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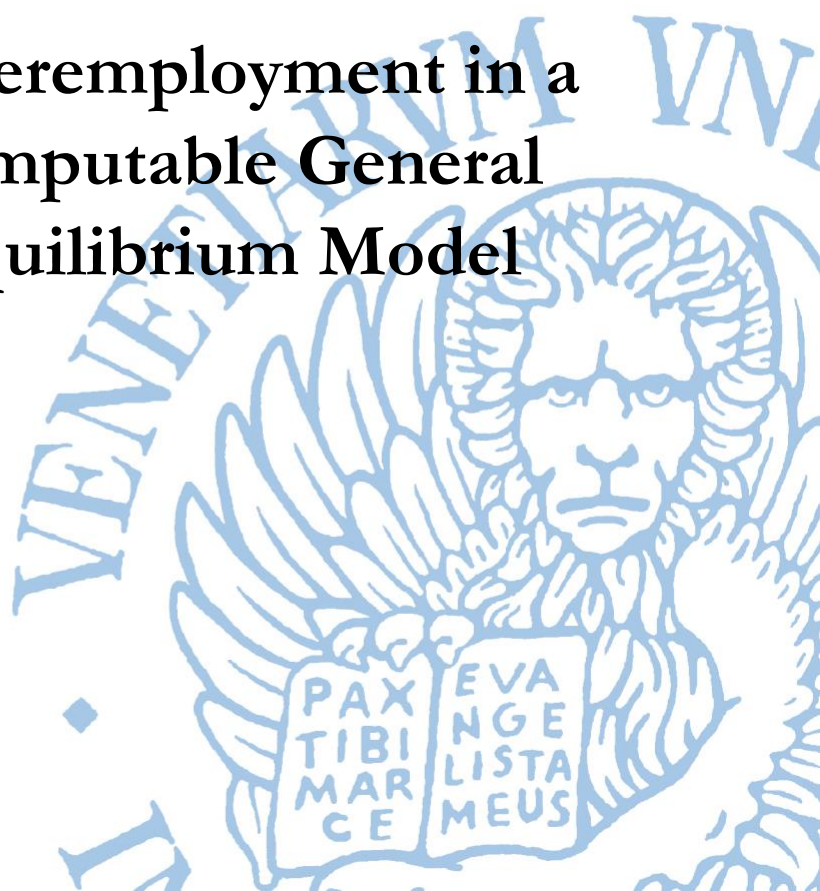
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Computable General  
Equilibrium Model**

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This paper presents a methodology to account, in a computable general equilibrium model, for the presence of underemployment in an economic system. The methodology is based on the estimation of a matrix, mapping different categories of workers to levels of educational attainment. A procedure is proposed, which allows recalculating the matrix after the realization of a simulation with a CGE model, when employment levels are varied. In this way, a new matrix is made consistent with the simulation results, identifying a new equilibrium in the labor market, which entails a different combination of unemployment and underemployment.

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Underemployment, Unemployment, Labor Market, Computable General Equilibrium Models

### **JEL Codes**

C68, C82, D58, E24, I20, J21, J24, J62, J82

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# Underemployment in a Computable General Equilibrium Model

Roberto Roson<sup>1</sup>

## ABSTRACT

This paper presents a methodology to account, in a computable general equilibrium model, for the presence of underemployment in an economic system. The methodology is based on the estimation of a matrix, mapping different categories of workers to levels of educational attainment. A procedure is proposed, which allows recalculating the matrix after the realization of a simulation with a CGE model, when employment levels are varied. In this way, a new matrix is made consistent with the simulation results, identifying a new equilibrium in the labor market, which entails a different combination of unemployment and underemployment.

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## 1. Introduction

According to the Merriam-Webster dictionary, underemployment is “the condition in which people in a labor force are employed at less than full-time or regular jobs or at jobs inadequate with respect to their training or economic needs”. There are, therefore, two distinct meaning of the word. One refers to a situation where a person does not work for the desired amount of time (let us call it “Type I” underemployment), the other one when a person does work, but covers a place somehow “inadequate” for her level of education, training, experience, or skill (“Type II”).

Of course, the two concepts share the common notion of under-utilization of human capital, and, in this respect, they have a nature not fundamentally different from that of unemployment. However, whereas unemployment is a much-studied feature of the labor market, for which regular statistics are produced and suitable attention is paid by policy makers, underemployment remains a rather elusive phenomenon. No data about underemployment levels is regularly produced by the national accounting agencies, and economic research is scant, as witnessed by the lack of a specific item “underemployment” in the standard bibliographic classification of the Journal of Economic Literature. This is surprising, as underemployment brings about the very same problems of unemployment: loss of economic potential, income, and welfare; obsolescence of knowledge, skills, lower individual productivity; decrease of self-esteem, social inclusion, identity; health problems; criminality (on this and other issues see Livingstone, 2018).

