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John Keill and the Pre-Critical Kant[*]

Marco Sgarbi

This paper focuses on John Keill's influence on Kant's pre-critical thought, as well as on his early understanding of Newtonianism. The first section reconstructs the spread of Newtonianism in Königsberg during Kant's university years. The second deals with Keill's method of philosophizing and its impact on Kant's methodological reflections in his early scientific writings. The third and fourth examine how Keill's conceptions of inverse square law, solidity, extension, and divisibility helped Kant find his own eclectic way in combining metaphysics and mathematics.

Keywords: *Kant, Keill, Newton, Metaphysics, Mathematics.*

1. *Newtonianism in Königsberg*

The eighteenth century is usually characterized as the age of Newton for the impact his thought had on many research fields from natural philosophy to ethics, from legal theories to literature, from mathematics to metaphysics. However, the "Newtonian moment" arrived very late in Germany[1], and in particular in Königsberg in comparison to other countries like France and Italy, and other university towns like Paris and Padua. Of Newton and his followers' reception and influence in Germany we know almost nothing. There are no serious studies like those of J.B Shank for the French Enlightenment, or of Maria Laura Soppelsa for Italy, or of that edited by Eric Jorink and Ad Maas for the Dutch Republic[2]. In spite of

[*] All references to Kant's works are cited in the body of the text according to the volume and page number, given in Arabic numerals separated by a colon, in the critical edition of *Kants gesammelte Schriften* (=KGS), edited by the Royal Prussian (later German, then Berlin-Brandenburg) Academy of Sciences (Georg Reimer, later Walter de Gruyter & Co. 1900). The one exception to this rule is the *Critique of Pure Reason*, where passages are referenced by numbers from "A," the first edition of 1781, and/or "B," the second edition of 1787. Unless otherwise noted, the translations of Kant's writings are from the *Cambridge Edition of the Works of Immanuel Kant* (Cambridge University Press, Cambridge 1992) and those of Aristotle's from the *Complete Works* (Princeton University Press, Princeton 1984).

[1] Mordechai Feingold, *The Newtonian Moment: Isaac Newton and the Making of Modern Culture*, Oxford University Press, Oxford 2004.

[2] Maria Laura Soppelsa, *Leibniz e Newton in Italia: il dibattito padovano, (1687-1750)*, LINT, Trieste 1989; John B. Shank, *The Newton Wars and the Beginning of the French Enlightenment*, University of Chicago Press, Chicago 2008; Eric Jorink, Ad Maas (eds.), *Newton and the Netherlands: How Isaac Newton Was Fashioned in the Dutch Republic*, Leiden University Press, Leiden

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the great effort of Thomas Ahnert in reconstructing the impact of Newton in the German-Speaking Lands, there are only feeble traces of interest in Newton's philosophy and mathematics before 1750, and most of these are in relation to the Leibniz-Clarke correspondence, to Christian Wolff's appropriation and rejection of Newton's ideas, or to Leonhard Euler's alleged Newtonianism [3].

The focus of my research is to assess Newton's impact in Königsberg, especially by reconsidering the role of the Scottish mathematician and natural philosopher John Keill. As far as the archives would indicate, there are no clues as to Newton's presence in Königsberg before 1745. The only trace that we can find of Newton in the *Vorlesungsverzeichnisse* comes from Johann Christoph Bohl, professor of medicine, who mentions Newton along with Mariotte and many others for his theories on vision. This was the period in which Immanuel Kant was a student at the Albertina and began the composition of his *Thoughts on the True Estimation of Living Forces* [4]. However, no other relevant university document reveals Newton's presence – hence no dispute, no dissertation, no academic program neither in mathematics nor in natural philosophy. None of the professors at Königsberg taught Newton in their classes, as far as we know.

The main textbooks for mathematics were Christian Wolff's *Elementa matheseos universae* (1713-1715) and *Auszug aus den Anfangsgründen aller mathematischen Wissenschaften* (1717), and this also in the period in which Wolffianism was temporarily banished from Königsberg. Wolff's mathematics dominated at the Albertina. For natural philosophy, or physics, in contrast, it is possible to list a range of manuals – from Christian Wolff's *Vernünfftige Gedancken von den Würkungen der Natur* (1723) to Johann Christoph Sturm's *Physi-*

2013; John B. Shank, *Before Voltaire: The French Origins of Newtonian Mechanics, 1680-1715*, University of Chicago Press, Chicago 2018.

[3] Ronald S. Calinger, *The Newtonian-Wolffian Controversy*, «Journal of the History of Ideas», 30 (1969), pp. 319-30; Thomas Ahnert, *Newtonianism in early Enlightenment Germany, c. 1720 to 1750: Metaphysics and the Critique of Dogmatic Philosophy*, «Studies in History and Philosophy of Science Part A», 35 (2004), pp. 471-91; Marius Stan, *Newton and Wolff: The Leibnizian Reaction to the Principia, 1716-1763*, «The Southern Journal of Philosophy», 50 (2012), pp. 459-81; Marius Stan, *Euler, Newton, and Foundations for Mechanics*, in Chris Smeenk, Eric Schliesser (eds.), *The Oxford Handbook of Newton*, Oxford University Press, Oxford 2017, pp. 1-22. Marius Stan, *Newton's Concepts of Force among the Leibnizians*, in Mordechai Feingold, Elizabethanne Boran (eds.), *Reading Newton in Early Modern Europe*, Brill, Leiden 2017, pp. 244-89; Thomas Ahnert, *Newton in the German-speaking Lands*, in Scott Mandelbrote, Helmut Pulte (eds.), *The Reception of Isaac Newton in* Europe, Bloomsbury, London 2019, pp. 41-58.

[4] Manfred Kuehn, *Kant. A Biography*, Cambridge University Press, Cambridge 2001, p. 86.

cae conciliatricis […] *conamina* (1684), to name only a few. Indeed, in natural philosophy the situation was somewhat fluid: Aristotelianism, Eclecticism and Wolffianism were frequently taught by the very same professor. Yet in mathematics there prevailed only one name, that of Wolff. Nonetheless, there is no clue of Newton. This is in opposition to what happened to many other members of the Royal Society like Robert Boyle or John Wallis, who enjoyed an almost immediate reception. This does not mean that Newton was not known in Königsberg, but rather that his mathematics and natural philosophy were not seriously and self-consciously engaged with during this period.

There is a long lasting myth according to which it was Martin Knutzen who introduced Newton to Königsberg[5], and in particular to Immanuel Kant. This myth has its origins in one of the earliest biographies of Kant, that penned by Ludwig Ernst von Borowski. He writes that Knutzen lent Newton's work to Kant[6]. However, there is no proof that what Borowski wrote actually happened, and Kant did not revise this portion of the text, while we know he read and rewrote other parts. More particularly, as Manfred Kuehn has pointed out, «Knutzen's understanding of scientific and mathematical matters was inadequate […] he did not belong to the small elite of scientists on the continent who understood the details of Newtonian physics […] his knowledge of calculus was especially deficient»[7]. It is difficult, therefore, to sustain the thesis that Knutzen was champion of Newtonianism in Königsberg.

