion.

[18] As Nina Emery pointed out, a glutty determinable-based approach also provides a basis for explaining what results when both slits are blocked.

no question of there being any sense in which *both* states obtain; again, it is determinate that only one of the states obtains. But if only one of the states obtains, then there's no physical basis for the interference characteristic of the double-slit pattern. In placing MI at the 'meta-level', as indeterminacy in which one of a range of precise options obtains, the supervaluationist does not have the resources needed to make sense of self-interference.

4 General objections to QMI

Thus far we have argued that a (glutty implementation of a) determinable-based approach to QMI offers a more illuminating treatment of the double-slit experiment than its primary competitor. We next turn to treating certain objections potentially bearing on this result, either to QMI in general, or to a determinable-based approach to QMI in particular.

4.1 The argument from the absence of fundamental QMI

Glick (2017) argues that there is no pressure to see QM as involving QMI. He first claims that, putting aside the orthodox interpretation (which he treats separately), prominent interpretations of QM don't involve *fundamental* QMI:

> First, and most straightforwardly, the Bohm theory endows particles with determinate positions and momenta at all times [...]. Second, the Everett interpretation, as developed by Wallace (2012), recognizes only the universal wavefunction in its fundamental ontology. The universal wavefunction is perfectly determinate at every time [...]. Finally, consider dynamical collapse theories such as versions of the GRW. The two versions of the GRW adopted by most contemporary defenders are the mass-density and flash-ontology varieties. Neither contain fundamental indeterminacy: the distribution of mass-density and the location of the flashes are both perfectly determinate. (2)

Glick then goes on to claim that if QMI occurs at the non-fundamental level, it is eliminable:

> [A]ny indeterminacy would occur at the non-fundamental level, and hence would be viewed as *eliminable*. (3)

He concludes that one may deny that there is any QMI.

We reply that both claims (premises) can be rejected. To start, though there is plausibly no fundamental QMI on a Bohmian interpretation, this need not be granted for Everettian or GRW interpretations. As Lewis (2016) notes, there are readings of the Everettian interpretation on which microscopic systems are fundamental and "can have indeterminate properties" (97), and readings of the GRW interpretation on which "fundamental particles like electrons typically lack determinate values for their physical properties [...]" (88–96).

To be sure, on the versions of these interpretations that Glick has in mind, microscopic particles or systems don't count as fundamental. But such conceptions are both controversial and problematic. Everettian wave-function fundamentalism faces notorious difficulties with regaining 3D macroscopic objects (for discussion, see Ney and Albert 2013). And as Lewis notes, “the postulation of a mass density in addition to the quantum state essentially makes massy GRW a hybrid collapse/hidden variable view, which seems unnecessarily complicated” (95); similarly for ‘flash’ GRW. Moreover, though considerations of space prevent detailed discussion here, there are other live interpretations of QM, including relational interpretations (see Rovelli 1996) and modal interpretations (see Dieks and Vermaas 1998), on which the QMI at issue is plausibly both metaphysical and fundamental, in ways moreover apt for determinable-based treatment.[19]

Second, one may deny that any non-fundamental QMI there might be is eliminable. Glick provides no argument to this effect, but argument is needed, not least because eliminativism about the non-fundamental would be wildly revisionary. Hence it is that such eliminativism is both rare and precisely targeted (e.g., as applying just to qualitative mental features, as in Churchland 1981, 1986). Far more common than eliminativism are treatments of derivative phenomena in ontologically reductive (i.e., identity-based) or non-reductive (e.g., functionalist or other realization-based) terms, as on physicalist accounts of special science entities and features.

Is there reason to think that there is something special about non-fundamental MI or QMI requiring that such MI be eliminated, unlike other non-fundamental goings-on? Though Glick doesn’t explicitly offer such reason, one might try to extract one from his claim that discussions of QMI assume that it is importantly different from other MI:

> This debate presupposes that quantum mechanics involves indeterminacy of a *particular sort*. It is this presupposition that I wish to challenge. (1)

If accounts of QMI take it to be importantly different from other MI, in virtue of being fundamental in particular, then a lack of fundamental QMI would be problematic. But the import of attention to QMI, at least in our work, is not directed at establishing a distinctive, much less fundamental, form of MI. Rather, what motivates this attention is that QMI represents perhaps the best case of MI of the sort not easily dismissed as being merely representational/semantic or epistemic. For our purposes, it doesn’t matter whether QMI is fundamental or non-fundamental.

