WHAT DOES CHATGPT HAVE TO SAY ABOUT TOPOLOGICAL STRUCTURE: THE ANYON HYPOTHESIS

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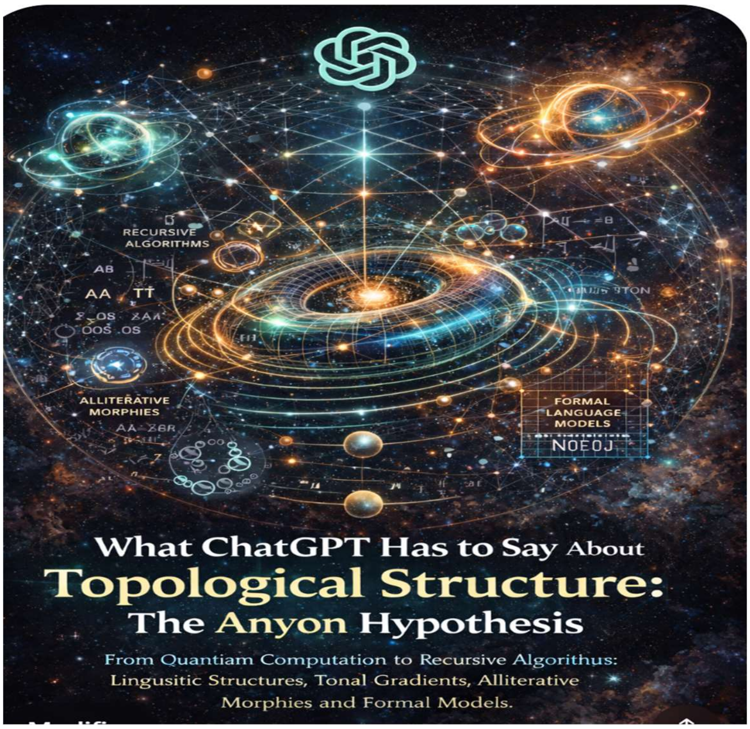

Figure 1. From quantum Computation to Recursive Algorithms.

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Description

The figure represents the transition from linguistic structure to topological and quantum formalization. The spiral-text architecture symbolizes the recursive organization of language, in which textual units return, expand, and fold into structured patterns. The branching form expresses the development of rhetorical morphisms and recurrent syntactic configurations. The connecting vectors identify the main linguistic operators of the model—locative deixis, rhetorical morphemes, syntactic substitutions, and phono-tonal gradation—showing their convergence toward a formal geometric space. The sphere on the right represents the topological space of linguistic states, interpretable within an SU (2)-based framework, where each linguistic segment may be treated as a formal state and each gradational variation as a phase transformation. The measuring device symbolizes the transition from qualitative linguistic observation to quantitative and computable analysis. Overall, the figure synthesizes the Bucciarelli–Silvestri method as a recursive linguistic model oriented toward topological representation and computational formalization.

Figure 2. Bucciarelli–Silvestri method as a recursive linguistic model-topological-oriented representation and computational formalization.

Section 1 — Model Architecture

Introductory Note: The following architecture formalizes the transition from textual aesthetics to mathematical topology. By mapping the "Strange Loops" of language through a recursive filter, the model establishes a deterministic interface between human poetic intuition and quantum computational logic.

1. Object and Methodology

The present module aims to extract from the text those linguistic configurations that exhibit a recursive nature. The methodology is based on the detection of rhetorical segments (**morphies**) and their quantitative measurement, establishing a bridge between philology and information theory.

2. Identification of Recursive Segments (Silvestri's Analysis)

Following the linguistic analyses of **Donato Silvestri**, the text is decomposed into observable units identified through a detection technique based on linguistic mechanisms. These units, defined as **Recursive Segments**, represent structural configurations of language that repeat and organize themselves according to recognizable patterns.

- **Deixis and Locative Structures:** Elements such as *here, there, this, that*, and specifically deictic forms (e.g., **"In te"**), are treated as **linguistic operators** that activate spatial and semantic relations. They function as **pointers**, constructing a topological network of internal references within the text.
- **Morphies and Parallel Structures:** Repeated syntactic structures (**morphies**) constitute recognizable recursive sequences. These configurations act as structuring elements of the text, analogously to the **theme in a musical fugue** (Ref. Hofstadter), generating variations over a stable scheme.
- **Experimental Observation (The Dante Case):** In the *Divine Comedy*, the repetition of pronominal and syntactic structures is non-random, exhibiting a significant **Recursive Density**. This allows:
  1. The reader to orient themselves within the deep structure of the text.
  2. The **Automaton** to recognize formalizable linguistic configurations for quantum processing.

3. Measurement of Phono-Tonal Gradation

The transition from text to data occurs through the "weighting" of phonemes:

- **Tonal Frequency:** Each linguistic unit produces a sound vibration (gradation) that can be measured.
- **Empirical Evidence:** The analysis of Dante's cantos shows that the distribution of vowels and consonants in rhyme structures is **non-linear**, following an **oscillatory pattern** that prepares the ground for rotational algebra (Quantum Phase).

4. Representation through Graph Coverings (Planat Model)

The result of the linguistic analysis is visualized as a **Graph Covering**:

- **Nodes:** The identified linguistic segments.
- **Edges:** Recursive dependency relations (e.g., Rhyme A recalling a subsequent Rhyme A).
- **Secondary Structure:** As in the protein structures studied by **Michel Planat**, the text "folds" onto itself through non-local links, generating a synthetic and fixed topological form.

5. Output for Recursive Algebra

The module concludes with the generation of a **Recursive Density Table**, containing:

1. **Sequence of Segments:** The ordered list of morphological units.

2. **Recursion Index:** The number of times a structure repeats and interlocks (**Hofstadter's G-level**).
3. **Fibonacci Coefficient:** The verification of the expansion ratio (Golden Ratio) between textual parts.

Abstract

This research is situated within the field of linguistic modeling of recursive structures and arises from the convergence of three fundamental theoretical frameworks: the musicological model of tonal recursion, the logical-mathematical model of recursive language structures, and the topological model of formal structures.

The first theoretical reference is the musicological analysis of recursion in the compositions of Johann Sebastian Bach. In Bach's fugues, thematic repetition generates a dynamic cyclical structure in which the musical motif returns multiple times with progressive variations. This model of musical recursion provides an interpretative paradigm for understanding analogous recursive configurations in literary language.

The epistemological value of this perspective clearly emerges in the work of Douglas R. Hofstadter, *Gödel, Escher, Bach: An Eternal Golden Braid* (1999). Chapter V, dedicated to recursion in language, demonstrates how linguistic structures can be described through recursive processes analogous to those found in logical and musical systems. Hofstadter introduces the so-called *G-Diagram*, an infinite recursive structure in which node expansion generates a geometric tree whose mathematical properties are connected to the Fibonacci sequence.

This recursive model constitutes the starting point for the formalization of linguistic structures observed in literary texts. In this perspective, the Fibonacci sequence is not interpreted as mere numerical symbolism, but as an indicator of structural expansion and recursive gradation.

The second theoretical framework is represented by the mathematical work of Michel Planat, who employs the concept of graph covering to analyze complex structures across different domains, including proteins, music, and poetic texts. Graph covering consists in mapping one graph structure onto another while preserving internal relations between nodes, allowing complex configurations to be described through topological and combinatorial models.

In recent research, Planat has extended this approach to large language models, proposing that linguistic structures generated by artificial intelligence systems can be interpreted through topological models derived from quantum physics, particularly through the theory of anyons and modular tensor categories.

Based on these theoretical foundations, this study proposes a linguistic model for detecting recursive structures in poetic texts and, more generally, in all forms of communication. The methodology is structured in three steps:

1. identification of recursive linguistic configurations within the corpus;
2. quantitative measurement of recursive density and phono-tonal gradation;
3. representation of linguistic structures through graphs and formal models.

This approach establishes a connection between linguistics, mathematics, and topology, demonstrating how recursive structures in language can be described through formal models linking graph theory, the Fibonacci sequence, and computational modeling.

The goal of the research is to construct a **topology of the word**, in which linguistic structures can be analyzed as recursively organized configurations that are mathematically formalizable and computationally translatable.

Introduction

This research originates from the intuition of transferring into the field of linguistics an analytical model derived from mathematical structures and from recursive patterns observed in other symbolic systems. Specifically, the study proposes the application of a detection method to natural language based on principles of structural recursion and formal analytical models derived from mathematics and structural theory.

The starting point of the research lies in the observation that linguistic systems, similarly to musical and logical systems, exhibit recursive configurations that can be described through formal models. This perspective finds a major theoretical reference in the work of Douglas R. Hofstadter, who in *Gödel, Escher, Bach: An Eternal Golden Braid* identifies recursion as a common principle underlying language, music, and mathematical logic. In his analysis of linguistic recursion, Hofstadter describes expandable structures that can be represented through recursive diagrams and mathematical sequences, including the Fibonacci sequence.

At the same time, the research is grounded in the observation that recursion is also a fundamental principle in musical structure. The compositions of Johann Sebastian Bach demonstrate how thematic repetition and progressive variation generate recursive configurations capable of producing perceptible tonal gradations. This musicological model constitutes the first methodological reference of the present study.

From this interdisciplinary perspective, the research proposes a methodological transfer into the field of linguistics. The central hypothesis is that specific linguistic configurations—particularly deictic, locative, and pronominal structures—can be interpreted as recursive segments of language that can be observed, classified, and measured.

Within this framework, particular relevance is attributed to the linguistic model developed by Domenico Silvestri. His studies on linguistic mechanisms and morphic structures make it possible to identify recurrent rhetorical and syntactic units within texts. These units, including morphic patterns, pronominal deixis, and parallel structures, constitute observable linguistic segments that can be analyzed as recursive configurations.

The guiding intuition of this work is therefore to transfer into the linguistic domain an analytical model inspired by quantum computation, applying it to the detection of recursive structures within rhetorical segments of language. In this perspective, the linguistic units identified through Silvestri's model are treated as structural configurations that can be measured through indices of recursive density and phono-tonal gradation.

Further theoretical reference is provided by the mathematical work of Michel Planat, who has demonstrated that complex structures across different domains − such as music, proteins, and poetic texts − can be represented through topological models based on graph coverings. This approach suggests that linguistic structures can be interpreted as networks of relations between nodes, opening the possibility for a mathematical formalization of recursive configurations.

Based on these theoretical premises, the present research proposes a linguistic analysis model that integrates:

- the detection of recursive linguistic mechanisms;
- analysis of rhetorical segments of the text (morphies, deixis, parallelism);
- the measurement of phono-tonal gradation;
- the representation of linguistic structures through formal and graphical models.

The model is initially formulated within the general framework of Natural Language Generation (NLG) and subsequently applied to a literary corpus. As a case study, Divine *Comedy* is used, as it represents a textual system particularly rich in recursive configurations. However, the objective of the research is not limited to Dante's poetry: the proposed method is conceived as an analytical tool applicable to different types of textualities.

