Journal of Philosophical Logic (2023) 52:643–668
https://doi.org/10.1007/s10992-022-09683-3

Extended Simples, Unextended Complexes

Claudio Calosi[1]

Received: 18 July 2021 / Accepted: 29 August 2022 / Published online: 5 October 2022

Abstract
Both extended simples and unextended complexes have been extensively discussed and widely used in metaphysics and philosophy of physics. However, the characterizations of such notions are not entirely satisfactory inasmuch as they rely on a mereological notion of extension that is too simplistic. According to such a mereological notion, being extended boils down to having a mereologically complex exact location. In this paper, I make a detailed plea to supplement this notion of extension with a different one that is phrased in terms of measure theory. This proposal has significant philosophical payoffs. I provide new characterizations of both extended simples and unextended complexes, that help re-evaluating the question of whether such entities are metaphysically possible. Finally, I advance several suggestions as to how different notions of extension relate, first, to one another and, second, to mereological structure.

Keywords Extension · Extended simples · Unextended complexes · Measure · Location · Mereological harmony

1 Introduction

Extended simples have been thoroughly discussed in metaphysics[1] and philosophy of physics.[2] Recently, unextended complexes have been investigated as well.[3] Despite the attention they have attracted, I find the characterizations of both extended simples and unextended complexes *not entirely* satisfactory inasmuch as they rely—for the most part—on a *mereological notion of extension* that is simplistic. According to

[1] See among others Scala [42], McDaniel [32], Gilmore [19], and Rettler [40].
[2] See Braddon-Mitchell and Miller [8], Baker [4], and Baron and LeBihan ([7]).
[3] See e.g., McDaniel ([33]: 239–242), and Pickup [39].

✉ Claudio Calosi
claudio.calosi@unige.ch; https://claudiocalosi.xyz/

1 Department of Philosophy, University of Geneve, Geneva, Switzerland

such a mereological notion, being extended boils down to having a mereologically complex exact location. In this paper, I make a detailed plea to introduce a different notion of extension that is phrased in terms of *measure theory*. A few caveats are in order. First, I don't mean to suggest that measure theoretic notions have been completely absent from the philosophical literature. For example, some solutions of Zeno's paradox of infinite divisibility crucially rely on such notions—starting from the classic [21].[4] By contrast, the literature on extended simples and unextended complexes has—at least explicitly—neglected measure theory. This paper represents a first step to remedying this situation. Second, I am not urging to *replace* the *mereological* notion of extension with the *measure-theoretic* one. Rather, my suggestion is to *keep them both*, side by side, in our logical and metaphysical toolkit. One can even go as far as claiming that, on a charitable reading, some philosophers implicitly recognized the need of something like a measure-theoretic notion of extension alongside the usual mereological notion, at least insofar as they talked about *size*.[5] However these philosophers provided hardly any detail on how to understand the relevant notion of extension. This paper is meant to provide such relevant details. The enriched formal framework that contains parthood, mereological, and measure-theoretic—or metrical—extension is useful for the sake of (i) providing a *measure* of different extensions, (ii) providing clear definitions of notions related to that of extension, such as the relation of "less extended than", (iii) providing different characterizations of both extended simples and unextended complexes, to mention a few. Furthermore, the metrical notion of extension may be fruitful for (many) other debates in the metaphysics of objects.

2 Preliminaries

2.1 Space

The following is the somewhat standard set-theoretic construction of space.[6] Space is "constructed out" of a set of elements called spatial *points*. Points are supposed to be mereologically *simple* and *unextended*. Spatial regions are *non-empty sets of points*,[7] and *any* non-empty set of points counts as a region.[8] Different structures can be defined on such a set. As of now, we need just to define its *topological structure*. This is usually taken to be $\mathbb{R}^3$. In effect, the main points of the paper can be made with respect to $\mathbb{R}^1$ so I will mostly stick to it.[9] I take $\mathbb{R}^1$ to have the standard

[4] For some recent papers that do consider measure theory at length see Arntzenius [2] and Lando and Scott [27].

[5] See e.g., Tognazzini ([47]: 122).

[6] Or spacetime. I will ignore this complication. See e.g., Baker [4] and Gilmore [19].

[7] This rules out that the empty-set counts as a region.

[8] Note that in the rest of the paper I will slightly abuse notation and write "region r is the union of two points p_1 and p_2" as $r = p_1 \cup p_2$ rather than $r = \{p_1\} \cup \{p_2\}$.

[9] See footnote 37 for some thoughts about generalizations to $\mathbb{R}^n$.

open-ball topology.[10] Any worry that this makes points, regions, and the entire space abstract entities will be met in the next section. As I pointed out this is the somewhat *standard* conception of space. This standard conception can be challenged. First, one might think that there are no atomic regions of space.[11] Second, one might think that there are atomic regions of space, but they are extended.[12] A detailed investigation of these unorthodox accounts and of the possible notions of extension definable in their terms goes beyond the scope of the paper—though I will briefly return to extended atomic regions. The main point of the paper is that even *within* the standard conception of space there is an alternative notion of extension—and extended simples, and unextended complexes—that has been overlooked.

2.2 Mereology

Following Cotnoir and Varzi [12], let $\sqsubseteq$ stand for a primitive two-place notion of parthood. In what follows I will use first order logic with identity and set-theory.[13] This is not mandatory.[14] But given that we already introduced set-theory in Section 2.1, and we will in fact use it again later on, we might use it here as well. Standard mereological definitions are as follows:

$$x \sqsubset y \equiv_{df} x \sqsubseteq y \wedge x \neq y \tag{1}$$

PROPER PART

$$x \circ y \equiv_{df} \exists z(z \sqsubseteq x \wedge z \sqsubseteq y) \tag{2}$$

OVERLAP

$$F(x, S) \equiv_{df} \forall y(y \in S \rightarrow y \sqsubseteq x) \wedge \forall w(w \sqsubseteq x \rightarrow \exists z(z \in S \wedge z \circ w)) \tag{3}$$

FUSION

Given the orthodox set-theoretic construction of space, and restricting variables to spatial regions, we can write:[15]

$$x \sqsubseteq y \leftrightarrow x \subseteq y \tag{4}$$

$$x \sqsubset y \leftrightarrow x \subset y \tag{5}$$

$$x \circ y \leftrightarrow x \cap y \neq \emptyset \tag{6}$$

[10] Let d be a metric on the set of spatial points. Define an open ball of radius r centered at point p as the set of points whose distance d from p is less than r. It is possible to show that open balls so defined induce a topology on the set of spatial points, the so-called open-ball topology. This view is *substantivalist* insofar as it does not try to reduce points and regions to something else, e.g. events, or material objects, and relations between those.

[11] Famously, Whitehead held this view. For an introduction see Gruszczynski and Pietruszczak [22].

[12] For a philosophically oriented introduction see e.g. Braddon-Mitchell and Miller [8].

[13] I am using *both* set-theory and mereology. Alternatively, one might want to develop a system that dispenses with set-theory altogether and only works with mereological notions. One possible step in this direction would be to look at Field [15] and his use of Hilbert's *segment arithmetic*. A significant development in this direction is in Arntzenius and Dorr [3].

[14] A widespread alternative in the literature uses *plural logic*.

[15] So that e.g., Eq. 4 should be read as: $x \sqsubseteq y \wedge \psi(x) \wedge \psi(y) \leftrightarrow x \subseteq y \wedge \psi(x) \wedge \psi(y)$, where $\psi(x)$ is the open formula "x is a region". The same goes for Eqs. 5–7. See e.g., Uzquiano [48].

$$F(x, S) \leftrightarrow x = \bigcup_{y_i \in S} y_i \tag{7}$$

Equivalences (4)–(7) just say that a (proper) part of a region is a (proper) subset of the region, two regions overlap iff their intersection is not empty, and the fusion of some regions is their union. They ensure that we can use only mereological vocabulary to talk about spatial regions. This should also alleviate the worry about spatial regions being abstract entities.[16] I will take parthood to be a partial order that obeys:

$$\neg x \sqsubseteq y \rightarrow \exists z(z \sqsubseteq y \wedge \neg z \circ x) \tag{8}$$

STRONG SUPPLEMENTATION

To keep things as simple as possible, I will also require two distinct fusion axioms. Let $\psi(x)$ and $\phi(x)$ be the open formulas: "x is a region" and "x is a material object" respectively. Then, the fusion axioms are:

$$S \neq \emptyset \wedge \forall y(y \in S \rightarrow \psi(y)) \rightarrow \exists x(F(x, S)) \tag{9}$$

REGION FUSION

$$S \neq \emptyset \wedge \forall y(y \in S \rightarrow \phi(y)) \rightarrow \exists x(F(x, S)) \tag{10}$$

OBJECT FUSION

The fusion axioms ensure that for any non empty set of material objects there is a fusion of those objects. The same goes for regions of space. The axioms *are silent* as to whether cross-categorical fusions exist. I am going to *assume* that they *do not*—but see footnote 34 for a *possible argument*. Together with strong supplementation the fusion axioms guarantee the *existence* and the *uniqueness* of the relevant mereological fusions.[17]

[16] One might hold the view that set-theoretic notions apply only to *abstract* objects. Yet, equivalences (4–7) ensure that set-theoretic talk can be translated into mereological talk for concrete spatial regions.

