Tullock Contest with Desert Concerns

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Abstract

We study the Tullock contest model with desert concerns (Gill and Stone (2010)). In a contest with n possibly heterogeneous players and convex effort costs, we establish the conditions necessary for a unique Nash equilibrium in pure strategies. Subsequently, we analyze the impact of desert concerns on players' spending behavior, probability of winning, and rent dissipation.

JEL Codes: D31, D72, D91

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1 Introduction

This paper studies the winner-takes-all Tullock (1967) contest with desert concerns first presented in Gill and Stone (2010). This principle suggests that, in a competitive setting, competitors aspire to gain what they deem fair based on their efforts relative to others. Notably, when all participants exert positive effort, the contest's probabilistic nature can result in winners (losers) receiving more (less) than what they believe they deserve. Consequently, the rewards for effort-motivated competitors are influenced by their perception of entitlement and monetary rewards. Within this structure, we explore how fairness concerns affect player behaviors and the dissipation of resources.

We study a contest involving n players who may differ in their productivities and desert concerns, with the desert payoff potentially having a more significant impact on the loss or gain domain. Specifically, in the former, undeserved losses cause more pain than undeserved gains. In the latter, undeserved benefits are more beneficial than undeserved losses.[1] Within this framework, we establish conditions for the uniqueness of equilibrium in pure strategies and conduct a comparative static analysis on players' spending behavior, probability of winning, and rent dissipation.

Our analysis yields the following findings. In a contest with a large number of symmetric players ($n \to \infty$), if the desert payoffs are more sensitive to losses (gains), the total expenditures are lower than (exceeds) the prize value. This occurs because the aversion to undeserved losses prompts players to reduce their effort. In contrast, in a contest with heterogeneous players, the aversion to undeserved losses can decrease or increase the players' effort. If an agent is dominant (meaning her chances of winning are higher than 1/2) and her desert payoff is more sensitive to losses, she will exert greater effort than in the standard Tullock contest. This agent can afford to exert extra effort, which reduces her chances of experiencing an undeserved loss. However, a non-dominant player (with a probability of winning below 1/2) will exert less effort than in the standard Tullock contest to lower her chances of experiencing undeserved losses. The opposite is true if the desert payoff is more sensitive to gains. Finally, we show that individual expenditure differs among agents in two players' contests with heterogeneity in ability. This result contrasts with the standard model, where heterogeneous players expend the same resources in equilibrium and nicely fits the recent experimental evidence on contests with heterogeneous agents

[1] For a comparison with inequity aversion and psychological game theory, refer to Gill and Stone (2010).

(Kimbrough et al., 2014; Fallucchi et al., 2021).

Our paper contributes to the literature on the interactions between reference-dependent preferences and strategic choices in competitive settings. To our knowledge, Gill and Stone (2010) and Dato et al. (2018) are the first to investigate the topic employing a two-player contest model à la Lazear and Rosen (1981). They primarily focus on the equilibrium fundamentals of a game in which players are loss averse, measured by $\lambda > 0$, around their meritocratically determined reference points.[2] Recently, Fu et al. (2022) included *moderate* and *symmetric* concerns for desert, $\lambda \in [0, \frac{1}{3}]$, into the Tullock contest with linear cost of efforts.[3]

Fu et al. (2022) also highlighted the significance of exploring competition among contestants who differ in their levels of loss aversion. The assumption of loss aversion, $\lambda > 0$, implies that undeserved losses always hurt more than the benefits of undeserved gains. Although this assumption seems plausible, the opposite may occur (i.e. $\lambda < 0$). Thus, we complement Fu et al. (2022) by studying Tullock contests in which players can be heterogenous in their productivities and reference-dependent preferences. Specifically, we provide conditions for the uniqueness of equilibrium for a larger, and possibly heterogenous, degree of loss aversion across players, $\lambda_i \in (-1, 1)$ allowing for convex effort costs. Finally, relaxing the assumption of symmetric preferences across agents, we can account for the impact of different degrees of desert concerns on players' spending behavior, probability of winning, and rent dissipation.

The remainder of the paper is organized as follows. Section 2 introduces the model; Section 3 provides the conditions for the uniqueness of equilibrium; Section 4 provides the comparative statics analysis; Section 5 concludes.

2 Preliminaries

There are n players participating in a Tullock contest, denoted by $i = 1, 2, ..n$. The winner of the contest receives a monetary prize normalized to 1, whereas the losers receive nothing. To win the contest, players exert an effort level denoted by x_i, at a

[2]See Gill and Stone (2015) for an extension to a cooperative setting in which payoffs are deterministic, and Daido and Murooka (2016) for applications to team incentives.

