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Table 1: List of symbols and acronyms used in Chapter 1

Symbol	Meaning	First occurrence
$\ln(C_t)$	Households consumption (log of)	Section 1.2
$\ln(Y_t)$	Households disposable income (log of)	Section 1.2
π_t^{GDP}	Inflation rate (GDP deflator)	Section 1.7
π_t^{PCE}	Inflation rate (Private Consumption Expenditure)	Section 1.2
y_t	Output gap	Section 1.7
i_t	Short-term interest rate	Section 1.7
u_t	Unemployment rate	Section 1.3.3
EU_t	Households Unemployment Expectations Index	Section 1.2
$N_t[\bullet]$	Professional forecasters expectation	Section 1.3.1
$M_t[\bullet]$	Population-mean value of households expectations	Section 1.3.1
$E_t^i[\bullet]$	Expectation of household i	Section 1.3.1
ϵ_t	Disturbance of the fundamental value	Section 1.3.1
η_t	Innovation in the fundamental value	Section 1.3.1
λ_t	Probability of being "infected" by the expert forecast	Section 1.3.1
α	Constant term of the fundamental value process	Section 1.3.2
β	Autoregressive coefficient of the fundamental value process	Section 1.3.2
μ	Fraction of *stubbornly pessimistic* households	Section 1.5.2
γ	Parameter determining the qualitative expectation	Section 1.5.2

Acronym	Meaning	
FIRE	Full Information Rational Expectation	Section 1.1
CSI	Common-source-infection	Section 1.1
EPU	Economic Policy Uncertainty Index	Section 1.4
GUI	Google Uncertainty Index	Section 1.4

Table 2: List of symbols and acronyms used in Chapter 2

Symbol	Meaning	First occurrence
$N_t[\bullet]$	Professional forecasters expectation	Section 2.2
$M_t[\bullet]$	Population-mean value of households	Section 2.2
$E_t^i[\bullet]$	Expectation of household i	Section 2.2
edu	Education level: LTHS, HS or College	Section 2.3
$IST^U_{edu,t}$	Households Unemployment Expectations Index	Section 2.3
u_t	Unemployment rate	Section 2.3
ϵ_t	Disturbance of the fundamental value	Section 2.2
η_t	Innovation in the fundamental value	Section 2.2
λ_{edu}	Probability of being "infected" by the expert forecast	Section 2.2
α	Constant term of the fundamental value process	Section 2.2
β	Autoregressive coefficient of the fundamental value process	Section 2.2
μ_{edu}	Fraction of *stubbornly pessimistic* households	Section 2.4
γ	Parameter determining the qualitative expectation	Section 2.4

Acronym	Meaning	First Occurrence
CSI	Common-source-infection	Section 2.1
LTHS	*Less than High School* education level	Section 2.3
HS	*High School* education level	Section 2.3
College	*College* degree of higher education level	Section 2.3

Table 3: List of symbols and acronyms used in Chapter 3

Symbol	Meaning	First occurrence
H	Horizon of the forecast	Section 3.2
T_H	Warming in period H	Section 3.2
T_t	Warming in period t	Section 3.2
θ_T	Lower bound of the support of the random variable T_H	Section 3.2
g_0	Constant growth rate of GDP (with no warming)	Section 3.2
g_t	Growth rate of GDP in period t (depends on warming)	Section 3.2
γ	Damage parameter (in growth rate)	Section 3.2
β	Damage parameter (in levels)	Section 3.2
η	Index of relative risk aversion	Section 3.2
δ	Discount rate	Section 3.2
τ	Level of warming to which limit the random variable T_H to	Section 3.2
$w^*(\tau)$	Willingness to pay to limit T_H to τ	Section 3.2
α	Exponent of the damage function (determines convexity)	Section 3.5.1

Acronym	Meaning	First occurrence
P12	Pindyck (2012)	Section 3.1
P09	Pindyck (2009)	Section 3.1
IPCC07	IPCC Fourth Assessment (2007)	Section 3.1
IPCC14	IPCC Fifth Assessment (2014)	Section 3.1

Abstract

This study is composed by three parts. The first two chapters develop a common-source infection model for explaining the formation of households expectations. The model is based on the work presented in "Macroeconomic Expectations of Households and Professional Forecasters" (Carroll, QJE, 2003). The extended framework is applied to study unemployment expectations for a selected group of European countries (France, Germany, Italy and the United Kingdom). Results show that: (*i*) the novel framework is supported by data; (*ii*) agent-based simulations confirm that the hypothesis of the model are reasonable in terms of replicating survey data; (*iii*) the probability of absorbing new information is (negatively) correlated with the level of uncertainty spread by media and the Internet; (*iv*) households expectations have a non trivial role in determining private consumption and the output gap. Furthermore, there are economically significant differences in expectations across different demographic groups and these differences may be explained through heterogeneous parameters of the agent-based model. In particular, education seems to be a driver of macroeconomic expectations and survey data are compatible with the less educated being less up-to-date and deviating from the rational expectation in a more pronounced fashion. In the third chapter, I present a replication and a robustness analysis of "Uncertain outcomes and climate change policy" (R. Pindyck, JEEM, 2012). The paper is concerned with the estimation of the willingness-to-pay of society to avoid climate change (and related economic damages). The replication part reproduces the original results in many cases and confirms the quality and interpretation of the work. Concerning the robustness analysis, on the one hand, re-estimating the model with more recent data on climate change, the willingness to pay does not vary much with respect to the original paper; on the other hand, changing the functional form produces much bigger and potentially problematic increments of the willingness to pay.

Introduction

"If we restrict ourselves to models which can be solved analytically, we will be modelling for our mutual entertainment, not to maximize explanatory or predictive power".

Harry M. Markowitz, Nobel laureate

The advent of computers have equipped researcher in the majority of fields, ranging from natural to social sciences, with enhanced tools to perform their work. Focusing in particular on the social sciences, like economics, computer estimation or simulation may provide reliable results beyond the ones attainable through the elegant, but limited, range of analytically tractable models (Helbing, 2012). For example Agent-Based Models, or the corresponding computational technique known as Multi-Agent Simulations, usually involve a substantial degree of heterogeneity and can be based on simple behavioural assumptions. Multi-Agent Simulations, which are computationally demanding, may be able to reproduce stylized facts starting from simple or even idealized assumptions, like heuristics involving little (or no) rationality. These simulation results are still comparable with empirical evidence without the need of a closed-form solution to be tested.

The apparently simple (complex) may be revealed to be complex (simple) (Epstein, 2008). Some macro-level results that are difficult to justify with a Representative Agent model may turn out to be reasonably (and convincingly) explained when Heterogeneous Agents are allowed for (see, for example, Krusell and Smith, 1998). At the very same time, even models which are analytically tractable and featuring clear qualitative economic intuitions may be extended or modified using computational methods. The use of computational tools may allow to obtain quantitative results that would be impossible to reach using "paper-and-pencil" methods (for example, because some functions are not analytically integrable) and nowadays this can be done routinely, even if in the past the task would have been considered computationally too demanding.

One of the beauty of computer simulations is that, once a model with good *explanuratory or predictive power* is formulated, this tool will facilitate the exploration of policy options and "parallel worlds". In Chapter 1, I start from a common-source-infection (CSI) theoretical model from "Macroeconomic expectations of household and professional forecasters", C.D. Carroll, *The Quarterly Journal of Economics*, 2003. I use a combination of econometric and simulation strategies to study the expectation formation process of European households, focusing on unemployment expectations, and its role on the aggregate economic activity. I find that (*i*) the novel common-source-infection framework is supported by data on unemployment expectations; (*ii*) micro-simulation results confirm that the hypothesis of the model are reasonable in terms of replication of the survey data from European Commission's Consumer Survey; (*iii*) the probability of absorbing new information is (negatively) correlated with the level of uncertainty spread by media and the Internet; (*iv*) households expectations have a non trivial role in determining private consumption and the output gap.

In Chapter 2, I extend the CSI model refined in Chapter 1 using an Agent-Based technique. The goal is to understand if empirical data concerning Italian households expectations, which appear *prima facie* irrational, could be explained assuming that households have heterogeneous expectations formation processes as a function of their education. In fact, according to other streams of literature, the more educated are also more informed (Lusardi and Mitchell, 2011a,b). Hence, for policy purposes, it is important to understand if there are demographic groups which are less aware of the actual state of the economy, since these are the groups more in need for specific education and information.

In Chapter 3, I conduct a replication and a robustness analysis, involving both new data and model extensions, of "Uncertain outcomes and climate change policy", R. Pindyck, *Journal of Environmental Economics and Management*, 2012. The paper is concerned with the estimation of the willingness-to-pay of society to avoid climate change (and related economic damages). The replication part reproduces the original results in many (although not all) cases and confirms the quality and interpretation of the work. Concerning the robustness analysis, on the one hand, (*i*) re-estimating the model with more recent data on climate change, the willingness to pay does not vary much with respect to the original paper; on the other hand, (*ii*) changing the functional form produces much bigger and potentially problematic increments of the willingness to pay.

Chapter 1

Expectations and uncertainty: A common-source infection model for selected European countries

Luca Gerotto* Antonio Paradiso†

Abstract

We present a common-source infection model for explaining the formation of expectations by agents. We start from the framework of "Macroeconomic expectations of household and professional forecasters", C.D. Carroll, *The Quarterly Journal of Economics*, 2003. We augment the original framework assuming that also uninformed individuals are able to update expectations according to a naive econometric process. In addition, we emphasize the role of the parameter measuring the probability of being infected in capturing the level of agents' uncertainty. The novel framework is applied to study the unemployment expectations for a selected group of European countries (France, Germany, Italy and the UK). Our results show that: (*i*) the novel framework is supported by data on unemployment expectations; (*ii*) micro-simulation results confirm that the hypothesis of the model are reasonable in terms of replicating the survey data from European Commission's Consumer Survey; (*iii*) the probability of being infected is (negatively) correlated with the uncertainty spread by newspapers and conveyed by Internet; (*iv*) households expectations have a non trivial role in determining private consumption and the output gap.

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At a general level, uncertainty is typically defined as the conditional volatility of a disturbance that is unforecastable from the perspective of economic agents.

Jurado et al. (2015)

1.1 Introduction

Expectations matter in the macroeconomy. Changes in expectations may lead to changes in economic activity, both at the individual level (i.e. firms and consumers) and at the aggregate level. For example, interest rates expectations enter into investment decisions of firms (Neumeyer and Perri, 2005), portfolio decisions of investors (Friedman and Roley, 1979), and bond issues of companies (Baker et al., 2003). Similarly, inflation expectations may impact on consumption behavior (D'Acunto et al., 2015; Duca et al., 2016), whereas stock price and output expectations may influence investment decisions (Lamont, 2000).

Expectations concerning unemployment are another important source of business fluctuations through their impact on consumption expenditure. Carroll and Dunn (1997) proxy income uncertainty, due to unemployment risk, with unemployment expectations. The authors find that unemployment expectations – the proxy of unemployment risk – are strongly correlated with consumer expenditure. Moreover, Carroll and Dunn (1997) show that the deterioration in unemployment expectations played an important role in explaining the 1990-1991 recession, and recent theoretical models emphasizes the role of perceived unemployment risk in amplifying business cycles;[1] see Sterk and Ravn (2017) and Beaudry et al. (2017). In addition, we run a very stylized macro VAR model – consumption, disposable income,[2] inflation and households unemployment expectations – on the set of countries studied in this paper. We take into consideration France and Germany, the two leading economies for the Euro area, Italy, one of the biggest countries among the ones suffering of low growth, and an important non-Euro country like the United Kingdom. As expected, a generalized impulse-response analysis highlights a common negative effect of unemployment expectations on consumption decisions. According to the results plotted in Figure 1.1, it appears that the more households are pessimistic, the less they choose to consume. This effect is highly negative and statistically significant for the above mentioned countries. These results give support to the idea of an important role of unemployment expectations on consumption/saving decisions.[3]

Although the recognized importance of unemployment expectations in generating business fluctuations, the way expectations are formed in macroeconomics still remains an open question. In general, most of empirical and theoretical models assume Full Information Rational Expectations (FIRE): agents have full-time access to all information, know the true model and use it to form predictions.

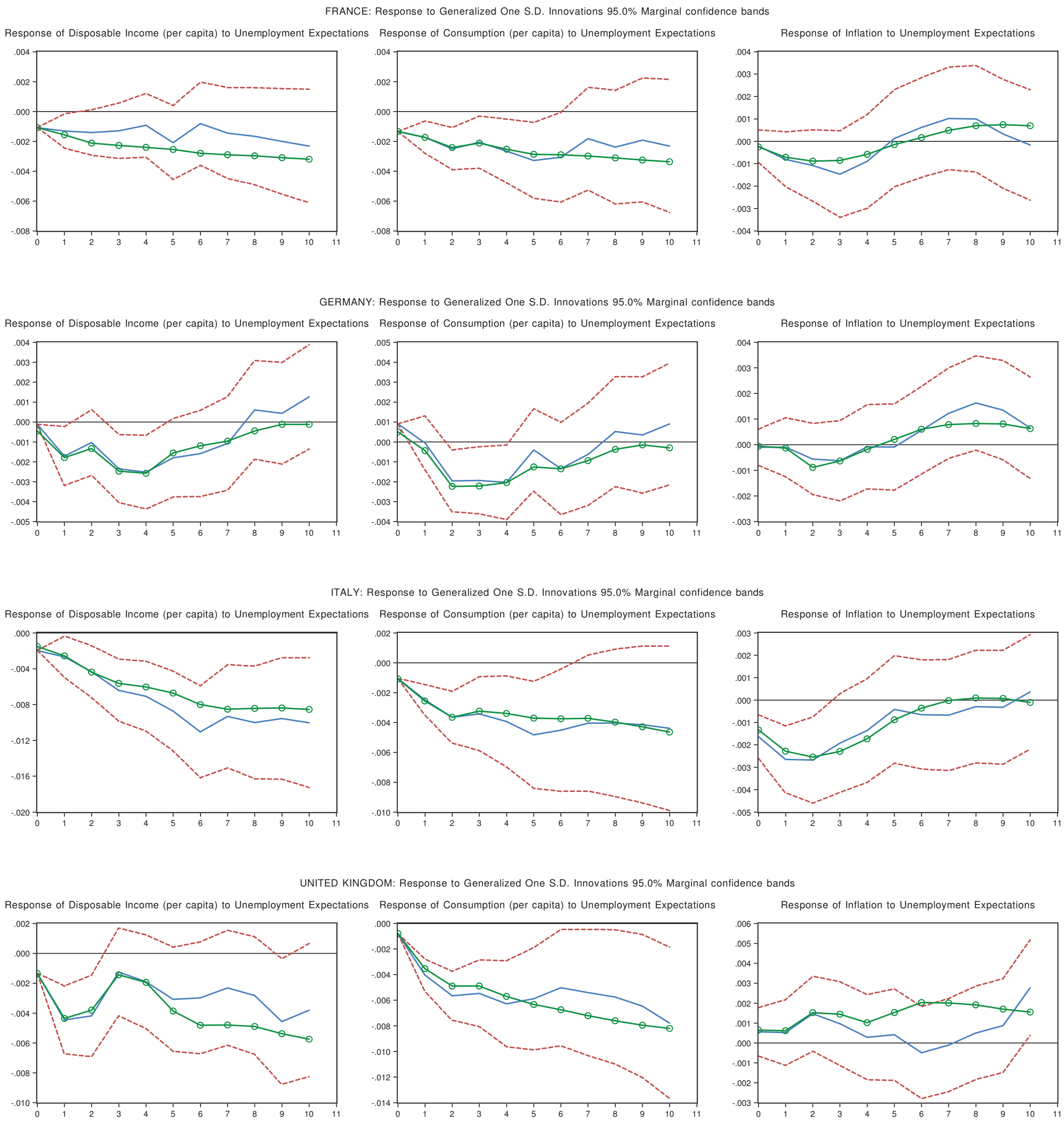
Even though the FIRE approach is an useful and theoretically strong starting point (Friedman, 1953; Muth, 1961), its actual empirical soundness has been repeatedly discussed in the last decades, as summarized in Curtin (2010). Simon (1959, 1978, 1979)

[1] For a more general analysis of the role of psychological factors and "less-than-fully-rational" shifts in expectations on business cycles, see Milani (2011).

[2] Disposable income does not include only labour income but also the other sources of income which could be promptly spent, like interest and dividend payments from financial assets, and rents and net profits from businesses.

[3] Possibly with the exception of Germany, where the effect is a bit weaker.

Figure 1.1: Impulse Response graph of disposable income per capita, consumption per capita and inflation to unemployment expectations (1991Q1-2016Q4).



Notes: Impulse response (blue) and confidence bands (red) are estimated according to the local projection method (Jordà et al., 2005; Jordà, 2009). Standard VAR estimates are in green.

casts doubts on the ability of theories based upon the rationality assumption to explain observed phenomena. Classical papers in behavioural economics have identified several cognitive biases (Kahneman et al., 1982; Earl, 1990; Thaler, 1994; Rabin and Schrag, 1999; Thaler, 2012) the presence of which makes expectations not so likely to be formed in a fully rational way. Actually, Roberts (1998) and Tortorice (2012) report that surveys reflect only an intermediate degree of rationality, and Ball (2000) proposes near-rationality in inflation expectations as a possible solution.

One of the main weaknesses of the FIRE is the assumption that all individuals have access to the same, complete set of information used to form expectations. Moreover, even if individuals have access to all information, not all of them may have the capacity and/or the willingness to absorb all the information available. If there are positive costs associated to collect and process information, the agents may find optimal to formulate less accurate expectations.

Examples in the direction of information rigidities are the "Sticky Information" (Mankiw and Reis, 2002) and "Noisy Information" models (Sims, 2003; Bacchetta and Van Wincoop, 2005; Woodford, 2003). "Sticky Information" (SI) models assume that agents are rational, but the presence of fixed costs in both updating and processing information induces agents to update their information set infrequently. Once they update, they acquire the FIRE. Conversely, "Noisy Information" (NI) models assume that agents update information every period,[4] but they are able to observe only one of many noisy signals rather than the true state. Being unable to disentangle the true innovation from the noise, they do not fully "trust" that signal. Rather, their new expectation is a weighted average of the signal and their prior belief. Despite the different underlying theoretical assumptions,[5] both SI and NI imply the same level of stickiness in aggregate expectations (Coibion and Gorodnichenko, 2015). For this reason, tests on aggregate empirical data cannot discriminate between NI and SI. Coibion and Gorodnichenko (2015) also point out that for NI, differently from SI, the weight put on the signal depends on (i) the persistence of the variable under consideration and (ii) the noisiness of the signal: the higher the variance of the noise, the less agents take the signal into consideration.

Similarly to SI, Branch (2004, 2007) assumes that agents are rational and are able to use sophisticated models to resolve uncertainty. However, sophisticated models are costly (in terms of both time and resources) and, for this reason, some agents may prefer to form their expectations using adaptive or naive models. Carroll (2003) has, instead, modelled the disagreement across people as the result of an "infection" process from a common source. He assumes that only a small fraction of agents (professional forecasters) form their own expectations. These professional opinions then spread across the population via news media like a virus. In any given period, each agent has a given probability of hearing the latest "official" forecast through newscasts. If this happens, he equalizes his expectation to this "professional" forecast, otherwise he maintains his previous expectation.

Whatever the cause generating disagreement across agents and staggered changes in expectations, one of the main differences between the above-mentioned approaches to modelize the expectations lies in the possibility for less informed agents to revise their

[4]In standard NI models, the underlying macroeconomic variable subject of expectations is formalized as an autoregressive process.

[5]According to the SI, the cross-sectional disagreement across people reflect the different choices to update information, while in NI it is the result of the different signals they observe.

expectations. While in Branch (2004, 2007), Woodford (2003) and Sims (2003) all agents revise their expectations, Mankiw and Reis (2002) and Carroll (2003) assume that only informed agents change their expectations. The uninformed (inattentive) group, instead, maintains the previous expectation. The hypothesis that inattentive agents do not revise at all their previous opinion may appear quite strong in practice. Even the more "discouraged" agents may take an effort to build an expectation.[6]

Starting from Carroll (2003),[7] we develop a common-source-infection (CSI) model applied to expected changes in the unemployment rate for a selected group of European countries, namely Germany, France, Italy, and UK.[8] This work is innovative in the framework of Carroll (2003) in three ways. First, we generalize the CSI framework, introducing the possibility that also the fraction of uninformed agents may change their expectations. In this regard, we assume that inattentive agents act as "naive" econometricians. More in detail, the idea is that the formulation of "sophisticated" expectations requires an investment of time and resources that only professional forecasters may sustain: non-professional agents rationally prefer not to spend time and resources to produce state-of-the-art forecasting models. As a consequence, if agents are "infected" by news, they embody professional expectations; otherwise, if agents are not "infected", they exploit the old information to build expectations using simple naive models, with a small effort in terms of time and resources. Second, we allow the key parameter measuring the probability of being infected to be time-varying, while Carroll (2003) estimates are based upon the assumption of a constant probability.[9] Third, we find a (negative) link between the time-varying infection probability and the level of uncertainty, both the one diffused by newspapers (proxied by the index introduced by Baker et al., 2016) and the one represented by web searches on economic uncertainty (proxied by Google searches on the topic). In this regard, we also suggest a measure of uncertainty which is captured by historical volatility of the "unexplained" part of agents' expectations (i.e., the difference between survey balance index and the model prediction), under the idea that a rise in the variability of the unexplained part is a signal of the high uncertainty faced by agents, as discussed for example in Jurado et al. (2015). The idea of using survey data to measure uncertainty is not new in the literature, and has been mainly focused on business surveys. Two recent examples are Bachmann et al. (2013) and Girardi and Reuter (2016).

[6]Easaw and Golinelli (2012) remove the assumption of fixed expectations by inattentive agents in Carroll (2003)'s framework by using the particular structure of UK survey. The authors assume that a fraction of uninformed agents use forecasts made in the previous period but over the same horizon (i.e. a multi-period ahead survey-based forecasts) and the remainder fraction is anchored to the previous forecast.

[7]The term "epidemiology" has different meanings in several different streams of literature. Carroll (2003) defines this as an epidemiological framework because the information is considered such as a virus spreading through the population. In order to obtain an estimable-closed-form solution of the model, the author assumes that: (*i*) only an unique common source of infection exists; (*ii*) no possibility of contagion among agents; (*iii*) no recovery from the virus. The above-mentioned assumptions deprive the model from characteristics which are considered as crucial for an epidemiological model in other streams of literature. In order to avoid any confusion in the reader, throughout the paper we prefer to label the model as "common-source-infection" model.

[8]The model is designed in terms of unemployment rates variations (i.e. in first-differences) since the formulation of survey question on unemployment expectations goes in this direction.

[9]In a different setup, a similar time-varying estimate is present also in Coibion and Gorodnichenko (2015). Anyway, considering the different aim of our work, our time-varying approach is totally model-based. We make this choice in order to avoid spurious correlation with the "news-based" indexes.

Bachmann et al. (2013) measure business-level uncertainty from business survey data for Germany and the United States. They construct measures based both on dispersion in ex-ante forecasts and dispersion in ex-post forecast errors, and the two measures turn out to be strongly correlated. Girardi and Reuter (2016) extend the work of Bachmann et al. (2013), adding as a further measure the inter-question dispersion, since uncertainty may impact differently the expectations on the various macroeconomic indicators.

Our main results are as follows. First, we find that the CSI model predictions track well the survey balances for unemployment expectations. Second, it appears that households spend less time in learning professional expectations when they perceive heightened uncertainty: the exact future value of unemployment becomes harder to forecast, even by professional forecasters. In this situation, it is highly likely that non-expert agents care less about expert opinions. Third, the measure of uncertainty obtained from the ”unexplained” part of our model has a similar cyclical pattern with respect to the news-based measure of uncertainty of Baker et al. (2016): in periods of heightened uncertainty, households expectations deviate more markedly from (bounded) rationality. Finally, we show that households expectations have a non trivial role on the business cycle; therefore, a transparent communication may be an useful policy instrument to influence the business cycle during booms or recessions.

The paper is organized as follows. Section 1.2 presents further empirical evidence on the importance of unemployment expectations at the macroeconomic level. Section 1.3 presents the theoretical framework. Sections 1.4 highlights the role of uncertainty in the CSI framework. Section 1.5 presents the estimation strategy and Section 1.6 the related output. Section 1.7 draws some policy considerations and Section 1.8 concludes.

1.2 The role of expectations on consumption

Before introducing the common-source-infection model, we shortly present further evidence on the role of unemployment expectations at the macroeconomic level, complementing the preliminary evidence of Figure 1.1. Expectations shape households behaviour: Carroll (1997) has shown that an agent which is both prudent and impatient may be induced to build up a "buffer stock" of savings to face periods of potentially low income (or, equivalently, potentially high expenses). The level of this "buffer" targeted by the household depends on his expectations about the future: the higher the uncertainty, the lower the income he expects,[10] the more he accumulates savings, inducing the reduction of current consumption levels. As examined in depth in Carroll and Dunn (1997), unemployment expectations are theoretically and empirically relevant, indeed they can be considered as a proxy for the (perceived) probability of having zero labour income, hence a deterioration of these expectations depresses the consumption level. In a recent paper, Carroll et al. (2012) analyse the US saving rate and find a positive effect of households expectations on the aggregate saving rate.

We look for comparable evidence for the countries under investigation. We start from an ARDL model with private consumption per capita ($\ln(C_t)$ as dependent variable, and disposable income per capita ($\ln(Y_t)$),[11] the unemployment expectations index (EU_t)

[10] Or, equivalently, the higher the expenses he expects to face.

[11] Disposable income does not include only labour income but also the other sources of income which

and the Private Consumption Expenditure (PCE) inflation rate (π_t^{PCE}) as independent variables:[12]

$$\begin{aligned}\ln(C_t) =&\gamma_{C1}\ln(C_{t-1}) + \gamma_{C2}\ln(C_{t-2}) + \gamma_{Y0}\ln(Y_t) + \gamma_{Y1}\ln(Y_{t-1}) + \gamma_{EU0}EU_t + \gamma_{EU1}EU_{t-1}\\ &+ \gamma_{EU2}EU_{t-2} + \gamma_{EU3}EU_{t-3} + \gamma_{EU4}EU_{t-4} + \gamma_{\pi0}\Delta\pi_t^{PCE} + \gamma_{\pi1}\Delta\pi_{t-1}^{PCE} + \epsilon_t \quad (1.1)\end{aligned}$$

Estimates of Eq. (1.1) are reported in Table 1.1. Moreover, it is possible to show that Eq. (1.1) could be rewritten in first differences,[13] incorporating the long-run solution ecm_t:

$$ecm_t = \ln(C_t) - (\alpha_Y \ln(Y_t) + \alpha_{EU}EU_t + \alpha_{EU}\pi_t^{PCE} + \alpha_0) \quad (1.2)$$

When the ecm_t, that is the error-correction term, is equal to zero, the consumption level, given the other variables, is in equilibrium. The estimates of Eq. (1.2) are presented in Table 1.2.

For France, Italy and the United Kingdom, unemployment expectations are significant regressors of both the ARDL and the error-correction term. The coefficients are highly significant and negative. This implies that changes in expectations have effects in the short run as well as in the long run. Short-run changes in expectations lead households to downwards adjust their consumption: in other terms, an increase of the index (i.e. a deterioration of expectations) depresses consumption in the short run. Unsurprisingly, unemployment expectations affect also the long-run equilibrium: a permanent increase in the index implies that households are permanently more pessimistic and, in light of Carroll (1997) Buffer Stock model, they save more. Therefore the "new" long-run equilibrium, keeping fixed income and inflation, is characterized by a lower consumption level. Until such a new equilibrium is reached, the deteriorated expectations leads households to downward adjust their consumption.

As far as Germany is concerned, evidence on the role of unemployment expectations is less clear.[14] The second, third and fourth lags have positive coefficients. The sum of the three coefficients is not statistically different from zero, implying that unemployment expectations are not significant in the error-correction term (Table 1.2).[15] Therefore, data confirm that also in Germany a change in the unemployment expectations has a short-term effect, but there is not enough evidence that expectations affect also the long-run equilibrium.

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could be promptly spent, like interest and dividend payments from financial assets, and rents and net profits from businesses.

[12] A single-equation in the spirit of the DHSY (Davidson et al., 1978) theoretically requires the presence an unique cointegrating relation and of weak exogeneity (Johansen, 1992) to provide unbiased and efficient results. The Johansen cointegration test fails to reject the presence of an unique cointegrating relation among consumption, income, inflation and unemployment expectations, but a weak exogeneity test rejects that these last three are exogenous. Still, as sustained in several empirical papers (see, for example, Constantinescu et al., 2017) the ARDL approach is robust to the presence of endogeneity. If the lags are selected by information criterion, the results are quite robust. For this reason, instead of starting from the classical specification of Davidson et al. (1978), we start from an ARDL model selected by the Schwarz Bayesian information criterion, as suggested by Constantinescu et al. (2017).

[13] $$\begin{aligned}\Delta\ln(C_t) =&\beta_{C1}\Delta\ln(C_{t-1}) + \beta_{Y1}\Delta\ln(Y_t) + \beta_{Y2}\Delta\ln(Y_{t-1}) + \beta_{EU1}\Delta EU_t\\ &+ \beta_{EU2}\Delta EU_{t-1} + \beta_{EU3}\Delta EU_{t-2} + \beta_{EU4}\Delta EU_{t-3} + \beta_{\pi1}\Delta\pi_t^{PCE} + \delta ecm_{t-1} + \epsilon_t\end{aligned}$$

[14] Fortunately, in Section 1.7 data on the output gap confirm that expectations influence the real activity also in Germany

[15] It can be shawn that $\alpha_{EU} = \frac{\gamma_{EU2}+\gamma_{EU3}+\gamma_{EU4}}{1-\gamma_{C1}-\gamma_{C2}}$.

Table 1.1: Estimates from ARDL model Eq. (1.1) (FRA-UK 1991q1-2016q4, GER 1992q1-2016q4, ITA 1991q3-2016q4)

	FRA	GER	ITA	UK
γ_{C1}	0.613***	0.845***	1.280***	0.923***
	(0.094)	(0.049)	(0.071)	(0.022)
γ_{C2}	0.205**		-0.350***	
	(0.080)		(0.065)	
γ_{Y0}	0.170***	0.652***	0.052***	0.070***
	(0.045)	(0.078)	(0.018)	(0.025)
γ_{Y1}		-0.505***		
		(0.076)		
$\gamma_{EU0} \cdot 100$	-0.014***		-0.011**	-0.022***
	(0.003)		(0.004)	(0.005)
$\gamma_{EU1} \cdot 100$				
$\gamma_{EU2} \cdot 100$		-0.029***		
		(0.006)		
$\gamma_{EU3} \cdot 100$		0.039***		
		(0.008)		
$\gamma_{EU4} \cdot 100$		-0.015**		
		(0.006)		
$\gamma_{\pi 0}$	-0.098**	-0.152***	0.130	-0.087*
	(0.046)	(0.050)	(0.093)	(0.044)
$\gamma_{\pi 1}$			-0.242***	
			(0.069)	
γ_0	0.089*	0.056	0.152**	0.064
	(0.051)	(0.072)	(0.074)	(0.060)

Notes: Newey-West (HAC) standard errors are reported in parentheses. Lag length selected by Schwarz Bayesian information criterion. For Germany, a dummy for 2007Q1 is included. When written in first differences, the coefficients δ on the ecm_{t-1} are: France -0.182^{***}, Germany -0.109^{**}, Italy -0.070^{***}, UK -0.077^{***}.

Table 1.2: Error-correction Long Run Coefficients Eq. (1.2)

	FRA	GER	ITA	UK
α_Y	0.933***	0.904***	0.739***	0.907***
	(0.039)	(0.077)	(0.134)	(0.100)
$\alpha_{EU} \cdot 100$	-0.077***	$-0.032^{\dagger}$	-0.151*	-0.284***
	(0.022)	(0.032)	(0.079)	(0.093)
α_π	-0.540*	-1.22***	-1.61***	-1.130*
	(0.314)	(0.361)	(0.402)	(0.627)
α_0	0.491	0.685	2.172*	0.838
	(0.34)	(0.657)	(1.131)	(0.862)

Notes: Newey-West (HAC) standard errors are reported in parentheses. † p-value=0.30. FRA-UK 1991q1-2016q4, GER 1992q1-2016q4, ITA 1991q3-2016q4

1.3 Theoretical framework

1.3.1 Carrol's CSI framework

Carroll (2003, 2006) introduced a CSI model to formalize households expectations. In this framework, the information propagates through the economy as a virus and each agent has a given probability to be infected. Denoting with x the variable of interest, the following points characterize Carroll (2003, 2006)'s model:[16]

I The typical person believes that x_t behaves like a *non-stationary stochastic model*:

$$x_t = x_t^* + \epsilon_t \tag{1.3}$$

$$x_{t+1}^* = x_t^* + \eta_{t+1}, \tag{1.4}$$

where x_t^* represents the "fundamental value" of x_t, and the disturbance ϵ_t and the innovation η_t are Gaussian independent processes.

II Only professional forecasters, a group of expert agents, are able to form expectations on x_{t+1}. These groups of experts have the ability to observe exactly x_{t+1}^*, so that the prediction of x_{t+1} corresponds to

$$N_t\left[x_{t+1}\right] = x_{t+1}^* = x_t^* + \eta_{t+1}, \tag{1.5}$$

where $N_t\left[x_{t+1}\right]$ indicates the professional forecasts prediction. In other words, the innovation η_{t+1} is always observed by expert agents in period t.[17].

III Professional forecasters expectations spread in the economy via news media (i.e., the so-called "common source of infection"). In each period, an agent i has a probability λ of being infected by the information and, then, to revise the expectation incorporating the professional forecasters prediction.[18]

IV $N_{t+k}\left[x_{t+k+1}\right]$ is a different "virus" with respect to $N_{t+k+h}\left[x_{t+k+h+1}\right]$ $\forall k \geq 0, h > 0$. The individual infected at a generic time t never recover from the "virus"; in other words, agents who acquire $N_{t+k}\left[x_{t+k+1}\right]$ never forget this information.

Under this set of assumptions, the expectation of x at time $t+1$ by a generic non-expert agent i can be written as:

$$E_t^i\left[x_{t+1}\right] = E_t^i\left[x_{t+1}^*\right] + \underbrace{E_t^i\left[\epsilon_{t+1}\right]}_{=0}. \tag{1.6}$$

If agent i is "infected" at time t, then Eq. (1.6) can be written as:

$$E_t^i\left[x_{t+1}\right] = N_t\left[x_{t+1}\right] = x_{t+1}^*. \tag{1.7}$$

[16]Carroll (2003, 2006) used these assumptions to develop a model describing the formation of inflation expectations. The framework introduced in Carroll (2003, 2006) is general enough to be extended to other kind of economic variables such as GDP, disposable income, consumption, and unemployment.

[17]It is important to note that future values of η beyond $t+1$ are unobservable for expert agents in period t.

[18]In terms of equation (1.5), this means that non-expert agents, if infected for example at time t, are able to observe directly the fundamental value x_{t+1}^*, without the ability to disentagle x_t^* from η_{t+1} (unless they have been infected also in period $t-1$).

If agent i is not infected in t, but was instead infected at time $t-1$, Eq. (1.6) is equal to

$$E_t^i\left[x_{t+1}\right] = N_{t-1}\left[x_{t+1}\right] = N_{t-1}\left[x_t\right] = E_{t-1}^i\left[x_t\right] = x_t^*. \tag{1.8}$$

According to these rules, the average expectation of x at time $t+1$ can be represented as:

$$M_t\left[x_{t+1}\right] = \lambda N_t\left[x_{t+1}\right] + (1-\lambda)\left\{\lambda N_{t-1}\left[x_t\right] + (1-\lambda)\left(\lambda N_{t-2}\left[x_{t-1}\right]\ldots\right)\right\}, \tag{1.9}$$

where $M_t\left[x_{t+1}\right]$ denotes the population-mean value of expectations of x_{t+1} made in t, $N_t\left[x_{t+1}\right]$ represents the professional forecasters expectation as reported by news media in t, and λ is the proportion of informed agents infected by news media.

Given the property of the lag polynomial (L), the right-hand side of (1.9) can be rewritten as:

$$\begin{aligned}\lambda N_t\left[x_{t+1}\right] + (1-\lambda)\left\{\lambda N_{t-1}\left[x_t\right] + (1-\lambda)\left(\lambda N_{t-2}\left[x_{t-1}\right]\ldots\right)\right\} &= \\ \left\{1 + (1-\lambda)L + (1-\lambda)^2 L^2 + \ldots\right\}\lambda N_t\left[x_{t+1}\right] &= \\ \frac{1}{1-(1-\lambda)L}\lambda N_t\left[x_{t+1}\right]&.\end{aligned} \tag{1.10}$$

Thus Eq. (1.9) can be expressed as:

$$M_t\left[x_{t+1}\right] = \frac{1}{1-(1-\lambda)L}\lambda N_t\left[x_{t+1}\right] \tag{1.11}$$

or

$$\left[1-(1-\lambda)L\right] M_t\left[x_{t+1}\right] = \lambda N_t\left[x_{t+1}\right] \tag{1.12}$$

which corresponds to

$$M_t\left[x_{t+1}\right] = \lambda N_t\left[x_{t+1}\right] + (1-\lambda) M_{t-1}\left[x_t\right]. \tag{1.13}$$

When the time is expressed in quarters and forecasts are made over the following year (i.e. from t to $t+4$), Eq. (1.13) can be written as:

$$M_t\left[x_{t+4}\right] = \lambda N_t\left[x_{t+4}\right] + (1-\lambda) M_{t-1}\left[x_{t+3}\right], \tag{1.14}$$

where $M_t\left[x_{t+4}\right]$ now indicates the population-mean value of expectations on x made in t over the quarter $t+4$ and $N_t\left[x_{t+4}\right]$ are the professional forecasters expectation as published by the news reports in t. More details on the derivation of (1.14) are reported in Appendix 1.A.1.

Carroll (2003, 2006) uses Eq. (1.14) to investigate the evolution of inflation and unemployment expectations in the US for the period after the second half of 1970s. The results show that people only occasionally pay attention to news reports: the fraction of updaters is, on average, equal to 0.25. This inattention generates high degree of "stickyness" in aggregate expectations, with important macroeconomic consequences.

One of the central implication in Carroll's model is the inability of inattentive agents to change expectations. This point is the result of the particular process assumed for x_t

(point I) and of the assumption that η_{t+1} is predictable only by professional forecasters (point II). The justification for point (II) is that observing η_{t+1} requires a costly activity (in terms of time and money spent to study how the economy works) for a typical person. Since news reports provide forecasts for free, an individual prefers to dedicate time to other activities such as work, family, hobbies, etc.

1.3.2 A new CSI framework allowing for changes of inattentive agents predictions

With respect to Carroll (2003, 2006)'s model, we modify point (I) as follows:

I' The typical person believes that x_t behaves like a *stationary stochastic model*:[19]

$$x_t = x_t^* + \epsilon_t \tag{1.15}$$

$$x_{t+1}^* = \alpha + \beta x_t^* + \eta_{t+1},\ 0 \leq \beta < 1 \tag{1.16}$$

where β represents the autoregressive coefficient of the fundamental value process, α is a constant term and the disturbance ϵ_t and the innovation η_t are Gaussian independent processes.

