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ORIGINAL ARTICLE

Definitions by abstraction and Leibniz's notion of quantity

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Abstract

This paper analyses the abstractionist account of quantity championed by Leibniz, especially in the 1680s. Leibniz introduced the notion of quantity in an indirect way, via an abstraction principle. In the first part of the paper, I identify the context in which this approach arose in light of Leibniz's criticism of his earlier dream of an 'alphabet of human thought'. Recognising the impossibility of such a project led him to realise that, when dealing with terms referring to abstract objects, we should always consider them within the true sentences in which they occur. In the second part, I describe this approach in detail. This allows us to look at some key concepts of Leibniz's theory of quantity. In particular, I raise the problem of the relationship between the two sides of the abstraction principle: how should we think of the relation between the claim that *a* and *b* are equal, and the claim that the quantity of *a* is identical to the quantity of *b*? I argue that we can find a positive answer to this problem in Leibniz.

KEYWORDS

abstract objects, abstraction principles, definitions by abstraction, Leibniz, nominalism

1 | INTRODUCTION

At the beginning of the 1680s, Leibniz conceived of Universal Mathematics (*mathesis universalis*)—the theory of the basic and more fundamental mathematical concepts—as a logic of imagination,[1] which mainly deals with two aspects: quality and quantity.[2] One remarkable

[1] 'Universal Mathemathics must provide a method of exact determination for those things that fall under the imagination, or so to speak a logic of imagination' (A VI 4, 513–1683).

[2] 'Imagination generally revolves around two things, quality and quantity, or magnitude and form; according to which things are said similar or dissimilar, equal or unequal. And certainly the consideration of similitude belongs to general mathematics no less than that of equality' (A VI 4, 514).

feature is the fact that, in a number of writings, the concepts of quality and quantity (along with the concept of *situs* to which is dedicated a special science called *analysis situs*) are introduced via definitions by abstraction, namely via some implicit definitions based on some previously defined equivalence relations.[3] Leibniz introduces equivalence relations such as similarity, equality or congruence and exploits them to define, respectively, what it means for two objects to have the same qualities, the same quantity or the same situs. This is strictly connected to the strategy of proving truths by reducing them to identities, and the theory of substitution *salva*. Similar things have the same qualities, and therefore they can be substituted one for the other *salva qualitate*. Equal things have the same quantity, and therefore they can be substituted one for the other *salva quantitate*. As Rabouin (2024) clearly explains, the famous theory of substitution *salve veritate* is only a special application of this general theory of abstraction.

My aim in this paper is to dig deeper into Leibniz's abstractionist approach, with a special focus on the notion of quantity. I have chosen the latter since it is one of the central notions of Universal Mathematics, which became increasingly important in Leibniz's late reflections.[4] However, my arguments can easily be generalised to the other concepts. I shall start by expounding the general logical and mathematical context in which the idea of definitions by abstraction arises (Section 2): this context includes the reassessment of Leibniz's original project of the analysis of truths via the analysis of notions. The impossibility of determining (a complete list of) primitive terms has the consequence of highlighting the key role of propositions in the explication of abstract concepts. The project of definitions by abstraction should be assessed in light of the general conception that abstract notions should be explained via the (true) propositions in which they appear. In Section 3, I shall expound in detail Leibniz's introduction of the notion of quantity via the equality relation. This section will give us a clear picture of how definitions by abstraction work. In Section 4, I shall raise the question of how to understand the relationship between the two propositions appearing in a definition by abstraction. In our case, the two propositions involved are 'objects *a* and *b* are equal' and 'the quantity of *a* is identical to the quantity of *b*'. I shall argue that, even though Leibniz never explicitly raised this question, it is possible to find in his work an answer to it: his explanation of why truth is not arbitrary gives him the tools to provide an account of the relationship between these two propositions. This explanation is clearly linked to the abstract nature of the concepts involved: as such Section 5 briefly deals with Leibniz's ontological reading of these principles. Section 6 concludes.

2 | FROM THE YOUNG PROJECT TO THE LOGICAL PRIMACY OF PROPOSITIONS

It is well known that Leibniz had dreamt of an alphabet of human thought that should serve as a basis for a combinatorial reconstruction of all notions and truths.[5] Roughly, the idea was to proceed with an analysis of truths based on the analysis of notions, aiming at the determination of the primitive ones (i.e., those simple elements which are not further analysable). Once these primitive notions have been identified, they should be expressed through a *Characteristica*

[3]The term 'definition by abstraction' was coined in 1894 in the Peano school (see Mancosu, 2016, p. 14). In the contemporary literature, definitions by abstractions are also referred to as 'abstraction principles'. In the following I shall use both expressions synonymously. An example of an abstraction principle is given by the introduction of the notion of cardinal number by the so-called Hume Principle: the number of Fs is identical to the number of Gs if and only if there is a bijection between the collection of the Fs and the collection of the Gs. An equivalence relation (a bijection) is here exploited to explain what it means for two collections to have the same number of elements. As we will see, a similar approach was championed by Leibniz with regard to other basic mathematical concepts. On this approach in contemporary philosophy see, for example, Cook (2009).

[4]In the present paper I won't deal with Leibniz's metaphysical conception of the notion of quantity. On this topic, see Costantini (2022).

[5]Much has been written on Leibniz's dream of the alphabet of human thought. One locus classicus is Couturat (1901). More modern interpretations can be found in Antognazza (2009), Arthur (2014), Rabouin (2015) and Rabouin (2024).

Universalis (a general symbolic language that should avoid any ambiguity), and then they should be recomposed following precise combinatorial rules: in this way we could achieve a more precise grasp on the arguments and different kinds of controversies could be resolved. In *Lingua Universalis*[6] (dated February 1678), Leibniz even proposed associating a different prime number to each primitive term, with the aim of associating each complex concept with the product of prime numbers. Since each integer has a unique decomposition into primes, this would have permitted the unique identification of a complex concept with a determined sequence of primitive concepts.

According to Leibniz, the categories of scholastic logic provided an ordering of terms (or concepts); his project wanted to produce a similar order of complex terms, that is, an order of true propositions.[7] What is crucial is the fact that the analysis of truths[8] required for their ordering is based on the analysis of the notions that compose them. The project may thus be described as a bottom-up approach to propositions: first we need to determine their primitive components, and then we should reconstruct propositions from the bottom up through strict combinatorial rules. The emphasis is clearly on the primitive notions.

