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Saving for retirement in Europe: the long-term risk-return tradeoff

Andrea Berardi[1] iD and Claudio Tebaldi[2]

[1]Department of Economics, Università Ca' Foscari, Venezia, Italy and [2]Department of Finance, Università Bocconi, Milano, Italy
Corresponding author: Andrea Berardi; Email: andrea.berardi@unive.it

Abstract
A comparison of the performances of pension products that ignores long-term trends might significantly overestimate the long-term impact of volatility risks while underestimating the impact of persistent, low-frequency trends. This paper proposes a comparison making use of projection models based on the long-term risk–return tradeoff proposed by Campbell and Viceira (2005) to explicitly take into account slow-moving economic trends. In order to illustrate the approach and its implications, we discuss the capital protection provided by life-cycle target-date fund strategies and minimum guarantee strategies.

1. Introduction

The process of accumulating wealth for retirement when young and transforming wealth into consumption when older involves relevant long-term financial commitments for most households. In a typical defined contribution plan, employees face various key choices: whether and how much to contribute; how to invest account balances; and at what rate to withdraw their accumulations at retirement. Therefore, the determination of an efficient retirement saving scheme is a major problem for households, as mistakes due to financial illiteracy are frequent and potentially very costly. Empirical evidence has shown that consumers face mounting difficulties in appropriately dealing with these decisions and several contributions to the household finance literature highlight the fact that more educated and richer households tend to behave closer to the prescriptions of normative models (see, e.g., Calvet *et al.*, 2009; Campbell *et al.*, 2011; Dahlquist *et al.*, 2018).

Recent policy actions, such as the Pan-European Personal Pension Product (PEPP) regulation promoted by the European Commission and adopted after the vote of the European Parliament, introduce a standardized tax-qualified funded defined contribution plan promoting public and private supply of a new wave of investment products that are expected to milden these costs and enhance financially illiterate workers' retirement security. To foster competition among providers, the PEPP establishes a level playing field allowing a number of different risk mitigation approaches and setting a standardized key information document that helps investors comparing different options. Clearly, the assessment of the individual product's performance is a critical step: on the one hand, it guides investor's choice, while, on the other hand, it is a non-trivial financial econometric exercise given the long-term nature of the plan's target.

To this aim, there exist advanced life-cycle consumption and portfolio choice models where investors have access to stocks, bonds, and tax-qualified retirement accounts that can be used to provide an accurate quantification of the welfare impact of different alternatives. Notable example of this approach can be found in recent analyses that explore the implications of money-back guarantees for individual retirement accounts (see Horneff et al., 2015; Horneff et al., 2019b).

This paper complements this literature by proposing a comparison between alternative investment strategies that makes use of projection models for the future performance taking explicitly into account slow-moving economic trends which may require intertemporal hedging portfolio policies.[1]

There is a consolidated strand of research analyzing the impact of long-run risk on strategic portfolio allocation (see, e.g., Campbell and Viceira, 2005) and on equilibrium asset prices (Bansal and Yaron, 2004; Ortu et al., 2013) highlighting that the precise determination of relevant risk exposures requires the difficult identification of long-term trends. Building on this broad and well-developed literature on strategic asset allocation and optimal retirement savings, we propose a simple, but realistic, model to simulate the dynamics of different forms of retirement investments for European individuals.

The model we present taken alone cannot produce a specific welfare assessment of different tax-qualified investment schemes. However, it allows us to state two important conclusions: first, the risk–return trade-off offered by different pension products that ignore long-term trends might significantly overestimate the long-term impact of volatility risks while underestimating the impact of persistent, low-frequency trends, such as the inflation one. Second, in order to avoid biasing consumers' choice, regulators' communication should include basic information about the impact that long-term trends have on the future performance of different products.[2]

To produce sensible dynamic scenario analyses, we assume that pension savings can be invested in a stylized European capital market for an accumulation period of 40 years. Following the Campbell and Viceira (2005) (hereafter, CV) approach to strategic asset allocation, dynamic simulations of asset returns are carried out by means of a conditional Vector Auto Regression (VAR) model (see also Gomes et al., 2022). We estimate the long-term risk–return tradeoff faced by a representative investor willing to allocate wealth in investment products mimicking those that are currently available to European pension savers. The simulations are based on historical returns data for a 53-year period from 1969 to 2021, which comprises both high and low interest rates and inflation rates.

We report the results of a comparison between different products currently available in the market. In particular, we compare the outcomes of life-cycle strategies with those of minimum return guarantee strategies. In the latter case, we develop a model of the asset-liability management procedure followed by an insurance company offering such products, where it is assumed that the insurer dynamically adjusts its balance sheet in order to service the policyholders, compensate its shareholders for providing sufficient funding, and comply with solvency requirements.

The result of our simulation study is that both life-cycle target-date fund strategies and minimum guarantee strategies provide satisfactory capital protection, but life-cycle strategies offer protection at lower costs. In this respect, our results are consistent with recent empirical evidence in Horneff et al. (2019a), Koijen and Yogo (2022), and Milevsky and Salisbury (2022).

The sample period of our analysis (1969–2021) comprises almost a decade of nearly zero interest rates, that is the period from 2013 to 2021, during which the European Central Bank (ECB) implemented a non-standard expansionary monetary policy based on different Asset Purchase Programmes.

In order to test the impact of this extreme interest rate scenario on the performances of both the minimum guarantee and life-cycle strategies, we also run the VAR simulation using a truncated sample running from 1969 to 2012, which thus avoids the issues related to the zero lower bound for nominal

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[1]This approach has been introduced by Berardi et al. (2018) to discuss the effective performance of different default options in a policy paper for the European Financial Asset Managers Association.

[2]As a common example, regulators should avoid the use of a geometric Brownian motion in modeling the dynamics of the underlying stock index, since it is well known (see Cornell, 2009) that the evaluation of a minimum guarantee over a realistic retirement time horizon based on this process of the underlying provides a poor approximation and usually overestimates the value of the guarantee.

interest rates (see, e.g., Campbell *et al.*, 2020). We find that also in this context minimum guarantee strategies are dominated by life-cycle strategies, but the difference in performance is less striking.

Finally, as a robustness test, we run our analysis for a shortened investment horizon based on a 20-year accumulation period. In relative terms, the results are no different from the 40-year case, as life-cycle strategies outperform the minimum guarantee strategy. However, the shortening of the accumulation period reduces both the level of protection and the return–risk profile of all strategies.

The paper is organized as follows. Section 2 reviews the basic principles of life-cycle investing and guaranteed investment. Section 3 reports the estimates of a conditional VAR model in the context of a stylized European capital market and produces dynamic scenarios. Section 4 contains the simulation analysis of the retirement investment products offered by financial intermediaries. Section 5 discusses the main empirical results and Section 6 illustrates a robustness test. Finally, Section 7 concludes.

2. Investment strategies for retirement savings

In this section, we survey the key predictions of theoretical life-cycle portfolio allocation models and investment strategies based on a guaranteed minimum rate that is usually embedded in insurance products.

2.1 Life-cycle investing

The goal of life-cycle portfolio allocation problems is to determine the optimal consumption and investment choices of an investor with total wealth consisting of human capital, financial wealth, and other real assets, such as housing property. Therefore, the selection of the optimal asset allocation in these settings incorporates also the illiquid and the real components of investors' wealth.

Formally, human wealth is the present value of all future (uncertain) labor income earned over the remaining lifetime. Alternatively, one can consider labor income as the liquid dividend of an illiquid stock of capital, which endows the newborn and eventually grows by investing in education. Given the inalienable, intangible, and at least partially idiosyncratic nature of human capital, the quantitative description of its risk and return profile is both a challenge and essential to shape the overall allocation strategy over the life-cycle.

One of the reasons to hold liquid financial wealth over the life-cycle is to partially hedge the systematic and predictable variations of human wealth. The benchmark advice that the amount of equity held in the portfolio should decline with age has a simple foundation in Samuelson (1969) and Merton (1971) model settings.

Bodie *et al.* (1992) study the hedging potential of financial wealth for risky human wealth under the assumption that exogenous labor income shocks are perfectly spanned by traded asset return shocks. In their model, households are able to fully hedge the effects of exogenous labor income shocks on their total wealth, if they are not constrained by short-selling or borrowing constraints. However, in reality, an important fraction of labor income risk is idiosyncratic and thus unhedgeable. Moreover, labor income and human capital investment are endogenous choice variables for households. Therefore, households may also actively react to weak financial wealth performance by adjusting their labor supply and retirement date.

The risk–return properties of endogenous labor income have been addressed by a second direction in the literature. Cocco *et al.* (2005) argue that labor income is similar to an implicit holding of safe assets, whereas Benzoni *et al.* (2007) document that labor and capital income are positively correlated in the long run. Under various assumptions on the correlation between liquid and illiquid wealth shocks, Schwartz and Tebaldi (2006) study optimal portfolio allocations in a model where human capital risk is not fully hedgeable. Importantly, housing property is both a consumption and an investment asset to the household. Moreover, its share of total wealth is extremely costly to adjust due to its intrinsic illiquidity. Kaplan and Violante (2014) introduce the notion of 'wealthy hand-to-mouth' investors to characterize households with substantial assets in the form of housing and retirement accounts, but with little liquid wealth or credit facilities to offset short-term income falls. Pension investment is the main component of financial investment for most households. Hence, it is an

essential instrument to diversify risk over the life-cycle, for example, by hedging intertemporal variations in human wealth or other illiquid components of households' wealth. From this perspective, it seems a very reasonable approach the one pioneered by Swanson (2012), which implies that households are allowed to adjust flexibly their pension allocations to risky assets during the last part of their life-cycle in accordance with their risk tolerance and their total wealth composition.