This assumption introduces an important change with respect to Carroll's version. Now, typical agents may form and change expectations by themselves, from one period to another, without relying on state-of-the-art professional forecasters estimates. A crucial implication is that, given the information set available, the expectation by a non-expert agent for x_{t+j} is different from the expectation for x_{t+j+1} $(\forall j \neq 0)$.[20]

An example similar to that presented in subsection 1.3.1 helps to clarify the different implications. Under the new assumption (I') and maintaining points $II - IV$ discussed in subsection 1.3.1, the expectation of x at time $t+1$ by a generic non-expert agent i can be written as:

$$E_t^i \left[x_{t+1}\right] = E_t^i \left[x_{t+1}^*\right] + \underbrace{E_t^i \left[\epsilon_{t+1}\right]}_{=0}. \tag{1.17}$$

If agent i is "infected" at time t, then Eq. (1.17) is equal to

$$E_t^i \left[x_{t+1}\right] = N_t \left[x_{t+1}\right] = x_{t+1}^*. \tag{1.18}$$

[19] From a mathematical point of view, a stationary process could be obtained with $-1 < \beta < 1$. Anyway, if β were negative, a fundamental shock η would imply an oscillatory pattern of the fundamental value of the variable of interest. Oscillatory pattern which has no confirmation on empirical data of the macroeconomic variables we are going to study and, more in general, to macroeconomic variables for which this model could be applied. The assumption on the autoregressive nature of the variable has been made also, in a different setup, by the "noisy information" model of Woodford (2003).

[20] Furthermore, on the one hand, under the random walk hypothesis of Eq. (1.4) informed agents have superior information also concerning the long-run horizon: in period t, the best guess for $x_\infty^* = x_{t+1}^* = x_t^* + \eta_{t+1}$. So, individuals who have learned about x_{t+1}^* (and implicitly about η_{t+1}) have more precise short and long-run expectations with respect to individuals who have read professional forecasts only one, or even more, periods before. On the other hand, there is no long-period advantage under the stationary process of (1.16), since $x_\infty^* = \frac{\alpha}{1-\beta}$: informed agents have a more precise short-run expectation, while the expectations of all agents (informed and uninformed) concerning the long-run horizon converge to the same steady level x_∞^*.

If agent i is not infected in t, but was instead infected at time $t-1$, he does not know the innovation η_{t+1} but, except for the disturbances, he is aware of the process, so Eq. (1.17) is equal to

$$E_t^i\left[x_{t+1}\right] = N_{t-1}\left[x_{t+1}\right] = \alpha + \beta N_{t-1}\left[x_t\right] = \alpha + \beta x_t^*. \tag{1.19}$$

According to these rules, the population-mean expectation of x at time $t+1$ can be represented as:

$$\begin{aligned}
M_t[x_{t+1}] &= \lambda N_t[x_{t+1}] + (1-\lambda)\{\lambda N_{t-1}[x_{t+1}] + (1-\lambda)(\lambda N_{t-2}[x_{t+1}] + (1-\lambda)(\lambda N_{t-3}[x_{t+1}]\ldots)\} \\
&= \lambda N_t[x_{t+1}] + (1-\lambda)\{\lambda[\alpha + \beta N_{t-1}[x_t]] + (1-\lambda)(\lambda[\alpha + \beta N_{t-2}[x_t]] \\
&\quad + (1-\lambda)(\lambda[\alpha + \beta N_{t-3}[x_t]]\ldots)\} \\
&= \lambda N_t[x_{t+1}] + (1-\lambda)\{\lambda[\alpha + \beta N_{t-1}[x_t]] + (1-\lambda)(\lambda[\alpha + \beta[\alpha + \beta N_{t-2}[x_{t-1}]]] \\
&\quad + (1-\lambda)(\lambda[\alpha + \beta[\alpha + \beta N_{t-3}[x_{t-1}]]]\ldots)\} \qquad (1.20) \\
&= \lambda N_t[x_{t+1}] + (1-\lambda)\{\lambda[\alpha + \beta N_{t-1}[x_t]] + (1-\lambda)(\lambda[\alpha + \beta[\alpha + \beta N_{t-2}[x_{t-1}]]] \\
&\quad + (1-\lambda)(\lambda[\alpha + \beta[\alpha + \beta[\alpha + \beta N_{t-3}[x_{t-2}]]]]\ldots)\}
\end{aligned}$$

where $M_t\left[x_{t+1}\right]$ denotes the population-mean value of expectation of x_{t+1} made in t, $N_t\left[x_{t+1}\right]$ represents the professional forecasters expectations as reported by news media in t, and λ is the proportion of informed agents infected by news media. Using the property of lag polynomials and rearranging terms as shown in Appendix 1.A.2, (1.20) corresponds to

$$M_t\left[x_{t+1}\right] = \lambda N_t\left[x_{t+1}\right] + (1-\lambda)\left(\alpha + \beta M_{t-1}\left[x_t\right]\right). \tag{1.21}$$

If the time is expressed in quarters and the forecast is over the next year (i.e. from t to $t+4$), Eq. (1.21) can be written as:

$$M_t\left[x_{t+4}\right] = \lambda N_t\left[x_{t+4}\right] + (1-\lambda)\left(\alpha + \beta M_{t-1}\left[x_{t+3}\right]\right). \tag{1.22}$$

Appendix 1.A.3 contains details on the derivation of Eq. (1.22).

While Eq. (1.22) may appear as a simple generalization of Eq. (1.14) (actually if $\alpha = 0$ and $\beta = 1$, (1.22) corresponds to (1.14)), it has very different implications. Hence, rather than a generalization, it has to be considered as an extension of Carroll (2003) model to variables which are characterized by a persistent, maybe even highly persistent, but not unit root process. Therefore, the question is: which version is applicable to a given variable? Our answer is: it depends on the statistical process of the variable under investigation.

1.3.3 Application of the CSI framework to unemployment expectations

Applying the CSI model to unemployment expectations requires us to study two important issues: first, the formulation of the question concerning unemployment expectations in the survey of households; second, the characteristics of the statistical process of the variable under investigation.[21] The first point allows us to identify how the variable is measured (i.e. level or growth rates). The second point is crucial to understand if the process is better described by:

1. a random walk, like inflation in US (Carroll, 2003), supporting the hypothesis that households do not change expectations if they do not learn about the innovation, leading to Eq. (1.14), or
2. a stationary autoregressive process, supporting the hypothesis that households may naively update their expectation multiplying the previous period value by a constant factor (and eventually adding another constant value), leading to Eq. (1.22)

In our analysis for France, Germany, Italy, and the UK, we consider survey data on unemployment expectations obtained from the European Commission's Joint Harmonised EU Programme of Consumer Surveys. The formulation of the question concerning unemployment expectations (Q7) is as follows:

Q7: How do you expect the number of people unemployed in this country to change over the next 12 months?
The number will: $(++)$ increase sharply; $(+)$ increase slightly; $(=)$ remain the same; $(-)$ fall slightly; $(--)$ fall sharply; (N) don't know.

Two aspects emerge analyzing the above question. First, it is clear that the survey question refers to a change in unemployment in the next year: i.e. the future number of unemployed people less the current one. Second, it is important to understand which kind of unemployment data the respondents have in mind: level or rate? In other words, do they reply to question Q7 in terms of a change in the level of unemployment or in terms of a change in unemployment rate? As a necessary premise, it has to be highlighted that both the number of unemployed people and the unemployment rate are very highly correlated, both in levels and in first differences. Furthermore, since usually newspapers and newscasts, communicating economic data, report data on unemployment expressed as a percentage of the labour force (i.e., the unemployment rate), we guess that agents have in mind this kind of data. A visual inspection between year-over-year change in the unemployment rate (i.e., a change in the unemployment rate with respect to the same period of the previous year) and survey data on unemployment expectations for all the countries under investigation confirm our view; see Figure 1.B.1 in Appendix 1.B.

Another important point concerns the unit used to measure households unemployment expectations. The time series of unemployment expectations are expressed by the European Commission as a balance index. The balance values range from -100 (all respondents choose the most positive option) to +100 (all respondents choose the most negative option).[22] For our purposes, this balance is firstly converted in quarterly time series and then,[23] following Carroll (2003), converted in the same unit of measure of the

[21] The order of investigation is important, since only after having identified how it is measured the expectation variable we are able to study its statistical process.

[22] For further details on aggregation and weighting of consumer surveys answers see European Commission (2016).

[23] More in details, survey data are published every month and are transformed in quarterly data (taking a simple average of the months) to fit with the frequency of the survey of professional forecasters. Full description of data is given in Appendix 1.G.

unemployment rate using the following auxiliary regression:[24]

$$u_{t+4} - u_t = \phi_0 + \phi_1 EU_t^U + \epsilon_t, \tag{1.23}$$

where u_{t+4} is the unemployment rate at time $t+4$, u_t is the unemployment rate at time t, and EU_t^U is the EU index of unemployment expectations. Using estimated values $\{\hat{\phi}_0, \hat{\phi}_1\}$, the forecast for the next year unemployment rates change can be constructed as:

$$\widehat{M}_t\left[\Delta_4 u_{t+4}\right] = \hat{u}_{t+4} - \hat{u}_t = \hat{\phi}_0 + \hat{\phi}_1 EU_t^U. \tag{1.24}$$

Looking at estimated coefficients from Table 1.3, it is interesting to note a negative and significant value of the intercept $\hat{\phi}_0$ for all selected countries. This negative value suggests a systematic overestimation of the level of future unemployment by households. As a further clue on the presence of a permanent bias, recall that the index is expressed as a balance: if the balance is positive, there are more individuals that expect the unemployment rate to increase, than the ones who expect it to decrease. The opposite if the balance is negative. Therefore we expect that, at least if we analyze a long period of time, the aggregate expectations of boundedly rational individuals are aligned with the realization of the variable. To check this, we regress the actual change of the unemployment rate on a constant term: for France, Germany and Italy this constant term is not statistically significant, while for the United Kindgom it is significant at the 5% level, but negative. Conversely, the same regression on the balance index returns statistically significant (p-value=0) positive coefficients: in a nutshell, a "pessimistic" intercept. The role of this overestimation will turn out to be relevant for the microsimulation presented in Section 1.5.2.

Table 1.3: Auxiliary regression $u_{t+4} - u_t = \phi_0 + \phi_1 EU_t^U + \epsilon_t$ (1986q1-2016q1)

	ϕ_0	ϕ_1
FRA	-0.5855***	0.0177***
GER	-0.3932***	0.0142***
ITA	-0.7320***	0.0283***
UK	-0.8291***	0.0267***

Notes: for Germany, 1991q1-2016q1. For the United Kingdom, 1986q1-2015q4

Having identified the variable under investigation, the second relevant point concerns the investigation of its statistical process. Does the year-over-year change in unemployment rate follow a process such as represented by Eqs. (1.3)-(1.4) or as represented by Eqs. (1.15)-(1.16)?

The usual way to clarify this dilemma consists in testing for a unit root in the year-over-year change of unemployment rate (i.e. $u_t - u_{t-4} \equiv \Delta_4 u_t$) for the countries under

[24] This auxiliary regression is known in the literature as the "regression approach" to qualitative surveys Pesaran (1984, 1987). This kind of approach may suffer from measurement errors, since it regresses ex-post actual change in the unemployment rate (x_t) with ex-ante expectations of the fundamental value x_t^*, which could be ex-post wrong due to the disturbance ϵ_t. Measurement errors cause attenuation bias in the estimated coefficients. In order to mitigate the possible attenuation bias problem we use IV instead of OLS (Sargan, 1958; Farmer et al., 2009).

investigation. We apply two types of tests: (1) a test with a unit root null (the Augmented Dickey-Fuller (ADF) of Dickey and Fuller (1979)) and (2) a test with a trend-stationary null (the Kwiatkowsky-Phillips-Schmidt-Shin (KPSS) test of Kwiatkowski et al. (1992)). Results are reported in Table 1.4. We find that, for all countries under investigation, the ADF test rejects the null while the KPSS test fails to reject the null. This implies that there is a strong evidence in favour of a stationary process of Δu_t for all countries.

Table 1.4: Unit root tests results (1986q1-2016q3)

	ADF		KPSS	
	Statistic	Lag	Statistic	k
$(\Delta_4 u_t)_{FRA}$	-2.963**	5	0.095	8
$(\Delta_4 u_t)_{GER}$	-3.896***	6	0.197	8
$(\Delta_4 u_t)_{ITA}$	-3.027**	6	0.135	8
$(\Delta_4 u_t)_{UK}$	-3.122**	5	0.109	8
Critical values	1%	-3.487		0.739
Critical values	5%	-2.886		0.463
Critical values	10%	-2.580		0.347

Notes: $u_t - u_{t-4} \equiv \Delta_4 u_t$. Since observed data does not exhibit an increasing or decreasing trend, in test equations only an intercept is considered as deterministic term. The H_0 in ADF is that the variable is I(1). The H_0 in KPSS is that the variable is I(0). The lag length in ADF is chosen using SIC. k is the bandwidth for the Newey-West HACC estimator with Bartlett weights. ***, ** and * denote significance at 1%, 5% and 10% levels, respectively.

Table 1.5: Unobserved component model estimation of $\Delta_4 u_t$ (1986q1-2016q3)

Model: $\Delta_4 u_t = \Delta_4 {u_t}^* + \epsilon_t$, $\epsilon_t \sim NID(0, \sigma_\epsilon^2)$
$\Delta_4 {u_{t+1}}^* = \alpha + \beta \Delta_4 {u_t}^* + \eta_{t+1}$, $\eta_t \sim NID(0, \sigma_\eta^2)$
(disturbances are uncorrelated)

	α	β	$\Delta_4 {u_t}^*$	Wald Test $\beta = 1$	$\sigma_\epsilon/\sigma_\eta$
FRA	-0.003	0.873***	(See Fig. 1.B.2)	p-value=0.009	1.72
GER	-0.010	0.874***	(See Fig. 1.B.2)	p-value=0.007	1.39
ITA	0.010	0.913***	(See Fig. 1.B.2)	p-value=0.022	1.38
UK	-0.020	0.908***	(See Fig. 1.B.2)	p-value=0.016	1.52

Notes: The estimation method is the Maximum Likelihood (ML) with BFGS optimization procedure with Marquardt step. The standard errors are computed using the negative inverse Hessian after convergence.*** indicates 1% significance level.

A more sophisticated alternative way to shed light on the above-mentioned dilemma consists in estimating the process of Δu_t via univariate unobserved component (UC) model. A UC allows us to decompose the change of the unemployment rate in a persistent component (Δu_t^*) and shocks elements (ϵ_t and η_t). The goal in this empirical exercise is to investigate the persistence of the fundamental value Δu_t^*.[25] Results of this estimation for France, Germany, Italy, and the UK are reported in Table 1.5. For all countries, the

[25] For a visual inspection of the dynamics between the fundamental value and the actual change in the unemployment rate, see Figure 1.B.2 in Appendix 1.B.

coefficient β, that measures the persistence of the fundamental component, is smaller than unity and the Wald test confirms this statistically. The unobserved component estimates allow us to check the central hypothesis of the CSI model, that changes in the unemployment rate move around a fundamental value proxied by the expert unemployment expectations. A correlation-based analysis in Appendix 1.C confirms this evidence giving an important support for this crucial assumption.

Following unit root and UC estimates, we assume households have some intuition that, in absence of new information, the best possible guess is that unemployment change is less-than-proportional to the previous one. On this basis, we can affirm that the most plausible version of the CSI model is that with a persistent (but stationary) fundamental value described in section 1.3.2. The final equation representing the aggregate change in unemployment expectation is the following:

$$M_t\left[\Delta_4 u_{t+4}\right] = \lambda N_t\left[\Delta_4 u_{t+4}\right] + (1-\lambda)\left(\alpha + \beta M_{t-1}\left[\Delta_4 u_{t+3}\right]\right), \tag{1.25}$$

which corresponds to the four-quarter unemployment rate change ($\Delta_4 u_t$) version of Eq. (1.22) described in section 1.3.2 for a generic macroeconomic or financial variable x.

1.4 CSI model and "news-based" uncertainty

In Carroll (2003, 2006), the parameter λ captures the probability of being infected by opinions diffused by news media and, in this way, it determines the aggregate expectation of the variable of interest. Given the relevance of households beliefs in influencing the pattern of economies, as presented in Section 1.2, it is important to understand which factors may influence λ and which is the channel of transmission of the virus (i.e. the professional forecasters expectations).

In general, non-expert agents adapt the level of attention they put on professional forecasters estimates in response to changes in the environmental conditions.

The very first intuition is that a more uncertain environment should induce economic agents to collect more information in order to avoid wrong decisions (Coibion and Gorodnichenko, 2015; Reis, 2006). Anyway, it is not the only effect involved. For example Moscarini (2004) presents a model in which agents update their information set infrequently, but absorbing information is more challenging (hence, more costly) when the environment is more uncertain.[26] This higher cost of collecting/processing information mitigates, and possibly outweights, the hunger for state-of-the-art information.

Furthermore, "noisy information" models (Sims, 2003; Woodford, 2003) emphasize that the weight agents put on the signal they receive depends on the level of noisiness of that signal. Similarly, in the CSI framework it is reasonable to assume that the level of economy-wide uncertainty perceived by non-expert agents may affect their decision to spend time in exploiting news media to “capture” the predictions of professional forecasters. For example, Heiner (1989), Beckert (1996), and Dequech (1999) claim that in moments of high uncertainty people adopt “rule of thumbs”. There is strong evidence in

[26] "For example, reading the Wall Street Journal every day in recent times of stock market turbolence is more time- and capacity-consuming because the quantity of information transmitted is higher for the given daily frequency, and less capacity is left for reading novels or thinking about dinner" Moscarini (2004).

experimental studies that people under uncertainty tend to use heuristics or intuitions deviating from full rationality (see, for example, Kahneman et al. (1974)). In our framework, this implies that uncertainty influences (negatively) the decisions of non-expert agents to look for information by reading newspapers, surfing the web and watching newscasts. In other words, agents, in presence of sustained uncertainty, are less confident on the capacity of experts to predict the future (actual) values of unemployment and may decide to use the rule of thumb updating expectation rule (i.e. Eq. (1.19) according to the CSI framework) instead of spending time to read newspapers. Hence, it would not be so surprising to observe a drop in parameter λ in periods of high uncertainty. It is important to emphasize that in the CSI framework this does not mean that agents may decide to "forget" and not to use the professional forecasts they are aware of;[27] conversely, they may not put a particular effort in capturing new forecasts. In a nutshell, this could imply that a typical agent continues to read newspapers but he may decide not to care about the financial section, which reports the updated forecasts. Furthermore, in periods in which agents pay less attention to expert forecasts, we expect to see in survey data waves of optimism or pessimism unrelated to the expert forecasts.

The mechanism described above is important because it helps to understand the transmission channel of the virus. Generally speaking, an agent may be infected through the “traditional” channel (print journalism and broadcast news) and the Internet channel (online versions of newspapers, plus online news blogs and social media). Whether the parameter λ is more sensitive to the level of uncertainty conveyed by the "traditional" press or to the one conveyed by the Internet, it is a relevant clue about which can be considered as the main channel of transmission of the virus. Obviously, it may happen that both channels influence agents decision to intercept the professional predictions.

As we describe more in detail in the data appendix (Appendix 1.G), the use of "news-based" indexes like the well-known Baker et al. (2016) Economic Policy Uncertainty Index (EPU), which is based upon newspaper articles content, and an index of uncertainty based on online search engines data from Google Trends (Google Uncertainty Index, GUI) may help to proxy the level of uncertainty spread out by the two transmission channels. One relevant difference between the two approaches is that while the traditional uncertainty index is based upon journalists' feeling about uncertainty,[28] the GUI focuses on agents perception of uncertainty counting the volume of searches for words containing the terms uncertain or uncertainty, economic or economy. The intensity of Internet searches, which are related to the above mentioned keywords, should reflect (proxy) a high level of uncertainty perceived among non-expert agents.

[27] Remember that in the model if you are infected you cannot recover from the infection (Assumption 4 in Section 1.3.1).

[28] Quoting from the methodology part of the EPU website `http://www.policyuncertainty.com/methodology.html`, "We count the number of newspaper articles containing the terms uncertain or uncertainty, economic or economy, and one or more policy-relevant terms".

1.5 Estimation strategy

1.5.1 Econometric strategy

We are interested in (i) estimating equation (1.25) together with the need to (ii) investigate the relationship between the parameter λ and the uncertainty in the economy (as explained in Section 1.4). In particular, the second point requires the adoption of a time-varying approach in estimating the parameters for comparing λ with the uncertainty index measure over time. The easiest way to satisfy the two point is to estimate equation (1.25) via a state-space approach. Equation (1.25) can be easily expressed as follows:

$$\begin{cases} \widehat{M_t}[\bullet] = \alpha_0 + \theta_t N_t[\bullet] + \varphi_t \widehat{M}_{t-1}[\bullet] + \epsilon_t^M \\ \theta_{t+1} = \omega_\theta \theta_t + \epsilon_{t+1}^\theta \sim NID(0, \sigma_\theta^2) \\ \varphi_{t+1} = \omega_\varphi \varphi_t + \epsilon_{t+1}^\varphi \sim NID(0, \sigma_\varphi^2) \end{cases} \tag{1.26}$$

where $\theta_t \equiv \lambda_t$ and $\varphi_t \equiv (1-\lambda_t)\,\beta_t$. The key parameter λ and the product of parameters $(1-\lambda)\,\beta$ are now expressed as AR(1) processes to study their evolution over time. With respect to a simple rolling window estimation, a state-space with time-varying coefficients has the advantage of not losing observations.[29]

In addition to the state-space model, as a robustness check, we run a GMM estimate of equation (1.25).[30] The choice of GMM, specifically IV, instead of OLS[31] lies in the presence of potential measurement errors in the non-expert agents expectations variable. These potential errors are due to the transformation needed to convert EU_t^U (Non-expert expectations expressed in balance terms) in the same metric of changes in the unemployment rate of $N_t[\bullet]$ (see Eq. (1.23) and Eq. (1.24)). In particular, as Sargan (1958) stressed, variables used for constructing the instrument need to be independent from the ones involved in the second-stage regression. This requirement excludes the use of the unemployment rate and lags of variables in the relationship. For our purposes we use (lagged) international variables and financial variables as instruments, which satisfy the requirement of Sargan (1958).

[29] Alternatively, it is possible to model the time-varying coefficient λ to be a function of exogenous factors related to uncertainty, such as NBER recessions (Coibion and Gorodnichenko, 2015) or uncertainty indexes (Easaw et al., 2017). Anyway, the main aim of our paper is instead first to investigate the time-varying proportion of people reading newspapers, then studying a relationship with uncertainty. For this reason, we prefer to avoid the approach suggested by the SDM (State Dependent Models) literature of considering volatility or uncertainty indexes as explanatory variables, since we would force a correlation and weaken our conclusions.

[30] As argued by Geary (1948) and Sargan (1958), and more recently by Fuller (2009, p.273), the instrumental variables is a suitable estimation technique in cases when the variables in the relationship are measured with errors.

[31] The measurement error may produce a downward bias in the estimated coefficients. Actually, OLS estimation produces estimates of λ which are much closer to zero and not significant at all:

$$\begin{array}{llll} \text{FRA} & \alpha(1-\lambda) = \underset{(0.016)}{-0.004} & \lambda = \underset{(0.045)}{0.071} & \beta = \underset{(0.046)}{0.861} \\ \text{GER} & \alpha(1-\lambda) = \underset{(0.014)}{-0.001} & \lambda = \underset{(0.034)}{0.011} & \beta = \underset{(0.038)}{0.868} \\ \text{ITA} & \alpha(1-\lambda) = \underset{(0.016)}{0.006} & \lambda = \underset{(0.052)}{0.054} & \beta = \underset{(0.051)}{0.918} \\ \text{UK} & \alpha(1-\lambda) = \underset{(0.020)}{-0.024} & \lambda = \underset{(0.049)}{0.047} & \beta = \underset{(0.042)}{0.951} \end{array}.$$

1.5.2 Micro-simulation specification

The derivation of an econometrically testable equation as in Section 1.5.1 requires some assumptions, like the presence of an unique source of infection, no interaction among agents and no heterogeneity across agents other than being "infected" or not in that particular period. Extending the model, relaxing one of more of these assumptions, requires a different approach than an OLS, GMM or state-space estimation. Carroll (2006) proposes the use of agent-based models in order to, for example, introduce heterogeneity in the infection rates (a point that will be deepened in Chapter 2), or to allow for interaction among agents instead of assuming a unique source of infection.

Although both the theoretical and the empirical analysis provide relevant support for the CSI model, the assumption that all agents are rational, even though boundedly rational, may be too strong in practice. There may be individuals who are permanently stuck to their overpessimistic or overoptimistic positions. In particular, the negative and significant estimates of $\hat{\phi}_0$ of the auxiliary regression (1.23) suggests the presence of a systematic overestimation in aggregate households expectations. Also Carroll (2003, 2006) finds as "the only real empirical problem" an highly significant constant term in the regression of Eq. (1.14), while theoretically there should be no intercept. He indicates as possible cause a misspecification of the model, like not allowing for social interaction, rather than "accepting" the presence of a permanent bias. As an alternative explanation of this bias, in the current model we add heterogeneity in households behaviour. We assume that there are two types of agents:

- a fraction $1-\mu$ of households develop expectations following the CSI framework
- the remaining fraction μ of households are not rational in any sense but are *stubbornly pessimistic.*

Given that the heterogeneous framework does not allow to apply the approach presented in Section 1.5.1, we adopt an Agent-Based model, in line with the suggestion of Carroll (2006). We will simulate individual expectations and then aggregate them, studying the fit with empirical survey balances.

We will avoid country subscripts for simplicity, but the following process is repeated independently for each selected country. We use the subscript i to denote a variable concerning agent i.[32] All agents are inizialized with $E_0^i[\Delta_4 u_4] = 0$. "Infection" of agent i, in any period t, is a Bernoulli random variable Inf_t^i with $\mathbb{P}(Inf_t^i = 1) = \lambda$, and *stubbornly pessimism* is modeled as a Bernoulli with $\mathbb{P}(Stub_t^i = 1) = \mu$. Therefore, we have:

$$E_t^i[\Delta_4 u_{t+4}] = \begin{cases} N_t[\Delta_4 u_{t+4}] & \text{if } Inf_t^i = 1 \text{ (Informed)}, \\ \alpha + \beta E_{t-1}^i[\Delta_4 u_{t+3}] & \text{if } Inf_t^i = 0 \text{ (Uninformed)}. \end{cases} \tag{1.27}$$

In such a way, we produce quantitative expectations. In order to compare the simulation results with the EC Consumer Surveys, we mimick Carlson and Parkin (1975) probability approach to qualitative surveys. We add the parameter γ to transform the simulated quantitative into qualitative estimates, namely the answer Ans_t^i to Q7 as reported in Section 1.3.3.

[32] User Guide for European Commission's Joint Harmonised EU Programme of Consumer Surveys reports the number of individuals interviewed per month in each country: 3300 for France and 2000 each for Germany, Italy and the United Kingdom. We simulate the (quarterly) model accordingly: 9900 agents for France and 6000 each for Germany, Italy and the UK.

$$Ans_t^i(E_t^i[\Delta_4 u_{t+4}]) = \begin{cases} ++ \ \text{(increase sharply)} & \text{if } E_t^i[\Delta_4 u_{t+4}] \geq \gamma \vee Stub_t^i = 1, \\ + \ \text{(increase slightly)} & \text{if } \frac{\gamma}{2} \leq E_t^i[\Delta_4 u_{t+4}] < \gamma \wedge Stub_t^i = 0, \\ = \ \text{(remain the same)} & \text{if } -\frac{\gamma}{2} < E_t^i[\Delta_4 u_{t+4}] < \frac{\gamma}{2} \wedge Stub_t^i = 0, \\ - \ \text{(fall slightly)} & \text{if } -\gamma < E_t^i[\Delta_4 u_{t+4}] \leq -\frac{\gamma}{2} \wedge Stub_t^i = 0, \\ -- \ \text{(fall sharply)} & \text{if } E_t^i[\Delta_4 u_{t+4}] \leq -\gamma \wedge Stub_t^i = 0, \end{cases}$$

The simulated index $\widehat{EU_t^U}$ is obtained as a balance of the simulated answers and compared with EU_t^U.

1.6 Estimation output

In this section we report the results of econometric estimates (Subsection 1.6.1) and simulation (Subsection 1.6.2). Furthermore, we present also an intepretation of the results in terms of uncertainty. A full description of data used is reported in Appendix 1.G.

1.6.1 Econometric results

The time-varying parameters pattern of state-space model (1.26) is plotted in Figure 1.2 and Figure 1.3. In particular, in Figure 1.2 we plot the evolution of λ_t, whereas in Figure 1.3 we plot the dynamics of aggregate $(1-\lambda_t)\,\beta_t$. From Figure 1.2 it emerges that in all countries λ fluctuates around an average value between 0.07 and 0.1. The dynamics are very similar for all countries. An important drop in the value of λ occurred in Germany and the UK in correspondence to the financial and sovereign crisis. This drop is less evident instead in Italy and France. Concerning Figure 1.3, the evolution of $(1-\lambda_t)\,\beta_t$ appears smoother for all countries. As a further consideration, the average values are smaller than unit as expected. The GMM estimates of Equation (1.25) are in Table 1.6. The values of the parameters are in line with the average values obtained via time-varying state-space model. In particular, France and the UK exhibit higher values of λ with respect to the other countries in accordance with the state-space estimates. More importantly, using the values of λ and β obtained from GMM, we obtain values very similar to the average values of $(1-\lambda_t)\,\beta_t$ in the state-space model.[33] Given the similarities of GMM and state-space model estimates, we can confirm the robustness of our results. Figure 1.4 compares the estimates of λ of various countries with the EPU of Baker et al. (2016). The λ seems to move clearly in opposite direction with respect to the EPU index for France and Italy;[34] for these two countries the correlation over the two series for the whole period (1997Q1-2016Q3) is -0.31 for France and -0.38 for Italy. The comovement of λ and the EPU is less clear for Germany and the UK; the correlation value is very low for both countries. These low values of correlation may suggest that a typical agent in Germany and the UK does not use print journalism and similar traditional media as the primary source of information (and then contagion). Figure 1.5 shows the dynamics of λ with

[33]In detail, the average values are: $[(1-\lambda)\,\beta]^{FRA} = 0.83$; $[(1-\lambda)\,\beta]^{GER} = 0.85$; $[(1-\lambda)\,\beta]^{ITA} = 0.87$; $[(1-\lambda)\,\beta]^{UK} = 0.84$.

[34]Note that in Figure 1.4 the uncertainty index is plotted on right axes with inverted scale.

Figure 1.2: Time-varying estimates of λ obtained via state space model (1986Q1-2016Q4)

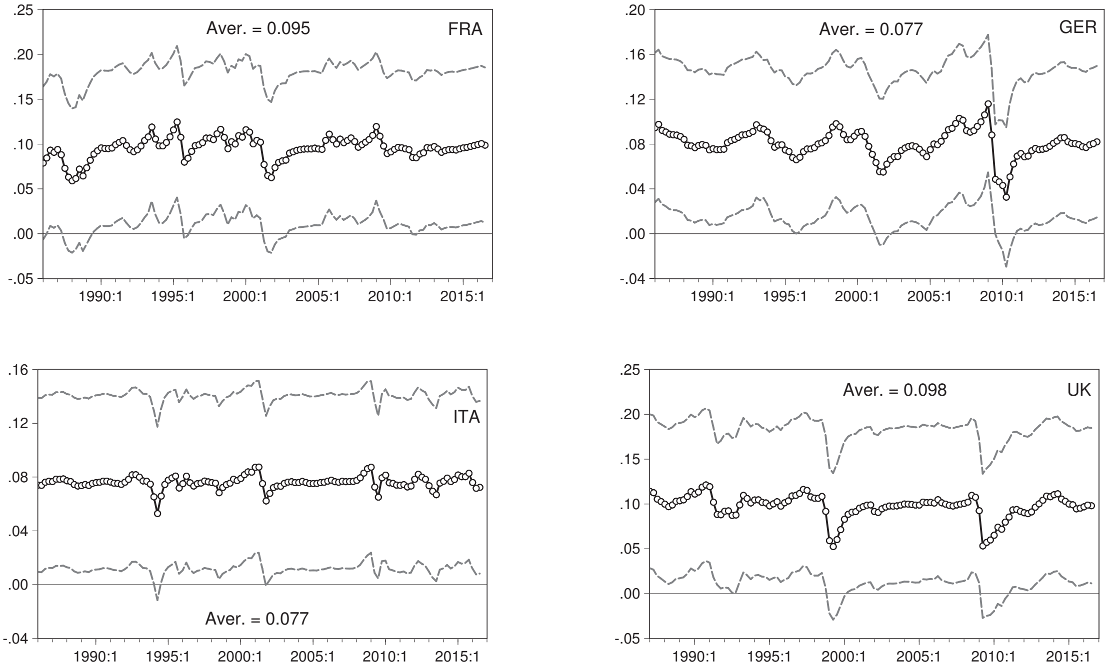


Figure 1.3: Time-varying estimates of $(1-\lambda)\beta$ obtained via state space model (1986Q1-2016Q4)

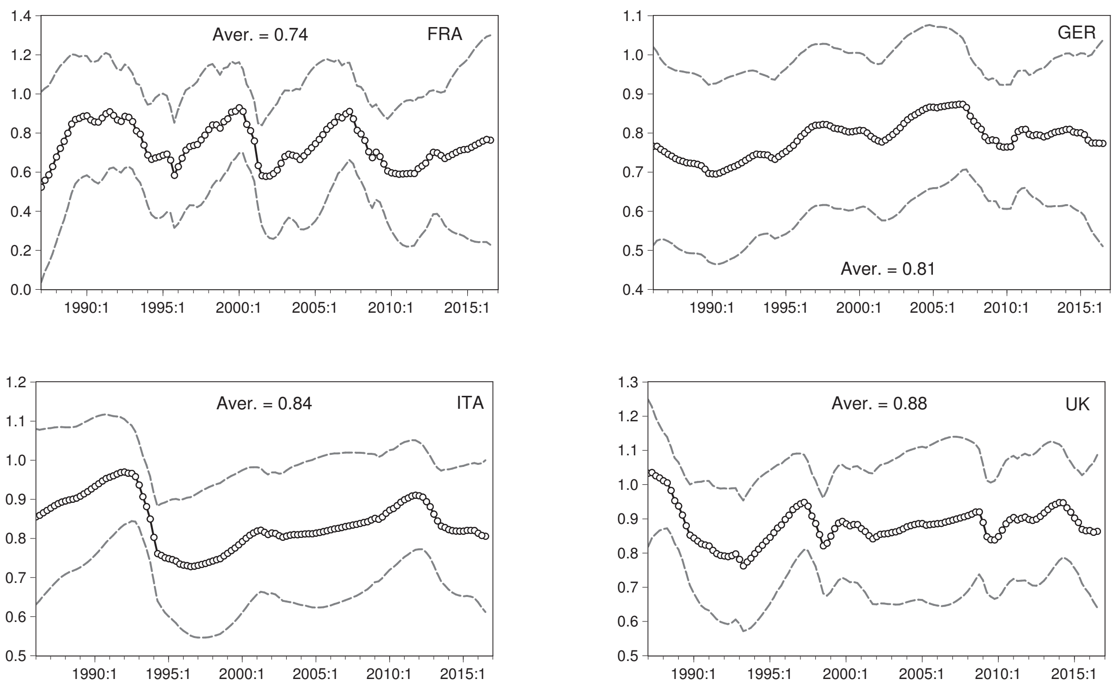


Figure 1.4: Time-varying estimates of λ *vs* Policy Uncertainty Index (`EPU`, inverted scale) (1997Q1-2016Q3)

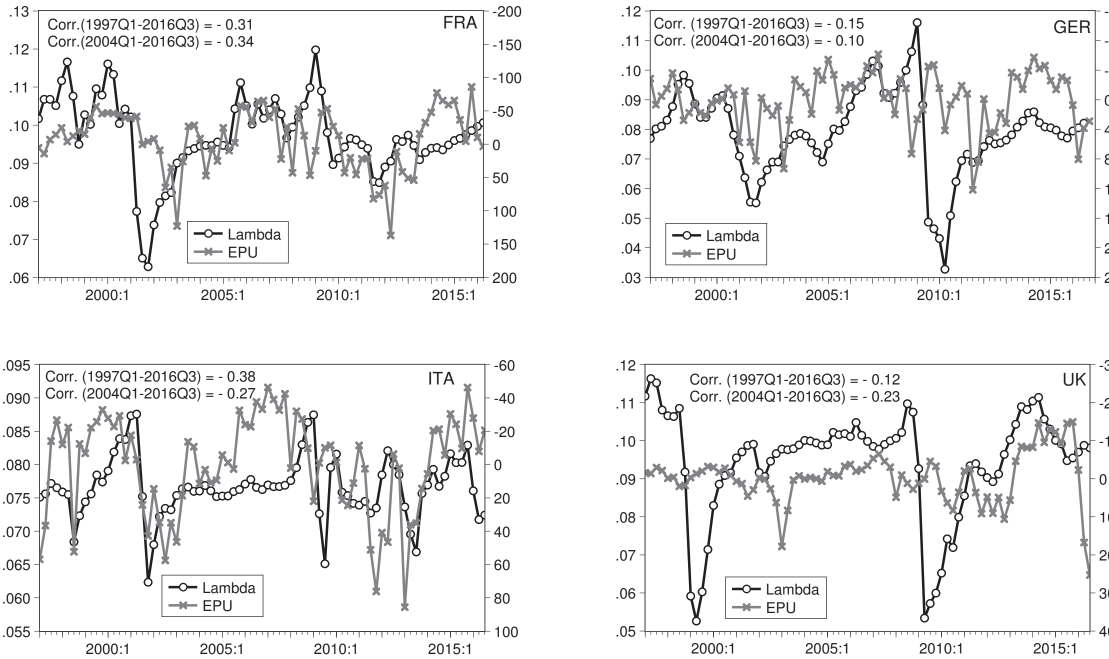


Figure 1.5: Time-varying estimates of λ *vs* Google Uncertainty Index (`GUI`, inverted scale) (2004Q1-2016Q3)

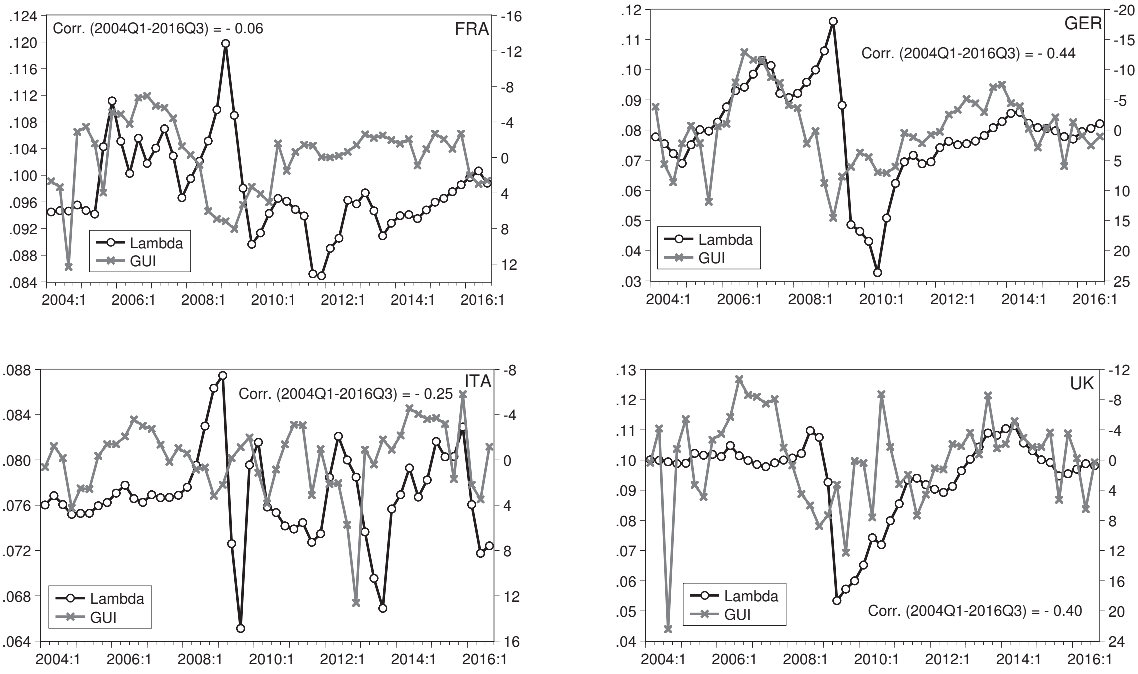
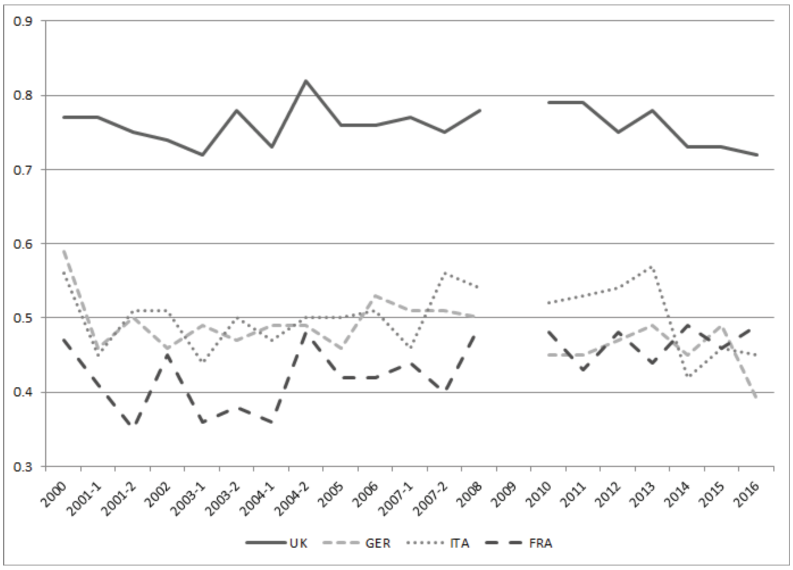


Table 1.6: GMM estimates of Eq. (1.25) (FRA-ITA 1987q2-2016q4, GER 1986q2-2016q4, UK 1987q3-2016q4)

Model: $\widehat{M_t}\left[\Delta_4 u_{t+4}\right] = \lambda N_t\left[\Delta_4 u_{t+4}\right] + (1-\lambda)\left(\alpha + \beta \widehat{M_{t-1}}\left[\Delta_4 u_{t+3}\right]\right.$				
	$\alpha(1-\lambda)$	λ	β	Prob (J-stat)
FRA	-0.014	0.135*	0.962***	0.448
	(0.015)	(0.077)	(0.071)	
GER	-0.003	0.080*	0.924***	0.378
	(0.009)	(0.042)	(0.058)	
ITA	0.011	0.093*	0.955***	0.810
	(0.007)	(0.050)	(0.037)	
UK	-0.052**	0.127**	0.962***	0.542
	(0.024)	(0.056)	(0.053)	

Notes: List of instruments used (in addition to the constant): FRA: $\sum_{j=1}^{4}\Delta_4 ln(y^{USA})_{t-j}$, $\sum_{j=1}^{2}\Delta_4 ln(sp)_{t-j}$, $\sum_{j=0}^{1}\Delta_4 ln(oil)_{t-1}$, $\sum_{j=0}^{1}\Delta_4 i_{t-j}$, $\sum_{j=1}^{3}\Delta_4 ln(hp^{USA})_{t-j}$; GER: $\sum_{j=1}^{2}\Delta_4 ln(y^{USA})_{t-j}$, $\sum_{j=0}^{1}\Delta_4 i_{t-j}$, $\sum_{j=1}^{2}\Delta_4 ln(sp)_{t-j}$, $\sum_{j=1}^{3}\Delta_4 ln(hp)_{t-j}$; ITA: $\sum_{j=1}^{2}\Delta_4 ln(y^{USA})_{t-j}$, $\Delta_4 ln(sp)_{t-1}$, $\sum_{j=0}^{2}\Delta_4 ln(oil)_{t-j}$, $\sum_{j=1}^{2}\Delta_4 ln(hp)_{t-j}$; UK: $\Delta_4 ln(sp)_{t-1}$, $\sum_{j=0}^{2}\Delta_4 ln(oil)_{t-1}$, $\sum_{j=1}^{3}\Delta_4 ln(hp^{USA})_{t-j}$, $\sum_{j=0}^{1}\Delta_4 i_{t-j}$, $\sum_{j=0}^{1} spread_{t-j}$.
$\widehat{M}[\bullet]$ indicates that the average non-expert agents expectation is built using the auxiliary regression estimates (1.24). Newey-West (HAC) standard errors are reported in parentheses. J-stat is the Sargan's J statistical test.

