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On Loss-Sensitive Rating Plans

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Abstract

Loss-Sensitive Rating Plans involve risk sharing between insurers and insured, providing mutual benefits such as loss control incentives. Unlike guaranteed-cost policies, these plans incorporate a variable premium component dependent on the insured's loss experience. This paper proposes a formalization of insurance premiums under retrospective rating and large deductible plans, offering a generalized approach. In retrospective rating, premiums adjust based on claims within the policy period, while large deductible plans entail fixed premiums with unknown actual costs below the deductible. The insurer's liability may be capped, but the insured's exposure remains unlimited. We introduce representations of insurer and insured burdens, emphasizing the importance of minimum and maximum premiums and adherence to the balance principle. Various mathematical formulations and properties are discussed, offering insights into premium calculation methodologies. This study underscores the need for other robust premium calculation criteria beyond mean values, ensuring risk-sharing practices between insurers and insured.

Keywords: Loss-Sensitive Rating Plans, Retrospective Rating Plan, Large Deductible Plan

1 Introduction

Loss-Sensitive Rating Plans are defined as insurance (or reinsurance) arrangements that involve risk sharing between the insured and the insurer, resulting in mutual benefits, such as an incentive for loss control. Unlike guaranteed-cost policies, where the premium paid by the insured is fixed at the start of the coverage (except for a small deductible), in these types of insurance, the premium has a variable component that depends on the insured's loss experience (G.K. Fisher et al., 2019, X. Chen et al., 2016, J.F. Walhin et al., 2001).

In this paper, we propose a formalization of the insurance premium that includes the premium under both a retrospective rating plan and a large deductible plan. This is a generalization as the proposed premium also includes a coverage limit. In the Retrospective Rating Plan, the insurance premium for a given policy period is adjusted based on the claims that occurred during that same policy period, unlike experience rating, which is based on the loss experience of previous periods (A. Campana, P. Ferretti, 2022).

In the Large Deductible Plan, the premium is generally fixed, but the actual cost of insurance for the insured is not known in advance, as the insured must reimburse the insurer for claims below the deductible.

In the case we examine, we assume a coverage limit is applied to the insurer's liability. Therefore, while the insurer's exposure is limited, the insured's exposure is not.

In the second paragraph, we propose a representation of the claims burden on both the insurer and the insured. In the third paragraph, we present the insurance premium by generalizing the retrospective premium.

2. Representation of the primary loss

Let S be a non-negative random variable (*rv*) which represents an aggregate amount of loss generated by a risk covered by an insurance guarantee and let $F_S(x) = \Pr\{S \leq x\}$ be its cumulative distribution function (*cdf*). The survival function is given by $G_S(x) = Pr\{S > x\} = 1 - \Pr\{S \leq x\}$.

Let $E(S)$ be the expected loss.

Let $L(\alpha, \beta, \gamma)$ be the *rv* defined as follows, with $0 \leq \alpha < \beta < \gamma$:

$$L(\alpha,\beta,\gamma) = \begin{cases} \alpha\mathrm{E}(S) & S \leq \alpha\mathrm{E}(S) \\ S & \alpha\mathrm{E}(S) < S \leq \beta\mathrm{E}(S) \\ \beta\mathrm{E}(S) & \beta\mathrm{E}(S) < S \leq \gamma\mathrm{E}(S) \\ S + \beta\mathrm{E}(\mathrm{S}) - \gamma\mathrm{E}(\mathrm{S}) & S > \gamma\mathrm{E}(S) \end{cases} \quad (1)$$

We denote by $S(\alpha, \beta, \gamma)$ the *rv* that expresses the difference $S - L(\alpha, \beta, \gamma)$:

$$S(\alpha,\beta,\gamma) = \begin{cases} S-\alpha\mathrm{E}(S) & S \le \alpha\mathrm{E}(S) \\ 0 & \alpha\mathrm{E}(S) < S \le \beta\mathrm{E}(S) \\ S-\beta\mathrm{E}(S) & \beta\mathrm{E}(S) < S \le \gamma\mathrm{E}(S) \\ \gamma\mathrm{E}(S) - \beta\mathrm{E}(S) & S > \gamma\mathrm{E}(S) \end{cases} \qquad (2)$$

