



Term premia and short rate expectations in the euro area

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ABSTRACT

Identifying the components of yields is a challenging task for monetary authorities. We use a term structure model with stochastic volatility and eurozone global macro factors to estimate time-varying term premia and short rate expectations for ten countries in the euro area. Unlike previous studies, we explicitly disentangle from these components the convexity effects that have substantial impact on long-term yields in turbulent times. The empirical evidence shows that term premia are significantly positively related to yield volatility across all countries, while term premia and expected short rates react in opposite directions to shocks in eurozone inflation and GDP growth expectations. A connectedness analysis based on variance decomposition suggests that there exist significant cross-country interconnections for the yield components, with the size of the links varying substantially over time and across countries.

1. Introduction

The Global Financial Crisis (GFC), the eurozone sovereign debt crisis and, more recently, the pandemic shock have all posed considerable challenges to European institutions, monetary authorities and investors. The European Central Bank (ECB) has been forced to adopt unconventional monetary policy measures, such as negative interest rates and asset purchases, to stabilize financial markets and stimulate economic growth. After more than a decade of near-zero interest rates, inflation has suddenly risen around the world and policymakers are currently debating whether the conditions exist to normalize interest rates. A clear understanding of the forces underlying the movements in interest rates has thus become a timely and very relevant issue for central bankers.

Bond yields are given by the sum of an expectation component that reflects the average of current and future expected short-term rates over the bond's maturity, and a risk premium component, defined as the term premium, which is the additional compensation required by the investors to hold a longer-term bond rather than a series of shorter-term bonds. Unfortunately, the decomposition of yields cannot be inferred directly from market prices, and identifying the drivers of yield curve movements in real time is a challenging task.

The literature on yield decomposition includes several empirical analyses for the U.S. (see, among the others, Kim and Wright (2005), Haubrich et al. (2012), Adrian et al. (2013), Abrahams et al. (2016)), the U.K. (Joyce et al. (2010), Malik and Meldrum (2016); Kaminska et al., (2018)), Japan (Imakubo and Nakajima (2015)), Australia (Jennison (2017)) and international comparisons (Jotikasthira et al., (2015), Moench (2019); Berardi and Plazzi (2022)).

Evidence for the euro area as a whole is presented in Dewachter et al., (2016), Cohen et al., (2018); Iskrev (2018); McCoy (2019) and Berardi and Plazzi (2022), while Chadha and Hantzschke (2018), Moessner (2018), Moench (2019), Corradin et al., (2021) and Moessner and de Haan (2022) provide a yield decomposition also for individual countries of the euro area. Term premia and short rate expectations in Cohen et al., (2018) are obtained by applying the Hordahl and Tristani (2014) and Monfort et al., (2017) models to

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eurozone data, while those in [Chadha and Hantzsche \(2018\)](#), [Moessner \(2018\)](#), [Moench \(2019\)](#) and [Moessner and de Haan \(2022\)](#) are all derived from the estimation of the [Adrian et al., \(2013\)](#) model.

Most of the studies on the decomposition of yields are based on the assumption that the yield curve can be modelled using a constant volatility affine term structure model (ATSM). However, the volatility of yields in the eurozone appears far from being constant over time. As an example, [Fig. 1](#) shows the time series of the 10-year yield volatility for the ten countries of the euro area that we consider in our empirical analysis (Germany, France, the Netherlands, Austria, Finland, Belgium, Italy, Spain, Portugal, and Ireland) over a 23-year sample period. We find that both realized volatility and a GARCH-based measure suggest a time-varying dynamics for the volatility of yields and, therefore, the need for a stochastic volatility framework in modeling yields and their components (see, for example, [Cieslak and Povala \(2016\)](#) and [Feldhutter et al. \(2018\)](#)).

Mis-estimating yield volatility has implications for yield decomposition, especially at long maturities. In fact, a significant variation in yield volatility implies that the “mechanical” convexity effect on a τ -maturity yield $Y_t(\tau)$, which is defined as $c_t(\tau) = -\tau^2/2 \times \text{Var}[dY_t(\tau)]$, can be substantial and, if not explicitly separated from term premia and short rate expectations, may induce excessive variability in the estimates of these two components (see [Berardi et al., \(2022\)](#)).¹

Unlike most previous studies on the euro area, in this paper we use a time-varying volatility ATSM to decompose yields into their term premium and short rate expectation elements and, in addition, we explicitly disentangle the convexity effect from those components.

Another innovative feature of the model compared to previous analyses is that it includes two global – eurozone-specific – macro factors among the state variables and therefore allows to study the impact of European macro conditions on the yield components in each country.

Although the model is estimated separately for each country, we provide a measure for the degree of connectedness in yield components among the eurozone countries by applying the variance decomposition technique of [Diebold and Yilmaz \(2012\)](#). A similar approach has been utilized by [Moench \(2019\)](#) in the context of multi-country term structure decompositions. This analysis also represents an additional contribution of our paper with respect to other studies on the decomposition of yields in the euro area and is to some extent related to the literature on contagion effects among euro sovereign bonds (see, for example, [Metiu \(2012\)](#), [Claeys and Vasicek \(2014\)](#) and [Caporin et al. \(2018\)](#)).

The empirical evidence is based on data for the euro area as a whole and for ten countries (Germany, France, the Netherlands, Austria, Finland, Belgium, Italy, Spain, Portugal, and Ireland) over the sample period ranging from January 2000 to September 2022. The model is fitted to: (i) yields with maturities from 2 to 10 years; (ii) the 10-year yield variance, calculated as the realized within-month variance of daily changes in yields; and (iii) macro expectations for the euro area, which are proxied by the average 1-year-ahead ECB survey forecasts of inflation and real GDP growth rates. A quasi-maximum likelihood method, with an approximate Kalman filter algorithm for the unobservable state variables, is used in estimation.

We find that term premia and expected short rates show a marked downward trend over the sample period and a sudden increase in the last two years, although there appear to be significant differences between “low-rate countries” (Germany, France, the Netherlands, Austria, Finland and Belgium) and “high-rate countries” (Italy, Spain, Portugal and Ireland) during turbulent times, such as the GFC, the 2010–2012 sovereign debt crisis and the 2020 outbreak of the pandemic.

The first main result of the paper is that term premia are significantly positively related to yield volatility across all countries. Despite this close link to volatility, our term premia show less variability than those in previous studies (see, for example, estimates reported in [Cohen et al., \(2018\)](#)), [McCoy \(2019\)](#) and [Moessner and de Haan \(2022\)](#)) or those obtained estimating a five-factor Gaussian (constant volatility) model.² This is because we calculate term premia net of time-varying convexity effects, which indeed have substantial impact on long-term yields in crisis periods. Therefore, modeling term premia by allowing for time-varying volatility and, at the same time, separating the convexity component can be crucial to obtain reliable estimates.

A second empirical result which is captured by the model’s specific factor structure is that expected short rates tend to increase in response to positive shocks in the eurozone expected inflation rate and GDP growth rate, while term premia react negatively to shocks in the global macro variables, with a U-shaped curve reaching a maximum at a time horizon between two and three years.

Although our model does not accommodate default or liquidity risks, through a simple regression analysis we study the relation between our estimated term premia and those risks (see, for example, [Krishnamurthy et al., \(2018\)](#) and [Corradin et al., \(2021\)](#)). In particular, we regress the term premium of a country against the corresponding CDS spread and a country-specific composite liquidity index (see [Poli and Taboga \(2021\)](#)) and find that liquidity risk has a similar impact on term premia for both low-rate and high-rate countries, while default risk affects term premia only in the case of high-rate countries.

Another novel contribution of the paper is the result of the connectedness analysis for yield components. Different from previous evidence on the eurozone based on variance decomposition (see [Claeys and Vasicek \(2014\)](#) and [Buse and Schienle \(2019\)](#)), our estimates show that total connectedness in yields, expected short rates and term premia appears to be relatively strong in stable times and decrease during crisis periods. For example, in the 2007–2013 sub-period, which includes the GFC and the sovereign debt crisis, we find only a few significant connections across low-rate and high-rate countries, while we observe stronger links within the two groups. This dichotomy is also evident in the 2014–2022 sub-period, which is strongly influenced by the effects of the ECB’s Asset Purchase Programme (see, for example, [Eser et al., \(2019\)](#), [De Santis and Holm-Hadulla \(2020\)](#) and [Altavilla et al., \(2021\)](#)) and by two crisis

¹ On the impact of the convexity effect on long-term yields, see [Brown and Schaefer \(2000\)](#) and [Balter et al. \(2021\)](#).

² The standard deviation of monthly changes in the 10-year term premium estimated for the euro area is equal to 52 basis points for the Gaussian model and 34 basis points for our stochastic volatility model.

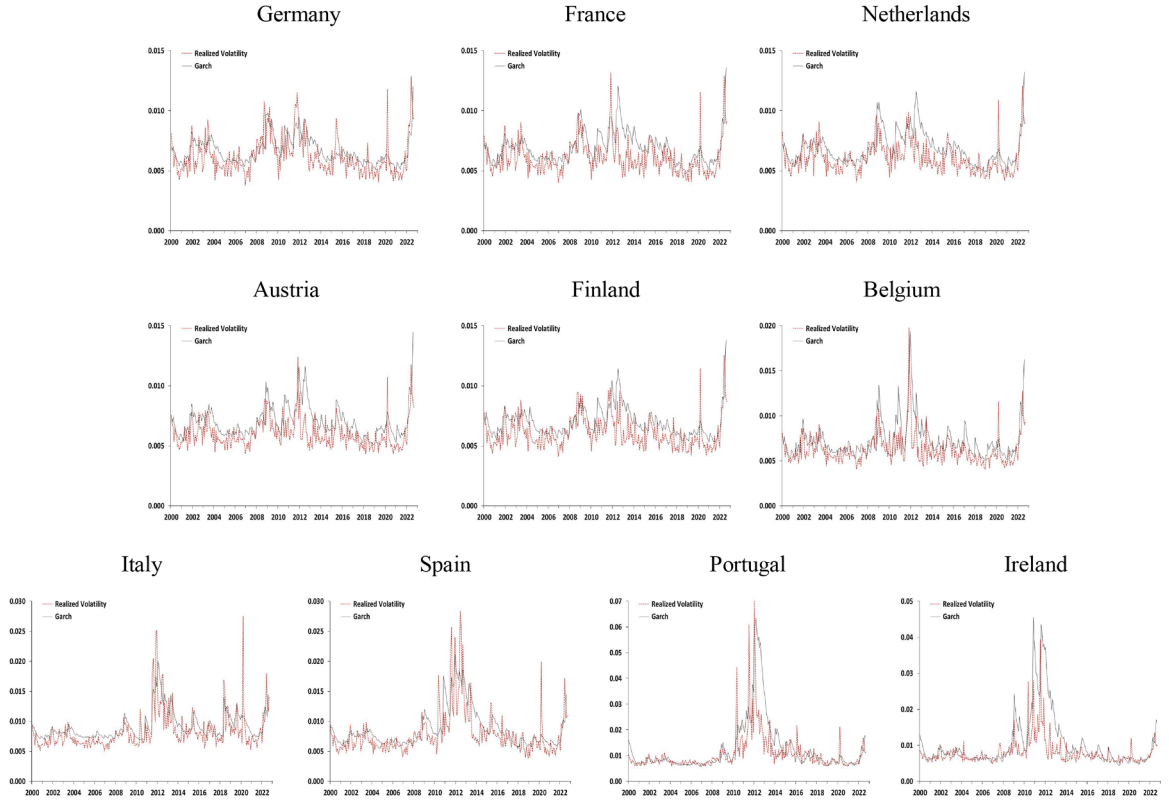


Fig. 1. Time-Varying Yield Volatility in the Euro Area

This figure shows the time series of yield volatility for the ten countries in our sample over the period January 2000 to September 2022. For each country, two measures of yield volatility are calculated: (i) the realized within-month standard deviation of daily changes in the 10-year yield, and (ii) the volatility predictor estimated by a GARCH(1,1) model applied to monthly changes in the 10-year yield.

events, such as the pandemic and the explosion in energy prices.

Overall, the evidence implies that there exist significant cross-country interconnections for both short rate expectations and term premia, but the size of these links can vary substantially over time and across low-rate and high-rate countries.

2. The model

We assume that the economy is driven by five latent factors, three of which are country-specific factors and two of which are linked to eurozone global factors. In particular, the country-specific factors are the following: (i) a variance factor v , which accounts for the dynamics of the conditional volatility of the other two country-specific variables; (ii) a factor ℓ , which represents a proxy for the level of yields; and (iii) a factor s , which is a proxy for the slope of the yield curve. The latent global factors are represented by the eurozone instantaneous expected inflation rate π and expected output growth rate μ . The state variables are collected in the state vector $X = (X_1 \ X_2)'$, where X_1 contains the country-specific factors, $X_1 = (v \ \ell \ s)'$, and X_2 the global macro factors, $X_2 = (\pi \ \mu)'$.

