



Universalism and extensionalism revisited

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Abstract

We present a new notion of mereological sum that is inequivalent to extant ones in the literature and does not fall prey to reasonable complaints that can be raised against some such notions. In light of this notion, we then revisit the relation between mereological universalism and extensionalism. In particular we argue that Varzi's claim to the point that universalism entails extensionalism is justified only insofar as one sticks to Varzi's notion of sum. In effect, we distinguish different versions of extensionalism and argue that universalism—when cashed out in terms of our new notion of sum—entails some versions but not others. Most significantly it does not entail extensionality of proper parthood. In the light of the above we set forth a new mereological system, Universalist Quasi-Supplemented Mereology, that can be considered a worthy alternative to different mereological systems in the literature.

Keywords Classical mereology · Mereological sums · Extensionalism · Universalism · Supplementation principles

1 Introduction

Mereological universalism (**U**) is the thesis that any plurality of entities has a *fusion*. It comes in different versions, depending on how the notion of fusion is characterized. *Mereological extensionalism* (**E**) is the thesis that composite entities having the same proper parts are identical. In this paper we address the relationship between **U** and **E** in light of a novel notion of *sum* that is not equivalent to extant ones in the literature. The attentive reader has probably noted that we just mentioned two notions, that of *fusion*, and that of *sum*. This is because, at this stage, we simply assume a distinction

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between a somewhat “intuitive notion” of *fusion* of a plurality of entities, and the various technical notions of *sum* that are introduced to capture it. Here is the plan. First, we introduce the debate on Universalism and Extensionalism (Sect. 2). We then put forward a notion of general sum that allows us to overcome a number of issues related to the problem of fusing pluralities of entities (Sect. 3). Finally, we show how this notion can be exploited to shed new light on the debate at hand (Sect. 4). The rest of this introduction is dedicated to setting the stage.

The debate on universalism and extensionalism focuses on two related questions:

Q1: What notion of fusion is available to non-extensionalists;

Q2: Whether, once a particular notion of fusion is adopted, **U** entails **E**.

In this respect, Varzi (2009) argues that, of the three notions of sum that are usually recognized in the literature, only one is sufficiently well-behaved so as to be used in the formulation of **U**. And, provided parthood obeys *Transitivity* and *Supplementation*, it turns out that **U** entails **E**.¹ Varzi’s argument has been challenged from two different perspectives. Cotnoir, in (2016), shows that, sticking to the notion of sum that Varzi uses, one can forgo the entailment from **U** to **E**, by changing the notion of proper parthood (specifically, by rejecting a basic principle like the antisymmetry of parthood). In addition, Cotnoir claims that this is in fact the only way to forgo such entailment. This is because, for the sake of argument, he grants Varzi that both *Transitivity* and *Supplementation* fix the meaning of the notion of parthood. As Varzi writes [p. 601]:

If one’s notion of parthood is so different from the standard notion as to violate either (1) [*Transitivity*] or (2) [*Supplementation*], then the meaning of (**U**) and (**E**) would change accordingly and the relationship between such theses might well be up for grabs.

By contrast, both Gilmore in (2009) and Loss in (forthcoming) suggest that replacing *Supplementation* with weaker principles is enough to undercut the entailment from **U** to **E**. As we will see shortly, what is crucial for our purposes is that *since the foregoing accounts depend crucially on the endorsement of the very notion of sum that Varzi adopts, they are forced either (i) to abandon the idea that parthood is a partial ordering or (ii) to accept mereological models where sums present some peculiar features*. Put differently, it seems to us that what Varzi has successfully shown is that, given a certain notion of *sum*, the only mereological systems where parthood is a partial ordering and sums do not present peculiar features are systems where **U** entails **E**. In the present paper we show that, by resorting to an alternative notion of sum, we can *both* preserve the standard account of parthood *and* avoid peculiar mereological models.² In doing so, we address the debate from a radically new perspective. In particular:

¹ We follow the nomenclature of the principles proposed in Varzi (2016), where *Supplementation* is identified with what is also known as *Weak Supplementation*. Varzi’s argument is discussed and criticized in Rea (2010), while (Calosi, 2020) provides a response to that criticism.

² It goes without saying that the models we find peculiar might be perfectly acceptable for some philosophers. In that case, the system we are going to propose can be viewed as a system featuring a notion of sum that, while being as similar as possible to the notion defined in classical extensional mereology, still allows us both to preserve an account of parthood as partial ordering and to reject the entailment from **U** to **E**.

- (1) We put forward a new definition of sum that Varzi, Cotnoir and Loss do not consider, and argue it is immune from the complaints in Varzi (2009);
- (2) We distinguish different principles of extensionality, namely *First Fusion Extensionality*, *Second Fusion Extensionality*, and *Proper Part Extensionality*—the latter coinciding with **E**.³
- (3) We propose a new mereological system that does not include *Supplementation* but includes both *Antisymmetry* and **U**—thus being different from the extant ones in the literature;
- (4) Finally, drawing on (1)–(3), we provide two new answers to the questions Q1 and Q2 above. In particular we argue that there is a notion of sum available to anti-extensionalists, and that, given that notion, **U** entails only some principles of extensionality but not others.

In view of the above, we can indeed reassure Varzi that the relationship between **U** and **E** is not *entirely* up for grabs. In effect, that relationship is *similar* to the one he envisaged, and yet *not identical* to it. On top of that, we contend that the reassessment of the debate provided in this paper is not only important in and on itself. Indeed, we will also be able to amend problematic claims in Cotnoir (2016), and show that the notion of sum in Sect. 3 and the mereological system in Sect. 4 are worth discussing on independent grounds because of their philosophical significance. Let us first introduce the basic formal frameworks we will work with.