A different concern with non-fundamental MI or QMI might advert to Barnes’s (2012) argument that any MI there may be must be fundamental, on pain of

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[19] On a relational interpretation, certain fundamental properties of a system prior to interaction with other systems correspond to undetermined determinables; after interaction these properties may become determinate, relative to these other systems. Hence there appears to be fundamental QMI on a relational interpretation. On a modal interpretation, there is a distinction between the dynamical state and the value state, where (on the usual gloss) properties in the dynamical state are properties that a system *might* have, whereas properties in the value state are properties that a system *has*. One might reasonably suppose that a given system actually has the determinable properties associated with the merely possible properties in its dynamical state; if some of these undetermined determinables are fundamental, then there is fundamental QMI on a modal interpretation.

contradiction. However, Barnes's argument presupposes (as per metaphysical supervaluationism and more generally, a meta-level approach) that MI involves its being indeterminate which of some range of perfectly determinate options obtains. In particular, Barnes's "simple argument that in order for there to be metaphysical indeterminacy at all there has to be indeterminacy in how things are fundamentally" (341) has as a premise that "For some complete description, *D*, of a way for things to be derivatively, it is indeterminate whether *D* is true". In our view, a meta-level approach to MI is unpromising both in general (see Wilson 2016) and again, as applied to quantum phenomena (see Calosi and Wilson 2018); but in any case Barnes's argument can't be used to support Glick's claim that in the absence of fundamental QMI there isn't any QMI at all. At best, Barnes's argument shows that any QMI of the meta-level, metaphysical supervaluationist variety would have to be fundamental, leaving open that QMI of an object-level, determinable-based variety may be non-fundamental.

4.2 The argument from the sparse view

Putting aside the Bohmian, Everettian, and GRW interpretations, Glick (2017) goes on to consider whether the so-called 'orthodox' interpretation should be understood as involving (fundamental) QMI, as we maintain, along with Darby (2010) and Skow (2010). As Glick observes, the orthodox interpretation has three main tenets—namely, the Eigenstate-Eigenvalue Link, the Schrödinger dynamics, and the collapse postulate—and so understood, there are many cases in which a system *S* lacks determinate properties. We take such cases to be indicative of QMI, in which the system in question has a determinable property but no unique determinate of that determinable. Glick objects that there is an alternative understanding of the cases, compatible with the orthodox interpretation, on which there is no QMI, as per:

> *The sparse view*: When the quantum state of a physical system *S* is not in an eigenstate of an operator *O* it lacks both the determinable and the determinate associated with *O*.

We have three replies.

First, a sparse approach is implausible in general, for cases where an entity fails to have a unique determinate are not generally cases where the entity also fails to have the associated determinable. Consider again an iridescent feather, where no determinate shade is non-arbitrarily taken to be 'the' unique determinate of the determinable instance of *colour*. Generalizing the sparse view to this case, it would follow, implausibly, that the feather is not coloured.

Second, the sparse view has highly counterintuitive implications, which are nicely illustrated by attention to double-slit interference. To start, observe that determinables admit of different levels of specification, so that the characterization of a property as determinable or determinate is relative to levels: *red* is a determinate of *colour*, but a determinable of *scarlet*, and so on. There are typically bounds in either direction, corresponding to maximal determinables and maximal determinates. For example, the spatiotemporal location (associated with a given trajectory) of a given particle might be more or less determinate: it might be in the apparatus, in the lab, in the university, in the city, or just in spacetime. Now consider a particle in the double-slit experiment, in the superposition state $|\psi\rangle = c_1|A\rangle + c_2|B\rangle$. When in this state, the particle does not have a determinate spatiotemporal location (trajectory)—so far, Glick and we agree. According to the sparse view, however, the particle also fails to have any determinable spatiotemporal location. But in that case, the particle is not located in the apparatus, or in the lab, or in the university, or in the city, or—with respect to the maximal determinable of spatiotemporal location—in spacetime. But surely the particle, if it exists at all, is located in spacetime. Where else could it be? Indeed, it is hard to see how the particle could produce the measurement results constituting the double-slit pattern were it not located in spacetime.[20] What a proponent of the sparse view must say, it seems, is that an act of measurement causes the particle to suddenly be located in spacetime—which one might reasonably interpret as entailing, for concreta like electrons, that an act of measurement causes the particle to pop into existence. That's implausible.