Even though scholars have often assumed that this project did not undergo any substantial change and remained the background project for Leibniz's entire life, Leibniz soon realised the limits of the analysis: not only are the primitive notions likely to be infinitely many, but it is also not clear that we are in a position to recognise a primitive term if we find one.[9] A possible solution to this situation is to *assume* that some terms are primitive and apply the method to them. Naturally, different situations may require the assumption of different primitive concepts, and the history of Leibniz's theory of quantity testifies to the fact that in different periods of his life, he assumed different concepts as primitive and/or derivative.[10] As Eco (1995, p. 276) recognises, 'the primitives need only be *postulated* as such for ease of calculation; it was not necessary that they truly be final, atomic and unanalysable'. However, the problem did not just concern the difficulties surrounding the primitive terms.[11] Most importantly, during his mathematical apprenticeship in Paris, Leibniz realised two fundamental things: first, that one of the goals of his project—the ordering of truths—already existed in the case of mathematical truths; second, that this order existed even though the basic mathematical concepts—such as that of prime numbers—are not completely understood by us. The fact that we do not have an algorithm that tells us where to find the primes on the scale of natural numbers, but we need to divide each integer into its prime factorisations, suggested to Leibniz that we are not in possession of a full analysis of the concept of prime numbers. But this does not prevent us from knowing many arithmetical truths. Therefore, Leibniz concluded that we do not need the analysis of notions to have a precise grasp of truths:

[6]'The best way to contract [expressions] will be that of reducing the thing to numbers multiplied among themselves, by positing that the elements of a character are all its possible divisors. […] Simple elements can be prime numbers, that is indivisibles' (A VI 4a, 65–66).

[7]'As a boy I learned logic […]. Seeing that there are categories for simple terms by which concepts are ordered, why should there not also be categories for complex terms, by which truths may be ordered?' (A VI 4, 538, in Leibniz, 1969, p. 229). Here 'simple terms' does not refer to terms not further analysable, but to terms that do not denote a proposition.

[8]Here and in what follows, I use the terms 'truth' and 'true proposition' synonymously.

[9]'We cannot easily recognise indefinable primary terms for what they are. They are like prime numbers, which we have hitherto been able to identify only by trying to divide them. Similarly, irresoluble terms could be recognised properly only negatively and provisionally' (A VI 3, 436–1676). On the changes that such a project underwent, see Rabouin (2015), section 1.

[10]Over the years, Leibniz struggled a lot with different presentations of his theory of quantity. For example, the abstractionist account that I am analysing here introduces quantity by means of other, more primitive notions, such as congruence and equality. By contrast, in *Initia rerum mathematicarum metaphysica*, quantity is introduced straight away, as a primitive notion, and equality is defined on the basis of it.

[11]Clearly, there were other factors that contributed to the amendment of the original project of the alphabet of human thought. A very important one was that the combinatorial theory of the *Ars Combinatoria* was explicitly restricted to necessary truths; but Leibniz would soon (around 1686) develop a theory of contingency which explicitly recognised that the analysis of contingent truths is a non-terminating one, so that contingent truths cannot be demonstrated a priori.

> a demonstration is perfect as soon as one can reach identical [propositions], which can happen even though everything is not analysed. For even notions which are not absolutely simple (like the parabola, or the ternary) can be stated from one another. (A VI, 3, 504, quoted from Rabouin, 2015, p. 54)

> It is very difficult to achieve the analysis of things, but it is not so difficult to achieve the analysis of truths needed. Since analysis of truth is achieved as soon as one has found a demonstration: and it is not always necessary to achieve the analysis of the subject or of the predicate to find the demonstration of a proposition. Most of the time, the beginning of the analysis of a thing is enough to reach the analysis or perfect knowledge of the truth that we get of this thing. (A VI 3, 671–1676, translation from Rabouin, 2015, p. 55)

The fact that we do not need a full analysis of notions to have a full analysis of truths means that the logical primacy should not be given to primitive terms, but rather to truths (true propositions). The immediate consequence is the centrality that the notion of proposition assumes in relation to the analysis of mathematical and philosophical terms. In particular, the centrality of proposition becomes evident when we are dealing with the analysis of 'abstract terms', that is, terms that denote abstract objects. In a wide variety of texts starting from the 1670s, Leibniz explicitly claims that the right evaluation of abstract terms requires us to consider the true propositions in which they compare. For example:

> Obliquities in incomplex terms cannot be fully analysed, without the explications of some sentences. Whoever thinks that everything concerning the incomplex terms can be explained before the complex terms is following a wrong method, in the same way as those who think that everything that regards the straight lines can be taught before the circles, or everything concerning the lines before the surfaces. (A VI 4A, 10, 1678, my translation)

In a much later text, we find the same thought:

> When abstracts are not beings, they are reducible to truths, for example, the rationality of a man is nothing but the truth of this proposition: "Man is rational." From this it is evident that the incomplex terms must often be grounded in the same complex terms, which nevertheless are posterior in nature to the incomplex terms themselves, of which they are a connection. (GP II 472, 1713–Leibniz (2007, p. 311), translation slightly modified)[12]

A few lines before this passage, Leibniz had written that 'abstract terms are likewise either beings or predicates. Thus virtue is a being, rationality is not a being. Abstract terms that are beings are absolute or relations' (GP II 471—Leibniz, 2007, p. 309). In this late text, the analysis of abstract terms through the true propositions in which they compare is restricted to a specific kind of terms, namely those abstracts expressed by predicates; it is therefore not meant to apply to all cases. However, with regard to abstract predicates, Leibniz here presents exactly the same analysis he developed in 1678. This analysis holds that there are terms (specifically, abstract predicates) that can be explicated only by looking at the true propositions in which they belong. In particular, the last quotation makes clear that this approach is consistent with the idea that these 'incomplex terms' are components of sentences, and therefore there is a sense

[12]The idea of evaluating abstract terms within the sentences in which they appear was already present in Hobbes's *De Corpore*, a text well known to Leibniz (see Di Bella, 2017, p. 210).

in which the meaning of the sentence depends on these components, but still the full meaning of the component can be clarified only by looking at the meaning of the whole proposition.[13]

The primacy of propositions in the explication of terms denoting abstracts implies the substitution of the analysis of notion with rigorous demonstrations. Here, Leibniz's famous programme consists in the reduction of propositions to identities: to prove a proposition means to reduce it to some identity (this can be either a proposition of the form A = A or a proposition stating an equivalence relation). The abstractionist approach to the notion of quantity is exemplary of this programme: the notion of quantity is explicated via a proposition which is reduced to an equivalence relation. In other words, definitions by abstraction clarify the meaning of a concept (for example, the concept of quantity) by looking at truths (true propositions) that contain it and by fixing the truth values of these propositions via some previously defined equivalence relation.

3 | DEFINITIONS BY ABSTRACTION

A concrete example of the difficulty in finding suitable primitive terms is given by Leibniz's attempts to establish the *Initia* (the foundations) of mathematics. Due to its highly abstract nature, mathematics should have been the easiest setting in which to perform the initial project of analysing notions; however, Leibniz struggled for his entire life with the analysis and reconstruction of the basic mathematical concepts. The difficulty was to individuate the most appropriate primitive terms from which to define and derive all the others. Over the years, he developed different proposals: here I shall focus on one in particular, the one which is based on definitions by abstraction (see Rabouin, 2024). Because of the difficulty in defining basic mathematical concepts, such as quality, quantity, or *situs*, Leibniz adopted a different approach: he introduced the basic concepts not directly, but by explaining what it means for two objects to have the same qualities, to have the same quantity, or to have the same *situs*. This explanation amounts to giving the truth-conditions for sentences of the form 'the quality/quantity/situs of A is the same as the quality/quantity/situs of B'. This is done via some equivalence relations: respectively, the relations of similarity, equality and congruence. In this paper, I will only focus on the notion of quantity, leaving the study of the other relations for another occasion.[14] An illustrative instance of this approach is the writing *De Quantitate* (GM VII 29–35),[15] which I shall briefly analyse here.