Under the physical probability measure $\mathbb{P}$, the state vector follows a $A_1(5)$ stochastic process of the Dai and Singleton (2000) type: $dX_t = K(\Theta - X_t)dt + \Sigma\sqrt{\Xi_t}dZ_t$, which can also be written as follows:

$$\begin{pmatrix} dX_{1t} \\ dX_{2t} \end{pmatrix} = \begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix} \begin{pmatrix} \Theta_1 - X_{1t} \\ \Theta_2 - X_{2t} \end{pmatrix} dt + \begin{pmatrix} \Sigma_{11} & 0 \\ 0 & \Sigma_{22} \end{pmatrix} \begin{pmatrix} \Omega_t & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} dZ_{1t} \\ dZ_{2t} \end{pmatrix} \quad (1)$$

where K_{11} , K_{12} , K_{21} , and K_{22} are matrices with dimension (3×3) , (3×2) , (2×3) , and (2×2) , respectively, Θ_1 and Θ_2 are (3×1) and (2×1) vectors, Σ_{11} and Σ_{22} are (3×3) and (2×2) diagonal matrices, respectively, Ω_t is a (3×1) diagonal matrix with elements equal to $\sqrt{v_t}$ and I is a (2×2) identity matrix. Finally, dZ_{1t} and dZ_{2t} are (3×1) and (2×1) vectors of independent Brownian motions, respectively.

In sum, we model the first factor v as a square-root process that enters the diffusion term of the other two country-specific, conditionally Gaussian factors, ℓ and s . Instead, the global factors π and μ in X_2 follow a Gaussian process and potentially interact with each other and with the country-specific factors through the drift term. We assume that they are linked to the exogenously given price level p and the real production output q through the following process:

$$dM_t = \Gamma_M X_t dt + \Sigma_M dZ_{Mt} \quad (2)$$

where $dM_t = \left(\frac{dp_t}{p_t} \quad \frac{dq_t}{q_t} \right)$, Γ_M is a (2×5) matrix of zeros apart from the fourth element of the first row and the fifth element of the second row, which are equal to one, Σ_M a (2×2) diagonal matrix and dZ_{Mt} a (2×1) vector of independent Brownian motions.

Using Ito's lemma, we derive the following expression for logarithmic changes in the price level and output over the time interval $[t, t + \bar{\tau}]$:

$$E_t \left[\ln \frac{M(t + \bar{\tau})}{M(t)} \right] = A_M(\bar{\tau}) + B_M(\bar{\tau}) X_t \quad (3)$$

where $B_M(\bar{\tau}) = \Gamma_M K^{-1} (I - e^{-K\bar{\tau}})$ and $A_M(\bar{\tau}) = -\frac{1}{2} \Sigma_M \Sigma_M' i \bar{\tau} + [\bar{\tau} \Gamma_M - B_M(\bar{\tau})] \Theta$, with $i' = (1 \ 1)$.

We characterize the dynamics under the risk-adjusted probability measure $\mathbb{Q}$ by using an “essentially affine” specification of the instantaneous market price of risk of the [Duffee \(2002\)](#) type:

$$\Psi_t = \sqrt{\Xi_t} (\Lambda_0 + \Lambda_1 X_t) \quad (4)$$

where Λ_0 is a (5×1) vector, Λ_1 a (5×5) matrix and $\sqrt{\Xi_t}$ a (5×5) diagonal matrix with the first three elements equal to $1/\sqrt{v_t}$ and the last two elements equal to 1.

This structure implies that the risk-adjusted process of the state vector X can be written as

$$dX_t = \tilde{K}(\tilde{\Theta} - X_t)dt + \Sigma \sqrt{\Xi_t} d\tilde{Z}_t \quad (5)$$

where $\tilde{K} = K + \Sigma \Lambda_1$, $\tilde{\Theta} = \tilde{K}^{-1} (K\Theta - \Sigma \Lambda_0)$ and $d\tilde{Z}_t = dZ_t + \Psi_t dt$.

The instantaneous interest rate r is affine in the state vector X :

$$r_t = \delta_0 + \delta_1' X_t \quad (6)$$

where δ_0 is a constant and δ_1 a (5×1) vector.

We impose on the model the admissibility constraint that the off-diagonal terms in the first row of matrix K are zero and the following identification constraints (see [Dai and Singleton \(2000, 2002\)](#) and [Duffee \(2002\)](#)): (i) $\theta_{\ell} = \theta_s = 0$ in vector Θ of equation (1); (ii) all elements in the first row of matrix Λ_1 in [Eq. \(4\)](#) equal to zero; (iii) $\delta_v = \delta_{\pi} = \delta_{\mu} = 0$ and $\delta_{\ell} = \delta_s = 1$ in vector δ_1 of [Eq. \(6\)](#), which means that the short rate is given by a constant (δ_0) plus the sum of the latent variables ℓ (level factor) and s (slope factor).

The time- t equilibrium price of a unit discount bond with time to maturity τ , $P_t(\tau)$, has an exponentially affine closed-form solution:

$$P_t(\tau) = \exp\{A(\tau) - B(\tau)' X_t\} \quad (7)$$

where $A(\tau)$ and $B(\tau)$ solve a system of ordinary differential equations (see, for example, [Dai and Singleton \(2000\)](#)). The term structure of interest rates is thus affine in the country-specific factors:

$$Y_t(\tau) = a(\tau) + b(\tau)' X_t \quad (8)$$

where $a(\tau) = -A(\tau)/\tau$ and $b(\tau) = B(\tau)/\tau$.

The diffusion term in the risk-adjusted dynamics of X is a function of v , which implies that yield volatilities are time-varying and are driven by that factor. In particular, the time- t instantaneous variance of changes in the τ -maturity yield, $V_t(\tau)$, is affine in v and is given by:

$$V_t(\tau) = b'(\tau) (\Sigma \Xi_t \Sigma') b(\tau) \quad (9)$$

The solution of the model implies that the time- t instantaneous forward rate for date $t+\tau$, $f_t(\tau) = \frac{1}{P_t(\tau)} \frac{\partial P_t(\tau)}{\partial \tau}$, can be expressed as:

$$f_t(\tau) = r_t + B'(\tau) K(\Theta - X_t) - B'(\tau) \Sigma (\Lambda_0 + \Lambda_1 X_t) - \frac{1}{2} B'(\tau) (\Sigma \Xi_t \Sigma') B(\tau) \quad (10)$$

or, equivalently, as:

$$f_t(\tau) = [r_t + G'(\tau) K(\Theta - X_t)] + [B'(\tau) - G'(\tau)] K(\Theta - X_t) - B'(\tau) \Sigma (\Lambda_0 + \Lambda_1 X_t) - \frac{1}{2} B'(\tau) (\Sigma \Xi_t \Sigma') B(\tau) \quad (11)$$

where $G'(\tau) = \delta_1' K^{-1} (I - e^{-K\tau})$. The first term in brackets on the right-hand side of [Eq. \(11\)](#) is the time- t instantaneous expected short rate at $t+\tau$ under the physical probability measure $\mathbb{P}$, $E_t^{\mathbb{P}}[r_{t+\tau}] \equiv [r_t + G'(\tau) K(\Theta - X_t)]$. Using a Gaussian framework, [Dai and Singleton \(2002\)](#) define the difference between $f_t(\tau)$ and $E_t^{\mathbb{P}}[r_{t+\tau}]$ as the “forward term premium”. However, in a stochastic volatility model, such as our model, there can be a significant and time-varying convexity effect, measured as $c_t = -\frac{\tau^2}{2} V_t(\tau) = -\frac{1}{2} B'(\tau) (\Sigma \Xi_t \Sigma') B(\tau)$, that must be removed from this expression. Therefore, we use the following definition for the time- t forward term premium on a τ -maturity

bond, $FTP_t(\tau)$:

$$FTP_t(\tau) = f_t(\tau) - E_t^{\mathbb{Q}}[r_{t+\tau}] - c_t(\tau) \quad (12)$$

Taking the integral of both sides of Eq. (12) and dividing by τ , we obtain an expression for the yield term premium: $TP_t(\tau) = \frac{1}{\tau} \int_0^{\tau} FTP_t(u) du$. Similarly, we define the average expected short rate between t and $t+\tau$ as $ES_t(\tau) = \frac{1}{\tau} \int_0^{\tau} E_t^{\mathbb{Q}}[r_{t+u}] du$ and the average convexity between t and $t+\tau$ as $CX_t(\tau) = \frac{1}{\tau} \int_0^{\tau} c_t(u) du$. Therefore, the yield on a τ -maturity zero coupon bond in Eq. (8) can also be expressed as:

$$Y_t(\tau) = ES_t(\tau) + TP_t(\tau) + CX_t(\tau) \quad (13)$$

i.e., as the sum of the τ -maturity expected short rate ES , term premium TP , and convexity CX .

3. Data and preliminary analysis

The model is estimated using monthly data on country-specific yields and yield volatilities and data on eurozone macroeconomic expectations from January 2000 to September 2022. Ten countries of the euro area are considered: Germany, France, the Netherlands, Austria, Finland, Belgium, Italy, Spain, Portugal and Ireland. Moreover, we provide estimates for the euro area as a whole. For each country, we use zero coupon yields with maturities between 2 and 10 years and the yield variance with maturity 10 years. The yield variance is obtained by calculating the realized within-month variance of daily changes in the 10-year yield. The data source is Bloomberg for the ten countries and the ECB website for the euro area as a whole.³ For macro expectations, we use the average 1-year-ahead forecasts of annual CPI growth and annual real GDP growth rates obtained from the ECB Survey of Professional Forecasters (SPF). These data are available on a quarterly basis.

Summary statistics for the data are reported in Table 1. We distinguish a group of low-rate countries and one of high-rate countries. The former is composed of Germany, France, the Netherlands, Austria, Finland and Belgium, for which average yields are below 1.6 % at the 2-year maturity and below 2.8 % at the 10-year maturity, while high-rate countries include Italy, Spain, Portugal and Ireland. The average term structure of yields is upward sloping in all countries, with the difference between the 10-year and 2-year yield ranging from 99 basis points (bps) for Germany to 155 bps for Italy. The average 10-year yield volatility is comprised between 60 and 63 bps for the group of low-rate countries and between 76 and 98 bps for the group of high-rate countries.

The table also shows that the average one-year-ahead SPF forecasts of the CPI inflation rate and real GDP growth rate in the euro area are both around 1.6 %, while the predicted GDP growth rate is about four times more volatile than the predicted inflation rate.

Panel A of Table 2 reports the result of a principal component (PC) analysis applied to monthly changes in yields and yield variances with maturities from 2 to 10 years.⁴ We find that, for each country, the cumulative percentage contribution of the first two PCs to the total variability of monthly changes in yields is around 98 % and, in all cases, these two PCs correspond to the level and the slope of the yield curve, i.e., the variables ℓ and s in our model.⁵

The PC analysis for monthly changes in yield variances shows that the first PC accounts for a significant proportion of the overall variability, with values ranging from 73 % (Finland) to 96 % (Ireland). Therefore, for each country, a single volatility factor (v in our model) can explain most of the dynamics of yield variances.

When we consider monthly changes of yields for all countries and all maturities – from 2 to 10 years for the ten countries, i.e., 90 series – the first PC accounts for 62 % of the total variability. This percentage becomes significantly higher if we restrict the analysis to low-rate countries (85 %), while it increases only marginally if we consider high-rate countries (64 %). In any case, these values show the existence of a significant interrelation between the yield curves of the ten countries. We conjecture that these cross-country effects extend to the yield components and try to capture them in our model through the two global macro factors, π and μ .

The fact that yields are substantially correlated across countries can also be observed in Panels B and C of Table 2, which contain the cross-country correlations between monthly changes in the 2-year and 10-year yields, respectively. We observe that correlations tend to be higher for the 10-year maturity (average correlation 0.67 vs. 0.54 for the 2-year maturity) and correlations between low-rate countries are generally stronger than those between high-rate countries (average correlation at 10-year maturity is 0.94 for low-rate and 0.60 for high-rate countries).

Overall, this analysis of the data seems to support our model specification, in that three country-specific factors (ℓ , s and v) can be used to fit the term structure of yields and the yield variance and one or more common factors (π and μ in our setting) could help capture cross-country effects in yield dynamics.