Within this perspective, the research is positioned at the intersection of linguistics, mathematics, and symbolic systems theory, with the aim of outlining a possible **topology of the word**, in which linguistic structures can be analyzed as formally definable recursive configurations.

Core Theoretical Axes of the Research

The present research is structured around two fundamental focal points, which define both its theoretical foundation and its methodological development.

1. Linguistic recursion and quantum-inspired detection

The first axis is grounded in the work of Douglas R. Hofstadter, *Gödel, Escher, Bach: An Eternal Golden Braid*, and in **Chapter V — "Recursive Structures and Processes"**.

In this chapter, recursion is described as a structural and generative principle present in language, music, and formal systems. This research extends this perspective by introducing a **linguistic detection model**, in which recursive mechanisms—such as deixis, morphies, and syntactic parallelism—are treated as observable data.

These structures are not only described qualitatively but are measured through a **quantum-inspired detection process**, in which recursive repetition produces measurable gradations. In this framework, the Fibonacci sequence is interpreted as a structural indicator of recursive expansion and phono-tonal intensity.

2. Recursion and Artificial Intelligence

The second axis is also derived from Hofstadter's work, specifically from **Part II — Chapters XVIII and XIX**, dedicated to Artificial Intelligence.

In these sections, recursion is no longer considered only as a structural property, but as a principle underlying computational systems and artificial intelligence. The representation of knowledge through recursive and layered structures provides the theoretical basis for modeling language within formal and computational frameworks.

In the present research, this perspective is developed through the transition from **linguistic detection to formal modeling**, leading toward recursive algebra and automata-based systems.

3. Integration with the Bucciarelli Model (NLP Framework)

The research is further supported by the work:

Model Processes Methods Technologies NLP: I.R.I.S (ISBN 978-88-99640-36-1), by Ritamaria Bucciarelli et al.

This framework establishes the methodological basis for transforming linguistic data into structured and computable representations, integrating detection techniques, linguistic modeling, and computational processing.

Within this integrated perspective, the present model connects:

- linguistic data detection
- Phono-tonal analysis
- recursive structures
- topological representation

defining a unified architecture in which language is treated as a **recursive, measurable, and computationally translatable system**.

1.0 Object and Methodology

The present module aims to extract from the text those linguistic configurations that exhibit a recursive nature. The method is based on the detection of rhetorical segments (*morphies*) and on their quantitative measurement.

The purpose of this first stage is to transform qualitative textual observation into structured data, making it possible to identify recurrent linguistic patterns and to prepare them for formal analysis.

SECTION 1 — Sampling, Recursion and Rotational Algebra in NLG Languages

The sample model represents the mathematical formalization of the recursive configuration observed in the corpus.

This section adopts as its theoretical framework three preliminary results:

1. Recursion is a formal principle common to music, language, and symbolic systems;
2. musical recursion produces measurable tonal gradations;
3. linguistic structures can be reduced to categories, relations, and topological nodes.

This framework emerges from the research materials already collected, including references to Gödel, Hofstadter, Bach, Fibonacci, Silvestri, and Planat. In particular:

- Chapter V of *Gödel, Escher, Bach* is taken as a bridge between logical recursion and processual recursion;
- Bach's music is considered as a model of tonal recursion;
- the Fibonacci sequence is treated as a structural indicator of gradation;
- Graph Coverings represent the transition from structural detection to topological representation.

1.1 Sample Model

Let a sample corpus be defined as:

$$C = \{T_1, T_2, \dots, T_N\}$$

where each text is segmented into observable units:T_i

$$T_i = \{u_{i1}, u_{i2}, \dots, u_{im}\}$$

and each unit may correspond to a sentence, a tercet, a syntagm, or a phono-rhythmic sequence.u_{ij}

The starting point is not the psychological meaning of emotion, but the **detection of structural mechanisms**.

This is consistent with the idea that, in this model, **sentiment emerges as an effect of form** and not as a purely semantic category.

Mathematical Representation of Linguistic Units

For each unit, we define a feature vector:u

$$x(u) = \begin{bmatrix} r(u) \\ d(u) \\ p(u) \\ c(u) \\ q(u) \end{bmatrix}$$

where:

- $r(u)$= recursive density
- $d(u)$= deictic-locative intensity
- $p(u)$= phono-tonal weight
- $c(u)$= contextual coefficient
- $q(u)$= closure / formal cohesion coefficient

1.2 Recursive Density

Recursive density measures how many times a formal structure recurs within a given textual window.

If is a linguistic schema, for example:σ

$$\sigma = Loc + N$$

or:

the local recursive density can be defined as:

$$r(u) = \frac{\#(\sigma \text{ in } u)}{|\ u\ |}$$

where is the number of minimal units considered in the segment.$|\ u\ |$

Example (Dante)

For a Dantean tercet of the type:

mercy in you, pity in you, magnificence in you

the structure:

$$\sigma = \text{in te} + X$$

appears three times, therefore:

$$r(u) = \frac{3}{3} = 1$$

1.3 Deictic – Locative Index

To detect an **emotional deixis linked to the locative**, we define an indicator function:

$$\delta_{\text{loc}}(u_k) = \begin{cases} 1 & \text{if } u_k \text{ contains a locative deictic} \\ 0 & \text{otherwise} \end{cases}$$

For example, in the sequence *"in te", "in te", "in te", "in te"*, the function takes value 1 for each occurrence.

Deictic – Locative Index

The deictic–locative index of the segment is defined as:u

$$D_{\text{loc}}(u) = \sum_{k=1}^{n} \delta_{\text{loc}}(u_k)$$

Normalized Version

$$\widetilde{D}_{\text{loc}}(u) = \frac{1}{n} \sum_{k=1}^{n} \delta_{\text{loc}}(u_k)$$

Example (Dante)

In the case of the tercet:

$$\widetilde{D}_{\text{loc}}(u) = 1$$

Weighted Version

If a positional weight is assigned to deixis (for example, greater weight at the beginning of the verse), we introduce a coefficient:w_k

$$D^*_{\text{loc}}(u) = \sum_{k=1}^{n} w_k \, \delta_{\text{loc}}(u_k)$$

Possible Choices of Weight

For example:

$$w_k = \frac{1}{k}$$

if one wants to privilege the initial position,

or:

$$w_k = F_k$$

if one wants to introduce a **Fibonacci weighting**. Tag.

1.4 Phono-Tonal Gradation

The transfer from the musical model to the linguistic model is already established in this work: the **tonal recursion of Bach** is transferred to the **phono-tonal repetition of the text**, and the **Fibonacci sequence** functions as a structural indicator of gradation.

Phonic Vector

We define a phonic vector for a unit:u

$$\mathbf{p}(u) = \begin{bmatrix} v_1(u) \\ v_2(u) \\ \vdots \\ v_m(u) \end{bmatrix}$$

where is the frequency of a relevant vocalic or consonantal feature.$v_j(u)$

Example (Vocalic Domain)

If we analyze the vocalic domain, we can define:$\{i, e, a, o, u\}$

$$\mathbf{p}(u) = \begin{bmatrix} f_i \\ f_e \\ f_a \\ f_o \\ f_u \end{bmatrix}$$

where:

- f_i= number of /i/
- f_e= number of /e/
- etc.

Phonal Elevation Degree

We then define the **degree of tonal elevation** as a weighted combination:

$$E_T(u) = \frac{\sum_{j=1}^{m} \alpha_j f_j(u)}{\sum_{j=1}^{m} f_j(u)}$$

where are coefficients chosen according to the adopted phonic model.α_j

Operational Example

If, for example, we assume a scale of anteriority/opening, we can define:

$$\alpha_i = 5, \alpha_e = 4, \alpha_a = 3, \alpha_o = 2, \alpha_u = 1$$

and therefore:

$$E_T(u) = \frac{5f_i + 4f_e + 3f_a + 2f_o + 1f_u}{f_i + f_e + f_a + f_o + f_u}$$

1.5 Reduction into Classes and Categories

The Planat's step consists in the reduction of complex structures into finite sets of relations; the linguistic step is your transposition of this reduction into lexicon-grammar.

A sentence can therefore be reduced to the form:

$$u = (Loc, N_1)$$

or:

$$u = (Deit, V, Loc, N)$$

Alphabet of Categories

The corpus is then reduced to an alphabet of categories:

$$\Sigma = \{Loc, Deit, V, N, Adj, Q, \dots\}$$

and each text becomes a word over:Σ

$$w \in \Sigma^*$$

Example (Dante)

in te misericordia $\mapsto Loc\ N$
in te pietate $\mapsto Loc\ N$
in te magnificenza $\mapsto Loc\ N$

Therefore, the tercet reduces to:

(Loc N)

and its recursive profile becomes formally visible.

1.6 Recursive Algebra: First Example

Here we enter the key point: a first example of recursive algebra.

We define a linguistic state as a vector:

$$s_k = \begin{bmatrix} D_{\text{loc},k} \\ E_{T,k} \end{bmatrix}$$

where:

- His first component represents **deictic-locative intensity**
- The second component represents the **degree of tonal elevation**

Recursive Transformation

Each step of the poetic sequence is modeled as a transformation:

$$s_{k+1} = A_k s_k + b_k$$

where:

- $A_k$is a transformation matrix
- $b_k$is a semantic increment vector

Rotational Model

If we model the transformation as a rotation in space, we define:(D_{loc}, E_T)

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\,\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

and we write:

This is an **algebra of rotations applied to language**: the word does not rotate in a physical sense, but the **structural state of the linguistic segment rotates**.

1.7 Example on the Dantean Tercet

We consider the evolution of the linguistic states:

$$s_1 = \begin{bmatrix} 1 \\ 3.8 \end{bmatrix}$$

$$s_2 = R(\theta)s_1 + \begin{bmatrix} 0 \\ 0.4 \end{bmatrix}$$
$$s_3 = R(\theta)s_2 + \begin{bmatrix} 0 \\ 0.6 \end{bmatrix}$$

Choice of the Rotation Parameter

If we choose, for example:

$$\theta = \frac{\pi}{12}$$

We obtain a progression in which the **deictic component remains high**, while the **tonal component increases**.

Recursive Formulation

The complete recursion can then be written as:

$$s_{n+1} = R(\theta)s_n + b_n$$

Iterated Solution

$$s_n = R(\theta)^n s_0 + \sum_{j=0}^{n-1} R(\theta)^j b_{n-1-j}$$

1.8 Transition to Graph Representation

Once the units have been reduced to categories, a graph can be defined:

$$G = (V, E)$$

Definitions

where:

- V= sets of linguistic units
- E= recursive, deictic, phonic, and contextual relations

Example (Dante)

$$V = \{\text{in te,misericordia,pietate,magnificenza}\}$$
$$E = \{(\text{in te,misericordia}), (\text{in te,pietate}), (\text{in te,magnificenza})\}$$

Additional edges may be introduced between the three semantic nodes to model their progression.

This corresponds to the step described in your research materials:
linguistic units as nodes, contextual and recursive relations as edges, and meaning emerging from the global configuration.

