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Three Essays in Macroeconomics, Climate Change and Inequality

Flavio Contrada

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Chapter 1

Optimal Carbon Tax, Labour Market Friction, And Equality Concerns

Flavio Contrada*†

Abstract

A profound and rapid transition toward carbon-neutral production is necessary to mitigate climate change. However, the economic costs of this transition are unlikely to be evenly shared: workers employed in polluting, energy-intensive industries are expected to face greater losses compared to those in cleaner sectors. This paper develops an environmental dynamic general equilibrium model with high- and low-skill workers and segmented labour markets to study how the optimal carbon tax addresses the market failure from carbon emissions while generating distributional and welfare effects. In particular, I examine how labour market frictions influence the optimal tax (with reference to the full labour mobility scenario) and how a uniform lump-sum rebate affects the welfare of different household types. Results show that, although the policy improves aggregate welfare, low-skill households experience welfare losses; a differentiated rebate scheme—where a larger share of tax revenues is directed to low-skill households—can enhance the welfare of both groups while maintaining aggregate efficiency. This is crucial for fostering political acceptability regarding climate policies.

Keywords: Climate change, optimal policy, optimal taxes, inequality, labour market frictions.

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1 Introduction

Climate change represents one of the most pressing challenges faced by global society, with profound implications for long-term welfare, economic growth, and intergenerational equity. Economists widely regard carbon pricing as one of the most powerful and cost-effective tools to address this problem, as it directly internalizes the environmental externality associated with greenhouse gas emissions (Nordhaus, 2013). Yet, despite its theoretical appeal, the effectiveness of a carbon tax (or a carbon budget) cannot be evaluated in isolation from its distributional consequences. Mounting evidence suggests that the burden of such policies is not evenly shared across agents in the economy, with heterogeneous effects on both the demand and the supply sides Drupp et al. (2024). The recent wave of social protests, such as the Yellow Vest movement in France, is a stark reminder that climate policy design and political acceptability are deeply intertwined.

A central reason for this heterogeneity lies in the labour market. A substantial fraction of the energy workforce is still employed in high-carbon-intensive firms (IEA, 2022), and the transition to a low-carbon economy requires massive labour reallocations away from polluting industries and towards green sectors. However, this reallocation is far from frictionless. On the one hand, many workers in carbon-intensive sectors possess skills that are only imperfectly transferable to clean sectors. On the other hand, the adoption of new green technologies often demands complementary and highly specific human capital (Popp et al., 2024). This raises the prospect of sectoral immobility: workers locked into brown industries may not easily transition to clean jobs, generating both efficiency losses and distributional tensions.

The present paper attempts to shed light on these dynamics by building an *Environmental Dynamic General Equilibrium Model* with sector-specific workers and segmented labour markets. The framework extends a standard Ramsey multi-sectoral growth model in which two intermediate energy goods—dirty and clean—are combined to produce a final consumption good. The dirty energy sector generates an externality that directly reduces the economy's Total Factor Productivity (TFP), while the clean sector is assumed emission-free. The economy is populated by two representative households: low-skill households that supply labour exclusively to the dirty-energy sector, and high-skill households that are employed only in the clean-energy sector. This stylized segmentation captures the empirical observation that occupational and educational heterogeneity strongly influences reallocation costs during the green transition.

Within this framework, numerical simulations reveal that while a carbon tax improves aggregate welfare by reducing damages from TFP losses, it disproportionately harms low-skill households. The gains in consumption made possible by reduced environmental damages do not fully offset the decline in wages experienced in the dirty-energy sector. This distributional asymmetry is the pivotal mechanism driving our policy results. We further compare the benchmark case of full labour immobility with the counterfactual of full mobility between energy sectors. In the standard calibration, the optimal tax instruments are remarkably similar. Yet sensitivity analysis, reported in the appendix, shows that under alternative parameterizations the divergence between the two cases can become quantitatively significant, underscoring the importance of modelling labour immobility explicitly.

The paper also explores policy design. We show that the adverse distributional effects of a carbon tax on low-skill households can be mitigated through alternative rebate mechanisms. In particular, if the carbon-tax dividend is not rebated uniformly across households, but instead redistributed more generously to low-skill workers, then both households' welfare can improve relative to the uniform rebate case. This result is robust across parameterizations and points to the importance of tailoring revenue-recycling schemes to the structure of the labour market.

This work contributes to a broader general-equilibrium literature on optimal carbon taxation, which seeks to characterize the tax path that restores efficiency during the transition to a low-carbon economy. Foundational contributions include Golosov et al. (2014), Dietz and N. Stern (2015), Barrage (2020), and Douenne, Hummel, and Pedroni (2022). However, theoretical papers that embed labour heterogeneity and limited mobility in the energy sector remain scarce. Notable exceptions include Calvacanti, Hasna, and Santos (2024) and Hafstead and Williams III (2018), who study the impact of a carbon tax in computational general-equilibrium models with labour frictions, but from a static perspective. Fremstad and Paul (2019) considers climate policy in a Real-Business-Cycle framework with heterogeneous households, emphasizing short-run dynamics, while Belfiori, Carroll, and Hur (2024) analyze long-run heterogeneity under carbon pricing but assume perfect labour mobility and a capital-less production structure.

Empirical evidence, by contrast, consistently documents the existence of significant frictions between brown and green jobs. For instance, Sato et al. (2023) and Vona et al. (2018) show that green occupations are systematically more skill-intensive than their brown counterparts, while Bluedorn et al. (2023) demonstrate that transitions from polluting firms to cleaner firms are particularly difficult for low-educated workers. These findings suggest that ignoring labour immobility may lead to overly optimistic predictions about the ease of decarbonisation and the distributional neutrality of carbon taxes.

In sum, this paper seeks to bridge a gap between theory and evidence by developing a dynamic general-equilibrium framework where immobile labour across energy sectors interacts with optimal carbon taxation. By explicitly modelling segmented labour markets, we highlight the role of skill heterogeneity in shaping both the efficiency and the equity of climate policy. The next section reviews the related literature in greater detail.

1.1 Related Literature

The Dynamic Integrated Model of Climate and the Economy DICE, (Nordhaus, 2010) provides a benchmark framework linking the global economy, energy use, greenhouse gas emissions, the carbon cycle, climate change, and the resulting damages. It is a versatile tool that has been extended in numerous ways to compute the optimal carbon tax and to quantify its environmental and economic impacts. Golosov et al. (2014) introduced this climate dimension into a richer dynamic stochastic general equilibrium model, in which the supply side of the energy sector is fully characterised. Their framework made it possible to assess the value and the time path of the optimal carbon tax in a tractable general equilibrium setting. Subsequent contributions have extended this "Golosovian" approach in multiple directions. For example, Barrage (2020) incorporate capital and income taxation to analyse how pre-existing distortions shape the First-Best carbon tax. Alongside these theoretical advances, new debates have emerged—including the double-dividend hypothesis, the green paradox, and the just transition—while empirical evidence has highlighted the heterogeneous burden of climate change, particularly its unequal distribution across households and regions. In particular, higher temperatures and climate shocks disproportionately affect lower-income groups, both through direct exposure to physical risks (Green and Healy, 2022, Burzyński et al., 2022, Tol et al., 2004, Hallegatte, 2016) and through their vulnerability to the redistributive effects of climate policy itself (Hsiang, Oliva, and Walker, 2019, Känzig, 2023).

Several integrated assessment models (IAMs) have sought to account for inequality across regions. For instance, the NICE model (Budolfson et al., 2017, Gazzotti et al., 2021) introduces distributional heterogeneity explicitly. Yet relatively few papers embed inequality into a rigorous economic-theory framework. A notable exception is Douenne, Hummel, and Pedroni (2022), who analyse optimal carbon taxation in an economy populated by heterogeneous agents differing in wealth and climate sensitivity. Our setup is close in spirit to theirs but departs in one critical dimension: labour is not assumed to be perfectly mobile across energy technologies. Instead, we explicitly introduce sector-specific labour markets. The transition from high- to low-carbon production requires the development of new skills and the accumulation of technology-specific human capital, reflecting the structural characteristics of clean energy production (Vona et al., 2018, Borissov, Brausmann, and Bretschger, 2019).

A growing empirical literature reinforces the importance of this assumption. A robust finding is that low-carbon jobs differ systematically from otherwise similar occupations in terms of skill intensity. Using near-universe U.S. vacancy data from 2010–2019, Saussay et al. (2022) show that low-carbon postings demand broader and deeper skill portfolios—including technical/engineering, managerial, IT, and cognitive domains—than both generic and high-carbon postings. This implies higher up-front re-skilling requirements and greater hiring frictions, even in the absence of explicit wage gaps. Wage-setting patterns exacerbate the problem: while green vacancies initially offered premia, by 2017–2019 these had eroded or turned negative, whereas brown-sector jobs continued to pay sizeable premia (especially in extraction and engineering). For workers contemplating a move, the private returns to reallocation are thus limited, rationalising observed resistance to climate policy. Spatial frictions compound these barriers: high-carbon manual jobs are geographically concentrated in fossil-resource hubs, whereas green jobs are more spatially dispersed and only weakly co-located with brown employment. This geographic mismatch makes local reallocation difficult without costly migration or targeted place-based policies (Saussay et al., 2022, Rud et al., 2024).

Theoretical advances echo these concerns. Nijs and Tyros (2023) develop a search-and-matching model with two-sided heterogeneity to study how skill mismatches interact with firms' adoption of green technologies. In their model, green capital vintages require technology-specific skills, and imperfect matching significantly slows down adoption, locking workers with green capabilities into brown jobs. These findings resonate with micro-level evidence: Bluedorn et al. (2023), for instance, show that transitions from polluting to clean firms are particularly challenging for low-educated individuals, due to a combination of geographical barriers, skill requirements, and sorting effects.

Taken together, these empirical and theoretical contributions provide strong support for relaxing the standard assumption of perfect labour mobility across sectors. They also motivate our primary research question: how do sector-specific labour markets shape the efficiency and distributional consequences of carbon taxation? Our model positions itself within this literature by embedding segmented labour markets into a dynamic general equilibrium framework. Moreover, our analysis complements other theoretical approaches that introduce sector-specific inputs in energy production (Carattini, Heutel, and Melkadze, 2023, Diluiso et al., 2021, Borissov, Brausmann, and Bretschger, 2019). Perfect labour mobility and full immobility can be viewed as two extreme benchmarks. Both deserve theoretical attention, and exploring their consequences for climate policy design is the aim of the present paper.

The remainder of this paper is structured as follows. Section (2) introduces the general model. Section (3) characterises the competitive equilibrium with taxes. Section (4) presents the social planner's problem and the Ramsey government problem. Section (5) discusses numerical simulations and results. Section (6) concludes.

2 Model

The present section proposes an overview of the economy. I build a multi-sector neoclassical growth model with two representative households (low-skill and high-skill households) and three goods. The economic structure is close to Golosov et al. (2014)

and Douenne, Hummel, and Pedroni (2022), i.e, a standard neoclassical growth model with an externality coming from the production of one of the three goods. The main departure from Golosov et al. (2014) resides in the presence of heterogeneous infinitely-lived households, while the departure from Douenne, Hummel, and Pedroni (2022) is the introduction of a labour market full-friction, in that the low-skill and high-skill households are not free to choose the sector to work in.

Time is discrete ($t = 0, 1, ..$) and infinite. The final good Y_t is produced by combining two intermediate energy goods, $E_{d,t}$ (dirty energy) and $E_{c,t}$ (clean energy), with a constant return to scale, constant elasticity of substitution (CES) production function. $E_{t,d}$ degrades the environment causing damages to the final good TFP via a damages function. $E_{t,d}$ and $E_{t,c}$ are produced through capital and labour, employing a standard Cobb-Douglas technology. The low-skill household can find employment only in the dirty energy sector and the high-skill household only in the clean energy sector. Final output can be either consumed by households or stored in a linear storage technology that transforms it into the next period generic capital. A graphical representation of the model is presented in Figure 1:

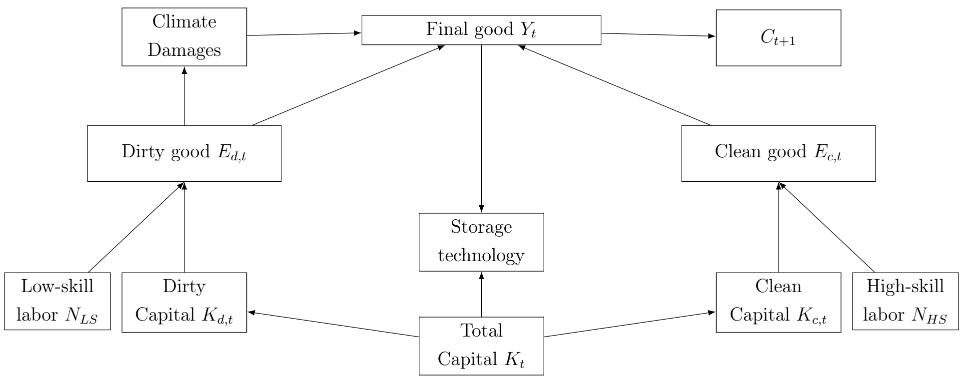

Figure 1: Overview of the model

2.1 Households

The utility function is a standard Constant Relative Risk Aversion (CRRA):

$$U^j \equiv \sum_{t=0}^{\infty} \beta^t \frac{C_{j,t}^{1-\sigma} - 1}{1-\sigma},$$

where j = {LS (low-skill), HS (high-skill)}. $C_{j,t}$ is consumption of the final good, β is the discount factor and $\sigma > 0$ is the inter-temporal elasticity of substitution. The representative households share the structural parameters β and σ. There is no dis-utility from labour; each time period the low-skill household is endowed with N_{LS} labour units, and the high-skill with N_{HS} labor units. Their budget constraints of the two households are respectively:

$$\sum_{t=0}^{\infty} q_t C_{HS,t} \leq \sum_{t=0}^{\infty} q_t W_{c,t} N_{HS} + p_0 K_0 + T_t(1-\xi), \tag{1}$$

$$\sum_{t=0}^{\infty} q_t C_{LS,t} \leq \sum_{t=0}^{\infty} q_t W_{d,t} N_{LS} + T_t \xi, \tag{2}$$

where q_t is the Arrow contingent claim on time t consumption (we focus on the time-0 equilibrium of this economy), $W_{c,t}$ and $W_{d,t}$ are the wages earned in the two energy sectors (clean, dirty) by the households, p0 is the price of the capital endowment, $T_t = \tau_{e,t} E_{d,t}$ are rebated carbon tax revenues and $\xi \in [0,1]$ is the share of carbon tax revenues given to the low-skill agents ($(1-\xi)$ is the share rebated to the high-skill agents). For the benchmark result, we assume $\xi = \frac{N_{LS}}{N_{HS}+N_{LS}}$ (i.e. uniform rebate scheme).

2.2 Intermediate Energy Goods Producers

Each intermediate energy sector is composed of a representative firm that combines capital and either low-skill (dirty energy sector) or high-skill labour (clean energy sector) to produce intermediate energy goods. They maximise two profit functions, given by equations (3) and (4):

$$\begin{aligned} \Pi_{d,t} = \max_{K_{d,t}, l_t^{LS}} \quad & (P_{d,t} - \tau_{e,t}) E_{d,t} - W_{d,t} l_t^{LS} - r_t K_{d,t}, \\ \text{subject to:} \quad & E_{d,t} = B_{d,t} K_{d,t}^{\alpha} (l_t^{LS})^{1-\alpha}, \end{aligned} \tag{3}$$

$$\begin{aligned} \Pi_{c,t} = \max_{K_{c,t}, l_t^{HS}} \quad & P_{c,t} E_{c,t} - W_{c,t} l_t^{HS} - r_t K_{c,t}, \\ \text{subject to} \quad & E_{c,t} = B_{c,t} K_{c,t}^{\alpha} (l_t^{HS})^{1-\alpha}. \end{aligned} \tag{4}$$

A Cobb-Douglas production function with constant returns to scale characterizes both intermediate energy sectors. Notice the full labour market friction (sector-specific labour): the low-skill household can only work in the dirty sector, whereas the high-skill household is employed in the clean one.$B_{d,t}$ and $B_{c,t}$ represent the Total-Factor-Productivity in the two intermediate energy sectors driven by exogenous deterministic processes. The representative energy producer is taxed with a per-unit carbon instrument τ_t. Generic capital can be employed in $E_{d,t}$ and in $E_{c,t}$ production, so that $K_{d,t}$ or $K_{c,t}$ are simply shares of the generic capital K_t. $P_{d,t}$ and $P_{c,t}$ are the energy goods prices paid by the final good producer to the energy producers; r_t is the interest rate paid to the storage firm that we will describe in few lines, and $W_{c,t}$ and $W_{d,t}$ are wages paid to the agents.

2.3 Final Good Producer

Y_t, the final output, is the numeraire of the economy, and it can be either consumed by households ($C_{j,t}$) or stored (I_t). Its production function is a standard constant elasticity of substitution (CES) with constant return to scale technology, having the two energetic goods as only inputs [1]:

$$Y_t = (1 - D(Z_t))A_t \left(E^{\rho}_{d,t} + E^{\rho}_{c,t}\right)^{\frac{1}{\rho}}$$

.

The ρ parameter measures the substitutability between $E_{d,t}$ and $E_{c,t}$ inputs, and A_t is the final sector exogenous TFP. The $E_{d,t}$ cumulative stock produced in the economy affects the final good sector negatively. The functional form of this damage engendered by $E_{d,t}$ is the one employed in Golosov et al. (2014), which is the exponential damage function

$$(1 - D(Z_t)) = e^{-\gamma(Z_t)},$$

where γ measures the intensity of damages. The link between the climate and the final good sector occurs through the variable Z_t. In our stylized framework it denotes cumulative emissions, i.e, the summation of $E_{t,d}$ produced in the economy between the first time period and the current one:

$$Z_t = Z_0 + \sum_{k=1}^{t} E_{d,k}, \tag{5}$$

with Z_0 being the starting cumulative stock of emissions. $E_{d,t}$ instantly flows into the atmosphere at time t, assuming no temperature inertia for simplicity ((Douenne, Hummel, and Pedroni, 2022), (Nordhaus, 2010)). To summarize, Z_t results in climate damage on the final TFP through the damage function. Its maximisation problem is:

$$\begin{aligned} \Pi^{FG}_t = \max_{E_{c,t}, E^{d,t}} \quad & Y_t - P_{d,t}E_{d,t} - P_{c,t}E_{c,t}, \\ \text{subject to:} \quad & Y_t = (1 - D(Z_t))A_t \left[E^{\rho}_{d,t} + E^{\rho}_{c,t}\right]^{1/\rho}. \end{aligned} \tag{6}$$

2.4 Storage Firm

The representative storage firm moves inter-temporally the capital good, through a linear technology:

[1] Following Acemoglu, Aghion, et al. (2012a), I let the production function of the final good depending only on energy inputs. Allowing for an additional, generic labour input employed in the final-good technology, or letting capital be used both in the intermediate energy sectors and in final-good production, would mainly complicate the model without materially affecting the key results or the distributional mechanisms highlighted in the thesis

$$\begin{aligned} \Pi_t^{SF} = \max_{I_t} \quad & p_{t+1}q_{t+1}K_{t+1} + r_t q_t K_t - p_t q_t K_t - q_t I_t, \\ \text{subject to:} \quad & K_{t+1} = I_t + (1-\delta)K_t, \end{aligned} \tag{7}$$

where r_t is the gross interest rate and p_t is the price of the capital good. The representative storage firm is active between two time periods. For instance, let us focus on the storage firm active between time t and $t+1$. It buys capital from the storage firm active between $t-1$ and t, renting the acquired capital to the intermediate energy goods producers. The revenues made through the interest rate and further output possibly bought in the market are the investment to be brought in the next period using the linear technology. The zero-profit conditions imply $p_{t+1}q_{t+1} = q_t$, and $p_t = r_t + (1-\delta)$.

2.5 Government

The Government sets a *per-unit* emission tax $\tau_{e,t}$ for the representative intermediate dirty energy producer, rebating the tax dividend to both households (uniformly or non-uniformly). The Government can fully commit to any given tax pattern and does not get into debt.

2.6 Feasibility Conditions

In this generic framework, four feasibility conditions have to be satisfied. The capital one, two labour ones and the final good one. Feasibility conditions for the inputs are the following:

$$l_t^{LS} \leq N_{LS} \tag{8}$$

$$l_t^{HS} \leq N_{HS} \tag{9}$$

$$K_{c,t} + K_{d,t} \leq K_t. \tag{10}$$

The aggregate resource constraint for the final good requires that Y_t has to be greater or equal to aggregate consumption and investment:

$$C_{LS,t} + C_{HS,t} + I_t \leq Y_t.$$

3 Decentralized Time-0 Equilibrium

We characterize the (deterministic) decentralized (time-0)equilibrium of the economy described in Section 2. Time is discrete, $t = 0, 1, \ldots$

Definition 1 (Competitive Equilibrium). *Given initial conditions* (K_0, Z_0)*, sequences of exogenous productivities* $\{A_t, B_{d,t}, B_{c,t}\}_{t=0}^{\infty}$*, and a sequence of carbon taxes* $\{\tau_{e,t}\}_{t=0}^{\infty}$*, a competitive equilibrium is a sequence of allocations* $\{C_{LS,t}, C_{HS,t}, E_{d,t}, E_{c,t}, K_{d,t}, K_{c,t}, K_t, I_t, l_t^{LS}, l_t^{HS}, Y_t, Z_t\}_{t=0}^{\infty}$ *and prices* $\{r_t, W_{d,t}, W_{c,t}, P_{d,t}, P_{c,t}, p_t, q_t\}_{t=0}^{\infty}$ *such that:*

(i) ***Households:***

- *Each representative household* $j \in \{LS, HS\}$ *solves:*

$$\max_{\{C_{j,t}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \frac{C_{j,t}^{1-\sigma} - 1}{1-\sigma}$$

subject to (1) and (2)

(ii) ***Intermediate Energy Producers***

- *solve their profits maximisation (3) and (4)*

(iii) ***Final Good Producer***

- *The final good producer chooses* $E_{d,t}, E_{c,t}$ *to solve the maximisation problem (6)*

(iv) ***Storage Firm:***

- *The storage firm chooses* I_t *to maximize equation (7) subject to its linear technology*

(v) ***Government:***

- *Collects* $T_t = \tau_t E_{d,t}$ *and rebates them lump-sum.*

(vi) ***Market Clearing:***

$$C_{HS,t} + C_{LS,t} + I_t = Y_t, \quad K_{c,t} + K_{d,t} = K_t, \quad l_t^{HS} = N_{HS}, \quad l_t^{LS} = N_{LS}.$$

(vii) ***Environmental Law of Motion:***

$$Z_{t+1} = Z_t + E_{d,t}.$$

3.1 Business-As-Usual Equilibrium (BAU)

Within this class of dynamic environmental general equilibrium models, it is standard to study the Business-As-Usual allocation (BAU henceforth), where the Planner does not internalise the externality, setting taxes equal to 0 ($\{\tau_{e,t} = 0\}_{t=0}^{\infty}$). In this equilibrium, the polluting industry is not affected by taxation; $W_{d,t}$ and the capital good rental rate are free to adjust in response to market forces. This feature of the BAU equilibrium drives the main results in terms of both aggregated and disaggregated welfare. Markets are competitive, so the prices of intermediate energy goods are determined by market conditions. Given the presence of full labour market frictions, households cannot choose the sector in which they work. This may lead to different wages if the structural parameters $B_{d,t}$, $B_{c,t}$, N_{HS}, and N_{LS} differ. The generic capital K_t is freely allocated across the two intermediate energy sectors, and therefore, the interest rate must be unique in the competitive market:

$$\frac{\partial R_{d,t}}{\partial E_{d,t}}\frac{\partial E_{d,t}}{\partial K_{d,t}} = r_t, = \frac{\partial R_{c,t}}{\partial E_{t,c}}\frac{\partial E_{c,t}}{\partial K_{c,t}}.$$

The representative final good producer demands $E_{t,d}$ and $E_{t,c}$ according to their marginal productivities:

$$\frac{\partial Y_t}{\partial E_{d,t}} = P_{d,t}, \quad \frac{\partial Y_t}{\partial E_{c,t}} = P_{c,t}.$$

The representative storage firm operates between periods t and $t+1$, paying $p_t q_t K_t$ to the previous storage firm (active between $t-1$ and t). It then rents the capital to the intermediate energy goods producers, earning $r_t q_t K_t$. After remunerating K_t, the storage firm invests through the linear technology described by equation (7)).

The household maximization problem yields the Euler condition:

$$\left(\frac{C_{j,t+1}}{C_{j,t}}\right)^{-\sigma} = \beta\left(\frac{q_{t+1}}{q_t}\right),$$

where $j \in \{\text{LS}, \text{HS}\}$. Given the functional forms of the utility functions, the individual Euler equations can be aggregated to obtain a single aggregate Euler condition.

The crucial insight is that environmental degradation is not mitigated by the emission tax $\tau_{e,t}$ between the returns on capital in the dirty and clean sectors. By lowering the net return on capital in the dirty energy sector, the tax would discourage investment in $K_{d,t}$ and shifts capital towards the clean sector $K_{c,t}$. This reallocation of resources would reduce the production of the polluting good $E_{d,t}$, thereby lowering emissions and the associated environmental damages. At the same time, the shift of capital would alter the marginal productivity of labor in both sectors, potentially leading to changes in wages $W_{d,t}$ and $W_{c,t}$. These combined effects on emissions, wages, and output composition will be examined in detail in the next section, where the Ramsey problem is characterised.

4 Ramsey Problem

In this section, I solve the optimal (constrained) Ramsey problem using a primal approach. The objective function of the Planner is a standard concave welfare function, where θ and $1-\theta$ represent the welfare weights attached to the brown (low-skill) and green (high-skill) representative households. The Planner maximizes this welfare function subject to technological constraints (production technologies for energy and final goods), as well as feasibility constraints (resource constraints, capital accumulation, and labour supply).

Importantly, the Planner has no access to lump-sum transfers. Redistribution between the two households can only occur through revenues generated by the available policy instrument, namely the emission tax $\tau_{e,t}$. As a result, the social welfare weights θ and $1-\theta$ must be endogenously consistent with the market wealth of the two households. Following Negishi's theorem (Negishi, 1960), the decentralized competitive allocation can be replicated by a social planner's allocation, where the social weights are functions of the agents' relative market incomes (wages and capital earnings). In other words, to retrieve the optimal allocation as long as the consistent (endogenous) thetas, the Planner must first solve for the optimal allocation for an initial guess of thetas, then, given the optimal allocation, reconstructs the decentralized economy (that is, the set of prices that supports the optimal allocation), and checks whether the agents' wealth is consistend with the initial thetas. If not, it updates thetas and replicate the same procedure until convergence. More details are given in the appendix. The Planner internalizes the environmental externality associated with the production of the dirty energy good $E_{d,t}$. In choosing the optimal allocation of resources across sectors and time, the Planner balances the benefits of energy production against the current and future damages induced by emissions. The remainder of this section characterizes the optimal allocation,particularly the production of $E_{d,t}$,and explains how it can be decentralized through the appropriate choice of $\tau_{e,t}$.

The Planner solves:

$$\max_{\{C_{LS,t},C_{HS,t},I_t,K_{i,t},E_{i,t},Z_t,l_t^j\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[\frac{\theta C_{LS,t}^{1-\sigma}}{1-\sigma} + \frac{(1-\theta) C_{HS,t}^{1-\sigma}}{1-\sigma} \right], \tag{11}$$

subject to, for all $t \geq 0$:

$$
\begin{aligned}
Y_t &= C_{LS,t} + C_{HS,t} + I_t, \qquad (\nu_{rc,t}) \qquad \text{(Aggregate Resource Constraint)} \\
Y_t &= (1 - D(Z_t))A_t \left[E_{d,t}^{\rho} + E_{c,t}^{\rho}\right]^{1/\rho}, \quad (\nu_{Y,t}) \quad \text{(Final Good Technology)} \\
E_{d,t} &= B_{d,t} K_{d,t}^{\alpha} (l_t^{LS})^{1-\alpha}, \quad (\nu_{E_{d,t}}) \quad \text{(Dirty Energy Production)} \\
E_{c,t} &= B_{c,t} K_{c,t}^{\alpha} (l_t^{HS})^{1-\alpha}, \quad (\nu_{E_{c,t}}) \quad \text{(Clean Energy Production)} \\
K_t &= K_{d,t} + K_{c,t}, \quad (\nu_{K,t}) \quad \text{(Capital Allocation)} \\
K_{t+1} &= I_t + (1-\delta)K_t, \quad (\nu_{I,t}) \quad \text{(Capital Accumulation)} \\
l_t^{LS} &= N_{LS}, \quad l_t^{HS} = N_{HS}, \quad (\nu_{L_j,t}) \quad \text{(Labour Feasibility)} \\
Z_{t+1} &= Z_t + E_{d,t}, \qquad (\nu_{Z,t}) \quad \text{(Cumulative Emissions Law)}
\end{aligned}
$$

with K_0, Z_0 and the TFP sequences $\{A_t, B_{d,t}, B_{c,t}\}_{t=0}^{\infty}$ given.

First-Order Conditions

The FOCs with respect to $C_{LS,t}$ and $C_{HS,t}$ are:

$$
\begin{aligned}
\beta^t \theta (C_{LS,t})^{-\sigma} &= \nu_{rc,t}, \\
\beta^t (1-\theta)(C_{HS,t})^{-\sigma} &= \nu_{rc,t},
\end{aligned}
$$

where $\nu_{rc,t}$ is the Lagrange multiplier on the aggregate resource constraint. Taking the ratio of these conditions and solving for $C_{LS,t}$ as a function of $C_{HS,t}$, I obtain:

$$
\begin{aligned}
C_{LS,t} &= \frac{\left(\frac{\theta}{1-\theta}\right)^{\frac{1}{\sigma}}}{1 + \left(\frac{\theta}{1-\theta}\right)^{\frac{1}{\sigma}}}(Y_t - I_t), \\
C_{HS,t} &= \frac{1}{1 + \left(\frac{\theta}{1-\theta}\right)^{\frac{1}{\sigma}}}(Y_t - I_t).
\end{aligned}
$$

The Planner thus determines aggregate consumption $C_t = C_{LS,t} + C_{HS,t}$ and splits it across households according to a non-linear function of their welfare weights.

Euler Equation. The FOC with respect to K_{t+1}, which determines I_t (aggregate investment), is:

$$
-\nu_{rc,t} + \nu_{rc,t+1}\left(\frac{\partial F_{t+1}^{FG}}{\partial K_{t+1}} + (1-\delta)\right) = 0.
$$

Substituting $\nu_{rc,t}$ with the consumption-based expression yields the standard Euler equation:

$$\left(\frac{1}{\beta}\right)^{\frac{1}{\sigma}} \frac{C_{t+1}}{C_t} = \left(\frac{\partial Y_{t+1}}{\partial K_{t+1}} + (1-\delta)\right)^{\frac{1}{\sigma}}. \tag{12}$$

Dirty Energy and Emissions. For $E_{d,t}$ and Z_t, the FOCs are:

$$[E_{d,t}]: \quad \nu_{rc,t}\frac{\partial Y_t}{\partial E_{d,t}} - \nu_{E_{d,t}} - \sum_{j=0}^{\infty} \nu_{Z,t+j} = 0, \tag{13}$$

$$[Z_t]: \quad \nu_{rc,t}\frac{\partial Y_t}{\partial Z_t} - \nu_{Z,t} = 0, \tag{14}$$

where $\nu_{E_{d,t}}$ is the multiplier on the dirty energy production constraint and $\nu_{Z,t}$ is the shadow value of emissions. Combining these conditions, we obtain:

$$\overbrace{\nu_{rc,t}\frac{\partial Y_t}{\partial E_{d,t}}}^{\text{Marginal benefit of dirty energy}} = \overbrace{\nu_{E_{d,t}}}^{E_{d,t}\text{ technology constraint}} + \overbrace{\sum_{j=0}^{\infty} \nu_{Z,t+j}}^{\text{Present value of externality damages}} \tag{15}$$

The left-hand side of (37) captures the *marginal benefit* of producing an additional unit of dirty energy $E_{d,t}$. This benefit arises because $E_{d,t}$ enters as an input in the final good production function Y_t, thereby increasing the quantity of output available for consumption and investment. The term $\nu_{rc,t}$ translates this additional output into utility units, reflecting the marginal utility of aggregate resources at time t.

The right-hand side of (37) accounts for the *full marginal cost* of producing $E_{d,t}$. The first component, $\nu_{E_{d,t}}$, is the shadow price associated with the dirty energy production technology. It represents the cost in terms of capital and low-skill labour required to produce $E_{d,t}$. The second component, $\sum_{j=0}^{\infty} \nu_{Z,t+j}$, captures the externality: since emissions generated by $E_{d,t}$ accumulate in Z_t and remain in the atmosphere indefinitely, each unit of $E_{d,t}$ creates a *persistent stock externality*. This implies that the Planner must internalize not only the immediate production costs but also the entire discounted stream of future damages arising from the higher stock of emissions.

To correctly value these future damages, each term $\nu_{Z,t+j}$ is weighted by the ratio of marginal utilities $\frac{\nu_{rc,t+j}}{\nu_{rc,t}}$, which adjusts future damages into present-value terms using the Planner's intertemporal utility structure. Thus, the externality term represents the present value of all future reductions in final good productivity $A_t(1-D(Z_t))$ caused by the incremental increase in Z_t today.

Given that low-skill and high-skill labour are fully segmented across sectors, the Planner cannot reallocate labour directly between the dirty and clean energy sectors. Therefore, the only margin of adjustment available is the allocation of the aggregate capital stock K_t. By reducing $K_{d,t}$ and increasing $K_{c,t}$, the Planner reduces the production of dirty energy $E_{d,t}$, mitigates emissions, and thereby lowers future climate damages. This reallocation also increases the marginal productivity of clean energy, facilitating the transition toward a cleaner energy mix while maintaining output.

The next section will show how this socially optimal allocation can be decentralized by introducing a tax policy (e.g., a Pigouvian tax on $E_{d,t}$) that replicates the Planner's internalization of the externality in a competitive market environment.

4.1 Decentralization of the Social Planner's Allocation

The Social Planner's allocation can be implemented in a decentralized competitive equilibrium by introducing a *carbon tax* $\tau_{e,t}$ that corrects the externality generated by dirty energy production $E_{d,t}$. The role of this tax is to make individual firms internalize the marginal social damage caused by emissions, which they would otherwise ignore.

Dirty Energy Producer. Consider the representative dirty energy producer. In the decentralized economy, it chooses capital $K_{d,t}$ and low-skill labor l_t^{LS} to maximize its profits:

$$\Pi_{d,t} = (P_{d,t} - \tau_{e,t})E_{d,t} - W_{d,t}l_t^{LS} - r_t K_{d,t},$$

subject to the production technology:

$$E_{d,t} = B_{d,t}K_{d,t}^{\alpha}(l_t^{LS})^{1-\alpha}.$$

Here, $P_{d,t}$ is the market price of dirty energy, $W_{d,t}$ the wage paid to low-skill workers, and r_t the rental rate of capital. The tax $\tau_{e,t}$ reduces the revenue the firm obtains per unit of $E_{d,t}$, effectively forcing the firm to consider part of the social damage from emissions. The corresponding first-order conditions (FOCs) for capital and labour are:

$$(P_{d,t} - \tau_{e,t})\frac{\partial E_{d,t}}{\partial K_{d,t}} = r_t, \quad (P_{d,t} - \tau_{e,t})\frac{\partial E_{d,t}}{\partial l_t^{LS}} = W_{d,t}.$$

These conditions equate the marginal revenue product of each input (adjusted for the tax) to its market price.

Final Good Producer. The representative final good producer chooses the energy inputs $E_{d,t}$ and $E_{c,t}$ (clean energy) to maximize:

$$\Pi_t^{FG} = Y_t - P_{d,t}E_{d,t} - P_{c,t}E_{c,t},$$

subject to the CES production function:

$$Y_t = (1 - D(Z_t))A_t \left[E_{d,t}^{\rho} + E_{c,t}^{\rho}\right]^{1/\rho}.$$

The FOCs of this problem are:

$$\frac{\partial Y_t}{\partial E_{d,t}} = P_{d,t}, \quad \frac{\partial Y_t}{\partial E_{c,t}} = P_{c,t},$$

which ensures that each energy input is used up to the point where its marginal product equals its price.

4.2 Optimal Tax Rule

To replicate the Social Planner's allocation in a competitive setting, we must determine the level of the carbon tax $\tau_{e,t}$ such that the decentralized equilibrium's choices of $E_{d,t}$, $K_{d,t}$, and l_t^{LS} coincide with the Planner's optimal allocation.

Planner's FOC for $E_{d,t}$. In the Planner's problem, the first-order condition with respect to $E_{d,t}$ reads:

$$\nu_{rc,t} \frac{\partial Y_t}{\partial E_{d,t}} = \nu_{E_{d,t}} + \sum_{j=0}^{\infty} \nu_{Z,t+j}, \tag{16}$$

where: $\nu_{rc,t}$ is the Lagrange multiplier on the aggregate resource constraint, which measures the marginal utility of an additional unit of final output, $\nu_{E_{d,t}}$ is the multiplier on the dirty energy technology constraint, reflecting the direct cost of increasing $E_{d,t}$ through capital and labour inputs,$\nu_{Z,t+j}$ captures the marginal external cost of emissions: one more unit of $E_{d,t}$ increases the stock Z_t, which leads to a reduction in future output via the damage function $D(Z_{t+j})$.

Equation (16) thus states that the marginal benefit of dirty energy in terms of final output (left-hand side) must equal its total marginal cost (right-hand side), consisting of the production cost and the full discounted stream of environmental damages. Dividing by $\nu_{rc,t}$, we can rewrite:

$$\frac{\partial Y_t}{\partial E_{d,t}} = \frac{\nu_{E_{d,t}}}{\nu_{rc,t}} + \sum_{j=0}^{\infty} \frac{\nu_{Z,t+j}}{\nu_{rc,t}}. \tag{17}$$

The second term on the right-hand side is the *Pigouvian correction*, which accounts for the externality that a decentralized firm would ignore.