The very first self-conscious appropriation of Newton seems to be with Kant. While there are many investigations of Kant's philosophy of science in relation to Newton in the critical period, first and foremost those of Michael Friedman[8], little attention has been given to the precritical period and focusing mainly on the impact

[5] Michael Friedman, *Kant and the Exact Sciences*, Harvard University Press, Cambridge (MA) 1992, p. 1.

[6] Ludwig Ernst Borowski, *Ueber Immanuel Kant*, Nicolovius, Königsberg 1804, pp. 163-4.

[7] Kuehn, *Kant. A Biography*, p. 84.

[8] Michael Friedman, *Kant and Newton: Why Gravity is Essential to Matter*, in Phillip Bricker, Robin I.G. Hughes (eds.), *Philosophical Perspectives on Newtonian Science*, MIT Press, Boston 1990, pp. 185–202; Michael Friedman, *Matter and Motion in the Metaphysical Foundations and the First Critique*, in Eric Watkins (ed.), *Kant and the Sciences*, Oxford University Press, Oxford 2001, pp. 53-69; Michael Friedman, *Kant on Science and Experience*, in Volker Gerhardt, Rolf-Peter Horstmann, Ralph Schumacher (eds.), *Kant und die Berliner Aufklärung*, De Gruyter, Berlin 2002, vol. I, pp. 233-45; Michael Friedman, *Newton and Kant: Quantity of Matter in the* Metaphysical Foundations of Natural Science, «Southern Journal of Philosophy», 50 (2012), pp.

of the English scientist regarding his conception of nature, forces, space and time[9]. Less attention has been given to reconstructing how Kant received Newton's philosophical and metaphysical lessons. The working hypothesis of this paper is that John Keill was the individual immediately responsible for Kant's understanding of Newtonianism in his precritical period[10].

2. *On Method*

Scholarship tends to identify Kant's early approach to Newton with the work *Thoughts on the True Estimation of Living Forces* (1749), which he wrote around 1744-1745 under Knutzen's influence and supervision. However, a close reading of Kant's very first writing shows that there is never an open approach to Newton, that the knowledge of Newton is mediated through his opponents, especially Leibnizians, or his followers[11]. The only quotation from Newton refers to the first law of motion and it is not based on a direct reading of the *Principia* since it contains substantial terminological variations[12].

482-503; Michael Friedman, *Kant's Construction of Nature: A Reading of the Metaphysical Foundations of Natural Science*, Cambridge University Press, Cambridge 2013.

[9] Giorgio Tonelli, *Elementi metodologici e metafisici in Kant dal 1745 al 1768*, Torino, Edizioni di Filosofia 1959; Martin Schönfeld, *The Philosophy of the Young Kant: The Precritical Project*, Oxford University Press, New York 2000; Eric Watkins, *Kant and the Metaphysics of Causality*, Cambridge University Press, Cambridge 2005; Eric Watkins, *The Early Kant's Newtonianism*, «Studies in History and Philosophy of Science Part A», 44 (2013), pp. 429-37; Michela Massimi, *The Legacy of Newton for the Pre-Critical Kant*, in Erich Schliesser, Chris Smeenk (eds.), *The Oxford Handbook of Newton*, Oxford University Press, Oxford 2017, https://doi.org/10.1093/oxfordhb/9780199930418.013.26.

[10] John Keill was born in 1671 in Edinburgh, where he studied under the supervision of David Gregory before attending lectures at Balliol College, Oxford. He became lecturer in experimental philosophy at Hart Hall, later Hertford College, focusing his study and teaching on Newton's philosophy. In 1698 he made his debut in the philosophical scene with *An Examination of Dr. Burnet's Theory of the Earth*, developing his first criticism of mechanical philosophy. His concise style and effective articulation made of him one of the most important popularizers of Newton's complex system. In 1712 he was appointed Savilian Professor of Astronomy in Oxford, becoming one of the most prominent scientists of his age. In 1702 Keill published *Introductio ad Veram Physicam seu Lectiones Physicae* divided into sixteen lectures, its 1739 edition used by Kant during his precritical period, and quoted explicitly in the *Physical Monadology*. Later he published a series of felicitous handbooks, including *Trigonometriae plane & sphaericae elementa* (1715), *De natura et arithmetica logarithmorum tractatus brevis* (1715), and *Introduction ad veram astronomiam seu lectiones astronomicae* (1718).

[11] *KGS*, I, pp. 58-9, 164-5.

[12] We can find a similar but not identical variation in Ludwig Philipp Thümmig, *Institutiones philosophicae Wolfianae*, Renger, Frankfurt-Leipzig 1740, p. 89; Antonio Genovesi, *Elementorum artis logico-criticae* Bettinelli, Venezia 1776, pp. 400-1.

Kant's Version	Newton's version
Corpus quodvis pergit in statu suo, vel quiescendi, vel movendi, uniformiter, in directum, nisi a causa externa statum mutare cogatur.	*Corpus omne perseverare in statu suo quiescendi vel movendi uniformiter in directum, nisi quatenus illud a viribus impressis cogitur statum suum mutare.*

Hence from this brief quotation there is no way of establishing whether Kant had read Newton directly or even understood him at this stage of his philosophical career. From SS 1755 to WS 1762/1763 Kant taught mathematics from Wolff's manual, just like any other professor of mathematics in Königsberg[13], but we also know from his student Johann Gottfried Herder that during his very last semester Kant was teaching "Newton's laws of nature" in class[14]. With little certainty, therefore, one may suppose that he was making Newton available in his teaching even in the preceding years.

1755 marks an important date in Kant's intellectual biography. Indeed, this is the year in which Kant seriously tackled Newtonianism with the composition of the *General History and Theory of the Heavens or an Essay on the Constitution and Mechanical Origin of the Whole Universe, Treated in Accordance with Newtonian Principles*. A thorough examination of the text, however, does not explain what treating a science in agreement with Newtonian principles means. Kant seems to suggest that the idea of writing this work came from a reading of Wright of Durham's *Original Theory of New Hypothesis of the Universe* (1750), from which he drew a lot of ideas and poetical inspiration, but not in a Newtonian sense. In Kant's text Newton is seldom mentioned and the subtitle seems to offer more a kind of future agenda than a project accomplished. In the very first occurrence he describes Newton's greatest achievement as follows

> Just as of all the tasks facing research into nature, none has been resolved with greater accuracy and certainty than the true constitution of the universe on the large scale, the laws of motion, and the internal mechanism of

[13] See Gottfried Martin, *Arithmetic and Combinatorics: Kant and His Contemporaries*, Southern Illinois University Press, Carbondale and Edwardsville 1985.

[14] Manfred Kuehn, *Kant's Teachers in the Exact Sciences*, in Eric Watkins (ed.), *Kant and the Sciences*, Oxford University Press, Oxford 2001, pp. 11-30.