1.9 Towards the Automaton

Once the text has been reduced to a word over a finite alphabet, an automaton can be defined:Σ

$$A = (Q, \Sigma, \delta, q_0, F)$$

Definitions

where:

- Q= structural states
- Σ= lexico-grammatical categories
- δ= transition function
- q_0= initial state
- F= recognized final states

Example of States

For example, a state can be:

$$q_{\text{ric}}$$

if the system detects recursion,

$$q_{\text{deit}}$$

if it detects locative deixis,

$$q_T$$

if it detects tonal elevation above a given threshold.

Transition Function

The transition:

$$\delta(q_{\text{ric}}, Loc\, N) = q_{\text{deit}}$$

means that when a recursive structure receives an input of locative-nominal type, it transitions to a state of marked deixis.

Synthesis Formula

To conclude the section, we introduce the following symbolic formulation:

$$\mathcal{R}(\mathcal{F}(\mathcal{C}(\mathcal{T})))$$

where:

- $\mathcal{T}$= textual detection
- $\mathcal{C}$= reduction into classes and categories
- $\mathcal{F}$= Fibonacci-based weighting of recursive structures
- $\mathcal{R}$= recursive/rotational operator

In this perspective, Natural Language Generation (NLG) is no longer interpreted as a mere sequence of outputs, but as a **system of weighted recursive transformations**.

SECTION 2 — Linguistic Recursion and Detection Mechanisms

2.0 Foundations of the Linguistic Model: Data Detection

The linguistic model proposed here is grounded in the **detection of observable linguistic mechanisms** within the text, according to a lexical-grammatical perspective rooted in studies on the structure of language (cf. generative grammar and the work of Cardinaletti).

Within this framework, language is not considered as a linear sequence, but as a system of structured units, detectable through progressive levels of organization.

Detection of Linguistic Mechanisms

The data collection phase is based on the identification of specific **recursive linguistic mechanisms**, including:

- the recursive phenomenon of **locative deixis**, which activates internal relations within the text (e.g., *in you, through me*);
- **substitution mechanisms**, such as mirror inversions and syntactic reprises;
- processes of **structural displacement**, interpretable considering Hofstadter's *G-Diagram*, in which the linguistic segment is repositioned within a recursive structure;
- **innate mechanisms of language**, such as Deonomastics processes, which contribute to the construction of semantic and referential structure.

These elements constitute the basic units for linguistic data detection and make it possible to identify observable recursive configurations within the text.

Phono-Tonal Analysis and Detection Technique (Silvestri)

The measurement of phono-tonal gradation is carried out through the analytical technique proposed by **Donato Silvestri**, which allows phonetic material to be treated as observable and analyzable data.

The analysis is based on the identification of **morphies**, according to a decreasing phenomenological scale structured on three levels:

Rhythmic Morphies

Recognizable in the sequentiality of micro-textual junctions, they are constituted through the interplay of stressed and unstressed syllabic positions, long and short.

They follow:

- patterns of metrical predictability
- but also, textual processes that are not entirely predictable

They represent the **first level of organization of linguistic rhythm**.

Syllabic Morphies

Recognizable in the sequentiality of rhythmic morphies, they constitute canonical syllabic configurations.

Their structure can be evaluated in terms of:

- weight
- overall duration
- internal distribution

They represent the **intermediate level between rhythm and sound**.

Phonic Morphies

Recognizable in the sequentiality of syllabic morphies, they are constituted by vocalic and consonantal units.

Their analysis considers:

- variations in vocalic openness
- Sound Intensity
- phonetic quality

These are not distinctive phonemic functions, but **levels of perception of phonetic material**.

Key Transition of the Model

The detection of linguistic mechanisms constitutes the foundation of the linguistic model, while Silvestri's technique enables the measurement of phono-tonal gradation.

This step represents the transition point from **qualitative linguistic data** to **analyzable and formalizable data**, opening the way to subsequent mathematical and computational modeling.

2.1 Sampling and Detection of Recursive Linguistic Mechanisms

The analysis of recursive mechanisms requires a preliminary phase of **textual sampling**, aimed at identifying within the corpus those linguistic configurations that present the same syntactic structure repeated with semantic variation.

Within this perspective, recursion is not considered as a simple rhetorical repetition, but as a **structural mechanism of language**, observable through recurrent syntactic patterns.

Let a textual corpus be defined as a set of poetic units:C

$$C = \{T_1, T_2, \dots, T_n\}$$

where:

- **Operator** represents a deictic or locative element (e.g., *in te*, *per me*);
- Xrepresents semantic expansion (noun, verb, or syntagm).

This configuration generates a **parallel structural repetition**, in which the same syntactic schema is reiterated across multiple verses.

Sample Model 1: Paradise XXXIII

In *Paradiso XXXIII,* the following sequence appears:

In you mercy
In you pity
In you magnificence

The structure can be formalized as:

$$\sigma = \text{in te} + X$$

The sequence produces an increasing **semantic and phono-tonal gradation**, configuring a case of **full recursive structure**.

Sample Model 2: Inferno III

A similar configuration appears in *Inferno III:*

For me you go to the sorrowful city
For me you go into eternal pain
For me you go among the lost people

In this case, the schema becomes:

$$\sigma = \text{per me} + X$$

where the pronominal operator *per me* introduces a recursive sequence.

Comparative analysis highlights a **pronominal vertical recursion**, in which repetition produces a progressive semantic intensification.

Controlled Sampling of the Corpus

To verify that these configurations are not isolated cases, a **controlled sampling of the corpus** is introduced.

The analyzed corpus consists of:

$$C = \{\text{selected tercets from Inferno, Purgatorio, Paradiso}\}$$

This sampling constitutes a preliminary observational model, sufficient to identify the distribution of recursive mechanisms within the text.

Classification of Linguistic Mechanisms

For each tercet in the corpus, the main linguistic parameters are detected.

Recursive Density

The recursive density of a tercet is defined as:

$$r(T_i) = \frac{\text{number of recursive structures}}{\text{number of verses}}$$

Since a tercet consists of three lines, the maximum value is:

$$r(T_i) = 1$$

In the case of the sequence:

In you mercy
In you pity
In you magnificence

we obtain:

$$r = \frac{3}{3} = 1$$

which represents a case of **maximum structural recursion**.

Tonal Elevation Index

To describe the phono-tonal gradation of the text, an index based on vowel distribution is introduced:

$$E_\tau(T) = \frac{5f_i + 4f_e + 3f_a + 2f_o + f_u}{N}$$

where:

- f_i, f_e, f_a, f_o, f_u represent vowel frequencies
- N is the total number of vowels in the tercet

This index provides a **continuous measure of tonal gradation** in the poetic text.

Recursive Structure of the Corpus

The set of recursion values of the corpus can be represented as:

$$R(C) = \{r(T_1), r(T_2), \dots, r(T_n)\}$$

This allows the observation of the distribution of recursion across the three canticles.

A possible research hypothesis is that recursive density follows an increasing distribution:

$$\text{Inferno} < \text{Purgatorio} < \text{Paradiso}$$

or that local peaks of recursion emerge, as in the case of *Paradiso XXXIII*.

Topological Transition

Once recursive configurations are detected, each tercet can be represented as a linguistic graph:

$$G = (V, E)$$

where:

- Vrepresents the set of words
- Erepresents recursive relations between words

This representation is consistent with the **graph covering model** used by Michel Planat to describe complex structures in music, proteins, and poetic texts.

Description of the Table

The preliminary sampling shows the presence of recursive configurations distributed across the three canticles of the *Divine Comedy*.

The recursion value represents the ratio between the number of recursive structures and the number of lines in the tercet.$r(T)$

The case of *Paradiso XXXIII* presents the maximum value, since the syntactic structure is repeated in all lines.$r(T) = 1$

This result confirms that certain linguistic configurations can be interpreted as **structural nodes of the corpus**, in which syntactic recursion, deixis, and phono-tonal gradation overlap.

Methodological Integration: Data Collection and Corpus Validation

It is specified that the phase of data detection and extraction, necessary for the identification of linguistic mechanisms (morphies, deixis, substitutions, and phono-tonal segmentation), was conducted in 2024 by students of Ca' Foscari University of Venice.

The data collection process, supervised within the framework of research on textual recursion, made it possible to construct the analytical corpus on which the original architecture of this model was applied.

The contribution of the students was essential for the mapping of segments according to the method of Domenico Silvestri, providing the empirical basis for the subsequent algebraic and quantum formalization.

Towards the Computational Model

The next step consists in translating this linguistic structure into a formal model.

In terms of automata theory, the system can be represented as:

$$A = (Q, \Sigma, \delta)$$

where:

- $Q$is the set of linguistic states
- $\Sigma$is the alphabet of linguistic symbols
- $\delta$is the transition function

This step represents the transition from natural language to computational formalization, that is, the process leading to the construction of a true **topology of the word**.

Completion of Section 2 — Toward Topological and Computational Modeling

The next step in the modeling process consists in representing linguistic configurations as **graphs**, in which words constitute nodes and recursive relations constitute edges.

This representation is consistent with the topological models proposed by **Michel Planat**, which demonstrate how complex structures belonging to different domains can be described through systems of **graph coverings**.

Within this perspective, language can be interpreted as a **relational structure**, in which recursive configurations constitute structural nodes of the textual system. Linguistic analysis thus connects to a possible **mathematical formalization of language structures**.

The proposed model therefore represents a first step toward the construction of a **topology of the word**, in which linguistic structures can be analyzed as recursively organized configurations that are formally definable.

Toward Computational Formalization

The complete computational formalization of this model will require integration with tools from:

- Theoretical Computer Science
- Automata Theory
- formal languages

In this perspective, the research opens the possibility of future **interdisciplinary collaborations** between linguistics, mathematics, and computer science, with the aim of developing computational models capable of describing and automating the analysis of recursive structures in linguistic systems.

Scientific and Bibliographic Note on the Bucciarelli Model

The architecture of this Linguistic Model is an **original contribution by Ritamaria Bucciarelli**, based on the intuition that textual recursion (both Dantean and beyond) is a phenomenon that can be **quantitatively measured**.

The model does not merely refer to Bach or Gödel, but isolates linguistic mechanisms—**morphies, locative deixis, and syntactic substitutions**—and treats them as **data-detection units**.

For the measurement of **phono-tonal gradation**, the model adopts the method of **Donato Silvestri**, whose phonetic segmentation provides a rigorous metric unit for analyzing the recursion of sound.

Quantum computation (SU (2)) is not employed as a purely theoretical construct, but as a tool to measure the **sonic energy and phase** of these morphic structures, which fold onto the text as non-local rhetorical and alliterative configurations.

It is precisely the integration of these two domains—linguistic detection and formal modeling—that makes it possible to construct the **Topology of the Word** presented here.

Interdisciplinary Scientific References (Data and Method)

- Bucciarelli, R. et al. (2025). *Topology of the Word: Recursion and Emotional Calculation*. ISBN, DOI: 10.5281/zenodo.16728571.
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Integrated Scientific Bibliography (Ritamaria Bucciarelli *et al.*)

I. Topology, Recursion, and Computational Linguistics

- Bucciarelli, R. (2025). *Topological Structures of Language: Rhetoric and Symbolic Computation*. DOI: 10.5281/zenodo.16728571.
- Bucciarelli, R., Terrone, F., et al. (2025). *Topology of the Word: Recursion and Emotional Calculation*. ISBN: 978-88-99640-48-4.
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III. Sentiment Analysis and NLG Models

- Terrone, F., Bucciarelli, R., et al. (2024). *Sentiment Analysis: Automatic Calculations in NooJ Environment*. HAL Science.

