Dialogues and Games of Logic 8

The Algebrization of Mathematics during the 17th and 18th Centuries

Dwarfs and Giants, Centres and Peripheries

Editors

Davide Crippa

Maria Rosa Massa-Esteve

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Volume 8

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Edited by

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and
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Introduction

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1 The algebrization of mathematics in 17– and 18–century Europe

Seventeenth century mathematics has been transformed through the interaction of three fundamental forces. The first one was the classical mathematical heritage of the sixteenth century, exemplified by the direct recovery in Greek and Latin translations of works by Euclid, Archimedes, Aristarchus and others; the second one was the "infinity revolution", that is, the extension of mathematics thanks to the use of infinite procedures and the study of geometric objects of infinite dimension; and the third one was the emergence of algebra and its use for solving problems.

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The emergence of symbolic algebra as a formal language in mathematics is commonly referred in the literature as the "algebrization" of mathematics (see [Massa-Esteve, 2006]; [Massa-Esteve, 2020]). The process of algebrization involved a cognitive shift, namely a change in the way of thinking about mathematics, marked by the transition from explicit constructions and geometric explanations to the reliance on formulas and equations as the main way to represent mathematical entities and reason upon them (see [Massa-Esteve, 2001]). This process has been called by the historian H. Bos [2001, p. 10] "degeometrization of analysis". Consequently, algebrization also prompted the emergence and the establishment during the eighteenth century of analytical geometry and calculus (or infinitesimal analysis) as two autonomous mathematical disciplines.

Building upon Jacob Klein's seminal work ([Klein, 1976/1936]), the historian M. Mahoney (see [Mahoney, 1980]) argued that changes in mathematics pedagogy played a crucial role in facilitating evolution towards an algebraic mode of thinking in the early modern period. The diffusion of Ramus' didactic model, which prioritized analysis as a more fruitful and intuitive teaching method compared to the synthesis, as shown in Euclid's proofs, contributed to the adoption of algebraic formulas. Therefore, according to Mahoney [1980], the inclination toward algebra can be seen as an outcome of the imposition of this "modern" teaching method.

This pedagogical emphasis on analysis is also evident in textbooks from the eighteenth century. For example, according to Christian Wolff, the author of a successful mathematics course, it was imperative for those seeking a solid foundation in mathematics to study analysis. In Wolff's context, "analysis" encompassed both algebra and calculus, which made use of a symbolic notation. It served as a framework that encompassed generalized arithmetic and provided a method for discovering new mathematical results and proofs. Wolff stressed that analysis should be learned because it engenders concepts that surpass ordinary imagination and enables the condensation of lengthy chains of reasoning ([Wolff, 1717]; *Elementa Analyseos Mathematicae*, *Praefatio*). The pedagogical virtues of analysis, in its algebraic form, were recognized by other eighteenth-century authors as well. For instance, Leonard Euler translated the geometric derivation of a cannonball's flight into analytical language because it was "simpler, clearer, and of greater utility" ([Alder, 2010, p. 72]).

However, teaching and learning algebraic analysis posed epistemic challenges that are still relevant in contemporary teaching practices (see [Massa-Esteve, 2020]). Such challenges stem from the chief characteristics that Mahoney [1980] attributed to the algebraic mode of thought: the use of symbolic operations, focussing on mathematical relationships rather than objects, and freedom from ontological commitments. These characteristics account for the fact that algebraic thinking does not align with intuitive understanding. Algebra allows for the introduction and manipulation of entities that are challenging to define or work within the framework of classical geometry. For example, negative and imaginary quantities or objects in dimensions beyond three have posed difficulties as they lacked an intuitive representation. Similarly, infinitesimal analysis, as seen in the Leibnizian and Newtonian calculi, employs a symbolic system that enables the manipulation of impossible objects such as infinitesimals, which do not fit the framework of Euclidean mathematics. Given this lack of proper foundations, how could mathematicians and teachers of mathematics be sure that algebraic reasoning led to a correct, i.e. non contradictory and meaningful results?

