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Quantum modal indeterminacy

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ABSTRACT

The paper investigates the existence and the nature of quantum indeterminacy in a particular realist interpretation of quantum mechanics, that is, the Modal Hamiltonian Interpretation (MHI)—together with an ontology of properties for quantum systems. In doing so, it serves a twofold purpose. First, it advances the debate on quantum ontology by showing precisely how and why indeterminacy arises in a quantum world—as described by the MHI. Second, it offers a naturalistic example of genuine metaphysical indeterminacy, an example coming from our best physics.

Metaphysical indeterminacy is, roughly, indeterminacy *in the world* as opposed to indeterminacy in our *knowledge* or *representation* of it. Until recently, the philosophical consensus was that there cannot be *metaphysical indeterminacy*. As Russell wrote in an unforgettable passage:

> *Apart from representation (…), there can be no such thing as vagueness* (…); things are what they are, and there is an end of it (Russell, 1923: 85, italics added).

Famously Dummett and Lewis thought that metaphysical indeterminacy was simply *unintelligible*.[1] And in the wake of Evan's and Salmon's arguments many agreed.[2] Things changed when detailed accounts of metaphysical indeterminacy were put forward.[3] Still, there is no consensus as to what constitutes a clear case of indeterminacy in the world. Candidates include—but are not limited to—*objects with fuzzy boundaries*, *future contingents*, and, finally, *quantum cases*. Indeed, it is a substantive question whether quantum mechanics, in any of its (realist) interpretations, offers examples of metaphysical indeterminacy. There is no agreement that this is in fact the case. Given the suspicion and skepticism that surrounded the very notion until very recently, one may feel motivated to find formulations of quantum theory that rule out metaphysical indeterminacy. An argument that starts from a realist interpretation of quantum theory and shows how this is committed to there being indeterminacy in the world would then serve a significant twofold purpose. On the one hand, it would provide a clear and precise reason of how and why metaphysical indeterminacy arises in quantum physics. On the other hand, it would provide a naturalistic example of genuine metaphysical indeterminacy as suggested by one of our best scientific theories. This is what the paper aims to do.

It argues that there is metaphysical indeterminacy according to a particular realist interpretation of quantum mechanics. In effect, the paper presents the first thorough investigation of *quantum metaphysical indeterminacy* (QMI) in the so-called *Modal Hamiltonian Interpretation* (MHI). In particular, it focuses on a recent proposal of an ontology of properties for the MHI, an ontology where physical systems are constituted by—if not identified with—"collections" of quantum observables.[4] It is not the aim of the paper to defend the MHI. Rather, the focus of the paper is on the interaction between the MHI and QMI, so to speak.[5] On the one hand, QMI provides a metaphysical framework that sheds light on crucial details of the MHI. On the other hand, MHI provides a significant example of a realist quantum interpretation that allows—if not outright requires—QMI. The rest of the paper is structured as follows. In §1 I provide a brief introduction to the MHI and to an ontology of properties. I then move on to discuss QMI and a particular account of it, the Determinable-Based Account (§2).[6] In §3 I discuss thoroughly QMI in

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[1] See Dummett (1975: 314), and Lewis (1986: 212).
[2] See Evans (1978), and Salmon (1981).
[3] See among others Akiba (2004), Barnes and Williams (2011), Wilson (2013), and Torza (2021).
[4] See e.g., Lombardi and Castagnino (2008), Lombardi et al. (2010), da Costa et al. (2013), Lombardi and Dieks (2016), and Lombardi (2019).
[5] For a similar project in the context of a different quantum interpretation, the relational interpretation, see Calosi and Mariani (2020).
[6] These sections contain all the details that are necessary to understand the following arguments. Yet, they do not provide an exhaustive picture. The interested reader is referred to Lombardi and Castagnino (2008) and Lombardi and Dieks (2021) for the MHI, and to Calosi and Mariani (2020) for QMI.

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the context of the MHI. In the conclusion (§4) I consider some implications and developments of the main arguments in the paper.

1. Modal interpretations of quantum mechanics

In general, modal interpretations of quantum mechanics distinguish between the *dynamical state* and the *value* state of a quantum system. The former is represented by the usual quantum state, and describes the properties the system *might* have—hence the term "modal". The quantum state always evolves unitarily, hence there is no collapse. By contrast, the latter describes the definite value properties the system *does* have. This is the main insight shared by all modal interpretations. It traces back to the original proposal in van Fraassen (1972, 1974).[7] The main difference between modal interpretations boils down exactly to the ascription of the value state. That is, different modal interpretations differ as to how they ascribe a set of *actualized* definite value properties to a physical system without violating quantum constraints given by no-go theorems such as e.g., the Kochen-Specker theorem. This general characterization of the main insight of modal interpretations already provides some details about this class of interpretations which will be crucial later on:

Realism: Modal interpretations are realist (no-collapse) interpretations of quantum mechanics.

"IF" eel: According to modal interpretations a quantum system can have an actualized definite value property even if the system is not in an eigenfunction of the corresponding operator. That is, the modal interpretations only retain one direction—the "if" direction—of the Eigenvalue-Eigenfunction Link (EEL)[8];

Limits to Joint Existence: According to the modal interpretations, non-commutativity of quantum observables, and in general no-go theorems such as the Kochen-Specker theorem,[9] impose serious constraints on the joint existence of definite valued properties that belong to the value state.

