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Three Essays in Financial Literacy

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Tesi di dottorato di **Yuri Pettinicchi** , Matricola 955406

Coordinatore del Dottorato	Tutore del dottorando
Prof. Agar Brugiavini	**Prof. Mario Padula**

The undersigned Yuri Pettinicchi, in his quality of doctoral candidate for a Ph.D. degree in Economics granted by the Università Ca' Foscari Venezia attests that the research exposed in this dissertation is original and that it has not been and it will not be used to pursue or attain any other academic degree of any level at any other academic institution, be it foreign or Italian.

Abstract

This dissertation presents three essays in financial literacy. In Chapter 1, I study the information acquisition process in a simple asset pricing model with heterogeneous beliefs about future prices. This is instrumental to investigate the effects of financial literacy on market stability. I posit that financial literacy affects the cost of acquiring information on the asset payoff and show that the effect on the market volatility is non-monotone and depends on the uncertainty of the fundamentals. I conclude that financial education programs increase price informativeness and, in a scenario with high uncertainty of the fundamentals, stabilize the market. However, when uncertainty is low, financial literacy improving policies increase the volatility of the markets.

Chapter 2 develops a tractable asset pricing model where individuals acquire financial information and face a fixed cost to participate to the market. I find out that, on the one hand, low participation cost increases the information acquired by the agents. On the other hand, it reduces the market participation and, therefore, the informativeness of the market price. Furthermore, with high participation cost, the information acquired also decreases. The effect on market price variance is non-monotone and depends on the uncertainty of the risky asset.
For increasing inequality in financial literacy, market participation of the less literate agents decreases, at a faster rate with low uncertainty in the fundamentals. Furthermore, the market friction prevents the market price to reveal all the private financial information acquired by the agents, reducing the information externalities.

In Chapter 3, that is a joint work with Mario Padula, we investigate asset pricing implications of letting individuals to decide how much financial literacy to acquire. We assume that individuals are born with the same amount of innate financial literacy, which they can improve upon by attending financial education programs. In a simple market trading model with noisy supply, a risky and a riskless asset, the benefit of

taking the financial training is the same for all individuals and pertains to the precision of the signal on the excess return. More financially literate individuals can buy more precise signals at a relatively lower cost, but individuals sharing the same level of financial literacy pay the same cost to buy the signal's precision. Conversely, the cost of attending the financial education program varies between individuals, depending on their cognitive abilities. This is instrumental to investigate how cognitive abilities relate to the decision to increase financial literacy and to the stability of financial markets. Moreover, the model allows to discuss the implications of policy affecting the productivity of financial education programs and show that general equilibrium effects make the share of literate individuals a non-monotone function of the productivity of financial education programs.

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Chapter 1

Financial Literacy, Information Acquisition, and Asset Pricing Implications

1.1 Introduction

In standard asset pricing models, agents maximize expected utility and choose optimally financial investments. Models posit that agents are fully informed, are able to correctly forecast future outcomes and to diversify appropriately investment risks.

Many stylized facts, such as low participation to the equity market by household (Mankiw and Zeldes (1991), Haliassos and Bertaut (1995)), low level of diversification of household portfolios (Curcuru *et al.* (2009)), or preferences for default options (Beshears *et al.* (2009)), cannot be explained by standard models. Campbell (2006) proposes that some assumptions should be dropped and some costs or other factors must be taken into account. Haliassos and Bertaut (1995) go further and propose to shift to non standard expected utility function.

A stream of household finance literature focus on lack of financial literacy; many

empirical studies (Lusardi and Mitchell (2009), Christelis *et al.* (2010), Lusardi and Tufano (2009)) document that low degree of financial literacy is widespread among households and suggest to improve it through financial education programs.[1] However, in this literature, up to my knowledge, general equilibrium models are missed. This kind of models allows the policy makers to investigate the effects of financial education programs.

Our work contributes to fill this gap. Specifically, our model explains the general equilibrium effects of financial literacy on market stability. The main intuition is that agents with lower degree of financial literacy face higher costs of being informed. This implies that they optimally choose to remain uninformed and they do not contribute to increase the informativeness of the market price.

The impact of financial education programs, aimed at improving the financial literacy of the agents, on the market stability is positive or negative, given uncertainty of the market fundamentals. Even if the price informativeness always increases, the market volatility decreases with high uncertainty in the fundamentals and it increases with low uncertainty.

Moreover, we identify a channel through which policy makers can affect the information market. We find out that the financial education programs are easily implementable and immediately effective only with high uncertainty of the fundamentals. With low uncertainty, the cost of a successful policy increases in the financial literacy inequality among the agents.

1.1.1 The role of Financial Literacy

In the last decades, in many industrialized countries, changes in the demographic patterns shifted the responsibility of saving choices on individuals. Financial mar-

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[1] We use the following definition of financial literacy: degree of knowledge of basic financial concept, ability to manage personal finances, and confidence of own choices made in a complex financial environment. For a review of financial literacy definitions Remund (2010), Hung *et al.* (2009).

kets are complex systems, and individuals face financial decisions subject to several behavioural biases, as it is well documented by Thaler and Benartzi (2004). Low degrees of financial literacy lead to suboptimal financial outcomes. Empirical and experimental studies assess the necessity to improve financial literacy among households.[2] Bernheim and Garrett (2003) show that the employer-based programs of financial education affect positively workers' saving choices. Lusardi and Mitchell (2007) find that planning abilities and financial education are negatively associated when they consider workers' retirement decisions. Guiso and Jappelli (2009) show that investors, who understand better investment products, hold more efficient portfolios. In a Dutch households survey, van Rooij *et al.* (2011) find out that households with low financial literacy are also less likely to hold risky portfolios.

Thus, policy makers provide education programs aimed at increasing individual financial literacy, in order to improve individual financial behaviour. Implicitly, they assume that social welfare would increase as well. Individuals can save more, and better manage risks, through insurance. At macro level, increased demand for financial services may improve risk-sharing and financial intermediation. The direct consequences would be financial development and competition in the financial services sector. It is still under debate the implication on markets volatility and economic growth. The key issue is to understand if the financial literacy improving policies lead to a more efficient allocation of capital.

Our approach takes into account feedback effects of these policies within a general equilibrium framework. We investigate the effect of a change in the distribution of the financial literacy among the agents on the stability of the market. As a proxy, we use the market price variance. Individuals make their own financial choices on the base of the information set they are able to handle. Lack of financial literacy affects negatively their ability to acquire and process information. We model explicitly how

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[2]Lusardi and Mitchell (2009), Christelis *et al.* (2010), Lusardi and Tufano (2009), Thaler and Benartzi (2004), and Agnew and Szykman (2005)

agents acquire information and how financial literacy affects this process. Our work follows the literature on financial information acquisition where traders are allowed to choose the precision of their private signal, paying a cost.[3] Given the CARA-Gaussian framework, in these models the financial information always improves the precision of subjective expectations. These model were used to explain inequalities among households, e.g. Verrecchia (1982) through heterogeneous risk aversion and Peress (2004) through heterogeneous initial wealth. The main implication is that wealthier households gain more from purchasing the private information, improving their Sharpe ratio. Thus, they end up to be more informed.

Our model differs in the source of heterogeneity. We consider heterogeneous financial literacy.[4] Agents can acquire costly information; namely, they can purchase an unbiased additive noisy signal on the payoff of the risky asset. They can choose the amount of the signal precision, and what they pay is proportional to their financial literacy. It is important to note that, in this model, policy makers can manipulate the degree of heterogeneity of individuals through financial education programs. Similar policies are not possible if we take into account heterogeneity in risk aversion or in other subjective characteristics.

Our results show that more financially literate agents are, more information they purchase and more revealing market prices are. The impact on market volatility is non-monotone and depends on the fundamentals uncertainty. We conclude that financial education programs increase the stability of the market only in a scenario

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[3]The framework was developed during the 80′s by Grossman and Stiglitz (1980), Hellwig (1980), Verrecchia (1982) and it was used to study static investment allocation and trade between agents. Traders receive unbiased private signals and form heterogeneous posterior beliefs such that no-trade theorems (Milgrom and Stokey (1982)) do not apply. Grundy and McNichols (1989), Wang (1993), and others developed a dynamic version of this framework to study the effect of public and private signals on the trade volume. It was common in this literature, to assume private unbiased normally distributed signals, uncorrelated errors across agents.

[4]We figure out a situation where agents face the same financial report and extract signals on the true payoff paying a cost in term of their utility. More expert they are, lower costs they have, more precise signals will be. Their ability to understand financial information is exogenous. Future research would be to take it endogenous, letting agents to choose optimally the amount of financial literacy they want to accumulate.

with high uncertainty. Conversely, with low uncertainty, increasing financial literacy of the agents leads to higher volatility in the markets.[5]

The chapter is organized as follow: in Section 1.2 we set up the model, in Section 1.3 we characterize the equilibrium and in Section 1.4 we discuss the implications of the model. Section 1.5 concludes and points out further research steps. All the proofs are collected in the Appendix.

1.2   Model

In this model agents face two choices: in the first period, they have to choose if and how much information they want to purchase. In the second period, if and how to trade in the market. There are two primitive assets available for trading. A riskless asset pays a rate of return r $(R = 1 + r)$ and it has a perfectly elastic supply. A risky asset, with price p, pays a payoff π with $\pi \sim N(\mu_\pi, \tau_\pi^{-1})$. Short selling is allowed.[6] The per capita supply of the risky asset is $\theta \sim N(\mu_\theta, \tau_\theta^{-1})$ which is interpreted as noise trading in the market.[7] We assume that π and θ are mutually independent random variables and their joint distribution is common knowledge.

Agents

We assume agents differ in their financial literacy. Heterogeneity is expressed by c, the financial literacy parameter. It affects the costly information acquisition process. We assume two types of agents. Type L (literate agents) with low costs and type H (illiterate agents) with high costs of acquiring financial information. Population of

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[5]In this chapter, even if we do not provide a welfare analysis, we focus on a reduction of the costs due to the volatility, having increasing price informativeness in both cases.

[6]Future research would consider margins and collateral to restrict short selling.

[7]The introduction of an exogenous aggregate risk allows to avoid the Grossman-Stiglitz paradox. With an extra noise, market price are not fully revealing, therefore there is still some incentives to purchase information. On the other hand, the introduction of an exogenous shock in the model is a quite strong assumption. Wang (1993) models the noise as investors' liquidity needs.

agents has mass one and, for each type, there are enough agents so that the law of large numbers applies.[8] We call $J = L \bigcup H$ the set of all agents. $G(j)$ denotes the distribution of the agents and λ the fraction of the literates.

They maximize the same concave utility function of their final wealth. For tractability, we assume CARA utility function with absolute risk aversion coefficient ρ: $U(W) = -\frac{1}{\rho}e^{-\rho W}$.[9]

Information structure

Once π is realized but not revealed, each agent can purchase an unbiased signal s on the risky asset payoff, and can observe a private realization:

$$s = \pi + \epsilon$$

where ϵ is independent of π, θ, and across agents. Its distribution is:

$$\epsilon \sim N(0, \frac{1}{x})$$

where x denotes the precision of the private signal.[10] Agents can purchase precision x paying a monetary cost $C(x, c)$. We can think they pay for financial advices. More effort the adviser needs to be understood, higher commission the advisor will ask. However, agents cannot acquire perfect precision, namely a signal with zero variance.

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[8]Within the group agents differ only for the realization of their private signal, if they purchase one. Moreover, we posit that they do not realize they can act strategically, affecting market price through their informative choice and their asset demand.

[9]The choice of this form for the utility function implies that it is always optimal to hold some risky assets for all agents. The assumed shape for the utility function fully satisfies the participation principle: given a positive equity premium, all agents invest money in the risky asset, regardless of the degree of risk aversion or the riskiness of the asset. Therefore, the agent does not choose to remain out of the market. Empirical studies show that the participation principle fails in reality (the stock-holding puzzle): limited market participation and heterogeneous portfolio behaviours characterize real financial markets (Guiso *et al.* (2003)). Further research will deal with this issue.

[10]In the appendix we solve the model taking into account that, when x is zero, the private signal is fully uninformative. Technically, we approximate a normal random variable with infinite variance to a uniform random variable with an infinite domain.

Formally, the cost of acquiring an amount x of precision is given by a continuous and twice differentiable function $C(x, c)$ over $x \in \mathbb{R}^+$ such that:

- $C(x, c)$ is at least twice differentiable in x and c with

$$C_x > 0, \ C_{xx} \geq 0 \qquad \text{and} \qquad C_c > 0, \ C_{cc} \geq 0$$

- C_x is increasing in c ($C_{xc} > 0$): acquiring information at the margin is more costly for less financially literate agents.

- $C(x, c)$ is continuous at $x = 0$: $C(0, c) = 0$. Moreover, $\lim_{x \to +\infty} C(x, c) = +\infty$. The last two properties imply that a totally uninformative signal is costless and a fully revealing signal is infinitely expensive.

These assumptions ensure the existence of a solution for the information choice. To illustrate the main intuition, we provide a simplified example, which we follow in each step of the model.

Example (Cost Function). *The information cost function is:* $C(x, c) = c(x^2 + x)$ *with* $c = \{c_L, c_H\}$ *and* $c_L < c_H$.

To solve the model, we focus on a partially revealing noisy rational expectation equilibrium. All agents have rational expectations in the sense of Hellwig (1980) and use the information revealed by the price while they form their posterior beliefs. Given that we assume unbiased private signals spread among agents and this is common knowledge, the agents know that the equilibrium price p contains some information about the payoff value. Therefore, they use it as an informative signal, where its precision is given by the aggregation of the private signal precisions through individual asset demands. Following the literature, we solve for an equilibrium in which the risky

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asset price is a linear function of π and θ: $pR = a + b\pi - d\theta$, where the coefficients a, b, d are determined in equilibrium.[11]

We denote agents' information set as $\mathcal{F} = \{s_j, p\}$ where s_j denotes the private signal observed by agent $j \in J$ and it is informative only if agent j acquires some information precision.

Timing

There are three periods. In period 1, the planning period, the agent can purchase a private signal s, and can choose its precision x. In period 2, the trading period, observing her private signal realization s and the market price p, the agent trades in a competitive market, choosing portfolio share α. In period 3, the consumption period, the agent consumes the proceeds from her investments.

Figure 1.1 provides the timeline of the model.
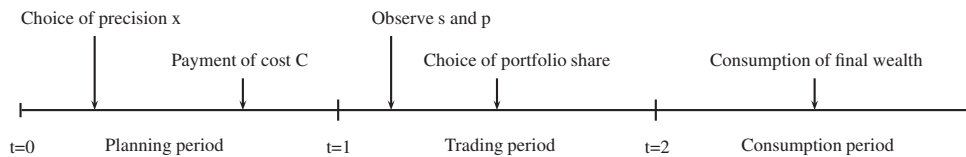
Figure 1.1: Timeline

1.3 Equilibrium

We solve the model by backward induction. In the third and last period, each agent consumes her final wealth. In the second period, the agent faces a portfolio allocation problem where she needs to choose the share of the portfolio invested in the risky asset, in order to maximize her expected utility, given the precision purchased x and the information cost paid C. She observes a private and a public signal (market

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[11]Linearity is a standard assumption in the literature when the aim of the research is to find a closed form solution for the price function. Non-linear price functions can provide a better approximation to the real price function. On the other hand, they lose in tractability and rely on numerical methods.

price) and she computes posterior beliefs about the final payoff value: $E[\pi|\mathcal{F}]$ and $Var[\pi|\mathcal{F}]$. In the first period, the agent chooses how much private signal precision x she wants and pays the monetary cost $C(x,c)$.

The portfolio share differs between types and within groups. It depends on the realization of the private signal. The information precision differs between types, but it is the same within groups.[12]

The trading period

In this period, each agent maximizes

$$\max_{\alpha} \quad E[U(W_2)|\mathcal{F}_j]$$

subject to

$$W_2 = (W_1 - C)R^p \tag{1.1}$$

$$R^p = \alpha(\frac{\pi - pR}{p}) + R \tag{1.2}$$

Equation (1.1) is the budget constraint and (1.2) is the return on the agent's portfolio. In this period C is a sunk cost and given by the choice made in the previous period.

The optimal share invested in risky assets differs between agents, depending on the signal observed and the precision purchased. In the trading period, all the information choices are already done and each trading agent transfers some of her purchased information to the market price through her asset demand. Therefore, private information is partially revealed by the market price. While they are forming their posterior beliefs and formulating their asset demands, agents take into account the price informativeness and transform the market price into an unbiased public signal.

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[12]Thus, $\forall j \in L$ the optimal precision is denoted by x_L and, $\forall j \in H$, by x_H.

The indirect utility for agent j's portfolio problem, is $E[U(W_2^*)|\mathcal{F}_j]$, which we note as $v(s_j, p; \Theta)$ where Θ is $\{R, W_1, \rho, \mu_\pi, \mu_\theta, \tau_\pi, \tau_\theta, \lambda\}$.

The planning period

In the planning period, each agent maximizes the indirect utility for the portfolio allocation problem with respect to the information choice. To simplify the notation, we drop the subscript j and write:

$$\max_{x \geq 0} E[v(s, p; \Theta)]$$

subject to

$$W_1 \geq C(x, c)$$

where the expected utility is computed over the joint probability distribution of s and p. Recall that signal precision x affects, by assumption, only the distribution of the private signal s. The optimal precision $x^*(c, \Theta)$ depends on the exogenous financial literacy cost. The agents are willing to purchase more information if there is less in the market (price is low informative) and they are not if the market price is more revealing. In the information market, it must hold the equilibrium condition, such that the price informativeness is given by all private information optimally acquired.

The equilibrium

A rational expectations equilibrium is given by individual asset demand function α_j and individual information demand function x_j, a price function p of π and θ, and a scalar I such that:

1. $x_j = x^*(c_j, \Theta)$ and $\alpha_j = \alpha^*(s_j, p; \Theta)$ solve the maximization problems, with $c_j = \{c_L, c_H\}$.

2. p clears the market for the risky asset:

$$\int_{j \in J} \alpha_j \frac{W_1 - C(x_j, c_j)}{p} \, dG(j) = \theta$$

3. The informativeness of the price I, implied by aggregating individual precision choices, equals to the level assumed in the agents' maximization problem:

$$I = \int_{j \in J} x_j \, dG(j)$$

In noisy rational expectations equilibrium models, investors make self-fulfilling conjectures about prices and the equilibrium is defined as the set of allocations, such that agents maximize their utilities, their conjectures hold true and markets clear.

The following three propositions describe the equilibrium allocations. Proposition 1 computes the price function and the optimal individual asset demand in the trading period, taking the information choices as given. Proposition 2 provides the optimal individual amount of precision. Proposition 3 claims that there exists a unique rational expectation equilibrium.[13]

Proposition 1. *The equilibrium price is given by:*

$$pR = a + b\pi - d\theta$$

where

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[13]Agents conjecture an amount of the price informativeness, such that, aggregating over their information choices, the market price ends up, in equilibrium, with that amount of aggregate informativeness. We substitute the informationally equivalent signal $\xi \equiv p$ in the information set $\mathcal{F}_j$.

$$a = \frac{\mu_\pi \tau_\pi + \frac{I}{\rho} \mu_\theta \tau_\theta}{\tau_\pi + I + \frac{I^2}{\rho^2} \tau_\theta} \qquad b = \frac{I + \frac{I^2}{\rho^2} \tau_\theta}{\tau_\pi + I + \frac{I^2}{\rho^2} \tau_\theta} \qquad d = \frac{\rho + \frac{I}{\rho} \tau_\theta}{\tau_\pi + I + \frac{I^2}{\rho^2} \tau_\theta}$$

The optimal portfolio share for agent $j \in J$ is given by:

$$\alpha_j^* = \frac{k_j \, p}{\rho[W_1 - C(x_j, c_j)]} (E[\pi | s_j, \xi] - pR)$$

Proof. See the appendix. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $\square$

We provide a sketch of the proof. In the first step, we guess a price linear function and we derive the informationally equivalent public signal ξ from the price function. In the second step, we compute the mean and the variance of the posterior beliefs given the two unbiased signals, ξ and s_j. In the third step, we derive the optimal asset demand. In the fourth step, we derive market clearing conditions and, in the last step, we impose rationality and determine the coefficients of the guessed linear price function.

The optimal portfolio share is the standard solution for the maximization problem of an agent with CARA utility function. It is always optimal to trade some risky assets, if the agent believes that the expected excess return is positive. For each agent $j \in J$, the optimal portfolio share α_j depends on the precision of the posterior belief ($k_j = \tau_\pi + x_j + \frac{I^2}{\rho^2} \tau_\theta$) and on the expected excess return conditional to the agent's informative set:

$$E[\pi | s_j, \xi] - pR = k_j^{-1}(\tau_\pi \mu_\pi + x_j s_j + \frac{I^2}{\rho^2} \tau_\theta \xi) - pR$$

where ξ is the informationally equivalent public unbiased signal derived by rational agents from the equilibrium price function:

$$\xi = \tfrac{1}{b}(pR - a + d\mu_\theta) = \pi - \tfrac{d}{b}(\theta - \mu_\theta)$$

We rewrite the optimal portfolio share to show how the private information affects the behaviour of the agent:

$$
\begin{aligned}
\alpha_j^* &= \frac{p}{\rho(W_1 - C_j)}\left[\tau_\pi(\mu_\pi - pR)(1 - \frac{I^2\tau_\theta}{\rho^2}\frac{1}{I + \frac{I^2\tau_\theta}{\rho^2}}) + x_j(s_j - pR)\right] \\
&= \frac{W_1}{W_1 - C_j}\,\alpha^*|_{x=0} + \frac{p}{\rho(W_1 - C_j)}\left[x_j(s_j - pR)\right]
\end{aligned}
$$

where the first term is the optimal share of an agent who follows only market feelings, i.e. all the public knowledge embodied in the prior beliefs and in the market price. Thus, the uniformed agent chooses her optimal portfolio share, herding on what the market partially reveals.[14]

The second term in bracket is the risky asset's premium as predicted by her private signal. It is the extra portfolio share of the informed agent. She follows her private knowledge and balances what the market suggests with a trading position consistent with her expectation. In case she gets a private signal realization different from the market feelings, she would like to bet against the market, in order to speculate on her private knowledge. Furthermore, more precise private signal she has, more aggressively she would like to trade.

In Figure 1.2 we plot the asset demand for both types. The steeper line is type L. They are more confident about the market, given that they are better informed.

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[14]When $I = 0$, no agents purchase private information. There are only uniformed and noisy traders in the markets. The market price just reflects the noisy supply and the agents hold risky assets in order to offset it. When $I \to \infty$, $(1 - \frac{I^2\tau_\theta}{\rho^2}\frac{1}{I + \frac{I^2\tau_\theta}{\rho^2}})$ goes to zero and agent j does not purchase any risky assets: $\alpha_j^* = 0$. The market price fully reveals the value of the fundamentals, therefore there are no reasons to trade.

Therefore, with the same signal, they would trade more aggressively. When $s_j = pR$ (point A), both types short sell to the noisy traders the same amount θ of risky assets. They do not know θ in advance. However, in equilibrium, they end up to hold exactly the per capita noisy supply. This result is derived endogenously from the conjectured price function by rational agents. In order to understand the role played by the market price, we rewrite it as:

$$pR = \frac{1}{\tau_\pi + I + \frac{I^2}{\rho^2}\tau_\theta}\left[\mu_\pi\tau_\pi + \frac{I}{\rho}\mu_\theta\tau_\theta + (I + \frac{I^2\tau_\theta}{\rho^2})(\pi - \frac{\rho}{I}\theta)\right]$$

Rearranging and considering that $K = \tau_\pi + I + \frac{I^2}{\rho^2}\tau_\theta$ is the posterior precision of the average agent's beliefs, the market price can be expressed as a linear function of the posterior mean and variance of the average investor:[15]

$$pR = K^{-1}\left[\mu_\pi\tau_\pi + \frac{I}{\rho}\mu_\theta\tau_\theta + \pi I + \frac{I^2\tau_\theta}{\rho^2}(\pi - \frac{\rho}{I}\theta)\right] - \frac{\rho\theta}{K} = \hat{\bar{\pi}} - \rho\theta K^{-1}$$

Market price is driven by two components: posterior belief of the average agent and the discount on the price demanded in order to be compensated for the risk due to the noisy supply. The latter term is weighted by the risk aversion coefficient.

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[15] We characterize the average agent as the one with posterior precision:

$$K = \int_j k_j dG(j) = \tau_\pi + I + \frac{I^2}{\rho^2}\tau_\theta$$

and posterior mean:

$$\hat{\bar{\pi}} = \int_j E[\pi|s_j,\xi]dG(j) = K^{-1}\left[\mu_\pi\tau_\pi + \pi I + \xi\frac{I^2\tau_\theta}{\rho^2}\right]$$

It is important to note that we do not mean there exists a real agent with these beliefs. We mean a fictitious agent with private signal precision equal to I and private signal realization equal to π. We can think about a shared-information economy where a central planner can observes all the private signals, normally distributed, and take the sample mean: $\int_j s_j dG(j) = \pi$, with precision $\int_j x_j dG(j) = I$.