A two-sorted first-order logic is assumed, containing constants and variables for individual entities (lowercase letters) and plural entities, or pluralities (uppercase letters).⁴ The following notation is used throughout⁵

$x : X$:	primitive	(x is one of the X)
$a \leq b$:	primitive	(a is a part of b)
$a \ll b$:	$a \leq b \wedge a \neq b$	(a is a proper part of b)
$a \dot{\leq} b$:	$a \not\leq b \wedge b \not\leq a$	(a is incomparable with b)
$X \leq a$:	$\forall x(x : X \rightarrow x \leq a)$	(all X s are parts of a)
$X \ll a$:	$\forall x(x : X \rightarrow x \ll a)$	(all X s are proper parts of a)
$a \circ b$:	$\exists x(x \leq a \wedge x \leq b)$	(a overlaps b)
$a \parallel b$:	$\neg(a \circ b)$	(a is disjoint from b)
$a \circ X$:	$\exists x(x : X \wedge a \circ x)$	(a overlaps some of the X s)
$a \parallel X$:	$\neg(a \circ X)$	(a is separated from all the X s)

There are three notions of sum that are routinely used in the mereological literature,⁶ and are exactly the ones Varzi considers in Varzi (2009):

Definition 1 Notions of sum and extensionality.

³ For the distinction between *Proper Part Extensionality* and *First Fusion Extensionality*, see Varzi (2008).

⁴ We use pluralities to introduce generalized notions of sum. This can be done in different ways, but nothing in what follows depends on the specific way we have chosen. For plural logic see Oliver and Smiley (2013).

⁵ Note Cotnoir, in (2016) uses another notion of proper parthood, that is different from the one we are about to define, namely: $a \ll^* b : a \leq b \wedge \neg b \leq a$. We will be assuming *Antisymmetry*, and in the presence of *Antisymmetry* the two notions are extensionally equivalent.

⁶ See Varzi (2016) and Cotnoir and Varzi (2021).

1. $Sum_1(a, X) := X \leq a \wedge \forall x(x \leq a \rightarrow x \circ X)$ a is a Sum_1 of the items in X if and only if all the items in X are parts of a and every part of a overlaps at least one item in X .⁷
2. $Sum_2(a, X) := \forall x(x \circ a \leftrightarrow x \circ X)$ a is a Sum_2 of the items in X if and only if a overlaps all and only the items that are overlapped by at least one item in X .⁸
3. $Sum_3(a, X) := X \leq a \wedge \forall y(X \leq y \rightarrow a \leq y)$ a is a Sum_3 of the items in X if and only if all the items in X are parts of a and a is part of all the items all the items in X are parts of.⁹

Extensionality (**E**): $\exists x(x \ll a) \wedge \forall x(x \ll a \leftrightarrow x \ll s) \rightarrow a = s$.

E states that composite entities having the same proper parts are identical.¹⁰ Finally, we call Minimal Mereology, **MM**, the mereological system in which the notion of parthood is characterized merely as a partial ordering, i.e., as a reflexive, transitive and antisymmetric relation.

2 The debate

In Varzi (2009) Varzi shows that **U** entails **E** in Supplemented Mereology, **SM**, which is minimal mereology plus *Supplementation*. *Supplementation* (**SP**): $x \ll s \rightarrow \exists y(y \leq s \wedge y \parallel x)$.

Since **U** can be formulated differently using different notions of sum, one obtains various universalist axioms by plugging in such notions in the following schema¹¹

Universalism_i (**U_i**):
 $\exists x(x : X) \rightarrow \exists s(Sum_i(s, X))$.

What Varzi shows is then that **U₁** entails **E**. Let us see how.

⁷ This is what Varzi in (2009) calls “f-fusion”.

⁸ This is what Varzi in (2009) calls “f’-fusion”.

⁹ This is what Varzi in (2009) calls “f”-fusion”.

¹⁰ **E** is a principle of extensionality of parthood. In the literature a principle of extensionality of overlap, stating that two entities overlapping the same entities are identical, is also considered. Here we focus on **E** for two main reasons: firstly, this is the principle discussed in the debate about the connection between Universalism and Extensionalism; secondly, it is not difficult to see that extensionality of overlap entails extensionality of parthood, and therefore arguing against extensionality of parthood implies arguing against that principle.

¹¹ We formulate **U**: as Varzi does, namely by a conditional with the antecedent specifying that the given plurality is not-empty. In many plural logics, such antecedent is indeed redundant.

2.1 From universalism to extensionalism

In **SM** + **U**₁ (the system obtained by adding axiom **U**₁ to **SM**), we can prove the so called *Proper Part Principle* (**PPP**), to the point that any composite entity having as proper parts the proper parts of *a* has *a* as part. And **PPP** ensures extensionality.¹²

$$\text{Proper Part Principle (PPP): } \exists x(x \ll a) \wedge \forall x(x \ll a \rightarrow x \ll s) \rightarrow a \leq s.$$

Proposition 1 *PPP is provable in SM + U*₁

Proof By contraposition. Suppose *a* is composite, and $a \not\leq s$. Then $s \ll a + s$, which is guaranteed to exist given **U**₁. So, for some *x*, $x \leq a + s$ and $x \parallel s$, by **SP**. Thus, $x \circ a$, since $x \parallel s$, so that *a*, being composite, has some proper part that is not a proper part of *s*, which proves the conclusion. □

Proposition 1 establishes that, provided **SM** is assumed, anyone who accepts **U**₁— and thus *Sum*₁ as the basic notion of fusion— is forced to accept extensionality, and therefore the full Classical Extensional Mereology.