Third, the proponent of the sparse view is not in position to endorse the natural explanation of double-slit interference offered by Dirac, Barrett, Lewis, and Maudlin. Again, on that explanation, double-slit interference is partly explained as reflecting that a single particle (or its wave function) can interfere with itself in cases where both slits are open. As above, a glutty implementation of a determinable-based view, in either relativized or degree-theoretic form, provides an intelligible basis for self-interference as reflecting concurrent (relativized or degree-theoretic) instantiation of multiple instances of the relevant determinates of position. But on the sparse view, from the particle's being in a superposition of position states it follows that the particle doesn't instantiate the determinable *position* at all, and so (as above) is not located in spacetime. But then there is no basis, much less an intelligible basis, for taking the particle (its wave function) to interfere with itself.[21]

Glick does offer an alternative account of the relation between the superposition state and the possible measurement outcomes, whereby this is a matter of brute nomological connection. As Glick describes it for the case of a particle in an infinite square well:[22]

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[20] See Calosi (2019) for a similar point, going beyond the quantum details.

[21] Relatedly, as Nina Emery pointed out, if particles in the double-slit experiment do not go through either slit (since not at all spatiotemporally located), then it is unclear why blocking both slits should change the result of the experiment, as in fact happens. Here again a glutty determinable-based approach comes out ahead, explanatorily speaking.

[22] The case of the infinite square well models a particle moving in one dimension inside a small region with impenetrable barriers, associated with the following potential:

$$V(x) = \begin{cases} 0, \text{if } 0 \leq x \leq a \\ \infty, \text{otherwise} \end{cases}$$

The particle is free to move in the potential $V(x)$ except at the two ends ($x = 0$ and $x = a$), where an infinite force prevents it from escaping.

> [T]he sparse view holds that the property of being in a superposition of x_1, x_2 is a determinate property that is nomologically related to the properties of being located at x_1 and being located at x_2 via the Born rule. (208)

We see three problems with this approach. First, such a brute connection between states and outcomes again fails to explain double-slit interference. Second, such an account of the observed interference pattern is unsystematic, for in classical cases of interference there is a clear basis for the pattern, as located in interacting waves, rather than a (merely) brute nomological connection between states and measurement outcomes. (By way of contrast, a determinable-based explanation of double-slit interference provides a systematic basis for the pattern, as located in interacting waves.) Third, on the face of it, the property *being in a superposition of* x_1 and x_2, where x_1 and x_2 are themselves spatiotemporal locations, is itself a spatiotemporal location property. After all, even if a particle is in a superposition of specific positions (or trajectories), it still would seem to have the property of being located in spacetime. Indeed, Glick himself refers to such a superposition as a "position state" (207).[23] But the sparse view forbids the assignment of *any* spatiotemporal location property to particles in cases of position superposition; hence a brute nomological explanation of double-slit interference appeals to a property whose attribution appears to be in direct tension with the sparse view.

These considerations suggest that the sparse view does not represent a viable alternative to an understanding of the orthodox (or any other) interpretation on which it is committed to (in particular, fundamental) QMI.

5 Objections to determinable-based QMI

The previous results show that attempts to undercut QMI as present on the Everettian, GRW, and orthodox interpretations are unsuccessful. We turn now to objections directed more specifically at a determinable-based treatment of QMI.

5.1 The argument from non-determinable superpositions

Glick (2017) objects to a determinable-based approach to superposition QMI on grounds that there is no clear way to make sense of the attribution of the associated determinable property. His line of thought is that said determinable would have to be the property of being in the associated superposition; but superposition properties fail to conform to our ordinary understanding of determinables; hence a determinable-based approach to MI cannot properly treat superposition QMI.