Decentralized Firm's Condition. In the absence of a tax, the dirty energy firm in a competitive equilibrium chooses $K_{d,t}$ and l_t^{LS} to maximize:

$$\Pi_{d,t} = P_{d,t} E_{d,t} - W_{d,t} l_t^{LS} - r_t K_{d,t},$$

with first-order condition:

$$P_{d,t} \frac{\partial E_{d,t}}{\partial K_{d,t}} = r_t. \tag{18}$$

Since the final good producer pays the marginal product of $E_{d,t}$, we have $P_{d,t} = \frac{\partial Y_t}{\partial E_{d,t}}$. This firm thus internalizes only the private benefit $\frac{\nu_{E_{d,t}}}{\nu_{rc,t}}$, but not the social cost term $\sum_{j=0}^{\infty} \frac{\nu_{Z,t+j}}{\nu_{rc,t}}$.

Introducing a Carbon Tax. In order for the firm's choice of $E_{d,t}$ to coincide with the Planner's, the effective price it perceives, $P_{d,t} - \tau_{e,t}$, must equal the Planner's measure of the marginal private benefit of dirty energy:

$$P_{d,t} - \tau_{e,t} = \frac{\nu_{E_{d,t}}}{\nu_{rc,t}}. \tag{19}$$

Optimal Tax Expression. From (17), we know that:

$$P_{d,t} = \frac{\nu_{E_{d,t}}}{\nu_{rc,t}} + \sum_{j=0}^{\infty} \frac{\nu_{Z,t+j}}{\nu_{rc,t}}.$$

Substituting this into (19), we obtain the expression for the optimal carbon tax:

$$\tau_{e,t} = \sum_{j=0}^{\infty} \frac{\nu_{Z,t+j}}{\nu_{rc,t}}. \tag{20}$$

Thus, $\tau_{e,t}$ exactly equals the present value of all future marginal damages caused by one additional unit of dirty energy produced at time t, measured in units of the final good.

The tax $\tau_{e,t}$ introduces a wedge between the market price of dirty energy $P_{d,t}$ and its private return, effectively forcing the firm to internalize the social cost of emissions. This leads to a reallocation of capital from the dirty sector $K_{d,t}$ toward the clean sector $K_{c,t}$, thereby lowering $E_{d,t}$ and the accumulation of emissions Z_t. As a result, the decentralized allocation with $\tau_{e,t}$ coincides with the Planner's first-best allocation.

5 Numerical Simulations

The model does not admit closed-form solutions, so I rely on numerical simulations to characterize the main allocations. Time is discrete, with each period representing 10 years (Golosov et al., 2014). I focus on the full transition path towards the balanced growth path (BGP), which spans 25 time periods, and compare the outcomes under the Business-As-Usual (BAU) allocation and the socially optimal allocation.

A key modeling assumption is that a backstop technology—capable of completely eliminating emissions from dirty energy production $E_{d,t}$—becomes available exogenously at the end of the 25th period. The emergence of this technology is necessary to achieve a fully decarbonized economy and thus reach a BGP. This assumption is consistent with the current literature (Golosov et al., 2014, Barrage, 2020, Douenne, Hummel, and Pedroni, 2022).

An additional focus of the simulations is the role of the carbon tax rebate scheme. I examine both a uniform rebate across households and a non-uniform rebate skewed toward the low-skill household, assessing the resulting welfare implications. This comparison highlights the distributional consequences of climate policy, particularly in the presence of segmented labour markets.

Finally, the quantitative results are sensitive to the calibration of the model parameters. I therefore provide a thorough discussion and sensitivity analysis of the chosen calibration values, ensuring the robustness of the results in the appendix. Moreover, I describe the computational algorithm employed to solve the model.

5.1 Parameterisation

This section outlines the (standard) parameterisation strategy used in the simulations, summarized in Table 1. The parameters are chosen to represent key features of the world economy, while remaining consistent with the relevant literature. However, changes in the standard parameters leave the results stable. The (annual) discount factor is set to $\beta = 0.985$, as in the DICE model (W. D. Nordhaus, 2017). The intertemporal elasticity of substitution is $\sigma = 1.5$, as in Barrage (2020). The parameters of the final good production function are drawn from Acemoglu, Aghion, et al. (2012b) and Golosov et al. (2014). The elasticity parameter $\rho = 0.66$ implies imperfect substitutability between $E_{d,t}$ and $E_{c,t}$ (with $\rho = 1$ indicating perfect substitutability).

This choice reflects the current technological limitations of clean energy, particularly with respect to storage and intermittency. [2]

Consistent with Golosov et al. (2014), both energy sectors combine capital and labour to produce energy inputs. However, the key novelty of this framework is the introduction of full labour market segmentation. Low-skill households supply labor exclusively to the dirty sector, while high-skill households are employed solely in the clean sector. This stylized skill mismatch is well documented in the empirical literature on energy transitions (Bluedorn et al., 2023).

The technological advantage of the dirty sector ($B_{t,d} > B_{t,c}$) reflects the path dependence of innovation: fossil-based technologies have been established for longer and thus benefited from more incremental improvements (Acemoglu, Aghion, et al., 2012b; Aghion et al., 2019). The climate damage parameter γ is based on the estimates of Peter H Howard and Sterner (2017a), which imply a 5% loss in TFP at 3°C global warming.

The values of K_0 and A_0 are chosen to remain consistent with the data, because we observe an increasing path for GDP and generic capital. Total labour endowment is

[2] An important difference from Barrage (2020) and Douenne, Hummel, and Pedroni (2022) lies in the interpretation of the clean sector: in the present model, the clean energy sector does not represent abatement technologies but instead corresponds to the production of a truly clean energy input $E_{c,t}$.

Description	Parameter	Value	Source / Notes
Discount factor (annual)	β	0.985	Nordhaus (2010)
Intertemporal elasticity of substitution	σ	1.5	Golosov et al. (2014)
Substitutability of $E_{d,t}$ and $E_{c,t}$	ρ	0.66	Acemoglu, Aghion, et al. (2012b)
Capital share in production	α	1/3	Acemoglu, Aghion, et al. (2012b)
Capital depreciation rate (annual)	δ	0.6513	W. D. Nordhaus (2017)
Initial dirty sector TFP	$B_{0,d}$	1.1	Acemoglu, Aghion, et al. (2012b)
Initial clean sector TFP	$B_{0,c}$	1.0	Normalization
Initial final good TFP	A_0	4.0	Normalization
Damage parameter	γ	5% loss at 3°C	Peter H Howard and Sterner (2017a)
Initial capital stock	K_0	1.0	Golosov et al. (2014)
Low-skill labor share	N_{LS}	0.5	IEA (2022)
High-skill labor share	N_{HS}	0.5	IEA (2022)
Energy sector TFP growth rate	λ_B	0.02	Golosov et al. (2014)
Final good TFP growth rate	λ_A	0.00	Golosov et al. (2014)

Table 1: Summary of Key Calibration Parameters

normalized to 1 (constant), with equal shares of low- and high-skill workers, in line with IEA (2022). Notwithstanding, the results are robust to significant twists of these two parameters, along with all the main paramaters here presented.

The next section examines the dynamic transition towards the balanced growth path (BGP), focusing on the trajectories of the optimal carbon tax, the energy input ratio $\frac{E_{d,t}}{E_{c,t}}$, the wage ratio $\frac{W_{d,t}}{W_{c,t}}$, and the Gini index. In the BAU equilibrium (with $\tau_{e,t} = 0$), the economy overproduces dirty energy because (i) low-skill labour is trapped in the dirty sector (labor market segmentation), and (ii) $B_{t,d} > B_{t,c}$ (technological advantage). This overproduction amplifies TFP losses through climate damages compared to the optimal allocation with a carbon tax.

I now turn to a comparison between the BAU and Planner allocations, highlighting both the efficiency gains and distributional implications of optimal climate policy.

5.2 Optimal Carbon Tax vs BAU allocations

The rising path of the tax reflects the increasing marginal damages of cumulative emissions and the economy's need to gradually reallocate resources from dirty to clean energy sectors.

In contrast, under the Business-as-Usual (BAU) scenario, where $\tau_{e,t} = 0$, no corrective mechanism exists, and emissions grow unchecked. During the transition toward a decarbonized steady state, economic agents in the optimal allocation are subject to an increasingly stringent carbon tax. This tax progressively reduces the production of the dirty energy good $E_{d,t}$, internalizing the externality caused by emissions that not only affect current total factor productivity (TFP) but also persist into the future through the accumulation of the emissions stock Z_t. As sections 4 and 5 show, this mechanism raises the marginal productivity of dirty energy $E_{d,t}$: with lower production, the remaining units of $E_{d,t}$ become more valuable in final goods production.

A key feature of the model is that capital is freely mobile across intermediate energy sectors. As a result, the return on capital is equalized across the dirty and clean sectors:

$$r_t = \frac{\partial R_{d,t}}{\partial K_{d,t}} = \frac{\partial R_{c,t}}{\partial K_{c,t}}.$$

When the carbon tax is introduced, capital reallocates from the dirty sector toward the clean sector, as the after-tax return to dirty capital $K_{d,t}$ falls relative to $K_{c,t}$. This shift not only reduces emissions but also expands the clean energy sector, increasing $E_{c,t}$ and boosting the productivity of high-skill labour employed there.

However, labour markets are fully segmented, meaning that low-skill and high-skill workers cannot switch sectors. Low-skill workers are locked into the dirty energy sector, while high-skill workers are tied to the clean energy sector. Even in the absence of a carbon tax, wages differ across sectors due to differences in the labour supply and technology. Since the supply of low-skill labor N_{LS} is relatively larger than the supply of high-skill labor N_{HS}, the marginal product of labor (and thus the wage) in the dirty sector, $W_{d,t}$, is lower than that in the clean sector, $W_{c,t}$. This pre-existing wage inequality is further exacerbated when the carbon tax is implemented.

The mechanism is straightforward. First, the carbon tax reduces the equilibrium price of the dirty energy good:

$$P_{d,t}^{\text{after-tax}} = P_{d,t} - \tau_{e,t}.$$

From the firm's first-order condition, this price directly affects the wage:

$$W_{d,t} = (P_{d,t} - \tau_{e,t}) \cdot (1 - \alpha) \frac{E_{d,t}}{N_{LS}},$$

implying that as $\tau_{e,t}$ increases, the share of revenue available for remunerating low-skill workers falls. Thus, low-skill workers face both lower employment demand (due to reduced $E_{d,t}$) and a lower price per unit of output, leading to a decline in their real wages.

Conversely, high-skill workers benefit from the carbon tax. As capital shifts toward the clean energy sector, the output of $E_{c,t}$ increases, raising its marginal product and thereby boosting $W_{c,t}$. This creates a redistributional effect: while the policy is welfare-enhancing in aggregate (due to the internalization of the environmental externality), it has asymmetric effects across skill groups. Low-skill households, whose incomes are tied to the shrinking dirty sector, are worse off relative to high-skill households, whose sector expands and becomes more productive under the policy.

In summary, the optimal carbon tax plays a dual role: An "Efficiency Channel£: It corrects the environmental externality, reducing emissions and improving long-term productivity by mitigating climate damages. A "Distributional Channel£: By shifting capital and production toward the clean sector, it redistributes income across households in a way that favours high-skill workers, while low-skill workers experience lower wages and reduced sectoral output.

These results highlight the importance of carefully designing tax rebates or complementary redistribution policies. In later sections, I explore uniform and non-uniform rebate schemes to assess whether they can mitigate the adverse distributional effects of the carbon tax while preserving its efficiency benefits.

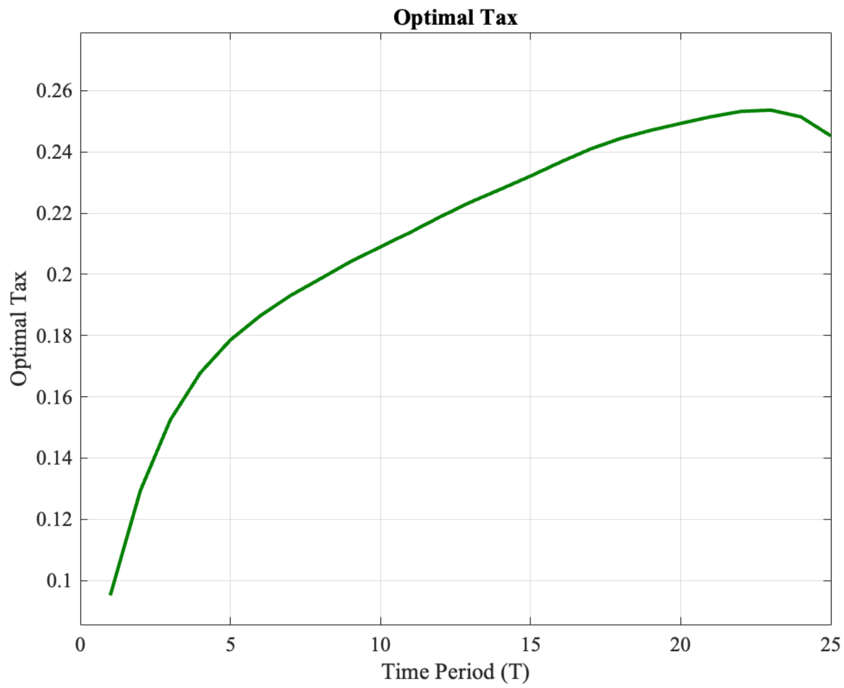

Figure 2: Optimal tax

Interestingly, the optimal carbon tax is not affected by labour immobility, compared to the case of perfect labour mobility: they basically overlap (Figure 3). This is mainly due to my parameterisation; when the endowments of low-skill (NLS) and high-skill (NHS) labour are close to each other, I casually obtain the same optimal τ_e, because the optimal share of labour between the two energy sectors in the full mobility economy, ceteris paribus for other parameters linked to production (ρ, $B_{0,d}$, $B_{0,c}$), is similar to the exogeneous labour share of the full labour immobility economy. However, I show in the appendix that by changing the relative size of the two classes of workers, or other relevant parameters, the two optimal taxes diverge significantly.

But the distributional effects of the carbon tax are clearly present compared to a frictionless labour market. These results highlight the importance of coupling the carbon tax with well-designed redistribution schemes.

The ratio $\frac{E_{d,t}}{E_{c,t}}$ declines over time under the optimal policy, in contrast to the Business-as-Usual (BAU) scenario, where it remains constant (figure 4).

This decline reflects the Planner's strategy of reallocating capital from the dirty to the clean energy sector to mitigate the damages caused by emissions. A similar effect is

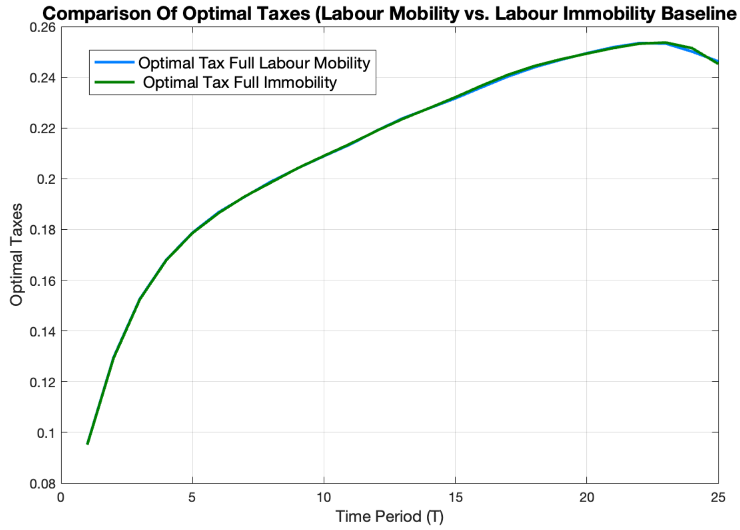

Figure 3: Optimal taxes

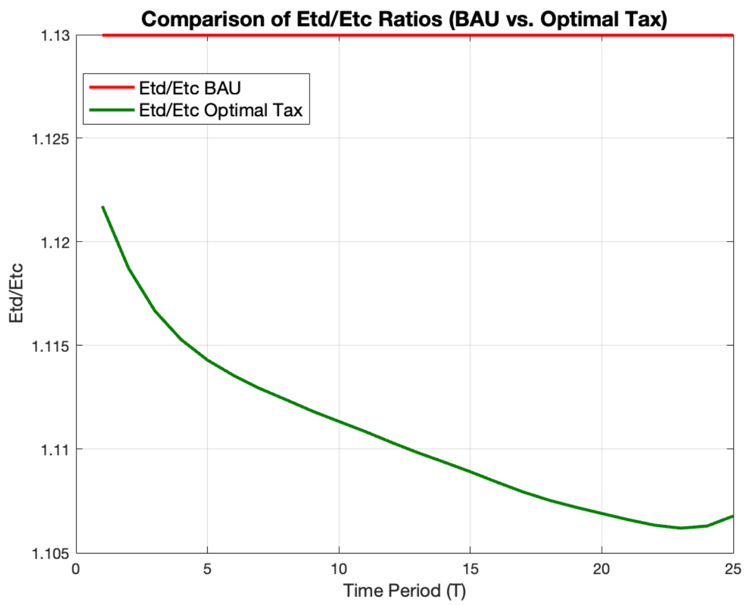

Figure 4: $\frac{E_{d,t}}{E_{c,t}}$ Bau vs Optimal Tax

achieved through the optimal carbon tax set by the benevolent government, which incentivizes a gradual shift toward cleaner energy production.

However, unlike in Golosov et al. (2014) and Barrage (2020), the Planner in this framework faces a crucial constraint: labour cannot be reallocated across energy sectors due to full labour market segmentation. While those models allow both capital and labour to flow freely between sectors, here only capital can be shifted from the dirty to the clean sector. This additional constraint limits the Planner's ability to fully offset the externality and amplifies distributional effects, leading to a reduction in the dirty sector wage $W_{d,t}$ and an increase in the clean sector wage $W_{c,t}$ compared to their BAU levels.

The relative $W_{d,t}$ reduction with reference to $W_{c,t}$ can be seen in figure 5:

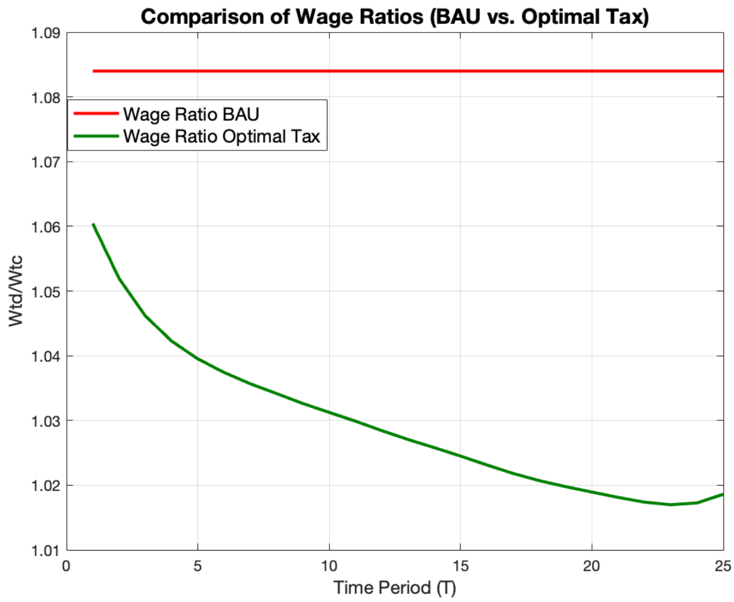

Figure 5: Wage ratios BAU vs Optimal Tax

Since the carbon tax is increasing with time, the intermediate dirty energy sector will be more and more impacted by the policy, and mostly in the future. Wages $W_{d,t}$ in the dirty sector are diminishing, meaning that there is a lower available consumption for the representative low-skill household. At the same time, $W_{c,t}$ is increasing, which is beneficial for the high-skill representative household in terms of consumption. Investigating the policy's impact on inequality is crucial to the paper. ìI analyze its dynamics using the Gini index. Since I have only two income classes, this can be easily defined as:

$$Gini_t = \frac{W_{t,c}N_{HS}}{W_{t,c}N_{HS} + W_{t,d}N_{LS}} - \frac{N_{HS}}{N_{HS} + N_{LS}}. \tag{21}$$

The Gini index is computed for each time period. A Gini index equal to one corresponds to maximum inequality. Hence, a higher Gini index indicates a greater income gap, with high-skill individuals receiving relatively higher remuneration compared to low-skill individuals.

Figure 6 illustrates the dynamics of the Gini index under both the Business-as-Usual (BAU) scenario and the optimal policy. Under the optimal policy, economic inequality rises due to changes in the wage structure. In the BAU economy, where the carbon tax is zero, the allocation of capital $K_{d,t}$ remains constant across periods, and the Gini index is stable over time.

When the time-varying carbon tax is introduced, capital progressively flows from the dirty sector to the clean intermediate energy sector. This reallocation reduces the marginal productivity of low-skill labour in the dirty sector, leading to a decline in $W_{d,t}$, while simultaneously raising the wage $W_{c,t}$ in the clean sector. The widening wage gap directly increases the Gini index, reflecting the growing income disparity between the two representative households.

This shift has important implications for welfare: as the low-skill household's relative income declines, the distributional effects of the policy become more pronounced. The Gini index thus captures not only the technological and capital reallocation effects of the optimal carbon tax but also the underlying trade-off between efficiency gains from emissions reduction and the resulting increase in income inequality.

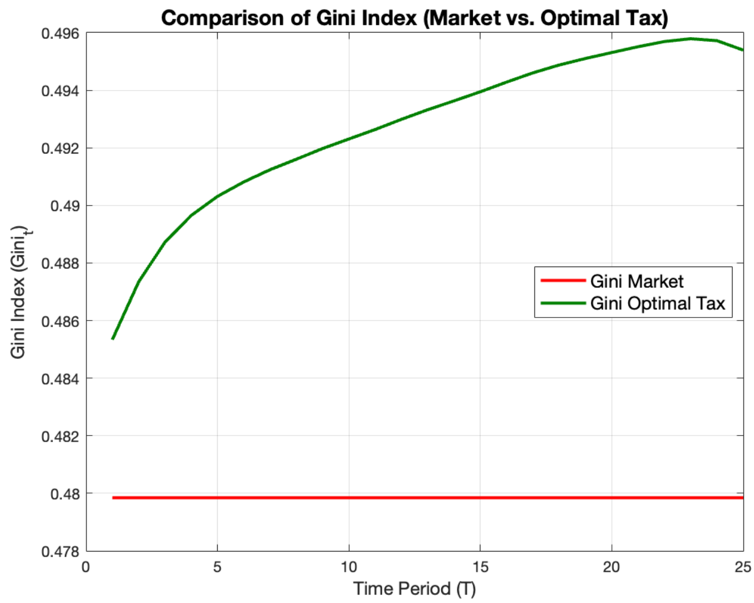

Figure 6: Gini index Business-as-usual vs Gini index Optimal Tax

Understanding the optimal carbon tax aggregated and disaggregated welfare impacts is important, and this will be properly discussed in the next section.

5.3 Welfare

The optimal carbon tax effectively mitigates the environmental externality and enhances aggregate welfare by internalizing the social cost of emissions. By raising the price of the polluting input, the policy incentivizes the economy to shift production towards the clean intermediate energy good $E_{c,t}$. This structural reallocation, however, comes with distributional consequences. The reduction in the production of the dirty intermediate energy good $E_{d,t}$ lowers both the return on capital allocated to the dirty sector and the wage $W_{d,t}$ of low-skill workers employed exclusively in this sector. As a result, while the policy improves efficiency and increases total output Y_t in the long run, these aggregate gains are insufficient to compensate the representative low-skill household for its loss in factor income.

Household	**BAU**	**Optimal Tax**
Low-skill Household	Welfare_{LS}= 6.3310	Welfare_{LS} = 6.2613
High-skill Household	Welfare_{HS} = 6.7727	Welfare_{HS}=6.8308
Aggregated	Welfare_{aggr}= 8.7947	$\text{Welfare}_{aggr.}$ = 8.7951

Table 2: Welfare outcomes for low-skill and high-skill households in the BAU economy and in the Optimal Tax economy

Under a uniform tax rebate, where the revenues from the carbon tax are distributed equally across all households, the asymmetry between the two groups of workers is further amplified. High-skill households, whose income is tied to the expanding clean energy sector $E_{c,t}$ and who enjoy rising wages $W_{c,t}$, receive an additional boost from the uniform rebate. Low-skill households, on the other hand, face a compounded effect: falling sectoral wages and relatively smaller benefits from the uniform transfer. As a result, while aggregate welfare improves compared to the Business-as-Usual (BAU) scenario, the welfare of the low-skill household is lower under the optimal policy with uniform rebates than it is in BAU.

However, this distributional imbalance can be addressed with a non-uniform rebate scheme. For example, if 89% of the tax revenues are rebated to the low-skill household and only 11% to the high-skill household, low-skill agents end up with the same Business-as-Usual welfare, while the high-skill household experiences a welfare gain again. The increased rebate compensates low-skill households for the wage losses induced by the carbon tax, while the high-skill household still benefits from rising capital returns and the growing clean sector. Importantly, this adjustment does not compromise the efficiency gains of the carbon tax, as the total rebate amount remains equal to the total tax revenue.

In summary, while the optimal carbon tax with uniform rebates exacerbates income inequality by hurting low-skill workers and disproportionately favouring high-skill workers, a carefully designed non-uniform rebate scheme can ensure that the low-skill representative household enjoys the same welfare compared to BAU, while maintaining the environmental and efficiency benefits of the policy (and a welfare gain for the high-skill household).

Household	**Uniform Rebate**	**89% Rebate to LS**
Low-skill Household	$\text{Welfare}_{LS} < \text{Welfare}_{LS,BAU}$	$\text{Welfare}_{LS} = \text{Welfare}_{LS,BAU}$
High-skill Household	$\text{Welfare}_{HS} > \text{Welfare}_{HS,BAU}$	$\text{Welfare}_{HS}>\text{Welfare}_{HS,BAU}$

Table 3: Welfare outcomes for low-skill and high-skill households under different rebate schemes. Green cells indicate welfare gains relative to BAU, red cells indicate welfare losses, while white cells designate unchanged welfare compared to BAU .

6 Conclusion

Introducing a carbon tax remains a central topic in both academic and policy debates. Key questions include: What is the optimal tax rate required to effectively internalize the environmental externality? How should the revenues from the carbon tax be distributed? And crucially, who benefits and who bears the costs of this transition? This paper contributes to these questions by examining the interplay between carbon taxation and labour market frictions in a two-sector economy with heterogeneous households.

My findings are nuanced. On the one hand, the optimal carbon tax improves aggregate welfare by reducing emissions, mitigating climate damages, and encouraging a reallocation of capital towards clean energy production. On the other hand, the policy has significant distributional consequences. Due to labor market segmentation, low-skill workers—who are trapped in the polluting sector—experience declining wages and reduced welfare. In contrast, high-skill workers, employed in the clean sector, benefit from increased investment and rising productivity. This asymmetry highlights that a "just transition" cannot be achieved by carbon taxation alone; without complementary measures, the policy risks widening pre-existing inequalities.

One contribution of this work is the exploration of carbon dividend allocation. With a uniform rebate, where tax revenues are equally distributed across households, the gains accrue primarily to high-skill households, while low-skill households remain worse off compared to the Business-as-Usual (BAU) scenario. However, our results show that a non-uniform rebate scheme, where a larger share of the tax revenue is directed to low-skill household, can reverse this outcome. For example, allocating 89% of the tax revenues to low-skill households and 11% to high-skill households raises the welfare of high-skill agents above BAU levels, while leaving unchanged the low-skill household's welfare (compared to the business-as-usual one). Importantly, this redistribution does not compromise the efficiency gains of the carbon tax, as the total rebate matches total revenues.

These results underscore the importance of combining carbon pricing with targeted redistribution mechanisms. By tailoring the rebate system, policymakers can maintain the efficiency of the carbon tax while addressing its regressive effects, ensuring that the transition to a low-carbon economy is both effective and equitable.

Several avenues remain open for future research. First, incorporating utility-based damages from climate change could provide a more comprehensive assessment of welfare impacts. Second, understanding how policies that enhance capital mobility or encourage labour reallocation across sectors might improve the efficiency-equity trade-off remains a key challenge. Finally, exploring the combination of carbon taxes with complementary policies, such as reskilling programs, wage subsidies, or progressive rebates, could yield policy packages that achieve both environmental and social objectives.

Appendix A. Sensitivity Analysis First Chapter

In this appendix, I demonstrate the robustness of the main results, changing one parameter at a time or multiple parameters together. I will show how the optimal tax changes compared to standard parameterisation (the one in the core paper). At the same time, I will give evidence that the aggregated and disaggregated welfare effects are robust to these parameters' twists. All the results for alternative parameterisation are always assuming a uniform tax rebate, (while a non-uniform tax rebate can achieve the same result as the one shown in the core paper).

In another section of this appendix, I also compare how the optimal carbon tax varies compared to the fully mobile labour economy, for different parameters settings, showing that the optimal taxes in the two economies can significantly diverge.

A third exercise is a primitive and imprecise relaxion of the full labour immobility assumption; I do assume an exogenous death rate for the low skill agents, outflowing into the high-skill workforce. Notwithstanding the sloppiness of this exercise, the negative welfare effect of the remaining low-skill agents is still present.

A1 Standard vs. Not Standard Parameters Settings

- Changing β from .985 to .99; the welfare effects are magnified, and the optimal carbon tax is higher.

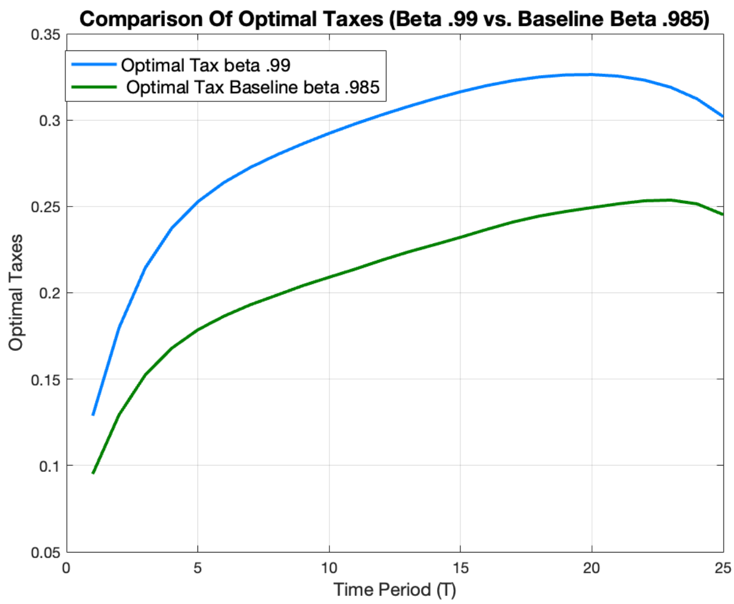

Figure 7: Optimal Taxes with different discount factors

Household	**BAU**	**Optimal Tax**
Low-skill Household	$\text{Welfare}_{LS}= 9.6726$	$\text{Welfare}_{LS} = 9.5342$
High-skill Household	$\text{Welfare}_{HS} = 10.0281$	$\text{Welfare}_{HS}=10.1517$
Aggregated	$\text{Welfare}_{aggr}= 12.9130$	$\text{Welfare}_{aggr.} = 12.9143$

Table 4: Welfare outcomes for low-skill and high-skill households in the BAU economy and in the Optimal Tax economy

- Changing λ_A from 0 (Golosov's benchmark) to 0.013 (Nordhaus (2010)).

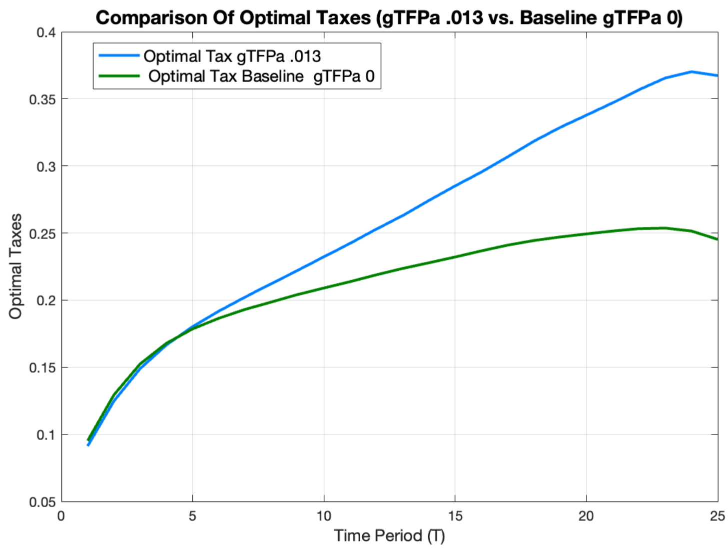

Figure 8: Optimal Taxes with different gTFPa

Household	**BAU**	**Optimal Tax**
Low-skill Household	Welfare_{LS}= 6.6540	Welfare_{LS} = 6.5907
High-skill Household	Welfare_{HS} = 7.1061	Welfare_{HS}=7.1585
Aggregated	Welfare_{aggr}= 9.0275	$\text{Welfare}_{aggr.}$ = 9.0278

Table 5: Welfare outcomes for low-skill and high-skill households in the BAU economy and in the Optimal Tax economy

- $B_{0,d}$ from 1.1 to 1.5 ($B_{0,c} = 1$).