Various authors have tried to characterize quantitatively the optimal portfolio strategies and the welfare implications of life-cycle allocations. Under different assumptions, Cocco *et al.* (2005), Gomes and Michaelides (2005), and Gomes *et al.* (2008) show that the empirically observed stock market participation rates and asset allocations can be reproduced by a calibrated life-cycle model with plausible specifications of uninsurable labor income risk and risk aversion. Fagereng *et al.* (2017) provide related empirical support on life-cycle portfolio allocations, by documenting a double (optimal) adjustment as households' age: a rebalancing of the portfolio composition away from stocks as they approach retirement and a stock market exit after retirement. In a life-cycle model similar to Cocco *et al.* (2005), but with flexible labor supply, Gomes *et al.* (2008) investigate optimal consumption, asset accumulation, and portfolio decisions. Importantly, they quantify the welfare costs of suboptimal life-cycle allocations that mimic popular default investment choices in defined-contribution pension plans. They document that life-cycle funds designed to match investor risk tolerance and investment horizon have small welfare costs, that is, a small reduction in the utility of the household. In contrast, all other policies induce substantial welfare costs.

In summary, this literature documents potentially large welfare costs of popular default investment choices in defined-contribution pension plans that deviate from optimal life-cycle allocations and instead rely on investments in fixed income or in other 'safe' instruments with low yields.

2.2 Guaranteed investment strategies

Various financial intermediaries, often insurers, offer pension products with upside performance linked to an equity benchmark and downside performance limited by a contractual long-term capital guarantee, in the form of a minimum nominal guaranteed rate of return.

Therefore, the payout of these products has a typical option-like form with respect to the benchmark investment: if the benchmark investment performance falls below a minimum floor level, the contract payoff equals the floor; if the benchmark investment produces a performance above the floor, then a fraction of the payout of the policy will raise with it. There are different categories of products explicitly designed to achieve this goal. The simplest one is an index-linked or unit-linked insurance contract, which guarantees to the policyholder a return that is the maximum between the index performance and a minimum guaranteed rate (see, e.g., Hipp, 1996).

From the point of view of the insurance, a policy with a minimum guaranteed rate induces a long-term liability whose present value depends on the level of interest rates. When interest rates are low, the price of the liability rises and the buffer of capital that can be invested in riskier assets is reduced. This problem, known with the name of 'lock-in', limits the performance of an index-linked strategy in periods of low rates, inducing a payout similar to the one offered by a long-term bond. Such an allocation is known to be optimal for an investor with arbitrary low-risk propensity (see Wachter, 2003).

A more advanced approach to structured products with minimum guarantees can be achieved by implementing an asset-liability management strategy (see, e.g., Consiglio *et al.*, 2006; Committee Global Financial System, 2011). Following the traditional actuarial approach, insurance companies actively use their balance sheet to reconcile the targets of policyholders, who play the role of insurance liability holders, and shareholders, who provide funding to keep adequate levels of risk capital required by regulators to minimize insolvency risk.[3] The provision of capital by shareholders is compensated through dividend distributions. The contributions paid by the policyholders of traditional guaranteed

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[3]In the Solvency II regulation, the risk capital required to limit the insurance's insolvency risk is determined by a VaR-type constraint.

products, which may be sold in different years and for different guarantee levels, are pooled together and invested into a corresponding portfolio of assets. The cash flows generated by the insurer's asset portfolio in any given year can then be used to fulfil the obligations deriving from the portfolio of outstanding policies. As long as these cash flows exceed the sum of all existing minimum guarantees, all insurance contracts participate to the asset portfolio excess performance.

From a broader perspective, the organization and regulation of the insurance sector are explicitly designed to optimize the diversification of individual risks through pooling and collective management. In the specific case of market risk, which is undiversifiable, whether such a collective management can be beneficial is still a subject of debate (see Hombert and Lyonnet (2022) and the references therein). When comparing index-linked and traditional participating life-insurance contracts, the latter allow for additional flexibility, as temporary provisions of (costly) capital may reduce the lock-in effect determined by low interest rates. Similarly, when the asset portfolio performance is high, a bonus reserve can be created, which may be used to re-inject capital when needed in the future. Maurer *et al.* (2016) discuss in depth how life insurers use accounting and actuarial techniques to smooth reporting of firm assets and liabilities, seeking to transfer surpluses in good years to cover benefit payouts in bad years. The final effect of these dynamic adjustments is to reduce the policyholder return volatility. This reduction is typically offered in a uniform way throughout the full policy life-cycle, not only in some proximity of the policy liquidation date. Therefore, these investment vehicles stand apart at least in part from some of the insights of traditional financial advices on life-cycle and retirement investing.

In our analysis, for simplicity, we will largely abstract from the economic reasons that may explain the policy provider's selection of a particular form of minimum guarantee investment and the corresponding asset portfolio.[4] Instead, we take the perspective of a potential policyholder who wants to quantify the welfare tradeoffs of a choice between a given minimum guarantee investment policy and other benchmark life-cycle investment strategies, such as a Poterba life-cycle strategy (see Poterba *et al.*, 2006). To achieve this purpose, we basically need to model plausible asset-liability management plans that are implementable by insurance companies in order to produce the contractual cash flows of minimum guarantee investment products.

3. A stylized model of the European capital market

In the empirical analysis, we compare costs and benefits of (i) minimum guarantee investments with a participation life insurance policy and (ii) life-cycle target-date investment funds. In such funds, each investor pays an upfront and a yearly management fee, in order to participate in the return of an asset allocation selected according to a corresponding life-cycle rule and investment mitigation scheme. We compare payout distributions assuming that insurance managers compensate equity capital at market prices and implement a regulatory solvency condition. They also require an upfront management fee from policy holders and pay a yearly asset management cost that is retroceded to policy holders.

Quantifying the relative efficiency of the various investment approaches under consideration also requires a model that reproduces the key properties of security return dynamics, such as the dependence of risks, returns, and correlations on the investment horizon, and their relation with relevant economic state variables. In order to produce a realistic dynamic scenario analysis, we assume that both the investment firm and the insurance company can invest in a common, stylized European capital market. Following the approach to strategic asset allocation pursued by Campbell and Viceira (2005), dynamic simulation of asset returns is performed using a conditional VAR model estimated on European data.[5]

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[4]In this way, we also abstract from broader economic issues related to the management of the actuarial risks in the life insurance business. See van Bilsen *et al.* (2014) for a recent analysis of this interesting issue.

[5]See also Bisetti *et al.* (2017) for an analysis based on US data, Fugazza *et al.* (2007) and Brière and Signori (2012) for an extended analysis including returns from real estate and commodities in the European market.

3.1 Basic specification of the VAR model

Consider the continuously compounded security market returns from time t to time $t+1$, $\mathbf{r}_{t+1}$, and denote with $\boldsymbol{\mu}$ the conditional expected log return given information up to time t:

$$\mathbf{r}_{t+1} = \boldsymbol{\mu} + \mathbf{u}_{t+1}, \tag{1}$$

where $\mathbf{u}_{t+1}$ is the unexpected log return. Then, the τ-period cumulative return from period $t+1$ through period $t+\tau$ is given by

$$\mathbf{r}_{t,t+\tau} = \sum_{i=1}^{\tau} \mathbf{r}_{t+i}. \tag{2}$$

The term structure of risk is defined as the conditional variance of cumulative returns, given the investor's information set, scaled by the investment horizon

$$\Sigma_r(\tau) \equiv \frac{1}{\tau} \mathrm{Var}(\mathbf{r}_{t,t+\tau} | D_t^{\mathrm{Mkt}}), \tag{3}$$

where $D_t^{\mathrm{Mkt}} \equiv \sigma\{z_\tau^{\mathrm{Mkt}} : \tau \leq t\}$ consists of the full histories of returns as well as predictors that investors use in forecasting returns. Following Barberis (2000) and Campbell and Viceira (2002), we describe asset return dynamics by means of a first-order vector autoregressive or VAR(1) model. We choose a VAR(1) as the inclusion of additional lags, even if easily implemented, would reduce the precision of the estimates:

$$\mathbf{z}_t^{\mathrm{Mkt}} = \Phi_0^{\mathrm{Mkt}} + \Phi_1^{\mathrm{Mkt}} \mathbf{z}_{t-1}^{\mathrm{Mkt}} + \nu_t^{\mathrm{Mkt}}, \tag{4}$$

where

$$\mathbf{z}_t^{\mathrm{Mkt}} = \begin{bmatrix} r_{0t} \\ \mathbf{x}_t^{\mathrm{Mkt}} \\ \mathbf{s}_t^{\mathrm{Mkt}} \end{bmatrix}$$

is a $(m \times 1)$ vector, with r_{0t} being the log real return on the asset used as a benchmark to compute excess returns on all other asset classes, $\mathbf{x}_t^{\mathrm{Mkt}}$ being the $(n \times 1)$ vector of log excess returns on all other asset classes with respect to the benchmark, and $\mathbf{s}_t^{\mathrm{Mkt}}$ is the $((m - n - 1) \times 1)$ vector of returns predictors. In the VAR(1) specification, Φ_0^{Mkt} is a $(m \times 1)$ vector of intercepts and Φ_1^{Mkt} is a $(m \times m)$ matrix of slopes. Finally, ν_t^{Mkt} is a $(m \times 1)$ vector of innovations in asset returns and returns' predictors for which standard assumptions apply:

$$\nu_t^{\mathrm{Mkt}} \sim \mathcal{N}(0, \Sigma_\nu^{\mathrm{Mkt}}), \tag{5}$$

where $\Sigma_\nu^{\mathrm{Mkt}}$ is the $(m \times m)$ variance–covariance matrix:

$$\Sigma_\nu^{\mathrm{Mkt}} = \begin{bmatrix} \sigma_0^2 & \sigma_{0x}' & \sigma_{0s}' \\ \sigma_{0x} & \Sigma_{xx} & \Sigma_{xs}' \\ \sigma_{0s} & \Sigma_{xs} & \Sigma_{ss} \end{bmatrix}.$$

