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# MANIPULATING INFORMATION REVELATION WITH RESERVE PRICES

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We introduce a novel motive for the use of a reserve price as an instrument to raise auction revenues in ascending auctions. The effect that we stress is of inducing coarser information aggregation. The reserve price may prevent information revelation because bidders cannot precisely observe at which price other bidders leave the auction. In simple settings where valuation functions are not symmetric, this may increase the expected revenue of the auction. To illustrate this motive, we exhibit an example in which the use of a reserve price increases revenue even though there are always at least two bidders active for prices higher than the reserve price.

*JEL Codes:* D44, D82.
*Keywords:* Auctions, Reserve Price.

## 1. INTRODUCTION

The standard effect of a reserve price in a second-price auction or an ascending auction pointed out in the literature (since Myerson (1981), Riley and Samuelson (1981)) is that it can increase expected revenue by avoiding to sell at a low price when the difference between the highest and the second highest valuation is large.

We provide a novel motive for the use of a reserve price in ascending auctions. The reserve price may be used in order to prevent the revelation of information that would otherwise occur during the auction process. By inducing coarser information aggregation, the auctioneer may raise expected revenue in situations where the linkage principle does not apply. Board (2009) already observed that when monotonicity and symmetry are not satisfied, the linkage principle does not apply and revealing information may decrease expected revenue. Hence, the seller may benefit from hiding information. Our contribution is to point at the reserve price as a practical instrument to achieve the desired manipulation of information. The idea that the information endogenously aggregated in an open ascending auction can be strategically manipulated is also present in Ettinger and Michelucci (2015). In that paper bidders have an incentive to do so using jump bids.

In order to illustrate our observation, this note introduces a simple example of an auction setting with asymmetric, yet reasonable, value functions in which the use of a reserve price increases expected revenue. This is the case even though at least two bidders are always active at a price strictly higher than the reserve price so that the standard motive for using reserve price does not apply in this context.

As in most of the literature on the ascending auction, we only consider Perfect Bayesian Equilibria with weakly dominant strategies when they do exist.

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## 2. AN ILLUSTRATIVE EXAMPLE

We consider a standard Japanese ascending auction in which the auctioneer has the possibility to impose a reserve price $R > 0$.[1] There are 3 bidders with private information. For $i = 1, 2, 3$, $t_i$ is bidder $i$'s private information. $F_i$, the distribution function of $t_i$ is common knowledge and the $F_i$ are i.i.d. and uniform on $[0, 1]$.

- $v_1 = 4t_1$
- $v_2 = 1 + 2t_1 + t_2$
- $v_3 = \frac{3}{2} + t_1 + t_3$

Notice that the environment we consider is specific to render the example neat and its proofs easy to catch. The main elements of the example are the following. Bidder 1's private information, $t_1$ matters for all the bidders. Bidder 1's valuation is the most sensitive to the value of $t_1$ but among other bidders, Bidder 2's valuation is more sensitive to the value of $t_1$ than Bidder 3's valuation. Hence, a low value of $t_1$ favors Bidder 3 and a high value of $t_1$ favors Bidder 2, in relative terms. $v_2$ and $v_3$ are also affected by a specific and private component, respectively $t_2$ and $t_3$. We add a fixed component in valuations functions to keep the expected value of $v_2$ and $v_3$ equal.

PROPOSITION 1 *Without reserve price, in any equilibrium of the game, Bidder 1 stays active up to* $4t_1$. *Denote* $\tilde{p}$, *the price at which Bidder 1 leaves the auction. If Bidder 1 is still active, Bidder 2 stays active up to* $2 + 2t_2$ *and if Bidder 1 has already left the auction, Bidder 2 stays active up to* $1 + \frac{\tilde{p}}{2} + t_2$. *If Bidder 1 is still active, Bidder 3 stays active up to* $2 + \frac{4t_3}{3}$ *and if Bidder 1 has already left the auction, Bidder 3 stays active up to* $\frac{3}{2} + \frac{\tilde{p}}{4} + t_3$.

**Proof**: It is a well-known result that in a second-price auction, staying active up to your valuation for the good is a weakly dominant strategy. Therefore, at the equilibrium, Bidder 1 will stay active up to $4t_1$. By observing at which price Bidder 1 leaves the auction, $\tilde{p}$, bidders 2 and 3 can perfectly infer the value of $t_1$ to be $\frac{\tilde{p}}{4}$ and consequently stay active up to their valuations for the good, respectively $1 + \frac{\tilde{p}}{2} + t_2$ and $\frac{3}{2} + \frac{\tilde{p}}{4} + t_3$. Therefore, the only part of the proposition which is not direct concerns the bidding behaviors of bidders 2 and 3 when Bidder 1 is still active and they do not know their valuations for the good.

