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Research paper

The social value of information uncertainty☆

Xue-Zhong (Tony) He [a],*, Lei Shi [b], Marco Tolotti [c]

[a] International Business School Suzhou (IBSS), Xi'an Jiaotong-Liverpool University, China
[b] Department of Applied Finance, Macquarie Business School, Macquarie University, Australia
[c] Venice School of Management, Ca' Foscari University of Venice, Italy

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ABSTRACT

We analyze the welfare implication of information acquisition uncertainty in a Grossman–Stiglitz economy with endowment shocks. Investors make optimal probabilistic information acquisition choices subject to an increasing and convex monetary cost. This uncertainty gives rise to an anticipatory benefit so that informed trading can improve social welfare. Although informed trading distorts risk-sharing and destroys trading opportunities, the welfare improvement can be significant when investors have weak risk-sharing incentives, the endowment shocks are small and less informative about the aggregate endowment, and the risky payoff information is more noisy. Moreover, with heterogeneous endowments, there can be a continuum of Pareto optimal information-acquisition equilibria. Therefore, regulations aiming to level the playing field must be exercised with caution.

1. Introduction

Advanced technologies have changed how information is produced and processed. However, multiple dimensions of information complexity and uncertainty can limit information processing capacity (e.g., Simon, 1995, Goldstein and Yang, 2015) and make information acquisition costly and uncertain.[1] Investors often need to make effort (and hence spend money) to acquire information; the more effort they make, the more likely they are able to acquire the information. Can such costly and uncertain information acquisition improve investors' welfare?

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* Correspondence to: Department of Finance, International Business School Suzhou (IBSS), Xi'an Jiaotong-Liverpool University, China.
E-mail addresses: xuezhong.he@xjtlu.edu.cn (X.-Z. He), l.shi@mq.edu.au (L. Shi), tolotti@unive.it (M. Tolotti).

[1] There is extensive evidence on costly information acquisition and limited capacity in processing information. French (2008) finds that, over the period 1980–2006 in the U.S., "*investors spend 0.67% of the aggregate value of the market each year searching for superior returns*". In monetary value, "*(T)he cost of active investing is 7.0 billion dollars in 1980, 30.5 billion in 1993, and 101.8 billion in 2006*".

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In this paper, we examine this question in a Grossman–Stiglitz economy with heterogenous endowment shocks among investors, where information acquisition is costly and uncertain. In the standard Grossman–Stiglitz economy (Grossman and Stiglitz, 1980), investor pays a (fixed) cost to be informed (for sure). It is well known that such informed trading improves information efficiency, but distorts risk-sharing and destroys trading opportunities, and therefore reduces investors' welfare (see, e.g., Allen, 1984; Kurlat and Veldkamp, 2015). Different from the *all-or-nothing* information acquisition decisions in the Grossman–Stiglitz economy, in this paper, we introduce a costly and uncertain information acquisition to characterize multiple dimensions of information complexity. Investors make optimal probabilistic information acquisition decisions subject to an increasing cost of the probability to be informed. That is, the more an investor pays, the more likely for the investor to be informed. We show that this information acquisition uncertainty gives rise to an ex-ante '*anticipatory benefit*' so that more informed trading can improve investors' welfare, particularly when the information acquisition level in the market is low.[2] Therefore, different from the standard Grossman–Stiglitz economy, the uncertain information acquisition can improve social welfare, even when investors are risk averse and have heterogenous valuation and endowments. Moreover, the welfare improvement can be substantial when investors' risk-sharing incentives are weak, the endowment shocks are small and less informative about the aggregate endowment, and the risky payoff information is more noisy. Otherwise, the information acquisition reduces investors' welfare, as in the standard Grossman–Stiglitz economy. Therefore, the uncertain information acquisition can lead to a *hump-shaped* welfare function in the informed trading. Moreover, with heterogeneous endowment shocks, there can be a continuum of Pareto optimal information-acquisition equilibria.

It is well documented in the standard Grossman–Stiglitz economy about the possibly detrimental effects of information acquisition (or disclosure) on investors' welfare. There exist two well-known channels through which information acquisition may cause negative externalities. The first channel is the *Hirshleifer effect* (Hirshleifer, 1971) of destruction of insurance opportunities. When investors want to hedge against their endowment shocks, information acquisition reduces future payoff uncertainty, bringing the asset price closer to its fundamental. This makes it more difficult to hedge, and thus reduces investors' willingness to trade. The second channel, the *risk-return effect*, is related to the erosion of trading opportunities (Kurlat and Veldkamp, 2015). Even without endowment shocks, information acquisition can be still harmful to investors, because the reduction in risk also reduces the expected return, and the net effect on welfare is negative. As explained by Goldstein and Yang (2017), a "*common theme of both channels is that disclosure harms investors through destroying trading opportunities*". Therefore, the Hirshleifer and risk-return effects can be combined into what we call the *informed-trading effect*, which is detrimental for investors' welfare.

This paper incorporates uncertain information acquisition to the standard Grossman–Stiglitz economy and presents a rational expectations equilibrium (REE) model to analyze the welfare consequences of the presence of costly and uncertain information acquisition. More explicitly, we consider a Grossman–Stiglitz pure-exchange economy of a continuum of investors (Grossman and Stiglitz, 1980) with two tradable securities, a risky asset and a risk-free asset, part of the risky payoff uncertainty can be resolved by acquiring a costly private signal. A crucial hypothesis of our model is that investors make '*probabilistic choices*' on information acquisition. That is, investors optimally choose a probability to observe the fundamental signal about the risky payoff subject to a quadratic (monetary) cost of the probability. This means that, to increase the probability of being informed, investors need to pay more.[3] To conduct welfare analysis, we follow Manzano and Vives (2011) and Bond and Garcia (2022) and consider investors with *heterogeneous endowment shocks*.[4] Therefore, investors in the economy are expected utility maximizers who make joint information acquisition and portfolio choices. In equilibrium, a fraction of the investors observe the fundamental payoff signal and become *informed*, while the rest of the investors remain *uninformed*.

We first show that, when the information acquisition is uncertain, the effort to acquiring information can bring an *anticipatory benefit*, giving rise to what we call an *opportunity effect*. This implies that, without any externality (i.e., a *fixed* investment opportunity set), the probabilistic information acquisition always leads to better welfare comparing to the standard Grossman–Stiglitz setup.

With heterogeneous endowment shocks, we decompose the marginal welfare into a positive opportunity effect and a negative informed-trading effect. We examine the conditions under which the positive opportunity effect dominates the negative externalities of informed trading such that the marginal welfare of information acquisition is positive. We show that investor's welfare either decreases monotonically or is humped-shaped with respect to the equilibrium level of informed trading. The former scenario is more likely to occur when either the risk-sharing incentives, or the endowment shocks, or the precisions of the payoff information and endowment are high. In these cases, investors are better off in the no-information equilibrium. When the risk-sharing incentives, the endowment shocks, and the precisions of the payoff information and endowment are low, the welfare function can be hump-shaped. Therefore, information acquisition improves investors' welfare in a market with low level of informed trading. With heterogeneous endowment shocks among investors, there can be a continuum of Pareto optimal information acquisition equilibria. The no-information equilibrium is the unique Pareto-optimal equilibrium only when investors have strong risk-sharing incentives.

Our findings provide two main policy implications. First, even in a pure-exchange economy where informed-trading results in only negative externalities, costly information acquisition uncertainty can be welfare-improving, especially for speculators who provide liquidity (to those who trade to hedge their endowment shocks). Regulations such as mandatory disclosure reduces investors'

[2] In equilibrium, investors' ex-ante and ex-post welfare are the same in the Grossman–Stiglitz economy after taking the information acquisition cost into account. With information acquisition uncertainty, ex-ante, information acquisition uncertainty provides investors chances to be informed, which is always better than being uninformed ex-post. This difference is highlighted in Corollary 13 and the related discussion for no-information equilibrium analysis.

[3] This is similar to playing a lottery game; the more tickets a player purchases, the better the chance of winning, and the higher is the cost of the action. In the context of financial markets, an investor may purchase, for example, an analyst report, hoping to obtain valuable information about the fundamental value of a firm. *Ex-ante*, investors expect a more valuable report by paying more.

[4] Most of the REE literature relies on exogenous noise traders, which are not suitable for conducting welfare analysis.

incentives to acquire information, particularly when the precisions of the private payoff signals and endowment shocks are low and investors have less incentive to share the risk.[5] Therefore, policymakers aiming to level the playing field should take great caution when imposing regulations which may discourage information acquisition activities. Such regulation can be costly and lead to unintended consequences. Discouraging information acquisition (by increasing the cost sensitivity) may become necessary only when the level of informed trading is excessively high, the endowment shocks are high, or when the payoff signals and endowment shocks are more informative.

Second, when active investors make probabilistic information acquisition choices due to information acquisition uncertainty, it could happen that, when the cost sensitivity is high, the majority of investors do not become informed, and they underperform compared to passive investors who did not expend attempting to acquire private information. However, this does not necessarily mean that average active investors are worse off, because they still experience the anticipatory benefit *ex-ante*. In fact, if we disregard the negative externalities of informed trading, all investors are better off with the opportunity to acquire information than without. Therefore, the standard investment recommendation offered, especially to retail investors, to adopt a low-cost *index-investing* strategy may be misguided. The equilibrium welfare consequence of indexing may be worse than we thought (Bond and Garcia, 2022).

Related literature

Our paper is closely related to the literature that examines the equilibrium outcomes of endogenous information acquisition and their implications for welfare, going back at least to Hirshleifer (1971).

By considering information acquisition uncertainty, we offer a novel anticipatory welfare benefit channel for welfare improvement of informed trading. In the Grossman–Stiglitz setup, it is well-understood that, by resolving uncertainty and destroying trading opportunities, information acquisition is welfare reducing (see, e.g., Allen, 1984; Kurlat and Veldkamp, 2015).

Some possible market regulations, including a tax on information acquisition (Allen, 1984) or mandatory information disclosure (Kurlat and Veldkamp, 2015) have been proposed to resolve this issue. Other literature has identified alternative channels for possible welfare improvement.[6] This paper deviates from the literature by introducing information acquisition uncertainty to the Grossman–Stiglitz equilibrium. We show that, even when investors have heterogenous endowments, information acquisition can improve social welfare. For market regulators who are aiming for a level playing field with respect to information in financial markets, this suggests that such costly financial reforms may not be necessary, can be beneficial for liquidity providers (speculators) but harmful for liquidity consumers (hedgers), and therefore not Pareto optimal.

This paper develops a new modeling framework for information acquisition in financial markets. Our probabilistic choice model is adopted from Mattsson and Weibull (2002). Different from Mattsson and Weibull (2002), we use monetary instead of utility cost, which is more suited to the Grossman–Stiglitz framework.[7] The framework naturally connects settings with and without information acquisition uncertainty, which helps to provide the economic mechanism for welfare improvement.

The costly information game setup in this paper also contributes to the literature on endogenous information equilibrium and limited attention.[8] Different from this literature, we consider an increasing and convex monetary cost for information that depends on the probability of being informed. The resulting welfare trade-off between the equilibrium level of information acquisition and information precision is in line with rational inattention described in Sims (2003); i.e., economic agents have limited ability to process information or to pay attention to it. Different from the rational inattention literature focusing on information precision as a decision variable, we focus on the probabilistic choice of information acquisition for given information precision. With increasing information complexity, investors are aware of the limited resources to grasp information and set the optimal level of effort. The greater the investor's effort, the higher the probability of being informed, and the higher the attention the investor pays to available signals.

Interestingly, Hoff and Stiglitz (2016) discuss the importance of advancing the economic modeling background to allow for endogenization of preferences and behaviors. They argue that an equilibrium in the economy is a joint (endogenous) outcome expressed in terms of *probability of types* and *market prices*. In this respect, our framework can be seen as the first attempt to introduce *endogenization of types* into an otherwise standard exchange economy.

The paper is organized as follows. We first develops the model of costly information acquisition uncertainty and characterize financial market and information acquisition equilibrium in Section 2. Section 3 examines trade off channels between opportunity and informed-trading effects and the underlying mechanism for welfare improvement. Section 4 extends the model to consider heterogeneous valuations. Section 5 conducts further robustness analysis with respect to preference for early resolution of uncertainty. Section 6 concludes. All the proofs are presented in the Appendix.

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[5] When the precision of the payoff signals is low, a mandatory disclosure of such information could reduce investors' incentive to acquire private information, making market less informative and reducing investors' anticipatory welfare, particularly for the speculators whose endowments are low. Examples of mandatory disclosure regulations include the Freedom of Information Act (FOIA), a law that allows for the disclosure unreleased information by the U.S. Government, and MiFID II in the EU that aims to increase transparency for trading costs and improve record keeping for transactions.

[6] These channels include risk-sharing among outsiders with stochastic liquidity (Bhattacharya and Nicodano, 2001), the feedback effect of investment policy (Dow and Rahi, 2003), social communication (Han and Yang, 2013), preventing market failure (Goldstein and Leitner, 2018), externality in the use of private information (Vives, 2017), and heterogeneous private valuations (Rahi, 2021). Because of private or heterogeneous valuation, Vives (2017) and Rahi (2021) do not consider endowment shocks, simplifying the welfare analysis significantly.

[7] Moreover, we use a quadratic cost function to obtain more explicit results without sacrificing its economic insights.

[8] See, e.g., Diamond and Verrecchia (1981), Admati (1985), Admati and Pfleiderer (1987), Veldkamp (2006), and Vives (2014).

2. The model

We extend the standard Grossman–Stiglitz model to incorporate information acquisition uncertainty under probabilistic choices. First, we follow Manzano and Vives (2011) and Bond and Garcia (2022) and consider the trading stage and exogenous equilibrium in the financial market in which the fraction of informed investors are given exogenously. We then endogenize information acquisition by assuming investors make optimal probabilistic information acquisition decisions before trading takes place. We emphasize that, instead of exogenous "noise" or "liquidity" trades, investors have heterogenous and privately observed endowment shocks.

2.1. Preferences and trading environment

Consider a static economy with a continuum of investors in the interval $[0,1]$ who maximize expected constant absolute risk aversion (CARA) utility over terminal wealth with a CARA coefficient, α. They trade a risky asset and a risk-free asset with interest rate normalized to zero. Both the risk-free and risky assets are in zero net supply. The risky asset has a price P and a payoff $D \sim \mathcal{N}(0, v_D)$. Moreover, investor-$i$ receives a fundamental signal, $\theta = D + \epsilon$ with $\epsilon \sim \mathcal{N}(0, v_\epsilon)$, about the payoff if he becomes informed, ϵ and D are uncorrelated, thus $v_\theta \equiv \mathbb{V}ar[\theta] = v_D + v_\epsilon$. To simplify the analysis, we first consider a homogeneous private signal as a baseline model. We extend the model to heterogenous and correlated private signals among investors, as in Manzano and Vives (2011), in Section 4.

Investor-i receives an endowment shock, e_i, privately known prior to trading, whereas its random payoff, $e_i D$, is realized at the end of trading. We assume the heterogeneous endowment shocks $e_i = z + u_i$, where $z \sim \mathcal{N}(0, v_z)$ and $u_i \overset{i.i.d}{\sim} \mathcal{N}(0, v_u)$ represent aggregate and idiosyncratic endowment shocks, respectively. Thus, $v_e \equiv \mathbb{V}ar[e_i] = v_z + v_u$. The endowment related random variables, z and u_i, are independent from the payoff related random variables, D and ϵ. As in Manzano and Vives (2011), the endowment shocks allow a channel for private learning when predicting the payoff and disentangling price from the aggregate endowment. Investors have not only information about the payoff of the asset (when they are informed), but also about the exposure to their hedging (or liquidity) needs, characterizing complex and multidimensional information in the market, which can make information acquisition uncertain.