The equilibrium market price follows a Gaussian distribution:

$$pR \sim N\left(\mu_\pi - \rho\mu_\theta K^{-1}, K^{-2}\left[\left(\frac{I^2\tau_\theta}{\rho^2} + I\right)^2 \frac{1}{\tau_\pi} + \left(\frac{I\tau_\theta}{\rho} + \rho\right)^2 \frac{1}{\tau_\theta}\right]\right)$$

When the price informativeness is zero ($I = 0$), it means nobody purchases private information, nobody observes informative private signals and nothing is revealed in the market price through the asset demand about the fundamental π (the coefficient b is zero). In this case, the market price reflects the prior mean plus the discount demanded for the presence of noisy asset supply ($pR = \mu_\pi - \frac{\rho}{\tau_\pi}\theta$). Moreover, the market price variance is just given by the variance of the noisy supply times the square of the prior variance and the square of the risk aversion coefficient: $\sigma_{pR}^2(I = 0) = (\frac{\rho}{\tau_\pi})^2\frac{1}{\tau_\theta}$.

Conversely, when the price informativeness tends to infinity ($I \to \infty$), the market price variance tends to the variance of the risky asset (τ_π^{-1}).[16] When the market price is fully revealing, we are back to the case where the supply of the risky asset is known by all the agents and the market price embodies only the uncertainty about the payoff of the asset.

In Appendix B, we check how fundamentals uncertainty, τ_π and τ_θ, and risk aversion ρ affect volatility. Decreasing uncertainty with respect to the asset payoff and the noisy supply decreases volatility. We note wider price fluctuations in Table 1.1(a) than what we observe in Table 1.1(b). This is due to the fact that the payoff uncertainty directly affects the market price, while the noisy supply uncertainty is weighted by the price informativeness and by the risk aversion.

Risk aversion does not monotonically affects volatility. With both high and low risk averse agents, the market price shows high sensitiveness to fundamentals uncer-

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[16]We have that the first part tends to one: $K^{-2}\left(\frac{I^2\tau_\theta}{\rho^2} + I\right)^2 \to 1$ and the second term tends to zero: $K^{-2}\left(\frac{I\tau_\theta}{\rho} + \rho\right)^2 \to 0$.

tainty. When agents are less risk averse, they trade more. Therefore, the market price is more informative given that the agents transfer more private information to the market through their larger asset demands. However, this also implies higher price sensitiveness with respect to the payoff. When agents are more risk averse, they trade less and the market price is less revealing. This implies lower market depth and higher price sensitiveness with respect to the noisy supply.

We distinguish between two scenarios, depending on the values of c_L and c_H. Table 1.1 report market price variance for a low degree of inequality between agents, $c_H = 3c_L$, while in Table 1.2, it is higher, $c_H = 10c_L$.

The next proposition shows the existence and the uniqueness of the optimal information choice and the endogenous threshold according to which is optimal to remain uninformed.

Proposition 2. *There exists a threshold $\overline{c}(\Theta)$ such that all agents with lower financial literacy cost purchase information.*

For all agents with $c < \overline{c}(\Theta)$, the optimal information choice x^* solves the following equation:

$$2\rho R C_x(x^*, c)\left(\tau_\pi + x^* + \tfrac{I^2}{\rho^2}\tau_\theta\right) = 1 \tag{1.3}$$

Proof. See the appendix. □

The proof follows three steps. First, we compute the indirect utility: $v(s, p; \Theta) = E[U(W_2^*)|\mathcal{F}_j]$. This is given by:

$$v(s, p; \Theta) = -\tfrac{1}{\rho}e^{-\frac{1}{2}k(E[\pi|s,\xi]-pR)^2-\rho R[W_1-C(x,c)]}$$

Then, we take expectation with respect to the joint probability distribution of the private and the public signal. It depends on the first two moments of the expected return of purchasing a unit of risky asset. In order to simplify the notation, we call

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f the expected excess return: $f = E[\pi|s_j, \xi] - pR$ is a normal random variable with mean μ_f and variance σ_f^2. The expected value of the indirect utility function is:

$$E[v(s, p; \Theta)] = -\tfrac{1}{\rho}\left(1 + k\sigma_f^2\right)^{-1/2} e^{\frac{-.5\mu_f^2}{k^{-1} - \sigma_f^2} - \rho R(W_1 - C(x,c))}$$

The last step provides conditions for the existence and the uniqueness of the optimal information choice.

To figure out a first feature of the result, we report the mean and the variance of the posterior beliefs:

$$E[E[\pi|s, \xi]] = \mu_\pi \qquad Var[E[\pi|s, \xi]] = \tfrac{1}{\tau_\pi} - \tfrac{1}{k}$$

The expectation is the prior mean of the risky asset for both types. The literate agents purchase more information and they end up with posterior beliefs close to the true value. The illiterate agents rely less on their private signal and their posterior beliefs are closer to the market price: they follow more market feelings. Therefore, the variance of the posterior expectation is increasing in the financial literacy.

This feature is important in order to understand the distribution of the expected excess return: $f \sim N(\mu_f, \sigma_f^2)$.

$$\mu_f = \rho\mu_\theta K^{-1} \qquad \sigma_f^2 = \tfrac{d^2}{\tau_\theta} + \tfrac{1}{K}\left(\tfrac{\tau_\pi}{K} - \tfrac{K}{k}\right)$$

The expected gain from being a trader is given by the mean of the noisy supply scaled by the risk aversion and the posterior beliefs precision of the average agent. It comes from the opportunity to use the informational advantages against those agents who need to trade for exogenous reasons. It is decreasing in the price informativeness. With fully revealing price ($I = \infty$), the expected gain is zero. With partial revealing price ($0 < I < \infty$), the opportunity to take advantage of noisy traders is shared with less traders, increasing the per capita expected gain.

The volatility of the expected excess return differs among types. There is a common part $\frac{d^2}{\tau_\theta}$ that refers to the volatility of the noisy supply scaled by the square of

market depth. Lower volatility of noisy supply and higher liquidity of the market imply lower volatility of the expected excess return.

The second part, $\frac{1}{K}\left(\frac{\tau_\pi}{K} - \frac{K}{k}\right)$, shows the informational gain achieved by one type with respect to the average agent. Those with more precise posterior beliefs take into account greater return due to the opportunity to use their informational advantages. However, the informational advantages decrease as the price informativeness increases. In this case, the literate agents reduce their investment in information and rely more on market price.

Differentiating equation (1.3) with respect to I, we can show that the optimal information choice is a non increasing function of the price informativeness. Formally, we have:

$$\frac{\partial x^*}{\partial I} = -\frac{C_x}{C_{xx}k}\frac{dk}{dI} = -\frac{C_x}{C_{xx}k + C_x}\frac{\partial k}{\partial I} \leq 0$$

which shows the strategic substitutability between private and public information. [17]

Another feature of the result is the threshold $\overline{c}(\Theta)$ that identifies the highest level of the financial literacy cost for which buying information is worthwhile. It is given implicitly by the following formula:

$$C_x(\overline{c}, 0) = \frac{1}{2\rho R(\tau_\pi + \frac{I^2}{\rho^2}\tau_\theta)}$$

The threshold $\overline{c}(\Theta)$ allows us to discriminate among informed and uninformed agents.[18] We continue to develop our example to get a closed form solution for the optimal information choice.

---

[17]We have that

$$\frac{dk}{dI} = \frac{\partial k}{\partial x}\frac{\partial x}{\partial I} + \frac{\partial k}{\partial I} \qquad \frac{\partial k}{\partial x} = 1 \qquad \frac{\partial k}{\partial I} = 2\frac{I}{\rho^2}\tau_\theta \geq 0$$

.

[18]For numerical results, we set the parameters of the model such that at least one type prefer to be informed.

Example, ctd (Optimal Information Choice)**.** *The domain of the agent is* $X = [0, \overline{x}(c)]$ *where* $\overline{x}(c) = \frac{1}{2}(-1 + \sqrt{1 + \frac{4W_1}{c}})$. *The agent decides to be informed if:*

$$c < \overline{c} = \frac{1}{2R\rho(\tau_\pi + \frac{I^2}{\rho^2}\tau_\theta)}$$

For any positive $c \leq \overline{c}$, *the FOC for the information choice is necessary and sufficient.* *Formally, the FOC implies that*

$$2\rho Rc(2x^* + 1)\left(x^* + \tau_\pi + \frac{I^2}{\rho^2}\tau_\theta\right) = 1$$

It is easy to check that x^* *is decreasing in the price informativeness* I.

The next proposition describes the equilibrium condition for the information market: the price informativeness is the sum of the private information of all agents.

Proposition 3. *There exists a unique rational expectation equilibrium.*

Proof. See the appendix. □

We prove the proposition checking that the conjectures made by the agents hold in equilibrium. We write the price informativeness I using the results of the proposition 2:

$$I = \int_{j \in J} x^*(c_j; \Theta)dG(j) \tag{1.4}$$

We use a fixed point argument to prove the existince of a solution for equation (1.4) and we apply the Leibniz's theorem to show uniqueness.

1.4 Policy implications

In Figure 1.3 we show the main results of the model. In Figure 1.3(a), we plot the demand and the supply in the information market; in Figure 1.3(b), we plot the optimal

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individual information choice given price informativeness: $x^*(I; c, \Theta)$. Note that it is decreasing in I for both types. This reflects the strategic substitutability between private and public information. Higher level of the price informativeness discourages agents to acquire private information, given that their informational advantages decrease. In Figure 1.3(c), we plot the optimal information choice for different degrees of the financial literacy of the illiterates. We keep c_L constant and we make c_H to increase. In equilibrium, the illiterate agent adjusts her optimal information choice $x^*(c_H, I; \Theta)$, taking into account the changes in the equilibrium price informativeness due to the information choice of the literate. The threshold $\bar{c}$ is implicitly given by $x(\bar{c}, I^*; \Theta) = 0$.

Policy evaluation

Now, we consider the role of policies aimed at improving the level of financial literacy of the illiterates. We show the following theorem:

Theorem 1. *The impact of the policy on the market stability is non-monotone, even if the policy always increases the price informativeness.*

As a proxy of the market stability, we use the market price variance. The policy affects the information choice of the illiterate, and indirectly also that of the literate, through its effect on the price informativeness. We decompose the effect to work out the region of the parameter space where the impact of the policy is well determined.

Recall that the formula for the market price variance is:

$$\sigma_{pR}^2(I, \Theta) = \frac{b^2}{\tau_\pi} + \frac{d^2}{\tau_\theta} = K^{-2} \left[\left(\frac{I^2 \tau_\theta}{\rho^2} + I \right)^2 \frac{1}{\tau_\pi} + \left(\frac{I \tau_\theta}{\rho} + \rho \right)^2 \frac{1}{\tau_\theta} \right]$$

Formally, the policy effect on the market stability is given by:

$$\frac{\partial \sigma_{pR}^2}{\partial c_H} = \frac{\partial \sigma_{pR}^2}{\partial I} \frac{\partial I}{\partial c_H} \tag{1.5}$$

with

$$\frac{\partial \sigma_{pR}^2}{\partial I} = 2\left[\frac{1}{K}\left(\frac{b}{\tau_\pi}\frac{\frac{\partial K}{\partial I}}{K} + \frac{d}{\rho}\right) - \frac{\frac{\partial K}{\partial I}}{K}\sigma_{pR}^2\right]$$

and

$$\frac{\partial I}{\partial c_H} = \frac{(1-\lambda)\frac{\partial x_H}{\partial c_H}}{\left(1-\lambda\frac{\partial x_L}{\partial I}-(1-\lambda)\frac{\partial x_H}{\partial I}\right)}$$

Hereafter, we study the sign of equation (1.5). We distinguish between two terms: the impact of the price informativeness on the market price variance and the impact of the policy on the price informativeness.

The latter, $\frac{\partial I}{\partial c_H}$, is always negative for any $c_H < \bar{c}(\Theta)$. Otherwise it is zero. This result is given by two effects: the first effect would drive up the price informativeness. This is because the policy allows the illiterate agents to acquire information at a lower cost and, therefore, they optimally acquire more information. The second effect would drive down the price informativeness: higher private information acquired by the illiterates implies higher price informativeness and this induces both types to reduce their acquisition of private information. We can note that the first effect dominates the second one and the policy leads to higher price informativeness.

The former term, $\frac{\partial \sigma_{pR}^2}{\partial I}$, is non-monotone. In order to study it, we collapse the behaviour of the two types into the fictitious average agent we described in footnote 15. The impact of the price informativeness on the market price variance is negative when the market price variance is greater than a threshold. This threshold is given by two terms: the first one is the posterior variance of the average agent's beliefs about the asset payoff, weighted by the equilibrium marginal effect of the payoff on the market price, i.e. $\frac{1}{K}\frac{b}{\tau_\pi}$. The second one is the effect of the price informativeness on the posterior belief variance, weighted by the equilibrium marginal effect of the noisy supply, i.e. $\frac{\partial K}{\partial I}\frac{1}{K^2}\frac{d}{\rho}$. Namely, we have:

$$\frac{\partial \sigma_{pR}^2}{\partial I} < 0 \Longleftrightarrow \sigma_{pR}^2 > \frac{1}{K}\frac{b}{\tau_\pi} + \frac{\partial K}{\partial I}\frac{1}{K^2}\frac{d}{\rho} > 0$$

where $\frac{1}{K}$ is the variance of the average agent with private information I (the posterior variance of the market belief), and $\frac{\partial K}{\partial I}\frac{1}{K^2}$ is the monotone change in the posterior beliefs when the average agent increases own private information: $\frac{\partial K}{\partial I} = 1 + 2\frac{I}{\rho^2}\tau_\theta$.

The effect is positive if the market price variance is sufficiently higher than $\frac{1}{K}$, depending on the sensitivity of the latter to changes in the price informativeness. If $\frac{1}{K}$ is highly responsive to price informativeness, for market stability to decrease as price informativeness increases, it must hold that the gap between the market price variance and the posterior belief variance ($\sigma^2_{pR} - \frac{1}{K}$), is high enough, i.e., the contribution of the noisy supply to the overall market volatility is large.[19] We report in Table 1.3, the market price variance and the partial derivative with respect to the price informativeness for several values of the fundamentals uncertainty: $\tau_\pi, \tau_\theta = [.1, .25, .5, 1, 5]$. The first element is the market price variance. It is decreasing as the fundamentals uncertainty decreases. The second element is the impact of the price informativeness on the stability of the market. With high fundamentals uncertainty, more aggregate information leads to lower instability.

We summarize the impact of the policy on market stability with the following formula:

$$\frac{\partial \sigma^2_{pR}}{\partial c_H} > 0 \iff \sigma^2_{pR} > \frac{1}{K}\frac{b}{\tau_\pi} + \frac{\partial K}{\partial I}\frac{1}{K^2}\frac{d}{\rho}$$

The policy makers can manipulate the financial literacy cost c_H and this intervention affects the price informativeness through the agents' behaviour. The behaviour of the fictitious average agent helps at illustrating the effect of the policy on market stability, which depends on the gap between the market price variance and the posterior belief variance, and on the sensitivity of the latter with respect to the price informativeness.

In Figure 1.4 we show the market price variance, computed in equilibrium, for different levels of the financial literacy of the illiterate. We keep constant the financial

---

[19]Recall that the market volatility reflects both the volatility of the asset and of the noisy supply, given the assumed market structure.

literacy of the other type. Figure 1.4 shows that when there is low fundamentals uncertainty, improving the financial literacy of the illiterate agents has a negative impact on the stability of the markets.

Policy effectiveness   Next, we turn to the effect of c_H, which we can see as the baseline level of financial literacy, e.g. the knowledge that an individual develops in the schooling period. Therefore, either increasing the mandatory years of schooling or introducing courses related to financial matters, affects the baseline level of financial literacy in the society. To understand the effectiveness of financial literacy improving policies, we focus on the endogenous threshold $\bar{c}(\Theta)$, derived from proposition 2, and we use it as a proxy of the access to the information market. With higher values for the threshold, agents are more likely to be informed. We show the following theorem:

Theorem 2. *Financial literacy improving policies provide incentives to become informed traders.*

From proposition 3, we derive the equilibrium price informativeness I^* that depends on the distribution of financial literacy among agents (c_L, c_H and λ). Therefore, we can rewrite it as: $I^* = I^*(c_L, c_H; \Theta)$. Substituting it back in $\bar{c}(I^*; \Theta)$, we derive the reduced equation for the threshold as:

$$\bar{c} = \bar{c}(I^*(c_L, c_H; \Theta); \Theta) = \bar{c}(c_L, c_H; \Theta)$$

The policy maker should take into account how the threshold changes when he provides financial education programs. Formally, the impact of c_H on $\bar{c}(c_L, c_H; \Theta)$ is:

$$\frac{\partial \bar{c}}{\partial c_H} = -\frac{C_x(0, \bar{c})^2 (4\rho R \frac{I}{\rho^2} \tau_\theta) \frac{\partial I}{\partial c_H} + C_{xx}(0, \bar{c}) \frac{\partial x}{\partial c_H}|_{x=0}}{C_{xc}(0, \bar{c})}$$

We can distinguish between two cases:

$$\frac{\partial \overline{c}(c_L, c_H; \Theta)}{\partial c_H} \begin{cases} = 0 & \text{if} \quad c_H > \overline{c} \\[2ex] > 0 & \text{if} \quad c_H < \overline{c} \end{cases}$$

When $c_H > \overline{c}$, the numerator is zero, given there is no effect on the price informativeness. Illiterate agents optimally choose to be uninformed and being more financially literate does not change their choice. On the other hand, when $c_H \leq \overline{c}$, improving financial literacy of the illiterate agents decreases the threshold.

Figure 1.4(c) shows that the market price variance remains constant for values of c_H greater than $\overline{c}$. It means that only literate agents acquire information and contribute to the price informativeness.

Let consider a financial education program aimed at making financially illiterate agents more informed and, therefore, more active traders. It is crucial to understand how far financially illiterate agents are from the threshold that triggers their status of being informed.

In Table 1.4, we keep c_L constant and we make c_H to increase, in order to compare different scenarios. It is interesting to measure the distance between the financial literacy cost of the illiterate and the triggering threshold. It is a measure of the per capita cost policy makers face to make the program effective. As we can see in Table 1.4, negative values mean that marginally decreasing the financial literacy cost of the illiterates has no effects on their behaviour: the illiterates still prefer to remain uniformed. In order to have positive policy outcomes, the financial literacy cost must be decreased up to the triggering threshold.

We distinguish between high and low fundamentals uncertainty. With low uncertainty, the model does not suggest to offer financial education programs to the illiterates: it would be not enough to provide basic programs because only sophisti-

cated ones would be effective. And, even if the program is effective, the policy maker should take into account the negative effects on the volatility of the market.

1.5 Conclusion

This work adopts a noisy rational expectations equilibrium with endogenous information acquisition to analyze heterogeneity in financial literacy and its impact on market stability. This model provides rationale for the existence of financial education programs, aimed at improving individual financial literacy.

Financial literacy affects individual information costs. Therefore, given their financial ability, they choose the information to acquire and the risky assets to hold. Within a general equilibrium framework, we derive the price informativeness and the market volatility. The key parameters are the endowments of financial literacy of the agents. We divide them into two types, literate and illiterate, and we let them trade against noisy traders. We point out that the more financially literate agents are, the more information they acquire and the more information is revealed by market price.

There are two distinct features of the current chapter. First, improving the financial literacy of the illiterate agents has a non-monotone effect on market stability. The effect depends on uncertainty in the market fundamentals. The policy implication is to provide financial education programs only when there is high uncertainty in the market fundamentals. On the other hand, when there is low uncertainty, improving literacy of the illiterate agents increases market volatility.

Second, we work out an endogenous channel through which we identify informed agents. This is instrumental to document the effectiveness of financial literacy programs on the financial behaviour, once it already has a positive effect on the financial knowledge of the agents.

Further research will take into account participation costs in order to explain the limited market participation and let the individual amount of financial literacy be endogenously chosen.

Appendix A - Proofs

Proof of proposition 1

The distribution of payoff, supply and signals is:[20]

$$
\begin{pmatrix} \theta \\ \pi \\ s \\ pR \end{pmatrix} \sim N \left\{ \begin{pmatrix} \mu_\theta \\ \mu_\pi \\ \mu_\pi \\ a + b\mu_\pi - d\mu_\theta \end{pmatrix}, \begin{pmatrix} \frac{1}{\tau_\theta} & 0 & 0 & d\frac{1}{\tau_\theta} \\ 0 & \frac{1}{\tau_\pi} & \frac{1}{\tau_\pi} & b\frac{1}{\tau_\pi} \\ 0 & \frac{1}{\tau_\pi} & \frac{1}{\tau_\pi} + \frac{1}{x} & b\frac{1}{\tau_\pi} \\ d\frac{1}{\tau_\theta} & b\frac{1}{\tau_\pi} & b\frac{1}{\tau_\pi} & b^2\frac{1}{\tau_\pi} + d^2\frac{1}{\tau_\theta} \end{pmatrix} \right\}
$$

The proof is given in five steps. In the first step, we guess a price linear function and we derive the informationally equivalent public signal ξ from the price function. In the second step, we compute the mean and the variance of the posterior beliefs given the two unbiased signals, ξ and s. In the third step, we derive the optimal asset demand. In the fourth step, we derive market clearing conditions and, in the last step, we impose rationality and determine the coefficients of the guessed linear price function.

Step 1 : Agents guess a price function linear in π (future payoff) and θ (noisy supply):

$$
pR = a + b \left(\lambda \int_{j \in L} s_j dG(j) + (1 - \lambda) \int_{j \in H} s_j dG(j) \right) - d\theta
$$

---

[20]Hellwig (1980) noted that a model of communication where agents are aware of the covariance between the price and their own signals but they act as price-taker is a bit schizophrenic. To remove this features he looked at the aggregation of information in a competitive sequence of economies. Verrecchia (1982) removed any potential "schizophrenia" on the part of traders assuming that the covariance between private signals (s_j) and price p does not depend on x. The solution to this kind of problem is either to assume a large economy (Verrecchia (1982) assumes "traders behave as if their decisions concerning how much information to acquire are independent of price") or to explicitly model strategic behaviour (Kyle (1989)).

Applying the law of large number within each group of traders, we have that $\int_j \epsilon_j dG(j) = 0$ with probability one. Therefore we can rewrite the price function as:

$$pR = a + b\pi - d\theta$$

Agents use the private signals to update their prior beliefs $\pi \sim N(\mu_\pi, \tau_\pi^{-1})$. The private signal is unbiased by construction $s|\pi \sim N(\pi, x^{-1})$ and conditionally independent from prior belief μ_π, $E[(\mu_\pi - \pi)(s - \pi)] = 0$. Rational agents use the price as a public signal. It is not unbiased: $E[pR|\pi] = a - d\mu_\theta + b\pi$. To apply Bayesian updating, agents need to transform the price in an informationally equivalent variable ξ:

$$\xi = \frac{pR - a + d\mu_\theta}{b} = \pi - \tfrac{d}{b}(\theta - \mu_\theta)$$

where

$$\xi|\pi \sim N(\pi, \tfrac{d^2}{b^2}\tfrac{1}{\tau_\theta})$$

Step 2 : Agent j observes $\mathcal{F} = \{s_j, p\} \equiv \{s_j, \xi\}$ and updates her prior beliefs with the two Gaussian signals. Using the well known formula for the multivariate normal distribution (Degroot (2004), p. 55), the posterior mean is given by:

$$
\begin{aligned}
E[\pi|s_j, \xi] &= \mu_\pi + \frac{1}{k}\left\{ x_j\left(s_j - E[s_j]\right) + \tfrac{b^2}{d^2}\tau_\theta(\xi - E[\xi]) \right\} \\
&= \tfrac{1}{k_j}\left(\tau_\pi \mu_\pi + x_j\, s_j + \tfrac{b^2}{d^2}\tau_\theta \xi \right)
\end{aligned}
$$

where the precision of the posterior belief k_j is given by is the sum of precisions of the prior, of the private signal and of the public signal.

$$k_j = \frac{1}{Var[\pi|s_j, \xi]} = \tau_\pi + x_j + \tfrac{b^2}{d^2}\tau_\theta$$

Step 3 : We maximize the CARA utility function with respect to the control variable α. Each agent solves the following:[21]

$$\max_{\alpha} E\left[-\tfrac{1}{\rho}e^{-\rho W_2(\alpha)}|s,\xi\right] = \max_{\alpha} -\tfrac{1}{\rho}e^{-\rho\{E[W_2(\alpha)|s,\xi]-\frac{\rho}{2}Var[W_2(\alpha)|s,\xi]\}}$$

where

$$E[W_2(\alpha)|s,\xi] = E\left[[W_1-C(x,c)]\alpha\tfrac{\pi-pR}{p}+R[W_1-C(x,c)]|s,\xi\right]$$

and

$$Var[W_2(\alpha)|s,\xi] = \tfrac{[W_1-C(x,c)]^2\alpha^2}{p^2}Var[\pi|s,\xi]$$

Substituting and deriving FOC, we get that:

$$-\rho[W_1-C(x,c)]\tfrac{E[\pi|s,\xi]-pR}{p}+\rho^2\alpha\tfrac{[W_1-C(x,c)]^2}{p^2}Var[\pi|s,\xi]=0$$

Therefore, the optimal risky asset demand for agent j is:

$$\alpha_j^* = \frac{1}{\rho[W_1-C(x_j,c_j)]}\frac{\frac{E[\pi|s_j,\xi]-pR}{p}}{\frac{Var[\pi|s_j,\xi]}{p^2}} = \frac{p}{\rho[W_1-C(x_j,c_j)]}\frac{E[\pi|s_j,\xi]-pR}{\frac{1}{k_j}}$$

The amount of wealth in risky assets depends on the precision of the posterior, the risk aversion coefficient and the expected excess return of the risky investment.