1.1. The Modal Hamiltonian Interpretation

As we saw, modal interpretations differ as to how they assign a value state to a given quantum system. Indeed, different interpretations single out a different *preferred contexts* for such an assignment.[10] Three such contexts are particularly significant:

Quantum State Alone: The preferred context for the assignment of definite value properties in the value state is given by the quantum state alone. That is, the value state depends solely from the dynamical (quantum) state of a system.[11]

Privileged Observables: The preferred context for the assignment of definite value properties in the value state is given by a set of privileged observables that always have a definite value, independently of the physical situation.[12]

Hamiltonian: The preferred context for the assignment of definite value properties in the value state is given by the Hamiltonian of the system. That is, the Hamiltonian fixes the preferred context in that it determines all the observables that will acquire definite values—thus figuring in the value state.[13]

The endorsement of Hamiltonian above yields exactly the MHI. According to such an interpretation one has the following First Actualization Rule[14]:

First Actualization Rule: Let s be a system with Hamiltonian $\hat{H}$. Then, the *definite valued properties* of s are $\hat{H}$ and all and only the properties that commute with $\hat{H}$.[15]

It is worth noting that the MHI provides a (viable) solution to the measurement problem, and is also able to account for many physical and chemical phenomena. As Lombardi (2019) writes:

> Besides the free hydrogen atom and the Zeeman effect, the MHI was applied to many other physical situations, leading to the results expected from a physical viewpoint; e.g., the free-particle with spin, the harmonic oscillator, the fine structure of atoms, the Born-Oppenheimer approximation (see Lombardi & Castagnino, 2008: section 5). Recently, the interpretation was applied to solve the problem of optical isomerism (Fortin, Lombardi, and Martinez Gonzalez 2018), which is considered one of the deepest problems for the foundations of molecular chemistry (Lombardi, 2019: 38).

In the light of the above, I submit, the MHI deserves to be taken seriously, and its metaphysical implications should be carefully considered.

1.2. An ontology of properties for the MHI

An interpretation of quantum mechanics may admit of several ontologies.[16] Recently, ontologies of properties and collections thereof have been explored for a number of interpretations, i.e., the relational interpretation championed by Rovelli[17] and the MHI. Let $\mathbb{O}$ be the space of self-adjoint operators acting on a Hilbert space $\mathcal{H}$. Then, for simple quantum systems there are a number of options available when setting forth an ontology of properties. For instance one may think of a system s as a "bundle" b of properties/observables $\hat{O}_i \subset \mathbb{O}$, and (some) eigenvalues of $\hat{O}_i$. Oldofredi (2021), as I read it, comes close to this view. In their development of the MHI, da Costa et al. (2013) claim that a quantum system is represented by $< \mathbb{O}, \hat{H} >$, where $\hat{H}$ is the time-dependent Hamiltonian. To be as general as possible, I will take a quantum system s to be represented by a collection of observables $\mathcal{C} = \{\hat{O}^i \subset \mathbb{O}\}$ and the set of (some of) its possible values $\{o^i_j\}$—that is, o^i_j represents a possible value of $\hat{O}^i$. This choice of notation will be clear shortly. A system s can be thus taken to be represented as follows: $< \mathcal{C} = \{\hat{O}^i \subset \mathbb{O}\}, \{o^i_j\} >$. I will also say—abusing terminology—that $< \mathcal{C}, \{o^i_j\} >$ "constitutes" quantum

[7] For the most detailed account by van Fraassen, see in particular, van Fraassen (1991: 279).

[8] In context, this is a formulation EEL: A system s is in an eigenfunction of $\hat{O}$ that corresponds to v iff s has in the value state a definite valued property that corresponds to the eigenvalue v of $\hat{O}$.

[9] This is a rough formulation of the theorem: in a Hilbert space $\mathcal{H}$ with dimension ≥ 3 it is impossible to assign definite value properties to all quantum observables while preserving the functional relations between commuting observables. The original paper is Kochen and Specker (1967). For an accessible introduction and references see Held (2018).

[10] I borrow the terminology from da Costa et al. (2013).

[11] These are known as Biorthogonal-decomposition and spectral-decomposition modal interpretations. They are usually associated with the work of Simon Kochen and Dennis Dieks. For an introduction and complete list of references, see Lombardi and Dieks (2021).

[12] Bub (1997) goes as far as claiming that Bohmian mechanics is really a modal interpretation where the set of privileged observables is the set of *positions*.

[13] One should note that it does not fix *which* value they will take. There is a clear sense in which Hamiltonian provides a list of privileged observables—as Privileged Observables does. The main difference is that the privileged observables in Hamiltonian change with the physical situation—apart from the Hamiltonian itself. By contrast, at least in some proposals, the privileged observables of Privileged Observables are independent of any physical situation. If one agrees with the diagnosis in Bub (1997), perhaps the best example of this view would be Bohmian Mechanics. Thanks to an anonymous referee here.

[14] See e.g., da Costa et al. (2013: 3675), Lombardi (2019: 35), and Lombardi and Dieks (2021: §9).