> the orbits of all the planets into which Newtonian philosophy can give such insights as can be found in no other part of philosophy[15].

Newton's authority in the field of philosophy seems to be indisputable. However, after this magnificent statement, Kant shifts the scope of his research. He declares «that of all the things in nature whose first cause we can investigate, the origin of the world system […] is the one which we might first hope to understand thoroughly and reliably»[16]. However, the search for the first cause is not properly speaking within the terrain of Newton's natural philosophy. Kant is not working within a Newtonian framework; rather, he is closer to Cartesian or Leibnizian investigations, whose objective was the exploration of the nature of things in a physical and metaphysical manner rather than with a view to mathematical description. For this reason, therefore, Kant's work cannot be classified as Newtonian, and Eric Watkins is right in saying that his attitude towards Newton in this writing is complex, if not even incoherent at times[17].

A few sentences after the narration of this great advancement, Kant becomes aware that his investigation is different from that of Newton. He bases his confidence in the statement «that the physical part of cosmology may in future hope for that completeness to which Newton raised its mathematical half»[18]. In saying this, Kant is clearly repeating the old claim of Cartesians and Leibnizians that Newton was pursuing a mathematical system, but not a physics. This was precisely Pierre-Sylvain Régis's criticism in his anonymous review of the *Principia*, in which he writes that Newton's demonstrations are merely mathematical, not physical, producing a hypothetical system and without any metaphysical commitment. Thus not even in this sense can Kant's work be considered Newtonian.

Kant emphasizes that in working on the laws governing the origin of the universe «the hand of a practised mathematician would cultivate fruitful fields»[19] but his own attempt is not mathematical, and in saying this he confessed to not being very familiar with mathematical and geometrical tools. Could Kant have been deceived himself and believed that he was a Newtonian? Could he have mis-

[15] *KGS*, I, p. 230.
[16] *Ibidem*, p. 229.
[17] Watkins, *The Early Kant's Newtonianism*, p. 431.
[18] *KGS*, I, p. 230.
[19] *Ibidem*.

interpreted Newton? Perhaps. Reading the controversial sentences of the "Fifth reflection" of *The Only Possible Argument in Support of a Demonstration of the Existence of God* (1763), Kant still seems to believe in the possibility of a physics or mechanics of the universe that might truly explain causes and origins according to universal laws, and attributes the lack of this in Newton's system not to the fact that the English scientist considered it from a methodological point of view an impossible or vain or useless attempt, but to purely mechanical reasons.

However, like many young scholars of the time, he felt an irresistible attraction to Newton, especially in the attempt to reconcile physics or even metaphysics with mathematics. The ideas of certainty, necessity and universality in Newton's mathematics were so powerful that the younger generation attempted to solve the problem as to how to apply them to physics and metaphysics. But this is not the ambition of the *General History and Theory of the Heavens*.

We can find a clue to this attitude in the *Physical Monadology*. The very first page makes clear the endorsement of Newton, explaining how his approach is helpful but limited, and the need to overcome his perspective by applying his ideas to the search for the first cause:

> Clear-headed philosophers, who are seriously engaged in the investigations of nature, unanimously agree, indeed, that punctilious care must be taken lest anything concocted with rashness or with a certain arbitrariness of conjecture should insinuate itself into natural science, or lest anything be vainly undertaken in it without the support of experience and without the mediation of geometry. Certainly, nothing can be thought more useful to philosophy, or more beneficial to it, than this rule[20].

Clear-headed philosophers are evidently Newtonians, who based their natural explanation on experience understood through mathematical tools and who were against the introduction of futile ungrounded hypotheses. Thus, Kant seems to appreciate this mathematical method for understanding nature and to criticize Cartesians and Leibnizians, who were on the wrong path in physics, that of pursuing reverie in order to explain natural effects. After this bold statement in favour of Newtonians, Kant the Newtonian approach's essential weakness and the need to overcome it:

[20] *KGS*, I, p. 475.

> if we follow this sound path, we can exhibit the laws of nature though not the origin and causes of these laws. For those who only hunt out the phenomena of nature are always that far removed from deeper understanding of the first causes. Nor will they ever attain knowledge of nature itself of bodies[21].

In this passage Kant makes explicit the limits of the Newtonians' philosophy of nature, which prevents knowledge of the causes of things. The importance of the search for «first causes» was already evident in the *General History and Theory of the Heavens* but the detachment from Newton's methodology was not so clearly expressed. In the *Physical Monadology* Kant shows a strong metaphysical commitment to the possibility of knowing the ultimate causes and principles of reality, even if he was perfectly aware of the potentialities of Newton's mathematical approach. He thought that metaphysics alone could provide illumination for complex issues such as the infinite divisibility of space, free motion in the void, and universal attraction without mechanical cause. There was no evident solution to these questions, so much so that for Kant a marriage of mathematics and metaphysics was more difficult than the mating of griffins with horses. For our present purpose, how Kant tried to solve these problems is not so important as that he thought of himself as complaint with Newton's principles. Furthermore, the general tone of his discussion regarding a polarity between geometry as the style of Newtonians and metaphysics as the style of Rationalists allows us to say that Kant's methodological reflections are borrowed or influenced by Keill, who is explicitly mentioned in the text[22]. It is well-known that Kant was sparing in citing his sources, and when he does so they are usually extremely significant for him.

One of the most important aspects of Keill's contribution in disseminating Newton's thought are his methodological reflections. This importance is due to the fact that Keill's textbook was written and published ahead of both Newton's *Opticks* (1704) and the second edition of the *Principles* (1713), which contain Newton's ideas on method in their most developed form.

[21] *Ibidem*. Translation has been slightly modified.

[22] *Ibidem*, p. 486. Kant owned John Keill, *Introductiones ad veram physicam et veram astronomiam. Quibus accedunt trigonometria, de viribus centralibus, de legibus attractionis. Editio novissima*, Verbeek, Leiden 1739. Quotations are from John Keill, *An Introduction to Natural Philosophy*, Longman, London 1745.

Newton's methodological lesson emerges in the *Preface* and in the first lecture, entitled *Of the Method of Philosophizing.* The chapter is designed mainly to refute what he calls «mechanical philosophy», and to show how its underlying metaphysics and ontological presuppositions are incompatible with a mathematical approach in the description of the world.

Mechanical philosophy is against the «laws of nature, and the established principles of mechanics», arguing in favour of miraculous solutions for explaining the phenomena of nature[23]. Their mechanical principles are more complicated than the works of nature that they aim to investigate, and for the most part they are incapable of deducing new phenomena. This is evident for Keill in the complete misunderstanding of the conception of gravity by the Cartesians and other mechanical philosophers. In general, their main error lies in the fact that they «presume to philosophize, and to give the causes of natural things» without knowing geometry, which is the only means of understanding the forces of nature[24]. This ignorance is evident also in Descartes, who was famous for being a skilled geometrician, but who completely neglected geometry in his philosophy. In so doing, Descartes and the Cartesians «have embraced the shadows of philosophy» [25], abandoning the idea of penetrating the laws of nature by means of mathematics, whereby the mechanical causes of things might be discovered[26]. By mechanical causes, Keill meant efficient causes.

