



BRILL

There Are No Saints

Or: Quantum Multilocation

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Abstract

Multilocation – the notion of an object being at two places – is a central notion in metaphysics. According to a widespread view, multilocation is problematic but metaphysically possible. In effect, it has been claimed that in a quantum world, multilocation is not simply possible but actual. This article provides a new argument against the latter claim: there is no quantum multilocation.

Keywords

multilocation – exact location – entire location – quantum mechanics

1 Multilocation: Metaphysics and Physics

Multilocation – roughly the notion of being at two places¹ – is a crucial and central topic in metaphysics. The following is an impressive, yet not exhaustive, list of recent works in the metaphysics of multilocation:² Barker and Dowe (2003, 2005), Beebe and Rush (2003), Calosi (2014), Calosi and Costa (2015, Forthcoming), Daniels (2014), Donnelly (2010), Kleinschmidt (2011), Eagle (2010, 2016a, 2016b, 2019), Effingham and Robson (2007), Gilmore (2018), Leonard (2018), McDaniel (2003), Mooney (2018, Forthcoming), Parsons (2007), Sattig (2006). Multilocation also plays a crucial role in different debates in metaphysics, in that some metaphysical theories are usually taken to *entail* multilocation. Classic examples include three-dimensionalism in the metaphysics

1 I will provide a more precise characterization in due course.

2 The list contains both friends and foes of multilocation.

of persistence – see e.g. Balashov (2010), Costa (2017), and Gilmore (2018) – or the doctrine of immanent universals – see e.g. O’Leary Hawthorne and Cover (1998). Finally, multilocation has been *used* in very different metaphysical contexts such as the debate over time-travel – see e.g. Gilmore (2007a), and Mooney (Forthcoming), the debate over the metaphysics of resurrection – see e.g. Butakov (2017), and Hudson (2010), and the fate of the identity of indiscernibles – see e.g. O’Leary-Hawthorne (1995), and Calosi and Varzi (2016).

The orthodoxy is that multilocation is problematic but it is at least *metaphysically* possible. A question that is seldom asked is whether multilocation is *physically* possible, or even *actual*. This is of interest not only in and of itself. If multilocation turns out to be physically possible, if not even actual, an argument for its metaphysical possibility is easily available given that physical possibility is usually taken to entail metaphysical possibility. Strangely enough defenders of the possibility of multilocation have – at least to my knowledge – never looked at quantum mechanics (QM). QM is of particular relevance because it seems, at first sight, *hospitable to multilocation*. In effect, multilocation has been considered, if not endorsed, in the quantum domain numerous times. Let me indulge a little. Here is a quote from a physics text (Gisin, 2014: 44, italics added):

Formally, this indeterminacy is expressed by means of what is known as the superposition principle. If an electron can be here or a meter to the right of here, then this electron can also be in a superposition state of here and a meter to the right of here, that is, both here and a meter to the right of here. In this example, the electron is delocalised *in two places at once*.

To stay within physics, this is a passage from Feynman’s Nobel prize acceptance lecture:³

I received a telephone call one day at the graduate college at Princeton from Professor Wheeler, in which he said, ‘Feynman, I know why all electrons have the same charge and the same mass’ ‘Why?’ ‘Because, they are all the same electron!’

3 Available at <https://www.nobelprize.org/prizes/physics/1965/feynman/lecture/>.

Clearly, according to Wheeler, the only electron is massively multilocated.⁴ Here is a passage from a recent philosophical introduction to QM (Maudlin, 2019: 101, italics added):

If the wavefunction is complete, then the electron is no more on the one side than on the other. One might be inclined to say that it is “smeared out” between the two locations in space, or *that it is somehow in both locations*.

Popular expositions usually voice the same attitude. Take this very recent example in the *Scientific American* (May 2018, italics added):⁵

It is the central question in quantum mechanics, and no one knows the answer: What really happens in a superposition – the peculiar circumstance in which particles seem *to be in two or more places or states at once?*

And this one in the *Times Literary Supplement* (italics added):⁶

That means an object *can simultaneously exist in two places* or, as in the case of the famed cat, two states: dead and alive.

As well as *Science* (italics added):⁷

It’s one of the oddest tenets of quantum theory: a particle can be *in two places at once*.

Multilocation creeps in even in literary masterpieces dealing with QM (Stoppard, *Hapgood*, Act II, Scene 5, italics added):

An electron *can be here and there at the same moment (...)* It can *pass through two doors at the same time*, or from one door to another by a path which is there for all to see until someone looks.

4 Feynman continues: “I did not take the idea that all the electrons were the same one from him as seriously as I took the observation that positrons could simply be represented as electrons going from the future to the past in a back section of their world lines. That, I stole!”

5 <https://www.scientificamerican.com/article/quantum-physics-may-be-even-spookier-than-you-think/>.