Step 4 : The equilibrium price clears the market for the risky asset. Aggregating over all traders yields the aggregate demand:

$$\lambda\int_{j\in L}\alpha_j^*\frac{W_1-C(x_j,c_j)}{p}dG(j)+(1-\lambda)\int_{j\in H}\alpha_j^*\frac{W_1-C(x_j,c_j)}{p}dG(j)=\theta$$

---

[21]We use the log-normal distribution properties and we drop subscript j to simplify notation. We will restore it when we aggregate individual asset demands.

We apply the weak law of large numbers for independent and identically distributed random variable with the same mean, such that $\int_j s_j dG(j) = \pi$. Therefore, imposing market clearing condition holds the following equation:

$$\left[\lambda \left(\mu_\pi \tau_\pi + x_L \pi + \tfrac{b^2}{d^2} \tau_\theta \xi - pRk_L \right) + (1-\lambda) \left(\mu_\pi \tau_\pi + x_H \pi + \tfrac{b^2}{d^2} \tau_\theta \xi - pRk_H \right) \right] = \theta$$

$$\mu_\pi \tau_\pi + \tfrac{b^2}{d^2} \tau_\theta \xi + \pi[\lambda x_L + (1-\lambda)x_H] - pR[\lambda k_L + (1-\lambda)k_H] = \theta$$

where x_L and k_L are the information choice and posterior precision of the literate agent (type L). Similarly, x_H and k_H for the illiterate agent (type H). Using the definition of price informativeness $I = \lambda x_L + (1-\lambda)x_H$, we can rewrite the price equation as

$$pR = (\tau_\pi + I + \tfrac{b^2}{d^2}\tau_\theta)^{-1} \left(\mu_\pi \tau_\pi + \tfrac{b^2}{d^2}\tau_\theta \xi + \pi I - \theta \right)$$

Step 5 : We impose rationality. ξ involves undetermined coefficients b, d. We substitute the expression for $\xi = \pi - \frac{d(\theta - \mu_\theta)}{b}$ and, rearranging the terms, we have:

$$pR = (\tau_\pi + I + \tfrac{b^2}{d^2}\tau_\theta)^{-1} \left(\mu_\pi \tau_\pi + \tfrac{b}{d}\tau_\theta \mu_\theta + \pi(I + \tfrac{b^2}{d^2}\tau_\theta) - \theta \left(\tfrac{b}{d}\tau_\theta + \rho \right) \right)$$

We derive $\frac{b}{d} = \frac{I}{\rho}$ and we substitute it back in the price function. We find out the following determined coefficients.

Coefficient of θ:

$$d = K^{-1} \left(\rho + \tfrac{I\tau_\theta}{\rho} \right)$$

Coefficient of π:

$$b = K^{-1} \left(I + \tfrac{I^2 \tau_\theta}{\rho^2} \right)$$

Constant term, a:

$$a = K^{-1} \left(\mu_\pi \tau_\pi + \tfrac{\mu_\theta I \tau_\theta}{\rho} \right)$$

where $K = \tau_\pi + I + \frac{I^2}{\rho^2}\tau_\theta$.

Proof of proposition 2

In order to solve for the information choice x^*, we need to compute the indirect utility: $v(s_j, p; \Theta) = E[U(W_2(\alpha_j^*))|\mathcal{F}_j]$.

Lemma 1. *For agent $j \in J$, the indirect utility function is:*

$$v(s_j, p; \Theta) = -\tfrac{1}{\rho} e^{-\frac{1}{2}k_j(E[\pi|s_j,\xi]-pR)^2 - \rho R[W_1 - C(x_j, c_j)]}$$

Proof. For each agent, the final wealth is $W_2(\alpha^*) = [W_1 - C(x, c)][R + \alpha^*(\frac{\pi - pR}{p})]$. In order to compute the indirect utility:

$$v(s, p; \Theta) = E[-\tfrac{1}{\rho} e^{-\rho W_2(\alpha^*)}|\mathcal{F}] = -\tfrac{1}{\rho} e^{-\rho\{E[W_2(\alpha^*)|s,\xi] - \frac{\rho}{2} Var[W_2(\alpha^*)|s,\xi]\}}$$

we need to compute the conditional mean $E[W_2(\alpha^*)|s, \xi]$ and the conditional variance $Var[W_2(\alpha^*)|s, \xi]$. At the trading period, the final wealth W_2 is given by normal random variables π, θ, s and p and a constant term $\rho R[W_1 - C(x, c)]$. Therefore, the conditional mean is:

$$
\begin{aligned}
E[W_2(\alpha^*)|s, \xi] &= E[[W_1 - C(x, c)]\alpha^* \tfrac{\pi - pR}{p} + R[W_1 - C(x, c)]|s, \xi] \\
&= [W_1 - C(x, c)]\alpha^* E[\tfrac{\pi - pR}{p}|s, \xi] + R[W_1 - C(x, c)] \\
&= [W_1 - C(x, c)]\tfrac{pk}{\rho[W_1 - C(x,c)]}(E[\pi|s, \xi] - pR)\tfrac{E[\pi|s,\xi] - pR}{p} + R[W_1 - C(x, c)] \\
&= \tfrac{k}{\rho}(E[\pi|s, \xi] - pR)^2 + R[W_1 - C(x, c)]
\end{aligned}
$$

and the conditional variance is:

$$
\begin{aligned}
Var[W_2(\alpha^*)|s, \xi] &= \tfrac{[W_1 - C(x,c)]^2(\alpha^*)^2}{p^2} Var[\pi|s, \xi] = \tfrac{[W_1 - C(x,c)]^2 p^2 k^2 (E[\pi|s,\xi] - pR)^2}{p^2[W_1 - C(x,c)]^2 \rho^2}\tfrac{1}{k} \\
&= \tfrac{k}{\rho^2}(E[\pi|s, \xi] - pR)^2
\end{aligned}
$$

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Thus, we can rewrite the indirect utility of agent j, as:

$$v(s_j, p; \Theta) = -\frac{1}{\rho} e^{-\rho\{\frac{k_j}{\rho}(E[\pi|s_j,\xi]-pR)^2 + R[W_1 - C(x_j,c_j)] - \frac{\rho}{2}\frac{k_j}{\rho^2}(E[\pi|s_j,\xi]-pR)^2\}}$$
$$= -\frac{1}{\rho} e^{-\frac{1}{2}k_j(E[\pi|s_j,\xi]-pR)^2 - \rho R[W_1 - C(x_j,c_j)]}$$

$\square$

Once we derive the indirect utility, we need to compute the expected value. It depends on the first two moments of the expected return of purchasing a unit of risky asset. For simplifying notation, we call f the expected return: $f = E[\pi|s_j,\xi] - pR$ that is normally distributed with mean μ_f and varianceσ_f^2.

Lemma 2. *For agent $j \in J$, the expected value of the indirect utility function is:*

$$E[v(s_j, p; \Theta)] = -\frac{1}{\rho}\left[(\tfrac{1}{k_j} + \sigma_f^2)k_j\right]^{-1/2} e^{\frac{-\frac{1}{2}\mu_f^2}{(\frac{1}{k_j}+\sigma_f^2)} - \rho R[W_1 - C(x_j,c_j)]}$$

Proof. The expected return $f = E[\pi|s,\xi] - pR$ is a linear function of two normally distributed random variables. Therefore, it is a normal random variable. In order to compute the expected value of the indirect utility function, we need to compute the mean, $\mu_f = E[E[\pi|s,\xi] - pR]$, and the variance, $\sigma_f^2 = Var[E[\pi|s,\xi] - pR]$, of the expected excess return. We start computing the expectation of the posterior belief:

$$E[E[\pi|s,\xi]] = E\left[\mu_\pi + \frac{1}{k}[x\,(s - E[s]) + \tfrac{I^2}{\rho^2}\tau_\theta(\xi - E[\xi])]\right]$$
$$= \mu_\pi$$

and the expectation of the market price:

$$E[pR] = E[a + b\pi - d\theta]$$
$$= a + b\mu_\pi - d\mu_\theta$$

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Therefore, the mean of the expected excess return is:

$$\mu_f \quad = \mu_\pi - a - b\mu_\pi + d\mu_\theta$$

$$= (1 - b)\mu_\pi - a + d\mu_\theta = \frac{\rho\mu_\theta}{K} = \mu_f$$

To compute the variance, we need the variance of the posterior belief, the market price variance and the covariance of two terms:

$$\sigma_f^2 = Var[E[\pi|s, \xi]] + Var[pR] - 2Cov[E[\pi|s, \xi], pR]$$

where the variance of the posterior belief is:[22]

$$Var[E[\pi|s, \xi]] \quad = \frac{x^2}{k^2}\left(\frac{1}{\tau_\pi} + \frac{1}{x}\right) + \frac{I^4\tau_\theta^2}{\rho^4 k^2}\left(\frac{1}{\tau_\pi} + \frac{\rho^2}{I^2\tau_\theta}\right) + 2\frac{xI^2\tau_\theta}{\rho^2 k^2}\frac{1}{\tau_\pi}$$

$$= \frac{1}{\tau_\pi k^2}\left(x^2 + x\tau_\pi + \frac{I^4}{\rho^4}\tau_\theta^2 + \frac{I^2}{\rho^2}\tau_\theta\tau_\pi + 2x\frac{I^2}{\rho^2}\tau_\theta\right)$$

$$= \frac{1}{\tau_\pi k^2}\left(x + \frac{I^2}{\rho^2}\tau_\theta\right)k$$

$$= \frac{1}{\tau_\pi} - \frac{1}{k}$$

The market price variance is:

$$Var[pR] = b^2\frac{1}{\tau_\pi} + d^2\frac{1}{\tau_\theta}$$

and the covariance between posterior beliefs and market price is:

$$Cov[E[\pi|s, \xi], pR] = b\frac{1}{\tau_\pi}$$

---

[22]We use the law of total variance: $Var[Y] = E[Var[Y|X]] + Var[E[Y|X]]$

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Thus, we can rewrite σ_f^2 as:

$$
\begin{aligned}
\sigma_f^2 &= \tfrac{1}{\tau_\pi} - \tfrac{1}{k} + b^2 \tfrac{1}{\tau_\pi} + d^2 \tfrac{1}{\tau_\theta} - 2b \tfrac{1}{\tau_\pi} \\
&= (1-b)^2 \tfrac{1}{\tau_\pi} + d^2 \tfrac{1}{\tau_\theta} - \tfrac{1}{k} \\
&= \tfrac{d^2}{\tau_\theta} + \tfrac{1}{K}\left(\tfrac{\tau_\pi}{K} - \tfrac{K}{k} \right)
\end{aligned}
$$

Once we have the first two moments of the excess return, we can compute the expected value of the indirect utility:

$$
\begin{aligned}
E[v(s,p;\Theta)] &= E[-\tfrac{1}{\rho} e^{-\frac{1}{2}kf^2 - \rho R[W_1 - C(x,c)]}] \\
&= -\tfrac{1}{\rho} e^{-\rho R[W_1 - C(x,c)]} E[e^{-\frac{1}{2}k\sigma_f^2 \left(\frac{f}{\sigma_f}\right)^2}]
\end{aligned}
$$

Recall that $f \sim N(\mu_f, \sigma_f^2)$ and $\left(\frac{f}{\sigma_f}\right)^2 \sim \chi_1^2$. Therefore, we can use the moment generating function of a non central χ_1^2. This is given by the following formula:

$$
M(t,h,\lambda) = E[e^{tz}] = \frac{e^{\frac{\lambda t}{1-2t}}}{(1-2t)^{h/2}}
$$

In our case, we have:

$$
\begin{aligned}
h &= 1 & t &= -\tfrac{1}{2}k\sigma_f^2 \\
\lambda &= \left(\tfrac{\mu_f}{\sigma_f}\right)^2 & 1-2t &= k(\tfrac{1}{k} + \sigma_f^2)
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
E\left[e^{-\frac{1}{2}k\sigma_f^2 \left(\frac{f}{\sigma_f}\right)^2} \right] &= \left[k(\tfrac{1}{k} + \sigma_f^2)\right]^{-1/2} e^{\frac{-\frac{1}{2}k\sigma_f^2 \left(\frac{\mu_f}{\sigma_f}\right)^2}{k(\frac{1}{k}+\sigma_f^2)}} \\
&= \left[k(\tfrac{1}{k} + \sigma_f^2)\right]^{-1/2} e^{\frac{-\frac{1}{2}\mu_f^2}{\frac{1}{k}+\sigma_f^2}}
\end{aligned}
$$

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Where the exponential term does not depend on x, given that $(\frac{1}{k} + \sigma_f^2)$ is constant and equal to $(1-b)^2 \frac{1}{\tau_\pi} + d^2 \frac{1}{\tau_\theta}$. Now, rearranging all the terms, the expected value of the indirect utility for agent j is:

$$E[-\tfrac{1}{\rho} e^{-\rho(W_2(\alpha_j^*))}] = -\tfrac{1}{\rho} \left[k_j (\tfrac{1}{k_j} + \sigma_f^2) \right]^{-1/2} e^{\frac{-\frac{1}{2}\mu_f{}^2}{(\frac{1}{k_j}+\sigma_f^2)} - \rho R[W_1 - C(x_j, c_j)]}$$

$\square$

Lemma 3. *Given $c > \underline{c}(\Theta)$ and $I < \infty$, there exists an endogenous thresholds $\overline{c}(\Theta)$. The optimal information choice is:*

$$x^*(I; c, \Theta) = \begin{cases} 0 & \text{if } c > \overline{c} \\ \hat{x} \mid 2\rho R C_x(\hat{x}, c) \left(\tau_\pi + \hat{x} + \frac{I^2}{\rho^2}\tau_\theta \right) = 1 & \text{if } c < \overline{c} \end{cases}$$

Proof. In order to prove the existence and the uniqueness of the solution, we apply the concave maximum theorem. We called γ the positive expression that is independent from the control variable x:

$$\gamma = \tfrac{1}{\rho} \left(\tfrac{1}{k} + \sigma_f^2 \right)^{-1/2} e^{-\frac{1}{2}\frac{\mu_f{}^2}{\frac{1}{k}+\sigma_f^2} - \rho R W_1}$$

and we rewrite the expected value of the indirect utility function as:

$$E[-\tfrac{1}{\rho} e^{-\rho W_2}] = -\gamma k^{-1/2} e^{\rho R C(x,c)} = -\gamma(\tau_\pi + x + I^2 \tfrac{\tau_\theta}{\rho^2})^{-1/2} e^{\rho R C(x,c)} \tag{1.6}$$

The objective function (1.6) is strictly concave and defined over a compact domain $[0, \overline{x}(c)]$ where $\overline{x}(c)$ solves $W_1 = C(\overline{x}, c)$. The concave maximum theorem guarantees the existence and the uniqueness of the solution. It could be an interior or a corner solution: $x^* = 0$ or $x^* = \overline{x}(c)$. We need to specify conditions over parameter space in

order to characterize the solution. First, we derive FOC and we compute it at $x = 0$:

$$\left.\frac{\partial E[v]}{\partial x}\right|_{x=0} = -\gamma\left(\tau_\pi + \frac{I^2}{\rho^2}\tau_\theta\right)^{-1/2}\left[\rho R C_x(0,c) - \frac{1}{2(\tau_\pi + \frac{I^2}{\rho^2}\tau_\theta)}\right]$$

When it is positive, agent has incentive to acquire information, $x^* > 0$. We call $\bar{c}(\Theta)$ the threshold over the financial literacy parameter space, such that the agent with this amount of financial literacy is indifferent between being informed or remain uniformed. Formally, it is implicitly given by:

$$\left[2\rho R(\tau_\pi + \frac{I^2}{\rho^2}\tau_\theta)\right]^{-1} = C_x(0; \bar{c})$$

Therefore, given strictly convexity of the cost function, $\forall c < \bar{c}$, $x^*(I; c, \Theta) > 0$.

For an interior solution, it is enough to show that there exists an $x \in [0, \bar{x}(c)]$ such that FOC is negative. Formally, we check when the following condition holds:

$$\frac{1}{2\rho R(\bar{x} + \tau_\pi + \frac{I^2}{\rho^2}\tau_\theta)} < C_x(\bar{x}, c)$$

We identify a second threshold $\underline{c}(\Theta)$ that it is implicitly given by:

$$\frac{1}{2\rho R(\bar{x}(\underline{c}) + \tau_\pi)} = C_x(\bar{x}(\underline{c}), \underline{c})$$

For all $\underline{c} < c < \bar{c}$, $x^*(I; c, \Theta)$ is an interior solution belonging to the set $[0, \bar{x}]$ and it is given by:

$$2\rho R C_x(x^*, c)\left(\tau_\pi + x^* + \frac{I^2}{\rho^2}\tau_\theta\right) = 1$$

We derive implicitly the amount of information $x^*(I; c, \Theta)$ that an agent optimally acquires. It depends on own financial literacy cost c and on the price informativeness I.

The agent is indifferent between being informed or uniformed when I goes to infinity (fully revealing market price) or if own financial literacy is equal to $\bar{c}$. For any $c > \underline{c}$, the optimal information choice is:[23]

$$x^*(I; c, \Theta) = \begin{cases} 0 & \text{if} \quad c > \bar{c} \\ \hat{x} \mid 2\rho R C_x(\hat{x}, c)\left(\tau_\pi + \hat{x} + \frac{I^2}{\rho^2}\tau_\theta\right) = 1 & \text{if} \quad c < \bar{c} \end{cases}$$

$\square$

We conclude describing the types specific optimal choice. In this model we characterize heterogeneity in financial literacy costs, distinguishing between literate and illiterate agents. The financial literacy cost can take only two values $c \in \{c_L, c_H\}$. Therefore, we can have three cases. If $\bar{c} < c_L < c_H$, then both types prefer to remain uninformed. If $c_L < \bar{c} < c_H$, then only the literate agents acquire information. If $c_L < c_H < \bar{c}$, there is incentive to purchase information for all the agents.

---

[23]We want to avoid the case where agents prefer to spend their whole initial wealth in the information market and nothing in the asset market. This case is possible given the form of the CARA utility function where agents take into account both the mean and the variance of the final wealth.

Proof of proposition 3

Let agents be distributed over J according a cdf G(j). Let c_j be the financial literacy of agent $j \in J$. Let assume that c_j can take any values greater than $\underline{c}(\Theta)$.[24] Let the compact set $\mathbf{Y} = [0, \frac{1}{2\rho RC_x(0,\underline{c})}] \subset \mathbb{R}^+$ the domain of the following mapping operator $F : \mathbf{Y} \to \mathbb{R}^+$:

$$F(y) = \int_j x^*(c_j, y; \Theta) dG(j)$$

where y is an element of $\mathbf{Y}$ and $x^*(c_j, y; \Theta)$ is given by:

$$x^*(c_j, I, \Theta) = \begin{cases} 0 & \text{if} \quad c_j > \bar{c} \\ \hat{x} \mid 2\rho RC_x(\hat{x}, c_j)\left(\tau_\pi + \hat{x} + \frac{I^2}{\rho^2}\tau_\theta\right) = 1 & \text{if} \quad c_j < \bar{c} \end{cases}$$

Continuity of $F(y)$ is guaranteed by the assumption that C_x is continuous. We need to prove that F maps into itself to apply fixed-point Brouwer's theorem $(F(y) = y)$. For all $c_j > \underline{c}$ and $y \in \mathbf{Y}$, $x^*(c_j, y; \Theta) \geq 0$. Moreover, given strictly convexity of the cost function, $C_x(0, \underline{c}) \leq C_x(x^*, c_j)$. This implies that:

$$x^* = \frac{1}{2\rho C_x(x^*,c)} - \left(\tau_\pi + \frac{y^2}{\rho^2}\tau_\theta\right) \leq \frac{1}{2\rho C_x(x^*,c)} \leq \frac{1}{2\rho C_x(0,\underline{c})}$$

For all $c_j > \underline{c}$ and $y \in \mathbf{Y}$,

$$0 \leq x^*(c_j, y; \Theta) = \max\{0, x^*\} \leq \frac{1}{2\rho C_x(0,\underline{c})}$$

Aggregating over j using the cdf $G(j)$ implies that:

$$0 \leq F(y) = \int_j x^*(c_j, y; \Theta) dG(j) \leq \frac{1}{2\rho C_x(0,\underline{c})}$$

---

[24]In the chapter, we assume that $G(j)$ is a discrete distribution with mass λ on $c = c_L$ and mass $(1-\lambda)$ on $c = c_H$. The proof provided easily applies to our case.

Thus, $F(y)$ maps into itself and we proved the existence of the equilibrium.

To prove uniqueness of the equilibrium we follow Peress (2004). We can rewrite $N \equiv \{j : c_j \in [\underline{c}, \overline{c}]\}$ and we know that $x^*(c_j, y; \Theta) > 0, \forall j \in N$. Therefore, $\int_{j \in N} x^*(c_j, y; \Theta) dG(j) = \int_{\underline{c}}^{\overline{c}} x^*(c_j, y; \Theta) dG(j)$.

Let $f(y) = y - \int_{\underline{c}}^{\overline{c}} x^*(c_j, y; \Theta) dG(j)$. The equilibrium value y^* is a root of $f(y)$ and, to be uniquely determined, we need monotonicity of $f(y)$.

Total differentiation of $f(y)$ yields:

$$ f'(y) = 1 - \left(\int_{\underline{c}}^{\overline{c}} \frac{\partial x^*(c_j, y; \Theta)}{\partial y} dG(j) + x^*(\overline{c}, y; \Theta) \frac{\partial \overline{c}}{\partial y} - x^*(\underline{c}, y; \Theta) \frac{\partial \underline{c}}{\partial y} \right) $$

The first term in brackets is the integral of the partial derivative of the optimal information choice with respect to the price informativeness. The second term in brackets is zero given the optimal choice x^* is zero for agents with $c_j = \overline{c}$. The last term is also zero given that we set $\underline{c}$ independent with respect to y.

Differentiating FOC we have:

$$ C''_{xx} \frac{\partial x_j^*}{\partial y} \left(x_j^* + \tau_\pi + \frac{y^2}{\rho^2} \tau_\theta \right) + C'_x (\frac{\partial x_j^*}{\partial y} + 2 \frac{y}{\rho^2} \tau_\theta) = 0 $$

$$ \frac{\partial x_j^*}{\partial y} \left[C''_{xx} \left(x_j + \tau_\pi + \frac{y^2}{\rho^2} \tau_\theta \right) + C'_x \right] + 2 C'_x \frac{y}{\rho^2} \tau_\theta = 0 $$

$$ \frac{\partial x_j^*}{\partial y} = - \frac{2 C'_x \frac{y}{\rho^2} \tau_\theta}{C''_{xx} \left(x_j + \tau_\pi + \frac{y^2}{\rho^2} \tau_\theta \right) + C'_x} \leq 0 $$

As long as our assumptions about the shape of the cost function hold and given $y \in \mathbf{Y} \subset \mathbb{R}^+$, we can conclude that $f'(y)$ is always non negative and $f(y)$ is monotone. Therefore, there exists a unique value of y such that the information market is in equilibrium.

Appendix B - Tables and Graphs

If it is not differently specified in the text, the model's parameters are the following: prior mean μ_π and prior precision τ_π are both equal to one. The noisy asset supply has mean zero and variance one tenth. The risk free asset is zero return ($R = 1$) and the risk aversion coefficient (ρ) is one. Initial wealth (W_1) is one. Literate agents are a fourth of the total ($\lambda = 0.25$). In short notation, we have:

$$\Theta = \{R = 1, W_1 = 1, \rho = 1, \mu_\pi = 1, \mu_\theta = 0, \tau_\pi = 1, \tau_\theta = 10, \lambda = 0.25\}$$

Table 1.1: Market price variance when $c_L = 0.01$ and $c_H = 0.03$. $R = 1$, $W_1 = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$, $\lambda = 0.25$.