The paper opens with the concepts of *determinant* (Leibniz's way of indicating a functional correspondence) and *coincidence*, which indicates those things that are identical but differ only in their denominations. After the concept of coincidence, we find the concept of congruence:

> Congruent are those things that, if they differ, they can be discerned only through something external [*respecta ad externa*] […], or when they are in different places at the same time, or when an object C is made of gold, the other D of silver. In this way a pound of gold and a pound of lead are congruent; today and yesterday are

[13]Here Leibniz is clearly dealing with what Frege would call the context principle, and its relation to the compositionality of meaning. On Frege and the context principle, see Linnebo (2018, ch. 7). The primacy of propositions, especially in contexts where we are dealing with 'abstract terms', has a direct precedent in Hobbes's book *De Corpore*: 'There is also this difference betwixt *concrete* and *abstract* names, that those were invented before propositions, but these after; for these could have no being till there were propositions, from whose *copula* they proceed' (Hobbes, 1992, p. 33). It is particularly significant for our current purposes that in these passages Hobbes explicitly speaks of quantity as a key example of an abstract notion. For instance, dealing with the abuse of abstract notions, Hobbes writes: 'and because quantity may be considered without considering body, they think also that quantity may be without body, and body without quantity; and that a body has quantity by the addition of quantity to it. From the same fountain spring those insignificant words, *abstract substance*, *separated essence*, and the like' (Hobbes, 1992, p. 34).

[14]A deep analysis of definition by abstraction is given by Rabouin (2024). See also De Risi (2007), especially for the notion of *situs*.

[15]Exactly the same approach can be found in *Characteristica geometrica* of 1679 (subsection 23, 24–GM V 150), and *Specimen ratiocinationum mathematicarum, sine calculus et figuris* (A VI 4a 417–423).

> congruent. Any point is congruent to whatever other point, and an instant with whatever other instant. (GM VII 29)

This is a clear case of what scholars have called a phenomenological characterisation. Congruence is a primitive concept that cannot be directly reduced to more basic notions. As a consequence, Leibniz introduces it by explaining how we come to know about it, that is, by comparing (congruent) objects with a third (and external) element that allows us to distinguish the two (congruent) objects. Two squares with the same area can only be distinguished by their different positions, that is, by recurring to an extrinsic denomination.[16] Having introduced the concept of congruence, Leibniz proceeds with the concept of equality:

> Equals are those things that are congruent […] or can be made congruent through a transformation. […] We can also define equal those things that can be decomposed into different (and disjoint) parts which are one by one congruent. (GM VII 29–30)

Congruent things are equal, and for this reason we cannot discern two congruent squares through their respective areas, since they have the same area. There are also equal objects which are not strictly speaking congruent, but which can nevertheless be made congruent via a transformation. Leibniz's own example regards a triangle with a certain area—let us say *m*—and a rectangle of the same area *m*. They are clearly not congruent since they differ in multiple aspects (one has three sides, the other four, etc.), but the triangle can be transformed into a rectangle which is congruent to the rectangle given. One can think of this transformation in the following way: the original triangle is decomposed into disjoint parts (sometimes Leibniz speaks of *minima*, in the sense that these parts can be taken as small as one pleases),[17] and then these parts are recomposed into a rectangle. Since the original triangle and the resulting rectangle are composed of the same parts, their area will be the same, and so equality is preserved under transformation.

The notion of transformation that appears in this definition is a technical notion. Any transformation must preserve both (topological) dimensionality and equality.[18] In other words, there can be a transformation only between objects of the same dimension (there can be a transformation between a square and a triangle, but not between a line and a square); moreover, there is no transformation between a square of area *m* and a rectangle of area *m*/2.[19] The concepts that are introduced at this point in the writing are the concepts of similarity ('similar objects are the things that cannot be discerned when taken singly'—GM VII 30), homogeneity ('homogeneous are those objects that are similar or that can be made similar through a transformation'), the part-whole relation (where the part is characterised in Leibniz's standard way as something which *is in* the whole—*inesse* relation—and is homogeneous to it), and the notion of quantity. On quantity, Leibniz writes: 'we can define the quantity of a thing as the property of the whole insofar as it has all its parts'. Such a characterisation establishes a strong relation between quantity and the part-whole relation: in fact, the mereological structure of an object is what allows its quantitative comparison with other objects. For example, just before this characterisation, Leibniz gives a definition of what it means for an object A to be less than another object B: A is said to be less than B if A is equal to a (proper) part of B ('the smaller is what is equal to a part of the other (the bigger)').

[16]'Extrinsic denomination' is a scholastic term which indicates a property of an object A, the ascription of which always involves reference to one or more individuals other than A.
[17]This happens, for instance, in *Specimen geometriae luciferae* (GM VII 260–295).
[18]'Magnitude [or quantity] is what is preserved under a transformation, or what can be distinguished in similar things' (A VI 873).
[19]For an analysis of such a notion, see De Risi (2007, ch. 2).

However, despite containing the word 'definition', this characterisation is hardly a proper definition of quantity[20]: Leibniz is just saying that whenever we have a whole with parts, we have some quantity, and vice versa, but he is not making clear exactly which property of a whole quantity is.

A better introduction of the notion of quantity can be found a few pages later, in a section where Leibniz explicitly derives consequences from the definitions he has presented:

> From these definitions we can derive the following sequences of axioms. […]. Given the same measure, the equals are expressed by the same number, or *they have the same quantity*. (GM VII 34, my emphasis)

Here we find a clear case of a definition by abstraction.[21] Leibniz had introduced the notion of equality via congruence (and transformation), and here he is deriving consequences from these notions. The consequence he is deriving is that equal things have the same quantity. This can directly be interpreted as an abstraction principle, where the notion of quantity is introduced via an equivalence relation (equality):

$$(\textbf{Def. of Quantity}): Qt(a) = Qt(b) \leftrightarrow E(a,b)$$

where $E(a,b)$ indicates that a and b are equals, and $Qt(a)$ denotes the quantity of a.

