The Review of Symbolic Logic
Volume 19, Number 1, March 2026

TO MAKE IT WHOLE: PARTS, FUSIONS, AND LOCATIONS

CLAUDIO CALOSI
Ca' Foscari University

Abstract. The present paper first distinguishes three different ways in which the logic of parthood and composition on the one hand, and the logic of location on the other might interact. It then goes on to explore several relations between location and composition. In doing so it (i) sheds new light on recent results and (ii) proves new substantive ones along the way.

I make it whole. This wholeness means that it has lost its power to hurt me; it gives me, perhaps because by doing so I take away the pain, a great delight to put the severed parts together.

V. Woolf, *The Waves.*

§1. Three grades of local involvement. There is space and spatial regions.[1] There is the material world and material objects. Both space and the material world have mereological structure. The mereological structure of spatial regions is often taken to be well understood. It's a model of classical extensional mereology.[2] By contrast, the mereology of material objects seems elusive. But material objects are *located* in space. What if we could use *location* to transfer, so to speak, the mereological structure of regions to objects? One initially tempting thought is that the mereological structures of regions and objects perfectly mirror each other. They are in complete and perfect harmony. Unsurprisingly, this is known in the literature as *Mereological Harmony*.[3] Mereological Harmony is but one example of how the logic of parthood and composition and the logic of location might interact.[4] We shall see examples of harmony principles throughout the paper, but as of now I want to distinguish *three grades of local involvement*, three ways in which locative notions play a somewhat

Received: February 12, 2025.
2020 *Mathematics Subject Classification*: Primary 08-XX.
Key words and phrases: composition, location, fusions, subregion theory of parthood, regionalism, partition.

1 Or spacetime. For the purpose of the paper this will not matter, so I will stick (mostly) to space.

2 For an introduction see Gilmore, Calosi, and Costa [11].

3 See, e.g., Leonard [14], Saucedo [18], and Uzquiano [19]. Varzi [21] calls it *Mirroring*.

4 I am using "logic" here in a very broad sense. Some may prefer to insist that there are only *theories* of parthood and location, not *logics* of them, insofar as parthood and location are relations rather than operators. I am fine with both terminological choices, as they will not affect the substance of the results in the paper.

doi:10.1017/S1755020325101020

substantive role in the mereology of objects. Abusing terminology, I will use "spatial mereology" and "material mereology" to refer to the mereology of space and the material world respectively. Let me stipulate that, e.g., a claim (principle, ...) is a purely mereological claim (principle, ...) iff it contains only mereological (and logical) vocabulary, and that a claim (principle, ...) is mereo-locational iff it contains mereological and locational (and logical) vocabulary. This might be somewhat vague but it is enough for our (initial) purposes. Then we can distinguish at least three grades of local involvement:[5]

> *First grade of local involvement*: the use of spatial mereology and location to *define* notions of material mereology.
>
> *Second grade of local involvement*: the use of spatial mereology and location to provide substantive *mereo-locational answers* to questions about material mereology.
>
> *Third grade of local involvement*: the use of location and locational principles to provide *purely mereological answers* to questions about material mereology.[6]

A few examples might help. These examples are not chosen randomly. They will keep us occupied for the rest of the paper. Arguably, the most debated example of *First Grade* is the so-called *subregion theory of parthood*.[7] This is roughly the following view:

> *Subregion Theory of Parthood*: For any material object x and y, x is part of y iff the location of x is part of the location of y.

But it is not the only example. Cartwright, in his classic paper on scattered objects,[8] suggests the following definition of *fusion* for material objects:

> *Locative Fusion*: For any material object x and set of material objects M, x fuses M iff "the region occupied by x is the smallest receptacle [i.e., location] that includes the receptacles occupied by the members of M" [7, p. 161].

And the literature on mereological harmony can be mined for other illustrative examples, as we shall see in due course. As for the second and third grades of involvement, it is instructive to recall the infamous special composition question.[9] One of its (many) formulations asks what are the necessary and sufficient conditions for material objects to compose.[10] Recently a particular answer has attracted significant attention. Said answer is known as *Regionalism*:

5 I do not mean to suggest that the following list is exhaustive.

6 A locational principle is simply a principle of a (given) theory of location. Note that many *locational principles* are *mereo-locational* claims, in that they feature both locational and mereological vocabulary.

7 The subregion theory of parthood has an impressive philosophical pedigree, starting with, arguably, Plato. In contemporary philosophy it can be traced back to the classic (Oppenheim and Putnam [16]).

8 Cartwright [7]. The following label is mine.

9 van Inwagen [20].

10 It is not the only formulation because some philosophers, e.g., Markosian [15], think of the special composition question as putting forward an *explanatory* request. It would not enough to provide biconditionals to answer it. One needs to provide, e.g., grounding claims to the effect that some material objects compose *because*—[insert your preferred answer here].

https://doi.org/10.1017/S1755020325101020 Published online by Cambridge University Press

Regionalism: Some material objects compose iff there is something located at the fusion of the regions at which the objects are located.

Even in this vague formulation, it should be clear that regionalism counts as a *mereo-locational* claim. It is a claim which crucially involves both mereological and locational vocabulary. As I said, regionalism is advertised as an answer to the special composition question, a paradigmatic question of material mereology, if any.[11] In the light of this, it seems a clear example of *Second Grade*.[12] By contrast, consider the following purely mereological answer to the special composition question:

Underlap: Some material objects compose iff there is something of which they are all parts.

Underlap is a purely mereological claim, in that it contains only mereological (and logical) vocabulary. An example of *Third Grade* would then be an example of a locational principle that entails Underlap. Once again: I did not choose these examples on a whim. I will argue that there is one such locational principle, the so-called *Arbitrary Partition* principle. This is, I contend, a substantive result on its own. In effect, once the distinction between different grades of local involvement is in place, a natural question arises. Suppose one endorses a "first grade thesis"—to abuse terminology once more. Does this force any second or third grade theses? The rest of the paper explores particular instances of this general question. In doing so not only it provides a critical assessment of some recent results in the literature, but it also proves new substantive (and surprising) ones along the way. All this sheds new light on the relations between the logic of parthood and composition and the logic of location.

Let me be upfront. I intend neither to defend nor to endorse any of the mereological and locational principles used in the arguments below. Gilmore and Leonard, in their recent discussion of *Regionalism*, write: "Our goal is to chart connections, not to proselytize" [12, p. 172]. I have the same goal.

§2. Formal framework. Let me then be explicit about the formal framework I will be using in the rest of the paper. This is basically the framework used in Gilmore and Leonard [12]—with minor alterations. This is mostly for pragmatic reasons. The next section provides a critical assessment of some of their results about regionalism. It's therefore easier to stick to their formalism as closely as possible. To that end, I will work with first order plural logic with (plural) identity, together with the definite description operator ι. Single signs such as x, o, and r stand for singular terms (constants and variables), whereas double signs such as xx, oo and rr stand for plural ones. To this we add a few special predicates. The first three such predicates are the plural logic predicate "being one of" ($\prec$, as in "x is one of the xx"), the mereological predicate "parthood" ($\leq$, as in "x is part of y"), and the locative predicate "exact location" (@, as in "x is exactly located at y"). Then there are two other special predicates, M for "being

[11] I will discuss (some) relations between *Locative Fusion*, the *Subregion Theory of Parthood*, and *Regionalism* in §3.5.