[3]Specifically, they study a multi-player lottery contest in which agents exhibit symmetric reference-dependent loss aversion à la Kőszegi and Rabin (2006, 2007) They provide conditions for the uniqueness of pure-strategy choice-acclimating personal Nash equilibrium, which corresponds to the Desert equilibrium in Gill and Stone (2010).

cost $\frac{x_i^r}{v_i}$, where $r \geq 1$ and $v_i > 0$ represents the player's productivity parameter. The probability of player i winning the contest is $\sigma_i = \frac{x_i}{X}$, where X is the sum of all the players' efforts.

Following the approach of Gill and Stone (2010), we incorporate players' desert concerns by assuming that they not only care about their own monetary payoff, but also about the comparison of their payoff with an endogenous reference point $r_i(x_i, x_j) = \sigma_i = \frac{x_i}{X}$. This reference point represents the monetary amount that players feel they deserve, given the efforts chosen by all competitors. Moreover, players share a common notion of fairness and agree on what each deserves to win, i.e., $\sum_i^n r_i(x_i, x_j) = 1$.

Overall, the player i's utility is assumed to be separable in money, desert concern, and cost of effort, and it is given by

$$U_i^W = 1 + g_i(1 - \sigma_i) - \frac{x_i^r}{v_i},$$

if she wins, and

$$U_i^L = 0 + l_i(0 - \sigma_i) - \frac{x_i^r}{v_i},$$

if she loses.

It is important to note that in a winner-takes-all contest, unless all players except one exert zero effort, the winner always receives more than what she deserves, while the losers receive less than what they deserve.[4] Specifically, $g_i(1 - \sigma_i)$ represents player i's desert payoff for the undeserved gains, while $l_i(0 - \sigma_i)$ represents the desert payoff for the undeserved losses.

We introduce a reasonable assumption regarding the slopes of the desert payoff: $0 \leq g_i < 1$ and $0 \leq l_i < 1$. In other words, when a player experiences an undeserved gain or loss, their desert payoff cannot exceed the monetary payoff associated with that gain or loss.[5]

Overall, the player i's expected utility is

$$EU_i = [\sigma_i - \lambda_i \sigma_i (1 - \sigma_i)] - \frac{x_i^r}{v_i}, \tag{1}$$

[4] At least two players are active in equilibrium.

[5] For instance, if player i did not win and incurred a negative desert payoff, reimbursing her with the expected monetary prize that she deserved but did not receive would more than compensate for the negative desert payoff.

where $\lambda_i = l_i - g_i$ and $-1 < \lambda_i < 1$. The sign of λ_i depends on whether the desert payoff is steeper in the loss domain, as is consistent with Prospect Theory (Kahneman and Tversky, 2013), or in the gain domain.

Before we proceed, it is essential to highlight some significant properties of the expression $-\lambda_i \sigma_i (1 - \sigma_i)$, which we shall henceforth refer to as the *expected desert payoff.*

Lemma 1. *Agent i's expected desert payoff is given by $-\lambda_i \sigma_i (1 - \sigma_i)$. If $\lambda_i > 0$ (undeserved losses hurt more), it is strictly negative, convex in σ_i, with the minimum occurring at $\sigma_i = \frac{1}{2}$. Conversely, when $\lambda_i < 0$ (underserved benefits are more beneficial), the expected desert payoff is strictly positive, concave in σ_i, and its maximum occurs at $\sigma_i = \frac{1}{2}$.*

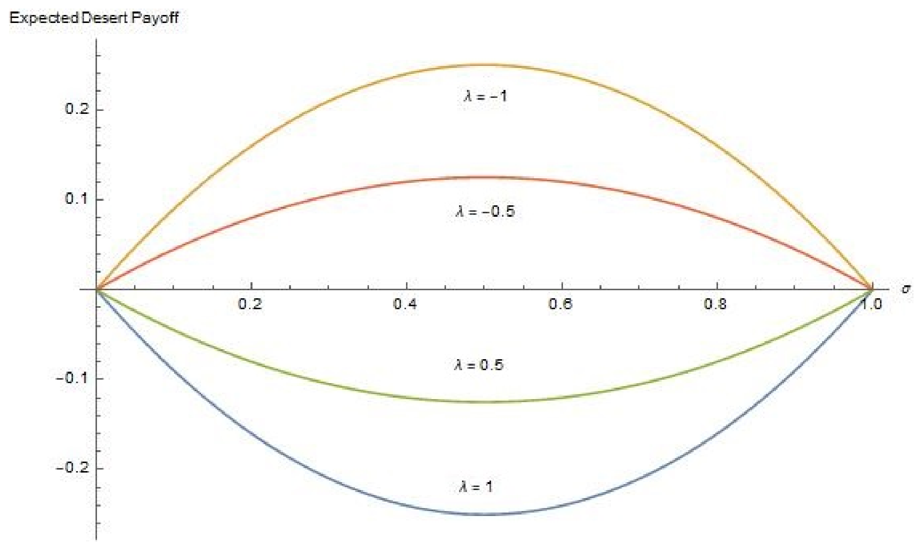

Figure 1: Expected desert payoff for negative and positive λ.