Assuming that the VAR is stationary and thus that the moments are well-defined, the unconditional mean and variance–covariance matrix of $\mathbf{z}_t$ can be represented as follows:

$$\mu_z^{\mathrm{Mkt}} = (I_m - \Phi_1^{\mathrm{Mkt}})^{-1} \Phi_0^{\mathrm{Mkt}} \tag{6}$$

$$vec(\Sigma_{zz}^{\text{Mkt}}) = (I_{m^2} - \Phi_1^{\text{Mkt}} \otimes \Phi_1^{\text{Mkt}})^{-1} vec(\Sigma_{\nu}^{\text{Mkt}}). \tag{7}$$

The conditional mean of the cumulative asset returns at different horizons are instead

$$
\begin{aligned}
E_t(z_{t+1}^{\text{Mkt}} + \ldots + z_{t+\tau}^{\text{Mkt}}) =& \left(\sum_{i=0}^{\tau-1} (\tau - i)(\Phi_1^{\text{Mkt}})^i \right) \Phi_0^{\text{Mkt}} \\
&+ \left(\sum_{j=0}^{\tau} (\Phi_1^{\text{Mkt}})^j \right) \mathbf{z}_t^{\text{Mkt}},
\end{aligned}
\tag{8}
$$

and their variance is:

$$
\begin{aligned}
\text{Var}_t(z_{t+1}^{\text{Mkt}} + \ldots + z_{t+\tau}^{\text{Mkt}}) =& \; \Sigma_{\nu}^{\text{Mkt}} + (I + \Phi_1^{\text{Mkt}})\Sigma_{\nu}^{\text{Mkt}}(I + \Phi_1^{\text{Mkt}})' \\
&+ (I + \Phi_1^{\text{Mkt}} + (\Phi_1^{\text{Mkt}})^2)\Sigma_{\nu}^{\text{Mkt}}(I + \Phi_1^{\text{Mkt}} + (\Phi_1^{\text{Mkt}})^2)' + \ldots \\
&+ (I + \Phi_1^{\text{Mkt}} + \ldots + (\Phi_1^{\text{Mkt}})^{\tau-1})\Sigma_{\nu}(I + \Phi_1^{\text{Mkt}} + \ldots + (\Phi_1^{\text{Mkt}})^{\tau-1})'.
\end{aligned}
\tag{9}
$$

Once the conditional moments of excess returns are available, the following selector matrix extracts, for each period, τ-period conditional moments of log real returns

$$M_r = \begin{bmatrix} 1 & 0_{1 \times n} & 0_{1 \times (m-n-1)} \\ \iota_{n \times 1} & I_{n \times n} & 0_{n \times (m-n-1)} \end{bmatrix} \tag{10}$$

which implies

$$\frac{1}{\tau} \begin{bmatrix} E_t(r_{0,t+1}^{\tau}) \\ E_t(\mathbf{r}_{t+1}^{\tau}) \end{bmatrix} = \frac{1}{\tau} M_r E_t(z_{t+1}^{\text{Mkt}} + \ldots + z_{t+\tau}^{\text{Mkt}}). \tag{11}$$

$$\frac{1}{\tau} \begin{bmatrix} \text{Var}_t(r_{0,t+1}^{\tau}) \\ \text{Var}_t(\mathbf{r}_{t+1}^{\tau}) \end{bmatrix} = \frac{1}{\tau} M_r \text{Var}_t(z_{t+1}^{\text{Mkt}} + \ldots + z_{t+\tau}^{\text{Mkt}}) M_r'. \tag{12}$$

Using the estimated VAR coefficients, it is possible to derive unconditional and conditional moments for returns and excess returns at all different investment horizons. These moments deliver the dynamics of returns and the risk of different assets across investment horizons. This information forms the input for portfolio allocation. Following the Campbell–Viceira methodology, we consider a benchmark portfolio to be obtained by attributing optimal weights to bonds, stocks and T-bills. Therefore, in $\mathbf{x}_t^{\text{Mkt}}$ we include excess returns on stocks and bonds, real returns on T-bills, while we include in $\mathbf{s}_t^{\text{Mkt}}$ three factors commonly recognized as good predictors of these assets' returns. In particular, the predictors are the nominal short-term interest rate, the dividend–price ratio, and the yield spread between long-term and short-term bonds.

3.2 Data and VAR estimates

The information set includes the returns of three reference securities: (i) the real short rate, *rtb*, obtained as the difference between the log of the total return on a strategy investing in three-month German T-bills and the one-year CPI inflation rate; (ii) the excess return on stocks, *xr*, given by the difference between the log of the total return of an investment in MSCI Europe and the log-return on the German three-month T-bill; (iii) the excess return on bonds, *xb*, calculated as the difference
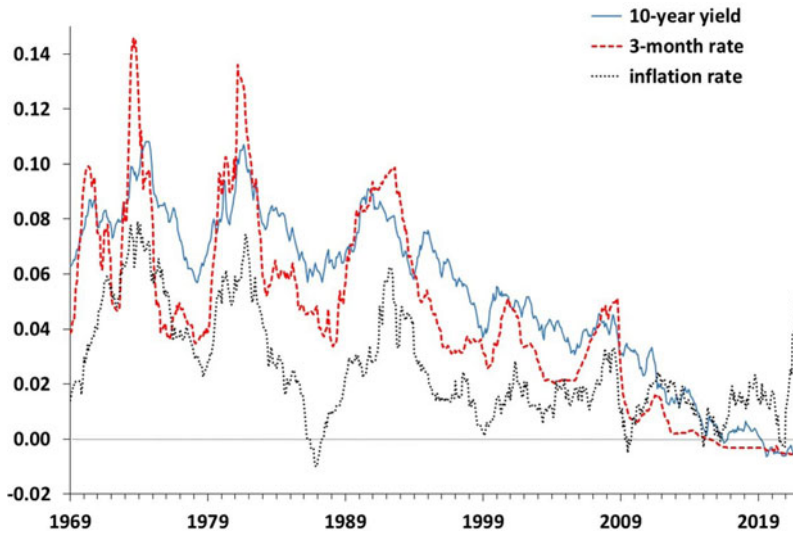
Figure 1. The evolution of interest rates and inflation in Germany. This figure plots monthly observations for the 10-year yield and 3-month rate on German government bonds and the annual inflation rate in Germany over the period January 1969 to December 2021.

between the log of the total return of an investment in 10-year German government bonds and the log-return on the German three-month T-bill. It also includes three conditioning state variables: (i) the German three-month T-bill rate, y; (ii) the log of the dividend–price ratio for a European benchmark securities portfolio, dp; (iii) the spread between the yield on the German 10-year government bond and the German three-month T-bill, spr.[6]

In order to include in the estimation also the period when the nominal short rate hit the zero lower bound (i.e., the interval between 2012 and 2021), we modify the original CV procedure by using as predictor for the real short-term rate the shadow rate[7] as computed in De Rezende and Ristiniemi (2023). Notice that the resulting VAR is observationally equivalent to the original one. In fact, by definition, the nominal short-term rate is given by the maximum between 0 and the shadow rate. On the other hand, the use of the shadow rate circumvents the estimation problem determined by the short-term nominal rate being at the zero lower bound (see Carriero et al., 2021).

The presence of the conditioning state variables is necessary to consider the modification of the long-term risk–return generated by asset return predictability. We define the vector $\mathbf{z}_t^{Mkt}$ as:

$$\mathbf{z}_t^{Mkt} := [rtb_t, xr_t, xb_t, y_t, dp_t, spr_t].$$

Our simulation is based on a sample of annual observations covering the period between 1969 and 2021. This 53-year sample period includes several different scenarios for interest rates and inflation rates. Figure 1 shows the time series of the 10-year yield and 3-month rate of German government bonds and the annual inflation rate in Germany from January 1969 to December 2021.[8] We observe that the three-month rate reaches a level close to zero around the end of 2012, becomes negative in 2015, and then remains below zero until 2021. The 10-year yield shows a strong downward trend over the

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[6]The source of data is Datastream.