[12] I took the *subregion theory of parthood* as an example of *First Grade* involvement, because I took it to be a *definition* of material parthood—on this see also footnote 16. But this is not the only possibility. One could take it as a *substantive principle* about material parthood that falls short of providing a definition. In this case it would rather be an example of *Second Grade* involvement. Thanks to a referee for this journal.

a material object," and R for "being a spatial region." Starting from the first special predicate. Standard plural logic with (i) no empty plurality, (ii) extensionality, and (iii) comprehension axioms is assumed throughout. As for the other two, I will discuss some mereological and locational details at length. As of now, we simply need to recall orthodox mereological definitions: (i) x is an extension of y ($\geq$) iff y is part of x, (ii) x is a proper part of y ($\ll$) iff it is a distinct part of y, (iii) x and y overlap ($\circ$) iff they share a part, and (iv) x and y are disjoint iff they do not overlap. The attentive reader probably noticed I did not talk about mereological fusion. It is well known that there are many inequivalent notions of fusion. Cotnoir and Varzi [10] distinguish three of them. Calosi and Giordani [4, 5] go even further and distinguish four. These differences will play a crucial role later on. At this stage it is worth recalling one particular definition. From now on, for the sake of readability, let "$xx \leq y$" abbreviate "$\forall x(x \prec xx \rightarrow x \leq y)$," and "$x \circ xx$" abbreviate "$\exists y(y \prec xx \wedge x \circ y)$." Then:

(D.1) $F_1(y, xx) \equiv_{\text{df}} xx \leq y \wedge \forall x(x \leq y \rightarrow x \circ xx)$. FUSION$_1$

As for location, I assume that @ is a *total injective function* from M to R.[13] We can thus assign to each material object M its unique exact location via:

(D.2) $\ell(x) \equiv_{\text{df}} \iota y(x@y)$. EXACT LOCATION OF x

Similarly for xx, that is, $\ell\ell(xx)$ stands for the exact locations of the xx. Besides these principles I assume that (i) everything is either a material object M or a region R; (ii) only objects are located at regions; (iii) both M and R are closed under parthood and fusions, i.e., parts and fusions of M are M—*ditto* for R; and (iv) regions are a model of *classical extensional mereology*, whereas material objects are a model of the *extensional mereology* EM. That is, $\leq$ is an extensional partial order. The difference between material and spatial mereology is that unrestricted composition holds for the latter. There is no fusion axiom for the former. A few words on two principles that will be used explicitly in the following proofs. The fact that $\leq$ is extensional means that it obeys:

(P.1) $\neg y \leq x \rightarrow \exists z(z \leq y \wedge \neg z \circ x)$. STRONG SUPPLEMENTATION

This yields that, when fusions exist, they are unique. The fact that @ is injective means that it obeys the following ANTI-COLOCATION principle:

(P.2) $x@y \wedge z@y \rightarrow x = z$. ANTI-COLOCATION[14]

[13] This is a substantive assumption. For *totality* entails that every material object has an exact location. This is famously problematic. For instance a point-particle in gunky space might fail to have an exact location (see, e.g., Parsons [17], Correia [8] and Gilmore, Calosi, and Costa [11]). This would put pressure on the *subregion theory of parthood* insofar as, presumably, that particle could be part of something else.

[14] ANTI-COLOCATION deserves a brief discussion. The possibility of colocaton might be taken to provide an objection to the subregion theory of parthood—or better, to its necessitation. Suppose x and y are distinct atoms exactly co-located at a given spatial point. The subregion theory of parthood entails $x = y$ *contra* our assumption. Markosian however notes that

> [T]he view is consistent with there being two colocated objects, as long as the two objects in question stand in all the same mereological relations to

https://doi.org/10.1017/S1755020325101020 Published online by Cambridge University Press

§3. From first to second grade: The debate over regionalism.

3.1. Formulations. The recent debate over regionalism boils down to whether regionalism follows from the subregion theory of parthood. This is why the debate can be thought of as an illustration of the general question introduced in §1. It amounts to asking whether a first grade thesis (here: the subregion theory of parthood) entails a second grade thesis (here: regionalism).

Markosian [15] claims the answer is in the positive: the relevant entailment holds. Korman and Carmichael [13] agree.[15] Gilmore and Leonard [12] recently challenged that conclusion. In the light of §2, the subregion theory of parthood translates into:[16]

(P.3) $x \leq y \leftrightarrow \ell(x) \leq \ell(y)$. STP

As for regionalism. Let me simplify the discussion if only slightly and take composition$_i$ to be the converse of fusion$_i$.[17] This is a simplification because one may take composition to be fusion by pairwise disjoint elements, as, e.g., Gilmore and Leonard do.[18] In which case it would be strictly stronger than composition as converse of fusion.[19] Let me then stipulate that "$C_1^{\exists}(xx)$"—the xx compose$_1$—abbreviates "$\exists y(F_1(y, xx))$," and that "$@L_1(xx)$"—some material object is located at something which is the fusion$_1$ of the locations of the xx—abbreviates "$\exists y \exists z(M(y) \wedge R(z) \wedge F_1(z, \ell\ell(xx)) \wedge y@z)$." Then regionalism can be formulated as follows:

(P.4) $C_1^{\exists}(xx) \leftrightarrow @L_1(xx)$. REGIONALISM$_1$

other objects. So what the STPer has to deny is the possibility of colocation without (what we can call) *comereology* [15].

Be that as it may, I will assume ANTI-COLOCATION throughout.

15 See also Baron, Miller and Tallant [1].

16 This is the same as Uzquiano's [19] principle *Part*, and Leonard's [14] $\mathrm{MH}_{1\leq}$. One may protest that **(P.3)** falls short to provide a definition. Point taken. One should strengthen it as follows: $x \leq y \equiv_{\mathrm{df}} \ell(x) \leq \ell(y)$. I take that this simply entails **(P.3)**, and I will stick to the weaker formulation.

17 I am using "schematic" formulations to be sensitive to different, (inequivalent) notions of fusion.

18 To be precise, Gilmore and Leonard [12] *first* define composition, *then* define fusion as F_1. And they actually use "fusion" explicitly only relative to regions. Their definition of composition—which they take from Markosian [15], and which traces back to van Inwagen [20]—is the following:

> "y is composed of $xx =_d f$ (i) every two of xx are disjoint, (ii) each of xx is a part of y, and (iii) each part of y overlaps at least one of xx" [12, p. 163].

Clause (ii) and (iii) are equivalent to "having an F_1." One gets different notions of composition whenever one replaces clauses (ii) and (iii) with different inequivalent ones. To anticipate, I will define composition$_2$ by replacing (iii) with with (iv): "y is part of everything that has all the xx as parts." The conjunction of clauses (ii) and (iv) is equivalent to "having an F_2" as defined in **(D.3)**. As we shall see in some detail, F_1 and F_2 are equivalent for regions because regions are a model of classical extensional mereology by assumption. But they are not equivalent for objects, because they are a model of a weaker mereology. Thus, for objects, different notions of composition diverge. And this is enough to run the arguments in §3.4.

19 This is actually crucial in some of the arguments I will *not* discuss. However, it does not matter for the arguments in the paper.

https://doi.org/10.1017/S1755020325101020 Published online by Cambridge University Press

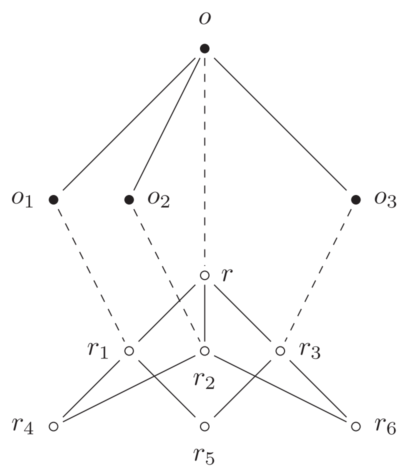

Figure 1. Countermodel 1.

