Online Appendix for "Mother's time allocation, child care and child cognitive development"

Ylenia Brilli

Appendix A. Analytical solution of the model

This Appendix derives analytically the closed-form solutions of the model. The process of backward induction involves the solution of the optimization problem in each period, starting from the last one, T. We first find the optimal child care and time input decisions at time T. The value function of the mother at period T can be written as:

$$V_T = max_{\tau_T, i_T, f_T}\ \alpha_1 ln(TT - h_T - \tau_T) + \alpha_2 ln(w_T h_T + I_T - p_{it} i_T - p_{ft} f_T) + \alpha_3 ln(A_T) + \\ + E_T \beta \{\tilde{V}_{T+1} + \rho \alpha_3 ln A_{T+1}\} \tag{A.1}$$

where the variables l_T for leisure and c_T for consumption have been already substituted using the time and budget constraints, the CAPF has been log-linearized for computational convenience, and the braces include the terminal period value function, as specified in Equation (6) in the paper.

The maximization of the value function at time T gives the following First-Order Conditions (FOCs):

$$\tau_T^c \Rightarrow \beta \rho \alpha_3 \left(\frac{\delta_{1T}}{\tau_T}\right) = \frac{\alpha_1}{TT - h_T - \tau_T} \tag{A.2}$$

$$i_T^c \Rightarrow \beta \rho \alpha_3 \left(\frac{\delta_{2T}}{i_T}\right) = \frac{p_{iT} \alpha_2}{w_T h_T + I_T - p_{iT} i_T - p_{fT} f_T} \tag{A.3}$$

$$f_T^c \Rightarrow \beta \rho \alpha_3 \left(\frac{\delta_{3T}}{f_T}\right) = \frac{p_{fT} \alpha_2}{w_T h_T + I_T - p_{iT} i_T - p_{fT} f_T} \tag{A.4}$$

Notice that the FOCs have the general form:

$$\frac{\partial V_{T+1}}{\partial ln A_{T+1}} \times \frac{\partial ln A_{T+1}}{\partial j_T} = \bar{V}_T' \tag{A.5}$$

where $\bar{V}_T = \alpha_1 ln(TT - h_T - \tau_T) + \alpha_2 ln(w_T h_T + I_T - p_{iT} i_T - p_{fT} f_T) + \alpha_3 ln(A_T)$ is the current utility in period T, $j_T = \{\tau_T, i_T, f_T\}$ represent the investment decisions of the mother, and the term on the left-hand side of the FOCs represent the marginal change in future utility associated with a variation in inputs.

The term on the right-hand side of Equation (A.2) ($\bar{V}_T' = \frac{\alpha_1}{TT - h_T - \tau_T}$) is the mother's marginal utility from leisure and indicates the marginal cost of maternal child-care time. This expression shows that the cost of maternal time investment increases with the mother's preferences for leisure α_1 and with the mother's labor supply h_T. Given that the mother's labor supply is positively associated with the mother's wage (see Equation (12) in the paper), a higher wage induces a larger cost of time investments. For non-working mothers, for which $h_T = 0$, the cost of time investments becomes $\bar{V}_T'(l) = \frac{\alpha_1}{TT - \tau_T}$, that is, it only depends on the mother's preferences for leisure. Similarly, Equations (A.3) and (A.4) indicate that the marginal cost of using informal and formal child care depends on

the price of each service and on forgone consumption; working mothers, in this case, face a lower cost.

By solving the FOCs, we obtain the demands for the three inputs at period T, conditional on labor supply h_T. These are given by:

$$\tau_T^c = \frac{\beta\delta_{1T}D_{T+1}}{\alpha_1 + \beta\delta_{1T}D_{T+1}}(TT - h_T) \tag{A.6}$$

$$i_T^c = \frac{\beta\delta_{2T}D_{T+1}}{p_{iT}(\alpha_2 + \beta\delta_{2T}D_{T+1} + \beta\delta_{3T}D_{T+1})}(w_T h_T + I_T) \tag{A.7}$$

$$f_T^c = \frac{\beta\delta_{3T}D_{T+1}}{p_{fT}(\alpha_2 + \beta\delta_{2T}D_{T+1} + \beta\delta_{3T}D_{T+1})}(w_T h_T + I_T) \tag{A.8}$$

where $D_{T+1} = \frac{\partial V_{T+1}}{\partial lnA_{T+1}} = \rho\alpha_3$.

By substituting Equations (A.6), (A.7) and (A.8) into (A.1), we obtain the value function at period $(T-1)$. By using the same procedure described for period T, and by computing the corresponding FOCs, we get the solutions for period $(T-1)$. The solutions for all the periods up to period $t = 1$ can be retrieved similarly. At the end, three sequences of optimal choices can be obtained. The sequence of optimal choices for time with the child, conditional on the mother's labor supply, is given by:

$$\tau_T^c = \frac{\beta\delta_{1T}D_{T+1}}{(\alpha_1 + \beta\delta_{1T}D_{T+1})}(TT - h_T) \tag{A.9}$$

$$\tau_{T-1}^c = \frac{\beta\delta_{1T-1}D_T}{(\alpha_1 + \beta\delta_{1T-1}D_T)}(TT - h_{T-1}) \tag{A.10}$$

$$\tau_{T-2}^c = \frac{\beta\delta_{1T-2}D_{T-1}}{(\alpha_1 + \beta\delta_{1T-2}D_{T-1})}(TT - h_{T-2}) \tag{A.11}$$

$$\vdots$$

$$\tau_t^c = \frac{\beta\delta_{1t}D_{t+1}}{(\alpha_1 + \beta\delta_{1t}D_{t+1})}(TT - h_t) \tag{A.12}$$

$$\vdots$$

$$\tau_2^c = \frac{\beta\delta_{12}D_3}{(\alpha_1 + \beta\delta_{12}D_3)}(TT - h_2) \tag{A.13}$$

$$\tau_1^c = \frac{\beta\delta_{11}D_2}{(\alpha_1 + \beta\delta_{11}D_2)}(TT - h_1) \tag{A.14}$$

Equation (A.12) is equal to Equation (7) in the text.