The answer to this question is not obvious, and algebra and infinitesimal calculus had a share of critiques as soon as they were circulated. For Hobbes and for Barrow, for instance, algebra was merely a stenographic notation to abbreviate arithmetical or geometrical proofs, but did not constitute a form of scientific knowledge on the same level of arithmetic and geometry ([Mancosu, 1996, pp. 86–88]). Likewise, the reception of the Leibnizian calculus in countries like France and Italy had to overcome an initial reticence due to the mistrust toward the obscurities of the new symbolism, especially when compared with the geometrical methods of the Greeks (see, in particular [Mazzone and Roero, 1997] for the Italian case, [Mancosu, 1989] for the French one). We thus wonder how pedagogical advantages associated with the formalism of algebra and infinitesimal analysis, in terms of the economy of thought and expansion of our imagination, were eventually able to overcome the corresponding cognitive and epistemic difficulties.

In this connection, and despite the considerable research conducted on the algebrization of mathematics and its historical development, there are further avenues for exploring how mathematicians, practitioners, students, and other learners gradually accustomed themselves to the advanced mathematics that made used of symbolic notations. While not providing definitive answers, the contributions in this book hope to provide lines of inquiry for understanding the fundamental shift that brought the process of algebrization and its cognate disciplines of algebra, analytic geometry, and calculus (or infinitesimal analysis) to the core of modern mathematics.

One significant theme discussed throughout this book is the role of institutions in shaping the teaching of analysis during the 18th century, with a particular focus on two main venues. On the one hand, the book delves into the impact of military and technical teaching (roughly speaking, the teaching of "military engineers") on mathematical education imparted outside universities. On the other hand, it delves into the influence of teaching connected to military and technical needs (e.g., practical geometry, fortification, and architecture) provided in universities. One example in this sense is given by the mathematics instruction offered by the Jesuits.

As a result of the necessity for a highly skilled and efficient military and state officers, the first half of the eighteenth century witnessed a remarkable proliferation of military schools across different European nations, often unrelated to preexisting academic traditions. Unlike the teaching of mathematics in European universities, mathematics in technical and military schools presented more diverse and specialized content, not strictly anchored to the kind of encyclopaedic knowledge sedimented in the traditional *Quadrivium*.

However, the immediate significance of mathematics instruction in the field of artillery was not as obvious as we might think. In his study of French technical education, historian Ken Alder shows that military engineers continued to rely heavily on practical rules and their expertise to teach the art of artillery, even when the formalism of calculus and algebra was available (see, for instance [Alder, 1999, p. 113ff.]). This preference stemmed from the belief that the implicit knowledge passed down by practitioners surpassed the accuracy of existing mathematical models in explaining various phenomena, such as the trajectory of a projectile moving through a resistant medium, for instance, a cannonball shot through the air.

Furthermore, there were debates about which aspects of mathematics were necessary to artillery. Several officers and military men believed that instruction that relied too heavily on abstraction was harmful and useless. Artillery professor B. Forest de Bélidor, whose textbooks were widely popular in French schools and circulated throughout the continent (see for example, M. R. Massa-Esteve's chapter in this volume), argued that engineers should avoid meaningless speculation or endless calculations without purpose. Instead of engaging in the speculations of "snobbish savants," Bélidor proposed that the engineers should focus on a mixture of practical skills and theoretical knowledge ([Alder, 1999, p. 116]).

Since the Renaissance, mathematics applied to physical quantities has been referred to as "mixed mathematics", distinguishing it from the "pure" disciplines of arithmetic and geometry.[1] Broadly speaking, in pre-18th century curricula, mixed mathematics dealt with quantities in conjuction with matter, and included a limited range of phenomena, spanning from mechanics to optics, whose aspects, such as positions, motions and forms could be quantified and measured. However, questions pertaining to the nature and causes of natural phenomena felt outside of the scope of mathematics into the realm of "physics", i.e. natural philosophy.

In the period covered in this book, the advancements in algebra and analysis enabled the mathematical treatment of a broader range of concrete entities and phenomena, referred to as "thick objects" by Ken Alder [2010, p. 71]. These included both natural processes and human activities, with notable examples being projectile motion and war-related artifacts. Although the distinction between "pure" and "mixed" mathematics persisted well into the 18th century, the development of mixed mathematics within artillery and technical schools transformed its original meaning. The new "mixed mathematics" of the Enlightenment aimed to apply mathematical formalism a diverse array of objects,

[1] [Massa-Esteve, 2011, p. 236]. Tartaglia's work on ballistic, the *Nova Scientia* (1537), was also relevant in this respect. See [Massa-Esteve, 2014].

real-world processes, and phenomena. This expanded scope sought to address the limitations of earlier mixed mathematics, which had been criticized for its inability to explain the majority of natural phenomena. Among the examples discussed in this book, the successes of analysis included investigating applications of algebra and infinitesimal calculus to solve optimization problems, which represented a crucial demand for military and technical professions. These included determining the number of cannonballs in a pile or displacing a mass of earth.[2] or applying mathematics to the dynamics of concrete bodies, such as to calculate the trajectory of a projectile in a resisting medium.