[17] Extensional Mereology is surely a controversial choice for material objects. The classic counterexample to extensionality is arguably that of a statue and the matter it is composed of. They are (allegedly) distinct, yet the share the same proper parts—the literature is literally too vast to mention. The interested reader could find an (almost) exhaustive list in Cotnoir and Varzi [12]. I chose classical mereology for the sake of simplicity, but nothing in the following arguments depends crucially on this choice. If one adopts a weaker mereology for material objects, there is one detail that can make a difference, and that is whether one takes the statue and the matter it is composed of to be *mereologically related*, by e.g., saying that the matter is part of the statue. If so, one can simply stick to the letter of the arguments to follow, and e.g., identify—to anticipate my proposal—the extension of both the statue and the matter with the extension of *their* exact locations. This is because the fusion of the statue and the matter would simply be the statue. However, if one believes that there are no mereological relations whatsoever between the statue and the matter—that is they are completely disjoint, as per *disjointism* (See Wasserman [51], and Limpan [29])—the fusion axioms entail that there is (at least) a *third object*, namely (one of) their fusion(s). Let's call one of the fusions of the statue and the matter, the *statter*. Arguably, the "statter" has the same (exact) location of the statue and the matter, which are, recall, its disjoint (co-located) proper parts. Then one would run in a similar problem, *mutatis mutandis*, I discuss in footnote 34. My recommendation is the same: distinguish between extensions of statues, hunks of matter, and their fusions. Thanks to an anonymous referee here.

2.3 Location

Let @ be a primitive notion of *exact location*. @ is supposed to represent, as Parsons [36] puts it, the "shadow" of a material object in substantival space, the region in which the object *exactly fits*. We can then define other locative notions in terms of @ and mereology (or set-theory):

$$x @_{\circ} y \equiv_{df} \exists z (x @ z \wedge z \circ y) \tag{11}$$

WEAK LOCATION

$$x @_{>} y \equiv_{df} \exists z (x @ z \wedge y \sqsubseteq z) \tag{12}$$

PERVASIVE LOCATION

That is, x is *weakly* located at y iff x is exactly located at a region that overlaps y, and x it is *pervasively* located at y iff it is exactly at a z that has y as a part. As an illustration, I am weakly located in my office, and I am pervasively located where my heart is exactly located. In what follows I will assume the following:

$$\exists y (x @_{\circ} y) \rightarrow \exists z (x @ z) \tag{13}$$

EXACTNESS

$$x @ y_1 \wedge x @ y_2 \rightarrow y_1 = y_2 \tag{14}$$

FUNCTIONALITY

Exactness guarantees that every spatial entity, i.e., everything that is at least weakly located in space, has an exact location.[18] Functionality dictates that everything has at most one exact location.[19]

I will also assume that *all* regions are located at themselves. From now on, I will then use r_1, ..., r_n as singular terms (constants and variables) for spatial regions. Given all this to each spatial entity—region or material object—we can associate its exact location. Thus we can set

$$L(x) \equiv_{df} \iota r (x @ r) \tag{15}$$

where ι is the Russell's operator. In the rest of the paper, three different principles of location will interest us:[20]

$$x @ r \wedge y @ r \rightarrow x = y \tag{16}$$

NON-COLOCATION

[18] As a matter of fact, there is no need to assume Eq. 13, insofar as it can be derived from Eq. 11. Alternatively, one could take *weak location* as primitive and define *exact location* in terms of it. If one uses the definition in Parsons [36] one ends up with *Functionality* being a theorem. Eagle [13] proposes another definition of exact location in terms of weak location that does not entail Functionality. For a critical discussion of Eagle's proposal see Calosi and Costa [10] and Payton [38].

[19] Exactness and Functionality are somewhat controversial axioms. I assume them for the sake of simplicity: the arguments in the rest of the paper would go through without any of them as well–though the arguments would need tweaking a little. I will suggest some tweaks myself in due course. Furthermore, whenever these axioms seem to do substantive metaphysical work, I will simply flag that out explicitly.

[20] See e.g. Casati and Varzi [11], Parsons [36], Varzi [49], and Saucedo [41].

$$x@_{>}r \rightarrow \exists y(y \sqsubseteq x \wedge y@r) \tag{17}$$

ARBITRARY PARTITION

$$x \sqsubseteq y \wedge y@r_1 \rightarrow \exists r_2(x@r_2 \wedge r_2 \sqsubseteq r_1) \tag{18}$$

EXPANSIVITY

The first principle, Non-Colocation, says that no two things can be exactly located at the same region.[21] The second, Arbitrary Partition says that things have parts at regions they pervade. Finally, Expansivity requires—roughly—that parts are located where wholes are.

3 Extended Simples and Unextended Complexes Defined

3.1 Extended Simples

The following provides a representative sample of definitions of extended simples in the literature—italics added:[22]

> [A] simple, in my sense, occupies a *greater than point-size region* of space and it is indivisible because it does not have, for instance, a right or a left half (Scala [42]: 394).

> [E]xtended simples are entities that are extended in space but have no (proper) parts (...) they would occupy a —*complex region* of space (Pickup [39]: 257).

> [A] simple is an entity that has no proper parts (...) Say that an entity is *extended* just in case it is a spatiotemporal entity and *does not have the shape and size of a point* (Gilmore [19]: 25–26).

> [W]e take an extended simple to be a mereologically simple entity that is *not point-like* (Calosi and Costa [10]: 1075–1076).

They seem to share the following picture. A mereologically complex region of space is extended; anything that is exactly located at an extended region is an extended entity. Pickup and Eagle are explicit:

> [O]ne natural way to understand what it is to be an extended region is as being composed of more than one point (Pickup [39]: 263).

[21] Strictly speaking there is a sense in which colocation occurs every time a material object x is exactly located at region r. For, x and r are indeed colocated at r in all such cases. By contrast, the Non-Colocation principle is meant to banish colocation between *material objects*—for, as I point out in Section 6, colocation of regions is ruled out by Functionality alone. Thus, in what follows Non-Colocation is indeed intended as Non-Colocation for material objects. I will stick to Eq. 18 for the sake of readability. Arguably, the most widely cited violation of NON-COLOCATION is the case of the statue and the matter it is composed of, as I discuss it in footnote 17

[22] Similar definitions are in McDaniel ([32, 33]: 131), Sider ([43]: 52), Simons ([45]: 63), and Rettler ([40]: 850).

> [I] will be understanding extendedness mereologically: a region is mereologically extended iff it has a proper subregion (Eagle [14]: 167).

Define *Atom* (or *Simple*, A) as something that does not have proper parts:

$$A(x) \equiv_{df} \neg\exists y(y \sqsubset x) \tag{19}$$

ATOM

We could then define *Being Extended*$_{\sqsubseteq}$ (E_L) and *Being Unextended*$_{\sqsubseteq}$ ($\neg E_L$):

$$E_{\sqsubseteq}(x) \equiv_{df} \neg A(L(x)) \tag{20}$$

EXTENDED$_{\sqsubseteq}$

$$\neg E_{\sqsubseteq}(x) \equiv_{df} A(L(x)) \tag{21}$$

UNEXTENDED$_{\sqsubseteq}$

"Being Extended$_{\sqsubseteq}$" boils down to having a *mereologically complex* (exact) location. This is why I have used the subscript "$\sqsubseteq$", to flag that this is a *mereological notion of extension*. Many of the arguments in this paper can be read as a suggestion to the point that mereological extension might be natural, as Pickup put it, but is not entirely satisfactory. In some cases at least it is simplistic. In any event, as of now, an extended simple $ES_{\sqsubseteq}$ is easily defined:[23]

$$\begin{aligned} ES_{\sqsubseteq}(x) &\equiv_{df} A(x) \wedge E_{\sqsubseteq}(x) \\ &\equiv_{df} A(x) \wedge \neg A(L(x)) \end{aligned} \tag{22}$$

EXTENDED SIMPLE$_{\sqsubseteq}$

I take Eq. 22 to be the definition of extended simples that is *widely* —if not universally—accepted in the philosophical literature.[24] Given that I am not entirely satisfied with Eq. 20, I am not entirely satisfied with Eq. 22 either.

