



Research papers

Microwave characterization of complex permittivity of magnetodielectric disks with resonant methods

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ARTICLE INFO

Keywords:

Microwave characterization
Complex permittivity
Split-post dielectric resonator
Hexaferrite
Ferromagnetic resonance
Magnetodielectric disks

ABSTRACT

Accurate determination of the complex permittivity of magnetodielectric materials at microwave frequencies remains challenging, particularly for ferrimagnetic samples exhibiting strong magnetocrystalline anisotropy. In resonant methods such as the Courtney technique, separation of dielectric and magnetic contributions requires magnetic saturation of the sample; however, tensor permeability effects under bias introduce systematic errors in the extracted permittivity. In this work, the applicability of split-post dielectric resonators (SPDR) to the characterization of small thin magnetodielectric disks is investigated. Measurements were performed at 5 GHz and 10 GHz on polycrystalline yttrium iron garnet (YIG) and W-type hexaferrite disks under static magnetic bias. The results show that SPDR measurements at higher frequencies provide significantly improved sensitivity and reduced uncertainty compared to lower-frequency operation. For the investigated samples, reliable permittivity values were obtained at 10 GHz, while 5 GHz measurements were limited by small frequency shifts and their associated uncertainty.

1. Introduction

Ferrimagnetic materials with both weak and strong magnetocrystalline anisotropy play a crucial role in modern microwave technology, particularly in nonreciprocal devices such as circulators, isolators and phase shifters operating in 5 G and emerging 6 G frequency bands. Hexagonal ferrites (hexaferrites), especially M-type $\text{BaFe}_{12}\text{O}_{19}$ and its doped derivatives, are of particular interest due to their high uniaxial anisotropy, relatively low dielectric losses, and potential for self-biased operation without external permanent magnets [1]. Any cost-effective design methodology and the performance of such devices depends critically on accurate knowledge of the complex permittivity ϵ^* and permeability μ^* , as well as on the ferromagnetic resonance (FMR) linewidth, which determine losses at microwave frequencies. However, reliable characterization of ϵ^* and μ^* in ferrimagnetic materials remains challenging [2,3]. Available methods can be divided into broadband and resonant ones. Coaxial line transmission/reflection methods can be employed in principle if it is possible to manufacture a disk with a hole for the inner coaxial conductor. However, the major problem in such measurements of both permittivity and permeability is limited resolution of the dielectric and magnetic loss measurements (about 0.01 for

the imaginary part of permittivity and permeability), as well as the presence of air gaps which introduce large systematic errors [4]. In the millimeter-wave range, employing coaxial methods becomes impractical. Broadband methods, such as quasi-optical ones have been demonstrated [5,6], however, they are hardly applicable to lower frequencies as the required sample lateral dimensions become prohibitively large [7]. Resonance techniques allow to overcome this difficulty and allow measurements of low loss magneto-dielectric materials. Needle-shaped ferrite samples can be characterized employing a TM cavity resonator and a TE dielectric resonator in tandem [8,9], however, manufacturing of samples of such aspect ratios is not an easy task. On the other hand, if large flat samples are available, split-post and single-post dielectric resonators (SPDRs and SiPDRs) can be employed to find the complex permittivity as well as the in-plane and out-of-plane initial permeabilities [10,11].

In particular, for materials exhibiting strong magnetocrystalline anisotropy, the permeability of a demagnetized sample is very different from unity at microwave frequencies [12], so the separation of dielectric and magnetic contributions to resonant frequencies of resonant systems loaded with such materials is necessary for the characterization of either of them, which is a non-trivial task.

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Received 21 February 2026; Received in revised form 21 July 2026; Accepted 19 August 2026

Available online 21 August 2026

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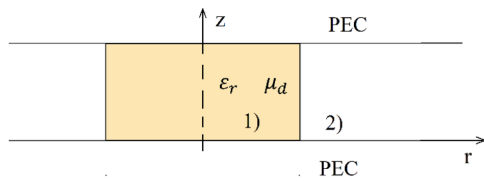


Fig. 1. Schematic of Hakki-Coleman [14] and Courtney [13] resonators.