[7]The shadow rate is defined as the (unobservable) short-term interest rate consistent with the long-term rates that would have been observed if the zero lower bound on interest rates had not been binding (see, among the others, Bauer and Rudebusch (2016) and Wu and Xia (2020)).

[8]These data are available on FRED economic data.

Table 1. Summary statistics

Panel A: full sample 1969–2021						
	rtb	*xr*	*xb*	*y*	*dp*	*spr*
Mean	0.0108	0.0552	0.0293	0.0345	−3.4818	0.0151
Std. dev.	0.0206	0.1974	0.0685	0.0285	0.3158	0.0108
Panel B: truncated sample 1969–2012						
	rtb	*xr*	*xb*	*y*	*dp*	*spr*
Mean	0.0169	0.0458	0.0296	0.0449	−3.4551	0.0158
Std. dev.	0.0168	0.2124	0.0724	0.0211	0.3383	0.0119

This table reports means and standard deviations for the three assets and the three predictors used in the estimation of the VAR model: rtb = ex-post real T-bill rate, xr = excess stock return, xb = excess bond return, dp = log dividend–price ratio, y = nominal T-bill yield, spr = yield spread. Panel A contains summary statistics for the sample 1969–2021, while panel B shows estimates for the truncated sample from 1969 to 2012.

sample period, especially starting from 1992. The rate declines to 2% for the first time in 2012, then keeps decreasing and becomes significantly negative in 2019, with a minimum at −65 basis points. The inflation rate also follows a declining trend with large fluctuations in the 1969–1992 period and significantly less pronounced variations in the 1992–2020 period, and a jump to about 5% in 2021.

In what follows, we analyze the performance of the minimum guarantee investment contracts (MG) and the life-cycle strategies (LC) over the 1969–2021 sample period.

We also run the VAR simulation using a truncated sample running from 1969 to 2012. This set of data therefore excludes the 2013–2021 extreme interest rate scenario driven by the Outright Monetary Transactions program carried out by the ECB since August 2012 and the subsequent Asset Purchase Programmes, that is, those non-standard monetary policy measures implemented by the ECB to support the monetary policy transmission mechanism (see, e.g., Eser *et al.*, 2019; Altavilla *et al.*, 2021). A similar approach is adopted in Campbell *et al.* (2020) where the sample period ends in 2011 in order to avoid the issues related to the zero lower bound for nominal interest rates.

Table 1 reports the unconditional risk and return characteristics for the asset classes and the predictors in the estimation samples.

We find that the main differences between the two samples regard the average level of the ex-post real T-bill rate and nominal T-bill yield, which are significantly higher for the 1969–2012 period excluding the zero lower bound regime, and the Sharpe ratio on stock returns, which is almost 30% higher for the longer sample period.

Figure 2 plots, for each sample, the term structure of volatilities and correlations, that is, the variation with the horizon of the riskiness, measured by the standard deviation of ex-post real rate of returns on a yearly basis, and of the correlation between the three asset classes under consideration. The estimated term structures of volatility are quite homogeneous across the two samples, while the term structures of correlation show some significant differences. In particular, the correlation between short-term and long-term bonds is relatively flat for long horizons in the truncated sample and instead increasing with the time horizon in the case of the full sample.

The estimated term structure of risk does not show relevant differences compared to the original CV results. As discussed by Campbell *et al.* (2009) and David and Veronesi (2013), the correlation between fixed income and equity investment returns may vary across different economic regimes and is difficult to capture within the present linear homoscedastic modeling approach. On the other hand, we use the VAR simply as a scenario generator, thus the heterogeneity of the correlation structure across different regimes could be used to test the robustness of the results with respect to different correlation structures.

Table 2 reports the estimated coefficients for the VAR specifications. Remarkably, the estimated coefficients show that the three state variables – the shadow rate, the dividend–price ratio, and the term spread – are all significant predictors for the three relevant asset classes in both samples.
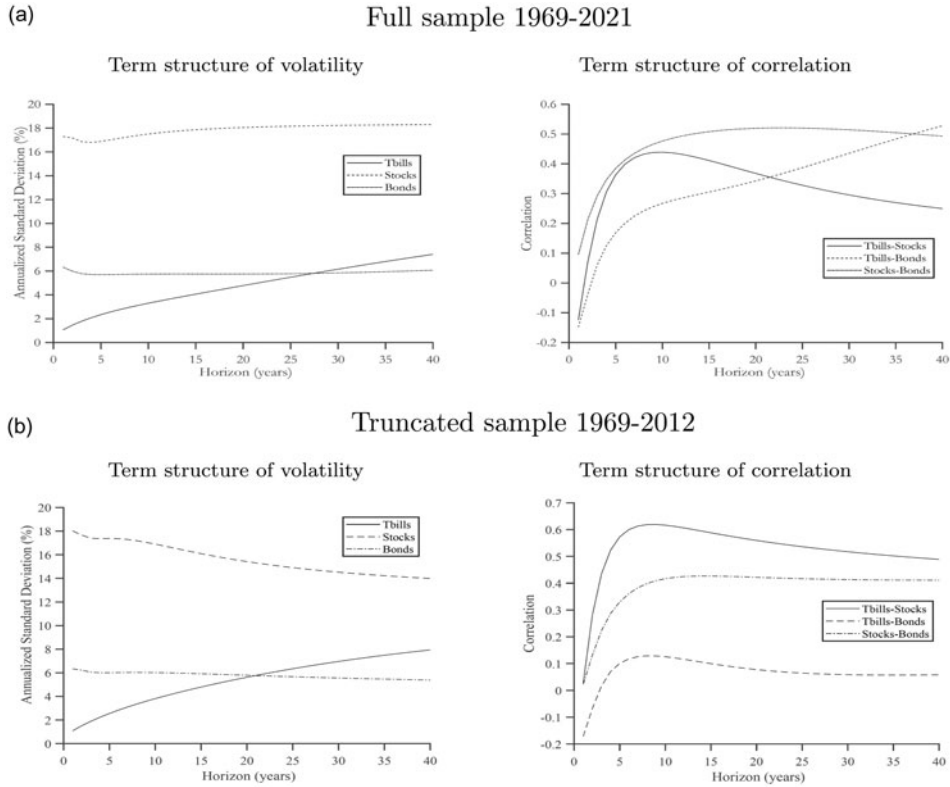
(a)   Full sample 1969-2021

Term structure of volatility   Term structure of correlation

(b)   Truncated sample 1969-2012

Term structure of volatility   Term structure of correlation

Figure 2. Term structure of volatility and correlations. This figure plots the term structure of volatilities and correlations, that is, the variation with the time horizon of the standard deviation of the ex-post annual real rate of returns and of the correlation between the three asset classes under consideration: short-term bonds, long-term bonds and stock market index. Panel A reports values for the full sample from 1969 to 2021, while Panel B shows values for the truncated sample from 1969 to 2012.

These predictors improve the accuracy of estimation of the long-term risk–return tradeoff. In fact, they capture the slow-moving trend and thus remove the volatility component that would be artificially induced by assuming a misspecified unconditional model with constant long-term mean that are still popular in the simulation analysis of pension products.

The differences that emerge between the coefficients in the two samples reflect the effect of the inclusion of the negative interest rate regime and will be used in our discussion to evaluate the robustness of the results with respect to changing economic conditions. In other words, we rely on the use of the different regimes to assess the potential costs that could arise in different long-term savings strategies as the underlying scenarios vary.

4. Simulation of payout distributions

Our scenario simulation reproduces the joint dynamics for the three tradable assets, that is, the return of a short-term cash investment (proxied by the 3-month German T-bill), the return of a long-term, 10-year maturity, investment (proxied by the 10-year German government bond), and the return of an aggregate stock market index (proxied by the MSCI Europe equity index).[9] It also generates the joint dynamics for the three economic state variables, that is, the dividend–price ratio of a portfolio equity index, the term spread between the 10-year and the 3-month yields on German T-bills, and the

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[9]VAR estimates obtained by using the 10-year French government bond return produce similar implications for our analysis.