2.2 Problems with weaker decomposition principles

It is important to note that having *Sum*₁ as a basic notion of fusion provides justification for accepting a supplementation principle like **SP**. To be sure, consider the weaker decomposition principles discussed in the literature:

Company (**C**):

$$a \ll b \rightarrow \exists x(x \ll b \wedge a \neq x)$$

Strong Company (**SC**):

$$a \ll b \rightarrow \exists x(x \ll b \wedge a \not\leq x)$$

Quasi Supplementation (**QS**):

$$a \ll b \rightarrow \exists x \exists y(x \leq b \wedge y \leq b \wedge x \parallel y)$$

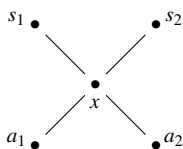
Minimal Supplementation (**MS**):

$$a \ll b \rightarrow \exists x(x \leq b \wedge x \wr a \wedge \exists y \exists z(y \leq a \wedge z \leq x \wedge y \parallel z))$$

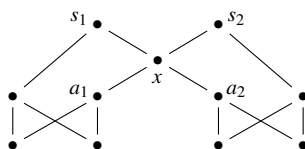
In plain English, **C** requires that any composite object has at least two proper parts; **SC** requires that any object having a part has another part that is not part of the first one; **QS** requires that any composite object has at least two disjoint proper parts; finally, **MS** requires that if something has a proper part it has another part that is incomparable with the first and such that some of their parts are separated. The last two principles are the ones that have been suggested in the literature as potential replacement for *Supplementation* in order to resist Varzi’s argument.¹³ At the present stage of the dialectic we need simply note that they are too weak to exclude the following models:

¹² See Simons (1987) **PPP** is slightly stronger than *Extensionality*. It is indeed not difficult to see that **E** follows from **PPP** by *Antisymmetry*, while there are models of **E** in which **PPP** fails. Here is one: {*a*₁, *a*₂, *b*₁, *b*₂}, where $b_1 \ll a_1$ and $b_1, b_2 \ll a_2$ are the only instances of \ll .

¹³ See Gilmore (2009) and Loss (Forthcoming) respectively.



Model 1a



Model 1b

As we can see, Model 1a satisfies both **C** and **QS**, while Model 1b satisfies all the principles we have introduced. Still, in these models x , s_1 and s_2 are all Sum_1 of all the entities that are below x —where “below” tracks the order induced by the proper parthood relation. This is the peculiar feature that strikes us as problematic: *two things that stand in the proper parthood relation count as sums of the same entities*. Indeed, we find natural to say that, if x is a sum of some items neither a proper part of x nor a proper extension of x should count as a sum of the *same* items.

Summing up what we take to be the two most important consequences of the discussion so far, we have that:

1. If the notion of fusion is appropriately captured by Sum_1 , then we are justified in accepting a system of Supplemented Mereology, given that decomposition principles weaker than **SP** do not rule out models in which Sum_1 behaves problematically.
2. If, in addition, we endorse *Universalism*—in its U_1 variant— then we are justified in accepting Classical Extensional Mereology, given that in Supplemented Mereology, U_1 entails **E**.

Hence, accepting *Universalism* forces us into the following dilemma¹⁴: either accept the full strength of Classical Extensional Mereology or reject the assumption that the notion of fusion is appropriately captured by Sum_1 . However, as we will see shortly, this second horn seems to be precluded.

2.3 From fusion to Sum_1

As we mentioned, one can be tempted to get out of the dilemma by rejecting Sum_1 as an appropriate characterization of fusion. We want to argue that, at the end of the day, this is not a viable option, *provided one only focus on the alternative notions of sum proposed in Sect. 1*. To do this, let us first define a relation of equivalence between pluralities.

Definition 2 $X_1 \equiv X_2 := \forall x(x \parallel X_1 \leftrightarrow x \parallel X_2)$.

Two pluralities are equivalent just in case they are disjoint from the same entities. This notion of equivalence is introduced to capture the specific connection that exists between two pluralities that intuitively amount the “same portion of reality”—to use Lewis’s suggestive phrase—.¹⁵ like the plurality of all cats and the plurality of the cells

¹⁴ Clearly this is a dilemma only for those that did not want to endorse the full extent of Classical Extensional Mereology in the first place.

¹⁵ See Lewis (1991). Here we set aside problems related to vagueness.

of the cats. The basic idea is that these two pluralities are equivalent because what overlaps a cat also overlaps some of its cells and vice-versa. So that what is disjoint from all the cats is also disjoint from all of their cells and vice-versa. It is important to note that the two pluralities—given orthodox assumptions about plural logic, e.g. the standard Plural Comprehension Principle—are *not identical*, and yet are, in a crucial sense, *identically related* so to speak, to the same portion of reality.

Armed with this, we can now distinguish different kinds of extensionalism (here *Fusion* stands for the intuitive notion of fusion that needs to be rigorously framed via different kinds of Sum_i , such as the ones proposed in Sect. 1).

First Fusion Extensionality (FE1):

$$Fusion(s_1, X) \wedge Fusion(s_2, X) \rightarrow s_1 = s_2.$$

Pluralities have unique fusions. This principle gives us a first desideratum on the conditions a fusion has to satisfy, since it excludes that fusing the same portion of reality might produce different entities.

Second Fusion Extensionality (FE2):

$$Fusion(s, X_1) \wedge Fusion(s, X_2) \rightarrow X_1 \equiv X_2.$$

Pluralities with the same fusion are equivalent. This principle gives us a second desideratum on the conditions a fusion has to satisfy, since it excludes that fusing different portions of reality might produce the same entity.