That is to say, the xx compose iff a material object is exactly located at the fusion of the xx's exact locations, as desired.[20]

The general argument is that REGIONALISM$_1$ does not follow from STP—not even in the presence of the substantive assumptions in §2. The following countermodels—where I used o-terms for objects and r-terms for regions for the sake of readability—show as much.

3.2. First countermodel. The first countermodel is presented in Figure 1. Full lines represent proper parthood (going uphill) and dashed lines represent exact locations (going downhill):

The argument is as follows. In Figure 1 the subregion theory of parthood is satisfied but regionalism is not, thus undermining the relevant entailment. To see this, note that r is the F_1 of r_1 and r_2, but o_1 and o_2 do not have an F_1 and therefore do not compose anything. Given that o is exactly located at r, there is an object exactly located at the region composed by the regions at which o_1 and o_2 are exactly located but o_1 and o_2 do not compose, contra the right-to-left direction of REGIONALISM$_1$.

3.3. Second countermodel. The second countermodel is depicted in Figure 2:

Similarly, in Figure 2, the subregion theory of parthood is satisfied but regionalism is not. Note that o is the F_1 of o_1 and o_2. o_1 and o_2 are exactly located at r_1 and r_2, respectively. Furthermore, r_4 is the F_1 of r_1 and r_2. Thus, o_1 and o_2 compose but there is nothing exactly located at the region that is composed by their exact locations, contra the left-to-right direction of REGIONALISM$_1$.

Gilmore and Leonard rightly note that the first countermodel violates[21]

(P.5) $x \circ y \leftrightarrow \ell(x) \circ \ell(y)$ OVERLAP

[20] Note that I focus on the "fusion" formulation of regionalism, rather than the "composition" formulation—for the distinction see Gilmore and Leonard [12]. This is in line with the simplification about composition and fusion being converses.

[21] Note that o_1 and o_2 are disjoint. By contrast, their exact locations, r_1 and r_2, respectively, overlap in that they share r_4.

https://doi.org/10.1017/S1755020325101020 Published online by Cambridge University Press

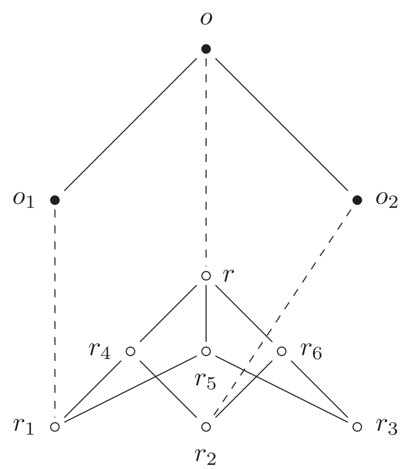

Figure 2. Countermodel 2.

whereas the second countermodel violates[22]

(P.6) $C(x) \wedge x@y \rightarrow$
$\forall yy(F_1(x, yy) \rightarrow$
$\forall w(w \leq y \rightarrow \exists z \exists v(z \prec yy \wedge z@v \wedge w \circ v)))$ STRONG DELEGATION

—where C stand for "being mereologically complex," i.e., $C(x) \equiv_{\text{df}} \exists y(y \ll x)$. OVERLAP is another clear example of *First Grade*. In effect, it is often enlisted as a core principle in different formulations of mereological harmony. STRONG DELEGATION looks complicated but the picture behind it is simple and reasonable enough.[23] It is supposed to capture the idea that complex objects fill their locations by "delegation," that is, with the help of their proper parts (see Gilmore and Leonard [12]). Interestingly, the second countermodel does *not* violate OVERLAP. Gilmore and Leonard go on to provide a new argument for the entailment between STP and REGIONALISM$_1$ based on those principles. I simply refer to Gilmore and Leonard [12] for the proof.

3.4. An assessment. As I noted F_1 is not the only notion of fusion in the literature. For the purpose of the paper, let me introduce just another one:

(D.3) $F_2(y, xx) \equiv_{\text{df}} xx \leq y \wedge \forall x(xx \leq x \rightarrow y \leq x)$. FUSION$_2$

Roughly, F_2 corresponds to the algebraic notion of *least upper bound*. Gilmore and Leonard run their argument from Countermodel 1 with only one notion of composition, the one that uses (albeit implicitly) F_1—as I did above. However, whilst it is true that in classical extensional mereology F_1 and F_2—and thus composition$_1$ and composition$_2$—are equivalent, EM is not strong enough to deliver the result. This is best appreciated as follows. Figure 3 is a model of EM. Yet there are F_2 that are not F_1.

For instance, it is easily checked that $F_2(o, [o_1, o_2])$ holds but $F_1(o, [o_1, o_2])$ doesn't—where $[o_1, o_2]$ represents the plurality oo whose sole members are o_1 and o_2. In other

[22] Focus on r_3. It is a proper part of the exact location of o, but o does not have any proper part that is even weakly located there.

[23] In plain English it says that if x is complex and located at y, then for every plurality yy that x fuses, each part of y overlaps a location of one of the yy.

https://doi.org/10.1017/S1755020325101020 Published online by Cambridge University Press

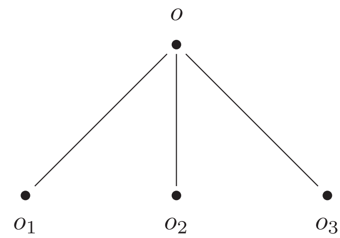

Figure 3. Model 3.

words, o counts as the F_2 of o_1 and o_2 but not as their F_1.[24] This might seem as an insignificant detail but it is not. This is because the "material object" part of Countermodel 1 *just is* Model 3. The reply to Gilmore and Leonard's first argument on behalf of the regionalist can then be put in the form of a dilemma.

Either (i) one sticks to EM as the background mereology or (ii) one moves to a stronger mereology in which F_1 and F_2 are indeed equivalent. First horn first. If one sticks to EM, to resist the original argument, regionalists simply need to switch from F_1 to F_2 in their formulation of regionalism. Let me elaborate. Let "$C_2^{\exists}(xx)$" abbreviate "$\exists y(F_2(y, xx))$," and let "$@L_2(xx)$" abbreviate "$\exists y \exists z(M(y) \wedge R(z) \wedge F_2(z, \ell\ell(xx)) \wedge y@z)$." Finally, claim that regionalism can be captured by:

(P.7) $C_2^{\exists}(xx) \leftrightarrow @L_2(xx)$. REGIONALISM$_2$

The gist is that now the original argument does not go through. Look back at Countermodel 1. r is the F_2 of r_1 and r_2 and o is still exactly located at r. However, now o_1 and o_2 compose$_2$ o. Thus, REGIONALISM$_2$ is not violated. Second horn then. If one takes the second horn one needs to change the mereology in such a way as to render F_1 and F_2 equivalent. But the argument in this section then establishes that one would therefore lose Countermodel 1.