3.2 Unextended Complexes

Unextended complexes have not attracted as much attention as extended simples. There are, however, notable exceptions. McDaniel [33] argues that unextended complexes are metaphysically possible. McDaniel [34] argues they are ruled out by reductive accounts of mereology.[25] Pickup [39] focuses exclusively on them. Both McDaniel and Pickup characterize them similarly:

[23] McDaniel [32, 33] distinguishes two types of extended simples, namely *spanners* and *multilocaters*. Similarly, Eagle [14] distinguishes between l-extended simples and f-extended simples—corresponding roughly to spanners and multilocaters respectively. Spanners are what I simply call "extended simples". Henceforth, I will work with a restricted notion of extended simples that does not include mutlilocaters.

[24] As I pointed out already, different notions of extension may be definable against the background of unorthodox constructions of space. Also, as I once again already pointed out, some philosophers explicitly talk about "size".

[25] I will return to this in Section 6.

> [M]ereologically complex point-sized objects are also possible (McDaniel [33]: 239).

> [T]here are two ways for an entity to be an extended complex, corresponding to two ways of being unextended. Something can be unextended (a) by having no location in space at all or (b) by being located at a simple part of space (Pickup [39]: 258).

Let us focus on spatial entities for the moment.[26] According to both McDaniel and Pickup unextended complexes ($UC_{\sqsubseteq}$) can be defined as follows:

$$\begin{aligned} UC_{\sqsubseteq}(x) &\equiv_{df} \neg A(x) \wedge \neg E_{\sqsubseteq}(x) \\ &\equiv_{df} \neg A(x) \wedge A(L(x)) \end{aligned} \tag{23}$$

UNEXTENDED COMPLEX$_{\sqsubseteq}$

Extended simples$_{\sqsubseteq}$ and unextended complexes$_{\sqsubseteq}$, as defined in Eqs. 22 and 23, challenge *mereological harmony*—roughly the view that the mereological structure of objects and the mereological structure of their exact locations perfectly mirror one another.[27] In fact, they provide counterexamples to some principles of location—the ones I presented in Section 2.3—that can be thought of as committing, to some extent, to such harmony.

3.3 Location and Extension

As I said, I find the mereological notion of extension not entirely satisfactory. Thus, I find the definitions of extended simples and unextended complexes not entirely satisfactory either. Let me briefly point out some of my perplexities. These perplexities should not be read as reasons to *discard* the mereological notion of extension. Rather, they should be taken as indicative of some limitations of that notion that are enough to motivate the search for alternatives.

The mereological notion can be used to discriminate between extended and unextended entities, but it is hardly of any use in providing a *measure* of that extension. We can say that x is extended but we cannot say *how much*. Then, *without recurring to any other primitives*, we cannot even say that x is less extended than y. We may think we can, in a few cases. For example we might want to say that when x is a proper part of y, x is less extended than y. This might actually be problematic, and I will return to this later on. But for the moment I just want to point out that even if we were to agree on that, this would be hardly enough for *defining* the relation of "being less extended then". For how are we supposed to handle cases in which x and y are mereologically disjoint—i.e. non-overlapping? Suppose we even introduce a new primitive relation to deal with such cases. What if I want to say that x is exactly n-times less extended

[26] That is, entities that are weakly located in space.

[27] For an introduction see Varzi [49], Uzquiano [48] and Leonard [28].

than y?[28] A natural question arises as to whether it is really *necessary to give a measure* of extension. Indeed, providing a measure of extension is crucial for both our everyday experience and our scientific practices. You don't want to know just that your bridal veil will be long. You want to know exactly *how long* it will be. In other words, you want to measure its extension. You don't want to know just that you have an internal bleeding that is not point-sized. You want to know exactly *how extended* it is. Your life might depend on its extension. Our scientific practices routinely involve *measures* of acceleration, velocity, pressure, cross-sections, and the like. They all *require to give a measure* of the extension of a spatial region. At this point one might think that a pure mereological framework can be easily expanded to provide such a measure. As a tentative suggestion, consider the following proposal: the measure of the extension of a region r is the number of proper parts of r. There are reasons to think that this suggestion won't do because it is not fine grained enough, at least for most of our purposes. Consider two intervals on $\mathbb{R}^1$, say $I_1 = (0, 1)$ and $I_2 = (2, 4)$. We want to be able to say that they have different extensions. In fact, we might want to say that I_1 is less extended than I_2. Yet, they have the same number of proper parts. To appreciate that note that they have the same cardinality. Hence their powers sets have the same cardinality. Thus, it seems that a pure mereological account should be supplemented with some other (primitive) notion(s). Perhaps a notion of *congruence* will do—as I mentioned already in footnote 28.

What I am about to suggest is that we can use measure theory in general, and Lebesgue measure in the particular case of the orthodox conception of space presented in Section 2.1. This raises the question: why this particular measure and not another? Many measures will arguably provide interesting metric notions of extension. Many but not all of them. Consider the *counting measure*: for any finite set S, the measure of S is the cardinality of S; for any infinite set S^* the measure of S^* is infinite. The problem is that this is not fine grained enough. According to the *counting measure* the sets I_1 and I_2 above have the same extension, namely an infinite extension. In the context that is assumed throughout the paper, once again, that of the orthodox conception of space, the Lebesgue measure has certain unique advantages. First, it allows us to provide a fine-grained measure for a vast number of subsets of $\mathbb{R}^1$—in fact on $\mathbb{R}^n$. Second, it has significant mathematical properties. It is the *unique* measure that is invariant under translations and send the "unit cube" to $+1$.[29] Because of this, it is routinely used in real analysis, and it is ubiquitous in empirical science.[30] Why shouldn't metaphysicians use it as well? Once we bought into set-theory, we should use set-theoretic constructions.[31] To further stress the point. Given the background assumptions I made in this paper, I will mostly be

[28] See Section 7 for some problematic attempts. I am not claiming it is impossible to find ingenious strategies to deal with the worries I just discussed. Perhaps we could take "being exactly n-times less extended than" as a primitive, and then work our way from there. Or perhaps a notion of *congruence* will do. However, we already have a detailed mathematical framework that gives us the resources to meet the challenges in Section 3.3 head on. It is the framework of *measure theory*, that I introduce in the following section.

[29] More precisely: (i) for any set S and any $x \in \mathbb{R}$, $\mu(S) = \mu(S+x)$; (ii) Let $U = (0, 1) \times ... \times (0, 1) \subset \mathbb{R}^n$. Then $\mu(U) = 1$.

[30] For quantum mechanics see e.g. Hughes ([25]: §1.11). For relativity see Wald ([50]: Appendix B).

[31] Those who want to eschew set theory altogether may develop the alternative mentioned in footnote 13.

concerned with Lebesgue measure. Yet, I will return to the more general notion of metrical extension—of which Lebesgue extension is but one example—a few times throughout the paper.

4 Measuring Extension

4.1 The Measure Theoretic Notion of Extension

In jargon, what we did in Section 2 was to define a topological space. We are going to define yet another structure over the set $\mathbb{R}^n$, a structure that will allow us to talk about extension in much greater detail.

Before we enter somewhat technical details let me convey the intuitive picture behind the Lebesgue measure (μ). We know how to assign extensions to particular regions. For instance we know how to assign a *length* to a line interval in $\mathbb{R}^1$, an *area* to a plane figure such as a rectangle in $\mathbb{R}^2$, and the *volume* to a solid figure such as a cube in $\mathbb{R}^3$. Let's call those regions, independently of their dimensionality, *boxes*–this is the technical term. Suppose now we want to assign an extension to an arbitrary plane figure x in $\mathbb{R}^2$, like the dotted figure x below. We can *cover* x *entirely* with *boxes* in such a way that the boxes are pairwise disjoint—unsurprisingly, this is known as a *disjoint covering*. We can then sum up the extensions of all the boxes we used to cover x, and obtain $n \in \mathbb{R}$. Clearly the extension of x is $\leq n$. We can repeat the process using coverings that are more and more fine-grained. They clearly approximate the extension of x better. We now take the *infimum* of all such extensions—i.e., of the extensions of the different coverings. Intuitively, we "minimize" such extension. We call it the *the outer measure* of x, $m^*(x)$. This is because we "measured" x from the outside so to speak. A dual approach measures x from the inside. We now take the *supremum* of the relevant extensions. Intuitively, we "maximize" such extension. We call it the *inner measure* of x, $m_*(x)$. This is because we measured x from the inside so to speak. The Lebesgue measure of x, $\mu(x)$ is now given by:

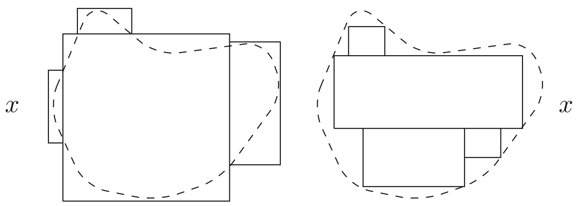
$$m^*(x) = \mu(x) = m_*(x) \tag{24}$$

In other words: the measurable regions of space—which are measurable sets—are those for which the outer measure is equal to the inner measure. We call such (equal) measure the Lebesgue measure. Figure 1 below provides a partial illustration.