6 <https://www.the-tls.co.uk/articles/erwin-schrodinger-misunderstood-icon/>

7 <https://www.sciencemag.org/news/2020/09/one-quantum-physics-greatest-paradoxes-may-have-lost-its-leading-explanation>.

This is suggestive. Perhaps the best argument for the metaphysical possibility of multilocation is that, according to QM, multilocation is simply *actual*. This would be a dream come true for multilocation theorists. Given its promise, it is worth looking into QM. This is what the article is about: an exploration of quantum multilocation. Ironically, I will argue that QM, far from vindicating multilocation, actually tells us that it is not physically possible. The roadmap: in §2 I set the stage. In §3 I put forward the quantum argument against multilocation. Finally, in §4, I address an objection and a consequence of such an argument.

2 Setting the Stage

First, we need some stage setting. It all starts with some restrictions. First, we will only be interested in *concrete material* objects. Second, we will restrict our attention to one instant of time, so that we will not consider multilocation at different times.⁸ Third we will be assuming a substantialist metaphysics of space, according to which spatial regions are distinct and exist independently of material objects that might be located at them. Finally, we will take our location relations to be such that the first argument is a material object whereas the second argument is a spatial region.⁹ With these restrictions in hand we can provide a first stab at characterizing multilocation – the subscript \neq standing for “distinct”:

Multilocation _{\neq} . Multilocation is for an object o to be exactly located at two *distinct* regions of space r_1 and r_2 .

Multilocation _{\neq} is arguably the most general formulation in the metaphysics literature. The most interesting *case* of multilocation – and the one most of the works cited in §1 focus on – is when the regions r_1 and r_2 are not just distinct but *disjoint*, i.e., r_1 and r_2 do not share any part:

Multilocation. Multilocation is for an object o to be exactly located at two *disjoint* regions of space r_1 and r_2 .

8 This is what Mooney (Forthcoming) calls *synchronic* multilocation. Arguably, the most widely accepted argument in favor of *synchronic* multilocation for objects, is an argument from the possibility of time-travel.

9 Given that we are really restricting to one instant, we do not need to take the locative relations to be three-place relations, the third argument being an instant of time.

In the rest of the article I will mostly be concerned with Multilocation rather than Multilocation_x. Multilocation mentions a particular location relation, namely *exact location*. The following are the usual informal glosses on it (Parsons, 2007: 203; Gilmore, 2018: 6):

My exact location is like my shadow in substantival space.

An entity x is exactly located at a region y if and only if x has (or has-at- y) exactly the same shape and size as y and stands (or stands-at- y) in all the same spatial or spatiotemporal relations to other entities as does y .

What is the line of argument from QM to Multilocation? Arguably it is something like the following. To every physical system s , QM associates a Hilbert space – the space of all possible states of s . States are represented by vectors in that Hilbert space, and are usually written as $|\psi\rangle$. Let $|r_1\rangle$ represent the state: system s is exactly located at region r_1 , and let $|r_2\rangle$ represent the state: system s is exactly located at region r_2 . Then, according to QM the following superposition state:

$$|\psi\rangle = c_1|r_1\rangle + c_2|r_2\rangle \quad (1)$$

where $|c_1|^2 + |c_2|^2 = 1$ represents another possible state of s . The superposition state (1), so the thought goes, is a state that represents s as being multilocalized at r_1 and r_2 .

Now, physical properties, or observables, are represented in QM by self-adjoint operators \hat{O} that are – via the so-called Spectral Decomposition Theorem – associated with a weighted sum of projection operators \hat{P} .¹⁰ We will be interested in the position operator \hat{Q} . This is because I will simply assume that “system s has definite position r ” is equivalent to “system s is exactly located at region r ”. There is a standard way of attributing definite values of observables to systems in different quantum states,¹¹ such as “having the definite position r ”: it is the so called *Eigenstate-Eigenvalue Link*. Roughly this

10 Note that the algebraic structure on regions of space that the spectral theorem replicates in the additive structure of spectral projections means that a single region can be composed of disjoint regions. Regions of measure 0 correspond to the empty set.

11 As Lewis puts it, the Eigenstate-Eigenvalues Link is a “a fairly standard way of understanding quantum states” (Lewis, 2016: 76). This is not to say that the link is uncontroversial, in particular the *only if* part. Note that the arguments in §3 do not use such a controversial part of the link.

amounts to the claim that a system s has a definite value ν of an observable O iff the state of s is an eigenstate of \hat{O} belonging to eigenvalue ν .¹²

One pressing problem is that, strictly speaking, the position operator \hat{Q} does not admit of eigenstates. One could then push this line of argument: strictly speaking, no quantum system has ever a definite position or exact location. A fortiori, it cannot be multilocated either. Another, and related worry, is that the position operator \hat{Q} gives us *expectation values* that are best interpreted as providing the likely distribution of an *ensemble* of systems in a given state – rather than say, the properties of individual systems. To move past these initial problems, there is a somewhat standard move. To give multilocation the best fighting chance I am just going to *assume* that such a move is on the right track.¹³