Figure 1.6: Confidence in the press, 2000-2016



Notes:Confidence in the press indicates the percentage of people who tend not to trust the press. Source: Eurobarometer survey (`http://ec.europa.eu/commfrontoffice/publicopinion/index.cfm/`).

respect to the GUI obtained via Google trends. Plots for Germany and the UK show high negative correlation with the GUI, equal to -0.44 and -0.40, respectively. These results are supported by other studies conducted on households habits in the European countries. In particular, the Eurobarometer survey data shows that British agents have a poor opinion about the quality and usefulness of the press.[35] The value is among the lowest in Europe. Figure 1.6 plots the percentage of people who do not trust the press for the period 2000-2016. From Figure 1.6 it emerges clearly that British agents are very skeptical about the reliability of information disseminated by press. Conversely, the French, the Germans and the Italians have a better consideration of press information content. This evidence may suggest that agents in the UK use as source of information other media such as blogs and social media. Figure 1.5 on the relation between λ and the GUI confirms this hypothesis. Similarly for Germany, λ is more correlated with the GUI than with the EPU; conversely, for France λ is almost uncorrelated with the GUI. The case of Italy, finally, is curious: it is the country with the highest correlation between λ and the EPU but, if we focus on the subperiod for which we have data for both the EPU and the GUI (i.e. since 2004), this correlation decreases and is almost equal to the one between λ and the GUI. It is like if Internet is partially substituting print journalism as a source of contagion. This insight is worth some future research.

1.6.2 Simulation results

The model, as highlighted in Section 1.5.2, is based on five parameters: λ, α, β, γ and μ.

The values of α and β are calibrated according to the macro-level estimates reported in Sections 1.3.3 and 1.6.1; According to Table 1.5 and Table 1.6, α is not statistically different from zero,[36] therefore we can safely assume $\alpha = 0$: hence, no permanent bias in the expectations of the boundedly rational individuals.[37]

Similarly, GMM estimates reported in Table 1.6 fail to reject the hypothesis that the value of β implied by survey data is different from the value of β related to the underlying fundamental process, as estimated in Table 1.5: in other words, we can not exclude that the level of persistence that people have in mind is the "true" one. At the very same time, GMM estimates reported in Table 1.6 are hardly significantly different from one. Summing up these considerations, in the baseline calibration we assume β equal to the estimates of the fundamental process (Table 1.5), and we provide as robustness checks in Appendix 1.D two alternative scenarios with $\beta = 0.8$ and $\beta = 1$.

λ is for the time being assumed to be constant (we will relax this assumption later) and equal to the average of the time-varying estimates reported in Figure 1.2. Once these three parameters have been fixed, γ and μ are estimated through the Method of Simulated Moments, in order to match the mean and the standard deviation of EU_t^U.

Table 1.7 reports the calibration of the micro-simulation parameters and the respective 95% confidence intervals. The estimates for μ suggest that empirical data are consistent with a non-trivial fraction of agents being "stubbornly" pessimistic. The number goes from about one fourth for Italy and the United Kingdom to about one third for France and Germany. Estimates for γ imply that changes in the unemployment rate beyond

[35] Available at http://ec.europa.eu/commfrontoffice/publicopinion/index.cfm.

[36] Except for the UK in Table 1.6.

[37] Which develop expectations according to the CSI framework.

Figure 1.7: Real and micro-simulated survey balances (constant λ) (1986Q1-2016Q2)

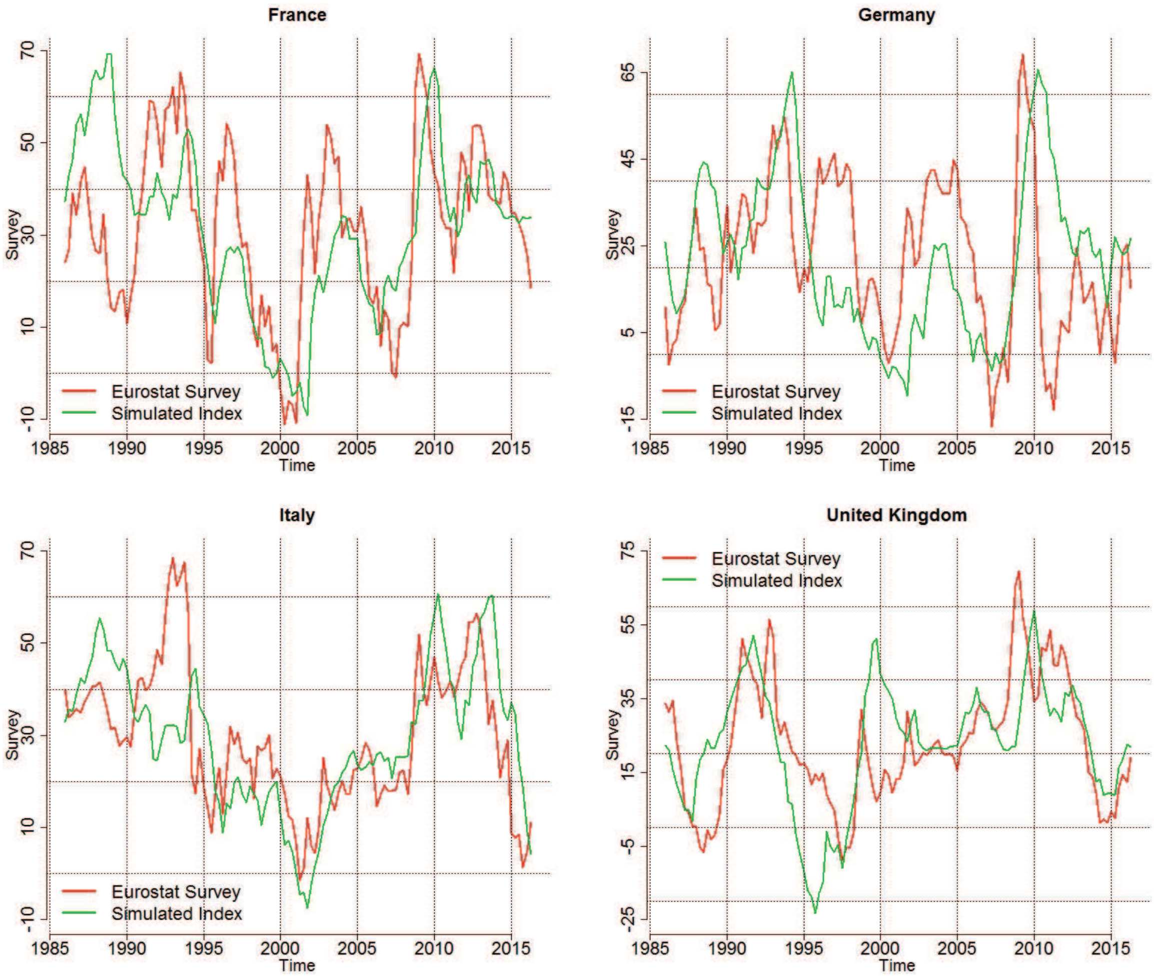


Figure 1.8: Real and micro-simulated survey balances (time-varying λ) (1986Q1-2016Q2)

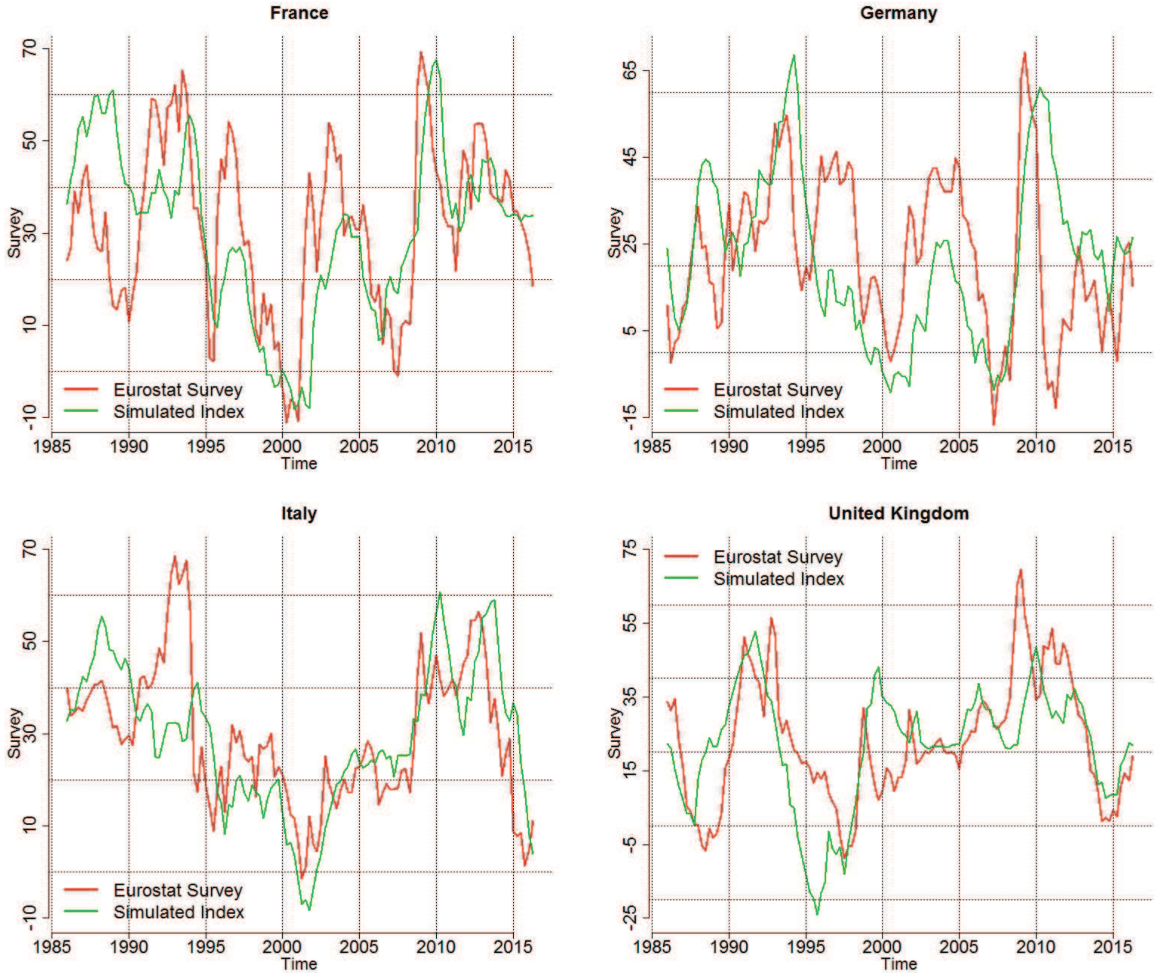


Table 1.7: Micro-simulation calibration (1986Q1-2016Q2)

Country	Fra	Ger	Ita	Uk
α	0	0	0	0
β	0.873	0.874	0.913	0.908
λ	0.095	0.077	0.077	0.098
μ	0.340 (0.338-0.341)	0.329 (0.326-0.332)	0.284 (0.282-0.286)	0.215 (0.213-0.216)
γ	0.449 (0.443-0.455)	0.261 (0.253-0.268)	0.388 (0.383-0.393)	0.455 (0.452-0.458)

0.25-0.45 percentage points are considered by agents as sharp changes. Noteworthy, γ estimates are of the same order of magnitude of the standard deviations of disturbances ϵ and η (see Table 1.8).

Table 1.8: Ratio σ_ϵ and σ_η (Table 1.5) to γ (Table 1.7) (1986Q1-2016Q2)

	Fra	Ger	Ita	Uk
σ_ϵ/γ	1.13	1.92	1.03	1.10
σ_η/γ	0.65	1.38	0.75	0.73

Figure 1.7 allows for a visual inspection of the fit between the real balance EU_t^U and the simulated balance $\widehat{EU_t^U}$ obtained through the calibration in Table 1.7. The correlation is significant and above 0.5 for all countries but for Germany, where it is around 0.25.

Figure 1.8 allows for a visual inspection of the fit between the real balance EU_t^U and the simulated balance $\widehat{EU_t^U}$ using the time-varying estimates for λ instead of the constant "infection probability" reported in Table 1.7. The correlation is a bit higher for all selected countries, in particular for France and Germany.

Table 1.9: Correlation of real balance and simulated balance (1986Q1-2016Q2)

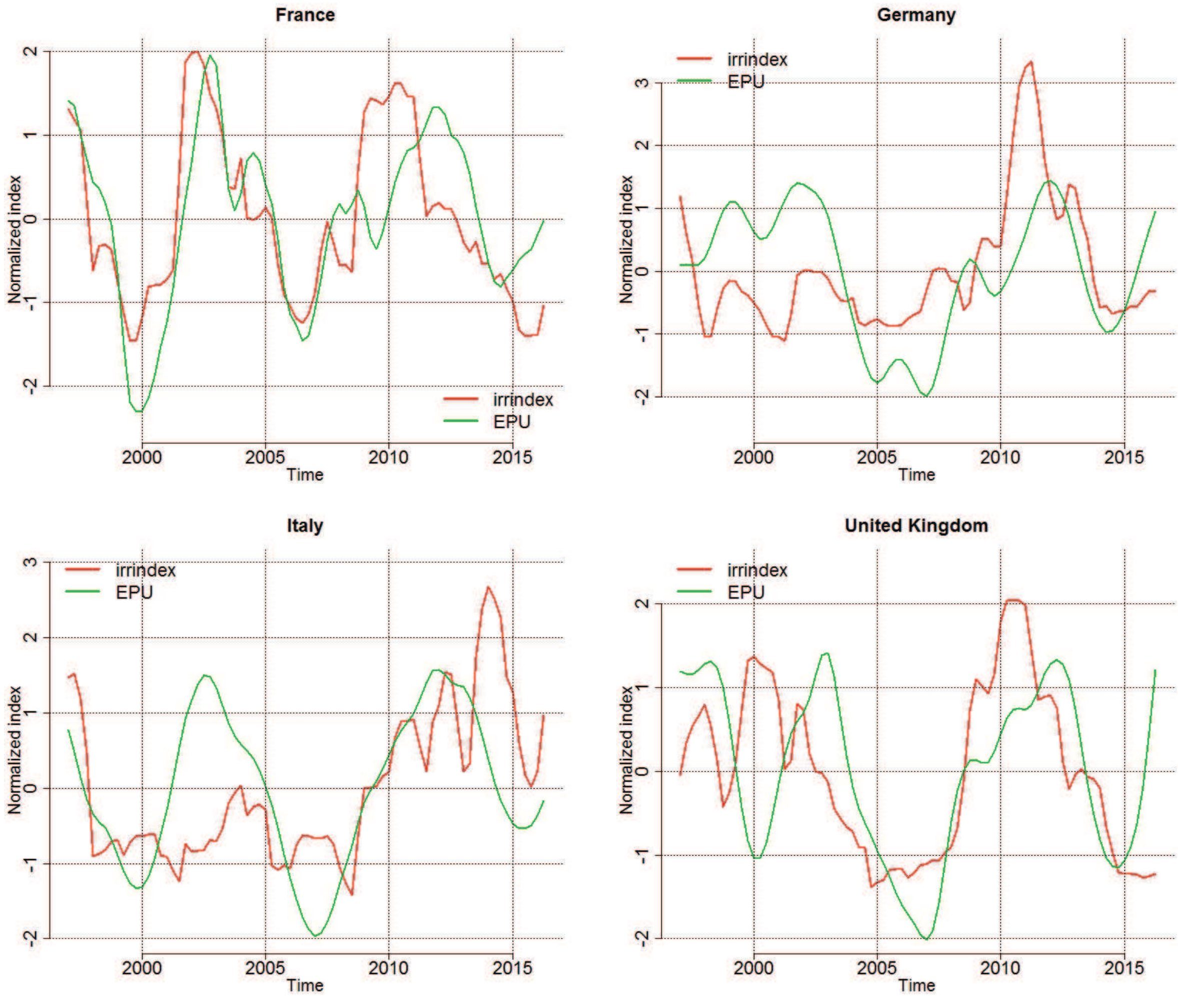
	Fra	Ger	Ita	Uk
Constant λ	0.49	0.27	0.59	0.55
Time-varying λ	0.55	0.33	0.61	0.57

As a further consideration, we develop an uncertainty measure based on the deviation of the real balance from the micro-simulated one. The idea is that, during periods of heightened (perceived) uncertainty, the actual change is expected to be a noisier representation of the its fundamental value. Hence, agents may think that experts are still able to correctly forecast the fundamental rate, but that the actual rate is likely to be much different. Therefore individuals care less about expert forecasts and their expectations may be influenced by alternative sources, leading to waves of optimism/pessimism unexplained by the expert forecasts. Our proposed uncertainty index (`irrindex`) is then based on the waves of optimism/pessimism that remain unexplained by the present model ($EU^U - \widehat{EU^U}$). Specifically, given that agents update approximately once every 10 quarters their information, we choose the 10-quarter rolling standard deviation:

$$\texttt{irrindex}_t = \sqrt{V(EU_t^U - \widehat{EU^U}_t, ..., EU_{t-9}^U - \widehat{EU^U}_{t-9})} \tag{1.28}$$

In Figure 1.9 we compare the `irrindex` with Economic Policy Uncertainty Index (Baker et al., 2016): on average, the higher the level of uncertainty, the more frequent the waves of optimism and pessimism unrelated to expert forecasts, complementing the interpretation of the time-varying λ from Section 1.6.1.

Figure 1.9: Rolling standard deviation of the component unexplained by the agent-based model (`irrindex`) *vs* policy uncertainty index (`EPU`) (1997Q1-2016Q2)



1.7 Policy considerations

Classical macroeconomic theory mostly concerns two policy instruments, the fiscal policy and the monetary policy. In the current macroeconomic setting, for most European countries these two weapons have become much less effect effective. Countries belonging to the Euro area have given up monetary policy - or, better rephrased, monetary policy is conducted at the European level, and the European central bank has to make a trade-off among the short-term needs of the different countries. Similarly, there are constraints in the use of fiscal policy. Exploring "alternatives" to these classical macro policy instruments is a relevant topic, in particular for these countries.

In this section, our aim is first to study the role of households (unemployment) sentiment on the output gap. Secondly, we will exploit the micro-simulation procedure to explore some "what-if" scenarios, assuming modifications in the degree of households attention and trust on professional forecasters opinions, and the consequences on the output gap during the financial and the sovereign crisis.

1.7.1 Output gap

We inspect the relation between the output gap and unemployment expectations. In order to do so, we write down an IS curve which links the output gap to the inflation rate and the nominal interest rate (Goodhart and Hofmann, 2005), augmented with unemployment expectations EU_t^U:

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 \Delta y_{t-1} + \alpha_3 (i_{t-1} - \pi_{t-1}^{GDP}) + \alpha_4 EU_t^U + \epsilon_t \tag{1.29}$$

where y_t is the output gap and i_t and π_t^{GDP} are the four quarter moving averages of the nominal short-term interest rate and of the inflation rate implied by the GDP deflator, respectively. Both the inflation and interest rates have been detrended using a quadratic trend. The output gap is calculated as the deviation from a quadratic trend.

Table 1.10: OLS estimates of the IS curve Eq. (1.29) (France and UK 1991q2-2016q4, Germany 1991q3-2016q4, Italy 1995q3-2016q4)

	FRA	GER	ITA	UK
α_0	0.246**	0.555***	0.182*	0.145**
	(0.096)	(0.162)	(0.099)	(0.058)
α_1	0.777***	0.657***	0.789***	0.825***
	(0.047)	(0.047)	(0.047)	(0.040)
α_2	0.359***	0.181**	0.549***	0.518***
	(0.089)	(0.066)	(0.120)	(0.153)
α_3	-0.034	-0.057	-0.016	0.000
	(0.036)	(0.146)	(0.065)	(0.020)
α_4	-0.008**	-0.024***	-0.008*	-0.006**
	(0.003)	(0.007)	(0.004)	(0.002)

Notes: Newey-West (HAC) standard errors are reported in parentheses. Nominal short-term interest rate and of the inflation rate are four quarter moving averages and have been detrended using a quadratic trend

OLS estimates for the IS curve are reported in Table 1.10. Results are a further evidence of the non-trivial role of households expectations; in particular, the role of those expectations on the output gap, not only on households consumption, is of remarkable interest for a policymaker. Appendix 1.E reports some robustness checks, namely (*i*) a version with a different detrending option (Hodrick-Prescott filter instead of quadratic trend), (*ii*) a fully backward IS curve with lagged expectations and (*iii*) an extended version of the IS curve. These robustness checks confirm the magnitude and relevance of households expectations on the business cycle.

1.7.2 "What if" scenarios

This "what if" scenario is grounded on the assumption that the policymaker might me able, devoting enough resources, to (at least temporarily) affect the expectation formation process. By "affect" we do not mean cheating citizens, lying on the current state of the economy. Conversely, we mean having a timely, direct, effective and sincere communication with households, influencing the way they develop expectations. This "transparent communication" policy might be to able to decrease the proportion of stubbornly pessimistic individuals (decrease μ), increase the frequency of update of information (increase λ) and decrease the use of alternative sources of information, that is the ones that lead the "unexplained" component of expectations.

We explore a "what if" scenario, in which we assume that the policymaker implemented a similar policy in the wake of the financial crisis (2008 Q1) for five years (until 2012 Q4). In periods of boom and recessions, like the one considered, stickiness in households expectations or the presence of individuals with expectations unrelated to reality might hinder a fast convergence to the equilibrium level of output.

For these 20 quarters, we consider as an effect of the policy a doubling in λ and halved μ,[38] together with halved "unexplained" component $EU^U - \widehat{EU^U}$.

A different level of household sentiment EU_t^U implies a different output gap y_t through eq. (1.29). The output gap is persistent and plausibly has an effect on the short-term unemployment forecasts of professionals, so we partially endogenize professional forecasts. Details are given in Appendix 1.F.

Formally, we consider as objective function the standard deviation of the output gap, like the study of Turner (2013) on the calibration of the Taylor rule. We assume that the aim of the policymaker is to keep the output gap as stable as possible, therefore minimizing the objective function. We consider the time period from 2005 Q1 to 2015 Q4, that is a 11-year time window centered around the policy implementation. As highlighted in Table 1.11, we achieve a lower value of the objective function in all countries under the "transparent communication policy" rather than without it.

Table 1.11: Standard deviation of the output gap (2005Q1-2015Q4)

	Fra	Ger	Ita	Uk
Baseline	1.08	1.94	1.58	1.55
Policy implementation	1.05	1.88	1.49	1.49

[38] For Germany, where the relation between output gap and households expectations is stronger, instead of a factor of 2 we consider a factor of 1.5.

We plot both the simulated consumer sentiment against the original survey series, and the effects on the output gap. From Figure 1.10, it is possible to note that, on the one hand, according to our framework the "transparent communication" policy has a short-term cost, reflected in a stronger negative value of the output gap in 2008. On the other hand, this policy would have allowed a faster return to values of the output gap oscillating around zero. In Figure 1.11, we can observe the decrease in households expectations that leads to this faster return to zero of the output gap.

Such simple simulation exercise, of course, has not the power to claim that a crisis, and in particular the financial and sovereign crisis, may be solved so trivially. The aim of this exercise is to shed light on the relevant role of consumer sentiment in mitigating or exacerbating business cycles, fostering further research on the routes through which a policymaker might apply a "transparent communication" policy.

Figure 1.10: Baseline and "what-if" (with policy implementation) output gap (2005Q1-2015Q4)

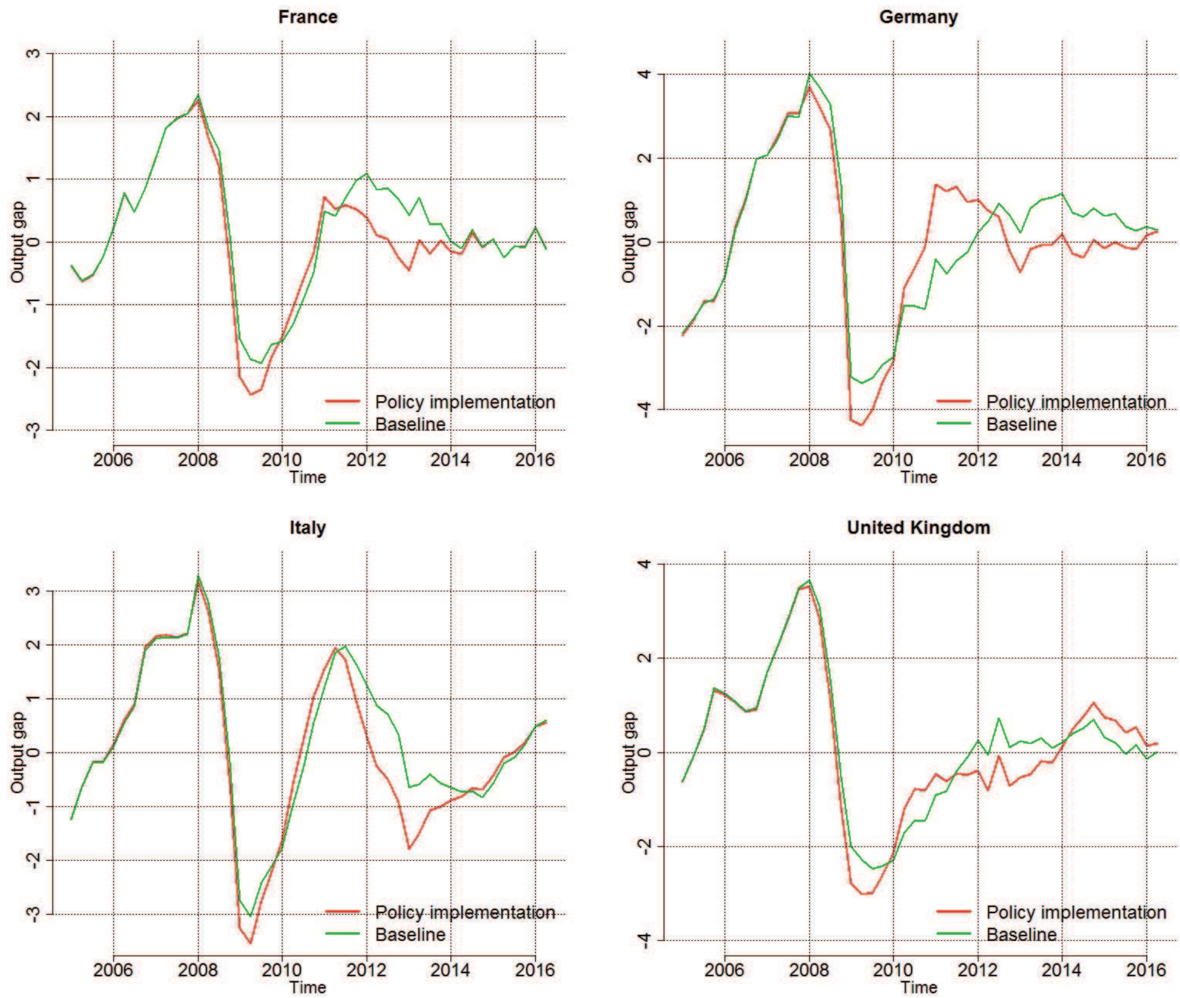
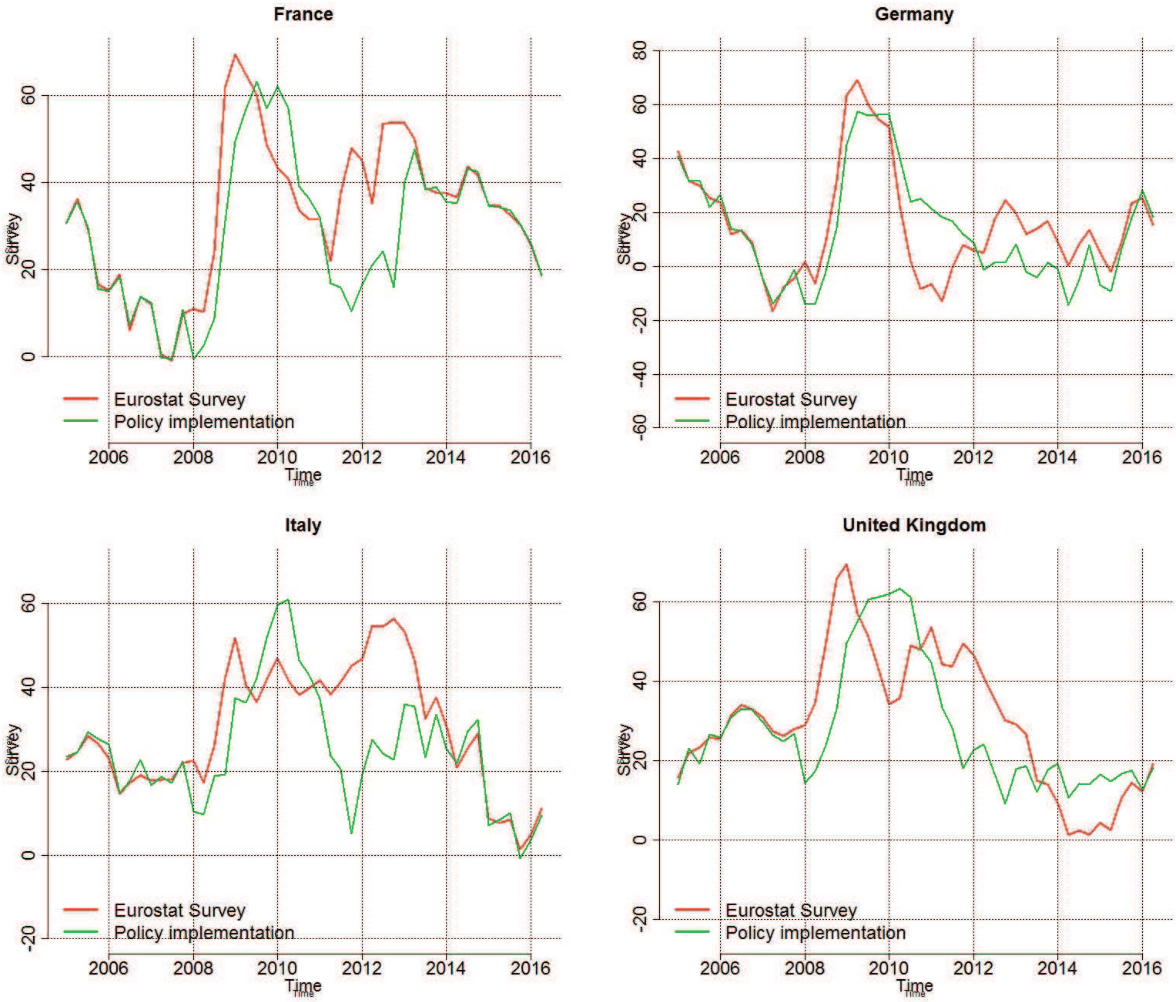


Figure 1.11: Baseline and "what-if" (with policy implementation) households expectations (2005Q1-2015Q4)



1.8 Conclusions

In the present work, we extend the "common-source-infection" (CSI) framework of Carroll (2003). This new formulation may allow researchers to apply the common-source-infection model to the study of macroeconomic and financial variables which are not governed by an unit root or quasi-unit root process. In particular, we have studied unemployment expectations from household surveys of selected European countries (France, Germany, Italy and the UK). Both a macro-level econometric approach and a micro-level agent-based approach have shown that a properly formulated CSI model, despite being relatively simple, is able to capture the main features of non-expert expectations. Data are compatible with a situation where a fraction of agents are boundedly rational, while the remaining are stubbornly and irrationally pessimistic. Among boundedly rational individuals, about one tenth of the population absorbs and processes new information (expert forecasts) in each quarter, whereas the remaining individuals behave as naive econometricians, updating their expectation using outdated information. Moreover, expectations seem to be related to the level of perceived uncertainty, proxied by newspaper coverage on economic uncertainty and by web searches on the topic: in periods of higher uncertainty, agents absorb new information less frequently and are partially influenced by waves of optimism/pessimism unrelated to expert forecasts. Finally, households expectations have a non-trivial role in determining the output gap, with an effect in the order of magnitude of tenths of a percentage point; moreover, a microsimulation has shown that a transparent and effective communication, which increases the awareness of the population about the current and the future state of the economy, may be an useful policy instrument.

Acknowledgements

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Appendix

1.A Technical Appendix

1.A.1 Derivation of Equation (1.14)

Under the hypothesis that data frequency is quarterly and the forecast horizon is one year (i.e. from t to $t+4$), the evolution of the variable x that people have in mind – in the case of Carroll (2003)'s CSI model – can be represented in the following way:

I''

$$x_t = x^*_{t-4,t} + \epsilon_t, \tag{1.30}$$

where $x^*_{t-4,t}$ denotes that fundamental value in period t, which is perfectly forecastable four periods in advance $(t-4)$ by professional forecasters.
In each period the fundamental value of the variable evolves according to the following process:

$$x^*_{t,t+4} = x^*_{t-1,t+3} + \eta_{t+4}. \tag{1.31}$$

II'' The professional forecasters expectation of the variable x at time $t+4$ corresponds to

$$N_t\left[x_{t+4}\right] = x^*_{t,t+4} = x^*_{t-1,t+3} + \eta_{t+4}, \tag{1.32}$$

where the subscript t is omitted from the notation since we are assuming from the beginning that the forecast horizon is of one year and it is already clear from the expectation operator $N_t\left[\bullet\right]$ that the starting period of forecasting is t.