(a) $\tau_\theta = 1$ & $\rho = 1$

τ_π	σ_{pR}^2
0.25	3.9104
0.50	1.9089
0.75	1.2408
1	0.9063
2	0.4028
5	0.1034

(b) $\tau_\pi = 1$ & $\rho = 1$

τ_θ	σ_{pR}^2
0.25	1.0950
0.50	0.9622
0.75	0.9232
1	0.9063
2	0.8887
5	0.8913

(c) $\tau_\pi = 1$ & $\tau_\theta = 1$

ρ	σ_{pR}^2
0.25	0.9658
0.50	0.9284
0.75	0.9051
1	0.9063
1.5	1.0085
2	1.2767

Table 1.2: Market price variance when $c_L = 0.01$ and $c_H = 0.1$. $R = 1$, $W_1 = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$, $\lambda = 0.25$.

(a) $\tau_\theta = 1$ & $\rho = 1$

τ_π	σ_{pR}^2
0.25	3.9094
0.50	1.9026
0.75	1.2299
1	0.8912
2	0.3771
5	0.0802

(b) $\tau_\pi = 1$ & $\rho = 1$

τ_θ	σ_{pR}^2
0.25	1.2669
0.50	1.0079
0.75	0.9278
1	0.8912
2	0.8473
5	0.8409

(c) $\tau_\pi = 1$ & $\tau_\theta = 1$

ρ	σ_{pR}^2
0.25	0.9510
0.50	0.8967
0.75	0.8703
1	0.8912
1.5	1.1130
2	1.6197

Market Price Variance

Market price function is $pR = a + b\pi - d\theta$. Given gaussian distribution of the fundamentals π and θ, the market price variance is:

$$\sigma_{pR}^2(I, \Theta) = b^2 \frac{1}{\tau_\pi} + d^2 \frac{1}{\tau_\theta}$$

Deriving with respect to the price informativeness holds:

$$\frac{\partial \sigma_{pR}^2}{\partial I} = \frac{2b}{\tau_\pi} \frac{\partial b}{\partial I} + \frac{2d}{\tau_\theta} \frac{\partial d}{\partial I}$$

with

$$\frac{\partial b}{\partial I} = \frac{\frac{\partial K}{\partial I}(1-b)}{K} \qquad \frac{\partial d}{\partial I} = \frac{\frac{\tau_\theta}{\rho} - \frac{\partial K}{\partial I} d}{K} \qquad \frac{\partial K}{\partial I} = 1 + 2 \frac{I \tau_\theta}{\rho^2}$$

Rewriting it, we have:

$$\frac{\partial \sigma_{pR}^2}{\partial I} = \frac{2b}{\tau_\pi} \left[\frac{\frac{\partial K}{\partial I}(1-b)}{K} \right] + \frac{2d}{\tau_\theta} \left(\frac{\frac{\tau_\theta}{\rho} - \frac{\partial K}{\partial I} d}{K} \right)$$

$$\frac{\partial \sigma_{pR}^2}{\partial I} = 2 \left[\frac{1}{K} \left(\frac{b}{\tau_\pi} \frac{\frac{\partial K}{\partial I}}{K} + \frac{d}{\rho} \right) - \frac{\frac{\partial K}{\partial I}}{K} \sigma_{pR}^2 \right]$$

that is negative when the market price variance is greater than the posterior variance of the average agent times the sum of the prior variance multiplied by the marginal effect of π on the market price and the inverse of the growth rate of the posterior precision of the average agent multiplied by the marginal effect of θ on the market price:

$$\frac{\partial \sigma_{pR}^2}{\partial I} < 0 \iff \sigma_{pR}^2 > \frac{1}{K} \left(\frac{b}{\tau_\pi} + \frac{d}{\rho} \frac{1}{K} \right)$$

We consider two extreme case: $I = 0$ and $I \to \infty$.

When $I = 0$, we have:

$$\frac{\partial \sigma_{pR}^2}{\partial I} \Big|_{I=0} < 0 \iff \frac{1}{\tau_\theta \tau_\pi} > \frac{1}{\rho^2}$$

When $I \to \infty$, we have:

$$\frac{\partial \sigma_{pR}^2}{\partial I}\big|_{I \to \infty} \to 0^- \iff \frac{1}{\tau_\pi} > 0$$

Price informativeness

The impact of the policy on the price informativeness is given by:

$$\frac{dI}{dc_H} = \lambda \frac{dx_L}{dc_H} + (1-\lambda)\frac{dx_H}{dc_H}$$

It is a linear combination of the effects on the information purchased by the two types. Formally, we have that improving financial literacy of type H has a positive direct effect, lowering their cost of information. This effect is mitigated by the price informativeness, that decreases their optimal information choice:

$$\frac{dx_H}{dc_H} = \frac{\partial x_H}{\partial c_H} + \frac{\partial x_H}{\partial I}\frac{\partial I}{\partial c_H} = -\frac{k_H C_{xc} + C_x \frac{\partial k_H}{\partial I}\frac{\partial I}{\partial c_H}}{k_H C_{xx} + C_x}$$

On the other hand, the policy has a negative indirect effect on type L, through the price informativeness:

$$\frac{dx_L}{dc_H} = \frac{\partial x_L}{\partial I}\frac{\partial I}{\partial c_H} = -\frac{C_x \frac{\partial k_L}{\partial I}\frac{\partial I}{\partial c_H}}{k_L C_{xx} + C_x}$$

Therefore, the total impact on the price informativeness is:

$$\frac{\partial I}{\partial c_H} = \lambda \frac{\partial x_L}{\partial I}\frac{\partial I}{\partial c_H} + (1-\lambda)\Big(\frac{\partial x_H}{\partial c_H} + \frac{\partial x_H}{\partial I}\frac{\partial I}{\partial c_H}\Big)$$

and, recollecting the terms, we have:

$$\frac{\partial I}{\partial c_H} = \frac{(1-\lambda)\frac{\partial x_H}{\partial c_H}}{\Big(1 - \lambda \frac{\partial x_L}{\partial I} - (1-\lambda)\frac{\partial x_H}{\partial I}\Big)}$$

It is easy to check that $\frac{\partial I}{\partial c_H} < 0$.[25] Similarly, thepartial derivative with respect to c_L is:

$$\frac{\partial I}{\partial c_L} = \frac{\lambda \frac{\partial x_L}{\partial c_L}}{\left(1 - \lambda \frac{\partial x_L}{\partial I} - (1 - \lambda) \frac{\partial x_H}{\partial I}\right)}$$

Information threshold

All the agents with lower financial literacy costs than the threshold $\bar{c}(\Theta)$ prefer to be informed. The threshold is given implicitly by the following formula:

$$C_x(0, \bar{c}) = \frac{1}{2\rho R(\tau_\pi + \frac{I^2}{\rho^2}\tau_\theta)}$$

We differentiate with respect to c_H

$$C_{xc}(0, \bar{c})\frac{\partial \bar{c}}{\partial c_H} + C_{xx}(0, \bar{c})\frac{\partial x}{\partial c_H}|_{x=0} = -C_x(0, \bar{c})^2(4\rho R\frac{I}{\rho^2}\tau_\theta)\frac{\partial I}{\partial c_H}$$

and we solve for the marginal effect of c_H on the threshold $\bar{c}$:

$$\frac{\partial \bar{c}}{\partial c_H} = -\frac{C_x(0, \bar{c})^2(4\rho R\frac{I}{\rho^2}\tau_\theta)\frac{\partial I}{\partial c_H} + C_{xx}(0, \bar{c})\frac{\partial x}{\partial c_H}|_{x=0}}{C_{xc}(0, \bar{c})}$$

---

[25]Numerator is negative $\frac{\partial x_H}{\partial c_H} = -\frac{k_H C_{xc}}{k_H C_{xx} + C_x} < 0$. Moreover, the denominator is the sum of two positive terms given that the linear combination of two negative terms is still negative.
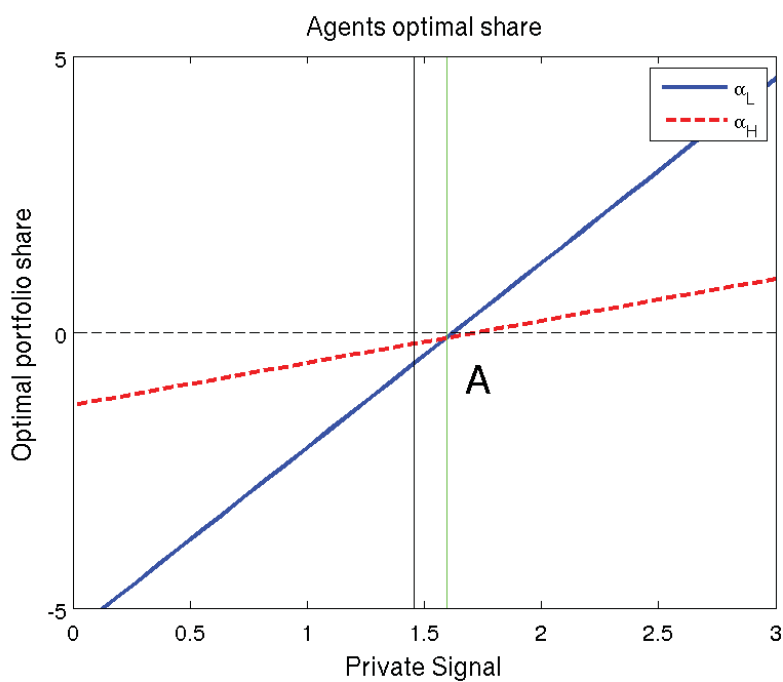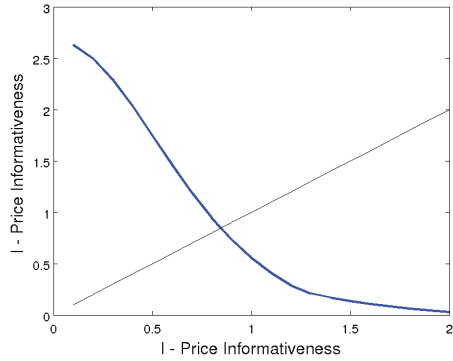
Figures and Tables

Figure 1.2: The optimal portfolio share as a function of the observed private signal for both types. The literate agents trade more aggressively than the illiterates: the solid line is steeper than the dashed line. The two vertical lines are the realized payoff (right) and the market price (left). $R = 1$, $W_1 = 1$, $\rho = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$, $\tau_\pi = 1$, $\tau_\theta = 10$, $\lambda = 0.25$, $c_L = 0.01$, $c_H = 0.05$.

(a) Information Market Equilibrium
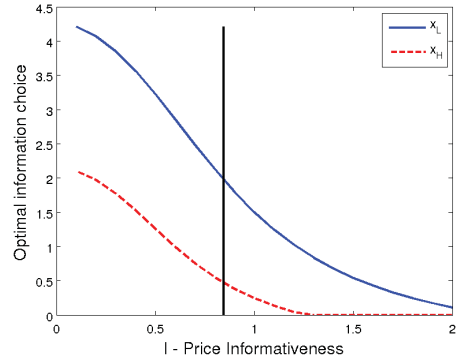
(b) Optimal Information Choice
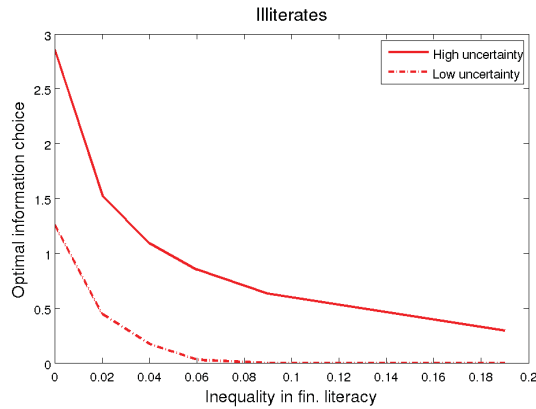
(c) Optimal Information Choice

Figure 1.3: In these three plots we report the market equilibrium allocations. We set $R = 1$, $W_1 = 1$, $\rho = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$, $\tau_\pi = 1$, $\tau_\theta = 10$, $\lambda = 0.25$. $c_L = 0.01$ for all figures, while in Fig. 1.3(a) and in Fig. 1.3(b) $c_H = 0.03$. In Fig. 1.3(c) we make c_H to increase and we compute the optimal information choice for the illiterate agents. We distinguish between two scenarios: high uncertainty with $(\tau_\pi, \tau_\theta) = (.5, .5)$ and low uncertainty with $(\tau_\pi, \tau_\theta) = (5, 5)$.
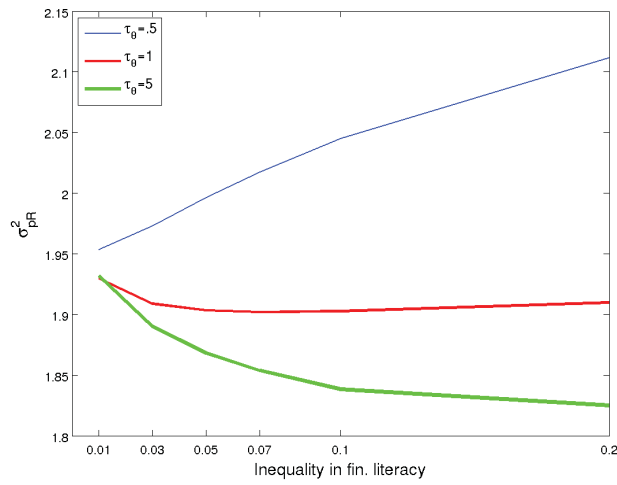
(a) Prior precision of the risky asset: $\tau_\pi = .5$
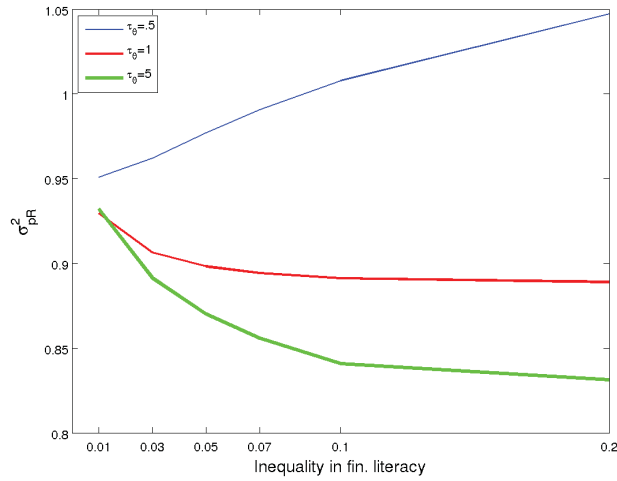
(b) Prior precision of the risky asset: $\tau_\pi = 1$
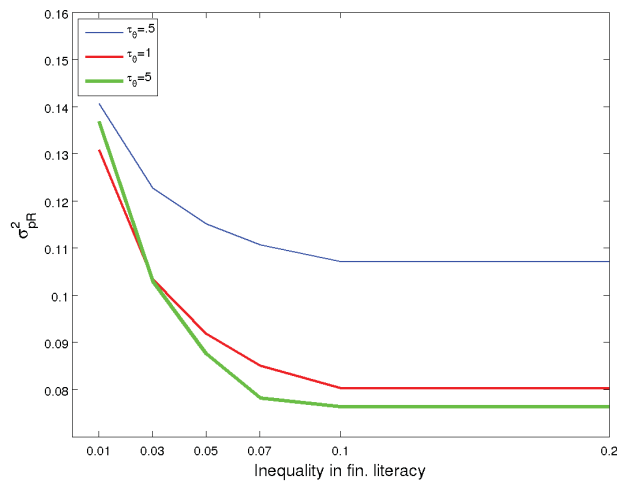
(c) Prior precision of the risky asset: $\tau_\pi = 5$

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Figure 1.4: In these three plots we report the market price variance, computed in equilibrium, as a function of financial literacy cost of type H. $R = 1$, $W_1 = 1$, $\rho = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$, $\lambda = 0.25$, $c_L = 0.01$.

Table 1.3: We report the equilibrium values for the market price variance σ^2_{pR} and its partial derivatives with respect to the price informativeness ($\frac{\partial \sigma^2_{pR}}{\partial I}$) and with respect to the financial literacy cost of the illiterates ($\frac{\partial \sigma^2_{pR}}{\partial c_H}$). $R = 1$, $W_1 = 1$, $\rho = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$, $\lambda = 0.25$, $c_L = 0.01$, $c_H = 0.05$.

τ_π	τ_θ	0.1	0.25	0.5	1.0	5.0
	σ^2_{pR}	11.0355	10.2567	10.0139	9.9085	9.8670
0.1	∂I	-1.0656	-0.4151	-0.1594	0.0013	0.2092
	∂c_H	2.1601	0.7115	0.2259	-0.0014	-0.1034
	σ^2_{pR}	4.9797	4.2385	4.0070	3.9065	3.8675
0.25	∂I	-0.9800	-0.3788	-0.1419	0.0079	0.2062
	∂c_H	1.9289	0.6332	0.1968	-0.0086	-0.1003
	σ^2_{pR}	2.8921	2.2099	1.9963	1.9034	1.8684
0.5	∂I	-0.8512	-0.3238	-0.1156	0.0178	0.2013
	∂c_H	1.5915	0.5181	0.1544	-0.0187	-0.0953
	σ^2_{pR}	1.7353	1.1586	0.9772	0.8982	0.8701
1.0	∂I	-0.6378	-0.2330	-0.0721	0.0336	0.1915
	∂c_H	1.0692	0.3392	0.0888	-0.0329	-0.0857
	σ^2_{pR}	0.3327	0.1679	0.1151	0.0917	0.0875
5.0	∂I	-0.0576	0.0028	0.0286	0.0499	0.1119
	∂c_H	0.0315	-0.0015	-0.0138	-0.0213	-0.0251

Table 1.4: The difference between the baseline financial literacy c_H and the triggering threshold $\bar{c}$. Negative values imply that financial literacy improving policies do not change the behaviour of the illiterates. $R = 1$, $W_1 = 1$, $\rho = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$, $\lambda = 0.25$, $c_L = 0.01$.

c_H	$\bar{c} - c_H$	
	$\tau_\pi = .5$ $\tau_\theta = .5$	$\tau_\pi = 5$ $\tau_\theta = 5$
.01	0.0990	0.0286
.1	0.2163	-0.0248
.2	0.1898	-0.1248
.3	0.1304	-0.2248
.4	0.0575	-0.3248
.5	-0.0277	-0.4248

Chapter 2

Financial Literacy and Limited Market Participation

2.1 Introduction

Households do not invest in either stocks or other financial assets. Rational agents in standard financial models would hold efficient and well diversified risky portfolio. Campbell (2006) points out that "textbook financial theory implies that all households, no matter how risk averse, should hold some equities as long as the equity premium is positive. It follows that limited participation in the equity market must be due to a failure of one of the standard assumptions."

In their paper, Haliassos and Bertaut (1995) describe limited market participation as a puzzle. According to the standard theory, the zero stockholding share violetes first order optimality. Several explanations are proposed. However, most of them fail to fully explain the nostockholding puzzle.

Participation and information costs are the most common reasons why households do not hold stocks. Several financial market barriers prevent households to hold financial assets in order to accumulate efficiently and to smooth consumption

with relevant effects on their welfare.[1] Campbell (2006) argues that the main reasons of nostockholding come from psychological factors that make stockownership uncomfortable for some households. Haliassos and Bertaut (1995) refers to the same reasons as "inertial" factors that are not usually taken into account in standard models. Agnew and Szykman (2005) provide experimental evidences about the role played by information overload on the choice of defined contribution plans and on the "path of least resistance" (Choi *et al.* (2001)). Wurman (2000) describe the main consequence of information overload as information anxiety: it is "produced by the ever-widening gap between what we understand and what we think we should understand. It is the black hole between data and knowledge, and what happens when information does not tell us what we want or need to know".

In this chapter we distinguish between participation cost and information cost. The former refers to all the opportunity costs such as time and effort spent in handling risky portfolio.[2] The latter refers to the costs of collecting and processing information, where these activities allow households to perform better in their market trading.

We think that financial literacy affects both participation and information costs. However, in the literature, its effect is not clearly identified. More experienced households' portfolio reflect higher participation, and higher overconfidence as well.[3] Korniotis and Kumar (2006) provide empirical evidence about the hypothesis that, "on the one hand, older investors would have relatively greater knowledge about the fundamental principles of investing, but on the other hand, effective application of those principles requires efficient processing of information, which they may lack".

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[1]Haliassos and Bertaut (1995) claim that "education and the free acquisition of information are important in overcoming the barrier to stockholding erected by ignorance and misperceptions".

[2]Luttmer (1999) provides a theoretical framework in order to compute the participation cost in monetary terms. He reports a lower bound of at least 3 per cent of the monthly per capita consumption.

[3]Odean (1998) describes over confident investors as those who over-estimate the precision of their private information and/or their ability to process that information. Consequently, they trade more frequently but earn lower net returns.

Households with lower degree of financial literacy could misperceive participation or information costs, leading to suboptimal performances.

Our paper provides a theoretical framework to understand the interaction between participation cost and information acquisition costs. We discuss how heterogeneous financial literacy affects the costs to purchase more informative signals, while the participation cost is homogeneous among the agents.[4] Trading choice is taken after agents learn the realization of their private signal. This allow us to take into account the behaviour of agents who make informed entry choices. A similar model provided by Peress (2005) assumes that the entry choice is made before the information choice and checks if the lowest level of wealth for being a trader changes. His source of heterogeneity is wealth and risk aversion is increasing with it. He found out that poorer agents stay out of the market and, among traders, only wealthier ones become informed.

Understanding the limited market participation phenomena could help policy makers to design adequate policies in order to increase household stockownership through financial education programs. The consequences of these policies have direct effects on individual welfare: Cocco *et al.* (2005) show an increase of available consumption from increased stockholding; Guvenen (2006) provides a framework where limited market participation creates substantial wealth inequality similar to the distribution of wealth between the stockholders and the non-stockholders in the data; Bernheim and Garrett (2003) claim that employer-based financial education significantly stimulates retirement saving among low and moderate savers.

Moreover, effects of such financial education policies on aggregate welfare are already studied in the literature: starting from the seminal paper of Mankiw and Zeldes (1991), researchers provide explanations of the equity premium puzzle through market participation rate and find out that the consumption of stockholders is more

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[4]In an extension of the model we focus on a participation cost that depends on financial literacy.

volatile and more correlated with the market excess return. Attanasio *et al.* (2002) get similar results for consumption growth. Pagano (1989) focus on the relation between market thinness and market price volatility. Multiple equilibria arise when the excess return depends on the depth of the market. Allen and Gale (1994) find out that thick markets can better absorb liquidity shocks reducing volatility of the market price.

Our paper explains, through a general equilibrium framework, the direct and the indirect effects on asset markets when agents become more financially literate. We focus either on the market participation rate and on the market stability. Increasing the inequality in the distribution of the financial literacy among agents implies a lower participation rate among the less literate agents.

The paper proceeds as follow: in Section 2.2, we review the literature about the stockholding puzzle. In Section 2.3, we set up the model and we define the equilibrium while in Section 2.4 we derive the equilibrium allocations. We discuss the main results in Section 2.5. In Section 2.6 we conclude and discuss further research steps.

2.2 Literature review

Households have limited participation in the financial market. Even around the pick of the dot.com bubble, no much more than half of the population hold directly or indirectly financial stocks. For US, Mankiw and Zeldes (1991) report a stock ownership percentage equal to $27,6\%$ of the sample of PSID. Similar estimation comes from SCF: $36,8\%$ from wave 1983 (Haliassos and Bertaut (1995)) and $37,2\%$ from wave 1992 (Poterba and Samwick (1995)). Recently, Bucks *et al.* (2009) report values around half of the SCF sample for the last decade.[5]

Many empirical papers find correlation between limited market participation and other factors than income and wealth. Bernheim *et al.* (2001) and Cole and Shas-

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[5]For wave 1998 a percentage of 48.9, for 2001, 52.2, for 2004, 50.2 and, for 2007, 51.1. See table 7, p. 27 of Bucks *et al.* (2009).

try (2009) point out that schooling affects market participation while the effect of school financial literacy programs is not significant. However, they argue that education affects cognitive abilities, which in turn increases participation as it has been shown by Christelis *et al.* (2010) in a cross country analysis. Participation is also increasing in measured financial literacy (Lusardi and Mitchell (2007) and van Rooij *et al.* (2011)). Notwithstanding the measurement of financial literacy is very sensitive to the wording of survey questions, van Rooij *et al.* (2011) reports empirical evidences of lower stock ownership among those households who have low financial literacy, while Lusardi and Mitchell (2007) show that financially literate households plan for retirement and behave more conscious than others, accumulating more assets in the retirement accounts. Bernheim and Garrett (2003) suggest to promote financial education in the workplace in order to significantly stimulate retirement saving among low and moderate saver. Malmendier and Nagel (2011) report that market experiences strongly affect market participation and risk aversion.

Haliassos and Bertaut (1995) test and reject risk aversion and belief heterogeneity as the main sources for explaining limited market participation.[6] They suggest some "inertial" factors such as cultural influences and costly information and to departure from expectation utility maximization.

Several papers refer to cultural influences in order to explain limited market participation. Guiso *et al.* (2008) introduce the concept of trust that affects the perception of the reality of the individual, in the updating beliefs process. Hong *et al.* (2004) refer to social connection in order to explain individual financial behaviour and the role played by network's information.