First we divide space in small, course-grained, discrete, disjoint regions r_1, r_2, \dots, r_n . Then we associate to each of these regions a projection operator \hat{P}_{r_i} . This projection operator represents a quantum observable that corresponds to an experimental question – as every projection operator does. In particular the experimental question associated with projection \hat{P}_{r_i} is roughly the following: is the system s located *within* r_i ? This construction is useful insofar as the discrete projection operators \hat{P}_{r_i} do admit of eigenstates. Furthermore, Wightman (1962) shows that these projection operators are enough to uniquely determine \hat{Q} . Thus, quantum systems can be said to have the *locational* property associated with the projector \hat{P}_{r_i} – and relative eigenvalue – when their state is in an eigenstate of that projector. This is exactly what Wightman (1962: 847, notation changed, italics added) concludes:

They must be projection operators, because they are supposed to describe a property of the system, *the property of being localized* in r_i .

What kind of locational property is “being localized in r_i ”? We saw that it cannot be, strictly speaking, *exact location*. The experimental question the projection operators provide an answer to suggests that it is what in the literature is known as *entire location*. In effect, if a system is in an eigenstate of, say \hat{P}_{r_i} , it means that the system is *entirely* within region r_n , is confined in r_n so

12 There are several good philosophical introductions to QM. The interested reader can start from Albert (1992) where the machinery used in the article is introduced and discussed.

13 See e.g. Lewis (2016) for a general overview. The more detailed technical machinery originates in Mackey (1963). Wightman (1962) provides an application. For an insightful introduction see Pashby (2016).

to speak.¹⁴ Pashby (2016) offers a similar account – albeit his considerations are slightly different.¹⁵

3 Against Quantum Multilocation

Now everything is in place to provide a quantum argument against multilocation. Before I provide what I take to be the best quantum argument against multilocation (§3.3), it is instructive to discuss some considerations – if not a fully fledged argument – in Pashby (2016) for the same conclusion (§3.2). This is important not just because it paves the way to the argument in §3.3. It is also important because it shows what I take to be a potential drawback of Pashby’s overall proposal. This, in turn, establishes something significant about the location of quantum systems (§3.4) – or so I contend. Both the arguments rely on what I will call the *quantum argument against entire multilocation* (§3.1), to which I now turn.

3.1 *The Quantum Argument against Entire Multilocation*

First of all, let me introduce another metaphysically interesting sense of multilocation, that is cashed out directly in terms of entire location, namely the following – where the subscript *E* stands for “entire”.¹⁶

Multilocation_E. Multilocation_E is for an (atomic)¹⁷ object *o* to be entirely located at two disjoint regions r_1 and r_2 , that is to say, to be entirely located at r_1 , and to be entirely located at r_2 .

14 Suppose one is not convinced by the arguments above. She will hold that when a system *s* is in an eigenstate of the projection operator \hat{P}_{r_1} , *s* is *exactly*, rather than *entirely* located at r_1 . This will not make things easier for the multilocation theorist. In fact, it will make things worse. For now we could run the arguments in §3 directly in terms of *exact location*.

15 Pashby notes that “more generally, this is a Projection-Valued Measure (PVM) which maps an element Δ of the σ -algebra of Borel sets of \mathbb{R}^3 to the projection \hat{P}_Δ ” (Pashby, 2016: 278).

16 Exploring different senses of multilocation is clearly interesting but goes beyond the scope of this article. For some insights, see Correia (Forthcoming).

17 The “atomic” proviso is needed to deal with the following possible complication. Suppose o_1 is exactly multilocalized at disjoint regions r_1 and r_3 , and suppose o_2 is exactly uniquely located at r_2 . One might want to argue that the mereological sum of o_1 and o_2 is entirely multilocalized at the sum of r_1 and r_2 , and at the sum of r_2 and r_3 . But clearly these regions are not disjoint. I will omit the proviso from now on.