To be fair, two things are worth noticing. First, F_1 is arguably the most widely used—and some say well-behaved—notion of fusion. One should also note that there are dissenting voices (see, e.g., Calosi and Giordani [5]). Interestingly, the new notion of fusion Calosi and Giordani define can be used to run a counterpart of Gilmore and Leonard's original argument. Second, F_2 is rarely used in the debates on "composition." One reason is that it might fail to capture some of our pre-theoretical intuitions about composition. For instance, an F_2 of the xx can have something that is completely disjoint from any of the xx as a part. One may also push the point that is also *biased* towards the existence of the so-called *arbitrary undetached parts*. And this is relevant, in this context, because of the relations between arbitrary parts and certain principles of location (see §4.3). Consider Countermodel 3. One may want to claim that something that is composed of o_1 and o_2 is an arbitrary undetached part of o. Given that the existence of such arbitrary parts is supposed to be *controversial*, one should not invoke a definition of composition that almost guarantees their existence. But, in the case at hand, such existence is guaranteed by F_2, insofar as o_1 and o_2 compose o. Therein lies the problem. Or so the thought goes. Here is a possible response. An arbitrary undetached part x of y is supposed to be a *proper part* of y. And given that—as I

[24] See also Calosi and Giordani [4, 5] and Cotnoir and Varzi [10].

https://doi.org/10.1017/S1755020325101020 Published online by Cambridge University Press

pointed out already—an F_2 of the xx could have parts that are disjoint from the xx, the fact that the xx have an F_2 might be necessary, but is *not sufficient* to qualify their F_2 as an arbitrary undetached part of y. Such F_2 should be distinct from y. In the case of Model 3, the F_2 of o_1 and o_2 thus fails to qualify as an arbitrary undetached part of o. More in general, the definition of F_2 does not seem to guarantee *by itself* the existence of arbitrary undetached parts.[25]

Whatever one thinks of the considerations above, the argument establishes that, contrary that what is assumed in the literature, the threat posed to regionalism by Countermodel 1 crucially depends on *both* very fine-grained details about the notion of fusion and the background mereology one works with. It also shows that the regionalist might have some leeway in both respects. The point can actually be strengthened. In §4.2 I will argue that one can prove the left-to-right direction of Regionalism$_2$ in the presence of STP alone. Let me be clear. This does not detract from Gilmore and Leonard's overall point. The argument from Countermodel 2 is still standing. But note that there seems to be a crucial difference between Countermodel 1 and Countermodel 2. As far as location is concerned Countermodel 1 does not seem problematic. The problem for regionalism stems from the fact that spatial mereology and material mereology are not aligned. In particular, material mereology is not a model of classical mereology, whereas spatial mereology is. By contrast, in Countermodel 2 location (rather than mereology) seems to be the source of the problem. In effect, both spatial and material mereology are models of classical mereology in Countermodel 2. What is problematic, intuitively, is that the composite object o extends further than its parts o_1 and o_2. One may think that any theory of location should rule that locative pattern out, independently of the background mereology of space and objects. Thus, one may push the following argument. Any satisfactory theory of location should rule out Countermodel 2 along with it. And those who want regionalism to follow from the subregion theory of parthood can undermine the argument from Countermodel 1. So there is at least hope for that entailment to emerge unscathed from the previous arguments. Rather than pushing this line of argument I want focus on a theory of location that does in effect rule out Countermodel 2 thanks to a locative principle that is different from Strong Delegation—the reason will be clear shortly. In the present context this is particularly relevant as it offers new, substantive, and unexplored examples of the interaction between the first and third grades of local involvement. More in general, it sheds new light on the relations between the logic of parthood and the logic of location.

3.5. Another argument. Interestingly, the logical relations between *Locative Fusion* and STP—the two examples of *First Grade* involvement I discussed in §1—provide another argument against the entailment between STP and Regionalism$_1$. In effect, they provide an argument against a particular direction of Regionalism$_1$, the left-to-right direction. That, as we shall see, is important. Suppose one defines (material) parthood in terms of fusion, using the standard definition of parthood for “fusion-first mereologies”:[26]

(P.8) $x \leq y \equiv_{\mathrm{df}} F(y, [x, y])$. Parthood

[25] Thanks to an anonymous referee for pushing this point.
[26] I used F here because in this case we are taking fusion as primitive and we are using it to define parthood.

And suppose one endorses Cartwrigth's locative fusion. It follows that $\ell(y)$ has the exact locations of x and y as parts. Thus $\ell(x) \leq \ell(y)$. This yields that $x \leq y \rightarrow \ell(x) \leq \ell(y)$. And $\ell(x) \leq \ell(y) \rightarrow x \leq y$ simply follows from *Locative Fusion*. This means that STP itself follows from *Locative Fusion*. Now for the argument. One may think that not every region of space(time) is *eligible* to be an exact location of a material object. For instance, three-dimensionalists *should* maintain that four-dimensional (spacetime) regions are not eligible exact locations for material objects. Say that a *receptacle* is a region of space that meets an *eligibility* requirement (E) to be an exact location for a material object:

(P.9) $REC(x) \equiv_{\text{df}} R(x) \wedge E(x)$. RECEPTACLE

There is no guarantee that REC is *closed under fusion*. That is, there is no guarantee that the fusion of two receptacles is a receptacle. This crucially depends on the eligibility condition E. Cartwright's own view of receptacles provides an example. Cartwright endorses the view that E is "being a non-null open domain," that is, a region that is identical with the interior of its closure [7, p. 156]. And as he notes, an open sphere cut by a plane is a fusion of open domains (the open regions on both sides of the plane) that is not an open domain. This is important because, given that STP follows from *Locative Fusion*, those who endorse the latter are committed to STP. But the "eligibility" argument above—as I shall call it—shows that things might have a fusion—and thus compose—even if there is nothing that is exactly located at the fusion of the exact locations of their parts, *contra* the left-to-right direction of REGIONALISM$_1$. For the sake of illustration let me use Cartwright's own example. Go back to the open sphere cut by a plane. Let o_l be the open region on the left of the plane and let o_r be the open region on the right. And let $x@o_l$ and $y@o_r$. Nothing prevents x and y to have a fusion, say $z = x \oplus_1 y$. z has an exact location that *is not* the fusion of the exact locations of its parts $o_l \oplus_1 o_r$, *contra* REGIONALISM$_1$. Therefore, regonalism cannot follow from STP *alone*. Something *else* needs to be added.[27]

§4. From first to third grade: Partitions and least upper bounds.

4.1. Partition and countermodels. Gilmore and Leonard rightly note that Countermodel 2 does not violate OVERLAP. Similarly, Countermodel 1 does not violate STRONG DELEGATION. Hence they assume *both* principles in their new argument. They also rightly claim that another locative principle in the vicinity of STRONG DELEGATION rules out Countermodel 2. This is ARBITRARY PARTITION from Casati and Varzi [6]. Informally, the principle says that an object has parts that are exactly located at every region it *pervades* (or fills—$x@_{\geq}y$):[28]

(P.10) $x@_{\geq}y \rightarrow \exists z(z \leq x \wedge z@y)$ ARBITRARY PARTITION

—where $x@_{\geq}y \equiv_{\text{df}} \exists z(x@z \wedge z \geq y)$. Interestingly, ARBITRARY PARTITION can be used to rule out *both* countermodels in one fell swoop. This is because in the presence of ARBITRARY PARTITION, OVERLAP follows from STP. OVERLAP, remember,