We now introduce three key parameters of the model. We denote $n \equiv v_D/v_\epsilon$ as the precision of the fundamental signal θ about risky payoff D, or *information precision* for short. Intuitively, by observing fundamental information θ investors can reduce a fraction of the payoff uncertainty, $n/(1+n)(= v_D/v_\theta)$, i.e., $\mathbb{V}ar[D|\theta] = [1-n/(1+n)]v_D$. The higher the information precision, the more reduction in the payoff uncertainty for being informed. Similarly, let $\Psi \equiv v_z/v_e$ be the precision of the heterogenous endowment e_i about the aggregate endowment z, or *endowment precision* for short. By observing the endowment shock e_i, investor-i can resolve a fraction, Ψ, of the aggregate endowment uncertainty, i.e., $v_{z|e} \equiv \mathbb{V}ar[z|e_i] = (v_u^{-1} + v_z^{-1})^{-1} = (1-\Psi)v_z$. Intuitively, the higher the endowment precision, the more reduction in the aggregate endowment uncertainty for the investor. Moreover, we denote $\xi_0 \equiv \alpha^2 v_D v_{z|e}$ as investors' *risk-sharing incentives*, which are stronger when investors are more risk averse, or the payoff and endowment are more noisy. These parameters play a key role in the welfare analysis.

2.2. Financial market equilibrium

We first assume that a given fraction λ of investors are informed. Then, we endogenize λ by modeling investors' optimal information acquisition decisions. For convenience, we assume all investors have zero initial wealth. Then, based on the information set, which is $\mathcal{F}_i = \{\theta, P, e_i\}$ if informed, and $\mathcal{F}_i = \{P, e_i\}$ if uninformed, investor-i makes the optimal portfolio choice to maximize $\mathbb{E}[-\exp(-\alpha W_i)|\mathcal{F}_i]$, where the portfolio wealth, combined with the endowment shock, is given by $W_i = (x_i + e_i)R + e_i P$, where $R = D - P$ is the return, the demand for the risky asset x_i has two different functional forms, depending on the information set of the investor: $x_i = x_I(\theta, P, e_i)$ for the informed, and $x_i = x_U(P, e_i)$ for the uninformed. That is,

$$x_I^*(\theta, P, e_i) = \arg\max_{x_i} \mathbb{E}[-\exp(-\alpha W_i)|\theta, P, e_i]$$

and

$$x_U^*(P, e_i) = \arg\max_{x_i} \mathbb{E}[-\exp(-\alpha W_i)|P, e_i].$$

Within the CARA-Normal framework, we can characterize the optimal portfolios of the informed and uninformed investors as follows,

$$x_i^* = \begin{cases} x_I^*(\theta, P, e_i) = \dfrac{\mathbb{E}[R|\theta, P, e_i]}{\alpha \mathbb{V}ar[R|\theta, P, e_i]} - e_i, & \mathcal{F}_i = \{\theta, P, e_i\}; \\ x_U^*(P, e_i) = \dfrac{\mathbb{E}[R|P, e_i]}{\alpha \mathbb{V}ar[R|P, e_i]} - e_i, & \mathcal{F}_i = \{P, e_i\}. \end{cases} \tag{2.1}$$

Note that informed investors observe θ and P, therefore the endowment shock e_i does not provide any additional information about the payoff D and hence the return R. In contrast, uninformed investors must extract information from price P and their endowment shocks e_i to form expectation and variance of the return R. We solve for investors' optimal portfolios in the following lemma.

Lemma 1 (*Optimal Portfolio*). *The optimal demand schedules for the informed and uninformed investors are given by*

$$x_I^*(\theta, P, e_i) = \frac{(1+n)^{-1} n\theta - P}{\alpha v_I} - e_i, \qquad v_I = (1+n)^{-1} v_D, \tag{2.2}$$

and

$$x_U^*(P, e_i) = \frac{-(1-\kappa)P - \kappa\beta_{e,P}e_i}{\alpha v_U} - e_i, \qquad v_U = v_D - \kappa\sigma_{D,P}, \tag{2.3}$$

respectively, where $\kappa = \sigma_{D,P}/\left(v_P - \frac{\sigma_{e,P}^2}{v_e}\right)$, $\beta_{e,P} = \sigma_{e,P}/v_e$, $\sigma_{D,P} = \mathbb{C}ov[D, P]$ *and* $\sigma_{e,P} = \mathbb{C}ov[e_i, P]$.

Note that $v_U = \mathbb{V}ar[R|P, e_i](< v_D)$ is the conditional variance of return of the uninformed investors after observing P and e_i. Moreover, $\kappa > 0$ since P acts as a positive signal for D, and $\beta_{e,P} < 0$ since e_i and z are positively related whereas z and P are negatively related.

A rational expectations equilibrium price P in the financial market is then determined by the following market clearing condition,

$$\int_0^1 \left[\lambda x_I^*(\theta, P, e_i) + (1-\lambda)x_U^*(P, e_i)\right] di = 0. \tag{2.4}$$

We conjecture a linear equilibrium price in the fundamental signal and aggregate endowment shock, i.e.,

$$P = b_\theta \theta - b_z z, \tag{2.5}$$

where the coefficients, b_θ and b_z, are determined in equilibrium by the market clearing condition. Note that the price coefficients are functions of the state variable, λ, which is then endogenized when discussing investors' probabilistic information acquisition. For given λ, we restate the following result in Manzano and Vives (2011) on the coefficients in the price function and the sufficient condition for uniqueness.[9]

Lemma 2 (*Financial Market Equilibrium, Manzano and Vives (2011)*). *For an exogenously given fraction of informed investors,* $\lambda \in [0, 1]$, *there exists a linear equilibrium price of the risky asset, given by (2.5), where* $\pi \equiv b_\theta/b_z$ *solves the fixed-point equation,*

$$\pi = f(\pi) \equiv \frac{1}{\alpha v_\epsilon}\left(\lambda + (1-\lambda)\frac{1}{\Psi^{-1} + v_\epsilon^{-1} v_u \pi^{-2}}\right), \tag{2.6}$$

and

$$b_\theta = 1 - \frac{v_D^{-1}}{\lambda v_I^{-1} + (1-\lambda)v_U^{-1}}, \qquad v_U = \left[v_D^{-1} + \frac{v_\epsilon^{-1}\pi^2(v_u^{-1} + v_z^{-1})}{v_\epsilon^{-1} + \pi^2(v_u^{-1} + v_z^{-1})}\right]^{-1}. \tag{2.7}$$

The ratio π in (2.6) measures price informativeness; the higher the ratio π, the more informative the price becomes. It is the root of the cubic Eq. (2.6) and thus can have multiple solutions. Manzano and Vives (2011) characterize the conditions for a unique or multiple equilibria, which we reiterate in the following lemma.

Lemma 3 (*Uniqueness of Equilibrium Price, Manzano and Vives (2011), Corollary 9*). *If* $\lambda \geq \Psi/(\Psi + 8)$, $\pi = b_\theta/b_z$ *is unique. Oppositely, if* $\lambda < \Psi/(\Psi + 8)$, *there exist two thresholds,* $\underline{\alpha}$ *and* $\overline{\alpha}$, *for the risk aversion coefficient such that (i)* π *is unique for* $\alpha < \underline{\alpha}$ *or* $\alpha > \overline{\alpha}$; *(ii) there are two equilibrium prices for* π *when* $\alpha = \underline{\alpha}$ *or* $\alpha = \overline{\alpha}$; *and (iii) there are three equilibrium prices for* π *when* $\underline{\alpha} < \alpha < \overline{\alpha}$.

The precise values of $\underline{\alpha}$ and $\overline{\alpha}$ are provided in Appendix A. Therefore, to guarantee price uniqueness for any $\lambda \in [0, 1]$, we assume throughout the paper that the risk aversion $\alpha < \underline{\alpha}$ or $\alpha > \overline{\alpha}$.

2.3. Endogenizing information acquisition

We now endogenize λ by allowing investors to make optimal information acquisitions that are *probabilistic*. Prior to trading, we assume each investor chooses a probability p_i to observe the fundamental signal θ at a quadratic cost μp_i^2, where $\mu > 0$ measures the *cost sensitivity*. This means that the higher the probability for an investor to become informed, the higher the cost the investor has to pay.[10] Afterwards, a Boolean random variable ω_i is drawn independently for each investor with $\mathbb{P}(\omega_i = 1) = p_i^*$ and $\mathbb{P}(\omega_i = 0) = 1 - p_i^*$. If $\omega_i = 1$, investor-i observes θ and becomes informed. Otherwise, $\omega_i = 0$; investor-i does not observe θ and remains uninformed. We assume investor-i chooses p_i^* conditional on e_i, which is known prior to trading, and also take λ as given. In line with rational expectations, the equilibrium λ is then determined by investors' probabilistic information choices.

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[9] In a more general setup, Manzano and Vives (2011) characterize financial market equilibrium for given information acquisition and we therefore apply their results to our model setup. They focus on multiplicity of equilibria in the financial market for a exogenous level of informed trading. However, different from Manzano and Vives (2011), we also characterize investors' optimal (probabilistic) information choice, thus the equilibrium level of informed trading, and their joint impact on investors' welfare.

[10] The analysis can be conducted for any increasing and convex cost function, at least numerically. For simplicity, we only consider a quadratic cost function in this paper.

Optimal information choice

By taking into account the associated cost, investor-i makes a probabilistic choice p_i to maximize their expected utility

$$\mathcal{W}(p_i;\lambda,e_i) \equiv \left[p_i V_I(\lambda,e_i) + (1-p_i)V_U(\lambda,e_i)\right] e^{\alpha\mu p_i^2}, \tag{2.8}$$

where

$$V_I(\lambda,e_i) = \mathbb{E}\left[-e^{-\alpha\left(x_I^*(\theta,P,e_i)R+e_iD\right)}\Big|e_i\right], \qquad V_U(\lambda,e_i) = \mathbb{E}\left[-e^{-\alpha\left(x_U^*(P,e_i)R+e_iD\right)}\Big|e_i\right]$$

are the expected utilities of the informed and uninformed investors attainable by their optimal portfolios $x_I^*(\theta,P,e_i)$ and $x_U^*(P,e_i)$, respectively. Note that $V_I(\lambda,e_i)$ and $V_U(\lambda,e_i)$ depend on λ since the equilibrium price P itself depends on λ.[11] Therefore, the expected utility (2.8) is '*ex-ante*' by taking the information acquisition uncertainty into account.

We now compute the expected utilities for informed and uninformed investors, i.e., $V_I(\lambda,e_i)$ and $V_U(\lambda,e_i)$. First, investor-i's expected utility conditional on his information set is given by

$$\mathbb{E}\left[-e^{-\alpha W_i}|\mathcal{F}_i\right] = -\exp\left\{-\alpha e_i P - \frac{1}{2}\frac{\chi_i^2}{v_i}\right\}, \tag{2.9}$$

where $\chi_i \equiv \mathbb{E}\left[R|\mathcal{F}_i\right]$ and $v_i \equiv \mathbb{V}ar\left[R|\mathcal{F}_i\right]$ are the expectation and variance of return conditional on investor-i's information set. Note that, conditional on the endowment shock e_i, the price P and conditional expected return χ_i follow a bivariate normal distribution. We can obtain the following expression for investor-i's expected utility with endowment shock.

Proposition 4 (*Expected Utility*)**.** *For $K = I, U$,*

$$V_K(\lambda,e_i) = V_K(\lambda)\exp\left\{-\frac{1}{2}A(\lambda)e_i^2\right\}V_0(e_i), \tag{2.10}$$

where

$$V_0(e_i) = -\exp\left\{\frac{1}{2}\alpha^2 v_D e_i^2\right\}, \tag{2.11}$$

$$A(\lambda) = \frac{\left[\beta_{e,P} + \alpha(v_D - \sigma_{D,P})\right]^2}{v_{P|e} + v_D - 2\sigma_{D,P}}, \tag{2.12}$$

$$V_K(\lambda) \equiv \frac{1}{\sqrt{1+\xi_K(\lambda)}}, \tag{2.13}$$

$\beta_{e,P}$ is defined in Lemma 1, and

$$\xi_K(\lambda) = \begin{cases} \dfrac{n(1+n)^{-1}v_D + v_{P|e} - 2\sigma_{D,P}}{(1+n)^{-1}v_D}, & K = I; \\ \dfrac{(1-\kappa)^2 v_{P|e}}{v_D - \kappa\sigma_{D,P}}, & K = U. \end{cases} \tag{2.14}$$

The decomposition (2.10) of the expected utility in Proposition 4 is in line with Bond and Garcia (2022). It helps to provide the economic mechanism for the welfare analysis in the following section. Note that $V_0(e_i)$ in (2.11) represents investor-i's expected utility without trading, namely, related to the end-of-period consumption e_iD from the endowment. Note that a large endowment shock (positive or negative) moves Investor-i's portfolio further away from the optimal holding. Thus, $V(e_i)$ can be described as disutility due to endowment shock. Of course, the greater the uncertainty, the larger the endowment shock is likely to be. The decomposition (2.10) shows that trading can improve $V_0(e_i)$ through two channels.

The first channel is the financial risk-sharing characterized by $V_K(\lambda)$ and hence $\xi_K(\lambda)$. In fact, $\xi_K(\lambda) > 0$ can be interpreted as the *squared* Sharpe ratio of the risky security. Essentially, investor-i is better off when financial market offers better risk-return trade-off, which increases $\xi_K(\lambda)$ and decreases $V_K(\lambda)$.[12]

The second channel is related to the exposure to endowment risk. Note that the term $-\exp\{-A(\lambda)e^2/2\}$ should be interpreted as the *hedging benefit of trading*, because investors trades partly to undo their exposure to endowment risk, thus reduce the utility cost of the endowment uncertainty. Examining the expression in (2.12), $A(\lambda)$ is increasing in $\beta_{e,P}$ and $\mathbb{C}ov[R,D] = v_D - \sigma_{D,P}$, and decreasing in $\mathbb{V}ar[R|e_i] = v_{P|e} + v_D - 2\sigma_{D,P}$.[13] Intuitively, it costs more to undo risk exposure by trading when price is more

[11] More precisely, in equilibrium, $P_\lambda = h_\lambda(\theta,z)$ is a random variable, where h_λ is a deterministic function depending on λ.

[12] The (squared) Sharpe ratio (as defined in (A.3)) $\xi_K(\lambda) \equiv \mathbb{V}ar[\chi|e]/\mathbb{V}ar[R|e]$, where $\chi = \mathbb{E}[R|\mathcal{F}_K]$. If investor-$i$ is informed, $K = I$, $\mathcal{F}_I = \{\theta,P,e\}$. Otherwise, investor-$i$ is uninformed, $K = U$, and $\mathcal{F}_U = \{P,e\}$. Therefore, ex ante, before the realization of the signal θ and price P, ξ_I (resp. ξ_U) is the squared Sharpe ratio anticipated by the informed (resp. uninformed) investors. Essentially, Sharpe ratio for the same risk asset can differ between investors due to asymmetric information about the asset payoff.

[13] Note that since $\beta_{e,P}$ is negative, A is not necessarily increasing in $\beta_{e,p}$. In fact, we can show that

$$\frac{\partial A}{\partial\beta_{e,P}} = \frac{2(\beta_{e,P} + \alpha(v_D - \sigma_{D,P}))}{\mathbb{V}ar[R|e]}$$

Thus, for $\partial A/\partial\beta_{e,P} > 0$, we require $\beta_{e,P} + \alpha(v_D - \sigma_{D,P}) > 0$, which is satisfied with all our parameterizations. Since we choose $\Psi = 0.01$, so individual endowment shocks contains very little information about the aggregate endowment shock, thus the equilibrium price.

negatively correlated to endowment shocks. More specifically, the correlation between e and z is measured by Ψ, i.e., the precision of the endowment signal. A higher Ψ makes $\beta_{e,P}$ more negative, which makes it more costly to undo endowment risk exposure. Thus, one would anticipate that A is decreasing in Ψ. Suppose investor-i receives a large positive endowment shock, if Ψ is high (close to one), it is likely that other investors also receive similar endowment shocks. This results in a more negative $\beta_{e,P}$ because many investors would attempt to short the risky asset together to hedge their endowment risk, which places downward pressure on the equilibrium price leading to large price impact, and hence higher hedging cost. Moreover, investor-i benefits when $\sigma_{D,P}$ is low, i.e., when price informativeness is relatively low, which makes speculators (those investors with small endowment shock and act as liquidity providers) more willing to trade. It is important to note that the improvement in the expected utility due to hedging, $A(\lambda)$, is the same for the informed and uninformed investors.