Under the new assumptions $(I'' - II'')$, and maintaining the points $III - IV$ discussed in Section 1.3.1, the expectation of x at time $t+4$ by a generic non-expert agent i can be written as:

$$E^i_t\left[x_{t+4}\right] = E^i_t\left[x^*_{t+4}\right] + \underbrace{E^i_t\left[\epsilon_{t+4}\right]}_{=0}. \tag{1.33}$$

If agent i is "infected" at time t, then Eq. (1.33) can be written as:

$$E^i_t\left[x_{t+4}\right] = N_t\left[x_{t+4}\right]. \tag{1.34}$$

If agent i is not infected in t, but was instead infected at time $t-1$, Eq. (1.34) is equal to

$$E^i_t\left[x_{t+4}\right] = N_{t-1}\left[x_{t+4}\right] = N_{t-1}\left[x_{t+3}\right]. \tag{1.35}$$

According to these rules, the average expectation of x at time $t+4$ can be represented as:

$$M_t\left[x_{t+4}\right] = \lambda N_t\left[x_{t+4}\right] + (1-\lambda)\left\{\lambda N_{t-1}\left[x_{t+3}\right] + (1-\lambda)\left(\lambda N_{t-2}\left[x_{t+2}\right]\ldots\right)\right\}. \tag{1.36}$$

Given the property of the lag polynomial, repeating the same arrangements described in section 1.3.1, it is easy to arrive at Eq. (1.14):

$$M_t\left[x_{t+4}\right] = \lambda N_t\left[x_{t+4}\right] + (1-\lambda)\, M_{t-1}\left[x_{t+3}\right].$$

1.A.2 Derivation of Equation (1.21)

Using the property of the lag polynomial, the right-hand side of (1.20) can be rewritten as:

$$\begin{aligned}
&= \lambda\{N_t[x_{t+1}] + (1-\lambda)\beta N_{t-1}[x_t] + (1-\lambda)^2\beta^2 N_{t-2}[x_{t-1}] + \ldots\} \\
&+ \lambda(1-\lambda)\alpha\{[1 + (1-\lambda) + (1-\lambda)^2 + \ldots] + (1-\lambda)\beta[1 + (1-\lambda) + \ldots] + (1-\lambda)^2\beta^2[1 + \ldots]\} \\
&= \lambda N_t[x_{t+1}]\{1 + (1-\lambda)\beta L + (1-\lambda)^2\beta^2 L^2 + \ldots\} \\
&+ \lambda(1-\lambda)\alpha\{1 + (1-\lambda)\beta + (1-\lambda)^2\beta^2 + \ldots\}\{1 + (1-\lambda) + (1-\lambda)^2 + \ldots\} \qquad (1.37) \\
&= \frac{1}{1-(1-\lambda)\beta L}\lambda N_t[x_{t+1}] + \frac{1}{1-(1-\lambda)\beta}\frac{1}{1-(1-\lambda)}\lambda(1-\lambda)\alpha \\
&= \frac{1}{1-(1-\lambda)\beta L}\lambda N_t[x_{t+1}] + \frac{1}{1-(1-\lambda)\beta}(1-\lambda)\alpha
\end{aligned}$$

Thus Eq. (1.20) can be expressed as:

$$M_t\left[x_{t+1}\right] = \frac{1}{1-(1-\lambda)\beta L}\lambda N_t[x_{t+1}] + \frac{1}{1-(1-\lambda)\beta}(1-\lambda)\alpha \tag{1.38}$$

or

$$[1-(1-\lambda)\beta L]M_t\left[x_{t+1}\right] = \lambda N_t[x_{t+1}] + \frac{1-(1-\lambda)\beta L}{1-(1-\lambda)\beta}(1-\lambda)\alpha \tag{1.39}$$

which corresponds to (1.21)

$$M_t\left[x_{t+1}\right] = \lambda N_t\left[x_{t+1}\right] + (1-\lambda)\left(\alpha + \beta M_{t-1}\left[x_t\right]\right).$$

1.A.3 Derivation of Equation (1.22)

Respect to the case presented in Appendix 1.A.1, point I'' changes as follows:

I'''. The typical person believes that x_t behaves like a *stationary stochastic model.* In quarterly terms, this means that we have:

$$x_t = x^*_{t-4,t} + \epsilon_t, \tag{1.40}$$

where the fundamental value of the variable evolves according to the following stationary process:

$$x^*_{t,t+4} = \alpha + \beta x^*_{t-1,t+3} + \eta_{t+4},\ 0 \leq \beta < 1, \tag{1.41}$$

where β represents the autoregressive coefficient of the fundamental value process, α is a constant term, and ϵ_t and η_t are Gaussian independent disturbances.

II'''. The professional forecasters expectation of the variable x at time $t+4$ corresponds to:

$$N_t\left[x_{t+4}\right] = x^*_{t,t+4} = \alpha + \beta x^*_{t-1,t+3} + \eta_{t+4}.^{39} \tag{1.42}$$

Under the new assumptions (I''') – (II'''), and maintaining points (III) – (IV) discussed in Subsection 1.3.1, the expectation of x at time $t+4$ by a generic non-expert agent i can be written as:

$$E^i_t\left[x_{t+4}\right] = E^i_t\left[x^*_{t+4}\right] + \underbrace{E^i_t\left[\epsilon_{t+4}\right]}_{=0}. \tag{1.43}$$

If agent i is "infected" at time t, then Eq. (1.43) is equal to

$$E^i_t\left[x_{t+4}\right] = N_t\left[x_{t+4}\right]. \tag{1.44}$$

If agent i is not infected in t, but was instead infected at time $t-1$:

$$E^i_t\left[x_{t+4}\right] = N_{t-1}\left[x_{t+4}\right] = \alpha + \beta N_{t-1}\left[x_{t+3}\right]. \tag{1.45}$$

According to these rules, the average expectation of x at time $t+4$ can be represented as:

$$M_t[x_{t+4}] = \lambda N_t[x_{t+4}] + (1-\lambda)\{\lambda N_{t-1}[x_{t+4}] + (1-\lambda)(\lambda N_{t-2}[x_{t+4}] + (1-\lambda)(\lambda N_{t-3}[x_{t+4}]\ldots)\} \tag{1.46}$$

Given the property of the lag polynomial, repeating the same arrangements described in Appendix 1.A.2, it is easy to arrive at Eq. (1.22):

$$M_t\left[x_{t+4}\right] = \lambda N_t\left[x_{t+4}\right] + (1-\lambda)\left(\alpha + \beta M_{t-1}\left[x_{t+3}\right]\right).$$

[39] The subscript t is omitted from the notation since we are assuming from the beginning that forecast horizon is of one year and it is already clear from the expectation operator $N_t\left[\bullet\right]$ that the starting period of forecasting is t.

1.B Additional Figures

Figure 1.B.1: Non-expert unemployment expectations index (`Unemp. Exp. Index`$=EU_t^U$) *vs* actual past unemployment change (`Unem. rate - Unem. rate(-4)`$=\Delta_4 u_t$) (1986Q1-2016Q3).

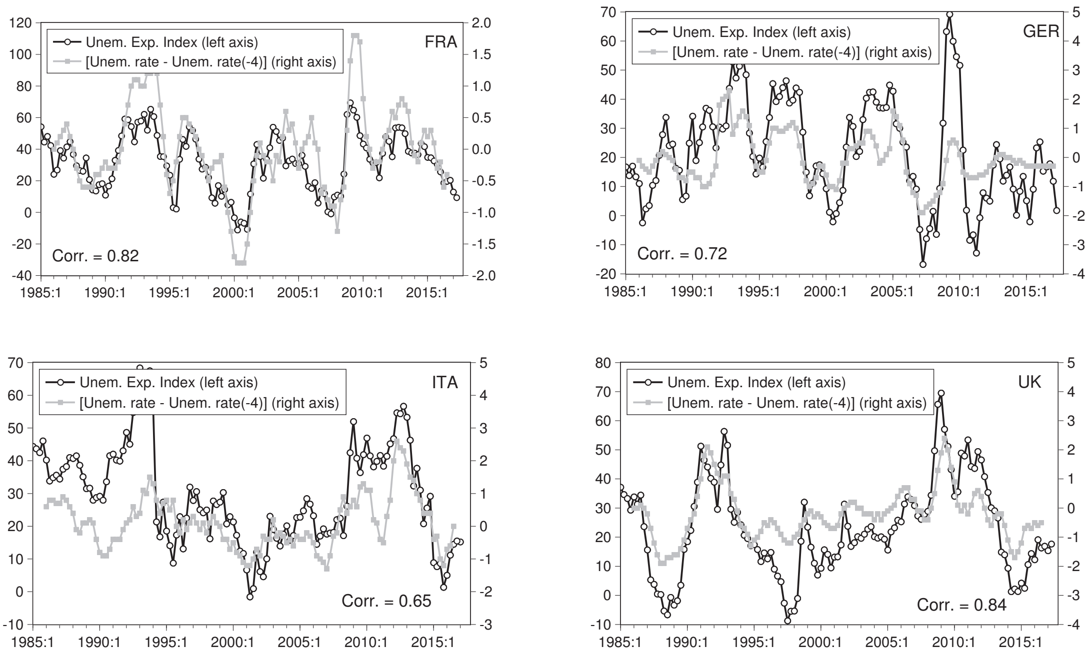
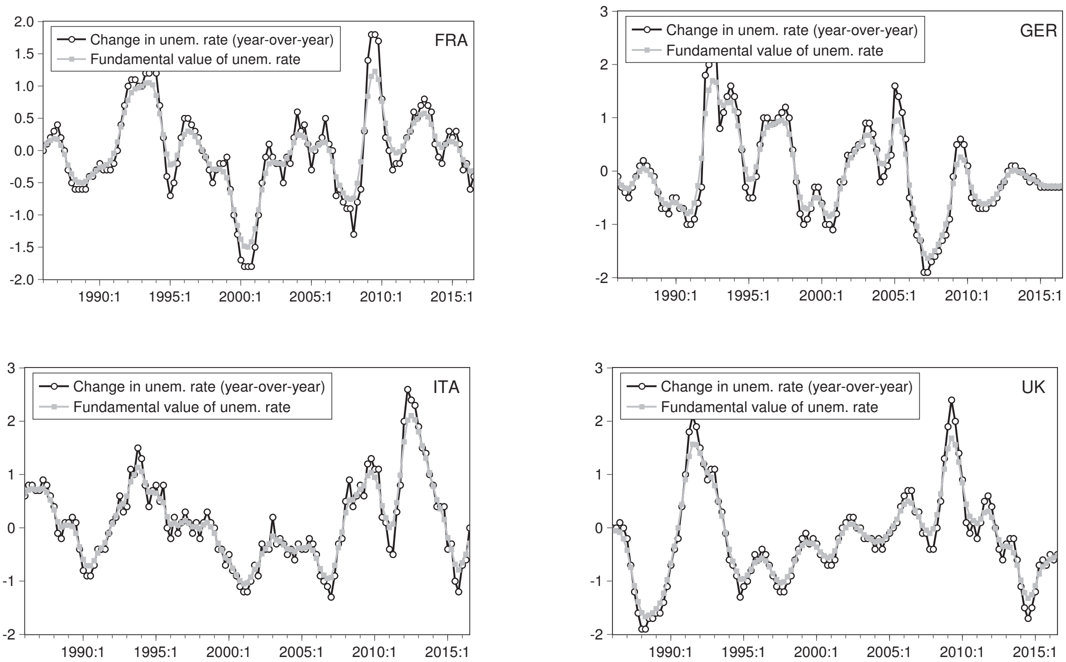


Figure 1.B.2: Fundamental value of change in unemployment rate ($\Delta_4 u_t{}^*$) *vs* actual change in unemployment rate ($\Delta_4 u_t$) (1986Q1-2016Q3).



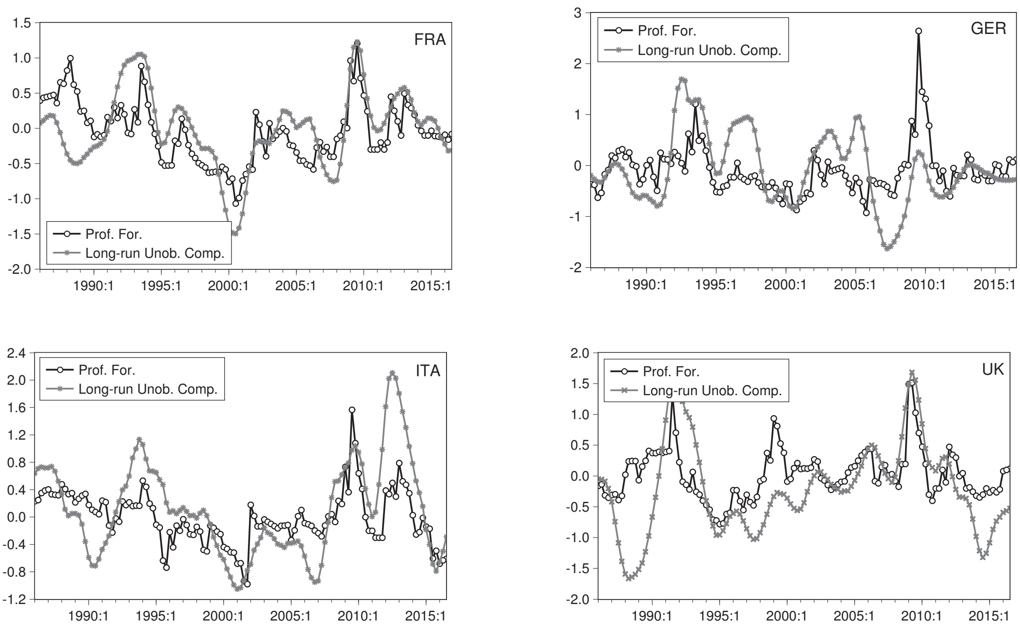
1.C Stylized facts: expert forecasts and (unobserved) long-run determinant in unemployment rate change

This Appendix presents a comparison between professional forecasts and the long-run component of change in unemployment rate $\Delta_4 u_t{}^*$, as estimated through Table 1.5. Figure 1.C.1 gives a visual inspection of the relation. The two series seem to move together over time. To give a statistical measure of this co-movement, we calculate the correlations, over the period 1986Q1-2016q3, between four lagged periods of professional forecasters $(N_{t-4}[\Delta u_t])$ and long-run component of change in unemployment rate (Δu_t) for each country.[40] Results are reported in the Table 1.C.1. It is important to emphasize that for all countries, the correlation is above 0.30. The exception is Germany, where the correlation is 0.15. The reason lies in the huge "outlier" observed in the professional forecasters predictions for the period 2009Q3-2010Q1. If these extreme values are excluded, the correlation is 0.30. These results confirm that, excluding for some anomaly predictions that may occur, the hypothesis that professional forecasters time series proxy the long-run component of change in unemployment rate is supported by data.

Table 1.C.1: Correlation of OECD forecasts and fundamental rate change (1986Q1-2016Q3)

$Corr. = (N_{t-4}[\Delta u_t], \Delta u_t)$			
Fra	Ger	Ita	Uk
0.34	0.15	0.31	0.50

Figure 1.C.1: Professional forecasts (`Prof. For`) *vs* (unobserved) long-run determinant of change in unemployment rate (`Long-run Unob. Comp.`) (1986Q1-2016Q3)



[40]Remember that professional forecasts predict the future value of change in unemployment rate at time $t + 4$.

1.D Alternative microsimulation calibrations

Table 1.D.1: Micro-simulation calibration (1986Q1-2016Q2) assuming $\beta = 0.8$

Country	Fra	Ger	Ita	Uk
α	0	0	0	0
β	0.8	0.8	0.8	0.8
λ	0.095	0.077	0.077	0.098
μ	0.339 (0.338-0.340)	0.321 (0.318-0.325)	0.282 (0.280-0.284)	0.218 (0.217-0.220)
γ	0.353 (0.348-0.359)	0.198 (0.189-0.206)	0.260 (0.257-0.264)	0.328 (0.323-0.333)

Figure 1.D.1: Real and micro-simulated survey balances ($\beta = 0.8$) (1986Q1-2016Q2)

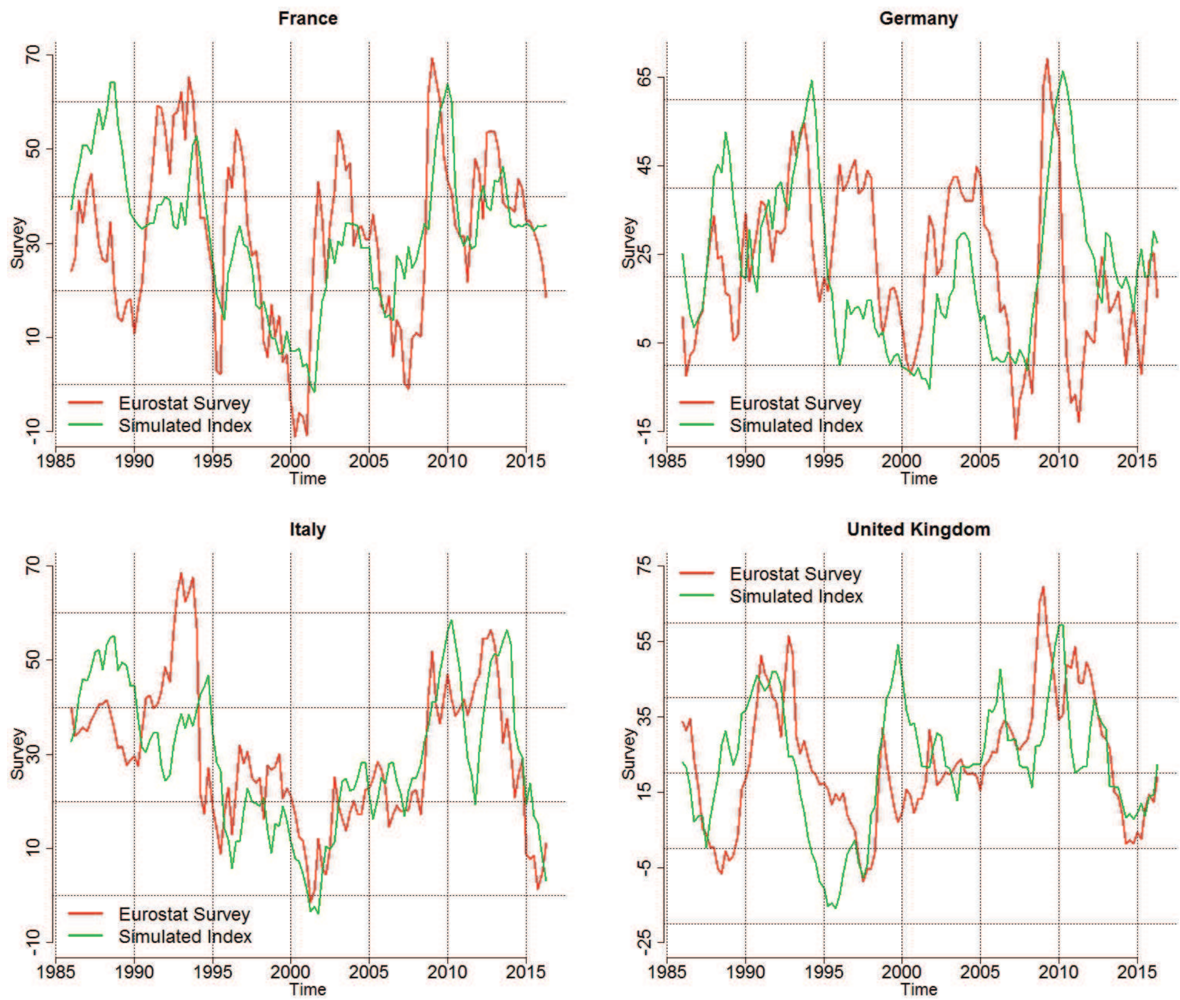
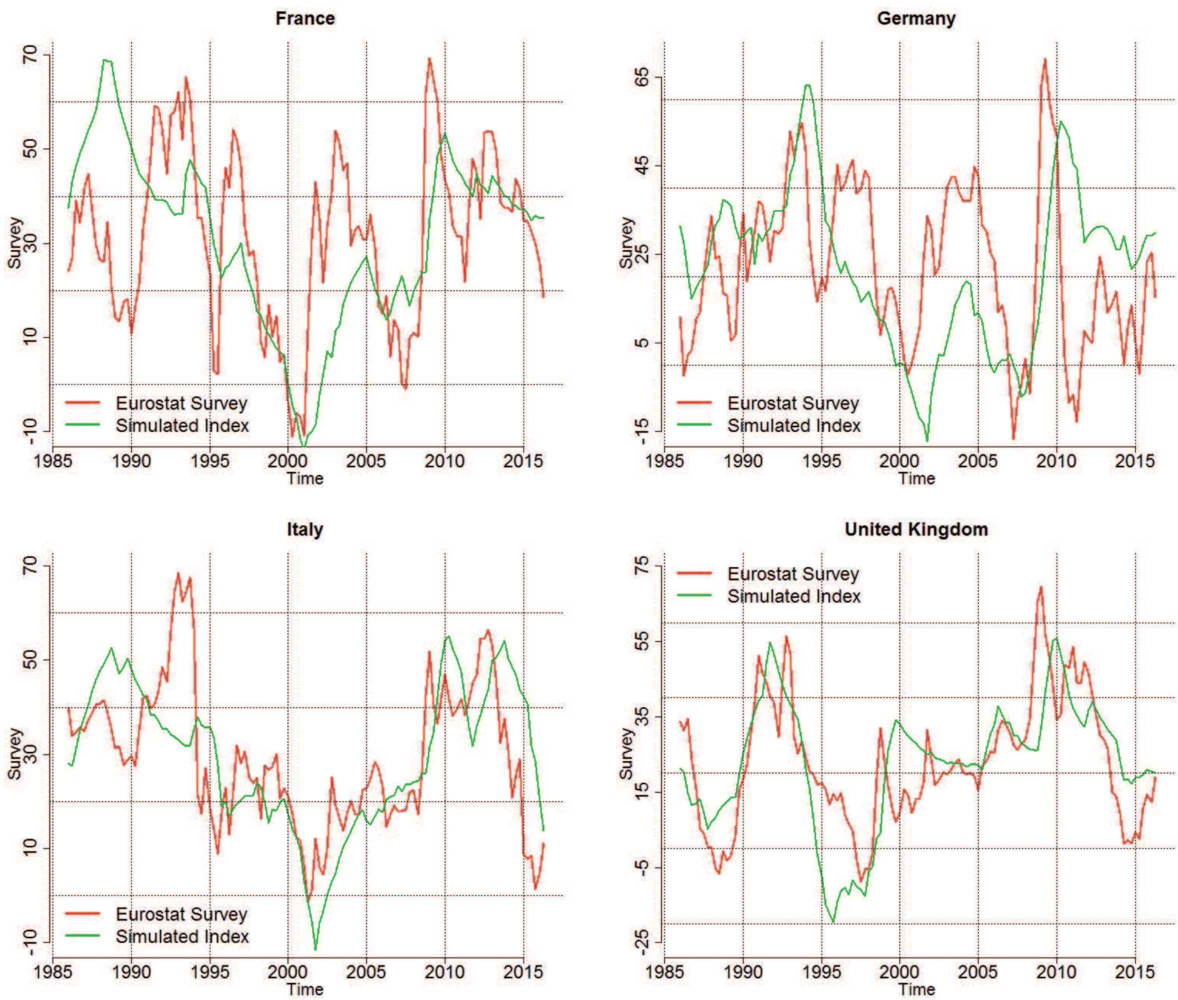


Table 1.D.2: Micro-simulation calibration (1986Q1-2016Q2) assuming $\beta = 1$

Country	Fra	Ger	Ita	Uk
α	0	0	0	0
β	1	1	1	1
λ	0.095	0.077	0.077	0.098
μ	0.341 (0.338-0.343)	0.349 (0.333-0.365)	0.276 (0.271-0.280)	0.206 (0.201-0.211)
γ	0.714 (0.701-0.728)	0.442 (0.392-0.492)	0.573 (0.540-0.606)	0.613 (0.607-0.619)

Figure 1.D.2: Real and micro-simulated survey balances ($\beta = 1$) (1986Q1-2016Q2)



1.E IS curve: robustess checks

1.E.1 Fully backward IS curve

In this robustness check, we take into account the presence of a potential simultaneity bias problem. Actually, as summarized in the introduction of this chapter, it is well-known in the literature that expectations theoretically and empirically influence the business cycle. Conversely, according to our CSI theoretical model, expectations are influenced only by professional forecasts on the topic subject of expectations, therefore excluding any role of contemporaneous macroeconomic indicators (like the output gap). Still, a potential criticism is that from a practical point of view, we can not exclude a priori the opposite direction of causality: even if official data on contemporaneous output (gap) are by definition not available, households may have some feelings about the current state of the economy and adjust their expectations accordingly. In econometrics handbooks, the simultaneity problem is solved through a 2SLS estimation, using as instruments exogenous variables that are explanatory variables of the contemporaneous households expectations, but are not explanatory variables of the contemporaneous output gap. Unfortunately, if we start from the idea that households expectations may be related to factors other than expert forecasts, to the best of our knowledge it is hard to imagine any variable that satisfies the requirement without making such an assumption as questionable as the initial assumption that expectations are influenced only by professional forecasts. Therefore, in this Appendix we try to overcome the simultaneity problem by not using contemporaneous variables.

We have shown in Section 1.6.1 that only a fraction of individuals update their information set each period, leading to a relevant degree of stickiness in aggregate expectations.[41] Stickiness implies that EU_t^U and EU_{t-1}^U are highly correlated,[42] therefore in this fully backward version we substitute contemporaneous expectations with lagged expectations. In this way, we miss the information on the innovation in expectations, likely weakening the explanatory power of EU^U, but we also exclude any casual link going from contemporaneous output gap to (lagged) expectations.

$$y_t = \psi_0 + \psi_1 y_{t-1} + \psi_2 \Delta y_{t-1} + \psi_3 (i_{t-1} - \pi_{t-1}^{GDP}) + \psi_4 EU_{t-1}^U + \epsilon_t \tag{1.47}$$

Estimates for the backward-looking IS are reported in Table 1.E.1. The estimates are slightly lower (in absolute values) than the ones of the baseline case (see Table 1.10), but maintain the same significance level, except for Italy. Actually, Italy is the only country for which ψ_4 is not significant. Anyway, if we follow the suggestion of Goodhart and Hofmann (2005) and we estimate again Eq. (1.47) excluding the variable with the highest p-value $(i_{t-1} - \pi_{t-1}^{GDP})$, ψ_4 turns out to be equal to -0.005 and significant at the 10% level.

Given the results of the backward-looking IS curve, we feel comfortable in stating that there is a causal link from households expectations to the output gap and, even if we do not formally take into account the simultaneity issue in Table 1.10, the results herein provided are not patently absurd.

[41] Actually, except for Italy, we fail to reject the hypothesis that the balance index EU_t^U follows a unit root process. Significance levels: France 1%, Germany 10%, the United Kingdom 5%.

[42] The country with the lowest correlation is Germany (75%).

Table 1.E.1: OLS estimates of the backward-looking IS curve (Eq. 1.47) (France and UK 1991q2-2017q1, Germany 1991q3-2017q1, Italy 1995q3-2017q1)

	FRA	GER	ITA	UK
ψ_0	0.199**	0.405***	0.087	0.125**
	(0.100))	(0.144)	(0.102)	(0.050))
ψ_1	0.768***	0.654***	0.807***	0.812***
	(0.052))	(0.055)	(0.044)	(0.046)
ψ_2	0.428***	0.302***	0.557***	0.530***
	(0.094))	(0.067))	(0.127)	(0.160))
ψ_3	-0.046	-0.058	-0.048	-0.007
	(0.041)	(0.114)	(0.078)	(0.018)
ψ_4	-0.006**	-0.018***	-0.004	-0.006**
	(0.003)	(0.004)	(0.003)	(0.002))

Notes: Newey-West (HAC) standard errors are reported in parentheses.

As a further experiment, we split the contemporaneous expectations EU_t^U into its lagged component EU_{t-1}^U plus the first difference $\Delta_1 EU_t^U$. The first difference may suffer of a simultaneity bias, while the lagged value does not. Estimates are reported in Table 1.E.2.

$$y_t = \chi_0 + \chi_1 y_{t-1} + \chi_2 \Delta y_{t-1} + \chi_3(i_{t-1} - \pi_{t-1}^{GDP}) + \chi_4 EU_{t-1}^U + \chi_5 \Delta_1 EU_t^U + \epsilon_t \qquad (1.48)$$

Table 1.E.2: OLS estimates of the IS curve. Contemporaneous expectations splitted in lag component and first difference (Eq. 1.48) (France and UK 1991q2-2016q4, Germany 1991q3-2016q4, Italy 1995q3-2016q4)

	FRA	GER	ITA	UK
χ_0	0.236**	0.443***	0.141	0.144**
	(0.104)	(0.114)	(0.096)	(0.067)
χ_1	0.784***	0.736***	0.806***	0.825***
	(0.047)	(0.044)	(0.036)	(0.081)
χ_2	0.352***	0.129	0.536***	0.518***
	(0.072)	(0.079)	(0.113)	(0.167)
χ_3	-0.035	-0.057	-0.024	0.000
	(0.037)	(0.129)	(0.049)	(0.024)
χ_4	-0.007**	-0.020***	-0.006*	-0.006
	(0.003)	(0.004)	(0.003)	(0.004)
χ_5	-0.009**	-0.038***	$-0.016^{\dagger}$	-0.007
	(0.004)	(0.014)	(0.010)	(0.009)

Notes: Newey-West (HAC) standard errors are reported in parentheses. † p-value 0.1172

The only country for which χ_4 and χ_5 are not significant is the United Kingdom. Even in this case, if we eliminate the variable with the highest p-value $(i_{t-1} - \pi_{t-1}^{GDP})$, χ_4 is equal to -0.007 and significant at the 5% level, while χ_5 remains not significant.

1.E.2 Hodrick-Prescott Filter

In the baseline model, the output gap is calculated as the deviation from a quadratic trend. In this robustness check, instead, it is calculated as the cyclical component obtained through a Hodrick-Prescott filter with a default smoothing parameter of 1600 (y_t^{HP}). Similarly, inflation (i_t^{HP}) and interest rates ($\pi_t^{GDP^{HP}}$) have been obtained through a Hodrick-Prescott procedure. Estimates are reported in Table 1.E.3.

$$y_t^{HP} = \zeta_0 + \zeta_1 y_{t-1} + \zeta_2 \Delta y_{t-1} + \zeta_3 (i_{t-1}^{HP} - \pi_{t-1}^{GDP^{HP}}) + \zeta_4 EU_t^U + \epsilon_t \tag{1.49}$$

Table 1.E.3: OLS estimates of the IS curve, using Hodrick-Prescott filter as detrending option (Eq. 1.49) (France and UK 1991q2-2016q4, Germany 1991q3-2016q4, Italy 1995q3-2016q4)

	FRA	GER	ITA	UK
ζ_0	0.252***	0.550***	0.181	0.143**
	(0.091)	(0.158)	(0.128)	(0.058))
ζ_1	0.780***	0.680***	0.820***	0.827***
	(0.051)	(0.055)	(0.055)	(0.044)
ζ_2	0.273***	0.153**	0.514***	0.517***
	(0.097)	(0.063)	(0.095)	(0.152)
ζ_3	-0.130**	-0.141	-0.182**	-0.006
	(0.056)	(0.189)	(0.082)	(0.029)
ζ_4	-0.008**	-0.024***	-0.007*	-0.006**
	(0.003)	(0.007)	(0.004)	(0.003)

Notes: Newey-West (HAC) standard errors are reported in parentheses.

We provide also the fully backward-looking IS with output gap, interest rate and inflation rate detrended through a Hodrick-Prescott filter. Estimates are reported in Table 1.E.4.:

$$y_t^{HP} = \eta_0 + \eta_1 y_{t-1} + \eta_2 \Delta y_{t-1} + \eta_3 (i_{t-1}^{HP} - \pi_{t-1}^{GDP^{HP}}) + \eta_4 EU_{t-1}^U + \epsilon_t \tag{1.50}$$

1.E.3 Extended IS curve

The elements added to the extended version of the IS curve are the real exchange rate rex_t, the change in housing price Δhp_t and the change in share price Δsp_t. As for the interest rate and inflation, we consider four-quarter moving averages.

$$y_t = \theta_0 + \theta_1 y_{t-1} + \theta_2 \Delta y_{t-1} + \theta_3 (i_{t-1} - \pi_{t-1}^{GDP}) + \theta_4 EU_t^U + \theta_5 rex_{t-1} + \theta_6 \Delta hp_{t-1} + \theta_7 \Delta sp_{t-1} + \epsilon_t \tag{1.51}$$

In comparison with the results of Table 1.10, the coefficients on households expectations θ_4 appear robust, both in magnitude and in significance. The only country for which the coefficient becomes insignificant is the United Kingdom. This is due probably to a multicollinearity problem, given the presence of an high number of correlated regressors.

Table 1.E.4: OLS estimates of the backward-looking IS curve, using Hodrick-Prescott filter as detrending option (Eq. 1.50) (France and UK 1991q2-2017q1, Germany 1991q3-2017q1, Italy 1995q3-2017q1)

	FRA	GER	ITA	UK
η_0	0.218**	0.402***	0.112	0.126**
	(0.083)	(0.146)	(0.081)	(0.050))
η_1	0.765***	0.682***	0.828***	0.819***
	(0.058)	(0.078)	(0.054)	(0.050)
η_2	0.349***	0.267***	0.529***	0.530***
	(0.103)	(0.051)	(0.121)	(0.156)
η_3	-0.136**	-0.160	-0.190*	-0.011
	(0.056)	(0.188)	(0.111)	(0.026)
η_4	-0.007**	-0.017***	$-0.005^{\dagger}$	-0.006**
	(0.003)	(0.004)	(0.003)	(0.003)

Notes: Newey-West (HAC) standard errors are reported in parentheses. † p-value=0.1533

However, if we follow the suggestion of the proposer of the extended IS curve (Goodhart and Hofmann, 2005) and we progressively eliminate the least significant variable until they are all significant, only y_{t-1}, Δy_{t-1}, EU_t^U and Δsp_{t-1} survive the selection, yielding for EU_t^U a coefficient of -0.005 significant at the 5% level.

As a further robustness check, we have substituted in 1.29 the long-run for the short-run interest rate and inflation based on the Private Consumption Expenditure for the one based on the GDP deflator. Results are not shown to save space, since the coefficients on households expectations are always significant and maintain the same order of magnitude of the baseline formulation. The only robustness check for which the coefficient appears to be much different is when we substitute the long-term for the short-term interest rate in Italy. Under this specification, the coefficient for EU_t^U jumps from $8 \cdot 10^{-3}$ to $13 \cdot 10^{-3}$, maintaining a 10% significance.

Table 1.E.5: OLS estimates of the IS curve Eq. (1.51) (France and UK 1991q2-2016q4, Germany 1991q3-2016q4, Italy 1995q3-2016q4)

	FRA	GER	ITA	UK
θ_0	-1.180	1.189	0.763	-0.427
	(0.749)	(1.811)	(0.606)	(0.547)
θ_1	0.769***	0.668***	0.795***	0.810***
	(0.042)	(0.055)	(0.042)	(0.084)
θ_2	0.240**	0.154*	0.478***	0.471***
	(0.098)	(0.083)	(0.094)	(0.017)
θ_3	-0.044	-0.018	-0.004	-0.005
	(0.045)	(0.126)	(0.055)	(0.029)
θ_4	-0.010***	-0.022***	-0.007**	-0.006
	(0.0025)	(0.006)	(0.003)	(0.005)
θ_5	0.016**	-0.007	-0.006	0.006
	(0.008)	(0.019)	(0.006)	(0.006)
θ_6	-0.006	-0.029	0.001	-0.015
	(0.027)	(0.064)	(0.031)	(0.020)
θ_7	0.003*	0.005	0.016	0.004*
	(0.002)	(0.003)	(0.011)	(0.002)

Notes: Newey-West (HAC) standard errors are reported in parentheses. Nominal short-term interest rate and of the inflation rate are four quarter moving averages and have been detrended using a quadratic trend. The real exchange rate, the change in housing price and the change in share price are four quarter moving averages

1.F Policy considerations: microsimulation details

There are three endogenous relations involved in the simulation. First of all, (a) the professional forecast is splitted into a component depending on the lags of the output gap and on an innovation ξ:

$$N_t[\Delta_4 u_{t+4}] = \rho_1 y_{t-1} + \rho_2 y_{t-5} + \xi_t \tag{1.52}$$

The other two endogenous relations are: (b) households expectations EU_t^U, which non-linearly depend on the current and past values of $N[\bullet]$ according to the CSI framework; and (c) the output gap described by Eq. (1.29) $y_t = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 \Delta y_{t-1} + \alpha_3(i_{t-1} - \pi_{t-1}^{GDP}) + \alpha_4 EU_t^U + \epsilon_t$. The interest and inflation rates are exogenous.

First of all, we run the regressions (1.52) and (1.29) and store residuals $\hat{\xi}_t$ and $\hat{\epsilon}_t$. Similarly, we simulate the agent-based model following the baseline calibration (Table 1.7) and we store the unexplained component $(EU_t^U - \widehat{EU^U}_t)$. The two residuals and the unexplained component are considered as exogenous factors, and we will add them back to the values obtained in the policy experiment.

This is the simulation strategy adopted since 2008 Q1, that is the first period with the different calibration implied by the "transparent communication" policy:

i Run the agent-based model and obtain the value $\widetilde{EU^U}_t$. To this value, add half of the unexplained component $(EU_t^U - \widehat{EU^U}_t)$,[43] obtaining $\widehat{\widehat{EU^U}}_t = \widetilde{EU^U}_t + (EU_t^U - \widehat{EU^U}_t)/2$.

ii Through (1.29), calculate $\hat{\hat{y}}_t = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 \Delta y_{t-1} + \alpha_3 (i_{t-1} - \pi_{t-1}^{GDP}) + \alpha_4 \widehat{\widehat{EU^U}}_t + \hat{\epsilon}_t$.

iii Through (1.52), calculate $\hat{\hat{N}}_{t+1}[\bullet] = \rho_1 \hat{\hat{y}}_t + \rho_2 y_{t-4} + \hat{\xi}_{t+1}$

iv Start back from point (i).

Note:

- Since 2013 Q1, the policy is not implemented any more and (i) is substituted for (i'): Run the agent-based model and obtain the value $\widetilde{EU^U}_t$. To this value, add the unexplained component $(EU_t^U - \widehat{EU^U}_t)$, obtaining $\widehat{\widehat{EU^U}}_t = \widetilde{EU^U}_t + (EU_t^U - \widehat{EU^U}_t)$.
- Of course, in point (ii) and (iii) we adopt the endogenously determined values of $\hat{\hat{y}}_t$ for y_t when they apply. That is, we use $\hat{\hat{y}}_{t-1}$ and $\Delta\hat{\hat{y}}_{t-1}$ in (ii) since 2008 Q2 and $\hat{\hat{y}}_{t-4}$ in (iii) since 2009 Q2.

1.G Data description

This appendix describes the data used in the empirical analysis for France, Germany, Italy, and the UK. All time series have quarterly frequency and cover different time periods according to their availability. All details are summarized in Table 1.G.1.

Data on the unemployment rate are expressed as year-over-year change (i.e. change respect to the same quarter of the previous year). Data are seasonally adjusted and are recovered from OECD and Federal Reserve Economic Data (FRED).

The non-expert unemployment expectations are the expectations on unemployment rate changes in the next 12 months taken from European Commission's Joint Harmonised EU Programme of Consumer Surveys. These expectations series are expressed as a balance index and are seasonally adjusted. Data are available at monthly frequency and are transformed in quarterly series taking the average of the corresponding monthly observations. Finally, the quarterly series are converted in the same unit of measure of the unemployment rate using an auxiliary regression. See Section 1.3.3 for more details.

The expert unemployment expectations are proxied by forecasts contained in the OECD Economic Outlook. The predictions refer to the seasonally adjusted unemployment rate in the next year. In our analysis we use the change in the unemployment rate expectations measured as the difference between the forecasted unemployment rate in the next four quarters and the unemployment rate of the current quarter.

The Economic Policy "news-based" Uncertainty index (EPU) is constructed counting the number of articles related to uncertainty and economy reported by the press.[44] The time series is then detrended using a quadratic trend. The source is Baker et al. (2016).

The Google Uncertainty Index (GUI) is built counting the volume of web searches containing the terms uncertain or uncertainty, economic or economy. The source is the

[43] In Section 1.7.2 we assume that during the policy implementation agents use less alternative sources of information, so we consider only half of the exogenous unexplained component.

[44] Quoting from the methodology part of the EPU website `http://www.policyuncertainty.com/methodology.html`, "We count the number of newspaper articles containing the terms uncertain or uncertainty, economic or economy, and one or more policy-relevant terms".

website Google Trends. We consider searches both in the native language of the country and in English. The intensity of Internet searches, which are related to the above mentioned keywords, should reflect (proxy) a high level of uncertainty perceived among non-expert agents. In this regard, Bontempi et al. (2017), in introducing a similar index based on Google Trends for US, presents a list of conditions necessary to make sure that online searches reflect perceived uncertainty and not mere general interest. First of all, there must be "a careful selection of the list of the specific search terms potentially related to uncertainty"; that is, it must be understood if there is an uncertainty-related common driver that leads to an increase or a decrease of these searches, while searches related to general interest can be considered as noise. The second condition is that this list "must be long enough to exploit the statistical averaging effect across many different queries". As an application of these two conditions, we opted for the keywords of Baker et al. (2016), while dropping the further very specific policy-related terms, since for our selected European countries there are too few data for several very specific searches, hindering the possibility to elaborate the related time series from Google Trends. The series are seasonally adjusted, converted in quarterly data (taking the average of montlhy observations), and detrended (using a quadratic trend).