Indeed the expansion of mixed mathematics in technical schools furthered its algebrization and contributed to the primacy of analytical thinking, steering the gradual transformation of "mixed mathematics" into "mathematical physics." This long process is discussed, for instance, in [Massa-Esteve, 2011], and in Massa-Esteve's chapter of this volume.

However, the algebrization of mathematics did not merely impose itself on the basis of its epistemic and cognitive virtues. As we have seen above, these virtues were sometimes the main reason for criticizing the usefulness of mathematics in artillery curricula. "Abstract" disciplines, such as finite and infinitesimal analysis, were often viewed with suspicion when cultivated for their own sake. However, in technical and military curricula, they may have contributed to inculcating a particular social role into artillery officers from the mid to late 18th century, who came to acquire a unique blend of practical skills, theoretical knowledge, and mathematical precision. This mix shaped the ideal image of the "engineer" in the late eighteenth century.[3]

This process happened through the essential role of people who acted as intermediaries, passing on and disseminating mathematical knowledge. To provide just one exemplary case, discussed in this volume, let us mention the mathematician and engineer Benjamin Robins (1706-1751). Despite receiving little attention from historians, Robins was an influential figure in the 18th century. He authored a work called "New Principles of Gunnery" in 1742, which was later translated into German

[2]This was a practical problem leading to original developments in mathematics. The problem is mentioned in Patergnani and Lugaresi's chapter in this book.

[3]This thesis is explored, in particular, in [Alder, 1999].

by Euler in 1745. Euler added commentaries that incorporated infinitesimal calculus. Robins' book, through Euler's contributions, became a widely used reference text in artillery schools in Germany, France, and other countries like Spain and Italy, contributing to demonstrate the importance of studying mathematics within technical education (see [Barrow-Green, 2010]). In a similar way, investigating the practices and contributions of other characters marginalized as "dwarfs" in many historic narratives, such as ordinary teachers, administrators, reformers, and textbook authors, can enrich our understanding of how mathematical knowledge was shared and circulated, and how the geography of disciplinary knowledge was modified.

2 Giants and dwarfs as historiographical categories

"Giants" and "dwarfs", "heroes" and "commoners", "luminaries" and "obscure" are metaphors commonly used in narratives of science to contrast great pioneers and innovators and less influential scientists, or those scientists who merely built upon existing knowledge. Are such categories useful in the history of science, especially, history of mathematics? How can they be employed to increase our knowledge of mathematics and its history? These are some of the questions raised by the contributions collected in this volume.

Narratives about the history of 18th century mathematics have often followed the structure of the "great-men narrative", focussing on the lives and work of a small number of canonical individuals considered "giants", "great minds" or "heroes", who made significant contributions to the discipline and towered over less original characters. For example, the account of mathematics in 18th century Europe offered by the renowned historian of mathematics D. Struik begins with a list of mathematicians forming a sort of "intellectual kinship" ([Struik, 1987/1948, p. 163]). Leibniz, Newton, the Bernoulli family, Euler, Lagrange, and a few other French mathematicians shaped, in Struik's narration, that has become quite standard, we find the main features of 18th century mathematics. These are an emphasis on the application of algebra and analysis to domains such as geometry and number theory, on free manipulations and experimentation with symbolic expressions over the rigorous study of the foundations, and the triumph of the "quantifying spirit," namely the idea that mathematics is a language applicable to disparate domains of the study of nature.

This narrative has been subject to scrutiny from various angles in recent years, but precisely those individuals whose achievements may have slipped through the cracks or failed to have a global impact on the discipline deserve our attention. Their significance lies in how they shaped their context, fostered connections among scientists, or played a pivotal role in the dissemination of knowledge, as in the case of teachers and intelligencers. In particular, historical studies of mathematics can play a crucial role in reviving the works of these alleged "dwarfs" by highlighting their contributions to the development of the field and reconstructing their biographies and social contexts. Additionally, shedding light on obscured contributors or groups can help us understand how knowledge is produced and transmitted and how historical canons are formed, including the one that shapes Struik's narration.