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This paper explores the use of Computable General Equilibrium models for the assessment of underemployment under specific scenarios, or policies. Admittedly, this may look as an overly ambitious objective. CGE models are based on neoclassic hypotheses of perfectly competitive markets, where supply matches demand, even in the labor market. The archetypal CGE model, typified by the standard GTAP model (Corong et al., 2017), normally assumes given endowments of primary factors, including labor. This means that the system is assumed to get to full employment, the only remaining unemployment being of frictional nature. However, model variants where employment is endogenously determined are also quite diffused. One simple solution is making the nominal or regional wage fixed, swapping this variable with labor demand (Burfisher, 2021). This amounts to assuming a horizontal labor supply function, rather than a vertical one. Labor supply can also be formulated as a positively sloped function, either through the introduction of a wage curve (e.g., Roson, 1998), or by considering a leisure-labor choice by the representative consumer (Boeters and Savard, 2013).

In any case, it is important to notice that a CGE model can – at best – estimate employment levels, not unemployment. To get unemployment, one needs to know the baseline unemployment rate, or the total workforce, thereby computing unemployment “out of the model”, that is, after a simulation run. As it will be shown later in this paper, a similar approach is proposed here for underemployment, meaning that estimates of the latter would not affect neither the model calibration, nor the counterfactual equilibrium, so that they can be computed ex post.

Bell and Blanchflower (2013, 2018a-b, 2020) have long investigated Type I underemployment, the one related to the time worked, and its implications. They propose a method to compute underemployment as the difference between the hours one person would like to work at constant wage rate, and the actual numbers of hours she works. As such, they also consider the possible existence of overemployment.

Underemployment implies workers are not on their labor supply curves, and they may be forced to agree to contracts which give employers rights to vary working time at short notice without varying pay rates. Having estimated underemployment measures for twenty-five European countries, Bell and Blanchflower show that, in contrast to the unemployment rate, underemployment in most countries has not returned to its prerecession levels after the big financial crisis of 2008-2009. As wage growth has been low around the world since that time, underemployment has become a more convincing explanation of lower wage growth than unemployment. This finding is confirmed by Hong et al. (2018), showing that involuntary part-time rate to total employment enters with a statistically significant negative sign in wage change equations.

Is it possible to consider Type I underemployment in a CGE setting? In principle it is, and in a quite simple manner. The key information used to calibrate employment in a CGE model is total salaries, and when a simulation is performed, it is possible to estimate the percentage variation in employment, independently of how it is measured: persons, efficiency units and, which is important here, worked hours. Consequently, to estimate Type I underemployment in a counterfactual scenario, one only needs to know what the baseline underemployment level would be, under the reasonable assumption that the total number of desired work hours can be taken as a constant. Baseline underemployment may be difficult to measure but, if you got this information, Type I underemployment in a CGE model could be handled the same way as unemployment.

The story is a little more complicated for Type II unemployment, which considers the presence of overqualified workers. For one thing: the data set used to calibrate parameters in a CGE model does not report information about educational attainments, or on-the-job experience. Therefore, any effort to this direction necessarily requires constructing first a mapping between, say, workers in the different categories and education levels.

To this end, the U.S. Bureau of Labor Statistics provides the composition by educational attainment (seven levels) of workers in 832 occupations.<sup>2</sup> Combining this data with employment by detailed occupation,<sup>3</sup> it is possible to get a matrix, with dimension 832x7, showing the joint distribution of workers by occupation and education level. The 832 categories are aggregated here to the five classes of workers considered in the GTAP database, which is the main source for calibration of the CGE model utilized in this paper. The five classes in the model are:

- Technicians and associated professionals (TEC)
- Clerks (CLE)
- Service and shop workers (SER)
- Officials and managers (MAN)
- Agricultural and low-skilled (AGR)

Whereas the seven levels of educational attainment considered by the BLS are:

- Less than high school diploma (LHS)
- High school diploma or equivalent (HSD)
- Some college, no degree (SCO)
- Associate's degree (ASD)
- Bachelor's degree (BAD)
- Master's degree (MAD)
- Doctoral or professional degree (DPD)

The result of the aggregation is displayed in Table 1. As expected, it is possible to notice a certain correlation between education levels and ranking (in terms of average salaries, prestige, etc.) of the different occupations. For example, most people with little education find jobs in “Agriculture and low-skilled”, whereas many workers in the classes “Technicians and associated professionals” and “Officials and managers” have a bachelor, master, or doctoral degree.