³ The tickers of the series are GDBR (Germany), GTFRF (France), GTNLG (The Netherlands), GTATS (Austria), GTFIM (Finland), GTBEF (Belgium), GBTPGR (Italy), GTESP (Spain), GTPTE (Portugal), and GIGB (Ireland). As the ECB data start only in September 2004, for the period January 2000 to August 2004, the yield curve for the euro area is calculated as the simple average of the yield curves of the ten countries considered in our dataset.

⁴ For each maturity, the yield variance is the realized within-month variance of daily changes in the yield.

⁵ For brevity, we do not report the structure of the eigenvectors. We observe that, for each country, the first is approximately constant across maturities, while the second is monotone decreasing with maturity.

Table 1
Summary Statistics

This table reports mean and standard deviation for the time series of yields, yield volatility and macro survey expectations. Data on yields are end-of-month observations with maturities from 2 to 10 years, while data on the 10-year yield volatility are annualised monthly realized volatilities calculated from daily changes in the 10-year yield. Data on macro expectations refer to the median of the ECB Survey of Professional Forecasters expectations of 1-year-ahead inflation and real GDP growth rates in the euro area. These data are available on a quarterly basis. The average value (Avg) is computed from the level of the variables. The standard deviation (S.D.) of yields and yield volatility is the annualised standard deviation of monthly changes in the variables, while the standard deviation of macro expectations is calculated from the level. All values are expressed in percentage terms. The sample period is January 2000 to September 2022.

	Maturity Years	Germany		France		Netherlands		Austria		Finland	
		Avg	S.D.	Avg	S.D.	Avg	S.D.	Avg	S.D.	Avg	S.D.
Yields	2	1.359	0.635	1.463	0.644	1.393	0.634	1.465	0.671	1.389	0.648
	3	1.475	0.668	1.601	0.678	1.557	0.636	1.641	0.645	1.550	0.648
	4	1.624	0.678	1.788	0.679	1.718	0.661	1.813	0.655	1.711	0.664
	5	1.776	0.669	1.965	0.689	1.875	0.673	1.974	0.673	1.872	0.661
	6	1.908	0.670	2.108	0.684	2.026	0.663	2.122	0.686	2.028	0.659
	7	2.040	0.659	2.243	0.674	2.169	0.655	2.259	0.696	2.174	0.663
	8	2.157	0.647	2.389	0.665	2.301	0.649	2.391	0.683	2.299	0.653
	9	2.257	0.641	2.521	0.668	2.413	0.657	2.511	0.669	2.417	0.658
	10	2.348	0.634	2.641	0.657	2.518	0.650	2.618	0.662	2.529	0.660
	10	0.619	0.433	0.610	0.421	0.605	0.383	0.601	0.395	0.606	0.401
Yield Vol											
	Maturity Years	Belgium		Italy		Spain		Portugal		Ireland	
		Avg	S.D.	Avg	S.D.	Avg	S.D.	Avg	S.D.	Avg	S.D.
Yields	2	1.541	0.807	2.035	1.280	1.934	1.195	2.721	2.947	2.214	2.332
	3	1.738	0.757	2.333	1.257	2.188	1.158	3.055	2.984	2.389	2.147
	4	1.928	0.754	2.560	1.222	2.373	1.097	3.269	2.708	2.579	1.934
	5	2.106	0.757	2.786	1.144	2.579	1.061	3.529	2.735	2.748	1.770
	6	2.265	0.763	2.980	1.100	2.760	0.995	3.734	2.592	2.925	1.620
	7	2.402	0.745	3.143	1.074	2.932	0.965	3.909	2.304	3.081	1.477
	8	2.539	0.730	3.296	1.033	3.084	0.940	4.050	2.060	3.242	1.343
	9	2.668	0.727	3.444	0.970	3.219	0.909	4.135	1.836	3.362	1.303
	10	2.783	0.721	3.589	0.921	3.347	0.872	4.180	1.628	3.472	1.231
	10	0.627	0.534	0.818	0.876	0.776	0.888	0.975	1.355	0.764	1.005
Yield Vol											
	Maturity Years	Euro Area									
		Avg	S.D.								
Yields	2	1.696	0.725								
	3	1.885	0.691								
	4	2.076	0.675								
	5	2.259	0.670								
	6	2.432	0.664								
	7	2.592	0.652								
	8	2.733	0.643								
	9	2.855	0.635								
	10	2.961	0.628								
	10	0.589	0.300								
Yield Vol	10	0.589	0.300								
Infl. Exp	1	1.622	0.395								
GDP Exp	1	1.675	1.692								

4. Empirical results

In this section, we first describe the methodology used for estimation and present the results for the goodness of fit and the estimated state variables and parameters. Then, we focus on the estimated yield components by analyzing their properties and studying their dynamic responses to factor innovations.

4.1. Estimation method

The parameters of the state-space representation of the model are estimated by the quasi-maximum likelihood method, with an approximate Kalman filter algorithm being used to calculate the values of the unobserved state variables and to accommodate the different frequencies of the series used in estimation. The use of approximate linear filtering is necessary in the cases in which the state vector has affine dynamics but is not Gaussian. In this scenario, an approximate transition equation can be obtained by exploiting the existence of an analytical expression of the first two conditional moments of the state vector (see, for example, [Christoffersen et al., \(2014\)](#)).

The estimation is performed in two steps. In the first step, we estimate the model for the euro area as a whole by fitting ten series with a monthly frequency, i.e., the 2- to 10-year yields ([Eq. \(8\)](#)) and the 10-year realized yield variance ([Eq. \(9\)](#)), and two series with a

Table 2

Factor Analysis and Cross-Country Correlation in Yields

This table reports a factor analysis and statistics on cross-country correlation in yields. Panel A shows: (i) for each country, the cumulative percentage contribution of the first two principal components (PC) to the total variability of monthly changes in yields with maturities from 2 to 10 years; (ii) for each country, the cumulative percentage contribution of the first PC to the total variability of monthly changes in yield variances with maturities from 2 to 10 years; (iii) the cumulative percentage contribution of the first PC to the total variability of monthly changes in yields with maturities from 2 to 10 years for all ten countries (90 time series of observations), for the low-rate countries, i.e., Germany, France, the Netherlands, Austria, Finland, and Belgium (54 time series), and for the high-rate countries i.e., Italy, Spain, Portugal, and Ireland (36 time series). Panels B and C contain the cross-correlation for monthly changes in 2-year yields and 10-year yields, respectively. All values are expressed in percentage terms. The sample period is January 2000 to September 2022.

Panel A: Principal components analysis										
	Ger	Fra	Net	Aus	Fin	Bel	Ita	Spa	Por	Ire
Yields: First two PCs	98.5	98.0	98.5	98.6	97.9	98.2	98.5	98.3	97.5	97.8
Yield Variances: First PC	83.6	81.5	79.8	85.4	72.6	92.6	90.0	92.8	95.4	96.2
	All countries			Low-rate countries				High-rate countries		
Yields: First PC	62.5			85.4				64.3		
Panel B: Correlation 2-year yields										
	Fra	Net	Aus	Fin	Bel	Ita	Spa	Por	Ire	
Ger	92.3	94.8	88.4	91.6	73.8	32.4	37.7	18.9	26.5	
Fra		90.7	89.0	87.4	81.0	42.3	46.2	24.1	26.8	
Net			89.3	89.9	72.7	33.8	39.4	23.9	29.9	
Aus				82.2	84.1	45.2	47.5	22.2	30.4	
Fin					69.7	32.6	38.0	21.9	29.2	
Bel						60.8	60.4	20.9	36.0	
Ita							83.0	46.1	53.1	
Spa								42.2	53.4	
Por									44.4	
Panel C: Correlation 10-year yields										
	Fra	Net	Aus	Fin	Bel	Ita	Spa	Por	Ire	
Ger	94.4	96.8	94.1	96.6	86.7	48.6	56.8	26.1	51.5	
Fra		95.4	95.1	95.0	91.8	60.2	63.2	30.4	56.3	
Net			95.9	96.8	88.7	52.3	59.8	25.1	54.5	
Aus				95.7	91.2	57.2	64.0	34.9	58.2	
Fin					89.3	51.7	58.8	29.0	53.4	
Bel						66.2	73.7	37.7	67.8	
Ita							78.6	43.1	59.6	
Spa								49.3	73.8	
Por									48.3	

quarterly frequency, i.e., the one-year ahead forecasts of inflation and real GDP growth (Eq. (3) with the forecast horizon $\bar{\tau}$ equal to 1).

In the second step, we keep fixed the parameters estimated for the stochastic processes of the global factors, i.e., the eurozone instantaneous expected inflation rate π and expected output growth rate μ (in Eq. (1)) and the related price level p and real production output q (Eq. (2)) and fit, separately for each country, the ten country-specific variables with monthly frequency, i.e., the 2- to 10-year yields and the 10-year realized yield variance.

In the econometric specification, the observation equations are written by adding normally distributed and homoskedastic error terms to the model-implied expressions for the yields, the yield variance and the one-year ahead forecasts of inflation and real GDP growth. The state equations are given by the first-order Vector AutoRegressive model derived from the continuous-time process in Eq. (1).

4.2. Estimation output

Panel A of Table 3 reports statistics on the goodness of fit of the model, where errors are defined as the difference between model estimates and actual values. The standard deviation of yield errors, across maturities from 2 to 10 years, ranges from 4.4 bps (Netherlands) to 18.1 bps (Portugal). The estimates of the 10-year yield volatility are very precise for all countries, with the standard deviation of estimation errors below 1 bp for most countries. The standard deviation of estimation errors for the 1-year-ahead inflation rate and GDP growth rate in the euro area is equal to 2.1 and 8.6 bps, respectively.

In addition, the panel reports estimated values for the maximal attainable Sharpe ratio (see Duffee (2010)), computed as $\sqrt{\Psi_t' \Psi_t}$, which can be interpreted as a diagnostic on the specification of the stochastic discount factor. We obtain reasonable estimates of this statistic for all countries, with average values ranging from 1.14 (Italy) to 3.60 (Ireland).

The Kalman filter technique allows us to obtain an estimate for the latent state variables. We find that the factors ℓ and s behave as

Table 3

Statistics on Estimation Outputs

This table reports some statistics on fitting errors and estimated state variables and parameters. Panel A reports the standard deviation (S.D.) of the estimation errors of (i) yields with maturities from 2 to 10 years, (ii) the 10-year yield volatility and, only for the euro area, (iii) 1-year-ahead expectations of inflation and real GDP growth rates. The panel also reports the average maximal Sharpe ratio calculated as $\sqrt{\Psi_t' \Psi_t}$. All values are expressed in basis points. In Panel B, a principal components analysis is applied, for each country, to monthly changes in yields (with maturities from 2 to 10 years) and monthly changes in yield variances (with maturities from 2 to 10 years). Then the correlation between the following variables is calculated: (i) monthly changes in the estimated state variable ℓ and the first principal component of yield changes (a proxy for the level of yields); (ii) monthly changes in the estimated state variable s and the second principal component of yield changes (a proxy for the slope of the yield curve); and (iii) monthly changes in the estimated state variable v and the first principal component of yield variance changes (a proxy for the level of yield variance). All values are expressed in percentage terms. The sample period is January 2000 to September 2022. Panel C shows, for each parameter, the average estimated value calculated across the ten countries and the euro area, and the corresponding standard error (in parenthesis).