2. Theory

One of the most accurate methods of the complex permittivity measurements of microwave ferrites is the Courtney method [13]. It is an extension of Hakki-Coleman method [14] which is commonly used for measurements of low loss dielectrics. In both techniques the sample under test creates a cylindrical dielectric or magnetodielectric resonator operating at the TE_{011} mode which is schematically shown in Fig. 1. For such a geometry and scalar magnetic and dielectric properties of the

sample rigorous solutions of Maxwell's equations are available making it possible to obtain a transcendental equation which relates resonance frequencies and Q-factors to the permittivity and permeability of the sample. Separation of the dielectric and magnetic properties in the Courtney method is obtained from two independent measurements of frequencies and Q-factors. The first one is performed on completely the demagnetized sample, while the other one is performed on the sample situated in a very strong static magnetic field directed along the cylinder axis of the sample (z-axis in Fig. 1). If the metal plates housing the sample are made from a material that is a perfect magnetic conductor (PMC) for DC magnetic fields, the external magnetic field in the z-direction is approximately equal to the internal magnetic field which determines the microwave resonance frequency, which has been assumed in the following theoretical considerations.

It is assumed that in the static magnetic field the permeability of the sample is equal to unity, so from this measurement the complex permittivity of the sample can be obtained. However, in the presence of a static magnetic field the permeability of the sample becomes a tensor