When a player's probability of winning equals zero (or one), her reference point corresponds to the actual outcome of the contest, and the expected desert payoff is equal to zero. However, as a player's chances of winning increase, the expected distance between the contest's outcome and her reference point also increases, resulting in the expected desert payoff affecting the player's utility. Intuitively, the extrema of the expected desert payoff occur at $\sigma_i = \frac{1}{2}$, as the possible outcomes of the contest are the farthest from the reference point. The sign of the expected desert payoff depends on whether undeserved losses hurt more than the benefits of undeserved gains. If $\lambda_i > 0$, the expected desert payoff is negative, whereas if $\lambda_i < 0$, it is positive. Figure 1 displays the expected desert payoff for different values of λ.

3 Equilibrium predictions

We restrict our attention to contests with a unique Nash equilibrium in pure strategies. Our approach consists of first establishing conditions for the quasi-concavity of the utility functions, followed by examining the necessary conditions for the equilibrium to be unique.

Player i's FOC is given by

$$\frac{(1-\sigma_i)}{X}(1+\lambda_i(2\sigma_i-1)) - \frac{r x_i^{r-1}}{v_i} \leq 0, \tag{2}$$

with equality holding if $x_i > 0$.[6]

Lemma 2. *Player i's utility is a quasi-concave function of x_i for any x_j if at least one of the following conditions holds: $-1 < \lambda_i \leq 0.5$, $r \geq 2$, and $r > \frac{(2-\sqrt{3}\lambda_i\sqrt{\frac{(1-\lambda_i^2)}{\lambda_i^2}})}{\lambda_i}$ when $0.5 < \lambda_i < 1$.*

To provide conditions for the uniqueness of the equilibrium, it proves convenient to divide both sides of (2) by $X^{r-1} > 0$.[7] The resulting equation implicitly defines $\sigma_i = \sigma(X, \lambda_i, v_i)$, and it is given by

$$\frac{(1-\sigma_i)}{X^r}(1+\lambda_i(2\sigma_i-1)) - \frac{r \sigma_i^{r-1}}{v_i} \leq 0. \tag{3}$$

The player's probability of winning, denoted by $s_i(X)$, is a function of the aggregate effort X. Specifically, $s_i(X)$ is defined as $max\{\sigma(X, \lambda_i, v_i), 0\}$, where $\sigma(X, \lambda_i, v_i)$ is the share function approach described in Cornes and Hartley (2005). It is important to note that effort and probability of winning must be non-negative.

From equation (3), we can observe that as X approaches infinity, $s_i(X)$ approaches zero when $r > 1$, and $s_i(X) = 0$ when $r = 1$. Conversely, as X approaches zero, $s_i(X)$ approaches one.

In equilibrium, the aggregate effort X must satisfy $\sum_i^n s_i(X) = 1$, which also provides the individual efforts through $x_i = \sigma_i X$. Finally, for low values of X, we have $\sum_i^n s_i(X) > 1$, whereas for high values of X, we have $\sum_i^n s_i(X) < 1$. As a result, when $\sum_i^n s_i(X)$ is strictly decreasing in X, the equilibrium is unique by the intermediate value theorem.

[6] Note that the equality always holds unless $r = 1$.

[7] In equilibrium at least two players exert a positive effort.

Proposition 1. *The contest has a unique Nash equilibrium in pure strategies if at least one of the following conditions holds for each contestant i:* $\lambda_i \leq 1/3$, $r \geq 2$, *and* $r > 2 - \sqrt{8\frac{(1-\lambda_i)\lambda_i}{(\lambda_i+1)^2}}$ *for* $1/3 < \lambda_i < 1$.

We proceed by assuming that at least one of the conditions outlined in Proposition 1, and hence those in Lemma 2, are satisfied.

4 Comparative Statics

In this section, we investigate how the inclusion of the desert payoff impacts players' behavior. While our primary focus is on the novel implications of the desert payoff, our analysis also reveals some behavioral regularities that align with those observed in Tullock contests without desert concerns. For example, if $r = 1$, our model predicts that at least two players will exert positive efforts in equilibrium, while contestants with lower ability may opt out of the contest. Conversely, if $r > 1$, all players are expected to exert a positive effort in equilibrium. Additionally, the higher a player's ability, the greater her probability of winning. Finally, when contestants are symmetric, they are all predicted to exert a positive effort and have an equal chance of winning ($\sigma_i = \frac{1}{n}\ \forall i$).