The sequences of the optimal informal and formal child care choices, conditional on the mother's labor supply, are given by:

$$i_T^c = \frac{\beta\delta_{2T}D_{T+1}}{p_{it}(\alpha_2 + \beta\delta_{2T}D_{T+1} + \beta\delta_{3T}D_{T+1})}(w_T h_T + I_T) \tag{A.15}$$

$$i_{T-1}^c = \frac{\beta\delta_{2T-1}D_T}{p_{iT-1}(\alpha_2 + \beta\delta_{2T-1}D_T + \beta\delta_{3T-1}D_T}(w_{T-1} h_{T-1} + I_{T-1}) \tag{A.16}$$

$$i_{T-2}^c = \frac{\beta\delta_{2T-2}D_{T-1}}{p_{iT-1}(\alpha_2 + \beta\delta_{2T-2}D_{T-1} + \beta\delta_{3T-2}D_{T-1})}(w_{T-2} h_{T-2} + I_{T-2}) \tag{A.17}$$

$$\vdots$$

$$i_t^c = \frac{\beta\delta_{2t}D_{t+1}}{p_{it}(\alpha_2 + \beta\delta_{2t}D_{t+1} + \beta\delta_{3t}D_{t+1})}(w_t h_t + I_t) \tag{A.18}$$

$$\vdots$$

$$i_2^c = \frac{\beta\delta_{22}D_3}{p_{i2}(\alpha_2 + \beta\delta_{22}D_3 + \beta\delta_{32}D_3)}(w_2 h_2 + I_2) \tag{A.19}$$

$$i_1^c = \frac{\beta\delta_{21}D_2}{p_{i1}(\alpha_2 + \beta\delta_{21}D_2 + \beta\delta_{31}D_2)}(w_1 h_1 + I_1) \tag{A.20}$$

$$f_T^c = \frac{\beta\delta_{3T}D_{T+1}}{p_{fT}(\alpha_2 + \beta\delta_{2T}D_{T+1} + \beta\delta_{3T}D_{T+1})}(w_T h_T + I_T) \tag{A.21}$$

$$f_{T-1}^c = \frac{\beta\delta_{23T-1}D_T}{p_{fT-1}(\alpha_2 + \beta\delta_{2T-1}D_T + \beta\delta_{3T-1}D_T}(w_{T-1} h_{T-1} + I_{T-1}) \tag{A.22}$$

$$f_{T-2}^c = \frac{\beta\delta_{3T-2}D_{T-1}}{p_{fT-2}(\alpha_2 + \beta\delta_{2T-2}D_{T-1} + \beta\delta_{3T-2}D_{T-1})}(w_{T-2} h_{T-2} + I_{T-2}) \tag{A.23}$$

$$\vdots$$

$$f_t^c = \frac{\beta\delta_{3t}D_{t+1}}{p_{ft}(\alpha_2 + \beta\delta_{2t}D_{t+1} + \beta\delta_{3t}D_{t+1})}(w_t h_t + I_t) \tag{A.24}$$

$$\vdots$$

$$f_2^c = \frac{\beta\delta_{32}D_3}{p_{f2}(\alpha_2 + \beta\delta_{22}D_3 + \beta\delta_{32}D_3)}(w_2 h_2 + I_2) \tag{A.25}$$

$$f_1^c = \frac{\beta\delta_{31}D_2}{p_{f1}(\alpha_2 + \beta\delta_{21}D_2 + \beta\delta_{31}D_2)}(w_1 h_1 + I_1) \tag{A.26}$$

Equation (A.18) is equal to Equation (8) in the main text, while Equation (A.24) corresponds to Equation (9) in the text. The sequence of values for D_{t+1} is reported in (10) in the paper.

Having found the solutions for the time allocation and non-parental child care decisions, the solution for the mother's labor supply can be computed using the same backward procedure. Equation (11) represents the optimal labor supply in each period as a function of τ_t, i_t, and f_t; substituting (7), (8) and (9) into (11) yields the optimal labor supply choice for each period t, as defined by (12) in the paper.

Appendix B. The PSID data and the CDS-TD supplements

The dataset used in this paper is composed of different supplements of the Panel Study of Income Dynamics (PSID) gathered in the period 1985-2007. Table B.1 summarizes the main information on availability and sources of data. To merge PSID and CDS data we exploit the information on the relationship of each CDS child with respect to the head of the household and the primary caregiver. The final sample is made up of all children aged 0-12 in 1997 without siblings and with both parents living in the household, without missing information on child's and parents' characteristics and with at least one test score observation. As summarized in Table B.2, children in this sample are born between 1984 and 1996, and the terminal period of the model ($T = 13$) corresponds to 1997 for those born in 1984 and to 2009 for those born in 1996. Table B.3 summarizes the available data for a child born in 1996. This table stresses the existence of a long time-gap of missing data because of the structure of the surveys and the timing of the interviews. In particular, data on maternal time, child's cognitive outcomes, and non-parental child care after kindergarten age are available only in the years of the TD and CDS supplements, i.e., 1997, 2002 and 2007.

Table B.4 shows the average characteristics of the sample used for the estimation ($N = 417$) and of the total sample of children in CDS, for whom it has been possible to derive information on their parents (3243 observations); this comparison sample includes both families with only one child and families with more children. Table B.5 reports the amount of time spent by children in the final sample in different categories of activity, by distinguishing between mothers with at least some college education (*high educated*) and mothers without a college education (*low educated*).

Appendix C. Estimation

The estimation is done in two stages: the parameters of the income process are estimated in the first stage, while all remaining parameters are estimated in the second stage. After computing the statistics defined in Table C.1 for the actual data, we proceed with the first-stage estimation of the income parameters. This involves the simulation of the income process, after drawing from a standard normal distribution $N \times R$ times, for every period, with $N = 417$ and $R = 5$. The statistics used to estimate these parameters are the average and standard deviation of income for all the periods, and the average other household income by a father's level of education, race and age. We compute these statistics for both the actual and the simulated income processes. The Method of Simulated Moments estimator for this first stage minimizes an objective function where each moment condition is the distance between the income data moments and their simulated counterparts. Each moment condition is weighted using the inverse of the corresponding statistics in the data.

The second stage involves the estimation of all remaining parameters using the same estimator. We simulate the data according to the data-generating process implied by the model, taking $N \times R \times T$ draws for wage, child-care prices, and income and $N \times R$ draws for the child's initial ability shock, the mother's skills, and the mother's preferences, with $N = 417$, $R = 5$ and $T = 13$. Following Keane and Moffitt (1998), we re-draw the errors

TABLE B.1
Availability and sources of data

Set of Variables	**Source**	**Survey Years**	**Additional Info**
Formal and informal child care	CDS	1997-2002-2007	Retrospective questions on the most used arrangements from birth until kindergarten and questions on the most used arrangements at the time of the survey
Child cognitive outcomes	CDS	1997-2002-2007	Only for children older than 3
Child demographic characteristics	CDS	1997-2002	Time-invariant (except *age*)
Maternal time with the child	CDS-TD	1997-2002	Available only for the year of the survey
Parents' hours of work	PSID	1985, 1986, 1987, 1988, 1989, 1990, 1991, 1992, 1993, 1994, 1995, 1996, 1997, 1999, 2001, 2003, 2005, 2007	Referred to the year before the survey
Parents' wages	PSID	1985, 1986, 1987, 1988, 1989, 1990, 1991, 1992, 1993, 1994, 1995, 1996, 1997, 1999, 2001, 2003, 2005, 2007	Referred to the year before the survey
Parents' non-labor income	PSID	1985, 1986, 1987, 1988, 1989, 1990, 1991, 1992, 1993, 1994, 1995, 1996, 1997, 1999, 2001, 2003, 2005, 2007	Referred to the year before the survey
Parents' demographic characteristics	PSID	1997	Time-invariant (except *age*)

to simulate the income distribution using the parameters estimated in the first stage. In each period, the values for the mother's labor supply, formal and informal child care and maternal time are derived using the optimal solutions implied by the model. Then, after having simulated the data for all the periods, we compute the statistics defined in Table C.1 from the simulated data.