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[23] To be clear, we take the suggestion that a superposition of position states is itself a position state to be independently intuitively plausible given the properties at issue, as opposed to following from a general principle to the effect that a superposition of states of a given type will also be of that type. As a referee points out, the general principle appears to be false (e.g., a superposition of z-spin properties will not be a z-spin property); but our purposes require only that a superposition specifically of position states is plausibly itself a position state.

Glick frames his concern by attention to the case of a particle in an infinite square well:

> Consider a simplified version of a particle in an infinite square well in which there are only two maximally-precise locations possible for the particle, x_1, x_2. If we measure the particle's momentum precisely, its position state will not be in an eigenstate of x_1 or x_2, but rather, in a superposition of the form $c_0|x_1\rangle + c_1|x_2\rangle$ where c_i is a complex number. There will be an operator associated with any such superposition, and the system will be in an eigenstate of that operator. It follows from the eigenstate-eigenvalue link that we should ascribe the system a property, but is this property the determinable with being located at x_1 and being located at x_2 as determinates? (207)

Qua eigenstate of the superposition operator, the property under discussion is presumably just the property of being in the relevant superposition—call it $|\psi\rangle$. But, Glick goes on, $|\psi\rangle$ is not properly taken to be a determinable, since, most significantly, while determinables continue to be instantiated when determined, a superposition property does not continue to be instantiated when one of its terms comes to be instantiated (e.g., upon measurement resolving the superposition).

Now, the first thing to say about this concern is that, as per our previous work and as Glick acknowledges, we maintain that the determinable at issue in a given case of superposition QMI is not appropriately taken to be the property of being in the relevant superposition, for just the reason Glick mentions. Hence in our (2018) discussion of QMI in the case of Schrödinger's cat, we characterize the relevant determinable not as the property of being in the relevant superposition, but rather as the property *having a certain life status*; and in the case of the double-slit experiment we characterize the determinable not as the property of being in the relevant superposition, but rather as something like the property of *having traveled from the source to the detector*. To be sure, the terms of the superposition are associated with possible determinates of the determinable, but from this it doesn't follow that the superposition is *itself* a determinable. Compare: we can disjoin the determinates of a given determinable, but deny that the resulting disjunction is properly identified with the determinable of these determinates.[24]

Glick maintains, however, that we still face a difficulty:

> The [determinable] property in question—whatever we choose to call it—is attributed on the basis of the system being in an eigenstate of an observable associated with a certain superposition (as per the eigenstate-eigenvalue link). After a measurement this is no longer the case, and hence, quantum mechanics provides no basis for thinking it still has the property. The indeterminist might claim that having a determinate entails having the corresponding determinable, but (a) they have already denied the inference in the other direction

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[24] Indeed, key to the determinable-based approach to MI is that determinables are not reducible to disjunctions or other constructions of determinates; see Wilson (2013), citing arguments for such irreducibility in Wilson (2012) and elsewhere.

> and (b) the determinable so entailed is not the same property as that associated with a particular superposition state. (208)

Why does Glick maintain that the determinable in a given case of superposition QMI must be "attributed on the basis of the system being in an eigenstate of an observable associated with a certain superposition (as per the eigenstate-eigenvalue link)"? So far as we can tell, Glick is assuming that the attribution of *any* property to a quantum system must proceed by way of what we will call the 'eigenstate criterion':

> *Eigenstate Criterion*: The only properties properly attributed to quantum systems are those associated with eigenstates of observables.

We deny the Eigenstate Criterion. To start, as we understand EEL, it is a criterion of what it is for a system to have a definite or determinate value of a given observable; it is not a criterion of what it is for a system to have any property whatsoever. That satisfaction of the criterion in EEL is not necessary for quantum property attribution follows from the fact that whenever a determinate property is instantiated, all of the associated determinables of the determinate are also instantiated: whenever a particle has a maximally specific position property, it thereby has the property of being located somewhere in space; when Schrödinger's cat is found to have the property *being alive*, it thereby has the property *having a life status*, and so on. This is par for the course for property attributions. Correspondingly, we have independent good reason to deny that property attribution in general proceeds by way of EEL, whether or not superpositions are at issue.