This is actually what Lebesgue himself originally did. We now use a somewhat different—yet provably equivalent—approach, starting with a general measure (m).[32] Consider a set S. A *sigma algebra* $\sigma(S)$ defined over S is a collection of subsets of S, i.e. $\sigma(S) \subseteq \mathcal{P}(S)$, such that: (i) $S \in \sigma(S)$; (ii) $\sigma(S)$ is closed under complement; (iii) $\sigma(S)$ is closed under countable unions. The pair $< S, \sigma(S) >$ is called a *measurable space*, and the sets $S_i \in \sigma(S)$ are called the *measurable sets*. A measure m on a measurable space $< S, \sigma(S) >$ is a map $m : \sigma(S) \to \mathbb{R}^{\geq 0} \cup \{\infty\}$ such that (i) $m(\emptyset) = 0$, and (ii) m is countably additive.

[32] For an introduction see e.g., Tao [46].

Fig. 1 Measuring x from the outside (left) and the inside (right)



In general, the Lebesgue measure μ on $\mathbb{R}^n$ is a measure on $< \mathbb{R}^n, \sigma(\mathbb{R}^n) >$—where $\sigma(\mathbb{R}^n)$ is the so called *Borel sigma algebra*—[33]defined as follows: $\mu([a_1, b_1) \times ... \times [a_n, b_n)) = (b_1 - a_1) \cdot .... \cdot (b_n - a_n)$, for all $[a_i, b_i) \in \mathbb{R}^n$ with $a_i < b_i$—note that this basically gives us the extension of a n-dimensional *box*, as we introduced it above. It is a substantive theorem that this map defines a *unique* measure for the *entire* $\mathbb{R}^n$.

Countable additivity is important. Roughly it states that, for any Lebesgue measurable set S, and any *countable* union of pairwise disjoint subsets S_n, such that $\bigcup S_n = S$, we have:

$$\mu(S) = \sum_{x \in S_n} \mu(x) \tag{25}$$

The Lebesgue measure on $\mathbb{R}^n$ gives us a precise way to talk about the *extension* of any measurable set $S \in \mathbb{R}^n$. The *extension* of the set S is just the Lebesgue measure of S. Indeed, we can prove that for particular sets the Lebesgue measure is exactly what we expect: it gives us the *length* of a line interval in $\mathbb{R}^1$, the *area* of a plane figure in $\mathbb{R}^2$, and the *volume* of a solid in $\mathbb{R}^3$. As I pointed out in the introduction, I will restrict here to $\mathbb{R}^1$ for it suffices to make the main points of the paper. With this restriction in place I suggest the following:[34]

$$Ext_\mu(x) = \mu(L(x)) = \mu(x) \tag{26}$$

EXTENSION$_\mu$

According to the (Lebesgue) measure-theoretic notion of extension[35]—hence the subscript μ—the extension of a spatial entity (in $\mathbb{R}^1$) is the (Lebesgue) measure

[33]This is the sigma algebra *generated* by the open sets of the standard topology of $\mathbb{R}^n$. A sigma algebra generated by a set S is defined as the smallest sigma algebra that includes S.

[34]This is where the ban on cross-categorical fusions enters the argument. Suppose x is exactly located at a region r, such that $\mu(r) \neq 0$. And suppose there is a cross-categorical mereological fusion of x and r. Call it w. Now, clearly, $L(w) = r$. Therefore, by Eq. 26, $\mu(w) = \mu(r)$. However, according to Lebesgue measure—under the assumption that x and r do not overlap, we have that $\mu(w) = \mu(x) + \mu(r) = \mu(L(x)) + \mu(r) = 2\mu(r) \neq \mu(r)$. Contradiction. If one wants to have cross-categorical fusions, perhaps because of the endorsement of full-blown unrestricted composition, one could—or perhaps should—insist that regions do not have exact locations, and then distinguish extension of regions, objects, and cross-categorical fusions of regions and objects.

[35]I will mostly omit the "(Lebesgue)" specification from now on.

(in $\mathbb{R}^1$) of its exact location.[36] Then, we can introduce the notions of Being Extended$_\mu$ and Being Unextended$_\mu$ as follows:

$$E_\mu(x) \equiv_{df} Ext_\mu(x) > 0 \equiv_{df} \mu(L(x)) > 0 \tag{27}$$

EXTENDED$_\mu$

$$\neg E_\mu(x) \equiv_{df} Ext_\mu(x) = 0 \equiv_{df} \mu(L(x)) = 0 \tag{28}$$

UNEXTENDED$_\mu$

In other words: an extended$_\mu$ entity is an entity that has Lebesgue measure $\mu > 0$.[37] It is immediately clear that this notion of extension does not suffer from the problems that were afflicting the mereological notion. We can easily "measure" the extension of a spatially extended entity. We can also easily define a general relation of *being less extended than* ($<^*_E$).[38] I am writing this down, for it will play a role in Section 7:

$$x <^*_E y \equiv_{df} \mu(L(x)) < \mu(L(y)) \tag{29}$$

Finally, we can also easily express that x is exactly n-times less extended than y, as: $\mu(L(y)) = n \cdot \mu(L(x))$.[39]

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[36] The definition assumes Functionality, which is controversial. Here is one way—not the only way—one can develop the main insight behind the definition in the absence of Functionality—that is, allowing for multilocated objects. For the sake of simplicity, let's stick to the case where x is exactly located at r_1 and r_2, with $r_1 \neq r_2$—the argument generalizes straightforwardly. One could *relativize* the attribution of the extension of x to its exact locations—indeed this is exactly the orthodox suggestion in the literature on multilocation. According to this proposal, x has an extension *relative* to r_1, and another *relative* to r_2. The most natural thing to do is (arguably) to identify the extension of x relative to r_1 with the extension of r_1, and the extension of x relative to r_2 with the extension of r_2. Let $\mu(x)_r$ stand for "the extension of x relative to (exact location) r". Then the suggestion is that $\mu(x)_{r_1} = \mu(r_1)$, and $\mu(x)_{r_2} = \mu(r_2)$. Thanks to an anonymous referee for pressing me on this point.

[37] We are working in $\mathbb{R}^1$. Generalizations to $\mathbb{R}^n$ might not be entirely straightforward. I am not considering them here for $\mathbb{R}^1$ is enough to make the main point of the paper, i.e. that there is a notion of extension, the measure theoretic one, that is extensionally not equivalent to the mereological one. But a general theory of extension should consider such generalizations. To see the challenges ahead, consider a one-dimensional region r, say the open interval (0,1). It is not difficult to see that r has Lebesgue measure $\mu = 0$ in $\mathbb{R}^2$. At this point, one might follow two strategies—I am not suggesting that one strategy is better than the other. Consider a spatial entity x and its exact location r. r has a particular dimension, say n. Then, according to the first strategy x has an extension only in $\mathbb{R}^n$, and in particular its extension in $\mathbb{R}^n$ is $\mu(x)_{\mathbb{R}^n}$. x is an extended entity iff its (only) extension is > 0. Go back to the example of a one-dimensional region r. Under the present proposal r simply does not have an extension in $\mathbb{R}^2$. It only has an extension in $\mathbb{R}^1$. In particular $\mu(r)_{\mathbb{R}^1} = 1$. Thus r is extended. According to the second strategy, x has an extension in every $\mathbb{R}^s = \mathbb{R}^1 \times ... \times \mathbb{R}^1$, where $s \geq n$. In particular, in $\mathbb{R}^s$, it has extension $\mu(x)_{\mathbb{R}^s}$. It is not difficult to see that for any $s > n$, $\mu(r)_{\mathbb{R}^s} = 0$. Thus, x will count as unextended in $\mathbb{R}^s$ with $s > n$. As a matter of fact $r = (0, 1)$ counts as unextended in $\mathbb{R}^2$. We could then define a notion of *extension simpliciter* along the following lines: $Ext(x) \equiv \exists y(y \sqsubseteq x \wedge \mu(y)_{\mathbb{R}^1} > 0)$. In other words, we say that x is extended iff it has at least a part that is extended in $\mathbb{R}^1$. $r = (0, 1)$ is unextended in $\mathbb{R}^2$, but it is extended *simpliciter*, insofar as it has a part whose extension in $\mathbb{R}^1$ is > 0. Developing these generalizations and alternatives goes beyond the scope of this paper.