The estimator used in this second stage minimizes an objective function where each moment condition is the distance between the data statistics and the simulated counterparts:

$$\hat{\theta} = arg\ min\ \hat{g}(\theta)' W \hat{g}(\theta) \tag{C.1}$$

where

$$\hat{g}(\theta) = \hat{m} - \hat{M}(\theta)$$

$\hat{m}$ is the vector of statistics defined from the actual data, while $\hat{M}(\theta)$ is the vector of simulated statistics according to the model that are functions of the structural parameters

TABLE B.2
Cohorts of children in the final sample

Year of Birth	Child's Age					
$t=0$	$t=1$	$t=2$	$t=3$	$\cdots$	$t=12=T-1$	$t=13=T$
1984	1985	1986	1987	$\cdots$	1996	1997
1985	1986	1987	1988	$\cdots$	1997	1998
1986	1987	1988	1989	$\cdots$	1998	1999
1987	1988	1989	1990	$\cdots$	1999	2000
1988	1989	1990	1991	$\cdots$	2000	2001
1989	1990	1991	1992	$\cdots$	2001	2002
1990	1991	1992	1993	$\cdots$	2002	2003
1991	1992	1993	1994	$\cdots$	2003	2004
1992	1993	1994	1995	$\cdots$	2004	2005
1993	1994	1995	1996	$\cdots$	2005	2006
1994	1995	1996	1997	$\cdots$	2006	2007
1995	1996	1997	1998	$\cdots$	2007	2008
1996	1997	1998	1999	$\cdots$	2008	2009

TABLE B.3
Available data for a child born in 1996

	Child's age (t)													Source	Survey Year
	1	2	3	4	5	6	7	8	9	10	11	12	13		
Non-parental child care	X	X	X	X	X		X					X		CDS	1997, 2002, 2007
Child cognitive outcomes						X					X			CDS	2002, 2007
Child demographic charact.	X					X					X			CDS	1997, 2002, 2007
Maternal time with the child	X					X								TD	1997, 2002
Parents' hours of work		X		X		X		X		X				PSID	1999, 2001, 2003, 2005, 2007
Parents' wages		X		X		X		X		X				PSID	1999, 2001, 2003, 2005, 2007
Parents' demographic charact.	X		X		X		X		X					PSID	1997, 1999, 2001, 2003, 2005, 2007

to be estimated (vector θ). W is a positive definite diagonal weighting matrix. The most efficient minimum distance estimator uses a weighting matrix whose elements are estimates of the inverse of the covariance matrix of the vector $\hat{m}$; this is the so-called optimal minimum distance (OMD) estimator (Cameron and Trivedi 2005, pag. 203). Since Altonji and Segal (1996) provide evidence of small sample biases in the OMD estimator, we use the diagonally weighted minimum distance estimator proposed by Blundell, Pistaferri, and Preston (2008). Given S number of moments, the weighting matrix is then defined as:

$$W = \begin{pmatrix} \hat{V}[\hat{m}_1]^{-1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \hat{V}[\hat{m}_S]^{-1} \end{pmatrix}$$

where $\hat{V}[\hat{m}]$ is estimated with non-parametric bootstrap and according to the formula (Davidson and MacKinnon 2003, p. 208):

$$\hat{V}\left[\hat{m}\right] = \left[\frac{1}{B}\right] \sum_{b=1}^{B} \left(\hat{m}_b^* - \bar{m}^*\right) \left(\hat{m}_b^* - \bar{m}^*\right)' \tag{C.2}$$

TABLE B.4
Mean characteristics of the sample with respect to PSID-CDS data

	PSID-CDS	Sample	T-test
Mother's hours of work	23.61	27.30	-10.71^{***}
Mother's time with the child	25.83	21.16	5.42^{***}
Formal child care	8.14	10.26	-6.99^{***}
Informal child care	4.94	5.84	-3.48^{***}
Mother's wage before child's birth	11.01	11.31	-1.25
Other household income	674.16	791.36	-7.56^{***}
Mother's education	12.99	13.27	-7.03^{***}
Mother's age at child's birth	26.99	28.20	-14.43^{***}
Mother's race: white	0.61	0.61	0.33
Child's gender: male	0.51	0.51	0.29
Child's birth weight	3315.53	3387.16	-7.77^{***}

[a] Monetary variables deflated into 1997 US$.

[b] Mother's wage before childbirth refers to the year before the child was born.

[***] Difference statistically significant at the $p < 0.01$ level.

NOTES: *PSID-CDS* refers to children in 1997 CDS for whom it was possible to retrieve information on the parents from the main PSID survey ($N = 3243$); *Sample* includes all children aged 0-12 in 1997 without siblings and with both parents living in the household, without missing information on child's and parents' characteristics and with at least one test score observation ($N = 417$).

TABLE B.5
Activities performed by the child with the mother, by a mother's level of education

	Low Educated	High educated	T-test
Household activities	0.76	0.79	-0.21
Care of other children	0.02	0.02	-0.26
Activities to obtain goods and services	1.94	1.71	0.76
Personal care	0.99	1.4	-1.79
Help and care to others	0.06	0.07	-0.41
Socializing activities	1.23	1.08	0.54
Computer-related activities	0.24	0.25	-0.13
Educational activities	1.84	1.99	-0.45
Sport and outdoor activities	0.99	0.75	1.17
Leisure: radio, TV, music	4.88	3.24	3.42***
Leisure: reading, being read to	0.38	0.67	-2.49***
Others (Eating, Sleeping, Traveling)	8.34	8.65	-0.39

NOTES: The table reports weekly hours spent by the child with the mother in each category of activities. The category *Household activities* include any activities performed at home, e.g. preparing meals, cleaning, gardening; *Care of other children* refers to child-care activities performed to other children; *Activities to obtain goods and services* includes any activity performed to obtain a good or a service, such as shopping at the grocery store; *Personal care* refers to the personal care of the child (washing hairs, taking a bath, dressing, etc); *Help and care to others* refers to any activity performed by the child with the mother to help or take care of other adult people; *Socializing activities* includes both the participation in groups or organizations, or the attendance to entertaining events; *Computer-related activities* refers to any activity performed with a personal computer; *Educational activities* include structured learning activities, such as doing homework; *Sport and outdoor activities* includes any sport or outdoor activity; *Leisure: radio, TV, music* refers to passive leisure time, e.g., listening to the radio or watching TV; *Leisure: reading, being read to* refers to leisure reading activities, either active or passive; the residual category *Others* mainly refers to eating, sleeping and traveling. A mother's level of education is defined as *high* if she has more than 12 years of education. *** indicates that the difference between the two subsamples is statistically significant at the $p < 0.01$ level. Source: own elaboration from Time Diary-CDS data.