Proper Part Extensionality (E):

$$\exists x(x \ll s_1) \wedge \forall x(x \ll s_1 \leftrightarrow x \ll s_2) \rightarrow s_1 = s_2.$$

This is the principle introduced before. It is not uncontroversial as a desideratum, but can be justified by **FE1** if one requires that any entity is identical with the fusion of its proper parts.

We will now argue that both Sum_2 and Sum_3 are controversial as regimentations of the notion of fusion, due to the fact that they do not satisfy the first two desiderata. Indeed, the following propositions are now provable.

Proposition 2 *In $SM + U_1$, Sum_1 satisfies FE1.*

Hence, $Sum_1(s_1, X) \wedge Sum_1(s_2, X) \rightarrow s_1 = s_2$.

Proof Let s be the Sum_1 of s_1 and s_2 , which exists by U_1 . Suppose $s_i \neq s$, where $i = 1, 2$. Since $s_i \leq s$, by the definition of U_1 , we have $s_i \ll s$. Thus, $x \leq s$ and $x \parallel s_i$, for some x , by **SP**. Still $x \leq s$ implies $x \circ s_i$, for all x , again by the definition of U_1 . Therefore, $s_i = s$, where $i = 1, 2$, and so $s_1 = s_2$. \square

Proposition 3 *In $SM + U_1$, Sum_1 satisfies FE2.*

Hence, $Sum_1(s, X_1) \wedge Sum_1(s, X_2) \rightarrow X_1 \equiv X_2$.

Proof Suppose $X_1 \not\equiv X_2$. Then, by the definition of \equiv , there is some x such that either $x \circ X_1$ and $x \parallel X_2$ or $x \parallel X_1$ and $x \circ X_2$. Without loss of generality, suppose the first. Since $x \circ X_1$, x has a part that is part of one of the X_1 , and so it has a part that is part of s , by the transitivity of \leq and $Sum_1(s, X_1)$. Since $x \parallel X_2$, all the parts of x are disjoint from X_2 , and so no part of x is part of s , by the definition of Sum_1 , contradicting $Sum_1(s, X_2)$. \square

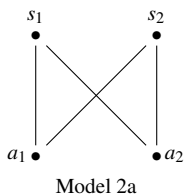
Proposition 4 *In $SM + U_2$, Sum_2 satisfies FE2.*

Proof Straightforward, by the definition of Sum_2 . □

Proposition 5 *In $SM + U_2$, Sum_2 does not satisfy FE1.*

Hence, it is possible that $Sum_2(s_1, X) \wedge Sum_2(s_2, X) \wedge s_1 \neq s_2$.

Proof Consider the following model



This model is supplemented, since every entity having a proper part x has another proper part that is disjoint from x . Furthermore, every plurality has a Sum_2 , since s_1 and s_2 are Sum_2 of any plurality. Finally, s_1 and s_2 are distinct Sum_2 of the same plurality, namely $X = \{a_1, a_2\}$. □

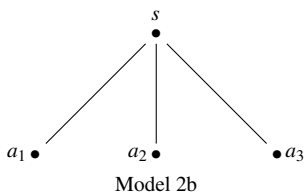
Proposition 6 *In $SM + U_3$, Sum_3 satisfies FE1.*

Proof Straightforward, by the definition of Sum_3 . □

Proposition 7 *In $SM + U_3$, Sum_3 does not satisfy FE2.*

Hence, it is possible that $Sum_3(s, X_1) \wedge Sum_3(s, X_2) \wedge X_1 \neq X_2$.

Proof Consider the following model



This model is supplemented, since every entity having a proper part x has another proper part that is disjoint from x . Furthermore, every plurality has a Sum_3 , since s is a Sum_3 of any plurality. Finally, s counts as a Sum_3 of at least two inequivalent pluralities, namely $X_1 = \{a_1, a_2\}$ and $X_2 = \{a_2, a_3\}$. □

The foregoing propositions show that Sum_2 and Sum_3 have serious drawbacks. As to Sum_2 , Model 1a shows that it is possible for a Sum_2 to not contain as parts some of the summands. Furthermore, it shows that it is possible for a Sum_2 to have none of the summands as parts, since s_1 counts as a Sum_2 of s_2 , which seems problematic,¹⁶

¹⁶ See Varzi (2009, p. 4): This is simply intolerable. No matter how exactly one defines the word, surely a fusion is supposed to include, among its parts, at least some of the things it fuses.

Similarly, Model 2b shows that a Sum_3 can have parts that are disjoint from its summands, since s counts as a Sum_3 of a_1, a_2 , and yet x has a part, a_3 , disjoint from both, which seems again problematic,¹⁷

Where are we? We have seen that, among the three usual definitions of sums, only Sum_1 seems to be acceptable when we work in a framework like $SM + U_i$.¹⁸ Unfortunately, Sum_1 is the worst possible notion for non-extensionalists, since it entails the full extensionalist package—by **Proposition 1-2-3**. What can anti-extensionalists do?

Let us take a new look at the lay of the land. The argument for extensionalism rests on three tenets:

- (i) The acceptance of **SM**;
- (ii) The acceptance of U_1 ;
- (iii) A crucial argument to the point that Sum_1 is the only well-behaved notion, so that it is the only notion to be used in the formulation of **U**.

Hence, provided we buy into the *partial ordering* axioms for parthood, it seems we do end up with full-blooded Classical Extensional Mereology. Is that the last word?