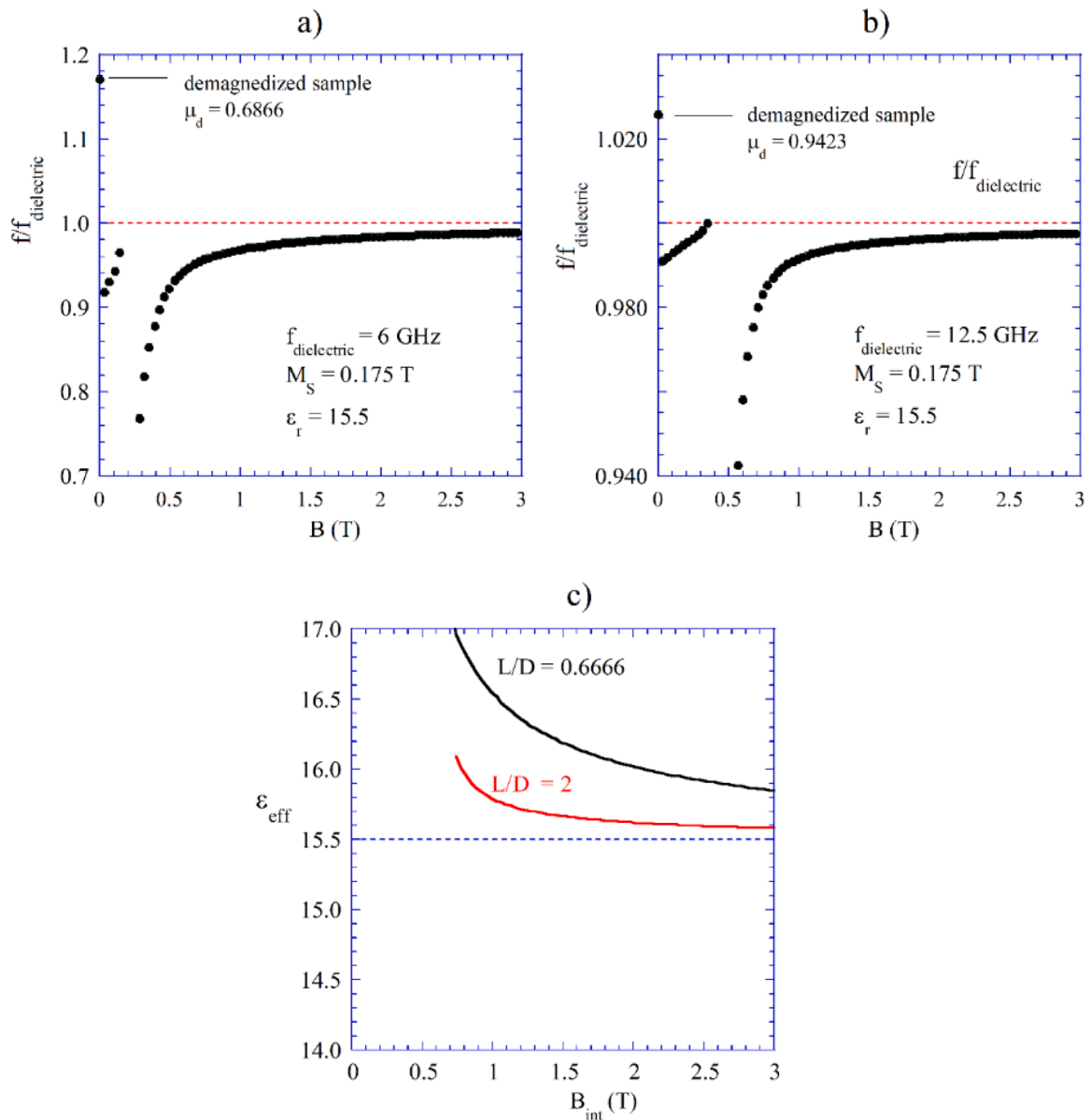


Fig. 2. Normalized the TE_{011} mode resonance frequencies versus static magnetic field for Courtney resonators made of polycrystalline yttrium iron garnet with $M_S = 0.175$ T, $\epsilon_r = 15.5$ a) $D = 15$ mm, $L = 10$ mm, $f_{\text{dielectric}} = 6$ GHz, b) $D = 5$ mm, $L = 10$ mm, $f_{\text{dielectric}} = 12.5$ GHz. c) Effective permittivity of the samples ϵ_{eff} obtained with the Courtney method versus static magnetic field bias.