One direction of research that will be pursued in this volume is to investigate how certain mathematicians who were once renown as "giants" have become "dwarfs", as they have been forgotten despite having played a non-negligible, or even outstanding public role. This trend follows a recent growing emphasis on rediscovering forgotten personalities in various fields, such as women in mathematics, mathematics teachers, and textbook authors. Emilie du Chatelet (1706-1749) and Maria Gaetana Agnesi (1718-1799) serve as prime examples of remarkable individuals highly acclaimed in their respective times, encompassing all three categories: women, mathematicians, and textbook authors.[4] While this book does not delve into their stories, the recent reassessment of their significance as mathematicians in 18th-century Europe provides a fundamental methodological lesson that we tried to follow in our book too. In the case of Agnesi, scholars such as P. Findlen [2011] and M. Mazzotti [2007] have reexamined archival resources, challenging the critical judgment that relegated Agnesi to the ranks of second-rate, unorigi-

[4]On E. du Chatelet, see, for instance, [Hagenbruger, 2011], and the project pursued by the group *History of Women Philosophers and Scientists* (`https://historyofwomenphilosophers.org/project/directory-of-women-philosophers/du-chatelet-emilie-1706-1749/`).

nal mathematicians of her time. As documented in the aforementioned studies, without a comprehension of Agnesi's biography in her social and intellectual context, one can be fooled into believing that she was not an "original" mathematician. On the contrary, she embodied an original career leading to the acknowledgment by the intellectual world of her time and to a university chair, albeit one she never practically occupied. Moreover, she stood out as a paradigmatic textbook author, representing a model for other female mathematicians.

In addition to recognizing the forgotten giants, historians can also acknowledge groups or individuals who had failed to receive public recognition in the past. As E. Robson and J. Stedall remarked "To limit the history of mathematics to the history of mathematicians is to lose much of the subject's richness" ([Robson and Stedall, 2009, p. 2]). A more comprehensive and richer study of practices should include "cloth weavers, accountants, instrument makers, princes, astrologers, musicians, missionaries, schoolchildren, teachers, theologians, surveyors, builders, and artists". As the various contributions in Stedall and Robson's book show, the categories listed above played a crucial role in the circulation of mathematical knowledge, yet their activities and their traces, whether written or not, have only recently started receiving exploration.

Understanding the role of these often-obscure characters in the production of mathematical knowledge becomes even more crucial when we consider their impact on the so-called "giants" of the field. It is important to recognize that even scientists who claimed to be self-taught or were seen as such did not acquire their expertise out of thin air. These individuals were not solitary figures; rather, they were immersed in a network of influences that contributed to the development and refinement of their mathematical expertise. Their knowledge was shaped by the correspondence networks they were part of, their access to books and journals, and the relationships they formed, including their interactions with fellow mathematicians and practitioners. By delving deeper into the intellectual environment surrounding prominent mathematicians, we gain a richer understanding of how their expertise was constructed and situated.[5]

While the biographical method has been widely used for “giants”, it can pose challenges when applied to the study of “dwarfs” who played significant and indispensable roles in the education of the former, for instance, but whose lives are often poorly documented due to limited available sources. To address this issue, one possible approach is to situate their lives and careers within larger groups, such as networks they were part of, more prominent colleagues, friends or correspondents, or the institutions where they may have been employed. A more comprehensive understanding of their contributions can be achieved by examining their connections and associations.

Studying individuals considered “dwarfs” from the perspective of standard histories of science offers more than a means to unravel the biographies of outstanding individuals and shed light on their life and scientific production. It also proves essential in comprehending some of the broad historical phenomena that shaped mathematics in the 18th century and beyond.