27 This entire discussion is massively indebted to a referee for this journal.

28 Taking @ as a primitive $@_{\geq}$ can be defined as follows: $x@_{\geq}y \equiv_{\text{df}} \exists z(x@z \wedge y \geq z)$. If there are no extended simple regions of space, ARBITRARY PARTITION rules out extended simple objects (see, e.g., Braddon-Mitchell and Miller [3]). By contrast STRONG DELEGATION is meant to be hospitable to them.

https://doi.org/10.1017/S1755020325101020 Published online by Cambridge University Press

is the following biconditional: $x \circ y \leftrightarrow \ell(x) \circ \ell(y)$. The left-to-right direction of that biconditional follows directly from STP. Assume $x \circ y$. Then there is a z such that $z \leq x$ and $z \leq y$. By STP, $\ell(z) \leq \ell(x)$ and $\ell(z) \leq \ell(y)$. Hence $\ell(x) \circ \ell(y)$. For the right-to-left, assume that $\ell(x) \circ \ell(y)$. Then, there is a region w such that w is part of both $\ell(x)$ and $\ell(y)$. Arbitrary Partition now entails that there is a z_1 such that $z_1 \leq x$ and $z_1 @ w$. It also entails that there is a z_2 such that $z_2 \leq y$ and $z_2 @ w$. It follows that $\ell(z_1) = \ell(z_2)$, which in turn entails that $\ell(z_1) \leq \ell(z_2)$ and $\ell(z_2) \leq \ell(z_1)$. By STP $z_1 \leq z_2$ and $z_2 \leq z_1$. But parthood is assumed to be a partial order and therefore anti-symmetric, which yields $z_1 = z_2$. x and y then overlap as desired. We already noted that Arbitrary Partition rules out Countermodel 2. It now rules out Countermodel 1 as well, by implying Overlap.[29]

Interestingly enough, Anti-Colocation and Arbitrary Partition can be used to derive yet another mereological harmony principle that might appeal to those who endorse STP. For obvious reasons I shall call it the *proper subregion theory of proper parthood* (PSTP):

$$\text{(P.11)} \quad x \ll y \leftrightarrow \ell(x) \ll \ell(y). \qquad \text{PSTP}$$

Suppose $x \ll y$. Then $x \leq y$ and $x \neq y$. By STP, $\ell(x) \leq \ell(y)$. There are two possibilities. Either $\ell(x) = \ell(y)$ or $\ell(x) \ll \ell(y)$. If $\ell(x) = \ell(y)$, then two distinct things are exactly located at $\ell(y)$, namely, x and y, against Anti-Colocation. As for the other direction assume $\ell(x) \ll \ell(y)$. Then $\ell(x) \leq \ell(y)$ and $\ell(x) \neq \ell(y)$. By STP, $x \leq y$. By Strong Supplementation, $\ell(y)$ has a part that is disjoint from $\ell(x)$, call it w.[30] y pervades w, so Arbitrary Partition entails that there is a part of y, say z that is exactly located at w. By Overlap z is disjoint from x. Thus x and y do not have the same proper parts, and are therefore distinct. Hence $x \ll y$ as desired.[31]

It should be clear that PSTP is an attractive principle by regionalist standards. Or better: it is attractive to those regionalists that want their regionalism to follow from STP. In effect, it should be how one cashes out the subregion theory of parthood if one takes $\ll$ as the mereological primitive. Now Arbitrary Partition is starting to look attractive, especially in the eyes of a regionalist. It (i) takes care of both the arguments from countermodels and (ii) it entails principles like Overlap and PSTP that are certainly in the same ballpark as STP. And it seems to get even better.

4.2. Partition and regionalism. In §3 I argued that Gilmore and Leonard's argument does not go through if one switches to F_2. I also claimed that the point can be strengthened. It turns out one can *prove* the right-to-left direction of Regionalism$_2$. To do that one needs plural versions of some principles we discussed. To be in line with extant literature,[32] I have formulated the subregion theory of parthood (and

[29] Note that one can use Anti-Colocation to derive the same conclusion. From $z_1 @ w$ and $z_2 @ w$, Anti-Colocation yields $z_1 = z_2$. Thanks to a referee for this journal.

[30] Note that Weak Supplementation—$x \ll y \rightarrow \exists z(z \leq y \wedge \neg z \circ x)$—is enough to deliver the proof. If $\leq$ is anti-symmetric, Weak Supplementation follows from Strong Supplementation.

[31] To be fair, one can derive PSTP with Anti-Colocation alone. Assume $\ell(x) \ll \ell(y)$. Then $\ell(x) \leq \ell(y)$ and $\ell(x) \neq \ell(y)$. By STP $x \leq y$. Thus, either $x = y$ or $x \ll y$. If $x = y$, Anti-Colocation yields that $\ell(x) = \ell(y)$. Contradiction.

[32] See among others Gilmore and Leonard [12], Leonard [14], Markosian [15], and Uzquiano [19].

https://doi.org/10.1017/S1755020325101020 Published online by Cambridge University Press

other principles) using exclusively singular terms. Plural versions are however readily available. For the purpose of the following argument we need the following:

(P.12) $xx \leq y \leftrightarrow \ell\ell(xx) \leq \ell(y)$. PLURAL STP

Given our agreed abbreviations, PLURAL STP just says that each of the xx is part of y iff the exact location of each of the xx is part of the exact location of y.[33] First let me prove that if the xx are part of y, their F_1 is also part of y:

(P.13) $xx \leq y \wedge F_1(w, xx) \rightarrow w \leq y$. PLURAL F_1-PART

Suppose there is a w that is the F_1 of xx but not part of y. By STRONG SUPPLEMENTATION w has a part that does not overlap y. Call it z_1. Given that w is an F_1 of the xx, all parts of w, z_1 included, overlap some xx. So there is a z_2 that is part of both z_1 and one of the xx, say x. But $\leq$ is transitive, so z_2 is part of y. Thus, z_2 is part of both y and z_1. That is, z_1 and y overlap. Contradiction.

Now for the argument from PLURAL STP to REGIONALISM$_2$. Suppose there is an object y that is exactly located at the fusion of the exact locations of some xx, $\ell\ell(xx)$. Then $\ell\ell(xx) \leq \ell(y)$. By PLURAL STP, $xx \leq y$. Let now w be an arbitrary extension of xx, that is $xx \leq w$. By PLURAL STP $\ell\ell(xx) \leq \ell(w)$. Therefore by PLURAL F_1-PART the fusion of $\ell\ell(xx)$ is part of $\ell(w)$.[34] But such fusion is nothing but $\ell(y)$. Thus, $\ell(y) \leq \ell(w)$. By STP, $y \leq w$. But w was an arbitrary extension of the xx. Thus xx are parts of y and y is part of any extension of xx. Hence $F_2(y, xx)$. This gives us the right-to-left direction of REGIONALISM$_2$.

One cannot prove the left-to-right direction. This is interesting. In §3.5 I actually argued that this is exactly the direction that cannot be derived from the subregion theory of parthood alone—be it singular or plural as it turns out. However, we *can* derive it in the presence of ARBITRARY PARTITION. Assume $F_2(y, xx)$. Then xx are parts of y. By PLURAL STP, $\ell\ell(xx) \leq \ell(y)$. Let w be the fusion of $\ell\ell(xx)$. By PLURAL F_1-PART, $w \leq \ell(y)$, and therefore y pervades w. ARBITRARY PARTITION now entails that there is a part of y that is exactly located at w. Therefore there is a material object that is exactly located at the fusion of the exact locations of the parts xx of y. Which gives us the left-to-right direction of REGIONALISM$_2$.