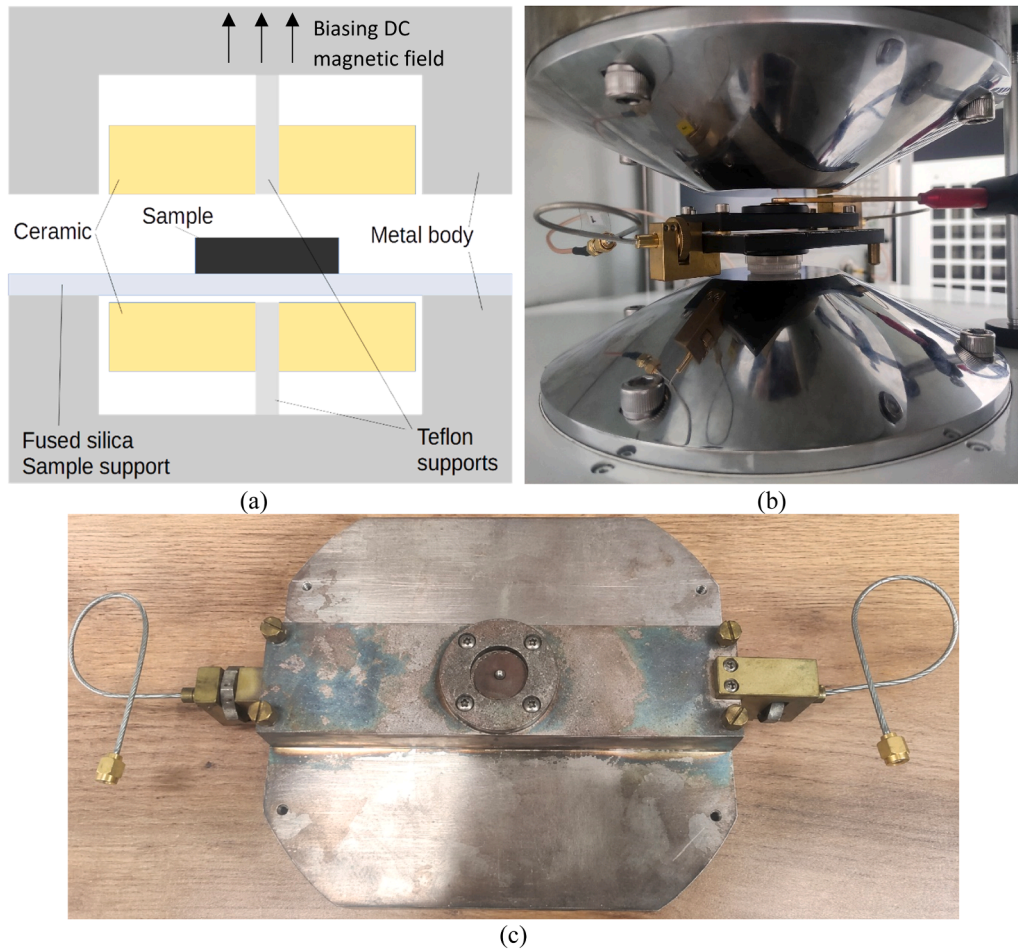


Fig. 3. a) Schematic of a disk placed on a fused silica support in a split-post dielectric resonator (SPDR). b) Experimental realization of a 10 GHz SPDR placed in a vertical electromagnet. c) 5 GHz SPDR resonator employed in experiments.

quantity described by Polder's tensor. Although the axial component of the permeability tensor saturates quickly to unity in the presence of a moderate static magnetic fields, the remaining components do not [15]. As a result, the permittivity of the sample obtained in this manner is always slightly different than the true permittivity of the sample.

Theoretical studies of such a relationship can be studied with advanced electromagnetic modelling. In Fig. 2a and Fig. 2b results of computations of normalized the TE_{011} mode resonance frequencies versus static magnetic field for Courtney resonators are presented obtained with Galerkin-Rayleigh-Ritz method [16]. The method allows to find the eigenfrequency of the resonant system shown in Fig. 1 at the TE_{011} mode of oscillation as a function of the static magnetic bias. In the figures, two samples made of polycrystalline yttrium iron garnet (YIG) with $M_s = 0.175T$, $\epsilon_r = 15.5$ are considered: sample a) with diameter $D = 15mm$, length $L = 10mm$ and sample b) with diameter $D = 5mm$, length $L = 10mm$ and resonance frequency assuming no magnetic properties (permeability $\mu = 1$) $f_{dielectric} = 12.5GHz$. The resonance frequency of sample a), assuming no magnetic properties (permeability $\mu=1$), is equal to $f_{dielectric} = 6GHz$, whereas for sample b) $f_{dielectric} = 12.5GHz$. Figs. 2a and 2b include the theoretical permeabilities μ_d of the samples in a demagnetized state ($B = 0T$), computed using Schlömann's formula [17]

$$\mu_d = \frac{1}{3} + \frac{2}{3} \sqrt{\frac{(f/\gamma)^2 - (H_a + M_s)^2}{(f/\gamma)^2 - H_a^2}}, \quad (1)$$

where $f = f_{dielectric}$, $\gamma \approx 2.8025$ MHz/Oe is the gyromagnetic ratio, H_a is the effective anisotropy field assumed to be $H_a = 66$ Oe, and M_s is the

saturation magnetization.

For sample a) and a static magnetic field value equal to 3 T the resonance frequency of the TE_{011} mode is still about 2% lower than that for dielectric sample. It should be emphasized that the static magnetic field required to saturate a hexaferrite material (which has a much higher anisotropy field and saturation magnetization M_s than YIG) to a similar level would be considerably higher. A much smaller discrepancy ($f/f_{dielectric}$ closer to unity) is obtained for sample b) because its higher aspect ratio (L/D) causes more electromagnetic energy to be stored in the magnetic field components in the axial direction [2]. Hypothetical effective permittivity values obtained with the Courtney method for these two YIG samples are presented in Fig. 2c. It can be concluded that permittivity errors significantly increase for a lower L/D ratio. Unfortunately, in Courtney's method there are limits for the maximum L/D ratio because electromagnetic fields in region 2 (Fig. 1) have to be evanescent to avoid radiation losses. For example, for permittivity $\epsilon_r = 10$ and infinite metal plates the maximum L/D ratio is $(L/D)_{max} \approx 0.5$.