In the GMM estimates we use as instruments the following exogenous variables: oil price changes, equity returns, housing price changes, short-run interest rate changes, spread between long-term and short-term interest rates, and US real GDP growth. All these data are recovered from the Federal Reserve website, with the exclusion of oil price which is taken from the OECD database.

Table 1.G.1: Data description and sources for France, Germany, Italy and the United Kingdom

Label	Description	Data measurement	Seas. Adj.	Period	Source
$\ln(C_t)$	Household per capita consumption	Logarithm of level	Yes	1985q1-2016q4 (FRA, UK) 1991q1-2016q4 (GER, ITA)	OECD, Eurostat (UK), Istat (ITA)
$\ln(Y_t)$	Household per capita income	Logarithm of level	Yes	1991q1-2016q4	OECD, Eurostat (UK), Dallas FED and Istat (ITA*)
π_t^{PCE}	Yearly inflation rate (Private Consumption Expenditure deflator)	Rate	Yes	1991q1-2016q4	Dallas FED
EU_t^U	Non-expert unemployment rate expectations	Balance Index	Yes	1986q1-2016q4	EU Commission
$\Delta_4 u_{t+4}$	Harmonised unemployment rate	Year-over-year change	Yes	1981q1-2016q3	OECD and FRED
$M_t\,[\Delta_4 u_{t+4}]$	Non-expert unemployment rate expectations*	Year-over-year change	Yes	1986q1-2016q4	EU Commission
$N_t\,[\Delta_4 u_{t+4}]$	Expert unemployment rate expectations	Year-over-year change	Yes	1986q1-2016q4	OECD Economic Outlook
EPU_t	News-based uncertainty index	De-trended using quadratic trend	Yes	1997q1-2016q3	Baker et al. (2016)
GUI_t	Google-based uncertainty index	De-trended using quadratic trend	Yes†	2004q1-2016q3	Google Trend
y_t	Output gap	Deviation of GDP from quadratic trend	Yes	1991q1-2016q4 1995q1-2016q4 (ITA)	OECD
π_t^{GDP}	Yearly inflation rate (GDP deflator)	Rate	Yes	1991q1-2016q4	OECD
i_t	Short-term interest rate	Rate	Yes	1991q1-2016q4	OECD
rex_t	Real effective exchange rate	Rate	Yes	1991q1-2016q4	Darvas (2012a,b)
$\Delta_4 hp_t$	Real housing price	year-over-year percentage change	Yes	1984q1-2016q4	Dallas FED
$\Delta_4 sp_t$	Real share price	year-over-year percentage change	Yes	1984q1-2016q4	Dallas FED
$\Delta_4 oil_t{}^{\ddagger}$	Oil price (US $)	year-over-year percentage change	NA	1987q1-2016q4	FRED
$\Delta_4 y_t^{USA\ddagger}$	US real GDP	year-over-year percentage change	Yes	1984q1-2016q4	FRED
$\Delta_4 hp_t^{USA\ddagger}$	US Real housing price	year-over-year percentage change	Yes	1984q1-2016q4	Dallas FED
$\Delta_4 i_t{}^{\ddagger}$	Short-term interest rate	year-over-year change	NA	1984q1-2016q4	OECD
$spread_t{}^{\ddagger}$	Spread between long-term and short-term interest rates	Percentage points difference	NA	1984q1-2016q4	OECD

Note: NA= Not Applicable. *Data expressed as a balance index and converted in the same unit of measure of unemployment rate (see Section 1.3.3 and Eqs (1.23) and (1.24)). * FED of Dallas (1991Q1-1998Q4); ISTAT (1999Q1-2017Q1). † The Internet-based uncertainty Index is seasonally adjusted by authors using X13-ARIMA procedure. ‡ Data used as instruments in GMM estimation

Chapter 2

Unemployment expectations in Italy: an Agent-based Model with education

Luca Gerotto*

Abstract

This chapter presents an agent-based model grounded on the common-source infection model for unemployment expectations developed in the first chapter. I relax the homogeneity assumption with the aim of understanding if the expectations are heterogeneous across different demographic groups. I find that there are economically significant differences that may be explained through heterogeneous parameters of the agent-based model for the different groups. In particular, I consider education as a driver of macroeconomic expectations and I find that survey data are compatible with a framework in which the less educated are also less up-to-date, meaning that they acquire new information less frequently and tend to be more pessimistic than historical data or professional forecasts may suggest.

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2.1 Introduction

Carroll (2003, 2006) CSI (Common Source Infection) model, for a detailed discussion of which I refer the reader to Chapter 1 of the current dissertation, is meant to capture the main features of the aggregate expectations formation process. The derivation of an econometrically testable equation required some assumptions, like the presence of an unique source of infection[1], no interaction among agents and no heterogeneity across agents other than being "infected" or not in that particular period. These assumptions, although a bit oversimplified, do not alter the main message of the model and allow to verify the empirical soundness of the theoretical implications of the underlying framework. Carroll (2003) finds that US inflation expectations from the Survey of Consumers (University of Michigan) are compatible with a fraction of about 0.25 of the population updating the information set each quarter. Using a slightly different model, the results of Chapter 1 show that unemployment expectations for selected European countries[2] are compatible with a fraction of about 0.1 of the population updating the information set each quarter and the remaining 0.9 changing the expectation following a simple heuristic. These results, in turn, imply that this simplified framework is able to capture the main features of the evolution of consumer sentiment.

Carroll (2003) also makes the hypothesis, left for further research, that there may be heterogeneity in absorption (i.e. infection) probabilities across agents. Such an heterogeneity would probably have a little role in the interpretation of the CSI model, since the assumed homogeneous parameter would be simply a weighted average of the "true" heterogeneous parameters, leading to tiny aggregate effects. Actually, Carroll (2006) proposes an agent-based models with heterogeneous absorption probabilities. He finds that the "average" absorption probability is not dissimilar from the one of the homogeneous case, but that the introduction of hetereogeneity allows to better reproduce some features of empirical data, like the cross-sectional dispersion of households expectations. However, he does not explicitly model the different absorption probabilities as a function the different demographic characteristics of the individuals, even though he proposes that demographic characteristics might actually be the driver. Highlighting differences among different demographic groups may turn out to be crucial for policy purposes. Knowing which are the groups having the more obsolete and possibly wrong information, and the channels through which they get these information, may allow the policy maker to take actions to foster their knowledge, preventing them to take bad economic decisions based on wrong grounds.

As reported in Pesaran and Weale (2006), in the literature there are several studies, mostly concerning the Michigan Survey, that try to understand whether there are systematic differences in survey expectations among different groups. Dominitz and Manski (2011) present summary statistics from the Michigan Survey concerning expectations of a positive nominal equity return. They find that men are on average more optimistic than women, and that optimism increases with education and decreases with age. Similar results have been obtained by Bryan et al. (2001), who study inflation expectations. For the UK, Blanchflower and MacCoille (2009) find that inflation expectations rise with age and decrease for the more educated and the home-owners; the more educated and the

[1]The media, homogeneously reporting the expectation from the Survey of Professional Forecasters.

[2]France, Germany, Italy and the UK. Data taken from the European Commission's Consumer Survey

home-owners also have more precise one-year expectations; namely, they have a lower ex-post forecast error. In this regard, Souleles (2004) tests for systematic demographic components in households' forecast error in the Michigan Survey.[3] He finds that demographic variables are jointly significant both from a statistical and from an economic point of view. He reports that "the inflation forecast error is about 0.4 percentage points larger in magnitude for those without high school education, relative to those with high school education". The error tends to decrease in income and age, and to be larger also for females with respect to males and for belongers to racial minorities with respect to whites. There are also some common patterns of heterogeneity in the degree of financial literacy around the world (Lusardi and Mitchell, 2011b): the more educated people are the more informed, women are less financially literate than men, and the older population tends to be overconfident on his knowledge.

A study that takes heterogeneity into account focusing on Italian data is Easaw et al. (2013). The authors analyse households inflation expectation from February 2003 to October 2010. They find that, as reported also in Malgarini (2009), "expected inflation decreases with age and education [...] and women expect higher rates of inflation than man". Moreover, in a CSI framework similar to the one of the present chapter, they find that also the absorption rate of professional forecasts is heterogeneous as a function of the demographic characteristics of the individual. The more educated have also the highest absorption rate. Similarly, the self-employed inform themselves more frequently, while there is mixed evidence on the working status: it depends on the interaction with the education level.

Empirical data from European Commission's Consumer Survey show that Italian households unemployment forecasts notably differ from forecasts produced by professionals, as proxied by OECD projections. In this sense they may be deemed irrational. However, the present paper argues that a simple model of bounded rationality, with staggered updates and some fraction of stubborn forecasters can replicate data quite well. Hence, rational or irrational this may look, data are consistent with a situation in which *i*) only some agents get the most recent professional forecast, while the others update expectations through heuristics; *ii*) some of them invariably and stubbornly say that the future is *always* going to be worse than the current situation. Furthermore, in the present paper I study if such systematic differences are present *among* Italian households. Section 2.2 looks at microdata to understand which could be the main demographic drivers of macroeconomic and financial expectations and summarize the CSI model[4]. Section 2.3 presents some summary statistics of households forecasts. Section 2.4 presents the simulation methodology. Section 2.5 calibrates the parameters and presents results. Finally, Section 2.6 concludes.

2.2 Microdata

In the present section, I look for systematic differences in expectations of Italian households with different demographic characteristics. I will use microdata taken from the

[3] The author is interested in the role of systematic errors on estimate of excess sensitivity in Permanent Income Hypothesis (PIH) tests. He believes that not controlling for demographic characteristics may lead to estimates of excess sensitivity even if each household is actually behaving according to the PIH.

[4] For an extensive illustration, see Chapter 1.

Survey on Households Income and Wealth (*SHIW*)[5], run by the Bank of Italy (*Banca d'Italia*). The survey collects information on each household income, saving and wealth levels, consumption patterns and habits of the previous year.

In the 2010 edition, there are some questions concerning the expectations of the individual towards macroeconomic indicators in the near future. These questions are:

R1.3 According to you, on a range from 0 to 100, what is the probability that in a year's time interest rates will be higher than today?

R1.4 (If you gave a figure for Question R1.3) What is the probability they will be more than 1 point higher?

R1.5 According to you, on a range from 0 to 100, what is the probability to make a profit in a year's time investing in the italian stock market today?

R1.6 (If you gave a figure for Question R1.5) What is the probability the investment will earn more than 10%?

R1.7 According to you, on a range from 0 to 100, what is the probability that in a year's time house prices will be lower than today?

R1.8 (If you gave a figure for Question R1.7) What is the probability that they will fall more than 10%?

Therefore, the survey concerns interest rates, stock market returns and housing prices expectations, asking implicitly for a figure concerning the aggregate economy rather than the specific household condition. We must be aware that differences among different groups may be influenced by biases related to the projection of one own situation at the aggregate level. For example, a more pessimistic opinion of an unskilled worker about national per-capita income may be related to his more uncertain job condition, projected at the aggregate level. According to this interpretation, different beliefs among demographic groups may be the "right" answer to the "wrong" question, rather than being related to different expectations formation processes.

Interest rates and housing prices answers may suffer of this drawback, since the respondent may reason about the interest rate charged by local banks for mortgages, or about local housing prices. Nevertheless, stock market returns is an almost neutral topic.

I regress the individual subjective probabilities on some demographic variables, such as gender, employment status, age, macroregion of residence (Northen Italy, Central of Italy or Southern Italy), dimension of the municipality of residence and education level. Regressions are presented in Table 2.1. Demographic regressors are jointly significant for all questions. Concerning the most neutral subject, that is stock market returns (R1.5 and R1.6) men are more optimistic than women, young people are more optimistic than old people and optimism is increasing in education. These findings are in line with the ones of Blanchflower and MacCoille (2009), Bryan et al. (2001), Easaw et al. (2013), Dominitz and Manski (2011) and Malgarini (2009); moreover, there is a role of municipality dimension and macroregion of residence.

These microeconometric results suggest that, even though the presence of psychological biases cannot be excluded *a priori*, they are not enough to explain different expectations

[5]Data available at the Bank of Italy website http://www.bancaditalia.it/statistiche/indcamp/bilfait/dismicro.

Table 2.1: Regression of expectations on demographic variables, *SHIW* 2011

	Dependent variable:					
	Subjective probability					
	(R1.3)	(R1.4)	(R1.5)	(R1.6)	(R1.7)	(R1.8)
Intercept	29.90***	29.26***	20.58***	12.40***	24.16***	17.89***
	(3.41)	(3.72)	(2.16)	(1.91)	(2.27)	(2.94)
Female	-5.77***	-2.74	-6.64***	-2.16**	-3.07***	-3.66***
	(1.51)	(1.68)	(0.96)	(0.86)	(1.02)	(1.40)
Self-Employed	5.86**	4.97**	0.95	-1.77	3.72**	4.39**
	(2.29)	(2.38)	(1.46)	(1.25)	(1.60)	(2.10)
Not employed	-5.18**	-5.73**	0.29	0.77	-0.61	1.59
	(2.15)	(2.32)	(1.37)	(1.20)	(1.47)	(2.04)
Age 35-44	2.79	-2.11	0.33	-2.04	2.42	6.63**
	(3.16)	(3.38)	(1.99)	(1.76)	(2.13)	(2.79)
Age 45-54	0.79	-4.83	-2.14	-3.18*	1.03	2.46
	(3.09)	(3.31)	(1.95)	(1.72)	(2.09)	(2.71)
Age 55-64	2.10	-0.22	-2.67	-4.64***	-2.29	0.27
	(3.20)	(3.43)	(2.01)	(1.78)	(2.15)	(2.86)
Age >65	-1.60	-1.92	-5.03**	-5.36***	-1.81	0.65
	(3.41)	(3.69)	(2.16)	(2.34)	(2.28)	(3.07)
Center of Italy	13.78***	10.66***	1.59	2.34**	2.74**	10.59***
	(1.80)	(2.01)	(1.14)	(1.02)	(1.24)	(1.71)
Southern Italy	11.80***	16.57***	-2.05*	1.75*	-1.43	6.41***
	(1.78)	(1.91)	(1.14)	(1.02)	(1.19)	(1.68)
Municipality 20000-40000	1.42	1.54	-3.40**	-2.61**	-4.84***	-6.70***
	(2.18)	(2.39)	(1.36)	(1.21)	(1.46)	(2.00)
Municipality 40000-500000	2.85	4.42**	-2.35**	-2.26**	-3.84***	-3.03*
	(1.76)	(1.99)	(1.14)	(1.02)	(1.21)	(1.65)
Municipality >500000	5.60**	1.69	2.17	-1.31	-5.98***	-11.33***
	(2.85)	(3.12)	(1.78)	(1.50)	(1.91)	(2.58)
High School	3.90**	-0.16	4.66***	2.88***	-1.64	-0.32
	(1.63)	(1.77)	(1.04)	(0.92)	(1.11)	(1.52)
College	6.14***	-4.54*	7.64***	3.41***	-0.40	-3.13
	(2.24)	(2.36)	(1.43)	(1.22)	(1.56)	(2.05)
Observations	2328	1628	1986	1332	2467	1431
R^2	0.071	0.075	0.069	0.037	0.024	0.061
Adjusted R^2	0.065	0.067	0.062	0.027	0.018	0.052
F Statistic	0.000	0.000	0.000	0.000	0.000	0.000

Notes: *p<0.1; **p<0.05; ***p<0.01.
The reference groups are: Male, Employee, Less than 35 years old, residence in Northern Italy, residence in a municipality with dimension lower than 20000, education level lower than high school.

among different groups. These different expectations may be the result of different expectations formation processes, and I will explore this hypothesis. Specifically, I consider the role of education to be worth of further analysis, since understanding whether the risk of having wrong expectations is related to schooling may help policymakers to target economic education and information efforts. A similar point is done in the financial literacy literature (see, for example, Lusardi and Mitchell (2011a,b)), where it is pointed out that less educated people are less informed. In some empirical applications, like Duca and Kumar (2014), education has been used as a proxy for the degree of financial literacy. In the following, I will therefore model the expectations formation process as a function of the education level of the individual. I will adopt the same CSI framework previously described in Section 1.3.2 of Chapter 1. Here I recall the main points:

I The typical person believes that $\Delta_4 u_t = u_t - u_{t-4}$, where u_t is the unemployment rate in period t, behaves like a *stationary stochastic model.* In quarterly terms, this means that we have:
$$\Delta_4 u_t = \Delta_4 u_t^* + \epsilon_t, \tag{2.1}$$
where $\Delta_4 u_t$ is the realized value, $\Delta_4 u_t^*$ the fundamental value and ϵ_t a Gaussian disturbance. The fundamental value of the variable, in particular, is determined four periods in advance and evolves according to the following stationary process:
$$\Delta_4 u_{t+4}^* = \alpha + \beta \Delta_4 u_{t+3}^* + \eta_{t+4},\ 0 \leq \beta < 1, \tag{2.2}$$
where β represents the autoregressive coefficient of the fundamental value process, α is a constant term, and η_t is Gaussian disturbance.[6]

II Only professional forecasters, a group of expert agents, are able to observe exactly $\Delta_4 u_{t+4}^*$ in period t, so that the prediction of $\Delta_4 u_{t+4}$ corresponds to
$$N_t \left[\Delta_4 u_{t+4}\right] = \Delta_4 u_{t+4}^* \tag{2.3}$$
where $N_t \left[\Delta_4 u_{t+4}\right]$ indicates the professional forecasters prediction. In other words, the disturbance η_{t+4} is always observed by expert agents.[7].

III Professional forecasters expectations spread in the economy via news media (i.e. the so-called "common source of infection"). In each period, an agent i has a probability λ of being infected by the information and, then, to revise the expectation incorporating the professional forecasters prediction.[8]

IV Agents who acquire $N_{t+k} \left[\Delta_4 u_{t+4+k}\right]$ never forget this information.

Under the assumptions (I) – (IV), the expectation of $\Delta_4 u$ at time $t+4$ by a generic non-expert agent i can be written as:
$$E_t^i \left[\Delta_4 u_{t+4}\right] = E_t^i \left[\Delta_4 u_{t+4}^*\right] + \underbrace{E_t^i \left[\epsilon_{t+4}\right]}_{=0}. \tag{2.4}$$

[6] ϵ_τ and η_κ are independent $\forall \tau, \kappa$.

[7] It is important to note that future values of η beyond $t+4$ are unobservable for expert agents.

[8] In terms of equation (2.3), this means that non-expert agents, if infected for example at time t, are able to observe directly the fundamental value $\Delta_4 u_{t+4}^*$, without the ability to disentagle $\alpha + \beta \Delta_4 u_{t+3}^*$ from η_t.

If agent i is "infected" at time t, then Eq. (2.4) is equal to

$$E_t^i\left[\Delta_4 u_{t+4}\right] = N_t\left[\Delta_4 u_{t+4}\right]. \tag{2.5}$$

If agent i is not infected in t, but was instead infected at time $t-1$:

$$E_t^i\left[\Delta_4 u_{t+4}\right] = N_{t-1}\left[\Delta_4 u_{t+4}\right] = \alpha + \beta N_{t-1}\left[\Delta_4 u_{t+3}\right]. \tag{2.6}$$

2.3 Summary statistics

Consumer confidence data are obtained from Italian National Institute of Statistics *Istat*. The formulation of the question concerning unemployment expectations (Question 6 of the questionnaire) is as follows:[9]

Q6: How do you expect the number of people unemployed in Italy to change over the next 12 months?
The number will: (++) increase sharply; (+) increase slightly; (=) remain the same; (−) fall slightly; (−−) fall sharply; (N) don't know.

Data are presented as a balance index of the qualitative answers received. The only difference with respect to the Consumer confidence data available in the *Eurostat* website is the different range: *Istat* balance index has a theoretical range of -200 to +200, while the *Eurostat* one has a theoretical range of -100 to +100. It can be easily verified that the *Istat* and the *Eurostat* balance indexes for Italy are perfectly correlated, so this different range is just a matter of scaling and have no influence on the interpretation of results.

Data are available on a monthly basis, and since 1995 are disaggregated at different demographic levels. Three education levels are considered:

- no education degree, primary and lower secondary school certificate. In short *Less than High School (LTHS)*
- upper and post secondary. In short *High School (HS)*
- tertiary (university, doctoral and specialization courses). In short *College*

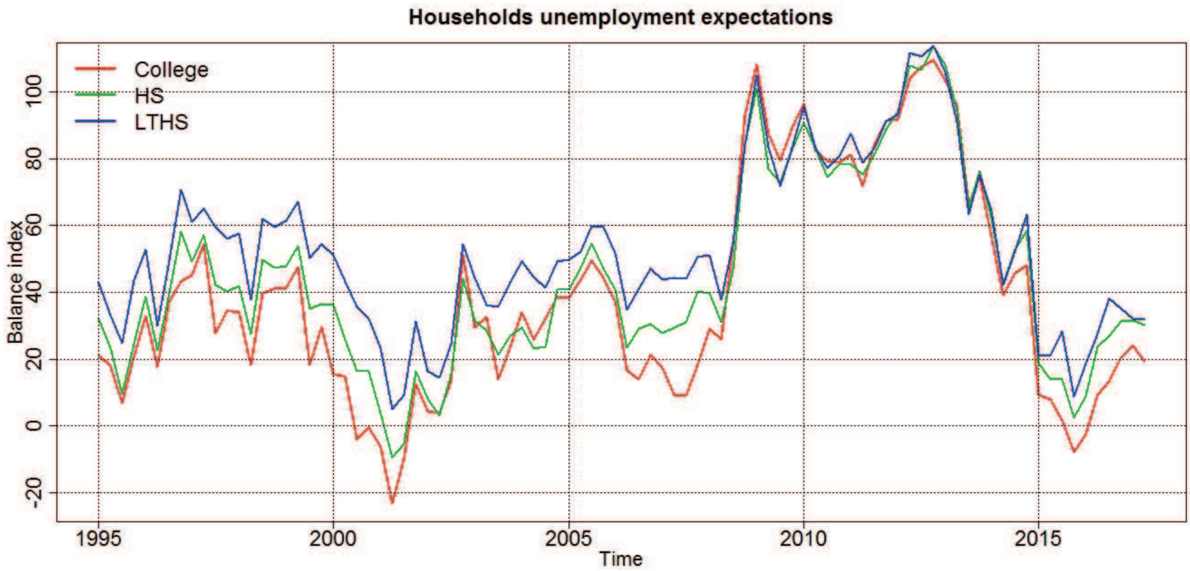
Table 2.1 reports some summary statistics for the balance indexes, disaggregated at the education level. It is immediately clear that the position indicators (mean and median) decrease as a function of education level, while dispersion indicators (standard deviation) increase as a function of education level. Moreover, less educated households balance index never reached negative values between 1995 and 2017, meaning that, on average, they always expected a rising unemployment rate: this is somewhat astonishing, in particular considering that between the very end of the 90ies and 2007 the unemployment rate has constantly decreased, almost halving (from about 12% to about 6%). Summarizing these findings, it appears that less educated individuals are more pessimistic (higher mean and median) and change less frequently their opinion (lower standard deviation). Figure 2.1 allows for a visual inspection of the series. The eyeball intuition is that, despite the different moments, they comove. Actually, the pairwise correlation among the series is always higher than 0.95.

[9] Being part on an harmonised European Commission project, the question asked by the Italian institute is the same we used in Chapter 1 for studying Consumer Confidence in selected European Countries.

Table 2.1: Summary statistics (1995q1-2017q2)

	Min	1st Qu	Median	Mean	3rd Qu	Max	St.Dev
LTHS	4.97	35.84	50.4	53.34	64.9	114	24.79
HS	-9.6	24.89	39.73	44.27	58.32	113.7	28.35
College	-23.47	15.77	32.7	39.09	54.32	109.5	32.39

Figure 2.1: Households unemployment expectations balance indexes by education level (1995Q1-2017Q2)



The aim, in the present paper, is to understand if unemployment expectations data are compatible with a CSI model with heterogeneous parameters as a function of the education level of the individual. Unfortunately, the GMM or the time-varying estimation at the aggregate macro level used in Chapter 1 ($\widehat{M_t}\,[\Delta_4 u_{t+4}] = \lambda N_t\,[\Delta_4 u_{t+4}] + (1-\lambda)\,(\alpha + \beta\widehat{M_{t-1}}\,[\Delta_4 u_{t+3}])$ are not suitable for this purpose. The auxiliary regression $\Delta_4 u_{t+4} + = u_{t+4} - u_t = \phi_{edu,0} + \phi_{edu,1} IST^U_{edu,t}$, by construction returns fitted values $\widehat{M_{edu,t}}\,[\bullet]$ which are perfectly correlated with $IST^U_{edu,t}$, have mean equal to $\mathbb{E}(\Delta_4 u_{t+4})$ and standard deviation equal to $SD(\Delta_4 u_{t+4}) \cdot COR(IST^U_{edu,t}, \Delta_4 u_{t+4})$. In other words, the differences in position and dispersion indicators of the original series by education would not be reflected in the fitted values by education and therefore on the estimated coefficients λ. Table 2.2 and Figure 2.2 confirm the intuition.

Table 2.2: Summary statistics of fitted values $\widehat{M_{edu,t}}\,[\bullet]$ (1995Q1-2016Q1)

	Min	1st Qu	Median	Mean	3rd Qu	Max	St.Dev
LTHS	-0.85	-0.27	-0.04	0.03	0.25	1.08	0.44
HS	-0.85	-0.3	-0.05	0.03	0.3	1.12	0.46
College	-0.88	-0.31	-0.06	0.03	0.26	1	0.46

Figure 2.2: Fitted values $\widehat{M_{edu,t}}\,[\bullet]$ by education level (1995Q1-2016Q1)

2.4 Simulation approach

In order to estimate the parameters, I adopt the same micro-simulation procedure of Chapter 1. In short, I assume that:

1. each individual with education *edu* has a constant probability λ_{edu} of absorbing the latest professional forecast.
2. among individuals with education *edu*, a fraction μ_{edu} of agents is *stubbornly pessimistic*, i.e. they always think unemployment is going to increase sharply, without caring about projections or historical values.

I will avoid education subscripts for simplicity, but the following process is repeated independently for each education level. We use the subscript i to denote a variable concerning agent i. All agents are initialized with $E_0^i[\Delta_4 u_4] = 0$. "Infection" of agent i, in any period t, is a Bernoulli random variable Inf_t^i with $\mathbb{P}(Inf_t^i = 1) = \lambda$, and *stubbornly pessimism* is modeled as a Bernoulli with $\mathbb{P}(Stub_t^i = 1) = \mu$. Therefore we have:

$$E_t^i[\Delta_4 u_{t+4}] = \begin{cases} N_t[\Delta_4 u_{t+4}] & \text{if } Inf_t^i = 1 \text{ (Informed)}, \\ \alpha + \beta E_{t-1}^i[\Delta_4 u_{t+3}] & \text{if } Inf_t^i = 0 \text{ (Uninformed)}. \end{cases} \tag{2.7}$$

And:

$$Ans_t^i(E_t^i[\Delta_4 u_{t+4}]) = \begin{cases} ++ \ \text{(increase sharply)} & \text{if } E_t^i[\Delta_4 u_{t+4}] \geq \gamma \vee Stub_t^i = 1, \\ + \ \text{(increase slightly)} & \text{if } \frac{\gamma}{2} \leq E_t^i[\Delta_4 u_{t+4}] < \gamma \wedge Stub_t^i = 0, \\ = \ \text{(remain the same)} & \text{if } -\frac{\gamma}{2} < E_t^i[\Delta_4 u_{t+4}] < \frac{\gamma}{2} \wedge Stub_t^i = 0, \\ - \ \text{(fall slightly)} & \text{if } -\gamma < E_t^i[\Delta_4 u_{t+4}] \leq -\frac{\gamma}{2} \wedge Stub_t^i = 0, \\ -- \ \text{(fall sharply)} & \text{if } E_t^i[\Delta_4 u_{t+4}] \leq -\gamma \wedge Stub_t^i = 0, \end{cases}$$

2.5 Results

The model has five parameters: α, β, γ, λ and μ. As in Chapter 1, I proxy the professional forecasts $N_t[\bullet]$ with forecasts from the OECD Economic Outlook. I assume α, β and γ to be homogeneous and equal to the calibration for Italy adopted in Chapter 1,[10] while λ_{edu} and μ_{edu} are heterogeneous, where *edu* denotes the education level. λ_{edu} and μ_{edu} are estimated through the Method of Simulated Moments. The moments to be matched are the mean and the standard deviation of the balance index (denoted $IST^U_{edu,t}$) of that education level.

Table 2.1: Micro-simulation calibration (1995Q1-2017Q2)

	μ	λ
LTHS	0.2956 (0.2923-0.2990)	0.0513 (0.0494-0.0533)
HS	0.2550 (0.2506-0.2595)	0.0605 (0.0578-0.0632)
College	0.2346 (0.2287-0.2405)	0.0721 (0.0684-0.0758)

Table 2.2: Summary statistics - Simulated and Original (1995Q1-2017Q2)

	Min	1st Qu	Median	Mean	3rd Qu	Max	St.Dev	Cor
LTHS - Simulated	5.27	36.52	52.25	53.18	69.68	107.40	24.68	0.68
LTHS - Original	4.97	35.84	50.40	53.34	64.90	114.00	24.79	
HS - Simulated	-8.60	26.86	42.28	43.97	63.82	103.40	27.51	0.72
HS - Original	-9.60	24.89	39.73	44.27	58.32	113.70	28.35	
College - Simulated	-24.00	16.80	37.80	37.82	58.02	109.00	32.83	0.73
College - Original	-23.47	15.77	32.70	39.09	54.32	109.50	32.39	

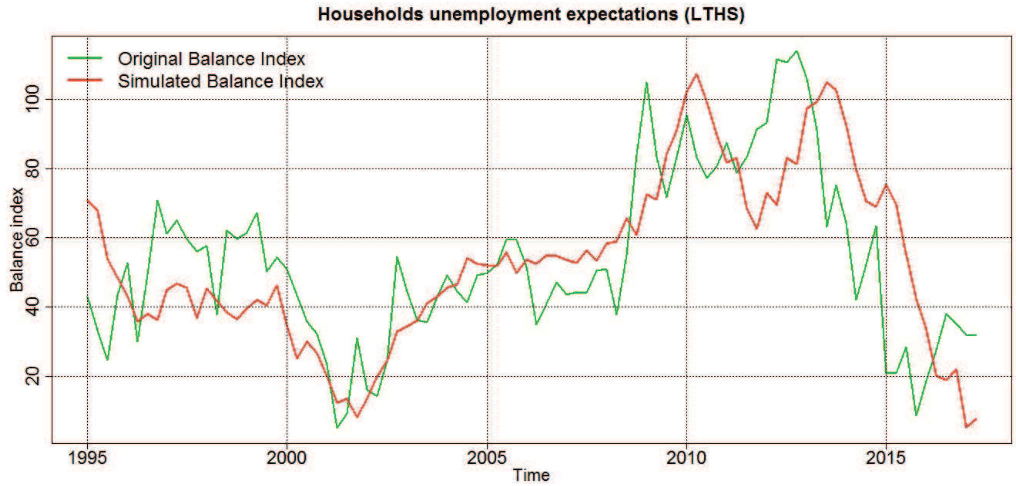
Table 2.1 reports the results of the estimation together with the corresponding 95% confidence intervals. Table 2.2 reports some summary statistics and compares them with the empirical ones, along with the correlation. Correlation is on purpose not a matched moment but turns out to be relevant (always higher than 0.68). Figure 2.1 allows for a visual inspection of the substantial fit. The interpretation of the parameters (Table 2.1) is intuitive and straightforward:

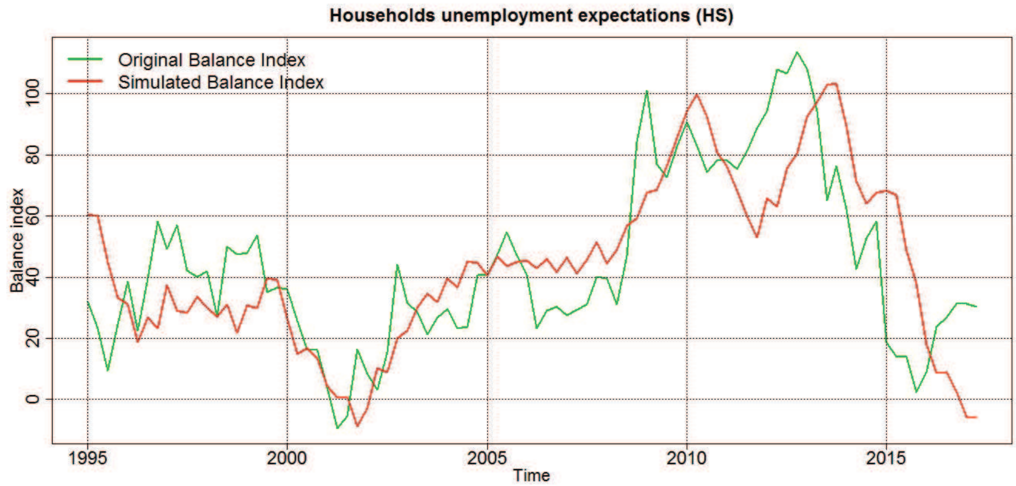
- The probability that a given individual has access to up-to-date professional forecasts (λ) is increasing in the education level.
- The proportion of individuals in the population that always think in a pessimistic way (μ), independently from forecast of the future or recent past trends, is decreasing in the education level. There are less *stubbornly pessimistic* individuals among more educated people. This finding is consistent with results reported in Table 2.1 and in Dominitz and Manski (2011) about expected returns on the Stock Market (an unambiguous sign of optimism).

According to this interpretation, the more educated can be considered therefore as more "rational", whereas also the less educated acquire information and update their forecasts, but much more infrequently.

[10] $\alpha = 0$, $\beta = 0.913$, $\gamma = 0.389$.

Figure 2.1: Households unemployment expectations balance for *LTHS*, *HS* and *College* (1995Q1-2017Q2)





The less educated are therefore more in need of frequent, transparent and easily accessible (and manageable) information in order to have a correct expectation.

About half of the Italian population is in the *Less than High School* group. It is striking to note that data for the this group are consistent not only with a really low updating probability (only 5 agents per quarter, out of one hundred, read the forecasts), but also with almost one third of agents formulating an opinion which is based on no empirical grounds. Not only they do not trust forecasts of national and supranational institutions, but they do not even care about the recent trend. Basically, they always claim that the future is going to be grim (or even awful!).

Moreover, I also estimate separately the parameters for two subperiods, that is until 2008, and from 2009 to 2012. There are two main reasons: the first one is that the three series related to different education levels appear to converge after 2009, and the second one is that for the period of heightened uncertainty (first financial crisis, then sovereign debt crisis) the model is not able to fit data.

Table 2.3: Micro-simulation calibration (1995Q1-2008Q4)

	μ	λ
LTHS	0.3135 (0.3097-0.3174)	0.0574 (0.0533-0.0616)
HS	0.2573 (0.2525-0.2622)	0.0627 (0.0579-0.0674)
College	0.2372 (0.2304-0.2441)	0.0748 (0.0669-0.0826)

Table 2.4: Micro-simulation calibration (2009Q1-2012Q4)

	μ	λ
LTHS	0.3310 (0.3211-0.3410)	0.0559 (0.0516-0.0603)
HS	0.3173 (0.3057-0.3290)	0.0553 (0.0501-0.0605)
College	0.3400 (0.3262-0.3537)	0.0491 (0.0424-0.0559)

It is possible to appreciate that, even if confidence intervals have widened, the estimation for the *pre-2009* subsample (Table 2.3) confirms the interpretation of λ and μ for the whole period; moreover, confidence intervals overlap with the corresponding ones of the baseline period, with the exception of μ_{LTHS}. Conversely, the intuition we got for the *post-2009* (Table 2.4) is the opposite: μ, the proportion of *stubbornly pessimistic* individuals, has increased for all education levels, up to 50% for *College*, and the estimates are basically the same for the three education groups. At the same time, the estimates of λ decrease and the confidence intervals widely overlap for all education levels, so the estimates of the absorption probabilities are not significantly different among education levels, too. Moreover, even if the Method of Simulated Moments is able to reproduce quite accurately the first two moments, the correlation coefficient is negative (around -0.2): this model is not able to explain the behaviour in the years from 2009 to 2012, while it has good explanatory power for the *pre-2009* period.

These estimates confirm the "eyeball" intuition that the expectation formation process may have changed and, in line with results of Chapter 1, in periods of heightened uncertainty agents are willing to spend less time to learn forecasts likely to turn out to be far from the actual value. What might be surprising is the high and homogeneous degree of pessimism under deepened uncertainty, which may have had effect on economic decisions of households (e.g. spending on durables, or saving).

2.6 Conclusions

In this work, I find that an important driver of expectations is the education level. I develop an agent-based model under the hypothesis that expectations may be partly not rational at all (*stubborn pessimism*) but may also simply be based on imperfect or old information and heuristic updating rules, leading to expectations that *prima facie* may appear totally irrational.

The CSI model is simple and is able to describe, with a sound story, the process that leads to expectations development as a function of the education level. The results are interpretable in a very simple and intuitive way: the higher the education level, the closer is the average expectation to the one of the professional forecasters. The probability of being informed of the latest professional forecast is increasing in the education level: the more educated read more newspapers, and so they have an higher probability of learning the new forecast. There is also a lower proportion of *stubborn pessimism* among the more educated. The fact that the average expectation of the more educated is the one closest to the professional forecasters prediction complements financial literacy studies on the importance of education to provide the ability to understand economic phenomena and react optimally. This has an effect on the business cycle, too: waves of excessive optimism and pessimism have frequently caused business cycle fluctuations (Milani, 2011), turning out to be self-fulfilling prophecies.

The CSI is one of the simplest infection models, which does not take into consideration, for example, interaction among agents.[11] Even if the fit of the CSI model is quite good, richer models can explain even more precisely the data and the underlying expectation formation process. For example, following the results of Chapter 1, an agent-based model may feature non-constant and endogenously defined parameters in the different periods. Or it could deviate from the unique-common-source assumption, allowing for multiple sources of information (including "fake news") and for communication between individuals. Again, it would be interesting to model the network of the different agents as a function of their characteristics: individuals are more likely to interact with their peers than with individuals belonging to different social and age classes.

Moreover, the methodology developed in Chapter 1 and in Chapter 2 could be adopted to study expectations development also for more sophisticated macroeconomic or financial indicators. One example is the long-term interest rate, which drives important household investment decisions, like housing expenditure or choice between fixed rate and variable rate mortgages. It would be very interesting to assess if the heterogeneity of consumers expectations process, as a function of education, increases in the degree of sophistication of the indicator considered.

[11] Carroll (2006) proposes an agent-based model with a fraction of individuals which are always informed, and all the others learn from the neighbour with the more recent information. He also proposes another variant of the common-source-infection, in which in each period a fraction of agents absorb the information from the common source, and a fraction of the remaining ones interact with a randomly chosen individual and share their information, i.e. the less informed of the two acquire the information of the more informed of the two.