We interpret $S(\alpha,\beta,\gamma)$ as the loss burden on the insurer, and consequently, $L(\alpha,\beta,\gamma)$ as the loss burden on the insured. In particular, $L(\alpha,\beta,\gamma)$ is the aggregate loss subject to a minimum ratable loss $\alpha E(S)$ and a maximum ratable loss $\beta E(S)$ within the limit $\gamma E(S)$. If $\alpha = 0$, it represents an insurance contract with an absolute deductible of $\beta E(S)$ and a coverage limit of $\gamma E(\mathrm{S})$.

Given $(x-a)_+ = max\{x-a,0\}$ it follows that:

$$S(\alpha,\beta,\gamma) = (S-\beta E(S))_+ - (S-\gamma E(S))_+ - (\alpha E(S) - S)_+ \qquad (3)$$

Denoting by $D(S;\beta E(S),\gamma E(S))$ the portion of loss S in the layer $[\beta E(S),\gamma E(S)]$, we can write (L.J. Halliwell, 2012):

$$S(\alpha,\beta,\gamma) = D\big(S;\beta E(S),\gamma E(S)\big) - (\alpha E(S) - S)_+$$

and consequently:

$$L(\alpha,\beta,\gamma) = (\alpha E(S) - S)_+ + S - D\big(S;\beta E(S),\gamma E(S)\big)$$

Let R be the ratio $S/E(S)$. The *rv* R is defined *entry ratio*.
Let $\bar{L}(\alpha,\beta,\gamma)$ be the ratio $L(\alpha,\beta,\gamma)/E(S)$. It turns out:

$$R = \bar{L}(\alpha,\beta,\gamma) + (R-\beta)_+ - (R-\gamma)_+ - (\alpha - R)_+) \qquad (4)$$

It is customary to estimate expected aggregate excess losses in terms of their entry ratio. The function

$$\varphi(t) = \mathrm{E}(R-t)_+, \qquad t \ge 0 \qquad (5)$$

is known as the Insurance Charge, Aggregate Loss Ratio, Excess Pure Premium Ratio or Table M charge (W.R. Gillam, 1991). It is:

$$\varphi(t)\mathrm{E}(\mathrm{S}) = \mathrm{E}(S - tE(S))_+$$

The function $\pi_S(t) = \varphi(t)\mathrm{E}(\mathrm{S})$ is known as the stop-loss transform of S or stop-loss premium.

Let $F_R(x)$ be the *cdf* function of the entry ratio R and $G_R(x)$ its survival function.

It is

$$\varphi(t) = \int_t^{\infty}(x-t)\, dF_R(x) = \int_t^{\infty} G_R(x)\, d(x) \tag{6}$$

Mathematical properties of $\varphi(t)$ are:

i. $\varphi(0) = 1$
ii. $\lim_{t\to+\infty} \varphi(t) = 0$
iii. $\varphi(t)$ is non-increasing: $\varphi'(t) = \frac{\mathrm{d}}{\mathrm{d}t}\varphi(t) = -G_R(t) \le 0$
iv. $\varphi''(t) = \frac{\mathrm{d}}{\mathrm{d}t}\varphi'(t) = f_R(t)$ where $\mathrm{f}_R(t)$ is the density function of the entry ratio R.