[38] The superscript "*" will become important later on.

[39] As I will discuss later, another limitation of the mereological notion of extension is that it seems impossible to define an extended simple region. I want to briefly sketch an argument to the point that, broadly speaking, a metrical notion does not suffer from the same limitation. Clearly, this is not intended to be a fully fledged account—which will have to wait for another occasion. For the sake of simplicity, imagine we only have two *simple extended* regions r_1 and r_2, and let S be the set containing those regions, i.e. $S =$

According to Eqs. 27 and 28, being extended or unextended is predicated only of spatial entities, regions or things that are exactly located at regions.[40] As we saw, Pickup thinks that things that are not in space count, by default, as unextended. I disagree: what we should say in those cases is that the entity in question *has no extension*. That is to say, that the predicate "being (un)extended" simply does not meaningfully apply to the entity. By contrast, unextended entities are entities to which the predicate *does* meaningfully apply. They just have extension $= 0$.[41] To sum up: things without extension are not unextended things.[42]

4.2 Extension$_\sqsubseteq$ and Extension$_\mu$

I have introduced two notions of "being extended" and "being unextended", mereological notions $E_\sqsubseteq$ and $\neg E_\sqsubseteq$ in Eqs. 20–21, and metrical notions E_μ and $\neg E_\mu$ in

$\{r_1, r_2\}$. A simple sigma algebra over S is the power-set of S, i.e. $\sigma(S) = \mathcal{P}(S) = \{\emptyset, \{r_1\}, \{r_2\}, \{r_1, r_2\}\}$. Then we define $m : \sigma(S) \to \mathbb{R}^{\geq 0} \cup \{\infty\}$ as follows: (i) $m(\emptyset) = 0$; (ii) $m(r_1) = a$, (iii) $m(r_2) = b$; (iv) $m(S) = a + b$, where $a, b \in \mathbb{R}^{>0}$, say $a = 1$ and $b = 2$. The reader can check that m does qualify as a measure according to the definition in Section 4.1. Then, we can still define "being extended" as "having an exact location with measure $m > 0$", and "being unextended" as "having an exact location with measure $m = 0$". This is exactly in line with Eqs. 27 and 28 above. m is not μ, but we still get a *metrical notion* of extension.

[40] In fact, it is meaningful to apply extension only at spatial entities that have exact locations corresponding to Lebesgue-measurable sets. Some might argue that this a drawback. I happen to think this is one elegant way to dissolve seeming paradoxes of extension, such as the Banach-Tarski paradox. For a different take, see Meyer [35]. This goes beyond the scope of the paper.

[41] Alternatively, one may introduce notions of "being extended*" and "being unextended*". The former notion *can* be predicated of spatial entities, whereas the latter can be predicated only of non-spatial entities. On this construal, entities in general are either extended* or unextended*, and no question arises as to the measure of such extension*. Only extensions can be measured—and thus, compared. A *spatial entity* always counts as extended*. I have nothing against introducing these notions and adopting such a terminology. The point would be to recognize that extended* entities could be *either extended* and *unextended*, and their extensions can be measured. In the paper I maintained that an unextended entity is an entity to which the extension predicate—rather that the extension* predicate—can indeed apply in general partly because this is the standard usage for other notions. Massless particles are entities to which the mass predicate can be applied, they just have 0 mass. The same goes for chargeless particles—see e.g. Balashov [5].

[42] I am not claiming that Lebesgue measure is without its conceptual difficulties. Infinite sets—both countable and uncountable—might have measure 0. Arguably the most infamous example of an uncountable set of measure 0 is the so called *Cantor set*. A useful way to understand the Cantor set is to think of it as the remainder of the interval [0, 1] after the iterative process of removing open middle thirds is taken to infinity. For an accessible introduction see Abbott ([1]: §3.1). Another difficulty is that there are sets that are not Lebesgue-measurable. A famous example is the so-called Vitali set. Non-measurable sets can give rise to paradoxes of extension such as the Banach-Tarski paradox. Roughly, the paradox has it that one can cut a solid sphere into finitely many pieces, shift some of them, then rotate all the pieces by different angles and obtain two perfect copies of the original sphere. For an insightful discussion see Meyer [35]. These difficulties should be acknowledged. One might think that this is evidence that Lebesgue measure is problematic as an explication of our naïve, pre-theoretical conception of extension. Granted. But, first, it is at least controversial to claim that our pre-theoretical notion of extension is applicable to complex mathematical objects such as the Vitali set, or the Cantor set. Second, my point is not that we should use Lebesgue measure to explicate our pre-theoretical notion. My point is that we should use Lebesgue measure to beef up metaphysical discussions of extension. As a matter of fact, I might even concede that the mereological notion of extension is closer to our pre-theoretical understanding. I am not trying to replace the mereological notion. I am trying to make a plea for enriching our basic metaphysical toolkit when dealing with extended simples and unextended complexes.

Eqs. 27–28. What are the relations between these notions? Under the "assumption" that atomic regions of space, i.e., points, have Lebesgue measure 0 we have that:[43]

$$\neg E_{\sqsubseteq}(x) \rightarrow \neg E_{\mu}(x) \tag{30}$$

Claim Eq. 30 tells us that if something is exactly located at a point (thus being unextended according to mereological extension), it has Lebesgue measure 0 (thus being unextended also according to metrical extension).[44] Contraposing (30) one obtains that every extended$_{\mu}$ entity is extended$_{\sqsubseteq}$, and therefore mereologically complex. Indeed, as we saw in Section 2.1, according to the orthodox conception of space, every region is just a set of points, and points have Lebesgue Measure 0. In view of Countable Additivity, any region with countably many points has Lebesgue measure 0. It then follows that any extended$_{\mu}$ region has at least *uncountably* many points—and we now know, post-Cantor, that e.g., every line-segment contains indeed uncountably many points.[45]

The crucial result is that Eq. 31 below *fails*:

$$\neg E_{\mu}(x) \rightarrow \neg E_{\sqsubseteq}(x) \tag{31}$$

This simply follows from Countable Additivity. To appreciate this, consider any finite union, or any countable union of regions that have Lebesgue measure 0. Countable Additivity dictates that the Lebesgue measure of such unions is 0 as well. Any entity that is exactly located at those regions—the regions themselves in the first place—will qualify as metrically unextended. Yet they will not qualify as mereologically unextended, for their exact location is (massively) complex. The simplest case would be that of a region r composed of only two distinct points p_1 and p_2, $r = p_1 \cup p_2$. For it follows that $\mu(r) = 0$ and $\neg A(r)$. This provides a counterexample to Eq. 31 and its contraposition.

All this plays a crucial role in the characterization of extended simples and unextended complexes. Before turning to that, let me discuss—albeit briefly—other examples in which the notion of metrical extension itself, and the distinction between mereological and metrical extension can be fruitful.[46]

4.3 Notions of Extension and the Metaphysics of Objects

There are other debates in the metaphysics of material objects (and beyond) that crucially depend on the notion of *extension*. One prominent example is the metaphysics of persistence. The following is the by now orthodox construction. It is mostly due to Gilmore [17] and Parsons [36]. For the sake of simplicity, we assume

[43] Given the definition of μ we could indeed prove that, for any point p, $\mu(p) = 0$. The assumption is that space is constructed out from such points.

[44] It may be worth noting that Eq. 30 will fail if atomic regions of space have a measure > 0. In that case, being exactly located at a simple region of space would not even be *sufficient* for being unextended.

[45] Here one sees that the distinction between regions with countably many and uncountably many points plays a crucial role.

[46] Thanks to an anonymous referee for prompting the following discussion. One can skip Section 4.3 if they so wish. A disclaimer: it is not the purpose of the section to offer a fully-fledged, exhaustive discussion. Rather it is to provide initial evidence for the potential fruitfulness of metrical extension.

a standard picture where time is a one-dimensional manifold, constructed out of simple, unextended temporal atoms called *instants* with the topology of $\mathbb{R}^1$. First we define the *path* of an object x to be the union of its (temporal) exact locations. Let $R = \{r_i | x@r_i\}$. Then:

$$path_x \equiv_{df} \bigcup_{r_i \in R} r_i \tag{32}$$

PATH

Object x persists iff x's path is not instantaneous. Given what we assumed about temporal instants, this is equivalent to the fact that x's path is not atomic. This in turn simply means that a persisting object is something with a *temporally extended*$_{\sqsubseteq}$ path:

$$Pers_{\sqsubseteq}(x) \equiv E_{\sqsubseteq}(path_x) \tag{33}$$

PERSISTENCE$_{\sqsubseteq}$

Clearly, the arguments in the paper can be used to define another notion of persistence according to which something persists$_{\mu}$ iff its path is *temporally extended*$_{\mu}$:

$$Pers_{\mu}(x) \equiv E_{\mu}(path_x) \tag{34}$$

PERSISTENCE$_{\mu}$

Everything that persists$_{\mu}$ persists$_{\sqsubseteq}$ but the converse does not hold. This is important. Recent arguments in the (meta)physics of persistence can be read as the claim that relativistic objects persist$_{\mu}$, and therefore persist$_{\sqsubseteq}$, whereas quantum objects persist$_{\sqsubseteq}$ without persisting$_{\mu}$.[47]

Another debate where the notion of metrical extension can play a crucial role is in answering what Markosian [30] calls the *Simple Question*: what are the necessary and jointly sufficient conditions for an object to be lacking proper parts, that is, to be simple or atomic. We will soon encounter two influential answers to the Simple Question in Section 5. As of now, all that matters is that e.g., Tognazzini [47] argues that virtuous answers to the Simple Question should be neutral as to whether space is discrete, and, more importantly for us, neutral on the possibility that regions of discrete space are of "non-uniform shape and size" (Tognazzini [47]: 123). But we already saw that the mereological notion of extension can hardly be of use (by itself) to assign a size to different regions, let alone compare different sizes. By contrast, we saw that the metrical notion of extension scores highly on both respects.