Leibniz is here introducing the notion of quantity through the equality relation. Equality allows the substitution of the relata a and b *salva quantitate*. What the substitution shows is that the quantity of one object is the same as the quantity of the other. This substantiates the idea of equivalence relations as special kinds of identity. In this way, he can introduce the notion of quantity without giving an explicit definition but instead by fixing the truth-conditions of a sentence like '$Qt(a) = Qt(b)$', by exploiting another proposition, namely $E(a,b)$. Therefore, we do not have an analysis that reduces quantity to more basic notions (it is not the case that by analysing the definition of quantity we find the notion of equality as a component of this definition); rather, we are introducing the concept of quantity by stating its *conditions of identity*, and this is enough to use it in mathematical contexts. This ensures two things: first, the right-hand side of the abstraction principle explains under which conditions the left-hand side is true (this is what is sometimes called the *explanatory role* of abstraction principles in the contemporary literature); second, the right-hand side also has an *epistemic role*: the knowledge of '$E(a,b)$' is sufficient for the knowledge of '$Qt(a) = Qt(b)$'.

The roots of the abstractionist approach are clearly to be found in the failure of the idea that we need to analyse concepts into their simplest elements to be able to perform exact derivations and proofs. In other words, to know what quantity is, we must look at true propositions (complex entities) that contain the notion of quantity: in particular, Leibniz's abstractionist approach suggests that we look at identity statements of the form 'the quantity of a is identical to the quantity of b'. And the truth-conditions of such a proposition are fixed by an equivalence relation (in this case the equality relation) between object a and object b.

There are two points of this approach that need to be investigated. The first is the relationship between the two sides of the bi-conditional. The second concerns the ontology of the abstract notions. I shall deal with these points in the next two sections.

[20]By 'proper definition', I mean a definition that explains a term via some more basic notions.

[21]One may object that I am treating as a definition what Leibniz calls here an axiom. There are two things to notice here. First, Leibniz claims that this axiom can be derived, that is, it can be proved. As was customary at that time, axioms were considered (evident) truths that should be demonstrated; second, I am using the term 'definition' in a very specific sense, as referring to definition by abstraction (and not to standard definition as when one introduces a concept via other, more basic concepts). I am claiming that, from a contemporary perspective, what Leibniz is doing here is exploiting an equivalence relation to explain when two things have the same quantity. In other words, he is making use of a definition by abstraction. Thanks to a referee for calling for the clarification.

4 | ABSTRACTION PRINCIPLES AND THE PROBLEM OF EXPRESSION

(Def. of Quantity) states an equivalence between two different propositions: 'the quantity of a is identical to the quantity of b', and 'a and b are equal'. Clearly, this does not just mean that the two propositions are materially equivalent; rather, they are necessarily equivalent. However, necessary equivalence can hardly account for the relation between the two propositions in an abstraction principle, since it cannot guarantee either the epistemic or the explanatory role that an abstraction principle should have.[22] Is Leibniz in a position to say more on this relationship? This is one of the most discussed points in contemporary debates on abstraction principles, and there are a number of proposals.[23] In contemporary post-Quinean terms, the main problem consists in the fact that the two propositions seem to express the same content, but they do so by referring to different (kinds of) entities. In particular, from $Qt(a) = Qt(b)$, within First-Order Logic, we can infer $\exists x(x = Qt(b))$, that is, there is something which is a quantity. In order for $Qt(a) = Qt(b)$ to be the case, it seems that we must acknowledge the existence of quantities. This is not the case for $E(a,b)$, from which we can only infer that $\exists x E(x,b)$, that is, there is something equal to b. In order for this claim to be true, it is not necessary that we commit ourselves to the existence of quantities. How can these claims be equivalent if, prima facie, they have different ontological commitments? It must be stressed that this is not a problem for Leibniz, who did not endorse anything like Quine's criterion of ontological commitment. It is well known that Leibniz preferred intensional over extensional interpretations of logic, that is, he preferred a conception of logic where logic deals with concepts (intensions) which do not require the existence of the individuals that instantiate them, rather than a conception of logic that deals with extensions, that is, collections of individuals.[24] However, it is still legitimate (and indeed fruitful) to ask whether we can find in Leibniz a suitable notion of content compatible with definitions by abstraction. The question is legitimate, not only because it arises as soon as definitions by abstractions are given, but also because Leibniz is committed to the idea that different propositions may express the same content. The most famous example is given by any general claim such as 'every human being is an animal'.[25] Leibniz held that this can be interpreted in an intensional way (the concept of human being includes—as a component—the concept of animality) or in an extensional way (the class of animal includes—as a subclass—the class of human beings). The two interpretations correspond to different propositions, which nevertheless seem to say the same thing. Moreover, Leibniz considers the two interpretations to be in some sense equivalent.[26] It is thus legitimate to ask which notion of content this idea presupposes. It seems to me that, with a bit of work, we can extrapolate a positive answer to this question. To do that, it is useful to look at his conception of the relation between characters (linguistic signs) and truth. Let us start by examining an early text (1677) where Leibniz, following the lesson of Johann Bisterfeld, criticises the extreme nominalistic position of Hobbes:

> I notice that if characters can be applied to reasoning, there must be some complex arrangement, some order which agrees with things, an order, if not in individual words (though that would be better), then at least in their conjunction and

[22]On this problem, see Linnebo (2018, p. 78).

[23]Linnebo (2018, ch. 4) provides a discussion of some of the most recent proposals.

[24]On this topic, see Mugnai (2016, section 2).

[25]'For when I say *Every man is an animal* I mean that all the men are included among all the animals; but at the same time I mean that the idea of animal is included in the idea of man. 'Animal' comprises more individuals than 'man' does, but 'man' comprises more ideas or more attributes' (Leibniz, 1996, p. 486).

[26]Despite having different ontological commitments (the intensional interpretation does not require the existence of individuals that instantiate the concepts), Leibniz believed that the intensional interpretation of logic is reciprocal to the extensional interpretation. The fact that the concept of human being contains the concept of animal means that the collection of animals contains the collection of human beings. On this idea and the problems that it raises, see Mugnai (2016).

> inflection. And a corresponding variegated order is found in all languages in one way or another. This gives me hope that we can avoid the difficulty. For though the characters are arbitrary, their use and connection have something that is not arbitrary, namely, a certain correspondence [*proportio*] between characters and things, and certain relations among different characters expressing the same things. And this correspondence or this relation is the ground of truth. For it brings it about that whether we use these characters or others, the same thing always results, or at least something equivalent, that is, something corresponding in proportion always results. This is true even if, as it happens, it is always necessary to use some characters for thinking. (A VI 4, 24/AG 271)

Hobbes (according to Leibniz's view) had defended the idea that truth depends on the definitions of words, and since these are arbitrary, truth will be arbitrary too: truth just regards our thoughts and not reality. Leibniz agrees that truth depends on definitions, and definitions are conventional, but he wants to resist Hobbes's conclusion. To do so he notices that, even though names and definitions may well be arbitrary, there is still a non-arbitrary element, which is individuated in a *proportion* that exists between characters and what they express. In the *Dialogue* from which the above passage is taken, Leibniz argues that characters are necessary for the expression of truths, but that different characters may well express the same truths. Notice that the correspondence between 'the things' and different systems of characters does not regard single words (which usually vary from system to system) but rather the connection between the words (in a sentence). Put otherwise, the correspondence lies at the level of the whole proposition and not at the level of its single components. Here it seems that Leibniz has mainly in mind the fact that the same truth can be expressed in different languages, that is, by means of different linguistic characters.[27]