Panel A: Fitting errors											
	Ger	Fra	Net	Aus	Fin	Bel	Ita	Spa	Por	Ire	E.A.
S.D. errors yields	5.30	5.03	4.41	5.39	5.36	6.37	6.49	6.27	18.12	11.61	4.31
S.D. errors yield volatility	0.59	1.17	0.37	0.39	2.43	0.46	0.64	0.78	1.97	0.78	0.31
S.D. errors inflation exp.											2.10
S.D. errors real GDP exp.											8.57
Avg max. Sharpe ratio	2.63	1.53	2.75	2.79	3.03	3.21	1.14	2.88	2.28	3.60	0.85
Panel B: Correlation between state variables and principal components											
	Ger	Fra	Net	Aus	Fin	Bel	Ita	Spa	Por	Ire	E.A.
ℓ – first PC yields	87.2	95.2	93.5	86.7	68.5	88.8	89.4	86.3	69.1	84.0	93.7
s – second PC yields	93.1	97.1	97.4	90.6	97.5	91.7	91.6	89.3	44.0	88.9	98.2
v – first PC yield variances	90.0	89.4	83.1	92.1	79.5	95.5	82.5	80.5	91.5	88.9	93.7
Panel C: Average estimated parameters											
	K				Θ		$\Sigma_{(ii)}$		$\Sigma_{M(ii)}$		
0.2445 (0.0270)	0 –	0 –	0 –	0 –	0.0038 (0.0030)	0.0635 (0.0052)	0.0019 –	0.0019 –	0.0019 –	0.0019 –	0.0019 –
0.0823 (0.0171)	0.1356 (0.0077)	0.0789 (0.0132)	-0.1207 (0.0149)	-0.0257 (0.0054)	0 –	0.2373 (0.0097)	0.0061 –	0.0061 –	0.0061 –	0.0061 –	0.0061 –
0.0378 (0.0070)	-0.0090 (0.0207)	0.3177 (0.0237)	-0.1012 (0.0065)	-0.1533 (0.0219)	0 –	0.1226 (0.0095)	0.0099 –	0.0099 –	0.0099 –	0.0099 –	0.0099 –
-0.0019 –	0.0542 –	0.0988 –	0.4265 –	0.0427 –	0.0214 –	0.0101 –	0.0101 –	0.0101 –	0.0101 –	0.0101 –	0.0101 –
0.0088 –	0.0312 –	0.1609 –	0.0332 –	0.6246 –	0.0257 –	0.0099 –	0.0099 –	0.0099 –	0.0099 –	0.0099 –	0.0099 –
0 –	0 –	Λ_1 0	0 –	0 –	Λ_0 -0.0515 (0.0068)	δ_0 0.0296 (0.0028)	δ_1 0 –	δ_1 0 –	δ_1 0 –	δ_1 0 –	δ_1 0 –
-0.0211 (0.0069)	-0.3618 (0.0200)	0.0947 (0.0239)	0.3590 (0.0689)	0.0425 (0.0247)	-0.0278 (0.0039)	0.0325 (0.0081)	0.0325 –	0.0325 –	0.0325 –	0.0325 –	0.0325 –
-0.0047 (0.0023)	-0.0494 (0.0067)	-0.1315 (0.0049)	0.8385 (0.0990)	1.1479 (0.2037)	0.0325 (0.0081)	0.0325 (0.0081)	0.0325 –	0.0325 –	0.0325 –	0.0325 –	0.0325 –
-0.0026 –	-0.0098 –	-0.0050 –	-0.0018 –	-0.0037 –	0.0001 –	0.0001 –	0.0001 –	0.0001 –	0.0001 –	0.0001 –	0.0001 –
-0.0008 –	-0.0027 –	-0.0013 –	-0.0049 –	-0.0007 –	0.0002 –	0.0002 –	0.0002 –	0.0002 –	0.0002 –	0.0002 –	0.0002 –

the “level” and the “slope” of the yield curve, respectively, with the “slope” defined as the difference between a short rate and a long rate. Indeed, Panel B of Table 3 shows that the correlation between monthly changes in ℓ and the first PC of monthly changes in the 2- to 10-year yields is very high for each country (on average, 86 %), and so is the correlation between monthly changes in s and the second PC of monthly changes in yields (on average, 89 %). The panel also shows that the estimated latent factor v is a good proxy for the level of the yield variance since, for each country, the correlation between monthly changes in v and the first PC of monthly changes in the 2- to 10-year realized yield variances is, on average, equal to 88 %. The estimated state variables π and μ are almost perfectly correlated with the observed 1-year-ahead expected inflation rate and GDP growth rate of the euro area.

Panel C of Table 3 contains the average value of each parameter, calculated across the ten countries and the euro area.⁶ We find that the level factor ℓ is relatively persistent, while the slope factor s is mean-reverting (average mean reversion coefficients equal to 0.14 and 0.32, respectively). The average mean reversion coefficient of the variance factor v is 0.24, while the latent macro factors π and μ exhibit a substantially higher mean-reverting behavior, with coefficients equal to 0.43 and 0.62, respectively.

The estimated average coefficients in matrix Σ imply that ℓ and s are more sensitive to innovations in v than v itself, while the (Gaussian) volatility coefficients for π and μ are both around 100 bps. The diagonal elements of matrix Λ_1 are negative, with the

⁶ For brevity, we do not report a separate table of the estimated parameters for each country.

coefficient on the level factor ℓ being about three times that on the slope factor s .

4.3. Yield components

Table 4 collects the results for the decomposition of the τ -maturity yield, $\tau = 2, 5, 10$ years, into the expected short rate $ES(\tau)$, the term premium $TP(\tau)$ and the convexity term $CX(\tau)$ (see Eq. (13)).

We find that the average term structure of ES is upward sloping for both low-rate countries and high-rate countries, with an average spread between the 10- and 2-year maturities around 110 bps. The volatility of ES is considerably lower at the 10-year than at the 2-year maturity, with a ratio ranging between 40 % (Ireland) and 65 % (Finland).

The average term structure of TP is also upward sloping for all countries, but with significant differences between low-rate and high-rate countries. In particular, the average 10-year TP of low-rate countries is negative, ranging from −16 bps (Germany) to −2 bps (Belgium), while in the case of high-rate countries the average 10-year TP is positive and ranges between 23 bps (Spain) and 93 bps (Portugal). We also observe that the volatility of estimated term premia is substantially higher for high-rate countries. The average 10-year CX is around −7 bps for all low-rate countries and about the double for Italy, Spain and Ireland, while for Portugal it reaches −26 bps.

Fig. 2 contains the time series estimates of the yield components at the 10-year maturity for all countries. It shows a marked downward trend for both ES and TP in the 2000–2020 sample period and a strong rebound in 2021–2022.

Panels A to F show that there are no significant differences between ES estimated for low-rate countries in the period between 2000 and 2008. However, with the GFC and the subsequent ECB quantitative easing program, ES in Germany becomes lower (on average, by about 20 bps) than the one estimated for the other low-rate countries. The minimum level of the series is reached during the pandemic shock in spring 2020, with values around only 60 bps. TP averages around 70 bps in the first part of the sample, decreases to zero in the period 2005–2007, and then bounces back to over 100 bps at the peak of the GFC. The subsequent downward movement is temporarily interrupted by the European sovereign bond crisis in 2010–2012, but after 2012, TP in low-rate countries becomes persistently negative and, apart from a couple of episodes – such as, for example, the sharp increase in correspondence with “Brexit” in 2016 – follows a downward trend until 2020. The minimum level of about −200 bps experienced in the last quarter of 2020 is followed by a marked rebound towards the zero level in 2021–2022. The sharp increase in both ES and TP in the final part of the sample is the effect of the sudden surge in inflation, that reaches unprecedented levels in the euro area, and the consequent tightening of the ECB monetary policy.

Panels G to J report time series estimates for the 10-year maturity ES , TP and CX in high-rate countries. We notice that in the 2000–2008 period all three components are similar for the four countries and fluctuate around the same values observed for low-rate countries. TP increases substantially with the outbreak of the GFC, especially in Portugal and Ireland, and reaches very high values during the European sovereign debt crisis between 2010 and 2012, with peak values around 250 bps for Italy and Spain, 480 bps for Ireland and above 800 bps for Portugal. In the same period, ES reaches almost 6 % for Italy and Spain and exceeds 8 % for Ireland and Portugal.

After the “whatever it takes” speech by ECB President Mario Draghi in July 2012 and with the implementation of the Asset Purchase Programme (APP), both ES and TP sharply decrease for high-rate countries.⁷ TP becomes negative in 2015 and, with the exception of Italy in 2018 and 2020,⁸ it remains below zero and reaches a minimum for all countries in November 2020, with a range between −203 bps (Ireland) and −127 bps (Italy). As in the case of low-rate countries, the rise in actual and expected inflation and the end of the ECB’s zero interest rate policy lead to a sharp increase in term premia in the final period of the sample.

The fact that the ten countries have in common a central bank (the ECB) implementing the monetary policy implies that, apart from differences (however small) in inflation expectations, expected short rates should be approximately equal in the various countries. Instead, during the 2010–2012 sovereign debt crisis, we observe significant differences between the estimated 10-year ES . This could be explained by the fact that in that period the creditworthiness of high-rate (and high-debt) countries deteriorates markedly and the market attributes to those countries a significant probability of default or exit from the eurozone. Fig. 3 reports, for the period from January 2008 to December 2014, the difference between the 10-year ES in Germany and in high-rate countries, i.e., Italy (Panel A), Spain (Panel B), Portugal (Panel C), and Ireland (Panel D), along with the corresponding difference in the 5-year CDS spreads. We find that, in all four cases, the two series display a strong correlation, which is equal, on average, to 0.94 in levels and 0.72 in monthly changes.

According to some empirical studies (see, among the others, Krishnamurthy et al., (2018) and Corradin et al., (2021)), term premia of eurozone government bonds are related to credit and liquidity risk premia. To test the influence of sovereign and liquidity risks on our TP estimates, we regress monthly changes in the 10-year TP on the corresponding monthly changes in the 5-year CDS spread and in the country-specific composite liquidity index developed by Poli and Taboga (2021). We run the regressions for two low-rate countries (Germany and France) and two high-rate countries (Italy and Spain) for the sample period from January 2010 to September 2022.⁹

Table 5 shows that credit risk is statistically significant only for high-rate countries, while liquidity risk has a similar impact on low-rate and high-rate countries. These results are in line with the evidence in Corradin et al., (2021), which shows that default risk premia

⁷ According to Eser et al. (2019), the APP has determined a persistent compression of about 100 bps in the eurozone 10-year term premium.

⁸ These two peaks can be explained by the results of the general elections in March 2018 and the outbreak of the pandemic in March 2020, which initially hit Italy above all.

⁹ The choice of the countries and the sample period are determined by the availability of data.

Table 4**Yield Components**

This table reports mean and standard deviation for the time series of estimated components of yields, i.e., expected short rate (ES), term premium (TP) and convexity (CX) with maturities 2, 5, and 10 years. The average value (Avg) is computed from the level of the variables, while the standard deviation (S.D.) is the annualised standard deviation of monthly changes in the variables. All values are expressed in basis points. The sample period is January 2000 to September 2022.

	Maturity Years	Germany		France		Netherlands		Austria		Finland	
		Avg	S.D.	Avg	S.D.	Avg	S.D.	Avg	S.D.	Avg	S.D.
ES	2	158	74	146	69	162	75	169	74	146	71
	5	211	59	200	57	220	61	227	60	201	58
	10	254	45	272	44	266	46	272	47	267	46
TP	2	-23	26	-1	8	-22	30	-21	28	-8	16
	5	-34	30	-6	18	-33	33	-30	33	-14	24
	10	-16	34	-6	32	-13	35	-7	38	-12	34
CX	2	0	1	0	0	0	1	0	1	0	0
	5	-2	4	-2	3	-2	3	-2	4	-2	3
	10	-7	12	-7	11	-7	9	-7	11	-7	10
	Maturity Years	Belgium		Italy		Spain		Portugal		Ireland	
		Avg	S.D.	Avg	S.D.	Avg	S.D.	Avg	S.D.	Avg	S.D.
ES	2	176	86	214	130	199	124	259	269	233	215
	5	235	68	290	99	267	93	311	188	281	141
	10	282	52	335	70	320	62	343	117	314	86
TP	2	-21	30	-7	38	-5	43	13	81	-14	37
	5	-26	34	-9	44	-8	46	45	137	-8	60
	10	-2	40	33	60	23	54	93	168	39	100
CX	2	-1	1	-1	3	-1	3	-2	7	-1	6
	5	-3	7	-5	17	-4	17	-10	34	-5	31
	10	-8	20	-15	54	-14	55	-26	92	-15	95
	Maturity Years	Euro Area									
		Avg	S.D.								
ES	2	194	78								
	5	253	61								
	10	291	45								
TP	2	-24	28								
	5	-27	32								
	10	6	34								
CX	2	0	0								
	5	-2	2								
	10	-6	7								

explain most of the yield variation in Italy and Spain, but are much less important sources of variation for yields in Germany and France. They are also consistent with the literature on the effects of the ECB's APP on term premia (see, for example, [Krishnamurthy et al., \(2018\)](#), [Moessner \(2018\)](#) and [Altavilla et al., \(2021\)](#)), which argues that asset purchase announcements improve credit and liquidity conditions especially in countries with higher sovereign risk and thus contribute to reducing term premia of those countries more than those of countries with lower sovereign risk.

Our *TP* estimates for the euro area show a pattern and an average level which are very similar to those observed in [Dewachter et al., \(2016\)](#) and [Berardi and Plazzi \(2022\)](#), while they are comparable to those provided by the [Monfort et al., \(2017\)](#) model (as reported in [Cohen et al., \(2018\)](#)) only for the post-GFC period and are very different from the *TP* values in [Iskrev \(2018\)](#), which are much more volatile and high in level.¹⁰

The *TP* estimates for France and Germany generated by our model are close to those in [Hordahl and Tristani \(2014\)](#) (as reported in [Cohen et al., \(2018\)](#)) and [McCoy \(2019\)](#), respectively, although the level of our *TP* is significantly lower in the first part of the sample (2000–2006) and moderately higher (less negative) in the 2015–2018 period. We also find the path of our estimated *TP* for Germany very similar to that in [Moench \(2019\)](#), but with a substantial difference in level of about 100 bps.