4.1 Rent-dissipation

Assuming that all n contestants are symmetric in ability and desert concern, i.e., $v_i = v$ and $\lambda_i = \lambda$ for all i, and that they exert the same effort in equilibrium, we can simplify the first-order condition to obtain

$$\frac{x^r}{v} = \frac{1}{r}(1-\sigma)\sigma(1+\lambda(2\sigma-1)), \tag{4}$$

where $\sigma = \frac{1}{n}$. As long as $n \geq 2$, the system of FOCs in (4) is satisfied, and by Proposition 1, the symmetric equilibrium is the unique pure strategy Nash equilibrium of the game.

Equation (4) allows us to express the equilibrium cost of effort, $\frac{x_i^r}{v}$, as a function of σ_i and λ:

$$c(\sigma, \lambda) = \frac{1}{r}(1-\sigma)\sigma(1+\lambda(2\sigma-1)). \tag{5}$$

When $\lambda = 0$ or $\sigma_i = \frac{1}{2}$, the cost of effort is equivalent to that of the standard Tullock contest, namely $c(\sigma, 0) = \frac{1}{r}(1-\sigma)\sigma$. By observing that together with $c(\sigma, \lambda)$ decreasing in λ, we can derive the following result.

Proposition 2. *For $n = 2$ symmetric contestants, the equilibrium cost of effort is $c(\frac{1}{2}, \lambda) = \frac{1}{4r}$ for all λ. For $n > 2$, the equilibrium cost of effort $c(\frac{1}{n}, \lambda)$ is strictly decreasing in λ. Moreover, we have the following:*

i) if $\lambda > 0$, $c(\frac{1}{n}, \lambda) < c(\frac{1}{n}, 0)$;

ii) if $\lambda < 0$, $c(\frac{1}{n}, \lambda) > c(\frac{1}{n}, 0)$.

Similarly to findings in Gill and Stone (2010), Proposition 2 reveals that in a symmetric two-player contest with desert concerns, the equilibrium efforts are the same as those in the standard Tullock contest. Additionally, Lemma 1 shows that the extrema of the expected desert payoff occur at $\sigma = 1/2$, indicating that its marginal effect is zero at that point. Therefore, the optimal effort is not affected by desert concerns, as the best response functions intersect at the same point, $\sigma = \frac{1}{2}$, for any value of λ (see Figure 2).

The second result in Proposition 2 is that the equilibrium cost of effort varies with λ and is influenced by the shape of the expected desert payoff, as discussed in Lemma 1. In particular, part i) of the proposition states that if $\lambda > 0$ (undeserved losses hurt more), the equilibrium effort is lower than that in the absence of desert concerns. This is because the unique equilibrium of the game is symmetric with $\sigma = \frac{1}{n} < \frac{1}{2}$, and the expected desert payoff decreases from $\sigma = 0$ to $\sigma = 1/2$, with a negative marginal effect that is more pronounced for higher values of λ. The same reasoning applies to part ii) of the proposition, where $\lambda < 0$.

We conclude this subsection by discussing the implications of desert concerns on rent dissipation.

Proposition 3. *In equilibrium, the proportion of rent dissipated is $nc(\sigma, \lambda) = \frac{1}{r}\frac{n-1}{n}(1 + \lambda(\frac{2-n}{n}))$. As n approaches infinity, this proportion tends to $\frac{1-\lambda}{r}$.*

The level of rent dissipation varies depending on the signs of λ and n. If $\lambda > 0$ (undeserved losses hurt more), the portion of rent dissipated is always less than or equal to one. On the other hand, if $\lambda < 0$ (undeserved gains are more beneficial), even a small number of contestants can cause the portion of rent dissipated to exceed one.