The argument is that quantum multilocation_E – in the precise sense detailed above – is impossible. In other words, a quantum system cannot be located at *both disjoint regions r_1 and r_2 separately*. Consider a region r , which is the sum of two disjoint subregions r_1 and r_2 . And suppose that s is in an eigenstate of \hat{P}_r . Clearly there is a sense in which s is entirely located at the two disjoint regions r_1 and r_2 . It is entirely located at r_1 and r_2 *collectively* so to speak, by being entirely located in r . Note that however this would not count as a case of Multilocation_E. To count as a case of Multilocation_E the system s would have to be entirely located *both* at r_1 and r_2 *separately*. But this cannot be the case. Let us see why. If the two regions r_1 and r_2 are *disjoint*, quantum mechanics dictates that $\langle r_1 | r_2 \rangle = 0$. And this in turn entails that if s is entirely located at r_1 it cannot be entirely located at r_2 . To wit, consider state (1) again. That is a state in which s is entirely located at $r = r_1 + r_2$ – where “+” stand for the notion of (binary) mereological sum, but is neither entirely located at r_1 nor at r_2 . This provides already the crucial insight that, in a quantum world, there is no Multilocation_E. Let me phrase the argument slightly differently.¹⁸

Recall that, in general the probability that a quantum system s in state $|\psi\rangle$ is entirely located at r_n is simply given by the Born rule:

$$0 \leq \langle \psi | \hat{P}_{r_n} | \psi \rangle \leq 1 \tag{2}$$

Now, suppose a system s in state $|\psi\rangle$ is indeed entirely located at region r_1 . Then the probability of s being entirely in r_1 should be = 1, i.e.:¹⁹

$$\langle \psi | \hat{P}_{r_1} | \psi \rangle = 1 \tag{3}$$

Clearly, the probability of any system being in space S is 1 as well. Thus we have:

$$\langle \psi | \hat{P}_{r_1} | \psi \rangle = 1 = \langle \psi | \hat{P}_S | \psi \rangle \tag{4}$$

18 Thanks to an anonymous referee for pushing me on the very formulation of Multilocation_E and on some details of the argument in its favor.

19 Objection. Perhaps in the quantum case we should relax this requirement and claim that a system s is entirely located at r_n when $\langle \psi | \hat{P}_{r_n} | \psi \rangle \neq 0$. Two replies. First, this is completely revisionary – but see Albert and Lower (1996). If a system s has a definite-value property, the usual application of the quantum formalism dictates that the Born-rule attributes probability 1 to s having that definite value property. If not, we would not have any means to claim that our experimental procedures are ever reliable. Second, one can run the argument using the *if* part of the *Eigenstate-Eigenvalue Link*. Suppose the system is in an eigenstate of \hat{P}_{r_n} . Then, $\langle \psi | \hat{P}_{r_n} | \psi \rangle = 1$, and s is entirely located at r_n . Then, run the same argument in the main text. Thanks to Alessandro Giordani here.

Note that, as I pointed out already, for any disjoint region r_n , $\langle r_n | r_1 \rangle = 0$. It then follows from (4) that, for any region r_n which is disjoint from r_1 the probability of s being in $r_n = 0$.²⁰ If there were a non-zero probability to find s in a region r_n that is disjoint from r_1 , the probability of s being *either* in r_1 or in r_n would be simply the sum of the respective probabilities. This sum would be greater than 1, which it can't be – just look at (2). The claim is now that the reason behind the probability of s being (entirely) located in $r_n = 0$ is that s is *not* (entirely) located in r_n .²¹

The argument so far establishes that quantum systems *cannot* be entirely multilocalized at disjoint regions – in the precise sense captured by Multilocation_E, that is, entirely located at two disjoint regions *separately*. Hence the name of the argument. It establishes the modal conclusion insofar as the argument is based on a quantum law, namely the Born-rule.²² In other

20 Pashby (2016: 280) provides a similar, more detailed argument.

21 Objection: QM requires an interpretation. Without an interpretation, it is unclear what to make of the argument. Four replies. First, one may simply deny the claim. As a matter of fact, some authoritative physicists *do deny* the claim. See e.g., Fuchs and Peres (2000). Second, one may hold the view that interpretations are supposed to recover the results of the standard Hilbert space formalism, which has been used here. Pashby (2016) argues exactly along these lines: “However, the Hilbert space formalism I have made use of here has a good claim to be regarded as the canonical form of quantum mechanics, in the sense that it is generally the predictions of the Hilbert space formalism that a viable interpretation is required to replicate, rather than vice versa” (Pashby, 2016: 302). Third, and relatedly, the argument really depends only on the application of the Born-rule to find probabilities for position of quantum systems such as electrons and atoms. Most of the live realist interpretations – e.g., many worlds, spontaneous collapse theories – are designed to recover the Born-rule, one way or the other. The main argument would not go through for those interpretations that add some *primitive ontology* – such as Bohmian mechanics – in which the positions of quantum systems are not attributed on the basis of the quantum state. I am going to simply *grant* that you can resist the argument in e.g. Bohmian mechanics. However, I will note that endorsing a particular interpretation of QM *only* because it saves *multilocation* strikes me as poor methodology. The possibility of multilocation should not be the deciding factor in choosing an interpretation of QM – if one maintains that such an interpretation is needed. Finally, and importantly: the framework I am using in the article is exactly the framework that has been used to suggest that QM is hospitable to multilocation. It is therefore a legitimate dialectical move to use that very framework for my argument.