As we show later, investor-i's information choice p_i^* depends only on the ratio $V_I(\lambda, e_i)/V_U(\lambda, e_i)$, and is hence independent of his endowment shock. It is therefore useful to introduce the relative utility gain of becoming informed,

$$\gamma(\lambda) \equiv 1 - \frac{V_I(\lambda, e_i)}{V_U(\lambda, e_i)}.$$

Thus, $\gamma(\lambda)$ measures the incentives for investors to acquire information. The following properties of $\gamma(\lambda)$ provide the underlying mechanism for the informed-trading effect documented in the literature.

Lemma 5. *The relative utility gain $\gamma(\lambda)$ of becoming informed satisfies $\gamma(\lambda) \in (0,1)$ and $\gamma'(\lambda) < 0$ for $\lambda \in [0,1]$. Moreover,*

$$\frac{V_I(\lambda, e_i)}{V_U(\lambda, e_i)} = \sqrt{\frac{v_I}{v_U}} \qquad \text{and} \qquad \gamma(\lambda) = 1 - \sqrt{\frac{v_I}{v_U}}. \tag{2.15}$$

Lemma 5 implies that $\gamma(\lambda)$ can be interpreted as the marginal reduction in the return standard deviation when a marginal investor becomes informed by observing θ_i. Consistent with Manzano and Vives (2011), Lemma 5 shows that the incentives for investors to acquire information decreases with the fraction of informed investors, λ, as in the Grossman–Stiglitz model. In other words, the market exhibits strategic substitutability in information acquisition. Intuitively, a higher λ increases price informativeness, which helps uninformed, but not informed, investors to resolve payoff uncertainty.

Back to utility $\mathcal{W}(p_i; \lambda, e_i)$ in (2.8), we assume investor-i takes λ as given when choosing optimal probability p_i^*. More precisely, each investor forms an expectation about the whole vector $(p_j)_{j\in(0,1)}$ and investors' probabilistic choices result in a non-cooperative strategic game. By the first order condition of (2.8), investor-i's probabilistic choice p_i^* satisfies

$$\alpha\mu p_i^* = \frac{1}{2}\frac{\gamma(\lambda)}{1 - p_i^*\gamma(\lambda)}. \tag{2.16}$$

Note that (2.16) can in general have multiple solutions. The following lemma provides a necessary and sufficient condition to guarantee the existence and uniqueness of the solution.[14]

Lemma 6. *For fixed $\lambda \in [0,1]$, a necessary and sufficient condition for $\mathcal{W}''(p_i; \lambda, e_i) < 0$ is given by $\gamma(\lambda) < \frac{2\alpha\mu + 1}{2\alpha\mu + 3}$.*

Based on Lemma 6, taking the level of informed trading, λ, as given, in order for the optimization problem to be solved by the first order condition (2.16), the information acquisition incentive $\gamma(\lambda)$ cannot exceed a threshold cost sensitivity, $\alpha\mu$, adjusted by risk aversion. Intuitively, when investors try to increase their chance to be informed, the relative gain of being informed, $\gamma(\lambda)$, at any given level of λ should not be more than the risk aversion adjusted relative cost (or cost sensitivity), $\alpha\mu$; otherwise all investors choose to become informed for sure (with $p_i = 1$), which is not optimal (due to the fact $V_I(1) < V_U(0)$). Therefore, we need the relative gain cost sensitivity, μ, to be large enough to guarantee concavity. Furthermore, we require the condition to hold in equilibrium, since λ depends on μ. We next show in Proposition 9 that the concavity of $\mathcal{W}(p_i; \lambda, e_i)$ can be guaranteed in equilibrium by placing an upper bound on the information precision n in Assumption 8.

Information choice equilibrium

When the information market is in equilibrium, we require $\lambda = \int_0^1 p_i^*\,di$, i.e., the level of informed trading must be consistent with investors' strategic probabilistic choices in a Nash equilibrium. Formally, we introduce the following definition of equilibrium for information choice.

Definition 7. The probabilities, $p^* = (p_i^*)_{i\in(0,1)}$, and the level of informed trading, λ, are in equilibrium if

(i) $p^* = (p_i^*)_{i\in(0,1)}$ is a Nash equilibrium, meaning that for every $i \in [0,1]$,[15]

$$\mathcal{W}(p_i^*; \lambda, e_i) \geq \mathcal{W}(p_i; \lambda, e_i) \qquad \text{for all } p_i \in [0,1];$$

[14] In Section 5, we explore a technical variant of this model, inspired by Breugem and Buss (2019) where concavity of $U(\cdot)$ is always granted and the equilibrium is therefore unique.

[15] With a slight abuse of notation, we write $\mathcal{W}(p_i;\lambda)$ in place of $\mathcal{W}(p_i; p_{-i}^*)$, where $p_{-i}^* = (p_j^*)_{j\neq i}$. Indeed, the only payoff-relevant variable for the information game is λ. Moreover, having a continuum of traders, the contribution of trader i on the realization of λ is negligible.

(ii) the following *consistency condition* is satisfied[16]

$$\lambda = \mathbb{E}\left[\int_0^1 \omega_i^* \, di\right] = \int_0^1 p_i^* \, di, \tag{2.17}$$

here ω_i^* is the random variable associated with the optimal probability p_i^*.

We now characterize the equilibrium level of informed trading, its existence and uniqueness. For clarity, we summarize the conditions for the existence and uniqueness of λ in the following modeling assumption. The conditions also guarantee that λ is monotonically decreasing in the cost sensitivity parameter μ. These are the conditions we assume when analyzing welfare implications.

Assumption 8. The following conditions on parameters hold true,

$$n \le 3; \quad \mu > \bar{\mu} \equiv \frac{1}{2\alpha}\frac{\gamma(1)}{1-\gamma(1)}.$$

The first condition, $n \le 3$, guarantees that an investor's utility, $\mathcal{W}(p_i; \lambda, e_i)$, is concave for any λ in equilibrium. It requires an upper bound on the information precision n. Together with Lemma 6, this implies an upper bound on the relative gain to be informed that ensures the concavity. The second condition, together with $n \le 3$, guarantees the existence of a unique equilibrium $\lambda \in (0, 1)$, where λ decreases in μ.[17] It indicates a low bound on the cost sensitivity for the unique Nash equilibrium on information acquisition in the following proposition.

Proposition 9 (*Information Acquisition Equilibrium*). *Under Assumption 8, the level of equilibrium informed trading, λ, satisfies*

$$\alpha\mu\,\lambda = \frac{1}{2}\frac{\gamma(\lambda)}{1-\lambda\gamma(\lambda)}, \tag{2.18}$$

where $\lambda \in (0, 1)$ is unique and decreases in μ. Moreover,

$$\gamma(\lambda) \le \frac{1}{2}. \tag{2.19}$$

Several features of Proposition 9 deserve comments. First, while the REE model of financial market is similar to that in the Grossman–Stiglitz model and Manzano and Vives (2011) in general, the information choice equilibrates differently. In the absence of information acquisition uncertainty, the Grossman–Stiglitz model requires $V_I(\lambda, e_i)e^{\alpha c} = V_U(\lambda, e_i)$, which is equivalent to $\gamma(\lambda) = 1 - e^{-\alpha c}$, where $c > 0$ is a fixed cost for acquiring information. Thus, the level of informed trading increases as information acquisition cost c decreases. In the current probabilistic-choice model, since investors are identical except for endowment shocks, they make the *same* optimal probabilistic choice, $p_i^* = p^*(\lambda) = \lambda$, which satisfies (2.18). Intuitively, the optimal probability choice p^* and hence the level of informed trading λ increases when the cost sensitivity μ decreases.[18] In other words, every investor chooses to pay the same cost for information acquisition, which equals to $\mu\lambda^2$ in equilibrium, but due to information uncertainty, only a fraction, λ, of the investors observe their private signal, θ, and become informed.

Second, the relative gain of being informed is bounded above by one half, $\gamma(\lambda) \le 1/2$. This is due to the fact that, in order to guarantee a symmetric equilibrium, i.e., $p_i^*(\lambda) = \lambda$, we require the concavity condition in Lemma 6 and the equilibrium condition in (2.18) to hold simultaneously. In Appendix A, we show that, when λ satisfies (2.18), $\gamma(\lambda) \le 1/2$ is sufficient for $\mathcal{W}''(p_i; \lambda, e_i) < 0$ for $p_i \in [0, 1]$, thus p_i^* is determined by the first order condition in (2.16). Moreover, the assumption $n \le 3$ is equivalent to $\gamma(0) < 1/2$, which in turn ensures that $\gamma(\lambda) \le 1/2$ since $\gamma'(\lambda) < 0$.

Third, note that in the Grossman–Stiglitz model, the expected utility of investor-i is given by $V_U(\lambda, e_i)$ in equilibrium. Compared to the Grossman–Stiglitz model, we have the following result.

Corollary 10. *The probabilistic information acquisition always leads to better welfare compared to the standard Grossman–Stiglitz model, i.e.,*

$$\mathcal{W}(p_i^*; \lambda, e_i) \ge V_U(\lambda, e_i), \qquad \textit{for} \qquad p_i^* \in (0, 1).$$

[16] At the equilibrium, the expectations are realized so that the fraction of informed, λ, exactly matches the value expected by the traders when using the revealed vector of probabilities p^*.

[17] For more details, see Appendix B. Note that, when $\mu \le \bar{\mu}$, λ is fixed at 1. In this case, welfare can be improved as the cost decreases further (as in the Grossman–Stiglitz model). However, we are interested in whether the decrease in the cost and hence the increase in informed trading can improve welfare. Moreover, we know that the full-information equilibrium $\lambda = 1$ is Pareto-inefficient, since it is always dominated by the no-information equilibrium $\lambda = 0$, even if information is costless ($\mu = 0$). Therefore, we do not consider the corner equilibrium $\lambda = 1$ in this paper.

[18] This holds true in particular under Assumption 8. We provide more general sufficient conditions for the uniqueness in Appendix B. In principle, there could be multiple equilibria in λ for the fixed point argument (2.18) even if $\mathcal{W}(p_i; \lambda, e_i)$ is concave in p^*. We leave this intriguing discussion on multiple equilibria for future research.

The welfare improvement in Corollary 10 is due to the fact that investors making probabilistic information choices is able to *anticipate* the welfare benefit of potentially becoming informed before information uncertainty is resolved. This anticipatory welfare benefit is *ex-ante*, which is different from *ex-post* due to investors' information acquisition uncertainty, while there is no difference between ax-ante and ex-post when investors make all-or-nothing information choices in the Grossman–Stiglitz model.

In summary, with information acquisition uncertainty, this section characterizes the financial and information market equilibrium and provides a condition for the existence and uniqueness of the equilibrium. The decomposition of the expected utility provides two channels via financial risk-sharing and hedging to improve the expected utility from trading, which are very helpful to the welfare analysis conducted in the next section.

3. Welfare analysis

This section examines investors' welfare in the economy from the viewpoint of a policymaker. More specifically, we assume the policymaker is able to control the level of informed trading, λ, by adjusting the cost sensitivity parameter, μ. Since λ is unique and decreasing in μ, we can substitute μ in terms of λ using the equilibrium condition (2.18).[19] Thus, from (2.8) and (2.10)–(2.13), investor-i's welfare, $\mathcal{W}(\lambda; e_i)$, can now be defined as a function of λ as follows,

$$\mathcal{W}(\lambda; e_i) \equiv \mathcal{W}(p^*, \lambda; e_i) = \bar{V}(p^*, \lambda) V_0(e_i) \exp\left\{ -\frac{1}{2} A(\lambda) e_i^2 + \frac{p^*}{2} \frac{\gamma(\lambda)}{1 - p^* \gamma(\lambda)} \right\}, \tag{3.1}$$

where $p^* \equiv p^*(\lambda) = \lambda$ is the optimal information choice which coincides with the level of informed trading in equilibrium, and

$$\bar{V}(p^*, \lambda) = p^* V_I(\lambda) + (1 - p^*) V_U(\lambda)$$

is the expected risk-sharing benefit. Moreover,

$$-\frac{1}{2} A(\lambda) e_i^2 \qquad \text{and} \qquad \frac{p^*}{2} \frac{\gamma(\lambda)}{1 - p^* \gamma(\lambda)}$$

represent the hedging benefit of trading, and the cost of information acquisition, respectively. In (3.1), λ is the endogenous state variable for the economy. The reason we keep the optimal probability, p^*, and the level of informed trading, λ, separately in the welfare definition, even though $p^*(\lambda) = \lambda$, is because it allows us to identify two separate effects on marginal welfare, namely, *opportunity effect* and *informed-trading effect*. More explicitly, the following proposition provides a decomposition of investor-i's marginal welfare.

Proposition 11. *Investor-i's marginal welfare can be decomposed as follows,*

$$\frac{\mathcal{W}'(\lambda; e_i)}{-\mathcal{W}(\lambda; e_i)} = \underbrace{\frac{1}{-\mathcal{W}} \frac{\partial \mathcal{W}}{\partial p^*}}_{\text{opportunity effect}} + \underbrace{\frac{1}{-\mathcal{W}} \frac{\partial \mathcal{W}}{\partial \lambda}}_{\text{informed trading effect}}, \tag{3.2}$$

where the opportunity effect is positive and given by

$$\frac{1}{-\mathcal{W}} \frac{\partial \mathcal{W}}{\partial p^*} = \frac{\gamma(\lambda)(1 - 2\lambda\gamma(\lambda))}{2(1 - \lambda\gamma(\lambda))^2} > 0, \tag{3.3}$$

and the informed trading effect is given by

$$\frac{1}{-\mathcal{W}} \frac{\partial \mathcal{W}}{\partial \lambda} = \underbrace{\frac{1}{2} A'(\lambda) e_i^2}_{\text{Hirshleifer effect}} + \underbrace{\left[\Gamma(\lambda) \frac{V_I'(\lambda)}{-V_I(\lambda)} + (1 - \Gamma(\lambda)) \frac{V_U'(\lambda)}{-V_U(\lambda)} \right]}_{\text{risk-return effect}}, \tag{3.4}$$

where

$$\Gamma(\lambda) \frac{\lambda(1 - 2\lambda\gamma(\lambda))(1 - \gamma(\lambda))}{2(1 - \lambda\gamma(\lambda))^2} \in (0, 1).$$

Since investors' welfare is measured ex-ante, the opportunity effect reflects the marginal anticipatory benefit in investor-i's welfare due to an increase in p^* while the level of informed trading λ is kept constant. In our economy, because of the information acquisition uncertainty, investors are expected to benefit from potentially becoming informed, as long as the expected benefit is more than the cost. Therefore, for given level of informed trading λ, the marginal anticipatory benefit of investors increases in their probability to be informed.

When the information acquisition is certain, as in the Grossman–Stiglitz model, this opportunity effect is shutdown. Intuitively, in the Grossman–Stiglitz model, investors know in advance about the outcome of information acquisition decision, an equilibrium with $\lambda \in (0, 1)$ requires investors to be *indifferent* between the two information choices, and investor-i's welfare is given by $V_0(e_i) V_U(\lambda) e^{-\frac{1}{2} A(\lambda) e_i^2}$. However, with information acquisition uncertainty, *ex-ante*, investors always prefer to be informed than

[19] In the welfare analysis, ideally we want to consider the effect of changing μ. However, this is complicated by the fact that (2.18) only provides an implicit solution for λ in terms of μ. Therefore, to simplify the welfare analysis, we consider $\mathcal{W}(\lambda)$, by making the following substitution using (2.18), $\mu = 1/(2\alpha\lambda)\gamma(\lambda)/(1 - \lambda\gamma(\lambda))$. Note that μ only has impact on welfare via λ. So, there are no confounding effects.

uninformed. Moreover, note that the hedging benefit of trading, $\frac{1}{2}A(\lambda)e_i^2$, does not depend on p^*. Therefore, all investors benefit from the positive opportunity effect, independent of endowment shocks.