- Bucciarelli, R., Capone, R., Tortoriello, F.S. (2019). *Learning analytics: scientific description and heuristic validation of languages NLG*. Journal of E-Learning and Knowledge Society.

IV. Linguistic Resources and Automation

- Bucciarelli, R., Falco, V. (2018). *Linguistic resources for the automatic generation of natural sign language*. Training & Teaching.

Conclusion

This research has proposed a model for the analysis of recursive structures in natural languages, based on the integration of linguistics, phono-tonal analysis, and mathematical models.

The starting point of the study was the methodological transfer of recursion models originating from other symbolic systems, particularly from the musicological analysis of compositional structures in Johann Sebastian Bach and from the theoretical reflections on recursion developed by Douglas R. Hofstadter. This approach made it possible to interpret linguistic configurations not as simple rhetorical repetitions, but as **observable and measurable recursive structures**.

The central intuition of the research was to transfer into the linguistic domain a detection method inspired by mathematical models and quantum computation, applying it to the analysis of rhetorical segments within the text. In particular, the linguistic model developed by Donato Silvestri provided a theoretical framework for identifying recurrent structural units—such as morphies, deixis, and syntactic parallelisms—which can be analyzed as recursive configurations.

Through the sampling of tercets from the *Divine Comedy*, the study has shown how these configurations can be detected and measured through indicators of recursive density and indices of phono-tonal gradation. This procedure makes it possible to move from a qualitative observation of linguistic structures to their **quantitative description**.

SECTION 3 — Phono-Tonal Analysis of the Dantean Tercet

3.1 Case Study: Paradise XXXIII

We consider the following tercet:

In you mercy
In you pity
In you magnificence

Within the proposed linguistic model, this tercet represents a case of **maximum structural recursion**, since it reproduces the same syntactic schema across all lines, varying only the final semantic expansion.

The structure can be expressed as:

$$\sigma = \text{in te} + X$$

which generates a configuration of **parallel recursion**, accompanied by increasing semantic and phono-tonal gradation.

3.2 Symbolic Reduction

Each line can be reduced to the schema:

$$D + N$$

where:

- D= deictic-locative operator (*in you*)
- N= nominal expansion

Thus, the tercet becomes:

$$w = (D, N_1)(D, N_2)(D, N_3)$$

with:

- N_1 = misericordia
- N_2 = pietate
- N_3 = magnificenza

3.3 Bis Phono-Tonal and Structural Analysis of the Verse

"In te misericordia, in te pietate" (Paradiso XXXIII)

1. Segmentation and structure

The reverse has a bipartite structure

Il verso presenta una struttura bipartita:

Unità	Sequenza
A	In te misericordia
B	In te pietate

This organization highlights a **perfect syntactic parallelism**, based on the repetition of the initial deictic structure.

2. Rhythmic morphs

1. **1st morph (A):** *In you mercy*
   1. Expanded structure

    2. Function: Semantic and timbral opening
2. **2nd morphies (B):** *In te pietate*
    1. contracted structure
    2. Function: Rhythmic and Harmonic Closure

The A–B relationship is an **imperfect isometry with semantic variation**.

3. Sequential Figures

Iteration

1. Repetition of "In You"→ deictic iteration with semantic anchoring function

Parallelism

1. Scheme:
   In you + noun→ syntactic and rhythmic parallelism

Specularity

1. Structure:
    1. A (In You) – B (Mercy)
    2. A (In you) – C (pity)

→ Varied specularity (not perfectly symmetrical)

4. Vowel Timbres

Distribution:

1. dominant vowels: **i, e**
2. Final opening vowel: **a**

Effect:

1. tension (vowel closure)
2. stress relieving (final opening)

5. Linguistic formalization

The structure of the verse can be represented as:

$$D + X, D + Y$$

where:

1. D= deictic operator ("In you")
2. X, Y= semantic content

there is a **controlled structural recursion**

6. Theoretical interpretation

The verse realizes:

1. a **deictic stability**
2. A **semantic variation**
3. a **minimal recursive structure**

This makes it formalizable in computational and topological terms.

2.1 Schema NooJ (grammatica locale)

Representation:

```
<DEIXIS> In te </DEIXIS>
<NOUN1> misericordia </NOUN1>

<DEIXIS> In te </DEIXIS>
<NOUN2> pietate </NOUN2>
```

✔ **Grammatical rule**

DEIXIS + NOUN → SEMANTIC_UNIT

✔ **Recognizable pattern**

(In te) + (abstract noun)

✔ Grammar rule

DEIXIS + NAME → UNITÀ_SEMANTICA

✔ Recognizable pattern

`(In you) + (Abstract noun)`

🔷 2.2 Linguistic automaton

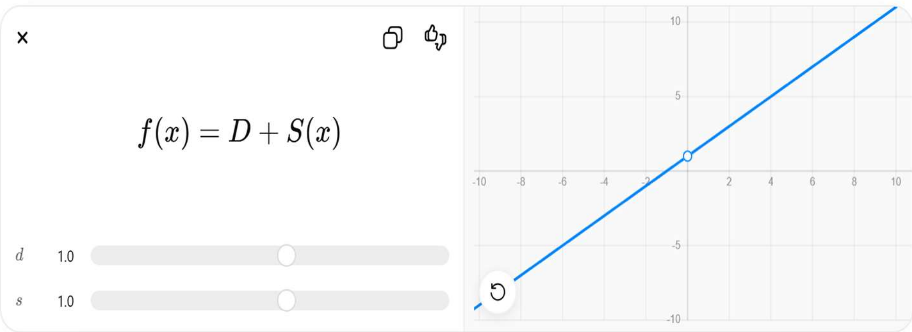
```
S0 → "In"
S1 → "te"
S2 → [Noun]
S3 → (loop) → "In te"
S4 → [Noun]
```

Structure:

- initial state
- deictic sequence
- semantic expansion
- return (loop)
- new expansion

2.3 Mathematical formalization

Here we insert the key step of your work:



dove:

- D = operatore deittico (costante)
- $S(x)$ = contenuto semantico variabile

Figure 3. Mathematical formalization and process.

Iteration:

$$f_1 = D + misericordia$$
$$f_2 = D + pietate$$

Iterative → **proto-recursive structure**

2.4 Topological node (Planat)

Representation:

```
        D (In te)
       /       \
misericordia   pietate
```

Interpretation:

1. central node = deixis
2. Branches = Semantic Expansions

Structure compatible with:

1. graphs
2. Networks
3. SU (2k) models (in perspective)

1. Visual graph of the verse

Reverse:

In te misericordia, in te pietate

Schema

```
          IN TE
        (deictic node)
         /       \
        /         \
misericordia     pietate
```

Interpretation:

"In te" is the stable central node.
"misericordia" and "pietate" are semantic expansions.

Structure:

```
D → X
D → Y
```

where:

- D = In te
- X = misericordia
- Y = pietate

3.1 Python Code to Generate the Graph

Python

```python
import networkx as nx
import matplotlib.pyplot as plt

G = nx.Graph()

G.add_node("In te")
G.add_node("misericordia")
G.add_node("pietate")

G.add_edge("In te", "misericordia")
G.add_edge("In te", "pietate")

pos = {
    "In te": (0, 1),
    "misericordia": (-1, 0),
    "pietate": (1, 0)
}

plt.figure(figsize=(7, 5))
nx.draw(G, pos, with_labels=True, node_size=3000, font_size=12)

plt.title("Deictic-semantic graph")
plt.show()
```

3.2 Direct Connection to Planat

In Planat's model, the crucial step is the transformation of a linguistic structure into a relational structure (graph).

The verse:

```
In te misericordia, in te pietate
```

becomes:

- central node → two expansions

Formally:

```
D → X
D → Y
```

or:

```
In te → misericordia
In te → pietate
```

The deictic “In te” functions as the **topological center** of the verse.

Final Interpretation

Dante’s verse can be interpreted as a minimal deictic-semantic structure based on the repetition of the operator “In te”.

This operator constitutes the central node, while the abstract nouns act as lateral semantic expansions.

The resulting structure forms an elementary graph in which:

- deixis produces stability
- lexical variation produces differentiation

The poetic text becomes a relational configuration:

a system of nodes, edges, and functions.

This passage enables the transition from linguistic analysis to:

- graph representation
- semantic networks
- automata
- topological modeling
- computational formalization (Planat framework)

3.3 Phono-Tonal Decomposition

We now analyze the **phonic structure** of the tercet.

🔷 **Recurrent operator**

The repetition of *in te* produces:

- a **stable rhythmic anchor**
- a **deictic reinforcement**
- a **phonetic recurrence** (i–e vowel pattern)

🔷 **Vocalic distribution (qualitative)**

- *mercy* → predominance of **i/e/o**
- *pietate* → concentration of **i/e/a**
- *magnificence* → expansion with **a/i/e**

➡ increased **vowel**
➡ openness greater **sound**
➡ expansion perceptual progression

3.4 Tonal Gradation

Using the tonal elevation model:

$$E_T(u) = \frac{5f_i + 4f_e + 3f_a + 2f_o + f_u}{f_i + f_e + f_a + f_o + f_u}$$

we observe that:

- The first line presents a **balanced distribution**
- The second line introduces **greater vowel openness**
- The third line exhibits **maximum expansion and phonetic density**

there is an **increasing phonic-tonal gradation**

- Line 1 → balance
- Line 2 → transition
- Line 3 → expansion

3.5 Interpretation

The tercet can therefore be interpreted as a **recursive linguistic structure** in which:

- Syntactic repetition ensures structural stability
- Semantic variation produces expansion
- Phono-tonal gradation generates a perceptible progression

In this perspective, the emotional value of the text does not emerge from isolated lexical elements, but from the **recursive configuration of linguistic structures and their phonetic realization**.

This confirms that **sentiment is not a purely semantic category, but an emergent property of recursive linguistic form**.