Not necessarily. Or at least this is what we think. We already argued that, in mereologies that are weaker than **SM**, Sum_1 suffers from drawbacks akin to the ones Sum_2 and Sum_3 face—this was the gist of our argument from Models 1a and 1b in Sect. 2.1 to the point that it is problematic that two entities related by proper parthood count as sum of the same entities. We think we should consider such drawbacks seriously—at least as seriously as we considered the drawbacks of Sum_2 and Sum_3 . By parity of reasoning, provided **SP** is not assumed, it seems that Sum_1 is unsatisfactory as well. But if we drop Sum_1 , we are not really doing the anti-extensionalist any favor. In effect, we simply deprived *everyone* of any useful notion of *Sum*, together with the possibility of formulating a thesis like **U**. And so we are left in the dark. But there's a glimmer of light in the corner, a glimmer we did not see because we were just staring into the darkness. If we go look in that corner we'll find a new notion of *Sum* which, maybe, can be of help.¹⁹

3 A novel notion of *Sum*

The new notion of *Sum* that was waiting for us in the corner is the following—we will call it *General Sum*:

Definition 3 General Sum.

$$Sum(a, X) := X \leq a \wedge \forall x(x \parallel X \rightarrow x \parallel a) \wedge \forall y(X \leq y \rightarrow a \leq y)$$

¹⁷ See Varzi (2009, p. 6): That is simply intolerable. No matter how exactly one defines the word, surely one should not get the same fusion from two wholly disjoint classes of atoms.

¹⁸ This is the conclusion Varzi draws in (2009) based on similar arguments.

¹⁹ In Kleinschmidt (2019) it is argued that every notion of fusion defined solely in terms of parthood and the plural logical relation of “being one of” would be unable to handle some problematic cases of composition. As we shall see, once a particular mereological system—based on the new notion of fusion we are about to introduce—is on the table, such cases can be somehow handled. We defer the discussion to footnote 25.

In plain English, a is the.²⁰ *Sum* of the items in X if and only if (i) all the X s are parts of a , (ii) a is separated from any thing which is separated from all the X s, and (iii) a is part of any thing which has all the X s as parts. Thus, our notion of *Sum* is simply the conjunction of three different requirements that one can impose on a notion of sum. The first requirement has it that the sum is inclusive enough to have all the things it is a sum of as parts. The second one requires that such a sum is not too inclusive, that is, does not admit as parts things that do not even overlap any of the things it is a sum of. According to the third and final requirement a sum is included in everything that includes the things it is the sum of. We do not want to defend our notion of *Sum* here.²¹ We just want to address the consequences of adopting such a notion for the debate on Universalism and Extensionalism. Before we do that we need to argue in favor of two related, yet separate claims. First, *Sum* is stronger than any Sum_i , if *no strong decomposition principle is assumed*. Second, *Sum* is immune to the complaints raised by both Varzi and us in Sect. 2.

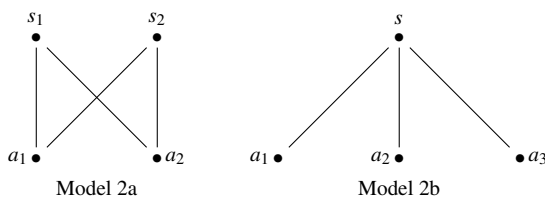
To see that *Sum* is in general stronger than any Sum_i we argue as follows.

Proposition 8 *If $Sum(a, X)$, then $Sum_i(a, X)$, for $i = 1, 2, 3$.*

First, note that $\forall x(x \parallel X \rightarrow x \parallel a)$ entails $\forall x(x \circ a \rightarrow x \circ X)$, by contraposition, and so $\forall x(x \leq a \rightarrow x \circ X)$, given that every part of a overlaps a . Furthermore, note that $X \leq a$ entails $\forall x(x \circ X \rightarrow x \circ a)$, given that every entity that overlaps one of the X also overlaps anything having the X as parts. So, suppose that $Sum(a, X)$. Since $X \leq a$ and $\forall x(x \parallel X \rightarrow x \parallel a)$, being a *Sum* entails both being a Sum_1 and being a Sum_2 . Since $X \leq a$ and $\forall y(X \leq y \rightarrow a \leq y)$, being a *Sum* also entails being a Sum_3 .

Proposition 9 *It is possible for something to be a Sum_i , for $i = 1, 2, 3$, without being a *Sum*.*

Consider again Models 2a and 2b:



It is not difficult to see that in these models s_1 is a Sum_1 and a Sum_2 of a_1 and a_2 respectively, while s is a Sum_3 of a_1 and a_2 . By contrast, neither s_1 nor s is a *Sum* of those pluralities. This establishes that *Sum* is indeed stronger than any Sum_i .

We can conclude that *Sum* is not subject to the aforementioned, reasonable complaints by Varzi and us. Let us unpack this last statement. In Model 1a, s_1 and x are

²⁰ Uniqueness is implied by the third conjunct given *Antisymmetry* See **Proposition 10**.

²¹ Suffice it to say that all the conditions in the definition of *Sum* are separately imposed in the Sum_i and that they can be recovered with respect to any useful notion of sum in systems of mereologies that are strong enough—see [Redacted].

both Sum_1 of a_1 and a_2 , and yet at most one should be, given that they are proper-parthood related. In Model 2a, s_1 is a Sum_2 of s_2 , and yet it should not be, given that s_2 is not part of s_1 . In Model 2b, s is a Sum_3 of a_1 and a_2 , and again it should not be, given that s has also a_3 as part. Sum is not subject to any of these complaints. In effect, neither s_1 nor s qualify as a Sum of the relevant entities. If so, we contend, we should consider what work Sum can do—if any—for anti-extensionalists. It is to this that we now turn to.