One might be tempted to interpret this passage as suggesting that the proportion/correspondence that guarantees truth is due to the *syntactic structure* of the propositions. The idea would be that there is a correspondence between the order of the world and the grammatical order of sentences that plays a key role in assuring the truths of propositions. This idea may work when we are dealing with sentences about substances, that is, things for which there exists a complete concept. Since the complete concept contains all the attributes of a substance, and a truth about a substance is just the claim that one of the attributes of the substance belongs to its complete concept, it makes sense, then, that there is correspondence between the syntax of the true proposition and the order of the things. In the same way in which the predicate is contained in the subject in the true proposition, so the correspondent attribute is in the substance. Puryear (2006) insists on this representational nature of language. Following his example, the sentence 'Socrates is a man' expresses the idea that the complete concept of Socrates includes the concept of man, 'thus the sentences relate the predicate to the subject in a way that parallels the way in which the proposition relates the concept of the predicate to the concept of the subject' (Puryear, 2006, p. 21). However, this idea can hardly work for abstract terms, of which there is no complete concept. For example, we already know that the general claim 'every human being is an animal' can be interpreted in both an intensional and an extensional way. According to the former, it expresses a conceptual containment (the concept of human being includes the concept of animality); for the latter, it expresses an inclusion between collections or extensions of concepts (the collection of animals includes the collection of men). The two interpretations correspond to different propositions (one about concepts, the other about extensions/collections of concepts), where the containment relation is inverted ('human being' contains 'animality' vs. 'animality' contains 'men'), and yet Leibniz considers these different propositions to be

[27]Clearly, Leibniz had in mind Descartes' answer to Hobbes, which consists in pointing out that we can express the same ideas in different languages (a discussion of this point can be found in Di Bella, 2017, p. 206).

equivalent.[28] Moreover, the idea of a syntactic representation of the order of the (external) things fails to give a notion of content which is adequate for our needs. In fact, if the syntactic structure enters into the determination of content, then the resulting notion of content is too fine-grained to explain the relationship between the two sides of an abstraction principle. The two sides have different syntactic structures and thus they express different contents.[29]

However, Leibniz's reply to Hobbes above suggests a further idea. Since the correspondence of which Leibniz speaks lies at the level of proposition and not at the level of words or objects, truth does not directly regard objects but the connection of objects. This opens up the possibility that the same truth can be described by reference to different (kinds of) objects. If this is the case, the proportion/correspondence that should guarantee truth is not merely the fact that the syntax should mirror the order between the objects, but that different propositions with different syntactic structures (and different subject matters) are made true by the same content. In other words, different propositions correspond to different perspectives on the same content: they *express* the same from different points of view.

The notion of expression clearly refers to Leibniz's mature theory of monadic expression.[30] The fact that this text may contain the seeds of such a theory is implicitly suggested by Di Bella (2017), who claims that Leibniz's reply to Hobbes is based on the recognition that 'all different expressions of one and the same proposition, though being conventional, must respect a certain order or share a common structure. This isomorphism [is] captured by the key technical notion of expression […]' (Di Bella, 2017, p. 206).[31]

My interpretation amounts to the following: the two statements in a definition by abstraction are equivalent because they have the same truth makers, that is, they are true in virtue of the same state of affairs. This requires that the correspondence (or proportion) between words and things is to be found neither at the level of the single words nor at the syntactic level, but at the level of the whole state of affairs described by a proposition. A state of affairs is something complex that can be described by attributing some properties or relations to one or more things. A state of affairs is thus a connection or relation of some kind. The first thing to notice is that, even though the expression 'state of affairs' is not a Leibnizian one, there are passages that suggest that he was not so far away from such a notion: 'Let us call "L" the state, by virtue of which A is B, and "M" the state, by virtue of which C is D' (A VI 4: 863, quoted from Di Bella, 2017, p. 213). Moreover, in the *Generales Inquisitiones*, Leibniz introduces what he calls 'logical abstracts'[32] that allow him to transform a categorical sentence of the form 'A is B' into a singular term such as 'A-being-B', which—in

[28]Another example that shows the inadequacy of this interpretation is given by different numerals (Roman numerals, Arabic numerals, etc.) which express the same content, that is, the number system. Thanks to a referee for pointing to this example.

[29]This is exactly the situation we face in the cases of structured propositions and/or Russellian propositions.

[30]A similar view can be found in Spinoza's *Ethics*. In the Scholium to Proposition 7, part II, Spinoza writes: 'the thinking substance and the extended substance are one and the same substance, which is comprehended now under this attribute, now under that. So also a mode of extension and the idea of that mode are one and the same thing, but expressed in two ways […] So whether we conceive Nature under the attribute of extension or under thought or under any other attribute, we shall find one and the same order, or one and the same connection of causes' (Spinoza, 2017, p. 25). The idea here is that the order of Nature will be expressed no matter under which attributes we perceive the substance. Therefore, different attributes give us different ways to express the same thing, which is exactly the idea that we found in Leibniz's passage (thanks to Noa Shein for calling my attention to Spinoza on this point).

[31]Here, Di Bella is using the word 'proposition' in a slightly different way from how I am doing so. According to his usage, an abstraction principle would express only one proposition in different ways, while according to my usage, the two sides express different propositions. This is only a nominal difference. On the interpretation of Leibniz's notion of expression through an isomorphism, see McRae (1976). De Risi (2007) and Swoyer (1995) relax the requirement and suggest that expression can be captured by a simple homomorphism. A concise discussion can also be founded in Arthur (2021, pp. 179–180).

[32]The expression 'logical abstracts' comes from *De Abstracto et Concreto* where Leibniz suggests that we can transform the philosophical abstracts, namely universals such as Wisdom, Rationality, and so forth, into logical ones (being wise, being rational, etc.). Notice that in logical abstracts, the propositional nature of abstracts (i.e., the fact that they have their origin in the connection between subject and predicate) is evident by the presence of the verb *esse*. A proposition such as 'A is B' is equivalent to the terms 'the B-ness of A' and/or 'A-being-B'. However, the latter is to be preferred because it dispenses with a philosophical abstract in favour of a state of affairs (a logical abstract). Section 139 of the *Generales Inquisitiones* claims that both kinds of abstract have their origin in proposition, making clear that abstract terms should be explained within the (true) propositions in which they are contained: '(139) In general, however, if we say that something is B, then this "something being B" is nothing else than the B-ness itself. Thus, "something being animal" is nothing else than "animality". Whereas "man's being animal" is the animality of man. From this originate both the abstract term and such an oblique' (Leibniz, 2021, p. 115).