[15] There is also a symmetry requirement to the point that commuting observables have at least the same symmetries of $\hat{H}$. I will leave it aside, as this requirement does not play a role in what follows—and it is satisfied in the examples I shall use.

[16] For a recent critical discussion about this point and its consequences, see Egg (2021).

[17] See e.g., Oldofredi (2021).

system *s*.[18] To wit, this raises the substantive question about the nature of this "constitution" relation. Arguably, the two candidates that have attracted the most attention in the literature are *compresence*, and *mereological composition*. One could also take constitution as a new *sui generis* primitive.[19] No matter how "constitution" is spelled out, the crucial thing to note here is that this is not just a *representation* relation. It is a *building* relation that constructs quantum systems out of properties, so to speak. Let me then provide some crucial details of an ontology of properties for the MHI. I will mostly follow the most-developed proposal in the literature, namely da Costa et al. (2013). However, my formulations are (sometimes) slightly different, and go beyond the original one, for reasons that will be obvious in due course. We are particularly interested in three claims. I will focus on two of them first[20]:

Existence of "Type Properties": There are "type properties" [A], [B], [C], (…), that have countless instances $[A^i]$, $[B^i]$, $[C^i]$, (…);

Existence of "Case Properties": There are "case properties" $[a^i_j]$, $[b^i_j]$, $[c^i_j]$, (…), that correspond to definite values of instances of "Type Properties"

thinsp; $[A^i]$, $[B^i]$, $[C^i]$, (…);

I take it that, in "metaphysical" jargon, type property [A] stands for a *universal* rather than a *trope*, in that it is a *repeatable* property. Indeed, as Existence of "Type Properties" explicitly acknowledges, type properties have countless instances.[21] We don't need to take a deep dive into the metaphysics of properties. We can just simply look, in our discussion, to instances of type properties. Consider now a particular quantum system *s* constituted by $< \mathcal{C} = \{\hat{O}^i \subset \mathbb{O}\}, \{o^i_j\} >$. It will have type properties $[A^1]$, $[B^1]$, (…), $[H^1]$ each of which will have possible values $[a^1_i], [b^1_j], (\ldots), [\omega^1_k]$. A type property $[A^1]$ is represented by $\hat{A}^1 \subset \mathbb{O}$, and the case property $[a^1_j]$ is represented by the *j*th eigenvalue of $\hat{A}^1$. That is, for type property $[A^1]$ and one of its case properties $[a^1_j]$ the classical eigenfunction equation holds—with self-evident notation:

$$\hat{A}^1 |a^1_j\rangle = a^1_j |a^1_j\rangle \tag{1}$$

All of this is actually silent on which properties of the relevant quantum system are *actual*, that is, in the terminology of §1, which definite value properties belong to the *value state* of a system *s* at a given time. For this we need another rule:

Second Actualization Rule: For any $[A^i]$, among its possible case properties $[a^i_j]$ *at most one* becomes actual (at a given time), i.e., figures in the value state of *s* at a given time.

The two actualization rules for the MHI give us the definite valued properties of the value state of system *s* at a given time: these are the case properties of type properties that commute with the Hamiltonian (First Rule). Of all the possible case properties of a given type property that commute with the Hamiltonian *only one* case property goes in the value state (Second Rule). Now, suppose that there is a type property [*O*] that *does not commute* with the Hamiltonian. By the first rule, the system *s* does not have any case property of that type property. da Costa et al. (2013) write:

> [A]n instance $[A^i]$ of the universal type-property [*A*] *may or may not actualize* [i.e., acquire a definite value]. This is a consequence of the Kochen-Specker theorem, which establishes one of the central differences between the quantum world and the classical world: in the quantum case, *omnimode determination does not hold*[22] (da Costa et al., 2013: 3676, italics added).

Let me now introduce a simple formal notation whose usefulness will become evident. Let $A^1(x)$ stand for "quantum system *x* has type property $[A^1]$".[23] Similarly $a^1_j(x)$ abbreviates "quantum system *x* has definite value case property $[a^1_j]$ of type property $[A^1]$". Then, using higher-order quantification, we are interested in the universal closure of the following principles[24]:

'Quantum' Requisite Determination $$O^i(x) \rightarrow \exists o^i_j(o^i_j(x)) \tag{2}$$

'Quantum' At Most One Determination $$o^i_j(x) \wedge o^i_k(x) \rightarrow o^i_j = o^i_k \tag{3}$$

'Quantum' Requisite Determination requires that all type-properties have a corresponding definite value property. I take that this is da Costa et al. (2013)'s *Omnimode Determination*. 'Quantum' At Most One Determination requires that actual definite value properties of a given type property in the value state are—*if* there are any—*unique*.[25]

Before we move on to (alleged) quantum examples of metaphysical indeterminacy, I should note that, as of now, the account is silent as to which "type properties"—beside the Hamiltonian—figure in the constitution of a quantum system. Perhaps some "type properties" are indeed *essential* to physical systems—that is, very roughly, properties a system has necessarily throughout its existence. Perhaps some are not, and can be gained or lost. As we shall see shortly, this will turn out to be important.[26]

2. Determinable quantum metaphysical indeterminacy

As I noted already, metaphysical indeterminacy is indeterminacy in the world, as opposed to our representation or knowledge of it.[27] Recently, the quantum failure of value definiteness has been investigated as one—if not *the*—paradigmatic example of metaphysical indeterminacy.[28] This is what is usually referred to as QMI. There are several accounts of quantum indeterminacy. In this paper we shall focus on one such account, namely the so-called determinable based account.