The heroes of the mathematical method as applied to the science of motion and mechanics are first and foremost Galileo, Torricelli, and Pascal, and then all the members of the Royal Society, among whom he mentions Huygens, Boyle, Wallis, and Halley. However, it was Isaac Newton, who opened up the «mysteries of nature» by means of mathematical analysis and scientific approach[27]. Keill, therefore, was in favour of an elaboration of mechanical philosophy based on the true laws of nature that was capable of grasping the efficient causes, while he firmly criticised those who confused mechanical philosophy with metaphysics.

[23] Keill, *An Introduction to Natural Philosophy*, Preface.
[24] *Ibidem*.
[25] *Ibidem*.
[26] *Ibidem*.
[27] *Ibidem*.

Keill provides a very clear picture of the advancement of Newtonian philosophy by distinguishing four methods for «investigating the causes of natural things». All of them are presented with their weaknesses and strengths. Their advocates comprise: (1) Pythagoreans and Platonists, (2) Aristotelians, (3) experimental philosophers, and (4) mechanical philosophers. Platonic positions are praised for their use of mathematics, capable of leading to certainty in the discovery of natural causes. However, they are also criticized for their obscure opinions. Keill does not even fully endorse the third method of investigation, that is experimental philosophy, which no doubt had led to new discoveries, but in many cases experiments were conceived in order to fit specific theories. Finally, he emphasizes once again his criticism towards mechanical philosophy for its erroneous combining of metaphysics with mathematics.

He is more interested in the second method, that of Aristotelian philosophy, so much so in fact that David B. Wilson characterizes Keill's natural philosophy as a form of «Aristotelian Newtonianism», in which «Aristotle enjoyed a surprising prominence»[28]. However, Keill is extremely critical of the Aristotelians' explanation of nature by means of matter and forms, virtues and occult qualities, faculties and attractions, because their prime purpose seems to be to give names to things rather than actively to explore the causes of things. Yet, for Keill this should not prevent the use of terms like quality, faculty, or attraction. These terms do not define true and physical causes and modes of action that pertain essentially to things, but actions that «may be intended and remitted», through which it is possible «to express the ratios of the forces or their augmentation and diminution»[29]. Ontological investigation of the nature of the quality is fruitless, because this law would work in any case, whatever the object is, as simple experiments can show. Before engaging in experimentation, in fact, one should observe three necessary rules in order to avoid all errors[30].

The first rule prescribes proceeding by setting definitions according to the method of geometers. To distinguish this position from that of the Aristotelians, Keill writes that these definitions are not to be thought of as logical, for they do not consist of genus and dif-

[28] David B. Wilson, *Seeking Nature's Logic: Natural Philosophy in the Scottish Enlightenment*, The Pennsylvania State University Press, University Park (PA) 2009, p. 44.

[29] Keill, *An Introduction to Natural Philosophy*, p. 4.

[30] *Ibidem*, p. 7.

ference, and nor do they reveal the intimate essence or the ultimate cause of things. This kind of investigation neither leads to scientific knowledge nor concerns physics, but rather it belongs to the realm of metaphysics. As with the majority of early Newtonians, Keill insists on the impossibility of knowing the intimate essences and causes of things (*intimae rerum naturae & causae*)[31], which lies within the scope of the metaphysicians. Indeed, the only possible way of acquiring reliable knowledge is via sensation. This kind of knowledge does not provide a definition that concerns the essence of things; rather, it furnishes a description, «whereby the thing described may be clearly and distinctly conceived and likewise be distinguished from everything else»[32].

The purpose of this process is to identify the simplest properties that belong to things in themselves, without any direct knowledge of the things in themselves. Then, having once determined these simple properties, by means of a «geometrical method», one may infer other properties of the same things[33]. In general, the objects of such knowledge are properties not essences. This according to Keill is very unsettling for many philosophers, who tend to believe that the main aim of natural investigations is to reveal the essence of things, which, however, «does not at all appear»[34]. Against the Cartesians, for instance, he defends the idea that these properties belong to things but are not their essence. Keill emphasizes the distinction between, on the one hand, properties that certainly pertain to things (*proprietates rebus ipsis certo competentes*) and, on the other, those that are really within things (*rebus ipsis revera insunt*). From the Newtonian standpoint, there are merely descriptions with regard to how things are experienced through properties via sensation, while from the Cartesian and Leibnizian perspective, there is an attempt to say something about the very nature of things – that is, how things are independently of experience. This is not possible for a loyal Newtonian.

The second rule of philosophizing for investigating natural truth specifies that the focus should be on the fewest possible conditions[35]. The limits of the human mind impede the consideration of

[31] *Ibidem*, pp. 7-8.
[32] *Ibidem*, p. 8.
[33] *Ibidem*.
[34] *Ibidem*.
[35] *Ibidem*, p. 9.

too many conditions at once, which would lead to fancy theories with no grounding in experience. Keill's third rule prescribes starting with the simplest cases and then incrementally adding conditions to investigate more complex phenomena. This is the most controversial rule because «most theorists», without being acquainted with the foundations of geometry, create a «weak superstructure». Keill applies all these rules in his natural investigations.

3. *Inverse Square Law*

In the previous section we saw how *Thoughts on the True Estimation of Living Forces* does not provide a smoking gun of Kant's direct knowledge of Newton. However, Kant elaborates very original ideas of Newtonian descent in paragraphs §9 and §10. Kant states that only if we conceive of a force in bodies that acts externally to them, are space and extension possible. In § 1, quoting Leibniz, he writes that «an essential force inheres in a body and belongs to it even prior to extension»[36]. Indeed, space, understood in relative terms, cannot exist without a connection taking place between bodies and this connection is provided by this force, which later Kant, following Leibniz, will call a living force. However, he confesses to being unable to explain whether and how the plurality of dimensions in space derives from this force or from another quality, virtue, or property. It is probable, he states, that «the three-dimensionality of space derives from the law according to which the forces of substances act on each other»[37]. § 10 makes this concept explicit:

> Because everything found among the properties of a thing must be derivable from what contains within itself the complete ground of the thing itself, the properties of extension and hence also its three-dimensionality

[36] *KGS*, I, p. 17. This is crucial in his discussion of the property of divisibility.

[37] *Ibidem*, p. 24. See Craig Callender, A*nswers in Search of a Question: "Proofs" of the Tridimensionality of Space*, «Studies in History and Philosophy of Modern Physics», 36 (2005), pp. 113-36; Francisco Caruso, Xavier Roberto Moriera, *On Kant's First Insight into the Problem of Space Dimensionality and Its Physical Foundations*, «Kant-Studien», 106 (2015), pp. 547-60; Silvia De Bianchi, James D. Wells, *Explanation and the Dimensionality of Space. Kant's Argument Revisited*, «Synthese», 192 (2015), pp. 287-303; Dimitria Gatzia, Rex Ramsier, *Dimensionality, Symmetry and the Inverse Square Law*, «Notes and Records: Royal Society Journal of the History of Science», 75 (2021), pp. 33-48.