In [Chadha and Hantzsche \(2018\)](#) are reported *TP* estimates for all the ten countries in our sample that are based on the estimation of the [Adrian et al., \(2013\)](#) (ACM) model. Such estimates are also used and updated in [Moessner \(2018\)](#) and [Moessner and de Haan \(2022\)](#). We find that while the paths of the time series are not too dissimilar to ours, the level and volatility of *TP* in these studies are very high, with an average difference of about 150 bps for low-rate countries and around 250 bps for high-rate countries. Interestingly, *TP* for Germany in [Chadha and Hantzsche \(2018\)](#) is over-estimated by about 50 bps also with respect to [Moench \(2019\)](#),

¹⁰ The *TP* estimates in [Dewachter et al. \(2016\)](#), [Berardi and Plazzi \(2022\)](#) and [Cohen et al. \(2018\)](#) are based on euro swap rates and OIS rates, while those in [Iskrev \(2018\)](#) are based on the curves for AAA-rated eurozone sovereign bonds provided by the ECB.

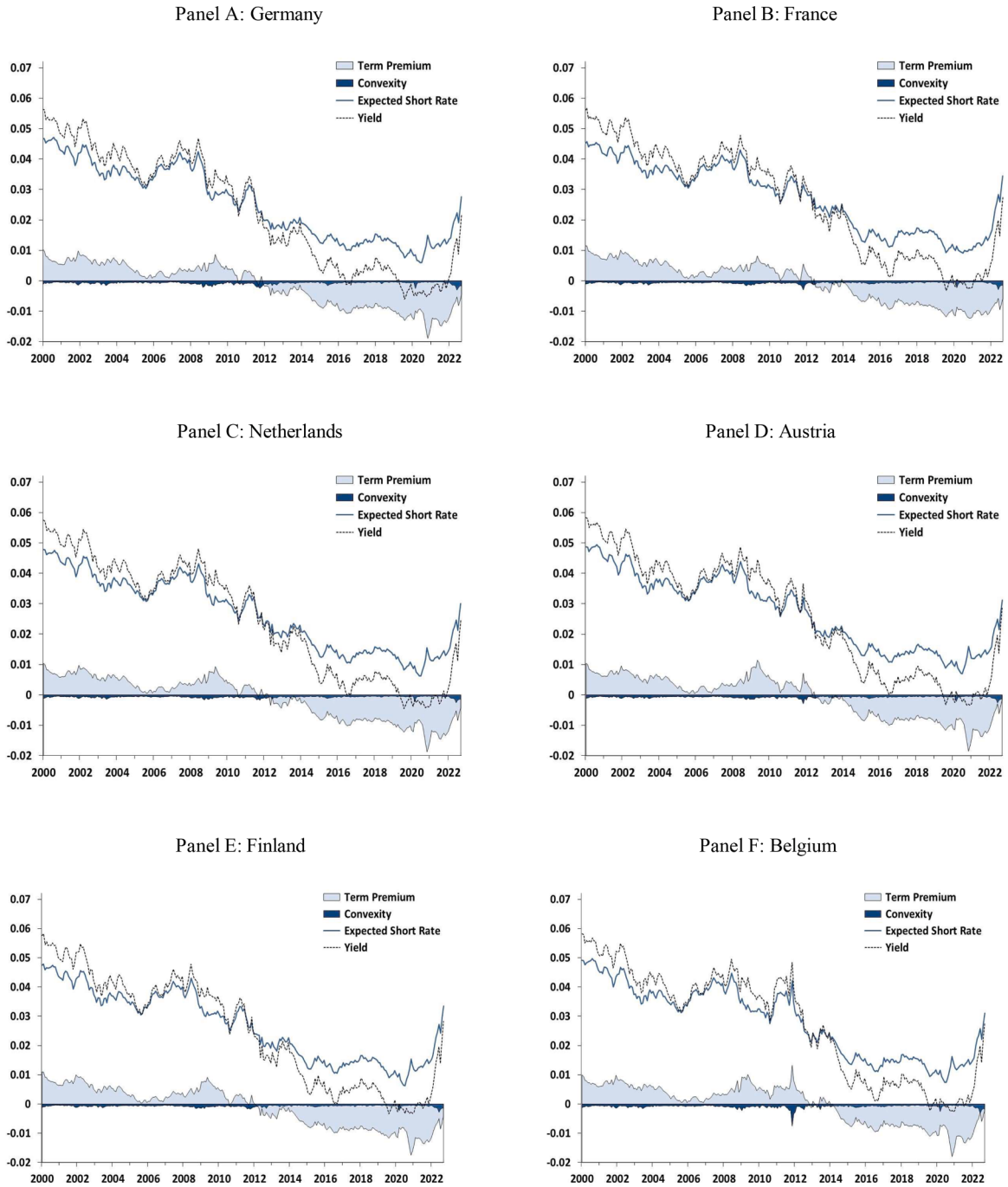


Fig. 2. Decomposition of Yields

This figure shows, for each country, the decomposition of the 10-year yield in its three components, i.e., expected short rate, term premium and convexity. The sample period is January 2000 to September 2022.

although both estimates are based on the ACM model.

As a robustness check, in [Appendix A](#) we compare our estimates of expected short rates and term premia for the euro area with those provided by a standard 5-factor Gaussian model estimated on our data sample. We find that, on average, the differences are small, but change significantly over time and become marked during crisis periods. We also notice that the Gaussian model produces expected short rates that are substantially less volatile than those of our model and, vice versa, term premia that are too volatile. This result is an effect of the fact that constant volatility models do not allow to separate the (time-varying) convexity effect, which can be substantial at long maturities, from the term premium.

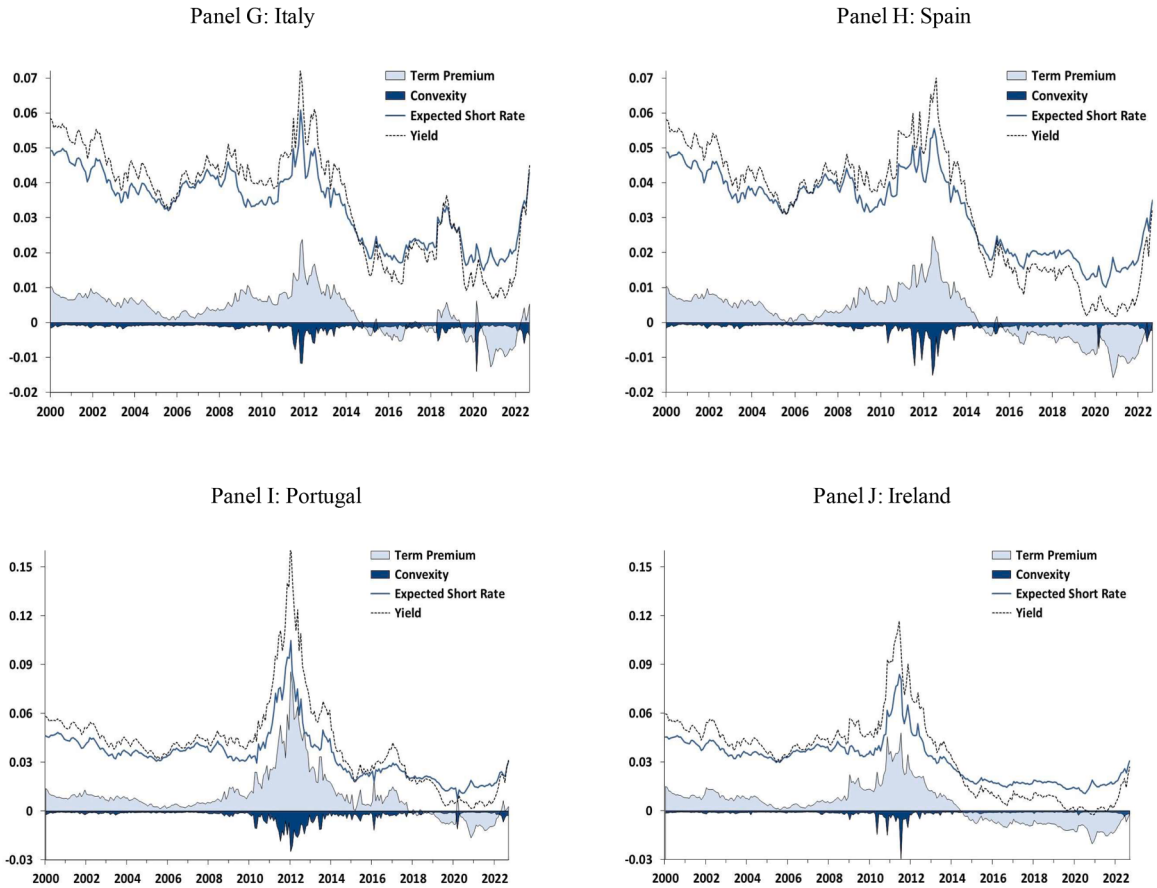


Fig. 2. (continued).

In fact, Fig. 2 shows that CX – which is proportional to yield variance – represents a significant component of long-term yields during turbulent times. In low-rate countries, the 10-year CX exceeds 20 bps, in absolute value, in correspondence with the four spikes in volatility in 2008 (GFC), 2012 (sovereign debt crisis), 2020 (pandemic shock) and 2022 (global energy crisis), while in high-rate countries it is well above 50 bps on several other occasions, and consistently during the debt crisis in 2010–2012 when it reaches almost 300 bps in Portugal and Ireland.

While the term structure of CX is, by definition, always (negatively) increasing with maturity, the slope of the term structure of ES and TP between the 2- and 10-year maturities changes significantly over the 2000–2022 sample period. This result is shown in Panels A and B of Fig. 4, which report, respectively, the time series of ES and TP at different maturities for the euro area as a whole.¹¹ We observe that the term structure of ES is relatively flat in the first part of the sample, becomes positively sloped since the GFC in 2008 and maintains a steep upward slope until the end of the sample period. Differently, TP increases with maturity between 2000 and 2014 and then, when it becomes negative, exhibits a U-shaped, sometimes inverted, term structure. Panel C shows that CX at the 10-year maturity is more than three times higher (in absolute terms) and substantially more volatile than that at the 5-year maturity.

4.4. Yield components and factor innovations

In this section, we look closer at the relationship between yield components and underlying state variables, i.e., the country-specific factors v , ℓ and s , and the global macro factors π and μ . In an essentially affine model, movements in the expected short rate, term premium and convexity depend on changes in the state variables Eqs. (8) and (13) and thus we can use model estimates to calculate the dynamic responses of yield components to factor innovations.

Panel A of Fig. 5 reports the instantaneous responses of monthly changes in the τ -maturity yield components, $\tau = 1, \dots, 10$ years, to innovations to the orthogonalized and normalized changes in the five state variables for Germany and Italy.¹²

¹¹ For brevity, we do not report the equivalent graphs for each country, as the results are qualitatively very similar.

¹² For the sake of space, we do not include the impulse responses for all countries, since the results for low- and high-rate countries are qualitatively similar to those for Germany and Italy, respectively.