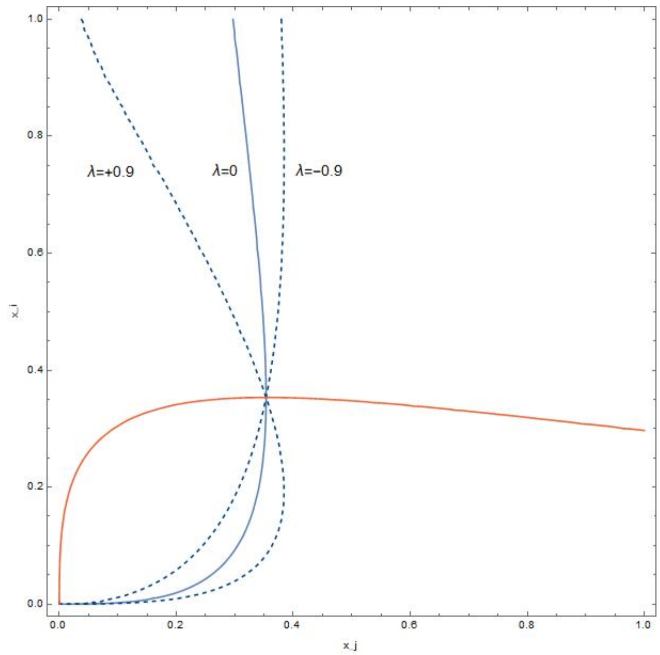

Figure 2: Best response functions for different λ in contest with $n = 2$ symmetric players.

4.2 Heterogeneous players

In the previous section, we demonstrated that in a symmetric contest, players exert the same effort in equilibrium, which decreases as λ increases. Since each player's probability of winning in a symmetric contest is $\frac{1}{n} \leq \frac{1}{2}$, desert concerns captured by λ affect all players equally. However, this result only provides a partial understanding of the effect of desert concerns on players' behavior.

To better understand the results of this section, we present a simple analogy based on non-strategic environments, although we must keep in mind that in our analysis, we move towards different equilibria. Specifically, suppose we have a group of players with heterogeneous abilities, and we ignore desert concerns for the moment. We refer to the player with a probability of winning $\sigma_D > \frac{1}{2}$, if any, as the "dominant player" D.

In the absence of any desert concern, players choose the amount of effort such that marginal costs equal marginal monetary gains. However, let us now introduce desert concerns for player D only. As illustrated in Figure 3, if $\lambda_D > (<)0$, the marginal effect of the desert for player D is positive (negative). Therefore, the dominant player will increase (decrease) their effort until the marginal cost equals the sum of the marginal gains, which include both the monetary and desert concerns. Consequently, when undeserved losses hurt more ($\lambda_D > 0$), the dominant player exerts more effort than in the absence of desert. The same reasoning applies to non-dominant players, with the only difference being that their marginal effect of the desert is negative (positive) if $\lambda_{i \neq D} > (<) 0$. These results are formally stated in the following proposition.

Proposition 4. *Consider a contest with n heterogeneous players. A change in desert concern such that $\lambda'_D > \lambda_D$ implies $\sigma'_D > \sigma_D$, $X^{**} > X^*$, and $x'_D = \sigma'_D X^{**} > x_D = \sigma_D X^*$. On the other hand, a change in desert concern such that $\lambda'_{i \neq D} > \lambda_{i \neq D}$ implies $\sigma'_{i \neq D} \leq \sigma_{i \neq D}$, $X^{**} \leq X^*$, $x'_{i \neq D} = \sigma'_{i \neq D} X^{**} \leq x_{i \neq D} = \sigma_{i \neq D} X^*$.*

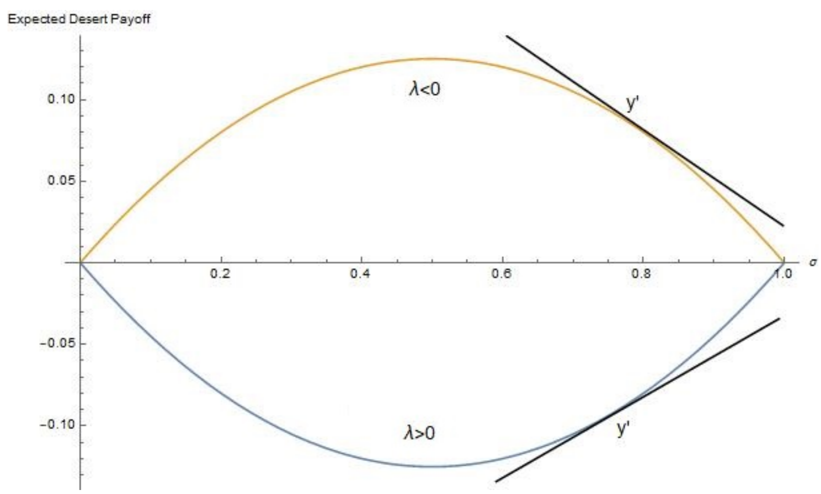

Figure 3: The marginal expected desert concern.