In a nutshell—and slightly abusing terminology—according to the determinable based account, there is metaphysical indeterminacy when an object instantiates a determinable property but no unique determinate of that determinable property (at some level of determination).[29]

This traces back to Wilson (2013), and has been developed for the quantum domain in e.g., Bokulich (2014), Wolff (2015), Calosi and Wilson (2018, 2021), Mariani and Torrengo (2021), Mariani (2021, Forthcoming), and Schroeren (2021) to mention a few. There are different reasons for this restriction. The first and foremost is that, as I will argue in §3, there is a significant parallel between determinable based indeterminacy and the MHI. Indeed, I will argue that the proposal for an ontology of

[18] This is an abuse of terminology insofar as $< \mathcal{C} = \{\hat{O}^i \subset \mathbb{O}\}, \{o^i_j\} >$ contains only *mathematical objects*. Strictly speaking, the mathematical objects—at least *some* of them—*represent* properties, and these properties constitute quantum systems.

[19] The choice between these alternatives is a substantive metaphysical issue that goes well beyond the scope of the present paper. I used the deliberately vague, general notion of "constitution" to signal that I do not want—nor need—to commit to a precise constitution relation.

[20] They correspond to "Proposition 1–3" in da Costa et al. (2013).

[21] Which I take to be identical, hence *universals*.

[22] The name of the principle comes from Wolff's *Philosophia Rationalis Sive Logica*. Indeed the principle was a cornerstone of modern philosophy, from Wolff himself to Bernoulli and Kant. See Lombardi and Dieks (2016).

[23] That is to say, A^1 is a predicate whose semantic value is a property $[A^1]$ represented by operator $\hat{A}^1$.

[24] I chose the following terminology for the analogy with principles of determination that will be introduced in the next section.

[25] This formulation actually hides some subtleties about the logic of the relevant determination relation. See Calosi (Forthcoming).

[26] Thanks to an anonymous referee for pressing me on this point.

[27] See e.g., Barnes (2010).

[28] See, among others, Darby (2010), Skow (2010), Calosi and Wilson (2018, 2021), Darby and Pickup (2019), Torza (2020), Mariani (Forthcoming), and Schroeren (2021). For a critique see e.g., Glick (2017).

[29] I will provide more details shortly.

properties in the MHI developed in §2 provides an example of QMI according to the determinable based account. Second, one of the most widely held alternative accounts, namely metaphysical supervaluationism, at least in its original formulation,[30] seems to run afoul of no-go theorems such as the Kochen-Specker theorem.[31] But modal interpretations in general take the conclusions of the no-go theorems extremely seriously, as witnessed by Limits to Joint Existence in §1.[32]

As I pointed out already, determinable based indeterminacy boils down to a particular pattern of instantiation of *determinable* and *determinate* properties. As Wilson (2017) writes,

> [D]eterminables and determinates are in the first instance type-level properties that stand in a distinctive specification relation: the "determinable determinate" relation (for short, "determination"). For example, *color* is a determinable having *red*, *blue*, and other specific shades of color as determinates; *shape* is a determinable having *rectangular*, *oval*, and other specific (including many irregular) shapes as determinates; *mass* is a determinable having specific mass values as determinates (Wilson, 2017: 1).

The determination relation is a primitive relation,[33] whose behavior is regimented by different principles.[34] Two different principles will be of interest to us, namely Requisite Determination, and At Most One Determination. According to the former, everything that has a determinable property has a determinate (at each level) of that determinable. According to the latter, no thing has more than one determinate (at the same level). Implicit in these formulations is the thought that determinables admit of different *levels of determination*, so that the characterization of a property as determinable and determinate is relative to levels. By way of illustration, red is a determinate of "color" but a determinable of "scarlet". To keep things manageable, I will assume that there are no *intermediate* levels of determination. That is, I will restrict my attention to two levels of determination. Thus, I will only consider *maximally unspecific determinables* (MUD) and *maximally specific determinates* (MSd). A *maximally unspecific determinable* is a determinable property that is not a determinate of any other property. Conversely, a *maximally specific determinate* is a determinate property that is not a determinable of any other property. Let now $D^i(x)$ abbreviate "x has the maximally unspecific determinable property $[D^i]$", and let $d^i_j(x)$ abbreviate "x has the maximally specific determinate property $[d^i_j]$ of $[D^i]$". Then, using higher order quantification once again, the two principles of determination we are interested in are given by the universal closure of the following:

Requisite Determination $$D^i(x) \rightarrow \exists d^i_j(d^i_j(x)) \quad (4)$$

At Most One Determination $$d^i_j(x) \wedge d^i_k(x) \rightarrow d^i_j = d^i_k \quad (5)$$

These principles are interesting for two reasons. The first one will be explored here, and the second one will be explored in the next section. The reason to be explored here is that these are exactly the principles that take center stage in the characterization of determinable based indeterminacy. In the words of Wilson:

> What it is for a state of affairs to be MI in a given respect *R* at a time *t* is for the state of affairs to constitutively involve an object (more generally, entity) *O* such that (i) *O* has a determinable property *P* at *t*, and (ii) for some level *L* of determination of *P*, *O* does not have a unique level-*L* determinate of *P* at *t* (Wilson, 2013: 366).