The requirement for a very strong static magnetic field can be mitigated by choosing a different resonance structure. Ref. [18] presents results of measurements of the effective permittivity of polycrystalline YIG samples employing Courtney and split post dielectric resonator methods (SPDR) [19] as a function of the static magnetic bias applied perpendicularly to the sample. For the split post dielectric resonator measurements presented in [18] the YIG sample had dimensions $50mm \times 50mm \times 0.503mm$. Results of the effective permittivity determination employing Courtney method are larger than the true permittivity value and are consistent with theoretical computations shown in Fig. 2c. However, in the SPDR method the effective

Table 1

Samples used in the experiments and their sizes.

Sample	Diameter D (mm)	Length L (μm)
Polycrystalline yttrium iron garnet (YIG)	5.9681	439.8
W-type hexaferrite	4.981	459

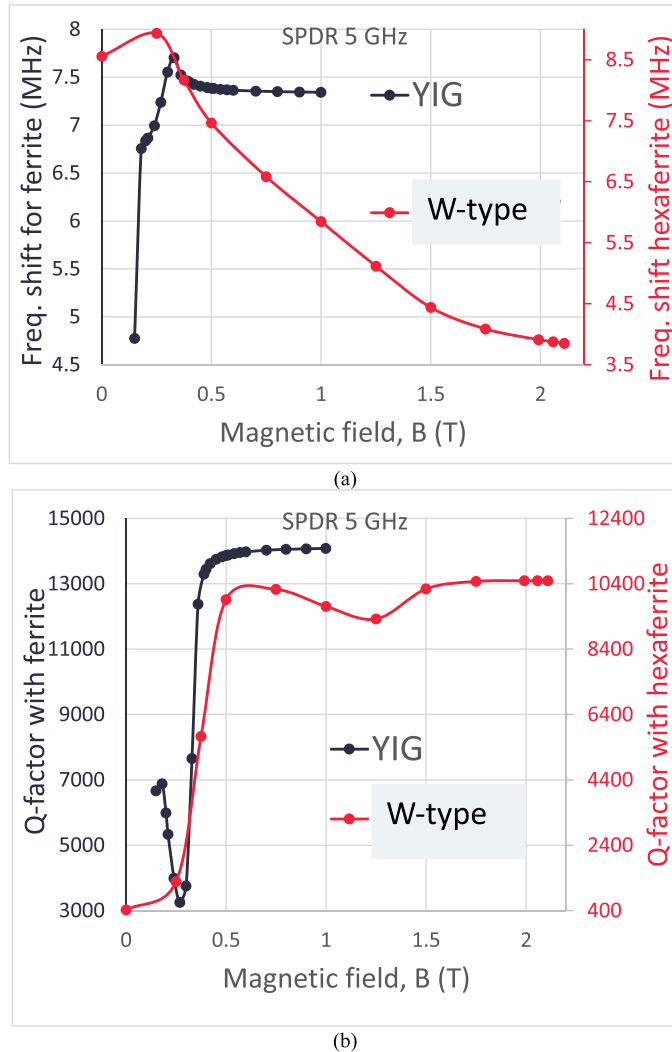


Fig. 4. a) Frequency shift vs. magnetic field B for the YIG sample in the 5 GHz SPDR (left y-axis) and the hexaferrite sample (right y-axis). The frequency shift was measured relative to the frequency of the SPDR loaded with each sample for $B = 0$. b) Q-factor vs. magnetic field for the YIG sample in the 5 GHz SPDR (left y-axis) and the W-type hexaferrite sample (right y-axis).

permittivity converges to the true value at much lower fields (ca. 0.8 T [18]). This due to the fact that the radial component of the magnetic field is zero on the symmetry plane of the split structure. Only the axial component of the magnetic field affects the effective permeability in practice, and it can become saturated to unity at bias fields that are not too high.

The main part of the work presented in this manuscript is to explore the applicability of SPDR resonators [20] to the measurement of not samples that fill the whole measurement area as in [18], but are in the form of small thin disks only.