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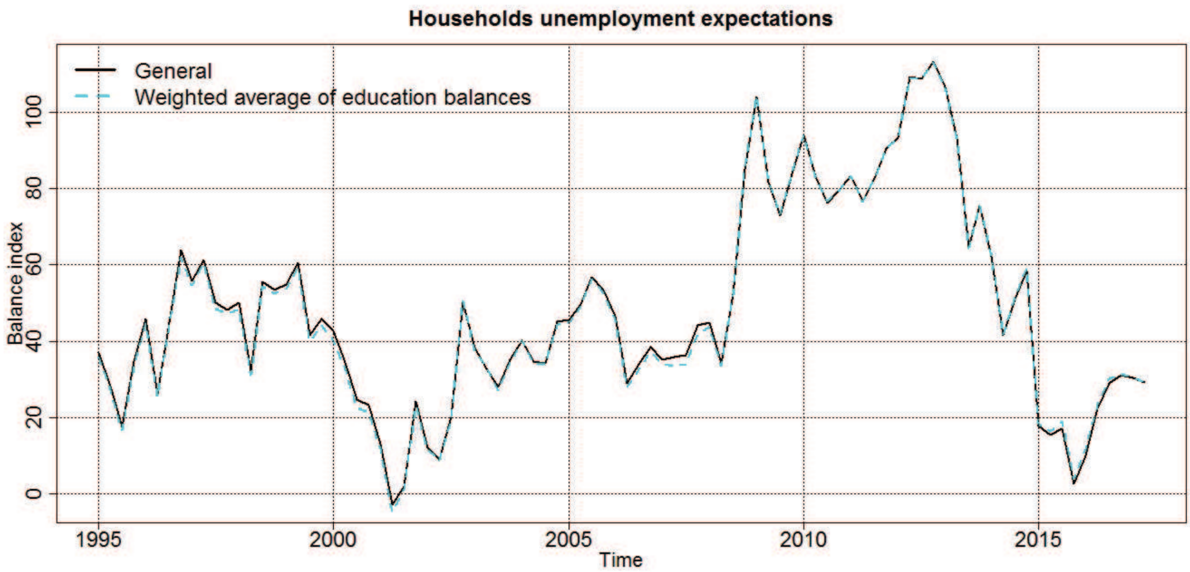
Appendix

2.A Number of agents

Istat reports to interview 2000 individuals per month, that is 6000 individuals per quarter. Given that about $\frac{1}{2}$ of the Italian population belongs to the *LTHS* group, about $\frac{1}{3}$ to the *HS* group and about $\frac{1}{6}$ to the *College* group, I simulate with 3000 agents per quarter for *LTHS*, 2000 for *HS* and 1000 for *College*. Moreover, in Figure 2.A.1 I to plot the general balance index, that is the one based on all the 6000 interviews, and the weighted sum of the three balances disaggregated by education. The two are almost not distinguishable, meaning that the used proportions $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{6}$ are a reasonable approximation.

Figure 2.A.1: Households unemployment expectations balance index (1995Q1-2017Q2)



2.B Alternative values of β and γ

β and γ are calibrated outside the simulation procedure, exploiting the results from Chapter 1. In the present sensitivity analysis, I explore if the interpretation of results (λ increasing and μ decreasing in education) is affected by the value of the homogeneous ones. $\beta = \{0.8, 0.85, 0.9, 0.95, 1\}$ and $\gamma = \{0.2, 0.4, 0.6\}$. Table 2.B.1 reports the estimated values of λ_{edu} and μ_{edu} for each of the 15 combinations of β and γ. λ is still increasing and μ still decreasing in education. The only exception is the scenario with $\beta = 1$ and $\gamma = 0.2$: in this scenario, the confidence intervals for μ_{HS} and $\mu_{College}$ overlap.

Table 2.B.1: Sensitivity analysis with alternative values of β and γ (1995Q1-2017Q2)

		α			μ		
β	γ	LTHS	HS	College	LTHS	HS	College
0.8	0.2	0.0485 (0.0469-0.0501)	0.0565 (0.0545-0.0585)	0.0663 (0.0633-0.0693)	0.3121 (0.3091-0.3152)	0.2738 (0.2696-0.2780)	0.2558 (0.2499-0.2616)
	0.4	0.0810 (0.0787-0.0832)	0.0944 (0.0915-0.0974)	0.1106 (0.1064-0.1148)	0.2958 (0.2934-0.2982)	0.2549 (0.2517-0.2581)	0.2343 (0.2301-0.2384)
	0.6	0.1310 (0.1276-0.1344)	0.1533 (0.1487-0.1578)	0.1806 (0.1736-0.1876)	0.2877 (0.2857-0.2897)	0.2456 (0.2431-0.2482)	0.2230 (0.2200-0.2260)
0.85	0.2	0.0415 (0.0401-0.0429)	0.0483 (0.0462-0.0504)	0.0567 (0.0540-0.0594)	0.3107 (0.3072-0.3142)	0.2721 (0.2674-0.2769)	0.2540 (0.2471-0.2608)
	0.4	0.0693 (0.0673-0.0713)	0.0810 (0.0783-0.0837)	0.0952 (0.0914-0.0990)	0.2947 (0.2920-0.2973)	0.2537 (0.2505-0.2570)	0.2330 (0.2287-0.2374)
	0.6	0.1153 (0.1122-0.1183)	0.1354 (0.1312-0.1396)	0.1603 (0.1535-0.1670)	0.2891 (0.2867-0.2915)	0.2471 (0.2442-0.2500)	0.2246 (0.2212-0.2280)
0.9	0.2	0.0330 (0.0315-0.0344)	0.0385 (0.0366-0.0403)	0.0458 (0.0431-0.0485)	0.3111 (0.3070-0.3151)	0.2723 (0.2666-0.2779)	0.2558 (0.2472-0.2643)
	0.4	0.0561 (0.0541-0.0581)	0.0659 (0.0632-0.0686)	0.0782 (0.0742-0.0822)	0.2942 (0.2912-0.2972)	0.2533 (0.2496-0.2570)	0.2328 (0.2273-0.2384)
	0.6	0.0948 (0.0916-0.0981)	0.1122 (0.1080-0.1163)	0.1344 (0.1285-0.1403)	0.2870 (0.2845-0.2895)	0.2450 (0.2420-0.2479)	0.2231 (0.2179-0.2282)
0.95	0.2	0.0239 (0.0225-0.0254)	0.0283 (0.0264-0.0302)	0.0344 (0.0313-0.0374)	0.3194 (0.3136-0.3252)	0.2824 (0.2748-0.2900)	0.2661 (0.2546-0.2775)
	0.4	0.0416 (0.0395-0.0437)	0.0494 (0.0467-0.0522)	0.059 (0.0548-0.0642)	0.2951 (0.2909-0.2993)	0.2544 (0.2485-0.2602)	0.2338 (0.2262-0.2414)
	0.6	0.0702 (0.0676-0.0729)	0.0840 (0.0803-0.0877)	0.1014 (0.0961-0.1066)	0.2835 (0.2806-0.2864)	0.2411 (0.2373-0.2449)	0.2184 (0.2128-0.2240)
1	0.2	0.0209 (0.0194-0.0223)	0.0252 (0.0235-0.0269)	0.0305 (0.0269-0.0340)	0.3447 (0.3307-0.3588)	0.3155 (0.3016-0.3294)	0.3043 (0.2846-0.3240)
	0.4	0.0316 (0.0299-0.0333)	0.0372 (0.0347-0.0396)	0.0444 (0.0405-0.0482)	0.3195 (0.3113-0.3278)	0.2802 (0.2716-0.2889)	0.2593 (0.2465-0.2721)
	0.6	0.0478 (0.0451-0.0505)	0.0573 (0.0534-0.0612)	0.0705 (0.0639-0.0772)	0.2922 (0.2866-0.2978)	0.2493 (0.2428-0.2558)	0.2259 (0.2173-0.2346)

2.C Alternative forecasts

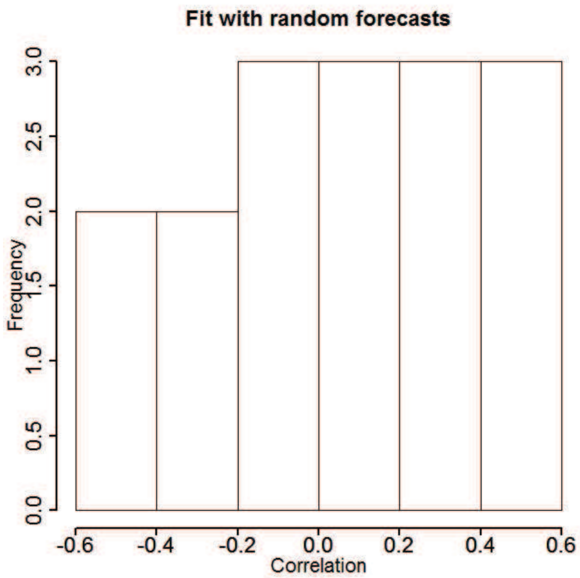
Does this model provide a good fit with empirical data whatever series of 126 numbers we substitute for the OECD forecasts? In order to check for this (unfortunate) possibility, I generate 80 random sequences from an AR(1) $x_{t+1} = \alpha + \beta x_t + \eta_t$ process with coefficients $\alpha = 0$, $\beta = 0.913$ and disturbance $\eta \sim N(\mu_\eta = 0, \sigma_\eta = 0.4)$ and estimate again the parameters, assuming these are the forecasts households could become aware of in each period. I limit my analysis to the *LTHS* case: given the high correlation among the three education balances, it is enough to study in depth one case, and I consider the case for which I got the lowest correlation (0.68).

Fortunately, the fit yield by the "random" forecasts is never higher than the 0.68 I got using the "true" OECD forecasts. As can be seen from Table 2.C.1 and the histogram in Figure 2.C.1, out of the 80 runs, the highest correlation is below 0.6 and about half of the correlations turn out to be negative. These findings confirm that the logic behind the present model could actually mimick reality, with households which update on average every eleven or twelve quarters their expectation by making reference to institutional forecasts, and in the meanwhile they "guess" which could be the unemployment change given the old information they are aware of.

Table 2.C.1: Summary statistics - Correlations of simulated balances using 80 batches of "random" professional forecasts with *LTHS* balance index (1995Q1-2017Q2)

	Min	1st Qu	Median	Mean	3rd Qu	Max
Correlation	-0.67	-0.32	0.09	0.00	0.28	0.55

Figure 2.C.1: Histogram of correlations of simulated balances using 80 batches of "random" professional forecasts with *LTHS* balance index (1995Q1-2017Q2)



Chapter 3

A replication of Pindyck's willingness to pay: on the sacrifice needed to obtain results

Luca Gerotto* Paolo Pellizzari‡

Abstract

We present a replication and a robustness check, involving both new data and model extensions, of "Uncertain outcomes and climate change policy", R. Pindyck, *Journal of Environmental Economics and Management*, 2012. As far as replication is concerned, we are able to reproduce the results provided in Pindyck's work in many cases and convincingly confirm the quality of the work. Some discrepancies are present, they are due to rounding or related to specific sets of parametric values and do not change the economic interpretation or significance of the results. The re-estimation of the model with more recent data on climate change made available in 2014 shows that temperature increments are now deemed to be higher in mean but less dispersed. As a consequence, the willingness to pay doesn't vary much with respect to the original paper. We also modify the functional form describing the impact of temperature increase on the growth rate of consumption, and we obtain much bigger and potentially problematic increments of the willingness to pay, or describing the pattern of temperature increase, and we get a modest increment in the willingness-to-pay.

Finally, the paper demonstrates that the numerical results are sensitive to a variety of technical settings used in the computations and suggests that great care is needed in obtaining estimates and employing results in policy discussions.

Keywords: Replication, environmental policy, climate change, economic impact, willingness to pay.

JEL codes: D81, Q51, O44.

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3.1 Introduction

This paper presents a replication and a robustness check of the work "Uncertain outcomes and climate change policy" by R. Pindyck, *Journal of Environmental Economics and Management* (Pindyck, 2012), P12 hereafter. Pindyck incorporates the distribution for the (uncertain) temperature change and the distribution of the (uncertain) impact of this change on the growth of consumption and computes the willingness to pay (WTP), i.e., "the fraction of consumption [...] that society would be willing to sacrifice, now and throughout the future, to ensure that any increase in temperature at a specific horizon H is limited to τ". These fractions are typically below 2% and it is stated in P12 that this is consistent with the adoption of a moderate abatement policy.

P12 is a sound paper tackling difficult questions with crystalline thinking and terse prose. The work was cited often (31 times on Scopus and 113 on Google scholar[1]) in a relatively short lifespan. Assumptions and methods are clearly spelled out, as indeed proven by the fact that most of the paper could be reproduced with no access to the original code or files. Most of important arguments are crucially based on figures numerically resulting from the model, and data and estimates are based on *IPCC* Fourth Assessment (IPCC, 2007c,a,b). We believe the results in P12 are important and insightful and summarize in a clever way a vast amount of knowledge on climate change. The computations involved in the model are technically demanding and, basically, require to evaluate many 3-dimensional non-trivial integrals (over a long span of time, over an estimated distribution of temperature changes, over an estimated distribution of an impact coefficient). Our replication was often facilitated by the working paper (Pindyck (2009), hereafter P09) which, we deem, was a detailed preparation of the contributions that were later streamlined and distilled in P12. The possibility to read two "versions" of the same work and access, when needed, alternative wording of the same procedures or descriptions is a fortunate circumstance, hence we hope more scholars will routinely publish in the future all the drafts of the papers that ultimately result in a "definitive" publication on a journal. The examination of multiple interrelated stages of development of a scientific research can illuminate technical passages, as well as clarify the logical path linking the original ideas to the final upshot.

Replication is of paramount importance in science and lies at the very heart of what differentiates science from cheap talk and non-scientific arguments. There is an increasing awareness of the need for more replication studies in social sciences (Duvendack et al., 2015) and too many scholars sadly admit that have attempted with faltering or no success to reproduce others' work (Baker, 2016). Replication, however defined, is in our opinion very important for another reason: boldly put, we believe that replicating a paper is the only way to (fully) understand it. This may be also true for theoretical works (say, reworking all the proofs) but we have no doubt that this is needed to master numerical, empirical or simulative work. The amount of effort needed is often inordinate but, even though the verification is never published as a standalone paper, the rewards are hefty and one of the authors of this article just believes that most, if not everything, of what he knows comes from the hard times spent in struggling with the details of papers to be replicated.

The term "replication" is actually used in the literature to refer to distinct procedures

[1] The number of citations was recorded on October 31, 2017.

and there is no widely agreed standard on its precise meaning. Clemens (2017) presents an excellent survey of replications in economic research and proposes a more rigid terminology to distinguish replication categories (*verification*, *reproduction*[2], *reanalysis*, *extension*). On the one hand, a *verification* has the aim of reproducing the results of the original paper using the *same specification of the model and the same data*. Therefore, a verification should not produce discrepant results unless there are plain errors or fraud in the original work. On the other hand, *reanalysis* and *extension* are robustness checks to the original work, having the aim of exploring the stability of results of the original work using different data and/or alternative model specifications. More specifically, a *reanalysis* uses the same data of the original work but with a different model specification, while an *extension* runs the original model using different data. Hence, if for a verification we expect the results to be the same of the original work, there is no reason to expect the same after an extension or a reanalysis. For additional clarity, we'll no longer use the word "replication" in what follows and stick to Clemens (2017) terminology.

In this paper, therefore, we present first a *verification*, then an *extension* and a *reanalysis* of P12. In the first part relative to verification, we aim at reproducing the results of P12 using the *same specification of the model and the same data*. Neither P12 nor P09 explicitly state the software used for the computations but Robert Pindyck, in a personal communication, made clear that MATLAB was used and provided us with some code. In what follows, we used R (R Core Team, 2015), a popular and reliable free software platform for statistical and numerical computing and data visualization. While we stress that some scholars, like Anderson et al. (2008), appear to require that a "verification" should use the same software (and perhaps the same hardware), we believe that the use of MATLAB, R or any other professionally trusted software (e.g. Octave, Mathematica) should not alter the substantial results of a research. In other words, if different results are obtained with different pieces of software, the case is indeed worth studying as done in McCullough and Vinod (2003).[3] Subsequently, we move forward and perform an extension using data from *IPCC* Fifth Assessment (IPCC, 2014) whereas P12 was based on the previous *IPCC* Fourth Assessment, (IPCC, 2007c,a,b). Finally, we have two different reanalysis: firstly, we assume the change in temperature convexly affects the consumption growth rate, in contrast with P12 where linear dependence is assumed. Secondly, we change the assumed pattern of temperature increase, opting for a different functional form more in line with recent climate scientists findings.

We obtained two main results from our verification. The first is that most of P12 can be reproduced quite accurately and even if discrepancies are present in some of our figures, they are small and do not affect the economic meaning or interpretation in any

[2]Reproduction is not implemented in the present chapter; however, for completeness, we report Clemens definition: "A reproduction test means resampling precisely the same population but otherwise using identical methods to the original study".

[3]From Axtell et al. (1996): "We have identified a few cases in which an older model has been reprogrammed in a new language, sometimes with extensions, by a later author. For example, Michael Prietula has reported reimplementing a model from Cyert and March (1963) and Ray Levitt has reported a reimplementation of Cohen, March and Olsen (1972). However, these procedures are not comparisons of different models that bear on the same phenomena. Rather they are "reimplementations", where a later model is programmed from the outset to reproduce as closely as possible the behaviour of an earlier model. Our interest is in the more general and troublesome case in which two models incorporating distinctive mechanisms bear on the same class of social phenomena, be it voting behaviour, attitude formation, or organizational centralization".

way.

The other outcome of the verification is more general and a bit troublesome: our results (and, hence, our ability to verify P12) are sensitive in many cases to choices of parameters used in the computation but otherwise having no deep relationships with the model. For instance, even though some integrals are naturally defined on the real (half) line, integration routines require to set an upper limit for the domain: while this should be intuitively irrelevant, it turns out that it can introduce non obvious and large biases. A more detailed discussion is deferred to Section 3.6 but our experience emphasizes that it may be difficult to select the "right" parameters leading to the "correct" results, especially if one has not an article, like P12, as a target for fine-tuning.

Regarding the extension, we change the data source and "redo the paper" to give the flavour of how our understanding and policy vary based on two subsequent *IPCC* reports. Essentially, more recent data support a temperature change distribution over next century that is higher on average and more concentrated. The effects on the WTP thus are opposite, as the higher mean would increase our willingness to pay but, at the same time, as extreme events are less likely, smaller risk tends to curb the WTP. The overall effect is a slight increase in the willingness to pay for strong mitigation and a slight decrease in the willingness to pay for moderate mitigation.

We then alter the specification of the model, still keeping the original data to be comparable with P12. First, we assume that the growth rate of consumption is convexly (as opposed to linearly) affected by the temperature change. This incorporates in the model a more cautious and risk-averse attitude as large (albeit rare) increments in the temperature can have drastic effects. This different specification produces (moderately) higher levels of the willingness to pay. The sensitivity of results to the choice of the damage function, about which "we know almost nothing" (Pindyck, 2013), suggests caution in the interpretation of results for policy making and sheds light on the need for further research on the economic implications of climate change. Second, we assume that the pattern of temperature increase is first convex then, after approximately one century, concave, while P12 assumes an always concave functional form. This modification produces a modest increase in the willingness to pay, both for moderate and for strong mitigation.

The paper is organized as follows. Section 3.2 briefly summarizes the model contained in Pindyck's work and describes the major conclusions of P12. In Section 3.3, we present our verification strategy and explain the functioning of the routines we have used in the verification. P12 is a rich paper with plenty of numerical results and robustness tests or discussions. We reproduced a vast body of outcomes, including pictures, key tables and robustness checks of the original paper. Section 3.4 and 3.5 are devoted to the extension and the reanalysis of the work, respectively. Section 3.6 discusses in detail some of the most relevant results of the previous parts. We then conclude with some final remarks and suggestions for future research.

3.2 The model

This section describes the model presented in P12. It is assumed that the temperature increase T_H at horizon H is distributed as a three-parameter displaced gamma density of the form:

$$T_H \sim f(x) = f(x; r = r_T, \lambda = \lambda_T, \theta = \theta_T) = \frac{\lambda^r}{\Gamma(r)}(x-\theta)^{r-1}e^{-\lambda(x-\theta)}, \quad x \geq \theta,$$

where θ is the displacement parameter and $\Gamma(r) = \int_0^\infty s^{r-1}\exp(-s)ds$ is the Gamma function. If T_H is the increase in temperature after H years, the increase at time t, T_t evolves according to

$$T_t = 2T_H[1-(1/2)^{t/H}], \tag{3.1}$$

so that, in particular, $T_t \to 2T_H$ as $t \to \infty$.

As done in many studies of climate change the effect of temperature increase is linked to the Gross Domestic Product (GDP) through the loss function $L(T) = \exp(-\beta T^2)$. The GDP (or consumption) at H is then $L(T_H)\mathrm{GDP}_H$, where GDP_H is the "would have been" GDP at $t = H$ with no warming. Clearly, the loss affects the level of GDP but it is argued in P12 that a model incorporating effects on the growth rate of GDP is more appropriate. Hence, assuming that in the absence of warming the GDP would grow at constant rate g_0 and that T_t decreases the instantaneous growth rate to

$$g_t = g_0 - \gamma T_t, \tag{3.2}$$

we obtain the path of the growth rate as

$$g_t = g_0 - 2\gamma T_H[1-(1/2)^{t/H}].$$

Hence, consumption (or GDP) $C_t = C_0\exp(\int_0^t g(s)ds)$ (with warming), can be computed as

$$C_0\exp\left(-\frac{2\gamma HT_H}{\ln(1/2)} + (g_0 - 2\gamma T_H)t + \frac{2\gamma HT_H}{\ln(1/2)}(1/2)^{t/H}\right), \tag{3.3}$$

for any t. Normalizing consumption C_0 at 1 and equating final consumption (3.3) at H with what would be obtained using a loss function on levels, one gets

$$\exp\left(-\frac{2\gamma HT_H}{\ln(1/2)} + (g_0 - 2\gamma T_H)H + \frac{2\gamma HT_H}{\ln(1/2)}(1/2)^{H/H}\right) = L(T_H)\exp(g_0H) = \exp(g_0H - \beta T_H^2),$$

and, subsequently, the relationship between β and γ:

$$\gamma = 1.79\frac{\beta T_H}{H}. \tag{3.4}$$

Typically, integrated assessment models in the literature provide estimates of β, which can be converted into values for γ that are finally used to fit a displaced gamma density $f_\gamma(y) = f_\gamma(y; r_\gamma, \lambda_\gamma, \theta_\gamma)$, for the random variable γ appearing in (3.2).

We define the social utility function

$$U(C_t) = \frac{C_t^{1-\eta}}{1-\eta},$$

where η is the index of relative risk aversion of the society. It is convenient in what follows to set $u(C_t) = C_t^{1-\eta}$ so that $U(C_t) = \frac{1}{1-\eta}u(C_t)$.

The willingness to pay $w^*(\tau)$ is the "fraction of consumption – now and thorough the future – society would sacrifice to ensure that an increase in temperature at a specific horizon H is limited to an amount τ", see p. 292 in P12. Pindyck's paper does not deal with the practically significant problem that $w^*(\tau)$ may not be enough to keep T below τ but assumes that the society is willing to sacrifice up to a fraction $w^*(\tau)$ of consumption to truncate the distribution $f(T)$, so that $T \leq \tau$. More formally, if no action is taken social welfare would be

$$\begin{aligned} W_2 &= \iiint U(\tilde{C}_t)e^{-\delta t} f(x) f_\gamma(y)\, dt\, dx\, dy \\ &= \frac{1}{1-\eta} \iiint u(\tilde{C}_t)e^{-\delta t} f(x) f_\gamma(y)\, dt\, dx\, dy \\ &= \frac{1}{1-\eta} G_\infty, \end{aligned} \tag{3.5}$$

where the tilde emphasizes the random nature of the quantity, $0 \leq t \leq \infty$[4], the uncertain temperature increase x spans the interval $\theta_T \leq x \leq \infty$ and the impact coefficient γ is in $\theta_\gamma \leq \gamma \leq \infty$.

If society sacrifices a fraction $w^*(\tau)$ of consumption, we have two effects in the computation of social welfare: firstly, only the remaining part of consumption, $C'_t = (1-w(\tau))C_t$, is used as an argument of the utility function; and, secondly, integration with respect to the variable x will be bounded to τ and be taken with respect to a truncated and renormalized density $f_\tau(x)$, where $f_\tau(x) = \mathbf{1}_{x\leq\tau} f(x)/F(\tau)$ and the normalizing constant is

$$F(\tau) = \int_{\theta_T}^{\tau} f(x)\, dx.$$

Hence, given the upper threshold τ for temperature increase, social welfare (under sacrifice) is

$$\begin{aligned} W_1(\tau) &= \iiint U(\tilde{C}'_t)e^{-\delta t} f_\tau(x) f_\gamma(y)\, dt\, dx\, dy \\ &= \frac{(1-w(\tau))^{1-\eta}}{1-\eta} \iiint u(\tilde{C}_t)e^{-\delta t} f_\tau(x) f_\gamma(y)\, dt\, dx\, dy \\ &= \frac{(1-w(\tau))^{1-\eta}}{1-\eta} G_\tau, \end{aligned} \tag{3.6}$$

where the integration domains are $0 \leq t \leq \infty$, $\theta_T \leq x \leq \tau$ and $\theta_\gamma \leq \gamma \leq \infty$, respectively, for the variables t, T and γ.

The willingness to pay $w^*(\tau)$ is then the solution of the equation $W_2 = W_1(\tau)$. Using (3.5) and (3.6) WTP can be written as

$$w^*(\tau) = 1 - \left[\frac{G_\infty}{G_\tau}\right]^{\frac{1}{1-\eta}}. \tag{3.7}$$

Observe that ultimately the WTP can be readily computed once the two 3-dimensional integrals G_∞ and G_τ are evaluated. It turns out that these computations are far from

[4] H is the forecasting horizon, but damages are evaluated also beyond that period.

trivial in a variety of parameters' constellations and require considerable care to be performed. Indeed, in Section 5.2 of P12 a simple case is examined in which no uncertainty is assumed on T and γ, which are replaced by a known T_H and by the mean $\overline{\gamma}$ of density f_γ (denoted as g in P12), see Figure 4 in P12. Technically speaking, this makes the previous integrals 1-dimensional and, more importantly, results can be contrasted with more general situations where uncertainty plays a role, spreading the set of feasible outcomes in ways depicted by the estimated densities for T and γ.

Our presentation of the model differs from the one in P12 as we emphasize the fact that relevant quantities are obtained taking 3-dimensional integrals whereas slightly more abstract mean operators are used to describe the very same objects in Pindyck's work. Incidentally, we hope that two equivalent descriptions may benefit different readers or clarify, if needed, both notations and their precise meanings.

Coming to the main concrete claims of P12, we believe it's fair to say that the author interprets his own results as an indication that "moderate abatement policies" should be pursued in the face of the large uncertainty surrounding the amount of future temperature increase and its unknown impact. This broad conclusion is stated in the abstract, in the introduction and elsewhere in the paper. In the concluding remarks, the argument takes an analogous flavour: asking "whether a stringent [abatement] policy is needed now", Pindyck says results "are consistent with beginning slowly". A similar lesson, we believe, can be drawn from the simplified example contained in Section 5.2 of P12: if the temperature increase is *known* to be $T_H = 6°\text{C}$ in $H = 100$ years under business as usual and *known* economic impact, then the willingness to pay to have no warming, $w^*(0)$, is *still only* 2.2% (italics are ours).

These considerations spurred us to assess the robustness of the model, using more recent data (Section 3.4) or changing part of the model specification the model (Section 3.5), in order to check how much can be retained of the gist of the original paper in different setups. However, in Section 3.3, we begin with a verification of most of the results contained in P12.

3.3 Verification

3.3.1 `R`

All the computations of this paper are obtained using the `R` platform, R Core Team (2015). R is a free software environment for statistical computing and graphics. Among the alternatives, we chose R since it is free, it is easy scriptable and, among the many existing packages, there is one (`cubature`, Johnson and Narasimhan (2013)) specifically developed to evaluate multiple-dimension integrals.[5]

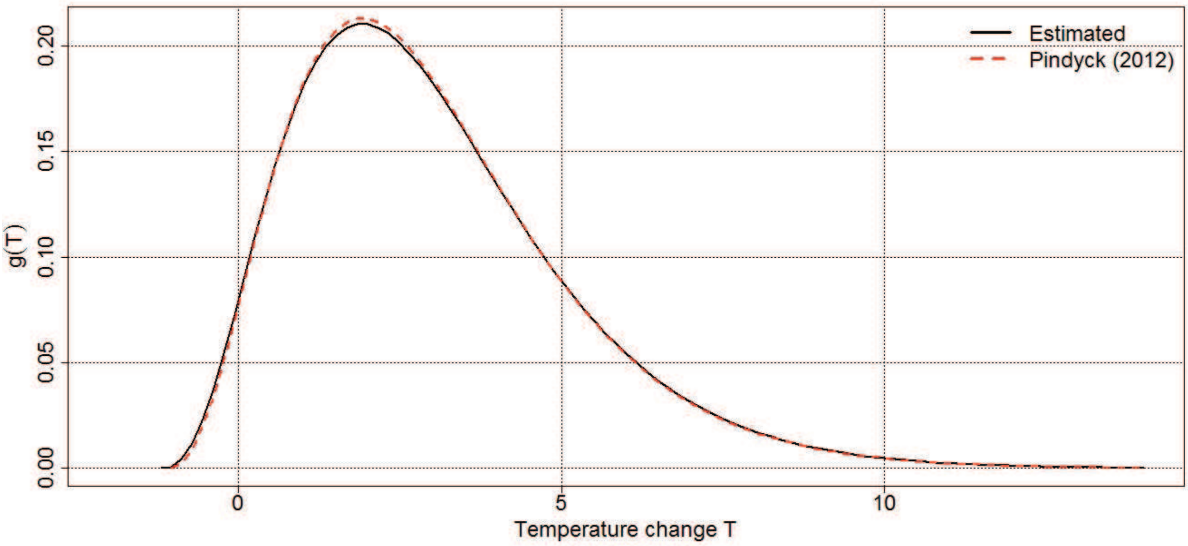
3.3.2 Estimation of the displaced gamma density

P12 assumes a displaced gamma density for both T_H and γ. Almost all the integrals for the calculation of the WTP involve these two random variables. Therefore the results of the paper heavily depend on this preliminary estimation. Like in P12, we fit the

[5] The code is available upon request.

parameters of a displaced gamma density for the random variable T_H with $\mathbb{E}(T_H) = 3°\text{C}$, $\mathbb{P}(T_H \leq 7°C) = 5\%$ and $\mathbb{P}(T_H \leq 10°C) = 1\%$. We obtain $r_T = 3.9, \lambda_T = 0.92$ and $\theta_T = -1.22$ that can be compared with the values reported in P12: $3.8, 0.92$ and -1.13, respectively. Figure 3.1 displays the two distributions which, despite the slightly different value of the parameters, appear to be almost indistinguishable. We decide to number our figures the same way they were numbered in P12: hence, to ease the comparisons for the readers, Figure X in this paper always corresponds to Figure X in P12 (of course, this may also be a bit perplexing as, say, there is no Figure 2 in this work and we jump from Figure 1 to 3). Along the same lines, we will retain the original numbering found in P12 in Sections 3.4 and 3.5, adding a literal suffix to the proper numeral.

Figure 3.1: Distribution of temperature change T_H.

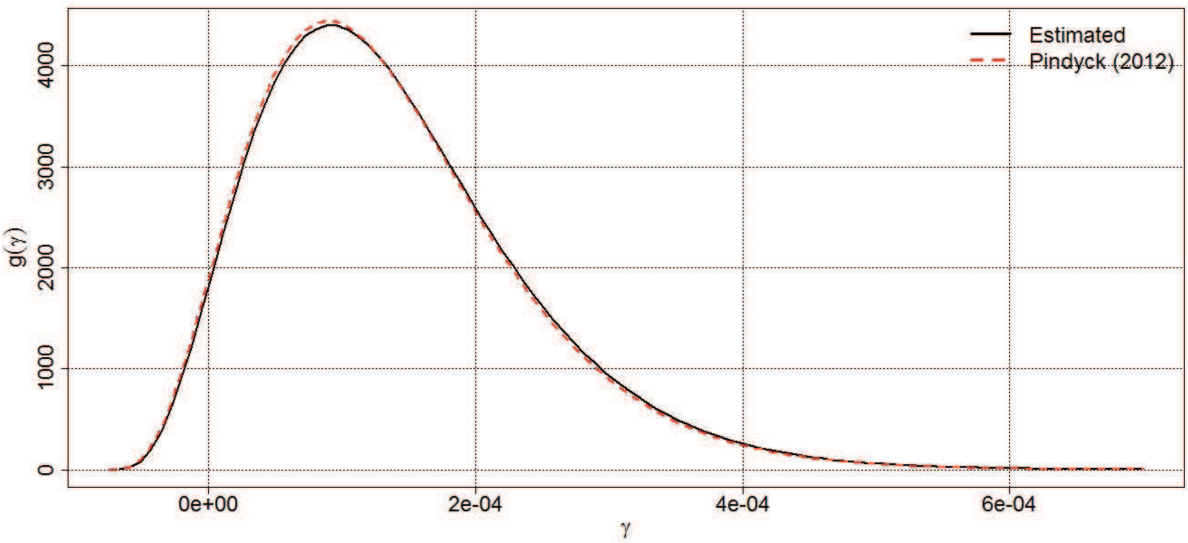


The distribution of the damage coefficient γ is calibrated in order to fit a displaced gamma density such that $\mathbb{E}(\gamma) = 0.0001363$, $\mathbb{P}(\gamma \leq 0.0000450) = 0.17$ and $\mathbb{P}(\gamma \leq 0.0002295) = 0.83.$[6].

We obtain $r_\gamma = 4.43, \lambda_\gamma = 20939$ and $\theta_\gamma = -7.28 \cdot 10^{-5}$. Again, our estimates are quite close to the values $r_\gamma = 4.50, \lambda_\gamma = 21431$ and $\theta_\gamma = -7.46 \cdot 10^{-5}$ reported in P12, and the plots corresponding to the two densities are basically undistinguishable (see Figure 3.3). The numerical approximation of the parameters for γ has been less trivial than it was needed for T_H, due to the different order of magnitudes of the three parameters; for details we refer the reader to Section 3.6.

[6]The procedure used to calculate these moments is not explicitly stated in P12, but can be inferred from a footnote of P09. These moments of the γ distribution are implied by the corresponding moments of β. IPCC (2007a) reports that, for a $4°C$ warming level, the expected production loss in levels $Loss_{T_H}$ is 3%, with a 66% confidence interval being 1-5%. We can obtain the values of β that are coherent with these projections using the equation $\exp(-\beta T_H^2) = 1 - Loss_{T_H}$. These estimates imply that $\bar{\beta} = 0.00190$, $\beta_{0.17} = 0.000628$ and $\beta_{0.83} = 0.00321$. The moments for γ are then obtained from (3.4).

Figure 3.3: Distribution of loss function parameter γ.



3.3.3 Estimation of Willingness to Pay

The computation of several WTPs with different values taken by key parameters is clearly one of the most important features of P12. The WTP is graphically displayed in Figures 4-7 as well as tabulated in Tables 1 and 2. In what follows, we reproduce the original Figures 4, 5 and 6 and recompute Table 1.

First, some benchmark WTPs are computed in a setup with no uncertainty on future temperature increase and economic impact, and Figure 3.4 depicts the WTP $w^*(0)$ to keep warming at zero as a function of a known T_H, assuming a fixed value for $\gamma = \bar{\gamma} = \mathbb{E}(\gamma) = 0.0001363$, and showing three possible scenarios for the growth rate $g_0 = 0.015$, 0.02 or 0.025 (the remaining parameters are the index of relative risk aversion $\eta = 2$ and discount rate $\delta = 0$). This is, therefore, a scenario with no uncertainty where the integrals needed to evaluate the WTP are 1-dimensional. It is, of course, formally impossible to test whether two figures are the same, but an eyeball test of the twin figures in P12 and in this paper (and lots of zooming!) show that they are essentially displaying the very same quantities. Perhaps more concretely, to exemplify the meaning of Figure 3.4 it is reported in P12 that when $T = 6$ and $g_0 = 0.02$ then $w^*(0)$ is about 0.022 or 2.2%. For comparison, our own computations produce 0.02156.

Second, WTPs are displayed in Figure 3.5, allowing for uncertainty. In the picture, the functions $w^*(\tau)$ are depicted in four scenarios combining different risk aversion η and baseline growth rate g_0. In this setup, full 3-dimensional integrals are involved and care is needed to set apparently irrelevant (technical) parameters, as detailed in Section 3.6. It is again hard to discern any differences in the two versions of Figure 3.5 of this paper and of Pindyck's one.

Third, we focus on Figure 3.6 where the dependence of $w^*(3)$, namely the WTP to limit the increase in temperature to 3°C, is plotted as a function of risk aversion η (under two different discount rates δ). This picture is interesting as it turns out that its replication is difficult, in particular, if η approaches 1 or when $\eta = 4$. In the first case, we have

Figure 3.4: $w^*(0)$, known temperature change T_H, $\eta = 2$, $g_0 = 0.015$, 0.020, 0.025, and $\delta = 0$.