Abusing terminology once more, I will say that for x to be indeterminate—with respect to a given property—is for x to have a maximally unspecific determinable without having a unique maximally specific determinate of that determinable. Given the restriction about two levels of determination, there are two ways some x can have a determinable without having a unique determinate of that determinable. Either it has *no determinate*—Wilson calls this case *gappy indeterminacy*—or it has *more than one determinate*—Wilson calls this case *glutty indeterminacy*.[35] This makes it clear why I focused on Requisite Determination and At Most One Determination above. Cases of *gappy indeterminacy* correspond to violations of Requiste Determination, whereas cases of *glutty indeterminacy* correspond to violations of At Most One Determination.

3. Modal determinable indeterminacy

In this section I will argue that the ontology of properties for the MHI developed in §1 provides us with an example of determinable based indeterminacy, in particular of *gappy indeterminacy*. I will then go on to investigate some features of said indeterminacy (§3.2). Before I am able to do that, I need a systematic way to compare claims about type properties and case properties on the one hand, and claims about determinable and determinate properties on the other. This is addressed in the next section (§3.1).

3.1. Translation

The ontology of properties for MHI in §1 mentions type and case properties, whereas determinable based indeterminacy mentions determinable and determinate ones. Luckily enough, translations between the two languages are easy. That is because in standard presentations of quantum mechanics (maximally unspecific) determinables are represented by self-adjoint operators and (maximally specific) determinates are represented by eigenvalues. Physics textbooks that feature such standard presentation include—but are not limited to—Baym (1969: 59–62), Gillespie (1970: 42–47), Beltrametti and Cassinelli (1981: 14–29), and Norsen (2017: 33–36). Among the texts in philosophy of physics one can mention Hughes (1989: 60), Albert (1992: 40-43), Lewis (2016: 72-74), and Maudlin (2019: 62-69). By way of illustration, consider the spin-operator in an arbitrary direction φ (for a spin $\frac{1}{2}$-particle), $\hat{S}_\varphi = \begin{pmatrix} z & x-iy \\ x+iy & -z \end{pmatrix}$ with eigenvalues $\pm\frac{1}{2}$,[36] or the momentum operator $\hat{P} = -i\hbar\frac{\partial}{\partial x}$ with eigenvalues $p = \hbar k$.

Determinables are represented by instances of the self-adjoint operator $\hat{S}_\varphi = \begin{pmatrix} z & x-iy \\ x+iy & -z \end{pmatrix}$ and the self-adjoint operator $\hat{P} = -i\hbar\frac{\partial}{\partial x}$, whereas determinates are represented by eigenvalues $\pm\frac{1}{2}$ and $p = \hbar k$ respectively. Fig. 1 above sums up such determination structure—where determination goes downward along the lines.[37]

[30] See e.g., Barnes and Williams (2011). For a development focusing on quantum indeterminacy see Mariani, Michels and Torrengo (Forthcoming).

[31] See e.g., Darby (2010), and Skow (2010). Very roughly, the argument is that metaphysical supervaluationsim requires all quantum observables to have a definite value at all times. And this is exactly what the Kochen-Speker theorem precludes.

[32] Note also that recent objections to the determinable based account in Fletcher & Taylor (Forthcoming) do not apply in the present context. This is because they rely on the full strength of the EEL. And we saw already that modal interpretations do not subscribe to the full force of the EEL.

[33] Indeed, one can construct the entire space of determinable and determinate properties out of determination alone. See Calosi (Forthcoming).

[34] See Wilson (2017)—from which I borrow the terminology.

[35] One needs to be reminded that advocates of glutty indeterminacy are crystal clear that the different determinates are either had relative to a given perspective, or to a different degree. That is, they are not had *simpliciter*. As it will be clear shortly this will play nor role in the paper.

[36] One gets a different operator for each direction φ. For a substantive discussion of different metaphysical options see Corti and Sanchioni (2021).

[37] Note that the figure is not intended to provide a complete, detailed graphic representation of the determination structure. Rather, it is supposed to provide a tentative visualizable guide.

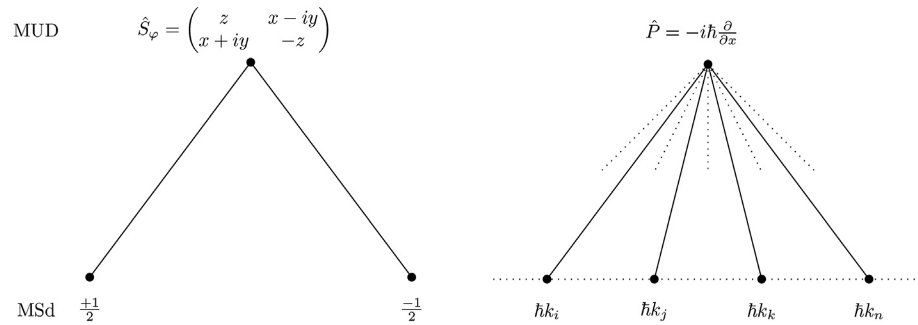

Fig. 1. Quantum determinables and determinates.