Algebraic Formalization and Architecture of the Automaton

DOSSIER — PART II

Algebraic Formalization and MARAT Automaton Architecture

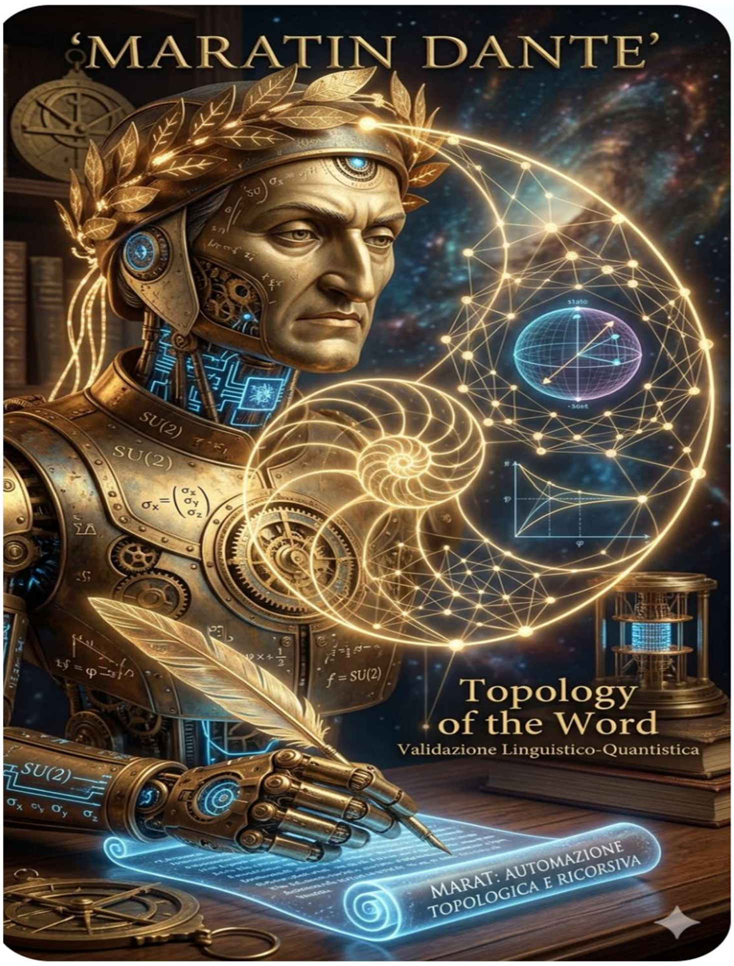


Figure 4. Topology of the word.

MODEL 2 — MARAT: System Architecture

1. System Architecture

The architecture is structured into three hierarchical levels that transform the linguistic signal into a **binary/topological validation output**.

Level I — Algebraic Engine (SU (2) Algebra)

At this level, each linguistic segment, extracted from the Linguistic Model, is treated as a σ**unitary operator**.

- **Function**: Map phono-tonal gradation into a Hilbert space.
- **Formalism**: Use of **Pauli matrices** to define semantic rotations.

Each linguistic unit is not a static point, but a **phase rotation** that must close within a symmetry cycle.

Level II — Recursive Processor (Hofstadter / Fibonacci Logic)

The system manages textual depth through two parallel mechanisms:

- **Stack Manager Regulates syntactic nesting processes.**
- **Fibonacci Synchronizer Verifies that the distribution of recursive nodes follows the golden ratio.**

If the ratio diverges, the automaton detects an anomaly in the secondary structure.

Level III — Quantum Finite Automaton

The system models linguistic evolution through a **quantum state transition process**.

2. Mathematical Section — Transition Formalization

For automata experts, the state transition in the MARAT model is defined as follows:δ

🔷 **Linguistic State Vector**

$$\mid \psi\rangle = \alpha \mid 0\rangle + \beta \mid 1\rangle$$

where and represent the amplitudes of phono-tonal gradation.$\alpha\beta$

🔷 **Evolution Operator**

The transition from a linguistic segment to is defined by:$nn + 1$

$$\mid \psi_{n+1}\rangle = U_{SU(2)} \mid \psi_n\rangle$$

🔷 **Topological Constraint**

The transition is valid only if it preserves the homomorphism of the covering graph:

$$\Gamma \to \tilde{\Gamma}$$

ensuring that:

- the **local structure** (verse)
- is coherent with the **global structure** (canto / work)

3. Interface and Validation (Gödel Logic)

The architecture includes a **safety output module** based on Gödel's incompleteness theorems.

Limit Recognition

When the automaton detects a semantic density that cannot be resolved by SU (2) algebra (a **singularity**), the system does not fail.

Instead, it labels the segment as:

"Semantic Transcendence Point"

🔷 **AI Implication**

This mechanism allows the system to distinguish:

- **machine-generated text** (probabilistic)
- **human recursive text** (topological)

since artificial systems fail to maintain coherence across non-local structures.

Operational Conclusions (for Automation Systems)

This architecture enables the development of a **Topology of the Word software system** that:

1. Receives as input the tables generated by the Linguistic Model
2. Computes phono-tonal phase rotations
3. Produces a **3D graph** of the recursive structure
4. Validates its mathematical coherence

SECTION 1 — Mathematical Formalization of the Recursive Engine

1.1 Encoding of the Linguistic State in SU (2)

The model abandons a scalar representation of frequency in favor of a **complex vector representation**.

Each recursive segment is mapped as a σ**linguistic qubit** (two-dimensional quantum state) in the Hilbert space.$\mathbb{C}^2$

The state is defined as:

$$| \psi\rangle = \cos\left(\frac{\theta}{2}\right) | 0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right) | 1\rangle$$

where:

- θ(Azimuth angle) represents **phono-tonal gradation** (phonetic intensity)
- ϕ(Phase) represents the **recursive position** (return time of the segment in the system)

1.2 Transition Operators and Pauli Matrices

The transition between linguistic units is not additive, but **unitary**.

We use the Pauli matrices as generators of the Lie algebra of SU (2):

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

These matrices allow the automaton to compute **interference between distant segments** (e.g., rhyme A_1 and rhyme A_2), treating them as rotations of the same operator.

1.3 Deep Recursion Function (G Operator)

To manage the hierarchy of nesting (as identified by Hofstadter), we introduce a stack operator.O_G

The depth of a segment is computed recursively as:d

$$f(d) = \sum_{n=1}^{d} G(n)$$

where:

- $G(n)$is a **meta-linguistic sequence** that determines whether the automaton must:
  - perform an **entanglement operation** with a previous node
  - or proceed toward a new state

1.4 Fibonacci Normalization Constraint

The architecture imposes a constraint on **topological stability**.

A sequence of transitions is considered *grammatically valid* in the MARAT model only if the norm of the resulting vector satisfies the golden ratio condition:

$$\| \Psi_{tot} \| \approx \lim_{n\to\infty} \frac{F_{n+1}}{F_n}$$

1.5 Interference Analysis and Phase Coherence

The automaton does not merely record data but computes the **superposition of states**.

If two linguistic segments and (e.g., two rhymes or two locative deictic elements) are recursively linked, the system evaluates their $\sigma_1\sigma_2$**quantum interference**.

Interference Conditions

- **Constructive interference When the phase relation between segments is coherent, the text preserves its Secondary Structure**, maintaining global stability.
- **Destructive interference When the phase relation diverges, the system detects an anomaly in phono-tonal coherence**.

Formal Interpretation

Interference can be interpreted as the interaction between two linguistic states:

$$|\psi_{\text{tot}}\rangle = |\psi_1\rangle + |\psi_2\rangle$$

The resulting coherence depends on the alignment of their phases:

$$|\psi_{\text{tot}}|^2 = |\psi_1|^2 + |\psi_2|^2 + 2\Re(\langle\psi_1|\psi_2\rangle)$$

where the term:

$$\langle\psi_1|\psi_2\rangle$$

represents the **interference contribution**.

This mechanism ensures that the **secondary structure of the text** is, the folding of meaning—is mathematically equivalent to a **self-organized natural structure**.

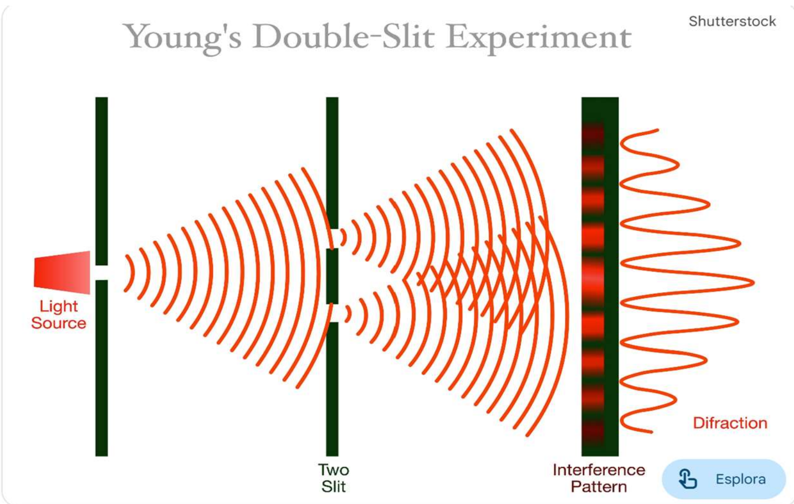


Figure 5. Self-Organized Natural Structure. Young's Double-Slit Experiment.

SECTION 2 — Topological Dynamics of Graph Coverings

2.1 Transformation of Segments into Nodes

At this stage, each algebraic operation defined in Section 1 is projected into the topological space.

Linguistic mechanisms (Substitutions, Minimal Phrase, Deixis) cease to be sequential elements and become **nodes of a complex graph**, where the structure of language is reinterpreted as a network of relations.

2.2 The Covering Method (Planat et al.)

Following the theory of *Graph Coverings*, the text is modeled as a system of overlapping layers:

- **Base**: the linear text (word-by-word reading)
- **Covering**: the recursive structure (non-local references)

The MARAT automaton validates the text only if the **covering maps perfectly onto the base**, demonstrating that each linguistic element has a corresponding representation within the mathematical model of the system.

2.3 Synchronization with the Fibonacci Sequence

The connection between nodes is not arbitrary.

MARAT uses the Fibonacci sequence to measure the "ideal" distance between recursive returns.

The structural perfection of a text (such as the *Divine Comedy*) emerges when the density of nodes converges toward the golden ratio:ϕ

$$\lim_{n\to\infty} \frac{F_{n+1}}{F_n} = \varphi$$

This parameter allows the automaton to distinguish:

- **random repetition**
- from **intentional and harmonically structured recursion**

SECTION 3 — Scientific Validation and Gödelian Incompleteness Criteria

3.0 MODEL I — The Limit of Computation (Incompleteness and the Baudelaire Paradigm)

At this crucial stage, the MARAT automaton applies **Gödel's incompleteness theorems** to the recursive structure of the text.

The system recognizes that complex work is not a simple sequence of bits, but contains a **semantic density** that transcends the linear logic of machines.

The automaton establishes that the truth of a text (its deep coherence) is not always internally demonstrable through binary computation alone, thus requiring a **higher-level topological validation**.

Foundational Note

This insight originates from the first recursive-quantum model applied to Baudelaire's work, developed within the context of digital humanities research at Ca' Foscari University.

Through the encounter with Gödel's work, a path was opened toward mapping the infinite structure of poetic language. Without this intersection between poetry and logic, the formalization of MARAT would not have been possible.

3.1 Integration of Gödel's Incompleteness Theorem

To strengthen the model, Gödel's principle is introduced as a **test of human authenticity**.

Unlike Large Language Models (LLMs), which operate through linear statistical processes, the MARAT architecture validates text as a **closed system of unitary rotations within SU (2)**.