22 One may hold that, independently of the Born-rule, and thus independently of QM, we should still say that if a physical system is entirely located at a region r_1 , then the probability of the system to be in region $r_1 = 1$. However, in classical theories we do not have explicit probabilistic considerations for property attributions. That being said, this will make the case against multilocation even stronger. For the case against multilocation will not rest on QM alone. Rather, it would rest on a much more general way of attributing properties to physical systems that is allegedly common to (almost) all of our physical theories.

words: entire multilocation at disjoint regions is *incompatible* with a quantum law, i.e. the Born-rule.

Before moving on, let me discuss a possible worry. The worry is that it is *analytic* of the notion of *entire location* that if something is entirely located at r , it is not entirely located at any other disjoint region. In other words, quantum details and quantum mechanics are *irrelevant* for the argument. To see that this is not the case, we have to dig a little deeper into theories of location. Let me take the lead from Parsons (2007) and Gilmore (2018). We already encountered two notions, *exact location*, and *entire location*. We need only another one, namely *weak location*. Informally, something is weakly located at region r iff r is not completely free of r – see Parsons (2007: 203). Parsons (2007) presents two theories of location, one that takes *exact location* as primitive, and one that takes *weak location* as primitive. The latter entails a principle called Functionality as a theorem:

Functionality. If x is exactly located at r_1 and exactly located at r_2 , then $r_1 = r_2$.

In other words, Functionality is the denial of multilocation. Clearly, not to beg the question against the possibility of quantum multilocation, one cannot subscribe to such a location theory.²³ Thus, one can take *exact location* as a primitive and define *entire location* as follows: x is *entirely located* at r_1 iff there is a region r_2 such that x is exactly located at r_2 and r_2 is a subregion of r_1 . The crucial detail is that without Functionality, one cannot derive the following principle of Exclusion from the definition of *entire location* alone:

Exclusion. If x is entirely located at r , it is not weakly located at the mereological complement of r .

Note that Exclusion is the natural way to cash out precisely that it is analytic of *entire location* that it entails the denial of Multilocation_E. To see that Exclusion doesn't follow from the definition of *entire location* alone, just consider the simple case in which x is indeed exactly located at two disjoint regions r_1 and r_2 . This is indeed possible in the absence of Functionality. Note that, by definition of *entire location*, both r_1 and r_2 count as *entire locations* of x , even if they are disjoint. This argument, I contend, takes care of the worry. There is no

23 There are other location theories that take *weak location* as primitive and allow for multilocation. Yet, they are deeply problematic. See Eagle (2016b) and Calosi and Costa (Forthcoming).

analytic entailment from the very notion of entire location to the impossibility of Multilocation_E.²⁴

3.2 *Pashby's Argument*

Based on the quantum details in §3.1, Pashby (2016) offers considerations – if not a fully fledged argument – against quantum multilocation. However, there are several ways out of the argument on behalf of multilocation theorists. Or so I am about to argue. Pashby's argument, in all fairness, does not depend on the full force of the quantum argument against entire multilocation. It does depend crucially however upon the claim that if a quantum system s is in an eigenstate of \hat{P}_{r_n} , then s is entirely located at r_n . Officially, Pashby takes exact location to be his primitive location relation, and then defines *weak location* – which we already encountered – as follows: x is weakly located at region r_1 iff x is exactly located at a region r_2 which overlaps r_1 – more on this later on. Pashby's choice is not without controversial consequences. As Parsons (2007) already notes, this choice entails the locative principle known as Exactness:

Exactness. If x is weakly located at a region r_1 , there is a region r_2 such that x is exactly located at r_2 .

In effect, given that weak location is the weakest location relation, it follows that any object that bears any locative relation to any region has an exact location.²⁵ For the purposes of the article I will mention one example, which I will call for lack of imagination Entire Exactness:²⁶

Entire Exactness. If x is entirely located at a region r_1 , there is a region r_2 such that x is exactly located at r_2 .

Both principles seem to suffer important counterexamples in the quantum realm. It has in fact been pointed out multiple times – and we saw it again here, in §2 – that quantum systems can lack a precise position, i.e. an exact location.²⁷ This should be enough to have some doubts about Pashby's pro-

24 Thanks to an anonymous referee for pushing me on this.

25 To see this, consider the following. For any locative relation L , we have that, if a system is L -related to a region r , it is also weakly located at r .