4 Quasi supplemented universalist mereology

Now that we have a robust, complaint-free notion of sum we are finally able to re-evaluate the relationship between **U** and **E**. Let us first highlight how having a new notion of sum allows us to break the chain of arguments leading from **U** to **E**. We saw that the notion of Sum_1 supports the choice of **SP** as an acceptable decomposition principle. This is because weaker principles admit of models that we would like to exclude, in that Sum_1 is not well-behaved in those models. However, since we are no longer forced to stick to Sum_1 , we are now free to select a decomposition principle which is weaker than **SP** and still powerful enough to prevent undesired consequences. In fact, it is not difficult to check that, in Models 1a and 1b, no entity above x counts as a Sum of any plurality of entities below x . Let us spend a few more words on the general dialectic here. The argument in Sect. 2.1 was that decomposition principles as strong as **SP** are required to rule out Models 1a and 1b because in these models Sum_1 is not well-behaved. Our claim is that Sum on the other hand *is well behaved in those models*. So there is no reason to rule them out.²² And this opens the possibility to accept weaker decomposition principles. Note that, from a more general perspective, this is also in line with different developments in the mereological literature that questioned *Supplementation* on independent grounds.²³

This already marks a significant difference with some other proposals in the literature. As we are going to see, Gilmore suggested in (2009) to replace *Supplementation* with *Quasi Supplementation*, and we will adopt the same strategy. Yet, the difference in the resulting systems is important, since we crucially rely on a stronger notion of sum. It is precisely this fact that allows us to avoid the problematic consequences of Models 1a and 1b that *Quasi Supplementation* is not strong enough to rule out. By contrast, Gilmore still uses Sum_1 , which entails that (i) he cannot use the argument we put forward, and (ii) he is committed to the problematic consequences derived from accepting those models. A similar argument—*mutatis mutandis*—applies for Loss's system in (forthcoming). This, we contend, is reason enough to explore thoroughly the alternative mereological system we are about to set forth. Indeed, we can go a step

²² Or at least there are no reasons coming from our notion of sum. We are not claiming that one cannot argue there are indeed other reasons.

²³ See e.g., Smith in (2009) and Cotnoir in (forthcoming). Cotnoir discusses *Supplementation* vis-à-vis *Quasi-Supplementation* in (2016). One of the main reason to discard *Supplementation* is that there are several metaphysical theses that are indeed committed to violations of it, ranging from Whitehead's mereotopology to Brentano's theory of accidents, from Fine's *qua-objects* to endurantism. Therefore, working in a framework where *Supplementation* is not imposed as an analytic principle makes room for different metaphysical projects. This is enough, we lay claim, for the purpose of the paper.

further and prove that these systems are also different with respect to the extensionality principles they entail, a fact that is plainly most relevant in the context at hand. In the light of the above, let us consider the mereological system that consists of the following axioms²⁴:

- Transitivity*: $x \leq y \wedge y \leq z \rightarrow x \leq z$
- Anti-Symmetry*: $x \leq y \wedge y \leq x \rightarrow x = y$
- Quasi-Supplementation*: $x \ll y \rightarrow \exists z \exists w (z \leq y \wedge w \leq y \wedge z \parallel w)$

Importantly, *all the models we used so far are models of such mereological system*. Let us now add **U**, using *Sum*:

$$\text{Universalism } \exists x(x : X) \rightarrow \exists s(\text{Sum}(s, X))$$

This system, call it Quasi Supplemented Universalist Mereology (**QSUM**), has never been considered in the debate.

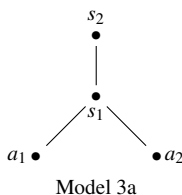
4.1 Comparison with existing systems

There are, in fact, significant differences distinguishing it from explored systems. First, Cotnoir insists that we should drop *Antisymmetry*. By contrast we retain it. Second, both Varzi and Cotnoir retain *Supplementation*, whereas we focus on the weaker *Quasi Supplementation*. Third, Loss suggests to replace *Supplementation* with *Minimal Supplementation* whereas we suggested to replace it with *Quasi Supplementation* instead. Finally, Cotnoir, Varzi, Gilmore and Loss all use Sum_1 in their formulation of **U**, whereas we use *Sum* instead.

Both the systems proposed by Gilmore and the system proposed by Loss are similar to the one we put forward, in that they maintain antisymmetry. Still, we claim that **QSUM** should be preferred to them. The main reason is that **QSUM** allows us to prove that our notion of sum satisfies the basic desiderata on the notion of fusion introduced above, thus avoiding the complaints we raised against Sum_1 —more on this shortly. On Gilmore’s system Let Quasi-Supplemented Mereology, **QSM**, be the system obtained by adding *Quasi-Supplementation* to minimal mereology. In (Forthcoming) Gilmore introduces a system which is equivalent to **QSM** + U_1 and shows that **E** is not derivable in it. Still, **QSM** + U_1 falls short of satisfying our desiderata, since it does not exclude models where the same entities give rise to different sums, i.e., it does not satisfy **FEI**. Let us show this.

Proposition 10 *In $\text{QSM} + \text{U}_1$, Sum_1 does not satisfy FEI.*

Proof Consider this model



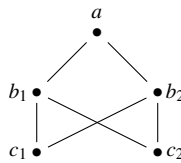
²⁴ Formulas are intended as universally closed.

Model 3a is quasi-supplemented: every composite entity has disjoint proper parts. It also satisfies U_1 , for every plurality has a Sum_1 . In fact, s_2 is a Sum_1 of any plurality. However, both s_1 and s_2 are Sum_1 of $\{a_1, a_2\}$. Hence, the model does not satisfy **FEI**. \square

On Loss' system Let *Minimally Supplemented Mereology*, (**MSM**), be the system obtained by adding *Minimal Supplementation* to minimal mereology. Then, Loss endorses a system which is equivalent to **MSM** + U_1 . The following is provable:

Proposition 11 *In **MSM** + U_1 , Sum_1 does not satisfy **FEI**.*

Proof Consider this model.