TABLE C.1
Statistics of actual and simulated data used for the estimation of the model

Mother's choices
Mean mother's hours of work, formal and informal child care and mother's time with the child by child's age
Std dev mother's hours of work, formal and informal child care and mother's time with the child by child's age
Proportion of mothers not working by child's age
Test scores
Mean test scores by child's age
Std deviation test scores by child's age
Correlation between mother's choices and exogenous variables
Corr mother's wage and mother's hours of work
Corr other household income and mother's hours of work
Corr mother's wage and mother's time with the child
Corr other household income and mother's time with the child
Corr mother's wage and formal child-care time
Corr other household income and formal child-care time
Corr mother's wage and informal child-care time
Corr other household income and informal child-care time
Correlation between mother's choices
Corr mother's hours of work and mother's time with the child
Corr mother's hours of work and formal child-care time
Corr mother's hours of work and informal child-care time
Productivity of inputs
Coefficient of mother's time with the child in $t-5$ in a OLS regression on test score in t, conditional on a dummy for LW
Coefficient of formal child care in $t-1$ in a OLS regression on test score in t, conditional on a dummy for LW
Coefficient of informal child care in $t-1$ in a OLS regression on test score in t, conditional on a dummy for LW
Coefficient of test score in $t-5$ in a OLS regression of test score in t on a dummy for LW and test score in $t-5$
Mother's education in the productivity of a mother's time with the child
Coefficient of a dummy for having a high-educated mother on a child's test score, conditional on child's age fixed effects, a dummy for LW and a mother's wage
Coefficient of a dummy for having a high-educated mother on mother's time with the child, conditional on child's age fixed effects and a mother's wage
Coefficient of a dummy for having a high-educated mother on mother's hours of work, conditional on child's age fixed effects and a mother's wage
Child's initial ability and test score specification
Variance of residuals from a child's test score OLS reg on a dummy for LW and child's age fixed effects
Average residuals from a child's test score OLS reg on a dummy for LW and child's age fixed effects by birth weight, gender and mother's age at birth
OLS regression of test score on a dummy for LW (coefficient)
Wage equation and other household income
Mean and std deviation of mother's wage
Average mother's wage by mother's level of education, race, age
OLS regression of log wage on a mother's cohort, area of residence and their interaction (coefficients)
Mean and std deviation of other household income
Average other household income by father's level of education, race and age
Price of formal and informal child care
Mean and std deviation of the price of formal child care
Mean and std deviation of the price of informal child care
OLS regression of formal child care price on the amount of state funding for pre-kindergarten
OLS regression of informal child care price on the number of family members present in the neighborhood
IV regression of formal child care hours on the price of formal child care, instrumented by the state funding for kindergarten
Mother's unobserved productivity and preferences
Variance of the residuals from a mother's wage OLS reg on mother's education, age, race, cohort, area of residence and their interaction
OLS reg of residuals from a mother's wage OLS reg on edu, age, race, cohort, area of residence and their interaction in t, on the residuals in $t-1$ (coefficient)
Variance of the residuals from a mother's time with the child OLS reg on child's age, mother's wage and other hh income
Variance of the residuals from a formal child care OLS reg on child's age, mother's wage and other hh income
Variance of the residuals from a informal child care OLS reg on child's age, mother's wage and other hh income
Variance of the residuals from a mother's hours of work OLS reg on child's age, mother's wage and other hh income
10th, 50th and 90th percentiles of a mother's hours of work, and a mother's time with the child
Corr between the residuals from a mother's wage OLS reg on mother's charact. with time with the child, formal and informal child care
Score transition probabilities
Prop of children with score in range p_y in years 1997 or 2002 and p_{y+5} in years 2002 or 2007

NOTES: These statistics are computed using PSID-CDS data on children aged 0-12 in 1997 without siblings, and simulated data according to the model defined in Section 3. Mother's time with the child is measured in 1997 and 2002; child's test scores are measured in 1997, 2002 and 2007, and refer to both the LW and the AP scores; from 1997 on, mother's hours of work, mother's wage and other household income are measured every two years and these variables refer to the year before the survey (see Section 4 and Appendix B for a description of the data). Child's age t ranges from 1 to 13. Ranges p_y, with $y = 1997, 2002, 2007$ are defined according to the following ranges of the score distribution: $1st - 25th$ perc, $25th - 50th$ perc, $50th - 75th$ perc, higher than $75th$ perc.

Non-parametric bootstrap (with replacement) is implemented following Wooldridge (2002, p. 379): we use a random number generator to obtain N integers, where $N = 417$ represents the sample size of the actual data, and these integers index the observations drawn from the actual distribution of data. Repeating this process B times, it yields B bootstrap samples on which the statistics defined in Table C.1 can be computed: $\hat{m}_b^*$

represents a statistic computed for the sample b, while $\bar{m}^*$ is the average of the statistics across the B samples.[1]

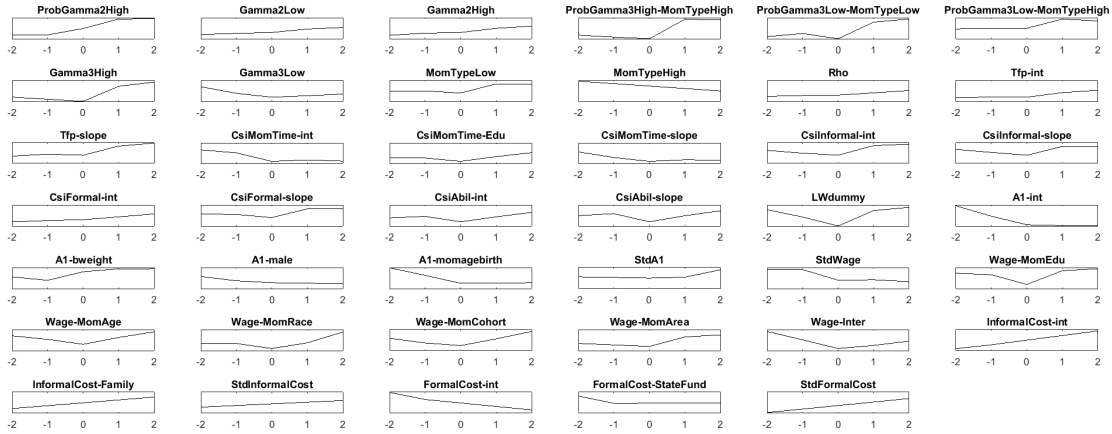
C.1. ***Standard errors.*** Non-parametric bootstrap with replacement is also used to compute the standard errors. After having drawn B_{se} samples from the actual data, we repeat the estimation of the parameters for each sample, by using different starting values for each bootstrap iteration.[2] This yields an empirical distribution of the parameters estimates, from which we can recover a bootstrap estimate of the variance, using the formula (Train 2009, pag. 201):