A further example comes from the debate in the *metaphysics of receptacles*. This is a debate as to whether there are any constraints for regions to be *receptacles*, i.e., possible exact locations of objects. The most permissive view is the so-called *liberal view of receptacles*, defended in Hudson [23, 24]. According to the *liberal view*, *any region whatsoever*, in particular any region of any size can be a receptacle. Once again, this presupposes that we can assign extensions to different regions, an easy task for the metrical notion, a difficult one for the mereological one.

[47] See Gilmore [18] for the relativistic case, and Pashby [37] for the quantum case.

Finally, there are all the arguments and debates where comparative claims about extension are crucial—for as we saw, this is another instance where the mereological notion shows its limits. I will defer this discussion to Section 7.

5 Extended Simples and Unextended Complexes Revised

5.1 Extended Simples, Again

Back to extended simples and unextended complexes. The measure theoretic notion of extension discussed in Section 4 can be used to provide a novel characterization of extended simples. We all agree that an extended simple is a mereological atom that is (spatially) extended. I am suggesting that we can also cash out the extension requirement in measure-theoretic terms. This gives us the following:

$$\begin{aligned} ES_{\mu}(x) &\equiv_{df} A(x) \wedge E_{\mu}(x) \\ &\equiv_{df} A(x) \wedge \mu(L(x)) > 0 \end{aligned} \tag{35}$$

EXTENDED SIMPLE$_{\mu}$

The results of Section 4 have now a profound consequence on the debate over extended simples. For the very same arguments establish that:

$$ES_{\sqsubseteq}(x) \rightarrow ES_{\mu}(x) \tag{36}$$

does not hold. To further stress the point: an *atomic* spatial entity that is exactly located at $r = p_1 \cup p_2$ counts as an extended simple$_{\sqsubseteq}$, but not as an extended simple$_{\mu}$, thus providing a counterexample to Eq. 32. We do however get:

$$ES_{\mu}(x) \rightarrow ES_{\sqsubseteq}(x) \tag{37}$$

This is because any spatial entity that has a Lebesgue measure > 0 is exactly located at a region r that certainly is mereologically (extremely) complex—as we saw already. Extended simples, that is, both extended simples$_{\sqsubseteq}$ and extended simples$_{\mu}$, violate Arbitrary Partition in Section 2.3. Thus, extended simples$_{\mu}$ challenge *mereological harmony* as much as extended simples$_{\sqsubseteq}$. To see this, just note that both extended *simples*$_{\sqsubseteq}$ and extended *simples*$_{\mu}$ have mereologically *complex* exact locations.[48] This might pave the way to the following worry. Given that being metrically extended suffices for being mereologically extended, the philosophical interest of metrical extension is exhausted by the mereological consequences of metrical extension. This worry is unfounded—or so I contend.

The worry is significant only inasmuch as the philosophical interest of metrical extension is limited to questions about mereological harmony. But I don't see any compelling reason why this should be the case. For example, looking a little beyond the orthodox conception of space in Section 2.1, one can appreciate another limitation

[48] It should be noted that this is a consequence of the fact that simple regions of space, according to the orthodox framework of Section 2.1, are both unextended$_{\sqsubseteq}$ and unextended$_{\mu}$. Braddon-Mitchell and Miller [8] argue that extended simples need not violate Arbitrary Partition if atomic regions of space are extended.

of mereological extension. If that were the only notion of extension at stake, *extended simple regions* would turn out to be impossible. This is because mereological extension boils down to mereological complexity for regions. Yet, various philosophical arguments crucially depend on the possibility of extended simple regions, e.g., the ones in Tognazzini [47] and Kleinschmidt [26].

Let me start from the first. Tognazzini argues that the *possibility of discrete* space provides an argument against several influential answers to the Simple Question, the *pointy view* answer (PV),[49] and the *maximally continuous view* answer (MaxCon).[50] He explicitly writes:

> On one picture, the space atoms are still point-sized. On the other, *the space atoms themselves are extended*. In what follows, I will be concerned only with this second picture of discrete space. It is with the possibility of this type of discrete space that MaxCon and PV are inconsistent (Tognazzini [47]: 119, italics added).

As for Kleinschmidt [26], her argument is that no theory of location with only one primitive can accommodate her *place cases*—as she labels them. The first such case, the *Almond in the Void*, features

> *[A]n extended simple region* r, which contains an almond (and its parts) which is smaller than r, and r is otherwise empty (Kleinschmidt [26]: 122, italics added).

It is clear that the arguments above crucially depend upon the possibility of extended simple regions. Indeed, some broader claims in those arguments depend on *comparative claims about size*, claims such as "simples are located at the *smallest regions of space*" (Tognazzini [47]: 121) or as "the almond is smaller than the region it is contained in" (Kleinschmidt [26]: 122). And, as I argued already, this presents a tremendous challenge for the mereological notion of extension. By contrast, this is not the case for the metrical notion of extension in general. Nothing prevents simple regions to be extended in the metrical sense, for mereological complexity is not a necessary condition for metrical extension.[51] In effect, *atomic measures* could be used to define extended simple regions. A measure m is *atomic* iff every measurable set of positive measure contains a "metrical atom", a positive-measure set S_1 that has only 0-measure subsets:[52]

$$m(S_1) > 0 \wedge \forall S_2(S_2 \subset S_1 \rightarrow m(S_2)) = 0 \tag{38}$$

METRICAL ATOM

[49] Roughly the claim that, necessarily, x is simple iff x is a point-like object. Note that unextended complexes provide an (alleged) counterexample to PV.

[50] Roughly the view that, necessarily, x is simple iff x is a maximally continuous object.

[51] One might worry that this trivializes the point. One can always find an atomic measure according to which a given region is extended. But this misses the point. The point is simply that the metrical notion of extension allows us to define extended simple regions, whereas the mereological notion does not.

[52] These sets are called "metrical atoms". Consider the *counting measure* in Section 3.3. Any singleton set $S = (\{n\}|n \in I_i)$ is a metrical atom.

A toy example of one such atomic measure is in footnote 39. It can be used to define extended simple regions.[53] The arguments in Tognazzini [47] and Kleinschmidt [26] could be run using metrical extension. This is one significant example where, in general, the divergence between the mereological and the metrical notion of extension has significant philosophical payoffs. And I will return to the case of comparative claims about different extensions in Section 7.

Furthermore, as we will see, the case of Unextended Complexes is significantly different from that of Extended Simples. In that case, the divergence of the metrical and mereological notion of extension plays a crucial role when assessing the metaphysical possibility of Unextended Complexes.

5.2 Unextended Complexes, Again

My take on unextended complexes parallels the one for extended simples. Unextended complexes are spatial entities that are mereologically complex and are (spatially) unextended. If we cash out spatial extension in measure-theoretic terms we have that:

$$\begin{aligned} UC_{\mu}(x) &\equiv_{df} \neg A(x) \wedge \neg E_{\mu}(x) \\ &\equiv_{df} \neg A(x) \wedge Ext_{\mu}(x) = 0 \equiv_{df} \neg A(x) \wedge \mu(L(x)) = 0 \end{aligned} \tag{39}$$

UNEXTENDED COMPLEX$_{\mu}$

The point is that the following does not hold:

$$UC_{\mu}(x) \rightarrow UC_{\sqsubseteq}(x) \tag{40}$$

That is to say, a spatial entity can be an unextended complex$_{\mu}$, without thereby being an unextended complex$_{\sqsubseteq}$. The simplest case in point is always the same. A *complex* spatial entity that is exactly located at $r = p_1 \cup p_2$ is an unextended complex$_{\mu}$ that is not an unextended complex$_{\sqsubseteq}$. On the other hand the converse of Eq. 40 holds:

$$UC_{\sqsubseteq}(x) \rightarrow UC_{\mu}(x) \tag{41}$$

The case of unextended complexes is different from the case of extended simples, for it turns out that unextended complexes$_{\sqsubseteq}$ and unextended complexes$_{\mu}$ violate very different principles of location. In effect, unextended complexes$_{\mu}$ do not violate *any* of these principles. This makes a substantive difference when it comes to their metaphysical possibility—as I am about to argue.