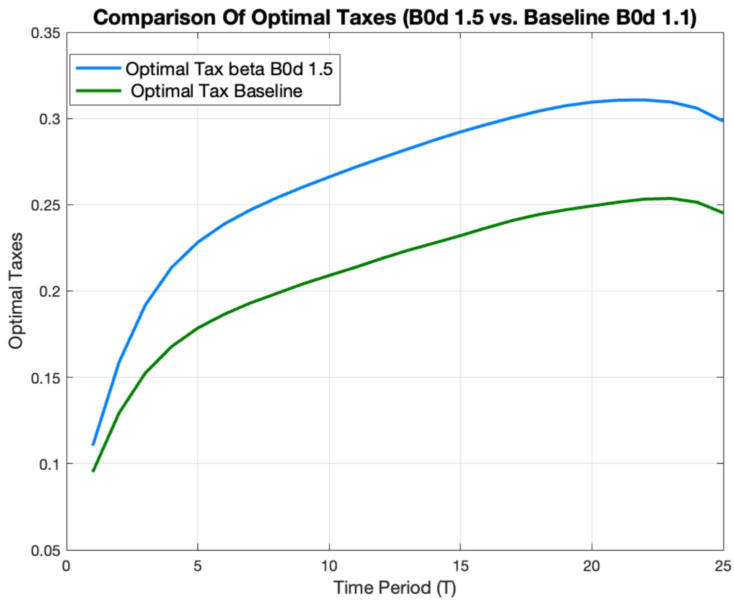

Figure 9: Optimal Taxes with different $B_{0,d}$

Household	**BAU**	**Optimal Tax**
Low-skill Household	$\text{Welfare}_{LS} = 7.5570$	$\text{Welfare}_{LS} = 7.4780$
High-skill Household	$\text{Welfare}_{HS} = 7.1497$	$\text{Welfare}_{HS} = 7.2430$
Aggregated	$\text{Welfare}_{aggr} = 9.3611$	$\text{Welfare}_{aggr.} = 9.3619$

Table 6: Welfare outcomes for low-skill and high-skill households in the BAU economy and in the Optimal Tax economy

- $B_{0,c}$ from 1 to 1.1 ($B_{0,d} = 1$)

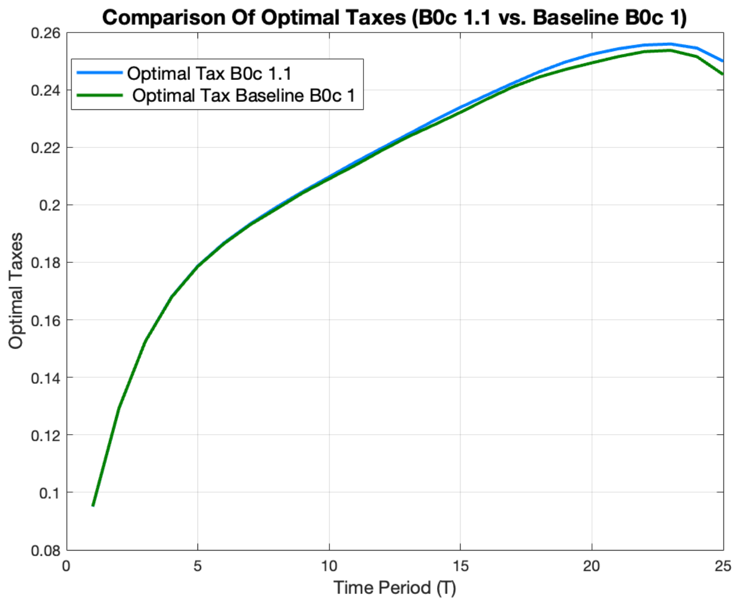

Figure 10: Optimal Taxes with different $B_{0,c}$

Household	**BAU**	**Optimal Tax**
Low-skill Household	Welfare$_{LS}$= 6.0165	Welfare$_{LS}$ = 5.6481
High-skill Household	Welfare$_{HS}$ = 7.0235	Welfare$_{HS}$=7.0713
Aggregated	Welfare$_{aggr}$= 8.8012	Welfare$_{aggr.}$ = 8.8016

Table 7: Welfare outcomes for low-skill and high-skill households in the BAU economy and in the Optimal Tax economy

- ρ (substitution parameter in the final good production function), from .66 to .90

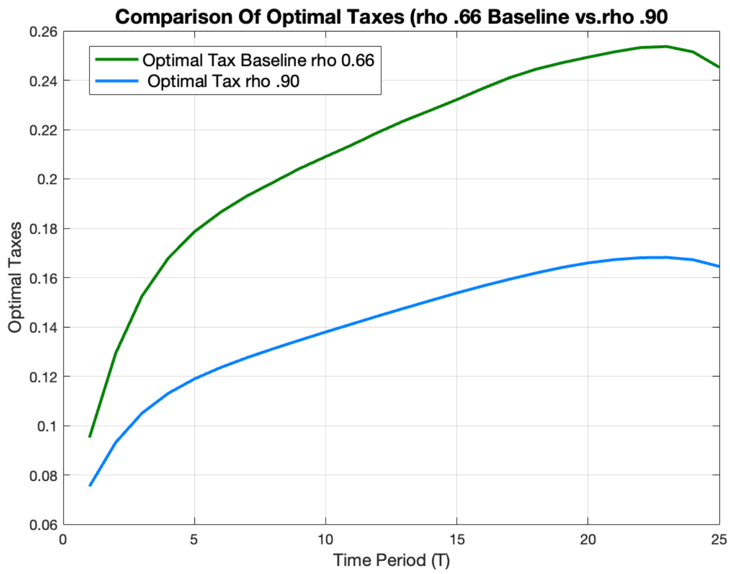

Figure 11: Optimal Taxes with different ρ

Household	**BAU**	**Optimal Tax**
Low-skill Household	Welfare_{LS}= 4.7897	Welfare_{LS} = 4.7981
High-skill Household	Welfare_{HS} = 5.1526	Welfare_{HS}=5.2243
Aggregated	Welfare_{aggr}= 9.3611	$\text{Welfare}_{aggr.}$ = 9.3619

Table 8: Welfare outcomes for low-skill and high-skill households in the BAU economy and in the Optimal Tax economy

- N_{LS} from .5 to .6 ($N_{HS} = .4$)

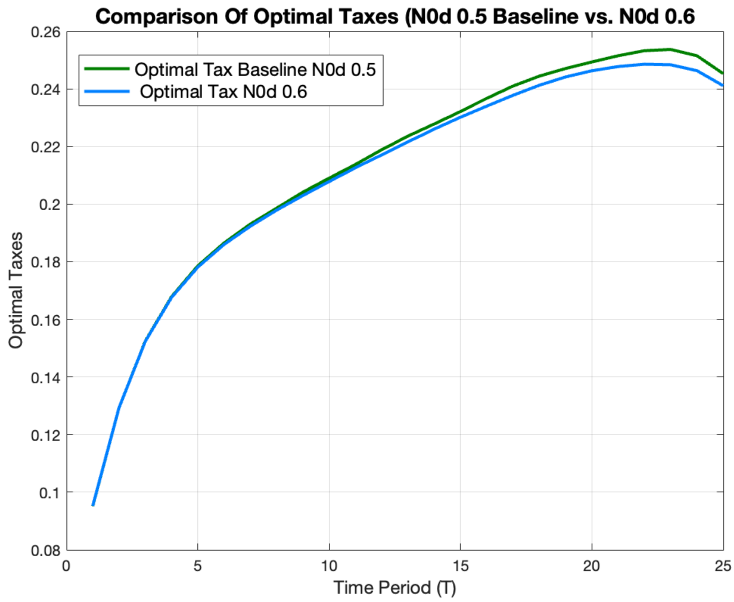

Figure 12: Optimal Taxes with different N_{LS}

Household	**BAU**	**Optimal Tax**
Low-skill Household	Welfare$_{LS}$= 6.7101	Welfare$_{LS}$ = 6.6586
High-skill Household	Welfare$_{HS}$ = 6.3691	Welfare$_{HS}$=6.4282
Aggregated	Welfare$_{aggr}$= 8.7833	Welfare$_{aggr.}$ = 8.7838

Table 9: Welfare outcomes for low-skill and high-skill households in the BAU economy and in the Optimal Tax economy

- N_{LS} from .5 to .8 ($N_{HS} = .2$)

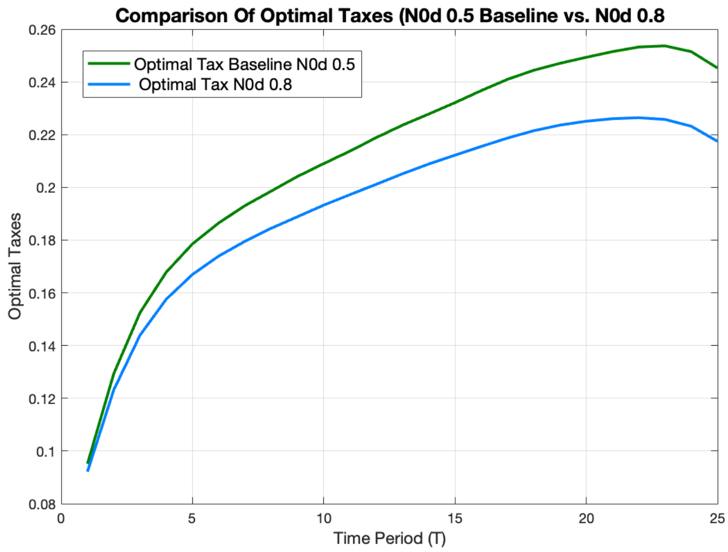

Figure 13: Optimal Taxes with different N_{LS}

Household	**BAU**	**Optimal Tax**
Low-skill Household	Welfare_{LS}= 7.1949	Welfare_{LS} = 7.1773
High-skill Household	Welfare_{HS} = 4.9236	Welfare_{HS}=4.9671
Aggregated	Welfare_{aggr}= 8.3696	$\text{Welfare}_{aggr.}$ = 8.6068

Table 10: Welfare outcomes for low-skill and high-skill households in the BAU economy and in the Optimal Tax economy

- N_{HS} from .5 to .6 ($N_{LS} = .4$)

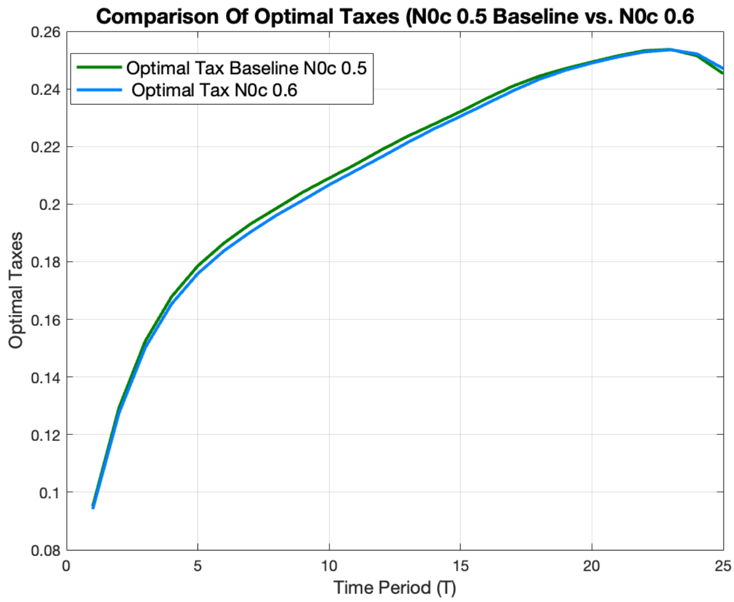

Figure 14: Optimal Taxes with different N_{HS}

Household	**BAU**	**Optimal Tax**
Low-skill Household	Welfare$_{LS}$= 5.8100	Welfare$_{LS}$ = 5.7156
High-skill Household	Welfare$_{HS}$ = 7.0702	Welfare$_{HS}$=7.1123
Aggregated	Welfare$_{aggr}$= 8.7631	Welfare$_{aggr.}$ = 8.7634

Table 11: Welfare outcomes for low-skill and high-skill households in the BAU economy and in the Optimal Tax economy

- N_{HS} from .5 to .8 ($N_{LS} = .2$)

Household	**BAU**	**Optimal Tax**
Low-skill Household	Welfare_{LS}= 4.3440	Welfare_{LS} = 4.2056
High-skill Household	Welfare_{HS} = 8.7120	Welfare_{HS}=8.8055
Aggregated	Welfare_{aggr}= 8.6031	$\text{Welfare}_{aggr.}$ = 8.5023

Table 12: Welfare outcomes for low-skill and high-skill households in the BAU economy and in the Optimal Tax economy

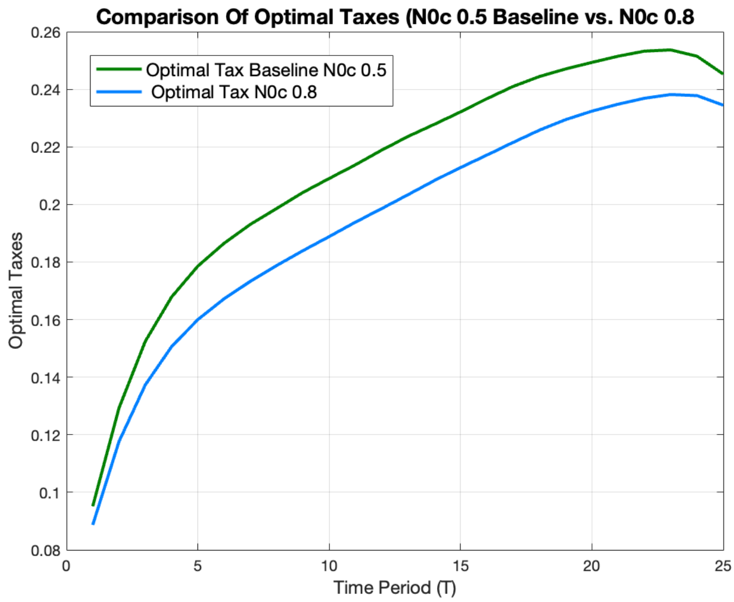

Figure 15: Optimal Taxes with different N_{HS}

A2 Mobility vs Immobility Optimal Taxes

This subsection compares the optimal carbon taxes in two benchmark economies: (i) a *full labour mobility* economy, where a unit mass of generic workers can freely reallocate between the dirty and clean energy sectors, and (ii) a *full labour immobility* economy, where labour is sector-specific with exogenous population shares N_{LS} and N_{HS}. Throughout, technology-side parameters $(\rho, B_{0,d}, B_{0,c})$ are held fixed, and damages operate through TFP as in the baseline model.

In the core calibration, the two optimal tax paths are almost indistinguishable: the optimal tax under mobility and immobility virtually overlap. The reason is that, under the standard parameterisation, the implied optimal sectoral labour allocations are very similar across the two environments, leaving only a small wedge between the associated marginal external damages.

The two optimal taxes can separate markedly when population shares are skewed, *ceteris paribus* on $(\rho, B_{0,d}, B_{0,c})$. Consider, for illustration, the cases:

(a) $N_{HS} = 0.8$ and $N_{LS} = 0.2$;

(b) $N_{LS} = 0.8$ and $N_{HS} = 0.2$.

In both configurations, the *mobility* economy features a higher optimal carbon tax than the *immobility* (sector-specific) economy. Figures 17 and 18 display these cases: $\tau_t^{*,\text{mob}}$ lies above $\tau_t^{*,\text{imm}}$ along the transition. With full mobility, labour equalises sectoral wages and is allocated to the highest return uses each period. Relative to the sector-specific benchmark (where one sector is mechanically capped by its exogenous labour endowment), this generates a stronger intensive-margin response: aggregate production is higher, and dirty output and emissions are also higher. As a consequence, the baseline emissions path in the mobility economy entails larger marginal external damages, which the planner offsets with a more aggressive Pigouvian tax.

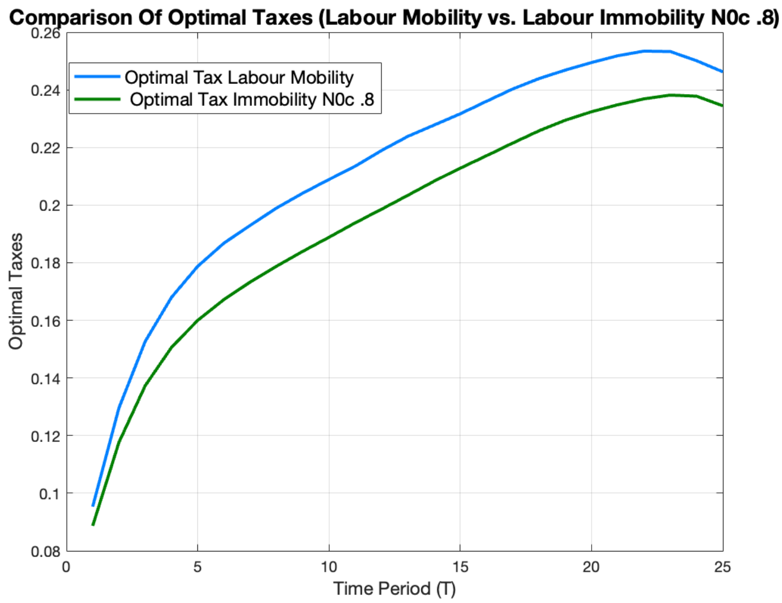

Figure 16: Optimal Taxes Mobility vs Immobility

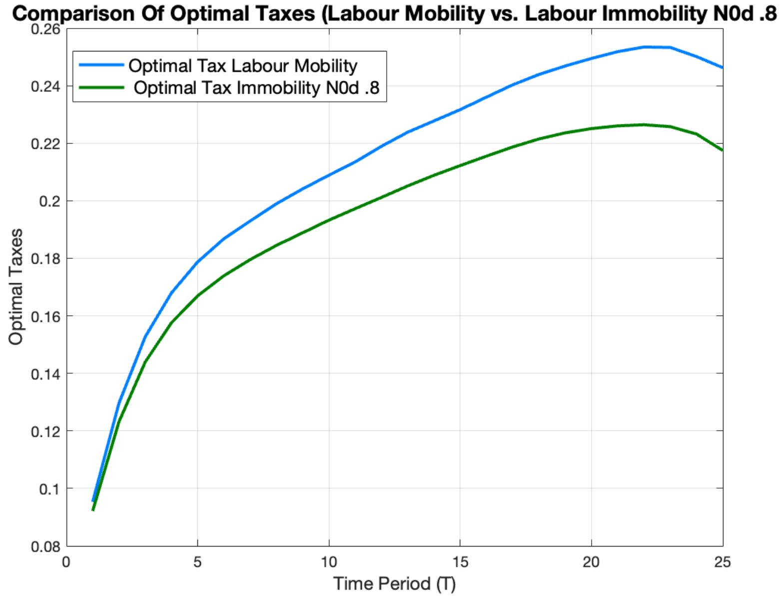

Figure 17: Optimal Taxes Mobility vs Immobility

The near-coincidence of optimal taxes in the baseline calibration hinges on balanced population shares and similar implied sectoral allocations. Once the labour endowment is skewed, the mobility economy's ability to reallocate workers magnifies productive capacity and the emissions base, calling for a higher optimal tax relative to the sector-specific benchmark. This highlights that conclusions about the level of the optimal carbon tax are sensitive to the labour-allocation environment even when technology-side parameters are held constant.

A3 Exogenous LS Outflow To HS Workforce

In this final section of the appendix, I analyse the implications for welfare and for the optimal climate policy, when the low-skill agents' labour endowment shrinks with time (due to institutional and political reasons). The death rate of low-skill agents is 1% every ten years (1 period of the model), given the standard parameterisation. I propose the optimal taxes and the usual table with welfare effect between the BAU economy and the Optimal Carbon Tax economy.

Household	BAU	Optimal Tax
Low-skill Household	Welfare$_{LS}$= 6.2147	Welfare$_{LS}$ = 6.1424
High-skill Household	Welfare$_{HS}$ = 6.8545	Welfare$_{HS}$=6.9104
Aggregated	Welfare$_{aggr}$= 8.7901	Welfare$_{aggr.}$ = 8.7905

Table 13: Welfare outcomes for low-skill and high-skill households in the BAU economy and in the Optimal Tax economy

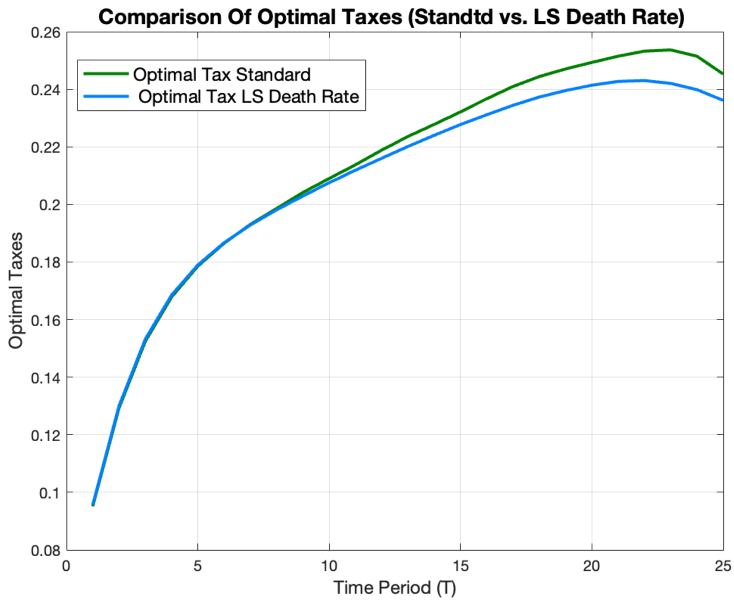

Figure 18: Optimal Taxes Mobility vs Immobility

Results are robust even for higher death rates.

A4 Computational algorithm for the planner's problem

In this chapter, the social planner's problem is solved numerically as a nonlinear program with constraints. The algorithm is implemented in MATLAB and is organized into three main files: (i) a *main file*, which contains the parametrization, constructs the vector of choice variables and calls the numerical solver; (ii) an `objective file` , which evaluates the planner's objective function; (iii) a `constraints file`, which implements the equilibrium constraints of the economy.

The time horizon of the simulation is finite and consists of two blocks:

- a transition phase of length $T = 25$ decadal periods;
- additional $P = 10$ continuation periods, which approximate the long-run balanced growth path.

Let $H = T + P$ denote the total number of simulated periods.

A4.1 Structure of the planner's problem

The planner chooses, for each period $t = 1, \dots, H$, the profiles of:

- capital allocation between the "dirty" and the "clean" energy sector, through the share $\xi_t \in (0, 1)$ invested in the dirty sector;
- the saving rate $s_t \in (0, 1)$;
- individual consumption of low-skilled ($C_{LS,t}$) and high-skilled ($C_{HS,t}$) workers;
- the tax rate on dirty energy production τ_t (defined for the transition periods $t = 1, \dots, T$, when the externality is active).

Given a vector of choices x, the economy evolves according to the capital accumulation equation, energy production in the dirty and clean sectors, final output with climate damages, and the pricing equations, as described by the model in Section 1.

The planner maximizes Negishi-weighted social welfare:

$$W = \theta \sum_{t=1}^{\infty} \beta^{t-1} u(C_{LS,t}) + (1 - \theta) \sum_{t=1}^{\infty} \beta^{t-1} u(C_{HS,t}),$$

where $\theta \in (0, 1)$ is the weight of the low-skilled group, β is the decadal discount factor and $u(\cdot)$ is a CRRA utility function (with risk-aversion coefficient σ), or logarithmic when $\sigma = 1$. In the code, the infinite-horizon problem is approximated using the H explicit periods plus a "tail" term that captures the present value after the balanced growth path, where we assume aggregate consumption grows at a constant rate gZBGP.

A4.2 Vector of choice variables

The *main file* builds a single choice vector of dimension $(5T + 2P + 2) \times 1$, collecting:

$$x = \Big(\underbrace{\xi_1, \dots, \xi_{T+1}}_{\text{capital shares}}, \underbrace{s_1, \dots, s_{T+1}}_{\text{saving rates}}, \underbrace{C_{LS,1}, \dots, C_{LS,T+P}}_{\text{LS consumption}}, \underbrace{C_{HS,1}, \dots, C_{HS,T+P}}_{\text{HS consumption}}, \underbrace{\tau_1, \dots, \tau_T}_{\text{carbon tax}}\Big).$$

In the code:

- the first $T + 1$ entries of x correspond to the dirty-sector capital shares $\{\xi_t\}_{t=1}^{T+1}$;
- the next $T + 1$ entries are the saving rates $\{s_t\}_{t=1}^{T+1}$;
- entries from $2T + 3$ to $3T + P + 2$ are the consumption levels of low-skilled workers $C_{LS,t}$, for $t = 1, \dots, H$;

- entries from $3T + P + 3$ to $4T + 2P + 2$ are the consumption levels of high-skilled workers $C_{HS,t}$, for $t = 1, \ldots, H$;
- entries from $4T + 2P + 3$ to $5T + 2P + 2$ are the tax rates τ_t, for $t = 1, \ldots, T$.

Simple bounds are imposed on some choice variables to guarantee numerical feasibility:

$$0.001 \leq \xi_t \leq 0.99, \qquad 0.001 \leq s_t \leq 0.99, \qquad \forall t,$$

while consumption and taxes are restricted via inequality constraints (e.g. non-negativity of consumption).

As an initial condition, the code uses:

- $\xi_t = \xi_0$ constant for $t = 1, \ldots, T + 1$;
- $s_t = \alpha\beta$ for $t = 1, \ldots, T + 1$;
- initial aggregate consumption $\{C_t\}$ implied by the corresponding equilibrium path;
- an initial split of consumption between LS and HS given by the weight θ_{ex} (the initial value for θ is chosen after a preliminary run of the code to let the code converge faster).

A4.3 Objective function: `objective file`

The file `Contrada_Objective_Final.m` takes as input a vector x and the model parameters, and returns the (negative) value of the planner's objective function. The main steps are:

1. **Reconstruction of the equilibrium path.** Given x, the code extracts:

$$\{\xi_t\}_{t=1}^{T+1}, \quad \{s_t\}_{t=1}^{T+1}, \quad \{C_{LS,t}\}_{t=1}^{H}, \quad \{C_{HS,t}\}_{t=1}^{H}, \quad \{\tau_t\}_{t=1}^{T}.$$

   Starting from K_0, from the labor endowments (N_{LS}, N_{HS}) and from the productivity processes $(A_t, B_{d,t}, B_{c,t})$, the code recursively determines:

   - aggregate capital K_t via:

$$K_{t+1} = s_t Y_t + (1 - \Delta) K_t;$$

   - dirty and clean energy production:

$$E_{d,t} = B_{d,t}(N_{LS})^{1-\alpha}(\xi_t K_t)^{\alpha}, \qquad E_{c,t} = B_{c,t}(N_{HS})^{1-\alpha}((1 - \xi_t)K_t)^{\alpha};$$

   - cumulative emissions Z_t as the sum of dirty emissions;

- final output with climate damages:

$$Y_t = A_t \exp(-\mu Z_t) \left(E_{d,t}^{\rho} + E_{c,t}^{\rho}\right)^{1/\rho}.$$

From Y_t and s_t, aggregate consumption is computed as $C_t = (1 - s_t)Y_t$, which is then compared to the sum of individual consumptions $C_{LS,t} + C_{HS,t}$ in the constraint file.

2. **Prices and rates of return.** Final-output demand implies prices for the two energy types (competitive markets first-order-conditions):

$$P_{d,t} = \frac{Y_t}{E_{d,t}} \frac{(E_{d,t})^{\rho}}{(E_{d,t})^{\rho} + (E_{c,t})^{\rho}}, \qquad P_t^c = \frac{Y_t}{E_{c,t}} \frac{(E_{c,t})^{\rho}}{(E_{d,t})^{\rho} + (E_{c,t})^{\rho}}.$$

The gross rates of return to capital in the two sectors are:

$$r_{d,t} = \frac{(P_{d,t} - \tau_t)\, \alpha E_{d,t}}{\xi_t K_t}, \qquad r_{c,t} = \frac{P_{c,t}\, \alpha E_{c,t}}{(1 - \xi_t) K_t},$$

for $t \leq T$. In the continuation periods ($t > T$), there is no tax and hence:

$$r_{d,t} = \frac{P_{d,t}\, \alpha E_{d,t}}{\xi_{T+1} K_t}.$$

3. **Intertemporal utility.** Instantaneous utility for each group is:

$$u(C_{i,t}) = \begin{cases} \dfrac{C_{i,t}^{1-\sigma} - 1}{1 - \sigma} & \text{if } \sigma \neq 1, \\ \log C_{i,t} & \text{if } \sigma = 1, \end{cases}$$

with $i \in \{LS, HS\}$. `Objective` computes the utility associated with the T transition periods (U_Cons) and the P continuation periods (U_Cont). For the last continuation period, a "tail" term is added that approximates the present value of utility along the balanced growth path, using the exogenous growth rate g_Y (denoted by `gZBGP` in the code):

$$\text{BGP term} = \frac{1}{1 - \beta(1 + g_Y)^{1-\sigma}}.$$

4. **Value of the objective function.** Overall social welfare is:

$$W = \theta \sum_{t=1}^{H} \beta^{t-1} u(C_{LS,t}) + (1 - \theta) \sum_{t=1}^{H} \beta^{t-1} u(C_{HS,t}) + \text{BGP tail term}.$$

The file returns $f = -W$, since `fmincon` minimizes the function.

A4.4 Equilibrium constraints: `constraints file`

The file `Contrada_Constraints_Final.m` implements the equilibrium conditions of the economy and returns:

$$c(x) \leq 0 \quad \text{(inequality constraints)}, \qquad \texttt{ceq}(x) = 0 \quad \text{(equality constraints)}.$$

Given the same decomposition of x, the file reconstructs $K_t, E_{d,t}, E_{c,t}, Y_t, C_t, P_{d,t}, P_{c,t} r_{d,t}, r_{c,t}$ as above and imposes:

- **Inequality constraints** (vector c):
  1. *Non-negativity of aggregate consumption*:
  $$C_t \geq \underline{C} \quad \Rightarrow \quad c_t = -(C_t - \underline{C}) \leq 0, \qquad t = 1, \ldots, H,$$
  where $\underline{C} = 0.0001$ in the code.
  2. *Feasibility of individual consumption*:
  $$C_t \geq C_{LS,t} + C_{HS,t} \quad \Rightarrow \quad c_{H+t} = -(C_t - C_{LS,t} - C_{HS,t}) \leq 0, \qquad t = 1, \ldots, H.$$
- **Equality constraints** (vector `ceq`):
  1. *Equalization of sectoral returns* (no-arbitrage condition):
  $$r_t^d = r_t^c \quad \Rightarrow \quad \texttt{ceq}_t = -(r_t^c - r_t^d) = 0, \qquad t = 1, \ldots, T.$$
  This condition ensures that capital is optimally allocated across the dirty and clean sectors.

If I have to solve for the Business-As-Usual scenario, I simply add a stream of equality constraints regarding the tax, i.e., $\text{tax}_t = 0 \ \ \forall t > 0$.

A4.5 Numerical solution and reconstruction of the economy

The *main file* calls `fmincon` with the SQP algorithm:

- first with a high maximum number of iterations and a tight tolerance on the objective function and constraints;
- then a second time, using the optimal vector just found as initial guess, to further refine convergence.

At each step, `fmincon`:

1. passes the current vector x to `objective file`, which returns $-W(x)$;

2. passes the same x to `constraints file`, which returns $c(x)$ and $\texttt{ceq}(x)$;

3. updates x until the first-order optimality conditions and constraints are satisfied within the prescribed tolerance.

Once the optimal vector $x^\star$ is obtained, the *main* file reconstructs the entire optimal equilibrium:

- the paths of $K_t, E_{d,t}, E_{c,t}, Z_t, Y_t$;
- prices $P_{d,t}, P_{c,t}$ and returns $r_{d,t}, r_{c,t}$;
- wages for the two groups:

$$W_t^{LS} = \frac{(P_{d,t} - \tau_t)E_{d,t}(1-\alpha)}{N_{LS}}, \qquad W_t^{HS} = \frac{P_{c,t}E_t^c(1-\alpha)}{N_{HS}};$$

- gross carbon tax revenues:

$$\text{Tax}_t = \tau_t E_{d,t}, \qquad \text{Tax\_gross}_t = q_t \, \text{Tax}_t.$$

By weighting wages and tax revenues with the discount factor q_t and multiplying by group sizes, the *main* file computes the intertemporal wealth of low-skilled and high-skilled agents (equations (1) and (2)). Comparing these wealth levels with the discounted consumption flows $\{q_t C_{LS,t}\}$ and $\{q_t C_{HS,t}\}$, the code checks that the solution is consistent with the Negishi weights used in the planner's problem: each group's wealth must equal the present value of its optimal consumption.

A4.6 Summary of the algorithm

To summarize, the computational algorithm proceeds as follows:

1. set technological, preference and climate-damage parameters, as well as the simulation horizon (T, P);

2. construct a feasible initial vector x_0 of planner's choices;

3. use `fmincon` (SQP) to solve:

$$\min_x \; -W(x) \quad \text{subject to} \quad c(x) \leq 0, \; \texttt{ceq}(x) = 0;$$

4. once the solution $x^\star$ is obtained, reconstruct the dynamics of the economy (quantities, prices, wages, taxes);

5. compute the intertemporal wealth of the two groups and verify that it is consistent with the planner's Negishi weight θ;
6. compute and report aggregate welfare and group-specific welfare.
7. converges if the wealth and consumption paths are consistent; update θ and restart from the point 3.

Chapter 2

Climate Policies, Inequality and Inputs' Markets Frictions

Flavio Contrada*†

Abstract

I develop a multisectoral neoclassical growth model with climate externalities, heterogeneous households, and input market frictions to examine the interplay between climate change and inequality. The final good is produced using two intermediate energy goods (each of them produced by sector-specific capital and labour). I investigate how input market frictions affect the per-unit optimal carbon tax and study the impact of this tax on both aggregate and disaggregated welfare. I highlight that the optimal carbon tax is not affected by market frictions in the inputs' market under the standard calibration; however, I show that low-skill households (employed in the dirty sector) experience welfare losses under environmental policy, even though aggregate welfare improves. Third, I analyse an alternative policy instrument (a tax on dirty capital goods rental rate instead of a per-unit carbon tax), showing that this policy mitigates the adverse effects on low-skill households. Finally, I demonstrate that implementing a non-uniform tax rebate scheme, where rebates are distributed asymmetrically between the two household types, can improve the welfare of both representative households under both taxation schemes. To conclude, a comparison between policies with sector-specific inputs and policies with fully mobile inputs is discussed.

Keywords: Climate change, optimal policy, optimal taxes, inequality, market frictions.

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1 Introduction

Climate policies, such as carbon taxation, stand at the centre of heated debates among economists and policymakers. While their necessity for mitigating climate change and protecting the planet is widely acknowledged, their societal impacts are less clear-cut. Who bears the greatest burden of these policies? Do they benefit everyone equally? Why are they often met with strong resistance from workers, as illustrated by the Yellow Vests movement in France? Although recent economic literature has begun addressing the connections between inequality and climate change (see Drupp et al., 2024 for a review), both empirical and theoretical analyses remain incomplete, especially when accounting for sectoral frictions and labour market heterogeneity.

This paper, building on the previous chapter, advances the literature on optimal carbon taxation by analyzing its distributional effects in the presence of inputs' immobility across energy sectors. We build a multisectoral neoclassical growth model with a climate externality, heterogeneous households, and fully immobile capital and labour. The final good is produced using two intermediate energy goods (dirty and clean), each relying on sector-specific capital and labour. Furthermore, the high-skill household works exclusively in the clean sector, while the low-skill household is employed in the dirty sector, which is responsible for climate damages that reduce the Total Factor Productivity (TFP) of the final good.

Our analysis addresses three core questions: (1) How is the optimal carbon tax modified when labour and capital are sector-specific rather than perfectly mobile? (2) What are the distributional consequences of this policy for households employed in different sectors? (3) Are there alternative taxation schemes that can achieve the same climate objectives while producing different welfare outcomes?

First, I show that, under the standard calibration, inputs' immobility does not affect the magnitude of the optimal carbon tax. Second, although the policy improves aggregate welfare, the representative low-skill household in the dirty sector experiences a wage cut and a decline in welfare, highlighting the regressive distributional effects of a uniform carbon tax under my assumptions. Third, by exploring an alternative policy (i.e, a tax on the rental rate of dirty capital), I show that it partially protects the low-skill household from wage losses while mitigating the externality.

Finally, I demonstrate that a non-uniform tax rebate scheme, with rebates tilted toward the low-skill household, can improve the welfare of both household types under both taxation regimes. This mechanism enhances the fairness and political acceptability of climate policies, reconciling environmental and distributional objectives without compromising climate mitigation.

At the same time, comparisons between the full immobile and the fully mobile scenarios are carried out in the appendix, showing that the results on the magnitudes of the instruments in the core paper are parameter sensitive.

This work contributes to the general-equilibrium literature on optimal carbon taxation, which seeks to characterize the efficiency-restoring tax path during the transition to a low-carbon economy. Foundational contributions include Golosov et al. (2014), Dietz and N. Stern (2015), Barrage (2020), and Douenne, Hummel, and Pedroni (2022). Yet theoretical models that embed labour heterogeneity alongside limited inter-sectoral mobility in the energy sector remain scarce. Notable exceptions include Calvacanti, Hasna, and Santos (2024) and Hafstead and Williams III (2018), who study the impacts of a carbon tax in computational general-equilibrium frameworks with labour frictions, albeit from a static perspective. Fremstad and Paul (2019) considers climate policy in a Real-Business-Cycle setting with heterogeneous households, emphasizing short-run dynamics, while Belfiori, Carroll, and Hur (2024) analyse long-run heterogeneity under carbon pricing but assumes perfect labour mobility and a capital-less production structure. In the next section, a detailed literature review is presented.

1.1 Related Literature

The DICE model (Nordhaus, 2010) has been widely used and extended to compute optimal carbon taxes and evaluate their economic and environmental impacts. While many integrated assessment models (IAMs) focus on linking economic activity to climate damages, a complementary strand of research explores carbon taxation from a more rigorous economic theory perspective. A seminal contribution is Golosov et al. (2014), which develops a representative-agent dynamic stochastic general equilibrium (DSGE) model of the energy sector to derive the optimal carbon tax and its time path under uncertainty.

Several studies build on this framework, focusing on different aspects of the carbon tax problem. Among the most relevant, Barrage (2020) introduces capital and income taxation to investigate how distortionary taxes affect the First-Best Carbon Tax. At the same time, new debates—such as the double-dividend hypothesis, the green paradox, and the just transition—have emerged, alongside growing empirical evidence on the unequal impacts of both climate change and climate policy. See (Green and Healy, 2022, Burzyński et al., 2022, Tol et al., 2004, Hallegatte, 2016, Hsiang, Oliva, and Walker, 2019, Känzig, 2023).

While several IAMs account for cross-regional inequality—most notably the NICE model (Budolfson et al., 2017, Gazzotti et al., 2021)—far fewer address it within a fully microfounded theoretical framework. A key exception is Douenne, Hummel, and Pedroni (2022), who study optimal carbon taxation in a heterogeneous-agent economy with wealth inequality and heterogeneous climate sensitivity. Our approach is close in spirit but departs in one crucial respect: we allow for imperfect mobility of labour and capital across sectors. The transition from a high- to a low-carbon economy requires building sector-specific human and physical capital, consistent with evidence on technological and occupational specificity between clean and dirty energy production (Vona et al., 2018, Bluedorn et al., 2023, Borissov, Brausmann, and Bretschger, 2019, Rud et al., 2024). This motivates relaxing the standard perfect-mobility assumption and focusing on the distributional implications of climate policy.

Moreover, the assumption of sector-specific capital for energy production is consistent with empirical findings and theoretical papers. Rozenberg, Vogt-Schilb, and Hallegatte (2014) develop a Ramsey-style model with two sector-specific capital stocks (dirty vs. clean) under irreversible investment. They show that optimal carbon pricing can rationally lead to under-utilization or early retirement of polluting capital (i.e., stranded assets), and that investment-based instruments (e.g., standards/feebates) can smooth short-run output by redirecting new investment toward clean capacity—albeit at higher intertemporal cost than a first-best carbon price. Complementing this, Van der Ploeg and Rezai (2020) synthesize how limited redeployability, adjustment costs, and irreversibility interact with policy credibility and technological shocks to determine the extent and distribution of stranding. Together, these studies underscore why theoretical models should treat capital as sector-specific: immobility creates lock-in, shapes optimal instrument choice and timing, adds a distinct "transition cost" component beyond technical abatement costs, and concentrates losses on owners and workers tied to carbon-intensive assets—central features for welfare and incidence analysis during decarbonization. Moreover, other papers do assume sector specific-capital in short-run models(Carattini, Heutel, and Melkadze, 2023, Diluiso et al., 2021), and thus, a more thorough analysis is needed also in long-run models.

This paper fills a gap by explicitly linking optimal carbon taxation to inequality through the lens of input immobility, highlighting how sector-specific labour and capital frictions shape both the magnitude of the optimal tax and its heterogeneous welfare effects. By comparing alternative taxation schemes and introducing non-uniform tax rebates, we provide new insights into how climate policy can be designed to achieve both environmental and distributional goals.

The remainder of the paper proceeds as follows: Section (2) introduces the general model, Section (3) characterizes the competitive equilibrium with taxes, and Section (4) studies the Social Planner's problem and the government's Ramsey problem. The final sections present numerical simulations and the main results.

2 Model

The present section proposes an overview of the economy. I build a multi-sectorial neoclassical growth model with climate externality and input market frictions. The economy is very similar to Golosov et al. (2014) i.e, a standard neoclassical growth model with an externality stemming from producing a dirty input good. I differentiate my set-up from theirs by introducing sector-specific capital and labour; the dirty capital good can only be employed in the dirty sector, while the clean capital good in the clean sector (Diluiso et al., 2021, Carattini, Heutel, and Melkadze, 2023). Regarding labour, two representative heterogeneous infinitely-lived households exist, a low-skill and a high-skill one. The low-skill one can only work in the dirty sector, instead the high-skill representative household supplies labour in the clean sector. Time is discrete ($t = 0, 1, ..$) and infinite.