$$\hat{V}\left[\hat{\theta}\right] = \left[\frac{1}{B}\right] \sum_{b=1}^{B} \left(\hat{\theta}_b^* - \bar{\theta}^*\right) \left(\hat{\theta}_b^* - \bar{\theta}^*\right)' \tag{C.3}$$

Taking the square root of (C.3) yields the bootstrap estimate of the standard errors $se_{\hat{\theta}}$.

C.2. ***Identification.*** This subsection provides evidence about the validity of the moment conditions used to identify the structural parameters of the model.

FIGURE C.1
Variation in the objective function around the estimated parameters



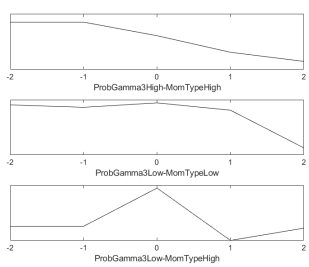
NOTES: This graph reports the values of the objective function that we obtain by perturbing each parameter by 2 standard deviations up and down with respect to the estimated value.

Figure C.1 shows the variation in the objective function (Equation (C.1)) induced by the perturbation of each estimated parameter in the vector $\hat{\theta}$. Figure C.2 reports the variation in the moment conditions used to identify the mother's unobserved productivity types in the labor market, by perturbing the estimated proportion of mothers in each group. Figure C.3 reports the variation in the moments used to identify the formal and informal child care cost equations: these moments represent the correlation between the cost of each child care type and the corresponding cost determinant, i.e., state funding for center-based child care for formal child care and presence of family members in the

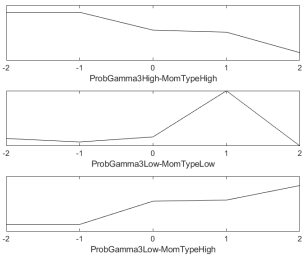
[1] $B = 200$.
[2] $B_{se} = 50$

neighborhood for informal child care. Figures C.4 and C.5 refer to the moments used for the identification of the parameters in the CAPF: Figure C.4 shows the variation in the moments used to identify the slope parameter in the elasticity of a child's ability with respect to a mother's child-care time, and the contribution of a mother's college education; Figure C.5 shows the variation in the moments used to identify the slope parameters in the elasticity of a child's ability with respect to informal and formal child care. Figure C.6 reports the variation in the moment conditions used to identify the relationship between the differential productivity of maternal child-care time induced by a mother's level of education and the mother's choices concerning child care and labor supply. Finally, Table C.2 and Figure C.7 provide evidence about the validity of the moment conditions used for the identification of parameters in the child's initial level of ability. Table C.2 reports the correlation coefficients between the child's test scores and the observable characteristics used to proxy the initial level of ability (see Equation (27)): for Column (1) we use as dependent variable the raw test scores, while for Column (2) we use as dependent variable the residuals from a regression of the first scores on child's age fixed effects and a dummy indicating whether the test is LW or AP. The results show that the specification in Column (2) gives more statistically significant coefficients and lower standard errors. Figure C.7 reports the variation in the moment used for the identification of the unobserved component of the initial ability, which considers the variance of the residuals previously described.

FIGURE C.2
Variation in the moment conditions used to identify a mother's unobserved productivity in the labor market, by perturbing the estimated parameters



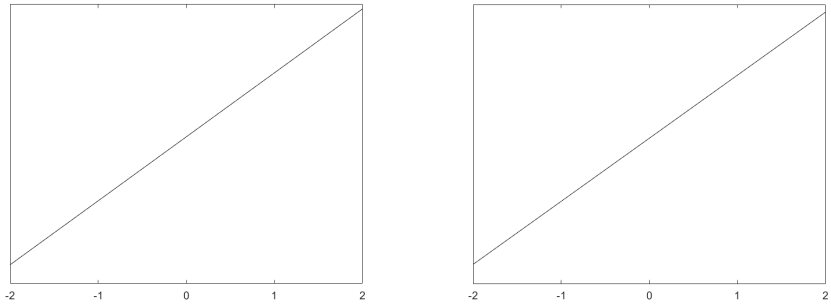
(A) Moment: Var. of Residuals Wage



(B) Moment: Serial Corr. of Residuals Wage

NOTES: This graph reports the values of the moment conditions obtained from the variance (Figure A) and serial correlation (Figure B) of the residuals from a OLS regression of a mother's wage on a mother's education, race, age, year of birth, area of residence and the interaction between the latter two, by perturbing the estimated parameters by 2 standard deviations up and down with respect to the estimated value. The parameters represent the proportion of mothers in each group identified by a level of unobserved skills in the labor market (*MomTypeLow* and *MomTypeHigh*) and a level of preference for a child's ability (*Gamma3Low* and *Gamma3High*).

FIGURE C.3
Variation in the moment conditions used to identify the parameters in the informal and formal child care cost equations, by perturbing the estimated parameters

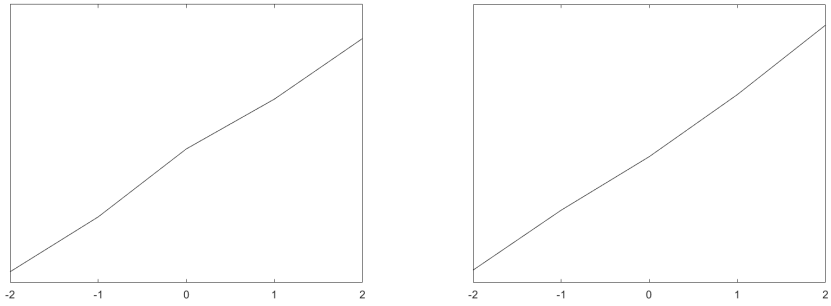


(A) Formal CC and Preschool State Funding

(B) Informal CC and Family Members in Neighborhood

NOTES: This graph reports the values of the moment conditions obtained from (i) the correlation between the formal child care cost and the state funding for center-based child care (Figure A), and (ii) the correlation between the informal child care cost and the presence of family members in the neighborhood (Figure B), by perturbing the estimated parameters by 2 standard deviations up and down with respect to the estimated values. The parameters represent the correlation between formal child care price and state funding for pre-kindergarten for Figure A, and the correlation between informal child care price and presence of family members in the neighborhood for Figure B.