The informed trading effect (3.4) represents the marginal welfare in informed trading λ while keeping the optimal probability p^* constant. In essence, informed trading affects welfare because it changes the investment opportunity set, which in turns affects the Sharpe ratio and hedging benefit of trading. To disentangle risk-return from hedging, we further decompose the informed trading effect (3.4) into the *Hirshleifer effect* and the *risk-return effect.*

The Hirshleifer effect measures the marginal cost of hedging endowment risk due to an increase in the equilibrium level of informed trading. Intuitively, informed trading reduces the incentives for speculators (investors with little or no endowment shocks) to trade, because the equilibrium price is closer to the fundamental. Thus it becomes more expensive for hedgers (investors with large endowment shocks) to execute their market orders. In other words, informed trading increases the *cost of liquidity*. Hence, we conjecture that $A'(\lambda) < 0$.

The risk-return effect characterizes the marginal impact of the informed trading on the Sharpe ratios, since

$$\frac{V_K'(\lambda)}{-V_K(\lambda)} = \frac{\xi_K'(\lambda)}{2(1+\xi_K(\lambda))}, \qquad K = I, U.$$

In general, informed trading reduces uncertainty about the payoff by making the equilibrium price more informative. Thus, as Kurlat and Veldkamp (2015) explain, "*decreasing risk lowers the equilibrium return and systematically raises the asset's average price. For welfare, this means that information reduces the asset's risk, but also implies lower return. With exponential utility and normally distributed payoffs, the return effect always dominates.*" Therefore, we conjecture that the marginal impact of the informed trading on the Sharpe ratios is negative, i.e., $\xi_K'(\lambda) < 0$.

In the absence of the opportunity effect (i.e., in the Grossman–Stiglitz model), when $A'(\lambda) < 0$ and $\xi_K'(\lambda) < 0$, the marginal welfare monotonically decreases in the informed trading. Hence, the *no-information equilibrium* ($\lambda = 0$) is the unique Pareto-optimal equilibrium that dominates all other equilibria with $\lambda > 0$. Because of the heterogenous endowments among investors, to show that $A'(\lambda) < 0$ and $\xi_K'(\lambda) < 0$ in general are challenging. To provide some insights to the welfare implications in general, in the following, we first consider some special situations to have analytical results. The study of the general case is then based on numerical analysis.

3.1. Analytical results

To conduct an analytical analysis, we consider two special cases: no-information equilibrium $\lambda = 0$, and full-information equilibrium $\lambda = 1$.

No-information equilibrium

We first consider the case of $\lambda = 0$, which corresponds to the no-information equilibrium. Since no-information equilibrium is the only Pareto optimal equilibrium in the Grossman–Stiglitz model, the welfare improvement from the no-information equilibrium can provide an insight to the welfare implication of information acquisition uncertainty.

Note that for $\lambda = 0$, the equilibrium price simplifies to $P = -\alpha v_D z$, and the uninformed investors' optimal portfolio is given by $x_U^*(P, e_i) = \frac{-P}{\alpha v_D} - e_i$. Thus, the welfare in the no-information equilibrium is given by

$$\mathcal{W}(0, e_i) = \frac{1}{\sqrt{1+\xi_0}} e^{-\frac{1}{2}A_0 e_i^2} V_0(e_i), \tag{3.5}$$

where

$$\xi_0 \equiv \alpha^2 v_D v_{z|e}, \qquad A_0 \equiv (1-\Psi)^2 \frac{\alpha^2 v_D}{1+\xi_0}.$$

This brings some of the key parameters, including ξ_0, Ψ, and n, introduced before to our welfare analysis.

The no-information equilibrium is equivalent to taking the limit when the cost sensitivity $\mu \to \infty$. Consequently, fundamental information is completely *locked up*. The following proposition provides a condition for the marginal welfare improvement at the no-information equilibrium, $\mathcal{W}'(0; e_i) > 0$ for investor-i with endowment shock e_i.

Proposition 12 (*No-Information Equilibrium*). *At $\lambda = 0$, investor-i's marginal welfare is given by*

$$\frac{\mathcal{W}'(0; e_i)}{-\mathcal{W}^{(0;e_i)}} = \underbrace{\frac{1}{2}\left(1 - \frac{1}{\sqrt{1+n}}\right)}_{\text{opportunity effect}} \underbrace{- \frac{n}{1+\xi_0}\left(\Psi + \xi_0\right)}_{\text{risk-return effect}} \underbrace{- \frac{n(1-\Psi)}{\Psi(1+\xi_0)^2}\left(\Psi + \xi_0\right)\frac{e_i^2}{v_e}}_{\text{Hirshleifer effect}}. \tag{3.6}$$

With the explicit decomposition in (3.6), Proposition 12 helps to provide key insight to the welfare analysis in general. At the no-information equilibrium, (3.6) provides explicit expressions for each of the three components that determine investors' marginal welfare. With respect to the three key parameters, we have the following three observations.

First, the opportunity effect in (3.6) depends solely on the information precision, n. It increases in n. Therefore, independent of endowment shocks, facing information acquisition uncertainty, investors can anticipate a higher welfare from the informed trading at the no-information equilibrium when the fundamental information, θ, is more precise.

Second, both the Hirshleifer and risk-return effects in (3.6) are strictly *negative*, as conjectured. They can be interpreted as the welfare costs of informed trading. Therefore, in the absence of information acquisition uncertainty, we have the following result.

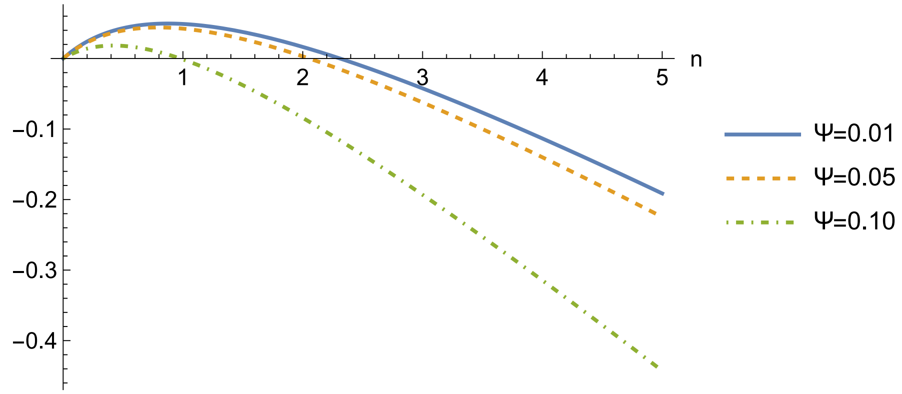

Fig. 3.1. Investors' marginal welfare, $\mathcal{W}'(0,e)/(-W(0,e))$, in the *no-information equilibrium* $\lambda = 0$ plotted against information precision, n, for different levels of endowment precision Ψ, here $\xi_0 = 0.01$, and $e^2/v_e = 0.04$.

Corollary 13. *At $\lambda = 0$, informed trading always reduces investor welfare in the Grossman–Stiglitz model.*

Moreover, both Hirshleifer and risk-return effects are more severe with higher information precision n and higher (squared) Sharpe ratio ξ_0. The Sharpe ratio can be used to measure risk-sharing incentives, which are higher when investors are more risk-averse and both the dividend risk, v_D, and aggregate endowment risk, $v_{z|e}$, are higher. Therefore, when investors have stronger risk-sharing incentives, distortion to risk-sharing due to the informed trading incurs a larger welfare cost.

Third, the marginal welfare effect of the endowment precision on informed trading is nonlinear. On the one hand, when n and ξ_0 are held constant, the risk-return effect is more severe when the endowment precision Ψ is higher. Intuitively, informed trading resolves more payoff uncertainty when individual investors' endowment shocks, e_i, are more informative about the aggregate endowment shock, z, corresponding to a high endowment precision Ψ. In the extreme case of $\Psi \to 1$, $e_i = z$ for all i, uninformed investors can extract the fundamental signal θ perfectly from the equilibrium P, reducing investors' incentive to acquire information. On the other hand, when e_i^2/v_e is held constant, increase in Ψ weakens the Hirshleifer effect. In the extreme case of $\Psi \to 1$, since $e_i = z$ for all i, Hirshleifer effect completely disappears. Although the risk-return and Hirshleifer effects are always negative, their joint effect, which is the informed-trading effect, is hump-shaped in the endowment precision Ψ, becoming more severe as Ψ increases initially and then less severe when Ψ increases further. Therefore, informed trading effect becomes more severe when the endowment precision is moderate.

Overall, the information precision n is positively related to all three effects. It can be verified from (3.6) that the marginal welfare is hump-shaped with respect to the information precision n, as illustrated in Fig. 3.1. It shows that, when $\Psi = 0.01$, i.e., endowment precision is relatively low, the opportunity effect tends to dominate, thus marginal welfare is positive, for a large range of n. In contrast, when $\Psi = 0.1$, marginal welfare is increasing in n only for relatively small values of n. Therefore, investors' welfare improves in the informed trading at the no-information equilibrium when the information precision n, the endowment precision Ψ, and the risk-sharing incentive ξ_0 are low, and the improvement can be substantial when the information precision is moderate.

Full-information equilibrium

Next, we consider the case of full-information equilibrium $\lambda = 1$, which is equivalent to $\mu \to \frac{1}{2\alpha}\frac{\gamma(1)}{1-\gamma(1)}$, the lower bound on the cost sensitivity. In this case, the price coefficients are given by $b_\theta = 1$ and $b_z = \alpha v_I$. We derive the following result.

Proposition 14 (*Full-Information Equilibrium*). *The full-information equilibrium $\lambda = 1$ is Pareto inefficient.*

This is due to the fact that investor-i with endowment shock e_i can experience a welfare improvement by moving from full-information, where $\lambda = 1$, to no-information equilibrium, where $\lambda = 0$, i.e., $\mathcal{W}(1; e_i) < \mathcal{W}(0; e_i)$ for $e_i \in \mathbb{R}$. Put differently, the full-information equilibrium is dominated by the no-information equilibrium, thus cannot be Pareto optimal. Therefore, even if investor-i's welfare is initially increasing in λ, i.e., $\mathcal{W}'(0, e_i) > 0$, eventually it decreases to the level $\mathcal{W}(1; e_i)$ as $\lambda \to 1$, which suggests that the welfare function, $\mathcal{W}(\lambda; e_i)$, can be decreasing or *hump-shaped* in λ.

We confirm this conjecture numerically in Fig. 3.2. We separate investor-i's log-welfare into two components, namely, the *Grossman–Stiglitz (GS) log-welfare*, and the *anticipatory welfare benefit*, as follows,

$$-\ln\left[-\mathcal{W}(\lambda, e_i)\right] = \underbrace{-\ln\left[-V_0(e_i)V_U(\lambda)\right] + \frac{1}{2}A(\lambda)e_i^2}_{\text{GS log-welfare}} + \underbrace{\ln\left[\frac{1}{1-\lambda\gamma(\lambda)}\right] - \frac{1}{2}\frac{\lambda\gamma(\lambda)}{1-\lambda\gamma(\lambda)}}_{\text{anticipatory welfare benefit}}, \tag{3.7}$$

by using the definition $\gamma(\lambda)$ in (2.15), the information equilibrium condition (2.18), and the definition of investor-i's welfare in (3.1).[20] Fig. 3.2 shows that, for low endowment precision Ψ, the welfare function is hump-shaped. This implies that investor-i's

[20] Note that investor-i's log-welfare is proportional to the certainty equivalent wealth, which is given by $-\frac{1}{\alpha}\ln\left[-\mathcal{W}(\lambda, e_i)\right]$.

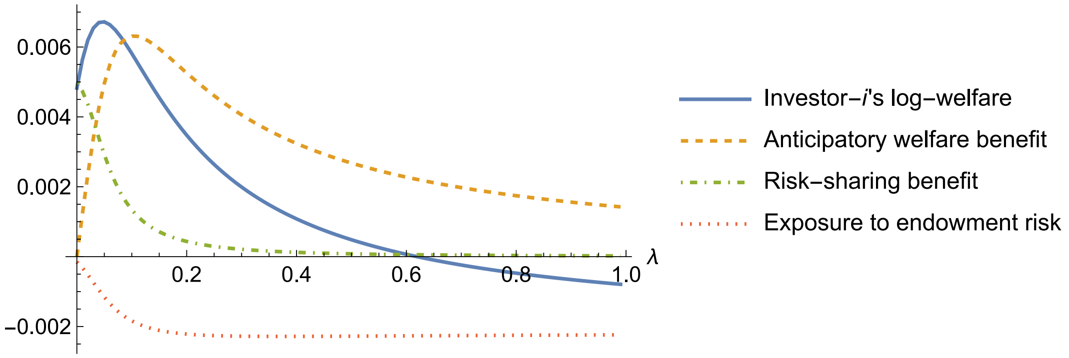

Fig. 3.2. Investor-i's log-welfare, $-\ln[-\mathcal{W}(\lambda,e)]$, anticipatory welfare benefit, $\ln\left[\frac{1}{1-\lambda\gamma(\lambda)}\right]-\frac{1}{2}\frac{\lambda\gamma(\lambda)}{1-\lambda\gamma(\lambda)}$, risk-sharing benefit, $-\ln\left[-V_U(\lambda)\right]$, and exposure to endowment risk, $\frac{1}{2}A(\lambda)e^2$, as a function the state variable λ. Parameters: $\xi_0=0.01$, $n=0.75$, $\Psi=0.01$, $\alpha=1$, $v_e=1$, $e=0.1$.

log-welfare can reach a maximum, $-\ln[-\mathcal{W}(\lambda_i^*;e_i)]$, at an optimal state, λ_i^*. Note that the optimal state differs between investors due to their heterogeneous endowment shocks. Investors with large endowments have their optimal state closer to zero, while investors with small endowments can have their optimal state away from zero.

Moreover, with the decomposition in Fig. 3.2, we also confirm that the two components, which make up the GS log-welfare, namely the risk-sharing benefit, $-\ln(-V_U(\lambda))$, and exposure to endowment risk, $-\frac{1}{2}(\alpha^2 v_D - A(\lambda)e^2)$, are indeed strictly decreasing, while the anticipatory welfare benefit is hump-shaped. Therefore, when condition (3.6) is satisfied, the opportunity effect dominates the risk-return and Hirshleifer effects initially when λ is small, which reverses eventually when λ becomes large enough. Therefore, investor-i's welfare follows a hump shape. On the other hand, when the endowment shock is relatively large, Hirshleifer effect may dominate, and the welfare function can be strictly decreasing rather than hump-shaped.

3.2. Numerical analysis

With the insights above about how the welfare improvement depends on the key model parameters, especially the analytical analysis at the no-information equilibrium $\lambda=0$, we now examine numerically the impact of the informed trading on investors' welfare improvement at the information equilibrium $\lambda\in(0,1)$.

In Fig. 3.3, for different level of informed trading λ, we identify the regions of (n,ξ_0), under which investor-i experiences a welfare improvement from more informed trading at the given information equilibrium. From Fig. 3.3, we have the following three observations.

First, as λ increases, the region is shrinking in both the risk-sharing incentives, ξ_0, and information precision, n. Intuitively, Hirshleifer and risk-return effects lead to greater distortion in risk-sharing when fundamental signal resolves a larger proportion of payoff uncertainty and when there are stronger incentives for risk-sharing. Thus the welfare cost of informed trading is more likely to outweigh its anticipatory welfare benefit. This is consistent with the analytical result at the no-information equilibrium $\lambda=0$.

Second, the regions shrink in the level of informed trading, λ, itself, which suggests that the marginal welfare is decreasing in λ, $\mathcal{W}''(\lambda;e_i)<0$, or investors' welfare function can be concave. This is potentially due to the fact that the positive opportunity effect, $\partial\mathcal{W}/\partial p^*$, on the marginal welfare is monotonically decreasing in λ,

$$\frac{d}{d\lambda}\left(\frac{\partial\mathcal{W}}{\partial p^*}\right)=\frac{\gamma'(\lambda)}{2(1-\gamma(\lambda))}<0.$$

Intuitively, informed trading makes the equilibrium price more informative about the fundamental signal, which helps the uninformed investors to resolve more payoff uncertainty. Therefore, the ratio between the return variances of the informed and uninformed investors, i.e., the ratio between $\mathbb{V}ar[R|\theta,P,e_i]$ and $\mathbb{V}ar[R|P,e_i]$, reduces. Hence, when the level of informed trading is relatively high, investors would anticipate a smaller welfare benefit from potentially becoming informed.