modern jargon—can be said to be a state of affairs. This point is worth emphasising. According to Leibniz's theory of truth, a proposition such as 'A is B' is true when the predicate B is contained in the subject A, otherwise it is false. What grounds the truth of propositions is the connection between ideas: for example, 'Socrates is wise' is a true proposition because the concept of wise is included in the concept of Socrates. But this can be *equivalently* restated by claiming that such a proposition is true in virtue of the obtaining of the state of affairs 'Socrates-being-wise'. Therefore, the notion of state of affairs can be seen as a central one in Leibniz's logic and philosophy. It must be remarked once more that an important feature of states of affairs such as 'Socrates-being-wise' is that they are abstract (indeed, Leibniz calls them logical abstract). They are abstract in the sense that a state of affairs like 'Socrates-being-wise' only regards an attribute of Socrates between the infinitely many attributes included in his complete concept. It is as if a state of affairs consisted in a selection of one or more attributes, while omitting all the others.[33]

At this point, the correspondence between characters and things should be interpreted to obtain at the level of state of affairs, which simply means that, despite characters being arbitrary, their connection in a true sentence manages to express an objective connection between objects or ideas.[34] If we follow this suggestion, Leibniz's proposal would amount to the claim that two equivalent propositions share the same proportion or correspondence because they are made true by the same state of affairs, that is, they express the same connection of ideas, and this connection actually obtains. Hobbes's mistake consists in deriving the arbitrariness of truth directly from the arbitrariness of characters, but the latter is not enough to conclude with the former because truth does not directly regard characters; rather, it regards the connection of ideas. It is therefore possible that the same truth is expressed by different systems of characters. Let us consider a couple of examples. First, consider again the concept of human being, which contains the concept of animality, and the collection of animals which has as a sub-collection the collection of human beings. The correspondent propositions are different (one is about concepts, the other about individuals), but it is clear that they are true in virtue of the same state of affairs, which in this case is the fact that human beings are animals (or the fact that the concept of human being contains as an ingredient the concept of animality). We may express this connection in different ways (the intensional or the extensional way), but in both cases, the correspondent proposition is made true by the obtaining of the same state of affairs (or logical abstract: human beings-being-animal). Their difference is accounted for by the fact that they provide different descriptions of the same state of affairs. Second, in the case of the definition of quantity analysed above, to claim that *a* and *b* are equal is to claim that there is a certain relation between *a* and *b*, that is, either they are congruent or they can be made congruent through a transformation. The subject matters are the objects *a* and *b* of which it is claimed that they are equal. On the contrary, the claim that the quantity of *a* is the same as the quantity of *b* has a different subject matter: the claim is not about *a* and *b*, but rather about an aspect of them, that is, their quantity. Of these quantities it is said that they are identical (and not that they are equal). Therefore, the two propositions are different because they have different subject matters, but they are linked by the fact that they are true in virtue of the same state of affairs. This state of affairs is the one obtaining between the lines *a* and *b* (*a* being equal to *b*). What makes true one proposition is exactly what makes true the other one, that is, the obtaining of *a*-being equal to-*b*. Such a notion of content (i.e., the idea that to express the same content means to be made true by the same state of affairs) is thus fine-grained enough to tell apart the two propositions

[33]It is important to stress that here states of affairs are not necessarily parts of reality. In particular, the state '*a* being equal to *b*' does not correspond to anything that exists for Leibniz, since for him there is no perfect equality in the world. Thus 'a being equal to b' is a state of affairs not about the world but about ideal objects. It does not describe a feature of reality *strictu* sensu.

[34]I will talk indifferently of connections of objects/things and ideas. This is because what we would usually consider a connection between objects or things such as the connection between Socrates and the property of wisdom is interpreted by Leibniz as a connection between ideas, that is, between the complete concept of Socrates and the concept of wisdom.

(they are different because they are about different subject matters), but it is coarse-grained enough to guarantee that necessarily they have the same truth value.

The passage from the right-hand side to the left-hand side of (Def. of Quantity) can be described as a reconceptualisation of the truth expressed by $E(a,b)$. Equivalence relations are special kinds of identity: to be equivalent with regard to a certain property *p* means to be *p-identical*: from the point of view of *p*, *p-equivalent* objects cannot be discerned. To be equal means to be indiscernible from the quantitative point of view, and so to have the same quantity. What was before only an aspect of objects *a* and *b*, namely the fact that they have a quantitative dimension, is now (in the left-hand side) considered *as if* it were an entity or a thing in its own right (the quantity of *a* and the quantity of *b*).[35] To claim that these two are identical amounts to claiming that the original two objects were equal: these are different ways of expressing the same connection of things or ideas. It is the same state of affairs described with reference to different subject matters.[36]

5 | THE ONTOLOGY OF ABSTRACTS

The abstraction principle (Def. of Quantity) allows us to introduce an abstract notion—quantity—via an equivalence relation between two objects. The equality between *a* and *b* shows that according to their quantitative aspect, these two objects are indiscernible. It is this indiscernibility that allows us to say that, under the quantitative aspect, they are identical, that is, they have the same quantity. The passage from right to left represents an abstraction: we are considering a feature of *a* and *b* independently from all other features of these objects, and we introduce a singular term referring to it. We are considering quantity not as an aspect or mode of a thing, but *as if* it were an entity in itself. Therefore, we have two equivalent propositions that seem to require, at least prima facie, different ontologies. How is it possible that these propositions are equivalent?

Leibniz's strategy of paraphrasing sentences with philosophical abstract terms into sentences with logical abstract terms (from 'Socrates possesses the property of wisdom' to 'Socrates is wise') requires us not to read out ontological commitments from the grammatical structure of a sentence. The idea that the grammar should not be given too much weight is even explicitly claimed in different texts, such as the following:

> The difference between substantives and adjectives can be omitted in the *Characteristica*; in fact, between *corpus* and *extensum* there is nothing else of what *corpus* already means: extended subject; this, however, already contains the notion of extension. In this way *homo* is nothing other than human subject or subject of humanity. (A VI 4a: 334–335, my translation)

A bit anachronistically, we may say that this implies the rejection of Quine's criterion of ontological commitment. In the present case, even if '$Qt(a) = Qt(b)$' is a true proposition, and '$Qt(a)$' is a singular term, we cannot conclude that quantity is a thing in the same sense in which *a* and *b* are things. Indeed, Leibniz denies that this is the case:

> Until now I have not seen another way of avoiding such difficulties than considering abstracts not as things, but as *compendia loquendi*, as when I name the heat, it

[35] Leibniz uses abstraction principles to provide an implicit definition of some mathematical concepts and not to introduce 'new' mathematical objects as happens with some contemporary theories such as that of Linnebo (2018).

[36] I would stress that the present proposal is not in contrast with Leibniz's idea that equal terms can be substituted with each other *salva quantitate*. Indeed, the substitution shows that equal objects have the same quantity, and so it is a way of exhibiting that 'being equal' is equivalent to 'having the same quantity'. However, it does not explain in virtue of what this is the case. Our proposal aims at offering such an explanation.