*Table 1. Employment by Education Matrix (U.S.A., 2021)*

	Employment 2021	LHS	HSD	SCO	ASD	BAD	MAD	DPD
<b>MAN</b>	13076.1	371.25	1846.57	2351.83	1096.61	4242.91	1970.67	1191.84
<b>TEC</b>	18179.1	152.32	1168.57	2387.58	2688.02	6529.23	3060.23	2189.35
<b>CLE</b>	19586.9	682.37	5318.47	6096.27	2554.4	3976.87	821.59	140.1
<b>SER</b>	69388.2	4942.59	17467.64	14629.29	6228.96	16392.21	7825.63	1896.97
<b>AGR</b>	37927.9	7630.68	14887.37	8352.72	2883.77	3441.4	600.4	139.52
<i>Total</i>	<i>158158.2</i>	<i>13779.21</i>	<i>40688.62</i>	<i>33817.69</i>	<i>15451.76</i>	<i>34582.62</i>	<i>14278.52</i>	<i>5557.78</i>
<i>Shares</i>	<i>100%</i>	<i>8.71%</i>	<i>25.73%</i>	<i>21.38%</i>	<i>9.77%</i>	<i>21.87%</i>	<i>9.03%</i>	<i>3.51%</i>

Source: own elaboration from BLS data.

Type II underemployment could be defined as the presence of people with high levels of education in low-ranked workplaces. The existence of underemployment in a specific year can be observed directly from an Employment by Education (EE) matrix, like that presented in Table 1. However, be aware that the interpretation of data in an aggregated EE matrix may be ambiguous. For example, the simple fact that somebody with doctoral degree works in agriculture does not automatically means that the person is underemployed. The same applies for the “Service and shop workers” group, which is highly heterogeneous.

<sup>2</sup> <https://www.bls.gov/emp/tables/educational-attainment.htm> .

<sup>3</sup> <https://www.bls.gov/emp/tables/emp-by-detailed-occupation.htm> .

Even if the original matrix has been aggregated here to five employment categories, to align it with the labor classification in the CGE model, there is no need for that. One could retain all the original 832 occupations, or aggregate them down to 22 groups, based on the first two digits of the BLS National Employment Matrix code.

In any case, it could be stated that an assessment of the effects on underemployment of some simulation, realized with a CGE model, essentially boils down to an estimation of a counterfactual EE matrix, consistent with the results of the model. To this end, the following Section 2 proposes a methodology, which considers a baseline EE matrix as an a-priori information, and a new matrix is estimated by minimizing a measure of distance from the original one. Section 3 illustrates the method, through a practical example. Some final remarks are provided in the concluding Section 4.

## 2. Methodology

Suppose that an EE matrix, like the one in Table 1, is available, and that a simulation with a CGE (or other model), estimating new levels of employment, has been carried out. To calculate a new EE matrix, consistent with the model results, one needs to ensure that the totals by row in the matrix (employment by category) are aligned with the variations obtained in the simulation.

What about the totals by column? They refer to employment by level of educational attainment. Should they vary, and how much? The model does not provide any specific information on this, but some reasonable assumptions could be introduced. According to OECD (2022) average unemployment levels, across all OECD countries, are 4.2% for people with tertiary education (university). This can be interpreted as frictional, that is, corresponding to full employment. The same figure, but for people with education below upper secondary (not possessing a high school diploma) is as much as 11%. This suggests that unemployment is a phenomenon mostly associated with low qualification, whereas the response to a weak labor demand by workers with a good level of education is accepting lower quality jobs: Type II underemployment.

To account for this asymmetric response in the estimation of a counterfactual EE matrix, one option is imposing that the sum by columns, for educational levels above a certain threshold, do not change. Therefore, the estimation of a new EE matrix can be framed as a modification of an original EE matrix (interpreted as a-priori information), with new totals for rows, and unchanged totals for some columns.