Third, note that in the Grossman–Stiglitz economy, investors do not experience any anticipatory welfare benefit. Due to the welfare cost of informed trading, the no-information equilibrium with $\lambda=0$ is the *unique* Pareto optimal equilibrium. In contrast, with the information acquisition uncertainty, there can be *a continuum of Pareto optimal equilibria* with $\lambda\in[0,\lambda_0^*]$, where λ_0^* is the unique value where $\mathcal{W}'(\lambda_0^*,0)=0$. It can be seen that when $\lambda\in[0,\lambda^*]$, it is impossible to improve welfare for one investor without harming the welfare of the other. For example, suppose the prevailing equilibrium is exactly $\lambda=\lambda_0^*$. If the policymaker increases μ (hence reducing the equilibrium level for λ), this would benefit all the investors with $e_i^2>0$ except for the pure speculators with $e_i=0$. Therefore, equilibrium with $\lambda=\lambda_0^*$ is Pareto optimal. On the other hand, $\lambda=0$ is always Pareto optimal, since we can always find an investor with a sufficiently large endowment shock such that $\mathcal{W}'(0,e_i)<0$. We illustrate this finding in Fig. 3.4 where we plot the level of welfare improvement from the no-information equilibrium given different sizes of the endowment shock. It is clear that the peak of the hump-shaped curve shifts to the left as e_i increases. This is because the Hirshleifer effect is stronger for investors with a larger endowment shock to hedge. For the pure speculator with $e_i=0$, we can numerically compute the optimal state, $\lambda_0^*\approx 0.06$. Therefore, any equilibrium with $\lambda\in[0,\lambda_0^*]$ is Pareto-optimal in this case.

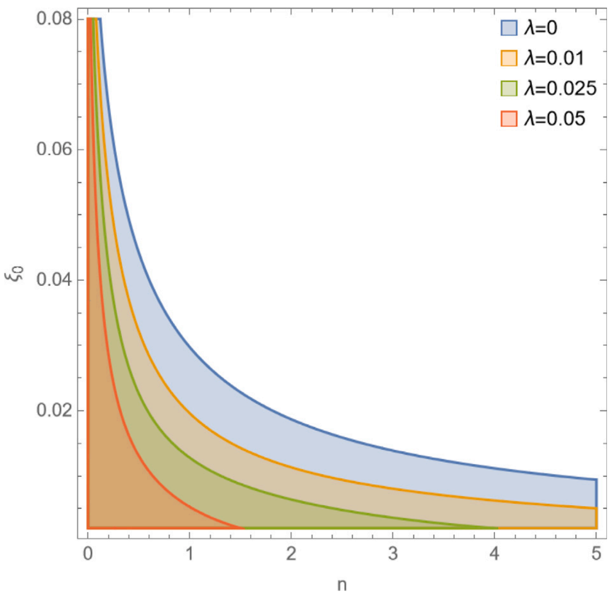

Fig. 3.3. Parameter regions marked by (n, ξ_0) in which $\mathcal{W}'(\lambda; e) > 0$ for different levels of informed trading, λ. Other parameters used are: $\alpha = 1$, $v_e = 1$, $\Psi = 0.01$, $e = 0.1$.

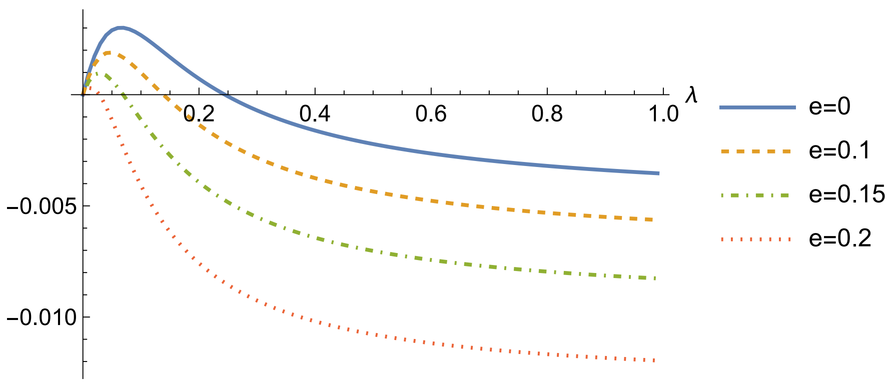

Fig. 3.4. Investor-i's welfare improvement from the no-information equilibrium benchmark, $1 - \mathcal{W}(\lambda; e)/\mathcal{W}(0; e)$, as a function the state variable λ for different size of endowment shock, e_i, here $\xi_0 = 0.01$, $n = 0.75$, $\alpha = 1$, $v_e = 1$, $\Psi = 0.01$.

Finally, we turn our attention to the probability distribution of investor-i's log-welfare, $-\ln[-\mathcal{W}(\lambda; e_i)]$, where $e_i \sim \mathcal{N}(0, v_e)$. In Fig. 3.5, we compare the log-welfare distribution between the no-information benchmark economy ($\lambda = 0$) and the Pareto-optimal economy ($\lambda = \lambda_0^*$) for a pure speculator. As the level of informed increases from $\lambda = 0$ to $\lambda = \lambda_0^*$, we have two observations. First, the *mode* of the log-welfare distribution shifts to right, which indicates that investors with endowment shocks sufficiently close to zero experience a welfare improvement. Second, the welfare distribution becomes more negatively skewed, which suggests that investors with relatively large endowment shocks experience a larger welfare reduction. Intuitively, there is a welfare trade-off when λ increases. To see this, we can write the log-welfare function as

$$-\ln[-\mathcal{W}(\lambda; e_i)] = \underbrace{-\ln[V_U(\lambda)] - \ln[1 - \lambda\gamma(\lambda)] - \frac{1}{2}\frac{\lambda\gamma(\lambda)}{1 - \lambda\gamma(\lambda)}}_{-\ln[-\mathcal{W}(\lambda;0)]} + \underbrace{\frac{1}{2}\left[A(\lambda) - \alpha^2 v_D\right]}_{\text{exposure to endowment risk}} e_i^2. \tag{3.8}$$

Eq. (3.8) shows that, while informed trading increases the welfare for a pure speculator (with zero endowment shock), due to the fact that $A(\lambda) < \alpha^2 v_D$, it also increases the exposure of non-speculators to the endowment risk, reducing their welfare levels. Therefore, in terms of Pareto-optimality, $\mathcal{W}(\lambda_0^*; e_i)$ does not dominate $\mathcal{W}(0; e_i)$ or vice-versa. In order to rank the two welfare distributions, one needs a stricter welfare criterion than Pareto-optimality.

In summary, without any externality, information acquisition uncertainty always leads to better welfare outcome when compared to the standard Grossman–Stiglitz setup. More importantly, despite that informed-trading distorts risk-sharing and destroys trading opportunity, information acquisition can be welfare-improving when the anticipatory benefit dominates the welfare cost of the

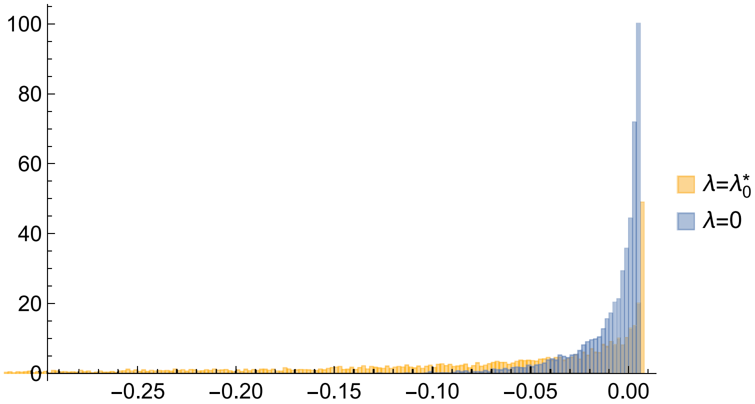

Fig. 3.5. Investor-i's log-welfare distribution measured by $-\ln[-\mathcal{W}(\lambda;e)]$ with $e \sim \mathcal{N}(0,v_e)$, for $\lambda=0$ and for $\lambda=\lambda_0^*$. Other parameters used are: $\xi_0=0.01$, $\alpha=1$, $v_e=1$, $\Psi=0.01$.

informed trading. Welfare improvement is more likely when investors have weak risk-sharing incentives, small endowment shocks, and when information precision is not too high, especially for speculators who provide liquidity. With heterogeneous endowment shocks among investors, there can be a continuum of Pareto optimal information-acquisition equilibria. The no-information equilibrium is the unique Pareto-optimal equilibrium only when investors have strong risk-sharing incentives.

4. Heterogenous private signals

Following Manzano and Vives (2011), this section conducts an analysis on the robustness of our findings by considering that informed investors receive heterogeneous private signals, θ_i, about the payoff, D, that are not perfectly correlated, i.e, $\theta_i = D + \epsilon_i$, where $\epsilon_i \sim \mathcal{N}(0, v_\epsilon)$ and $\mathbb{C}ov[\epsilon_i, \epsilon_j] = \rho v_\epsilon$ with $\rho \in [0,1]$ for $i \neq j$. The average signal $\bar{\theta}$ satisfies

$$\bar{\theta} = \frac{1}{\lambda}\int_0^\lambda \theta_i di = D + \tilde{\epsilon}, \qquad \tilde{\epsilon} = \frac{1}{\lambda}\int_0^\lambda \epsilon_i di \sim \mathcal{N}(0, \rho v_\epsilon),$$

where $\tilde{\epsilon}$ represents the residual uncertainty about the payoff and ρ parameterizes the common component of the endowment shock in the signal residual uncertainty. As noted in Manzano and Vives (2011), this setup nests most of the REE models in the literature, including Grossman and Stiglitz (1980) with $\rho = 1, v_u = \infty$, Diamond and Verrecchia (1981), Hellwig (1980), and Admati (1985) with $\rho = 0, \lambda = 1, v_u = \infty$, and Ganguli and Yang (2009) with $\rho = 0$. Also, note that when $\rho = 1$, the private signal becomes homogeneous, we thus recover the baseline model in the previous sections.

Financial market equilibrium

For given fraction of informed investors, Manzano and Vives (2011) solve for the symmetric linear equilibrium price which we restate below.

Lemma 15 (*Financial Market Equilibrium, Manzano and Vives (2011), Proposition 1*). *For an exogenously given fraction of informed investors, $\lambda \in [0,1]$, there exists a linear equilibrium price of the risky asset,*

$$P = b_\theta \bar{\theta} - b_z z,$$

where $\pi \equiv b_\theta / b_z$ solves the fixed-point equation,

$$\pi = f(\pi) \equiv \frac{1}{\alpha v_\epsilon}\left[\lambda \frac{v_\epsilon^{-1} + v_u^{-1}(1-\rho)\pi^2}{v_\epsilon^{-1} + (v_z^{-1} + v_u^{-1})\rho(1-\rho)\pi^2} + (1-\lambda)\frac{\pi^2 v_u^{-1}}{v_\epsilon^{-1} + \pi^2 \rho (v_u^{-1} + v_z^{-1})}\right],$$

and

$$\begin{cases} b_\theta = 1 - \dfrac{v_D^{-1}}{\lambda v_I^{-1} + (1-\lambda) v_U^{-1}}, \\ v_I = \mathbb{V}ar\left[D|\theta_i, P, e_i\right] = \left[v_D^{-1} + \dfrac{v_\epsilon^{-1}\left(v_\epsilon^{-1} + \pi^2 (v_u^{-1} + v_z^{-1})(1-\rho)\right)}{v_\epsilon^{-1} + \pi^2 (v_u^{-1} + v_z^{-1})\rho(1-\rho)}\right]^{-1}, \\ v_U = \mathbb{V}ar\left[D|P, e_i\right] = \left[v_D^{-1} + \dfrac{\pi^2 (v_u^{-1} + v_z^{-1})}{v_\epsilon^{-1} + \pi^2 (v_u^{-1} + v_z^{-1}\rho)}\right]^{-1}. \end{cases} \tag{4.1}$$

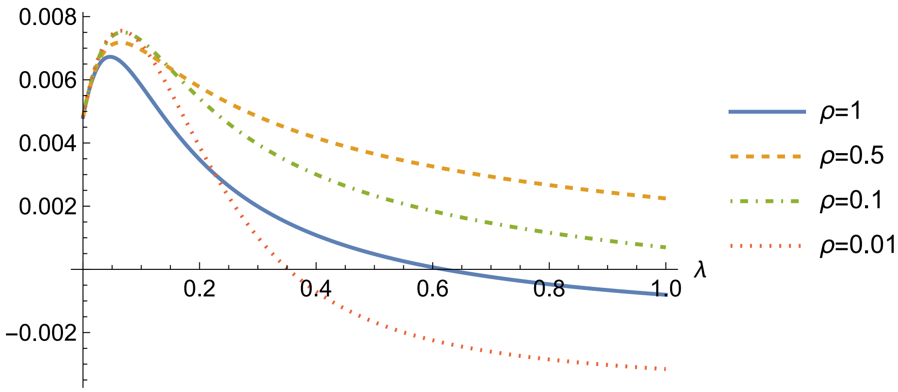

Fig. 4.1. Investor-i's log-welfare function, $-\ln[-\mathcal{W}(\lambda, e)]$ with correlation between private signals $\rho \in (0, 1]$. Other parameters used are: $\xi_0 = 0.01$, $n = 0.75$, $\alpha = 1$, $v_e = 1$, $\Psi = 0.01$, $e = 0.1$.

Concerning the uniqueness of the equilibrium, Manzano and Vives (2011) Proposition 3 shows that if $\rho > \frac{27}{64}\frac{n\Psi^2}{\xi_0}$, then $f'(\pi) < 1$, and there exists a unique symmetric linear equilibrium. Hence, the uniqueness can be guaranteed when the private signals are sufficiently correlated. Note that this condition does not depend on the level of informed trading. Moreover, it can be more easily satisfied when the endowment precision, Ψ, is relatively low.

Welfare analysis

Investors' welfare function and marginal welfare follow the same decomposition as in (3.1) and (3.2) of the baseline model, respectively. Furthermore, the expected utilities for informed and uninformed follow the same form in (2.10), where $V_K(\lambda) = \sqrt{v_K/(v_{P|e} + v_D - 2\sigma_{D,P})}$ for $K = I, U$ and v_I and v_U are given by (2.7). The following proposition explicitly characterizes investors' marginal welfare at the no-information equilibrium $\lambda = 0$.

Proposition 16 (*No-Information Equilibrium*). *For $\lambda = 0$ and $\rho \in (0, 1]$, investor-i's marginal welfare is the same, given by (3.6), as in the baseline model.*

Surprisingly, Proposition 16 shows that the behavior of the marginal welfare at the no-information equilibrium is independent of the correlation between private signals, ρ. Numerically, we calculate and plot the log-welfare function, $-\ln[-\mathcal{W}(\lambda, e_i)]$, for different values of ρ in Fig. 4.1. When the private signals are not perfectly correlated, i.e., $\rho < 1$, we still observe the hump-shape welfare function. This shows the robustness of the findings in the baseline model. Also, at low informed-trading equilibrium, i.e., when λ is small, the hump-shape welfare function becomes more peaked when the correlation is low. This implies that the welfare improvement becomes less significant for high correlation. Intuitively, increasing the correlation in investors' private signals make the equilibrium price more informative, making the informed trading effect more severe. Therefore, although log-welfare is the same at the no-information equilibrium $\lambda = 0$, for relatively low level of informed trading (i.e., λ close to zero), the welfare improvement becomes less significant when the private signals are more correlated.

5. Preference for early resolution of uncertainty

With the information uncertainty, we have shown in Section 2 that the analysis of equilibrium information acquisition can be complicated. With uncertainty aversion, investors may have a preference for early resolution of uncertainty. In this case, we show in this section that the analysis on information acquisition equilibrium can become easier and our findings are robust with respect to preference for early resolution of uncertainty.