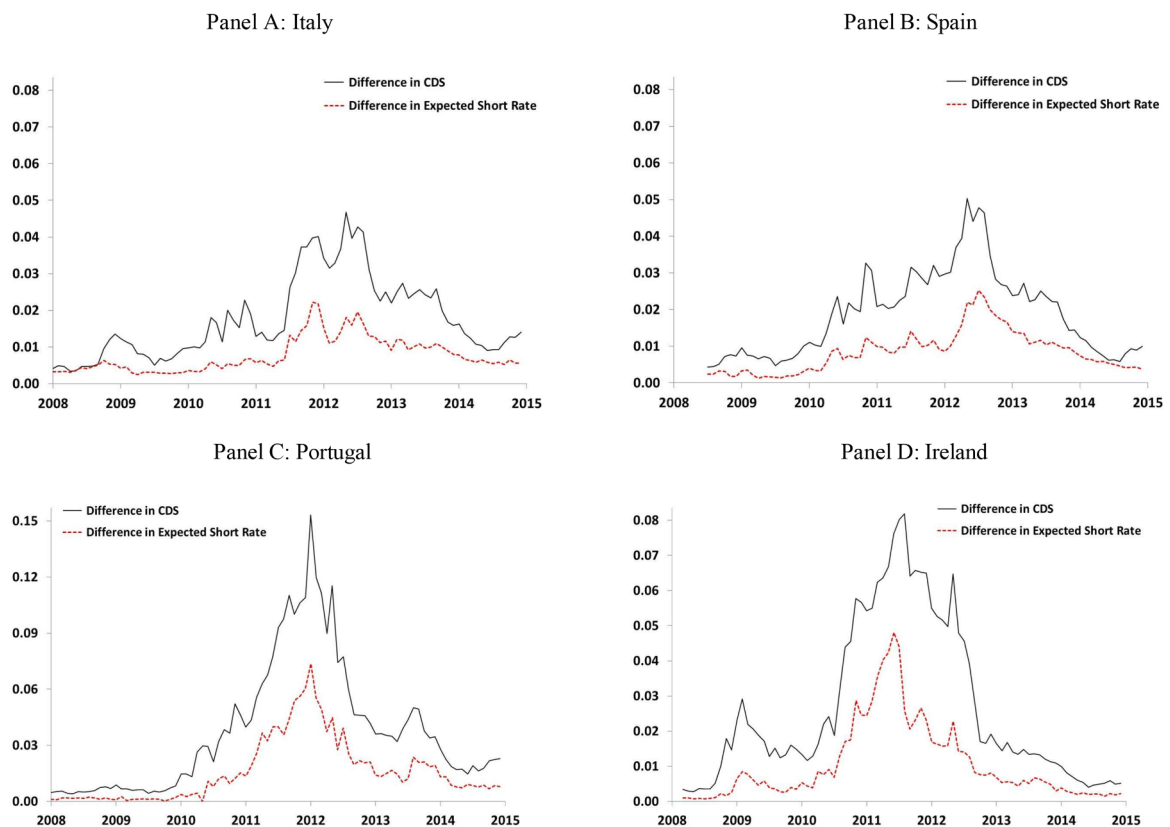


Fig. 3. Differences in Expected Short Rates and Credit Spreads

This figure shows the time series of the differences in expected short rates and market CDS spreads for high-rate countries and Germany. The figure covers the period January 2008 to December 2014, which is centered on the 2010–2012 sovereign debt crisis. Panel A reports the time series for Italy, while Panels B to D for Spain, Portugal (different scale) and Ireland, respectively.

Table 5

Term Premia vs. Credit and Liquidity Risk

This table reports the results of the regression of monthly changes in the estimated 10-year term premium on monthly changes in the 5-year CDS spread and a country-specific composite liquidity index (see [Poli and Taboga \(2021\)](#)). Panel A contains the results for two low-rate countries (Germany and France), while Panel B for two high-rate countries (Italy and Spain). Newey-West adjusted standard errors in parentheses. The sample period is January 2010 to September 2022.

Panel A: Low-rate countries					
	Germany			France	
CDS spread	-0.216 (0.123)	-0.039 (0.127)		-0.129 (0.077)	0.050 (0.075)
Liquidity index	0.189 (0.038)		0.169 (0.037)	0.130 (0.024)	0.112 (0.022)
R ²	0.14	0.00	0.13	0.16	0.15
Panel B: High-rate countries					
	Italy			Spain	
CDS spread	0.099 (0.036)	0.280 (0.047)		0.083 (0.039)	0.229 (0.046)
Liquidity index	0.151 (0.012)		0.165 (0.012)	0.120 (0.012)	0.130 (0.012)
R ²	0.60	0.19	0.58	0.48	0.46

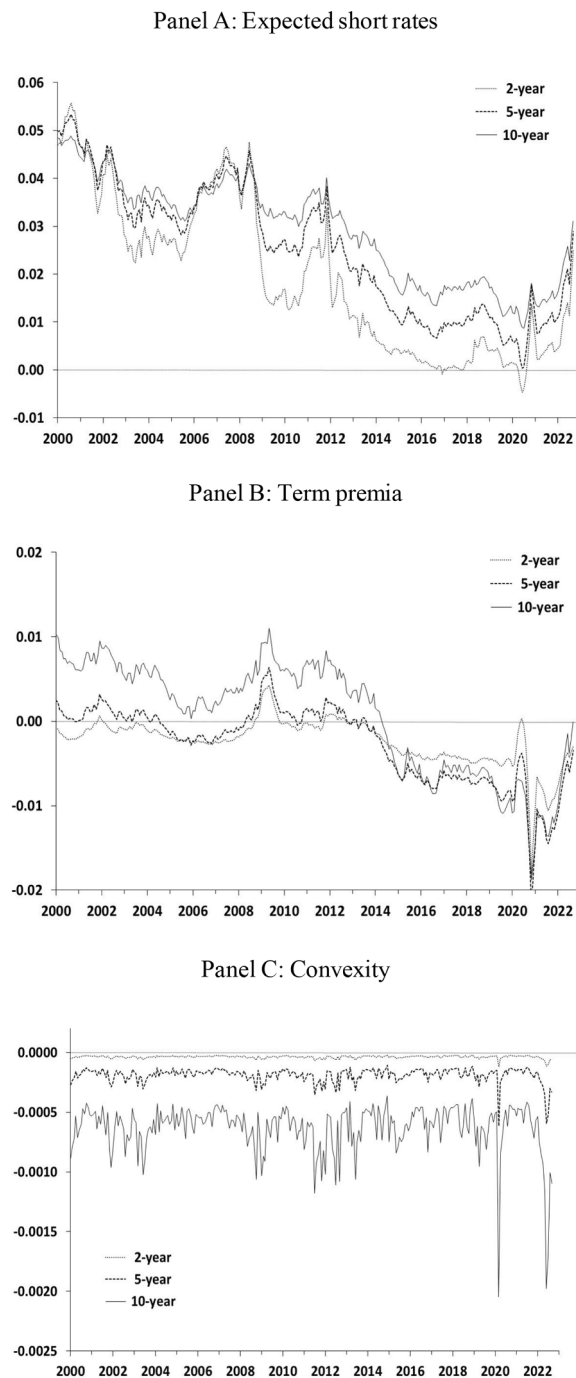
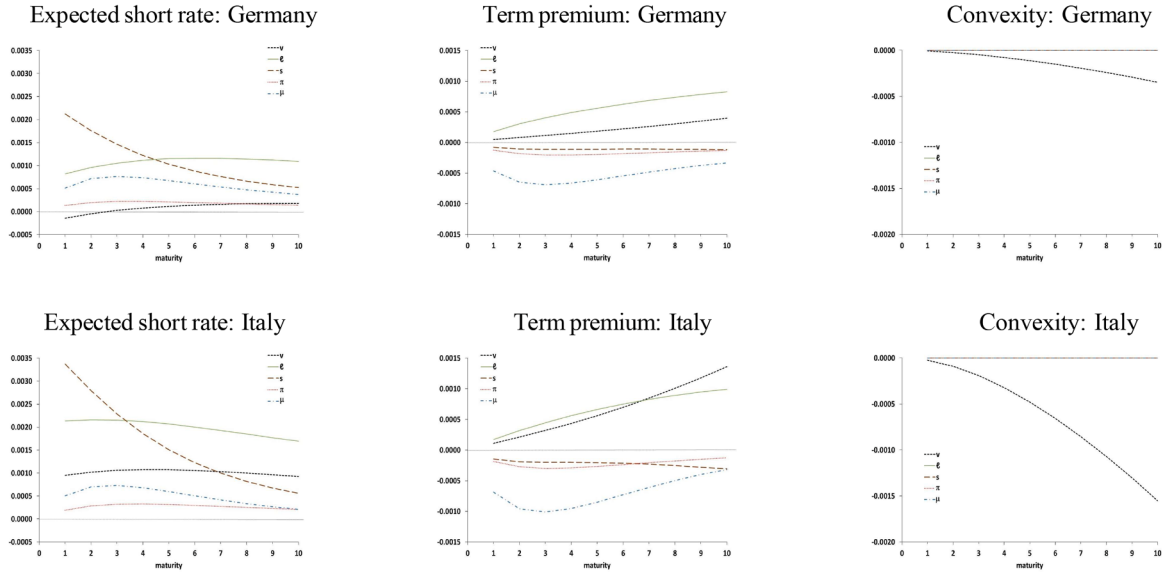


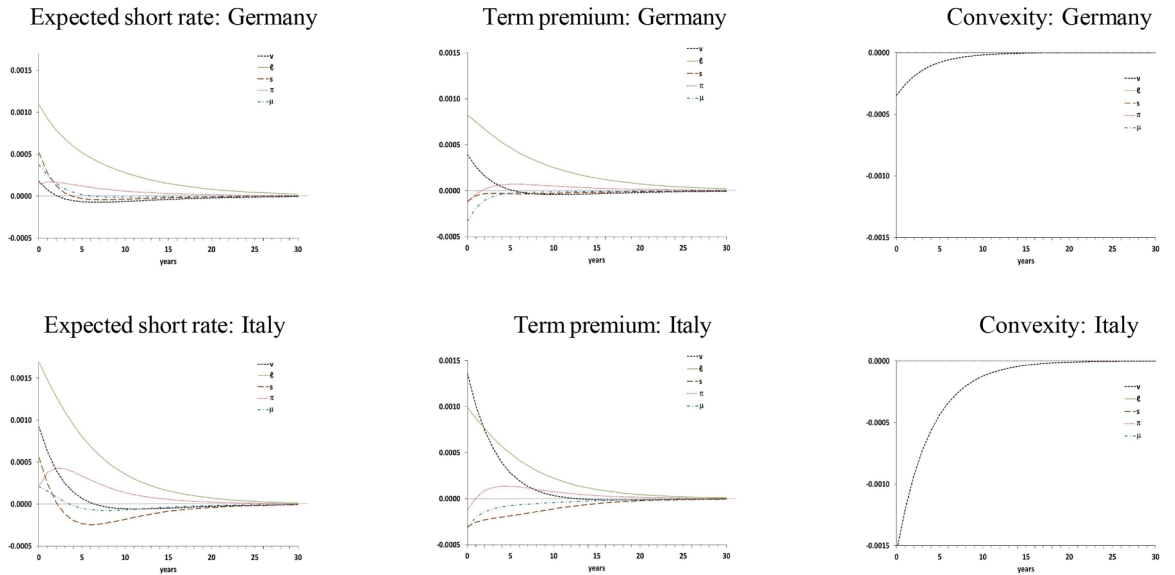
Fig. 4. The Term Structure of Yield Components in the Euro Area

This figure shows the time series behavior of the term structure of yield components in the euro area. Panel A contains the time series of the expected short rate at the 2-, 5-, and 10-year maturities, while Panels B and C report the time series of the term premium and the convexity, respectively, at the same maturities. The sample period is January 2000 to September 2022.

Panel A: Responses of yield components to shocks in the factors



Panel B: Impulse-response function for 10-year yield components

**Fig. 5.** Impulse Responses of Yield Components to Factor Innovations

This figure shows the dynamic responses of yield components to innovations in the five latent factors (i.e., the country-specific factors v , ℓ and s , and the global macro factors π and μ) for Germany and Italy. Panel A reports the instantaneous responses of monthly changes in τ -maturity expected short rates, term premia and convexities, with $\tau = 1, \dots, 10$ years, to innovations to the orthogonalized and standardized monthly changes in the five state variables. Panel B shows the dynamic responses of the 10-year expected short rate, term premium and convexity to a one-standard-deviation shock in each standardized state variable.

We observe that for both countries the expected short rate (ES) at any maturity is positively influenced by shocks in the level factor ℓ and in the slope factor s , with significantly higher sensitivities (nearly double) for Italy. The variance factor v plays a significant role only in the case of Italy and shows a constant impact on all maturities, while innovations to the global macro factors π and μ have positive effects on ES for both countries, with a hump-like structure that peaks around the 3-year maturity.

The sensitivities of the term premium (TP) with respect to shocks in the variance factor v and the level factor ℓ are positive, increase substantially with maturity, and are stronger for high-rate countries (Italy). Shocks to the slope factor s (which is defined as short rate minus long rate, i.e., as *minus* the slope of the yield curve) have a negative impact on TP that is almost flat across maturities. Therefore, increases in the level and volatility of interest rates lead to a rise in TP especially on long maturities, while a steeper yield curve causes TP to rise over all horizons.

Shocks to the global macro factors π and μ negatively affect TP , with U-shaped functions reaching their maximum level, in absolute terms, between the 2- and 3-year maturities. We estimate that the response to innovations in output growth is almost three times stronger than that to innovations in expected inflation.

Overall, for both low-rate and high-rate countries, we observe that increases in eurozone expected inflation and output growth push ES higher and TP lower at all maturities, with offsetting effects.

Finally, we find that, as expected, only innovations to the variance factor v affect convexity (CX), with the size of the effect increasing (non-linearly) with maturity and being considerably larger for Italy (i.e., high-rate countries).

Panel B of Fig. 5 contains the impulse responses of the 10-year yield components to the five structural shocks in our model. Again, we report only results for Germany and Italy, which are a good proxy for low-rate and high-rate countries, respectively. Specifically, we measure the reaction of the 10-year ES , TP and CX to a positive one-standard-deviation shock to each of the five factors (v , ℓ , s , π , μ).