> must also be based on the properties of substances possess in respect of the things with which they are connected [38].

But this force must be understood according to a principle or a law «that manifests in its mode of action» [39]. This law, which emerges from the essential virtue of bodies, determines the relations between bodies, and hence space or extension. All bodies and substances seem to act on each other for Kant «in such a way that the strength of the action is inversely proportionate to the square of distance» [40]. Therefore, he concludes:

> that substances in the existing world, of which we are a part, have essential forces of such a kind that they propagate their effects in union with each other according to the inverse-square relation of the distances; secondly, that the whole to which this gives rise has, by virtue of this law, the property of being three-dimensional; thirdly, that this law is arbitrary, and that God could have chosen another, e.g., the inverse-cube, relation; fourthly, and finally, that an extension with different properties and dimensions would also have resulted from a different law [41].

Eric Watkins correctly remarks that «stated in this form, the law is original to Kant and applies to various types of fields» [42], and he explains Kant's way of thinking as follows:

> One could represent a substance as a point, a three-dimensional space as a sphere enclosing it, and the propagation of force as lines extending from the point into the sphere. The lines in the sphere would pierce through, as they would another, larger sphere enclosing the point. Visualizing the pierced nested spheres shows that the lines move ever farther apart and that radiation – force acting in space – decreases with distance. The rate of decrease is supplied by basic geometry: the surface area of a sphere is determined by the square of the radius; surface areas on expanding spheres increase as the squares of their radii; hence radiation weakens as the inverse square of the distance from the center [43].

This very same procedure was envisaged by Keill in the first lecture *Of the Method of Philosophizing*, as Giorgio Tonelli first sug-

[38] *KGS*, I, p. 23.
[39] *Ibidem*, p. 24.
[40] *Ibidem*.
[41] *Ibidem*.
[42] Eric Watkins, *Notes*, in Immanuel Kant, *Natural Science*, Cambridge University Press, Cambridge 2012, p. 691.
[43] *Ibidem*, pp. 691-2.

gested, and Michael Friedman later confirmed[44]. Keill introduces the problem when discussing the use of terms like quality, faculty, and attraction for describing natural phenomena. Keill states that it is impossible to use these terms «to define the true and physical causes and modus of action», yet it is possible to determine how «these actions may be augmented and diminished», and therefore to express the law and ratios of these forces. For instance, according to Keill it is always possible to characterize gravity as a quality

> whereby all bodies are carried downwards, whether its cause arises from the virtue of the central body, or is innate to matter itself, or whether it proceeds from the action of the ether by a centrifugal force and so tending upwards, or finally, whether it is produced after any other manner whatsoever[45].

Keill makes clear that the investigation of the nature of causes is not as important as mechanical philosophers believed. The search for this nature of causes seemed to him to be a quite impossible enterprise for the limits of human mind. What was at stake was their applicability for the description of natural phenomena. They could be considered occult qualities in the sense that mathematical equations designate unknown quantities with letters like x, but not in the sense that they could not be investigated in their intensions and remissions. Keill applies this conception to the forces of bodies (*conatus*) tending mutually towards one another – in particular, that of attraction. It is not important to know the cause of attraction, «whether it proceeds from the action of the bodies tending mutually to one another, or from their being agitated by effluvia emitted etc…»[46]. Indeed, «however ignorant we are of the nature of qualities, and how much soever the modus of operations is concealed from us»[47], it is always possible to demonstrate the theorem concerning their intension and remission, according to which every quality that «is propagated every way in right lines from a center, is diminished in a duplicate proportion of distance from that center»[48]. This theorem is mathematically demonstrable in a univer-

[44] See Tonelli, *Elementi metodologici e metafisici*, pp. 66-9; Friedman, *Kant and the Exact Sciences*, p. 1.

[45] Keill, *An Introduction to Natural Philosophy*, 4.

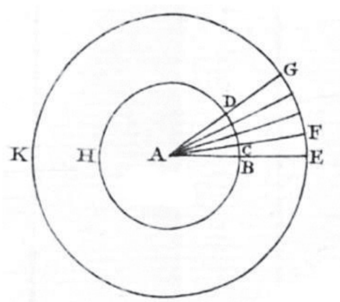
[46] *Ibidem*.

[47] *Ibidem*.

[48] *Ibidem*, p. 5.

sal way, «whatever is the nature of the quality» [49]. Hence there is no quality or virtue that does not follow this law. Keill provides a geometrical demonstration:

> Let a be a point or center, whence every way the quality is diffused, in the right Lines AB. AC. AD. And innumerable others spread indefinitely through the whole Space. I say, that the intension of the quality decreases in ratio duplicate of that, whereby the distances increase; or, which is the same thing, its intension at a distance equal to AB is to its intension at a distance equal to the right line AE, reciprocally in a duplicate ratio of the distance AB, to the distance AE; that is, directly as the square of AE to the square of AB.



> Since, from the hypothesis, the quality is propagated in a sphere every way by right lines, its intension at any distance from the center, will be proportionable to the spissitude or density of the rays at the same distance. By rays I here mean the rectilineal ways by which the quality is diffused: now the rays that at the distance AB are diffused thro' the spherical superficies BCDH, will be dispersed at the distance AE, thro' the whole spherical superficies EFGK: but the spissitudes of any given rays are reciprocally as the spaces they take up; namely, if the superficies EFGK is double the superficies BCDH, the rays at the superficies BCDH, will have double the density, that the same rays will have at the superficies EFGK: and if the superficies EFGK is triple the superficies BCDH, the rays likewise at the superficies BCDH will have triple the density that the same rays will have at the superficies EFGK; and universally whatever proportion the superficies EFGK has to the superficies BCDH, the same proportion will have the density of the rays at the superficies BCDH to the density of the same rays, at the superficies EFGK. And from *Archimed. of the Sphere and Cylinder*, it appears that all spherical superficies are in a duplicate ratio of their diameters or semidiameters: the spissitude therefore, or density of the rays by which the quality is propagated to a distance equal to the Distance AB, will be to the density of the same rays at a distance equal to AE, reciprocally in a duplicate ratio of the semidiameter or distance AB, to the semidiameter or distance AE. But

49 *Ibidem*, p. 6.

> as it has been said, the intension of the quality at any given distance, will always be as the spissitude of the rays by which it is propagated to that distance; therefore the intension of a quality at a distance equal to AB, will be to the intension of the same quality at a distance equal to AE, reciprocally in a duplicate ratio of the distance AB to the distance AE; that is, directly as the Square of AE to the Square of AB [50].

The theorem is independent of the nature of the quality, «so that it acts in right lines; and it hence follows, that the intensions of light, heat, cold, perfumes, and the like qualities, will be reciprocally as the squares of their distances from the point whence they proceed» [51]. But what is more important, according to Keill, is that the discovery of the laws is fruitful because then, in a second phase, it is possible to apply them for the description of other phenomena. Indeed, for Keill this specific law is the very same that rules «the actions of the Sun on different planets» [52]. In other words, relations between bodies are determined by the inverse square law.