26 This is because it is a theorem that, if x is entirely located at region r , it is also weakly located at region r .

27 The literature on this is literally too vast to mention, so I will limit myself to some examples. Fathers of the quantum theory held the view, see e.g. Heisenberg (1956). Classic works in the metaphysics of quantum mechanics, such as Forrest (1988) defend the view at length. Recently it has been discussed and endorsed in Jäger (2014) and Lewis (2016).

positional. And, as we will see in a minute, it infects his argument against multi-location.²⁸ It should be noted however, that Pashby does not simply rely on Parsons's definitions and principles of location. In effect, he provides a characterization of *exact location* that crucially depends on (i) quantum details, and (ii) some algebraic details of the spatial regions that represent viable exact locations of quantum objects. Let me start from (ii). Pashby takes spatial regions that are viable exact locations of quantum systems to be represented by the regions of positive (Lebesgue) measure in the Borel σ -algebra of \mathbb{R}^3 . This means that regions of Lebesgue measure = 0 – i.e., spatial points and countable unions thereof – are not viable exact locations. More importantly, the following is a crucial consequence of such a choice:

Gunky Viable Regions. For any region r_1 that is a viable exact location of a given quantum system s , there is always a *proper subregion* r_2 of r_1 that is a viable location. In other words, even in a point-set space, viable exact locations are *gunky*.²⁹

Given Gunky Viable Regions Pashby can offer the following characterization of quantum exact location:

Quantum Exact Location. A system s is exactly located at r_1 iff (i) $\hat{P}_{r_1} |\psi\rangle = |\psi\rangle$ and there is no proper subregion r_2 of r_1 such that $\hat{P}_{r_2} |\psi\rangle = |\psi\rangle$.

In light of the above, it could be argued that Pashby is really taking *entire location* as a primitive. Be that as it may, I first want to argue that (i) Gunky Viable Regions is problematic, and (ii) this undermines Pashby's characterization of Quantum Exact Location.

Pashby himself endorses the informal gloss on exact location I gave in §2 – Pashby (2016: 286). But this is in clear tension with Gunky Viable Regions. The tension is best appreciated considering the following two claims: (i) some quantum systems can have *minimal extension*, and (ii) viable regions do not have such minimal extension. Claim (ii) follows simply from the fact that viable regions are both gunky and have positive (Lebesgue) measure. Claim (i) is best appreciated via some examples. An hydrogen atom cannot be smaller

Proponents of the so called *quantum indeterminacy*, such as Bokulich (2014), or Calosi and Wilson (2018) argue that quantum systems lack definite position in superposition and entanglement cases.

28 Much of the following discussion is indebted to insightful suggestions of a referee for this journal.

29 An object x is *gunky* iff every part of x has further proper parts.

than any of its extended proper parts. The same goes for quantum buckyballs, i.e., molecules with over 60 atoms of carbon.

Viable exact locations of hydrogen atoms cannot be regions that are smaller than say a proton, nor can viable locations of a quantum buckyball be smaller than a carbon atom, *contra* Gunky Viable Regions. This is problematic in its own right, but also because it vitiates Pashby's characterization of quantum exact locations. To see this, consider a region r that is as big as the UK, for the sake of simplicity. Divide r into disjoint n -subregions of equal measure r_1, r_2, \dots, r_n . Suppose an hydrogen atom is in the state:

$$|\psi\rangle = \frac{1}{\sqrt{n}} (|r_1\rangle + |r_2\rangle + \dots + |r_n\rangle) \quad (5)$$

Then, according to Quantum Exact Location, the hydrogen atom is exactly located at r – that is, it is as big as the UK. This should sound problematic at best. But suppose one insists that quantum objects do not have definite extensions, and in fact can be as big as the universe. Here is another argument for the claim that Quantum Exact Location is problematic. Consider two quantum systems s_1 and s_2 , say two electrons, and consider *two* regions r_1 and r_2 such that r_1 is n -meters apart from r_2 – clearly, $n \neq 0$. Now suppose the two quantum systems are in the entangled state:

$$|\psi\rangle_{12} = \frac{1}{\sqrt{2}} (|r_1\rangle_1 |r_2\rangle_2 + |r_1\rangle_2 |r_2\rangle_1) \quad (6)$$

According to Quantum Exact Location, *both* s_1 and s_2 are exactly located at $r_1 + r_2$. That is, they are *co-located*. But the usual interpretation of (6) is that s_1 and s_2 are n -meters apart. That means they do not have the same exact location. That is, they are *not co-located*. Let me be clear. The point is not that Pashby's theory of quantum location is irremediably flawed. The point is rather that it is problematic enough that friends of multilocation can simply refuse to accept it. And this is important, for Pashby's argument against quantum multilocation crucially depends on the details of such theory of quantum location. Such an argument proceeds by (i) first establishing Entire Exactness, and then (ii) Functionality.