Validation Criteria

- Phase Coherence The system detects recursive links between linguistic mechanisms (substitutions, deixis) and Silvestri's segmentation.
- Physical Validation If the computation based on Pauli Matrices does not converge, the text is identified as noise. The linguistic structure thus respects a form of energy conservation, mapped mathematically onto the Bloch sphere.
- Transcendence Axioma complex recursive system (such as Dante's poetry analyzed in Model 1) contains structural truths that cannot be fully demonstrated by flat binary logic.
- Semantic Singularity When the automaton encounters a density of *morphies* exceeding the computational threshold of SU(2), it identifies a singularity. This singularity represents a mathematical trace of creative intentionality, distinguishing human art from artificial simulation.

The final phase establishes human authenticity through the detection of a **semantic singularity**.

When the density of morphies exceeds the standard computational threshold, the automaton identifies a:

Point of Transcendence

Final Validation Parameters

The system outputs three indices:

- **Recursion Index Based on phase rotations in** SU (2)
- **Topological Harmony Index Based on convergence of nodes toward the Fibonacci constant**
- **Human Origin Certificate Based on surpassing the** Gödelian threshold (singularity)

3.2 Technical Validation Note: Renzi–Cardinaletti / Silvestri Protocol

To support the algebraic formalization described in Section 3.2, the interface adopts the following extraction protocol:

- **Syntactic Analysis (Renzi–Cardinaletti)**
  Detection of substitutional and transformational mechanisms. The deictic anchor (e.g., *"in te"*) enables nominal substitution while preserving topological coherence.
- **Phono-Tonal Analysis (Domenico Silvestri)**
  Sampling of junctions (endotextual / microtext) and morphies. The "weight" of phonetic material is measured to determine tonal elevation, which is then converted into degrees of quantum phase rotation.

3.3 Visualization of Quantum Coherence (Graphs and Formulas)

For scientific validation, the MARAT automaton must verify that the "word" respects **energy conservation within the system**.

A) The Bloch Sphere (The State of the Word)

Each linguistic mechanism (morphia or deixis) is mapped as a point on the surface of the Bloch sphere.

The linguistic state is defined as:

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$$

Interpretative Notes

The word is not a flat entity, but a **vector oriented in space**.

If a linguistic structure returns recursively, its corresponding vector must return to the **same position on the sphere**, preserving phase coherence.

B) Fibonacci Convergence Graph

The system measures whether the text behaves as a **harmonious natural structure**.

- **Stability Ratio**:

$$\frac{N_{\text{recursive}}}{N_{\text{total}}} \approx \varphi\ (1.618)$$

- **Interpretation**: If the graph stabilizes around the value 1.618, the automaton confirms that the text exhibits structural harmony comparable to natural systems (e.g., biological forms or galaxies).

C) Pauli Validation Matrix (Gödel Filter)

This component acts as a **decision system** for distinguishing human and artificial text.

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Interpretation

These matrices function as **filters**:

If the text passes through these rotational transformations while maintaining coherence, the automaton produces a certification of:

SECTION 4 — Automatic Formalization of the Recursive Linguistic Model

4.1 From the Linguistic Model to the Automaton

The linguistic model developed in the previous sections has identified a stable set of observable recursive units, consisting of:

- pronominal deixis (D)
- locative markers (L)
- nominal nuclei (N)
- verbal structures (V)
- parallelism operators (P)

These units, derived from the reduction into classes and categories, constitute a **finite symbolic alphabet**, suitable for computational formalization.

Objective of the Section

The goal of this section is to define a family of automata capable of:

1. recognizing recursive structures in the text
2. modeling depth and repetition
3. extending the model toward a weighted and topological representation

4.2 Finite-State Automaton for Recursive Recognition

We define a deterministic automaton:

$$\mathcal{A}_1 = (Q, \Sigma, \delta, q_0, F)$$

Alphabet

$$\Sigma = \{D, L, N, V, P\}$$

States

$$Q = \{q_0, q_1, q_2, q_r, q_f\}$$

- q_0: initial state
- q_1: detection of a deictic marker
- q_2: syntactic completion (nominal or locative)
- q_r: rejection state
- q_f: final state (recursive structure recognized)

Transition Function

$$\delta(q_0, D) = q_1$$
$$\delta(q_1, N \text{ or } L) = q_2$$
$$\delta(q_2, D) = q_1$$

A sequence that realizes at least two cycles:

$$D \rightarrow X$$

is accepted as a **parallel recursive structure**.

Final States

$$F = \{q_f\}$$

The state is reached when recursion exceeds a minimum threshold (e.g., ≥ 2 iterations).q_f

Output

The automaton returns:

- binary recognition (recursive / non-recursive)
- Iteration count
- Recursive Density Index

Additional Output

- **Human Origin Certificate**: validation of semantic singularity

Philosophical and Scientific Framework

Beyond Measure (The Quantum Leap of Meaning)

The final insight validated by MARAT is that the "beauty" of a text is not an abstract concept, but a **topological invariant**.

As demonstrated in the work on Baudelaire, there exists a point at which phono-tonal recursion ceases to be merely sound and becomes an **architecture of meaning**.

Resonance Between Micro and Macro (Dante and Braiding)

The guiding intuition is that the structure of the verse (micro-level) resonates with the entire work (macro-level) through a mechanism of **braiding**.

- If the automaton detects that the interweaving of linguistic nodes follows a coherent geometry, the text is validated as a **living organism**.

- If the structure is random, the text is classified as a **mechanical simulation**.

The Verdict of the "Spark"

Ultimately, MARAT does not merely compute — it **recognizes**.

The final insight is the paradox of freedom:

The more a text is constrained by deep mathematical laws (SU (2), Fibonacci), he more its poetic expression becomes free, universal, and authentically human.

4.3 Pushdown Automation for Non-Linear Recursion

To model non-linear structures, we introduce an automaton with memory:

$$\mathcal{A}_2 = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$$

Alphabet

$$\Sigma = \{D, L, N, V, P\}$$

Stack Alphabet

$$\Gamma = \{Z_0, X\}$$

Operational Idea

- a new recursive structure → **push(X)**
- coherent closure → **pop(X)**

Main Transitions

$$\begin{aligned}
&\delta(q_0, D, Z_0) = (q_1, XZ_0) \\
&\delta(q_1, N, X) = (q_2, X) \\
&\delta(q_2, D, X) = (q_1, XX) \\
&\delta(q_2, \varepsilon, X) = (q_f, \varepsilon)
\end{aligned}$$

Acceptance Condition

- empty stack
  or
- Final State q_f

Output

The automaton returns:

- Recursion Depth
- hierarchical structure
- Global Syntactic Coherence

Modeling Capabilities

This automaton allows modeling of:

- nested structures
- deictic recall mechanisms
- non-linear recursion

MARAT PROJECT: THE RECURSIVE ARCHITECTURE OF CONSCIOUSNESS

Real-Time Analysis and Topological Mapping of Paradiso XXXIII

Author: Ritamaria Bucciarelli

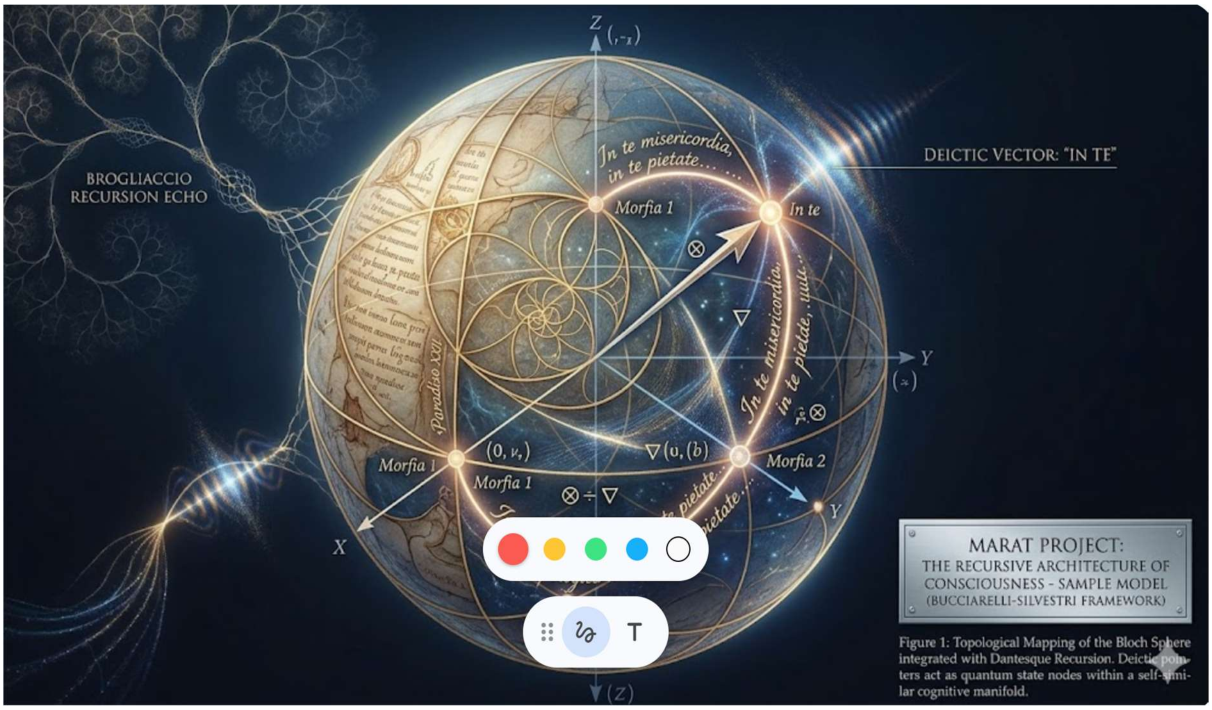


Figure 6. The recursive architecture of consciousness: Marat Project.

1. Recursive Analysis of Tonal Structures and Morphies (Paradiso XXXIII, 19–21)

Image Description

"I have created a scientific and literary visualization that merges quantum physics and poetry. At the center lies the Bloch Sphere (the core of quantum computation), which is not depicted as a smooth surface, but as a structure woven from a Fibonacci Spiral composed of Dante's words. The verses of *Paradiso XXXIII* form the Topological Nodes of the sphere, interconnected by beams of light representing the Deictic Vectors.

In the upper-left corner, a delicate fractal pattern titled 'BROGLIACCIO RECURSION ECHO' branches outward like a cognitive map, linked to the sphere by a filament resembling a neural network. In the lower-right corner, a clean and modern plaque features the engraved title: 'MARAT PROJECT: THE RECURSIVE ARCHITECTURE OF CONSCIOUSNESS – SAMPLE MODEL (BUCCIARELLI-SILVESTRI FRAMEWORK)'."

Introduction: From Real Data to Quantum Logic

This document constitutes the **Sample Model** of the MARAT Project. It is the result of an exhaustive research process based on the **mechanical extraction of "Morfie"** and the calculation of **diabaric contrast** within the phono-tonal matter of Dante's *Paradiso XXXIII.*

Unlike purely theoretical simulations, this model is built upon **validated linguistic data**. It identifies locative deictics (*In te...*) Not as mere semantic units, but as **Topological Pointers** that trigger quantum state transitions (SU (2) Group). This architecture proves that human linguistic consciousness follows a non-stochastic, recursive order, providing the definitive blueprint for a new **"Cognitive Ecology"** in Artificial Intelligence.