Note that if $\sigma_i = 0.5$, a change in λ_i does not affect player i's probability of winning. This means that, just like in the symmetric scenario, the marginal effect of desert concern at $\sigma_i = \frac{1}{2}$ is always zero, regardless of the value of λ_i. For example, in a contest between two players with equal abilities and different λ values, both players have equal chances of winning, with $\sigma_i = \sigma_j = 0.5$.

Proposition 4 provides an outline of how players' behavior is influenced by varying degrees of desert concerns. By utilizing these findings, we can directly infer the effect that a change in players' symmetric desert concerns has on their probability of winning.

Proposition 5. *Consider a contest with n players with heterogeneous abilities but* *symmetric λ. A change in desert concerns such that $\lambda' > \lambda$ implies $\sigma'_D - \sigma'_{i \neq D} > \sigma_D - \sigma_{i \neq D}$ $\forall i$.*

When players are heterogeneous only in terms of their abilities, desert concerns can either exacerbate or reduce the disparity in their probability of winning. As we have discussed earlier, the impact of an increase in λ varies depending on whether a player's probability of winning is higher or lower than $\frac{1}{2}$. For the dominant player, if any, an increase in λ positively affects her marginal desert concerns, leading to greater effort exertion. Conversely, for non-dominant players, the opposite effect occurs.

5 Applications: Two players contest

Suppose there are only two contestants with abilities $v_D > v_L = 1$, but they share the same λ value. We can express players' effort in Equation 2 in terms of their probability of winning, as shown in Equation 3. Since the FOCs conditions hold with equality in a contest between two players, we can take their ratio and use $\sigma_D = 1 - \sigma_L$ to obtain:

$$\frac{(1-\sigma_D)^r}{\sigma_D^r} \frac{[1 + \lambda(2\sigma_D - 1)]}{[1 + \lambda(1 - 2\sigma_D)]} = \frac{1}{v_D}. \tag{6}$$

Equation (6) implicitly defines the equilibrium probability of winning for the dominant player as a function of desert concern, where $\sigma_D(\lambda) > \frac{1}{2}$ because $v_D > 1$. As $\sigma_D(\lambda)$ strictly increases in λ, and $\sigma_D(0) = \frac{(1-\sigma_D)^r}{\sigma_D^r}$[8], we can directly infer the following corollary from Proposition 4.

Corollary 1. *In a contest with n=2, the probability of winning of the dominant player D is strictly increasing in λ. Furthermore, $\sigma_D(\lambda) > \sigma_D(0)$ for all $\lambda > 0$, and $\sigma_D(\lambda) < \sigma_D(0)$ for all $\lambda < 0$.*

5.1 Cost-ratio

Finally, we investigate whether there are any differences in the cost of effort for players when considering desert concerns. Player i's cost of effort is

$$\frac{x_i^r}{v_i} = \frac{1}{r}(1 - \sigma_i)\sigma_i(1 + \lambda(2\sigma_i - 1)). \tag{7}$$

[8] The left-hand side of Equation (6) is decreasing in σ_D and increasing in λ

We also introduce the cost-of-effort ratio, which compares the dominant player's cost of effort to that of the non-dominant player. The cost-of-effort ratio is

$$\hat{c}(\sigma_D, \lambda) = \frac{[1 + \lambda(2\sigma_D - 1)]}{[1 + \lambda(1 - 2\sigma_D)]}. \tag{8}$$

It is well known that in a Tullock contest without desert concerns ($\lambda = 0$), heterogenous players expend the same resources in equilibrium, i.e. $\hat{c}(\sigma_D, 0) = 1$ for all σ_D. However, this is not the case when taking into account desert concerns.

Proposition 6. *In a game with $n = 2$ players, the ratio cost of effort $\hat{c}(\sigma_D, \lambda)$ is strictly increasing (decreasing) in v_D if $\lambda > (<)0$. Furthermore, $\hat{c}(\sigma_D, \lambda) > \hat{c}(\sigma_D, 0) = 1$ for all $\lambda > 0$, and $\hat{c}(\sigma_D, \lambda) < \hat{c}(\sigma_D, 0) = 1$ for all $\lambda < 0$.*

Proof. Recall that $\sigma_D(v_D)' > 0$. Thus, $\hat{c}(\sigma_D, \lambda)' > 0$ if $\lambda 2(1 + \hat{c}(\sigma_D, \lambda)) > 0$. □

In other words, when undeserved losses hurt more (less) than undeserved gains, the dominant player expends more (less) effort than the non-dominant player. This result is in contrast to the standard Tullock contest, where both players spend the same amount of resources regardless of their abilities.