Departures from expectation utility maximization focus on behavioral biases such as loss and ambiguity aversion (Knox (2003) and Ang *et al.* (2005)), optimism (Puri and Robinson (2007)), subjective beliefs (Dominitz and Manski (2005)), and model uncer-

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[6]Other tested possible explanations are habit persistence, time-nonseparability, borrowing constraints, differential borrowing and lending rates, and minimum-investment requirements.

tainty (Cao *et al.* (2005)). However, these biases do not fully explain the stockholding puzzle (Barberis *et al.* (2006)).

Costly information has been studied under several aspects: simple monetary costs to participate in the stock market or more related to the information set available to the individual. Luttmer (1999) provides a theoretical framework in order to measure the amount of the participation cost that is able to explain the current market participation rate. He calibrates his model on US economy and he works out that the implied amount is 3 percent of monthly per capita consumption. Vissing-Jorgensen (2003) and Paiella (2001) apply the econometric framework to the PSID, finding a median value around 350\$, and to the CEX ($95 - 175\$$). While, for Italy (SHIW), Guiso and Jappelli (2005) report a median of 750 euro.[7] These estimations give the degree of the impact of the participation cost on household's welfare. For poor households, facing a participation cost is a consistent explanation for their not-stockholding. However, the welfare effect decreases its impact once we focus on wealthier households, leaving unsolved the nostockholding puzzle.

From the point of view of the information acquisition, familiarity (Huberman (2001)), local knowledge (Coval and Moskowitz (2001)), limited information (Merton (1987)) and stock awareness (Guiso and Jappelli (2005)) represent factors that affect the performance of the individual, no matter how much rich she is. However, on the one hand, we know that wealthier households can easily afford costly information provided by financial advisor in order to deal better with stocks and bonds. On the other hand, even if they can afford the advisor's fees, they have to trust them. Guiso *et al.* (2008) discuss the role of trust in the stock market and find, in Dutch

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[7]Luttmer (1999) estimates "lower bounds on the fixed cost required to ensure that a hypothetical consumer with a consumption process equal to U.S. per capita consumption of nondurables and services does not choose to deviate from that consumption process when given the opportunity to trade in U.S. Treasury bills and in stocks on the NYSE".

and Italian micro data, empirical evidences that lack of trust implies limited market participation, even for wealthier households.[8]

We include all these complementary features in the concept of financial literacy, without making distinctions. We synthesize them in the costs of the information acquisition process. Our aim is to check the effects of the inequality in financial literacy on welfare distribution and our simplification does not affect the outcomes of the model.

2.3 Model

Agents face two choices: in the first period, they have to choose if and how much costly information purchase and, in the second period, if and how much to trade in the market, after having paid a participation cost. Agents can trade a riskless and a risky asset. The former pays a rate of return of r (we set $r = 0$) and its supply is perfectly elastic. The latter has a normalized price p and a payoff $\pi \sim N(\mu_\pi, \tau_\pi^{-1})$. Agents can short sell it. The risky asset supply is $\theta \sim N(0, \tau_\theta^{-1})$ which is interpreted as noise trading in the market.[9] We assume that π and θ are mutually independent random variables and their joint distribution is common knowledge.

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[8]They define trust as the subjective probability individuals attribute to the possibility of being cheated. In the absence of any cost of participation, a low level of trust can explain why a large fraction of individuals does not invest in the stock market. Moreover, their model shows that lack of trust amplifies the effect of costly participation.
A similar approach (Cao *et al.* (2005)) introduces model uncertainty as a key issue in order to explain market participation. When uncertainty dispersion is large, investors with high uncertainty choose not to participate in the stock market.

[9]The introduction of an exogenous aggregate risk allows to avoid the Grossman-Stiglitz paradox. With an extra noise, market price are not fully revealing, therefore there is still some incentives to purchase information. On the other hand, the introduction of an exogenous element in the model is a quite strong feature. Wang (1993) models the noise as investors' liquidity needs.

Agents

We assume a costly information acquisition process, where agents differ in their financial literacy cost, c. We assume two types of agents. Type L (literate) with low costs and type H (illiterate) with high costs of acquiring financial information. Agents are continuously distributed according to a cumulative density function $G(j)$ over $J = [0, 1]$ and the fraction of the literates is denoted by λ.

They maximize the same concave utility function. For tractability, we assume CARA utility function and risk neutrality $U(W) = -e^W$.[10] To study the impact of the financial literacy on the market participation, we introduce a friction in the model. All the agents have to pay a homogeneous participation cost F in order to trade. Limited market participation arises when a fraction of agents optimally decides to stay out of the risky asset market.

Information structure

Once π is realized but not revealed, each agent can purchase an unbiased signal s of the risky payoff and can observe a private realization $s = \pi + \sqrt{\frac{1}{x}}\epsilon$, where ϵ is a white noise, independent with respect to π, θ, and across agents.[11] For positive amount of precision x, she gets an informative signal on the payoff of the risky asset. While for zero precision, she gets a completely uninformative signal.

In this framework, we think about a situation where agents need to pay for financial advices. We assume there is an external adviser who shows a detailed report.

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[10] We assume wealth homogeneity. Therefore, the limit of the CARA utility function, i.e., optimal portfolio share independent from wealth, does not affect our analysis.

[11] Actually, we distinguish between two cases: if the agent acquires some information, then the private signal has a normal distribution. While, if the agent is uninformed, she observes a fully uninformative signal:

$$s = \begin{cases} \pi + \sqrt{\frac{1}{x}}\epsilon & \text{if} \quad x > 0 \\ \{\emptyset\} & \text{if} \quad x = 0 \end{cases}$$

For $x \to 0$, the distribution of the beliefs of an informed agent converges to that of an uninformed one.

The degree of technicality is chosen by the agent. However, the literate agents need to spend less time in order to understand it, or the adviser needs to spend less effort in order to make the report understood. Therefore, they pay lower fee than the illiterate agents.[12]

To model this feature of the information acquisition process, we introduce a cost function that is affected either by the degree of technicality and by the financial literacy. Formally, the cost of acquiring an amount x of precision is given by a continuous and twice differentiable function $C(x, c)$ over $x \in \mathbb{R}^+$ such that $C(x, c)$ is at least twice differentiable in x and c with $C_x > 0$, $C_{xx} \geq 0$ and $C_c > 0$, $C_{cc} \geq 0$. The marginal cost C_x is increasing in c: $C_{xc} > 0$. This means that acquiring information at the margin is more costly for less financially literate agents. The cost function $C(x, c)$ is continuous at $x = 0$: $C(0, c) = 0$, and $\lim_{x \to +\infty} C(x, c) = +\infty$. The last two properties imply that a totally uninformative signal is costless and a fully revealing signal is infinitely expensive. These assumptions ensure the existence of a solution for the information choice.

To illustrate the main intuition, we provide a simplified example.

Example (Cost function). *The information cost function is $C(x, c) = c(x^2 + x)$ with $c = \{c_L, c_H\}$ and $c_L < c_H$.*

To solve the model, we focus on a partially revealing noisy rational expectation equilibrium.[13] Following the literature, we solve for an equilibrium in which the price is a linear function of π and θ: $p = a + b\pi - d\theta$, where the coefficients a, b, d are

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[12]Let's take two extreme cases: a genius and a stubborn agent. No matter what the adviser does, the former perfectly understands the most technical report and ends up being fully informed. While the latter is not able to understand at all the report and remains uniformed.

[13]All agents have rational expectations in the sense of Hellwig (1980) and use the information revealed by the price, while they form their posterior beliefs. Moreover, given that we assume unbiased private signals observed by the agents and this is common knowledge, agents know that the equilibrium price p contains some information about the payoff. Therefore, they use it as an informative signal, where its precision is given by the aggregation of the private signal precisions through the individual asset demands.

determined in equilibrium.[14]

We denote agents' information set as $\mathcal{F} = \{s_j, p\}$ where s_j denotes the private signal observed by agent $j \in J$ and it is informative only if agent j acquires some information precision.

Timing

We assume that agents observe their private signal before their trading choice. They submit a limit order that includes their participation choice.[15] Given the presence of the participation cost, the agent optimally chooses to stay out of the market for a range of realizations of the signals, belonging to her information set $\mathcal{F}$.

There are three periods. In the first one, called the planning period, agents can purchase a private signal s and choose its precision x. In the second one, called the trading period, after having observed the own private signal realization s and the market price p, the agent submits a limit order in a competitive market and chooses the optimal portfolio share α. In the last period, called the consumption period, the agent consumes the proceeds from her investments. Figure 2.1 provides the timeline of the model.
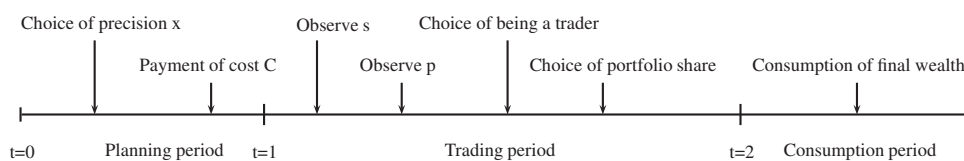

Figure 2.1: Timeline with entry choice.

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[14]Linearity is a standard assumption in the literature when the aim of the research is to find a closed form solution for the price function. On the one hand, guessing a non linear function can provide a better approximation. On the other hand, the model loses tractability and we should rely on numerical methods. We consider this extension in further research.

[15]A limit order is an order placed with a brokerage to buy or sell a set number of shares at a specified price or better

2.4 Equilibrium

We solve the model by backward induction. In the consumption period, each agent consumes her final wealth. In the trading period, the agent faces a portfolio allocation problem where she needs to choose α, share of the portfolio allocated in the risky asset, in order to maximize her expected utility, given x, the precision purchased, and C, the information cost paid. She observes a private and a public signal (the equilibrium price) and she computes posterior beliefs about the final payoff value: $E[\pi|\mathcal{F}]$ and $Var[\pi|\mathcal{F}]$. If she trades, she pays participation cost F. In the planning period, the agent chooses how much precision x of her private signal she wants and pays the monetary cost $C(x, c)$.

The risky asset price p clears the market and partially reveals private information of the traders. When agents submit their limit orders, they take into account how many traders are in the market. They identify a region over signals space according to which it is not optimal to trade. Given that the joint distribution of the two signals is common knowledge, agents are able to compute the probability to be a trader, for any agent. Therefore, they are able to compute the size of the market for any equilibrium market price.[16] An implication of this model is that the market size is endogenous. We call $M = \{j \in J : \alpha_j \neq 0\}$ the set of traders and Q_L (Q_H) the fraction of literate (illiterate) agents who become traders.

The trading period

In the trading period, each agent maximizes

$$\max_{\alpha} \quad E[U(W_2)|\mathcal{F}] \tag{2.1}$$

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[16]Agents conjecture two thresholds on private signal domain, for given market price, such that only those agents with lower or higher private signal realizations become traders.

subject to

$$W_2 = (W_1 - C)R^p - F \, \mathbb{1}_{[\alpha \neq 0]} \tag{2.2}$$

$$R^p = \alpha(\frac{\pi - p}{p}) + 1 \tag{2.3}$$

The budget constraint is characterized by the participation cost, F, that the agent pays in order to trade, $\alpha \neq 0$. The information costs, C, are already paid in the previous period. The portfolio share invested in risky assets differs between types and within groups. It depends on the realization of the private signal.

The optimal share differs between agents, depending on the signal observed and on the precision purchased. In the trading period, all the information choices are already done and each trading agent transfers some of her private information to the market price through her asset demand. Therefore, private information is partially revealed by the market price. While they are forming their posterior beliefs and formulating their asset demand, agents take into account price informativeness and transform the market price into an unbiased public signal. We substitute the informationally equivalent signal $\xi \equiv p$ into the information set $\mathcal{F}$.

The indirect utility for agent j's portfolio problem is $E[U(W_2^*)|\mathcal{F}_j]$, which we note as $v(s_j, p; F, \Theta)$ where Θ is $\{W_1, \mu_\pi, \tau_\pi, \tau_\theta, \lambda\}$.

The planning period

In the planning period, each agent maximizes the indirect utility of the portfolio allocation problem with respect to the information choice. To simplify the notation, we drop the subscript j and write:

$$\max_{x \geq 0} E[v(s, p; F, \Theta)] \tag{2.4}$$

subject to

$$W_1 \geq C(x, c)$$

where the expected utility is computed over the joint distribution of s and p. Recall that signal precision x affects, by assumption, only the distribution of the private signal s. The optimal precision $x^*(c, F, \Theta)$ depends on the exogenous financial literacy cost and on the participation cost.[17] Agents are willing to purchase more information if there is less in the market (price is little informative) and they are not if market price is more revealing. This feature reflects substitutability between the two sources of information.

The equilibrium

We define the concept of equilibrium with the participation cost. In noisy rational expectations equilibrium models, investors make self-fulfilling conjectures about prices and the equilibrium is defined by the set of allocations such that agents maximize their utilities, their conjectures hold and markets clear.

Therefore, a rational expectations equilibrium with the participation cost is given by individual demand function α_j and information demand function x_j, a price function p of π and θ, and a scalar I such that:

1. $x_j = x^*(c_j, F, \Theta)$ and $\alpha_j = \alpha^*(s_j, p; F, \Theta)$ solve the maximization problem, with $c_j = \{c_L, c_H\}$.

2. p clears the market for the risky asset:

$$\int_{j \in J} \alpha_j \frac{W_1 - C(x_j, c_j)}{p} \, dG(j) = \theta$$

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[17] The information precision differs between types, but it is the same within groups. Thus, we have $\forall j \in L$ the optimal precision will be x_L and $\forall j \in H$ it will be x_H.

3. The informativeness of the price I implied by aggregating individual precision choices equals to the level assumed in agents' maximization problem

$$I = \int_{j \in M} x_j \, dG(j)$$

The following three theorems describe the equilibrium allocations. Theorem 3 computes the price function and the optimal individual asset demand. Theorem 4 provides the optimal individual information choice. Theorem 5 describes the equilibrium in the information market. The agents assume an amount of price informativeness, such that, aggregating over the individual information choices, market price has, in equilibrium, that amount of aggregate informativeness.

Theorem 3. *The equilibrium price is given by:*

$$p = a + b\pi - d\theta$$

where

$$a = K^{-1}(\mu_\pi \tau_\pi) \quad b = K^{-1}(I + I^2 \tau_\theta) \quad d = K^{-1}(1 + I\tau_\theta)$$

and $K = \tau_\pi + I + I^2 \tau_\theta$.
The optimal portfolio share of risky asset for agent $j \in J$ is given by

$$\alpha_j^* = \begin{cases} 0 & if \quad s_j \in [\underline{s}, \overline{s}] \\ \frac{k_j \, p}{[W_1 - C(x_j, c_j)]} (E[\pi | s_j, \xi] - p) & otherwise \end{cases}$$

where thresholds $\{\underline{s}, \overline{s}\}$ are defined, respectively, by 2.7 and 2.8.

Proof. See the appendix. □

We provide a sketch of the proof. In the first step, agents guess a price linear function and they derive the informationally equivalent public signal ξ from the market

price. In the second step, they compute the mean and the variance of their posterior beliefs, given the two unbiased signals, ξ and s. In the third step, they derive their optimal asset demand and submit it to the market. In the fourth step, market clears and, in the last step, rationality condition must hold and the coefficients of the guessed linear price function are determined.

The optimal portfolio share depends on the realizations of the signals. We call the trading region, the subset of the signals space such that the agent becomes a trader. We plot it in Figure 2.2. If the agent receives signals belonging to the trading region, she updates her beliefs such that her expected excess return is greater than the participation cost. When the agent trades, α^* depends on the precision of the posterior belief ($k = \tau_\pi + x + I^2 \tau_\theta$) and on the expected excess return, conditional to agent's informative set:

$$E[\pi|s, \xi] - p = k^{-1}(\tau_\pi \mu_\pi + xs + I^2 \tau_\theta \xi) - p$$

where ξ is the informationally equivalent public unbiased signal derived by rational agent from the equilibrium price function: $\xi = \pi - \frac{1}{I}\theta$.

Participation   We characterize the market participation when the asset market is in equilibrium. We distinguish between the participation of the uninformed ($x = 0$) and that of the informed ($x > 0$) agents.

The uninformed agents trade only for public signal realization $\xi \notin [\underline{\xi}, \overline{\xi}]$ where

$$\underline{\xi}(F, \Theta) = \mu_\pi - \frac{K}{I\tau_\pi}\sqrt{2F(\tau_\pi + I^2\tau_\theta)} \tag{2.5}$$

$$\overline{\xi}(F, \Theta) = \mu_\pi + \frac{K}{I\tau_\pi}\sqrt{2F(\tau_\pi + I^2\tau_\theta)} \tag{2.6}$$
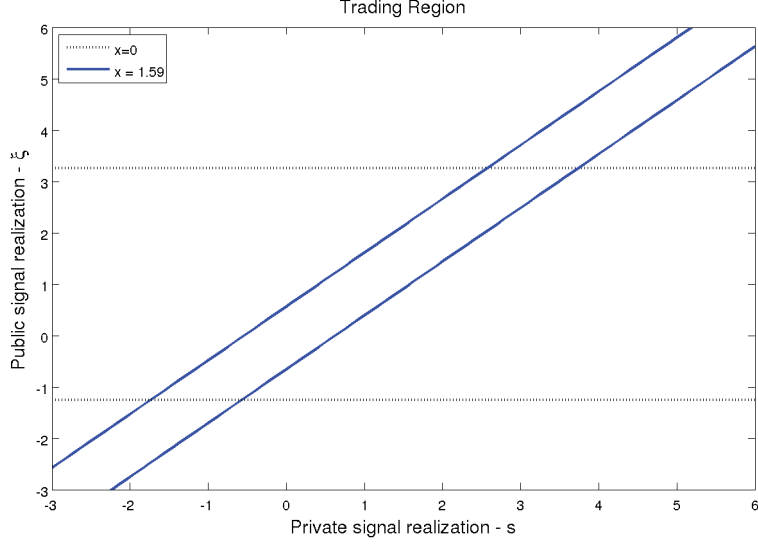
Figure 2.2: The trading region for the uniformed (dotted line) and the informed (solid line) agent.

These two thresholds are the horizontal lines in Figure 2.2.

The informed agents trade only for private signal realization $s \notin [\underline{s}(\xi), \overline{s}(\xi)]$ where

$$\underline{s}(\xi; F, \Theta) = \frac{1}{x} \left\{ (\frac{k}{K} - 1)\mu_\pi \tau_\pi + \xi \left[(\frac{k}{K} - 1)I^2 \tau_\theta + I\frac{k}{K} \right] - \sqrt{2Fk} \right\} \tag{2.7}$$

$$\overline{s}(\xi; F, \Theta) = \frac{1}{x} \left\{ (\frac{k}{K} - 1)\mu_\pi \tau_\pi + \xi \left[(\frac{k}{K} - 1)I^2 \tau_\theta + I\frac{k}{K} \right] + \sqrt{2Fk} \right\} \tag{2.8}$$

The trading region of an informed agent who acquires really small private information $(x \rightarrow 0^+)$ converges to the trading region of the uniformed. In Table 2.1, we report the behaviour in the limits of the threshold functions. This imply that, for any public signal ξ, the optimal portfolio share function is continuous in $x = 0$.

We plot functions (2.7) and (2.8) in Figure 2.3. When the public signal belongs to one of the two external ranges, $\xi < \underline{\xi} \vee \xi > \overline{\xi}$, and the private information vanishes, the no trading region collapses. Otherwise, it enlarges, similar to the trading behaviour of the uniformed agents.

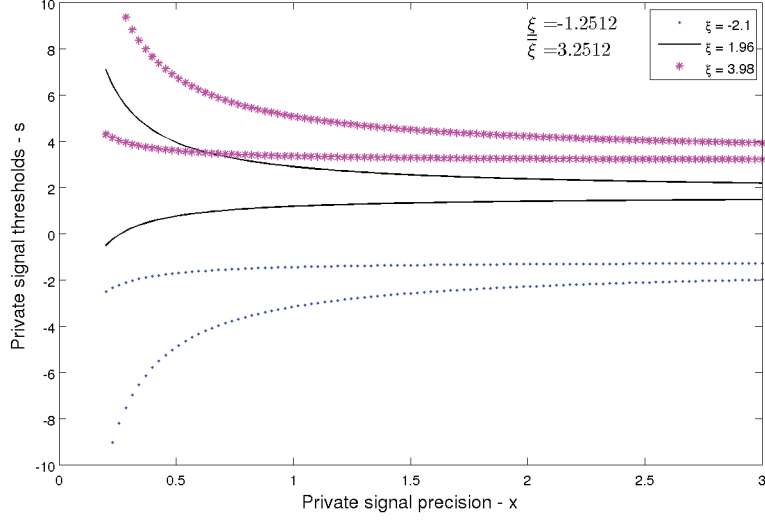With the following two lemmas we derive and characterize the participation probability function $q(\xi)$.

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Figure 2.3: The trading region for given public signal.

Lemma 4. *Conditional to the market price, the probability of being a trader is:*

$$
q(\xi; F, \Theta) = \begin{cases} 1 - \mathbb{1}_{[\xi \in [\underline{\xi}, \bar{\xi}]]} & if \quad x = 0 \\ 1 - [\Phi_{\mu_{s|\xi}, \tau_{s|\xi}}(\bar{s}) - \Phi_{\mu_{s|\xi}, \tau_{s|\xi}}(\underline{s})] & if \quad x > 0 \end{cases} \tag{2.9}
$$

where $\mu_{s|\xi}$ and $\tau_{s|\xi}$ are the mean and the precision of the conditional distribution function $h(s|\xi)$.

Proof. See the appendix. □

Lemma 5. *Let X be a compact set such that, for a given positive $\bar{x} < +\infty$, $X = [0, \bar{x}]$. Function $q(\xi; F, \Theta)$ is differentiable with respect to $x \in X$.*

Proof. See the appendix. □

In Figure 2.4, we plot the participation rate, conditional to the market price, where we keep constant the price informativeness and we let the private information vary. When the market price is close to the expectation (the dot-dashed line), the uniformed agent prefers to stay out of the market. Becoming informed, the agent adjusts her beliefs, using own private information, and the probability to trade increases.

With market price either too low (the solid line) or too high (the dashed line), the uniformed agent always trades. She follows market feelings. However, increasing the amount of information acquired, her participation rate does not behave monotonically. With low precision, the agent adjusts her beliefs, becoming more skeptical about the extreme market signals and reduces her participation rate. With high precision, she trades against the extreme market feelings, in order to spoil profitable opportunities given her informational advantages.

In Figure 2.5, we plot the participation rate, conditional to the market price where we keep constant the price informativeness and we let the public signal vary. The market participation rate for an uniformed agent is strongly affected by the market price (curved line), while, for more informed agent, it is less affected (flatter line). This feature reflects the ability of the informed to recognize profitable opportunities, for any kind of market scenarios. The uniformed agent prefers to stay out of the market when the public signal is not clear enough.
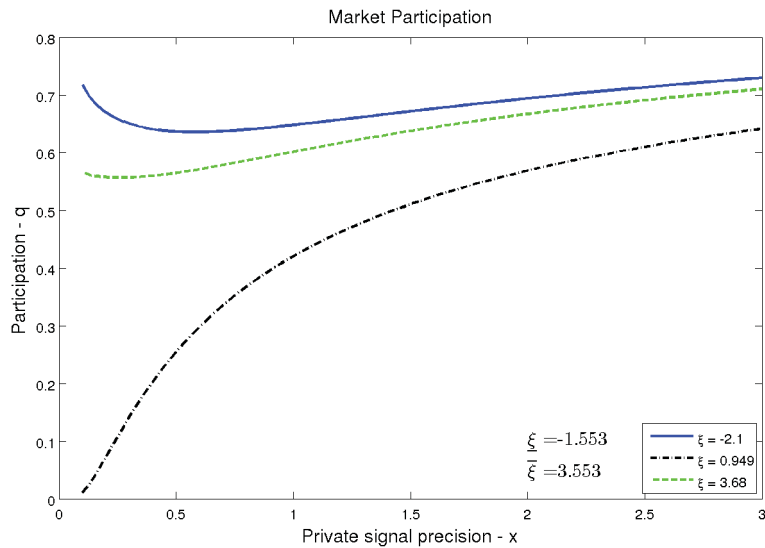

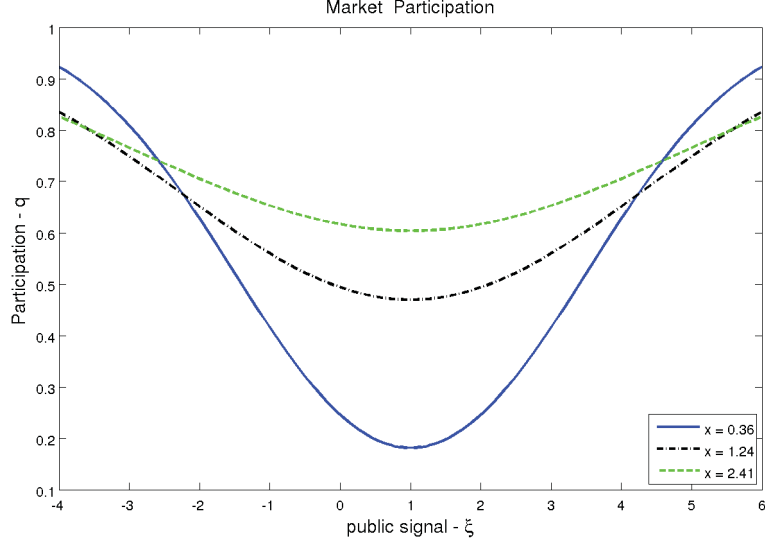Figure 2.4: Market participation rate for different public signals.