The supply block of the economy is characterized by a final good technology employed to produce Y_t. Two intermediate energy goods, the dirty one ($E_{t,d}$) and the clean one ($E_{t,c}$), are used in the final good technology. However, $E_{t,d}$ damages the economy because it affects the climate; its impacts are imposed on the final good, Total-Factor-Productivity (TFP henceforth), through a damage function. Two different representative households populate the economy: a low-skill household and a high-skill household. At the beginning of each time period, the low-skill household is endowed with N_{LS} units of labour and the high-skill household with N_{HS}. The representative high-skill household additionally possesses the initial endowment of dirty capital ($K_{0,d}$) and clean capital ($K_{0,c}$). The dirty intermediate energy good $E_{t,d}$ employs dirty capital $K_{t,d}$ and low-skilled labour; meanwhile, the intermediate clean energy good only uses high-skilled labour and clean capital ($K_{t,c}$). The final output can be consumed by households or stored in two different linear storage technologies that transform it into the next period of clean/dirt capital. A graphical representation of the model is presented in Figure 1:

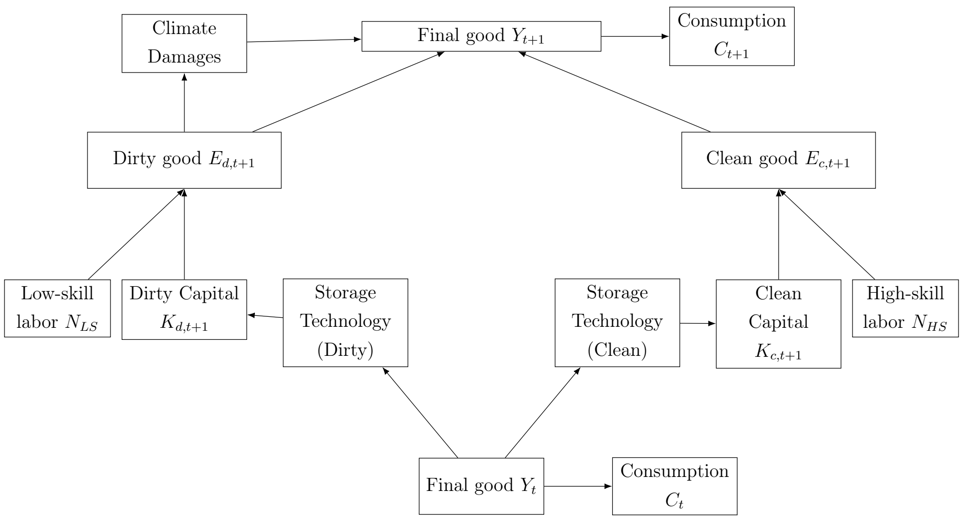

Figure 1: Overview of the model

2.1 Households

The utility function is a standard Constant Relative Risk Aversion (CRRA):

$$U^j \equiv \sum_{t=0}^{\infty} \beta^t \frac{C_{j,t}^{1-\sigma} - 1}{1-\sigma},$$

where j = {LS (low-skill), HS (high-skill)}. $C_{j,t}$ is consumption of the final good, β is the discount factor and σ is the inter-temporal elasticity of substitution. The representative households share the structural parameters β and σ. There is no dis-utility from labour; each period the low-skill household is endowed with N_{LS} labour units, and the high-skill household with N_{HS} labour units.

2.2 Intermediate Energy Goods Producers

Two representative intermediate energy firms, dirty and clean, combine sector-specific capital and labour through a Cobb-Douglas technology with constant return to scale:

$$E_{d,t} \leq B_{d,t} F_d(K_{d,t}, l_t^{LS}) = B_{d,t}(K_{d,t})^{\alpha}(l_t^{LS})^{1-\alpha} \quad (1)$$

$$E_{c,t} \leq B_{c,t} F_c(K_{c,t}, l_t^{HS}) = B_{c,t}(K_{c,t})^{\alpha}(l_t^{HS})^{1-\alpha}. \quad (2)$$

$B_{d,t}$ and $B_{c,t}$ represent the TFP in the two intermediate energy sectors driven by exogenous parameters. The main departure from Golosov et al. (2014) lies in the full inputs' market frictions. For what concerns labour, the low-skill household can only work in the dirty sector; vice versa, the high-skill household is employed in the clean one. With regards to capital, $K_{d,t}$ and $K_{c,t}$ are the two sector-specific capital goods supplied by two different storage firms to the representative firms.

2.3 Final Good Producer

The final good production function is a standard constant elasticity of substitution (CES) with constant return to scale technology, having the two energetic goods as inputs:

$$Y_t \leq F^{FG}(Z_t, E_{d,t}, E_{c,t}) = (1 - D(Z_t))A_t \left(E_{d,t}^{\rho} + E_{c,t}^{\rho}\right)^{\frac{1}{\rho}}. \quad (3)$$

The ρ parameter measures the substitutability between $E_{d,t}$ and $E_{c,t}$ inputs, and A_t is the final sector exogenous TFP. The $E_{t,d}$ cumulative stock produced in the economy affects the final good sector negatively. The functional form of this damage engendered by $E_{d,t}$ is the one employed in Golosov et al. (2014), which the following exponential damage function

$$D(Z_t) = 1 - e^{-\gamma(Z_t)},$$

where γ measures the intensity of damages. The link between the climate and the final good sector occurs through the variable Z_t, denoting cumulative emissions, i.e, the summation of $E_{t,d}$ produced in the economy between the first time period and the current one:

$$Z_t = Z_0 + \sum_{k=1}^{t} E_{d,k}, \tag{4}$$

where Z_0 is the starting cumulative stock of emissions. $E_{d,t}$ instantly flows into the atmosphere at time t, assuming no temperature inertia for simplicity (Douenne, Hummel, and Pedroni (2022), Nordhaus (2010)). To conclude, Z_t results in climate damage on the final TFP A_t through the damage function. Y_t is the numeraire of the economy.

2.4 Storage Firms

The agents can either consume Y_t ($C_{j,t}$) or invest it in two different linear technologies that transform the final good into next-period $K_{c,t+1}$ and $K_{d,t+1}$:

$$K_{i,t+1} \leq I_{i,t} + (1 - \delta_i)K_{i,t}, \tag{5}$$

with i = {d (dirty), c (clean)}. The two representative storage firms are active between two time periods. The dirty and clean storage firms active between time t and $t + 1$ buy dirty and clean capital goods from the respective storage firms active between $t - 1$ and t. Then, they rent the acquired capital to the dirty and clean intermediate energy goods producers.

The revenues made through the rental rate, and further output possibly bought in the market are the investments to be brought in the next period using their linear technology.

2.5 Feasibility Conditions

In this generic framework, seven feasibility conditions must be satisfied in equilibrium. Two capital good feasibility conditions, two labour feasibility conditions, two intermediate energy feasibility conditions and the final good feasibility conditions:

$$l_t^{LS} \leq N_{LS} \tag{6}$$

$$l_t^{HS} \leq N_{HS} \tag{7}$$

$$K_{c,t} \leq I_{c,t-1} + (1 - \delta_c)K_{c,t-1} \tag{8}$$

$$K_{d,t} \leq I_{d,t-1} + (1 - \delta_d)K_{d,t-1}. \tag{9}$$

The aggregate resource constraint for the final good requires that Y_t has to be greater or equal to aggregate consumption and the two investments:

$$C_{LS,t} + C_{HS,t} + I_{d,t} + I_{c,t} \leq Y_t.$$

At time $t+1$, $I_{d,t}$ and $I_{c,t}$ determine the amount of $K_{d,t+1}$ and $K_{c,t+1}$ (along with the undepreciated part of $K_{d,t}$ and $K_{c,t}$) used to produce $E_{d,t+1}$ and $E_{c,t+1}$, which are agglomerated again to produce the final output Y_{t+1}. With the updated variable Z_{t+1} (resulting in TFP damages), the final output is either consumed or invested in the dirty or clean storage technologies. The following section describes the decentralized equilibrium.

3 Decentralized Time-0 Equilibrium

We now analyze the deterministic time-0 competitive equilibrium. All markets are competitive, and all agents have access to the complete set of Arrow-Debreu contingent claims. Labour endowments, denoted by N_{HS} and N_{LS}, are fixed and do not vary over time. The total tax revenue T collected by the Government is rebated lump-sum to households. Unless otherwise specified, this rebate is uniform across households.

3.1 Households

The high-skill (HS) representative household chooses consumption $\{C_{HS,t}\}_{t=0}^{\infty}$ to maximize lifetime utility:

$$\begin{aligned} \max_{\{C_{HS,t}\}_{t=0}^{\infty}} \quad & \sum_{t=0}^{\infty} \beta^t \frac{C_{HS,t}^{1-\sigma} - 1}{1-\sigma} \\ \text{s.t.} \quad & \sum_{t=0}^{\infty} q_t C_{HS,t} \leq \sum_{t=0}^{\infty} q_t W_{c,t} N_{HS} + p_{d,0} K_{d,0} + p_{c,0} K_{c,0} + T(1-\xi), \end{aligned} \tag{10}$$

where q_t is the price of the Arrow-Debreu claim for period t, $W_{c,t}$ is the wage in the clean energy sector, $p_{i,0}$ denotes the price of the clean ($i = c$) or dirty ($i = d$) capital good at time 0, and $T(1-\xi)$ is the share of the total tax rebate going to the high-skill household. Under a uniform rebate scheme, $(1-\xi) = \frac{N_{HS}}{N_{HS}+N_{LS}}$. In the business-as-usual (BAU) case, $T = 0$.

The low-skill (LS) representative household solves:

$$\begin{aligned} \max_{\{C_{LS,t}\}_{t=0}^{\infty}} \quad & \sum_{t=0}^{\infty} \beta^t \frac{C_{LS,t}^{1-\sigma} - 1}{1-\sigma} \\ \text{s.t.} \quad & \sum_{t=0}^{\infty} q_t C_{LS,t} \leq \sum_{t=0}^{\infty} q_t W_{d,t} N_{LS} + T\xi, \end{aligned} \tag{11}$$

where $W_{d,t}$ is the wage in the dirty energy sector and ξ is the share of the tax rebate going to the low-skill household.

Both households participate in the Arrow-Debreu contingent claims market, enabling them to smooth consumption intertemporally. Since there is no disutility from labour, both households supply their entire labour endowment to their respective energy sectors.

3.2 Producers

The final good Y_t serves as the numeraire of the economy and can be stored through a linear storage technology. The representative final-good producer operates under perfect competition and maximizes profits:

$$\begin{aligned} \Pi_t^{FG} = \max_{\{E_{d,t}, E_{c,t}\}} \quad & Y_t - P_{c,t}E_{c,t} - P_{d,t}E_{d,t} \\ \text{s.t.} \quad & Y_t \leq (1 - D(Z_t))A_t \left(E_{d,t}^{\rho} + E_{c,t}^{\rho}\right)^{\frac{1}{\rho}}, \quad \forall t \geq 0, \end{aligned} \tag{12}$$

where $D(Z_t)$ represents climate-induced damages to productivity, $P_{i,t}$ is the price of energy type $i \in \{c, d\}$, and A_t is the productivity parameter.

Dirty Energy Producer. The dirty intermediate energy producer uses dirty capital $K_{d,t}$ and low-skill labor l_t^{LS} under a Cobb-Douglas technology:

$$E_{d,t} \leq B_{d,t}(K_{d,t})^{\alpha}(l_t^{LS})^{1-\alpha}, \tag{13}$$

where $\alpha \in (0, 1)$. In the case of a *per-unit carbon tax* τ_t on dirty energy, the profit function is:

$$\Pi_{d,t} = \max_{\{K_{d,t}, l_t^{LS}\}} \quad (P_{d,t} - \tau_{e,t})E_{d,t} - W_{d,t}l_t^{LS} - r_{d,t}K_{d,t}. \quad \text{s.t} \quad (13) \tag{14}$$

The tax τ_t reduces the returns to dirty capital and low-skill labour, altering the decentralized equilibrium. Since capital markets ensure a unique rental rate across sectors ($r_{d,t} = r_{c,t}$), the tax induces a dynamic reallocation of capital from the dirty to the clean sector. Low-skill workers, unable to switch sectors, suffer from lower wages.

Alternative Taxation Scheme. If the Government instead taxes dirty capital at rate $\tau_{k_{d,t}}$, the dirty energy firm maximizes:

$$\Pi_{d,t} = \max_{\{K_{d,t}, l_t^{LS}\}} \quad P_{d,t}E_{d,t} - W_{d,t}l_t^{LS} - (r_{d,t} + \tau_{k_{d,t}})K_{d,t}. \quad \text{s.t} \quad (13) \tag{15}$$

Here, the tax increases the rental cost of dirty capital, reducing $E_{d,t}$ indirectly. Unlike the per-unit tax, the wage of low-skill workers is not directly penalized.

Clean Energy Producer. The clean intermediate energy producer solves:

$$\begin{aligned} \Pi_{c,t} = \max_{\{K_{c,t}, l_t^{HS}\}} \quad & P_{c,t}E_{c,t} - W_{c,t}l_t^{HS} - r_{c,t}K_{c,t} \\ \text{s.t.} \quad & E_{c,t} \leq B_{c,t}(K_{c,t})^{\alpha}(l_t^{HS})^{1-\alpha}. \end{aligned} \tag{16}$$

3.3 Government

The Government chooses between a *per-unit carbon tax* $\tau_{e,t}$ on dirty energy or a tax $\tau_{k_{d,t}}$ on dirty capital rental. In both cases, tax revenues T are rebated lump-sum to households, with distribution determined by the parameter ξ. The Government can fully commit to any tax policy path and runs a balanced budget (no debt).

3.4 Storage Firms

The representative storage firms operate between t and $t+1$, maximizing:

$$\begin{aligned} \Pi_t^{SF} = \max_{\{I_{i,t}\}} \quad & p_{i,t+1}q_{t+1}K_{i,t+1} + r_{i,t}q_tK_{i,t} - p_{i,t}q_tK_{i,t} - q_tI_{i,t} \\ \text{s.t.} \quad & K_{i,t+1} \leq I_{i,t} + (1-\delta)K_{i,t}, \quad \forall t \geq 0, \end{aligned} \tag{17}$$

where $i \in \{c, d\}$. Each storage firm purchases capital goods $K_{i,t}$ at price $p_{i,t}$ from the previous period's operator. The initial capital endowments $K_{i,0}$ are transferred to storage firms at $t = 0$.

The price $p_{i,t}$ differs from the price of the final good because capital goods can be rented or stored, while the final good can only be stored. Zero-profit conditions in equilibrium imply:

$$p_{i,t+1}q_{t+1} = q_t,$$
$$p_{i,t} = r_{i,t} + (1-\delta_i).$$

By the no-arbitrage condition, $p_{d,t} = p_{c,t}$ in equilibrium. I define a competitive equilibrium for this economy as follows:

3.5 Definition of the Equilibrium

We characterize the (deterministic) decentralized (time-0)equilibrium of the economy described in Section 2. Time is discrete, $t = 0, 1, \ldots$

Definition 2 (Competitive Equilibrium)**.** *Given a stream of taxes* $\{\tau_{e,t}$ *or* $\tau_{k_{d,t}}\}_{t=0}^{\infty}$*, initial total factor productivity* A_0*, initial environmental state* Z_0*, initial capital endowments* $\{K_{d,0}, K_{c,0}\}$*, and labour endowments* $\{N_{HS}, N_{LS}\}$*, a competitive equilibrium is a set of prices* $\{r_{d,t}, r_{c,t}, p_{d,t}, p_{c,t}, W_{d,t}, W_{c,t}, P_{d,t}, P_{c,t}, q_t\}_{t=0}^{\infty}$*, tax revenues* $\{T_t\}_{t=0}^{\infty}$*, and an allocation*

$\{C_{HS,t}, C_{LS,t}, Y_t, I_{d,t}, I_{c,t}, K_{d,t}, K_{c,t}, l_t^{LS}, l_t^{HS}, E_{c,t}, E_{d,t}, Z_t\}_{t=0}^{\infty}$ such that low-skill and high-skill households choose $\{C_{j,t}\}_{t=0}^{\infty}$ to maximize utility (10)–(11), taking taxes and prices as given; the final good producer and the intermediate energy producers $\{c, d\}$ maximize profits as in (12), (14) (or (15)) and (16); the storage firms solve (17); the government's budget constraint $T_t = \tau_t E_{d,t}$ or $T_t = \tau_{k_{d,t}} K_{d,t}$ holds with rebate parameter ξ; all markets clear, and the environmental state $\{Z_t\}_{t=0}^{\infty}$ evolves according to its law of motion.

In the next section, I solve the Ramsey Problem for this economy, and show how it can be decentralized in the market.

4 Ramsey Problem

In this section, I solve the optimal (constrained) Ramsey problem using a primal approach. The objective function of the Social Planner is a standard utilitarian welfare function, where θ and $1-\theta$ represent the welfare weights assigned to the brown (low-skill) and green (high-skill) representative households, respectively. The Planner maximizes this welfare function subject to the technological constraints—namely, the CES final good production technology, the Cobb-Douglas intermediate energy production functions with sector-specific capital and labor—and the feasibility constraints, which include the economy's resource constraint, the laws of motion for clean and dirty capital, and labor endowments.

A key difference from Golosov et al. (2014), Barrage (2020) and Douenne, Hummel, and Pedroni (2022) is that the economy features two sector-specific capital goods, $K_{d,t}$ and $K_{c,t}$, which are not freely interchangeable across the dirty and clean intermediate energy sectors. This adds a second margin of adjustment: the Planner must determine not only the allocation of output between consumption and investment, but also the optimal accumulation of capital in each sector, taking into account the distinct returns and the environmental impact of the dirty sector.

Importantly, the Planner has no access to lump-sum transfers. Redistribution between the two households can only occur through the revenues generated by policy instruments—either a per-unit carbon tax $\tau_{e,t}$ on the dirty energy good $E_{d,t}$, or an alternative tax $\tau_{t,K_{d,t}}$ on the rental rate of dirty capital $K_{d,t}$. Consequently, the social welfare weights θ and $1-\theta$ must be interpreted consistently with the income distribution determined by wages and sectoral capital earnings. As in the classic Negishi approach (Negishi, 1960), the decentralized competitive allocation can be mapped to a Social Planner's problem where the welfare weights are functions of relative market incomes.

The Planner fully internalizes the environmental externality generated by dirty energy production $E_{d,t}$, which accumulates into emissions Z_t and depresses final–good total factor productivity A_t through the damage function. In choosing the optimal allocation of resources across time and sectors, the Planner therefore trades off the benefits from energy use and capital in the dirty sector against the present and future climate damages that such production entails. From an aggregate perspective, this problem is independent of the particular policy instrument used to decentralize the social optimum: with a single externality and a single corrective instrument, both a per–unit tax on dirty energy and a tax on the rental rate of dirty capital implement the same socially optimal allocation of production and consumption. However, as we show below, the two instruments have distinct distributional implications across agents. In addition, the presence of sector–specific capital goods introduces a non–trivial trade–off between current production efficiency and the future costs of reallocating capital. By adjusting the paths of $K_{d,t}$ and $K_{c,t}$, the Planner jointly determines the optimal mix of clean and dirty energy inputs while smoothing the transition toward a low–emissions equilibrium.

The remainder of this section characterizes the optimal allocation (including the optimal paths for $E_{d,t}$, $K_{d,t}$, and $K_{c,t}$) and explains how it can be decentralized through either a per-unit carbon tax $\tau_{e,t}$ or a dirty capital tax $\tau_{t,K_{d,t}}$, possibly combined with a non-uniform tax rebate scheme that addresses the welfare losses of the low-skill household without compromising aggregate efficiency.

The Planner solves (with K_0, Z_0 and the TFP sequences $\{A_t, B_{d,t}, B_{c,t}\}_{t=0}^{\infty}$ given):

$$\max_{\{C_{LS,t},C_{HS,t},I_{i,t},K_{i,t},E_{i,t},Z_t,l_t^j\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[\frac{\theta(C_{LS,t}^{1-\sigma}-1)}{1-\sigma} + \frac{(1-\theta)(C_{HS,t}^{1-\sigma}-1)}{1-\sigma} \right], \tag{18}$$

subject to, for all $t \geq 0$:

$$\begin{aligned}
Y_t &= C_{LS,t} + C_{HS,t} + \sum_{i=c,d} I_{i,t}, \qquad (\nu_{rc,t}) \qquad \text{(Aggregate Resource Constraint)}\\
Y_t &= (1-D(Z_t))A_t\left(E_{d,t}^{\rho}+E_{c,t}^{\rho}\right)^{1/\rho}, \quad (\nu_{Y,t}) \quad \text{(Final Good Technology)}\\
E_{d,t} &= B_{d,t}K_{d,t}^{\alpha}(l_t^{LS})^{1-\alpha}, \quad (\nu_{E_{d,t}}) \quad \text{(Dirty Energy Production)}\\
E_{c,t} &= B_{c,t}K_{c,t}^{\alpha}(l_t^{HS})^{1-\alpha}, \quad (\nu_{E_{c,t}}) \quad \text{(Clean Energy Production)}\\
K_{i,t+1} &= I_{i,t} + (1-\delta)K_{i,t}, \quad (\nu_{I_{i,t}}) \quad \text{(Capital Accumulation}, i \in \{c,d\})\\
l_t^{LS} &= N_{LS}, \quad l_t^{HS} = N_{HS}, \quad (\nu_{L_j,t}) \quad \text{(Labour Feasibility)}\\
Z_{t+1} &= Z_t + E_{d,t}, \qquad (\nu_{Z,t}) \quad \text{(Cumulative Emissions Law)}
\end{aligned}$$

Taking the First-Order-Condition for consumption we have:

$$\nu_{rc,t} = \beta^t \theta MU(C_{LS,t}) \tag{19}$$

$$\nu_{rc,t} = \beta^t (1-\theta) MU(C_{HS,t}), \tag{20}$$

where MU is the marginal utility of consumption, $\nu_{rc,t}$ is the lagrangian multiplier associated to the time-t resource constraint and θ is the Pareto weight attached to the representative low-skill household.

If we differentiate the objective function for $K_{d,t+1}$ and $K_{c,t+1}$ given the constraints:

$$-\nu_{rc,t} + \nu_{rc,t+1}\left((1-\delta_d) + \frac{\partial Y_{t+1}}{\partial K_{d,t+1}}\right) = 0 \tag{21}$$

$$-\nu_{rc,t} + \nu_{rc,t+1}\left((1-\delta_c) + \frac{\partial Y_{t+1}}{\partial K_{c,t+1}}\right) = 0. \tag{22}$$

I obtain two Euler equations describing the law of motion of dirty/clean capital goods accumulation process. Rearranging (21) and (22) we obtain the following:

$$\frac{\nu_{rc,t}}{\nu_{rc,t+1}} = \left((1-\delta_d) + \frac{\partial Y_{t+1}}{\partial K_{d,t+1}}\right)$$

$$\frac{\nu_{rc,t}}{\nu_{rc,t+1}} = \left((1-\delta_c) + \frac{\partial Y_{t+1}}{\partial K_{c,t+1}}\right),$$

where the ratio $\frac{\nu_{rc,t}}{\nu_{rc,t+1}} = \frac{MU(C_t)}{\beta MU(C_{t+1})}$. The utility functional form (CRRA) implies that representative households' consumptions (from (19) e (20)) are functions of their Pareto weights and of aggregate consumption C_t. In the ratio $\frac{\nu_{rc,t}}{\nu_{rc,t+1}}$, the Pareto weights disappear, and thus the Euler equations must hold in aggregate terms. Interestingly, this implies that for the same discount factor δ_i, the price of the two capital goods and their rental rates have to be the same at the optimum.

4.1 First Best vs Business-As-Usual

It is important to understand that in the Business-As-Usual (BAU)allocation, the climate effects of $K_{d,t+1}$ on Z_{t+j} are not taken into account; meanwhile, in the First-Best allocation, the Social Planner internalizes all the future TFP damage stream that the dirty energy intermediate good imposes on the society. Using the chain rule, we can rewrite $\frac{\partial Y_{t+1}}{\partial K_{d,t+1}}$ as $\frac{\partial Y_{t+1}}{\partial E_{d,t+1}}\frac{\partial E_{d,t+1}}{\partial K_{d,t+1}}$. By taking the first order conditions for $E_{d,t+1}$ and Z_{t+1} we have:

$$[E_{d,t+1}]: \quad \nu_{rc,t+1}\frac{\partial Y_{t+1}}{\partial E_{d,t+1\}}} - \nu_{E_{d,t+1}} - \sum_{j=1}^{\infty} \nu_{Z,t+j} = 0 \tag{23}$$

$$[Z_{t+1}]: \; \nu_{rc,t+1}\frac{\partial Y_{t+1}}{\partial Z_{t+1}} - \nu_{Z,t+1} = 0, \tag{24}$$

Manipulating (23) and (24), it is straightforward to show that the marginal benefits of producing an extra-unit of $E_{d,t+1}(= \frac{\partial Y_{t+1}}{\partial E_{d,t+1}})$ have to be equalized to the future climate

TFP damages that it imposes to the society). In mathematical terms:

$$\overbrace{\frac{\partial Y_{t+1}}{\partial E_{d,t+1}}}^{E_{d,t}\text{marginal benefits}} = \overbrace{\frac{\nu_{E_{d,t+1}}}{\nu_{rc,t+1}}}^{E_{t,d}\text{ marginal economic costs}} + \overbrace{\sum_{j=1}^{\infty} \frac{\nu_{rc,t+j}}{\nu_{rc,t+1}} \frac{\partial Y_{t+j}}{\partial Z_{t+j}}}^{\text{streams of climate damages}} \tag{25}$$

In the BAU allocation $\frac{\partial Y_{t+1}}{\partial K_{d,t+1}}$ is simply equal to $\frac{\alpha Y_{t+1}}{K_{d,t+1}} \frac{E^{\rho}_{d,t+1}}{E^{\rho}_{d,t+1}+E^{\rho}_{c,t+1}}$. In the FB allocation, the Planner also considers the climate part, which is the weighted stream of future climate TFP damages. Equation (25) describes thus the pivotal mechanism driving the model's results. On the left-hand side, the benefits are coming from the production of $E_{d,t+1}$ since it raises the final good quantity. On the right-hand-side, instead, there are the costs associated from the production of $E_{d,t+1}$ along with the marginal externality cost associated to it , i.e: $\sum_{j=1}^{\infty}(\frac{\nu_{rc,t+j}}{\nu_{rc,t+1}} \frac{\partial Y_{t+1}}{\partial Z_{t+1}})$. Producing one unit of $E_{d,t+1}$ will affect the variable Z_{t+1} (equation (4)), and Z_{t+1} damages the final good TFP through the damage function. Since $E_{d,t+1}$ is going to stay in the atmosphere forever, you have to consider all the stream of future climate damages stemming from its production, the social planner has to weight them with the ratio of resource constraint lagrangian multipliers $(\sum_{j=1}^{\infty}(\frac{\nu_{rc,t+j}}{\nu_{rc,t+1}}))$, i.e, the discounted marginal utility of aggregate consumption divided by the time t marginal utility of aggregate consumption. In the next part, I introduce the Ramsey problem for the economy.

4.2 Decentralization of the Social Planner's Allocation

In this section, I carefully analyze how the Social Planner's optimal allocation—obtained from the Ramsey problem—can be decentralized in a competitive market equilibrium by means of appropriate price signals and tax instruments. The aim is to show that the Planner's allocation can be replicated by private agents (households, firms, and the government), provided that the government implements either of two alternative taxation schemes on the dirty energy sector.

Two taxation schemes are considered:

1. A per-unit tax $\tau_{e,t}$ levied directly on the production of the dirty intermediate energy good $E_{d,t}$, effectively reducing the net price received by the dirty energy producer.

2. A tax on the rental rate of dirty capital, denoted by $\tau_{K_{d,t}}$, which increases the cost of using dirty capital $K_{d,t}$ for the dirty energy firm.

The decentralization approach proceeds as follows: First, I examine the behavior of the representative final good producer and derive the relationship between the prices of intermediate goods ($P_{d,t}$ and $P_{c,t}$) and their marginal contributions to final output Y_t.

Then, I show how the dirty energy producer's optimality conditions differ from the Planner's optimality conditions due to the presence of the environmental externality.

Finally, I determine the level of taxation ($\tau_{e,t}$ or $\tau_{K_{d,t}}$) required to close this gap, forcing firms to internalize the social cost of dirty energy production.

Final Good Producer. Consider the representative final good firm, which takes as inputs the two intermediate energy goods, $E_{d,t}$ and $E_{c,t}$, produced by the dirty and clean energy sectors respectively. Its profit maximization problem is:

$$\max_{E_{d,t},E_{c,t}} (1 - D(Z_t))A_t \left(E_{d,t}^{\rho} + E_{c,t}^{\rho}\right)^{1/\rho} - P_{d,t}E_{d,t} - P_{c,t}E_{c,t}.$$

$(1 - D(Z_t))A_t \left(E_{d,t}^{\rho} + E_{c,t}^{\rho}\right)^{1/\rho}$ represents the CES (constant elasticity of substitution) production function for the final good Y_t. $P_{d,t}$ and $P_{c,t}$ are the market prices of dirty and clean intermediate energy, respectively.

The first-order conditions (FOCs) of this problem determine the equilibrium prices of the intermediate energy goods:

$$P_{d,t} = \frac{\partial Y_t}{\partial E_{d,t}}, \quad P_{c,t} = \frac{\partial Y_t}{\partial E_{c,t}}. \tag{26}$$

This means that, in a competitive equilibrium, the price of each energy input equals its marginal product with respect to the final good Y_t. This property is crucial for decentralization: it ensures that the final good producer's demand for energy is aligned with the Planner's marginal valuation of these inputs.

Link with the Planner's FOCs. From the Planner's optimal allocation, the marginal product of dirty energy can be decomposed into two components:

$$\frac{\partial Y_t}{\partial E_{d,t}} = \frac{\nu_{E_{d,t}}}{\nu_{rc,t}} + \sum_{j=1}^{\infty} \frac{\nu_{Z_{t+j}}}{\nu_{rc,t}}.$$

Here: $\nu_{rc,t}$ is the Lagrange multiplier on the aggregate resource constraint, representing the marginal utility of final output. $\nu_{E_{d,t}}/\nu_{rc,t}$ represents the *private marginal benefit* of dirty energy, reflecting its contribution to final output through capital and labour inputs in the dirty sector. The summation $\sum_{j=1}^{\infty} \frac{\nu_{Z_{t+j}}}{\nu_{rc,t}}$ captures the *present value of the external cost* (future damages) due to emissions $E_{d,t}$, since emissions accumulate in Z_t and reduce future total factor productivity via the damage function $D(Z_t)$.

A competitive dirty energy firm, however, does not internalize the future damage term $\sum_{j=1}^{\infty} \frac{\nu_{Z_{t+j}}}{\nu_{rc,t}}$. It only sees the private marginal benefit $\frac{\nu_{E_{d,t}}}{\nu_{rc,t}}$. This divergence from the Planner's condition justifies the introduction of a corrective tax.

Pigouvian Tax on Dirty Energy. To ensure that the decentralized equilibrium coincides with the Planner's allocation, the government introduces a *per-unit* Pigouvian tax

$\tau_{e,t}$ on dirty energy $E_{d,t}$, defined as:

$$\tau^*_{e,t} = \sum_{j=0}^{\infty} \frac{\nu_{Z_{t+j}}}{\nu_{rc,t}} \tag{27}$$

This tax is exactly equal to the present value of all future marginal damages caused by one additional unit of $E_{d,t}$. By lowering the effective price received by the dirty energy producer from $P_{d,t}$ to $P_{d,t} - \tau_{e,t}$, the firm's FOC:

$$(P_{d,t} - \tau_{e,t})\frac{\partial E_{d,t}}{\partial K_{d,t}} = r_{d,t},$$

is modified to fully internalize the externality, thus replicating the Planner's optimal choice for $E_{d,t}$.

Alternative Dirty Capital Tax. An alternative decentralization scheme is to tax the rental rate of dirty capital, instead of directly taxing dirty energy. In this case, the dirty energy firm solves:

$$\max_{K_{d,t}, l_t^{LS}} P_{d,t}E_{d,t} - W_{d,t}l_t^{LS} - (r_{d,t} + \tau_{K_d,t})K_{d,t},$$

where $\tau_{K_d,t}$ increases the effective cost of capital $K_{d,t}$.

The first-order condition with respect to $K_{d,t}$ is now:

$$P_{d,t}\frac{\partial E_{d,t}}{\partial K_{d,t}} = r_{d,t} + \tau_{K_d,t}. \tag{28}$$

To match the Planner's condition, the tax must be set as:

$$\tau_{K_{d,t}} = \sum_{j=0}^{\infty} \frac{\nu_{Z_{t+j}}}{\nu_{rc,t}} \frac{\partial E_{d,t}}{\partial K_{d,t}}. \tag{29}$$

Both $\tau_{e,t}$ and $\tau_{K_d,t}$ introduce the same wedge between the private and social returns to dirty energy. However, the incidence of the tax differs. Under $\tau_{e,t}$, the tax directly reduces the revenue from each unit of dirty energy, affecting both capital and labour returns in the dirty sector. Under $\tau_{K_d,t}$, the tax is levied on the rental of dirty capital, indirectly reducing $E_{d,t}$ through the firm's cost minimization, while leaving the wage $W_{d,t}$ less affected.

This difference has important distributional consequences, as the burden on low-skill households (who supply labour to the dirty sector) may be mitigated under the dirty capital tax compared to the per-unit tax. In the following section, I analyze the numerical simulations and properly discuss the main results of the paper.

5 Numerical Simulations and Transition Path

Numerical simulations are employed to compute the main allocations of the model, given the absence of closed-form solutions. Time is discrete, with each period corresponding to 10 years, following the approach in Golosov et al. (2014). The focus is on the full transition path toward the balanced growth path (BGP), which spans 30 periods and captures the gradual reallocation of resources between the dirty and clean energy sectors. This transition is analyzed by comparing two benchmark scenarios: the Business-As-Usual (BAU) allocation—where no corrective policies are implemented—and the socially optimal allocation derived from the Planner's problem (with uniform and non-uniform tax rebates).

A crucial assumption is the emergence of a backstop technology, which becomes available exogenously at the end of the 30th period. This backstop technology fully eliminates the externality generated by dirty energy production $E_{t,d}$, allowing the economy to transition to a completely decarbonized state. The assumption of such a technology is consistent with the established climate-economy modeling literature (see Golosov et al. (2014); Barrage (2020); Douenne, Hummel, and Pedroni (2022)), where the introduction of a backstop energy source is necessary to achieve a long-run BGP with zero emissions.

The dynamic paths of the key variables—such as energy inputs, capital stocks, and relative wages—are computed over the full horizon, highlighting how the optimal carbon policy induces a structural shift from the dirty to the clean energy sector. Special attention is given to the temporal dynamics of the optimal carbon tax, the energy ratio $\frac{E_{t,d}}{E_{t,c}}$, and the wage ratio $\frac{W_{d,t}}{W_{c,t}}$, as these variables reveal both the environmental and distributional consequences of the policy.

Finally, the results are inherently sensitive to the chosen calibration of model parameters, which determine the relative strength of climate damages, substitutability between energy inputs, and the productivity of each sector. A dedicated section discusses the calibration strategy in detail and examines the sensitivity of the model's outcomes to variations in key parameters. This sensitivity analysis is particularly important for understanding the robustness of the optimal policy path and the welfare implications for both low-skill and high-skill households.

5.1 Parameterization

This section presents the main parameters, summarized in Table 1. The aim of the calibration is to replicate key features of the world economy, with particular attention to energy sector dynamics and climate-related externalities. Each parameter is chosen either from well-established references in the literature or based on reasonable normalization assumptions.

For the (annual)preference discount factor, we adopt $\beta = 0.985$, in line with the calibration used in the DICE model ((Nordhaus, 2013)). The intertemporal elasticity of substitution is set to $\sigma = 1.5$ (Barrage, 2020, Douenne, Hummel, and Pedroni, 2022). This choice is common in long-term climate-economy models and ensures moderate curvature in households' utility, balancing consumption smoothing with responsiveness to intertemporal trade-offs.

The parameters of the final good production function are based on Acemoglu, Aghion, et al. (2012b) and Golosov et al. (2014). A key parameter is the elasticity of substitution between dirty and clean energy, captured by $\rho = 0.66$. This value reflects the empirical reality that clean and dirty energy sources are imperfect substitutes—significant differences exist in their availability, reliability, and cost. In particular, renewable energy sources, such as wind and solar, suffer from intermittency and imperfect energy storage technologies, which limit their ability to fully replace fossil-based energy in the short run. Thus, $\rho < 1$ captures this imperfect substitutability (with $\rho = 1$ corresponding to perfect substitution).

Our setup differs from Barrage (2020) and Douenne, Hummel, and Pedroni (2022), who incorporate abatement technologies into the production structure. In our model, the clean energy good $E_{t,c}$ represents actual production of clean energy rather than abatement or offsetting mechanisms. This distinction allows us to study structural capital and labour allocations between clean and dirty sectors in greater detail.

In line with Golosov et al. (2014), each energy sector combines capital and labour inputs via a Cobb–Douglas production function. However, our main novelty lies in the introduction of complete frictions in both labour and capital markets. Low-skill households can only supply labor to the dirty energy sector, while the clean sector exclusively employs high-skill households. This segmentation reflects the skill mismatch between energy sectors, which is well-documented in the empirical literature (Bluedorn et al. (2023)). Similarly, the capital stock used in each energy sector is assumed to be sector-specific, reflecting the technological specialization of assets used for fossil-based versus renewable energy production. This assumption relies on Acemoglu, Aghion, et al. (2012b), Diluiso et al. (2021), Carattini, Heutel, and Melkadze (2023).

The technological advantage of the dirty sector, $B_{d,t} > B_{c,t}$, is consistent with the concept of path dependency in innovation (Acemoglu, Aghion, et al. (2012b); Aghion et al. (2019)). Fossil-based technologies have historically benefited from a longer period of incremental improvements, allowing them to achieve higher productivity levels compared to relatively newer renewable technologies. The damage parameter γ is calibrated following Peter H Howard and Sterner (2017a), implying a 5% reduction in TFP at $3°C$ of global warming.