And now ARBITRARY PARTITION is starting to look *really* attractive. We saw in §4.1 that (i) it takes care of both the arguments from countermodels and (ii) it entails principles like OVERLAP and PSTP that are certainly in the same ballpark as STP. In this section we even saw that it actually entails regionalism—at least REGIONALISM$_2$. But...

4.3. Partition and composition. In the light of the results above a natural questions arises: Why don't regionalists simply endorse ARBITRARY PARTITION?[35] The problem is that it also entails composition principles that are different from regionalism. Or so I am about to argue.

33 Note that STP is just a special case of PLURAL STP.

34 Recall that regions of space are models of classical mereology. Notions of fusions are equivalent, and fusions are unique.

35 Regionalists might have *some* reasons. First, ARBITRARY PARTITION entails the existence of arbitrary undetached parts—that were discussed in §3.3. And these might be problematic, in view of, e.g., measure theoretic paradoxes, such as the Banach–Tarski Pardox. Second, in the absence of extended simple regions, it rules out extended simple objects—as I mentioned already in footnote 28.

Let me start from a "finitary" principle as a warm up. Say that x and y underlap ($\odot$) iff they have a common extension. Let me then abuse notation and write $\oplus_1$ as a short-hand for binary fusion$_1$. ARBITRARY PARTITION entails the following:[36]

(P.14) $x \odot y \to \exists z(z = x \oplus_1 y)$. BINARY UNDERLAP

First, note that the following is a finitary consequence of PLURAL F_1-PART:

(P.15) $x \leq z \wedge y \leq z \wedge w = x \oplus_1 y \to w \leq z$. F_1-PART

Now for the argument in favor of BINARY UNDERLAP. Suppose x and y underlap. Then there is a z such that $x \leq z$ and $y \leq z$. By STP we get that $\ell(x) \leq \ell(z)$ and $\ell(y) \leq \ell(z)$. There is a region $\ell(x) \oplus_1 \ell(y)$. By F_1-PART, $\ell(x) \oplus_1 \ell(y) \leq \ell(z)$. By ARBITRARY PARTITION there is a w such that $w \leq z$ and $\ell(w) = \ell(x) \oplus_1 \ell(y)$. We show that $w = x \oplus_1 y$. Clearly $\ell(x) \leq \ell(w)$ and $\ell(y) \leq \ell(w)$. By STP, $x \leq w$ and $y \leq w$. Consider an arbitrary part v of w. By STP, $\ell(v) \leq \ell(w) = \ell(x) \oplus_1 \ell(y)$. It follows that, for an arbitrary part v of w, its exact location overlaps either the exact location of x or the exact location of y. By OVERLAP, an arbitrary part of w overlaps either x or y. Thus, by definition of F_1, $w = x \oplus_1 y$. This gives us BINARY UNDERLAP.

As I mentioned, BINARY UNDERLAP is a finitary principle. However, the result could be generalized to cover infinite pluralities. Say that the xx underlap ($U(xx)$) iff they have a common extension. Then set:

(P.16) $U(xx) \to \exists z(F_1(z, xx))$. GENERAL UNDERLAP

GENERAL UNDERLAP is logically stronger than BINARY UNDERLAP.[37] The argument from ARBITRARY PARTITION to GENERAL UNDERLAP is similar to the one one we just saw

Suppose the xx underlap. Then there is a z such that $xx \leq z$. By PLURAL STP we get that $\ell\ell(xx) \leq \ell(z)$. There is a region, call it s, such that $F_1(s, \ell\ell(xx))$. By PLURAL F_1-PART, $s \leq \ell(z)$. By ARBITRARY PARTITION there is a w such that $w \leq z$ and $\ell(w) = s$. We show that $F_1(w, xx)$. Clearly $\ell\ell(xx) \leq \ell(w)$. By PLURAL STP again $xx \leq w$. Consider an arbitrary part v of w. By STP, $\ell(v) \leq \ell(w) = s$. This entails that for an arbitrary part v of w, its exact location overlaps an exact location of the xx—s being the fusion of the xx's exact locations. By OVERLAP, an arbitrary part of w overlaps one of the xx. Thus, by definition of F_1, $F_1(w, xx)$. This gives us GENERAL UNDERLAP.

For the sake of completeness let me note that that the subregion theory of parthood entails another locative principle that provides a counterpart, to speak loosely, of principles such as ARBITRARY PARTITION and STRONG DELEGATION. The idea behind such principles is that wholes should *not go further* than their parts. If a whole goes somewhere, so does one of its parts. Conversely, it seems plausible to require that wholes go *at least as far as* their parts. If one of their parts goes somewhere, so does the whole it is part of. There are different ways to capture this formally, one of which is given by the following principle:

[36] I use F_1 here because, recall, the argument from the first Countermodel crucially relies on the fact that a given plurality, namely, $[o_1, o_2]$, does not have an F_1. However, in the presence of ARBITRARY PARTITION, F_1 and F_2 turn out to be equivalent—I will argue for this shortly.

[37] GENERAL UNDERLAP has played some role in mereological discussions. Bostock [2] has proposed something similar, although with a different notion of fusion. For an interesting discussion of some mereological consequences of GENERAL UNDERLAP see Cotnoir [9].

https://doi.org/10.1017/S1755020325101020 Published online by Cambridge University Press

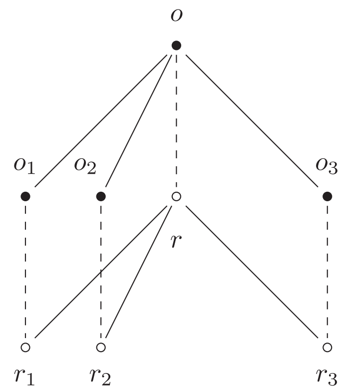

Figure 4. Countermodel 3.

(P.17) $x \leq y \wedge x@w \wedge y@z \to w \leq z$. EXPANSIVITY

EXPANSIVITY follows from STP. To see this assume the antecedent, i.e., $x \leq y$, $x@\ell(x)$, and $y@\ell(y)$. STP, recall, is the following biconditional, $x \leq y \leftrightarrow \ell(x) \leq \ell(y)$. The left-to-right directly yields the consequent of EXPANSIVITY. Interestingly, EXPANSIVITY can be used to run a different argument in favor of, e.g., BINARY UNDERLAP—which I relegate to a footnote.[38]

The arguments in favor of BINARY UNDERLAP and GENERAL UNDERLAP actually show how strong the mereology of regions needs to be for the point to go through. As I noted, spatial mereology is a model of *classical extensional mereology*. In effect, the following countermodel shows that a weaker mereology would not support the arguments.

In Figure 4, Countermodel 3 above both ARBITRARY PARTITION and EXPANSIVITY are satisfied but BINARY UNDERLAP—and therefore GENERAL UNDERLAP—is not: o_1 and o_2 underlap but they do not have an F_1. The significant feature of the countermodel is that regions obey a weaker mereology than before. For instance, "universal fusion for regions" fails. This, in turn, makes it possible that there is no $r_1 \oplus_1 r_2$, and therefore ARBITRARY PARTITION does not entail that something is exactly located *there*.