One example, which is also a transversal theme of our book, is the process of habituation to analytic formulas and the transformation of algebra (or, as it used to be known in the 17th and 18th centuries, “analysis of finite quantities”) and differential and integral calculus (also known as “infinitesimal analysis”), from methods applied to geometry and arithmetic into a new branch of mathematics, simply called “analysis”, during the 18th century. In the words of the historian of mathematics Henk Bos:

> Explicit construction as basis for understanding the objects of mathematics was replaced by a trust in the formula, based on a gradually established conviction that the equations of analysis always, explicitly or implicitly, defined an object, and that therefore that object could be accepted as given or as existent. A process of habituation to the world of formulas and equations finally eliminated the demand for a geometri-

[5] Among recent examples in the literature, see the case of Herschel examined by Winterburn [2014], Lagrange studied by Borgato and Pepe [1990], and Bolzano investigated by Fuentes Guillén and Crippa [2021].

cal explanation.[6]

The essential steps in this transformation were the contributions of Viète and Descartes, as highlighted, for instance in [Panza, 1997]. What Bos' and Panza's investigations (see also [Bos, 2001]) do not fully clarify is the question about the conditions, both material and social, that made possible the process through which algebraic language imposed itself as the ground and language of mathematics, and analysis became an autonomous discipline at the very heart of modern scientific education.

A general thesis underlying the contributions in this volume is as follows: in order to fully understand the phenomena of habituation and the emergence of new disciplinary fields in mathematics (and, more generally in science), it is necessary to examine how the "great minds" learned algebra and calculus, often from lesser-known teachers and correspondents, how their new ideas circulated and became established, and how mathematical innovations were adopted and adapted by wider communities of mathematicians, often indirectly related to the original sites of their production. As some recent works have demonstrated (for instance [Warwick, 2003], [Ehrhardt, 2010]), the historical development of ideas in mathematics is not solely the work of "giants" but is shaped by a complex interplay of social and intellectual constraints. Even prominent mathematicians go through education and learn from their teachers, who have often been overlooked in the traditional histories of mathematics.

3 Centres and peripheries

In this book, we also examine the concept of algebrization through the lens of "geographies of knowledge" (for a general presentation of this issue, see [Livingstone, 2013]). This perspective encompasses the dissemination of novel theories and methodologies, particularly those associated with the emergence of algebra and calculus. Thus, we will explore the locations where these theories and methods were taught and acquired, as well as the individuals responsible for imparting and acquiring this knowledge. Additionally, we consider the social and political contexts that influenced knowledge production in this field.

[6] [Bos, 1996, p. 17].

Apart from the above-mentioned [Livingstone, 2013], [Secord, 2004] and the collections edited by Gavroglu [1999] and Blanco and Bruneau [2020] represent reference works for our investigation. In particular, Gavroglu [1999] and Blanco and Bruneau [2020] have emphasized the appropriation of knowledge as an active process, moving away from the idea of transmission as passive reception. These researchers have considered the circulation of knowledge products, such as manuscripts and textbooks, and the relevance of translations for the transfer of knowledge. They also considered travelers' biographies, namely practitioners and scholars who crossed national borders. The circulation of scientific knowledge is now understood as more complex than simply receiving and reproducing it.

The contributions presented in this book align with these trends and investigate the transmission of mathematical knowledge through the circulation of manuscripts, textbooks, their translations, and the itineraries of single scholars and traveling mathematicians such as J. Wendligen (1715-1790), discussed in J. Berenguer's chapter or P. Calbó Caldès (1752-1817), studied by A. Roca Rosell.

A common denominator among the various chapters of the book is the circulation of mathematics from the viewpoint of certain European peripheries, such as Spain and Italy, during the 18th century. These peripheries can be considered "dwarfs" in contrast to "giants" such as centers of knowledge production in the 18th century like France or Prussia. Even if at the time the dichotomy was not stated in these terms, there were undoubtedly places recognized as centers exporting models of academic knowledge. For example, artillery schools in France were funded by Louis XIV and became models to be imitated by other nations, such as Italy (as studied in E. Patergnani and M. G. Lugaresi's contribution) and England. Later, Napoleonic campaigns in Europe disseminated French educational models based on the Polytechnic School.

This book aims to show that the relationship between institutional "dwarfs" and "giants" was not that of simple imitation or passive reception. A noteworthy example of this is the flourishing of artillery schools in various Italian states. Although France stood out as a model, local conditions shaped the organization and syllabi of various military academies. In this regard, it is instructive to compare the place of geometry and analysis in the syllabi of military schools in Turin and Verona.[7] In other local contexts such as Naples, synthetic and analytical methods reflected conflicting social and political undercurrent (see the recent [Mazzotti, 2023]).