More explicitly, we follow Breugem and Buss (2019) and assume investor-i maximizes

$$\mathbb{E}[u_1(\mathbb{E}[u_2(W_i|\mathcal{F}_i)])], \tag{5.1}$$

where the inner utility function $u_2(x)$ governs risk aversion and the outer utility function $u_1(x)$ governs the preference for the timing of the resolution of uncertainty. The preference for timing of the resolution of uncertainty was firstly introduced to distinguish attitudes toward risk from behavior toward intertemporal substitution in optimal consumption problem (e.g., Kreps and Porteus (1978), Weil (1990)). Kreps and Porteus (1978) show that agents exhibit a preference for early (respectively, late) resolution of uncertainty depending on u_1. Specifically, investors are indifferent about the timing when u_1 is linear, while have a preference for early (late) resolution of uncertainty when u_1 is a convex (concave) function. Intuitively, Weil (1990) argues that, '*lotteries in which uncertainty resolves early are less risky than late resolution lotteries with the same distribution of prizes. Early resolution lotteries, however, feature certainty equivalent fluctuation of utility over time... Therefore, agents who dislike risk "more" than intertemporal fluctuations prefer ... early resolution.*' In the following, we model investors' aversion to uncertainty and choose $u_1(x) = -\ln(-x)$ for a preference of early resolution. Thus, while investor-i's portfolio choice, x_i^*, remains the mean–variance type as in Lemma 1, the information choice p_i^* is affected by $u_1(\cdot)$.

More specifically, we can derive investor-i's objective function corresponding to (5.1) as follows,

$$\mathcal{W}(p_i;\lambda,e_i) \equiv -p_i \ln[-V_I(\lambda;e_i)] - (1-p_i)\ln[-V_U(\lambda;e_i)] - \alpha\mu p_i^2, \tag{5.2}$$

where $V_K(\lambda;e_i)$, $K = I,U$, are given in (2.13). Note that, since $u_1(\cdot)$ is convex, investor-i prefers the fundamental signal θ to be more informative, so that the difference between $V_I(\lambda;e_i)$ and $V_U(\lambda;e_i)$ increases. A more informative signal also means that a larger proportion of payoff uncertainty is resolved before making the optimal portfolio choice. Hence, investor-i has a preference for early resolution of uncertainty.

In the following, we show that the welfare results in Section 3 are robust to the alternative model specification proposed in (5.2). In particular, we show in the marginal welfare decomposition that the Hirshleifer and risk-return effects have similar behavior compared to our baseline model; the opportunity effect shows a different expression but remains strictly positive. For convenience, we refer to investors who prefer early resolution of uncertainty as *Recursive-Utility* (RU) investors, and investors who are indifferent to the timing of resolution of uncertainty as *Expected-Utility* (EU) investors.

Equilibrium information choice

From (5.2), since $\mathcal{W}''(p_i;\lambda,e_i) < 0$, by the first order condition, $\mathcal{W}'(p_i^*;\lambda,e_i) = 0$, we can derive the optimal probabilistic information choice for an RU investor,

$$p_i^* = \frac{1}{2\alpha\mu}\ln\left(\frac{1}{1-\gamma(\lambda)}\right). \tag{5.3}$$

Therefore, similar to the baseline model, RU investors make the same optimal probabilistic choice given the investment opportunity set, $\gamma(\lambda)$. Thus, equilibrium requires that $p^*(\lambda) = \lambda$, from which we can obtain the equilibrium level of informed trading λ.

Proposition 17 (*Information Choice Equilibrium*). *Assume $\mu > 1/(2\alpha)\ln(1/(1-\gamma(1)))$, the equilibrium level of informed trading satisfies*

$$\alpha\mu\lambda = \frac{1}{2}\ln\left(\frac{1}{1-\gamma(\lambda)}\right), \tag{5.4}$$

where $\lambda \in (0,1)$ is unique and decreasing in μ.

In an economy with RU investors, since the concavity condition $\mathcal{W}''(p_i;\lambda,e_i) < 0$ is guaranteed, the condition $n \le 3$ from Assumption 8 is no longer required to ensure that (5.3) characterizes the unique information choice for all investors. Therefore, information equilibrium is always *unique* in an economy with RU investors.

Welfare analysis

From (5.2) and (5.4), investors' welfare function can be written as

$$\mathcal{W}(\lambda;e_i) \equiv \mathcal{W}(p^*,\lambda;e_i) = -\ln[-V_0(e_i)] - p^*\ln[V_I(\lambda)] + (1-p^*)\ln[V_U(\lambda)] + \Phi(p^*,\lambda;e_i), \tag{5.5}$$

where $V_0(e_i)$, $V_K(\lambda)$, $K = I,U$ are as given in Proposition 4, $p^* \equiv p^*(\lambda) = \lambda$, and

$$\Phi(p^*,\lambda;e_i) = \frac{1}{2}A(\lambda)e_i^2 - \frac{p^*}{2}\ln\left(\frac{1}{1-\gamma(\lambda)}\right).$$

As in the baseline model, we can separate marginal welfare into two components,

$$\mathcal{W}'(\lambda;e_i) = \underbrace{\frac{\partial\mathcal{W}(p^*,\lambda;e_i)}{\partial p^*}}_{\text{opportunity effect}} + \underbrace{\frac{\partial\mathcal{W}(p^*,\lambda;e_i)}{\partial\lambda}}_{\text{informed trading effect}},$$

where the opportunity effect of anticipatory benefit is given by

$$\frac{\partial\mathcal{W}}{\partial p^*} = \frac{1}{2}\ln\left(\frac{1}{1-\gamma(\lambda)}\right), \tag{5.6}$$

and the informed-trading effect of welfare cost is given by

$$\frac{\partial\mathcal{W}}{\partial\lambda} = \underbrace{\frac{1}{2}A'(\lambda)e_i^2}_{\text{Hirshleifer effect}} + \underbrace{\left[\Gamma(\lambda)\frac{V_I'(\lambda)}{-V_I(\lambda)} + (1-\Gamma(\lambda))\frac{V_U'(\lambda)}{-V_U(\lambda)}\right]}_{\text{risk-return effect}}, \qquad \Gamma(\lambda) = \frac{\lambda}{2}. \tag{5.7}$$

Therefore, we have a very similar marginal welfare decomposition as in the baseline model. In the special case of $\lambda = 0$, i.e., in the no-information equilibrium, we can write down the marginal welfare explicitly,

$$\mathcal{W}'(0;e_i) = \underbrace{\frac{1}{2}\ln\left(\sqrt{1+n}\right)}_{\text{opportunity effect}} - \underbrace{\frac{n(1-\Psi)(\Psi+\xi_0)}{\Psi(1+\xi_0)^2}\frac{e_i^2}{v_e}}_{\text{Hirshleifer effect}} - \underbrace{\frac{n}{1+\xi_0}\left(\Psi+\xi_0\right)}_{\text{risk-return effect}}. \tag{5.8}$$

By comparing this expression with the marginal welfare computed for the baseline model (see Proposition 12), we see that RU investors derive a stronger opportunity effect than EU investors, since it can be shown that $\ln(y) > 1 - 1/y$ where $y = \sqrt{1+n}$.

Thus, given the same endowment shock, RU investors are more likely to experience a welfare improvement than EU investors at the no-information equilibrium.

Intuitively, as aforementioned, RU investors prefer a more informative signal than EU investors. A large n helps to resolve a large fraction of the payoff uncertainty before making an optimal portfolio choice. Therefore, an increment in the optimal probabilistic choice, p^*, leads to a greater improvement in marginal welfare for RU investors than for EU investors.

6. Conclusion

In this paper, we consider information uncertainty and examine the welfare consequences of *probabilistic* information acquisition in a pure-exchange economy with rational expectations and heterogeneous endowments. Instead of making all-or-nothing information choices, investors optimally choose the probability to observe a fundamental payoff signal given a quadratic cost function. We derive regularity conditions for a symmetric information equilibrium, where investors make the same optimal probabilistic choice, which in equilibrium matches the level of informed trading in the market.

As for the welfare analysis, we find that information acquisition has two offsetting effects, a positive opportunity effect due to investors *anticipating* the potential benefit of becoming informed, and a negative informed trading effect due to the destruction of trading opportunities with more informed trading. We show that the former effect can prevail, thus making information acquisition welfare-improving, for those investors with relatively small endowment shocks, especially when risk-sharing incentives are weak. For robustness, we show the same results hold in a general REE model setup when the private signals of the informed investors are correlated, and under the preference for early resolution of uncertainty. Overall, our results suggest that information acquisition, despite being costly, can provide a substantial welfare benefit for a significant portion of investors. Therefore, with information acquisition uncertainty, regulations aiming to level the playing field by minimizing the information asymmetry between investors must be exercised with caution.

Declaration of competing interest

As an author of the paper entitled "The social value of information uncertainty", I hereby declare that I have no relevant or material financial interests that relate to the research described in this study.

Appendix A. Proofs

To simplify the notation, we drop the trader-specific subscript i for all proofs in this appendix.

A.1. Proof of Lemma 1

If an investor is informed, the information set is $\mathcal{F} = \{\theta, P, e\}$, then the expected and variance of return, $R = D - P$, are given by $\mathbb{E}[R|\theta, P, e] = \frac{n}{1+n}\theta - P$ and $\mathbb{V}ar[R|\theta, P, e] = \frac{1}{1+n}v_D$. If an investor is uninformed, the information set is $\mathcal{F} = \{P, e\}$, then the expected and variance of return are given by $\mathbb{E}[R|P, e] = \mathbb{E}[D|P, e] - P$ and $\mathbb{V}ar[R|P, e] = \mathbb{V}ar[D|P, e]$. Let $X = (D, P, e)^\top$, then X follows a multivariate normal distribution with mean $\mu_X = \mathbf{0}$ and covariance matrix

$$\Sigma_X = \begin{pmatrix} v_D & \sigma_{D,P} & 0 \\ \sigma_{D,P} & v_P & \sigma_{e,P} \\ 0 & \sigma_{e,P} & v_e \end{pmatrix}.$$

Next, let $Y = (P, e)^\top$, we can partition Σ_X into the following components, $\Sigma_{DD} = v_D$, $\Sigma_{DY} = (\sigma_{D,P}, 0)$, $\Sigma_{YD} = \Sigma_{DY}^\top$, and

$$\Sigma_{YY} = \begin{pmatrix} v_P & \sigma_{e,P} \\ \sigma_{e,P} & v_e \end{pmatrix}.$$

Then, we can obtain the following, $\mathbb{E}[D|Y] = \Sigma_{DY}\Sigma_{YY}^{-1}Y$ and $\mathbb{V}ar[D|Y] = v_D - \Sigma_{DY}\Sigma_{YY}^{-1}\Sigma_{YD}$. Therefore,

$$\mathbb{E}[D|Y] = \underbrace{\left(\frac{\sigma_{D,P}}{v_P - \sigma_{e,P}^2/v_e}\right)}_{\kappa}\left[P - \underbrace{\left(\frac{\sigma_{e,P}}{v_e}\right)}_{\beta_{e,P}} e\right]$$

and

$$\mathbb{V}ar[D|Y] = v_D - \left(\frac{\sigma_{D,P}^2}{v_P - \sigma_{e,P}^2/v_e}\right) = v_D - \kappa\sigma_{D,P}.$$

Hence, $\mathbb{E}[R|Y] = -(1-\kappa)P - (\kappa\beta_{e,P})e$ and $\mathbb{V}ar[R|Y] = v_D - \kappa\sigma_{D,P}$.

A.2. Proof of Lemma 2

See the proof of Proposition 1 in Manzano and Vives (2011), pp. 361–364, and Example 3 for the case of $\rho = 1$ on p.357.

A.3. Proof of Lemma 3

See the proof of Corollary 9 in Manzano and Vives (2011), pp. 366–367. Moreover, assume $\lambda < \Psi/(8+\Psi)$, the values for $\overline{\alpha}$ and $\underline{\alpha}$ are given by

$$\underline{\alpha} = \sqrt{\frac{\Psi^2(1-\lambda)^2 + 20\Psi\lambda(1-\lambda) - 8\lambda^2 + (\Psi - \lambda(\Psi+8))\left(\sqrt{\Psi(1-\lambda)(\Psi - \Psi\lambda - 8\lambda)}\right)}{8\lambda(\frac{1}{v_z} + \frac{1}{v_u})\frac{1}{v_e}(\Psi + \lambda - \Psi\lambda)^3}}$$

and

$$\overline{\alpha} = \sqrt{\frac{\Psi^2(1-\lambda)^2 + 20\Psi\lambda(1-\lambda) - 8\lambda^2 - (\Psi - \lambda(\Psi+8))\left(\sqrt{\Psi(1-\lambda)(\Psi - \Psi\lambda - 8\lambda)}\right)}{8\lambda(\frac{1}{v_z} + \frac{1}{v_u})\frac{1}{v_e}(\Psi + \lambda - \Psi\lambda)^3}}.$$

A.4. Proof of Proposition 4

Note that each trader's expected utility conditional on the information set is given by

$$V_K(\lambda; e) \equiv \mathbb{E}\left[-\exp\{-\alpha W\}\Big|e\right] = \mathbb{E}\left[-\exp\left\{-\alpha e P - \frac{1}{2}\frac{\chi^2}{v}\right\}\Big|e\right], \qquad K \in \{I, U\}, \tag{A.1}$$

where $\chi \equiv \mathbb{E}[R|\mathcal{F}]$ and $v \equiv \mathbb{V}ar[R|\mathcal{F}]$, respectively. First, given the endowment shock e, χ and P follow a bivariate normal distribution with mean vector and covariance matrix given by

$$\mu = \begin{pmatrix} \mu_{\chi|e} \\ \mu_{P|e} \end{pmatrix} \quad \text{and} \quad \Sigma = \begin{pmatrix} v_{\chi|e} & \sigma_{(\chi,P)|e} \\ \sigma_{(\chi,P)|e} & v_{P|e} \end{pmatrix}, \tag{A.2}$$

where $\mu_{\chi|e} \equiv \mathbb{E}[\chi|e]$, $\mu_{P|e} \equiv \mathbb{E}[P|e]$, $v_{\chi|e} \equiv \mathbb{V}ar[\chi|e]$, $v_{P|e} \equiv \mathbb{V}ar[P|e]$ and $\sigma_{(\chi,P)|e} \equiv \mathbb{C}ov[\chi, P|e]$. Thus, we can establish the following result,

$$V_K(\lambda; e) = -\exp\left\{-\frac{\mu^2_{\chi|e} + \alpha e\left[2\nu\mu_{P|e} + 2\mu_{\chi|e}\sigma_{(\chi,P)|e} + \alpha e\left(\sigma^2_{(\chi,P)|e} - \nu v_{P|e}\right)\right]}{2\nu}\right\}\sqrt{\frac{1}{1+\xi_K}}, \tag{A.3}$$

where $\nu \equiv \mathbb{V}ar[R] = v + v_{\chi|e}$, and $\xi_K \equiv \mathbb{V}ar[\chi|e]/v$. If the trader is informed, i.e., $\mathcal{F} = \{\theta, P, e\}$, since $\chi = \frac{n}{1+n}\theta - P$ and $v = v_D/(1+n)$, we obtain that

$$\mu_{\chi|e} = -\beta_{e,P}e, \quad \mu_{P|e} = \beta_{e,P}e, \quad v_{\chi|e} = \left(\frac{n}{1+n}\right)^2 v_\theta + v_{P|e} - 2\underbrace{\frac{n}{1+n}\sigma_{\theta,P}}_{\sigma_{D,P}}, \quad \sigma_{(\chi,P)|e} = \sigma_{D,P} - v_{P|e}. \tag{A.4}$$

Substituting (A.4) into (A.3) leads to the expected utility of an informed trader in (2.10) with $v = v_D/(1+n)$. On the other hand, if the trader is uninformed, i.e., $\mathcal{F} = \{P, e\}$, since $\chi = (1-\kappa)(-P) - \kappa\beta_{e,P}e$ and $v = v_D - \kappa\sigma_{D,P}$, $\kappa = \sigma_{D,P}/v_{P|e}$, we obtain that

$$\mu_{\chi|e} = -\beta_{e,P}e, \quad \mu_{P|e} = \beta_{e,P}e, \quad v_{\chi|e} = (1-\kappa)^2 v_{P|e}, \quad \sigma_{(\chi,P)|e} = -(1-\kappa)v_{P|e}. \tag{A.5}$$

Substituting (A.5) into (A.3) leads to the expected utility of an uninformed trader in (2.10) with $v = v_D - \kappa\sigma_{D,P}$.