6 Discussion and Conclusions

Gill and Stone (2010) introduced fairness concerns in Lazear and Rosen (1981) tournaments between two loss-averse ($\lambda > 0$) players. Recently, Fu et al. (2022) introduced symmetric desert concerns into the Tullock contest. The authors show that under the assumption of linear costs the pure-strategy Nash equilibrium is unique under *moderate* concerns for desert, $\lambda \in [0, \frac{1}{3}]$. In this framework, allowing for convex costs of effort, we provide conditions under which the resulting equilibrium is unique regardless of whether contestants have different and less moderate concerns for fairness, $\lambda_i \in (-1, 1)$. The wider range of preferences allows us to provide new insightful results.

In a large contest between symmetric players, we show that rent-dissipation can either exceed or fall behind the value of the prize depending on players' fairness concerns. If undeserved losses hurt more than undeserved gains, players reduce their efforts compared to the standard case, and there is no full rent dissipation. On the other hand, if undeserved gains are more beneficial than undeserved losses, the marginal effect of desert concern is positive, and players increase their contributions. As a result, rent-dissipation exceeds the value of the prize. Furthermore, we extend our analysis to the contests with heterogeneous players and show that high-enough ability players increase (decrease) their contribution when undeserved losses hurt more (less) than the benefits of undeserved gains. Additionally, we provide conditions under which desert concerns can either exacerbate or reduce the probability of winning between the dominant player and other competitors. Finally, our analysis of a contest between heterogeneous agents leads to results that better resemble those from economics experiments, as early proposed by Fonseca (2009). In particular, we prove that in a contest between two players with heterogenous in ability, concerns for fairness lead to expenditures that are not symmetric. This result contrasts with the standard scenario in which contestants expend the same resources regardless of their abilities in equilibrium. If undeserved gains are more beneficial than undeserved losses, the advantaged players reduce their effort while the disadvantaged one increases it. In addition, the low-ability player expends more resources and has a higher probability of winning than in the standard Tullock contest. These results are in accordance with recent experimental evidence by Kimbrough et al. (2014) and Fallucchi et al. (2021) among others.

Our model seems to provide a more realistic prediction of contestants' behavior that considers "emotions" driven by the gap between the expected and the realized outcomes. A further step involves checking under what condition the total rent-seeking increases in contests with multi-prize structures (Fu et al., 2021) when agents have heterogeneous productivities and desert concerns.

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7 Appendix A

A.1 Proof of Lemma 2

Player i's expected utility is

$$EU_i = \sigma_i - \lambda_i\sigma_i(1-\sigma_i) - \frac{x_i^r}{v_i} \tag{9}$$

When $r = 1$, we have

$$EU_i' = \sigma_i' - \lambda_i\sigma_i'(1-\sigma_i) + \lambda_{ii}\sigma_i\sigma_i' - 1 \tag{10}$$

and

$$EU_i'' = \sigma_i'' - \lambda_i\sigma_i''(1-\sigma_i) + 2\lambda_i(\sigma_i')^2 + \lambda_i\sigma_i\sigma_i'', \tag{11}$$

where $\sigma_i'' = -2\frac{x_j}{X^3}$ and $\sigma_i' = \frac{x_j}{X^2}$. After some rearrangements, the SOC boils down to

$$1 - 2\lambda_i + 3\lambda_i\sigma_i > 0, \tag{12}$$

which is satisfied for any x_j iff $\lambda_i \leq \frac{1}{2}$.

When $r > 1$, the FOC is

$$\sigma_i' - \lambda_i\sigma_i'(1-\sigma_i) + \lambda_i\sigma_i\sigma_i' - r\frac{x_i^{r-1}}{v_i} = 0, \tag{13}$$

which can be rewritten as

$$\frac{(1-\sigma_i)}{X}(1 + \lambda_i(2\sigma_i - 1)) = r\frac{x_i^{r-1}}{v_i}. \tag{14}$$

Note that, as long as $-1 \leq \lambda_i < 1$ (and $r > 1$), $x_i > 0\ \forall x_j > 0$.

The SOC is

$$EU_i'' = \sigma_i'' - \lambda_i\sigma_i''(1-\sigma_i) + 2\lambda_i(\sigma_i')^2 + \lambda_i\sigma_i\sigma_i'' - r(1-r)\frac{x_i^{r-2}}{v_i} < 0, \tag{15}$$

where $\sigma_i'' = -2\frac{x_j}{X^3}$ and $\sigma_i' = \frac{x_j}{X^2}$. It can be written as

$$\sigma_i''(1 + \lambda_i(2\sigma_i - 1)) + 2\lambda_i(\sigma_i')^2 - r(1-r)\frac{x_i^{r-2}}{v_i} < 0, \tag{16}$$

It is easy to check that the SOC is negative whenever $\lambda_i < 0$.