Model 3b

Model 3b is minimally supplemented. It also satisfies U_1 , for every plurality has a Sum_1 . In fact, a is a Sum_1 of every plurality. However both b_1 and a are Sum_1 of $\{c_1, c_2\}$. Hence the model does not satisfy **FEI**. \square

In what follows we show that, by contrast, **FEI** is derivable in **QSUM**. Note that this is exactly the reason why *Sum* does not fall prey to the arguments against Sum_1 . It is not possible for a given plurality to have distinct *Sum*-s, let alone distinct *Sum*-s that are related by proper parthood—as it happens with Sum_1 .²⁵

²⁵ As we anticipated in footnote 19, in Kleinschmidt (2019) some cases are presented that allegedly push towards a mereology that takes “fusion” as a primitive and defines parthood and other mereological notions in terms of it. Such cases are characterized as follows (p. 696):

In each of the problematic cases we’re describing, an object has multiple, disjoint proper parts, and the object does not seem to be made up entirely out of the collection of all of its proper parts. Further, there is no single proper part of the object that is a fusion of all of the others.

As a paradigmatic example, Kleinschmidt considers the *Revised T-Rex* case, where a T-Rex is described as having only two parts—two disjoint tiny flecks of green scale—whose fusion is plainly different from the entire reptile. A detailed comparison of the view we are advocating and Kleinschmidt’s would deserve an independent scrutiny. However, it is interesting to see that **QSUM** offers an interesting, novel way to handle such cases. First of all, as Kleinschmidt herself concedes, every mereological system that features some form of universalism among its axioms rules the cases out simply because it is not true that there is no single part of the object that is a fusion of all the others. Still, significantly, in **QSUM** we can do something more, since **QSUM** allows for composite entities that are not the *Sum* of their proper parts. As a way of illustration, consider s_2 in Model 3a. In such a case, it is true both that s_2 has two parts and that it is not the sum of these parts. However, it is not true that it has only two parts, since their sum s_1 is also a part of s_2 , and it is not true that it is the sum of them, since s_1 is different from s_2 . Model 3a shows how one can understand Kleinschmidt’s *Revised T-Rex* case: a_1 and a_2 are the two disjoint tiny flecks of green scale, s_1 is their *Sum*, and s_2 is the T-Rex. Evidently, this is not a model of *Supplementation* but, as we saw, we have independent reasons to drop this principle.

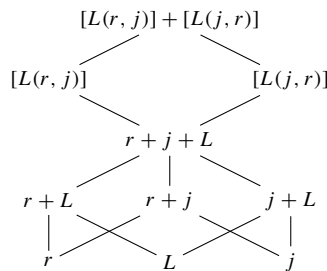
4.2 Strength of QSUM

We are going to show that **FE1** is derivable in **QSUM**. More generally, the key point is that **QSUM** is both universalist, having **U** among its axioms, and strong enough to derive both **FE1** and **FE2**.

As we pointed out in Sect. 2.2, we want to distinguish three different versions of extensionalism, namely:

- **FE1**: Things that are *Sum* of the same plurality are identical;
- **FE2**: Pluralities that have the same *Sum* are equivalent;
- **E**: Composite things that have exactly the same proper parts are identical.

To shed light on the entailments from **U** to any of these variants, let’s have a look at the following model, where *r* is Romeo, *j* is Juliet, *L* is the relation of loving, [*L*(*r*, *j*)] is the state of affairs that Romeo loves Juliet, [*L*(*j*, *r*)] is the state of affairs that Juliet loves Romeo, and + is the operation of binary sum²⁶:



Model 4

This model of **QSUM** is interesting not only for clarifying the connection of **U** and **E**, but also for advancing at least one application of the present system, thus showing its significance to the current debate on the metaphysical structure of the world—in this case for capturing the relations involved in a world of states of affairs without introducing non-mereological notions.

This should not be underestimated. It will open the possibility of endorsing *mereological monism*—roughly the thesis that there is but *one notion of composition*, namely *mereological composition*—and still accept the existence of states of affairs—that were supposed to provide alleged counterexamples to such a metaphysical thesis.²⁷

Upon inspection, it is plain that **E** fails, as there are distinct things in the model with the same proper parts, namely [*L*(*r*, *j*)] and [*L*(*j*, *r*)]. By contrast, **QSUM** has **FE1** and **FE2** as theorems, since they both follow from the definition of *Sum* and *Antisymmetry*

²⁶ We are not committed here with a particular position about the existence of state of affairs. What we want to do is just to use an example from a metaphysics of state of affairs to illustrate how the present system helps us to provide a mereological view of philosophical frameworks that are hardly understandable if a standard Extensional Mereology is assumed.

²⁷ We should be explicit here in admitting that more needs to be said at this juncture. It’s true that there is a standard mereological relation holding between *r + j + L* and [*L*(*r*, *j*)], namely *proper parthood*. But one may well ask how to “pass”, so to speak, from the mereological sum to state of affairs that has that sum as a part. The model is silent on that. Providing such details is an intriguing task, but goes beyond the scope of the present paper.

alone. It is instructive to see that this is in fact the case. We restrict ourselves here to **FE1** and leave the proof of **FE2** to the reader.

Proposition 12 *FE1 is provable in QSUM*

Proof Assume the antecedent of **FE1**, that is $Sum(s_1, X)$ and $Sum(s_2, X)$. Then, $X \leq s_1$, and $X \leq s_2$, by the definition of *Sum*. Thus, $s_1 \leq s_2$ and $s_2 \leq s_1$, again by the definition of *Sum*, and so $s_1 = s_2$, by *Antisymmetry*. \square

In light of this, we can now provide new, detailed answers to the questions Q1 and Q2 about the relationship between **U** and **E** in the *present context*.