FIGURE C.4
Variation in the moment conditions used to identify the elasticity of a child's cognitive ability with respect to a mother's time with the child and the contribution of a mother's education, by perturbing the estimated parameters



(A) Mother's time prod. - Slope

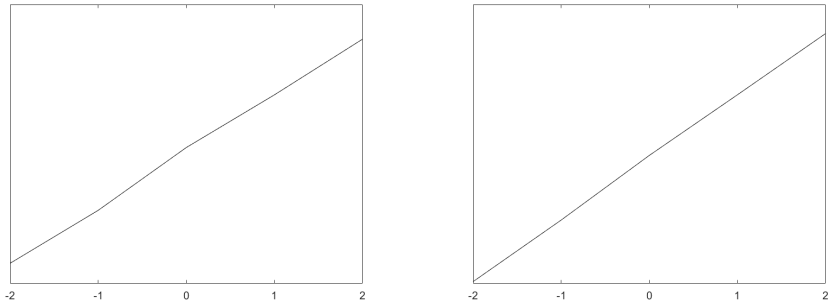
(B) Mother's time prod. - Education

NOTES: This graph reports the values of the moment conditions obtained from (i) the correlation between a mother's time with the child in t and the child's scores in $t + 5$, conditional on whether the score is LW or AP (Figure A), and (ii) the correlation between a mother's education and a child's score, conditional on whether the score is LW or AP and on a mother's wage (Figure B), by perturbing the estimated parameters by 2 standard deviations up and down with respect to the estimated values. The parameters represent the elasticity of a child's ability with respect to a mother's time with the child (Figure A) and the contribution of a mother's education to such elasticity (Figure B).

APPENDIX D. ADDITIONAL RESULTS

Figure D.1 reports the time-varying elasticity of a child's cognitive ability with respect to the level of ability in the previous period and the estimated total factor productivity. Table D.1 reports the untransformed parameters in the mother's utility function (Panel A), and in the child's cognitive ability production function (Panel B). Table D.2 reports

FIGURE C.5
Variation in the moment conditions used to identify the elasticity of a child's cognitive ability with respect to informal and formal child care, by perturbing the estimated parameters



(A) Formal child care prod. (B) Informal child care prod.

NOTES: This graph reports the values of the moment conditions obtained from the correlation between informal (Figure A) and formal (Figure B) child care hours in t and the child's scores in $t+1$, conditional on whether the score is LW or AP, by perturbing the estimated parameters by 2 standard deviations up and down with respect to the estimated values. The parameters represent the elasticity of a child's ability with respect to informal (Figure A) and formal child care (Figure B).

the estimated parameters in the other income function (Panel A), and the estimated parameters in the initial level of ability of the child and in the test score specification (Panel B). Table D.3 reports the fit for the targeted unconditional moments used for the estimation of the model. Finally, Figure D.2 represents the marginal cost of maternal child-care time, defined in Section 3.2 in the paper, as a function of a mother's preferences for leisure by a mother's employment status. The *Baseline* value is defined by using the simulated data after the model estimation, while the *Wage subsidy policy* value is defined by using the data obtained after the simulation of the wage subsidy policy (Policy A) described in Section 7.1 in the paper. The wage subsidy policy B described in Section 7.1 induces a similar variation in the cost of maternal child-care time, while the policies regulating and subsidizing the non-parental child care market considered in Section 7.2 determine a limited increase in labor supply, which translates into a very small variation in the marginal cost of maternal child-care time.[3]

[3]Results on the wage subsidy policy B and policies regulating and subsidizing the non-parental child care market are the available upon request to the author.

Appendix E. Sensitivity analysis

This section presents the results from a sensitivity analysis that we perform in order to understand the implications of omitting the father's time with the child from the specification of the CAPF in the baseline model. In fact, according to the baseline specification, only the mother's time is productive for the child cognitive development, while the father's contribution only comes through his labor income that affects the mother's investment decisions. However, it could be the case that fathers become more involved in the child-care activities, especially as the child grows up, and that this time also contributes to the cognitive development of the child later on. In addition, fathers married with more educated women may be more likely to be involved with the child, as a consequence of assortative mating. Both these channels may result in a biased estimate for the parameters of the elasticity of a child's ability with respect to maternal child-care time (especially for high-educated mothers) and of the alternative forms of care.

In order to understand how the omission of a father's child-care time in the CAPF affects the estimated parameters, we re-estimate the model by using an alternative measure of time investments, that includes both mother's and father's time with the child. The estimated parameters for the maternal/parental and non-parental child care inputs are reported in Figure E.1. By comparing Figure E.1-Left with Figure 3-Left, it can be observed that the estimated elasticity of a child's ability with respect to time investments is hardly affected. However, Figure E.1-Right shows a less relevant difference between the productivities of formal and informal child care. This result seems to suggest that if fathers' time is also considered in the time investments received by the child at home, high-quality non-parental child care play a less important role for the child's cognitive development. Thus, the absence of a father's time as an input in the CAPF is likely to generate an upward bias in the estimated elasticity of a child's ability with respect to formal and informal non-parental child care. Interestingly, the estimation that includes a father's time in the home time investments received by the child also leads to a lower estimated total factor productivity at older ages,[4] which is in line with previous findings from Del Boca et al. (2014) showing that a father's child-care time becomes important from age 10 onward.

References

Altonji, J. G. and L. M. Segal (1996). Small-sample bias in GMM estimation of covariance structures. *Journal of Business & Economic Statistics 14*(3), 353–366.

Blundell, R., L. Pistaferri, and I. Preston (2008). Consumption inequality and partial insurance. *American Economic Review 98*(5), 1887 – 1921.

Cameron, A. C. and P. K. Trivedi (2005). *Microeconometrics. Methods and applications*. New York, USA: Cambridge University Press.

Davidson, R. and J. G. MacKinnon (2003). *Econometric theory and method*. New York: Oxford University Press.

[4]Results available upon request to the author.

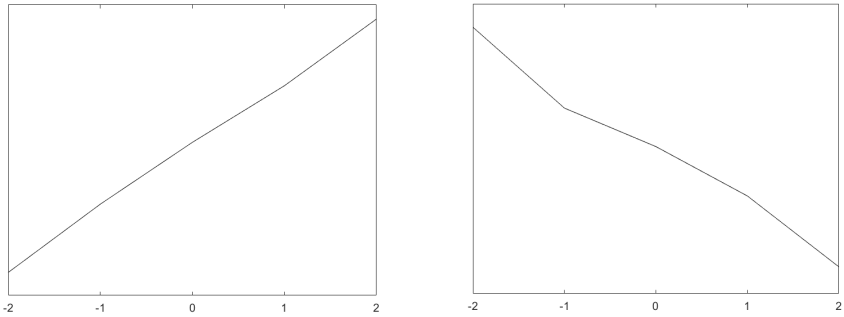
Del Boca, D., C. J. Flinn, and M. Wiswall (2014). Household choices and child development. *Review of Economic Studies 81*(1), 137–185.