> is not necessary that I mention a certain vague subject; or that I say that something is hot, and to that extent I am a nominalist [*et eatenus sum nominaliis, saltem per provisionem*], at least for the moment (or provisionally). […] It is enough that only the substance is considered as a thing (*res*), and to enunciate truths about it. Also the geometers do not use the definition of abstracts, *but they reduce them to the concretes*, so Euclid does not use the definition of a ratio, which he nevertheless had, but he explains it through the things that are said to have a ratio that is identical, greater or smaller. (*De Realitate accidentis*, A VI 4 996, my emphasis)

The abstraction process introduces some terms which are merely abbreviations for expression, namely a shorter way to express the same content of the right-hand side of (Def. of Quantity). This must be understood in the sense explained in the previous paragraph: we are expressing the same state of affairs, that is, the same connection of ideas, but using a different terminology. Again, notice the primacy of proposition over terms: having considered the substances, to explicate the abstract terms it is enough to consider truths about substances. Even if, from a grammatical point of view, we have singular terms that play the part of the subject, they do not really refer to something existing, but they can be fully dispensed with.

Particularly interesting in the last quotation is the reference to Euclid and the fact that Euclid defines ratios by explicating what it means for two things to have *the same ratio*.[37] Therefore, Leibniz is clearly speaking of definitions by abstraction. And he describes this definitional strategy by saying that 'geometers do not use the definition of abstracts, *but they reduce them to the concretes*', which shows Leibniz's nominalistic reading of such definitions.

This nominalistic interpretation of abstraction principles consists in a double move: first, the idea that the left-hand side of a definition by abstraction is true (in our case, '$Qt(a) = Qt(b)$' expresses a true proposition); second, that this truth does not require a further ontological commitment with regard to what is required by the truth of the right-hand side ('$E(a,b)$').[38] This is possible because the terms 'quantity of *a*' and 'quantity of *b*' do not really refer to further objects (with regard to *a* and *b*), but they are only abbreviations for expression. Alternatively, we may say that their references are *fictional objects*: 'fictional' here indicates that we feign that there are such objects, and that we speak as if they existed, even though their existence is only a fiction of the mind. Notice that the term 'fictional' here is a relative term: $Qt(a)$ is a fictional object *in relation to a* (and the same for $Qt(b)$ and *b*).[39] In fact, by moving from left to right in (Def. of Quantity), we could dispense with $Qt(a)$ and work only with *a*. But what kind of thing is *a*? Leibniz's passage above, where he speaks of 'reducing them to concretes' may suggest that *a* and *b* are concrete items, in the sense that *x* is concrete if it is in space–time and has causal power. Yet, however natural this reading may be, it is a gross mistake. In fact, in the quotation above Leibniz mentions Euclid's characterisation of *ratio* where Euclid explains what it means for two geometrical magnitudes (e.g., lines, surfaces, etc.) to have the same ratio. The 'concretes' in that quotation are geometrical objects, which are in fact abstract objects. The same happens with (Def. of Quantity): '$E(a,b)$' describes an abstract state of affairs. We have argued above that every state of affairs is abstract, since it singles out only some attributes of an object, not the whole of its features. In Leibniz's terminology, a state of affairs is a logical abstract. What makes true the two sides of a definition by abstraction is thus an abstract state of affairs, that is, an abstract connection of things (or ideas). The reduction of which Leibniz speaks is thus a reduction of certain abstract entities to other abstract notions. There is thus no way of fully getting rid of abstracts via Leibnizian abstraction principles. This is exactly the relative

[37] 'To conclude, I have done here much like Euclid, who, not being able to make his readers well understand what ratio is absolutely in the sense of geometers, defines what are the same ratios. Thus, in like manner, in order to explain what place is, I have been content to define what is the same place' (Leibniz & Clarke, 2000, p. 47).

[38] It is this second condition that makes Leibniz's interpretation of definitions by abstraction nominalistic.

[39] On this specific sense of 'fictionality', see Rabouin (2022).

sense of fiction mentioned above: the dispensability of some abstracts (the quantities of *a* and *b*) is only relative to the acceptance of other abstract entities: the state of affairs *a*-being-equal-to-*b*. This is the idea that Leibniz nicely summarised in the *New Essays*:

> The Scholastics hotly debated *de constantia subjecti*, as they put it, i.e. how a proposition about a subject can have a real truth if the subject does not exist. The answer is that its truth is a merely conditional one which says that if the subject ever does exist it will be found to be thus and so. But it will be further asked what the ground is for this connection, since there is a reality in it which does not mislead. The reply is that it is grounded in the linking together of ideas. In response to this it will be asked where these ideas would be if there were no mind, and what would then become of the real foundation of this certainty of eternal truths. This question brings us at last to the ultimate foundation of truth, namely to that supreme and Universal Mind who cannot fail to exist and whose understanding is indeed the domain of eternal truths. (Leibniz, 1996, p. 477)

A true proposition may be about a subject matter that does not exist. This is possible because the proposition states how things would be if the subject existed. A proposition thus expresses some connection between objects or ideas, and therefore what makes the proposition true is the existence of a certain connection (what we have called the obtaining of a state of affairs). This connection is abstract, but also objective because it is ultimately grounded on ideas in the divine intellect.

Leibniz's nominalist interpretation of definitions by abstraction is thus embedded in a realist philosophy. What grounds the truth of the statements that appear in such definitions are abstract concepts (connections of ideas) that exist in the mind of God. For this reason, mathematical truths are eternal truths. But these connections may be described in different ways, by resorting to a different set of terms, and these terms must not be thought of as requiring the existence of the objects to which they seem to refer. At this point, one may wonder why so much emphasis should be put on the paraphrase if we can never fully dispense with all abstracts. Moreover, is there a difference between abstracts that we can dispense with and those that we cannot?

We have already seen that the left-hand sides of definitions by abstraction—the quantity of *a* is the same as the quantity of *b*; the qualities of *a* are the same as the qualities of *b*; the situs of *a* is the same as the situs of *b*, and so forth, —treat as an entity in itself, a *res*, what is only a mode or accident of individual things. The dichotomy that Leibniz has in mind here is that between the individual thing (*res*) as the ontological subject of inherence and predication, and the accidents, that is, the properties that can be said of an individual.[40] The abstracts that we should dispense with are thus the abstracts that refer to modes or accidents of individual things considered as things in themselves: 'since I have excluded abstract terms, I have also excluded those substantive names that signify not substance, but accidents' (A VI 4574). In the same way in which the passage from the right- to the left-hand side represents the process of abstraction or reification of modes and accidents into individual things, so the passage from left to right gives us a way of dispensing with those abstracts. The paraphrase is important to avoid the mistake of treating accidents and modes as individual things. On the contrary, what we cannot dispense with are abstract concepts and ideas, and the connections among them. These are objective ideas in the mind of God, and they lie at the ground of the same idea of truth as the containment of (the idea expressed by) the predicate in (the idea expressed by) the subject. To dispense with them would require dispensing with truth itself, something Leibniz would rightly regard as absurd. However, even when we are dealing with dispensable abstracts, it should be stressed that Leibniz has nothing to complain about with the use of them. He is in fact not proposing to always paraphrase propositions that contain them through propositions that do not

[40]On the distinction between *res* and modes, Di Bella (2018) is very instructive.

contain them. His position is more accommodating of this kind of abstracts: we may use them provided that we can, in principle, always dispense with them.