Scientific novelties also travelled from centres beyond the alps to communities in Italy and Spain. Cases at point are represented by the circulation of the Leibnizian calculus in Italy, studied by Mazzone and Roero [1997], or the Newtonian calculus and physics (see [Mazzotti, 2019] and, more generally, the whole collection of essays containing that publication). Newton, together with French authors such as Clairaut and La Caille, were also influential in Spain.[8]

Similarly, pedagogical novelties also circulated in 18th century Europe through multi-volume mathematical courses such as Wolff' *Elementa matheseos*, reprinted in numerous editions and abridgements.

Moreover, new mathematical courses were created in peripheral regions by adapting and using materials from other books. Among the examples discussed in this book, Tomàs–Vicent Tosca (1651-1723) composed nine volumes between 1707 and 1715, and Benet Baïls (1730-1797) published 11 volumes between 1779 and 1784.[9] These texts also served as models for other authors.

Another example, untreated in the book but neverteless worth mentioning is Agnesi's *Istituzioni Analitiche* published in 1748 in Milan, at the time part of the Habsburg monarchy. Despite being produced in a place far from the European cultural and economic centers and by a woman (thus by a member of a class of alleged "dwarfs", at least according to a wrong-headed view in common historiography), it was successful throughout Europe, even in translations. In fact, Agnesi's book circulated both in published translations (such as the partial French edition) and unpublished ones, such as Colson's "The Plan of the Lady's System of Analyticks", a two-volume draft translation of the *Istituzioni*.[10]

[7]This comparison is addressed in this volume by Patergnani and Lugaresi.

[8]See [Navarro Loidi, 2020] and Navarro Loidi's chapter for this volume. On the influence of Newton in Spain see [Berenguer, 2021].

[9]In fact, the volumes were written in 1772 and published in 1779. Baïls shows a high level of mathematics in his course. See [Martínez-Verdú et al., 2023].

[10]Traces of the circulation of Agnesi's book are visible through discussions of some passages in her books. For instance, the problem in the area of the oblique cone was

In conclusion, by examining different contexts of knowledge production and acquisition, this volume offers several arguments to confirm that the transfer of knowledge is not a straightforward process of information flow from centres to peripheries, since there were networks that overruled the simple centre/periphery division and new knowledge was created in alleged peripheries through the adaptation of new texts to local contexts, or through the reverse circulation of texts from so-called "peripheries" to "centres". We might instead resort to the notion of "dialogue", which involves interaction and interchange: a dialogue between centres and peripheries, between giants and dwarfs. It is in the editors' hope that the question raised and addressed, and the methodologies here deployed could be applied to other case studies, other "peripheries" and other "dwarfs", in Europe and worldwide.

4 The contributions in this volume

The original idea of this book as well as most of its chapters come from a symposium organized by the editors during the latest ICHST-2021 meeting entitled: "Giants and dwarfs in the transformations of mathematics in the XVIII century".[11] The goal of this symposium was to study two interconnected and relevant changes in mathematics that occurred between the middle of 17th to the end of 18th century: the passage of analysis from a method to a discipline regularly taught in colleges and sometimes in universities by the second half of the 18th century, and the transformation of "mixed" mathematics into "mathematical physics" and later "applied" mathematics. In both cases, the perspective of "dwarfs" was chosen as the privileged viewpoint.

The chapter written by F. Gómez García, P. José Herrero Piñeyro, A. Linero Bas, and A. Mellado Romero on Ozanam, as well as E. Dorrego's chapter on Lambert and Legendre study cases of 18th century "giants" who have fallen into oblivion until the recent past. Lambert was deemed as a reference scientific figure by eighteenth century mathematicians and scholars, such as Gauss and Kant, but until a recent

the subject of a monograph printed in Spain in 1755 ([Massa-Esteve, 2011, p. 246]), just a few years after the publication of Agnesi's *Istituzioni* (1748).