[38] Suppose x and y underlap. Then there is a z such that both x and y are parts of it. Either $z = x \oplus_1 y$ or $z \neq x \oplus_1 y$. In the first case, we get the consequent of BINARY UNDERLAP and we are done. In the second case, note that z has a proper part w that is disjoint from *both* x and y. To see this, suppose it is not the case. This means that each part of z overlaps either x or y. But given that x and y are both parts of z this would make z an F_1 of x and y which is is not by assumption. Therefore w is disjoint from both x and y. Given that @ is a total function, we get $\ell(w)$, $\ell(x)$, and $\ell(y)$. By OVERLAP, given that w is disjoint from both x and y, $\ell(w)$ is disjoint from both $\ell(x)$ and $\ell(y)$. There is a region $\ell(x) \oplus_1 \ell(y)$. By STP both $\ell(x)$ and $\ell(y)$ are parts of $\ell(z)$. By F_1-PART their F_1, $\ell(x) \oplus_1 \ell(y)$, is part of $\ell(z)$ as well. By ARBITRARY PARTITION z has a part that is exactly located at $\ell(x) \oplus_1 \ell(y)$. Given that @ is a *function*, this cannot be x, y or w. Nor it can be z itself. Suppose it is something that has w as a part. Then EXPANSIVITY would dictate that $\ell(w)$ is a part of $\ell(x) \oplus_1 \ell(y)$ which is not. Hence, whatever part of z is exactly located at $\ell(x) \oplus_1 \ell(y)$ does not have w as a part. This leaves out $x \oplus_1 y$ as the only possibility—note that ANTI-COLOCATION prevents other parts of z to be exactly located at $\ell(x)$ and $\ell(y)$, or any other material object for that matter.The argument is general and gives us BINARY UNDERLAP.

BINARY UNDERLAP—and therefore GENERAL UNDERLAP—rules out Model 3 and thus Countermodel 1 and Countermodel 3 as well. Indeed, we can now close the circle, so to speak, and show that the addition of GENERAL UNDERLAP to EM results in a mereology in which F_1 and F_2 are indeed equivalent. A little more precisely. Let EMGU be the mereology that is obtained by adding GENERAL UNDERLAP to EM. Then, the following is a theorem of EMGU:[39]

(P.18) $F_1(y, xx) \leftrightarrow F_2(y, xx)$. F_i-EQUIVALENCE

The left-to-right direction is actually provable in EM alone. Suppose this is not the case. Then there is a plurality xx such that $F_1(y, xx)$ but not $F_2(y, xx)$. Given that y is not an F_2 of the xx, there is some z such that $xx \leq z$ but y is not part of z. By STRONG SUPPLEMENTATION, there is a part of y that is disjoint from z. But each part of y overlaps some of the xx, given that y is an F_1 of the xx. And given that the xx are all part of z, each part of y overlaps z.[40] Contradiction.

The right-to-left relies on GENERAL UNDERLAP. Suppose once again this is not the case. Then there is a plurality xx such that $F_2(y, xx)$ but not $F_1(y, xx)$. By GENERAL UNDERLAP and our assumption, there is a z such that $F_1(z, xx)$ and $z \neq y$. Given that y is an F_2 of the xx, it is part of everything that have the xx as parts. In particular $y \leq z$. The xx are parts of y. Then, by F_1-PART, their F_1 is part of y. Thus, $z \leq y$, and $z = y$ by anti-symmetry. Contradiction. This establishes F_i-EQUIVALENCE.[41]

F_i-EQUIVALENCE is particularly interesting in this context, for different reasons. First, it addresses the first horn of the dilemma we introduced in the discussion of regionalism. Second, what GENERAL UNDERLAP claims is that pluralities that have an *upper bound* have a fusion. But F_2 is basically the definition of *least upper bound* induced by $\leq$. Thus, what GENERAL UNDERLAP tells us is that pluralities that have an upper bound have a least upper bound—something which is known to be algebraically important. Finally, together with the results in §4.2, it also shows that ARBITRARY PARTITION entails REGIONALISM$_1$ as well.

The following figure (Figure 5) sums up the logical consequences of ARBITRARY PARTITION—arrows indicating entailment.[42] It also serves as a reminder of how powerful that principle is.

---

39 This result is in the same spirit as the results in Cotnoir and Varzi [10]. They prove minimal requirements for different notions of fusions to be equivalent (see footnote 34). They do not mention GENERAL UNDERLAP. This is why I am fleshing the argument in some detail.

40 This follows from the fact that $x \circ y \wedge y \leq z \rightarrow x \circ z$. This in turn follows from Transitivity of $\leq$.

41 Cotnoir and Varzi [10] discuss several *purely mereological* principles that will deliver that every F_2 is an F_1. In particular they prove that the following FILTRATION principle will do: $F_1(y, xx) \wedge z \leq y \rightarrow \exists x(x \prec xx \wedge x \circ z)$ (Cotnoir and Varzi [10, p. 159], notation adapted). In effect, a weaker principle, namely, $\exists$-FILTRATION would be enough for non-empty fusions. $\exists$-FILTRATION is just FILTRATION conditionalized to the existence of an $x \prec xx$. Under the assumption that there are no empty pluralities—an assumption I did make in §2—the two principles are equivalent. Cotnoir and Varzi also prove that one can replace STRONG SUPPLEMENTATION with the following REMAINDER principle: $\neg x \leq y \rightarrow \exists z \forall w(w \leq z \leftrightarrow (w \leq x \wedge \neg w \circ y)))$. Then F_i-EQUIVALENCE becomes provable.

42 Together with STP, ANTISYMMETRY, and ANTI-COLOCATION I should add. Note that a little care is needed in reading Figure 5. For instance, it should not be forgotten that OVERLAP does not entail GENERAL UNDERLAP on its own.

https://doi.org/10.1017/S1755020325101020 Published online by Cambridge University Press

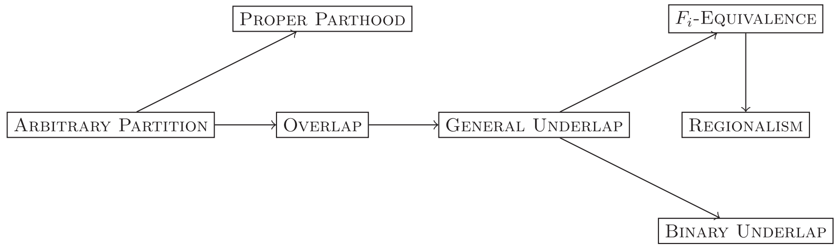

Figure 5. Logical consequences of Arbitrary Partition.

These results are not only important on their own. They have substantive consequences for the debate over the special composition question. For it is now immediate to prove:[43]

(P.19) $U(xx) \leftrightarrow \exists z(F(z, xx))$. Underlap[44]

This vindicates the claim I made at the beginning, namely, that a locative principle, Arbitrary Partition, entails (together with STP) a purely mereological answer to the special composition question, Underlap. It also shows that this is an example of an entailment between a first grade (here: the subregion theory of parthood) and a third grade thesis (here: arbitrary partition and underlap)—as advertised. In effect, interestingly enough, Arbitrary Partition provides both an example of an entailment between a first and a third grade theses, and another example of an entailment between a first and second grade thesis—given that it entails regionalism as well.

4.4. Underlap and regionalism. As we saw, Arbitrary Partition entails both regionalism and the underlap principles. What are the relations between the two? First, one should note that they are *logically independent*. For a model of General Underlap that is a countermodel to both Regionalism$_1$ and Regionalism$_2$ one needs to look no further than Countermodel 2 in Figure 2. And for a model of both Regionalism$_1$ and Regionalism$_2$ that is also a countermodel to underlap principles one needs to look no further than Countermodel 3 in Figure 4. However, as I noted, that model is also a countermodel of classical mereology for regions. The one below is a model of both classical mereology for regions and Regionalism$_1$, but a countermodel to both Binary and General Underlap: See Figure 6 below.