The initial capital stocks, $K_{d,0}$ and $K_{c,0}$, as well as the initial final good TFP A_0, are chosen to normalize initial world GDP in 2015 to unity. Growth rates for sectoral TFPs follow the baseline calibrations of Golosov et al. (2014) and Barrage (2020), which are

Description	Parameter	Value	Sources and Notes
Discount factor (annual)	β	0.985	W. D. Nordhaus (2017)
Intertemporal elasticity of substitution	σ	1.5	Golosov et al. (2014)
Substitutability of $E_{d,t}$ and $E_{c,t}$	ρ	0.66	Acemoglu, Aghion, et al. (2012b)
Capital share in energy production	α	1/3	Acemoglu, Aghion, et al. (2012b)
Capital depreciation rate	δ	0.6513	W. D. Nordhaus (2017)
Initial dirty sector TFP	$B_{d,0}$	1.1	Normalization
Initial clean sector TFP	$B_{c,0}$	1.0	Normalization
Initial final good TFP	A_0	4.0	Normalization
Damage parameter	γ	5% TFP loss at $3°C$	Peter H Howard and Sterner (2017a)
Initial dirty capital	$K_{d,0}$	0.52	Own calibration
Initial clean capital	$K_{c,0}$	0.48	Own calibration
Low-skill labor share	N_{LS}	0.5	IEA (2022)
High-skill labor share	N_{HS}	0.5	IEA (2022)
Energy sector TFP growth rate	λ_B	0.02	Golosov et al. (2014)
Final good TFP growth rate	λ_A	0.00	Golosov et al. (2014)

Table 1: Summary of Key Calibration Parameters

standard in the climate-economy literature. The total labour force is normalized to 1, with equal shares for low-skill and high-skill workers, consistent with IEA (2022), which reports that the distribution of workers across energy sectors is approximately balanced.

6 Optimal Carbon Taxes vs BAU allocations

The following section presents the transition towards the balanced growth path for the Optimal Carbon Taxes, $\frac{E_{d,t}}{E_{c,t}}$, $\frac{W_{d,t}}{W_{c,t}}$ along with the Gini index in the BAU allocation and in the two different taxation schemes allocations. In the competitive equilibrium without tax, there will be more production of $E_{d,t}$. This overproduction of dirty energy input will generate more significant TFP damages via climate change than those hitting the allocation with the optimal carbon tax. We now focus on a comparative analysis of the BAU and Planner's allocations, emphasizing their main differences. Both the optimal per-unit carbon tax (Figure (2)) and the dirty capital rental rate tax (Figure (3)) are time-varying and exhibit a rising trajectory over time:

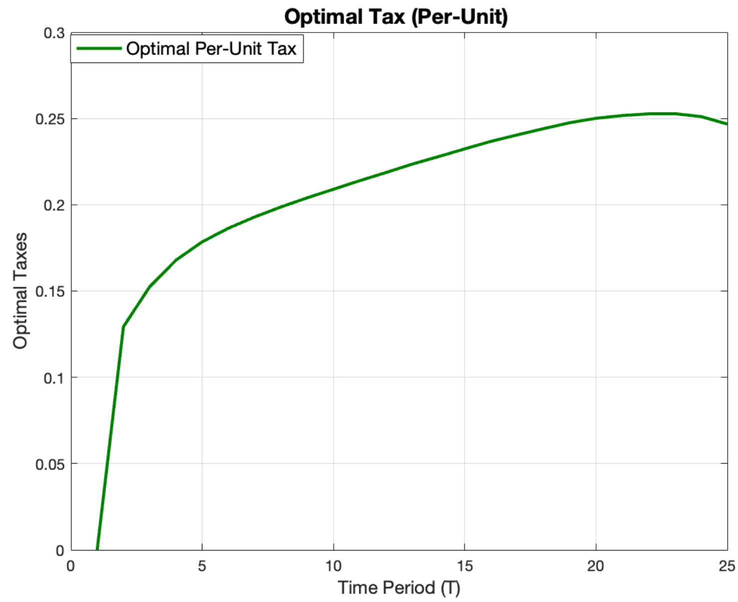

Figure 2: Optimal Per-Unit Tax

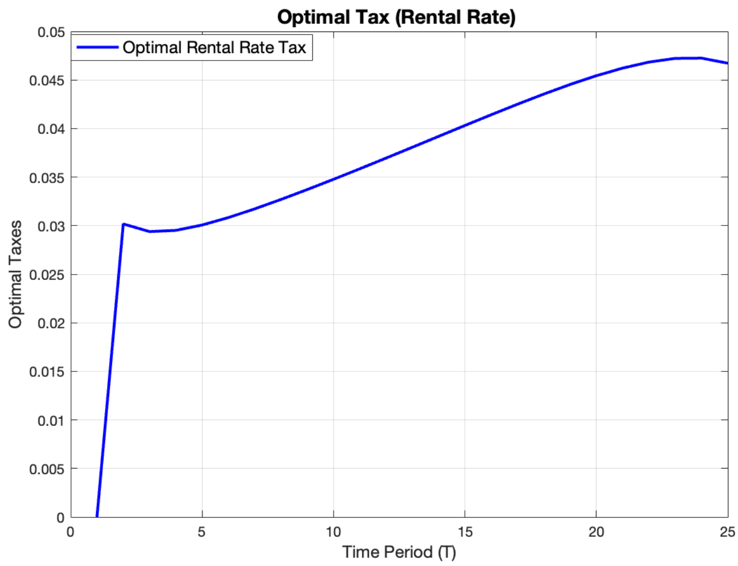

Figure 3: Optimal Rental Rate Tax

In both taxation schemes, the production of dirty energy $E_{d,t}$ decreases, yet the two instruments generate markedly different distributional effects. The *per-unit* tax τ_t directly reduces the net price of the dirty energy good $P_{d,t}$, which, through the firm's first-order condition, translates into lower wages for workers employed in the dirty energy sector. Conversely, high-skill workers benefit from the policy-induced intertemporal reallocation of capital toward the clean energy sector. This sector becomes more productive following the introduction of the tax, leading to higher wages and returns for high-skill households.

An interesting finding is that the dirty capital rental rate tax $\tau_{t,K_{d,t}}$ is significantly lower than the *per-unit* tax τ_t, even though it induces the same optimal reduction in $E_{d,t}$. This is because the capital tax operates through a different channel: by increasing the cost of renting dirty capital goods rather than directly reducing the dirty energy price. As a result, the adverse impact on dirty-sector wages is considerably softer compared to the *per-unit* taxation case.

In the optimal allocation, the $\frac{E_{d,t}}{E_{c,t}}$ ratio is declining over time, differently from the Business-as-usual scenario where it is constant. The Planner, internalising the climate damages, must reduce the $E_{t,d}$ production. The optimal carbon taxes obtains the same beneficial effect. However, in Golosov et al. (2014) or in Barrage (2020) the Planner does not have a constraint on the inputs that can reallocate from the dirty to the clean intermediate energy sector. In this environment, instead, the constrained Planner cannot mitigate the externality by moving intertemporally capital and labour from one intermediate sector. This additional constraint induced by the inputs' market full friction reduces $W_{d,t}$ and increases $W_{c,t}$ compared to the Business-as-usual wage rates.

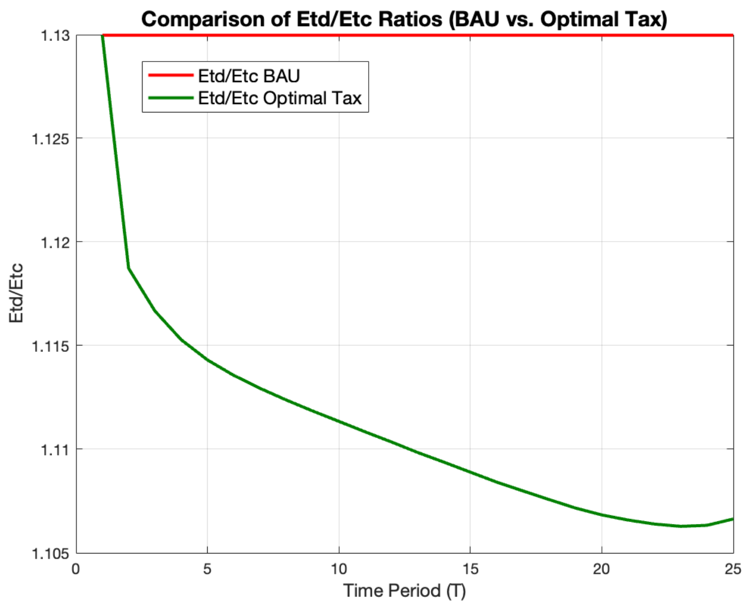

Figure 4: $\frac{E_{t,d}}{E_{t,c}}$ Bau vs Optimal Taxes

The relative $W_{d,t}$ reduction with reference to $W_{c,t}$ in the three scenarios can be seen in Figure 5:

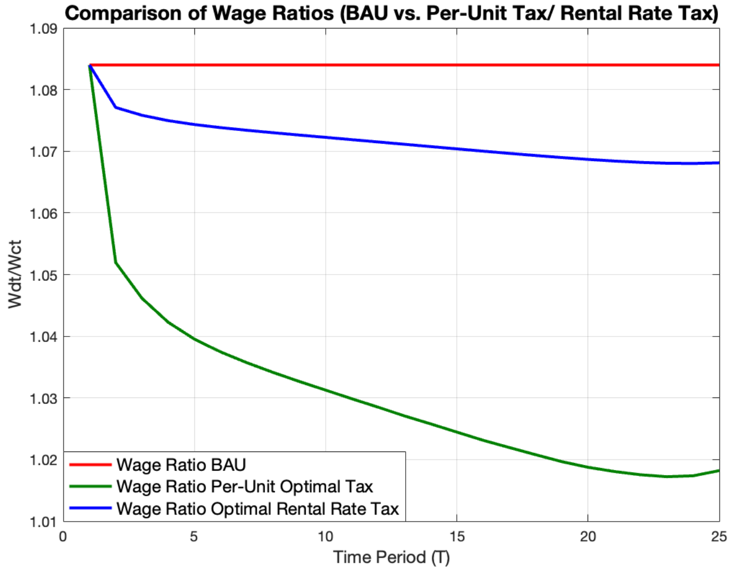

Figure 5: Wage ratios BAU vs Optimal Taxes

In the BAU allocation, the relative wage ratio $\frac{W_{d,t}}{W_{c,t}}$ remains constant over time, as no corrective policy is implemented and both sectors evolve symmetrically according to their technological parameters. In contrast, under the two taxation schemes, the dynamics of relative wages change significantly due to the introduction of time-varying carbon taxes.

Since both the *per-unit* tax τ_t and the dirty capital rental rate tax $\tau_{K_{d,t}}$ optimally increase over time, the dirty energy sector becomes progressively more affected by these policies, especially in the medium to long term. As a result, wages in the dirty sector $W_{d,t}$ exhibit a declining trend, reflecting the reduction in profitability and the lower marginal product of labour in that sector. This decline directly translates into a reduction in available consumption for the representative low-skill household, which exclusively supplies labour to the dirty energy sector.

Conversely, the clean energy sector benefits from an intertemporal reallocation of capital and resources, which boosts its marginal productivity. This is reflected in rising wages $W_{c,t}$, improving the consumption prospects and welfare of the representative high-skill household.

Household	**BAU**	**Optimal Per-Unit Tax**
Low-skill Household	Welfare_{LS}= 6.3767	Welfare_{LS} = 6.3116
High-skill Household	Welfare_{HS} = 6.8201	Welfare_{HS}=6.8747
Aggregated	Welfare_{aggr}= 8.8487	$\text{Welfare}_{aggr.}$ = 8.8492

Table 2: Welfare outcomes for low-skill and high-skill households in the BAU economy and in the Per-Unit Optimal Tax Economy

Household	**BAU**	**Optimal Per-Unit Tax**
Low-skill Household	Welfare_{LS}= 6.3767	Welfare_{LS} = 6.3657
High-skill Household	Welfare_{HS} = 6.8201	Welfare_{HS}=6.8305
Aggregated	Welfare_{aggr}= 8.8487	$\text{Welfare}_{aggr.}$ = 8.8492

Table 3: Welfare outcomes for low-skill and high-skill households in the BAU economy and in the Rental Rate Optimal Tax Economy

It is worth emphasising that the impact on wages is significantly milder under the dirty capital rental rate tax $\tau_{K_{d,t}}$ compared to the *per-unit* tax τ_t. This difference arises because the capital tax operates by increasing the cost of using dirty capital rather than directly reducing the price of the dirty energy good $P_{d,t}$. As a result, while both instruments achieve the desired reduction in dirty energy $E_{d,t}$, the distributional consequences—particularly for low-skill households—are less severe when the tax is levied on dirty capital rather than on output.

Investigating the policy's impact on inequality is crucial to the paper. I analyze its dynamics using the Gini index, defined as:

$$\text{Gini}_t = \frac{(W_{c,t}N_{HS})}{W_{c,t}N_{HS} + W_{d,t}N_{LS}} - \frac{N_{HS}}{N_{LS} + N_{HS}}. \tag{30}$$

The Gini index is computed for each time period, and tax revenues are uniformly rebated to representative households. A Gini index equal to one corresponds to full inequality; the higher the Gini index, the higher the relative remuneration for the high-skill individuals compared to the low-skill ones. In Figure 5, the Gini indices dynamics are plotted for the Business-as-usual economy and the economy with the optimal policies. As you can see, both sustainability policies increase economic inequality through the wage channel. However, the dirty capital good taxation $\tau_{K_{d,t}}$ has a much lower impact on the inequality. Figure 5 shows the Gini indices in the three environment:

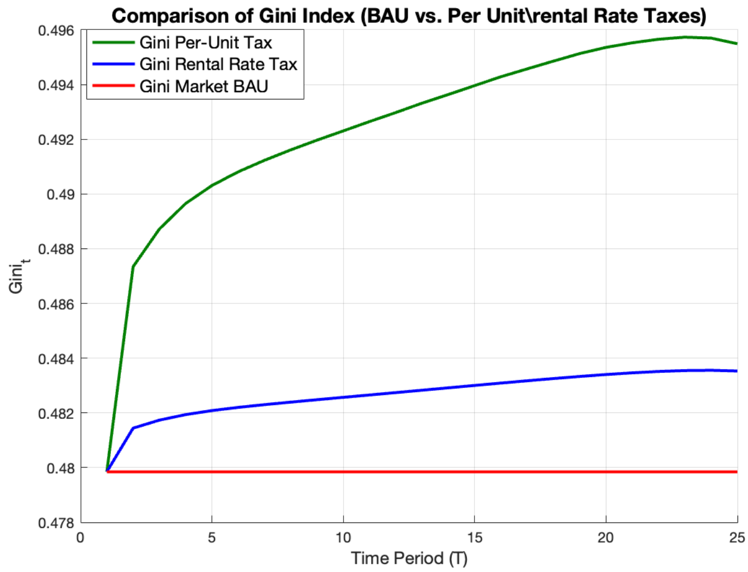

Figure 6: Gini index Business-as-usual vs Gini index Optimal Taxes

Understanding the optimal carbon tax aggregated and disaggregated welfare impacts is important, and I will discuss it in the next section.

6.1 Distributional Effects of Carbon Tax Rebates

The implementation of an optimal carbon tax effectively mitigates the climate externality and improves aggregate welfare compared to the Business-as-Usual (BAU) allocation. However, a closer examination of the disaggregated welfare dynamics reveals that the policy does not enhance the welfare of both representative households equally. While the high-skill household gains, the low-skill household suffers from the redistributional effects of the policy under a uniform tax rebate.

It is straightforward to understand the mechanism behind this result. The optimal carbon tax is designed to reduce the use of the dirty intermediate energy good $E_{d,t}$, thereby incentivizing a shift towards the cleaner intermediate energy $E_{c,t}$. This structural change has profound implications for income distribution: as $E_{d,t}$ contracts, the demand for low-skill labour and dirty capital—both specific to the dirty energy sector—declines. As a result, both the wage $W_{d,t}$ of low-skill workers and the rental return to dirty capital fall over time. From the perspective of the representative low-skill household, the negative income effect outweighs the positive general equilibrium effects stemming from increased aggregate productivity and final output Y_t. In other words, although aggregate welfare under the optimal policy exceeds BAU aggregate welfare, the welfare of the low-skill household is *lower* under the optimal carbon tax compared to BAU. The uniform rebate mechanism, which returns tax revenues proportionally to population shares, is insufficient to compensate for the welfare loss of the low-skill household, as it does not account for the asymmetric incidence of the policy across the two groups.

A crucial insight of our analysis is that the welfare outcomes can be substantially improved by adjusting the tax rebate scheme. If the government reallocates the carbon tax revenues (both the per-unit and the dirty capital tax dividends) asymmetrically, favouring the low-skill household, it is possible to offset the regressive nature of the policy. In particular, my simulations show that, in the case of a per-unit tax, with a $\xi_t = 91\%$, i.e, the 91% of the total tax dividend is given to the low-skill household (9% to the high-skill household) compared to the uniform rebate benchmark (where $\xi = \frac{N_{LS}}{N_{HS}+N_{LS}}$) improves the welfare of high-skill household, while maintaining unchanged the low-skill household's welfare compared to the Business-As-Usual one. If I consider the rental rate tax, with a $\xi_t = 71\%$ I obtain the same result, that is, we improve aggregate welfare without negatively affecting the low-skill household.

This finding highlights the importance of coupling carbon taxation with well-designed redistributional mechanisms. A carefully tailored non-uniform rebate does not undermine the efficiency of the optimal carbon tax. Instead, it ensures that both representative households can share the gains from improved aggregate efficiency and reduced climate damages, thereby enhancing the political feasibility and fairness of climate policies.

In summary, while the optimal carbon tax maximises total welfare, its *distributional design* critically determines whether all households benefit. A uniform rebate exacerbates inequality by disproportionately hurting low-skill households, whereas a skewed rebate (92% more for the low-skill group) is sufficient to achieve *Pareto improvements*, with both households better off relative to BAU.

Household	**Uniform Rebate**	**91% Rebate to LS**
Low-skill Household	$\text{Welfare}_{LS} < \text{Welfare}_{LS,BAU}$	$\text{Welfare}_{LS} = \text{Welfare}_{LS,BAU}$
High-skill Household	$\text{Welfare}_{HS} > \text{Welfare}_{HS,BAU}$	$\text{Welfare}_{HS} > \text{Welfare}_{HS,BAU}$

Table 4: Welfare outcomes for low-skill and high-skill households under different rebate schemes. Green cells indicate welfare gains relative to BAU, while red cells indicate welfare losses. White cells indicate unchanged welfare compared to the BAU (Per-Unit Tax)

Household	**Uniform Rebate**	**71% Rebate to LS**
Low-skill Household	$\text{Welfare}_{LS} < \text{Welfare}_{LS,BAU}$	$\text{Welfare}_{LS} = \text{Welfare}_{LS,BAU}$
High-skill Household	$\text{Welfare}_{HS} > \text{Welfare}_{HS,BAU}$	$\text{Welfare}_{HS} > \text{Welfare}_{HS,BAU}$

Table 5: Welfare outcomes for low-skill and high-skill households under different rebate schemes. Green cells indicate welfare gains relative to BAU, while red cells indicate welfare losses. White cells indicate unchanged welfare compared to the BAU (Rental Rate Tax)

7 Full Mobility Vs. Full Immobility Optimal Taxes

This section compares the two optimal climate instruments considered in the paper—a per-unit tax on dirty output (or emissions) and a tax on the rental rate of dirty capital—across two benchmark environments. The first is the setting developed here, where labour and capital are *sector-specific* and cannot reallocate between the dirty and clean energy sectors (full immobility). The second follows Golosov et al. (2014) and Barrage (2020), where both factors *freely reallocate* across sectors (full mobility). Under the standard calibration, the optimal policy paths in the two environments almost overlap: both the per-unit tax and the rental-rate tax track each other closely over the transition. The intuition is that the calibrated economy implies very similar sectoral allocations and marginal external damages across the two settings, leaving little scope for differences in the Pigouvian signal. Moreover, the rental-rate tax is tuned to mimic the per-unit instrument's effects on dirty activity, so when allocations align, the implied rental wedge does as well.

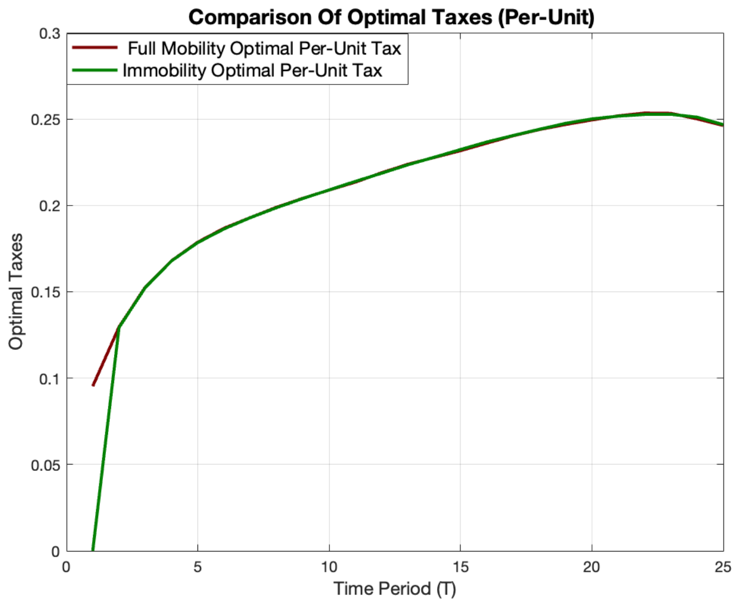

Figure 7: Optimal Per-Unit Taxes in the two economies

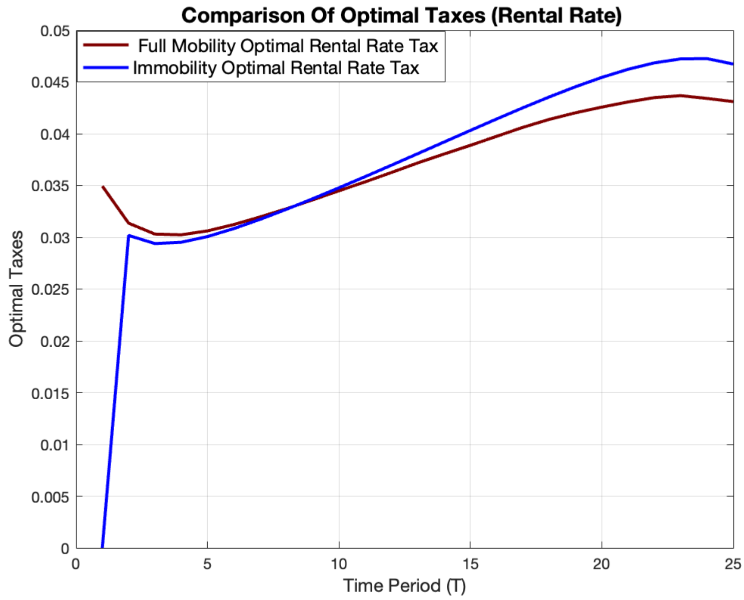

Figure 8: Optimal Rental Rate Taxes in the two economies

First of all, it is important to note that in the full immobility case, it is natural to have in the first period of the simulation null taxes, because both the inputs are pre-determined by the initial values of capital and labour endowments. Only after the first period of the simulation the instruments start to be significant and relevant to correct the externality. However, the optimal climate policies do not differ much from the full mobility case (the rental rate tax is slightly higher in the full immobility economy); this is mainly due to the standard parameterisation, where the optimal labour share (and production parameters) is very close to the optimal labour share in the full mobility case.

Despite this baseline near-coincidence, the two environments can yield distinct optimal taxes when the calibration tilts the economy away from balanced allocations (for example, with skewed labour endowments across skill groups, asymmetric sectoral productivities, or sharper damage curvature). Suppose that the low-skill labour endowment is considerably higher than the high skill one ($N_{LS} = 0.8$ and $N_{HS} = 0,2$), and remember that in the full mobility economy, this labour endowment difference is not affecting the optimal allocation since there is no high/low skill labour, but only an omogenous fully mobile labour:

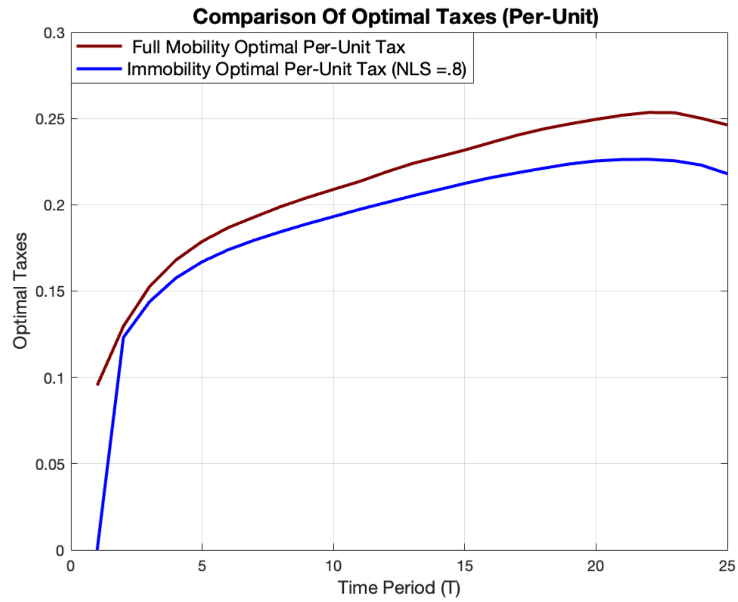

Figure 9: Optimal Per-Unit Taxes in the two economies with $N_{LS} = .8$

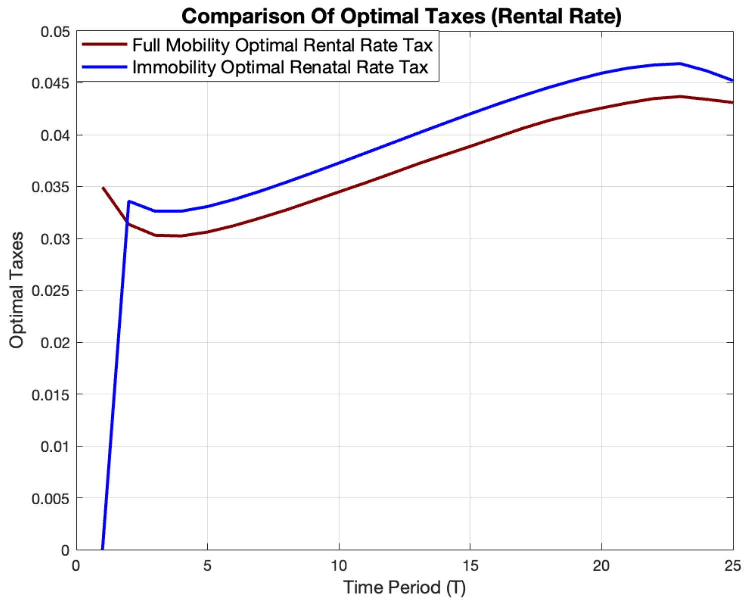

Figure 10: Optimal Rental Rate Taxes in the two economies with $N_{LS} = .8$

When labour and capital are mobile, resources reallocate toward the most productive uses at each point in time. This stronger intensive-margin response raises aggregate production capacity and, absent policy, expands dirty output and emissions along the baseline path and higher baseline emissions imply larger marginal damages. A per-unit (Pigouvian) tax targets the externality directly; consequently, the planner sets a higher per-unit rate in the full mobility case to offset the larger marginal damages generated by the higher emissions base. Mobility also makes the per-unit instrument less distortionary: because factors can move away from the taxed activity, a stronger Pigouvian signal achieves the environmental objective at a lower output cost, further justifying a higher optimal rate.

A tax on the rental rate of dirty capital is an indirect instrument: it raises the user cost of an input rather than pricing emissions themselves. Its effectiveness hinges on how elastically dirty production contracts when that input becomes more expensive. Under full immobility, labour and capital are effectively pinned to the dirty sector, making emissions less responsive to changes in the cost of capital. The reallocation channel is muted, so the same change in the rental rate produces a smaller reduction in emissions. To deliver a given correction of the externality, the planner therefore needs a larger wedge on the rental rate in the immobile economy than in the mobile one. With full mobility, by contrast, resources can flow out of the dirty sector when the user cost rises, so a smaller rental-rate tax suffices.

Think of the per-unit tax as a direct price on the externality, and the rental-rate tax as a price on one of the inputs that generates the externality. When factors are mobile, the externality is larger in equilibrium (hence the need for a stronger direct price), but the response to input prices is also more elastic (hence a weaker input tax suffices). When factors are immobile, the externality is smaller in equilibrium, yet adjustments via input prices are harder to achieve, so the indirect instrument must work harder. This combination produces the pattern observed in the simulations: the optimal per-unit tax is higher with mobility, while the optimal rental-rate tax is higher with immobility (see Figures 9–10). However, more detailed robustness checks on these comparisons need to be further analysed.

8 Conclusion

This paper developed a multisectoral neoclassical growth model with heterogeneous households, sector-specific capital, and climate externalities to study the interplay between carbon taxation, income distribution, and welfare. By introducing full input market frictions—where low-skill labour and dirty capital are tied exclusively to the dirty energy sector, while high-skill labour and clean capital are confined to the clean sector—our model captures a crucial dimension of the energy transition that is often overlooked in the literature.

First, while the optimal per-unit carbon tax improves aggregate welfare by internalising the environmental externality and promoting a cleaner energy mix, its benefits are not evenly distributed. Low-skill households, whose incomes are directly tied to the dirty energy sector, suffer welfare losses compared to the Business-as-Usual allocation. The uniform tax rebate, which returns revenues proportionally to population shares, fails to offset this regressive effect.

Second, my analysis of alternative tax instruments shows that a dirty capital tax ($\tau_{K_{d,t}}$)

achieves the same environmental target but with less adverse impact on low-skill wages. This highlights the importance of the tax base when designing climate policies. +

Third, I demonstrated that the distributional outcome can be further improved through a non-uniform tax rebate. Specifically, allocating 91% of tax revenues to the low-skill household leads to unchanged welfare for the low-skill household relative to BAU, and to a welfare gain for high-skill agents; this is a crucial result for fostering climate policies' political feasibility.

Fourth, the magnitudes of the two instruments can or cannot diverge with reference to the instruments of the full mobility case. More detailed intuitions on this topic would increase the value of this paper.

These results have clear policy implications. Climate policies that neglect distributional effects risk creating winners and losers, potentially undermining social and political support (as illustrated by the Yellow Vests protests). My findings suggest that pairing optimal carbon pricing with carefully designed redistribution mechanisms not only addresses inequality concerns but can also enhance the political feasibility of climate policy.

Future research could extend our model by incorporating endogenous technological change in clean energy, transitional unemployment effects, or global trade channels. Nevertheless, the present work underscores a key message: optimal carbon taxation must be evaluated not only in terms of efficiency but also in terms of its distributional consequences and the design of accompanying fiscal policies.

Appendix B. Sensitivity Analysis Second Chapter

In this appendix, I demonstrate the robustness of the main result, changing one parameter at a time or multiple parameters together. I will show how the optimal taxes chang compared to standard parameterisation (the one in the core paper) for both the per-unit tax and the rental rate tax. At the same time, I will give evidence that the aggregated and disaggregated welfare effects are robust to these parameters' twists. All the results for alternative parameterisation are always assuming a uniform tax rebate, (while a non-uniform tax rebate can achieve the same result as the one shown in the core paper).

Secondly, I continue the exercise of the "imprecise" relaxion of the full labour immobility assumption as done in the first chapter appendix; I do assume an exogenous death rate (more than one) for the low skill agents, outflowing into the high-skill workforce; the negative welfare effect of the remaining low-skill agents is still present.

B1 Standard vs. Not Standard Paramaters Settings

- Changing β from .985 to .99; the welfare effects are magnified, and the optimal carbon taxes are higher.

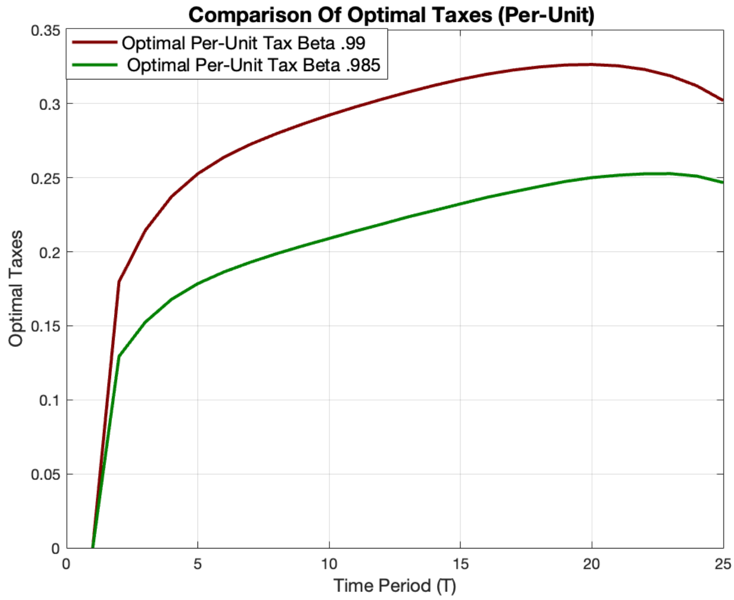

Figure 11: Optimal Taxes with different discount factors (per-unit)

Household	**BAU**	**Optimal Tax**
Low-skill Household	Welfare_{LS}= 9.6719	Welfare_{LS} = 9.5396
High-skill Household	Welfare_{HS} = 10.0287	Welfare_{HS}=10.1472
Aggregated	Welfare_{aggr}= 12.9130	$\text{Welfare}_{aggr.}$ = 12.9143

Table 6: Welfare outcomes for low-skill and high-skill households in the BAU economy and in the Optimal Tax economy

Household	**BAU**	**Optimal Tax**
Low-skill Household	Welfare_{LS}= 9.6719	Welfare_{LS} = 9.6464
High-skill Household	Welfare_{HS} = 10.0549	Welfare_{HS}=10.1472
Aggregated	Welfare_{aggr}= 12.9130	$\text{Welfare}_{aggr.}$ = 12.9143

Table 7: Welfare outcomes for low-skill and high-skill households in the BAU economy and in the Optimal Tax economy

- Different TFP growth rates (from 0 to 0.013 Nordhaus (2010))

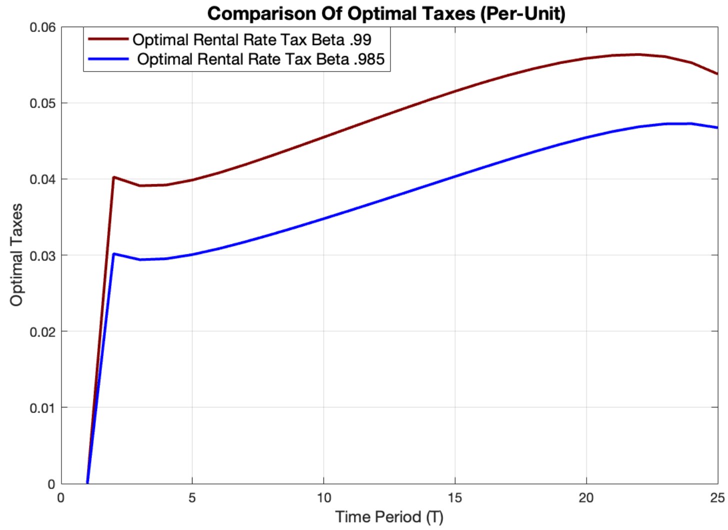

Figure 12: Optimal Taxes with different discount factors (Rental Rate Tax)

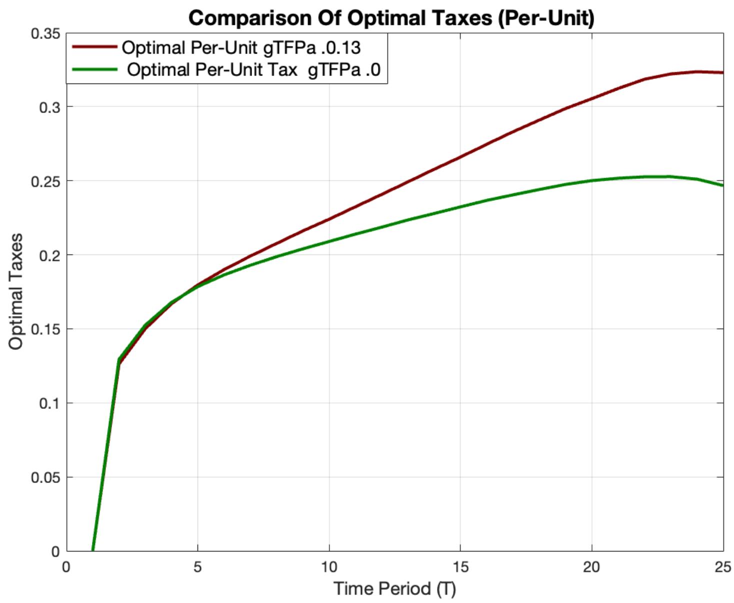

Figure 13: Optimal Taxes with different gTFPa (per-unit)

Household	**BAU**	**Optimal Tax**
Low-skill Household	Welfare$_{LS}$= 6.5495	Welfare$_{LS}$ = 6.4888
High-skill Household	Welfare$_{HS}$ = 6.9993	Welfare$_{HS}$=7.0497
Aggregated	Welfare$_{aggr}$= 8.9525	Welfare$_{aggr.}$ = 8.9529

Table 8: Welfare outcomes for low-skill and high-skill households in the BAU economy and in the Optimal Tax economy