Those who have worked with estimation of blocks in input-output or social accounting matrices may notice the strong resemblance of the problem with the estimation of sub-matrices through RAS or entropy-maximizing techniques (see, e.g., Robinson, Cattaneo, and El-Said, 2001). Another possibility is casting the question as a minimum distance optimization problem:

$$\min \frac{1}{2} \sum_{i,j} \left( \frac{x_{i,j}}{\bar{x}_{i,j}} - 1 \right)^2 \quad (1)$$

s.t.

$$\sum_j x_{i,j} = \bar{X}_i \quad (2)$$

$$\sum_i x_{i,j} = \bar{X}_j \quad \forall j \in \Omega_j \quad (3)$$

Where:  $x_{i,j}$  refer to cells in the new matrix,  $\widetilde{x}_{i,j}$  those in the old matrix,  $\overline{X}_i$  are constrained row totals,  $\overline{X}_j$  are constrained column totals<sup>4</sup> in the subset  $\Omega_j$ .

The measure of distance utilized in (1) is the square of the relative (percentage change) variation. This kind of measure allows preserving as much as possible the relative proportions between flows in the matrix. It is also thinkable to interpret it as a measurement of disutility, associated with the relocation of workers. This would mean that, for example, joining a category where there are already quite some other workers with the same education level would be less unpleasant than joining a group without a sizeable presence of peers.

The constrained minimization problem (1) gives rise to three sets of first order conditions. In addition to (2) and (3):

$$\left(\frac{x_{i,j}}{\widetilde{x}_{i,j}} - 1\right) \frac{1}{\widetilde{x}_{i,j}} + \lambda_i + \lambda_j = 0 \quad (4)$$

Which can also be written as:

$$\frac{x_{i,j}}{\widetilde{x}_{i,j}} + \widetilde{x}_{i,j} \lambda_i + \widetilde{x}_{i,j} \lambda_j = 1$$

Taken together, (2) (3) and (4) defines a linear system, whose solution identifies the cells in the new matrix  $x_{i,j}$ , as well as the Lagrange multipliers  $\lambda$ . Being a linear system, it can also be expressed as a single matrix equation, like:<sup>5</sup>

$$\begin{bmatrix} \widetilde{x}_{1,1}^{-1} & 0 & 0 & 0 & \widetilde{x}_{1,1} & 0 & \widetilde{x}_{1,1} \\ 0 & \widetilde{x}_{1,2}^{-1} & 0 & 0 & \widetilde{x}_{1,2} & 0 & 0 \\ 0 & 0 & \widetilde{x}_{2,1}^{-1} & 0 & 0 & \widetilde{x}_{2,1} & \widetilde{x}_{2,1} \\ 0 & 0 & 0 & \widetilde{x}_{2,2}^{-1} & 0 & \widetilde{x}_{2,2} & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_{1,1} \\ x_{1,2} \\ x_{2,1} \\ x_{2,2} \\ \lambda_1 \\ \lambda_2 \\ \lambda_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ \overline{X}_1 \\ \overline{X}_2 \\ \overline{X}_1 \end{bmatrix} \quad (5)$$

### 3. Example

Let us look at the results of a relatively simple simulation exercise, realized with the standard GTAP CGE model (Corong et al., 2017). Five regions are considered (EU, USA, China, Russia, Rest of the World), and ten sectors, including "Extraction". The latter comprehends coal, oil, gas, and minerals. Real wage is kept exogenous and fixed, whereas employment (by labor category) is endogenously computed.

A reduction by 90% of imports of extraction products from Russia to EU and USA is simulated.<sup>6</sup> The shock produces a GDP reduction of -1.82% for Russia, -0.66% for the EU, -0.32% for the USA, and a slight increase in China and the Rest of the World.<sup>7</sup> Employment is reduced when GDP shrinks. Most notably, in the USA, employment falls by -0.34% in all labor categories, except "Managers and Officials" (-0.33%) and "Agriculture and Low-Skilled" (-0.32%).

<sup>4</sup> Based on the discussion above, one could expect the totals to be fixed at the same levels of the original matrix. However, this is not strictly necessary. Other totals may be imposed, if additional information is available, or if alternative hypotheses are adopted.

<sup>5</sup> This would correspond to a 2x2 matrix with only one constraint at the first column.

<sup>6</sup> This is made possible by swapping imported quantities with tariffs, before shocking the imports.

<sup>7</sup> The simulation is inspired by the economic consequences of the war in Ukraine, yet it does not pretend to be realistic. More information about the exercise is available from the author, upon request.

Results for employment in the USA can be used to generate an updated EE matrix, from the baseline one shown in Table 1, using the methodology presented in the previous section. The outcome, expressed as percentage variations from the baseline, is displayed in the Table 2 below.