Let us first consider Bidder 2's suggested equilibrium strategy conditional on Bidder 1 being active: staying active up to $2 + 2t_2$. Note that this is the price at which Bidder 2 would break even if Bidder 1 also quit at the same price and Bidder 2 was allocated the object.

We will show that leaving the auction at a price strictly lower than $2 + 2t_2$ or strictly higher than $2 + 2t_2$ yields a strictly lower expected profit to Bidder 2 than leaving the auction when the price is equal to $2 + 2t_2$.

Suppose that, conditional on Bidder 1 being active, Bidder 2 leaves the auction at a price $\hat{p} < 2 + 2t_2$ rather than leaving at a price $2 + 2t_2$. This will only a make a difference if $t_1 \in (\frac{\hat{p}}{4}, \frac{1}{2} + \frac{t_2}{2})$.[2] In that case, if Bidder 2 leaves the auction at price $\hat{p}$, he will lose the auction with probability 1. Instead, if Bidder 2 stays active up to $2 + 2t_2$, he will observe the price $\tilde{p} < 2 + 2t_2$ at which Bidder 1 leaves the auction and will infer his valuation: $1 + \frac{\tilde{p}}{2} + t_2$. Since $\tilde{p} < 2 + 2t_2$, we have that $\tilde{p} < 1 + \frac{\tilde{p}}{2} + t_2$ so that Bidder 2's valuation

[1] See, for instance, Krishna (2010) for a description of the auction rules for this format.

[2] In order to shorten the proof, we do not consider the boundaries of the interval. There is not technical issue in these cases. Besides the probability that $t_1$ is equal to a boundary of the interval is equal to zero.

for the good is strictly higher than the price at which Bidder 1 leaves the auction. Hence, Bidder 2 never derives a negative profit if he stays active up to $2 + 2t_2$, and he derives a strictly positive profit if $v_3 < v_2$, which arises with a strictly positive probability if $v_2 > 0$. We can then conclude that leaving the auction for a price equal to $2 + 2t_2$, conditional on Bidder 1 being active, is a better response to Bidder 1 and 3's strategies than leaving the auction for a price strictly lower than $2 + 2t_2$.

Now, suppose that, conditional on Bidder 1 being active, Bidder 2 leaves the auction at a price $\hat{p} > 2 + 2t_2$ rather than leaving at a price $2 + 2t_2$. This will only a make a difference if $t_1 \in (\frac{1}{2} + \frac{t_2}{2}, \frac{\hat{p}}{4})$.[3] In that case, if Bidder 2 leaves the auction at price $2 + 2t_2$, he will lose the auction with probability 1. Instead, if Bidder 2 stays active up to $\hat{p}$, he will observe the price $\tilde{p} > 2 + 2t_2$ at which Bidder 1 leaves the auction and will infer his valuation: $1 + \frac{\tilde{p}}{2} + t_2$. Since $\tilde{p} > 2 + 2t_2$, we have that $\tilde{p} > 1 + \frac{\tilde{p}}{2} + t_2$ so that Bidder 2's valuation for the good is strictly lower than the price at which Bidder 1 leaves the auction. Hence, Bidder 2 never derives a positive profit if he stays active for a price strictly higher than $2 + 2t_2$, and he derives a strictly negative profit if he is the only active bidder at price $\hat{p}$, event which happens a strictly positive probability. We can then conclude that leaving the auction for a price equal to $2 + 2t_2$, conditional on Bidder 1 being active, is a better response to Bidder 1 and 3's strategies than leaving the auction for a price strictly higher than $2 + 2t_2$.

The argument is exactly the same for Bidder 3's strategy conditional on Bidder 1 being active. We only need to replace $2 + 2t_2$ by $2 + \frac{4t_3}{3}$, "Bidder 2" by "Bidder 3" and "Bidder 3" by "Bidder 2".

Q.E.D.

We now show that the introduction of a reserve price increases revenue even though the price is never equal to the reserve price. The reserve price chosen is such that at least two bidders always participate in the auction and always have a value strictly higher than the reserve price.