Keane, M. P. and R. A. Moffitt (1998). A structural model of multiple welfare program participation and labor supply. *International Economic Review 39*(3), 553–589.

Train, K. (2009). *Discrete Choice Methods with Simulation* (2 ed.). Cambridge University Press.

Wooldridge, J. (2002). *Econometric Analysis of Cross Section and Panel Data*. Cambridge: MA: MIT Press.

FIGURE C.6
Variation in the moment conditions used to identify the relationship between a mother's level of education and her choices, by perturbing the estimated parameter for a mother's education in the CAPF



(A) Mother's education & Child-care time

(B) Mother's education & Labor supply

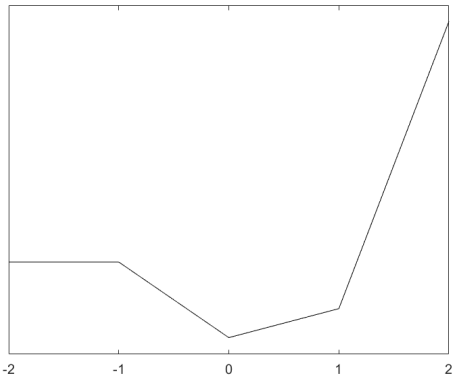
NOTES: This graph reports the values of the moment conditions obtained from (i) the correlation between a mother's level of education and her child-care time, conditional on a mother's wage (Figure A), and (ii) the correlation between a mother's level of education and her labor supply, conditional on a mother's wage (Figure B), by perturbing the estimated parameter for a mother's level of education ξ_{1Edu} in the CAPF by 2 standard deviations up and down with respect to the estimated value.

TABLE C.2
Correlation between test scores and observable characteristics used to proxy the initial child's ability

	(1) Raw Test Scores	(2) Residuals
Child is male	-0.547	-0.504
	(0.484)	(0.361)
Mom Age at childbirth	0.115***	0.122***
	(0.042)	(0.034)
Birth weight$\leq$ 2500 grams	-1.558	-1.316*
	(1.026)	(0.729)

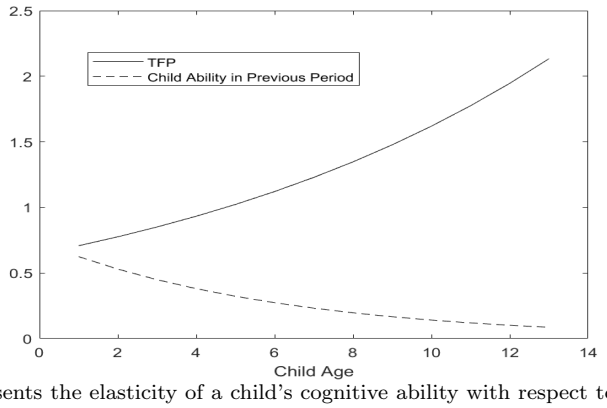
NOTES: OLS regression in column (1) uses as dependent variables the raw test score and controls for child's age fixed effects and a dummy indicating whether the test score is LW or AP. OLS regression in column (2) uses as dependent variable the residuals of a regression of raw test scores on a dummy indicating whether the score is LW or AP and child's age fixed effects, and only consider the first test score observed for each child. The regressions are computed using PSID-CDS data on children aged 0-12 in 1997 without siblings. Child's test scores refer to both the LW and the AP scores. Significance levels: *** p<0.01, ** p<0.05, * p<0.1.

FIGURE C.7
Variation in the moment conditions used to identify the intercept and shock in the child's initial ability, by perturbing the estimated parameters



NOTES: This graph reports the values of the moment condition obtained from the variance of the residuals from a OLS regression of a child's first test score observation on a dummy indicating whether the test is LW or AP and a child's age fixed effects, by perturbing the estimated parameter by 2 standard deviations up and down with respect to the estimated values. The parameter represents the standard deviation of the shock in the initial level of ability of a child.

FIGURE D.1
Elasticity of a child's cognitive ability with respect to the level of ability of the child in the previous period, and estimated total factor productivity (TFP)



NOTES: This graph represents the elasticity of a child's cognitive ability with respect to the level of ability of the child in the previous period (A_t), and the estimated total factor productivity parameter, as a function of child's age $t = 1, 2, 3, \ldots 13$. The specification of the parameters is reported in Equations (22) and (26) in the paper.

TABLE D.1
Estimated untransformed parameters in the mother's utility function and the child's cognitive ability production function

		Estimate	Std. Errors
Panel A. Utility function			
γ_{2l}	Utility from consumption Type I	-0.0218	0.2020
γ_{2h}	Utility from consumption Type II	-0.0179	0.2872
γ_{3l}	Utility from child ability Type I	-0.6952	0.2052
γ_{3h}	Utility from child ability Type II	-0.1238	0.1391
Panel B. Cognitive ability production function			
ξ_{0tfp}	Total factor productivity. Intercept	-0.4371	0.1384
ξ_{1tfp}	Total factor productivity. Slope	0.0919	0.0160
$\xi_{0\tau}$	Mother's time with the child. Intercept	0.2623	0.2981
ξ_{1Edu}	Mother's time with the child. Effect of a mother's education	0.6135	0.6779
$\xi_{1\tau}$	Mother's time with the child. Slope	-0.3036	0.0393
ξ_{0i}	Informal child care. Intercept	-0.0060	0.2740
ξ_{2i}	Informal child care. Slope	-0.6362	0.0648
ξ_{0f}	Formal child care. Intercept	0.3470	0.3305
ξ_{3f}	Formal child care. Slope	-0.6709	0.0501
ξ_{0A}	Child's ability in the previous period. Intercept	-0.3047	0.0667
ξ_{4A}	Child's ability in the previous period. Slope	-0.1653	0.0312

NOTES: Standard errors are estimated with non-parametric bootstrap, by changing the starting values in each bootstrap iteration.