4. *Impenetrability*

Kant goes on to develop his conception of living force in a further direction in the *Physical monadology*, where he explicitly mentions Keill as his source. In the "Preliminary considerations" he explains that:

> The force, which is inherent in the elements, must be a moving force, and one, indeed, which operates in an outward direction, since it is present to what is external; and since we are unable to conceive of any other force for moving that which is co-present than one which endeavours to repel or attract; and since, furthermore, if we posit only the repulsive force, we shall not be able to conceive the conjunction of elements so that they form compound bodies, but only their diffusion, whereas if we posit only an attractive force we shall only be able to understand their conjunction, but not their determinate extension and space […] [53].

[50] *Ibidem*, pp. 5-6.
[51] *Ibidem*, pp. 6-7.
[52] *Ibidem*, p. 7.
[53] *KGS*, I, p. 476.

In addition to the *Thoughts on the True Estimation*, therefore, Kant gives much emphasis of the existence of two forces for the constitution of a body.

The force through which bodies act externally to themselves, and without which no space and extension would be possible, is understood as a form of impenetrability. Impenetrability is the «force by which the simple element of a body occupies its space», or is «that property of a body, in virtue of which a thing in contact with it is excluded from the space which the body occupies» [54]. Kant adds that by virtue of this force of impenetrability bodies would not have a determinate extension or volume. However, this repulsive force is not sufficient to determine the terminations of the bodies: indeed, «if it were this force alone which existed, bodies would have no cohesive structure», and «no body would have a volume which was circumscribed by a determinate limit» [55]. There must be another force of attraction, without which bodies «would have no determinate volume» [56]. This is the context in which Kant openly discusses the inverse square law exposed by Keill, contesting its applicability for the description of the force of impenetrability:

> If one imagines a force emanating in straight lines from a given surface, as light does, or even in Keill's view, the attractive force itself, the force exercised in this way will be in proportion to the number of the lines which can be drawn from this surface, that is to say in proportion to the surface of the active being. Thus, if the surface is infinitely small, the force will be infinitely small, as well; and if finally, it is a point, the force will be nothing at all. A force spreading along lines diverging from a point cannot, therefore, have a specifiable value at certain specifiable distance. And, therefore, its exercising an effect can only be ascertained if it fills the whole space in which it acts. But spherical spaces are in proportion to the cube of their radii. Therefore, since the same force diffused throughout a larger sphere is diminished in a ratio which is the inverse of the volume of their spaces, the force of impenetrability will be in inverse ratio of the cubes of the distances from the centre of their presence. On the other hand, since attraction is, of course, the action of the same element, albeit in the opposite direction, the spherical surface towards which the attraction is exercised at a given distance will be the limit from which it is exercised. Since the multitude of the points, from which lines extending to the centre can be drawn, is determinate, and since, therefore, the magnitude of the attraction is also determinate, it follows that the attractive force can be assigned a definite value: it will decrease in the

[54] *Ibidem*, p. 482.
[55] *Ibidem*, p. 484.
[56] *Ibidem*.

> inverse ratio of the spherical surfaces, that is to say, with the inverse square of distance. If, therefore, it is established that the repulsive force decreases according to the inverse cube and thus at a far greater rate than the attractive force, there must be some point on the diameter where attraction and repulsion are equal. This point will determine the limit of impenetrability […] it will determine the volume; for the repulsive force, once it has been overcome by attraction ceases to act any further[57].

Thus, Kant accepts the inverse square law for the force of attraction, as he did in the *Thoughts on the True Estimation*, but introduces the inverse cube ratio for the force of repulsion. Since one force is calculated in relation to the surface, and the other in relation to the volume, at a specific distance from the center of the monad the two forces enter a state of balance. One of the consequences of this is that the ratio between these two forces is a constant: that is, for every monad they always become balanced at the same distance from the center. In conclusion, all monads, which together constitute all the various bodies, always have identical volumes.

There is nothing like a living force in Keill, yet there is something prior to extension and that inheres in all bodies. According to Keill, it is methodologically wrong to provide a definition of a body according to its nature or essence (*natura seu essentia*); rather, it is better to characterize it by some of its properties, in particular those of solidity, extension, and motion. Solidity for Keill represents that essential property of a body that Kant, in another context, calls living force: «Solidity is proper to bodies only, and so essential to all of them, that you cannot so much as separate it from them in your imagination, but at the very same time you destroy that very idea which you had formed of body»[58].

It is so essential that «if the essence and intimate nature of body is to be placed in some one attribute, solidity certainly has a much better pretence to be that attribute than» all the others[59]. Solidity is that virtue according to which a body «resists all other bodies that press it on every side». It is a force that acts outward and hinders all other bodies from entering into the place the body possesses. Unlike Kant though, Keill does not openly suggest a law that describes the force of repulsion.

[57] *KGS*, I, pp. 484-5.
[58] Keill, *An Introduction to Natural Philosophy*, p. 14.
[59] *Ibidem*, p. 15.

This property is what prevents contact between two bodies if there is a third body between them, whatever this third body comprises: «for a drop of water or a particle of air», which seem not solid at all, «does not less hinder the contact of those two bodies than the hardest metal or a diamond»[60]. Keill distinguishes this kind of solidity from the impenetrability of the Aristotelians and also from that of the mathematicians, which characterized it as what has three dimensions but which can be penetrable.

Solidity for Keill involves impenetrability and three-dimensionality. Keill borrows this idea from John Locke. In particular, he learns from Locke that solidity is «in the minutest particle of matter, that can exist», is «inseparably inherent in body, where-ever, or however modified»[61], and is a principle of «mutual impulse, resistance, and protusion» among bodies[62]. Solidity also provides the idea that a body fills a space, and therefore is distinguished from space. Pure space, indeed, is characterized as that place a body with its solidity has deserted and whereinto another body may enter without resistance[63]. Indeed, solidity «consists in repletion, and so an utter exclusion of other bodies out of the space it possesses»[64]. Therefore, it is through solidity that pure space and the extension of bodies are conceivable. But the most important conclusion Keill drew from Locke is the following:

> by this idea of solidity, is the extension of body distinguished from the extension of space. The extension of body being nothing, but the cohesion or continuity of solid, separable, moveable parts; and the extension of space, the continuity of unsolid, inseparable, and immoveable parts[65].

According to Keill, the «idea of extension into its triple dimension» is characterized by the distances (depth, breadth, and length) taken together among the various terminations of a body[66]. All bod-

[60] *Ibidem*, p. 13.

[61] John Locke, *The Clarendon Edition of the Works of John Locke: An Essay Concerning Human Understanding*, Oxford University Press, Oxford 1975, p. 123. Locke states that solidity is like Aristotelian impenetrability, though he prefers to use the word solidity «because it carries something more of positive in it, than impenetrability, which is negative, and is, perhaps, more a consequence of solidity, than solidity itself».