We find shocks in the level factor ℓ to be very persistent and the most important drivers of ES in both countries, with a one-standard-deviation shock in ℓ leading to a short-term increase in ES of about 11 and 17 bps for Germany and Italy, respectively. The instantaneous response of TP to the shock in ℓ amounts to 8 bps for Germany and 10 bps for Italy.

Innovations to the variance factor v have significant short-term effects on all three yield components. In the case of Italy – and, in general, high-rate countries – shocks in v are the main determinants of TP , with an instantaneous positive response of about 14 bps and half-life around two years.

The effects of innovations to the slope factor s are different in the two countries, as they tend to quickly disappear for Germany and be relatively persistent for Italy, with a positive shock to the slope of the yield curve (i.e., a negative shock to s) leading to a short-term increase of 3 bps in TP and decrease of 5 bps in ES .

As regards the global macro factors, we observe that shocks in expected output growth (μ) lead to an increase in ES and a decrease in TP and, interestingly, these reactions are significantly stronger for Germany (low-rate countries) than for Italy (high-rate countries). On the contrary, innovations to expected inflation (π) determine stronger responses for low-rate countries. These innovations are relatively persistent and display a hump-like function with a peak between two and three years, both for ES and TP . Shocks in π positively affect ES , while the response of TP is initially negative and then becomes positive at horizons longer than one year. Therefore, in the long run, positive shocks to inflation expectations determine a rise in TP , which is more significant for high-rate countries.

4.5. Connectedness analysis

In this section, we analyze the degree of connectedness between yields and yield components in the ten countries.

First, we analyze the contemporaneous cross-country correlation in the estimated components of the 10-year yield. Panels A and B of Table 6 show that monthly changes in the 10-year ES and TP are highly correlated within low-rate countries (average correlation around 90 %), while the correlation within high-rate countries is lower (54 %, on average, for ES and 49 % for TP), with a significant exception being the correlation between Italy and Spain. Panel C contains the corresponding cross-correlation for monthly changes in the convexity component of the 10-year yield and shows values around 76 %, on average, within low-rate countries and only 41 % within high-rate countries. The average cross-correlation between low- and high-rate countries ranges from 46 % (ES) to 50 % (TP and CX) for all three components.

Next, we consider the lead-lag dimension of the relation between yield components in the ten countries by calculating different measures of connectedness based on the Diebold and Yilmaz (2012 and 2014) variance decomposition methodology (see Appendix B for a brief description).

We first apply a static full-sample analysis and compute the pairwise directional connectedness for monthly changes in the 10-year yield and each of its components (ES , TP and CX) over the full 2000–2022 sample period. These are the coefficients $d_{ij}, i = 1, \dots, 10, i \neq j$, in Eq. (B.2), which are then used to obtain the total directional connectedness to other countries from the i th country ($D_i, i = 1, \dots, 10$, in Eq. (B.3)) and the total directional connectedness from other countries to the i th country ($D_i, i = 1, \dots, 10$, in Eq. (B.4)). Finally, we take the difference between these two values and derive the *net* total directional connectedness for each country ($D_i^*, i = 1, \dots, 10$, in Eq. (B.5)).

Table 7 (rows 1 to 10) shows that low-rate countries appear to be “exporters” of shocks with regard to yields and their components, with Austria and Belgium being the main transmitters. On the opposite, high-rate countries – in particular, Portugal – are mainly “importers” of shocks.

The last row of the table reports the total connectedness measure for monthly changes in the 10-year yield and its components ($D(H)$ in Eq. (B.6)) over the 2000–2022 sample period. We find a total connectedness around 80 % for yield, ES and TP , which means

Table 6**Correlation in Yield Components**

This table reports the cross-country correlation in the estimated components of the 10-year yield. Panel A contains the cross-correlation in monthly changes of the 10-year expected short rate. Panels B and C show the same correlation for monthly changes in the 10-year term premium and convexity, respectively. All values are expressed in percentage terms. The sample period is January 2000 to September 2022.

Panel A: Expected short rate									
	Fra	Net	Aus	Fin	Bel	Ita	Spa	Por	Ire
Ger	90.8	96.2	93.6	96.2	85.8	44.4	55.9	21.1	43.3
Fra		92.2	91.9	93.1	87.6	53.6	60.6	23.5	44.8
Net			94.5	96.6	87.3	49.0	59.9	15.3	44.1
Aus				94.9	91.3	56.0	63.1	26.6	49.0
Fin					88.6	51.4	60.5	19.4	44.4
Bel						66.6	74.6	26.9	58.6
Ita							83.1	35.5	58.5
Spa								33.6	69.6
Por									44.6
Panel B: Term premium									
	Fra	Net	Aus	Fin	Bel	Ita	Spa	Por	Ire
Ger	88.6	95.8	92.7	94.6	86.5	53.8	62.9	24.1	44.2
Fra		87.6	87.9	89.9	84.3	57.6	61.7	25.2	42.6
Net			91.9	94.3	85.1	55.4	66.1	23.3	43.1
Aus				92.5	92.5	60.4	62.5	38.1	47.1
Fin					87.0	56.7	63.1	28.8	42.6
Bel						65.7	67.4	39.9	59.1
Ita							79.2	38.0	48.7
Spa								31.4	54.8
Por									44.0
Panel C: Convexity									
	Fra	Net	Aus	Fin	Bel	Ita	Spa	Por	Ire
Ger	66.4	75.6	65.8	72.2	71.3	60.6	67.8	52.5	44.0
Fra		76.5	89.5	66.2	87.6	44.2	53.4	49.6	30.6
Net			76.1	86.7	79.8	79.5	65.7	44.0	36.9
Aus				63.7	87.6	38.2	50.5	51.7	24.6
Fin					70.2	81.8	56.2	34.8	42.5
Bel						48.5	54.8	52.9	32.0
Ita							60.0	30.1	40.2
Spa								70.1	29.8
Por									16.6

that about four-fifths of the variation in the variables in a country can be explained by shocks from other countries and only about one-fifth of the variation by domestic factors. This result is consistent with previous evidence at the international level (see Longstaff et al., (2011) and Moench (2019)) and, more specifically, for the euro area (see Claeys and Vasicek (2014)). Total connectedness for CX is also relatively strong, with a value of about 74 %.

The static full-sample analysis provides a picture of the links between countries. However, the degree of connectedness may change

Table 7**Net Directional Connectedness in Yield Components**

This table shows the net total directional connectedness in the 10-year expected short rate, term premium and convexity. The “net” total directional connectedness is calculated as the difference between the total directional connectedness “to” others and the total directional connectedness “from” others. Therefore, positive values of net connectedness indicate that the country is a net exporter of shocks in the yield, or yield component, for the other countries, while negative values mean that the country is a net importer of shocks from the other countries. The total connectedness measure at the bottom of the table is obtained as the average of all the “to” others (or “from” others) total directional connectedness values. All values are expressed in percentage terms and refer to the period January 2000 to September 2022.

	Yield	Exp. Short Rate	Term Premium	Convexity
Germany	4.97	8.34	8.54	4.29
France	13.53	9.74	4.25	17.25
Netherlands	9.19	10.33	10.33	3.17
Austria	15.94	15.88	15.16	13.95
Finland	9.96	11.59	9.92	5.57
Belgium	15.02	15.88	16.23	1.86
Italy	-15.64	-16.38	-11.70	5.76
Spain	-2.36	-4.01	-3.57	-8.80
Portugal	-33.15	-30.20	-28.98	-26.53
Ireland	-17.46	-21.17	-20.18	-16.52
Total Connectedness	81.20	78.97	79.02	73.69

significantly over time and, to study its evolution, we run a dynamic rolling-sample analysis. Panel A of Fig. 6 reports rolling estimates of the total connectedness measure for monthly changes in the 10-year yield and its estimated components. The rolling window comprises 36 months, which means that the first estimation window ranges from February 2000 to January 2003 and the last window from October 2019 to September 2022. The estimated degree of connectedness is high (close to 90 %) for all components in the first part of the sample and then decreases sharply, as a consequence of both the GFC and the European sovereign debt crisis. Since 2015, we observe a rebound of total connectedness above 80 % for *ES* and *TP*, another drop around 2018, and again an increase towards 90 % in the final part of the sample. This result is in contrast with [Claeys and Vasicek \(2014\)](#) reporting a total spillover index that rises after 2008, while it is consistent with the evidence in [Caporin et al., \(2018\)](#), which shows a divergent path of sovereign yields in the eurozone as a consequence of the GFC.¹³

In Panel B, we compare the total connectedness for monthly changes in the 10-year yield with a default risk measure, represented by the average difference between the 5-year CDS spreads of Italy and Germany observed in each 36-month window.¹⁴ We find that the

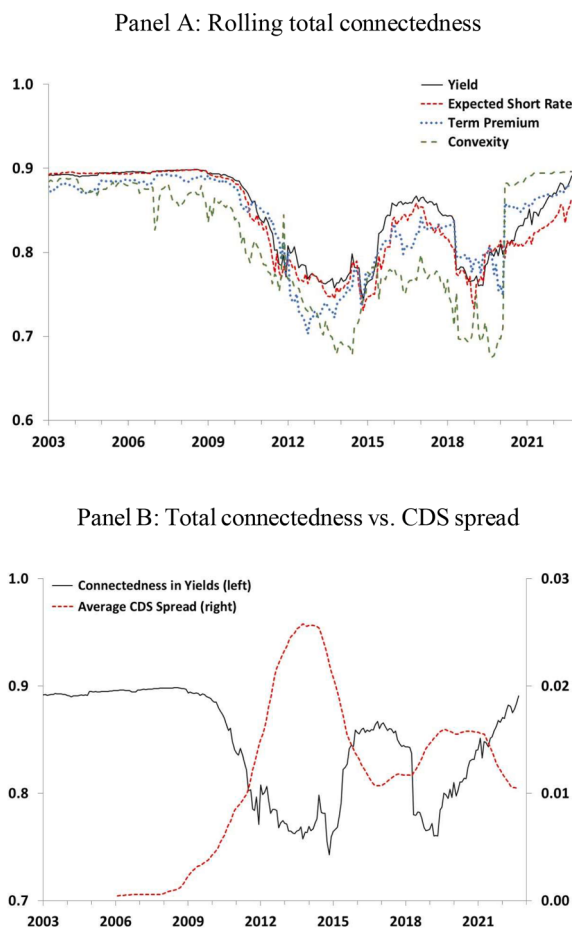


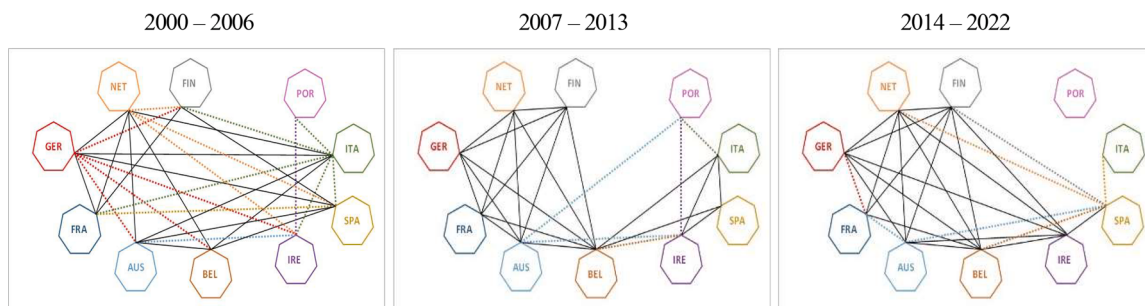
Fig. 6. Total Connectedness

This figure contains evidence on the total connectedness in yields and their components. Panel A reports rolling estimates of the total connectedness measure – defined as the average of all the “to” others (or “from” others) total directional connectedness values – for monthly changes in the 10-year yield and its estimated components. The size of the rolling window is 36 months, which means that the first estimation window ranges from February 2000 to January 2003 and the last window ranges from October 2019 to September 2022. Panel B compares the total connectedness measure for monthly changes in the 10-year yield (left axis) with the average difference between the 5-year CDS rates of Italy and Germany observed in each 36-month window (right axis). Since data on CDS rates start in March 2003, the first window ranges from March 2003 to February 2006.