The function

$$\psi(t) = \mathrm{E}(t-R)_+ \qquad t \geq 0 \tag{7}$$

is known as the Insurance Savings, Aggregate Minimum Loss Factor or Table M savings and it is so defined

$$\psi(t) = \int_0^t (t-x)\, dF_R(x) = t - \int_0^t \mathrm{G}_R(x)\, d(x) \tag{8}$$

Mathematical properties of $\psi(t)$ are:

i. $\psi(0) = 0$
ii. $\lim_{t\to+\infty} \psi(t) = +\infty$
iii. $\psi(t)$ is increasing: $\psi'(t) = \frac{\mathrm{d}}{\mathrm{d}t}\psi(t) = 1 - G_R(t) \ge 0$
iv. $\psi''(t) = \frac{\mathrm{d}}{\mathrm{d}t}\psi'(t) = f_R(t)$.

It holds $R = t + (R-t)_+ - (t-R)_+$ and then

$$\varphi(t) - \psi(t) = 1 - \mathrm{t} \tag{9}$$

From (4) we obtain:

$$1 = E(\bar{L}(\alpha,\beta,\gamma)) + \varphi(\beta) - \varphi(\gamma) - \psi(\alpha) \tag{10}$$

From (10), multiplying both sides by $E(S)$, we obtain:

$$E\big(L(\alpha,\beta,\gamma)\big) = E(S) + [\psi(\alpha) + \varphi(\gamma) - \varphi(\beta)]E(S) \tag{11}$$

In the absence of a coverage limit, we define the *rv* $L(\alpha,\beta)$ as

$$L(\alpha,\beta) = \begin{cases} \alpha\mathrm{E}(S) & S \le \alpha\mathrm{E}(S) \\ S & \alpha\mathrm{E}(S) < S \le \beta\mathrm{E}(S) \\ \beta\mathrm{E}(S) & S > \beta\mathrm{E}(S) \end{cases} \tag{12}$$

and the corresponding *rv* $S(\alpha, \beta)$ follows the definition

$$S(\alpha, \beta) = (S - \beta E(S))_+ - (\alpha E(S) - S)_+ \quad (13)$$

$L(\alpha, \beta)$ is the aggregate loss subject to a minimum ratable loss $\alpha E(S)$ and a maximum ratable loss $\beta E(S)$ underlying the insurance premium calculated according to the retrospective method (G. Meyers, 2014).

3. Generalization of retrospective rating

We represent the insurance premium with $\Pi\big(S(\alpha, \beta, \gamma)\big)$, a function of the *rv* $S(\alpha, \beta, \gamma)$.
Let Ψ be a functional representing the financial standing of the insurer in response to risk. Let's assume that the following condition must hold for the insurer

$$\Psi\big(\Pi(\alpha, \beta, \gamma)\big) = \Pi_{min}(S(\alpha, \beta, \gamma))$$

where $\Pi_{min}(S(\alpha, \beta, \gamma))$ is the minimum premium that the insurer is willing to receive to assume the risk that S generates.
Let Φ be a functional that represents the financial standing of the insured in response to risk. We assume that the following condition must hold for the insured

$$\Phi\big(\Pi(\alpha, \beta, \gamma)\big) = \Pi_{max}(S(\alpha, \beta, \gamma))$$

where Π_{max} $(S(\alpha, \beta, \gamma))$ is the maximum premium that the insured is willing to pay to enter into the contract.

For the insurance contract to be stipulated it must be:

$$\Pi_{min}(S(\alpha, \beta, \gamma)) \leq \Pi_{max}(S(\alpha, \beta, \gamma))$$

Suppose the following holds:

$$\Pi_{min}\big(S(\alpha, \beta, \gamma)\big) \leq \Pi\big(S(\alpha, \beta, \gamma)\big) \leq \Pi_{max}(S(\alpha, \beta, \gamma)) \quad (14)$$

$\Pi(S(\alpha, \beta, \gamma))$ can be a *rv*, while $\Pi_{min}\big(S(\alpha, \beta, \gamma)\big)$ and $\Pi_{max}(S(\alpha, \beta, \gamma))$ are deterministic quantities.