Glick suggests, above, that since we deny that the having of a determinable entails the having of a (unique) determinate, it would be unprincipled to maintain (as per what Wilson, 2017, calls 'Determinable Inheritance') that the having of a determinate entails the having of its associated determinables. But there's nothing unprincipled about rejecting one direction of entailment while accepting the other. Consider the relation between identity and the sharing of properties (a.k.a. 'indiscernibility'): most accept the indiscernibility of identicals, while most reject the identity of indiscernibles.

The Eigenstate Criterion is both implausible and unsupported. But in that case we have no reason to accept Glick's claim that in cases of superposition QMI, the only available candidate for the determinable property is the superposition property which is "attributed on the basis of the system being in an eigenstate of an observable associated with a certain superposition".

Again, in the case of the double-slit experiment, for example, we can take the determinable at issue to be *having traveled from the source to the detector*, or yet more generally, position. Importantly, when such determinable properties come to be determined as a result of measurement, they 'stick around', as is required if they are to be determinables. Perhaps relatedly, it is worth noting that ordinary quantum observables (or associated properties) are typically treated as determinables. These are the determinable properties that enter into a determinable-based account of QMI, and they are in good standing from the point of view of both QM and the metaphysics of determinables and determinates.

We can say a bit more, generalizing upon our previous account of the determinables and determinates at issue in our treatment of double-slit QMI. For on standard presentations of QM, determinables are represented by *operators*, not vectors in the Hilbert space, and maximally specific determinates are represented by *eigenvalues* of the associated operator. This understanding is often implicit in physics textbooks (see, e.g., Baym 1969, 59–62; Gillespie 1970, 42–47; Beltrametti and Cassinelli 1981, 14–29; Norsen 2017, 33–36) and philosophy of physics expositions (see, e.g., Albert 1992, 40–43), including expositions by philosophers skeptical about the very usefulness of the notion of a quantum observable (as in Maudlin 2019, 62–69). The most explicit formulation is perhaps in Hughes (1989), 69 (Table 2.1), where Hughes identifies observables with operators on state space (that is, Hilbert space), and identifies the possible (maximally) determinate values of a given observable with the possible eigenvalues of its associated operator. Hughes writes:

> The radical differences between classical mechanics and quantum mechanics appear with the representation of observables. Instead of the real-valued functions of classical theory, quantum mechanics uses Hermitian operators in the Hilbert space to represent observables. (63)

Correspondingly, we are inclined towards a view on which observables are (typically) determinables, and their possible values, i.e., their eigenvalues, are maximal determinates.[25]

These considerations confirm that we need not restrict ourselves to eigenstates when casting about for the relevant determinable in a given case of superposition QMI. Eigenstates are important, not least in tracking facts about measurement outcomes; but they are not the only way for things to be. Correspondingly, we need not take the determinable in a given case of superposition-based QMI to be the associated superposition property. Rather, we can take this determinable to correspond to the relevant quantum observable in the case at hand, consonant with the standard treatment of observables as determinables, by both physicists and philosophers of physics.

5.2 The argument from revisionism

We turn now to an objection specifically directed at a determinable-based approach to QMI, due to Torza (2020). Torza's focus is on the status of the observable *position*, though his argument, were it to go through, would generalize in an obvious

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[25] For example, the determinable *momentum* is associated with the operator $\hat{p} = -i\hbar\frac{\partial}{\partial x}$; its eigenfunctions are plain waves $|\psi\rangle = e^{ipx/\hbar}$ with eigenvalues $p = \hbar k$. The determinable *spin* of a $\frac{1}{2}$-particle along a given direction α is associated with the general operator

$$\hat{S_\alpha} = \begin{pmatrix} z & x - iy \\ x + iy & -z \end{pmatrix}$$

of which the Pauli matrices for spin in the x, y and z directions, having eigenvalues $\pm\frac{1}{2}$, are specific examples. And so on.

way. Following Torza we write '$\phi(e, x_i)$' for 'electron e has position x_i', where the x_i range over possible positions $x_i \in I$. Torza observes that the following claims are inconsistent:

1. A disjunction, such as $\bigvee_{i \in I} \phi(e, x_i)$, cannot be true without any of its disjuncts $\phi(e, x_i)$ being true;
2. $\bigvee_{i \in I} \phi(e, x_i)$ is logically equivalent to $\exists z \phi(e, z)$;
3. The claim that an electron e has a position is regimented by $\exists z \phi(e, z)$;
4. According to the determinable-based account, $\exists z \phi(e, z) \equiv \bigvee_{i \in I} \phi(e, x_i)$ can be true without any of associated disjuncts $\phi(e, x_i)$ being true.