A.5. Proof of Lemma 5

From Eq. (2.10) in Proposition 4, the ratio between the conditional expected utilities is given by

$$\frac{V_I(\lambda, e)}{V_U(\lambda, e)} = \sqrt{\frac{1+\xi_I}{1+\xi_U}} = \sqrt{\frac{\mathbb{V}ar\{\mathbb{E}[R|\theta, P, e]\} + \mathbb{V}ar[R|\theta, P, e]}{\mathbb{V}ar[R|\theta, P, e]}\left(\frac{\mathbb{V}ar[R|P, e]}{\mathbb{V}ar\{\mathbb{E}[R|\theta, P, e]\} + \mathbb{V}ar[R|P, e]}\right)}.$$

By the law of total variance,

$$\mathbb{V}ar\{\mathbb{E}[R|\theta, P, e]\} + \mathbb{V}ar[R|\theta, P, e] = \mathbb{V}ar\{\mathbb{E}[R|P, e]\} + \mathbb{V}ar[R|P, e] = \mathbb{V}ar[R].$$

Therefore, we have $V_I(\lambda, e)/V_U(\lambda, e) = \sqrt{\mathbb{V}ar[R|, \theta, P, e]/\mathbb{V}ar[R|P, e]} < 1$.

Next, we show that the quantity, $\gamma(\lambda) = 1 - \frac{V_I(\lambda,e)}{V_U(\lambda,e)}$, is strictly decreasing for $\lambda \in [0, 1]$, which is equivalent to showing $v'_U(\lambda) < 0$, where $v_U(\lambda) \equiv \mathbb{V}ar[R|P, e] = v_D - \kappa\sigma_{D,P}$. Moreover, it can be shown that

$$\kappa\sigma_{D,P} = \frac{b^2_\theta v^2_D}{b^2_z v_{z|e} + b^2_\theta v_\theta} = \frac{v^2_D}{v_\theta + (1/\pi^2)(v_u^{-1} + v_z^{-1})^{-1}}.$$

Therefore, to prove $\gamma'(\lambda) < 0$ we only need to show that $\pi'(\lambda) > 0$. Recall the fixed-point equation $\pi - f(\pi) = 0$, since $f'(\pi) < 1$ for a unique equilibrium, we can apply the Implicit Function Theorem to obtain $\pi'(\lambda) = (\partial/\partial\lambda)f(\pi)/(1 - f'(\pi))$. Using the fact that $(\partial f/\partial\lambda) > 0$, it follows that $\pi'(\lambda) > 0$, hence $\gamma'(\lambda) < 0$.

A.6. Proof of Lemma 6

Let $\bar{V}(\lambda,e) \equiv \lambda V_I(\lambda,e) + (1-\lambda)V_U(\lambda,e)$ and $\bar{V}(p;\lambda,e) \equiv pV_I(\lambda,e)+(1-p)V_U(\lambda,e)$, then

$$\mathcal{W}'(p;\lambda,e) = e^{\alpha\mu c(p)}[\alpha\mu c'(p)\bar{V}(p;\lambda,e) + [V_I(\lambda,e)-V_U(\lambda,e)]], \qquad c(p)=p^2;$$
$$\mathcal{W}''(p;\lambda,e) = \alpha\mu c'(p)e^{\alpha\mu c(p)}\left[\left(\alpha\mu c'(p)+\frac{c''(p)}{c'(p)}\right)\bar{V}(p;\lambda,e) + 2[V_I(\lambda,e)-V_U(\lambda,e)]\right].$$

Therefore, the necessary and sufficient condition for $\mathcal{W}''(p;\lambda) \le 0$ is

$$\left(\alpha\mu c'(p)+\frac{c''(p)}{c'(p)}\right)\bar{V}(p;\lambda,e) + 2[V_I(\lambda,e)-V_U(\lambda,e)] \le 0,$$

or, put differently,

$$\left(\alpha\mu c'(p)+\frac{c''(p)}{c'(p)}\right)V_U(\lambda,e)[1-p\gamma(\lambda)] - 2V_U(\lambda,e)\gamma(\lambda) \le 0.$$

Let $K(p) = (\alpha\mu c'(p) + c''(p)/c'(p))$, then such condition is equivalent to

$$K(p)[1-p\gamma(\lambda)] - 2\gamma(\lambda) \ge 0 \iff \gamma(\lambda) \le \frac{K(p)}{2+K(p)p}, \tag{A.6}$$

where the latter inequality must hold true for any p. Now, since $c(p)=p^2$, the r.h.s. in (A.6) is decreasing in p. Therefore, (A.6) is equivalent to

$$\gamma(\lambda) \le \frac{K(1)}{2+K(1)} = \frac{2\alpha\mu+1}{2\alpha\mu+3}.$$

A.7. Proof of Proposition 9

Note that if the concavity condition, $\mathcal{W}''(p;\lambda,e) \le 0$, is satisfied, the Nash equilibrium for the choice of probability p to observe the fundamental signal θ must be symmetric, since traders are homogeneous, i.e., $p^* = \lambda$ for all traders, from which we obtain

$$\alpha\mu = -\frac{1}{2\lambda}\frac{V_I(\lambda,e)-V_U(\lambda,e)}{\bar{V}(\lambda,e)} = \frac{1}{2\lambda}\frac{\gamma(\lambda)}{1-\lambda\gamma(\lambda)}.$$

Substitute this expression into

$$\gamma(\lambda) \le \frac{2\alpha\mu+1}{2\alpha\mu+3},$$

then it can be written as

$$\gamma(\lambda)^2(3\lambda^2-1) + \gamma(\lambda)(1-3\lambda-\lambda^2) + \lambda \ge 0.$$

A tedious algebraic derivation shows that this is satisfied as soon as $\gamma(\lambda) \le 1/2$. Moreover, since $\gamma(\lambda)$ is a decreasing function, it is sufficient to have $\gamma(0) \le 1/2$. Finally, recall that $\gamma(0)$ can be written as a function on n. Specifically, we obtain

$$\gamma(0) = 1 - \frac{1}{\sqrt{1+n}} \le \frac{1}{2} \iff n \le 3.$$

In other words, for $c(p)=p^2$, $n \le 3$ guarantees $\gamma(\lambda) \le 1/2$, which is a sufficient condition for $\mathcal{W}''(p;\lambda,e) \le 0$ given any equilibrium $\lambda \in [0,1]$, hence $p^* = \lambda$, and λ in equilibrium is given by (2.18).

The proof on the existence and uniqueness of the Nash equilibrium p^* with respect to parameter μ is provided in Appendix B.

A.8. Proof of Corollary 10

Note that in the Grossman–Stiglitz model, the expected utility of investor-i is given by $V_U(\lambda,e_i)$ in equilibrium, thus the ex-ante welfare is $V_U(\lambda,e_i)$. By contrast, in the probabilistic-choice model, the optimal information acquisition decision of investor i's implies that

$$\begin{aligned}\mathcal{W}(p_i^*(\lambda);\lambda,e_i) &= \left[p_i^*(\lambda)V_I(\lambda,e_i) + (1-p_i^*(\lambda))V_U(\lambda,e_i)\right]e^{\alpha\mu(p_i^*)^2} \\ &\ge \mathcal{W}(0;\lambda,e_i) = V_U(\lambda,e_i).\end{aligned}$$

Therefore, the probabilistic information acquisition always leads to better welfare compared to the standard Grossman–Stiglitz model.

Alternatively, in the Grossman–Stiglitz model, the expected utility of investor-i is given by $V_U(\lambda,e_i)$ in equilibrium, thus the ex-ante welfare is $V_U(\lambda,e_i)$. By contrast, in the probabilistic-choice model, by using the definition of $\gamma(\lambda)$ in (2.15) and the first order condition (2.16), we can show that investor-i's ex-ante welfare can be written as

$$\frac{\mathcal{W}(p_i^*;\lambda,e_i)}{V_U(\lambda,e_i)} = (1-p_i^*\gamma(\lambda))e^{\frac{1}{2}\frac{p_i^*\gamma(\lambda)}{1-p_i^*\gamma(\lambda)}}. \tag{A.7}$$

Moreover, from (A.7) it can be shown that, since $\gamma(\lambda) \le \frac{1}{2}$ in equilibrium,[21]

$$\frac{\mathcal{W}'(p_i^*;\lambda,e_i)}{V_U(\lambda,e_i)} = -\frac{1}{2}\frac{1-2p_i^*\gamma(\lambda)}{1-p_i^*\gamma(\lambda)}e^{\frac{1}{2}\frac{p_i^*\gamma(\lambda)}{1-p_i^*\gamma(\lambda)}} \le 0.$$

Therefore, for any level of informed trading λ, we have $\mathcal{W}(p_i^*;\lambda,e_i) \ge V_U(\lambda,e_i)$ for $p_i^* \in (0,1)$.

A.9. Proof of Proposition 11

From the welfare definition in (3.1), if we take partial derivative w.r.t p^*, we obtain the following,

$$\frac{1}{-\mathcal{W}}\frac{\partial\mathcal{W}}{\partial p^*} = \frac{V_I(\lambda)-V_U(\lambda)}{-\bar{V}(p^*,\lambda)} + \frac{\gamma(\lambda)}{2(1-p^*\gamma(\lambda))^2},$$

where the first term is from taking derivative of $\bar{V}$. We use the fact that $V_I(\lambda) = (1-\gamma(\lambda))V_U(\lambda)$ to simplify the first term, i.e.,

$$\frac{V_I(\lambda)-V_U(\lambda)}{-\bar{V}(p^*,\lambda)} = \frac{\gamma(\lambda)}{1-p^*\gamma(\lambda)}$$

Thus, by substituting $p^* = \lambda$, we obtain

$$\frac{1}{-\mathcal{W}}\frac{\partial\mathcal{W}}{\partial p^*} = \frac{\gamma(\lambda)}{1-\lambda\gamma(\lambda)} + \frac{\gamma(\lambda)}{2(1-\lambda\gamma(\lambda))^2},$$

which leads to (3.3).

Next, we take partial derivative w.r.t λ and obtain

$$\frac{1}{-\mathcal{W}}\frac{\partial\mathcal{W}}{\partial \lambda} = \frac{p^*V_I'(\lambda)+(1-p^*)V_U'(\lambda)}{-\bar{V}(p^*,\lambda)} + \frac{1}{2}e^2A'(\lambda) - \frac{(p^*)^2\gamma(\lambda)\gamma'(\lambda)}{2(1-p^*\gamma(\lambda))^2} - \frac{p^*\gamma'(\lambda)}{2(1-p^*\gamma(\lambda))}.$$

We are able to simplify the above expression by using the fact that

$$\gamma'(\lambda) = (1-\gamma(\lambda))\left(\frac{V_I'(\lambda)}{-V_I(\lambda)} - \frac{V_U'(\lambda)}{-V_U(\lambda)}\right),$$

which give us the following,

$$\frac{1}{-\mathcal{W}}\frac{\partial\mathcal{W}}{\partial \lambda} = \frac{1}{2}e^2A'(\lambda) + \frac{p^*(1-\gamma(\lambda))(1-2p^*\gamma(\lambda))}{2(1-p^*\gamma(\lambda))^2}\left(\frac{V_I'(\lambda)}{-V_I(\lambda)} - \frac{V_U'(\lambda)}{-V_U(\lambda)}\right) + \frac{V_U'(\lambda)}{-V_U(\lambda)}.$$

Hence, substituting $p^* = \lambda$ leads to (3.4). Therefore, in equilibrium, investor-i's marginal welfare has the following decomposition,

$$\frac{\mathcal{W}'(\lambda;e)}{-\mathcal{W}(\lambda,e)} = \underbrace{\frac{\gamma(\lambda)(1-2\lambda\gamma(\lambda))}{2(1-\lambda\gamma(\lambda))^2}}_{\text{opportunity effect}} + \underbrace{\frac{1}{2}A'(\lambda)e^2}_{\text{Hirshleifer effect}} + \underbrace{\left[\Gamma(\lambda)\frac{V_I'(\lambda)}{-V_I(\lambda)} + (1-\Gamma(\lambda))\frac{V_U'(\lambda)}{-V_U(\lambda)}\right]}_{\text{risk-return effect}}, \tag{A.8}$$

where

$$\Gamma(\lambda) = \frac{\lambda(1-2\lambda\gamma(\lambda))(1-\gamma(\lambda))}{2(1-\lambda\gamma(\lambda))^2}.$$

A.10. Proof of Proposition 12

Based on (A.8), we evaluate each component at $\lambda = 0$.

Anticipatory effect:

$$\frac{\gamma(0)}{2} = \frac{1}{2}\left(1 - \frac{1}{\sqrt{1+n}}\right).$$

Hirshleifer effect:

$$\frac{1}{2}A'(0)e^2 = \frac{1}{2}e^2\left[\frac{\partial A}{\partial b_\theta}(0)b_\theta'(0) + \frac{\partial A}{\partial b_z}(0)b_z'(0)\right]$$

Recall from Eq. (2.12) in Proposition 4 that

$$A(\lambda) = \frac{\left[\beta_{e,P} + \alpha(v_D - \sigma_{D,P})\right]^2}{v_{P|e} + v_D - 2\sigma_{D,P}} = \frac{\left(\alpha(v_D - b_\theta v_D) - \frac{b_z v_z}{v_e}\right)^2}{b_\theta^2 v_\theta - 2b_\theta v_D + b_z^2 v_{z|e} + v_D}.$$

[21] From an individual investor-i's viewpoint, cost sensitivity μ is fixed. In this case, $\mathcal{W}'(p^*;\lambda) = 0$. However, here we take the view point of the regulator (social planner), who is able to *set* the value of μ, hence control investors' probability of becoming informed p^*. Therefore, for a fixed level of informed trading λ, (2.20) shows that investors' welfare improves if regulator lowers μ which increases p^*.

Then, using the fact that $b_\theta(0) = 0$ and $b_z(0) = \alpha v_D$, we can obtain

$$\frac{\partial A}{\partial b_\theta}(0) = -\frac{2\alpha^2 v_D v_u \left(\alpha^2 v_D v_e v_{z|e} + v_z\right)}{\left(\alpha^2 v_D v_e v_{z|e} + v_e\right)^2} \tag{A.9}$$

and

$$\frac{\partial A}{\partial b_z}(0) = -\frac{2\alpha v_u \left(\alpha^2 v_D v_e v_{z|e} + v_z\right)}{\left(\alpha^2 v_D v_e v_{z|e} + v_e\right)^2}, \qquad v_{z|e} = (v_u^{-1} + v_z^{-1})^{-1}. \tag{A.10}$$

Next task is to compute $b'_\theta(0)$ and $b'_z(0)$. First, we recall Eq. (2.6) in Lemma 2, by differentiating both side of the equation w.r.t λ we can obtain

$$\pi'(\lambda) = \frac{1}{\alpha v_\epsilon}\left[1 + \frac{2v_u v_\epsilon (1-\lambda)\Psi^2 \pi(\lambda)\pi'(\lambda)}{\left(v_u \Psi + v_\epsilon \pi(\lambda)^2\right)^2} - \frac{v_\epsilon \Psi \pi(\lambda)^2}{v_u \Psi + v_\epsilon \pi(\lambda)^2}\right],$$

and, since $\pi(0) = 0$, we have

$$\pi'(0) = \frac{1}{\alpha v_\epsilon}. \tag{A.11}$$

Moreover, from Eq. (2.7) in Lemma 2, we can rewrite b_θ and b_z as functions of π,

$$b_\theta = \frac{v_D \pi (v_z \alpha + \pi)}{v_z + v_D v_z \alpha \pi + (v_D + v_\epsilon)\pi^2}, \qquad b_z = \frac{v_D (v_z \alpha + \pi)}{v_z + v_D v_z \alpha \pi + (v_D + v_\epsilon)\pi^2} \tag{A.12}$$

by using the fact that

$$\lambda = \frac{\alpha v_\epsilon \pi - \left[\Psi^{-1} + \pi^{-2} v_\epsilon^{-1} v_u\right]^{-1}}{1 - \left[\Psi^{-1} + \pi^{-2} v_\epsilon^{-1} v_u\right]^{-1}}.$$