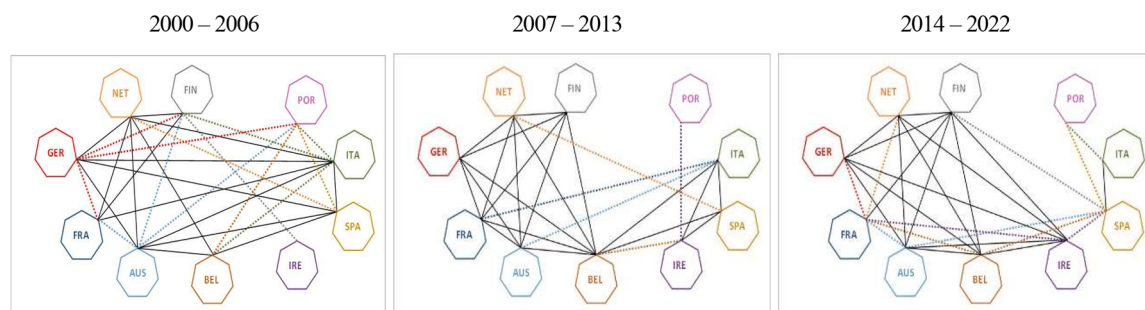
¹³ A similar result can also be observed in [Moench \(2019\)](#) for the expected short rates and term premia calculated from a large panel of both developed and emerging market sovereign bonds.

¹⁴ Since data on CDS spreads start in March 2003, the first window ranges from March 2003 to February 2006.

Panel A: Sub-samples pairwise directional connectedness for expected short rate



Panel B: Sub-samples pairwise directional connectedness for term premium

**Fig. 7.** Pairwise Directional Connectedness for Yield Components

This figure shows the network structure for monthly changes in the estimated components of the 10-year yield. Panel A reports the pairwise directional connectedness for the 10-year expected short rate calculated over three separate sub-sample periods: (i) January 2000 to December 2006, (ii) January 2007 to December 2013, and (iii) January 2014 to September 2022. Panel B contains the equivalent measure for the 10-year term premium. The lines in the figures represent the most important connections among the pairs of the ten euro area countries and correspond to values greater than the fiftieth percentile of all pairwise directional connections in the sample period under consideration. The black line indicates a mutual relation, i.e., that the connection is significant in both directions (for example, a black line between Germany and France denotes a significant directional connectedness “from” Germany “to” France, and vice versa). The dotted coloured line indicates a one-way direction, with the color equal to that of the “transmitting” country (for example, a red dotted line between Germany and France denotes a significant directional connectedness “from” Germany “to” France, but not in the opposite direction).

correlation between the two series is highly negative (correlation coefficient is -0.88) and this implies, again, that total connectedness in yields tends to be relatively high during stable periods and decrease in turbulent times.

To study the evolution of the cross-country relationships for *ES* and *TP*,¹⁵ in Fig. 7 we report the pairwise directional connectedness for the 10-year *ES* (Panel A) and *TP* (Panel B) computed for three non-overlapping sub-periods: (i) 2000–2006, (ii) 2007–2013 and (iii) 2014–2022. In the 2000–2006 sub-period, which is characterized by relatively stable spreads between yields and yield components in high-rate and low-rate countries, we observe statistically significant pairwise directional connectedness for *ES* and *TP* within low-rate countries and between low-rate countries and Italy and Spain. Italy and Spain are also mutually connected, while Portugal and Ireland do not affect other countries and rather receive shocks from them (Portugal for *TP* and Ireland for *ES*).

The 2007–2013 sub-period, which includes both the GFC and the European debt crisis, shows a clear dichotomy between low-rate and high-rate countries, as there seem to be only a few significant connections across the two groups for both *ES* and *TP*. A notable exception is the link between Belgium and high-rate countries and, in particular, the mutual dependence between Belgium, Italy and Spain. Claeys and Vasicek (2014) find an analogous result and explain it as an effect of the large exposures of the banks of those countries to the sovereign debt of the other two countries.¹⁶ Finally, we observe a relatively strong interdependence between *ES* and *TP* in the high-rate countries, a result that is consistent with some studies on the presence of contagion effects in that period (see, for example, Metiu (2012)).

In the period 2014–2022, when yields and their components are strongly influenced first by the quantitative easing of the ECB (see, for example, Altavilla et al., (2019) and Eser et al., (2019)) and then, in the last period, by the tightening of the ECB monetary policy, the pairwise connectedness from high-rate to low-rate countries is almost totally absent. A notable exception is represented by Ireland,

¹⁵ For brevity, we do not include the results for *CX*, which are less significant.

¹⁶ Similarly, Ang and Longstaff (2013) consider Belgium, Italy and Spain as systemic bond markets.

which becomes highly integrated with the network of low-rate countries. The other three high-rate countries, i.e., Italy, Spain and Portugal, appear to be interconnected in terms of *TP*, but not *ES*.

To sum up, our evidence shows that the degree of connectedness between expected short rates and term premia in the euro area varies substantially over time, can be very different for low-rate and high-rate countries, and tends to weaken during crisis periods.

5. Concluding remarks

Separating the expectation and risk premium components of yields is crucial for monetary authorities. We address this issue using a term structure model with stochastic volatility and global macro information and estimate time-varying term premia and short rate expectations for ten countries in the euro area over the 2000–2022 sample period.

The empirical analysis shows that term premia and expected short rates follow a downward trend over the sample period, but increase sharply in the last two years as a result of rising global inflation and central banks monetary policy tightening. We observe significant differences in the dynamics of the estimated yield components across countries, especially during turbulent times, such as the GFC, the European sovereign debt crisis and the outbreak of the pandemic.

We find that in all countries term premia are strongly related to yield volatility. Nonetheless, our term premia exhibit less variability than those obtained estimating Gaussian (constant volatility) models, because we explicitly disentangle from term premia the time-varying convexity effects, which substantially affect long-term yields and their components during high volatility periods.

The estimation of the responses of term premia and expected short rates to shocks in eurozone macro variables, which is allowed by the specific factor structure of our model, shows that term premia react negatively to shocks in global expected inflation and GDP growth, while expected short rates are positively influenced by those innovations.

The study of the cross-country connectedness between the yield components of the ten countries, carried out by means of a variance decomposition technique, reveals the existence of significant interconnections for both term premia and short rate expectations. However, the size of these links varies substantially across countries and over time. Departing from previous empirical evidence on the euro area, we observe that total connectedness in term premia and expected short rates is relatively strong in stable times and decreases in high volatility periods.

CRediT authorship contribution statement

Andrea Berardi: Conceptualization, Methodology, Validation, Formal analysis, Data curation, Writing – review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix

A. Comparison with estimates from a Gaussian model

In this appendix, we report the estimates of short rate expectations and term premia for the euro area obtained from a standard 5-factor Gaussian model (see Dai and Singleton (2000)). The model assumes, as in Adrian et al., (2013), that the state variables are the first five principal components of yields and is estimated using the two-step approach proposed by Joslin et al., (2011) (see also Wright (2011)). In particular, we first run a principal components analysis on yields, take the first five principal components as state variables, and estimate a vector autoregression (VAR) for the dynamics of the five factors to obtain the coefficients of their processes under the physical measure. We then keep these coefficients fixed and estimate the remaining parameters – i.e., those that determine the market price of risk – by imposing the cross-sectional no-arbitrage restrictions implied by the model. The estimation method is based on maximum likelihood, assuming that the differences between the observed yields and the corresponding model-implied values are i.i.d. measurement errors. The empirical analysis is for the euro area as a whole, and the sample period is January 2000 to September 2022.

Fig. A.1 reports the time series of the difference between the 10-year expected short rate (*ES*) and term premium (*TP*) estimated by the Gaussian model and those estimated by our model. We find that the Gaussian model significantly over-estimates *TP* (and under-estimates *ES*) in the period preceding the GFC and during the 2010–2012 sovereign debt crisis, while, on the contrary, in the final part of the sample the Gaussian model tends to substantially under-estimate *TP* (and over-estimate *ES*). Overall, the comparison shows that, with the exception of the peak in correspondence with the pandemic outbreak in 2020, the differences in the estimated *ES* and *TP* range in the $-70/+70$ bps interval and are on average close to zero.

However, when we consider the volatility of the original series of *ES* and *TP* generated by the two models, a relevant difference

emerges. Indeed, the standard deviation of *ES* estimated by the Gaussian model is substantially lower than that of the corresponding estimates provided by our model, and vice versa for *TP*. In particular, the volatility of *TP* is 51 % higher than that of our model, a result which is consistent with previous evidence on the excessive volatility of *TP* estimated by Gaussian models (see, for example, [Bauer et al., \(2014\)](#)). This result might be explained by the fact that constant volatility Gaussian models do not explicitly separate the convexity component, which is relevant at long maturities, and is thus mainly incorporated in the *TP* component of yields.

B. Connectedness measures

We calculate different measures of connectedness using the variance decomposition methodology of [Diebold and Yilmaz \(2012 and 2014\)](#).

For the 10-year yields and for the corresponding estimated components, we estimate a covariance stationary VAR(1) of the form $x_t = \Phi x_{t-1} + \varepsilon_t$, where x_t is a vector that contains the 10-year yields (or the 10-year expected short rates, term premia, convexities) of the ten countries and $\varepsilon_t \sim (0, \Gamma)$ is a vector of independently and identically distributed disturbances. The moving average representation is $x_t = \sum_{h=0}^{\infty} Q_h \varepsilon_{t-h}$, where the $(N \times N)$ matrices Q_h ($N = 10$ in our case) follow the recursion $Q_h = \Phi Q_{h-1}$, with Q_0 being the identity matrix and $Q_h = 0$ for $h < 0$.

The fraction of the *H*-step-ahead error variances in forecasting the variable x for the i th country that are due to shocks to the variable x for the j -th country, $i, j = 1, \dots, 10, i \neq j$, is computed as:

$$\tilde{d}_{ij}(H) = \frac{\gamma_{jj}^{-1} \sum_{h=0}^{H-1} (e_i' Q_h \Gamma e_j)^2}{\sum_{h=0}^{H-1} (e_i' Q_h \Gamma Q_h' e_i)} \quad (\text{B.1})$$

where γ_{jj} is the standard deviation of the error term for the j -th equation, e_i (e_j) is a vector with one as i th (j -th) element and zeros otherwise, and the denominator is the variance of the *H*-step-ahead forecast error. Since the variables are not orthogonalized, the sum of the entries of the variance decomposition, $\tilde{d}_{ij}(H)$, might be different from one. Therefore, we normalize each element by the sum of the elements:

$$d_{ij}(H) = \frac{\tilde{d}_{ij}(H)}{\sum_{j=1}^N \tilde{d}_{ij}(H)} \quad (\text{B.2})$$

The fractions $d_{ij}(H)$, $i \neq j$, define the “pairwise directional connectedness *from* the j -th country *to* the i th country” and, similarly, $d_{ji}(H)$, $j \neq i$, the “pairwise directional connectedness *from* the i th country *to* the j -th country”. Therefore, the sum

$$D_i(H) = \sum_{j=1, j \neq i}^N d_{ji}(H) \quad (\text{B.3})$$

indicates the “total directional connectedness *to* other countries *from* the i th country”, and the sum

$$D_i(H) = \sum_{j=1, j \neq i}^N d_{ij}(H) \quad (\text{B.4})$$

the “total directional connectedness *from* other countries *to* the i th country”. The difference between $D_i(H)$ and $D_i(H)$, i.e., the difference between the shocks transmitted to and the shocks received from all other countries, gives the “*net* total directional connectedness for the i th country”:

$$D_i^*(H) = D_i(H) - D_i(H) \quad (\text{B.5})$$

The “total connectedness” measure is obtained as the average of the *from* values $D_i(H)$ or the *to* values $D_i(H)$, $i = 1, \dots, N$:

$$D(H) = \frac{1}{N} \sum_{i=1}^N D_i(H) = \frac{1}{N} \sum_{i=1}^N D_i(H) \quad (\text{B.6})$$

In the empirical analysis, we set $H = 1$, i.e., a one-month forecast horizon. However, estimates obtained using a higher value for H produce very similar results.

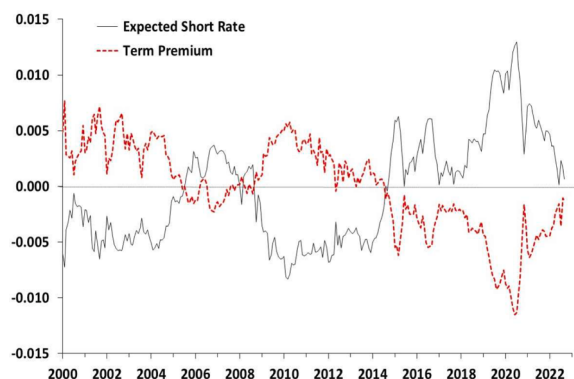


Fig. A.1. Comparison with Yield Components from a Gaussian Model

This figure reports the differences between the 10-year expected short rate and term premium in the euro area estimated by a 5-factor Gaussian model and by our model. The sample period is January 2000 to September 2022.

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