PROPOSITION 2 *With a reserve price $R = 1$, in any equilibrium of the game, Bidder 1 participates in the auction and then stays active up to his valuation for the good, if $t_1 \geq \frac{1}{4}$; and does not participate, otherwise. Bidder 2 always participates in the auction. If Bidder 1 participates in the auction, Bidder 2 follows the same behavior as the one described in Proposition 1. If Bidder 1 does not participate, Bidder 2 stays active up to $\frac{5}{4} + t_2$. Bidder 3 always participates in the auction. If Bidder 1 participates in the auction, Bidder 3 follows the same behavior as the one described in Proposition 1. If Bidder 1 does not participate, Bidder 3 stays active up to $\frac{13}{8} + t_3$.*

**Proof**: Bidder 1's strategy is standard. He participates when his valuation is higher than the reserve price and stays active up to his valuation for the good. Bidders 2 and 3 participation decisions are also trivial as their valuations are always higher than the reserve price. If Bidder 1 participates, Bidders 2 and 3 strategies are optimal for the same reason as in the proof of Proposition 1. If Bidder 1 does not participate in the auction, they stay active up to their expected valuations conditional on Bidder 1's not participating in the auction, which is optimal for the same reason that bidding up to the own valuation

[3]See supra.

is under the private value case.

Q.E.D.

The auction is no longer efficient because the use of a reserve price prevents bidders 2 and 3 from inferring the value of $t_1$, which is a necessary information to achieve an efficient allocation. To verify this, note that when $t_1$ is low (below $\frac{1}{8}$, the expected value of $t_1$ conditional on Bidder 1's non participation), Bidder 2 may win the auction even though $v_2 < v_3$ and when $t_1$ is higher (in the interval $(\frac{1}{8}, \frac{1}{4})$), Bidder 3 may win the auction even though $v_3 < v_2$.

PROPOSITION 3 *The expected revenue of the auction is strictly higher with a reserve price 1 than without reserve price.*

**Proof**: If $t_1 \geq 1/4$, the outcome of the auction is the same with a reserve price 1 or without reserve price. Therefore, we only need to evaluate what happens when $t_1 < 1/4$.

Without reserve price, the expected revenue of the auction, conditional on $t_1 < 1/4$ is:

$$\begin{aligned} V_N = {} & 4\int_0^{\frac{1}{4}}\int_{\frac{1}{2}-t_1}^{1}\int_0^{t_1+t_2-\frac{1}{2}}\left(\frac{3}{2}+t_1+t_3\right)dF(t_3)dF(t_2)dF(t_1) \\ & + 4\int_0^{\frac{1}{4}}\int_0^{\frac{1}{2}+t_1}\int_0^{\frac{1}{2}-t_1+t_3}\left(1+2t_1+t_2\right)dF(t_2)dF(t_3)dF(t_1) \\ & + 4\int_0^{\frac{1}{4}}\int_{\frac{1}{2}+t_1}^{1}\int_0^{1}\left(1+2t_1+t_2\right)dF(t_2)dF(t_3)dF(t_1) \end{aligned}$$

With a reserve price 1, the expected revenue of the auction, conditional on $t_1 < 1/4$ is:

$$V_R = \int_0^{\frac{5}{8}}(\frac{13}{8}+t_3)(\frac{5}{8}-t_3)dF(t_3) + \int_0^{\frac{3}{8}}(\frac{5}{4}+t_2)dF(t_2) + \int_{\frac{3}{8}}^{1}(\frac{5}{4}+t_2)(\frac{11}{8}-t_2)dF(t_2)$$

Simple computations show that $V_N < V_R$ so that the expected revenue is strictly higher with the reserve price than without it.

Q.E.D.

For $R = 1$, both Bidders 2 and 3 participate in the auction and the price is always higher than $R$. Hence the positive effect of the reserve price on the expected price does not derive from the fact that the winner of the auction pays $R$ rather than the bid of an opponent. Here, the seller exploits the coarseness of the information when Bidder 1 does not take part in the auction - Bidder 2 and Bidder 3 only know that $t_1 \in [0, 1/4]$ - in order to raise his expected revenue. The intuition for the revenue effect is as follows. If the value realization of the signal $t_1$ is aggregated by Bidder 2 and Bidder 3, lower values of $t_1$ relatively advantage Bidder 3 and higher values advantage Bidder 2. Hence, the knowledge of $t_1$ increases the difference between the valuations of those two bidders. However, in a second price auction the seller prefers that the variation in values is ceteris paribus smaller rather than larger because she only gets the lowest of the two values. Hence, the benefit of using a reserve price derives from ensuring less variation in the expected values of Bidders 2 and 3.

## 3. CONCLUSION

We have shown, by means of a simple example, that a seller may strategically use a reserve price in order to manipulate the information revelation process during the auction in order to increase expected revenue. The information that is hidden because of the reserve price is the precise signal of the bidders deciding not to participate. Our result provides additional support for the use of reserve prices.

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