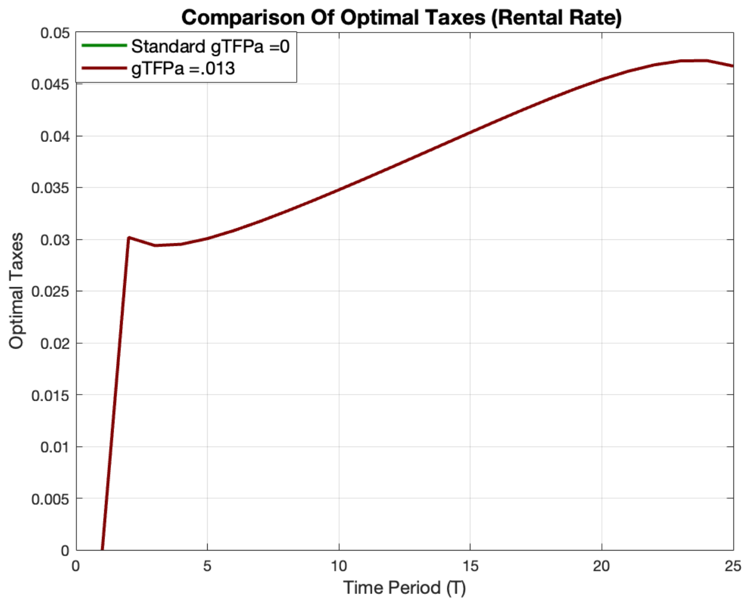

Figure 14: Optimal Taxes with different gTFPa (Rental Rate Tax)

Household	**BAU**	**Optimal Tax**
Low-skill Household	Welfare$_{LS}$= 6.5495	Welfare$_{LS}$ = 6.5380
High-skill Household	Welfare$_{HS}$ = 6.9993	Welfare$_{HS}$=7.0098
Aggregated	Welfare$_{aggr}$= 8.9525	Welfare$_{aggr.}$ = 8.9529

Table 9: Welfare outcomes for low-skill and high-skill households in the BAU economy and in the Optimal Tax economy

- Different N_{LS} low-skill labour endowment (from .5 to .8 with N_{HS} = .2)

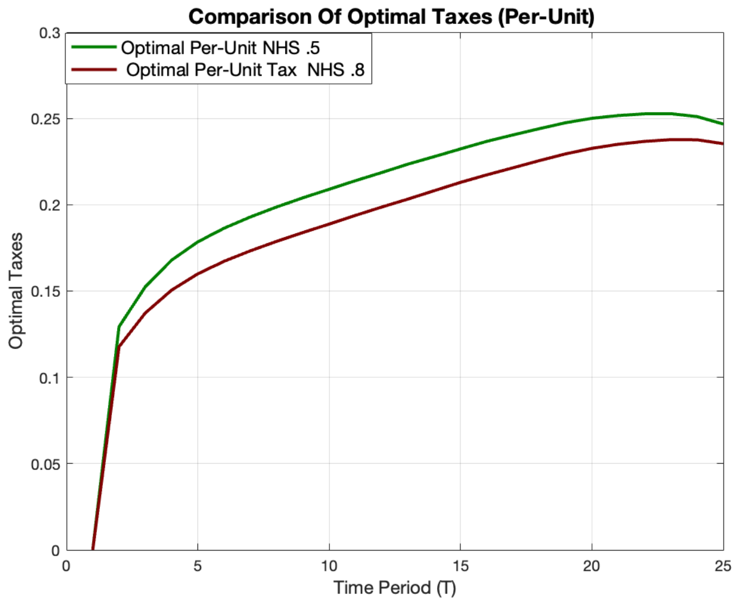

Figure 15: Optimal Taxes with $N_{LS} = .8$ (per-unit)

Household	**BAU**	**Optimal Tax**
Low-skill Household	Welfare$_{LS}$= 7.1957	Welfare$_{LS}$ = 7.1748
High-skill Household	Welfare$_{HS}$ = 4.9217	Welfare$_{HS}$=4.9728
Aggregated	Welfare$_{aggr}$= 8.6061	Welfare$_{aggr.}$ = 8.6063

Table 10: Welfare outcomes for low-skill and high-skill households in the BAU economy and in the Optimal Tax economy

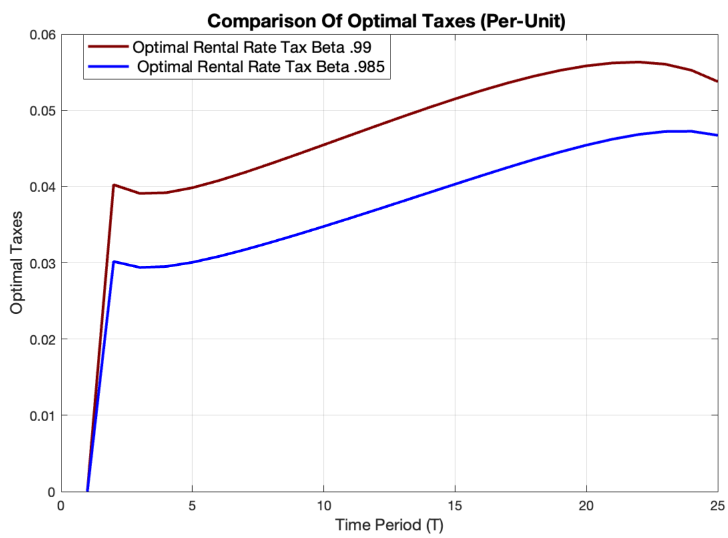

Figure 16: Optimal Taxes with $N_{LS=}.8$(Rental Rate Tax)

Household	BAU	Optimal Tax
Low-skill Household	Welfare_{LS}= 7.1957	Welfare_{LS} = 7.1868
High-skill Household	Welfare_{HS} = 4.9217	Welfare_{HS}=4.9378
Aggregated	Welfare_{aggr}= 8.6061	$\text{Welfare}_{aggr.}$ = 8.6063

Table 11: Welfare outcomes for low-skill and high-skill households in the BAU economy and in the Optimal Tax economy

- 10% climate damages (from 5%, at 2 degrees increase from pre-industrial temeperature)

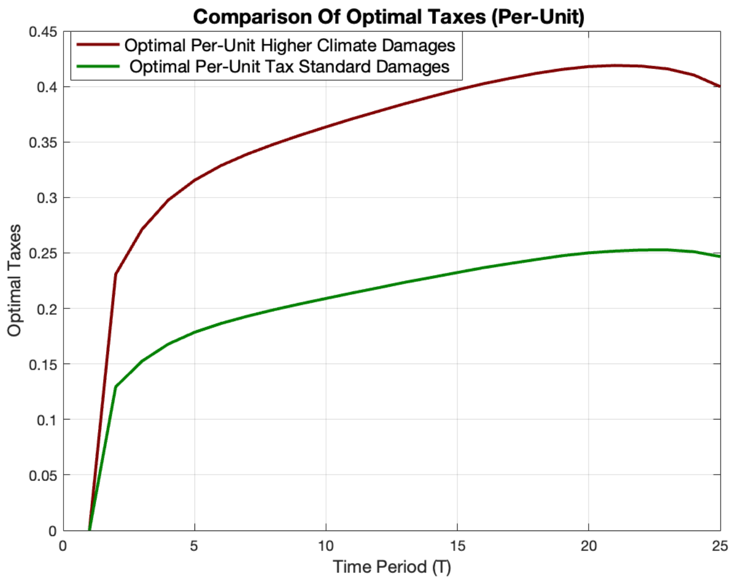

Figure 17: Optimal Taxes Climate Damages 10% (per-unit)

Household	BAU	Optimal Tax
Low-skill Household	Welfare_{LS}= 6.7090	Welfare_{LS} = 6.1514
High-skill Household	Welfare_{HS} = 6.2688	Welfare_{HS}=6.8071
Aggregated	Welfare_{aggr}= 8.7501	$\text{Welfare}_{aggr.}$ = 8.7514

Table 12: Welfare outcomes for low-skill and high-skill households in the BAU economy and in the Optimal Tax economy

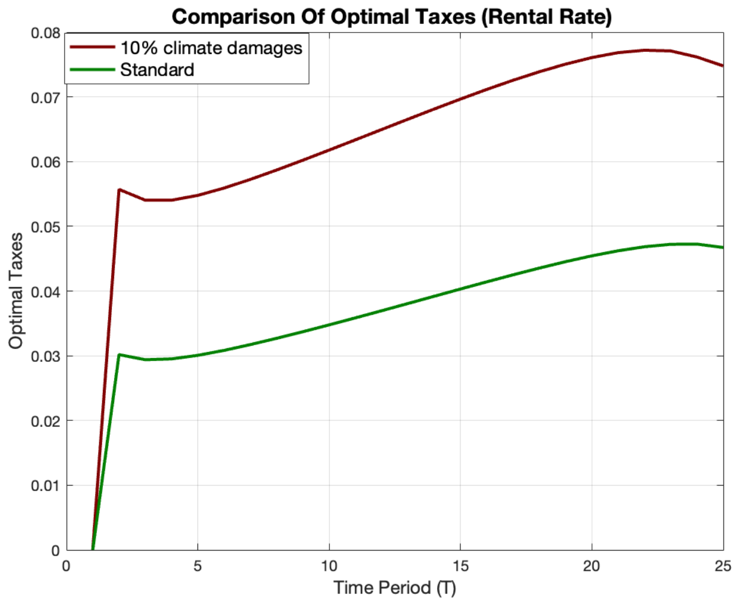

Figure 18: Optimal Taxes with 10% Climate Damages (Rental Rate Tax)

Household	**BAU**	**Optimal Tax**
Low-skill Household	Welfare_{LS}= 6.7090	Welfare_{LS} = 6.2463
High-skill Household	Welfare_{HS} = 6.2688	Welfare_{HS}=6.7311
Aggregated	Welfare_{aggr}= 8.7501	$\text{Welfare}_{aggr.}$ = 8.7514

Table 13: Welfare outcomes for low-skill and high-skill households in the BAU economy and in the Optimal Tax economy

- Multiple Twists; $\beta = .990$, gTFPA $= .013$, $\sigma = 2$.

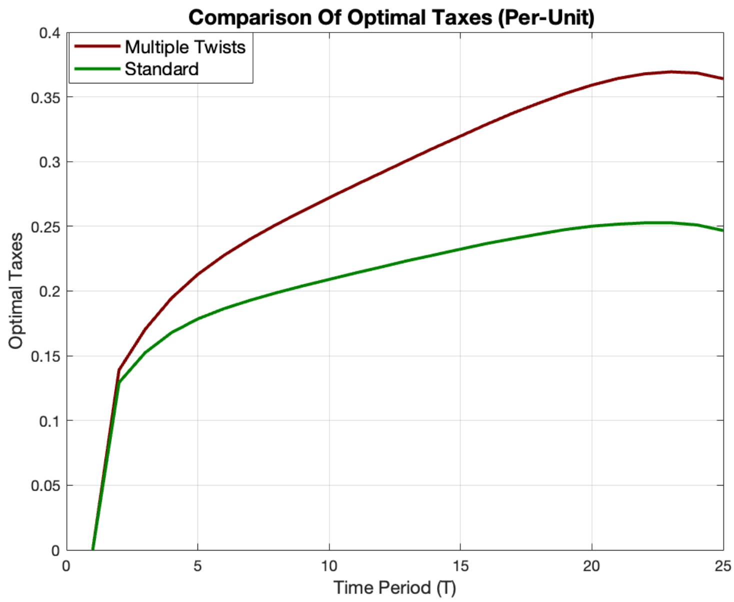

Figure 19: Optimal Taxes Multiple Twists (per-unit)

Household	**BAU**	**Optimal Tax**
Low-skill Household	Welfare_{LS}= 7.4433	Welfare_{LS} = 7.3920
High-skill Household	Welfare_{HS} =7.7281	Welfare_{HS}=7.7689
Aggregated	Welfare_{aggr}= 8.8708	$\text{Welfare}_{aggr.}$ = 8.8711

Table 14: Welfare outcomes for low-skill and high-skill households in the BAU economy and in the Optimal Tax economy

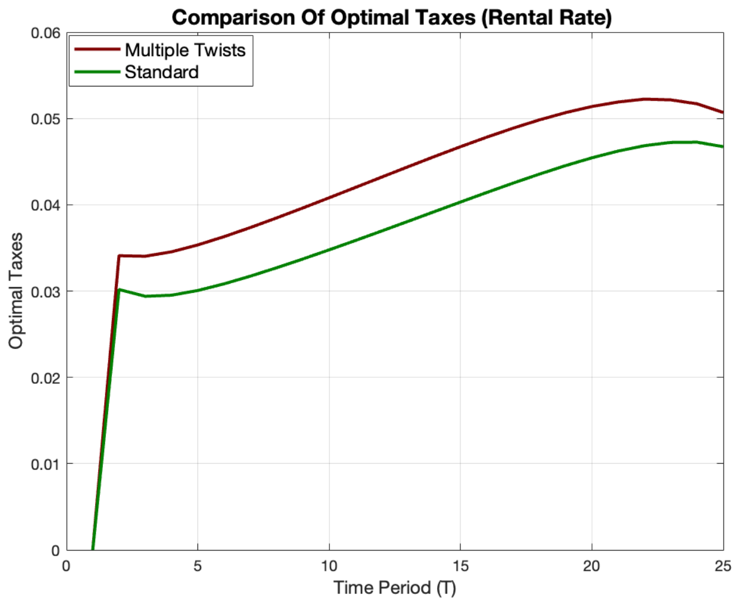

Figure 20: Optimal Taxes Multiple Twists (Rental Rate Tax)

Household	**BAU**	**Optimal Tax**
Low-skill Household	Welfare$_{LS}$= 7.4433	Welfare$_{LS}$ = 7.4350
High-skill Household	Welfare$_{HS}$ =7.7281	Welfare$_{HS}$=7.7358
Aggregated	Welfare$_{aggr}$= 8.8708	Welfare$_{aggr.}$ = 8.8711

Table 15: Welfare outcomes for low-skill and high-skill households in the BAU economy and in the Optimal Tax economy

B2 Exogenous LS Outflow To HS Workforce

Again, as in the previous chapter, I assume an exogenous "death" rate, meaning that the N_{LS} shrinks with time. Suppose a death rate of 5%.

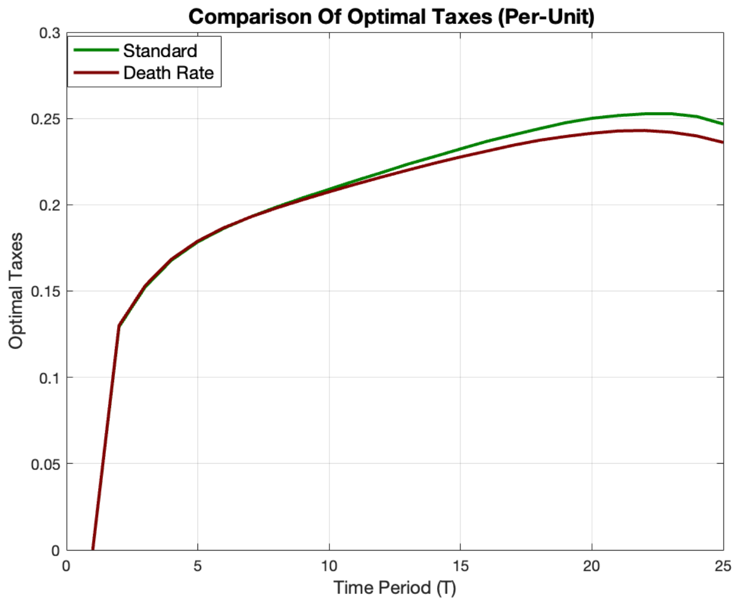

Figure 21: Optimal Taxes Multiple Twists (per-unit)

Household	**BAU**	**Optimal Tax**
Low-skill Household	Welfare$_{LS}$= 5.9822	Welfare$_{LS}$ = 5.9078
High-skill Household	Welfare$_{HS}$ =6.6745	Welfare$_{HS}$=7.0246
Aggregated	Welfare$_{aggr}$= 8.7698	Welfare$_{aggr.}$ = 8.7699

Table 16: Welfare outcomes for low-skill and high-skill households in the BAU economy and in the Optimal Tax economy

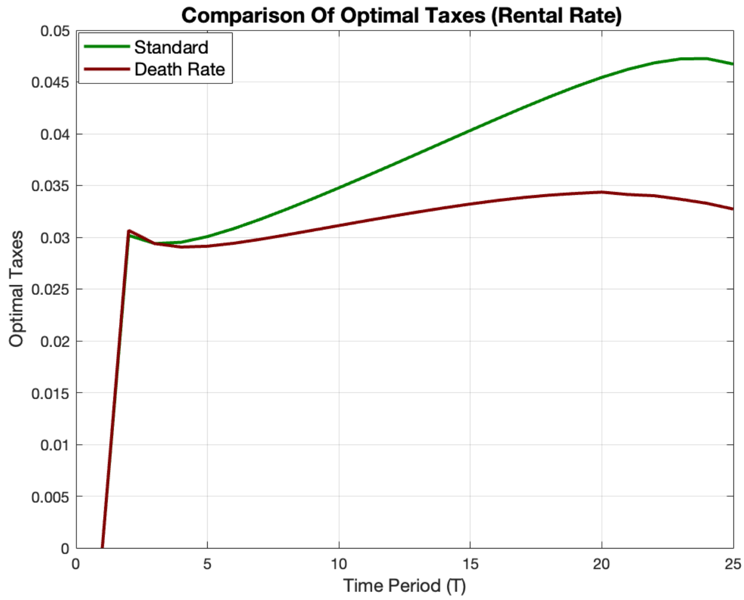

Figure 22: Optimal Taxes Multiple Twists (Rental Rate Tax)

Household	**BAU**	**Optimal Tax**
Low-skill Household	Welfare_{LS}= 5.9822	Welfare_{LS} = 5.9666
High-skill Household	Welfare_{HS} =6.6745	Welfare_{HS}=6.9858
Aggregated	Welfare_{aggr}= 8.7698	$\text{Welfare}_{aggr.}$ = 8.7699

Table 17: Welfare outcomes for low-skill and high-skill households in the BAU economy and in the Optimal Tax economy

B3 Computational algorithm for Chapter 2

The numerical solution of the planner's problem in Chapter 2 follows the same structure and solution method as in Chapter A4. The model, for each taxation scheme, is implemented in MATLAB using a *main file* plus two auxiliary files, `objective` and `constraints`, and is solved with `fmincon` (SQP algorithm) over a finite horizon of $T = 25$ transition periods and $P = 10$ continuation periods.

Each taxation scheme (i.e., the per-unit tax and the rental rate tax) is simulated separately; at the aggregated level, the two instruments will result in the same allocation, affecting only the distribution of resources between households.

The key difference with respect to Chapter 1 concerns the set of control variables. In Chapter 2, the planner's choice set is reduced to:

- the saving rates into dirty and clean capital, $\{s_{b,t}\}_{t=1}^{T+1}$ and $\{s_{g,t}\}_{t=1}^{T+1}$, which jointly determine the separate capital stocks in the two energy sectors;

- the consumption profiles of low-skilled and high-skilled agents, $\{C_{LS,t}\}_{t=1}^{H}$ and $\{C_{HS,t}\}_{t=1}^{H}$, with $H = T + P$.

- The per-unit tax τ_t or the rental rate tax $\tau_{K_{d,t}}$ (as separate control variables in different Main.m files)

There is no longer a separate capital share control (as in Chapter 1) the allocation between dirty and clean capital is entirely driven by the two saving rates $s_{b,t}$ and $s_{g,t}$.

Given a candidate vector of controls, the `objective` file reconstructs the paths of dirty and clean capital, sectoral energy outputs, aggregate output, with the related optimal taxes and evaluates the discounted welfare of the two groups (plus the balanced growth path continuation term), exactly as in Chapter 1. The `constraints` file enforces the feasibility of aggregate and individual consumption, the usual no-arbitrage conditions across sectors, and the dynamic equilibrium relationships. The Business-As-Usual allocation is obtained adding an extra flow of constraints on the climate instruments, equalizing them to 0.

An outer loop over the Negishi weight θ is used, as in Chapter 1, to ensure that the implied intertemporal wealth of low- and high-skilled agents is consistent with their discounted consumption paths. The algorithmic structure and numerical solution strategy are identical to those described in Section A4.

Chapter 3

Climate Policies, Financial Frictions, And Inequality

Flavio Contrada[*†]

Abstract

In this paper, I provide a heterogeneous-agents environmental dynamic general equilibrium model with segmented labour/capital markets and financial frictions. I consider a closed economy operating in discrete time, comprising two representative households, a brown household working in the polluting industry and a green household working in the clean firm (or in the banking sector), a government, and a final good sector relying on energy inputs. The economy features two externalities: climate change and financial frictions à là Gertler and Karadi (2011). I investigate the interaction between financial frictions, climate policies and inequality in the transition toward a balanced growth path. A comparison between the welfare effects of a per-unit tax and a loan capital tax reveals that both measures have similar effects on mitigating the externality, improving aggregate welfare; however, the brown representative household experiences a welfare loss compared to the Business-As-Usual allocation if tax revenues are rebated uniformly across brown and green households and the emission tax is implemented. Ceteris paribus regarding the uniform rebate, the loan rate tax induces welfare gains for both households.

Keywords: green financial policy, climate policy, carbon tax, transition risk, inequality

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[†]This is a joint work with Pietro Dindo (Ca' Foscari University) and Alessandro Spiganti (University of Genoa).

1 Introduction

Policies aimed at supporting the transition to a carbon-neutral economy are high on the policy agenda. The effectiveness of these policies in reducing emissions—together with their distributional consequences—is actively debated. On the one hand, the necessity of climate policy is well understood; recent updated estimates on the relation between rising temperatures and GDP damages are disheartening Bilal and Känzig (2024), calling for stronger intervention. On the other hand, their societal impacts remain unclear. Who is most affected? Does everyone benefit? Why have climate policies faced protests from workers, as in the Yellow Vests movement in France? The economic literature exploring the connections between inequality and climate change is expanding Drupp et al. (2024), but further empirical and theoretical work is needed.

This paper contributes to the literature on optimal carbon taxation and financial frictions with a novel focus on the link between inequality and optimal policy when (i) inputs in the energy sector are sector-specific, (ii) credit frictions are present, and (iii) households are heterogeneous. We develop a multisector neoclassical growth model with a climate externality, heterogeneous households, input markets, and credit frictions to address the following questions: How is the optimal carbon tax affected when financial frictions impede the economy and when labor and capital used to produce intermediate energy goods are sector-specific? What happens to households working in the "brown" (polluting) sector? Are there alternative taxation schemes that mitigate the externality while delivering different—and potentially more equitable—distributional effects? Are there rebate schemes that deliver more neutral disaggregated welfare effects when lump-sum transfers across types are unavailable?

First, numerical simulations show that the magnitude of the optimal carbon tax along the transition is lower than in the frictionless benchmark: with financial frictions, a smaller tax restores efficiency (Question 1). Second, although a standard per-unit carbon tax raises aggregate welfare, under a uniform tax rebate it reduces wages in the brown sector and lowers the welfare of low-skill households (Question 2). Third, we study an alternative instrument—a tax on the loan (rental) rate of brown capital ("loan-rate tax"). Even under a uniform rebate, this loan-rate tax both mitigates the externality from brown energy production and makes both household types better off relative to the per-unit tax with uniform rebates (Question 3). In other words, holding the uniform recycling rule fixed, the loan-rate tax dominates the per-unit tax in terms of distributional performance while preserving efficiency along the transition. Fourth, when lump-sum transfers across types are unavailable, a non-uniform recycling rule that directs (almost) the entire carbon dividend to the brown household can neutralize the inequality generated by climate policy (Question 4).

This work is related to Douenne, Hummel, and Pedroni (2022) and Fried, Novan, and

Peterman (2024), who analyze optimal carbon taxation with heterogeneity in productivity and climate sensitivity. Our model instead imposes full frictions in labor (and capital) mobility across sectors, allowing us to study inequality between workers in different sectors rather than only across income classes—an assumption supported by evidence that the green transition requires new skills and the accumulation of human and physical capital Vona et al. (2018),Borissov, Brausmann, and Bretschger (2019), Bluedorn et al. (2023), Rud et al. (2024). We also incorporate financial frictions, contributing to a growing literature that embeds credit-market imperfections in environmental macro models to study macro-financial stability under climate policy Diluiso et al. (2021), Carattini, Heutel, and Melkadze (2023), Comerford and Spiganti (2023),Schuldt and Lessmann (2023). Unlike much of that literature, the presence of heterogeneous households lets us study the interplay among financial frictions, climate policy, and inequality. The full transition dynamics and the associated optimal carbon-tax paths are computed numerically.

The next section presents the model, equilibrium concept, the Business-As-Usual allocation, and the social optimum, and concludes with the simulation results.

2 Model

I consider a deterministic closed economy in discrete and infinite time. This economy consists of two representative households, a government, and several types of firms. It features five sectors: i) a competitive final sector producing a homogeneous final good using brown and green inputs, ii) a green and a brown competitive sector, producing green and brown inputs using labour and sector-specific capital, iii) a competitive capital-producing sector, providing sector-specific capital to brown and green firms and buying back undepreciated capital, and iv) a competitive financial sector, collecting deposits from households and providing loans to brown and green firms:

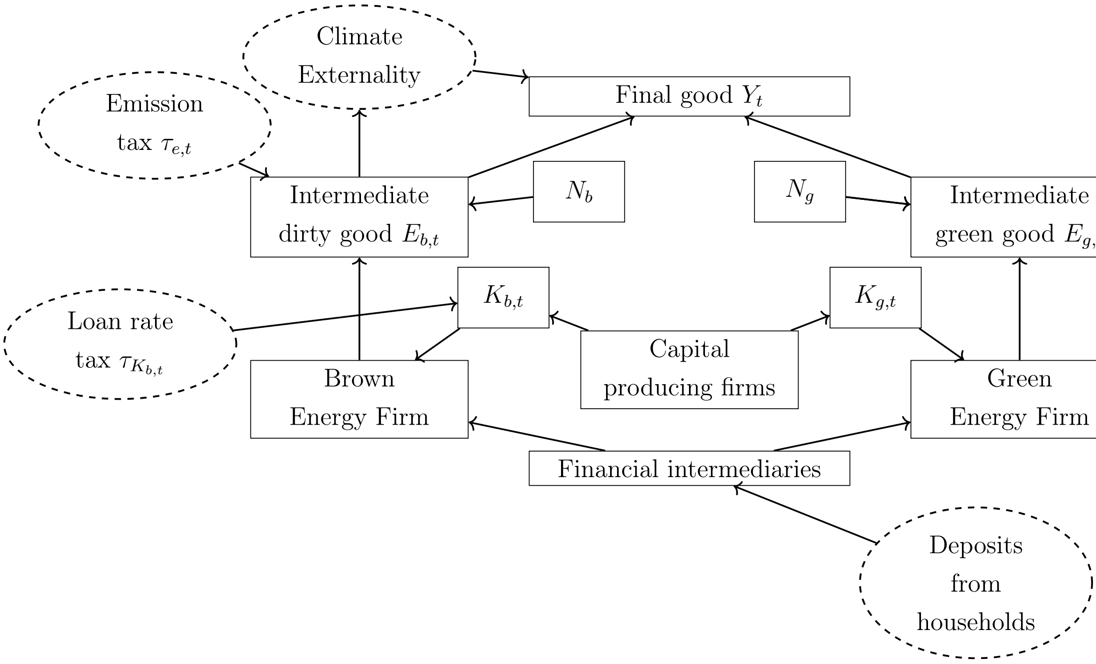

Figure 1: Overview of the model

2.1 Households

There are two representative households, which we label brown (b) and green (g). The members of the g household can be workers in the clean sector or bankers. Members of the b household can only be workers in the brown sector. Concerning the members of the g household, a fraction $1-\iota$ are workers, while the remaining fraction ι are bankers. Workers of the i household supply on aggregate N_i (labour endowments are constant at each time period, $\bar{N}_i$) units of labour inelastically to firms in sector i in exchange for wages, which are returned to the household. The representative household of type i derives utility from household consumption $C_{i,t}$.

The net present value of the lifetime utility of this representative household is

$$\sum_{t=0}^{\infty} \beta^t \frac{C_{i,t}^{1-\sigma} - 1}{1-\sigma}, \tag{1}$$

where $\beta \in (0,1)$ is the discount factor, and $\sigma > 0$ is the inverse of the elasticity of intertemporal substitution. Considering the budget constraint of the g household, it is the sum of sector-specific wage bills $W_{g,t}N_g$, of tax rebate $T_{g,t}$ from the government and of a share of total profits from good-producing firms and from banks ($\Pi_{g,t}$ and $\Xi_{g,t}$). This representative household can save in the form of bank deposits $D_{i,t}$, a one-period risk-free asset paying a return R_{t+1}:

$$C_{g,t} + D_{g,t} = W_{g,t}N_g + R_t D_{g,t-1} + \Pi_{g,t} + \Xi_{g,t} + T_{g,t}. \tag{2}$$

The representative household b is hand-to-mouth, consuming what earning through wages (plus the uniform tax rebate),

$$C_{b,t} = W_{b,t} N_b + T_{b,t}. \tag{3}$$

Letting $u'(C_{g,t}) = C_{g,t}^{-\sigma}$ denote the marginal utility of consumption in period t, the following first-order conditions (FOCs) ensue,

$$u'(C_{g,t}) = \lambda_{g,t} \tag{4a}$$

$$\Lambda_{g,t,t+1} R_{t+1} = \frac{1}{\beta}, \tag{4b}$$

where $\lambda_{i,t}$ is the shadow price of relaxing the budget constraint (i.e. the Lagrangian multiplier) and $\Lambda_{g,t,t+1} \equiv u'(C_{g,t+1})/u'(C_{g,t})$, so that $\beta\,(\Lambda_{i,t,t+1})$ is household g's real discount factor. Since the only household using the banking sector is the green one, for now on $\Lambda_{g,t,t+1} = \Lambda_{t,t+1}$

2.2 Final Good Production

Households consume a unique final good Y_t which is produced competitively by a representative firm that combines brown and green inputs, $E_{b,t}$ and $E_{g,t}$, according to the following constant elasticity of substitution (CES) technology,

$$Y_t = (1 - D(Z_t))\, A_t \left(E_{b,t}^{(\epsilon-1)/\epsilon} + E_{g,t}^{(\epsilon-1)/\epsilon} \right)^{\epsilon/(\epsilon-1)}, \tag{5}$$

where ϵ is the elasticity of substitution between the brown and green inputs, A_t is the exogenous total factor productivity (TFP) of the final sector, and $D(Z_t)$ is a damage function capturing the effect of climate change on productivity, in the spirit of the dynamic integrated climate-economy model Nordhaus (2010). Following Golosov et al. (2014), we calibrate this to $D(Z_t) \equiv 1 - \exp(-\rho Z_t)$, where ρ measures the intensity of damages and Z_t is cumulative emissions. More about the externality will be said in subsection 2.4.

Final Good Producer maximizes:

$$\begin{aligned} \max_{E_{b,t},\, E_{g,t}} \quad & Y_t - p_{b,t} E_{b,t} - p_{g,t} E_{g,t} \\ \text{subject to} \quad & Y_t = A_t (1 - D(Z_t)) \left(E_{b,t}^{\frac{\varepsilon-1}{\varepsilon}} + E_{g,t}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}. \end{aligned} \tag{6}$$

First-Order-Conditions for $E_{b,t}$ and $E_{g,t}$ are given by:

$$[E_{b,t}] : P_{b,t} = (1 - D(Z_t)) A_t (E_{b,t}^{\frac{\epsilon-1}{\epsilon}} + E_{g,t}^{\frac{\epsilon-1}{\epsilon}})^{\frac{1}{\epsilon}} E_{b,t}^{-\frac{1}{\epsilon}} \tag{7}$$

$$[E_{g,t}] : P_{g,t} = (1 - D(Z_t)) A_t (E_{b,t}^{\frac{\epsilon-1}{\epsilon}} + E_{g,t}^{\frac{\epsilon-1}{\epsilon}})^{\frac{1}{\epsilon}} E_{g,t}^{-\frac{1}{\epsilon}} \tag{8}$$

The optimality conditions for the final good producer imply that the relative demands for the intermediate inputs are inversely related to their prices,

$$\frac{E_{gt}}{E_{bt}} = \left(\frac{P_{bt}}{P_{gt}}\right)^{\epsilon}. \tag{9}$$

Without loss of generality, I normalise the price of the final good to one at each date, i.e. $\left(P_{gt}^{1-\epsilon} + P_{gt}^{1-\epsilon}\right)^{1/(1-\epsilon)} \equiv 1$.

2.3 Intermediate Energy Good Producers

A representative firm in each sector $i \in \{b, g\}$ produces competitively the intermediate good $E_{i,t}$, using labour N_i and sector-specific capital $K_{i,t-1}$. Both representative firms rely on funds from the financial sector to purchase capital from capital-producing firms. Whereas green production does not create carbon emissions, the production of the brown input entails carbon emissions as a negative externality. The stock of carbon dioxide in the atmosphere evolves according to $Z_t = Z_0 + \sum_{\tau=1}^{t} E_{b,\tau}$ [1]

Both types of firms operate a constant returns to scale technology,

$$E_{i,t} = A_{i,t} K_{i,t-1}^{\alpha} N_i^{1-\alpha}, \tag{10}$$

where $\alpha_i \in (0,1)$ is the capital share and $A_{i,t}$ denotes sector-specific stochastic TFP. At the end of period t, firms in the brown and green sectors buy capital $K_{i,t}$ to be used in production at time $t+1$ from capital-producing firms at market price $Q_{i,t}$. This capital acquisition is financed by borrowing an amount $Q_{i,t}K_{i,t}$ from banks. After production takes place in $t+1$, firms repay financial intermediaries at rate $R_{i,t+1}$, resell undepreciated capital $(1-\delta_i)\,K_{i,t}$ at price $Q_{i,t+1}$ to capital-producing firms, and purchase capital that will be employed in the subsequent period.

As a consequence, realized profits in t in the brown sector are, respectively, depending on the different taxation schemes

$$\max_{\{N_b, K_{b,t-1}\}} \quad (P_{b,t} - \tau_{e,t})\,E_{b,t} - W_{b,t}N_b - R_{b,t}Q_{b,t-1}K_{b,t-1} + (1-\delta_b)Q_{b,t}K_{b,t-1} \tag{11}$$
$$\text{s.t.} \quad E_{b,t} = A_{b,t}K_{b,t-1}^{\alpha}N_b^{1-\alpha}$$

[1] I do not incorporate a carbon cycle following insights in atmospheric science, arguing that warming is linear in cumulative carbon emissions Allen et al. (2009), where Z_0 is the pre-industrial stock of carbon emissions. Therefore, the representative brown firm may be subject to a carbon tax $\tau_{e,t}$ imposed by the government, or to a rental rate taxation on the brown capital (τ_{t,K_b}).

or

$$\max_{\{N_b, K_{b,t-1}\}} \quad P_{b,t}E_{b,t} - W_{b,t}N_b - \left(R_{b,t} + \tau_{t_{K_b}}\right) Q_{b,t-1}K_{b,t-1} + (1-\delta_b)Q_{b,t}K_{b,t-1} \tag{12}$$
$$\text{s.t.} \quad E_{b,t} = A_{b,t}K^{\alpha}_{b,t-1}N_b^{1-\alpha}.$$

In equation (11), the taxation is a *per-unit* tax $\tau_{e,t}$ on the emission, assuming $E_{b,t}$ impact on the cumulative stock of emissions linearly and for all of its magnitude (i.e., $Z_t = Z_{t-1}+E_{b,t}$); in equation (12), the tax is loan rate taxation on the $K_{b,t}$ utilization, increasing its cost up to $\tau_{t_{K_b}}$.

Green producer's profit maximization is instead,

$$\max_{\{N_g, K_{g,t-1}\}} \quad P_{g,t}E_{g,t} - W_{g,t}N_g - R_{g,t}Q_{g,t-1}K_{g,t-1} + (1-\delta_g)Q_{g,t}K_{g,t-1} \tag{13}$$
$$\text{s.t.} \quad E_{g,t} = A_{g,t}K^{\alpha}_{g,t-1}N_g^{1-\alpha}$$

The representative brown firm, in the case of the *per-unit* taxation on emissions $\tau_{e,t}$, chooses labour and capital according to the following FOCs :

$$W_{b,t} = (1-\alpha)\frac{E_{b,t}}{N_b}\left(P_{b,t} - \tau_{e,t}\right) \tag{14a}$$
$$R_{b,t}Q_{b,t-1} = \alpha\frac{E_{b,t}}{K_{b,t-1}}\left(P_{b,t} - \tau_{e,t}\right) + (1-\delta_b)Q_{b,t}. \tag{14b}$$

Alternatively, in the case of the brown capital loan rate taxation, the relative FOCs are:

$$W_{b,t} = (1-\alpha)\frac{E_{b,t}}{N_b}P_{b,t} \tag{15a}$$
$$(R_{b,t} + \tau_{t_{K_b}})Q_{b,t-1} = \alpha\frac{E_{b,t}}{K_{b,t-1}}P_{b,t} + (1-\delta_b)Q_{b,t}. \tag{15b}$$

Likewise, the representative green firm chooses labour and sector-specific capital to satisfy:

$$W_{g,t} = (1-\alpha)\frac{P_{g,t}}{P_t}\frac{E_{g,t}}{N_g} \tag{16a}$$
$$R_{g,t}Q_{g,t-1} = \alpha P_{g,t}\frac{E_{g,t}}{K_{g,t-1}} + (1-\delta_g)Q_{g,t}. \tag{16b}$$

2.4 Externality

The Z_t cumulative stock produced in the economy affects the final good sector negatively. The functional form of this damage engendered by $E_{b,t}$ is the one employed in

Golosov et al. (2014), which is the following exponential damage function,

$$D(Z_t) = 1 - e^{-\rho(Z_t)},$$

where ρ measures the intensity of damages. The link between the climate and the final good sector occurs through the variable Z_t, denoting cumulative emissions, i.e, the summation of $E_{b,t}$ produced in the economy between the first time period and the current one:

$$Z_t = Z_0 + \sum_{k=1}^{t} E_{b,k}, \tag{17}$$

where Z_0 is the starting cumulative stock of emissions. $E_{b,t}$ instantly flows into the atmosphere at time t, assuming no temperature inertia for simplicity ((**douenne2023**), (W. D. Nordhaus, 2017)). To conclude, Z_t results in climate damage on the final TFP A_t through the damage function.