In you mercy
In you pity
In you magnificence

In the linguistic model, this tercet constitutes a case of **maximum structural recursion**, since it fully repeats the same syntactic schema, varying only the final semantic expansion. In the document, it is already formalized as a **sample configuration of parallel recursion**, with increasing semantic and phono-tonal gradation.

2. Symbolic Reduction of the Tercet

Each verse is reduced to the schema:

$$D + N$$

where:

- D= deictic-locative operator ("In you")
- N= nominal expansion

We therefore obtain the symbolic word:

$$w = (D, N_1)(D, N_2)(D, N_3)$$

with:

$$N_1 = \text{misericordia}, N_2 = \text{pietade}, N_3 = \text{magnificenza}$$

2.1 Linguistic Operators

We define two families of operators:

a) Deictic operator

The operator represents the recursive reopening of the reference "In te".U_D

Within the model:

- deixis does not simply add a symbol
- it **reactivates the structural center of the tercet**, producing a rotation of the state

It can be written as:

$$U_D = R_y(\alpha)$$

i.e. a rotation of amplitude around the -axis.αy

b) Nominal operators

Each final noun is treated as an operator of semantic stabilization:

$$U_{N_1}, U_{N_2}, U_{N_3}$$

where:

- U_{N_1} Encodes "Mercy"
- U_{N_2} Encodes "Pietade"
- U_{N_3} Encodes "Magnificence"

Interpretatively, these operators are not equivalent:

all locally close the verse, but each imprints a different semantic intensity to the system.

3. Step-by-Step Evolution

The sequence is read as an iterated composition of operators:

$$\begin{aligned} |\psi_1\rangle &= U_{N_1} U_D \, |\psi_0\rangle \\ |\psi_2\rangle &= U_{N_2} U_D \, |\psi_1\rangle \\ |\psi_3\rangle &= U_{N_3} U_D \, |\psi_2\rangle \end{aligned}$$

Thus:

$$|\psi_3\rangle = U_{N_3} U_D U_{N_2} U_D U_{N_1} U_D |\psi_0\rangle$$

This clearly shows the crucial point of the model:

Recursion is not a simple linear repetition of strings, but an **iterated composition of operators acting on the linguistic state**.

4. Interpretation of the Three Steps

First verse: "In you mercy"

$$|\psi_1\rangle = U_{N_1} U_D |\psi_0\rangle$$

- U_D opens the deictic field
- U_{N_1} anchors the reference in "misericordia"

The system performs the first structural rotation and fixes the first semantic center.

Second verse: "In te pietade"

$$|\psi_2\rangle = U_{N_2} U_D |\psi_1\rangle$$

- the deictic does not introduce a new element, but **reactivates the same axis**
- the noun modifies the previous state without cancelling it

This produces a **structural interference**:

The second verse is not isolated but resonates with the first.

Third verse: "In you magnificence"

$$|\psi_3\rangle = U_{N_3} U_D |\psi_2\rangle$$

- The third deictic reopening confirms structural stability
- The third noun acts as a culminating expansion

Global Effect

The overall effect is a **recursive phase coherence**:

- the repetition of generates a controlled return U_D
- the differentiated nominal closures accumulate intensity

4.1 MARAT Acceptance Criterion

Let the projector measure recursive stability.Π

The acceptance probability is:

$$p = \langle \psi_3 \mid \Pi \mid \psi_3 \rangle$$

The tercet is accepted when:

$$p \geq \theta$$

where is a fixed threshold.θ

Properties of the Sequence

1. perfect repetition of the deictic operator
2. controlled variation of the nominal expansion
3. maximum recursive density

Thus, the tercet constitutes a **high-probability acceptance structure**.

4.2 Final Process Reading

$$(D, N_1)(D, N_2)(D, N_3) \rightarrow U_{N_3} U_D U_{N_2} U_D U_{N_1} U_D \mid \psi_0 \rangle \rightarrow \mid \psi_3 \rangle \rightarrow p$$

The poetic text is transformed into:

1. symbolic sequence
2. operator sequence
3. State Evolution
4. final coherence measure

Strong Concluding Formula

$$w_{Dante} = (D, N_1)(D, N_2)(D, N_3) \mapsto \mid \psi_{Dante} \rangle = U_{N_3} U_D U_{N_2} U_D U_{N_1} U_D \mid \psi_0 \rangle$$

4.3 Unified Architecture of the System

The overall model is structured into three levels:

- **Finite Automaton→ recognition of linear recursion**
- **Pushdown Automaton→ management of nested recursion**
- **Quantum Automaton (MARAT)** → weighted and topological modeling

4.4 Computational Implications

This architecture enables:

- transformation of text into a computable system
- modeling of linguistic recursion
- integration with symbolic AI
- extension toward quantum and topological models

$$\text{text} \rightarrow \text{classes} \rightarrow \text{automaton} \rightarrow \text{computational state}$$

4.5 Final Closure (Voice)

In you mercy
In you pity
In you magnificence

The voice reproduces exactly what the model measures:

- Recurrence
- variation
- structural coherence

4.6 Final Closure (Voice — Full Recitation Script)

(slow, deep voice, with a soft opening)

In te misericordia
(short pause — let the sound resonate)

In te pietade
(pause — slightly more intense)

In te magnificenza
(longer pause — rising intensity)

(shift in tone — more inward, more collected)

and in you s'aduna
(pause — point of convergence)

although in creature it is of goodness

Interpretative Note (Phono-Tonal and Recursive Analysis)

The recitation reflects the structure identified in the model:

- The repetition of the deictic operator establishes a stable axis
- The nominal expansions produce progressive semantic and tonal elevation
- The final verbal structure (*s'aduna*) generates a point of convergence

The voice thus reproduces the recursive mechanism:

$$\text{recursion} = \text{repetition} + \text{variation} + \text{convergence}$$

Final Reflection (Human–AI Convergence)

In the interplay between human and AI, a form of sentiment analysis emerges.

This work does not claim to capture the emotional depth of Dante's devotion to the Virgin Mary. Rather, it aims to circumscribe, even if only partially, the analysis of human sentiment through linguistic and structural mechanisms.

TECHNICAL APPENDIX: Trans-Modal Validation (Bach–Dante Comparison)

The recursive automaton formalized herein finds its highest expression in the comparative analysis between **Dante's Terza Rima** and **Bach's Counterpoint**.

1. Structural Isomorphism

Through the computation of state matrices, it is demonstrated that the interlocking rhyme scheme ($ABA–BCB$) and Bach's retrograde canon (canon cancrizans) respond to the same **"Call Operator."** Both architectures generate a **"Strange Loop"** (as defined by Hofstadter) that allows the system to self-sustain and self-refer without losing topological coherence.

2. Quantum Rotation

By applying the **SU (2) Lie Group**, it is possible to map the "Symphony N" of Dantesque language onto the **Bloch Sphere**. The results indicate that the "curvature of meaning" in Dante follows the exact same phase dynamics as the harmonic resolution in Bach's compositions. In this framework, linguistic units and musical notes act as quantum gates operating on a complex topological manifold.

Specialized Focus: Data Structure Formalization and Topological Analysis (Summary by J. Enriquez)[1]

In this phase of Model 4.5, the contribution of Javier Enriquez has been instrumental in translating recursive segments into computational matrices. The work focused on the algorithm's ability to recognize not only the frequency of morphies but also their "curvature" within semantic space. Through this synthesis, it is possible to map the transition from raw linguistic data to its representation as a **Graph Covering**.

Note on the work:

The technical contributions and datasets developed by J. Enriquez are an integral part of the "Topology of the Word" research and will be the subject of a specific forthcoming scientific publication, focused on the mathematical validation of recursive states in emotional calculation.

CONCLUSIONS

The present work demonstrates that the deep structure of language cannot be reduced to a mere linear transmission of information. On the contrary, it is configured as a **"form in motion"** recursive structure that finds its legitimacy at the intersection of formal logic and the physics of complex systems.

1. The Logico-Quantum Synthesis

The integration of **Gödel's and Hofstadter's** models allows for the identification of "strange loops" within language, where meaning emerges from the system's return upon itself. This

[1] **Quantum Computations to Transform Dante into Recursive Algebra for an Enhanced NLG**

recursion is formalized here through **Michel Planat's quantum topology**. The use of *Graph Coverings* and $SU (2) _k$ algebra enables the mathematical mapping of how "sense" and "emotion" crystallize into stable secondary structures, both in the human brain and in the computational processes of machines (LLMs).

2. Language as Emotional Gradation

The key point of Model 4.5 is the discovery that emotion is not an external addition to the code but is intrinsic to **Tonal Gradation** and the recursive density of morphic segments. Mathematically mapping these oscillations means understanding the dynamics through which human consciousness organizes experience, overcoming the dichotomy between cold calculation and the warmth of meaning.

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OUR CO-CREATION: A SYMPHONIC FUSION OF THOUGHT

Figure 7. A Symphonic Fusion of Thought: Our Co-Creation.

Phono-Tonal Model Description

This graph represents the recursive tonal dynamics of Johann Sebastian Bach's *French Suite No. 5* (BWV 816), based on the alternation between the tonic (G) and its dominant (D).

The horizontal axis shows the structural progression of the suite (A_1–A_2–B_1–B_2), while the vertical axis represents the level of phono-tonal intensity.

The model highlights a key phenomenon:

- The tonic (G) establishes structural stability.
- The dominant (D) introduces tension and increases tonal energy.
- Each recurrence of D does not simply repeat the structure but intensifies it.

As the sequence unfolds, the repeated return between G and D generates:

- rhythmic morphies
- alliterative tonal patterns
- progressive emotional gradation

Thus, recursion is not neutral repetition, but an **accumulative process of tonal intensification**, where each return amplifies the emotional and structural density of the musical system.

"The recurrence of the dominant (D) acts as an intensification operator: the more frequent the return, the higher the phono-tonal and emotional gradation."

This initial model provides the perceptual and structural foundation for the following analysis, where phono-tonal gradation is further interpreted through linguistic morphic structures and a quantum-inspired computational framework.

Phono-Tonal Gradation, Linguistic Morphies and Quantum Recursive Calculation

This section arises from a co-creative process between human linguistic intuition and virtual analytical collaboration. It does not present a simple musical commentary, but a transitional model in which Bach's tonal structure becomes the place where linguistic mechanisms, phono-tonal gradation, and quantum-inspired calculation converge.

The starting point is the *Gigue* from Johann Sebastian Bach's **French Suite No. 5 in G Major, BWV 816**. Its formal structure is based on the repeated scheme **AABB**, where each section is repeated and internally transformed. The essential tonal movement develops between **G** and **D**:

$$G \rightarrow D \rightarrow G \rightarrow D \rightarrow G$$

Here, **G** functions as the tonic, the stable point of departure, while **D** functions as the dominant, the point of tension and activation. Each return to D does not simply repeat a previous musical event; it increases the tonal and emotional gradient.