Table 2. Estimated coefficients of the VAR model

Panel A: full sample 1969–2021

	rtb_t	xr_t	xb_t	y_t	dp_t	spr_t	R^2	$adj.R^2$
rtb_{t+1}	0.566	0.028	−0.004	0.292	−0.005	0.150	0.659	0.614
	(3.92)	(3.18)	(−0.20)	(2.49)	(−0.93)	(0.74)		
xr_{t+1}	3.905	−0.033	0.865	−3.176	0.204	1.059	0.213	0.108
	(1.32)	(−0.23)	(3.09)	(−1.64)	(1.82)	(0.38)		
xb_{t+1}	−0.792	−0.015	−0.094	0.464	−0.011	−2.315	0.139	0.024
	(−1.20)	(−0.31)	(−0.56)	(0.93)	(−0.36)	(−2.96)		
y_{t+1}	0.099	0.014	−0.041	0.907	−0.001	0.749	0.904	0.891
	(0.70)	(2.09)	(−2.14)	(10.49)	(−0.19)	(4.56)		
dp_{t+1}	−2.688	0.014	−0.874	3.266	0.545	0.215	0.538	0.476
	(−0.76)	(0.091)	(−2.17)	(1.52)	(4.01)	(0.06)		
spr_{t+1}	−0.020	−0.012	0.046	0.012	0.005	0.450	0.326	0.236
	(−0.17)	(−2.02)	(2.37)	(0.18)	(0.89)	(2.76)		

Panel B: truncated sample 1969–2012

	rtb_t	xr_t	xb_t	y_t	dp_t	spr_t	R^2	$adj.R^2$
rtb_{t+1}	0.722	0.017	0.009	0.273	−0.009	0.387	0.596	0.526
	(4.32)	(2.32)	(0.45)	(2.89)	(−1.93)	(1.90)		
xr_{t+1}	4.054	0.063	0.807	−4.114	0.284	−1.363	0.245	0.115
	(1.29)	(0.39)	(2.44)	(−1.97)	(2.68)	(−0.37)		
xb_{t+1}	−0.787	−0.038	−0.067	−0.147	0.003	−3.080	0.187	0.048
	(−1.11)	(−0.71)	(−0.35)	(−0.19)	(0.06)	(−2.84)		
y_{t+1}	0.155	0.016	−0.039	0.917	0.001	0.863	0.807	0.774
	(0.96)	(2.22)	(−1.77)	(7.48)	(0.16)	(4.38)		
dp_{t+1}	−2.675	0.182	−1.010	3.084	0.657	−0.710	0.620	0.555
	(−0.65)	(1.38)	(−2.09)	(1.31)	(5.14)	(−0.14)		
spr_{t+1}	−0.116	−0.010	0.038	−0.044	0.006	0.237	0.305	0.186
	(−0.79)	(−1.50)	(1.79)	(−0.56)	(0.87)	(1.38)		

This table reports summary statistics for the estimated coefficients of the VAR specification. The variables included in the model are: rtb = ex-post real T-bill rate, xr = excess stock return, xb = excess bond return, dp = log dividend–price ratio, y = nominal T-bill yield, spr = yield spread. Panel A reports estimates based on the full sample from 1969 to 2021, while panel B shows estimates for the truncated sample from 1969 to 2012. t-statistics in parentheses.

real 3-month yield of German T-bills, obtained by deflating the nominal yield by German CPI inflation.

A key feature of the simulation approach is represented by the fact that it takes into account the dependence of risks and correlations on the time horizon. Moreover, the inclusion of an inflation proxy in the estimated VAR dynamics allows us to obtain relevant information about the effective degree of long-term inflation protection offered by the risk mitigation schemes under scrutiny.

4.1 The simulation approach

Contribution scheme. We fix a contribution (wealth accumulation) phase of 40 years, that is, the worker joins the defined-contribution (hereafter DC) pension plan at age 25 and leaves it after retirement at age 65. The following assumptions of pension contributions and wage evolution further underly our computations: (i) an annual initial wage of 18,000 euro, which corresponds approximately to the current euro area average net income; (ii) an annual wage growth rate of 2%, consistent with a deterministic wage growth rate compatible with euro area inflation and productivity growth rates;[10] (iii) a monthly wage contribution of 10% to the DC pension plan.

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[10]Assuming that the inflation rate and the productivity growth rate are stochastic or that wages increase at a different annual rate does not alter the main results of our analysis. Therefore, for the sake of simplicity, we keep the growth rate of wages fixed at 2%, which is approximately equal to the sum of the average inflation rate in the euro area since 1999 (1.5%) and the current productivity growth rate in advanced economies, which has been estimated at a value around 0.3% (see Lagarde, 2017).

Under these assumptions, the worker/investor starts contributing 150 euro, that is, 10% of her initial monthly wage, to the DC pension plan. Every following month, the wage increases by 2% on an annual basis, implying a final annual salary at the retirement date of about 40,000 euro. Therefore, at the retirement date the fix percentage contributions of 10% of monthly salaries give rise to total contributions of about 110,000 euro for the DC pension plan.

We exemplify the performance of different allocation strategies by comparing the performance of different pension products using the same set of initial conditions and the same set of simulation scenarios.

Asset return dynamics. The first step to set up the simulation approach is common to both life-cycle and life-insurance investment analyses. In each simulation run, we use the VAR model to generate 5,000 possible scenarios for the random evolution of the market returns and the state variables. Our input data will be the annual rates of returns produced by the set of tradable instruments for a number of years sufficient to determine the final performance of the alternative strategies. The state variables determine information on the inflation rate, the dividend–price ratio, and the yield spread.

From each simulation, we compute the time series of real returns for each of the three tradable asset classes:

$$R_{t+1} := (r_{t+1}^{EQ}(\omega), r_{t+1}^{SB}(\omega), r_{t+1}^{LB}(\omega)),$$

where $\omega = 1, \ldots, NoPaths$, $t = 0, \ldots, T-1$ and EQ, SB, and LB denote Equity, Short-term Bonds, and Long-term Bonds, respectively.

This comparison will be performed in real terms in order to take into account the erosion of the portfolio value driven by inflation.

Portfolio strategies. An asset allocation strategy $\Pi_t = (\pi_t^{EQ}, \pi_t^{SB}, \pi_t^{LB})$ produces a return between time t and time $t+1$ along the path ω, $R_{t+1}^{\Pi}(\omega)$, that is given by:

$$R_{t+1}^{\Pi}(\omega) = \pi_t^{EQ} r_{t+1}^{EQ}(\omega) + \pi_t^{SB} r_{t+1}^{SB}(\omega) + \pi_t^{LB} r_{t+1}^{LB}(\omega), \quad t = 0, \ldots, T-1.$$

We assume that the account balance is regulated on a yearly basis, so that the return $R_{t+1}^{\Pi}(\omega)$ is capitalized in the account on a yearly basis. Administrative costs are charged up-front and correspond to 0.5% of the contributions and for each year the asset management fees are assumed to be equal to 0.5%.

4.2 Implementation of strategies

In what follows, we sketch the main steps that are necessary to compute the investment performance for each path of the asset returns and for each investment approach. First, we detail the asset-liability management procedure adopted by the insurance company to dynamically adjust the balance sheet in order to (i) service the policy holders, (ii) fulfill the regulatory requirements, and (iii) compensate the shareholders. In essence, this procedure aims at calculating the minimum guaranteed rates that an insurer can offer to comply with these three objectives. Then, we describe the life-cycle investment strategy adopted in the simulations, which is based on the 'Poterba age-based scheme' (see Poterba *et al.*, 2006).

4.2.1 Participating life insurance with capital guarantee

We quantify the performance of the minimum guarantee strategy implementing a traditional participating life insurance contract. We concentrate solely on the analysis of the financial risk, thus abstracting from the actuarial risk to avoid the need of using biometric data. The following procedure builds on the approach proposed by Graf *et al.* (2011) and Eling and Holder (2013) and combines a risk-neutral and an historical-risk measure to simultaneously analyze the performance of the strategy from the point of view of the insurance company, the policyholder, and the regulator.

In order to compute the benefit distributed to the policyholder, $L_T(\omega)$, it is necessary to simulate a simplified model for the balance sheet of the insurer and for its evolution.

When a policy of this type is sold, the policyholder agrees to pay the annual contribution and in exchange the intermediary, that is, an insurance company, commits to credit the policyholder's account with a minimum nominal interest rate G each year. Since we intend to analyze the real return to the policyholder, the effective guaranteed real rate is the level G net of the ex-post inflation rate π_t measured during year t, .

Both policyholders and shareholders participate in the investment performance and receive a fraction of the annual surplus. To align the interests of shareholders to those of policyholders, the annual dividend d_t credited to the shareholders is usually assumed to be equal to a fraction of the surplus credited to the policyholders. The evolution of the single period asset performance is determined by the equation:

$$A_{t+1}^- = (1 + R_{t+1}^\Pi(\omega))A_t^+,$$

where A_t^- and A_t^+ are the time t assets prior and posterior, respectively, to the distribution of the dividends and the investment of the collected premium:

$$A_t^+ = \max\left\{\underset{-}{\overset{t}{A}} - d_t, L_t\right\} + P_t + c_t,$$

with P_t defined as the individual contribution at time t net of management fees and c_t as a capital injection. Notice that if the accumulated asset, net of dividends, falls short of the liabilities, the following capital injection is required

$$c_t = \max\{L_t - A_t^- + d_t, 0\}$$

and the shareholders must be compensated for its provision.

The liabilities evolve as follows:

$$L_{t+1} = (1 + G_t)L_t + sur_t,$$

where sur_t is an additional distribution, whose exact expression is detailed in Eling and Holder (2013), which implements the German legal requirements and the traditional surplus distribution policy described below.

The exact value credited by the insurance company to the policyholder is determined by a target rate z as long as the reserve quota x_t, $x_t := ((A_t^- - d_t - L_t)/L_t)$, lies within a predetermined region $x_t \in [a, b]$. If the reserve quota falls below a, the interest rate credited to the policyholder is the maximum between the guaranteed interest rate and the level of the participation rate that restores x_t to the minimum capital requirement value a. If crediting the target rate z leads to a reserve quota above b, then the company credits exactly the rate necessary to set $x_t = b$. In our analysis, we will assume $a = 0.05$ and $b = 0.30$, while in the simulations the target rate z is set in the range 3–5%.