The two are different also when it comes to their relation with *extreme* answers to the Special Composition Question. *Both* are compatible with *both* extreme answers, *mereological universalism* and *mereological nihilism*. This should not come as a surprise. Usually, (allegedly) restricted answers to the special composition question single out necessary and sufficient conditions φ for some plurality xx to compose. But they usually do not tell us whether and when the conditions φ are satisfied. At least *not by themselves*. Consider one of the most controversial restricted answers, namely, van Inwagen's *organicism*, roughly the view that the xx compose iff they constitute a life. Organicism alone—as an answer to the special composition question—does not tell us what it takes for something to constitute a life. Thus, restricted answers usually do not tell us whether conditions are always or never satisfied—which renders such allegedly restricted answers compatible with extreme ones. This does not mean

43 Given F_i-Equivalence we need not add any subscript to F.

44 That entails the finitary version $x \odot y \leftrightarrow \exists z(z = x \oplus y)$.

https://doi.org/10.1017/S1755020325101020 Published online by Cambridge University Press

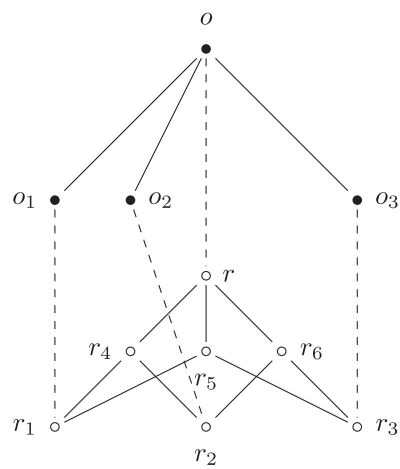

Figure 6. Model of regionalism, countermodel of underlap.

there are no differences in that respect. For instance, in the presence of the *universe*, that is, the mereologically maximal element of which everything is part, GENERAL UNDERLAP is *equivalent* to *mereological universalism*. By contrast, neither versions of regionalism are. In effect, *mereological universalism* entails only the left-to-right direction of regionalism—noting that REGIONALISM$_1$ and REGIONALISM$_\mathrm{I}$ are equivalent in the presence of *mereological universalism*. *Mereological Nihilism* on the other hand, together with the assumption that @ is a total injective function from M to R, entails the right-to-left direction of both regionalism and the underlap principles. Relatedly, regionalism and the underlap principles may diverge in their composition verdicts. *Assume* that I exist and consider my *clavibula*, the fusion of my left clavicle and my right fibula. Given that they are both part of me, the underlap principles—even the weaker finitary one—entail its existence. By contrast, regionalism is silent over the existence of clavibulas. Note that I had to *assume* my existence in the "clavibula argument" above, for the underlap principles are not enough to deliver that I exist. As we saw, this is not uncommon.

A final difference between the principles relates to their grade of local involvement, as they were distinguished in §1. Regionalism is a *mereo-locational* answer—hallmark of the *second grade*, whereas the underlap principles are purely *mereological*—hallmark of the *third grade*. It follows that regionalism informs us of the *locations* of parts and wholes. By contrast, the underlap principles are silent when it comes to locational issues.

To conclude. This paper addressed several issues related to different kinds of "local involvement," as I phrased it. In doing so, it provided new results about the relations between the logic of composition and the logic of location. At the cost of sounding repetitive. My goal was just to chart logical and metaphysical connections, not to defend different principles or different metaphysics of parthood and location—let alone convince anyone of their truth. A cartographer is not a prophet.

Acknowledgments. For discussions on previous drafts of the paper I want to thank Fabrice Correia, Kit Fine, Alessandro Giordani, Cody Gilmore, Achille Varzi and Lisa Vogt. I also would like to thank three anonymous referees, and the editors for this journal—especially Gabriel Uzquiano—for their careful reading and insightful suggestions.

BIBLIOGRAPHY

[1] Baron, S., Miller, K., & Tallant, J. (2022). *Out of Time*. Oxford: Oxford University Press.

[2] Bostock, D. (1979). *Logic and Arithmetic: Rational and Irrational Numbers*. Oxford: Clarendon Press.

[3] Braddon-Mitchell, D., & Miller, K. (2006). The physics of extended simples. *Analysis*, **66**, 222–226.

[4] Calosi, C., & Giordani, A. (2023a). Atoms, combs, syllables and organisms. *Philosophical Studies*, **180**, 1995–2024.

[5] ———. (2023b). Universalism and extensionalism revisited. *Synthese*. **201**, article number 106. Available at: https://link.springer.com/article/10.1007/s11229-023-04091-9.

[6] Casati, R., & Varzi, A. (1999). *Parts and Places*. Cambridge: MIT Press.

[7] Cartwright, R. (1975). Scattered objects. In Lehrer, K., editor. *Analysis and Metaphysics*. Dordrecht: Reidel, pp. 153–171.

[8] Correia, F. (2022). A general theory of location based on entire location. *Journal of Philosophical Logic*, **51**, 555–582.

[9] Cotnoir, A. (2014). Universalism and junk. *Australasian Journal of Philosophy*, **92**(4), 649–664.

[10] Cotnoir, A., & Varzi, A. (2021). *Mereology*. Oxford: Oxford University Press.

[11] Gilmore, C., Calosi, C., & Costa, D. (2024). Location and mereology. In *Stanford Encyclopedia of Philosophy* (E. N. Zalta, and U. Nodelman, editors).

[12] Gilmore, C., & Leonard, M. (2020). Composition and the logic of location. *Mind*, **129**, 159–178.

[13] Korman, D., & Carmichael, C. (2016). Composition. In *Oxford Handbooks*. Oxford: Oxford University Press. https://doi.org/10.1093/oxfordhb/9780199935314.013.9.

[14] Leonard, M. (2016). What is mereological harmony? *Synthese*, **193**, 1949–1965.

[15] Markosian, N. (2014). A spatial approach to mereology. In Kleinschmidt, S., editor. *Mereology and Location*. Oxford: Oxford University Press, pp. 69–90.

[16] Oppenheim, P., & Putnam, H. (1958). Unity of science as a working hypothesis. In Fiegl, H., Scriven, M., & Maxwell, G., editors. *Concept, Theories and the Mind-Body Problem*. Minneapolis: Minnesota University Press, pp. 3–36.

[17] Parsons, J. (2007). Theories of location. *Oxford Studies in Metaphysics*, **3**, 201–232.

[18] Saucedo, R. (2011). Parthood and location. *Oxford Studies in Metaphysics*, **6**, 223–284.

[19] Uzquiano, G. (2011). Mereological harmony. *Oxford Studies in Metaphysics*, **6**, 199–224.

[20] van Inwagen, P. (1990). *Material Beings*. Ithaca: Cornell University Press.

[21] Varzi, A. (2007). Spatial reasoning and ontology: Parts, wholes and location. In Aiello, M., Pratt-Hartmann, I., & van Bentham, J., editors. *Handbook of Spatial Logic*. Berlin: Springer, pp. 945–1038.

CA' FOSCARI UNIVERSITY
VENICE
ITALY
E-mail: claudio.calosi@unive.it

https://doi.org/10.1017/S1755020325101020 Published online by Cambridge University Press