6 | CONCLUSION

In this paper, I described Leibniz's use and conception of definitions by abstraction, looking at one specific case, namely that of quantity. I first identified as the background view the failure of his initial project of an alphabet of human thought with the consequence of the primary role that the notion of proposition acquires with regard to its components. Definitions by abstraction are a case in point of the idea that in cases where we cannot directly define some abstract notions, we should look at true propositions about them, and use them to rigorously introduce these notions via equivalence relations.

Secondly, I dealt with the problem of the relation between the two sides of an abstraction principle: I argued that the statements differ because they are about different subject matters, but they are equivalent because they are made true by the same state of affairs. The two statements express the same content by referring to different objects. Indeed, the mature theory of monadic expression will have each monad express the whole universe from its own unique perspective: the same content is expressed in infinitely many ways.

Finally, I argued that while Leibniz's theory of definitions by abstraction is nominalist, it is embedded in a realist philosophy of mathematics. It is nominalistic because it allows us to accept the truth of propositions such as '$Qt(a) = Qt(b)$' without committing ourselves to the existence of quantities; moreover, by moving from left to right in (Def. of Quantity) we have a straightforward way of dispensing with quantities treated as objects on their own in favour of relations between objects a and b. However, Leibniz's theory of definitions by abstraction is embedded in a realist philosophy of mathematics because the state of affairs that makes true both propositions in (Def. of Quantity) is abstract, along with all other states of affairs. They in fact represent connections of ideas in the mind of God. Mathematical truths are thus eternal, because they express (obtaining) relations between ideas in the mind of God.

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REFERENCES

Antognazza, M.R. (2009) *Leibniz: an intellectual biography*. Cambridge: Cambridge University Press.
Arthur, R.T.W. (2014) *Leibniz*. Cambridge: Polity Press.
Arthur, R.T.W. (2021) *Leibniz on time, space, and relativity*. Oxford: Oxford University Press.
Cook, R. (2009) New waves on an old beach: Fregean philosophy of mathematics today. In: Bueno, O. & Linnebo, Ø. (Eds.) *New waves in philosophy of mathematics*. London: Palgrave Macmillan, pp. 13–34.
Costantini, F. (2022) Quantity as limit. Leibniz on the metaphysics of quantity. *The Leibniz Review*, 32, 1–23. Available from: https://doi.org/10.5840/leibniz2022321
Couturat, L. (1901) *La logique de Leibniz d'après des documents inédits*. Paris: Ancienne Librairie Germer Baillière.
De Risi, V. (2007) *Geometry and Monadology: Leibniz's Analysis situs and philosophy of space*, Vol. 33. Basel: Springer Birkhäuser.
Di Bella, S. (2017) Some perspectives on Leibniz's nominalism and its sources. In: di Bella, S. & Schmaltz, T.M. (Eds.) *The problem of universals in early modern philosophy*. Oxford: Oxford University Press, pp. 198–219.
Di Bella, S. (2018) La res e i termini. Leibniz e la questione della cose tra semantica e ontologia. *Quaestio*, 18, 239–251.
Eco, U. (1995) *The search for the perfect language*. Oxford: Blackwell Publishing.
Hobbes, T. (1992) In: Molesworth, W. (Ed.) *The collective works of Thomas Hobbes*, Vol. I. London: Routledge Thoemmes Press.
Leibniz, G.W. (1849) In: Gerhardt, C.I. (Ed.) *Leibnizens mathematische Schriften*, Vol. 1-7, [cited by series, volume, and page, e.g. GM VII 4, 283]. Berlin and Halle: Asher and Schmidt.
Leibniz, G.W. (1875–90) In: Gerhardt, C.I. (Ed.) *Philosophische Schriften*, Vol. 7. Berlin: Weidmann. [cited by series, volume, and page, e.g. GP. II 471].
Leibniz, G.W. (1923) Sämtliche Schriften und Briefe. In: Leibniz, G.W. (Ed.) *Akademie der Wissenschaften der DDR*. Berlin: Akademie-Verlag. [cited by series, volume, and page, e.g. A VI 4, 552-557].
Leibniz, G. W. (1969). *Philosophical papers and letters*, ed. and trans. by L. Loemker. D. Reidel. Dordrecht: Springer.
Leibniz, G.W. (1996) *New essays on human understanding*, ed. and trans. by P. Remnant and J. Bennett. New York: Cambridge University Press.
Leibniz, G.W. (2007) In: Look, B.C. & Rutherford, D. (Eds.) *The Leibniz-Des Bosses correspondence*. New Haven: Yale University Press.
Leibniz, G.W. (2021) In: Mugnai, M. (Ed.) *General inquiries on the analysis of notions and truths*. Oxford: Oxford University Press.
Leibniz, G.W. & Clarke, S. (2000) In: Ariew, R. (Ed.) *Leibniz and Clarke: correspondence*. Indianapolis, IN: Hackett Publishing.
Linnebo, Ø. (2018) *Thin objects*. Oxford: Oxford University Press.
Mancosu, P. (2016) *Abstraction and infinity*. Oxford: Oxford University Press.
McRae, R. (1976) *Leibniz: perception, apperception, and thought*. Toronto: University of Toronto Press.
Mugnai, M. (2016) Leibniz e la logica. In: *Matematica, Cultura e Società. Rivista dell'Unione Matematica Italiana, Serie 1*, Vol. 1, pp. 241–257.
Puryear, S.M. (2006) *Perception and representation in Leibniz*. Doctoral dissertation. Pittsburgh: University of Pittsburgh.
Rabouin, D. (2015) The difficulty of being simple: on some interaction between mathematics and philosophy in Leibniz's analysis of notions. In: Goethe, N.B., Beeley, P. & Rabouin, D. (Eds.) *G.W. Leibniz, interrelations between mathematics and philosophy*. Dordrecht: Springer.
Rabouin, D. (2022) Can one be a Fictionalist and a Platonist at the same time?Lessons from Leibniz. *Noésis*, 36, 161–194.
Rabouin, D. (2024) *Mathématiques et Philosophie chez Leibniz. Au fil de l'analyse et des vérités*. Vrin.
Spinoza, B. (2017) In: Bennett, J. (Ed.) *Ethica*. Available from: https://www.earlymoderntexts.com/assets/pdfs/spinoza1665.pdf
Swoyer, C. (1995) Leibnizian expression. *Journal of the History of Philosophy*, 33(1), 65–99.

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