Now, according to Torza, giving up (1) or (2) requires being revisionary about classical logic, whereas giving up (3) requires being revisionary about the quantum formalism. As Torza notes, (1) fails on some understandings of quantum logic; hence one way to avoid contradiction would be to retreat to non-classical logic.[26] While this option is open to a proponent of *Determinable-based MI*, we prefer to retain classical logic, whether quantum or non-quantum MI is at issue; hence by our lights (1) and (2) are non-negotiable. Our targets will rather be (3) and (4).

According to (3), the claim that an electron e has a position is regimented by $\exists z \phi(e, z)$. Given (2), the relevant existential statement is equivalent to a disjunction. Now, is (3), so understood, true? Interestingly, the answer depends on the indefinite article 'a'. We are happy to allow that the claim that an electron e has *a* position is regimented as an existential claim, which in turn is logically equivalent to a disjunction. But in that case we will deny (4); for it is no part of a determinable-based account to maintain that, e.g., the claim that an electron has *a* position can be true without any of the associated disjuncts (each expressing the electron's having a given specific position) being true, for on a natural reading of the claim that something has *a* position, this claim entails that that thing has some or other specific position.

What a determinable-based approach to QMI requires is that it make sense for the claim that an electron e has *position* to be true, even if no claim registering that e has a specific position is true. Correspondingly, if (3) is to be relevant to assessing our view, it should be revised to say (3*): 'The claim that an electron has *position* is regimented by $\exists z \phi(e, z)$'. We will then deny (3*) (and relatedly again deny (4) as stated), since we deny that attributions of determinable properties such as *position* are properly regimented as attributions of the having of some or other determinate property. As per Wilson (2012) and elsewhere, determinables are not analyzable as disjunctions (or indeed, as any construction of maximal determinates); and as per Wilson (2013) and Calosi and Wilson (2018), it is core to a determinable-based approach to MI that determinables are not so reducible.[27]

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[26] On these non-classical understandings, quantum disjunctions, unlike classical disjunctions, are not equivalent to existential statements.

[27] See also Calosi (2019) for reasons to think that *position* should not be regimented in disjunctive terms.

Now, Torza contends that giving up (3) is revisionary, on grounds that *position* is understood in QM as inducing a

> partial function ϕ from particles to position values [...] Therefore, for *e* to have a position is for it to have assigned some position value *z* under the function ϕ, to be such that $\exists z \phi(e, z)$. (4263)

Again, what is at issue is not whether (as per 3) the attribution to *e* of *a* position should be understood as an attribution of a specific position, but rather whether (as per 3*) the attribution to *e* of *position* should be understood as an attribution of a specific position.

We maintain that an understanding of *position* in QM as not conforming to (3*) is not revisionary. To start, note that this observable is standardly represented by the position operator $\hat{x} : \mathcal{H} \rightarrow \mathcal{H}$, which is a *total* function from the Hilbert space to itself. And while this operator can be associated with a function from particles to position values, such a function is not plausibly seen as exhausting the understanding of *position* in QM. As Torza acknowledges, given EEL, a function from particles to position values cannot be total, for such a function is defined only for those particles in an eigenstate of position. Hence taking at face value the suggestion that for something to have position is for it to be (in a state corresponding to) an argument of the partial function in question, it would follow that particles not in an eigenstate of position do not have position, *simpliciter*.[28] But as previously argued in discussing Glick's sparse view, this is deeply implausible; for to have position is to be located in spacetime, and particles in a superposition of position eigenstates are surely in spacetime. Correspondingly, it is not revisionary to reject an understanding of *position* in QM having this implausible consequence.[29]

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[28] This claim presupposes that the operative logic is classical, which some deny for quantum contexts. But first, note that Torza's original objection also rests on the endorsement of classical logic—hence it was that we challenged 3 and 4 rather than 1 and 2. And second, if one endorses quantum logic, then nothing prevents one from identifying a quantum determinable with a quantum disjunction of determinates, since the logic will allow that such a disjunction can be true without any of its disjuncts being true, and will also rule out a disjunction's being equivalent to an existential statement. So while endorsing quantum logic would block our specific complaint here, doing so would also undercut the original argument from revisionism against the determinable-based account. Thanks to a referee for discussion.