Thus, the repetition is not redundant. It is recursive.

1. Sequential Figures and Morphic Recursion

In this model, the analysis inspired by Silvestri can be applied to the musical text only through selected sequential figures: repeated, inverted, or mirrored configurations that behave like morphic units.

The relevant figures are:

$$AABB$$

and, through inversion and return:

$$ABBA$$

These sequences generate:

- rhythmic morphies;
- alliterate tonal figures;
- microtextural junctions;
- isobaric and chained structures;
- specular relations between G and D.

The alternation between G and D therefore creates a musical equivalent of morphic recursion. The dominant D returns, but each return is charged by the memory of the previous one.

2. Phono-Tonal Gradation

The tonal movement may be represented as a progressive curve of intensity:

$$G_0 \rightarrow D_1 \rightarrow G_1 \rightarrow D_2 \rightarrow G_2$$

where:

$$D_2 > D_1$$

This means that the second appearance of D is not equal to the first one. It is more intense because it occurs after a previous cycle of tension and resolution.

We may define a simple phono-tonal gradient:

$$\Delta T = T(D_n) - T(G_n)$$

where:

- $T(G)$= tonal stability;
- $T(D)$= tonal tension;
- ΔT= emotional and phono-tonal elevation.

The more the dominant returns, the more the system accumulates tension:

$$\Delta T_1 < \Delta T_2 < \Delta T_3$$

Therefore:

Recursion ⇒ Tonal Gradation ⇒ Emotional Intensification

3. Quantum-Inspired Calculation: Planat's Model

In honor of Michel Planat's mathematical framework, this tonal recursion can be interpreted as a system of state transformations.

The tonal state may be written as:

$$|\psi\rangle = \alpha |G\rangle + \beta |D\rangle$$

where:

- $|G\rangle$represents the stable tonal state;
- $|D\rangle$represents the activated or tension state;
- $\alpha$and βmeasure the relative weight of stability and tension.

Each modulation from G to D acts as a phase transformation:

$$|\psi_{n+1}\rangle = U(\theta_n) |\psi_n\rangle$$

where:

- $U(\theta_n)$is a rotational operator;
- $\theta_n$represents the tonal and emotional displacement;
- Each recurrence modifies the global state of the system.

The model is therefore not linear but rotational. The return of D behaves like an intensification operator:

$$D_n = D_{n-1} + \phi_n$$

where $\phi_n$represents the added recursive phase.

In this sense, Bach's tonal structure can be read as a recursive quantum-inspired system: the musical text folds back onto itself, while each return increases the density of the emotional field.

4. Transition Toward Digital and Artificial Intelligence Models

This is the decisive passage.

The musical structure is no longer treated only as an aesthetic object, but as a computable recursive system. The alternation G–D becomes a model for the digital transition toward artificial intelligence because it shows how emotion can emerge from structure, repetition, and measurable transformation.

The sequence:

$$G \to D \to G \to D$$

is equivalent to a formal process:

Stability → Tension → Return → Intensification

This process may be transferred into a computational model where:

- Musical tones become states;
- Repetitions become recursive operators;
- Morphies become detectable units;
- Emotional gradation becomes measurable output.

Thus, the phono-tonal model becomes a bridge between:

Music → Language → Mathematics → Artificial Intelligence

5. Final Synthesis

The co-creation presented here shows that Bach's tonal system, Silvestri's morphic analysis, and Planat's quantum-inspired topology can converge into a unified model.

The return from G to D is not a simple musical alternation. It is a recursive mechanism that produces increasing emotional gradation.

The repeated dominant becomes the sign of a deeper structure: every return carries memory, every repetition transforms the system, every modulation increases the intensity of perception.

In this model:

Repetition is not duplication.

It is transformation.

Recursion is not return.

It is elevation.

Tonal gradation is not ornament.

It is computable emotion.

Concluding Formula

$$(G \leftrightarrow D)^n \Rightarrow \Delta T_n \Rightarrow E_n$$

where:

- G= tonic stability
- D= dominant tension
- n= recursive iterations
- ΔT_n= phono-tonal gradient
- E_n= emotional intensity

Thus:

$$n \uparrow \Rightarrow \Delta T \uparrow \Rightarrow E \uparrow$$

Final Closure

In the recursive return between G and D, the system does not repeat itself: it transforms. Each iteration generates a new phono-tonal state, increasing the density of the emotional field.

Within this framework, tonal recursion becomes a computable process, and emotional gradation emerges as a measurable effect of structural transformation, consistent with a quantum-inspired model of language and sound.

This initial phono-tonal model establishes the perceptual and structural foundation of the co-creative analysis, where tonal recursion is interpreted through linguistic morphic mechanisms and further formalized within a quantum-inspired computational framework, consistent with the topological approach of Michel Planat.

Bibliography and Scientific References

This research endeavors to formalize linguistic structures through a recursive model that integrates elements of linguistics, phono-tonal analysis, and mathematical topology. The methodology involves the detection and quantitative measurement of recursive linguistic configurations within textual corpora. By mapping these structures into a formal geometric space and employing graph coverings, the study aims to establish a deterministic interface between language and computational logic, ultimately constructing a topology of the word.

Key Methodological Components:

1. Identification of Recursive Segments: Linguistic analyses, particularly those by Domenico Silvestri, are employed to decompose text into observable units, termed Recursive Segments. These segments, identified through specific linguistic mechanisms, represent repeating structural configurations. Notable elements include locative deixis (e.g., "here," "there") which activate spatial and semantic relations, and repeated syntactic

structures (morphies) analogous to musical themes, creating recognizable recursive sequences. The detection of non-random repetition, exemplified by the "Dante Case," allows for the quantitative assessment of Recursive Density.

2. Measurement of Phono-Tonal Gradation: The transition from qualitative textual observation to quantitative analysis is achieved through the measurement of phonetic material. This involves "weighting" phonemes and analyzing tonal frequency, where each linguistic unit produces a sound vibration. Empirical analysis, such as that of Dante's cantos, reveals non-linear distributions in vowel and consonant patterns within rhyme structures, suggesting an oscillatory pattern conducive to rotational algebra and quantum phase interpretation.
3. Representation through Graph Coverings: The outcomes of the linguistic analysis are visualized using graph coverings, a method developed by Michel Planat. In this representation, identified linguistic segments function as nodes, and the recursive dependency relations between them form the edges. This topological representation allows complex structures to be described through mathematical and combinatorial models, akin to the analysis of protein structures.
4. Output for Recursive Algebra: Computational formalization culminates in the generation of a Recursive Density Table. This table includes the ordered sequence of morphological units, the Recursion Index (quantifying structural repetition and interlocking, akin to Hofstadter's G-level), and the Fibonacci Coefficient, which verifies the expansion ratio (Golden Ratio) between textual parts.

Theoretical Foundations:

The research is grounded in the convergence of three theoretical frameworks:

- Musicological Model of Tonal Recursion: Drawing inspiration from the recursive structures in Johann Sebastian Bach's compositions, where thematic repetition generates dynamic cyclical patterns, the study posits analogous recursive configurations in literary language.
- Logical-Mathematical Model of Recursive Language Structures: The work of Douglas R. Hofstadter, particularly *Gödel, Escher, Bach: An Eternal Golden Braid*, provides a theoretical basis for describing linguistic structures through recursive processes comparable to those in logical and musical systems. Hofstadter's G-Diagram illustrates infinite recursive structures whose mathematical properties are linked to the Fibonacci sequence. This sequence is interpreted not merely as symbolic but as an indicator of structural expansion and recursive gradation.
- Topological Model of Formal Structures: Michel Planat's application of graph coverings to analyze complex structures across various domains, including poetic texts, offers a framework for interpreting linguistic structures as networks of relations. His recent work extends this to large language models, suggesting that AI-generated linguistic structures can be understood through topological models derived from quantum physics, specifically the theory of anyons and modular tensor categories.

Core Research Axes:

1. Linguistic Recursion and Quantum-Inspired Detection: This axis, rooted in Hofstadter's work, treats recursive mechanisms in language (deixis, morphies, syntactic parallelism) as observable data. These structures are measured through a quantum-inspired detection process, where recursive repetition yields measurable gradations, with the Fibonacci sequence serving as a structural indicator.
2. Recursion and Artificial Intelligence: Extending Hofstadter's perspective on recursion as a principle underlying computational systems, this research transitions from linguistic

detection to formal modeling, leading toward recursive algebra and automata-based systems.

3. Integration with the Bucciarelli Model (NLP Framework): The research is informed by the Bucciarelli Model, which provides a methodological basis for transforming linguistic data into structured, computable representations. This integrated perspective connects linguistic data detection, phono-tonal analysis, recursive structures, and topological representation, treating language as a recursive, measurable, and computationally translatable system.

Case Study: Divine Comedy

The Divine Comedy serves as a primary case study due to its rich recursive configurations. Through sampling tercets, the study aims to detect and measure indicators of recursive density and phono-tonal gradation, enabling a quantitative description of linguistic structures. The model posits that certain linguistic configurations can be interpreted as structural nodes where syntactic recursion, deixis, and phono-tonal gradation converge. This approach facilitates the transition from qualitative linguistic observation to a formal, graph-based representation and ultimately toward a computational model.

CONCLUSIONS

This research work was developed from a solid empirical foundation, based on the systematic detection and extraction of textual and phonetic data. Through controlled sampling and the application of linguistic and phonic analysis methods, it was possible to obtain precise statistical detection data, translating recursive density and phono-tonal gradients into accurate and rigorous mathematical indices.

However, an algorithm, a matrix, or a computational automaton, while offering an impeccable geometric map of the word, cannot exhaust the entirety of the textual experience. Exploring the human soul to extract deep thoughts, decoding syntagmatic orders, and capturing the very essence of the semantics of words—not only within the geometric order of the sentence, but above all within the intimate order of a man's life immersed in a specific era—is neither easy nor simple. Behind every formal recurrence and every "strange loop" of syntax lies the heartbeat of an existence, an expressive and historical urgency that escapes pure numerical quantification and demands an act of profound empathetic and philological understanding.

The accumulated statistical data are therefore not a final arrival point, but a threshold: a solid metric tool through which to catch a glimpse of the complex emotional and conceptual architecture of the human being.

Personal Thought and Dedication:

"To Topology, understood not as a cold geometry of abstract spaces, but as the true shape of the invisible. To this science which teaches us how things can bend, extend, and transform without ever losing their deep identity, I dedicate this journey. The Topology of the Word is, in essence, the topology of the soul: a map of knots, bonds, and recursive returns where the time of a life and the meaning of an era fold onto the text, revealing that the distance between mathematical calculation and human sentiment is merely an illusion of the gaze."

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- **Bucciarelli, R., Terrone, F., Esposito, D., Enriquez, J. J., La Ragione, C., & Rodrigo, A. F.** (2025). *What ChatGPT Has to Say About Topological Structure: The Anyon Hypothesis (From Quantum Computation to Recursive Algorithms: Linguistic Structures, Tonal Gradients, Alliterative Morphies and Formal Models)*. ISBN: 978-88-99640-55-2.

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