Table 1
The translation table.

Mathematics	Physics	Metaphysics$_1$	Metaphysics$_2$
Self-adjoint Operator $\hat{O}^i$	Observable	Type property $[O^i]$	Maximally Unspecific Determinable $[O^i]$
Eigenvalue o^i_j	Definite-valued property	Case Property $[o^i_j]$	Maximally Specific Determinate $[o^i_j]$

Now, recall that self-adjoint operators and eigenvalues are exactly the mathematical objects that represent type and case properties. Therefore, it stands to reason to simply identify type properties with (maximally unspecific) determinables, and case properties with (maximally specific) determinates, as in Table 1 below.[38]

Indeed, quantum type properties are but an *example* of determinables, and quantum case properties are but an *example* of determinates. With this in hand, it is easy to see the usefulness of the simple formal notation I used to phrase principles (2)–(5). This is because it is clear upon inspection that Quantum Requisite Determination (2) is but an example of Requisite Determination (4), and that Quantum At Most One Determination (3) is an example of At Most One Determination (5). One obtains the quantum examples by restricting the higher-order variables of the general principles (3) and (5) to quantum observables and their eigenvalues. This is important not only on its own. It also gives us a way to translate determinable indeterminacy in the context of the MHI. Here is my proposal:

Quantum Metaphysical Indeterminacy—for MHI: A quantum system s is metaphysically indeterminate with respect to observable $\hat{O}^j$ iff $< \mathcal{C} = \{\hat{O}^i \subset \mathbb{O}\}, \{o^i_j\} >$ constitutes s, and $< \mathcal{C} = \{\hat{O}^i \subset \mathbb{O}\}, \{o^i_j\} >$ contains the type property $[O^j]$ but no unique case property $[o^j_k]$ of $[O^j]$.

In general, even in this case there are two ways a system s can fail to have a unique case property $[o^i_j]$ of type property $[O^i]$: either it has no case property, or it has more than one. Given our translation scheme, these correspond to violations of Quantum Requisite Determination and At Most One Determination respectively. Going back the quote from da Costa et al. (2013) in §1.2, one sees that they expect violations of Quantum Requisite Determination, but endorse At Most One Determination. Indeed, the latter follows from Second Actualization Rule. This rules out cases of *glutty* QMI. By contrast, if we could find violations of the former, there would be quantum indeterminacy of the *gappy* variety. I am going to argue next that this is in fact the case.

3.2. Gappy indeterminacy in the MHI

One of the most common argument in favor of the existence of QMI crucially depends on the "only if" part of the EEL. Very roughly the argument has it that, given the "only if" part of the EEL, a quantum system has a definite value property $O = v$ only if its state is an eigenfunction of $\hat{O}$ that corresponds to value v. Thus, when the state is not such an eigenfunction the system, so the thought goes, is metaphysically indeterminate with respect to O.[39] But, as we saw in "IF EEL", the "only if" part is exactly the part of the EEL that is jettisoned in the MHI. Therefore, the EEL-argument in favor of the existence of QMI is not available in the present context. I contend that the key to argue in favor of the existence of QMI in the MHI is the First Actualization Rule. According to such a rule the only definite valued properties—that is, maximally specific determinate properties in the value state—are the ones that commute with the Hamiltonian.[40] The argument is best appreciated by focusing on an example. Consider the commutator of the Hamiltonian and the Momentum operator:

$$[\hat{H}, \hat{P}] = \left[\frac{\hat{P}^2}{2m} + V, \hat{P}\right] = i\hbar\frac{\partial V}{\partial x} \tag{6}$$

Given (6), in any case in which the potential V does not depend on x, $\hat{H}$ and $\hat{P}$ commute, as $\frac{\partial V}{\partial x} = 0$. Let me now consider one simple such case, that of a quantum *free particle*—for which one simply sets $V(x) = 0$. In this case plane waves

$$\psi(x) = e^{ikx/\hbar} \tag{7}$$

are eigenfunctions of both the Momentum and the Hamiltonian operator. Quantum mechanics now dictates that the Hamiltonian *does not commute with the Position operator* $\hat{X}$. It follows then from the First Actualization Rule that the particle does not have any definite value position property. That is, it does not have any case-maximally specific determinate position.

It remains to be seen that the particle has the type-maximally unspecific determinable property of position, that is, *it is somehow located somewhere in space.*[41]

[38] The first three columns in Table 1 are similar to Lombardi and Castagnino (2008: 397) and da Costa et al. (2013: 3674).

[39] See e.g., Calosi and Mariani (2021).

[40] The First Actualization Rule entails that the Hamiltonian itself is always determinate, that is determinate for any instant t. Yet, nothing in the MHI entails that it is determinate which value the Hamiltonian will have at a time t^* later than t. One might push the point that this represents another form of *indeterminacy*, closely related to future-contingent indeterminacy. It is a substantive question whether determinable-based indeterminacy can account, in general, for future-contingent indeterminacy.

[41] The claim is defended in e.g., Calosi and Wilson (2021): §4.2). As for another example, consider the following passage from Schroeren (2021):[T]he thesis that particles which are not in an eigenstate of position nonetheless instantiate a determinable of position is a natural way to cash out the claim that the particles are in space despite lacking precise positions (Schroeren, 2021: §5.4).