2.5 Capital Producing Firms

The capital-producing sectors follow closely the specification in Gertler and Karadi (2011). Capital is sector-specific and immobile across sectors, and two representative capital-producing firms build competitively green and brown capital goods.

After production takes place in any period t, the i-type capital-producing firm buys back undepreciated capital $(1-\delta_i)\,K_{i,t-1}$ from current period producers in sector i at price $Q_{i,t}$, and refurbish it at no cost. They then decide how much new sector-specific capital to produce $I_{i,t}$, and sell the aggregate level of new and refurbished capital $K_{i,t} = I_{i,t} + (1-\delta_i)\,K_{i,t-1}$ to be used in production in $t+1$ at the same price $Q_{i,t}$, commonly known as Tobin's Q.

In particular, the competitive representative capital good producers then choose the stream of $I_{i,t}$ to maximize:

$$\begin{aligned} \max_{\{I_{i,t}\}} \quad & \sum_{t=0}^{\infty} \sum_{i\in\{b,g\}} \Lambda_{0,t} \left[Q_{i,t}K_{i,t} - I_{i,t} - Q_{i,t}(1-\delta_i)K_{i,t-1} \right] \\ \text{s.t.} \quad & K_{i,t} = (1-\delta_i)K_{i,t-1} + I_{i,t}, \quad \forall t,\ i\in\{b,g\} \end{aligned} \tag{18}$$

Plugging the law of motion of capital into the objective function, we obtain, the unconstrained maximization problem:

$$\max_{\{I_{i,t}\}} \quad \sum_{t=0}^{\infty} \sum_{i\in\{b,g\}} \Lambda_{0,t} \left[Q_{i,t}((1-\delta_i)K_{i,t-1} + I_{i,t}) - I_{i,t} - Q_{i,t}(1-\delta_i)K_{i,t-1} \right].$$

The zero-profits condition consistent with the equilibrium implies:

$$Q_{i,t} = 1 \ \ \forall t, \, i \in \{b, g\}$$

Without adjustment costs, the capital goods prices are unitary and are equal to the final good price.

2.6 Banks

I examine a representative financial intermediary, denoted by j, that functions within the economy to channel household savings into productive investments. At the end of each period t, this intermediary issues claims $S_{b,t}(j)$ and $S_{g,t}(j)$, representing investment positions in capital allocated to the brown and green sectors, respectively, for utilization in period $t+1$. These claims are employed to purchase physical capital at a market price $Q_{i,t}$ from firms producing capital goods. The financial intermediary is owned by the green household, with a fraction ι of the household's members serving as bankers. Following Gertler and Karadi (2011), the banker faces a moral hazard problem: they have the option to divert an exogenous fraction κ of the funds raised through deposits for personal use (i.e., transferring these funds to their household). Depositors can shut down the bank after recovering the remaining $(1-\kappa)$ fraction of the assets. Then, they will only lend to the banker if she has sufficient incentives to act honestly. In each period, the intermediary contends with an exogenous exit shock. With probability $(1-\gamma)$, the intermediary exits the market at the end of the period, leading to liquidation and the distribution of its remaining net worth as dividends to the green household. The j-th bank's balance sheet is given by:

$$Q_{b,t}S_{b,t}(j) + Q_{g,t}S_{g,t}(j) = NW_t(j) + D_t(j) \tag{19}$$

Net worth evolves according the following equation, where $R_{i,t+1}$ are the realized loan rates on asset $S_{i,t}$ and R_t is the deposit rate:

$$NW_{t+1}(j) = \sum_{i=\{b,g\}} R_{i,t+1}Q_{i,t}S_{i,t}(j) - R_t D_t(j). \tag{20}$$

Substituting $D_t(j)$ obtained in (19) into (20), the net worth evolution process becomes:

$$NW_{t+1}(j) = \sum_{i=\{b,g\}} (R_{i,t+1} - R_t)Q_{i,t}S_{i,t}(j) + R_t NW_t(j) \tag{21}$$

I can define the franchise value $V_t(j)$ recursively as:

$$V_t(j) = \max_{\{S_{b,t}(j), S_{g,t}(j)\}} \sum_{i\in\{b,g\}} \Lambda_{t,t+1}[(1-\gamma)NW_{t+1}(j) + \gamma V_{t+1}(j)]. \tag{22}$$

If the banker exits the business with probability $(1-\gamma)$, she withdraws the entire net worth; otherwise, with probability γ, the bank continues with continuation (or franchise) value $V_t(j)$. The incentive compatibility constraint commands the franchise value to be greater or equal to the fraction of the banker's "divertable" funds, i.e.:

$$V_t(j) \geq \kappa(Q_{b,t}S_{b,t}(j) + Q_{g,t}S_{g,t}(j)) \;\; \forall t. \tag{23}$$

The complete j-th banker's problem (in recursive terms) becomes:

$$\begin{aligned} V_t(j) = \max_{\{S_{b,t}(j), S_{g,t}(j)\}_{t=0}^{\infty}} & \sum_{i\in\{b,g\}} \Lambda_{t,t+1}[(1-\gamma)NW_{t+1}(j) + \gamma V_{t+1}(j)] \\ \text{s.t.} \quad & V_t(j) \geq \kappa(Q_{b,t}S_{b,t}(j) + Q_{g,t}S_{g,t}(j)) \;\; \forall t \\ & Q_{b,t}S_{b,t}(j) + Q_{g,t}S_{g,t}(j) = NW_t(j) + D_t(j) \;\; \forall t \\ & NW_{t+1}(j) = \sum_{i=\{b,g\}} (R_{i,t+1} - R_t)Q_{i,t}S_{i,t}(j) + R_t NW_t(j) \;\; \forall t \end{aligned} \tag{24}$$

A simple guess-and-verify approach is utilized to show that franchise value $V_t(j)$ is linear in net worth $NW_t(j)$, where $V_t(j) = \phi_t NW_t(j)$, with ϕ_t being a time-varying parameter:

$$V_t(j) = \max_{\{S_{b,t}(j), S_{g,t}(j)\}} \sum_{i\in\{b,g\}} \Lambda_{t,t+1}[(1-\gamma)NW_{t+1}(j) + \gamma V_{t+1}(j)]$$

$$V_t(j) = \max_{\{S_{b,t}(j), S_{g,t}(j)\}} \sum_{i\in\{b,g\}} \Lambda_{t,t+1}\left[(1-\gamma)NW_{t+1}(j) + \gamma\phi_{t+1}NW_{t+1}(j)\right]$$

$$V_t(j)(= \phi_t NW_t(j)) = \max_{\{S_{b,t}(j), S_{g,t}(j)\}} \sum_{i\in\{b,g\}} \Lambda_{t,t+1}[((1-\gamma) + \gamma\phi_{t+1})]NW_{t+1}(j)$$

Now, define variables χ_t^b, χ_t^g and ν_t as:

$$\chi_t^b = \Omega_{t+1}(R_{b_{t+1}} - R_t) \tag{25}$$

$$\chi_t^g = \Omega_{t+1}(R_{g_{t+1}} - R_t) \tag{26}$$

$$\nu_t = \Omega_{t+1}R_t, \tag{27}$$

where $\Omega_{t+1} = \beta^t\Lambda_{t,t+1}((1-\gamma) + \gamma\phi_{t+1})$.

Using χ_t^b, χ_t^g and ν_t, the incentive compatibility constraint becomes:

$$\chi_t^b Q_{b,t} S_{b,t}(j) + \chi_t^g Q_{g,t} S_{g,t(j)} + \nu_t NW_t(j) \geq \kappa(Q_{b,t} S_{b,t}(j) + Q_{g,t} S_{g,t}(j)). \tag{28}$$

The franchise value $V_t(j)$ is instead:

$$V_t(j) = \chi_t^b Q_{b,t} S_{b,t}(j) + \chi_t^g Q_{g,t} S_{g,t}(j) + \nu_t NW_t(j) \tag{29}$$

The full constrained maximization problem can be written using the lagrangian function. Defining μ_t as the lagrangian multiplier associated with the incentive compatibility constraint, the lagrangian function is:

$$\max_{S_{b,t}(j),\, S_{g,t}(j)} \mathcal{L}_t = \left(\chi_t^b Q_{b,t} S_{b,t}(j) + \chi_t^g Q_{g,t} S_{g,t}(j) + \nu_t NW_t(j)\right)(1+\mu_t) - \mu_t \kappa \left(Q_{b,t} S_{b,t}(j) + Q_{g,t} S_{g,t}(j)\right)$$

First-Order-Conditions for assets $S_{i,t}$ are:

$$[S_{b,t}(j)] : (1+\mu_t)\chi_t^b = \mu_t \kappa \tag{30}$$
$$[S_{g,t}(j)] : (1+\mu_t)\chi_t^g = \mu_t \kappa. \tag{31}$$

First-Order-Condition for μ_t:

$$[\mu_t] : \mu_t \left[\chi_t^b Q_{b,t} S_{b,t}(j) + \chi_t^g Q_{g,t} S_{g,t}(j) + \nu_t NW_t(j) - \kappa(Q_{b,t} S_{b,t}(j) + Q_{g,t} S_{g,t}(j))\right] = 0, \text{with } \mu_t \geq 0 \tag{32}$$

(30) and (31) combined lead to the non-arbitrage condition on the prices of the assets:

$$\Omega_{t+1}(R_{b,t+1} - R_t) = \Omega_{t+1}(R_{g,t+1} - R_t). \tag{33}$$

Arbitrage forces require the assets to have the same expected excess return for the banker. The lagrangian multiplier μ_t can be obtained by (30)

$$\mu_t = \frac{\chi_t^b}{\kappa - \chi_t^b}$$

The incentive-compatibility constraint binds whenever $\mu_t > 0$ or when $0 < \chi_t^b < \kappa$. I focus on a transition toward the Balanced-Growth-Path featuring a binding incentive compatibility constraint. By equation (32), $\mu_t > 0$ leads to:

$$\left[\chi_t^b Q_{b,t} S_{b,t}(j) + \chi_t^g Q_{g,t} S_{g,t}(j) + \nu_t NW_t(j) - \kappa(Q_{b,t} S_{b,t}(j) + Q_{g,t} S_{g,t}(j))\right] = 0$$

Optimality conditions (30) and (31) imply $\chi_t^b = \chi_t^g$, so that we can write equation (28) as:

$$(Q_{b,t}S_{b,t}(j) + Q_{g,t}S_{g,t}(j)) = \frac{\nu_t}{\kappa - \chi_t^b} NW_t(j).$$

It is now straightforward to show that the franchise value is linear in the banker's net worth:

$$V_t(j) = \phi_t NW_t(j) = \frac{\nu_t}{\kappa - \chi_t^b} NW_t(j) + \nu_t NW_t(j) = \frac{\kappa\nu_t}{\kappa - \chi_t^b} NW_t(j),$$

with $\phi_t = \frac{\kappa\nu_t}{\kappa - \chi_t^b}$.

ϕ_t does not depend on firm-specific factors, so that we can aggregate over all the banks (via incentive compatibility constraint) to obtain:

$$\sum_{i=\{b,g\}} S_{i,t}Q_{i,t} = \phi_t NW_t, \tag{34}$$

where $\sum_j S_{i,t}(j) = S_{i,t}$ and $NW_t = \sum_j N_t(j)$. Moreover, on the aggregate level, (for arbitrage forces) I obtain:

$$Q_{i,t}S_{i,t} = Q_{i,t}K_{i,t} \text{ for i} = \{g, b\}. \tag{35}$$

2.7 Government

The government transfers net revenues from the carbon tax to households uniformly or not uniformly, $T_{it} = \tau_{e,t}E_{b,t}, \forall i$, or the brown capital loan rate tax, $T_{it} = \tau_{K_{b,t}}K_{b,t}, \forall i$. The two different taxation schemes are studied one at a time, to focus on their different allocation' impacts in terms of welfare.

2.8 Competitive Equilibrium

Definition 3. *Given initial conditions for sector-specific capital goods* $\{K_{b,0}, K_{g,0}\}$, *initial banking sector net worth and pre-industrial cumulative stock of emissions* $\{NW_0, Z_0\}$, *given exogenous TFP sequences* $\{A_t, A_{b,t}, A_{g,t}\}_{t=0}^{\infty}$, *and a sequence of emission taxes* $\{\tau_{e,t}\}_{t=0}^{\infty}$ *or brown capital loan rate taxes* $\{\tau_{t_{K_b}}\}_{t=0}^{\infty}$, *a competitive equilibrium is a sequence of allocations* $\{C_{i,t}, D_{g,t}, E_{i,t}, K_{i,t+1}, S_{i,t},$
$N_i, NW_t, D_t, Z_t, I_{i,t}\}_{t=0}^{\infty}$, *of prices* $\{P_{i,t}, W_{i,t}, Q_{i,t}, R_t, R_{i,t}\}_{t=0}^{\infty}$ *and transfers* $\{T_t = \tau_{e,t}E_{b,t}\}_{t=0}^{\infty}$ *(or* $\{T_t = \tau_{t,K_b}K_{b,t-1}\}_{t=0}^{\infty}$ *such that:*

(i) ***Households:***

- *Brown households consume their entire labor income and transfers.*
- *Green households solve:*

$$\max_{\{C_{g,t},D_{g,t}\}} \sum_{t=0}^{\infty} \beta^t \frac{C_{g,t}^{1-\sigma} - 1}{1-\sigma}$$

subject to their intertemporal budget constraint (2), a transversality condition and given initial capital holdings.

(ii) ***Firms:*** *Intermediate energy producers maximize profits under two tax regimes:*

(a) Under Emissions Tax $\tau_{e,t}$:

$$\max_{\{N_b, K_{b,t-1}\}} \quad (P_{b,t} - \tau_{e,t}) \, E_{b,t} - W_{b,t} N_b - R_{b,t} Q_{b,t-1} K_{b,t-1} + (1-\delta_b) Q_{b,t} K_{b,t-1}$$
$$\text{s.t.} \quad E_{b,t} = A_{b,t} K_{b,t-1}^{\alpha} N_b^{1-\alpha}$$

(b) Under Brown Capital Loan Rate Tax τ_{t,K_b}:

$$\max_{\{N_b, K_{b,t-1}\}} \quad P_{b,t} E_{b,t} - W_{b,t} N_b - (R_{b,t} + \tau_{t,K_b}) \, Q_{b,t-1} K_{b,t-1} + (1-\delta_b) Q_{b,t} K_{b,t-1}$$
$$\text{s.t.} \quad E_{b,t} = A_{b,t} K_{b,t-1}^{\alpha} N_b^{1-\alpha}.$$

(c) Green Sector Energy Producer:

$$\max_{\{N_g, K_{g,t-1}\}} \quad P_{g,t} E_{g,t} - W_{g,t} N_g - R_{g,t} Q_{g,t-1} K_{g,t-1} + (1-\delta_g) Q_{g,t} K_{g,t-1}$$
$$\text{s.t.} \quad E_{g,t} = A_{g,t} K_{g,t-1}^{\alpha} N_g^{1-\alpha}$$

(iii) ***Banks:*** *Banks maximize their expected discounted franchise value:*

$$V_t(j) = \mathbb{E}_t \sum_{s=1}^{\infty} (1-\gamma) \gamma^{s-1} \Lambda_{t,t+s} NW_{t+s}(j)$$

subject to:

$$Q_{b,t} S_{b,t}(j) + Q_{g,t} S_{g,t}(j) = NW_t(j) + D_t(j)$$
$$NW_{t+1}(j) = \sum_i (R_{i,t+1} - R_t) Q_{i,t} S_{i,t}(j) + R_t NW_t(j)$$
$$V_t(j) \geq \kappa \left(Q_{b,t} S_{b,t}(j) + Q_{g,t} S_{g,t}(j) \right)$$

(iv) ***Government:*** *The government sets either $\tau_{e,t}$ or $\tau_{K_b,t}$ and collects:*

$$T_t = \begin{cases} \tau_{e,t} E_{b,t}, & \textit{under emission tax regime} \\ \tau_{t,K_b} K_{b,t}, & \textit{under capital rental tax regime} \end{cases}$$

and redistributes uniform lump-sum transfers T_t to households.

(v) ***Market Clearing:***

$$\text{Final good: } C_{b,t} + C_{g,t} + \sum_i Q_{i,t} I_{i,t} = Y_t,$$

$$\text{Capital: } \sum_j S_{i,t}(j) = S_{i,t} = K_{i,t},$$

$$\text{Labour: } N_i = \bar{N}_i$$

$$\text{Deposits: } D_t = D_{g,t}$$

(vi) ***Environmental Constraint:***

$$Z_{t+1} = Z_t + E_{b,t}$$

2.9 Business-As-Usual Equilibrium

Within this class of dynamic environmental general equilibrium models, it is standard to study the Business-As-Usual allocation (BAU henceforth), where the Planner does not internalize the externality, setting taxes equal to 0 ($\{\tau_{e,t} = 0\}_{t=0}^{\infty}$ or $\{\tau_{t_{K_b}=0}\}_{t=0}^{\infty}$). In this equilibrium, the polluting industry is not affected by taxation (FOCs relative to wages and the sector-specific capital good, as shown in equations (14a), (14b), (15a), and (15b)); the different taxation schemes do not impact brown wages, $W_{b,t}$, or the sector-specific capital good rental rates; they are free to adjust in response to market forces.

Both intermediate energy firms must obtain funds through financial intermediaries to purchase sector-specific capital goods from capital-producing firms. At the end of any generic time t, the capital producing firms buy undepreciated capital and invest in final good to create period $K_{i,t}$ capital goods, to be used in $t+1$ [2]. Green household deposits resources in the banking sector, which are lent to intermediate energy good firms to buy capital goods used in period $t+1$ production, at price $Q_{i,t}$ (with $i = b, g$). When lending resources to intermediate energy good firms in exchange for loan rates, bankers must satisfy the incentive compatibility constraint, as outlined in equation (23), leading to an under-provision of funds for the brown and green industries. This results in a spread between deposit rates and loan rates, driving the evolution of net worth, capital accumulation and lending dynamics.

When taxes are null, the equalization of excess returns of the brown and green assets (equation (25), (26)) must be satisfied, and results in a constant wage ratio ($\frac{W_{g,t}}{W_{b,t}}$). The first-best allocation, however, implies a different mechanism for the brown industry, generating a cascading effect on the entire economy. In the next section, the constrained Ramsey problem is fully characterised.

[2]Timing is crucial; technological constraints for the intermediate good producers are $E_{i,t+1} = A_{i,t+1} N_i^{1-\alpha_i} K_{i,t}^{\alpha_i}$, so that $K_{i,t}$ is utilized at time $t+1$

3 Optimal Ramsey Problem

In this section, I solve for the optimal (constrained) Ramsey problem utilising the primal approach. The objective function of the Planner is a standard utilitarian welfare function, where θ and $(1-\theta)$ are the welfare weights attached to, respectively, the brown representative household and the green representative household. This function has to be optimised under technological, feasibility, and financial constraints. As I mentioned before, the Planner does not have at its disposal lump-sum transfers, aside from the two instruments (emission tax $\tau_{e,t}$, or a brown capital loan rate tax τ_{t,K_b}). The only way the Planner can redistribute is through their revenues. This constrains the Planner on the social weights it can attach to the two representative households; θ and $1-\theta$ must be consistent with their market wealth, since for Negishi's theorem (Negishi, 1960) the decentralized allocation is equivalent to the Planner's allocation where the social weights it link to the agents are function of their market's wealth (wages and capital income for instance).

The Planner internalises the externality stemming from the $E_{b,t}$ production when choosing optimal quantities, and the following parts of the paper show how the Planner picks up the optimal allocation in terms of $E_{b,t}$ production, and, moreover, how to decentralize this optimal allocation through the two different taxation schemes.

3.1 Planner's Problem

The Ramsey problem consists of finding the allocation path that maximizes the representative household's intertemporal utility, subject to the full set of equilibrium constraints of the decentralized economy. The planner takes as given the initial conditions and the structural constraints of production, emissions, and financial markets, including the presence of frictions. Formally, the problem is characterized as a dynamic constrained optimization where the policy-maker internalizes all externalities (aside from the financial friction), and chooses allocations directly rather than policy instruments.

In this environment, households are heterogeneous in sectoral attachment. The planner assigns distributional weights to each group through the parameter θ. The presence of emissions generated by brown production introduces an intertemporal environmental externality, as these accumulate into a pollution stock that affects economic outcomes via damages to aggregate productivity. The planner internalizes the effect of current emissions on future damages through forward-looking constraints.

In addition to environmental concerns, the economy features financial market imperfections: capital allocation across sectors is subject to collateral constraints, and banks face incentive compatibility conditions that limit leverage. These financial frictions introduce wedges between the social and private returns to capital, which the planner must respect. Consequently, the Ramsey problem becomes constrained not only by technological feasibility and resource availability, but also by market-based constraints reflecting decentralized incentives.

Importantly, the planner does not choose taxes or prices directly, but instead selects allocations—consumption, investment, capital stocks, emissions, and financial positions—that are consistent with a competitive equilibrium under optimal policy. In this sense, the problem is "Ramsey" in nature: it seeks a second-best allocation that decentralizes to a competitive outcome through a set of optimal policy instruments, such as emission taxes and financial regulations.

I now proceed to formalize the problem. The planner maximizes the representative households' lifetime utility, taking into account the full intertemporal structure of constraints described above, ie

$$\max_{\{C_{b,t},C_{g,t},I_{i,t},Z_t,E_{i,t},NW_t,D_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[\frac{\theta C_{b,t}^{1-\sigma}}{1-\sigma} + \frac{(1-\theta)C_{g,t}^{1-\sigma}}{1-\sigma} \right], \tag{36}$$

subject to:

$$
\begin{aligned}
Y_t(=f(E_{i,t})) &= C_{b,t} + C_{g,t} + \sum_{i\in\{b,g\}} Q_{i,t}I_{i,t}, \qquad (\lambda_t) \;\; \text{(Aggregate Resource Constranit)} \\
K_{i,t} &= (1-\delta_i)K_{i,t-1} + I_{i,t}, \qquad (\bar{L}_{i,t}) \;\; \text{(Capital Goods Accumulations)} \\
E_{b,t} &= A_{b,t}K_{b,t-1}^{\alpha}N_b^{1-\alpha} \quad (\xi_{i,t}) \; \text{(Tecnhological Constraint On Energy Productions)} \\
Z_{t+1} &= Z_t + E_{b,t} \qquad (\zeta_t) \;\; \text{(Cumulative Emissions LoM)} \\
D_t(j) &= Q_{b,t}S_{b,t}(j) + Q_{g,t}S_{g,t}(j) - NW_t(j) \qquad (\Delta_t) \qquad \text{(Balance Sheet identity)} \\
NW_{t+1} &= \sum_i (R_{i,t+1} - R_t)Q_{i,t}K_{i,t} + R_t NW_t \quad (\eta_t) \; \text{(Net Worth Dynamic)} \\
\phi_t NW_t &\geq \sum_i \kappa Q_{i,t}K_{i,t} \quad (\upsilon_t) \; \text{(Incentive Compatibility Constraint)} \\
N_b &= \bar{N}_b, \quad N_g = \bar{N}_g. \quad (\omega_{i,t}) \;\; \text{(Labour Feasibility)}
\end{aligned}
$$

Let the Lagrange multipliers associated with the planner's optimization problem be defined as follows. The multiplier λ_t corresponds to the resource constraint, capturing the marginal utility of relaxing the economy's aggregate feasibility condition. The sector-specific capital accumulation has its own lagrange multiplier $\bar{L}_{i,t}$ ($i \in \{b, g\}$), and reflects its shadow value. The term $\xi_{i,t}$, i $\in \{b, g\}$ is linked to the brown energy production technology and represents the shadow value of producing an additional unit of dirty energy. The sequence $\{\zeta_t\}$ refers to the multipliers on the emissions accumulation equation, measuring the discounted marginal disutility of cumulative emissions over time.

In the financial sector, Δ_t is the multiplier attached to the bank's balance sheet identity. η_t denotes the multiplier associated with the law of motion of net worth, reflecting the intertemporal impact of financial returns on household wealth. The multiplier υ_t enforces the incentive compatibility constraint, which ensures that lending remains consistent with collateral requirements and market frictions. Finally, $\omega_{b,t}$ and $\omega_{g,t}$ are the Lagrange multipliers attached to labor feasibility conditions in the brown and green sectors, respectively, representing the marginal utility of relaxing sectoral labor constraints.

The Planner's first-order-condition with respect to $E_{b,t}$:

$$\begin{aligned} \underbrace{\lambda_t \frac{\partial Y_t}{\partial E_{b,t}}}_{\text{marginal benefits}} = \underbrace{\xi_{b,t}}_{\text{technological constraint}} - \underbrace{\sum_{s=0}^{\infty} \zeta_{t+s}}_{\text{externality damages}} \\ - \underbrace{\left(\sum_{s=0}^{\infty} (\eta_{t+s} + \upsilon_{t+s}\,\phi_{t+s}) \prod_{j=0}^{s-1} R_{t+j} + \Delta_t \right) \frac{\partial NW_t}{\partial E_{b,t}}}_{\text{net worth effects: dynamic, IC, and balance sheet}} . \end{aligned} \tag{37}$$

where I define the cumulative effects of NW_t on NW_{t+j} is given by :

$$\frac{\partial NW_{t+s}}{\partial NW_t} = \prod_{j=0}^{s-1} R_{t+j},$$

Equation (37) represents the planner's first-order condition with respect to the polluting intermediate input $E_{b,t}$. On the left-hand side, $\lambda_t \cdot \frac{\partial Y_t}{\partial E_{b,t}}$ captures the marginal utility-weighted benefit of increasing dirty energy input, reflecting how additional emissions raise final output and, in turn, household utility. This term internalizes the production benefits from using $E_{b,t}$ in the current period.

On the right-hand side, three distinct cost components arise. The first, $\xi_{b,t}$, is the Lagrange multiplier associated with the brown energy production technology constraint. It represents the technological or resource cost of increasing $E_{b,t}$, capturing the implicit price of expanding dirty energy within the production process, given the underlying capital and labour constraints.

The second component, $\sum_{s=0}^{\infty} \zeta_{t+s}$, aggregates the present value of marginal external damages caused by an additional unit of emissions today. Since emissions accumulate over time, this term accounts for the entire future stream of environmental degradation induced by marginally increasing $E_{b,t}$ in period t. Each ζ_{t+s} reflects how today's emissions raise the future pollution stock Z_{t+s}, which in turn lowers output through the climate damage function. As such, the planner fully internalizes the dynamic intertemporal externality associated with pollution, weighing it against the short-run production gains.

The third component captures the marginal effect of $E_{b,t}$ on financial conditions through its impact on the net worth of financial intermediaries. Because dirty energy output directly affects the return on brown capital, it alters the level of internal funds in the banking sector. A marginal increase in $E_{b,t}$ raises NW_t, which yields utility benefits through three channels. First, it improves the intertemporal allocation of resources via the net worth accumulation constraint, with marginal utility weight η_{t+s} in each future period. Second, it relaxes the banking sector's incentive compatibility constraint, valued at $\upsilon_{t+s}\phi_{t+s}$, by increasing the financial intermediary's capacity to intermediate. Third, it influences the static bank balance sheet identity at time t, with utility weight Δ_t, by reducing reliance on external deposits.

Although each of these effects provides utility benefits to the planner, they enter the FOC as subtracted terms because they represent the opportunity cost of using marginal dirty energy output to generate financial slack, rather than immediate consumption. These dynamic and static effects are aggregated in the following term:

$$\left(\sum_{s=0}^{\infty}\left(\eta_{t+s}+\upsilon_{t+s}\phi_{t+s}\right)\Theta_{t,s}+\Delta_t\right)\cdot\frac{\partial NW_t}{\partial E_{b,t}},$$

where $\Theta_{t,s} = \prod_{j=0}^{s-1} R_{t+j}$ denotes the marginal propagation of today's net worth into period $t+s$. This term captures how a marginal increase in NW_t, induced by expanding dirty energy, propagates across time, relaxing financial constraints and improving future allocations, but at the cost of reducing immediate productive efficiency.

The externality term can be expressed as:

$$\sum_{s=0}^{\infty}\zeta_{t+s}=\sum_{s=0}^{\infty}\beta^s\lambda_{t+s}\frac{\partial D(Z_{t+s})}{\partial Z_{t+s}}\frac{\partial Z_{t+s}}{\partial E_{b,t}}Y_{t+s}=\sum_{s=0}^{\infty}\beta^s\lambda_{t+s}\left(-\frac{\partial(1-D(Z_{t+s}))}{\partial Z_{t+s}}\right)\frac{\partial Z_{t+s}}{\partial E_{b,t}}Y_{t+s}$$

Since $Z_{t+s} = Z_t + \sum_{u=0}^{s-1} E_{b,t+u}$, we have:

$$\frac{\partial Z_{t+s}}{\partial E_{b,t}}=1,\quad \forall s\geq 0$$

Hence, the full stream of marginal damages accounts for all future effects of a marginal increase in $E_{b,t}$. That is, equation (37) becomes:

$$\begin{aligned}\lambda_t\frac{\partial Y_t}{\partial E_{b,t}}=\xi_{b,t}&+\sum_{s=0}^{\infty}\beta^s\lambda_{t+s}\frac{\partial D(Z_{t+s})}{\partial Z_{t+s}}\frac{\partial Z_{t+s}}{\partial E_{b,t}}Y_{t+s}\\&-\left(\sum_{s=0}^{\infty}\left(\eta_{t+s}+\upsilon_{t+s}\,\phi_{t+s}\right)\Theta_{t,s}+\Delta_t\right)\frac{\partial NW_t}{\partial E_{b,t}}\,.\end{aligned}\tag{38}$$

3.2 Decentralization And Optimal Emission Tax

We now turn to the decentralization of the planner's allocation through an appropriate tax scheme. In the decentralized economy, competitive firms take prices and policy instruments as given and choose input allocations to maximize profits. Our objective is to demonstrate how the planner's first-order condition with respect to polluting energy inputs can be supported as a decentralized equilibrium under an appropriately chosen emission tax $\tau_{e,t}$. This equivalence establishes the environmental tax as a Pigouvian instrument that aligns private incentives with the socially optimal allocation.

The brown energy firm solves a two-step problem:

Given a required energy output $E_{b,t}$, the firm chooses capital and labor inputs $\{K_{b,t-1}, N_b\}$ to minimize total costs:

$$\min_{K_{b,t-1},N_b} \quad W_{b,t}N_b + (R_{b,t}Q_{b,t-1} - (1-\delta_b)Q_{b,t})\, K_{b,t-1} \tag{39}$$

subject to the brown energy production function:

$$E_{b,t} = A_{b,t}K^{\alpha}_{b,t-1}N_b^{1-\alpha} \tag{40}$$

Let $\xi^{\text{firm}}_{b,t}$ be the Lagrange multiplier on this constraint. Then the FOCs of the cost minimization are:

$$W_{b,t} = \xi^{\text{firm}}_{b,t} \cdot \frac{\partial E_{b,t}}{\partial N_b} = \xi^{\text{firm}}_{b,t}(1-\alpha)\frac{E_{b,t}}{N_b} \tag{41}$$

$$(R_{b,t}Q_{b,t-1} - (1-\delta_b)Q_{b,t-1}) = \xi^{\text{firm}}_{b,t} \cdot \frac{\partial E_{b,t}}{\partial K_{b,t-1}} = \xi^{\text{firm}}_{b,t}\alpha\frac{E_{b,t}}{K_{b,t-1}} \tag{42}$$

Solving this yields the minimised cost function, as a function of $E_{b,t}$ and pins down $\xi^{\text{firm}}_{b,t}$ as the marginal cost of producing energy, i.e., the private marginal cost. Substituting in the objective function the optimised inputs' prices, we obtain the cost function to utilise in the profits maximisation problem:

$$C(E_{b,t}) = \xi^{\text{firm}}_{b,t}(1-\alpha)\frac{E_{b,t}}{N_b}N_b + \xi^{\text{firm}}_{b,t}\alpha\frac{E_{b,t}}{K_{b,t-1}}K_{b,t-1} = \xi^{firm}_{b,t}E_{b,t}$$

Taking prices and tax as given, the firm chooses $E_{b,t}$ to maximize profits:

$$\max_{E_{b,t}} \quad (P_{b,t} - \tau_{e,t})E_{b,t} - C(E_{b,t})(= \xi^{firm}_{b,t}E_{b,t}) \tag{43}$$

The FOC is:

$$P_{b,t} - \tau_{e,t} = \xi^{\text{firm}}_{b,t} \tag{44}$$

In the planner's problem, $\xi^{\text{planner}}_{b,t}$ represents the marginal utility value of dirty energy, being the Lagrange multiplier on the production constraint of the brown energy firm. In the decentralized setup, $\xi^{\text{firm}}_{b,t}$ represents the marginal cost of producing one more unit of energy, in monetary terms. Since utility and prices live in different spaces (utils vs. monetary units), the two are related through the marginal utility of consumption λ_t.

Specifically, we have:

$$\xi^{\text{firm}}_{b,t} = \frac{\xi^{\text{planner}}_{b,t}}{\lambda_t} \tag{45}$$

This allows us to express the planner's condition in terms that are directly comparable to the decentralized firm's FOC, and ultimately solve for the tax level that closes the gap between private and social valuation.

Hence, the planner FOC:

$$\begin{aligned} \frac{\partial Y_t}{\partial E_{b,t}} &= \frac{\xi^{\text{planner}}_{b,t}}{\lambda_t} + \sum_{s=0}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t} \frac{\partial D(Z_{t+s})}{\partial Z_{t+s}} Y_{t+s} \\ &\quad - \frac{1}{\lambda_t} \left(\sum_{s=0}^{\infty} \big(\eta_{t+s} + \upsilon_{t+s}\, \phi_{t+s}\big)\, \Theta_{t,s} + \Delta_t \right) \frac{\partial NW_t}{\partial E_{b,t}} . \end{aligned} \tag{46}$$

can be rewritten using the decentralized identity:

$$\begin{aligned} \frac{\partial Y_t}{\partial E_{b,t}} &= \xi^{\text{firm}}_{b,t} + \sum_{s=0}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t} \frac{\partial D(Z_{t+s})}{\partial Z_{t+s}} Y_{t+s} \\ &\quad - \frac{1}{\lambda_t} \left(\sum_{s=0}^{\infty} \big(\eta_{t+s} + \upsilon_{t+s}\, \phi_{t+s}\big)\, \Theta_{t,s} + \Delta_t \right) \frac{\partial NW_t}{\partial E_{b,t}} . \end{aligned} \tag{47}$$

From cost minimization we have:

$$\begin{aligned} \frac{\partial Y_t}{\partial E_{b,t}} &= P_{b,t} - \tau_{e,t} + \sum_{s=0}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t} \frac{\partial D(Z_{t+s})}{\partial Z_{t+s}} Y_{t+s} \\ &\quad - \frac{1}{\lambda_t} \left(\sum_{s=0}^{\infty} \big(\eta_{t+s} + \upsilon_{t+s}\, \phi_{t+s}\big)\, \Theta_{t,s} + \Delta_t \right) \frac{\partial NW_t}{\partial E_{b,t}} . \end{aligned} \tag{48}$$

From which we solve for the optimal emission tax, since in competitive markets the final good producer sets $\frac{\partial Y_t}{\partial E_{b,t}} = P_{b,t}$:

$$\tau^*_{e,t} = \sum_{s=0}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t} \frac{\partial D(Z_{t+s})}{\partial Z_{t+s}} Y_{t+s} - \frac{1}{\lambda_t} \left(\sum_{s=0}^{\infty} \left(\eta_{t+s} + \upsilon_{t+s}\phi_{t+s}\right) \Theta_{t,s} + \Delta_t \right) \frac{\partial NW_t}{\partial E_{b,t}} \tag{49}$$

The optimal emission tax deviates from the textbook Pigouvian rule due to the presence of financial frictions. The first term in the expression, $\sum_s \beta^s \frac{\lambda_{t+s}}{\lambda_t} \frac{\partial D(Z_{t+s})}{\partial Z_{t+s}} Y_{t+s}$, captures the conventional dynamic externality associated the discounted stream of marginal damages generated by additional emissions.

The second term reflects the planner's internalization of how changes in dirty energy production affect the financial system. An increase in $E_{b,t}$ raises the profitability of brown-sector firms, which in turn increases the net worth NW_t of financial intermediaries. Higher net worth brings utility gains to the planner, as it improves the efficiency of capital allocation through three key channels: it relaxes the net worth accumulation constraint (via η_{t+s}), it weakens the tightness of the incentive compatibility constraint (via $\upsilon_{t+s}\phi_{t+s}$), and it reduces reliance on household deposits through the static balance sheet identity (via Δ_t).

These effects propagate recursively across time with weights $\Theta_{t,s} \equiv \prod_{j=0}^{s-1} R_{t+j}$, which represent the marginal impact of today's net worth on future net worth. As a result, increasing $E_{b,t}$ yields indirect financial benefits that partially offset the environmental externality.

Thus, the optimal emission tax is not only a function of climate damages but also of financial frictions. In this setting, restoring the social optimum requires under-taxing relative to the Pigouvian benchmark, unless financial constraints are irrelevant. The emission tax reflects a trade-off: internalizing the climate externality while preserving or enhancing financial intermediation capacity.

3.3 Decentralization and Optimal Loan Rate Tax

In this section, we consider an alternative policy instrument: a loan rate tax on brown capital, τ_{t,K_b}. The rationale for this instrument lies in the presence of financial frictions, which may distort capital allocation by limiting access to external financing or creating incentives for overinvestment in polluting technologies due to underpriced credit risk. A tax on the loan rate charged to brown firms effectively raises their cost of capital and allows the planner to correct both the environmental externality and the financial inefficiencies, thus implementing the optimal allocation in a decentralized setting.