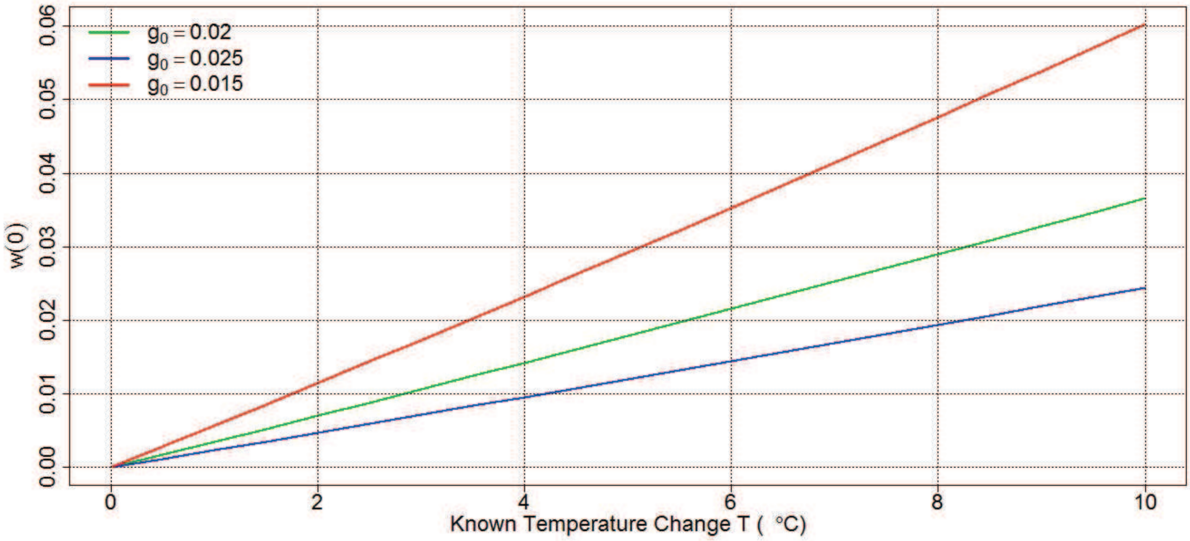
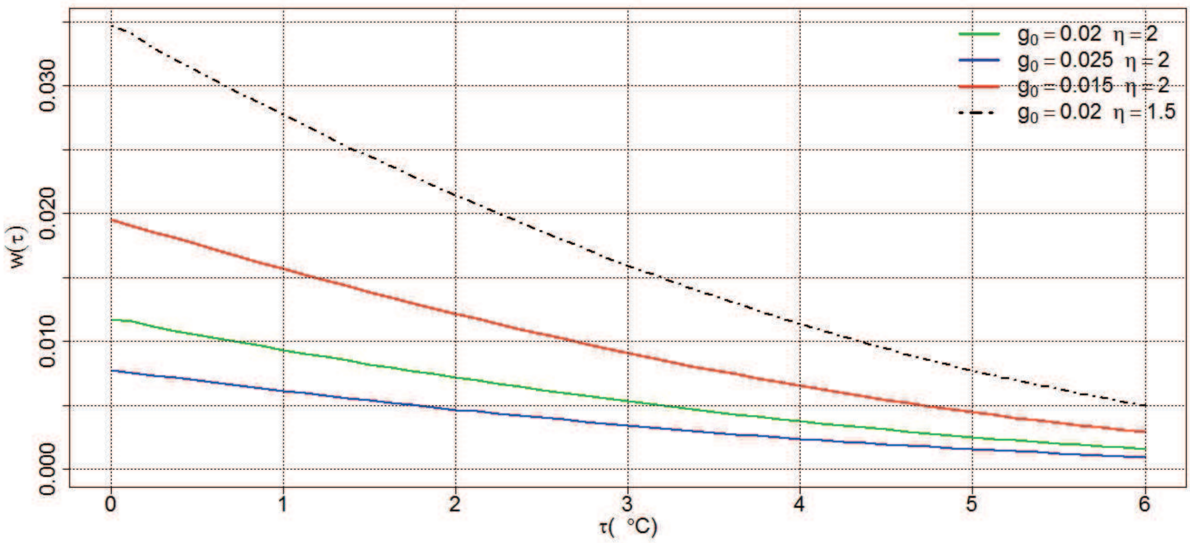
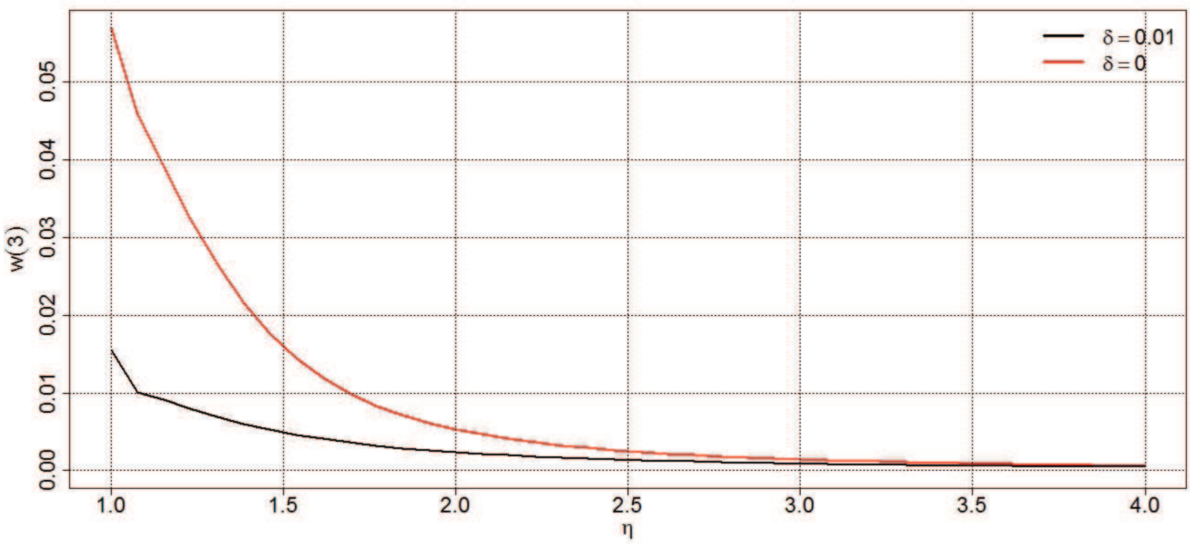


Figure 3.5: $w^*(\tau)$, T_H and γ uncertain, $\eta = 1.5, 2$, $g_0 = 0.015$, 0.020, 0.025, and $\delta = 0$.



an evident singularity in the definition of $U(\cdot)$ and can resort to the fact that, in this situation, the utility function is – up to constant – a logarithmic function.[7] It is less clear why high values of η prove to be relatively ill-posed for the integration routine `cubature`. While additional details are deferred to Section 3.6, we observe that our Figure 3.6 is extremely similar to the one found in P12.

Figure 3.6: $w^*(3)$ versus η. $g_0 = 0.020$ and $\delta = 0, 0.01$



We finally move to Table 1, which lists 19 pairs of WTPs and allows for a more rigorous comparison of numeric figures obtained tilting the reference values of some parameters. In particular, the two WTPs $w^*(0)$ and $w^*(3)$ are tabulated and, unless otherwise indicated, $\delta = 0$, $\eta = 2$, $g_0 = 0.02$, $\mathbb{E}(T) = 3°C$, $\mathbb{E}(\gamma) = 0.0001363$ and social utility is computed on a span of time of 5 centuries ($t_{max} = 500$).

Generally speaking, our estimates of the WTP (in the second and fourth columns) match very well the ones in P12 (placed side by side in the third and fifth columns). For instance, in the first row relative to the baseline case the values of $w^*(0)$ and $w^*(3)$ differ by about $5 \cdot 10^{-4}$. In many cases, we have similar gaps that are insignificant from the practical economic point of view but give the flavour of the "numeric noise" that affect (accurate) estimates obtained by different authors, with distinct software and code. This (small) noise can be attributed to slightly different computational methods being used in different packages, or to dissimilar settings of an abundance of default parameters that are used in standard routines for numerical computations. To give an example: there may be different defaults for stopping criteria; or finer/coarser grids are used when the user is not providing optional specifications.

However, some noteworthy discrepancies can be spotted in rows 8, 16, 18 and 19. Observing preliminarily that two such cases are related to the position $\eta = 4$, in row 8 the WTPs computed in P12 setting $g_0 = 0.01$ are 30 or 60-fold larger than ours. The previous row contains the same WTPs when the growth rate is 0.02 and, in agreement with intuition, halving the growth rate of the economy inflates the WTP to reduce the

[7] A detailed calculation of the willingness-to-pay with log utility is presented in Appendix 3.A.

Table 3.1: WTPs with alternative parameter values.

	Cases	$w^*(0)$	P12 $w^*(0)$	$w^*(3)$	P12 $w^*(3)$
1	Base case	0.0118	0.0113	0.0053	0.0059
2	$t_{max} = 300$	0.0112	0.0110	0.0050	0.0056
3	$t_{max} = 1000$	0.0118	0.0113	0.0053	0.0059
4	$g_0 = 0.010$	0.0369	0.0372	0.0179	0.0190
5	$g_0 = 0.005$	0.0761	0.0775	0.0384	0.0407
6	$g_0 = 0$	0.1432	0.1463	0.0750	0.0791
7	$\eta = 4$	0.0015	0.0014	0.0007	0.0008
8	$\eta = 4,\ g_0 = 0.010$	0.0060	0.1844	0.0029	0.1820
9	$\varepsilon(T_H) = 5°C$	0.0187	0.0189	0.0103	0.0105
10	$\varepsilon(T_H) = 5°C,\ g_0 = 0.010$	0.0596	0.0599	0.0350	0.0350
11	$\varepsilon(T_H) = 5°C,\ g_0 = 0.005$	0.1223	0.1232	0.0746	0.0749
12	$\varepsilon(\gamma) = 0.0002726$	0.0240	0.0243	0.0112	0.0116
13	$\varepsilon(\gamma) = 0.0002726,\ g_0 = 0.015$	0.0402	0.0401	0.0194	0.0198
14	$\varepsilon(T_H) = 5°C,\ \varepsilon(\gamma) = 0.0002726$	0.0384	0.0373	0.0218	0.0211
15	$g_0 = 0,\ \delta = 0.01$	0.0369	0.0372	0.0179	0.0190
16	$g_0 = 0,\ \delta = 0.02$	0.0118	0.0074	0.0053	0.0039
17	$g_0 = 0.005,\ \delta = 0.01$	0.0196	0.0195	0.0091	0.0098
18	$\eta = 4,\ g_0 = 0.005,\ \delta = 0.01$	0.0089	0.0315	0.0045	0.0178
19	$\varepsilon(T_H) = 5°C,\ g_0 = 0.010,\ \delta = 0.01$	0.0187	0.0599	0.0103	0.0350

Notes: unless otherwise indicated $\delta = 0$, $\eta = 2$, $g_0 = 0.020$, $\varepsilon(T_H) = 3°C$, $\varepsilon(\gamma) = 0.0001363$, $t_{max} = 500$ years.

expected wounds inflicted by climate change to a frail economic growth. While, say, according to our computations, $w^*(0)$ moves from 0.0015 in row 7 to 0.0060 in row 8, a four-fold increase, in P12 we have a spectacular jump from 0.0014 to 0.1844. The same occurrence is visible for $w^*(3)$.

In row 18, again with $\eta = 4$, our $w^*(0)$ and $w^*(3)$ are quite smaller than the WTPs in P12 (differences exceed 2 and 1 percentage point, respectively).

Finally, the last row of Table 3.1 portrays a large difference in both WTPs. We feel that, nonetheless, there may be a simple material typing error in P12 as the entries in Pindyck's Table are *exactly* the same as in row 10, whereas we expected the same figure of row 9. This is due to the fact that, as it is possible to notice by looking at (8) of P12,[8] with $\eta = 2$ an increase in δ compensates for a decrease in g_0 of the same absolute value, implying that a scenario with $\varepsilon(T_H) = 5°C$ and $g_0 = 0.02$ is actually the same as a scenario with $\varepsilon(T_H) = 5°C$ and $g_0 = 0.01$ and $\delta = 0.01$. For the same reason, the entries in rows 1 and 16 in Table 3.1 should be the same but this does not happen in P12.

All in all, with some exceptions possibly related to low growth rates and extreme values for η, our estimates are often close to the ones obtained by Pindyck (some reasons for what is happening in cases 8 and 18 are discussed in Section 3.6).

3.4 Extension

This section provides an extension of the original paper, where we change the data used to estimate the density for the temperature increase, without altering the theoretical structure of the model. The fact that we change the data source makes clear that the results shown here cannot be expected to resemble the ones in P12, but should be used to appraise how the original results are affected by the availability of new data.

The Fifth *IPCC* Assessment Report released in 2014 (IPCC, 2014) contains new data that can be used to estimate fresh densities for the uncertain quantities used in P12. In particular, IPCC (2014) describes four GHG (GreenHouse Gases) possible scenarios, called Representative Concentration Pathways (RCP), which are named after a possible range of radiative forcing values in the year 2100 relative to pre-industrial values. The one without any GHG emissions[9] mitigation effort beyond current legislation, which can be considered as the baseline path for the present analysis, is RCP8.5[10]: the alternative

[8] Equations (7), (8), (9) of P12 report that in the simple case where T_H and γ are known,

$$W = \frac{1}{1-\eta} \int_0^{+\infty} e^{\omega - \rho t - \omega (1/2)^{t/H}} \, dt$$

where:

$$\rho = (\eta - 1)(g_0 - 2\gamma T_H) + \delta,$$

$$\omega = 2(\eta - 1)\gamma H T_H / \ln(1/2).$$

When $\eta = 2$, the value of ρ depends on the sum $g_0 + \delta$. Hence, decreases in the growth rate can be offset by equal increases in the discount rate leaving ρ unaffected.

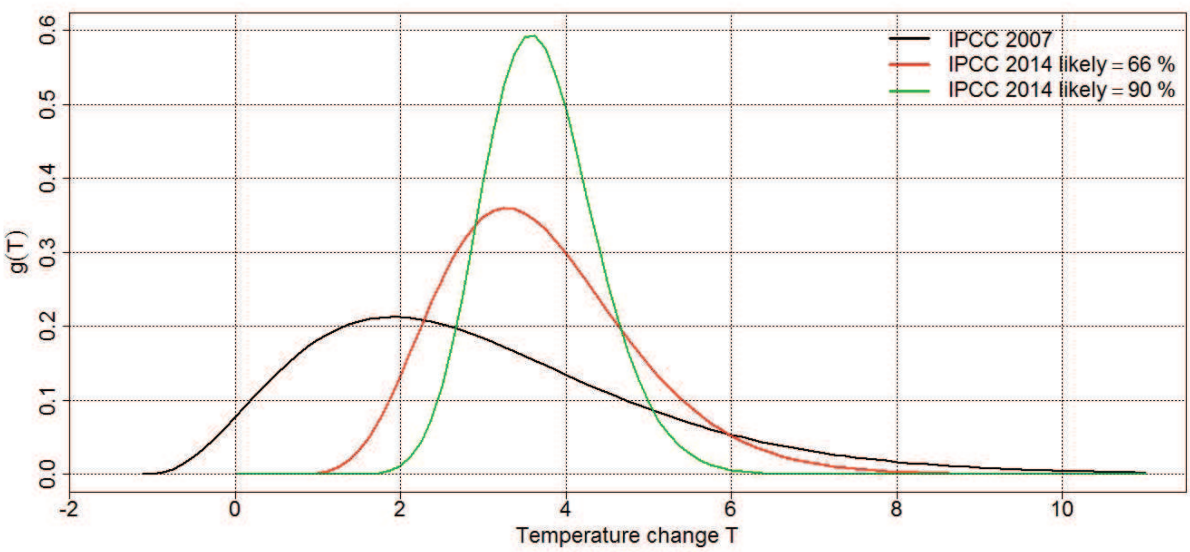
[9]Please note that an high GHG emission scenario does not necessarily imply an high "pollution" scenario: pollution depends on the emissions of other substances, like for example SO_2.

[10]Representative Concentration Pathways 8.5, therefore, assumes an increase of 8.5 W/m^2 radiative forcings with respect to preindustrial levels.

scenarios, with increasing level of mitigation and, therefore, decreasing level of emissions, are RCP6.0, RCP4.5 and RCP2.6.

Under scenario RCP8.5 the forecast is of a $3.7°C$ temperature increase from 1986-2005 to 2081-2100, with a "*likely*" range of $2.6°C$ to $4.8°C$. In Figure 3.1a, we plot the "old" density for T_H and the "new" density interpreting the term "likely" as a 66% or a 90% confidence interval (in P12 the term "likely" is associated to 66%). Using for the estimation the information $\mathbb{E}(T_H) = 3.7°C$, $\mathbb{P}(T_H \leq 2.6°C) = 17\%$ and $\mathbb{P}(T_H \leq 4.8°C) = 83\%$, the results for the parameters of the gamma displaced density $f_{2014}(x)$ are: $r_{2014} = 7.82$, $\lambda_{2014} = 2.38$ and $\theta_{2014} = 0.42$. Figure 3.1a shows that the distributions computed with more recent data shift to the right and are more concentrated around the mean of 3.7°C (and, in particular, the right tail is clearly much thinner than in P12 in either of the two versions obtained from 2014 data).

Figure 3.1a: Distribution of temperature change T_H, 2014 data.



Observe that with the new parameters, being θ_{2014} greater than zero, it is impossible to keep the temperature increment at 0 and, hence, $w^*(0)$ cannot be estimated. In Table 3.1a we report the WTPs for the same cases listed in Table 3.1, replacing $w^*(0)$ with $w^*(1)$, under the two interpretations of "likely". In the third (sixth) column, we display $w^*(1)$ ($w^*(3)$) for the various cases based on IPCC07 and in the fourth and fifth (seventh and eighth) columns the values obtained from 2014 data.

Looking at $w^*(1)$, a scenario related to a stricter abatement policy, WTPs most often grow moving from 2007 to 2014 assessments, meaning that the newer *IPCC* report implies an upward revision of the WTP to curb warming to 1° C. The differences in the WTPs are anyway modest and generally are around 0.2-0.3% or smaller. Scenarios which assume $\varepsilon(T_H) = 5°C$ imply a lower WTP under 2014 assessment. This is related to the lower standard deviation underlying the 2014 projections: keeping the same expected value, a lower standard deviation implies a lower WTPs, as explained in P12. In a few other instances, such as the ones where $\eta = 4$ (cases 7, 8, 19), the new values are equal or smaller than the ones in P12. The inspection of the columns 4 and 5 also shows that the WTPs are virtually the same regardless of the chosen interpretation of "likely".

Table 3.1a: WTPs with alternative parameter values, IPCC (2014) data

	Cases	$w^*(1)$ 07	$w^*(1)$ 14 66%	$w^*(1)$ 14 90%	$w^*(3)$ 07	$w^*(3)$ 14 66%	$w^*(3)$ 14 90%
1	Base case	0.0094	0.0102	0.0101	0.0053	0.0048	0.0037
2	$t_{max} = 300$	0.0089	0.0097	0.0096	0.0050	0.0045	0.0035
3	$t_{max} = 1000$	0.0094	0.0103	0.0101	0.0053	0.0048	0.0037
4	$g_0 = 0.010$	0.0300	0.0320	0.0312	0.0179	0.0154	0.0119
5	$g_0 = 0.005$	0.0625	0.0656	0.0639	0.0384	0.0322	0.0250
6	$g_0 = 0$	0.1191	0.1227	0.1190	0.0750	0.0616	0.0476
7	$\eta = 4$	0.0012	0.0013	0.0013	0.0007	0.0006	0.0005
8	$\eta = 4$, $g_0 = 0.010$	0.0048	0.0052	0.0051	0.0029	0.0025	0.0019
9	$\varepsilon(T_H) = 5°C$	0.0158	0.0150	0.0149	0.0103	0.0087	0.0081
10	$\varepsilon(T_H) = 5°C$, $g_0 = 0.010$	0.0512	0.0475	0.0469	0.0350	0.0284	0.0262
11	$\varepsilon(T_H) = 5°C$, $g_0 = 0.005$	0.1064	0.0974	0.0959	0.0746	0.0598	0.0550
12	$\varepsilon(\gamma) = 0.0002726$	0.0193	0.0208	0.0204	0.0112	0.0097	0.0076
13	$\varepsilon(\gamma) = 0.0002726$, $g_0 = 0.015$	0.0326	0.0347	0.0340	0.0194	0.0166	0.0129
14	$\varepsilon(T_H) = 5°C$, $\varepsilon(\gamma) = 0.0002726$	0.0327	0.0307	0.0304	0.0218	0.0180	0.0166
15	$g_0 = 0$, $\delta = 0.01$	0.0300	0.0320	0.0312	0.0179	0.0154	0.0119
16	$g_0 = 0$, $\delta = 0.02$	0.0094	0.0102	0.0101	0.0053	0.0048	0.0037
17	$g_0 = 0.005$, $\delta = 0.01$	0.0157	0.0169	0.0166	0.0091	0.0080	0.0062
18	$\eta = 4$, $g_0 = 0.005$, $\delta = 0.01$	0.0073	0.0075	0.0073	0.0045	0.0036	0.0028
19	$\varepsilon(T_H) = 5°C$, $g_0 = 0.010$, $\delta = 0.01$	0.0158	0.0150	0.0149	0.0103	0.0087	0.0081

Notes: unless otherwise indicated, for 2014 estimates $\delta = 0$, $\eta = 2$, $g_0 = 0.020$, $\varepsilon(T_H) = 3.7°C$, $\varepsilon(\gamma) = 0.0001363$, $t_{max} = 500$ years. For 2007 estimates $\varepsilon(T_H) = 3°C$ instead of $\varepsilon(T_H) = 3.7°C$. "07" ["14"] represents estimates based on the *IPCC* 2007 [2014].

The examination of the columns relative to $w^*(3)$, instead, reveals that more recent data "suggest" a lower WTP for a moderate abatement policy (namely, limiting the temperature increment to $3°C$), the only exception being case 16 with an high discount rate ($\delta = 0.02$). With respect to the WTP estimated according to 2007 data, differences range from 0.1 to 1.8%. If "likely" is associated to 90% confidence intervals then, typically, WTPs are further reduced by an amount ranging between 0.1 and 0.7%.

Figures 3.5a and 3.6a are the counterparts (with more recent data) of Figures 5 and 6 in P12. A careful inspection confirms the previous findings and comments but, perhaps more importantly, may suggest that the inclusion of fresh 2014 data appear to have not changed the gist of the conclusions and lessons in P12. It is true that temperature cannot be kept at the present level, and that strict (moderate) abatement policies are slightly more (less) worthwhile, but changes are perhaps minor in size in many circumstances of practical importance. This was somehow to be expected after the marginal analysis in P12 already pointed out that a hike in the mean temperature would have been offset by a reduction in the standard deviation.

3.5 Reanalysis

This section provides a reanalysis of the original paper, where we change part of the structure of the model. In particular, in Section 3.5.1 we modify the functional form of (3.2) defining how much a temperature increase would affect the growth rate g_t, while in Section 3.5.2 we modify the functional form of (3.1) specifying how the temperature

Figure 3.5a: $w^*(\tau)$, T_H and γ uncertain, $\eta = 1.5, 2$, $g_0 = 0.015, 0.020, 0.025$, and $\delta = 0$, 2014 data. Observe that WTP cannot be plotted when $\tau = 0$ as $\theta_{2014} = 0.42 > 0$.

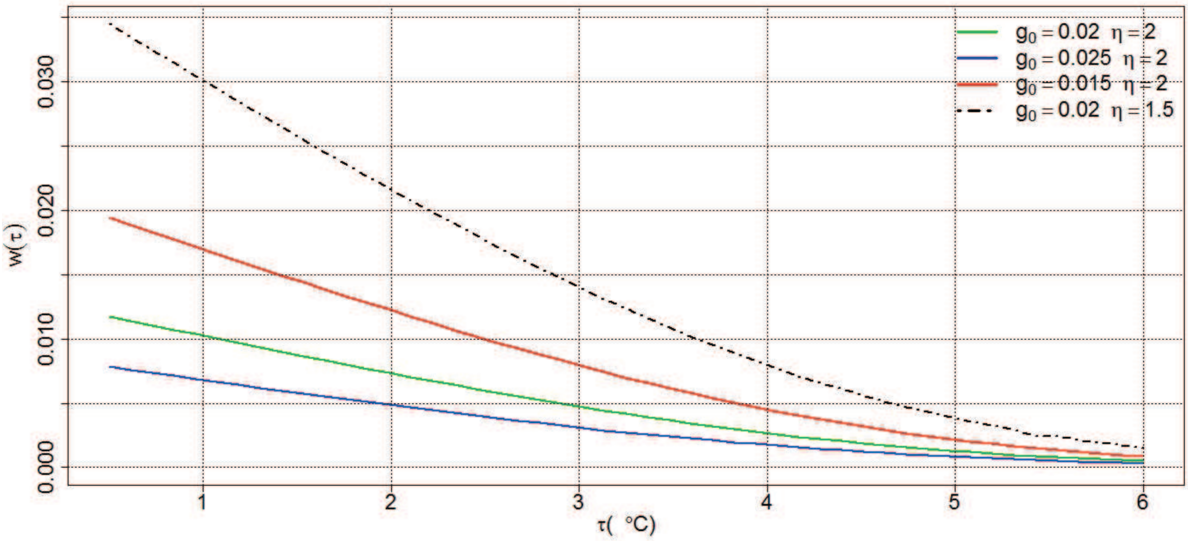
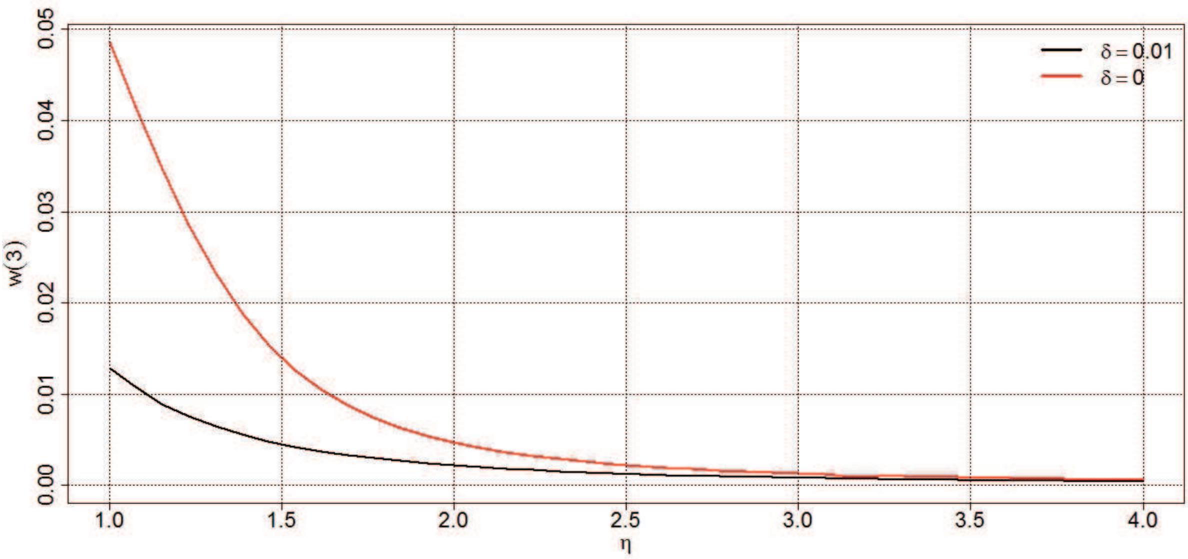


Figure 3.6a: $w^*(3)$ versus η. $g_0 = 0.020$ and $\delta = 0, 0.01$, 2014 data



increase evolves. As in every reanalysis, the adoption of a different theoretical model makes clear that the results shown here are not expected to coincide with the ones in P12. Conversely, it is possible to appreciate if and how much the estimated willingness to pay changes after modification of key parts of the assumptions.

3.5.1 Convex damage function

"When assessing climate sensitivity, we at least have scientific results to rely on, and can argue coherently about the probability distribution that is most consistent with those results. When it comes to the damage function, however, we know almost nothing".

Pindyck (2013)

The linear way the increment in temperature affects the economic growth rate is the most speculative part of the analysis, since there is not enough empirical or theoretical support for any specific damage function, and arbitrary choices of the damage function (Weitzman, 2010) or, more generally, on the structure of the model (Pindyck, 2013, 2017) may have non-trivial effects on related policy considerations. Indeed, as far as we would like to understand the consequences of unprecedented warming, we may be tempted to assume (say, for precautionary reasons) super-linear, i.e. convex, damages. Hence, we reanalyse the willingness to pay assuming a convex relationship, in place of the linear specification in (3.2), between the growth rate g_t of GDP and the level of warming. If we suppose that

$$g_t = g_0 - \gamma' T_t^{\alpha}, \tag{3.8}$$

where the parameter $\alpha \geq 1$ can shape different degrees of convex impact and γ' is a constant coefficient. As $T_t = 2T_H[1 - (1/2)^{t/H}]$, we obtain:

$$g_t = g_0 - \gamma' \Big\{ 2T_H \Big[1 - \Big(\frac{1}{2}\Big)^{\frac{t}{H}} \Big] \Big\}^{\alpha}$$

As for the linear case, the value of γ' is obtained equating the consumption at horizon H along the path of the growth rate determined by (3.8) with what will be obtained using a loss function on levels. Given that the path of the growth rate is different with respect to the baseline case exposed in P12, γ' has to be estimated again. We have:

$$\begin{aligned} C_H &= C_0 \exp\Big(\int_0^H g_0 - \gamma' (2T_H)^{\alpha} \Big[1 - \Big(\frac{1}{2}\Big)^{\frac{t}{H}} \Big]^{\alpha} dt \Big) && (3.9) \\ &= 1 \cdot \exp\Big(g_0 H - \gamma' (2T_H)^{\alpha} \int_0^H \Big[1 - \Big(\frac{1}{2}\Big)^{\frac{t}{H}} \Big]^{\alpha} dt \Big) && (3.10) \end{aligned}$$

Therefore, equating the result obtained through the effect on the growth rate to the one on the level, we get:

$$\exp\Big(g_0 H - \gamma' (2T_H)^{\alpha} \int_0^H \Big[1 - \Big(\frac{1}{2}\Big)^{\frac{t}{H}} \Big]^{\alpha} dt \Big) = \exp(g_0 H - \beta T_H^2)$$

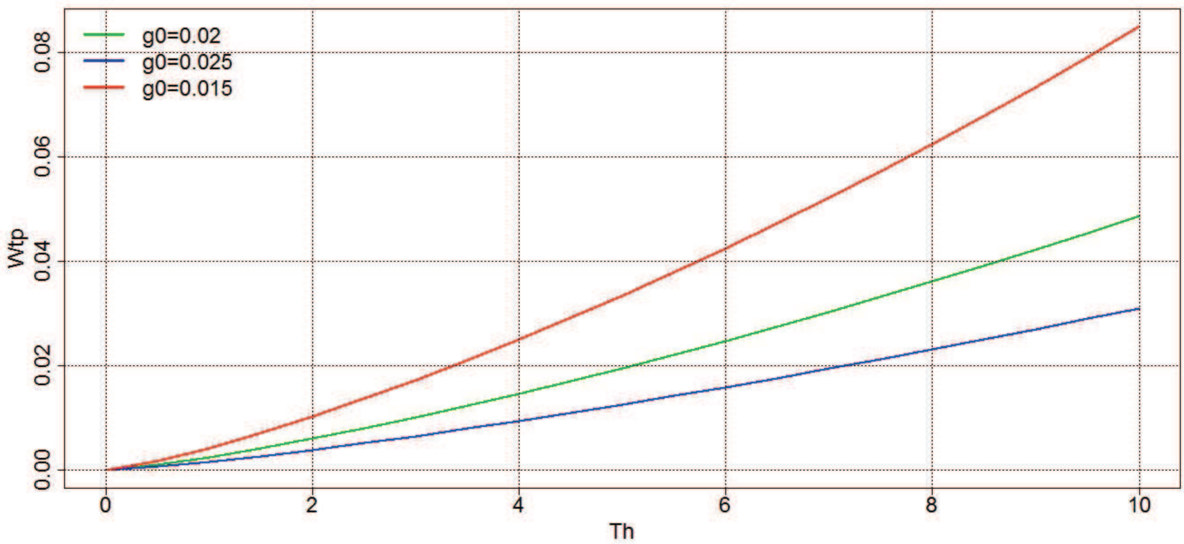
And, finally:

$$\gamma' = \frac{\beta T_H^{2-\alpha}}{2^\alpha \int_0^H \left[1 - \left(\frac{1}{2}\right)^{\frac{t}{H}}\right]^\alpha dt} \tag{3.11}$$

Note from (3.11) that γ' is a decreasing function of α. The lower value of γ' under convex damages, with respect to the γ estimated for linear damages, is necessary to have, after 100 periods, the same consumption loss implied by the loss function in levels $\exp(-\beta T_H^2)$. This, in turn, implies that when $\alpha > 1$ we would initially have smaller losses for low increases of T_H to move to greater losses for large increments of the temperature.

To provide some insight, we reanalyse the WTP assuming $\alpha = 1.25$, which implies a modest increase in convexity with respect to the baseline linear case. We obtain $\mathbb{E}(\gamma') = 0.0001068$ which, as just said, is lower than the standard case $\mathbb{E}(\gamma) = 0.0001363$. Figure 3.4b depicts the situation in which there is no uncertainty on T_H or γ' (and, hence, it should be compared with Figures 4 in this paper or in P12). As expected, the lower γ' produces lower WTPs for low or moderate levels of warming (below 3-4°C), but eventually convex damage takes place for higher warming levels (when T_H is about 8-10°C) inflating the WTP.

Figure 3.4b: $w^*(0)$, known temperature change T_H, $\eta = 2$, $g_0 = 0.015, 0.020, 0.025$, and $\delta = 0$, convex damage function $g_t = g_0 - \gamma' T_t^{1.25}$

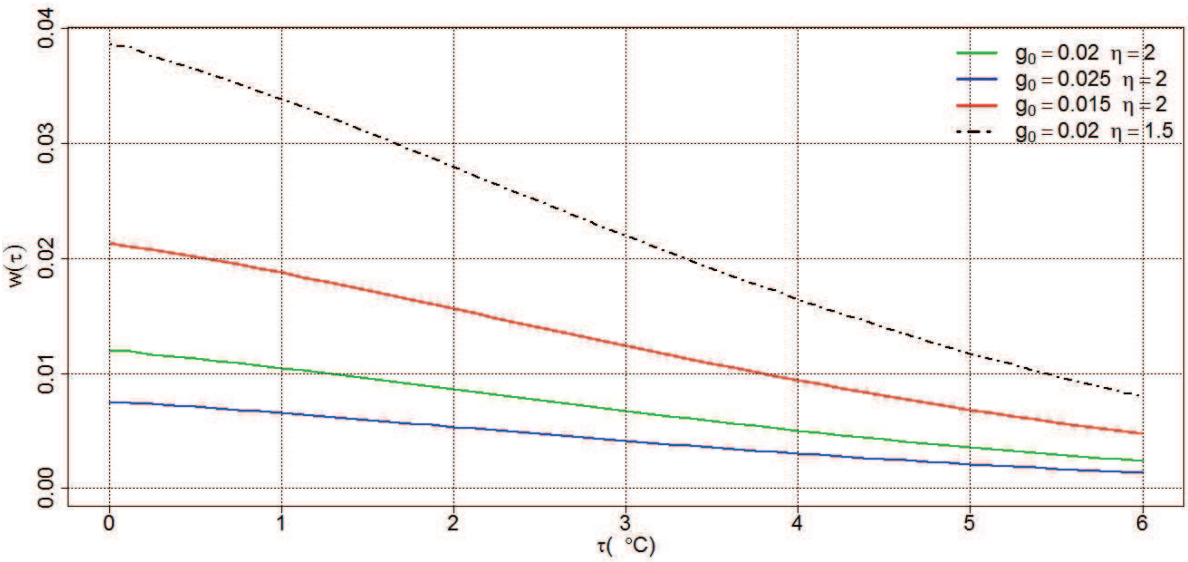


Once uncertainty is introduced again, WTPs are depicted in Figures 3.5b, 3.6b and displayed in Table 3.1b. While in Figure 3.5b, which shows WTPs as a function of τ, the modifications due to $\alpha = 1.25$ appear to be minor, Figure 3.6b draws the attention of the significant effect at least for low values of the parameter η: when $\eta = 1$, the WTP are very close to 2 and 8% when the discount rate δ is 0 or 0.01, respectively . The same numbers in Figure 3.6 are about 1.5 and 5.5%. Due to the decreasing trend of the WTP as a function of η, differences fade for medium to large values of η.

The first three rows of Table 3.1b show that little changes, if any, are observed in the baseline case or varying t_{max} while keeping fixed the values of the other parameters.

More interestingly, it also discloses that when $\alpha = 1.25$ large effects are caused by the reduction of the growth rate g_0. Just to provide an example, in row 6 WTPs for $w^*(0)$ and $w^*(3)$ jump to 0.1935 and 0.1316, with increments about 5 percentage points with respect to the standard case where $\alpha = 1$. Even more spectacular hikes are visible in rows 8 and 18, which feature a combination of low g_0 and high η. The explosion of some WTPs seen in the third and fourth columns of the table can be related to the peculiar effects on the utility generated by low growth rates, $\alpha > 1$ and high values for η. Indeed, as $g_t = g_0 - \gamma' T_t^{\alpha}$, consumption can decrease to infinitesimal levels for combinations of parameters that make the growth rate negative. Consequently, the utility of nearly null consumption can attain very low and negative values, effectively approaching $-\infty$ at fast speed for large values of η. The examination of the fifth and sixth columns shows that setting $\alpha = 1.25$ generally produces relatively small effect on $w^*(3)$ which, we recall, may corresponds to a situation in which moderate abatement is sought for.

Figure 3.5b: $w^*(\tau)$, T_H and γ uncertain, $\eta = 1.5, 2$, $g_0 = 0.015, 0.020, 0.025$, and $\delta = 0$, damage function $g_t = g_0 - \gamma' T_t^{1.25}$



In order to give the reader the flavour of what would be the scenario assuming an even more convex damage function, say $\alpha = 1.5$, we plot the equivalent of Figure 4 and 5 of P12 (Fig. 3.4c and Fig. 3.5c). Comparing Figures 3.4, 3.4b and 3.4c, it appears that the more convex the damage function, the more convex the relationship between expected warming and the WTP to keep the warming at zero. Moreover, from a visual inspection of Figures 3.5, 3.5b and 3.5c, $w^*(\tau)$ becomes flatter for increasing values of α, since high-warming events are potentially catastrophic and the willingness-to-pay to avoid them is substantial. If $\alpha = 1.5$, there is also a curiouos difference in the pattern of $w^*(\tau)$ if we assume an higher risk aversion $\eta = 1.5$: there is an higher $w^*(\tau = 0)$ but the willingness to pay, moving towards moderate and light abatement policies, decreases more steeply than for less risk-averse individuals.

This analysis, further, suggests that the use of large α (say, a quadratic damage function would be obtained when $\alpha = 2$) is likely to results in problematic estimates of WTPs close to 1 (i.e., 100%) for some choice of other parameters of the model. Some

Figure 3.6b: $w^*(3)$ versus η. $g_0 = 0.020$ and $\delta = 0, 0.01$, damage function $g_t = g_0 - \gamma' T_t^{1.25}$

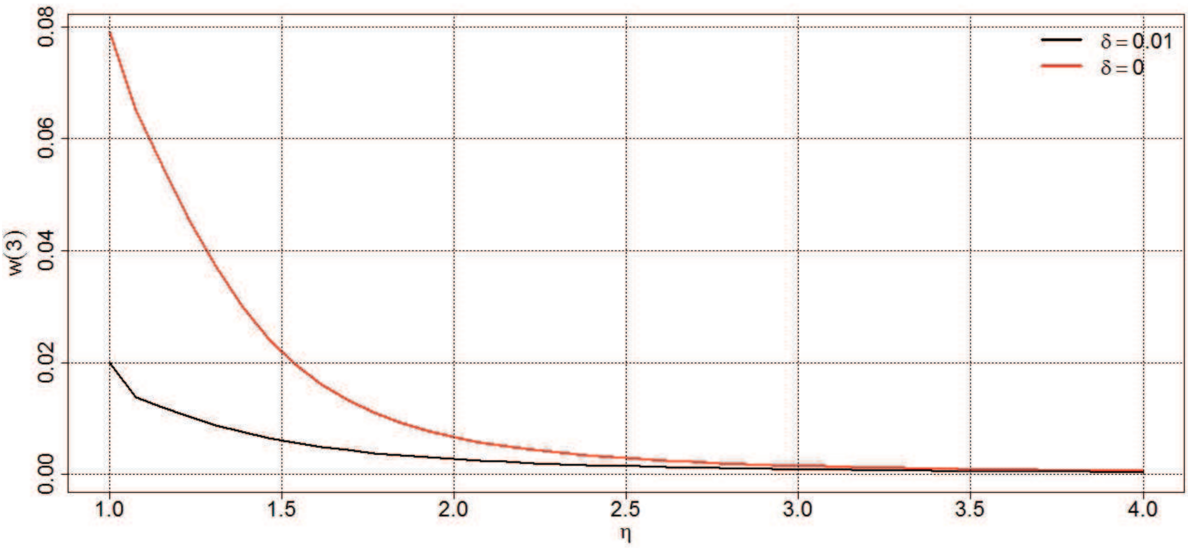


Table 3.1b: WTPs with alternative parameter values, damage function $g_t = g_0 - \gamma' T_t^{1.25}$

	Cases	$w^*(0)$		$w^*(3)$	
		$\alpha = 1.25$	$\alpha = 1$	$\alpha = 1.25$	$\alpha = 1$
1	Base case	0.0120	0.0118	0.0067	0.0053
2	$t_{max} = 300$	0.0112	0.0112	0.0062	0.0050
3	$t_{max} = 1000$	0.0135	0.0118	0.0083	0.0053
4	$g_0 = 0.010$	0.0436	0.0369	0.0267	0.0179
5	$g_0 = 0.005$	0.0977	0.0761	0.0633	0.0384
6	$g_0 = 0$	0.1935	0.1432	0.1316	0.0750
7	$\eta = 4$	0.0012	0.0015	0.0007	0.0007
8	$\eta = 4$, $g_0 = 0.010$	0.7230	0.0060	0.7223	0.0029
9	$\varepsilon(T_H) = 5°C$	0.0214	0.0187	0.0141	0.0103
10	$\varepsilon(T_H) = 5°C$, $g_0 = 0.010$	0.0801	0.0596	0.0570	0.0350
11	$\varepsilon(T_H) = 5°C$, $g_0 = 0.005$	0.1776	0.1223	0.1328	0.0746
12	$\varepsilon(\gamma) = 0.0002136$	0.0250	0.0240	0.0147	0.0112
13	$\varepsilon(\gamma) = 0.0002136$, $g_0 = 0.015$	0.0453	0.0402	0.0276	0.0194
14	$\varepsilon(T_H) = 5°C$, $\varepsilon(\gamma) = 0.0002136$	0.0450	0.0384	0.0305	0.0218
15	$g_0 = 0$, $\delta = 0.01$	0.0436	0.0369	0.0267	0.0179
16	$g_0 = 0$, $\delta = 0.02$	0.0120	0.0118	0.0067	0.0053
17	$g_0 = 0.005$, $\delta = 0.01$	0.0213	0.0196	0.0124	0.0091
18	$\eta = 4$, $g_0 = 0.005$, $\delta = 0.01$	0.8683	0.0089	0.8678	0.0045
19	$\varepsilon(T_H) = 5°C$, $g_0 = 0.010$, $\delta = 0.01$	0.0214	0.0187	0.0141	0.0103

Notes: unless otherwise indicated, $\delta = 0$, $\eta = 2$, $g_0 = 0.020$, $\varepsilon(T_H) = 3.7°C$, $\varepsilon(\gamma) = 0.0001068$, $t_{max} = 500$ years.