[29] One might still want to hear more about how to express the having of *position*, insofar as the associated predicate will still need to be a formula in one free variable—namely, one that can be predicated of a particle *x* just in case *x* has position. Here we suggest that the proponent of determinables can and should avail themselves of predicates for relevant determinables as well as relevant determinates in their language. One such predicate will be *DP*, representing the maximally unspecific determinable *position*; the property of *x* having position can then be represented by the formula $DP(x)$. Hence in a language with higher-order quantification and identity, using '*D*' as a variable ranging over maximally unspecific determinables, '*DP*' as a predicate representing (the property of having) *position*, and '*M*' as a predicate representing (the property of being a) *material object*, that all material objects have *position* can be expressed as follows: $\exists D((D = DP) \wedge \forall x(M(x) \rightarrow DP(x)))$.

6 Concluding remarks

On the face of it, and on several live interpretations, QM involves genuine MI, attaching to the presupposed fundamental or non-fundamental quantum ontology. Here our focus has been on the case that, as Feynman put it, lies at the heart of, and encodes the mystery of, quantum mechanics—namely, the case of superposition MI at issue in the double-slit experiment.

We have argued that attention to double-slit interference provides powerful support for *Determinable-based MI*, as accommodating the QMI at issue in a way rendering intelligible how it could be that a single particle (or its wave function) can interfere with itself, as a natural explanation of double-slit interference suggests. The explanation more specifically reflects that the determinable *having traveled from the source to the detector* (or some more determinable variant thereof) may be multiply determined, in either relativized or degree-theoretic fashion, by the determinates associated with the more specific trajectories through slit *A* or slit *B*, respectively. That these position determinates are jointly instantiated by a particle on a given run of the experiment—not *simpliciter*, but again, in relativized or degree-theoretic fashion—provides an intelligible basis for self-interference to occur. We have moreover argued that available alternative accounts or treatments of QMI, including a metaphysical supervaluationist account or the deflationary 'sparse' view, do not render self-interference intelligible.

To be sure, there are versions of certain interpretations of QM, as discussed above, on which there is no QMI, or at any rate no fundamental QMI. The existence of non-fundamental QMI would suffice for our purposes—non-fundamental QMI is still QMI. In any case, and more to the dialectical point, the underlying motivations for such deflationary positions plausibly reflect the assumption that there is no way to make good sense of MI, whether fundamental or non-fundamental. Hence Lewis (2016) says

> Perhaps the existence of quantum indeterminacy renders these different versions of quantum mechanics (GRW, Bohm, Everett) unnecessary in the first place. One way of understanding the role of these alternatives is to rescue determinacy at the macroscopic level. But perhaps the world is more indeterminate than we take it to be. (79)

It may be that a supposed need to rescue determinacy was pressing prior to the advent of *Determinable-based MI*—especially in light of the failure of a metaphysical supervaluationist approach to accommodate the 'deep' indeterminacy at issue in QM. But a determinable-based account properly treats QMI on its own insuperable terms. Correspondingly, that some versions of some interpretations of QM do not involve QMI carries little independent dialectical weight in the present context.[30]

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[30] As Nina Emery observed, similar remarks may attend to recently popular high-dimension ontologies for QM (e.g., wave-function realism à la Albert 1996), to the extent that the attraction of such views reflects concerns that QMI could not be reconciled with more familiar particle ontologies.

Natural next steps lie in considering further applications of *Determinable-based MI* to the quantum context. Questions remaining include: Is all superposition QMI best treated in glutty terms? What about cases of entanglement and incompatible observable QMI? Would it make sense, for example, to treat entanglement QMI in glutty terms, where the instantiation of the multiple determinates in one entangled system is relativized to the relevant state of the other system? As in the case of the double-slit experiment, answers to these and other questions may shed light not just on the indeterminacy at issue, but on other 'mysterious' aspects of quantum reality.

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