Figure 2.5: Market participation rate for private signal precision and price informativeness.

The market price aggregates the whole information available in the market. We rewrite it as:

$$p = \frac{1}{\tau_\pi + I + I^2\tau_\theta} \left[\mu_\pi\tau_\pi + (I + I^2\tau_\theta)(\pi - \tfrac{1}{I}\theta) \right]$$

Rearranging and considering that $K = \tau_\pi + I + I^2\tau_\theta$ is the posterior precision of the average agent's beliefs, the market price can be expressed as a linear function of the posterior mean and variance of the average trader:[18]

$$p = K^{-1}\left[\mu_\pi\tau_\pi + I\pi + I^2\tau_\theta(\pi - \tfrac{\theta}{I}) \right] - \tfrac{\theta}{K} = \hat{\bar{\pi}} - \theta K^{-1}$$

[18]We characterize the average agent as the one with posterior precision

$$K = \int_{j \in M} k_j dj = \tau_\pi + I + I^2\tau_\theta$$

and posterior mean

$$\hat{\bar{\pi}} = \int_{j \in M} E[\pi|s_j, \xi] dj = K^{-1}\left(\mu_\pi\tau_\pi + I\pi + I^2\tau_\theta\xi \right)$$

It is important to note that we do not mean there exists a real agent with these beliefs. We mean a fictitious agent with private signal precision equal to I and private signal realization equal to π. We can think about a shared-information economy where a central planner can observes all the private signals, normally distributed, and take the sample mean: $\int_j s_j dG(j) = \pi$, with precision $\int_j x_j dG(j) = I$.

The market price is driven by two components: the posterior belief of the average agent and the discount on the price demanded in order to compensate for the risk due to the presence of noisy supply.

The equilibrium market price follows a Gaussian distribution:

$$p \sim N\left(\mu_\pi, K^{-2}\left[\left(I^2\tau_\theta + I\right)^2 \frac{1}{\tau_\pi} + \left(I\tau_\theta + 1\right)^2 \frac{1}{\tau_\theta}\right]\right)$$

When the price informativeness is zero, market price reflects the prior beliefs plus the discount demanded for the presence of noisy asset supply ($p = \mu_\pi - \frac{1}{\tau_\pi}\theta$). Price variance fully incorporates the whole variance of the fundamentals.

When the market price is fully revealing, we are back to the case where the supply of the risky asset is known by all the agents and the market price incorporates only the uncertainty about the payoff of the asset.

The next theorem shows the existence and the uniqueness of the optimal information choice and the endogenous threshold according to which is optimal to remain uninformed.

Theorem 4. *There exists a threshold $\overline{c}(F, \Theta)$ such that all agents with lower financial literacy cost purchase information.*

For all agents with $c < \overline{c}(F, \Theta)$, optimal information choice x^* is:

$$x^*(c, F, \Theta) = \arg\max\{z(x; c, F, \Theta) | x \in X(c)\} \qquad (2.10)$$

Proof. See the appendix. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\square$

In order to solve for the information choice x^*, we need to compute the agent's indirect utility: $v(s, p; F, \Theta) = E[U(W_2(\alpha^*))|\mathcal{F}]$. Then, we need to take expectation with respect to s and ξ, $z(x; F, \Theta) = E[v(s, p; F, \Theta)]$. The last step is to find the

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maximum for the expectation, $z(x; F, \Theta)$, with respect to the private signal precision, x.

The market participation $Q(c, F, \Theta)$ is the probability that an agent with financial literacy cost c becomes a traders, as we show in the following lemma.

Lemma 6. *The probability of being a trader for an agent with financial literacy cost c is:*

$$Q(c, F, \Theta) = \begin{cases} 1 - [\Phi_{\mu_\xi, \tau_\xi}(\overline{\xi}) - \Phi_{\mu_\xi, \tau_\xi}(\underline{\xi})] & if \quad c \geq \overline{c} \\ \int q(\xi; F, \Theta) g(\xi) d\xi & if \quad c < \overline{c} \end{cases} \qquad (2.11)$$

Function (2.11) is decreasing with respect to the participation cost F, as we can see in Figure 2.7. When there is low fundamentals uncertainty, the illiterate agent has less incentives for trading in the market. However, it is optimal for her to acquire some information, because she prefers to make informed participation choice.

The endogenous size of the market is reported in Figure 2.8. With low fundamentals uncertainty, the market size shrinks more with respect to the case with high volatility. This is due to the relative higher impact of the market friction on the expected excess returns.

The next theorem claims the existence of a noisy rational expectation equilibrium and identifies the equilibrium condition for the information market: the price informativeness is the sum of the private information of all the agents.

Theorem 5. *There exists a rational expectation equilibrium.*

Proof. See the appendix. □

In the proof of theorem 5, we check that the price informativeness I solves:

$$I = \lambda Q_L x_L^* + (1 - \lambda) Q_H x_H^*$$

where $Q_L = Q(c_L, F, \Theta)$ is the market participation rate of the agent with financial literacy cost c_L and $x_L^* = x(c_L, F, \Theta)$ is the optimal information choice for the agent

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with financial literacy cost c_L. Similarly for the agent with financial literacy cost c_H. In the proof we use a fixed point argument. In order to get a clear insight of the limited market participation, we compare the optimal information choice, the price informativeness, and the market price variance with different levels of participation costs (none, low and high), with just one type of agent.

We observe a non-monotone effect of the participation cost on the optimal information choice. Low participation cost provides incentives to acquire more private information. From Tables 2.2(a) and 2.2(b), we can see that agents purchase more information. This is due to two main factors. First, they need more precise expectations because they face a market friction. Second, some private precision is wasted because it is not transferred into the market price. Therefore, in order to compensate the lower price informativeness, they end up to purchase more private information. However, when there is low uncertainty about the risky payoff, high participation cost balances these two effects, decreasing the probability to trade and, therefore, the benefit derived by being literate. With zero probability to trade, there is no incentive to acquire information.

The limited market participation affects either the price informativeness and the market price variance. Even if the individual information choice increases, the participation cost leads to a small market size and this implies lower price informativeness (Tables 2.3(a) and 2.3(b)). The effect on the market price is ambiguous: from Tables 2.4(a) and 2.4(b), we can see that market price variance increases when the uncertainty of the fundamentals is high and decreases when the uncertainty is low.

2.5   Discussion of the results

In this section we discuss the results of the model and we focus on different degrees of inequality among agents. The proxy for the inequality is given by the difference

between types' financial literacy cost: $c_H - c_L$. We fix the amount of c_H and we let c_L decrease. The economic intuition of this exercise is to compare the effects on individual and market behaviour when the literates are increasingly more sophisticated than the illiterates. The goal is to check if inequality in financial literacy leads to inequality in welfare distribution and to market instability.

The benchmark of the analysis is the case where both types are equally financially literate ($c_L = c_H$). We set the participation cost equal to the ten percent of the initial wealth ($F = 0.1$).[19]

Individual information choice   In the benchmark, agents optimally acquire some information. When inequality increases, the optimal information choice of the two types diverges. As we can see in Figure 2.9, the agent who is more literate acquires more information than who is illiterate. This is due to the relative lower information cost. As a consequence, the market price reveals more information and this result reinforces the illiterate's preferences to acquire less private information and to rely more on public signals. With low inequality, the illiterate is more likely to be an informed trader. While, with high inequality, she prefers to remain uniformed and out of the market.

The literate agent takes into account price informativeness and is affected by the choice of the other type. Notwithstanding her optimal information choice is always increasing in her financial literacy, we distinguish between two cases: when the illiterate is informed and when the illiterate is uninformed. In the former case, the illiterate contributes to the price informativeness, the literate balances own acquisition of private information and partially rely on the public information. In the latter case, the literate is the only supplier of information and changes in her financial literacy lead to wider changes in her optimal information choice.

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[19]Results with different parameter combinations are available on request.

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When the economy is in a regime of low uncertainty, we observe the same dynamics. Ceteris paribus, all the agents need to acquire lower amount of information in order to trade optimally.

Price informativeness and market participation Although information choice of the literate increases exponentially as she becomes more sophisticated, the price informativeness does not increase with the same growth rate. In Figure 2.10, we report the price informativeness on a logarithmic scale. Either with high and low fundamentals uncertainty, the price informativeness is increasing in financial inequality. However, its growth rate is higher in the low uncertainty scenario. This is due to the different behaviour of the two types. While with high uncertainty, the participation rate is high for both types; with low uncertainty, there are less expected profitable opportunities and, therefore, agents are more likely to stay out of the market. See Figure 2.11. This implies that even with more sophisticated and well informed agents, the individual information does not pass to the market price and the price informativeness is kept low as we discuss previously.

Another feature of the presence of more sophisticated agents is that the illiterates optimally prefer to stay out of the market. In Figure 2.11, we can note that the participation rates diverge for higher levels of financial inequality. With low uncertainty, the literate increases her participation rate and this is due to the fact that price informativeness is still low and the market price does not fully reveal the risky payoff. Therefore, even with low uncertainty, the literate finds optimal to trade given that her expectation is concentrated around the true payoff and the market price is still anchored to the prior mean. On the other hand, the expectation of the illiterate is driven by the market price. This leads her to stay out of the market, given that her expected excess return is not enough to recover the participation cost. With low

uncertainty and financial literacy inequality, the literates and the noisy traders trade in the market .

Individual and market stability   Financial literacy inequality affects market stability. The presence of more sophisticated literate agents implies that the market price is more volatile when there is low uncertainty in the fundamentals. In Figure 2.12 we report the market price variance on a logarithmic scale. We can note that with high inequality, the instability increases with low uncertainty (the dot-dashed line) and remains stable with high uncertainty (the solid line).

The main intuition is that, in the latter case, the illiterate trades in the market and mitigates the market position of the literate. The market price reflects the expectation of the illiterate which is mainly based on the public information. Therefore, the volatility of the market is stable because the behaviour of the agents does not change with more inequality.

In the former case, the participation cost induces the illiterate to stay out of the market as the inequality increases. The literate, becoming more sophisticated, acquires more information and is more likely to trade. Therefore, the market price is heavily affects by the expectations of the literates, close to the true payoff. The market price variance reflects all the variability due to the noisy trading and to the noise of the literate private information.

The market excess return $(f_m = \pi - p)$ is another proxy for the stability of the market. Its variance $\sigma_{f_m}^2$ is given by:

$$
\begin{aligned}
\sigma_{f_m}^2 &= \tfrac{1}{\tau_\pi} + b^2 \tfrac{1}{\tau_\pi} + d^2 \tfrac{1}{\tau_\theta} - 2b \tfrac{1}{\tau_\pi} \\
&= \tfrac{(1 + I\tau_\theta)^2}{K^2 \tau_\theta} + \tfrac{\tau_\pi}{K^2}
\end{aligned}
$$

It negatively depends on the price informativeness. When the market price is more revealing, the covariance between market price and asset payoff increases and this

reduces the volatility of the excess return. With high inequality, the variance of the market excess return decreases (see Figure 2.13).

The impact of the financial inequality on the variance of the expected excess return differs among types. We characterized the expected excess return as $f_j = E[\pi|s_j, \xi] - p$ and we compute the variance:

$$
\begin{aligned}
\sigma^2_{f_j} &= \frac{1}{\tau_\pi} - \frac{1}{k_j} + b^2 \frac{1}{\tau_\pi} + d^2 \frac{1}{\tau_\theta} - 2b\frac{1}{\tau_\pi} \\
&= \frac{(1 + I\tau_\theta)^2}{K^2 \tau_\theta} + \frac{1}{K}\left(\frac{\tau_\pi}{K} - \frac{K}{k_j}\right)
\end{aligned}
$$

In the right graph of Figure 2.14, we can note that when the inequality increases, the variance of the expected excess return diverges between types. The illiterate becomes uniformed and relies only on public information embodied in the market price. Therefore, her expectation follows the market feelings.

The literate, given their informational advantages ($\frac{K}{k} < 1$), forms expectations close to the true value and different from the market price. Therefore, the variance of the expected excess return increases given the low covariance between the expectation and the market price. With high uncertainty, the price informativeness is high and the market price and beliefs are more correlated. This implies that the variance of the expected excess return decreases.

Individual welfare All the effects of the financial inequality on the agents' welfare can be measured with respect to their expected utility (Figure 2.15). The wedge between the expected indirect utilities of the two types enlarges as the literate becomes more sophisticated. This is the premium of being more literate. The expected utility of the literate always increases in financial inequality and the expected indirect utility of the illiterate decreases when there is high uncertainty. While, with low uncertainty, it is stable, close to the level of a non trader. The literate will benefit more

from market participation than the illiterate and the barrier of the participation cost exacerbates the effects of this market mechanism.

2.6 Conclusion

This paper develops a tractable asset pricing model with information acquisition, which contemplates the decision to participate to the financial market. The model examines the relationship between financial literacy costs, affecting the information acquisition process, and the participation cost, paid to be a trader in the risky market. We underline that participation costs, such as banking fees or financial asset taxation, are relevant for policy makers. When they design financial literacy improving policies, they take into account the role played by market frictions.

Our model provides a general equilibrium framework through which it is possible to work out policy implications of financial education programs in the presence of market barriers that mitigate the policy effectiveness. We conclude that, on the one hand, low participation cost increases the information acquired by the agents. On the other hand, it reduces the market participation and limits the informativeness of the market price. Furthermore, with high participation cost, even the information acquired decreases. The effect on market price variance is non-monotone and depends on the uncertainty of the risky asset.

We focus on another policy implication of the model. For given participation costs, we compare the behaviour of the agents and that of the market for increasing inequality in financial literacy, i.e., the literate agents become more sophisticated. We find out that the market participation rates of the two types diverge, pushing the illiterates out of the market, especially with low uncertainty in the fundamentals. Furthermore, the market friction prevents the market price to reveal private information. Even if the literate increases her private information, the price informativeness

does not proportionally increase. The increased financial literacy inequality, on the one hand does not affect the market stability and, on the other hand, accentuates the differences between the two types.

Appendix A

Proof of Theorem 3

The distribution of payoff, supply and signals is:

$$
\begin{pmatrix} \theta \\ \pi \\ s \\ p \end{pmatrix} \sim N \left\{ \begin{pmatrix} 0 \\ \mu_\pi \\ \mu_\pi \\ a+b\mu_\pi \end{pmatrix}, \begin{pmatrix} \frac{1}{\tau_\theta} & 0 & 0 & d\frac{1}{\tau_\theta} \\ 0 & \frac{1}{\tau_\pi} & \frac{1}{\tau_\pi} & b\frac{1}{\tau_\pi} \\ 0 & \frac{1}{\tau_\pi} & \frac{1}{\tau_\pi}+\frac{1}{x} & b\frac{1}{\tau_\pi} \\ d\frac{1}{\tau_\theta} & b\frac{1}{\tau_\pi} & b\frac{1}{\tau_\pi} & b^2\frac{1}{\tau_\pi}+d^2\frac{1}{\tau_\theta} \end{pmatrix} \right\}
$$

The proof is given in five steps. In the first step, we guess a linear price function and we derive the informationally equivalent public signal ξ from the price function. In the second step, we compute the mean and the variance of the posterior beliefs given the two unbiased signals, ξ and s. In the third step, we derive the optimal asset demand. In the fourth step, we derive market clearing conditions and, in the last step, we impose rationality and determine the coefficients of the guessed linear price function.

Step 1 : Agents guess a price function linear in π (future payoff) and θ (noisy supply):

$$
p = a + b \left(\lambda \int_{j \in L \cap M} s_j dj + (1-\lambda) \int_{j \in H \cap M} s_j dj \right) - d\theta
$$

Applying the law of large number within each group of traders, we have that $\int_j \epsilon_j dj = 0$ with probability one. Therefore we can rewrite the price function as:

$$
p = a + b\pi - d\theta
$$

Agents use the private signals to update their prior beliefs $\pi \sim N(\mu_\pi, \tau_\pi^{-1})$. The private signal is unbiased by construction $s|\pi \sim N(\pi, x^{-1})$ and conditionally inde-

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pendent from prior belief μ_π, $E[(\mu_\pi - \pi)(s - \pi)] = 0$. Rational agents use the market price as a public signal. It is not unbiased: $E[p|\pi] = a + b\pi$. To apply Bayesian updating, agents need to transform the price in an informationally equivalent variable ξ:

$$\xi = \frac{p - a}{b} = \pi - \tfrac{d}{b}\theta$$

where

$$\xi|\pi \sim N(\pi, \tfrac{d^2}{b^2}\tfrac{1}{\tau_\theta})$$

Step 2 : The agent observes $\mathcal{F} = \{s, p\} \equiv \{s, \xi\}$, where s is uninformative when $x = 0$. She updates her prior beliefs with the two Gaussian signals. Using the well known formula for the multivariate normal distribution (Degroot (2004), p. 55) the posterior mean is given by:

$$E[\pi|s, \xi] = \mu_\pi + \frac{1}{k} \left\{ x \left(s - E[s]\right) + \tfrac{b^2}{d^2}\tau_\theta(\xi - E[\xi]) \right\}$$
$$= \tfrac{1}{k} \left(\tau_\pi \mu_\pi + x\, s + \tfrac{b^2}{d^2}\tau_\theta \xi \right)$$

where the precision of the posterior belief k is given by the sum of the precisions of the prior, of the private signal and of the public signal.

$$k = \frac{1}{Var[\pi|s, \xi]} = \tau_\pi + x + \tfrac{b^2}{d^2}\tau_\theta$$

Step 3 : The agents maximizes a CARA utility function with respect to the control variable α. Using the properties of the log-normal random variables, we have that:

$$\max_\alpha E[-e^{-W(\alpha)}|s, \xi] = \max_\alpha e^{-\{E[W(\alpha)|s,\xi] - \frac{1}{2}Var[W(\alpha)|s,\xi]\}}$$

where

$$E[W(\alpha)|s,\xi] = E[[W_1 - C(x,c)]\alpha\frac{\pi-p}{p} + [W_1 - C(x,c)] - F\,\mathbb{1}_{[\alpha\neq 0]}|s,\xi]$$

and

$$Var[W(\alpha)|s,\xi] = \frac{[W_1-C(x,c)]^2\alpha^2}{p^2}Var[\pi|s,\xi]$$

Substituting and deriving FOC, we get that:

$$[W_1 - C(x,c)]\frac{E[\pi|s,\xi]-p}{p} - 2\alpha\frac{[W_1-C(x,c)]^2}{p^2}Var[\pi|s,\xi] = 0$$

Therefore, the optimal risky asset demand for a trader is:

$$\alpha^* = \frac{1}{[W_1 - C(x,c)]}\frac{\frac{E[\pi|s,\xi]-p}{p}}{\frac{Var[\pi|s,\xi]}{p^2}} = \frac{p}{[W_1 - C(x,c)]}\frac{E[\pi|s,\xi]-p}{\frac{1}{k}}$$

The amount of wealth invested in risky assets depends on the precision of the posterior belief, on the risk aversion coefficient and on the expected excess return of the risky investment.

The agent compares the expected utility of the optimal portfolio share of being a trader in the risky market, with the expected utility of holding only risk-free assets. She does not trade if:

$$E[e^{-W(\alpha^*,F)}|s,\xi] < E[e^{-W(0)}|s,\xi]$$

and this holds when:

$$(E[\pi|s,\xi] - p)^2 > 2Fk^{-1}$$

Formally, we need to distinguish between two cases: when the agent is uniformed ($x = 0$) and when she is informed ($x > 0$). In the former case, we compute the thresholds

$\{\underline{\xi}, \overline{\xi}\}$ over the public signal domain, such that, for those public signal realizations, the agent would be indifferent between being or not being trader. In the latter case, we solve the previous inequality for given public signal ξ in order to find two thresholds $\{\underline{s}, \overline{s}\}$ over private signal domain. An informed agent is indifferent between trading or staying out of the market, when she observes private signal realizations, such that the following equations are satisfied:

$$\begin{cases} E[\pi|\underline{s}, \xi] = p - \sqrt{2Fk^{-1}} \\ E[\pi|\overline{s}, \xi] = p + \sqrt{2Fk^{-1}} \end{cases}$$

We substitute the expected value $E[\pi|s, \xi]$ and we solve for the thresholds $\{\underline{s}, \overline{s}\}$. The economic intuition is that agents submit market order such that, given market price, they would like to trade only for certain values of private signal. After some algebra, we derive the following linear functions:

$$\begin{aligned} \underline{s}(\xi; F, \Theta) &= \tfrac{1}{x}(\tfrac{k}{K} - 1)\mu_\pi \tau_\pi + \tfrac{1}{x}\left[(\tfrac{k}{K} - 1)\tfrac{b^2}{d^2}\tau_\theta + \tfrac{b}{d}\tfrac{k}{K}\right]\xi - \tfrac{1}{x}\sqrt{2Fk} \\ \overline{s}(\xi; F, \Theta) &= \tfrac{1}{x}(\tfrac{k}{K} - 1)\mu_\pi \tau_\pi + \tfrac{1}{x}\left[(\tfrac{k}{K} - 1)\tfrac{b^2}{d^2}\tau_\theta + \tfrac{b}{d}\tfrac{k}{K}\right]\xi + \tfrac{1}{x}\sqrt{2Fk} \end{aligned}$$

with $K = \tau_\pi + \tfrac{b}{d} + \tfrac{b^2}{d^2}\tau_\theta$.

For an informed agent, the optimal portfolio choice is given by the following asset demand function:

$$\alpha^*(s, \xi; \Theta) = \begin{cases} 0 & \text{if } s \in [\underline{s}(\xi), \overline{s}(\xi)] \\ \frac{kp}{[W_1 - C(x,c)]}(E[\pi|s, \xi] - p) & \text{otherwise} \end{cases} \tag{2.12}$$

Similarly, we derive the optimal portfolio choice of the uniformed:

$$\alpha^*(\xi; \Theta) = \begin{cases} 0 & \text{if } \xi \in [\underline{\xi}, \overline{\xi}] \\ \frac{kp}{W_1}(E[\pi|\xi] - p) & \text{otherwise} \end{cases}$$

where

$$\underline{\xi} \;\; = \;\; \frac{ak - \tau_\pi \mu_\pi - \sqrt{2Fk}}{\frac{b^2}{d^2}\tau_\theta - bk}$$

$$\overline{\xi} \;\; = \;\; \frac{ak - \tau_\pi \mu_\pi + \sqrt{2Fk}}{\frac{b^2}{d^2}\tau_\theta - bk}$$

Step 4 : Each agent $j \in J$ observes own private signal s_j and submits an asset demand α_j given own private signal precision x_j.

The equilibrium price clears the market for the risky asset. Aggregating over all traders yields the following market condition:

$$\lambda \int_{j \in L \cap M} \alpha_j^* \frac{W_1 - C(x_j, c)}{p} dj + (1 - \lambda) \int_{j \in H \cap M} \alpha_j^* \frac{W_1 - C(x_j, c)}{p} dj = \theta$$

For any given public signal ξ, we denote the amount of traders for each type with, respectively, $q_L = \#\{j : j \in L \cap M\}$ and $q_H = \#\{j : j \in H \cap M\}$.

Let assume, for the moment, that q_L and q_H are greater than $\overline{q}$, the minimum amount of agents such that the assumption of competitive markets still holds.[20] Thus, we apply the weak law of large numbers for independent random variable with the same mean, such that $\int_j s_j dj = \pi$. Moreover, given homogeneity within types, limited participation means that only a fraction of acquired information is revealed in the market and this fraction is q in our framework. Therefore, imposing market clearing condition, we have:

$$\left[\lambda \left(\mu_\pi \tau_\pi + q_L x_L \pi + \tfrac{b^2}{d^2} \tau_\theta \xi - pk_L \right) + (1 - \lambda) \left(\mu_\pi \tau_\pi + q_H x_H \pi + \tfrac{b^2}{d^2} \tau_\theta \xi - pk_H \right) \right] = \theta$$

$$\mu_\pi \tau_\pi + \tfrac{b^2}{d^2} \tau_\theta \xi + \pi[\lambda q_L x_L + (1 - \lambda) q_H x_H] - p[\lambda q_L k_L + (1 - \lambda) q_H k_H] = \theta$$

---

[20]We check which conditions on parameters must hold in order to keep competitive markets.