[62] *Ibidem*, p. 126.

[63] *Ibidem*, p. 124.

[64] *Ibidem*, p. 125.

[65] *Ibidem*, p. 126.

[66] Keill, *An Introduction to Natural Philosophy*, p. 11.

ies have this triple dimension, otherwise they would be either a point or line or surface. A line or surface has extension, but not triple dimension. The triple dimension of the extension of a body is provided by its solidity. Finally, Keill distinguishes solidity from space, which is penetrable, immovable, and wherein all bodies are placed and moved[67]: it «is a universal receptacle, wherein all bodies are contained and moved»[68]. Therefore «we have, or at least imagine we have, an idea of space distinct from the idea of body» – that is, of the extension of space as opposed to the extension of a body[69]. Extension is a common, essential attribute to space and body, but as attributes «they are very different things»[70]. Kant is therefore aware of Keill's reflections since the *Thoughts on the True Estimation*, but he finds them insufficient for his metaphysical investigations in the *Physical Monadology*.

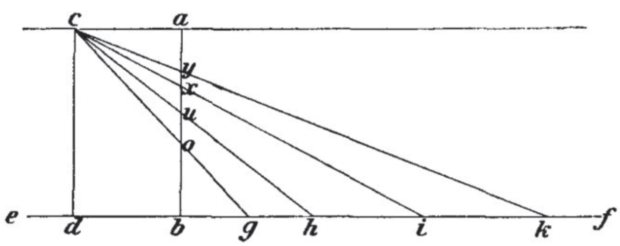
5. *Infinite Divisibility*

One of the central themes of the *Physical Monadology* is the existence of simple indivisible substances called monads. At the same time, Kant defends the ideas that «space which bodies fill is divisible to infinity»[71]. In supporting these two tenets, Kant takes an explicit stance against Cartesians who argued in favour of a perfect state of coincidence between extension and the body. For the indivisibility of space Kant provides a geometrical demonstration:

> Let there be given a line ef which is indefinitely extended, that is to say, a line which is such that it can always be extended further; and let there be given another line ab, a physical line, that is to say, a line which, if the reader will permit, is composed of the fundamental parts of matter, and which intersects ef at a right angle. To the side of ab let another line cd be erected, which is equal to ab and parallel to it. This, it will not be disputed, can be done not only in the geometrical sense but also in the physical sense. Let arbitrary points, g, h, i, k, and so indefinitely, be marked on the line ef. First of all, no one will dispute that between any two points, or, if will, between any two given monads, it is possible to draw a physical straight line. Thus, let a line cg be drawn, and let the point where it intersects the perpendicular

[67] *Ibidem*, p. 19.
[68] *Ibidem*.
[69] *Ibidem*, p. 14.
[70] *Ibidem*.
[71] *KGS*, I, p. 478.

ab be called o. Now imagine another physical line drawn between points c and h: the place u, which is common to both ch and ab, will be closer to point a. Continuing in this way, let there be drawn from the same point c lines to whatever points you wish on line ef extended indefinitely, such as i, k, etc. Their points of intersection get closer and closer to the point a, as is self-evident even to those who are completely ignorant of geometry. And if you suppose that these physical lines will eventually be too close together, so that they will no longer be able to continue to exist next to each other, the lower lines can be removed. Nonetheless, it is obvious that the points of intersection must get closer and closer to a, the further and further along the line ef you place the point. Since this distance can be extended to infinity, the point of intersection can be moved closer and closer to a by addition of infinitely many parts. But the intersection will never coincide with a in this way. For, in fact, since the points c and a are equidistant from ef, no matter how far you extend the line which joins points c and a, it will always be the same distance from the line ef beneath it; nor can they ever meet, for this would be against the hypothesis. Thus, by continuously dividing the line oa, we shall never arrive at simple parts, which cannot be divided further. That is to say, space is divisible to infinity and it does not consist of simple parts[72].



As we have mentioned, Kant's conception is based on the idea that bodies consist of parts, which have a separate permanent existence. These monads are not infinitely small particles or atoms of a body. Kant explains this impossibility through a methodology that combines mathematics with metaphysics. Indeed, if a body is a compound of infinite particles and the composition is accidental, then no substance could exist. But, since substances must exist, there must be a number of particles which lead to the minimal and essential composition of these substances. If bodies are composed of particles, then these particles or simple elements will be limited and determinate in number. Therefore, Kant concludes that it is absurd

[72] *Ibidem*.

to admit an infinite division[73]. Kant's position is clear: the space a body occupies is divisible at infinite, while the body not. But all this reasoning works only by admitting monads, the reality of which is not in question.

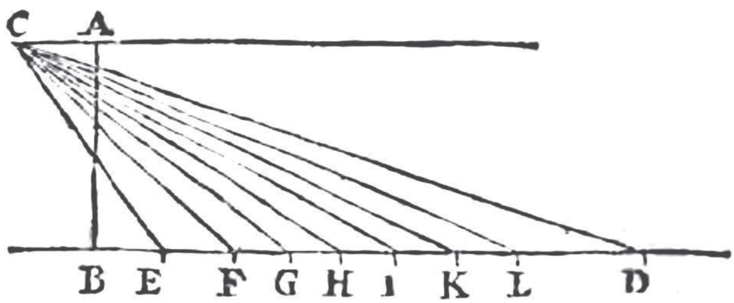
Kant's demonstration is borrowed by Keill, even if the Scottish mathematician proceeds from very different tenets. Kant and Keill agree that there is a distinction between extension and body, but they disagree on the nature of space and on the infinite divisibility of matter. Keill would have it that there are no monads. In other words, there are no indivisible particles and thus matter is infinitely divisible.

Keill discusses the possibility of divisibility for every kind of magnitude. He points out, first of all, that there is a distinction to be drawn between actual separation of the whole into parts and divisibility. Divisibility is understood only in a purely geometrical sense, but this does not mean that it is applicable only to geometrical objects, but rather that all extensions, whether corporeal or incorporeal, are infinitely divisible, or are conceivable as being constituted of an infinite number of parts. Most of the third lecture explains how geometrical demonstrations are applicable to physics, positioning itself against many philosophers, who tended to separate the two disciplines. His geometrical argument runs as follows:

> Let AB represent a right Line, I say it is divisible into parts exceeding any finite number whatever. Through A let be drawn any right line AC, and parallel to it let be drawn through B the right line BD, and in AC let there be taken any point as C: if therefore the right line AB is not divisible into an infinite number of parts, let it be divisible only into a finite number of parts; and let that number, for example, be six. In the line BD on the side opposite to C, let there be taken any number of points exceeding six; for example, the points E, F, G, H, I, K, L, and let there be drawn by the first postulate of Euclid, CE, CF, CG, CH, CI, CK, CL. These thus drawn, divide the right line AB into as many parts as there are right lines; for if they do not, then some of the right lines intersect AB in one and the same point: but all of them intersect one another in the common point C, whence some two right lines will cut one another twice, or will have the same common segment; both which is contrary to an axiom in the *Elements*. AB is therefore divided into as many different parts, as there are right lines; but there are as many right lines, as there were points taken in the right line BD: wherefore since there were taken more points than six, the right line AB is divisible into more parts than six. After the same manner, how great soever the number as-

[73] *Ibidem*, p. 479.

sumed shall be, it may be shown that the line AB is divisible into a number of parts greater than that number; namely, by taking in the right line BD a greater number of points (which may be easily done, since no finite number is so great, but a greater may be assumed, and that in any given ration of a greater inequality) and by drawing right lies from the point C to the points taken in the right line BD: for these right lines will divide the right line AB into as many parts, as there are right lines, and therefore into more parts than the number first assumed (how great soever it was) contains units; and consequently the right line AB is divisible into more parts than can be expressed by any finite number, and therefore it is divisible in infinitum[74].