Given a required energy output $E_{b,t}$, the firm chooses capital and labor inputs $\{K_{b,t-1}, N_b\}$ to minimize total costs:

$$\min_{K_{b,t-1},N_b} \quad W_{b,t}N_b + \left((R_{b,t} + \tau_{t,K_b})Q_{b,t-1} - (1-\delta_b)Q_{b,t}\right)K_{b,t-1} \tag{50}$$

subject to the brown energy production function:

$$E_{b,t} = A_{b,t}K_{b,t-1}^{\alpha}N_b^{1-\alpha}, \quad \text{with multiplier } \xi_{b,t}^{\text{firm}}. \tag{51}$$

The first-order condition with respect to capital is:

$$\xi_{b,t}^{\text{firm}} = \frac{(R_{b,t} + \tau_{t,K_b})Q_{b,t-1} - (1-\delta_b)Q_{b,t}}{\frac{\partial E_{b,t}}{\partial K_{b,t-1}}} \tag{52}$$

Since the final good producer is competitive, it sets the marginal value of brown energy equal to its price: $P_{b,t} = \frac{\partial Y_t}{\partial E_{b,t}}$.

Optimal Loan Rate Tax

As in the emissions tax case, we aim to match the planner's FOC with that of the decentralized firm's problem. However, here the distortion is introduced not through the price of the polluting energy directly, but via the cost of capital used to produce it. This means the planner's condition must account for how changes in brown capital use affect both environmental externalities and financial constraints through their impact on net worth.

The planner's FOC with respect to brown capital, after substituting the chain rule via $E_{b,t} = A_{b,t} K_{b,t-1}^{\alpha} N_b^{1-\alpha}$, is:

$$\begin{aligned}\lambda_t \frac{\partial Y_t}{\partial E_{b,t}} \frac{\partial E_{b,t}}{\partial K_{b,t-1}} &= \xi_{b,t} \frac{\partial E_{b,t}}{\partial K_{b,t-1}} + \sum_{s=0}^{\infty} \zeta_{t+s} \frac{\partial E_{b,t}}{\partial K_{b,t-1}} \\ &\quad - \left(\sum_{s=0}^{\infty} \left(\eta_{t+s} + \upsilon_{t+s} \phi_{t+s} \right) \Theta_{t,s} + \Delta_t \right) \frac{\partial NW_t}{\partial K_{b,t-1}} .\end{aligned} \tag{53}$$

Using the relationship $\xi_{b,t}^{\text{firm}} = \frac{\xi_{b,t}}{\lambda_t}$, we divide both sides by λ_t to express in decentralized terms:

$$\begin{aligned}\frac{\partial Y_t}{\partial E_{b,t}} \frac{\partial E_{b,t}}{\partial K_{b,t-1}} &= \xi_{b,t}^{\text{firm}} \frac{\partial E_{b,t}}{\partial K_{b,t-1}} + \sum_{s=0}^{\infty} \frac{\zeta_{t+s}}{\lambda_t} \frac{\partial E_{b,t}}{\partial K_{b,t-1}} \\ &\quad - \frac{1}{\lambda_t} \left(\sum_{s=0}^{\infty} \left(\eta_{t+s} + \upsilon_{t+s} \phi_{t+s} \right) \Theta_{t,s} + \Delta_t \right) \frac{\partial NW_t}{\partial K_{b,t-1}} .\end{aligned} \tag{54}$$

Using the firm's cost minimization result:

$$\xi_{b,t}^{\text{firm}} \cdot \frac{\partial E_{b,t}}{\partial K_{b,t-1}} = (R_{b,t} + \tau_{t,K_b}) Q_{b,t-1} - (1 - \delta_b) Q_{b,t} \tag{55}$$

we substitute back and solve for the optimal tax:

$$\begin{aligned}\tau_{t,K_b}^{*} = \frac{1}{Q_{b,t-1}} &\left[\sum_{s=0}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t} \frac{\partial D(Z_{t+s})}{\partial Z_{t+s}} Y_{t+s} \frac{\partial E_{b,t}}{\partial K_{b,t-1}} \right. \\ &\quad \left. - \left(\sum_{s=0}^{\infty} \left(\eta_{t+s} + \upsilon_{t+s} \phi_{t+s} \right) \Theta_{t,s} + \Delta_t \right) \frac{\partial NW_t}{\partial K_{b,t-1}} \right] .\end{aligned} \tag{56}$$

The optimal loan rate tax must correct both the environmental externality and the intertemporal financial distortions created by brown capital use. The first term in the expression reflects the marginal damages from increased emissions, passed through the dependence of emissions on capital. The second term captures the marginal utility benefit of the additional net worth generated by allocating capital to the brown sector. Higher net worth improves financial intermediation through increased lending capacity, relaxed incentive compatibility constraints, and reduced reliance on household deposits.

Hence, the planner internalizes this benefit by reducing the optimal tax below the purely Pigouvian level. In doing so, the planner balances environmental goals with the necessity of maintaining financial sector function. As a result, the optimal loan rate tax ensures that decentralized firms face a capital cost that aligns their private decisions with the social optimum, incorporating both environmental and financial frictions.

In the next section quantitative experiments are carried out and results carefully discussed.

4 Quantitative Illustrations

This section presents the quantitative results of the model, focusing on the transition dynamics under optimal climate policy. The economy is simulated over a finite horizon of 25 periods (each period corresponds to 10 years), with special attention given to how financial frictions affect the intertemporal path of policy instruments, production allocations, and distributional outcomes.

All simulations converge toward a 10-period Balanced Growth Path (BGP) in which the externality is no longer active, and allocations stabilize. The BGP serves as the long-run anchor of the economy, ensuring consistency between transitional dynamics and steady-state behavior. A key assumption in the numerical setup is the introduction of an exogenous *backstop technology* that becomes available in period 25. This technology absorbs the polluting energy input entirely, fully eliminating emissions and rendering the externality inactive from that point onward. The inclusion of such a backstop is common in the climate-economy literature (e.g., Golosov et al., 2014, Barrage, 2020, Douenne, Hummel, and Pedroni, 2022) and provides a natural closure mechanism to the decarbonization transition.

A central focus of the analysis is the comparison between two institutional settings: one with financial frictions (the baseline model), and one in which the financial constraint is removed. I compute the optimal emission tax schedule $\{\tau_{e,t}\}_{t=0}^{T}$ and the optimal loan rate tax $\{\tau_{t,K_b}\}_{t=0}^{T}$ in both environments. The results show that the presence of financial frictions lowers the optimal tax level throughout the transition. This is because dirty-sector capital contributes positively to bank net worth and thereby supports future financial intermediation. The planner internalizes this benefit and reduces the tax pressure on the polluting sector to avoid an excessive contraction in credit supply. As a consequence, the tax schedule in the frictionless model is uniformly higher, but also induces sharper sectoral reallocation and stronger wage divergence.

In addition to comparing the frictional and frictionless economies, I contrast the optimal transition path with the Business-As-Usual (BAU) scenario, where all taxes are set to zero: $\tau_{e,t} = 0$ and $\tau_{t,K_b} = 0$ for all t. This benchmark allows me to isolate the total effect of optimal taxation on economic allocations, emissions, and welfare. The BAU scenario is characterized by persistent reliance on brown energy, rising cumulative emissions, and an inefficient allocation of resources due to the uninternalized externality. By comparing outcomes under optimal taxation with the BAU baseline, I quantify the environmental and distributional improvements brought about by policy intervention. In particular, I highlight how the welfare of households, especially those tied to the polluting sector, evolves under each scenario. Additionally, a comparison between a uniform and a non uniform tax rebates is implemented, to show that redistribution schemes of the carbon dividend are crucial to ensure welfare gains for both representatitive households.

All quantitative results rely on a calibration of structural parameters guided by the literature. Preference and production parameters are chosen to match empirical estimates of substitution elasticities, capital shares, depreciation rates, and damage intensities. The elasticity between green and brown energy inputs, in particular, plays a key role in shaping the marginal cost of abatement. The calibration also determines the baseline productivity gap between sectors and the scale of climate damages at benchmark temperature levels. Parameter values are provided in Table 1.

To assess the robustness of our findings, we perform a sensitivity analysis on key parameters: the elasticity of substitution ϵ, the damage function curvature, and the strength of financial frictions (captured by the IC parameter κ). Results show that higher substitutability leads to faster decarbonization and lower transitional costs, while tighter financial constraints amplify the welfare gap between frictional and frictionless scenarios. These findings emphasize the importance of incorporating realistic financial structures in the design of climate policy.

4.1 Calibration

Table 1: Parameter Values

Description	Parameter	Value	Source
Discount factor (annual)	β	0.985	W. D. Nordhaus (2017)
Relative risk aversion	σ_c	1.5	Golosov et al. (2014)
Elasticity (brown-green)	ϵ	3	Acemoglu, Aghion, et al. (2012b)
Capital shares	α	0.33	Acemoglu, Robinson, and Verdier (2017)
Capital depreciation rates (decadal)	δ_i	0.65	Golosov et al. (2014)
Damage parameter	ρ	5% TFP loss at $3°C$	Peter H. Howard and Sterner (2017b)
Initial dirty capital	$K_{d,0}$	0.5	Normalization
Initial clean capital	$K_{c,0}$	0.5	Normalization
Low-skill labor share	N_{LS}	0.5	*World energy employment* (2022)
High-skill labor share	N_{HS}	0.5	*World energy employment* (2022)
Energy sector TFP growth rate	$gTFP_e$	0.02	Golosov et al. (2014)
Final good TFP growth rate	$gTFP_A$	0.0	Golosov et al. (2014)
Percentage of divertible funds	κ	0.2	Own Calibration
Survivor rate	γ	0.8	Own Calibration

Calibrated parameters are in Table 1. Each period in the model represents 10 years. First, we calibrate the preferences parameters. In particular, we set the annual discount factor to $\beta = 0.985$ W. D. Nordhaus (2017) and the constant relative risk aversion parameter to $\sigma = 1.5$ Golosov et al. (2014).

Second, we consider the production side of the economy. I set the capital depreciation rates to 5% per year for the energy sectors. The capital shares are set to $\alpha = 0.33$, as in Acemoglu, Aghion, et al. (2012a). We assume that clean and dirty inputs are imperfect substitute, and I set the elasticity of substitution between them to $\epsilon = 3$.[3] Third, I calibrate the climate change module. The damage parameter ρ is set to lead to damages approximately equal to 4.58% at an increase in global average temperature of 2 degree celsius relative to the pre-industrial period Peter H. Howard and Sterner (2017b).[4]

Fourth, I calibrate the parameters governing the banking and financial intermediation sector. Rather than following Gertler and Karadi (2011), I set the fraction of funds that bankers can divert to personal use to $\kappa = 0.2$ and the survival rate of bankers to $\gamma = 0.8$, ensuring persistence in net worth accumulation and numerical stability. This can

[3]Elasticities used in integrated assessment and macroeconomic models have ranged between 1 and 10. For example, Acemoglu, Aghion, et al. (2012a) provide simulations for elasticities equal to 3 and 10, Golosov et al. (2014) set it to approximately 1, Hart (2019) to 4, Carattini, Heutel, and Melkadze (2023) set it to 2, Lemoine (2024) uses 1.8, and Spiganti, Campiglio, Wiskich, et al. (2024) use 3. Most empirical estimates range between 0.5 and 3 D. I. Stern (2012)

[4]Note that this is consistent with Kalkuhl and Wenz (2020), where damages are approximately 7.5% at 3-degree. We assume that the 2-degree increase in temperature corresponds to cumulative emissions from the start of the simulation of 1350GtCO_2, which is the estimated remaining carbon budget calculated from the beginning of 2020 to achieve a warming of 2°C with a 50% probability (IPCC, 2021, Table 5.8).

be seen as a benchmark scenario, since these parameters jointly determine the tightness of financial constraints, the spread between deposit and lending rates, and the evolution of net worth in the intermediary sector. I preferred to have my own calibration and play then with the financial parameter values in the appendix. In this model, each time period corresponds to ten years, and tuning the parameters to the values chosen in Gertler and Karadi (2011), where time periods correspond to quarters is not a winning strategy. The calibration targets a realistic size of the financial wedge given my calibrated model, and ensures that the incentive constraint is binding during the transition path and in the long-run equilibrium. This setup is critical to examine how financial frictions interact with climate policy and to quantify their influence on the optimal tax schedule. Numerical stability also played a crucial role in determining the choice of the parameters' values.

4.2 Optimal Taxes

In this section, I compare the optimal tax paths under two regimes: one in which financial frictions are present (the baseline case), and one in which they are absent (the frictionless benchmark). Two types of climate policy instruments are considered: the per-unit emission tax $\tau_{e,t}$ and the loan rate tax on brown capital τ_{t,K_b}. The analysis focuses on the dynamic path of each instrument, the interaction with financial constraints, and the resulting implications for transition policy.

Figure 2 displays the trajectory of the optimal emission tax $\tau_{e,t}$ in both regimes. In the absence of financial frictions, the planner sets a high emission tax from the beginning, in line with standard Pigouvian logic. This immediate pricing of the externality triggers a swift decline in emissions and accelerates the transition to a cleaner energy mix. However, in the presence of financial frictions, the optimal emission tax is set to zero in the initial periods. This is because capital invested in the dirty sector contributes to bank net worth, which in turn relaxes the incentive compatibility constraint and improves the efficiency of financial intermediation. The planner values this benefit and postpones environmental taxation in order to allow the financial sector to rebuild balance sheets. As net worth accumulates and the financial friction weakens, the tax is gradually increased but it does not converge to its Pigouvian counterpart, always being lower than its first best level.

A similar pattern emerges for the loan rate tax τ_{t,K_b}, shown in Figure 3. In the frictionless setting, the tax is front-loaded to raise the cost of polluting capital and discourage investment in the brown sector. Without financial frictions, the planner faces no downside apart for lower production from taxing dirty capital early. In contrast, when financial frictions are active, the optimal loan tax is initially set to zero. By keeping the cost of capital low in the dirty sector, the planner supports investment and bank net worth. As in the emission tax case, the loan rate tax is introduced only gradually, once the banking sector has accumulated sufficient internal funds to support lending without violating the

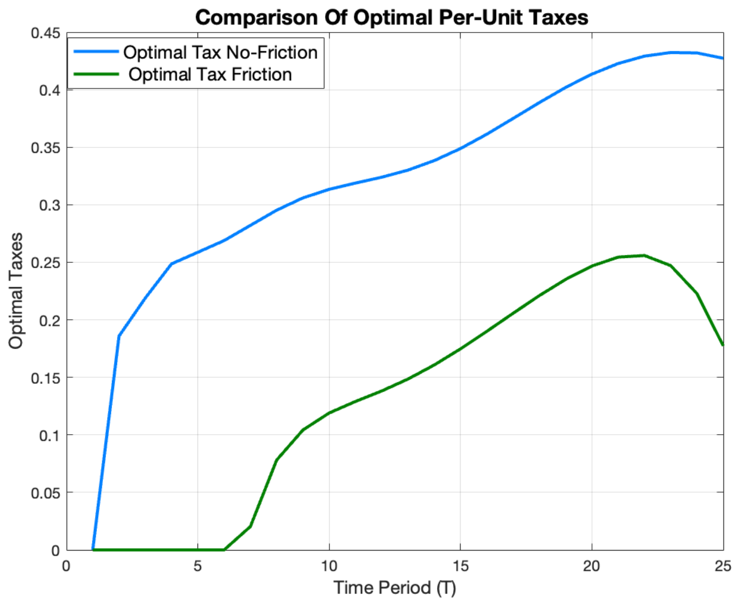

Figure 2: Optimal Emission Taxes

incentive compatibility constraint. The tax increases over time. Again, it remains below its Pigouvian value for all the transition periods.

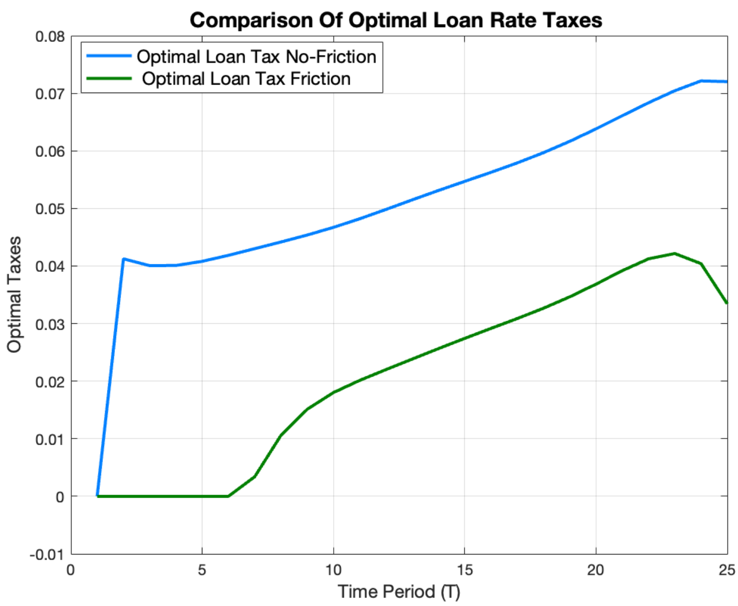

Figure 3: Optimal Loan Taxes

The comparison between the two policy instruments highlights a central mechanism of the model: in the presence of financial frictions, the planner uses the dirty sector not only as a source of energy, but also as a temporary tool to relax financial constraints and smooth the transition. Both taxes are strategically delayed in the early periods to facilitate net worth accumulation. As the economy evolves and the financial constraint becomes less binding, the planner begins to deploy the tax instruments more actively to address the environmental externality. In the frictionless case, by contrast, taxes are deployed immediately and at higher levels, since there are no intertemporal trade-offs related to banking dynamics.

Finally, the results illustrate that optimal tax design in climate policy cannot be separated from macro-financial considerations. When financial frictions are present, the timing and intensity of environmental regulation must take into account the role of capital allocation in supporting future intermediation. Optimal policy therefore balances two goals: reducing emissions and sustaining financial stability during the transition.

4.3 Inequality

In this section, I study the distributional consequences of optimal climate policy by comparing the evolution of inequality under different tax instruments. Specifically, I analyze how inequality changes under the optimal emission tax and the optimal loan rate tax, both implemented in economies with financial frictions, relative to the Business-As-Usual (BAU) scenario where no taxes are applied.

I analyze its dynamics using the Gini index, defined as:

$$\text{Gini}_t = \frac{(W_{g,t}N_g)}{W_{g,t}N_g + W_{b,t}N_b} - \frac{N_g}{N_g + N_b}. \tag{57}$$

The results show that inequality increases in both policy cases compared to the BAU allocation. This occurs because, in the presence of financial frictions, workers in the polluting sector—who cannot transition to the clean sector—suffer from lower relative wages as the transition unfolds. These wage effects are amplified by sector-specific capital and labour immobility, as well as the absence of lump-sum transfers in the planner's instruments.

Interestingly, the first periods of the simulations show that, when the optimal taxes are 0, inequality is at the Business-As-Usual level, since wages are not affected by taxes.

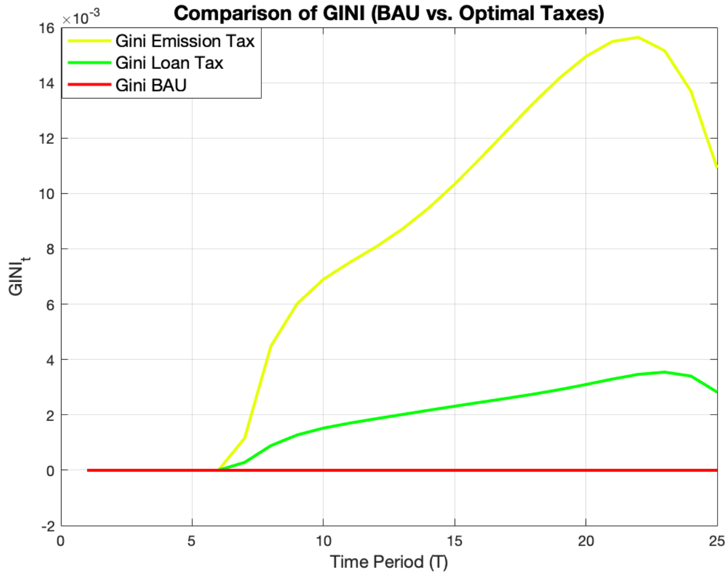

Figure 4: Gini Indeces

However, the increase in inequality is not symmetric across instruments. In the case of the optimal loan rate tax, the planner initially suppresses taxation to avoid undermining financial intermediation, and as a result, the wage structure in the early periods is less distorted. The slower reallocation of capital and labour from the brown to the green sector implies a more gradual shift in wage differentials. In contrast, under the optimal emission tax, the planner targets the externality directly, and the resulting contraction in the brown sector causes a more pronounced decline in wages for workers tied to polluting industries.

I quantify these distributional effects using the Gini index, which is computed for each simulated economy along the transition path. The Gini index captures inequality in household consumption and indirectly reflects the heterogeneous exposure of workers to both taxation and sectoral dynamics. The comparison reveals that while both policy regimes lead to an increase in inequality relative to the BAU case, the rise in the Gini index is systematically lower under the loan tax scenario. This highlights a central result of the model: when redistribution is constrained, the choice of climate policy instrument has important implications for inequality. Instruments that operate through capital channels (like the loan rate tax) can mitigate the wage compression experienced by vulnerable households, even when achieving similar aggregate environmental targets.

4.4 Welfare

In this section, I assess the welfare implications of the optimal climate policies and conclude the paper by highlighting the trade-offs between environmental effectiveness and distributional outcomes. I consider both aggregate and disaggregated welfare changes relative to the Business-As-Usual (BAU) scenario, where no taxes are applied and the externality remains unpriced.

Both the optimal emission tax and the optimal loan rate tax improve aggregate welfare relative to the BAU benchmark. This result holds despite the presence of financial frictions and constrained redistribution. The welfare gain stems from the internalization of the climate externality and the efficient reallocation of capital and labour toward the cleaner sector. However, the disaggregated welfare consequences vary substantially depending on the instrument used.

In the case of the emission tax, the brown sector household suffers a substantial welfare loss. The tax directly reduces the demand for polluting energy, leading to a contraction in the brown sector. Because labour is immobile and workers cannot switch sectors, the wage paid to low-skill households falls sharply. Although aggregate output improves and damages are mitigated, the resulting decline in labour income more than offsets the gains for this group, leaving them worse off than under the BAU scenario. The green sector household, by contrast, benefits from the expansion of the clean sector and from the general equilibrium gains associated with reduced emissions. Their wages rise, and they enjoy higher consumption throughout the transition.

In contrast, the loan rate tax delivers a more balanced welfare outcome. Since the planner initially delays taxation to support net worth accumulation and minimizes financial distortions, the brown sector is not immediately compressed. As a result, the wage in the polluting sector remains relatively stable in the early transition years. This cushioning effect prevents the sharp welfare loss for the brown household seen under the emission tax. Over time, however, the tax increases, and only the green household group experiences consumption gains, due to expansion of the clean sector and lower future damages. Importantly, the brown household is worse off than under BAU, but not as in the per-unit tax scenario, where the welfare loss is greater.

To summarise these results, Table 2 reports the direction of welfare changes across household types and policy regimes.

Table 2: Welfare Effects Relative to Business-As-Usual

Household Type	**Emission Tax**	**Loan Rate Tax**
High-skill (Green sector)	Welfare gain	Welfare gain
Low-skill (Brown sector)	Welfare loss	Welfare loss

This comparison highlights a central message of the paper: while both climate policies succeed in improving overall welfare, their distributional consequences may differ. The emission tax, though effective in addressing the environmental externality, imposes substantial losses on vulnerable groups in the presence of frictions and rigidities. In contrast, the loan rate tax allows the planner to align climate goals with intertemporal financial stabilization, delivering a smoother adjustment path and a less inequal welfare outcome.

These findings underscore the importance of considering both macro-financial conditions and heterogeneous agent impacts when designing climate policy. In second-best environments without lump-sum transfers, the choice of policy instrument matters not only for efficiency but also for fairness.

5 Conclusion

This paper studies optimal climate policy in a multisector economy with a climate externality, heterogeneous households, sector-specific inputs, and financial frictions. Three core results emerge from the analysis of inequality, welfare, and optimal tax paths along the transition.

In second-best environments with constrained redistribution and limited factor mobility, both an optimal emission tax and an optimal loan-rate tax raise inequality relative to Business-As-Usual (BAU). The mechanism runs through wages in the brown sector: when labor and capital are sector-specific, workers trapped in polluting industries face comparatively lower wages as the transition unfolds. However, the increase in inequality is systematically smaller under the loan-rate tax. By initially suppressing taxation to preserve financial intermediation and allow net-worth accumulation, the planner induces a slower reallocation of factors, which tempers early wage compression in the brown sector. Consistently, the Gini index rises under both regimes but less so with the loan-rate instrument.

Both instruments improve aggregate welfare relative to BAU by internalizing the externality and reallocating resources toward cleaner production. Distributionally, the two policies differ starkly. With a *uniform* rebate, the per-unit emission tax lowers brown-sector wages enough to leave low-skill households worse off than under BAU, while high-skill households gain; the loan-rate tax delivers a more balanced outcome: delaying the tax at the start cushions brown wages and avoids large early losses; as the tax is phased in however, only the green household experiences welfare gains. Hence, under a uniform rebate, no lump-sum transfers, and financial frictions, I obtain the same welfare effects as in the previous chapters.

Financial frictions fundamentally reshape optimal timing. In both cases, the planner initially sets taxes to zero to rebuild bank net worth, relax incentive-compatibility constraints, and stabilize intermediation. Taxes are then ramped up as the constraint weakens. Relative to the frictionless benchmark—where taxes are front-loaded—the frictional planner back-loads intervention. For the emission tax, this can even lead to higher levels later in the transition as the planner catches up on environmental pricing once financial stability is secured. The loan-rate tax displays an analogous pattern: zero early on to support dirty-sector balance sheets, then gradually increasing toward levels consistent with marginal climate damages. The results speak to the design of politically feasible climate policy under financial constraints:

Instrument choice matters for equity. When redistribution is constrained and rebates must be uniform, a loan-rate tax dominates a per-unit emission tax on distributional grounds while still achieving environmental goals. Timing is a policy lever. With binding financial frictions, optimal policy is deliberately gradual: early forbearance facilitates net-worth accumulation and smooths the transition. If non-uniform recycling is available, directing (most of) carbon dividends toward vulnerable brown-household types can neutralize inequality without sacrificing efficiency. Relaxing sectoral immobility (training, re-skilling, and easing capital reallocation) directly attenuates the wage channel that drives inequality.

The analysis abstracts from international spillovers, endogenous innovation, and richer labour-market frictions (e.g., search and matching). Extending the framework to allow for partial mobility, sector-specific learning-by-doing, and alternative financial imperfections would further test the robustness of the instrument ranking and the optimal timing results. Embedding political-economy constraints could also help quantify how distribution-aware design—such as loan-rate taxes with uniform rebates—improves policy acceptability.

Appendix C. Sensitivity Analysis Third Chapter

In this technical appendix, I present some sensitivity analysis regarding the financial parameters, since modifying parameters related to the real block of the economy would lead to the same robustness as the previous chapter. In the end, I present the computational algorithm used to solve the model.

C1 Financial Parameters Values

Since I decided to set κ equal to .2 in the benchmark calibration, it is reasonable to understand what happens to the optimal instruments when I lower it to .1 (ceteris paribus for $\gamma = .8$), and to compare these results to a non-friction environment. Higher values for κ instead would result in very low values for the optimal climate instruments, since the financial component of equation (38) would prevail.

Welfare effects are robust to twists in financial parameters, but the closer we are to the non-friction scenario (i.e., $\kappa = 0$ and $\gamma = 1$), the more we go back to the second chapter quantitative results.

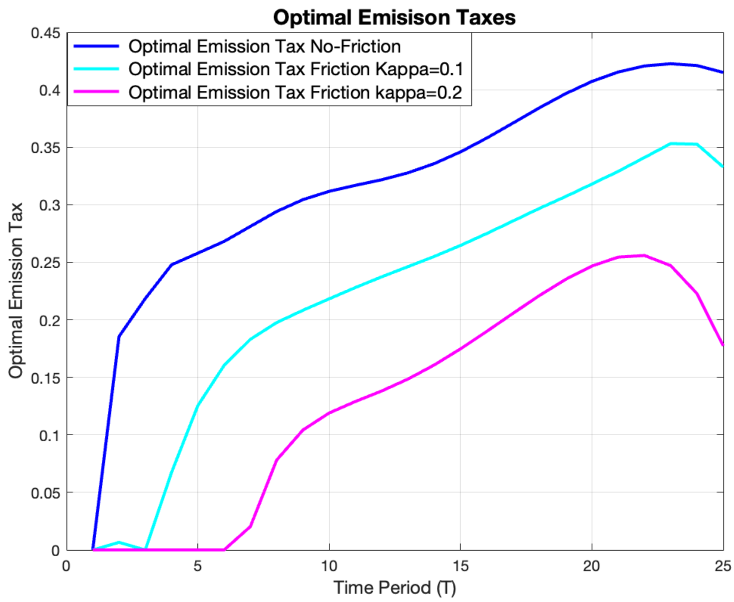

Figure 5: Optimal Emission Taxes Changing κ

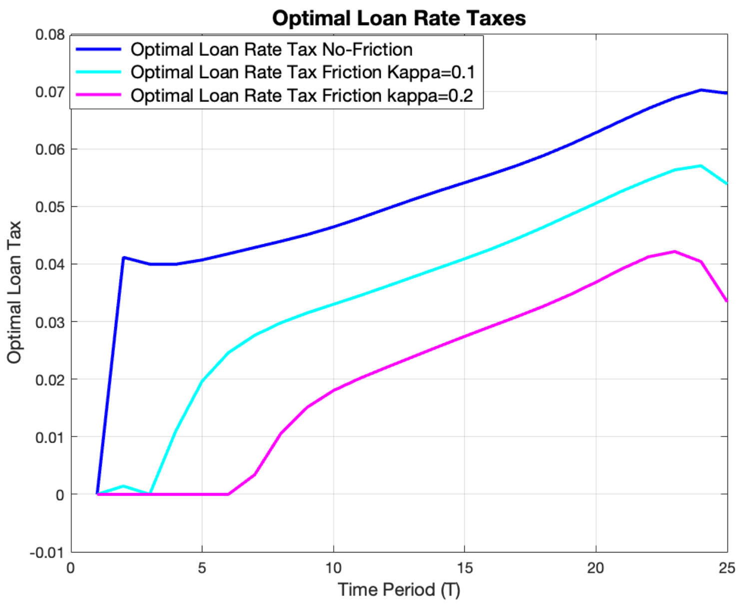

Figure 6: Optimal Loan Rate Taxes Changing κ)

I obtain more interesting results when I change γ for a given value of κ ($= .2$, as in the standard calibration). Raising γ from .8 to .9 generates non-linear results for the optimal climate instruments; in particular, the optimal instruments with higher γ are below the ones with no frictions at the beginning, but, in the final periods, they become higher. This is due to the initial 0 taxes periods, when the financial friction term in equation (38) prevails. Later on, due to this delay in the environmental taxation, the cumulative stocks of climate emissions in the final part of the simulations are so higher that the Social Planner has to set higher taxes to internalize the climate externality optimally.

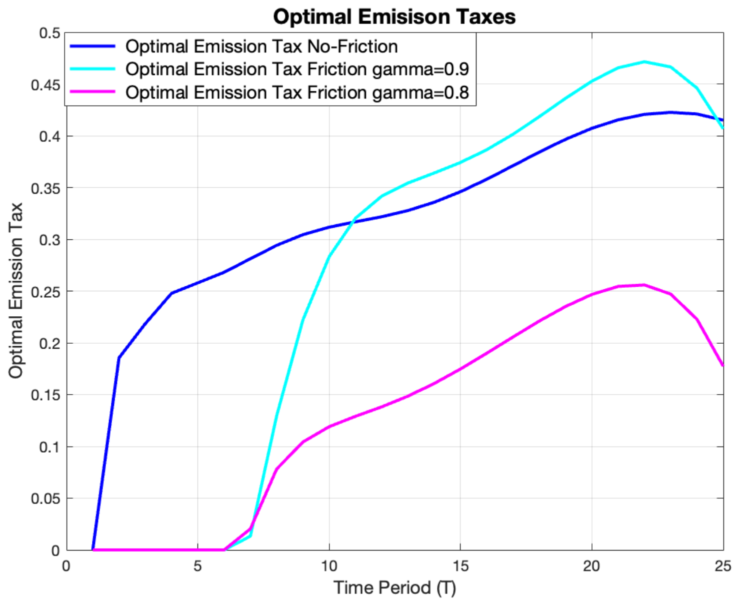

Figure 7: Optimal Emission Taxes Changing γ

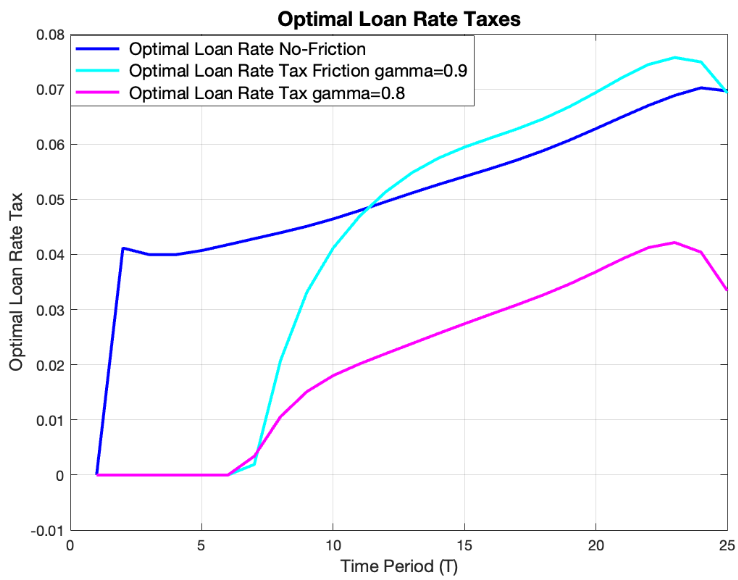

Figure 8: Optimal Loan Rate Taxes Changing γ)

C2 Computational algorithm for Chapter 3

The numerical solution in Chapter 3 follows the same structure as in the previous chapters: a MATLAB *main file* builds the vector of choice variables, calls an objective function and a constraint file, and solves the social planner's problem using `fmincon` with the SQP algorithm over a finite horizon of $T = 25$ transition periods and $P = 10$ continuation periods.

As in Chapter 2, the planner chooses aggregate saving and its allocation between dirty and clean capital. In particular, the control vector includes:

- the aggregate saving rate $\{S_t\}_{t=1}^{T+P}$;
- the share of investment allocated to dirty (brown) capital, $\{\text{brownshare}_t\}_{t=1}^{T+P}$, which determines the split between I_t^b and I_t^g;
- the per–unit tax on brown energy $\{\tau_t\}_{t=1}^{T}$ or the loan rate tax on brown capital;
- (in the objective) the consumption paths of heterogeneous agents, as in the previous chapters.

Given these controls, the main and objective files reconstruct the quantities, prices and welfare in the real block exactly as before, with sector-specific capital stocks $K_{b,t}$ and $K_{g,t}$, dirty and clean energy $E_{b,t}$ and $E_{g,t}$, emissions Z_t, climate damages, and final output Y_t.

The key difference in Chapter 3 lies in the constraint file `constraints`, which now embeds both the real and the financial blocks and enforces their joint consistency in a second–best environment with financial frictions.

Real block

For any candidate vector x, the constraint file:

- computes dirty and clean energy, $E_{b,t}$ and $E_{g,t}$, from the two capital stocks and sectoral productivities;
- updates emissions Z_t and output Y_t using the damage function, as in previous chapters;
- defines aggregate investment $I_t = S_t Y_t$ and splits it into dirty and clean components via the portfolio share:

$$I_t^b = \text{brownshare}_t I_t, \qquad I_t^g = (1 - \text{brownshare}_t) I_t;$$

- updates sector-specific capital:

$$K_{b,t+1} = I_t^b + (1 - \delta_b) K_{b,t}, \qquad K_{g,t+1} = I_t^g + (1 - \delta_g) K_{g,t};$$

- computes energy prices $P_{b,t}, P_{g,t}$ and the corresponding gross returns on dirty and clean capital, $r_{b,t}$ and $r_{g,t}$, taking into account the tax on dirty energy.

As before, the goods market clearing condition implies $C_t = Y_t - I_t$, and non–negativity of consumption is imposed as an inequality constraint.

Financial block and second–best constraint

In addition, the `constraints file` introduces an explicit banking sector that intermediates between deposits and sectoral capital. Banks hold the two capital stocks as assets and issue one–period deposits as liabilities:

- aggregate bank assets are given by $A_t = K_{b,t+1} + K_{g,t+1}$;
- net worth NW_t evolves according to a retained–earnings rule:

$$NW_t = \gamma\big[r_{b,t}K_{b,t} + r_{g,t}K_{g,t} - R_t^D L_{t-1}\big],$$

  where R_t^D is the gross return on deposits and L_t is the stock of deposits (liabilities);
- liabilities satisfy $L_t = A_t - NW_t$.

The return on deposits R_t^D is pinned down by the consumption-based stochastic discount factor,

$$R_t^D = \frac{1}{\Lambda_{t,t+1}}, \qquad \Lambda_{t,t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma},$$

so that the financial block is fully induced by the real allocation.

A backward recursion is then used to compute a leverage multiplier $\{\phi_t\}$, starting from a terminal condition and using the parameters (γ, κ) and the spread between capital returns and the deposit rate (equations (25), (26), (27)). The resulting implementability condition

$$A_t = \frac{\phi_t}{\kappa}NW_t$$

is imposed as an additional set of equality constraints, ensuring that the bank balance sheet and the leverage constraint are jointly satisfied in every period. This captures the second–best nature of the problem: the planner cannot implement the first–best allocation because financial frictions restrict the admissible paths of capital accumulation and portfolio choice.

Alongside the financial constraint, the file also enforces:

- a no–arbitrage condition $r_{b,t} = r_{g,t}$ for $t \leq T + P$;

Overall, the main and objective files in Chapter 3 follow the same logic as in the previous chapters, but the constraint file now integrates the real and financial blocks so that any candidate allocation must be both goods-market feasible and compatible with the banking sector's balance sheet and leverage constraint in this second–best environment. Since I simplified the household's structure, I do not need an outer loop to find the market thetas since the brown househeold consumes what it earns in the labour market plus the tax(es) rebate(s).

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