Using the definition of aggregate informativeness $I = \lambda q_L x_L + (1-\lambda) q_H x_H$, we can rewrite the price equation as:

$$p = \left(\tau_\pi + I + \tfrac{b^2}{d^2}\tau_\theta\right)^{-1} \left(\mu_\pi \tau_\pi + \tfrac{b^2}{d^2}\tau_\theta \xi + \pi I - \theta\right)$$

Step 5 : Public signal ξ involves undetermined coefficients b, d. We substitute the expression for $\xi = \pi - \tfrac{d}{b}\theta$ and, rearranging terms, we have:

$$p = \left(\tau_\pi + I + \tfrac{b^2}{d^2}\tau_\theta\right)^{-1} \left[\mu_\pi \tau_\pi + \pi(I + \tfrac{b^2}{d^2}\tau_\theta) - \theta\left(\tfrac{b}{d}\tau_\theta + 1\right)\right]$$

We can derive $\tfrac{b}{d} = I$ and we substitute it back into the price function.
Coefficient of θ:

$$d = K^{-1}\left(1 + I\tau_\theta\right)$$

Coefficient of π:

$$b = K^{-1}\left(I + I^2\tau_\theta\right)$$

Constant term, a:

$$a = K^{-1}\mu_\pi\tau_\pi$$

where $K = \tau_\pi + I + I^2\tau_\theta$.

Proof of lemma 4

Conditional to market price, the probability of being a trader is

$$q(\xi; F, \Theta) = \begin{cases} 1 - \mathbb{1}_{\left[\xi \in [\underline{\xi}, \overline{\xi}]\right]} & \textit{if} \quad x = 0 \\ 1 - \left[\Phi_{\mu_{s|\xi}, \tau_{s|\xi}}(\overline{s}) - \Phi_{\mu_{s|\xi}, \tau_{s|\xi}}(\underline{s})\right] & \textit{if} \quad x > 0 \end{cases}$$

where $\mu_{s|\xi}$ and $\tau_{s|\xi}$ are the mean and the precision of the conditional distribution function $h(s|\xi)$.

Proof. To compute the market participation probability of an agent, we distinguish between informed and uniformed agents. When she is informed, the joint distribution between the public and the private signal is:

$$
\begin{pmatrix} s \\ \xi \end{pmatrix} \sim N \left\{ \begin{pmatrix} \mu_\pi \\ \mu_\pi \end{pmatrix}, \begin{pmatrix} \frac{1}{\tau_\pi} + \frac{1}{x} & \frac{1}{\tau_\pi} \\ \frac{1}{\tau_\pi} & \frac{1}{\tau_\pi} + \frac{1}{I^2 \tau_\theta} \end{pmatrix} \right\}
$$

Following Degroot (2004), p. 55, the conditional distribution $h(s|\xi)$ is still a normal density function:

$$
h(s|\xi) = \sqrt{\frac{\tau_{s|\xi}}{2\pi}} e^{-\frac{1}{2}\tau_{s|\xi}(s - \mu_{s|\xi})^2}
$$

with mean:

$$
\mu_{s|\xi} = \mu_\pi + \frac{Cov(s,\xi)}{Var[\xi]}(\xi - \mu_\pi) = \mu_\pi + \frac{I^2 \tau_\theta}{\tau_\pi + I^2 \tau_\theta}(\xi - \mu_\pi) = \mu_\pi + \gamma(\xi - \mu_\pi)
$$

and variance:

$$
\frac{1}{\tau_{s|\xi}} = Var[s] - \frac{Cov(s,\xi)^2}{Var[\xi]} = \frac{x + \tau_\pi}{x \tau_\pi} - \frac{I^2 \tau_\theta}{\tau_\pi(\tau_\pi + I^2 \tau_\theta)}
$$

Therefore, we can compute the probability to be a trader, conditional to the market price, $q(\xi; F, \Theta)$, as the area below the density function over the set $(-\infty, \underline{s}) \cup (\overline{s}, +\infty)$.[21] Thus, we have:

$$
\begin{aligned}
q(\xi; F, \Theta) &= \int_{-\infty}^{\underline{s}(\xi;F,\Theta)} \sqrt{\frac{\tau_{s|\xi}}{2\pi}} e^{-\frac{1}{2}\tau_{s|\xi}(s - \mu_{s|\xi})^2} ds + \int_{\overline{s}(\xi;F,\Theta)}^{\infty} \sqrt{\frac{\tau_{s|\xi}}{2\pi}} e^{-\frac{1}{2}\tau_{s|\xi}(s - \mu_{s|\xi})^2} ds \\
&= 1 - [\Phi(\overline{s}) - \Phi(\underline{s})]
\end{aligned}
$$

For uniformed agents, the participation probability function assumes value equal to one when public signal is lower than the threshold $\underline{\xi}$ or greater than the threshold $\overline{\xi}$.

---

[21]Given the assumptions over the two types, the conditional probability to be a trader for an agent of one type corresponds to the fraction of the agent of the same type to receive a signal, such that they will be willing to trade at that price. Therefore, we can say that $q_L(\xi; F, \Theta) = \int_{j \in L \cap M(\xi)} dG(j)$ and $q_H(\xi; F, \Theta) = \int_{j \in H \cap M(\xi)} dG(j)$.

It is easy to see that, when $x \to 0^+$, the thresholds $\underline{s}$ and $\overline{s}$ diverge to ∞, as it is report in table 2.1. The sum of the two integrals is zero if $\xi \in [\underline{\xi}, \overline{\xi}]$ and it is one if $\xi \notin [\underline{\xi}, \overline{\xi}]$. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $\square$

Proof of lemma 5

Let X be a compact set such that, for a given positive $\overline{x} < +\infty$, $X = [0, \overline{x}]$. Function $q(\xi; F, \Theta)$ is differentiable with respect to $x \in X$

Proof. To prove it we apply Leibniz's formula. We use the generalized formula to integrals with unbounded intervals of integration.

For given ξ, $q(\xi; F, \Theta) : \mathbb{R}^+ \to [0, 1]$. It is continuous with respect to $x \in \mathbb{R}^+$ by continuity of the integrand function $h(s, \xi, x)$ and of the boundary functions $\underline{s}(\xi; F, \Theta)$ and $\overline{s}(\xi; F, \Theta)$: $h(x)$, $\underline{s}(x)$ and $\overline{s}(x)$ are $\mathcal{C}^1$. In $x = 0$, $q(\xi; F, \Theta) = \{0, 1\}$ but we know that $\lim\limits_{x \to 0^+} q(\xi; F, \Theta) \to q(\xi; F, \Theta)|_{x=0}$ by proof of lemma 4. Therefore, we can conclude that $q(\xi; F, \Theta)$ is continuous in $\mathbb{R}^+$.

To prove differentiability we need further conditions in order to apply Leibniz's formula. For any positive x, partial derivative $h'_x(s, \xi, x)$ exists and it is continuous in $\mathbb{R}^+$.[22] Integrals $\int_{-\infty}^{\underline{s}} h(s, \xi, x) ds$ and $\int_{\overline{s}}^{\infty} h(s, \xi, x) ds$ converge for each positive x, by boundedness of cumulative distribution function. Furthermore, $h'_x(s, \xi, x)$ is integrably bounded.[23]

Therefore, by Leibniz's formula, $q'_x(\xi; F, \Theta)$ does exist and it is equal to

$$q'_x(\xi; F, \Theta) = -\left[h(\overline{s}, \xi, x)\overline{s}'_x - h(\underline{s}, \xi, x)\underline{s}'_x + \int_{\underline{s}}^{\overline{s}} h'_x(s, \xi, x) ds \right]$$

---

[22] $h'_x(s, \xi, x) = \frac{\partial h}{\partial \tau_{s|\xi}} \frac{\partial \tau_{s|\xi}}{\partial x} = \frac{1}{2} \tau_{s|\xi} h \left(\tau_{s|\xi}(s - \mu_{s|\xi})^2 - \frac{1}{2\pi} \right) \left(-\frac{1}{x^2} \right)$

[23] In the sense that there exists a function $f(s)$, independent of x, such that $|h'_x(s, \xi, x)| \leq f(s)$ for all $s \in \mathbb{R}$ and for all $x > 0$ and such that $\int_{-\infty}^{+\infty} f(s) ds$ converges.

Moreover, $\lim_{x \to 0^+} q'_x(\xi; F, \Theta) \to q'_x(\xi; F, \Theta)|_{x=0} = 0$. This implies that $q(\xi; F, \Theta)$ is differentiable with respect to $x \in X$. $\qquad\square$

Proof of Theorem 4

In order to solve for the information choice x^*, we need to compute the indirect utility: $v(s, p; F, \Theta) = E[U(W_2(\alpha_j^*))|\mathcal{F}_j]$. Then, we need to take the expectation of the indirect utility function with respect to s and ξ, $z(x; F, \Theta) = E[v(s, p; F, \Theta)]$. The last step is to find the maximum, $x^* = \arg\max z(x; F, \Theta)$. We compute it numerically.

Lemma 7. *For each agent, the indirect utility function is*

$$
v(s, p; F, \Theta) = \begin{cases} \begin{cases} -e^{-W_1} & \textit{if } \ \xi \in [\underline{\xi}, \overline{\xi}] \\ -e^{-W_1 - \frac{1}{2}k(E[\pi|\xi] - p)^2 + F} & \textit{otherwise} \end{cases} & \textit{if } \ x = 0 \\ \begin{cases} -e^{-[W_1 - C(x,c)]} & \textit{if } \ s \in [\underline{s}, \overline{s}] \\ -e^{-[W_1 - C(x,c)] - \frac{1}{2}k(E[\pi|s,\xi] - p)^2 + F} & \textit{otherwise} \end{cases} & \textit{if } \ x > 0 \end{cases}
$$

(2.13)

Proof. Final wealth is $W_2 = [W_1 - C(x, c)][1 + \alpha^*(\frac{\pi - p}{p})]$ with α^* given by 2.12. In order to compute the indirect utility:[24]

$$
v(s, p; F, \Theta) = E[e^{-W_2(\alpha^*)}|\mathcal{F}] = -e^{-\{E[W_2(\alpha^*)|s,\xi] - \frac{1}{2}Var[W_2(\alpha^*)|s,\xi]\}}
$$

we need to compute the mean $E[W_2(\alpha^*)|s, \xi]$ and the variance $Var[W_2(\alpha^*)|s, \xi]$ of the final wealth (W_2). At the trading period, wealth $W_2(\alpha^*)$ is given by normal random

---

[24]We use the properties of the log-normal distribution.

variables π, s, p and a constant term $[W_1 - C(x,c)]$. Therefore, the mean is:

$$
\begin{aligned}
E[W_2(\alpha^*)|s,\xi] &= [W_1 - C(x,c)]\alpha^* E[\tfrac{\pi - p}{p}|s,\xi] + [W_1 - C(x,c)] \\[2mm]
&= \begin{cases}
[W_1 - C(x,c)] & \text{if} \quad s \in [\underline{s}, \overline{s}] \\[2mm]
[W_1 - C(x,c)]\frac{pk}{[W_1 - C(x,c)]}(E[\pi|s,\xi] - p)\frac{E[\pi|s,\xi]-p}{p} + [W_1 - C(x,c)] - F \\
\end{cases} \\[2mm]
&= \begin{cases}
[W_1 - C(x,c)] & \text{if} \quad s \in [\underline{s}, \overline{s}] \\[2mm]
k(E[\pi|s,\xi] - p)^2 + [W_1 - C(x,c)] - F & \text{otherwise}
\end{cases}
\end{aligned}
$$

and the variance is:

$$
\begin{aligned}
Var[W_2(\alpha^*)|s,\xi] &= \frac{[W_1 - C(x,c)]^2(\alpha^*)^2}{p^2}Var[\pi|s,\xi] \\[2mm]
&= \begin{cases}
0 & \text{if} \quad s \in [\underline{s}, \overline{s}] \\[2mm]
\frac{[W_1 - C(x,c)]^2 p^2 k^2 (E[\pi|s,\xi]-p)^2}{p^2[W_1 - C(x,c)]^2}\frac{1}{k} & \text{otherwise} \\
\end{cases} \\[2mm]
&= \begin{cases}
0 & \text{if} \quad s \in [\underline{s}, \overline{s}] \\[2mm]
k(E[\pi|s,\xi] - p)^2 & \text{otherwise}
\end{cases}
\end{aligned}
$$

Thus, for $x > 0$, the indirect utility is:

$$
\begin{aligned}
v(s,p;F,\Theta) &= \begin{cases}
-e^{-[W_1 - C(x,c)]} & \text{if} \quad s \in [\underline{s}, \overline{s}] \\[2mm]
-e^{-[W_1 - C(x,c)]+k(E[\pi|s,\xi]-p)^2 - \frac{k}{2}(E[\pi|s,\xi]-p)^2 + F} & \text{otherwise}
\end{cases} \\[2mm]
&= \begin{cases}
-e^{-[W_1 - C(x,c)]} & \text{if} \quad s \in [\underline{s}, \overline{s}] \\[2mm]
-e^{-[W_1 - C(x,c)] - \frac{k}{2}(E[\pi|s,\xi]-p)^2 + F} & \text{otherwise}
\end{cases}
\end{aligned}
$$

Similarly, for $x = 0$, indirect utility is:

$$v(p; F, \Theta) = \begin{cases} -e^{-W_1} & \text{if} \quad \xi \in [\underline{\xi}, \overline{\xi}] \\ -e^{-W_1 - \frac{k}{2}(E[\pi|\xi] - p)^2 + F} & \text{otherwise} \end{cases}$$

$\square$

In Figure 2.6, we plot the indirect utility function, for given values of market price, price informativeness and private signal precision. We let vary the participation cost, F, in order to note how the no trading region enlarges.
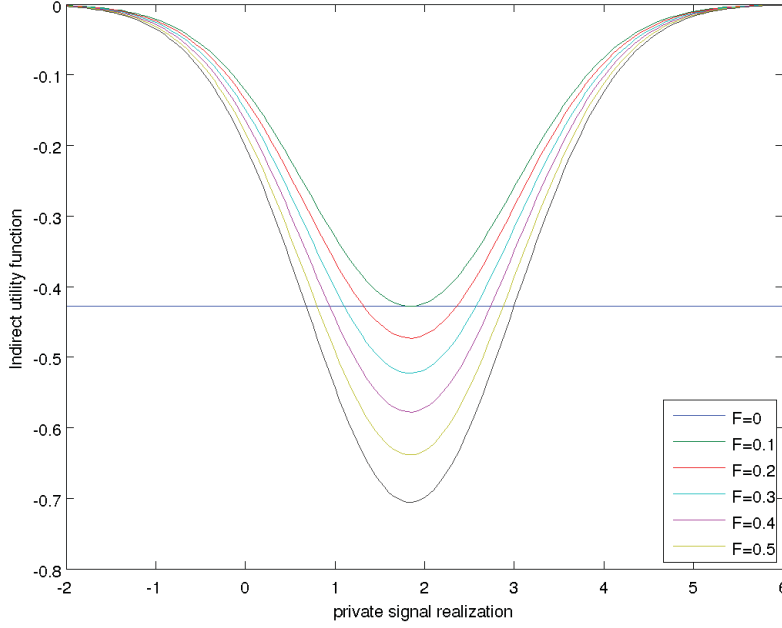

Figure 2.6: The indirect utility function with the participation cost.

Lemma 8. *The expected indirect utility function is:*

$$z(x; F, \Theta) = \begin{cases} -e^{-W_1}(g_1 + g_2 + g_3) & \textit{if} \quad x = 0 \\ -e^{-[W_1 - C(x,c)]}(f_1 + f_2 + f_3) & \textit{if} \quad x > 0 \end{cases} \tag{2.14}$$

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where f_1, f_2 and f_3 are defined by equations (2.15) and g_1, g_2 and g_3 are defined by equations (2.16).

Proof. We double integrate (2.13) over the domain of the two signals, given their joint distribution $f(s, \xi)$:

$$E[v(s, p; F, \Theta)] = \int \int v(s, \xi; F, \Theta) f(s, \xi) d\xi ds$$

$$= -e^{-[W_1 - C(x,c)]} \left\{ \int \left[\int_{-\infty}^{\underline{s}(\xi; F, \Theta)} e^{-\frac{1}{2}k(E[\pi|s,\xi]-p)^2 + F} f(s, \xi) ds \right] d\xi \right.$$
$$+ \int \int_{\underline{s}(\xi; F, \Theta)}^{\overline{s}(\xi; F, \Theta)} f(s, \xi) ds d\xi$$
$$\left. + \int \left[\int_{\overline{s}(\xi; F, \Theta)}^{\infty} e^{-\frac{1}{2}k(E[\pi|s,\xi]-p)^2 + F} f(s, \xi) ds \right] d\xi \right\}$$

We compute the integrals by steps.

1. First of all, we focus on the first one. We rewrite the joint distribution as the product of the conditional and the marginal:

$$e^F \int \left[\int_{-\infty}^{\underline{s}(\xi; F, \Theta)} e^{-\frac{1}{2}k(E[\pi|s,\xi]-p)^2} h(s|\xi) ds \right] g(\xi) d\xi$$

2. We open the square and we rewrite it as:

$$-\tfrac{1}{2}k(E[\pi|s,\xi]-p)^2 = -\tfrac{1}{2}k\left\{ \tfrac{1}{k} \left[x(s - \mu_\pi) + I^2\tau_\theta(\xi - \mu_\pi) \right] - (p - \mu_\pi) \right\}^2$$
$$= -\tfrac{1}{2} \left[\beta_1(s - \mu_\pi)^2 - 2\beta_2(s - \mu_\pi)(\xi - \mu_\pi) + \beta_3(\xi - \mu_\pi)^2 \right]$$

where

$$\beta_1 = \frac{x^2}{k} \qquad \beta_2 = x(b - \tfrac{I^2\tau_\theta}{k}) \qquad \beta_3 = k(b - \tfrac{I^2\tau_\theta}{k})^2$$

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Moreover, we explicit the conditional density:

$$h(s|\xi) = \sqrt{\frac{\tau_{s|\xi}}{2\pi}} e^{-\frac{1}{2}\tau_{s|\xi}[(s-\mu_\pi)-\gamma(\xi-\mu_\pi)]^2}$$

3. The integrand becomes:

$$e^{-\frac{1}{2}k(E[\pi|s,\xi]-p)^2} h(s|\xi) =$$

$$= \sqrt{\frac{\tau_{s|\xi}}{2\pi}} e^{-\frac{1}{2}\left\{\beta_1(s-\mu_\pi)^2-2\beta_2(s-\mu_\pi)(\xi-\mu_\pi)+\beta_3(\xi-\mu_\pi)^2+\tau_{s|\xi}[(s-\mu_\pi)^2-2\gamma(s-\mu_\pi)(\xi-\mu_\pi)+\gamma^2(\xi-\mu_\pi)^2]\right\}}$$

$$= \sqrt{\frac{\tau_{s|\xi}}{2\pi}} e^{-\frac{1}{2}\left[(\beta_1+\tau_{s|\xi})(s-\mu_\pi)^2-2(\beta_2+\tau_{s|\xi}\gamma)(s-\mu_\pi)(\xi-\mu_\pi)+(\beta_3+\tau_{s|\xi}\gamma^2)(\xi-\mu_\pi)^2\right]}$$

4. We want to complete the square in the exponential term, in order to write the integrand as a normal density function of s conditional to ξ.

$$(\beta_1+\tau_{s|\xi})(s-\mu_\pi)^2-2\frac{(\beta_2+\tau_{s|\xi}\gamma)(s-\mu_\pi)(\xi-\mu_\pi)(\beta_1+\tau_{s|\xi})}{(\beta_1+\tau_{s|\xi})}\pm\frac{(\beta_2+\tau_{s|\xi}\gamma)^2(\xi-\mu_\pi)^2}{(\beta_1+\tau_{s|\xi})}+(\beta_3+\tau_{s|\xi}\gamma^2)(\xi-\mu_\pi)^2 =$$

$$= \frac{\left\{s-\mu_\pi+\left[\frac{(\beta_2+\tau_{s|\xi}\gamma)(\xi-\mu_\pi)}{(\beta_1+\tau_{s|\xi})}\right]\right\}^2}{\frac{1}{(\beta_1+\tau_{s|\xi})}} - \frac{(\beta_2+\tau_{s|\xi}\gamma)^2(\xi-\mu_\pi)^2}{(\beta_1+\tau_{s|\xi})} + (\beta_3+\tau_{s|\xi}\gamma^2)(\xi-\mu_\pi)^2$$

$$= \frac{\left\{s-\mu_\pi+\left[\frac{(\beta_2+\tau_{s|\xi}\gamma)(\xi-\mu_\pi)}{(\beta_1+\tau_{s|\xi})}\right]\right\}^2}{\frac{1}{(\beta_1+\tau_{s|\xi})}} - (\xi-\mu_\pi)^2\left[\frac{(\beta_2+\tau_{s|\xi}\gamma)^2}{(\beta_1+\tau_{s|\xi})} - (\beta_3+\tau_{s|\xi}\gamma^2)\right]$$

5. The first term can be defined as the new conditional density function of the private signal $\tilde{h}(s|\xi)$ after we change the measure, to take into account the risk of being a trader. The new conditional mean is:

$$\tilde{\mu}_{s|\xi} = \mu_\pi + \frac{(\beta_2+\tau_{s|\xi}\gamma)(\xi-\mu_\pi)}{(\beta_1+\tau_{s|\xi})}$$

and new conditional variance is:

$$\tilde{\sigma}^2_{s|\xi} = \frac{1}{\beta_1+\tau_{s|\xi}}$$

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6. The second part of the integral does not depend on s. Then, we can take it out from the inner integral.

7. The first double integral can be rewritten as:

$$
\int \left[\int_{-\infty}^{\underline{s}(\xi;F,\Theta)} e^{-\frac{1}{2}k(E[\pi|s,\xi]-p)^2} h(s|\xi)ds \right] g(\xi)d\xi =
$$

$$
= \int \sqrt{\frac{\tau_{s|\xi}}{2\pi}} e^{\frac{1}{2}(\xi-\mu_\pi)^2 \left[\frac{(\beta_2+\tau_{s|\xi}\gamma)^2}{(\beta_1+\tau_{s|\xi})} - (\beta_3+\tau_{s|\xi}\gamma^2) \right]} \sqrt{2\pi\tilde{\sigma}_{s|\xi}^2} \int_{-\infty}^{\underline{s}(\xi;F,\Theta)} \tilde{h}(s|\xi)dy \ g(\xi)d\xi
$$

8. We can compute the inner cumulative distribution function. It depends on ξ. The first integral becomes:

$$
\sqrt{\tau_{s|\xi}\tilde{\sigma}_{s|\xi}^2} \int e^{\frac{1}{2}(\xi-\mu_\pi)^2 \left[\frac{(\beta_2+\tau_{s|\xi}\gamma)^2}{(\beta_1+\tau_{s|\xi})} - (\beta_3+\tau_{s|\xi}\gamma^2) \right]} \Phi(\underline{s}(\xi;F,\Theta);\tilde{\mu}_{s|\xi},\tilde{\sigma}_{s|\xi}^2) \ g(\xi)d\xi
$$

9. Similarly, the third integral becomes:

$$
\sqrt{\tau_{s|\xi}\tilde{\sigma}_{s|\xi}^2} \int e^{\frac{1}{2}(\xi-\mu_\pi)^2 \left[\frac{(\beta_2+\tau_{s|\xi}\gamma)^2}{(\beta_1+\tau_{s|\xi})} - (\beta_3+\tau_{s|\xi}\gamma^2) \right]} \left[1 - \Phi(\overline{s}(\xi;F,\Theta);\tilde{\mu}_{s|\xi},\tilde{\sigma}_{s|\xi}^2) \right] \ g(\xi)d\xi
$$

10. The middle part of the double integral tells us what the unconditional expected participation rate of the agents is.

$$
\begin{aligned}
\int \int_{\underline{s}(\xi;F,\Theta)}^{\overline{s}(\xi;F,\Theta)} f(s,\xi)dsd\xi &= \int g(\xi) \int_{\underline{s}(\xi;F,\Theta)}^{\overline{s}(\xi;F,\Theta)} h(s|\xi) \ ds \ d\xi \\
&= \int 1 - [\Phi(\overline{s};\mu_{s|\xi},\sigma_{s|\xi}^2) - \Phi(\underline{s};\mu_{s|\xi},\sigma_{s|\xi}^2)] \ g(\xi) \ d\xi \\
&= \int q(\xi;F,\Theta) \ g(\xi) \ d\xi = Q(x;F,\Theta)
\end{aligned}
$$

For the first equality, we write the joint as the product between the conditional and the marginal. For the second one, we use the previous result of the conditional participation rate. Then, we integrate over the public signal ξ.