Then, from (A.12), we can compute derivative of b_θ and b_z w.r.t λ at $\lambda = 0$, which are given by

$$b'_\theta(0) = \frac{\partial b_\theta}{\partial \pi}(0)\pi'(0) = \alpha v_D \left(\frac{1}{\alpha v_\epsilon}\right) = \frac{v_D}{v_\epsilon} = n \tag{A.13}$$

and

$$b'_z(0) = \frac{\partial b_z}{\partial \pi}(0)\pi'(0) = v_D \left(\frac{1}{v_z} - \alpha^2 v_D\right)\left(\frac{1}{\alpha v_\epsilon}\right) = n\left(\frac{1}{\alpha v_z} - \alpha v_D\right). \tag{A.14}$$

Putting it altogether we obtain

$$\frac{1}{2}A'(0)e^2 = -\frac{n(1-\Psi)}{\Psi(1+\xi_0)^2}(\Psi + \xi_0)\left(\frac{e^2}{v_e}\right), \qquad \xi_0 = \alpha^2 v_D v_{z|e} \text{ and } \Psi = \frac{v_z}{v_e}. \tag{A.15}$$

Risk-return effect:

$$\frac{V'_U(0)}{-V_U(0)} = \frac{1}{2}\frac{\xi'_U(0)}{1+\xi_0}.$$

Recall from Eq. (2.13) in Proposition 4 that

$$\xi_U(\lambda) = \frac{(1-\kappa)^2 v_{P|e}}{v_D - \kappa \sigma_{\theta,P}} = \frac{\left(b_\theta^2 v_\theta - b_\theta v_D + b_z^2 v_{z|e}\right)^2}{v_D\left(b_\theta^2 v_\epsilon + b_z^2 v_{z|e}\right)}.$$

Then

$$\xi'_U(0) = \frac{\partial \xi_U}{\partial b_\theta}(0) b'_\theta(0) + \frac{\partial \xi_U}{\partial b_z}(0) b'_z(0),$$

where, by using the fact that $b_\theta(0) = 0$ and $b_z(0) = \alpha v_D$,

$$\frac{\partial \xi_U}{\partial b_\theta}(0) = -2, \qquad \frac{\partial \xi_U}{\partial b_z}(0) = 2\alpha v_{z|e}, \tag{A.16}$$

and $b'_\theta(0)$ and $b'_z(0)$ are given by (A.13) and (A.14), respectively. Therefore, putting it altogether we obtain

$$\frac{V'_U(0)}{-V_U(0)} = -\frac{n}{1+\xi_0}(\Psi + \xi_0). \tag{A.17}$$

A.11. Proof of Proposition 14

To prove that the full-information equilibrium ($\lambda = 1$) is dominated by the no-information equilibrium ($\lambda = 0$), we write the log-welfare functions as

$$-\ln[-\mathcal{W}(0; e)] = -\ln[V_U(0)] + \frac{1}{2}[A(0) - \alpha^2 v_D]e^2,$$

and

$$-\ln[-\mathcal{W}(1;e)] = -\ln[V_I(1)] + \frac{1}{2}[A(1) - \alpha^2 v_D]e^2 - \underbrace{\alpha\mu}_{\text{quadratic cost}}.$$

Moreover, using Proposition 4, Eq. (2.12), and the fact that $b_\theta = 0$, $b_z = \alpha v_D$ when $\lambda = 0$, and $b_\theta = \frac{n}{1+n}$, $b_z = \alpha \frac{v_D}{1+n}$ when $\lambda = 1$, we can compute that

$$A(0) = (1-\Psi)^2 \frac{\alpha^2 v_D}{1+\xi_0}, \qquad A(1) = (1-\Psi)^2 \frac{\alpha^2 v_D}{1+n+\xi_0},$$

and $\xi_U(0) = \xi_0, \xi_I(1) = \xi_0/(1+n)$. Thus, $A(1) < A(0)$ and $\xi_I(1) < \xi_U(0)$ for $n > 0$. Hence, $\mathcal{W}(1;e) < \mathcal{W}(0;e)$ for $n > 0$ and $e \in \mathbb{R}$.

A.12. Proof of Lemma 15

See Manzano and Vives (2011), the proof of Proposition 1, on pp. 361–364.

A.13. Proof of Proposition 16

Given the financial market equilibrium in Lemma 15, we re-valuate the three components of investor-i's marginal welfare when $\lambda = 0$. We first note that when $\lambda = 0$, $v_I = v_D/(1+n)$, $v_U = v_D$, thus $b_\theta = 0$, $\pi = 0$ and $b_z = \alpha v_z$, which are the same as in the baseline model.

Anticipatory effect:

$$\frac{\gamma(0)}{2} = \frac{1}{2}\left(1 - \frac{1}{\sqrt{1+n}}\right).$$

Hirshleifer effect:

$$\frac{1}{2}A'(0)e^2 = \frac{1}{2}e^2\left[\frac{\partial A}{\partial b_\theta}(0)b'_\theta(0) + \frac{\partial A}{\partial b_z}(0)b'_z(0)\right].$$

From Manzano and Vives (2011), p.365, the proof of Corollary 5, we can write

$$A(\lambda) = \frac{\alpha^2\left(-b_\theta^2\rho\left(v_u^{-1}+v_z^{-1}\right) - \frac{b_z^2}{v_\epsilon} + \frac{b_z\left(2(b_\theta-1) + \frac{b_z}{\alpha v_D v_u\left(v_u^{-1}+v_z^{-1}\right)}\right)}{\alpha v_u v_\epsilon}\right)}{\left(v_u^{-1}+v_z^{-1}\right)\left(\frac{b_\theta^2\rho}{v_D} + \frac{(b_\theta-1)^2}{v_\epsilon}\right) + \frac{b_z^2}{v_D v_\epsilon}}.$$

We can then compute $\frac{\partial A}{\partial b_\theta}(0)$ and $\frac{\partial A}{\partial b_z}(0)$ to obtain the same expressions as (A.9) and (A.10).

Next, from Lemma 15, we can compute $\pi'(\lambda)$. For brevity, we do not report the full expression. Then, substituting $\lambda = 0$ and $\pi(0) = 0$ into the expression for $\pi'(\lambda)$ leads to $\pi'(0) = 1/(\alpha v_\epsilon)$. Moreover, from Eq. (4.1) in Lemma 15, by substituting

$$\lambda = \frac{\alpha v_\epsilon \pi - \frac{\pi^2}{v_u\left(\pi^2\rho\left(\frac{1}{v_u}+\frac{1}{v_z}\right)+\frac{1}{v_\epsilon}\right)}}{\frac{\frac{\pi^2(1-\rho)}{v_u}+\frac{1}{v_\epsilon}}{\left(\pi^2(1-\rho)\rho\left(\frac{1}{v_u}+\frac{1}{v_z}\right)+\frac{1}{v_\epsilon}\right)} - \frac{\pi^2}{v_u\left(\pi^2\rho\left(\frac{1}{v_u}+\frac{1}{v_z}\right)+\frac{1}{v_\epsilon}\right)}},$$

we can rewrite b_θ and b_z as functions of π as follows,

$$b_\theta = \frac{v_D\pi(\alpha v_z + \pi)}{v_z + \alpha v_D v_z \pi + \pi^2(v_D + v_\epsilon\rho)}, \qquad b_z = \frac{v_D(\alpha v_z + \pi)}{v_z + \alpha v_D v_z \pi + \pi^2(v_D + v_\epsilon\rho)}. \tag{A.18}$$

Then, from (A.18), we can compute $b'_\theta(\lambda)$ and $b'_z(\lambda)$ at $\lambda = 0$ by using the fact that $\pi(0) = 0$ and $\pi'(0) = 1/(\alpha v_\epsilon)$, it turns out that $b'_\theta(0)$ and $b'_z(0)$ given by the same expressions as in (A.13) and (A.14). Therefore, putting altogether leads to the expression for $\frac{1}{2}A'(0)e^2$, which coincides with (A.15).

Risk-return effect: From Manzano and Vives (2011), p.365, the proof of Corollary 5, we can write

$$V_U(\lambda) = \left[\frac{\frac{v_D v_\epsilon}{v_U}\left(\left(\frac{1}{v_u}+\frac{1}{v_z}\right)\left(\frac{b_\theta^2\rho}{v_D} + \frac{(1-b_\theta)^2}{v_\epsilon}\right) + \frac{b_z^2}{v_D v_\epsilon}\right)}{\frac{1}{v_u}+\frac{1}{v_z}}\right]^{-\frac{1}{2}}. \tag{A.19}$$

We can then compute

$$\frac{V'_U(0)}{-V_U(0)} = \frac{1}{-V_U(0)}\left(\frac{\partial V_U}{\partial b_\theta}(0)b'_\theta(0) + \frac{\partial V_U}{\partial b_z}(0)b'_z(0)\right),$$

where

$$\frac{1}{-V_U(0)}\frac{\partial V_U}{\partial b_\theta}(0) = -\frac{1}{1+\xi_0}, \qquad \frac{1}{-V_U(0)}\frac{\partial V_U}{\partial b_z}(0) = \frac{\alpha v_z(1-\Psi)}{1+\xi_0}.$$

Putting altogether we obtain the same expression as in (A.17).

A.14. Proof of Proposition 17

The equilibrium condition can be written as

$$\mu = F(\lambda), \qquad F(\lambda) = \frac{1}{2\alpha\lambda}\ln\left[\frac{1}{1-\gamma(\lambda)}\right].$$

since $F(1) = 1/(2\alpha)\ln[1/(1-\gamma(1))]$, $F(0)\to\infty$, and

$$F'(\lambda) = -\frac{1}{2\alpha\lambda^2}\left(\ln\left[\frac{1}{1-\gamma(\lambda)}\right] - \frac{\lambda\gamma'(\lambda)}{1-\gamma(\lambda)}\right) < 0,$$

assuming $\mu > F(1)$, there is a unique $\lambda \in (0,1)$ that satisfies the equilibrium condition. Moreover, for $\lambda = \lambda(\mu)$, taking the derivative w.r.t μ, we have $1 = F'(\lambda)\lambda'(\mu)$, thus $\lambda'(\mu) = 1/F'(\lambda) < 0$.

Appendix B. Existence and uniqueness of Nash equilibrium

This appendix examines the existence and uniqueness of the Nash equilibrium with respect to parameter μ, which measures the sensitivity to the cost of information acquisition. For convenience, we define $\xi_1 = \alpha^2 v_I(v_z^{-1}+v_u^{-1})^{-1} = \alpha^2 v_I v_z(1-\Psi)$. Note that $\xi_1 = \xi_I(1)$, representing the squared Sharpe ratio of informed traders when $\lambda = 1$. Intuitively, in equilibrium, $\lambda\to 0$ as $\mu\to\infty$; $\lambda = 1$ when μ is small enough; otherwise $\lambda\in(0,1)$. This is demonstrated as follows.

Proposition 18. *Assume $c(p) = p^2$ and $n \le 3$. Then*

(i) $\lambda = 0$ as $\mu\to\infty$;
(ii) $\lambda = 1$ when $\mu \le \bar\mu := \frac{1}{2\alpha}\frac{\gamma_1}{1-\gamma_1}$, where $\gamma_1 \equiv \gamma(1) = 1-\sqrt{\frac{n+\xi_1}{n+\xi_0}}$;
(iii) there exists a unique $\lambda\in(0,1)$ when $\mu > \bar\mu$;
(iv) λ is decreasing in μ.

Moreover, the equilibrium price P satisfies (2.5) with the coefficients b_θ and b_z evaluated at the equilibrium λ.

Proof. Note that $\gamma(\lambda)\in(0,1)$. With $c(p) = p^2$, from the equilibrium condition $2\alpha\mu\lambda = \gamma(\lambda)/[1-\gamma(\lambda)]$, it is easy to see that $\lambda\to 0$ as $\mu\to\infty$. For $\lambda = 1$, we have $\mu = \bar\mu = \frac{1}{2\alpha}\frac{\gamma(1)}{1-\gamma(1)}$. It remains to discuss the case $\mu > \bar\mu$. To this aim, note that, in case of $c(p) = p^2$, the fixed point (2.18) is equivalent to

$$\lambda^2 - \frac{1}{\gamma(\lambda)}\lambda + \frac{1}{2\alpha\mu} = 0. \tag{B.1}$$

By defining

$$F_1(\lambda) = \frac{1}{2\gamma(\lambda)} - \frac{1}{2\gamma(\lambda)}\sqrt{1-\frac{2\gamma^2(\lambda)}{\alpha\mu}}; \quad F_2(\lambda) = \frac{1}{2\gamma(\lambda)} + \frac{1}{2\gamma(\lambda)}\sqrt{1-\frac{2\gamma^2(\lambda)}{\alpha\mu}},$$

(B.1) can be rewritten as $[\lambda - F_1(\lambda)][\lambda - F_2(\lambda)] = 0$. Assuming $\mu \ge 2\gamma^2(\lambda)/\alpha$ (otherwise the fixed point has no solution and $\lambda = 1$), F_1 and F_2 are well-defined. It is not difficult to show that $0 < F_1(\lambda) \le F_2(\lambda)$. Therefore, since $F_1(0) > 0$, one solution to (B.1) exists if and only if $F_1(1) < 1$. This condition is exactly $\mu > \bar\mu$. Finally, concerning uniqueness, note that $dF_1(\lambda)/d\lambda < 0$. Indeed,

$$\frac{dF_1(\lambda)}{d\lambda} = -\frac{\gamma'(\lambda)}{2\gamma^2(\lambda)}\left(1-\sqrt{1-\frac{2\gamma^2(\lambda)}{\alpha\mu}}\right) + \frac{\gamma'(\lambda)}{\alpha\mu\sqrt{1-\frac{2\gamma^2(\lambda)}{\alpha\mu}}} = \frac{\gamma'(\lambda)}{\gamma(\lambda)}\frac{F_1(\lambda)}{\sqrt{1-\frac{2\gamma^2(\lambda)}{\alpha\mu}}} < 0.$$

Negativity is due to the fact that $\gamma'(\lambda) < 0$, $\gamma(\lambda) > 0$, and $F_1(\lambda) > 0$. By monotonicity, $\lambda = F_1(\lambda)$ provides at most one solution. Therefore, if a second solution $\tilde\lambda$ to the fixed point exists, it must solve $\tilde\lambda = F_2(\tilde\lambda)$. By definition $F_2(\lambda) > \frac{1}{2\gamma(\lambda)}$; therefore, as soon as $\gamma(\lambda) < 1/2$, we have $\tilde\lambda = F_2(\tilde\lambda) > 1$, which is not feasible. This proves that the solution to the fixed point is unique as soon as the sufficient condition for (strict) concavity, $\gamma(\lambda) < 1/2$, is satisfied.

Next we prove that the equilibrium $\lambda = \lambda(\mu)$ is decreasing in μ. In equilibrium, $\alpha\mu c'(\lambda) = -\frac{V_I(\lambda,e)-V_U(\lambda,e)}{\bar V(\lambda,e)} = G(\lambda) \equiv \frac{\gamma(\lambda)}{1-\lambda\gamma(\lambda)}$. For $\lambda = \lambda(\mu)$, taking the derivative w.r.t. μ, we have $\alpha c'(\lambda) = -\lambda'(\mu)[G'(\lambda) - \frac{c''(\lambda)}{c'(\lambda)}G(\lambda)]$. Therefore $\lambda'(\mu)\le 0$ if and only if

$$\frac{G'(\lambda)}{G(\lambda)} \le \frac{c''(\lambda)}{c'(\lambda)}, \qquad G(\lambda) = \frac{\gamma(\lambda)}{1-\lambda\gamma(\lambda)}; \tag{B.2}$$

or equivalently

$$\frac{\gamma^2(\lambda)+\gamma'(\lambda)}{1-\lambda\gamma(\lambda)} \le \frac{c''(\lambda)}{c'(\lambda)}. \tag{B.3}$$

Since $c(p) = p^2$, condition (B.3) becomes

$$\lambda[\gamma^2(\lambda) + \gamma(\lambda) + \gamma'(\lambda)] \leq 1, \tag{B.4}$$

which is satisfied due to the fact that $\lambda \leq 1$, $\gamma(\lambda) \leq 1/2$, and $\gamma'(\lambda) < 0$. □

Data availability

No data was used for the research described in the article.

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