According to the German legislation, at least $\delta = 90\%$ of the earnings on book values must be distributed to the policyholders. Therefore, we assume that a fraction $\eta = 0.7$ of the market value $A_{t+1}^- - A_t^+$ is distributed to the policyholders and the final participation to the surplus is given by:

$$\delta\eta(A_{t+1}^- - A_t^+),$$

while the dividend distributed to the shareholders is a fraction $\alpha = 0.05$ of the surplus:

$$\alpha\delta\eta(A_{t+1}^- - A_t^+).$$

Pricing fairness is guaranteed by ensuring that the risk-neutral present value of the dividend distributed to the shareholder $d_t(\omega)$ is sufficient to compensate the capital provisions $c_t(\omega)$, that is, it must be larger than (or equal to) the risk-neutral discounted value of future capital provisions net of the total change in capital reserves:

$$E^{\mathbb{Q}}\left[\sum_T^{t=1}\frac{d_t-c_t}{B_t}\right]+\left\{E^{\mathbb{Q}}\left[\frac{e_T}{B_T}\right]-e_0\right\}\simeq 0, \tag{13}$$

where $e_t = A_t^- - L_t - d_t$ denotes the residual value of the reserve account at time t.

In order to compute the risk-neutral probability of the different scenarios, we assume a stochastic discount factor driven by the VAR innovations for the excess returns on the stock market index and the long-term bond:

$$-\log(m_{t+1}) = r_{0,t} + \frac{1}{2}[\Lambda_{EQ}, \Lambda_{LB}]\Omega_{2x2}[\Lambda_{EQ}, \Lambda_{LB}]'$$
$$+ [\Lambda_{EQ}, \Lambda_{LB}][\varepsilon_{t+1}^{xr}, \varepsilon_{t+1}^{xb}]',$$

where $r_{0,t}$ is the rate of return on the short-term bond as determined by the VAR dynamics; Λ_{EQ} and Λ_{LB} are the market prices of risk for stocks and for long-term bonds, respectively, which are assumed to be constant and set equal to the historical values in the sample used to estimate the VAR; Ω_{2x2} is extracted from the variance–covariance matrix of the VAR residuals selecting the variances and covariances between equity and long-term bond excess returns.

The risk that the management strategy of the guarantee is unsuccessful and the account balance does not break even is then quantified considering two risk measures: the Probability of Shortfall (PS)

$$PS := \mathbb{P}(A_T < L_T), \tag{14}$$

and the Expected Shortfall (ES)

$$ES := \mathbb{E}^{\mathbb{P}}[\mathbb{1}_{\{A_T<L_T\}}(L_T - A_T)]. \tag{15}$$

To simplify the analysis, we will consider as acceptable a management strategy if the relative ES, that is, the ratio between the ES and the present value of future contributions, NPV_C, is smaller than 0.5%:

$$\frac{ES}{NPV_C} < 0.5\%. \tag{16}$$

The probability of shortfall is considered as a reference measure to assess the frequency of the paths where the insurance management of the policy does not break even.

The level of the minimum guarantee is computed imposing: (i) condition (13), that is, contract fairness and (ii) condition (16), that is, a solvency condition.

4.2.2 Life-cycle Poterba-style investment strategy

The Poterba age-based scheme dictates a percentage wealth allocation to stocks which decreases with the age of the investor. So, for example, if the wealth allocation to stocks at age 25 is 75%, the stock allocation at the retirement age of 65 is 35%, if we decrease the wealth allocation by 1% each year. In this investment strategy, the individual account A_t is updated following the same rule of total assets under management:

$$A_{t+1} = (1 + R_{t+1}^{\Pi}(\omega) - apf)(A_t + P_t),$$

where P_t has been defined above as the individual contribution at time t net of the management fee and *apf* is a management fee on the annual performance which accounts for the costs of trading. We apply *apf* = 0.5%.

In order to reduce the disinvestment risk, we assume that the pensioner stops the contributions at the age of 65 and then may choose either to liquidate the investment, if the amount is higher than the money back amount, or wait until the age of 72. This simplified scheme is assumed to mimic the role of more sophisticated disinvestment options that are known in the literature.[11]

4.3 Asset allocation strategies

This section specifies the structure of (i) minimum guarantee investments, with a participation life insurance policy that can be offered at market prices, and (ii) Poterba-style life-cycle target-date investment funds.

We assume that the asset allocation of the minimum guarantee contracts is 5% in equity and 95% in long-term bonds:[12]

$$\pi^{EQ} = 5\%; \quad \pi^{LB} = 95\%, \tag{17}$$

where π^{EQ} and π^{LB} are constant percentage wealth allocations to equities and long-term bonds, respectively.

Note that in practice there are structural reasons that force an insurer who sells a guaranteed product to allocate most of the wealth in long-term bonds and thus to reduce the diversification of the asset allocation. First, the principle of duration matching, which implies that to lower the exposure of the balance sheet to interest rate volatility risk, the duration of assets must be close to the duration of liabilities, which is high due to the presence of the guarantee on the liability side. Second, capital requirements for equity are higher than those for bonds according to Solvency II regulation.

For comparison with the minimum guarantee investment strategy supported by the above allocation, we consider three Poterba-style life-cycle strategies with the following time-varying allocations to equities and long-term bonds only:

$$\text{Low equity: } \pi_t^{EQ} = \frac{85 - \tau}{100},$$

$$\text{Medium equity: } \pi_t^{EQ} = \frac{100 - \tau}{100}, \tag{18}$$

$$\text{High equity: } \pi_t^{EQ} = \frac{115 - \tau}{100}$$

and $\pi_t^{LB} = 1 - \pi_t^{EQ}$ in all cases. In equation (18), τ is the age of the life-cycle investor, ranging from an initial age of 25, when starting the life-cycle strategy, and an age of 65 at retirement. The choice to focus on zero allocations to cash also in the life-cycle strategy is taken in order to make the comparison more consistent with the implications of the allocation for minimum guarantee investment products. In this way, we can focus on a comparison with the long-term risk–return tradeoff resulting only from a time-varying allocation to equity and long-term bonds in the life-cycle strategy.

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[11]See, for example, Di Giacinto and Vigna (2012). During the disinvestment period, pension funds are invested in financial assets after retirement and the pensioner withdraws periodic amounts until annuitization occurs (if ever). This option is named 'income drawdown option' in the UK, 'phased withdrawals' or 'programmed withdrawals' in the US. By taking it, reinvestment risk and annuitization risks are shifted from the retirement date to the post-retirement period.

[12]See Graf *et al.* (2011) and Hieber *et al.* (2015), for an extended discussion on the optimality conditions used to set the allocation strategy. Our allocation is broadly in line with most of the portfolios adopted in the insurance industry. We also estimate the model assuming a moderate increase in the equity component, that is, 10% in equity and 90% in long-term bonds. However, the empirical results do not differ substantially from those obtained with the $5 - 95\%$ allocation.

4.4 Computation of minimum guarantees

For simplicity, we assume an insurer's balance sheet that is regulated on a yearly basis. Therefore, the random yearly return R_{t+1}^{II} on the insurer's asset portfolio is capitalized in the account with a yearly frequency, for each of the allocation scenarios introduced in the previous section. Moreover, a fee of 50 basis points is charged up-front on every policy contribution to cover the sales costs. Finally, the yearly asset return credited to policy holders is reduced by 50 basis points to cover the asset management fees.

The minimum guaranteed rate depends on market conditions and is determined using an iterative numerical procedure, which implies that the level of the minimum guarantee is lowered (raised) until the condition on contract fairness (13) and the solvency constraint (16) are simultaneously satisfied. We find that the minimum annual guaranteed rate affordable by the insurance company is equal to $G^F = 2.25\%$ for the full sample period 1969–2021 and $G^T = 4.25\%$ for the truncated sample 1969–2012.

5. Performance assessment

Given the results of the VAR estimation, the asset allocations, and the minimum guaranteed rate, we simulate the asset-liability management strategy implemented by the insurance company to compute the simulated distribution of the total final payout produced by the investment policy with minimum guarantee (hereafter MG). We build the same distribution by running the simulation also for each of the three life-cycle Poterba-style strategies (hereafter LC). To ensure a meaningful comparison, the payouts of the MG and the LC are simulated path-by-path under identical economic conditions.

Then, as a proxy for the distribution of the final effective strategy payouts, we compute the distribution of the ratio between the inflation-deflated total strategy payouts and the nominal total contributions paid.

We define the ratio between the total strategy payouts and the total contributions paid as the Payoff/MoneyBack (*PMB*) ratio.[13] A *PMB* ratio higher than 1 implies that the nominal rate of return guaranteed each year by the investment of the contributions is at least sufficient to compensate the loss in value due to the ex-post observed inflation rate. In other words, a unit *PMB* ratio ensures that the investment strategy generates a final wealth equal to the inflation-indexed value of the total contributions paid during the accumulation period.

In Figure 3, we plot the simulated distribution of the *PMB* ratio for the MG versus the simulated distribution for each of the LC (see equation (18)): LC–L (low equity), LC–M (medium equity), and LC–H (high equity).

Panel A shows the distributions obtained for the full sample period from 1969 to 2021. We observe a key difference in the distribution of the *PMB* ratio in the MG and the LC: the LC produce a more positively skewed distribution, that is, a thicker right tail and a practically absent left tail because of the risk-mitigation, together with a higher upside return potential for a wide range of simulated economic scenarios.

The estimates for the truncated sample 1969–2012 in panel B show that the distribution of the *PMB* ratio shifts to the right in all cases. Therefore, excluding the decade of nearly zero interest rates improves the performance of all strategies, as we will illustrate in more detail below, discussing the results in Table 3.