6 The Metaphysical Possibility of Unextended Complexes

As we saw, according to Pickup there are two kinds of unextended complexes$_{\sqsubseteq}$: (a) mereological complexes that don't have any location in space —e.g., mereologically

[53] The Lebesgue measure is not atomic. However, I will put forward a suggestion that uses the Lebesgue measure in Section 7.

complex abstract entities, and (b) mereologically complex spatial entities that are exactly located at spatial points. These are "pointy complexes".

I briefly pointed out what I take to be misleading about case (a). I think that the right thing to say in such a case is that the entities in question lack extension, not that they are unextended. An unextended entity is *not* an entity without an extension: in fact, it has a very precise metrical extension. This leaves case (b), i.e., that of pointy complexes. Here is McDaniel:

> [T]he argument is as follows: (1) co-located point-sized objects are possible; (2) if co-located point-sized objects are possible complex point-sized objects are also possible (McDaniel [33]: 239).

McDaniel is explicit in grounding the metaphysical possibility of unextended complexes$_\sqsubseteq$ in the metaphysical possibility of co-location. This already rules out unextended complex$_\sqsubseteq$ *regions* for Functionality entails Non-Colocation for regions. Pickup [39] discusses co-location as well, but he also adds a new interesting spin:

> [H]ow does the pointy complex occupy the point it is at? (...) The point is occupied by each of the proper parts of the entity: these parts are all exactly located at the point. On this alternative the parts are co-located. Or, secondly, the point could be *spanned*: the whole pointy complex could be located at the point without any of the parts of the entity having locations at all (...) On this alternative, the parts have no location (Pickup [39]: 260).

In the passage above Pickup notes yet another possibility for a pointy complex to occupy a spatial point: by having parts that have *no* exact location. It is interesting to note that, on this second alternative, Non-Colocation is *not* violated. Rather, both Exactness and Expansivity are.[54] Now, at this point one might suspect that a theory of location that features Exactness begs the question against this possibility. Exactness may be problematic, so that we might indeed prefer a theory of location that does not have it among its axioms/theorems. But, even in the absence of Exactness, Pickup's second alternative still violates Expansivity, for crucially the pointy complex has an exact location, as Pickup explicitly acknowledges.

It is difficult to evaluate the arguments in favor of the possibility of unextended complexes$_\sqsubseteq$ *vis-a-vis* the possible violations of different principles of location. I will just note that whereas Non-Colocation and Exactness are subject to possible serious counter-examples, to my knowledge almost nobody in the literature is ready to give up Expansivity.[55] I don't want to get into these details here, for they would lead us astray.

[54] Assuming that proper parts are *in* space, that is, at least *weakly located* at a region. This could of course be denied. Markosian [31] endorses a principle according to which every material object has an exact location. The case at hand would violate such a principle too. Note that, if x has an exact location, it also has a weak location. This assumes that the parts of the pointy complex are themselves material objects.

[55] Saucedo [41] might be the only relevant exception. In the absence of Exactness one might formulate a weaker version of Expansivity, WEAK EXPANSIVITY as follows: $x \sqsubseteq y \wedge y @_\circ r \rightarrow \exists r_2 (x @_\circ r_2 \wedge r_2 \sqsubseteq r_1)$. Note that the scenario described by Pickup [39] does not violate Weak Expansivity. I owe this suggestion to an anonymous referee for this journal.

This discussion is however useful in dispensing with yet another argument in favor of the possibility of unextended complexes$_\sqsubseteq$ in Pickup [39]:

> [H]ow could it be that something located at a single point of space has proper parts? But I contend that it is no stranger than the extended simple case: (...) until a reason is given why pointy complexes are worse off than extended simples, we should treat their possibility equally (Pickup [39]: 259).

I find this wanting. Extended simples[56] and unextended complexes$_\sqsubseteq$ violate very different principles of location. Extended simples violate Arbitrary Partition; unextended complexes$_\sqsubseteq$ violate either Non-Colocation, or Exactness, or Expansivity. One might have very different attitudes towards these principles. And different attitudes towards these principles will warrant different attitudes towards the metaphysical possibility of the relevant entities that constitute a counterexample to them.

As I pointed out in Section 5, unextended complexes$_\mu$ need not be unextended complexes$_\sqsubseteq$. What about *their* metaphysical possibility? The simplest argument I can think of is the following: if the orthodox construction of space in Section 2.1 is on the right track, they are *actual*, therefore they are metaphysically possible.

Consider any countable union of regions with Lebesgue measure 0. Call it r: r is an example of an unextended complex$_\mu$, given Countable Additivity. And the existence of r is guaranteed by the existence of Lebesgue measure 0 regions, and the fusion axioms in Section 2.2.

What about unextended complexes$_\mu$ that are material objects? An argument in favor of their metaphysical possibility runs as follows: (i) material objects that are exactly located at regions of Lebesgue measure 0 are metaphysically possible; (ii) mereological fusions of such objects are metaphysically possible; therefore unextended complexes$_\mu$ that are material objects are metaphysically possible. Claim (ii) follows from the fusion axioms in Section 2.2, and the claim that existents are metaphysically possible.[57] So, the crux of the argument lies in premise (i). Now, one may think that material objects that are exactly located at (some) regions of Lebesgue measure 0 are *physically impossible*. Here is Simons:

> [H]owever such point particles are *physically impossible*, because they would have to have infinite density, being a finite mass in a zero-volume (...) Therefore there can be no point-particles (Simons [44]: 373, italics added).

This would not yet tell against their *metaphysical* possibility. I am not sure what novel argument I can give in favor of *that*. One such argument has been provided by Hudson [23] in his defense of the *liberal view of receptacles*, which we already encountered.[58] He writes:

[56]That is, both extended simples$_\sqsubseteq$ and extended simples$_\mu$.

[57]Even in the absence of these axioms, I suspect that one should grant such a possibility. As far as I can see, only those who believe in the *metaphysical necessity* of *mereological nihilism* are in a position to object to (ii).

[58]For an argument against the metaphysical possibility of point-sized objects, see Giberman [16]. Thanks to an anonymous referee here.

> Since I believe that any region is a receptacle, I am willing to acknowledge the possibility of open, closed, and partially-open material objects of *all sizes, shapes, and surfaces*—including 3-dimensional solids, 2-dimensional plane-walls and sphere-shells, 1-dimensional ribbons and poles, and 0-dimensional grains and fusions-of-countably-many-grains (Hudson [23]: 432–433, italics added).

Here, I shall be content with just pointing out that the arguments in the literature already *assume* that e.g., point-sized particles are indeed metaphysically possible. They then go on to claim that *co-located* point particles are possible. Insofar as the argument in favor of the possibility of unextended complexes$_\mu$ that are material objects is not hostage of the (controversial) possibility of co-location, it is a much stronger argument. This is yet another instance of the philosophical significance of the divergence between the mereological and metrical notion of extension.

Now, unextended complexes$_\sqsubseteq$ present a challenge to mereological harmony: they are *complex* entities with a *simple* exact location. Unextended complexes$_\mu$ on the other hand do not. They are complex entities, and their exact location is complex as well. There might be a worry that this renders unextended complexes$_\mu$ metaphysically *less interesting* than their mereological counterparts. If the metaphysical interest of unextended complexes were exhausted by their alleged challenge to mereological harmony that would perhaps be the case. But, once again, I don't think that it is. Unextended complexes$_\mu$ present a formidable challenge to our naïve conception of extension. Consider the set S of rational numbers between 0 and 1, i.e. $S = \{x \in \mathbb{Q} | 0 \leq x \leq 1\}$. This set is *dense* (in $\mathbb{R}$). And yet it has measure 0. Let me try to convince you that this is in fact challenging. Imagine you could take a walk—this is just a metaphor—on the *rational* line—from 0 to 1. S is dense: you will always step on a rational number, you will never have to jump. You can just *leisurely stroll* along the rationals from 0 to 1. When you get to 1, you look back, and you wonder how long is the path you took, the answer is 0! In fact, you can walk the entire infinite rational line, and you still would have walked a path of length 0, for $\mu(\mathbb{Q}) = 0$. How is this challenge to our naïve notion of extension not interesting? Note that, in this respect, it is unextended complexes$_\sqsubseteq$ that are *less interesting*. For it is arguably neither interesting nor surprising that an object that is exactly located at a spatial point is unextended.