[11] See: https://www.ichst2021.org.

reappraisal in the literature, it was little known even among specialists. The reasons for Lambert's neglect are partly explained by José Ferreiros' foreword in [Dorrego López, Fuentes Guillén, 2023]. According to Ferreiros' argument, Lambert was a character imbued with the spirit of Enlightenment:

> He was no specialist, but rather the opposite: a philosopher as much as a scientist, he contributed to all the sciences of his time; while active in the Academies of Munich and Berlin, he contributed to all the different «classes» or areas of work. It has been said, that, for bad and good, Lambert was the perfect example of the eighteenth-century erudite, who wrote about God and the world, about all possible topics: mathematics, experimental science, philosophy, languages, and history.[12]

Ozanam represents a similar case of a scholar considered as a second-order mathematician, at least until the recent past.[13] Did his contemporaries also share this judgment? The chapter published in this book offers arguments to consider the opposite. In the late 17th century, Ozanam was a well-known author who enjoyed the protection of important patrons, he was esteemed by his peers and his students, and received public praises in journals. Leibniz even considered Ozanam worthy of being part of the circle of expert mathematicians, as he was one of the most skilled and experienced in performing ordinary calculations effortlessly.

The reputation of Ozanam had changed in the space of a few generations. In his famous *Histoire des mathématiques*, E. Montucla did not spare criticism when he remarked on how Ozanam had to curb to the whims of his public, which distracted him from serious mathematical contributions (in [Schaaf, 1970-1991]). Just as Lambert, Ozanam embodied the character of a "polymath" distant from the idea of the

[12][Dorrego López, Fuentes Guillén, 2023, p. viii].

[13]According to the *Dictionary of scientific biographies*: "By almost any criterion Ozanam cannot be regarded as a first-rate mathematician, even of his own time" ([Schaaf, 1970-1991]).

"mathematician as a specialist" that would gain prominence and acceptance during the next century.[14]

The contributions of Berenguer, Navarro Loidi, Massa-Esteve, Roca Rosell, and Patergnani and Lugaresi shed light on the biographies of teachers, reformers, and officers by examining the academic context in which they worked. This context was, for instance, that of technical education in private colleges analyzed by Roca Rosell, that of military academies studied by Navarro Loidi, Massa-Esteve, and Patergnani and Lugaresi, or that of Jesuit scientists studied by Navarro Loidi and Berenguer.

Mateo Calabro and Pedro Lucuce, professors at the Royal Military Academy of Barcelona, Tomas Morla and Pietro Giannini, teachers at the Royal Military College of Knights Cadets of Segovia, are the key figures analyzed by Navarro Loidi as instrumental for the establishment of a modern technical curriculum in Spain during the second half of the eighteenth century. During that period, algebra and calculus became essential components of the curricula in technical schools. An intriguing aspect of the story addressed by Navarro Loidi is that these disciplines were not accepted into teaching without opposition. Among the examples studied by Navarro Loidi, the gunner and teacher Tomás Morla recognized, in his Treatise on Artillery (1784-86), that mere practice was "blind and servile" when "divested of principles and theory." However, Morla opposed the excessive use of mathematics in military schools, arguing that military men were not "astronomers" and did not require the same type of theoretical instruction as students of "pure mathematics." Morla's target was Pietro Giannini's course written for the Military Academy of Segovia. The course was deemed too theoretical, and its author, Giannini, was accused of being a "mathematician" but not a "gunner". Finally, the combination of scientific progress, political support, and institutional leadership helped overcome resistance to the inclusion of algebra and calculus in military education in Spain during the 18th century.

By studying the beginnings of the Royal Military Academy of Mathe-

[14]Such a change in the image of the mathematician is examined by A. Alexander ([2006]). Broadly speaking, Alexander relates changes in the ways mathematicians were perceived as public figures to changes in mathematical practices that occurred at the turn of the nineteenth century.

matics of Barcelona (1720) using original sources, M. Rosa Massa-Esteve sheds light on the key transformations of mathematics between the 17th and 18th century: the transition of analysis from being a method to becoming a discipline regularly taught in colleges and universities, the transformation of “mixed” mathematics into “physic mathematics”, and the advancements in the classification of mathematical disciplines during this period. In addition, the story of Jorge Prospero Verboom, creator of the Spanish Corps of Military Engineers, which was officially approved in 1711, stands out as yet another example of a “dwarf” who, despite being downplayed in the history of mathematics, were important for the aforementioned transformations. Another character explored in Massa-Esteves chapter is Pedro Lucuce, who was appointed as director and mathematics teacher at the Royal Military Academy of Mathematics of Barcelona from 1738 to 1756 and from 1760 to 1779.[15] Such as the case of other mathematics teachers populating this volume, Lucuce prepared his courses selecting the content from various works circulating in Europe and adapting them to his audience, looking for a balance between innovation and adherence to classical authorities.[16]

The objections raised against the introduction of abstract and higher mathematics are also discussed in Patergnani and Lugaresi’s chapter on the history of Italian artillery schools. The authors mention the course offeed by Lagrange at Turin’s artillery school in 1755 and how his approach was perceived as too abstract and overly challenging for future officers’ needs. Lagrange’s course entered deep discussions on infinitesimal calculus and analytic geometry, which were considered irrelevant for military education.