3. Experiments

The frequency and Q-factor vs. static magnetic field of an SPDR

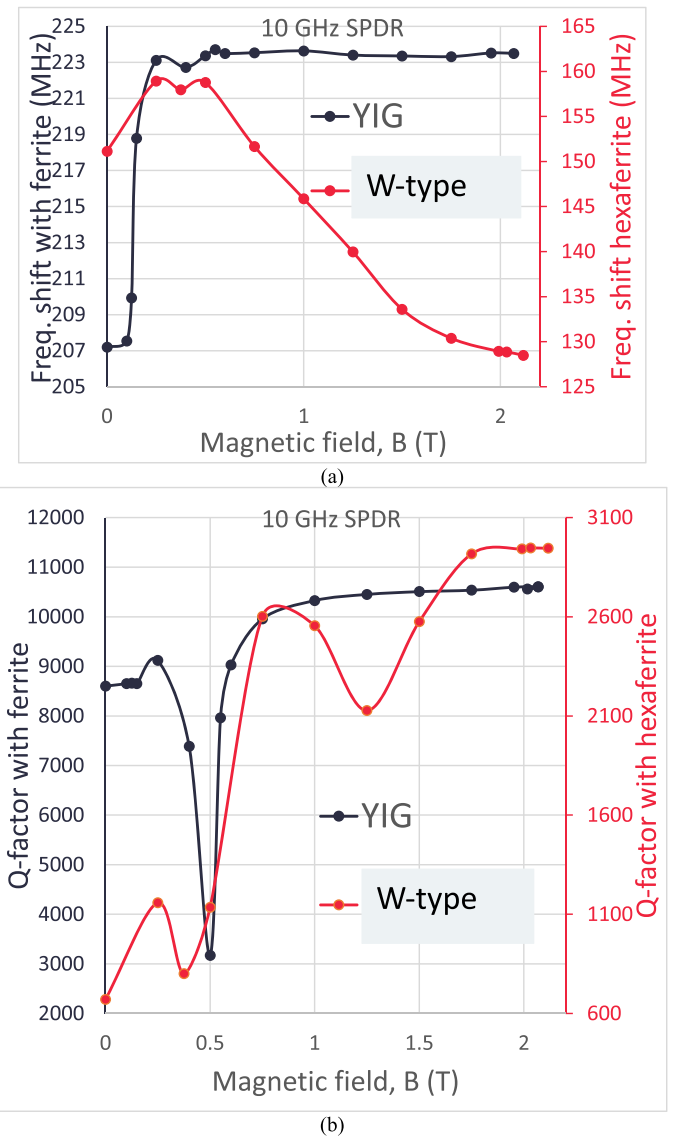


Fig. 5. a) Frequency shift vs. magnetic field B for the YIG sample in the 10 GHz SPDR (left y-axis) and the hexaferrite sample (right y-axis). The frequency shift was measured relative to the frequency of the SPDR loaded with each sample for $B = 0$. b) Q-factor vs. magnetic field for the YIG sample in the 10 GHz SPDR (left y-axis) and the W-type hexaferrite sample (right y-axis).

resonator loaded with a ferrite disk, shown schematically in Fig. 3a, has been measured for two different disks listed in Table 1. One of the disks was made from polycrystalline yttrium iron garnet with a small anisotropy field compared to the other disk made from W-type pressed polycrystalline hexaferrite.

Two kinds of SPDRs have been used, one operating at ca. 5 GHz and the other at 10 GHz. The resonators are shown in Figs. 3b) and 3c), respectively. The key differences between them are the employed ceramic pills (ca. 25 mm vs ca. 15 mm), which largely determine the resonance frequencies, as well as the maximum thickness of the materials under test (2 mm vs 1 mm). The samples were loaded into the SPDRs in the following way: the two halves of the SPDR were separated, and a fused silica substrate was placed onto the lower half, followed by the ferrite disk sample and a small piece of styrofoam with properties close to the properties of vacuum which helped to fix the disk in position. Finally, the upper half of the SPDR was carefully put back. The resonance frequency and Q-factor of such a structure was measured vs. the axial magnetic field as shown in Fig. 3 b) employing a N5241A PNA-

X vector network analyzer. The results are shown in Figs. 4 and 5.