There is one final point I'd like to discuss—albeit briefly—concerning the difference between unextended complexes$_\sqsubseteq$ and unextended complexes$_\mu$. Recently some philosophers have explored—if not endorsed—reductive accounts of mereology, roughly along the following lines: $x \sqsubseteq y \equiv_{df} L(x) \subseteq L(y)$. They include Markosian [31], McDaniel [34] and Calosi [9]. As McDaniel explicitly acknowledges such accounts are incompatible with the existence of unextended complexes$_\sqsubseteq$. However they are not incompatible with unextended complexes$_\mu$, for the obvious reason that unextended complexes$_\mu$ are not a threat to mereological harmony. By distinguishing the mereological and metrical notion of extension, those who endorse such reductionist accounts can accept a substantive notion of unextended complexes.

7 On "Being Less Extended Than"

In the broadest sense, the paper offers a plea to introduce a notion of metrical extension along with the mereological one. This notion can be used to do some real work—as in the case of extended simples and unextended complexes, and in the cases discussed in Section 4.3. It also sheds new light on the limits of the mereological notion. Recall Section 3.3. There I claimed that, even if we were to introduce the notion of "being less extended than" ($<_E$), it was even problematic to claim that if x is a proper part of y, then x it less extended than y:

$$x \sqsubset y \rightarrow x <_E y \tag{42}$$

To see this, suppose we *define* $<_E$ simply as $<_E^*$ in Eq. 29:

$$x <_E y \equiv_{df} \mu(L(x)) < \mu(L(y)) \equiv_{df} x <_E^* y \tag{43}$$

Given definition (43), it is easy to find counter-examples to Eq. 42. The counterexample is in fact, always the same. Take two distinct points p_1 and p_2. Clearly we have that $p_1 \sqsubset p_1 \cup p_2$,[59] and yet $\mu(p_1) = 0 = \mu(p_1 \cup p_2)$, *contra* (42).

This is already enough to see that some recent claims in the literature are problematic. For example, Baron [6] contains an argument—the *Argument from Size*—against the thesis that all non-fundamental physical objects, regions included, are composed of fundamental physical objects. The argument crucially relies on the following:

> [S]maller Than: For any x and y, x is smaller than y iff there is a region r at which x is exactly located that is a proper subregion of the region r^* at which y is exactly located (Baron [6]: 391).

One immediate problem is that the left-to-right direction of "Smaller Than" entails that this speck of dust fluttering over my desk is not smaller than the Cliffs of Dover, for they are mereologically disjoint. Now, perhaps "Smaller than" is more charitably interpreted as a conditional claim, the condition being that x is part of y. If so, it shows that we cannot really use it to provide a *definition* of the "being less extended than" relation. Be that as it may, the argument above spells also trouble for the right-to-left direction. Depending on the definition of "being less extended than", it provides a counterexample to such direction. Suppose that x is exactly located at p_1 and y, the fusion of x and z, is exactly located at $p_1 \cup p_2$. Then, x is exactly located at a proper subregion of the exact location of y but they have the same extension.

This discussion is also significant for another argument we already encountered. Recall place-cases in Kleinschmidt's [26]. A crucial element in one such case, the *Almond in the Void*, is that the almond (a) is "smaller" than the extended simple region (r) it is contained in. Kleinschmidt does not provide any detail on how to characterize the notion of "smaller than", and relies instead on a somewhat intuitive understanding. I want to suggest something based on the arguments in the paper. First, endorse something like the following principle of *Duplicate Extension*: For any x and y, is x is a a duplicate of y, then x has the same extension as y. Next, one

[59] This is equivalent to $p_1 \subset p_1 \cup p_2$.

considers a duplicate of the almond, call it a^*, located in a space described by the orthodox construction in Section 2. The same for a duplicate of r, r^*—modulo details about their mereological structures. Then, one uses Eq. 42 to claim that the $a^* <^*_E r^*$ insofar as $\mu(L(a^*)) < \mu(L(r^*))$. Finally, one uses *Duplicate Extension* to claim that it follows that $a <^*_E r$, which is exactly what we were after.

What seems clear at this juncture is that we do have two notions of "being less extended/smaller than", $<_E$ and $<^*_E$. Once again, there seems to be (at least) two options. One option is to *discard* the mereological notion of extension altogether, and then stick to definition (39) for the relation of "being less extended than". The other is to retain the mereological notion alongside the metrical one, and consequently *two* notions of "being less extended than", $<_E$, and $<^*_E$.

As I already pointed out a few times, I favor the latter option.[60] In this case we use Eq. 29 to define only $<^*_E$. Then we could e.g., claim that Eq. 42 is an axiom that regiments one notion of "being less extended than" ($<_E$) but not the other ($<^*_E$). It then becomes a substantive question what is the interaction between these notions. To conclude, I want to discuss such interaction. First I want to argue that

$$x <_E y \rightarrow x <^*_E y \tag{44}$$

does *not* hold. Let me first introduce another property of the Lebesgue measure, namely *Monotonicity*. For any two sets S_1 and S_2, such that $S_1 \subset S_2$ we have:

$$\mu(S_1) \leq \mu(S_2) \tag{45}$$

with equality holding *iff* S_1 and S_2 differ for a set of 0 measure. Given all this, it is clear that we can have counterexamples to Eq. 44. Let $\bar{x}_y$ stand for the relative mereological complement of x with respect to y, that is, the mereological fusion of the parts of x that do not overlap y. Now consider x and y such that (i) $x \sqsubset y$ and (ii) $\mu(\bar{x}_y) = 0$ both hold. Given Eq. 42 and (i), it follows that $x <_E y$. Yet, given *Monotonicity* $\mu(x) = \mu(y)$, so that $x <^*_E y$ does not hold, *contra* (44).

Does the converse of Eq. 44, i.e.,

$$x <^*_E y \rightarrow x <_E y \tag{46}$$

hold? Interestingly enough, this cannot be answered in full generality. All we know about $<_E$ is that it obeys (42). This is enough to conclude that, in the case in which

[60] As a matter of fact, I am open to the possibility that we should introduce *even further* notions of extension. One possible such notion is based on *metric spaces*. In a nutshell, the thought is the following. A *metric space* (S, d) consists of a non-empty set S and a function $d : S \times S \rightarrow \mathbb{R}^{\geq 0} \cup \{\infty\}$ such that d respects the following conditions: (i) d is *positive*, that is, for all x and y either $d(x, y) > 0$ or $x = y$; (ii) d is *symmetric*, i.e. $d(x, y) = d(y, x)$; and (iii) d respects the *Triangle Inequality*, that is, for all x, y, and z, $d(x, y) \geq d(x, z) + d(z, y)$. Then we could define a *metric-space* notion of extension in the following way: a region r is *extended$_{met}$* iff there are two points $p_1, p_2 \in r$ such that $d(p_1, p_2) \neq 0$. A fully-fledged development of this notion of extension and its relations to both the locational and the measure theoretic notion clearly deserves an independent scrutiny. Baron and LeBihan [7] is not completely explicit, but it makes use of metric-spaces. Relatedly, Goodsell et al. [20] use a somewhat similar notion of extension to define an explicitly *extrinsic* notion of extended simple region. Note that my proposed definition is *intrinsic* instead. A comparison between these notions of metric-spaces notions of extension will have to wait for another occasion. Note that neither Baron and LeBihan [7] nor Goodsell et al. [20] even mention measure theory.

$x \sqsubseteq y$, Eq. 46 holds. For it follows from *Monotonicity* and $x <^*_E y$ that x is a *proper part* of y—not just a part—whose complement has positive measure. This is enough to entail $x <_E y$, by Eq. 42. But it is not enough to derive Eq. 46 in its full generality. One needs to provide more details on $x <_E y$, and show that they are enough to prove Eq. 46. Alternatively, one can assume it as an axiom—a quite plausible one. All this should be taken into account in the development of a truly *general theory of extension*.

Acknowledgments I would like to thank two anonymous referees for this journal for their insightful suggestions which improved the paper greatly. For discussions on previous drafts of the paper I would like to thank Fabrice Correia, Alessandro Giordani, Gonzalo Rodriguez-Pereyra, and Achille Varzi. This work has been funded by the Swiss National Science Foundation, (SNF), project number PCEFP1_181088.

Author Contributions I am the sole author of the manuscript.

Funding Open access funding provided by University of Geneva. Swiss National Science Foundation, (SNF), project number PCEFP1_181088.

Availability of Data and Material There is no data material.

Code Availability There is no code.

Declarations

Ethical Approval N/A

Competing Interests I have no competing interests.

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