One possible argument in favor of that claim is simply that this is actually *definitional of material objects* such as particles. The philosophical pedigree of such a view is nothing short of impressive. It is a central tenet of e.g., both Descartes's and Hobbes's metaphysics of material objects. In contemporary philosophy the view is endorsed in Quine (1976), and has been recently defended in Markosian (2000), Hudson (2005) and Schaffer (2009) to mention just a few.[42]

However one may be unconvinced by the previous argument, particularly because it rests upon a very broad and general metaphysical view. But there are other arguments available—interestingly related to that one. For instance, suppose one defines an operator $\hat{O}$ that projects onto the subspace of Hilbert space associated with a location anywhere in space. One could then use the argument before to actually claim that the particle *determinately* has the property represented by that operator. Granted. But now, suppose we define a set-theoretic partition over spatial regions. That is, we divide the entire space into disjoint regions that sum up to the entire space. In fact, let us define a partition—idealizing just slightly—into "minimal regions", regions r_i such that the particle cannot occupy any proper subregion of any r_i. The we could define operators $\hat{P}_i$ such that every $\hat{P}_i$ projects onto the subspaces associated with region r_i.[43] Now, the different projection operators $\hat{P}_i$ seem to qualify as determinates of the determinable $\hat{O}$.[44] To see that, note that they are *specific* ways of "being anywhere in space"—exactly as the determinate "red" is a specific way of being colored. Moreover, the construction guarantees that for any $i \neq j$, $\hat{P}_i$ and $\hat{P}_j$ project onto orthogonal subspaces. That is, they represent incompatible properties—exactly as determinates at the same level of determination represent. Recall that the free-particle Hamiltonian does not commute with $\hat{X}$. Hence, it does not commute with any of the $\hat{P}_i$-s either. Thus, the particle does not have any specific position-location associated with any $\hat{P}_i$—and their eigenvalues. We already granted that the particle has the property represented by $\hat{O}$. Thus, the particle has the property represented by the determinable $\hat{O}$ but no unique determinate of it at the level of the $\hat{P}_i$-s—and their eigenvalues. The desired conclusion now follows: according to Quantum Metaphysical Indeterminacy—for the MHI, a quantum free particle is metaphysically indeterminate with respect to $\hat{X}$ ($\hat{O}$). In particular, this is an example of *gappy* indeterminacy, in that it provides a counter-example to Quantum Requisite Determination.[45]

Let us look at another example. Equation (6) also tells us that any Hamiltonian where the potential V depends non-trivially on x—and indeed, for any potential V that is not invariant under translations—does not commute with the momentum operator $\hat{P}$. Now, suppose a particle is moving in such a non-trivial potential V. Then, it follows from the First Actualization Rule that it has no determinate eigenvalue of $\hat{P}$. But it is definitional of moving particles that they have momentum. As we saw, $\hat{P}$ and its eigenvalues are the paradigmatic examples of determinable-determinate properties. This is yet another case in which a system has a determinable and no (unique) determinate of that determinable according to the MHI. We can actually provide a specific example of such a quantum system. Consider the simple *quantum harmonic oscillator*. The Hamiltonian is given by:

$$\hat{H} = \frac{\hat{P}^2}{2m} + V(x) = \frac{\hat{P}^2}{2m} + \frac{1}{2}m\omega^2\hat{X}^2 \qquad (8)$$

The potential in (8) is not invariant under spatial translations and thus the Hamiltonian does not commute with $\hat{P}$. Hence the system does not have any precise eigenvalue of momentum according to the MHI, given its First Actualization Rule. Yet, the system is *surely in motion*. These is because the allowed energies are given by:

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega \qquad n = 0, 1, 2, 3, (\ldots) \qquad (9)$$

These are all greater than 0, the energy ground-state E_0 included. This provides a clear physical example of a quantum system that has the determinable momentum $\hat{P}$—given that, as we saw, it moves—but no precise determinate eigenvalue of momentum—because the Hamiltonian does not commute with $\hat{P}$. The same conclusion we drew for the free-particle holds for the quantum harmonic oscillator as well. According to Quantum Metaphysical Indeterminacy—for the MHI, a quantum harmonic oscillator is metaphysically indeterminate with respect to $\hat{P}$. In particular, this is an example of *gappy* indeterminacy, in that it provides a counter-example to Quantum Requisite Determination.

Once the existence of QMI has been established, an interesting question arises as to whether this indeterminacy is *fundamental* or *derivative*. This arguably depends on subtleties about the notion of fundamentality.[46] There are (at least) two views on fundamentality that are relevant here. According to the first view, the fundamental entities determine (or fix) everything else.[47] I shall refer to this view as the *All Determining* view of fundamentality. According to the second view, the fundamental entities are not determined (or fixed) by anything else. I shall refer to this second view as the *Undetermined* view of fundamentality.[48] Under the first view, the *All Determining* view, the indeterminacy in MHI is arguably fundamental: one needs to fix which collection of properties constitute quantum systems (and which do not) to determine (or fix) everything else. And we saw that some such collections display indeterminacy. Under the second view, the *Undetermined* view, there is more leeway to argue that indeterminacy is derivative. This is because one can insist that whether a particular collection of properties constitutes a given quantum system is (at least partially) determined (or fixed) by the *existence* of type/determinable properties and case/determinate properties in the collection. In any event, what seems clear is that the indeterminacy at hand is *not eliminable*. Indeterminacy is a consequence of one of the constitutive rules for property attribution to quantum systems. Indeed, it is a consequence of the very *metaphysical constitution* of quantum systems in the first place.[49] This is important because it provides a challenge to a recent argument due to Glick. Glick (2017)—but see also Glick (Forthcoming)—argues that realist interpretations of quantum mechanics do not feature fundamental indeterminacy. And that derivative indeterminacy is *eliminable*. Glick focuses

[42] On certain views about essence and definition this translates into the claim that it is *essential* to a material object to have a certain maximally unspecific position—that is location in spacetime.