4. Discussion

The measurement results show that for the YIG sample, the resonance mode stabilizes for moderate values of the static magnetic field at 5 GHz, which is expected and consistent with previous work [18]. As it can be noticed in Fig. 4a) and 5 a), as the magnetic field increases, the frequency of the resonator loaded with the YIG sample increases, and decreases for the resonator loaded with the hexaferrite sample. This is due to the fact that in the first case the permeability of YIG increases, saturating at 1, while the permeability of the hexaferrite also saturates to 1 but it decreases from a value greater than 1. This can be justified in the following way. According to Schlomann's model, given in (1), the real part of the initial permeability of a demagnetized ferrite is smaller (higher) than unity if the frequency is higher (smaller) than the natural ferromagnetic resonance frequency. Several models for the permeability tensor components for partially magnetized ferrites have been subsequently proposed [21–23]. In particular, the real part of the permeability tensor component parallel to the static magnetic biasing field (μ_z) is known to tend to unity both for sufficiently high frequencies and the biasing field [22], although it should be emphasized that the changes need not to be monotonic, which is confirmed by the measurement results presented in Fig. 4. and 5.

Unfortunately, the frequency shifts in Fig. 4a) and 5 a) are relatively small (several MHz) and therefore the complex permittivity extracted from the 5 GHz SPDR measurements is burdened with a high uncertainty. The results at 5 GHz are 15.0 ($\tan\delta = 1e-4$) for the YIG disk and 14.5 ($\tan\delta = 1.19e-2$) for the W-type hexaferrite.

The frequency offsets due to the samples are much higher in the case of the 10 GHz SPDR and amount to 223 MHz (YIG) and 130 MHz (hexaferrite), therefore it can be expected that the accuracy of the results at 10 GHz is much higher. For the 10 GHz SPDR, the extracted permittivity of the YIG sample is $\epsilon = 14.0$ ($\tan\delta = 1e-4$), and for the W-type hexaferrite $\epsilon = 12.85$ ($\tan\delta = 8.1e-3$).

5. Conclusions

This study examined resonant methods for determining the complex permittivity of small magnetodielectric disks, with particular emphasis on ferrimagnetic materials characterized by different anisotropy fields. The analysis confirms that in classical Courtney-type measurements, incomplete magnetic saturation and tensor permeability effects introduce systematic deviations in the extracted permittivity, especially for materials with strong magnetocrystalline anisotropy.

Experimental investigations using split-post dielectric resonators demonstrated that the accuracy of permittivity extraction strongly depends on operating frequency. At 5 GHz, the small frequency shifts induced by thin disks resulted in elevated uncertainty of the extracted permittivity. In contrast, operation at 10 GHz produced substantially larger frequency offsets and correspondingly improved measurement reliability. For the investigated YIG and W-type hexaferrite disks, consistent permittivity values were obtained at 10 GHz.

The method may be extended to other electrically insulating, low-loss magnetodielectric materials provided that the planar specimen can be driven into a sufficiently uniform magnetic state saturated perpendicular to its surface within the available bias-field range. In this state, the longitudinal permeability component relevant to the SPDR mode approaches unity, while the resonator geometry minimizes sensitivity to the remaining transverse permeability components. Potential candidates include ferrite–ceramic [24] and ferrite–polymer [25] composites. Materials with large in-plane magnetic anisotropy or high electric/magnetic losses would not be suitable for the presented method.

The results provide practical guidance for microwave characterization of small ferrite disks and indicate that higher-frequency SPDR measurements constitute a more robust approach for accurate

permittivity determination of low-loss magnetodielectric materials.

CRediT authorship contribution statement

Piotr Bogorodzki: Supervision, Resources. **Bartłomiej Salski:** Supervision, Funding acquisition. **Jerzy Krupka:** Writing – review & editing, Supervision, Software, Resources, Methodology, Formal analysis, Conceptualization. **Robert C. Pullar:** Resources, Investigation. **Adam Pacewicz:** Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgement

This work was supported by the OPUS 2024/53/B/ST7/01848 scientific project financed by the National Science Centre.

Data Availability

Data will be made available on request.

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