*Table 2. Simulated changes in the US EE matrix*

	LHS	HSD	SCO	ASD	BAD	MAD	DPD	Var %
<b>MAN</b>	-0.07%	-0.34%	-0.44%	-0.12%	-0.47%	-0.22%	-0.07%	<b>-0.33%</b>
<b>TEC</b>	-0.03%	-0.19%	-0.38%	-0.23%	-0.54%	-0.25%	-0.07%	<b>-0.34%</b>
<b>CLE</b>	-0.06%	-0.50%	-0.58%	-0.06%	-0.07%	-0.01%	0.01%	<b>-0.34%</b>
<b>SER</b>	-0.28%	-0.99%	-0.83%	0.10%	0.32%	0.15%	0.13%	<b>-0.34%</b>
<b>AGR</b>	-0.28%	-0.55%	-0.31%	0.10%	0.14%	0.02%	0.01%	<b>-0.32%</b>
<i>Var %</i>	<b>-0.26%</b>	<b>-0.71%</b>	<b>-0.60%</b>	<b>0.00%</b>	<b>0.00%</b>	<b>0.00%</b>	<b>0.00%</b>	

The variations in total employment by category are aligned with those obtained by the model, whereas no changes in the totals are assumed for workers with a degree. The new matrix reveals that much of the unemployment would be borne by people with a high school diploma, or some college, and especially in the services. Instead, workers with higher education, especially those with a bachelor's degree, move away from good (and possibly better paid) occupations, replacing them with jobs requiring less qualification.

Therefore, the estimation of a counterfactual EE matrix, consistent with the results of the CGE simulation, allows sorting out the effects of a weaker labor demand into unemployment and underemployment. It also makes it possible to identify which categories of workers are mostly affected, and how much.

Notice that the results are not being driven by unequal variations in total employment, but rather by the initial distribution of workers in the baseline matrix. In fact, the same kind of effects would have been noticed if an identical change in employment would have been applied across all worker's categories. This also means that, to apply the methodology proposed in this paper, it is not necessary to operate with a macro model, where different typologies of labor are distinguished.

Finally, the example analyzed here refers to a negative macroeconomic shock, but a positive one could well have been considered. In this case, the method would estimate some reductions in both unemployment and underemployment.

#### 4. Conclusion

Underemployment is a key factor in the labor market, and a central objective for economic policy. Nonetheless, there are only limited data and modeling tools which can help in monitoring and assessing the phenomenon. To partly overcome these limitations, a method is proposed in this paper, which allows estimating the impact of variations in the labor market, after a simulation with a computable general equilibrium model, in terms of both unemployment and underemployment.

When a macroeconomic model produces estimates of changes in employment levels, the unemployment rate can be computed as the difference between workforce and employment. A similar procedure can be followed to compute Type I underemployment, if employment is defined in terms of time (e.g., hours) and, instead of the workforce, potential labor supply is expressed as the total desired working time, for the aggregate of all workers. Of course, the latter is not normally available in official national accounts, and it should therefore be estimated, through surveys, along the lines suggested by Bell and Blanchflower (2018b).



Type II underemployment refers to a mismatch between education and skill levels, on one hand, and qualification requirements in some workplaces, on the other hand. An evaluation of this kind of underemployment, therefore, needs some information about the distribution of workers with given characteristics (like education attainment) and occupations.

The method proposed here to assess Type II underemployment is based on the estimation of a counterfactual “employment by education” matrix, where a baseline matrix is used as a-priori information and some constraints are imposed. These constraints line up the matrix with the results of a numerical simulation. As people with a sufficiently high education level are supposed to always find (sooner or later) some job, even if not satisfactory, the method allows to separately distinguish the effects on unemployment from those on underemployment.

Some obstacles to the implementation of the methodology can be devised, though. First, the example illustrated in this paper is based on a baseline EE matrix, obtained from original data by the US Bureau of Labor Statistics. A similar kind of accounts is not readily available in other countries. In these cases, the base matrix could be estimated in a more imprecise and indirect way, for example by applying the US educational structure to different national employment levels.

Even if the example presented in this paper is based on a dichotomy between workers with a degree and workers without a degree, this is not necessary for the implementation of the method. In principle, the column constraints could be set in more flexible way if, for instance, the elasticity of occupation (by education) to total labor demand would be known. However, I am not aware of any study specifically addressing this subject.

Finally, one useful extension would be considering other dimensions for worker’s qualification, like years of experience. In principle, that could be done by conceiving a three-dimensional matrix EEE (employment-education-experience), on which further constraints on the marginals could be imposed, exactly as in the case of education. The key precondition would then be, of course, the availability of suitable data.

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