TABLE D.2
Estimated parameters for the other household income function, the child's initial ability and the test score specification

		Estimate	Std. Errors
Panel A. Other household income function			
μ_{inc0}	Intercept	-0.3759	0.3067
μ_{inc1}	Coefficient for father's years of education	0.1263	0.0145
μ_{inc2}	Coefficient for father's race	0.2162	0.0529
μ_{inc3}	Coefficient for father's age	0.0102	0.0054
σ_{inc}	Std deviation income shock	0.6185	0.0366
Panel B. Initial ability and test score specification			
η_0	Intercept	-17.1175	9.2067
η_1	Coefficient for birth weight	-13.2826	22.0854
η_2	Coefficient for gender	-20.8972	18.8766
η_3	Coefficient for a mother's age at birth	-18.2699	6.6867
σ_v	Std deviation initial ability shock	16.0095	0.8058
κ	Coefficient for LW test scores (vs AP)	0.1748	0.0317

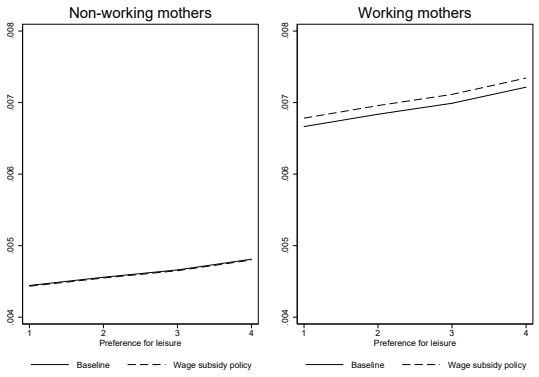
NOTES: Standard errors are estimated with non-parametric bootstrap, by changing the starting values in each bootstrap iteration.

TABLE D.3
Fit for targeted unconditional moments

	Data	Simulation
Corr mother's wage and mother's hours of work	0.0054	0.0858
Corr other hh income and mother's hours of work	−0.3147	−0.7119
Corr mother's wage and mother's time with the child	0.2665	0.2168
Corr other hh income and mother's time with the child	−0.0598	−0.0423
Corr mother's wage and formal child-care time	0.7460	0.3814
Corr mother's wage and informal child-care time	0.3898	0.2263
Corr other hh income and formal child-care time	0.9965	0.2364
Corr other hh income and informal child-care time	0.5115	0.1919
Corr mother's hours of work and mother's time with the child	−0.0447	−0.5757
Corr mother's hours of work and formal child-care time	0.4393	0.0838
Corr mother's hours of work and informal child-care time	0.2420	0.0704
Coefficient of mother's time with the child in $t-5$ in a OLS reg on test score in t, cond. on a dummy for LW	0.5880	0.4109
Coeff of a dummy for high-educated mother on child's test score, cond. on child's age FE, a dummy for LW and mother's wage	1.5746	2.5506
Coeff of a dummy for high-educated mother on mother's time with the child, cond. on child's age FE and mother's wage	1.5311	8.6370
Coeff of a dummy for high-educated mother on mother's hours of work, cond. on child's age FE and mother's wage	−1.4386	−7.4585
Coeff of formal child care in $t-1$ in a OLS regression on test score in t, cond. on a dummy for LW	0.3443	0.0091
Coeff of informal child care in $t-1$ in a OLS regression on test score in t, cond. on a dummy for LW	0.6979	0.0088
Var of residuals from child's test score OLS reg on a dummy for LW and child's age FE	39.9555	35.5324
Mean mother's wage	14.3659	4.0003
Std deviation mother's wage	10.2725	18.0704
Var of the residuals from a mother's wage OLS reg on mother's charact.	0.2199	0.2314
Coeff of residuals from a mother's wage OLS reg on mother's charact. in t on the residuals in $t-1$ (autocorr)	0.8739	0.5174
Mean price formal child care	1.0769	2.9485
Std deviation price formal child care	3.5989	4.8544
Mean price informal child care	0.2788	2.4187
Std deviation price informal child care	1.2928	3.5448
Corr price formal child care and state funding for center-based child care	0.4572	0.6029
Corr price informal child care and family in neighborhood	−0.0409	0.0886
IV reg of formal child-care hours on the price of formal child care, instrumented by state funding for center-based child care	−1.0439	−1.2683
Mean other household income	7.9136	7.9395
Std deviation other household income	6.4406	6.4411

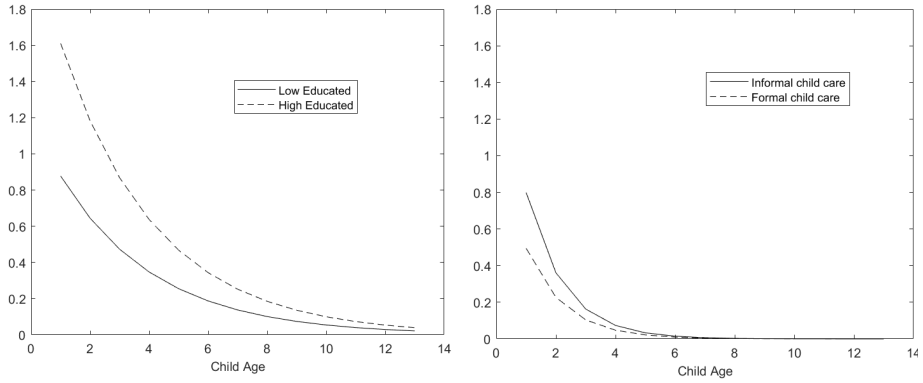
NOTES: Actual data represent PSID-CDS data on children aged 0-12 in 1997, without siblings. Simulated data represent the data obtained simulating the model described in Section (3) and setting the parameters at the estimated values.

FIGURE D.2
Cost of maternal child-care time by a mother's employment status and a mother's preferences for leisure



NOTE. The figure reports the cost of maternal child-care time as a function of the mother's preference for leisure and by a mother's employment status. The cost of maternal time is defined as $\frac{\alpha_1}{(TT-h_t-\tau_t)}$ for each child's age t (see Section 3.2 in the paper). The estimated values for the parameters α_1 are reported in Table 2 in the paper. *Baseline* refers to the data simulated after the model estimation, and is obtained by setting the mother's labor supply h and child-care time τ at their average values for working and non-working mothers. *Wage subsidy policy* is obtained by setting the mother's labor supply h and childcare time τ at their average values for working and non-working mothers after the simulation of the wage subsidy policy A described in Section 7.1, which increases wages by 20 percent for all mothers.

FIGURE E.1
Elasticity of a child's cognitive ability with respect to parental time investments and non-parental child care, obtained when including a father's child-care time in the time investments measure.



(A) Parents' child-care time (B) Non-parental child care

NOTE. This graph represents the elasticity of a child's ability with respect to parental child-care time (τ_t) and non-parental child care (i_t and f_t), as a function of a child's age $t = 1, 2, 3, \ldots 13$. Parental child-care time includes the time spent by the child with the mother and/or the father, and the estimated parameters are reported by a mother's level of education. The specification of the parameters is reported in Equations (23), (24) and (25) in the paper.