Let's consider a retrospective rating plan and let's assume that the insurance premium is given by:

$$\Pi\big(S(\alpha, \beta, \gamma)\big) = B + cL(\alpha, \beta, \gamma) \quad (15)$$

where B is referred to as the basic premium amount, reflecting fixed charges that do not vary with actual losses, c is called the loss conversion factor, and $L(\alpha, \beta, \gamma)$ represents the *ratable losses* used to calculate the retrospective premium.[1]
According to the balance principle, it is usually required that the plan is balanced to the guaranteed cost premium, i.e. the average value of the premium should be equal to the standard premium (G. Meyers, 2014).
Therefore, let's assume it holds:

$$E(\Pi(S(\alpha, \beta, \gamma))) = e + E(S) \tag{16}$$

where e represents the total expenses, known as the expense ratio underlying the guaranteed-cost premium.
From equations (11) and (15), we obtain:

$$B = e + (1 - c)E(S) + c[\varphi(\beta) - \varphi(\gamma) - \psi(\alpha)]E(S) \tag{17}$$

In the case of retrospective rating without a coverage limit γ, given the premium

$$\Pi\big(S(\alpha, \beta)\big) = B + cL(\alpha, \beta)$$

similarly to equation (17), the basic premium can be written as:

$$B = e + (1 - c)E(S) + c[\varphi(\beta) - \psi(\alpha)]E(S) \tag{18}$$

The difference $\varphi(\beta) - \psi(\alpha)$ is called Net Aggregate Loss Factor or Net Table M charge (G.K. Fisher et al., 2019).
In this case, the premium is constrained both at the lower and upper bounds

$$P_{min}\,(S(\alpha, \beta)) \leq \Pi\big(S(\alpha, \beta)\big) \leq P_{max}\big(S(\alpha, \beta)\big)$$

where $P_{min}\,\big(S(\alpha, \beta)\big) = B + c\alpha E(S)$ and $P_{max}\big(S(\alpha, \beta)\big) = B + c\beta E(S)$.

From (18) we obtain:

$$P_{min}\,\big(S(\alpha, \beta)\big) = e + (1 - c)E(S) + c[\varphi(\beta) - \psi(\alpha) + \alpha]E(S)$$

By substituting $-\psi(\alpha) + \alpha = 1 - \varphi(\alpha)$ (see (9)), we obtain:

$$P_{min}\,(S(\alpha, \beta) = e + E(S) + c[\varphi(\beta) - \varphi(\alpha)]E(S)$$

[1] In the case of the retrospective method, the premium is given by $[B + cL(\alpha, \beta)]T$ with T being the tax multiplier, which we neglect here for simplicity.

Given the minimum premium and the maximum premium, the values of α and β must satisfy the following equations (referred to as the *balance equations for aggregate losses*):

- $\beta - \alpha = \frac{P_{max}\,(S(\alpha,\beta)) - P_{min}\,(S(\alpha,\beta))}{cE(S)}$
- $\varphi(\alpha) - \varphi(\beta) = \frac{1}{cE(S)}[e + E(S) - P_{min}\,(S(\alpha,\beta)]$

If the coverage limit γ is present, the premium is not capped by any amount since the exposure $L(\alpha,\beta,\gamma)$ of the insured is not limited upwards [2], and we only have:

$$B + c\alpha E(S) \leq \Pi\big(S(\alpha,\beta,\gamma)\big)$$

4. Concluding remarks

The application of the balance principle in calculating the premium B entirely neglects the variability (and thus the risk) of the stochastic component of the insurance premium. The amounts $P_{min}\,(S(\alpha,\beta))$ and $P_{max}\,(S(\alpha,\beta))$ in the retrospective premium do not assume the role and significance attributed to the minimum and maximum premiums mentioned in inequality (14).

A valid alternative could be to consider a calculation criterion not based on the mean value, such as the variance criterion or expected utility. The values α,β,γ must meet the requirements of both parties (insurer and insured): minimum premium and maximum premium, and the balance principle represent some reference conditions.

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