Figure 3.4c: $w^*(0)$, known temperature change T_H, $\eta = 2$, $g_0 = 0.015, 0.020, 0.025$, and $\delta = 0$, damage function $g_t = g_0 - \gamma' T_t^{1.5}$

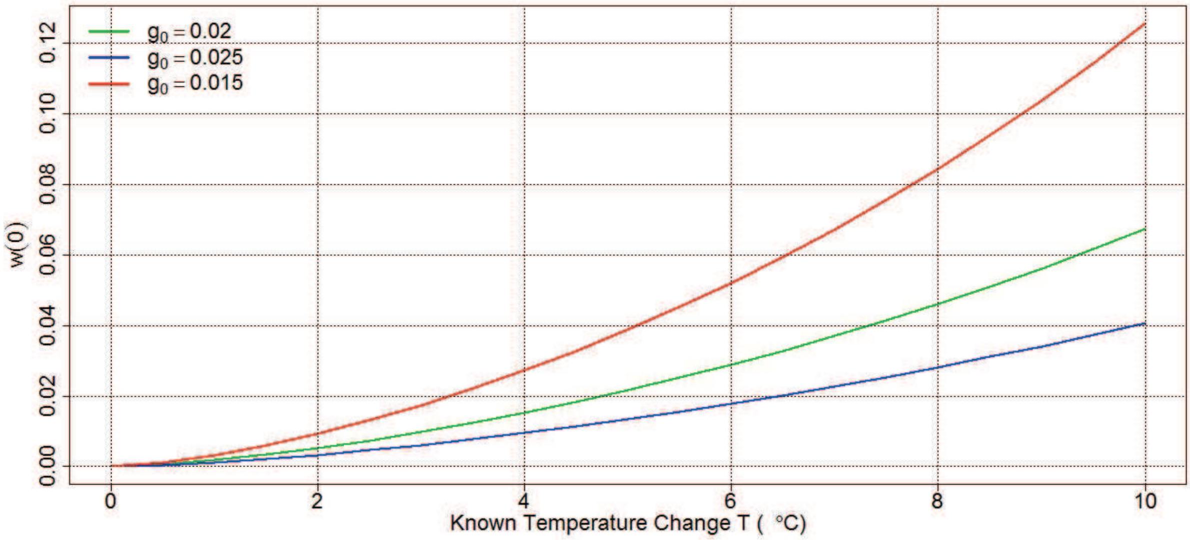
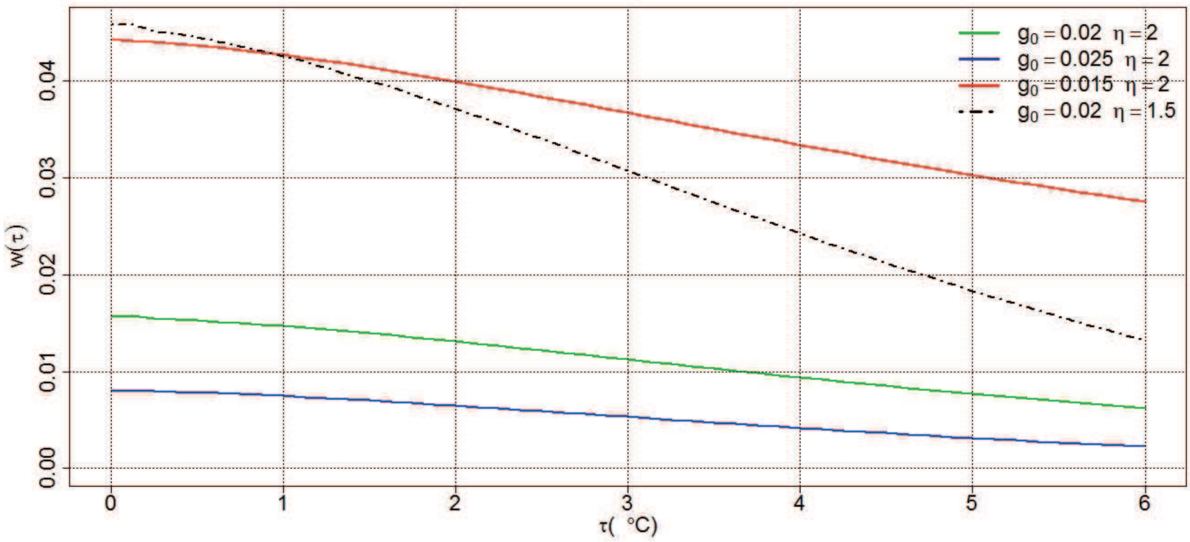


Figure 3.5c: $w^*(\tau)$, T_H and γ uncertain, $\eta = 1.5, 2$, $g_0 = 0.015, 0.020, 0.025$, and $\delta = 0$, damage function $g_t = g_0 - \gamma' T_t^{1.5}$



reflections on the technical difficulties and practical implications of the computation of the WTP in this and other cases form the bulk of next section.

3.5.2 Non-concave pattern of temperature increase

P12 has a very elegant formulation for the pattern of T_t, reported in Equation (3.1), which allows for an elegant closed-form solution for C_t (Equation (3.3)). T_t is an always concave function. Anyway, the more recent IPCC (2014), in particular figure 2.8 b p. 74, suggests that the pattern of temperature increase is not forecasted to be always concave, at least as far as the scenarios with less mitigation (RPC8.5 and RCP6.0) are concerned. A functional form in line with recent climate scientists projections should be first convex and then should change convexity after more or less one century, actually approacing approximately $2T_H$ as t becomes large. A good candidate is the general form of a sigmoid function ($f(x) = \frac{a}{1+bc^{-x}} + d$):

$$T'_t = \frac{aT_H}{1 + bc^{-(t-H)}} + dT_H \tag{3.12}$$

This formulation, with T_H multiplying both a and d, makes sure that the pattern depends on the value of the forecast at horizon H and that, as T_H goes to zero, T_t goes to zero for all possible values of t

There are four parameters to be determined: a, b, c and d. We identify them through a sistem of four unknown and four non-linear equations. From the *IPCC* 2014 we have mean forecasts under the no-mitigation scenario for 2046-2065 (2.0^oC) and for 2081-2100 (3.7^oC), therefore $T'_{65} = 2.0$ and $T'_H = T'_{100} = 3.7 = T_H$. We keep from P12 the assumption that $T'_{+\infty} = 2 \cdot T_H = 7.4$. The last equation is, trivially, $T'_0 = 0$. Like for P12, $H = 100$. The analytical derivation of the proper values of a, b, c and d is reported in Appendix 3.B.

As for the case of convex damages of Section 3.5.1, it is necessary to estimate a new distribution for the damage coefficient γ''. Fortunately, there is a closed-form solution for $\int_0^t g''_t dt = \int_0^t g_0 - \gamma'' T'_t dt$. Unfortunately, it is less elegant than the original formulation involving T_t:

$$C_t = C_0 \exp\Big(g_0 t - \gamma'' T_H \Big(\frac{a}{\log c} \log\Big(\frac{bc^H + c^t}{bc^H + 1}\Big) + d \cdot t\Big)\Big)$$

We calculate the new damage coefficient equating the loss in levels at horizon $H = 100$ with the loss implied by the damage-in-growth function:

$$g_0 H - \gamma'' T_H \Big(\frac{a}{\log c} \log\Big(\frac{c^H(b+1)}{bc^H + 1}\Big) + dH\Big) = g_0 H - \beta(T_H^2) \tag{3.13}$$

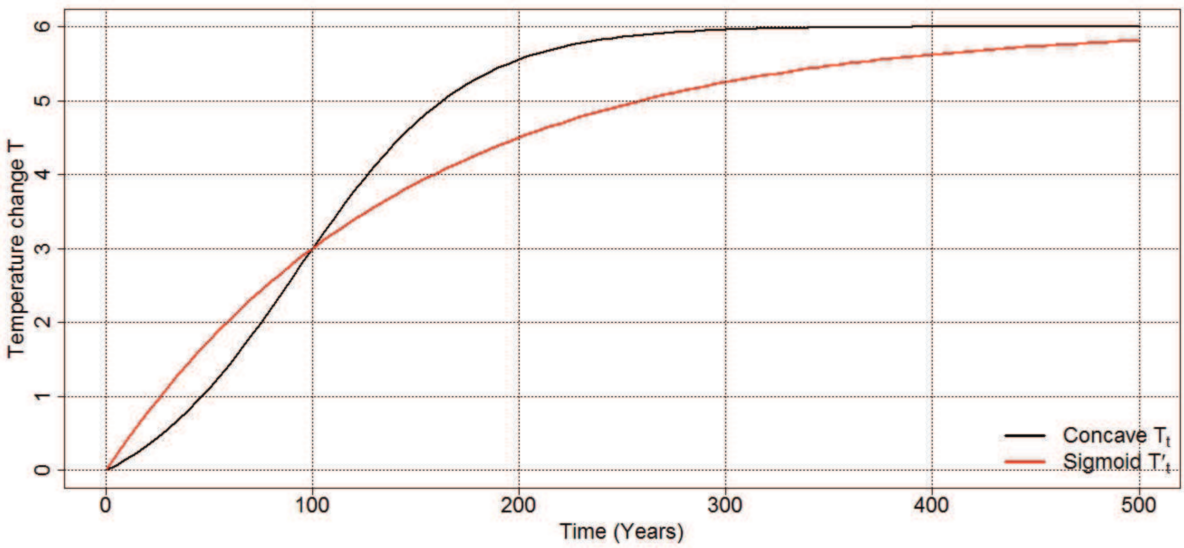
$$\gamma'' \Big(\frac{a}{\log c} \log\Big(\frac{c^H(b+1)}{bc^H + 1}\Big) + dH\Big) = \beta T_H \tag{3.14}$$

$$\gamma'' = \frac{\beta T_H}{\frac{a}{\log c} \log\Big(\frac{c^H(b+1)}{bc^H+1}\Big) + dH} \tag{3.15}$$

We obtain $\mathbb{E}(\gamma'') = 0.0001828$ which is higher than the standard case $\mathbb{E}(\gamma) = 0.0001363$. In the range $0 < t < 100$, T_t is concave, while T'_t is convex. $T_0 = T'_0$ and $T_{100} = T'_{100}$, implying $T_t > T'_t \; \forall t : 0 < t < 100$. Even if damages are assumed to be linear in both cases, an higher expected value of the damage coefficient is needed to produce an expected 3% loss in levels after 100 years.

There are two effects that influence the willingness to pay. The first one is that a sigmoid pattern is linked to a distribution for the damage coefficient with higher mean, which unambiguously increases the willingness to pay. The consequences of the second effect are less trivial: as it is possible to appreciate from Figure 3.2d, for $t < H \; T_t \geq T'_t$, while for $t > H \; T_t \leq T'_t$. This implies that the effect of this modified assumption on the willingness to pay will depend on the relative importance of the present and of the future. The relative importance, in turn, depend on preferences (the index of relative risk aversion η, the discount rate δ, the time span considered) and on the growth rate of the economy: the stronger the growth of GDP, the less society has to worry about wounds from climate change in the far future. In order to have a quantitative estimate of the overall effect, we reanalyse the WTP assuming a sigmoid pattern for T *but* keeping $\mathbb{E}(T_H) = 3.0^oC$ as in P12. Visual inspection of Figures 3.4d, 3.5d and 3.6d and, more formally, the analysis of Table 3.1d confirms that the upward pressure dominates the downward one, leading to an increase in the willingness to pay under basically all scenarios, possibly with the exception of the ones with $\eta = 4$. This increase is stronger, both in absolute and relative values, for scenarios in which the economy has a weak growth, or in which the considered time span is longer, or in which the future is discounted less heavily.

Figure 3.2d: Pattern comparison



3.6 Discussion

This section is devoted to the analysis of three important issues faced in verifying, extending or reanalysing P12. Firstly, we describe why care is needed in the estimation of the coefficients of the gamma displaced densities estimated for T_H and γ. Secondly, we highlight how the statement "*with the other moments of the distribution unchanged*" has

Figure 3.4d: $w^*(0)$, known temperature change T_H, $\eta = 2$, $g_0 = 0.015, 0.020, 0.025$, and $\delta = 0$, sigmoid pattern $T'_t = \frac{aT_H}{1+bc^{-(t-H)}} + dT_H$

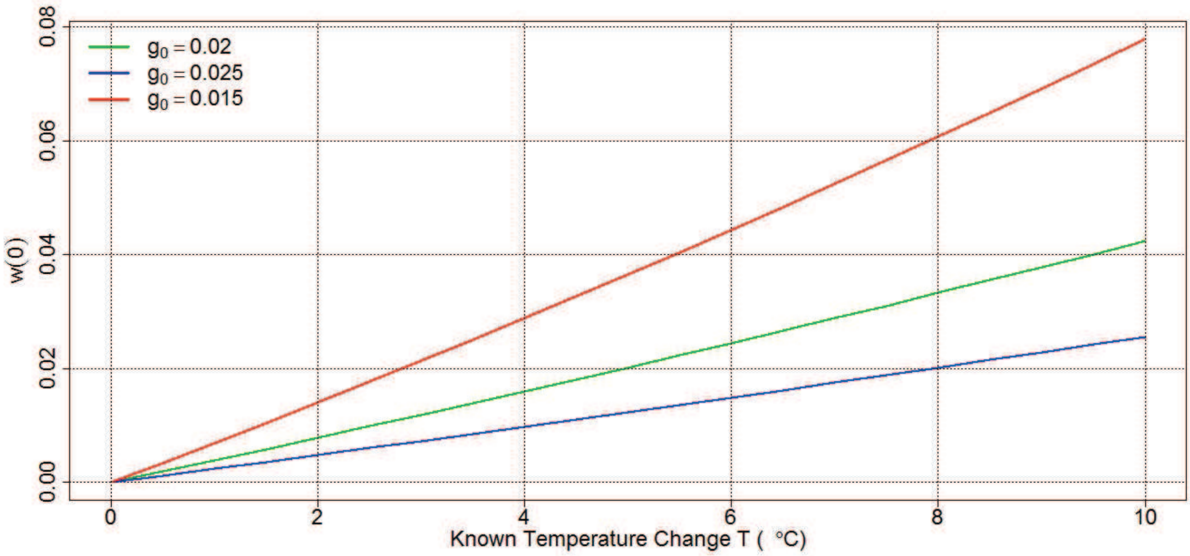


Figure 3.5d: $w^*(\tau)$, T_H and γ uncertain, $\eta = 1.5, 2$, $g_0 = 0.015, 0.020, 0.025$, and $\delta = 0$, sigmoid pattern $T'_t = \frac{aT_H}{1+bc^{-(t-H)}} + dT_H$

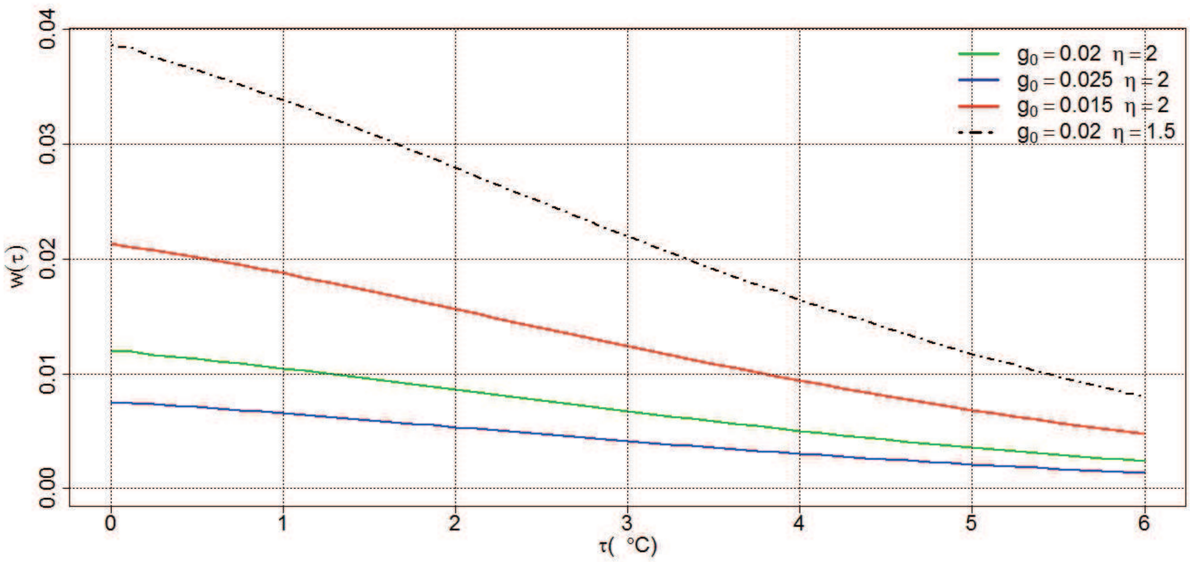


Figure 3.6d: $w^*(3)$ versus η. $g_0 = 0.020$ and $\delta = 0, 0.01$, sigmoid pattern $T'_t = \frac{aT_H}{1+bc^{-(t-H)}} + dT_H$

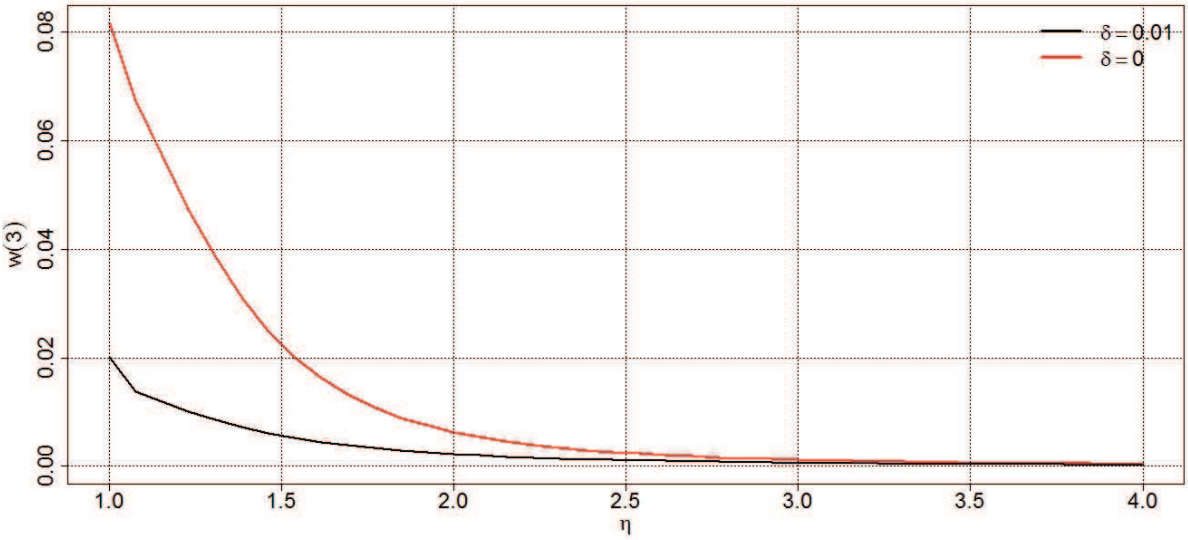


Table 3.1d: WTPs with alternative parameter values, sigmoid pattern $T'_t = \frac{aT_H}{1+bc^{-(t-H)}} + dT_H$

	Cases	$w^*(0)$		$w^*(3)$	
		Sigmoid	Concave	Sigmoid	Concave
1	Base case	0.0135	0.0118	0.0063	0.0053
2	$t_{max} = 300$	0.0125	0.0112	0.0058	0.0050
3	$t_{max} = 1000$	0.0136	0.0118	0.0064	0.0053
4	$g_0 = 0.010$	0.0530	0.0369	0.0275	0.0179
5	$g_0 = 0.005$	0.1179	0.0761	0.0650	0.0384
6	$g_0 = 0$	0.2262	0.1432	0.1315	0.0750
7	$\eta = 4$	0.0010	0.0015	0.0004	0.0007
8	$\eta = 4$, $g_0 = 0.010$	0.1094	0.0060	0.1070	0.0029
9	$\varepsilon(T_H) = 5^oC$	0.0218	0.0187	0.0125	0.0103
10	$\varepsilon(T_H) = 5^oC$, $g_0 = 0.010$	0.0881	0.0596	0.0553	0.0350
11	$\varepsilon(T_H) = 5^oC$, $g_0 = 0.005$	0.1936	0.1223	0.1286	0.0746
12	$\varepsilon(\gamma) = 0.0003656$	0.0263	0.0240	0.0119	0.0112
13	$\varepsilon(\gamma) = 0.0003656$, $g_0 = 0.015$	0.0533	0.0402	0.0277	0.0194
14	$\varepsilon(T_H) = 5^oC$, $\varepsilon(\gamma) = 0.0003656$	0.0462	0.0384	0.0275	0.0218
15	$g_0 = 0$, $\delta = 0.01$	0.0530	0.0369	0.0275	0.0179
16	$g_0 = 0$, $\delta = 0.02$	0.0135	0.0118	0.0063	0.0053
17	$g_0 = 0.005$, $\delta = 0.01$	0.0251	0.0196	0.0123	0.0091
18	$\eta = 4$, $g_0 = 0.005$, $\delta = 0.01$	0.4011	0.0089	0.3983	0.0045
19	$\varepsilon(T_H) = 5^oC$, $g_0 = 0.010$, $\delta = 0.01$	0.0218	0.0187	0.0125	0.0103

Notes: unless otherwise indicated, $\delta = 0$, $\eta = 2$, $g_0 = 0.020$, $\varepsilon(T_H) = 3.7°C$, $\varepsilon(\gamma) = 0.0001828$, $t_{max} = 500$ years.

to be implemented. Finally, we investigate the effect of seemingly irrelevant and technical positions related to the upper extremes of integration, as $+\infty$ cannot materially be used in (most of) numerical routines routinely available. The next three subsections explore one issue at a time.

3.6.1 Estimation of the densities

P12 assumes that the two most important uncertain quantities of the model are distributed as displaced gamma distributions, which offer the bonus of a remarkable analytical tractability. However, most of what we say would hold, with simple and obvious changes, for any distribution. Essentially, as $f(x|r,\lambda,\theta)$ depends on three unknown parameters to be estimated, three equations should suffice for the estimation and in Section 3.3.2 we have minimized, say for T_H, the sum of squared deviations from the given "moments", which were in turn extracted from the literature. As the mean of a displaced gamma is known and equal to $r/\lambda + \theta$ the sum of squared errors is:

$$\left(\frac{r}{\lambda} + \theta - 3\right)^2 + \left(\int_\theta^7 f(x|r,\lambda,\theta)\, dx - 0.95\right)^2 + \left(\int_\theta^{10} f(x|r,\lambda,\theta) dx\ - 0.99\right)^2,$$

where the conditions $E(T_H) = 3°C$, $Pr(T_H \leq 7°C) = 0.95$ and $Pr(T_H \leq 10°C) = 0.99$ can be easily seen. The previous function of r, λ, θ can obviously be optimized but accurate results were obtained only after the selection in the R command `optim` of the numerical method "BFGS" rather than the default choice. Even more crucially, most optimization packages, including the one we have used, implicitly assume by default that the sensitivity of the target function with respect to changes in the variables (or parameters, in our case) are of the same magnitude.

While this is roughly true in the estimation of the density for T_H, it is definitely not the case for the parameters of the density of impact γ, which differ by several orders of magnitude and are reported in P12 to be $r = 4.5, \lambda = 21341$ and $\theta = -7.46 \cdot 10^{-5}$. Typically, in similar cases the default choice of the numerical method may fail to be the correct one, possibly resulting in inaccurate results. To tackle this "scaling" problem (Nash, 2014) the user can provide optional information to the algorithm, basically giving the correct magnitudes as an (additional) input. Specifically, in our code we use the option `parscale` to feed the optimizer with the proper `par`ametric `scales`.

To summarize, it should be kept in mind that in this case other than the defaults computational methods and scaling coefficients were provided to the optimizer in R. Obviously, care and some experience are needed to customize some details prior to minimization and additional scrutiny and checks of the adequacy of the results, with analytical or graphical methods, are advised. The task is hugely facilitated when replicating an existing paper that can be inspected and appear to be much harder if no hint can be guessed, say on starting points or sizes, from other sources.

3.6.2 On sensitivity analysis

Among the several sensitivity analysis reported in Table 3.1, some concern an increase in the mean μ of T_H or γ or in both of them. To facilitate the reader in "verifying our verification", we briefly outline how to obtain the same results of P12.

The statement "*with the other moments of the distribution unchanged*" means that the value of θ must not be altered, and that the values of the parameters r and λ have to be chosen in such a way to leave σ^2 unchanged, get the desired μ and let the moments beyond the second to vary. Exploiting the properties of the displaced gamma distribution, it can be shown that there is a closed-form solution for r and λ as a function of μ, σ^2 and θ:

$$r = \frac{(\mu - \theta)^2}{\sigma^2}$$

and

$$\lambda = \frac{(\mu - \theta)}{\sigma^2}.$$

We adopted this procedure and got estimates for cases 9–14 in Table 3.1 virtually identical to the ones contained in P12.

3.6.3 On the upper limits of integration

The quantification of the WTPs entirely relies, as we have seen in previous sections, on the computation of two (multi-dimensional) integrals in (3.7). In principle, these integrals are to be computed over intervals reaching $+\infty$. However, for practical reasons, the support of T_H and γ is truncated and the upper limits in the computations are instead taken to be large numbers, which we denote by T_{max} and γ_{max}. We recall that the same computational shortcut is explicitly mentioned in P12 when, say, the utility of consumption is integrated up to $t_{max} = 500$ (or 1000 in the robustness check of row 3 in Table 3.1). All the computation in this work used $T_{max} = 15\ °C$ and $\gamma_{max} = 0.0007$. Intuitively, this is justified by the fact that truncating the distributions should not have a large effect provided that T_{max} and γ_{max} are "big enough".[11]

Table 3.2 shows the WTP $w^*(0)$ for the baseline combination of parameters (corresponding to row 1 of Table 3.1), as a function of the upper limits of integration for T_H and γ. Our reference value $w^*(0) = 0.0118$, obtained when $\gamma_{max} = 0.0007$ and $T_{max} = 15$, is singled out and boldfaced in the table. It is clear that replacing $+\infty$ in the integrals with smaller values has little consequences and, unless really too small T_{max} or γ_{max} are chosen, results appear remarkably robust, *for the given set of parametric values* used in the table.

Quite a different behaviour is documented in Table 3.3, which shows $w^*(0)$ when η increases to 4 and the growth rate g_0 decreases to 0.01 (see the parameters of row 8 of Table 3.1). Even a cursory look at the table reveals that the WTP dramatically depends on T_{max} and γ_{max}, despite the intuitive belief that they should only play a technical role in the computations, bounding the integration domain. It is quite clear that $w^*(0) = 0.0060$ which, as argued in previous sections, is notably different from the figure shown in P12, is not a robust estimate and its wild fluctuations cast serious doubts on any related policy suggestions. Generally speaking, the increase of T_{max}, as well as γ_{max} appear to fuel the WTP till 100%. This value is reached because there are combinations of T and γ in the integration domain for which consumption "grows" at a negative rate and rapidly approaches infinitesimal levels, due to the small g_0 and to the term $-\gamma T$ in (3.2),

[11] Limiting the upper extremes to 15 and 0.0007, respectively, we are excluding from the analysis an area amounting to probability $2.3 \cdot 10^{-8}$.

Table 3.2: Case 1 (baseline) of Table 3.1. Sensitivity analysis of $w^*(0)$ with several combinations of γ_{max} ($x10^{-4}$) (horizontal axis) and T_{max} (vertical axis)

	5	7	9	11	13	15	17	19
12	0.0101	0.0102	0.0102	0.0102	0.0102	0.0102	0.0102	0.0102
13	0.0110	0.0111	0.0111	0.0111	0.0111	0.0111	0.0111	0.0111
14	0.0114	0.0115	0.0116	0.0116	0.0116	0.0115	0.0116	0.0116
15	0.0116	**0.0118**	0.0118	0.0118	0.0118	0.0118	0.0118	0.0118
16	0.0117	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0118
17	0.0118	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119
18	0.0118	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119
19	0.0118	0.0119	0.0119	0.0119	0.0120	0.0119	0.0119	0.0119
20	0.0118	0.0119	0.0119	0.0120	0.0119	0.0120	0.0119	0.0119
21	0.0118	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119
22	0.0118	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119
23	0.0118	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0120
24	0.0118	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119
25	0.0118	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119
26	0.0118	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119
27	0.0118	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119
28	0.0118	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0121
29	0.0118	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119

Notes: $\delta = 0.02$, $\eta = 2$, $g_0 = 0$, $\varepsilon(T_H) = 3°C$, $\varepsilon(\gamma) = 0.0001363$, $t_{max} = 500$ years.

Table 3.3: Case 8 ($\eta = 4$, $g_0 = 0.01$) of Table 3.1. Sensitivity analysis of $w^*(0)$ with several combinations of γ_{max} ($x10^{-4}$) (horizontal axis) and T_{max} (vertical axis)

	5	7	9	11	13	15	17	19
12	0.0054	0.0055	0.0055	0.0055	0.0056	0.0058	0.0064	0.0079
13	0.0057	0.0058	0.0058	0.0059	0.0064	0.0083	0.0162	0.0469
14	0.0058	0.0059	0.006	0.0065	0.0093	0.0268	0.1167	0.3613
15	0.0059	**0.0060**	0.0062	0.0081	0.0251	0.1490	0.4792	0.7474
16	0.0059	0.0061	0.0066	0.0140	0.1033	0.4687	0.7757	0.9098
17	0.0060	0.0061	0.0076	0.0362	0.3279	0.7372	0.9083	0.9682
18	0.0060	0.0062	0.0099	0.1097	0.5974	0.8762	0.9628	0.9888
19	0.0060	0.0063	0.0161	0.2778	0.7785	0.9422	0.9850	0.9961
20	0.0060	0.0065	0.0319	0.4960	0.8805	0.9731	0.9940	0.9986
21	0.0060	0.0068	0.0701	0.6762	0.9359	0.9875	0.9976	0.9995
22	0.0060	0.0073	0.1502	0.7977	0.9656	0.9942	0.9990	0.9998
23	0.0060	0.0082	0.2808	0.8747	0.9816	0.9973	0.9996	0.9999
24	0.0060	0.0098	0.4369	0.9226	0.9902	0.9988	0.9998	1
25	0.0060	0.0126	0.5816	0.9523	0.9948	0.9994	0.9999	1
26	0.0060	0.0175	0.6970	0.9706	0.9972	0.9997	1	1
27	0.0060	0.0261	0.7803	0.9819	0.9985	0.9999	1	1
28	0.0060	0.0408	0.8453	0.9889	0.9992	0.9999	1	1
29	0.0060	0.0653	0.8899	0.9931	0.9996	1	1	1

Notes: $\delta = 0.02$, $\eta = 2$, $g_0 = 0$, $\varepsilon(T_H) = 3°C$, $\varepsilon(\gamma) = 0.0001363$, $t_{max} = 500$ years.

generating (very) negative utility. The effect is enhanced by the relatively large value taken by η, and we have already observed that several WTP are hard to compute when $\eta = 4$, the largest value examined in P12.

The previous discussion demonstrates that reliable computations of WTP may be hard under some circumstances or, if you wish, that the model appears to be fragile, being the results too sensitive to "internal" inputs of the numerical software. Clearly, changes in the specification of the utility function may remove some forms of ill-posedeness. Additional stability, of course, would be obtained tolerating positive values for δ but we are well aware that the proper level of the intertemporal discount rate is at the heart of a (moral) debate among scholars and economists (Dasgupta, 2008; Pindyck, 2013)[12].

3.7 Conclusions

In this paper we present a verification, an extension and a reanalysis of the incisive paper Pindyck (2012). Retracing the path followed by the author has, to a large extent, allowed to verify the accuracy of the estimates of the willingness to pay in order to limit the temperature increase below some threshold τ. This was possible with no access to the original code under a variety of parametric instantiations and critical discrepancies from the results of P12 are present but uncommon, possibly due to one material typing error and likely to be related to a few specific combinations of values taken by the parameters η, related to the risk aversion of the society, intertemporal discount rate δ and growth rate of consumption g_0.

Our extension corroborate the main message of Pindyck (2012): we have shown that using more recent data from *IPCC* 2014 does not change the value of the statement that the willingness to pay, given what we know and its sheer uncertainty, is consistent with a moderate abatement policy.

A functional form for the temperature increase which is more in line with recent climate scientists assessment yields an higher willingness to pay, but does not alter the substantial message of a moderate abatement policy consideration.

As we know little about the damage function that relates the temperature increase to the decrement in the growth rate of consumption, our reanalysis also investigates how convex damages affect the results, implicitly assuming that rare (and catastrophic) events are greatly valued in the computation of the utility function. While in standard cases this is not changing much the results, in other circumstances the willingness to pay increases in outstanding ways, hinting at some fragility of the model. This reanalysis suggests caution in the interpretation of policy decisions that may be driven by the model.

We believe that another important outcome of this work was the demonstration that some results critically depend on the values of technical parameters of the numeric algorithms at work in the evaluation, such as the upper extremes of integration. The fragility of this version of the model exemplifies why an oftentimes excruciating effort is needed to verify the results obtained by other scholars, even in this case where abundant information was available in an extended and detailed working paper written by Pindyck on the very same model.

[12] According to Dasgupta (2008) and Pindyck (2013), the different calibration of the discount rate and other crucial parameters is at the very heart of the ten-fold difference between Nordhaus (1994, 2014) and Stern (2007, 2008) in their estimated Social Costs of Carbon.

While we hope that our sacrifice makes a contribution to the ongoing discussion on the usefulness of verifying and reanalysing scientific works, we are well aware that even more sacrifice is surely needed to understand and reduce adverse effects of climate change.

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Appendix

3.A Log utility

When η approaches to 1, economists usually approximate the CRRA utility function with the logarithmic utility function. W_1 thus becomes

$$\begin{aligned}
W_1(\tau) &= \iiint u(\tilde{C}_t')e^{-\delta t} f_\tau(x) f_\gamma(y)\ dt\ dx\ dy \\
&= \iiint \log\big((1-w_{log}(\tau))(\tilde{C}_t)\big)e^{-\delta t} f_\tau(x) f_\gamma(y)\ dt\ dx\ dy \\
&= \iiint \log\big(1-w_{log}(\tau)\big)e^{-\delta t} f_\tau(x) f_\gamma(y)\ dt\ dx\ dy + \iiint \log\big(\tilde{C}_t\big)e^{-\delta t} f_\tau(x) f_\gamma(y)\ dt\ dx\ dy \\
&= \log\big(1-w_{log}(\tau)\big)\int e^{-\delta t}\ dt + G_\tau, \qquad (3.16)
\end{aligned}$$

The last step follows from $\big(1-w_{log}(\tau)\big)e^{-\delta t}$ not depending on x and y, so

$$\begin{aligned}
\iiint \log\big(1-w_{log}(\tau)\big)e^{-\delta t} f_\tau(x) f_\gamma(y)\ dt\ dx\ dy &= \int \log\big(1-w_{log}(\tau)\big)e^{-\delta t}\ dt \iint f_\tau(x) f_\gamma(y)\ dx\ dy \\
&= \int \log\big(1-w_{log}(\tau)\big)e^{-\delta t}\ dy \int f_\tau(x)\ dx \int f_\gamma(y)\ dy \\
&= \int \log\big(1-w_{log}(\tau)\big)e^{-\delta t}\ dt \cdot 1 \cdot 1 \\
&= \int \log\big(1-w_{log}(\tau)\big)e^{-\delta t}\ dt \\
&= \log\big(1-w_{log}(\tau)\big)\int e^{-\delta t}\ dt \qquad (3.17)
\end{aligned}$$

Keeping the notation G_∞ for the expected utility with unbounded potential warming, we obtain

$$\begin{aligned}
\log\big(1-w_{log}(\tau)\big)\int e^{-\delta t}\ dt + G_\tau &= G_\infty \\
\log\big(1-w_{log}(\tau)\big) &= \frac{G_\infty - G_\tau}{\int e^{-\delta t}\ dt} \\
\big(1-w_{log}(\tau)\big) &= exp\Big(\frac{G_\infty - G_\tau}{\int e^{-\delta t}\ dt}\Big)
\end{aligned}$$

And finally

$$w_{log}(\tau) = 1 - exp\Big(\frac{G_\infty - G_\tau}{\int e^{-\delta t}\, dt}\Big) \tag{3.18}$$

3.B Analytical derivation of parameters of Equation (3.12)

$$T'_0 = \frac{3.7 \cdot a}{1 + bc^{-(0-100)}} + 3.7 \cdot d = 0 \tag{3.19}$$

$$T'_{65} = \frac{3.7 \cdot a}{1 + bc^{-(65-100)}} + 3.7 \cdot d = 2.0 \tag{3.20}$$

$$T'_{100} = \frac{3.7 \cdot a}{1 + bc^{-(100-100)}} + 3.7 \cdot d = \frac{3.7 \cdot a}{1+b} + 3.7 \cdot d = 3.7 \tag{3.21}$$

$$T'_{+\infty} = \lim_{t \to +\infty} \frac{3.7 \cdot a}{1 + bc^{-(t-100)}} + 3.7 \cdot d = 3.7 \cdot a + 3.7 \cdot d = 7.4 \tag{3.22}$$

In order to obtain an analytic solution for the values of the parameters, we start from Equation 3.22, from which we trivially obtain

$$3.7 \cdot (a + d) = 7.4 \implies d = 2 - a.$$

Then, from 3.21, we have

$$\frac{\cancel{3.7} \cdot a}{1+b} + \cancel{3.7} \cdot (2 - a) = \cancel{3.7}.$$

After a bit of rearrangement, we obtain

$$b = \frac{1}{a-1}.$$

As a third step, we substitute the so obtained b and d into 3.19 and ignoring the costant 3.7, we have:

$$\begin{aligned}
\frac{a}{1 + \frac{1}{a-1}c^{100}} + 2 - a &= 0 \\
\frac{a(a-1)}{a - 1 + c^{100}} + 2 - a &= 0 \\
a(a-1) + (2-a)(a - 1 + c^{100}) &= 0 \\
(a-2)c^{100} &= a - 1 \\
c &= \sqrt[100]{\frac{a-1}{a-2}} \text{ for } a < 1 \text{ or } a > 2
\end{aligned}$$

Finally, we substitute back the identities $b = \frac{1}{a-1}$, $c = \sqrt[100]{\frac{a-1}{a-2}}$ and $d = 2 - a$ into 3.20, we can find the unique root of the function $g(a) = T'_{65}(a) - 2$, that is $a = 2.209882$, leading to $b = 0.8265268$, $c = 1.02475$ and $d = -0.2098821$.

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