The chapter discussed by Lugaresi and Patergnani charts the evolution of technical teaching institutions in Italy from the early 18th century to the second half of the 19th century, and shows that the presence of military schools became a crucial factor in understanding the circulation of mathematics during this period. Italy’s military schools were fashioned after the French system, with the Napoleonic era playing a particularly

[15] In the years from 1757 to 1760, Lucuce was director of the Royal Military Society of Mathematics for elaborating and printing a Mathematical Course. Eventually, the project of this Society failed. See [Blanco and Puig-Pla, 2020].

[16] Lucuce’s course was used until 1803, when the Royal Military Academy of Mathematics of Barcelona was closed.

influential role in shaping the country's own military education models. This period witnessed a geopolitical transformation in Italy, triggered by Napoleon's actions, resulting in the removal of rulers from the *Ancien Régime* and the establishment of "sister republics" modelled after French institutions. Among these institutions, the Royal Polytechnic and Military School of Naples adopted the French model. Similarly, the school in Pavia also experienced the impact of French influence during the Napoleonic period. One noteworthy consequence of this influence was the dissemination of educational resources, textbooks, and teaching methods, that extended beyond military education and made their way into broader technical education. This influence continued to shape engineers training at universities in the following century.

The subject of Berenguer's article is another alleged "dwarf", the bohemian teacher Johannes Wendlingen. Wendlingen's career sheds light on the efficient jesuit academic networks that facilitated the mobility of teachers between various European and non-European countries.

In mid-eighteenth century Spain, the Society of Jesus was one of the intellectual groups in the service of the Crown in the process of modernizing the country. Science was an essential factor in promoting a renewal movement aimed at consolidating the Monarchy, and the Society of Jesus became one of the institutions capable of implementing it and guiding the establishment of the new science.

Berenguer's chapter explores Johannes Wendlingen's main achievements as a mathematician, astronomer and cosmographer in the service of the Spanish Crown. Wendlingen was entrusted with the task of writing a complete mathematics course to be employed as a textbook at the Imperial College in Madrid. Of particular significance is the section of the treatise dedicated to differential calculus, which reveals that Wendlingen was instrumental in its introduction to Spain - a field that was relatively unfamiliar in the country at the time. Berenguer's chapter also provides a detailed comparative analysis showing how Wolff's *Elementa* was used as a guide by Wendligen to teach arithmetic, practical geometry, and infinitesimal calculus. Berenguer's discussion can be taken as an example illustrating how the reception of a text is determined by both local contexts and the purposes for which it is reused. This can vary among mathematical communities and individuals. In

Berenguer's case study, there was a need to adapt Wolff's exercises in geometry to meet the practical needs of topographers in Spain.

Finally, the lesser-known teacher and textbook author Calbó Caldés is studied in A. Roca Rosell 's chapter. Pasqual Calbó Caldés was an artist-scientist from Minorca who taught a course in mathematics in the late 18th century. The manuscript of Calbó's course, written in Minorcan Catalan and analyzed by Roca-Rosell, provides us with valuable insight into the history of private technical education and the role of mathematics in it during the Enlightenment.

Furthermore, Calbó was a living example of how knowledge circulated not only through books but also through people, spending nine years among Venice, Rome, and Vienna. In these cities, he trained as an artist, which may have influenced his own activity as a mathematics teacher. Calbó's manuscript contains a wealth of knowledge on pure and mixed mathematics, showing an interest in experimental physics, sundials, perspective, architecture, and shipbuilding.

All in all, Roca Rosell's chapter is an example of the importance of researching non-academic environments, such as private teaching settings and "workshop cultures," namely the private environments where soon-to-be engineers and architects received their training, often via the contact of experts in these professions, before the ultimate setting up of dedicated schools in the 19th century.

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