Answer to Q1. If non-extensionalists rest content with the failure of **E**, then the notion of *Sum* will suit their purposes;

Answer to Q2. If *Sum* is used in the formulation of **U**, then **U** still entails **FE1**, and **FE2**, but does not entail **E**.

One might be skeptical about one detail of our answer to Q2. There, we seem to suggest that it is really **U** that does the heavy lifting in deriving **E**. But the proof of **Proposition 12** clearly shows this is not the case. *Sum* and *Antisymmetry* alone are more than capable of doing the job. This is correct, but clearly does not detract from our argument. On top of this, **U** can be used crucially to derive another extensionalist principle, *Universal Unique Fusion*:

Universal Unique Fusion (UUF): $\exists x : X \rightarrow \exists!s(Sum(s, X))$

Note that **UUF** is strikingly similar to Tarski's axiom of *Unrestricted Unique Fusion* in his axiomatization of Classical Mereology.²⁸ It is easy to see that:

Proposition 13 *UUF is provable in QSUM*

Proof By **U** any non empty plurality has a *Sum*. By **FE1** such a *Sum* is unique. \square

It is now clear that this time it is indeed **U** that does the heavy lifting. However, having changed the notion of sum in **U** is still crucial. Indeed, Tarski's axiom, being formulated in terms of Sum_1 , is strong enough, given *Transitivity*, to derive all the principles characterizing Classical Extensional Mereology, whereas our axiom is consistent with rejecting **E**.

Let us now highlight a crucial consequence of the previous discussion. We have focused on different mereological systems that accept at least **MM**, that is, systems that endorse the partial ordering axioms for parthood, namely **CEM**, defended in Varzi (2009), **QSM + U₁**, defended in Gilmore (2009), **MSM + U₁**, defended in Loss (Forthcoming), and the one we just suggested as a worthy alternative, that is, **QSUM**. We can now classify these three mereological systems with respect to the extensionality principles we distinguished. The result is summed up in Table 1 below:

Before concluding the section, we want to use what has been established so far to respond to a problematic claim by Cotnoir in (2016). Cotnoir writes:

²⁸ See Tarski (1923). It is not quite the very principle because Tarski uses a different notion of sum in its formulation.

Table 1 Mereological systems and extensionality principles

	CEM	QSUM	QSM + U ₁	MSM + U ₁
FE2	✓	✓	✓	✓
FE1	✓	✓		
E	✓			

[H]ere is a quite general result: if parthood is a partial order, then any model in which a F'' -type fusion [our Sum_3] is unrestricted simply cannot have a counterexample to extensionality. Why? Because in the presence of the partial order axioms, particularly antisymmetry,²⁹ F'' -type fusions guarantees the existence of a *least* upper bound. These must be unique (Cotnoir, 2016, p. 123).

So far so good. But immediately before he wrote:

[E]xtensionality states that composed objects with the same proper parts are identical (Cotnoir, 2016, p. 122).

We can now see that, given that definition of extensionality, the conclusion drawn by Cotnoir does not follow, since there are systems of mereology that feature **U** among their axioms, that is the universalist thesis in terms of Sum , but not **E** among their theorems. This notwithstanding, we still have that **U** entails both **FE1** and **FE2**. Thus, it seems that the two passages above do not sharply distinguish between **E** and **FE1**. By making such distinction, we are in a position to claim that there is a sense in which any system obtained by adding **U** to Minimal Mereology is extensional, in that it at least provides extensionality of fusions.

5 Conclusion

Let us take stock. We have introduced a novel notion of sum and used it to provide a better understanding of the relation between **U** and **E**. First, we have noted that the argument from **U** to **E** is strongly supported in a framework where Sum_1 is the only acceptable notion of sum among the available ones. In fact, accepting Sum_1 justifies the assumption of **SP** as an appropriate decomposition principle. And, working in a system of supplemented mereology, like **SM**, allows us to prove that **U** entails **E**. Second, we have introduced different extensionality theses and highlighted that the main drawbacks concerning alternative conceptions of fusion do not stem from violations of **E**, but rather, from violations of **FE1** and **FE2**. Third, we have casted doubts on the acceptability of Sum_1 in a framework where **SP** is *not* assumed, since in such a framework we are not able to exclude some models that admit of sums that strike us as problematic, especially in light of some basic desiderata on the notion of fusion. Finally, we have advanced a new notion of sum not suffering from the problems afflicting Sum_1 and proved that this notion enables us to accept decomposition principles that are weaker than **SP**. Having done that, we think we have brought the debate

²⁹ See our argument in the main text.

on the connection between **U** and **E** back to its pure form: either to accept **SP**, thus buying the full package of Classical Extensional Mereology, or to reject **SP** in favor of principles like **QSP**, thus allowing for mereologies where it is possible for diverse entities to be composed of the same elements. The main result in this respect is that system **QSUM**, obtained by enriching minimal mereology—the mereological system comprising only the partial ordering axioms for parthood—via **QSP** and universalism of *Sum*, is powerful enough to discard all the problematic models we encountered, while preserving at the same time **FE1** and **FE2**, thus granting us the largest amount of extensionality consistent with the rejection of **E**. More than that, **QSUM** enables us to describe interesting and independently motivated structures in mereological terms, like the structure of states of affairs or of similar structured wholes, thus opening a fascinating and virtually unexplored line of research.

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Declarations

Conflict of interest The authors declare that they have no conflicts of interest.

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