[43] Indeed, Whitman (1962) shows how to construct $\hat{X}$ out of such projection operators. For a contemporary construction along these lines, see Pashby (2016).

[44] Nothing so far entails that they are maximally specific determinates. The following argument does not require this further step. If one were to be nit-picking one would have to formalize different axioms of determination quantifying over levels of specifications, and modify formulations accordingly. See Calosi (Forthcoming) for the logical details.

[45] And a fortiori, a counterexample to Requisite Determination (4).

[46] Many such details go way beyond the scope of the paper, and I will rest content with a very general picture.

[47] I used a deliberately vague notion of "determining" or "fixing" for I don't want to commit to specific relations such as supervenience, grounding, ontological dependence, and the like.

[48] For a discussion of these two notions of fundamentality, see e.g., Bennet (2017). Note that the two notions can come apart. See e.g., Leunberger (2020).

[49] Recall the discussion in §1.2. Talk of "constitution" should be taken metaphysically seriously. The claim that a collection $< \mathcal{C}, \{o_j^i\} >$ constitutes a quantum system *s* is not *just* a "representational claim". Now, as I pointed out in footnote 18 strictly speaking the mathematical objects–the operators—represent properties and these properties constitute quantum systems. One may try to argue that, insofar as not every operator represents a genuine physical property, there is still wiggle room to argue that the indeterminacy at hand is representational in nature. The response here would be that the cases discussed in the main text are exactly cases in which almost anyone agrees that the operators—position and momentum—do in fact represent genuine physical properties.

only on three realist quantum interpretations, the most widely discussed ones: Bohmian mechanics, spontaneous collapse theories, and many worlds. No matter what stance one takes on Glick's original argument, there is no reason to limit oneself to those realist interpretations. Indeed, modal interpretations are explicitly realistic interpretations—see Realism in §1—that seem to challenge Glick's argument. They offer an example (at least some of them) of a realist interpretation of quantum mechanics where indeterminacy is fundamental—if one endorses the *All Determining* view of fundamentality—or it is derivative—if one endorses the *Undetermined* view—but ineliminable.[50]

4. Conclusion

Summing up, I argued that there is ineliminable metaphysical indeterminacy according to one significant interpretation of quantum theory—supplemented with an ontology of properties. As I pointed out in the Introduction this is crucial for (at least) two reasons. First, it advances the debate on quantum ontology by showing precisely how and why indeterminacy arises in a quantum world, as described by the MHI. Second, it offers a naturalistic example of genuine metaphysical indeterminacy, an example coming from our best physics.

To conclude let me discuss briefly some implications and developments of the arguments in the paper. If the arguments in the paper are on the right track, they show that yet another quantum interpretation suggests that there is quantum indeterminacy.[51] Arguably, there are ways to avoid the existence of indeterminacy in at least some such interpretations.[52] Yet, the sheer amount of interpretations that are hospitable to—or even entail—the existence of QMI should give us pause: perhaps indeterminacy is indeed a crucial feature of quantum worlds—be it at the fundamental or derivative level. Relatedly, the arguments also show that the determinable based account of QMI is flexible enough to provide an account of QMI in many different interpretations. This, I contend, provides a (defeasible) argument in its favor.

As for possible developments, two issues seem to naturally arise from the discussion in the paper. The first one is whether its conclusions carry over to *other modal interpretations*, in particular the ones that endorse Quantum State Alone rather than Hamiltonian—see §1.1. Arguably, the first thing to do is to see whether different actualization rules entail the existence of QMI. The second one is whether the arguments carry over to *other ontologies* of properties for different quantum interpretations, at least those that endorse Limits to Joint Existence—see again §1.1.[53] These are the proverbial stories for another time.

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[50] One may simply protest that this entire discussion is somewhat misleading. Surely, the thought goes, non-relativistic quantum mechanics is not fundamental. Therefore, any indeterminacy that stems from it is derivative. I agree. But, in context, it is clear that the question is whether *relative to the description of the world offered by non-relativistic quantum mechanics*, the indeterminacy at hand is fundamental or derivative.

[51] See Schroeren (2021) for the orthodox interpretation, Mariani (Forthcoming) for GRW, Wilson (2020) and Calosi and Wilson (2021) for Everettian many worlds interpretations, and Calosi and Mariani (2021) for relational quantum mechanics.

[52] See Glick (2017) for the orthodox interpretation, and Glick(Forthcoming) for GRW.

[53] See Oldofredi (2021) for relational quantum mechanics.

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