Keill defends this demonstration within geometry, differently from Kant, who slips from geometry into physics by speaking of a «physical line» and of monads as «physical points». Like many Newtonians, Keill ignores and is seemingly completely indifferent to Kant's metaphysical theme of substantiality. There is no need to introduce substances to explain how the world is constituted. Kant argues in favour of the indivisibility of monads following Johann Joachim Darjes's argument against Keill, precisely in relation to this specific demonstration. In his *Elementa metaphysices* (1743), Darjes's demonstration is rooted in metaphysics and based on the definition of what is thinkable (*cogitabile*). The thinkable is either possible or impossible: if possible, then it is something (*aliquid*); if impossible, it is nothing (*nihil*)[75]. Thus, there is for Darjes the concept of an object in general, the thinkable, prior to any distinction regarding its possibility or impossibility, and before establishing whether

74 Keill, *An Introduction to Natural Philosophy*, pp. 26-7.
75 Johann J. Darjes, *Elementa metaphysices*, Cuno, Jena 1743, p. 34.

it is something or nothing[76]. Darjes adds that every instance of the thinkable has an essence or form (*essentia*, *forma*, *ratio formalis*, *quidditas*), through which it can be an object of the mind[77].

The thinkable – or, better, its essence – is either simple or complex. If it is complex, then it can be resolved into simple parts; otherwise, it is already simple. The resolution into simple parts is necessary because otherwise it would be impossible to establish the essence of a thinkable object. These simple parts cannot be further resolved, and without them it would be impossible to think of an object as such. This indivisibility and hence unity of the substance or essence is clearly metaphysical and, according to Darjes and authors like Christian Wolff, metaphysics has a clear expression or reflection in physics and in reality[78].

Darjes in fact misunderstands Keill, attributing to him the idea that points and surfaces cannot be infinitely resolved[79]. However, in this particular passage Keill was exposing the contradictions of philosophers who believe that each and every body is mathematically divisible in infinitum, but not in physical terms. For Keill, there is a perfect overlap between physics and mathematics, and no quantity can consist of indivisibles, as Darjes and Kant maintained:

> If quantity consisted of indivisibles, it would follow, that all motion would be equally swift, nor would a slow snail pass over a less space in the same time than the swift-footed Achilles. For let us suppose Achilles to run very swiftly, and the snail to creep sluggishly along; if extension consisted of indivisibles, the snail could not in any given time pass over less space than Achilles: for if in a moment's time Achilles passes over an indivisible space, the snail cannot in the same moment of time pass over less space; by reason, from the hypothesis, there cannot be a less. For one indivisible cannot be less than another, therefore it will pass over an equal space. The same may be said of any other moment of time: therefore, the spaces passed over by them both will be equal; and consequently, the swift-footed Achilles cannot pass over more space than the slowest snail: which is absurd. Other absurdities of the like sort, may be deduced from the same hypothesis of indivisibles […][80].

[76] *Ibidem*.
[77] *Ibidem*, p. 37.
[78] *Ibidem*, p. 40.
[79] *Ibidem*, p. 45.
[80] Keill, *An Introduction to Natural Philosophy*, pp. 31-2.

Methodologically Kant, like Darjes, in his mating of horses with griffins, or of mathematics with metaphysics, falls conceptually into error in order to preserve what for him constituted a necessary element of thought, that is a minimal and primitive substance, without which nothing could be thinkable.

6. *Conclusion*

Keill helped to popularize Newton's idea in very simple terms, and Kant no doubt was struck by his epistemological assumptions, making it possible to say something truthful about reality by means of mathematics, without defining the nature and essence of things. This seemed to be a perfect compromise between experience and theory, missing in Cartesian or Leibnizian metaphysics. Kant deploys aspects of Keill's argument, but he is not always successful in their application for his original attachment to metaphysical thinking.

Kant submitted the *Physical Monadology* on 23 March 1756, and it was discussed on 10 April, the day on which, in the *Wöchentliche Königsbergische Frag- und Anzeigungs-Nachrichten*, there appeared the first part of Kant's *Continued Observations on the Earthquakes That Have Been Experienced for Some Time*. At the very beginning of this text, Kant reiterates Newton's greatest achievement as having being the purging of physics of foolish ideas, the banishing of miracles and hypotheses from the investigation of natural phenomena[81]. The same adhesion to Newton's epistemology is emphasized in *The Only Possible Argument*, in which Kant states that it is in vain for any metaphysical attempt to demolish what has been established on the basis of empirical observations and mathematical deduction by introducing fancy definitions[82].

Kant's confidence in Newton's epistemology was not without limitations at this stage of his philosophical development[83]. Indeed, as Keill himself pointed out, Newton's approach was perfect in providing mathematical descriptions, but did not answer genuine metaphysical questions – which in Kant's view had extreme relevance for

[81] *KGS*, I, p. 466.
[82] *KGS*, II, p. 139.
[83] See Marco Sgarbi, *The Age of Epistemology: Aristotelian Logic in Early Modern Philosophy 1500-1700*, Bloomsbury, London 2023, pp. 223-37.

the advancement of scientific and philosophical thought. Already in *The General Natural History and Theory of the Heavens*, he had aimed to integrate metaphysics and mathematics. In the following years, Kant emphasizes the necessity of overcoming the gap between the two disciplines. He desperately tries to find a solution to this momentous problem. However, in the *Inquiry Concerning the Distinctness of the Principles of Natural Theology and Morality* (1763), in his acknowledgement that metaphysics (as it was conceived) and mathematics had two very different methods and epistemologies, his attempt was destined to fail. It took more than eighteen year for Kant to introduce mathematics within the fold of metaphysics, even with the help of friends more mathematically skilled in mathematics such as Johann Schultz[84]. However, from the early years, Keill introduced Kant to a new epistemological platform that the Königsberg philosopher came to elaborate in the *Critique of Pure Reason* (1781) and then the *Metaphysical Foundations of the Science of Nature* (1786).

[84] See Marco Sgarbi, *Matematica e filosofia trascendentale in Kant. Nota a margine di una fonte dimenticata della* Kritik der reinen Vernunft, «Philosophical Readings», 1 (2010), pp. 209-24.

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