Given such strong asymmetries in the payout distributions, a risk–return tradeoff comparison based on symmetric measures of average return and risk, such as, for example, means and volatilities, is inappropriate.[14] Therefore, we follow Antolin *et al.* (2009) by adopting quantile-based measures of risk and performance.

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[13]The denominator of the *PMB* ratio is equal to the contributions capitalized at a zero ex-post real rate.

[14]Symmetric measures of dispersion such as volatility do not distinguish between losses and gains. Similarly, the sample mean does not measure a typical investment performance in the presence of strongly skewed distributions. To illustrate this point, consider an investment of 10,000 euro for one year in a security producing each week a capital gain of 80% or a loss of 60% with equal probability. The mean weekly payout is $10,000 \times 0.5 \times (1.8 + 0.4) = 11,000$ euro, which may naively suggest
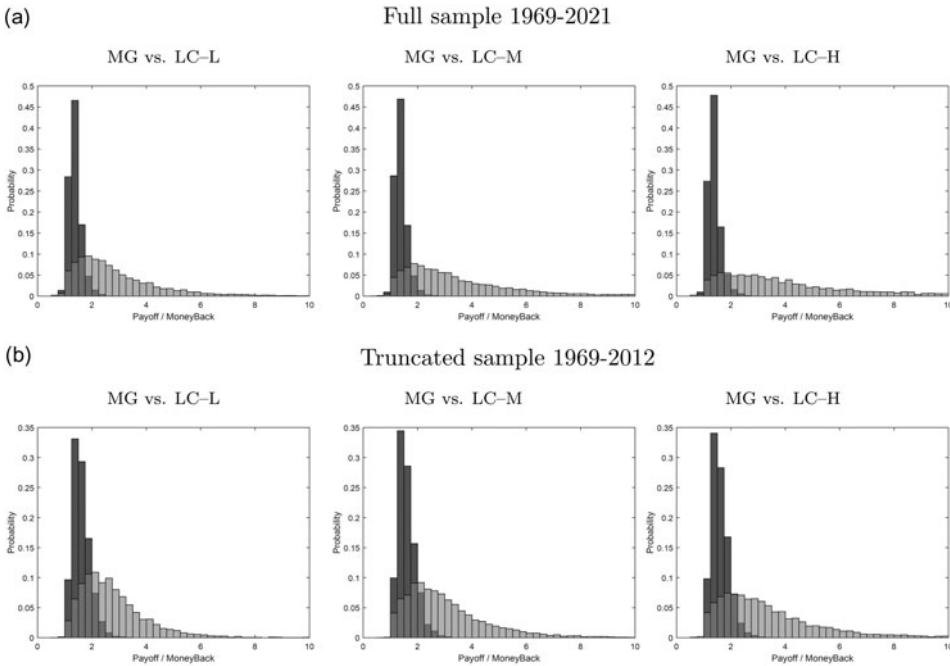
Figure 3. Payoff distribution of minimum guarantee vs. life-cycle strategies. This figure reports a comparison of the payoff distribution of the minimum guarantee strategy (MG) vs. the different life-cycle strategies (LC) on a 40-year accumulation period. The MG is based on the allocation $\pi^{EQ} = 5\%$; $\pi^{LB} = 95\%$, where π^{EQ} and π^{LB} are constant percentage wealth allocations to equities and long-term bonds, respectively. The nominal guaranteed rate is $G^F = 2.25\%$ for the full sample from 1969 to 2021 and $G^T = 4.25\%$ for the truncated sample from 1969 to 2012. The time-varying allocations of the three LC are (i) LC−L:$\pi_t^{EQ} = ((85 - \tau)/100)$, (ii) LC−M:$\pi_t^{EQ} = ((100 - \tau)/100)$, (iii) LC−H:$\pi_t^{EQ} = ((115 - \tau)/100)$, and $\pi_t^{LB} = 1 - \pi_t^{EQ}$ in all cases; τ is the age of the life-cycle investor, ranging from 25 to 65 years. Panel A compares the distributions obtained for the MG (dark grey area) and the LC (light grey area) using the full sample from 1969 to 2021, while panel B compares the same distributions for the truncated sample from 1969 to 2012.

We measure the 'typical' payout of the payout distribution with the median internal rate of return (IRR), denoted as *Med*,[15] while we compute the downside risk of that distribution using the IRR corresponding to the lower 5% quantile, defined as $LP_{5\%}$. We also introduce a 'reward–risk ratio', *RR*, which allows us to compute the level of performance offered to investors (in terms of median IRR) for a given level of payout performance granted to 95% of the population sample: $RR := (Med/LP_{5\%})$.[16]

These indicators of return performance and downside risk are reported in Table 3. The evidence for the full sample, in panel A, shows that the *Med* generated by the MG is significantly lower than the *Med* under each of the three LC, including the case of the 5% worse-off individuals. In particular, we find that the *Med* of the three LC ranges between 2.8 and 4%, while the *Med* for the MG is 1.5%. At the same time, the $LP_{5\%}$ is 1.2% for the MG and between 1.3 and 1.4% for the LC. This implies that the reward–risk ratio *RR* of the three LC is significantly above 2 in all cases (from 2.13 for LC–L to 2.80 for LC–H), while that of the MG is equal to only 1.25. In the last row of the table, we observe that in about 90% of the simulated values, the retirement balance provided by the LC is higher than in the MG. Therefore, the MG is dominated by the LC with respect to any metric.

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an attractive investment opportunity. However, the most likely end-of-year payout after reinvesting the initial capital every week is: $10,000 \times (1.8)^{26} \times (0.4)^{26} = 1.95$ euro, that is, less than 0.02% of the initial capital.

[15]Since payouts are deflated, also the IRR in this section provides information depurated from the effect of inflation.

[16]Given two pension plans, say x and y, y has to be preferred to x if $LP_{5\%}(y) \geqslant LP_{5\%}(x)$ and $RR(y) \geqslant RR(x)$.

Table 3. Guaranteed vs. life-cycle strategies

Panel A: full sample 1969–2021

	MG	LC–L	LC–M	LC–H
Med	1.50	2.81	3.33	3.99
$LP_{5\%}$	1.20	1.32	1.37	1.42
_RR_	1.25	2.13	2.43	2.80
	(0.0048)	(0.0153)	(0.0358)	(0.0366)
MBI	99.50	99.49	99.44	99.36
_PS_	8.52			
_LC > MG_		89.1	91.2	92.5

Panel B: truncated sample 1969–2012

	MG	LC–L	LC–M	LC–H
Med	2.03	3.60	4.49	5.12
$LP_{5\%}$	1.52	1.51	1.56	1.49
_RR_	1.34	2.38	2.87	3.43
	(0.0054)	(0.0495)	(0.0488)	(0.0482)
MBI	99.99	99.84	99.62	99.54
_PS_	3.72			
_LC > MG_		77.8	79.6	81.6

This table reports statistics for the risk–return profile of the minimum guarantee strategy (MG) vs. the different life-cycle strategies (LC) on a 40-year accumulation period. _Med_ is the median IRR of the payout distribution, $LP_{5\%}$ is the IRR corresponding to the lower 5% quantile of the payout distribution, and _RR_ is the reward/risk ratio, defined as $RR = Med/LP_{5\%}$. Standard deviations of _RR_ are in parentheses. _MBI_ denotes the 'MoneyBack' indicator, which is defined as the probability that the final policy payouts exceed the total contributions paid, in real terms, _PS_ is the probability of shortfall (equation (14)), and _LC > MG_ indicates the share of cases for which the payout in the LC is higher than in the MG. Apart from the _RR_ ratio, all variables are expressed in percentage terms. The MG is based on the allocation $\pi^{EQ} = 5\%$; $\pi^{LB} = 95\%$, where π^{EQ} and π^{LB} are constant percentage wealth allocations to equities and long-term bonds, respectively. The nominal guaranteed rate is $G^F = 2.25\%$ for the full sample from 1969 to 2021 (panel A) and $G^T = 4.25\%$ for the truncated sample from 1969 to 2012 (panel B). The time-varying allocations of the three LC are (i) LC–L: $\pi_t^{EQ} = ((85 - \tau)/100)$, (ii) LC–M: $\pi_t^{EQ} = ((100 - \tau)/100)$, (iii) LC–H: $\pi_t^{EQ} = ((115 - \tau)/100)$, and $\pi_t^{LB} = 1 - \pi_t^{EQ}$ in all cases; τ is the age of the life-cycle investor, ranging from 25 to 65 years.

Consistent with the evidence in Figure 3, we find that all strategies offer a real capital protection with a very high probability. Indeed, both for the MG and the LC we observe that the 'MoneyBack' indicator (_MBI_), which is defined as the probability that the final policy payouts exceed the total contributions paid, in real terms, is close to 99.5%. Therefore, the risk mitigation approach underlying both the MG and the LC appears as quite effective in truncating the lower tail of the distribution of the _PMB_ ratio.