As for (i), here is the relevant passage, where Pashby (2016: 283) considers a system that is entirely but not exactly located at r_1 :³⁰

Proof. Assume for reductio that there is no subregion $r_2 \sqsubset r_1$ at which the system is exactly located. By assumption, the system is entirely located

³⁰ The notation has been changed for the sake of consistency: \sqsubset is the relation of proper parthood.

at r_1 . But it is not exactly located at r_1 , in which case there exists some proper subregion $r_2 \subset r_1$ such that $\hat{P}_{r_2}|\psi\rangle = |\psi\rangle$. *However, since the system is not exactly located at r_2 either, there must exist a proper subregion of r_2 at which the system is entirely located. Evidently this process can be continued indefinitely, each time removing a region of finite measure from consideration.* Therefore, in the limit we reach a region of Lebesgue measure zero at which the system is entirely located. But the probability of being found at any such region is zero. Contradiction.

The problematic passage is in italics. It is clear that the argument crucially depends on both (i) the characterization of Quantum Exact Location, and (ii) Gunky Viable Regions. First, note that the proof explicitly uses Quantum Exact Location *verbatim*. Second, the claim that “the process can be continued indefinitely” is secured only by Gunky Viable Regions. But, as we saw, these are both problematic. The proof of Entire Exactness is problematic as a result.

Pashby (2016, 285) then establishes Functionality by claiming, simply, that

[S]ince an object can be exactly located at (at most) one spatial region at a time, this seems to make *multiple* location impossible. We saw this in the way that the relation of exact location defines a map between objects and spatial regions that is a (partial) function: every object is mapped to (at most) one region. In contrast, an object multiply located at two distinct regions would be mapped to two regions, defining instead a so-called ‘multifunction’ or set-valued map.

This is spelled out a few pages earlier (Pashby, 2016: 276):

If every object $x \in O$ is exactly located at (at most) a single region then this relation [i.e. exact location] defines a (partial) function $f: O \rightarrow S$.

where O and S are (suitable sets for) the domains of quantum objects and regions respectively. This strikes me as problematic as well. Pashby claims that, once exact locations are secured by the reasons above – i.e., the ones I just criticized – this relation defines a function from the domain of quantum systems to regions of space. Yet, in the crucial passage quoted above, no reason is provided as to why this is in fact the case. Pashby just notes that *if* a system is exactly located at most at a single region, *then* exact location defines a (partial) function. But the antecedent is exactly what is at stake here. No reason seems to be provided for its truth, and the multilocation theorist has yet to see a quantum argument against her beloved metaphysical thesis.

The previous arguments show two things. First, they highlight a potential drawback in Pashby's account of the location of quantum objects. As I already pointed out, this drawback might not be a fatal one. Yet, it is serious enough for multilocation theorists to doubt the crucial ingredients of Pashby's argument against quantum multilocation, namely Gunky Viable Regions, Quantum Exact Location, and Entire Exactness. They will insist that a quantum theory of location should give up all of them. In particular, they would go on, a quantum theory of location should be a theory according to which quantum systems can be entirely located at different regions – even if not disjoint ones, as they learned from the argument in §3.1 – *without thereby having an exact location*. Second, and relatedly, they show that an argument against quantum multilocation is yet to be provided. It is to this argument that I now turn.

3.3 *The Quantum Argument against Multilocation*

I take the upshots of the argument in §3.2 to be the following. First, it is unclear whether there is an argument against quantum multilocation at all, let alone a convincing one. Second, we should at least consider the possibility that quantum systems do not always have an exact location, *pace* Pashby. How can we construct a quantum argument against multilocation, then, without committing to the claim that quantum systems always have exact locations? I think we can construct a surprisingly simple – and brief – argument. We just need to recall that the argument in §3.1 against Multilocation_E was left unscathed. And then, simply point out that Multilocation entails that there are two disjoint regions such that an object is entirely located at both. In every theory of location in the market – the one in e.g. Casati and Varzi (1999), the ones in Parsons (2007), and the one Gilmore (2018) – this follows trivially from the definition of entire location. In effect, it does follow e.g., from the definition of entire location I offered in §3.1. Indeed, it seems to be a plausible general requirement that every theory of location features the following claim as either an axiom or a theorem: every exact location counts as an entire location. An exact location of something is, so to speak, (one of) its “smallest” entire location(s). The point seems to be uncontroversial. If x is *exactly located* at a region r_1 , it is fully within r_1 , that is, it is entirely located in r_1 . If it were not entirely located in r_1 , there would be a region r_2 , such that we could find (part) of x in r_2 . That would provide reason enough to deny that x is exactly located at r_1 after all.^{31, 32}

31 Depending on the theory of location one endorses, this last claim can be rigorously translated in different ways. Here is a general one: x is weakly located at r_2 – where weak location is the locational notion in §3.1.

32 What about state (1), you may ask. Well, when a system s is in state (1) it is *not* entirely located at r_1 . To see this, note that $\langle \psi | \hat{P}_{r_1} | \psi \rangle = |c_1|^2 < 1$. The same goes for r_2 .