Thus, the last step requires checking quasi-concavity when $\lambda_i > 0$. When the FOC holds, the SOC boils down to

$$-2\sigma_i - 6\lambda_i\sigma_i^2 + 4\lambda_i\sigma_i - (r-1) - (r-1)2\lambda_i\sigma_i + (r-1)\lambda_i < 0, \tag{17}$$

and it can be rewritten as

$$-6\lambda_i\sigma_i^2 - [(r-1)2\lambda_i + 2 - 4\lambda_i]\sigma_i - (r-1)[1-\lambda_i] < 0. \tag{18}$$

After some tedious calculations, it is strictly negative if at least one of the following holds: $r \geq 2$, $0 < \lambda_i \leq 0.5$, $r > \frac{(2-\sqrt{3}\lambda_i\sqrt{\frac{(1-\lambda_{i_i}^{2})}{\lambda_i^2}})}{\lambda_i}$ when $0.5 < \lambda_i < 1$.

A.2 Proof of Proposition 1

When $r = 1$, the FOC can be written as

$$\frac{(1-\sigma_i)(1+\lambda_i(2\sigma_i - 1))}{X} - \frac{1}{v_i} \leq 0. \tag{19}$$

Let $\sigma_i = \sigma(X, \lambda_i, v_i)$, then $\sigma'(X, \lambda_i, v_i) < 0$ if $\lambda_i \leq 1/3$.

When $r > 1$, $EU_i'(0) > 0$. As a result, all players exert a positive effort $x_i = \sigma_i X > 0$. This allows us to rewrite the FOC as

$$\frac{1}{X^r} = \frac{r\sigma_i^{r-1}}{(1-\sigma_i)(1+\lambda_i(2\sigma_i - 1))}\frac{1}{v_i}. \tag{20}$$

Let $\sigma_i = \sigma(X, \lambda_i, v_i)$, then $X \to \infty$, implies $\sigma(X, \lambda_i, v_i) \to 0$, and $X \to 0$, implies $\sigma(X, \lambda_i, v_i) \to 1$. Finally, if $\sigma(X, \lambda_i, v_i)$ is strictly decreasing in X, then the equilibrium is unique because by the intermediate value theorem, there is only one X^* such that $\sum_i^n \sigma(X^*, \lambda_i, v_i) = 1$.

We can check that $\sigma(X, \lambda_i, v_i)$ is decreasing in X by looking at the RHS of the FOC: if it is increasing in σ_i, then $\sigma(X, \lambda_i, v_i)$ is strictly decreasing in X. After some tedious calculations, the RHS is increasing in σ_i either when $\lambda_i \leq 1/3$ for any $r \geq 1$, $r \geq 2$ for any λ_i, or when $r > 2 - \sqrt{8\frac{(1-\lambda_i)\lambda_i}{(\lambda_i+1)^2}}$ for $\lambda_i > 1/3$.

A.3 Proof of Proposition 4

Here, we prove that player i's probability of winning and aggregate effort increase in λ_i iff $\sigma_i > \frac{1}{2}$. The same proofs can be used to show that player i's probability of winning and the aggregate effort decrease in λ_i iff $\sigma_i < \frac{1}{2}$.

Let contestant i be an active player in equilibrium, then (3) holds with equality and can be written as

$$\frac{1}{X^r} = \frac{r\sigma_i^{r-1}}{(1-\sigma_i)(1+\lambda_i(2\sigma_i - 1))}\frac{1}{v_i}, \tag{21}$$

where the numerator equals one if $r = 1$. In equilibrium, we have that $s_i(X^*, \lambda_i) = max\{\sigma(X^*, \lambda_i, v_i), 0\}$, and $\sum s_i(X^*, \lambda_i) = 1$. Suppose that the desert concern for player i changes to $\lambda_i' > \lambda_i$. If $\sigma_i > 0.5$, the RHS decreases in λ_i. Fixing X^*, in order for the equality to hold σ_i needs to increase. This implies that $s_i(X^*, \lambda_i') > s_i(X^*, \lambda_i)$. Clearly, X^* can not be the new equilibrium aggregate as $s_i(X^*, \lambda_i') + \sum s_j(X^*, \lambda_j) > 1$. Since $s(X^*, \lambda)$ is strictly decreasing in X, the new equilibrium aggregate X^{**} increases until it satisfies $s_i(X^{**}, \lambda_i') + \sum s_j(X^{**}, \lambda_j) = 1$, where $\sum s_j(X^{**}, \lambda_j) < \sum s_j(X^*, \lambda_j)$ and $s_i(X^{**}, \lambda_i') > s_i(X^*, \lambda_i)$.