We observe that $LP_{5\%}$ increases with the riskiness of the investment strategy. This is due to the fact that, by raising the equity component, the expected return increases and, therefore, the distribution shifts to the right without widening too much. In fact, long-term predictability implies that, as we lengthen the time horizon, the volatility of equity becomes less and less relevant relative to the volatility of the risk-free asset and, in the long run, the two are comparable. The result that the reward–risk ratio _RR_ increases for the riskier allocations indicates that incrementing the equity component in the portfolios improves the performance even after adjusting for risk. Therefore, in the simulation we observe that the effect of increasing the average of returns dominates the effect of increasing the dispersion (on the left) of the returns. This result critically depends on long-term stationarity: long-term predictability allows us to reduce the long-term volatility that is otherwise over-estimated by integrated models.

The period of near-zero interest rates (from 2013 to 2021) included in the full sample has a negative impact on both the MG and the LC. In fact, the low level of the short rates reduces the ability of either strategy to protect the capital. This effect is relevant only for 'unfortunate' investors in the case of the LC, while it worsens dramatically the overall performance in the case of the MG, since it reduces substantially the minimum guaranteed rate (2.25% in our case) and the target rate. As a result, we observe in Table 3 that the probability for the insurer to have a loss, _PS_, is relatively high at 8.5%.

The comparison with the results in panel B, which refer to the truncated sample 1969–2012, provides a quantitative measure of the decrease in performance determined by the long period of low interest rates. Indeed, this alternative scenario is expected to be more favorable for an asset liability
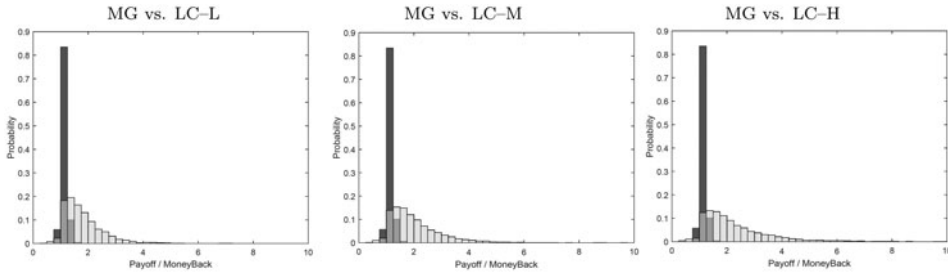
Figure 4. Payoff distribution on a 20-year accumulation period. This figure reports a comparison of the payoff distribution of the minimum guarantee strategy (MG) vs. the different life-cycle strategies (LC) on a 20-year accumulation period. The portfolio allocations are defined as in Figure 3, with the difference that in this case τ, the age of the life-cycle investor, ranges from 45 to 65 years. The distributions obtained for the MG (dark grey area) and the LC (light grey area) are based on the full sample from 1969 to 2021, with a nominal guaranteed rate equal to $G = 1.25\%$.

management strategy that may profit from a higher level of long-term rates to offer higher guaranteed rates (4.25% in our case).

In the truncated sample case of panel B, we find that the MG increases the *Med* by about 33% (from 1.5 to 2%), with a reward–risk ratio *RR* growing to 1.34, and exhibits a significant drop in the shortfall probability *PS* (from 8.5 to 3.7%).

Improvements in performance can be observed also for the LC, with the *Med* ranging from 3.6 to 5.1% and the reward–risk ratio *RR* from 2.4 and 3.4.

Overall, the difference in the risk–return comparison between the MG and the LC is relatively less striking for this truncated sample excluding the period of very low interest rates. In fact, the share of cases for which the payout in the LC is higher than in the MG, which is around 90% for the full sample, decreases to values around 80%.

However, even though under this scenario the affordable nominal minimal guarantee at fair market conditions is equal to a much higher 4.25%, the truncation of the upside for the MG remains so relevant that its performance is significantly lower than that of the more conservative LC scenario (LC–L), which grants an equivalent level of capital protection.

6. Robustness test

In this section, we test the robustness of the results with respect to the length of the period of accumulation.

Starting pension savings at age 25 and accumulating wealth for 40 years may be too optimistic in reality because of low liquid wealth at early ages and conflicting purposes of saving, such as buying a house or starting a family. Therefore, in what follows, we repeat the same analysis of the previous section for a shortened investment horizon based on a 20-year accumulation period.

For this shorter accumulation period, we adopt a starting allocation of equity which is 7.5% higher than the corresponding 40-year version, a change that raises the protection with respect to inflation.

We run the simulation only for the full sample period 1969–2021, with a nominal guaranteed rate for the MG equal to 1.25%.

Figure 4 shows that the distribution of the *PMB* ratio for the LC is significantly more skewed than for the MG, which means a thicker right tail and thus a higher upside potential for returns.

In Table 4, we observe that the return protection offered for the 95% of the sample by the MG and the LC are comparable ($LP_{5\%}$ equal to about 0.45% in all cases), while the median IRR of the payout distribution (*Med*) offered by the MG is substantially lower than the one granted by the LC: 0.49% vs. values ranging between 0.71 and 0.84%. The median return performances are only a third (MG) or a fourth (LC) of those achieved using the equivalent 40-year accumulation period investment strategy.

The reward–risk ratio *RR* is equal to 1.12 for the MG, while it is in a range between 1.57 and 1.86 for the LC, that is, an improvement between 40 and 66% with respect to the MG performance. We find

Table 4. Shorter (20 years) accumulation period

	MG	LC–L	LC–M	LC–H
Med	0.49	0.71	0.79	0.84
$LP_{5\%}$	0.44	0.45	0.45	0.45
RR	1.12	1.57	1.75	1.86
	(0.0027)	(0.0087)	(0.0201)	(0.0235)
MBI	94.28	96.88	97.06	96.94
PS	9.52			
LC > MG		87.3	89.3	89.7

This table reports statistics for the risk–return profile of the minimum guarantee strategy (MG) vs. the different life-cycle strategies (LC) on a 20-year accumulation period. The variables and the portfolio allocations are defined as in Table 3, with the difference that in this case τ, the age of the life-cycle investor, ranges from 45 to 65 years. The results refer to the full sample from 1969 to 2021, with a nominal guaranteed rate equal to $G = 1.25\%$.

that the payout in the LC–L is higher than the payout in the MG in about 87% of the cases, while this percentage increases to about 90% for the LC–H.

The shortening of the accumulation period reduces the level of protection, which in the case of the MG is signaled by a relatively high probability of shortfall, *PS*, at a level of 9.5%. The level of protection offered by the LC is about 97%, while that offered by the MG is only 94.3%, a value that is well below the benchmark level set by the Solvency II regulation, which prescribes a safety level of 99.5%.

To sum up, the risk–return profiles obtained for the MG and the LC over the shorter accumulation period confirm the main findings discussed in comparing the performances of the two strategies for the longer 40-year investment horizon.

7. Conclusions

Contributors to a pension plan must make a decision on how to allocate assets across various investment vehicles. Simulation analysis highlights that the capital protection provided by a life-cycle target-date fund strategy or a minimum guarantee strategy is comparable, while the cost of the protection is lower for life-cycle strategies.

The role of long-term predictable trends changes substantially the risk–return tradeoff scenarios faced by investors. In particular, investors modify the relative importance of volatility and inflation risks altering the fair price of volatility risk mitigation approaches, such as minimum guarantees.

Not unexpectedly, life-cycling in a low interest rates scenario achieves a better performance due to the larger stake of equity that young households allocate in their portfolio. In contrast, the insurers' asset allocation is tilted toward fixed income securities reflecting empirical evidence on the allocations that are conventionally applied to back minimum guarantee products. It is fair to say that this allocation choice is driven by the design of capital charges and regulatory constraints that penalize equity risk exposures in the attempt to lower the impact of market volatility on the insurer balance sheet. On the other hand, the action of the monetary stimulus raising prices of fixed income securities raises also the implicit cost determined by the shift in the allocation to the policyholder.

In summary, our simulations highlight a number of important aspects in the comparison between the minimum guarantee (MG) and the life-cycle (LC) strategies. First, MG-type investments are likely to produce weak performance results in a persistently low interest rate scenario. Here, the LCs appear to offer both a high level of capital protection and a high expected return, thanks to the potential return that equity investment can generate over the savers' life-cycle. Second, in economic scenarios characterized by higher inflation and interest rate levels, when MG-type investments are able to offer a sizeable minimum nominal guaranteed return rate, the LCs also score better in terms of risk–reward ratios. The limited capacity of the MG to capture upside performance is structurally related to the limitations imposed on the asset allocations of the insurance company to keep under control the risk of insolvency.

Our results show clearly that a proper asset allocation that delivers a good long-term performance in real terms requires investment flexibility and, in particular, the possibility that the investor takes an

exposure with respect to long-term systematic risks that are remunerated by the market. At the same time, the determination of the effective long-term risk–return tradeoff requires a careful modeling of the long-term trends that is often overlooked. In this respect, it will be important to analyze whether the role of these trends is fully endogenized by the current solvency regulation, as constant capital charges in presence of time-varying trends might generate a procyclicality of allocations detrimental for both financial stability and financial performance of the pension products.

The approach proposed in this paper is a step toward a better assessment of the influence of long-run trends on pension welfare and provides an empirical analysis of the direct impact of those trends on the effective cost and the risk–return tradeoff offered by these products. It paves the way for the design of new products that are optimized so that the welfare assessment – along the lines, for example, of Horneff *et al.* (2019*b*) – takes into account the indirect impact of trends on beliefs. This is a matter for future research.

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