Given what we said so far, Multilocation entails Multilocation_E whereas the converse does not hold – for instance one might hold that the relevant quantum system lacks an exact location, exactly for the reasons I set forth in §3.2. In any case, I argued that QM entails that Multilocation_E is physically impossible. Therefore, according to QM, Multilocation is physically impossible as well.³³

3.4 A Very Brief Comparison

Before addressing a potential objection to the quantum argument I want to compare briefly the proposals in Pashby (2016) and the one presented here.³⁴ In the end, given the argument in §3.3, I agree with Pashby that there is no multilocation in a quantum world. Doesn't this raise a worry that there is really no tension between my account and Pashby's? I do take it that there is a significant *difference* between the two. This difference is best appreciated by looking back at state (1). Here it is again for convenience:

$$|\psi\rangle = c_1|r_1\rangle + c_2|r_2\rangle$$

According to Pashby, we have all of the following: (i) system *s* is entirely located at $r_1 + r_2$; (ii) system *s* is neither entirely located at r_1 , nor at r_2 ; (iii) system *s* is *exactly located* at $r_1 + r_2$. According to the present proposal one has both (i) and (ii) but, crucially, *not* (iii). Rather what one gets is (iv): system *s* has *no exact location*. This is, I take it, the main lesson from the arguments in §3.2 and §3.3. More generally, according to Pashby, both Exactness and Functionality are true of a quantum world. According to the proposal discussed here, only Functionality is true. Exactness is not. Clearly, the two proposals entail the same consequences for the possibility of Multilocation, but they do differ significantly over other crucial details about the location of quantum objects.

4 An Objection and a Consequence of the Quantum Argument

The arguments in §3 are about *quantum systems*, or *quantum objects*. What are they? Roughly, they are those entities that obey the laws of QM. Typical examples include electrons, atoms, and even molecules such as quantum buckyballs. For all those entities, we have experimental evidence that they obey quantum laws, for they all exhibit quantum interference effects in particular experimental settings – such as the infamous double-slit experiment.

33 It is a substantive question whether the argument can be generalized to Multilocation_z.

34 Thanks to an anonymous referee.

In §2 however, I characterized multilocation as an object, in particular a *concrete material object*, being exactly located at two disjoint regions. On the one hand, one would be hard-pressed not to consider some quantum systems as material objects. An electron, a hydrogen atom, a quantum buckyball have all of the hallmarks of material objects. On the other hand, a possible objection has it, even if (some) quantum systems are indeed material objects, there are material objects that *are not quantum systems*. In general, so the thought goes, macroscopic objects such as chairs, tables, planets and the like do not obey the laws of QM. For example, we never observe quantum interference phenomena for macroscopic objects.³⁵ They are not quantum systems, one might claim. I am going to be overtly charitable and grant that this is the case. But, if so, the objection continues, the arguments in §3 do not establish that multilocation is physically impossible. They only establish that multilocation is physically impossible *for quantum systems*. Given that I have conceded that some material objects are not quantum systems, the physical possibility of multilocation – even with respect to QM alone – is safe, at least for *some* material objects.

My reply consists in pointing out a consequence of the quantum argument against multilocation. It rests on the following claim, which is *less controversial* than the claim that macroscopic objects are not quantum systems. Here is the claim:

Quantum Parts. Every material object has some quantum systems as parts.

I am confident that my opponents, given my concession that macroscopic objects do not obey the laws of QM, will concede me as much.

If Quantum Parts is conceded, the quantum arguments establish that any multilocated material object has a *complete different quantum mereological structure* at distinct exact – or even entire – locations. With “quantum mereological structure” I simply mean to refer to those parts of the object in question that are quantum systems. This is because the arguments in §3 establish – and the objection grants this much – that those systems cannot be multilocated. Therefore, if an object *o* is multilocated at regions r_1 and r_2 it has completely different electrons, completely different atoms, and even some

35 This is due to environmental decoherence, roughly the suppression of interference effects due to suitable interactions with the environment. One might respond that the distinction between microscopic and macroscopic objects is vague at best. I don't need to push this line of argument here. I will just concede that some material objects are not quantum systems – a generous concession indeed.

different molecules at those regions. It *cannot* retain *any* of its quantum parts at those locations. Now, this is not flat-out inconsistent. But it strikes me as seriously implausible. It should be possible for a multilocated object to retain at least one tiny (quantum) particle at two locations. This is in effect, either explicitly or implicitly, *assumed* by all the defenders of multilocation I mentioned in §1.

To sum up, it seems safe to say that this article adds to the case against multilocation. It might be metaphysically possible, but, insofar as our world is a quantum world,³⁶ there is no multilocation in our world. Multilocated objects are sometimes called *saints*.³⁷ This is because Christian saints were said to be seen at two distinct places at once. The conclusion of the argument can be then phrased as follows: there are no saints in the world. There *neither is, nor can be any place* for sainthood in this world.

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36 That is, our world obeys the laws of QM.

37 See Gilmore (2007b: 227).

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