For positive x, the expected value of the indirect utility is given by:[25]

$$z(x; F, \Theta) = E[v(s, p; F, \Theta)] = -e^{-[W_1 - C(x,c)]}(f_1 + f_2 + f_3)$$

where:

$$f_1 = e^F \sqrt{\tau_{s|\xi}\tilde{\sigma}_{s|\xi}^2} \int e^{\frac{1}{2}(\xi - \mu_\pi)^2 \left[\frac{(\beta_2 + \tau_{s|\xi}\gamma)^2}{(\beta_1 + \tau_{s|\xi})} - (\beta_3 + \tau_{s|\xi}\gamma^2)\right]} \Phi(\underline{s}(\xi; F, \Theta); \tilde{\mu}_{s|\xi}, \tilde{\sigma}_{s|\xi}^2) g(\xi) \, d\xi$$

$$f_2 = \int 1 - [\Phi(\overline{s}; \mu_{s|\xi}, \sigma_{s|\xi}^2) - \Phi(\underline{s}; \mu_{s|\xi}, \sigma_{s|\xi}^2)] g(\xi) \, d\xi \qquad (2.15)$$

$$f_3 = e^F \sqrt{\tau_{s|\xi}\tilde{\sigma}_{s|\xi}^2} \int e^{\frac{1}{2}(\xi - \mu_\pi)^2 \left[\frac{(\beta_2 + \tau_{s|\xi}\gamma)^2}{(\beta_1 + \tau_{s|\xi})} - (\beta_3 + \tau_{s|\xi}\gamma^2)\right]} \left[1 - \Phi(\overline{s}(\xi; F, \Theta); \tilde{\mu}_{s|\xi}, \tilde{\sigma}_{s|\xi}^2)\right] g(\xi) \, d\xi$$

Similarly, for $x = 0$, we integrate (2.13) over the domain of public signal $\xi \sim N(\mu_\pi, \tau_\xi)$. We tilt the public signal distribution and we get a new measure that takes into account the market risk, $\xi \sim N(\mu_\pi, \tilde{\tau}_\xi^{-1})$ where: $\tilde{\tau}_\xi = \left(\frac{I\tau_\pi}{kK}\right)^2 + \frac{\tau_\pi I^2 \tau_\theta}{k}$.

Therefore, the expected value of the indirect utility is:

$$z(0; F, \Theta) = E[v(p; F, \Theta)] = -e^{-W_1}(g_1 + g_2 + g_3)$$

where

$$g_1 = e^F \sqrt{\frac{\tau_\xi}{\tilde{\tau}_\xi}} \Phi(\underline{\xi}; \mu_\pi, \tilde{\tau}_\xi^{-1})$$

$$g_2 = 1 - [\Phi(\overline{\xi}; \mu_\pi, \tilde{\tau}_\xi^{-1}) - \Phi(\underline{\xi}; \mu_\pi, \tilde{\tau}_\xi^{-1}] \qquad (2.16)$$

$$g_3 = e^F \sqrt{\frac{\tau_\xi}{\tilde{\tau}_\xi}}[1 - \Phi(\overline{\xi}; \mu_\pi, \tilde{\tau}_\xi^{-1})]$$

$\square$

---

[25]For the results reported in tables and graphs, we compute the expected indirect utility function through MonteCarlo numerical integration methods.

Lemma 9. *(Existence)*

Given continuity of the objective function and compactness of the domain, there exist an optimal solution for the problem (2.4).

Proof. Recall that the expected indirect utility is:

$$z(x; c, F, \Theta) = \begin{cases} -e^{-[W_1 - C(x,c)]}(f_1 + f_2 + f_3) & \text{if} \quad x > 0 \\ -e^{-W_1}(g_1 + g_2 + g_3) & \text{if} \quad x = 0 \end{cases}$$

We need continuity of the objective function in order to apply Weierstrass theorem to prove the existence of a solution. It can be seen that, for $x > 0$, the first term is continuous given continuity of cost function and of the term in bracket. The latter is the sum of three continuous functions: the integral operator preserves continuity of the integrand.

We need to check continuity in $x = 0$. We can show that:

$$\lim_{x \to 0^+} z(x; F, \Theta) \to z(0; F, \Theta)$$

given that:

$$\left[\frac{(\beta_2 + \tau_{s|\xi}\gamma)^2}{(\beta_1 + \tau_{s|\xi})} - (\beta_3 + \tau_{s|\xi}\gamma^2)\right] \longrightarrow_{x \to 0^+} \left(\frac{I\tau_\pi}{kK}\right)^2$$

and

$$\sqrt{\tau_{s|\xi}\tilde{\sigma}^2_{s|\xi}} \longrightarrow_{x \to 0^+} 1$$

Moreover, f_1 goes to g_3 and f_3 goes to g_1.

$\square$

Lemma 10. *(Uniqueness)*

Given strictly concavity of the objective function over the compact set $X(c)$, there exists a unique optimal solution for the problem (2.4). *It is an interior solution for all $c \in (\underline{c}, \overline{c})$.*

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Proof. To prove it, we need to show that the objective function (2.14) is strictly concave in $x \in X$. Then we apply the maximum theorem and we characterize the optimal solution with respect to the financial literacy cost c.

We know that the indirect utility function is given by the pointwise maximum of two functions:

$$v(s, p; F, \Theta) = \max\{v_{in}, v_{out}\}$$

where

$$v_{in} = -exp\left\{-[W_1 - C(x, c)] - \tfrac{1}{2}k(E[\pi|s, \xi] - p)^2 + F\right\}$$

and

$$v_{out} = -exp\left\{-[W_1 - C(x, c)]\right\}$$

The functions v_{in} and v_{out} are both strictly concave in x, $\forall\{s, \xi, F\} \in \mathbb{R}^2 \times \mathbb{R}^+$, given the assumptions of the cost function and the properties of the exponential function: the argument is always negative.[26]

The max of two strictly concave functions is strictly concave. Thus, $v(s, p; F, \Theta)$ is strictly concave with respect to x, $\forall\{s, \xi, F\} \in \mathbb{R}^2 \times \mathbb{R}_+$.

We compute the expected value of the indirect utility function and we use the property that the integral operator preserves convexity. Given that $v(s, p; F, \Theta)$ is strictly concave in x for each $\{s, \xi\} \in \mathbb{R}^2$ and the joint density function $f(s, \xi)$ is greater than or equal to zero for each $\{s, \xi\} \in \mathbb{R}^2$, we can say that the function $z(x; c, F, \Theta)$, defined as:

$$z(x; c, F, \Theta) = \int\int v(s, \xi; F, \Theta)f(s, \xi)dsd\xi$$

is strictly concave in x for any given $F \in \mathbb{R}_+$ (provided the integral exists).

---

[26]The problem is subject to the budget constraint $W_1 \geq C(x, c)$ and the incentive compatibility constraint:
$$\tfrac{1}{2}k(E[\pi|s, \xi] - p)^2 \geq F$$

.

Applying maximum theorem to a strictly concave objective function and recalling that $X(c) = \{x | x \in [0, \overline{x}(c)]\}$ has a convex graph, we have the following results: the solution for the problem (2.4) is unique and $x^*(c, F, \Theta) = \arg\max\{z(x; c, F, \Theta) | x \in X(c)\}$ is continuous in c.[27]

Furthermore, given that is $X(c)$ monotonically enlarging in c, $x^*(c, F, \Theta)$ is non increasing in c: an agent with high financial literacy cost optimally purchases less information than a more financially literate agent. We can see that, when c increases and tends to infinity, $x^*(c, F, \Theta)$ is zero, the lower bound of the set $X(c), \forall c$. This is due to the fact that financial literacy cost c affects only the marginal cost and does not the marginal benefit. Therefore, when the marginal cost increases, the optimal action of the agents is to decrease the information to purchase, up to zero. When the financial literacy cost c vanishes, the marginal cost for the information is null and the agent would acquire it as much as she can, given the positive marginal benefit. In this case, the optimal information choice would be $x^* = \overline{x}(0) \to \infty$. We set a lower bound over the domain of the financial literacy cost, $\underline{c}(F, \Theta) | x^*(\underline{c}, F, \Theta) < \overline{x}(\underline{c}) < \infty$, such that none of the agents in the market has the incentive to spend the whole initial wealth in information.

Given the described behaviour of $x^*(c, F, \Theta)$, there exists at least one c, that we denote $\overline{c}(F, \Theta)$ such that $x^*(\overline{c}, F, \Theta) = 0$ and $x^*(\overline{c}^-, F, \Theta) > 0$. By monotonicity, $x^*(c, F, \Theta) = 0, \forall c > \overline{c}$. $\qquad\qquad \square$

---

[27]$\overline{x}(c)$ is the maximum precision that an agent can purchase, given her budget constraint. It is implicitly given by $W_1 = C(\overline{x}, c)$.

Proof of Theorem 5

To prove the existence of a rational expectation equilibrium, we need to check if the there exists a distribution of beliefs according to which all the agents optimally behave, the stock market clears and it is consistent with the behaviour of the agents. The agents conjecture about the price informativeness. If it is equal to the aggregate amount of their private information passed to the price through individual asset demands, then the conjecture is true and the equilibrium exists.

Formally, we need to show the existence of a fixed point of the following function:

$$F(I) = \int_{j \in M} x^*(I; c_j, F, \Theta) dG(j)$$

where M is the set of the agents who trade in the market.

Recall that λ characterizes the distribution of the literate agent and that $c_L < c_H$ are the financial literacy costs of the two types of agents. Moreover, recall that $\overline{x}(c)$ is the maximum feasible amount of the information that an agent with financial literacy cost c can acquire.

Let the compact set $\mathcal{I} = [0, \overline{x}(c_L)] \subset \mathbb{R}^+$ be the domain of the following mapping operator $F : \mathcal{I} \to \mathbb{R}^+$:

$$F(I) = \lambda \, Q(I; c_L, F, \Theta) \, x^*(I; c_L, F, \Theta) + (1 - \lambda) \, Q(I; c_H, F, \Theta) \, x^*(I; c_H, F, \Theta)$$

where $I \in \mathcal{I}$, $x^*(I; c, F, \Theta)$ is given by equation (2.10) and $Q(I; c, F, \Theta)$ is given by equation (2.11). We can note that either Q and x^* are continuous function with respect to I. Therefore, $F(I)$ is continuous in I. If F maps into itself, then, by Brouwer's theorem, a fixed-point, such that $F(I) = I$, exists.

Note that $0 \le Q \le 1$ and $0 \le \lambda \le 1$, by construction. We know that for any $c \in \{c_L, c_H\}$ and for all $I \in \mathcal{I}$, $x^*(I; c, F, \Theta) \ge 0$. Thus, $F(I) \ge 0$, given that it is a

convex combination of two non-negative terms.

Moreover, $\overline{x}(c_L) \geq x^*(I; c_L, F, \Theta) \geq x^*(I; c_H, F, \Theta)$, for any $c_L < c_H$, where the first inequality holds because $x^* \in X(c)$ for all $I \in \mathcal{I}$ and the second one by monotonicity of x^* with respect to c for all $I \in \mathcal{I}$. Thus,

$$
\begin{aligned}
F(I) \ &= \lambda\, Q_L\, x_L^* + (1 - \lambda)\, Q_H\, x_H^* \\
&\leq \lambda\, x_L^* + (1 - \lambda)\, x_H^* \\
&\leq \lambda\, \overline{x}(c_L) + (1 - \lambda)\, \overline{x}(c_L) \\
&\leq \overline{x}(c_L)
\end{aligned}
$$

where $\overline{x}(c_L)$ is bounded for all $c_L > \overline{c}$, as we describe in the proof of the lemma 10. Thus, $F(I)$ maps into itself and we prove the existence of an equilibrium.

Appendix B

Figures and Tables

Table 2.1: Limits of the functions $\underline{s}$ (2.7) and $\overline{s}$ (2.8).

	ξ		
$x \to 0^+$	$\underline{\xi}$		$\overline{\xi}$
$\overline{s}$	$-\infty$	$+\infty$	$+\infty$
$\underline{s}$	$-\infty$	$-\infty$	$+\infty$

Table 2.2: Optimal information choice: $W_1 = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$, $\lambda = 0.25$, $c_L = c_H = 0.03$.

(a) $\tau_\theta = 1$

τ_π	$F = 0$ x	$F = 0.1$ x	$F = 0.5$ x
0.25	1.5274	1.8082	2.0307
0.50	1.4863	1.7442	1.9468
0.75	1.4452	1.6950	1.8191
1	1.4043	1.6389	1.7148
2	1.2419	1.4166	1.3165
5	0.7961	0.7455	0

(b) $\tau_\pi = 1$

τ_θ	$F = 0$ x	$F = 0.1$ x	$F = 0.5$ x
0.25	1.8038	1.8913	1.9099
0.50	1.6197	1.7780	1.8258
0.75	1.4961	1.7017	1.7785
1	1.4043	1.6487	1.7385
2	1.1795	1.4995	1.5706
5	0.9002	1.2665	1.3836

Table 2.3: Price informativeness: $W_1 = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$, $\lambda = 0.25$, $c_L = c_H = 0.03$.

(a) $\tau_\theta = 1$

τ_π	$F = 0$ I	$F = 0.1$ I	$F = 0.5$ I
0.25	1.5274	1.2199	0.9074
0.50	1.4863	1.1735	0.8388
0.75	1.4452	1.1195	0.7780
1	1.4043	1.0699	0.7119
2	1.2419	0.8631	0.4337
5	0.7961	0.3347	0

(b) $\tau_\pi = 1$

τ_θ	$F = 0$ I	$F = 0.1$ I	$F = 0.5$ I
0.25	1.8038	1.4519	1.1004
0.50	1.6197	1.2676	0.9145
0.75	1.4961	1.1474	0.7780
1	1.4043	1.0574	0.6989
2	1.1795	0.8535	0.5547
5	0.9002	0.6262	0.3810

Table 2.4: Market price variance: $W_1 = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$, $\lambda = 0.25$, $c_L = c_H = 0.03$.

(a) $\tau_\theta = 1$

τ_π	$F = 0$ σ^2_{pR}	$F = 0.1$ σ^2_{pR}	$F = 0.5$ σ^2_{pR}
0.25	3.9063	3.9156	3.9813
0.50	1.9029	1.9058	1.9512
0.75	1.2331	1.2305	1.2553
1	0.8970	0.8892	0.8970
2	0.3889	0.3660	0.3272
5	0.0879	0.0614	0.0401

(b) $\tau_\pi = 1$

τ_θ	$F = 0$ σ^2_{pR}	$F = 0.1$ σ^2_{pR}	$F = 0.5$ σ^2_{pR}
0.25	1.1671	1.2787	1.4671
0.50	0.9798	1.0208	1.1069
0.75	0.9228	0.9334	0.9758
1	0.8970	0.8891	0.8979
2	0.8667	0.8214	0.7632
5	0.8630	0.7854	0.6563

Figure 2.7: Market participation rate for the literate and illiterate agents. We distinguish between two scenarios: high uncertainty with $(\tau_\pi, \tau_\theta) = (.5, .5)$ and low uncertainty with $(\tau_\pi, \tau_\theta) = (5, 5)$. $W_1 = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$, $\lambda = 0.25$, $F = 0.1$, $c_L = 0.01$, $c_H = 0.03$.
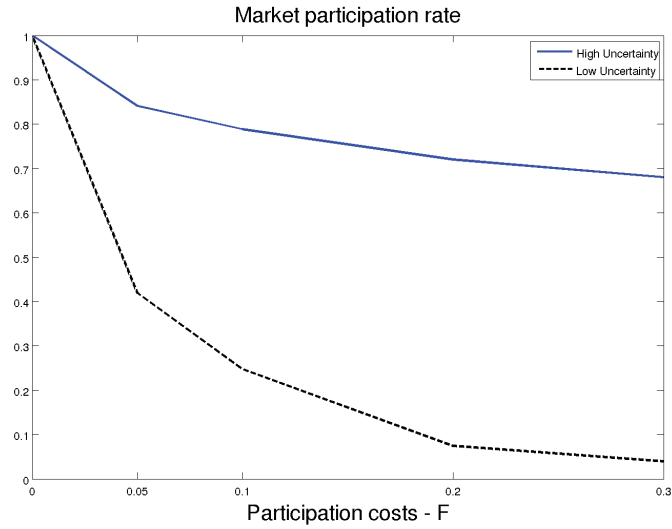
Figure 2.8: Size of the risky asset market. We distinguish between two scenarios: high uncertainty with $(\tau_\pi, \tau_\theta) = (.5, .5)$ and low uncertainty with $(\tau_\pi, \tau_\theta) = (5, 5)$. $W_1 = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$, $\lambda = 0.25$, $F = 0.1$, $c_L = 0.01$, $c_H = 0.03$.
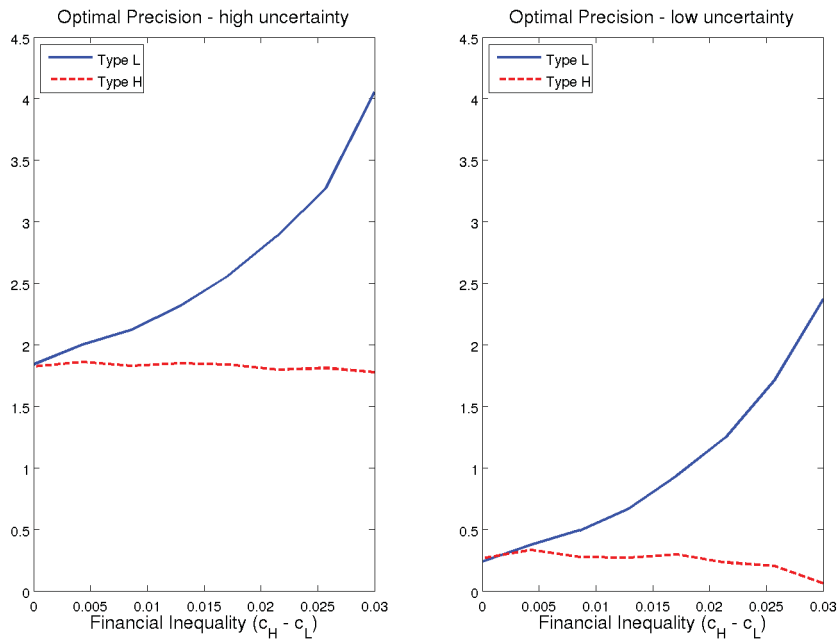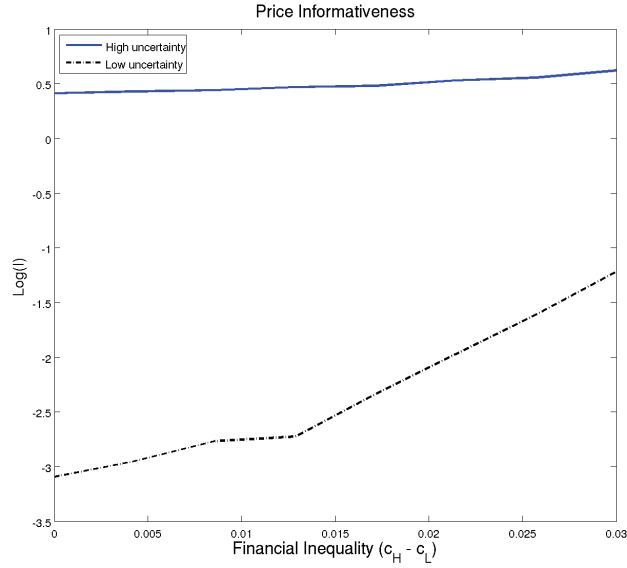

Figure 2.9: Optimal information choice for both types. We make c_L to decrease, starting from $c_L = 0.045$. We distinguish between two scenarios: high uncertainty with $(\tau_\pi, \tau_\theta) = (.5, .5)$ and low uncertainty with $(\tau_\pi, \tau_\theta) = (5, 5)$. $W_1 = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$, $\lambda = 0.25$, $F = 0.1$, $c_H = 0.045$.
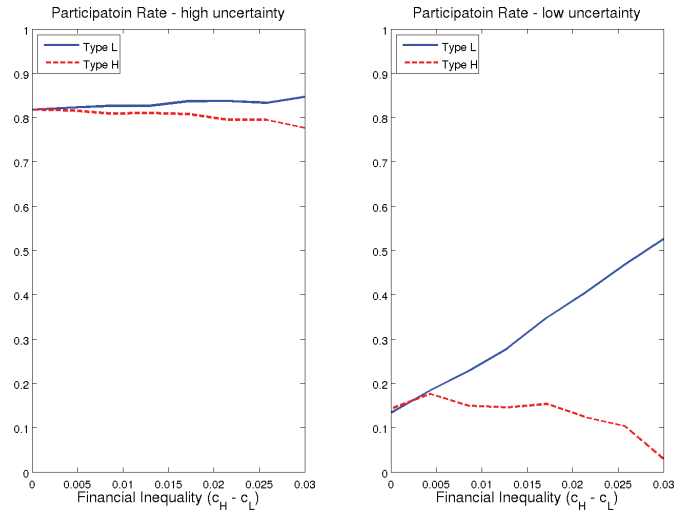
Figure 2.10: Logarithm of the price informativeness. We make c_L to decrease, starting from $c_L = 0.045$. We distinguish between two scenarios: high uncertainty with $(\tau_\pi, \tau_\theta) = (.5, .5)$ and low uncertainty with $(\tau_\pi, \tau_\theta) = (5, 5)$. $W_1 = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$, $\lambda = 0.25$, $F = 0.1$, $c_H = 0.045$.

Figure 2.11: Market participation rate for both types. We make c_L to decrease, starting from $c_L = 0.045$. We distinguish between two scenarios: high uncertainty with $(\tau_\pi, \tau_\theta) = (.5, .5)$ and low uncertainty with $(\tau_\pi, \tau_\theta) = (5, 5)$. $W_1 = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$, $\lambda = 0.25$, $F = 0.1$, $c_H = 0.045$.
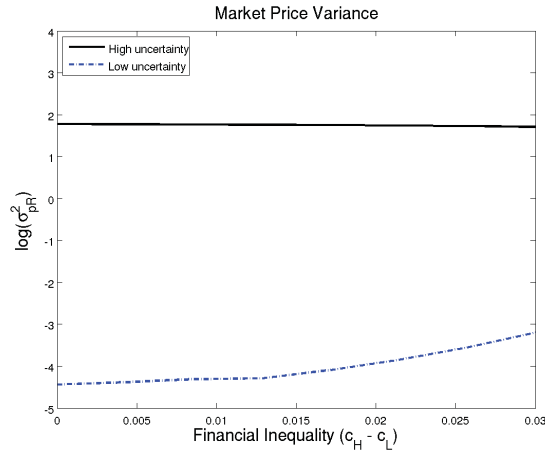
Figure 2.12: Market price variance, computed in equilibrium. We make c_L to decrease, starting from $c_L = 0.045$. We distinguish between two scenarios: high uncertainty with $(\tau_\pi, \tau_\theta) = (.5, .5)$ and low uncertainty with $(\tau_\pi, \tau_\theta) = (5, 5)$. $W_1 = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$, $\lambda = 0.25$, $F = 0.1$, $c_H = 0.045$.
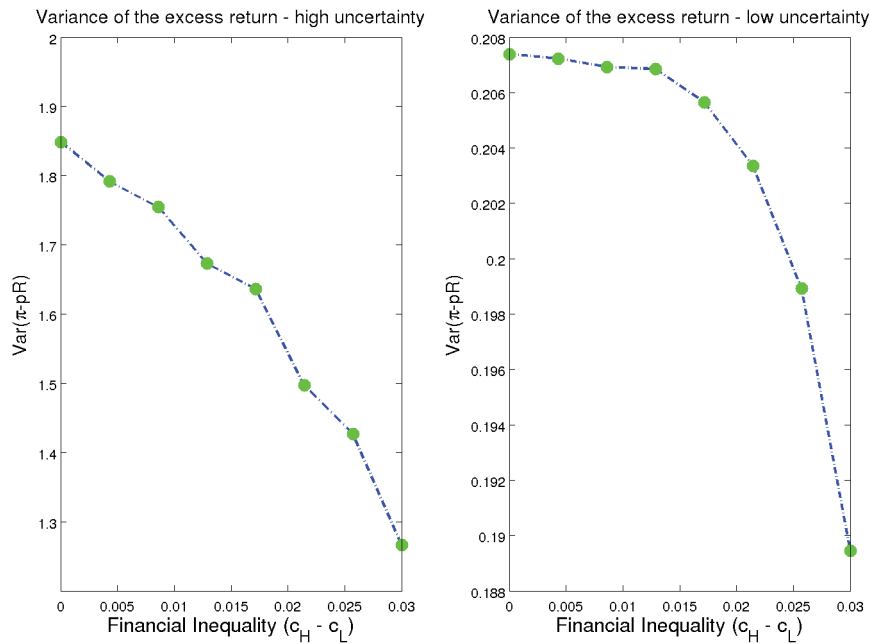

Figure 2.13: Variance of the market excess return. We make c_L to decrease, starting from $c_L = 0.045$. We distinguish between two scenarios: high uncertainty with $(\tau_\pi, \tau_\theta) = (.5, .5)$ and low uncertainty with $(\tau_\pi, \tau_\theta) = (5, 5)$. $W_1 = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$, $\lambda = 0.25$, $F = 0.1$, $c_H = 0.045$.
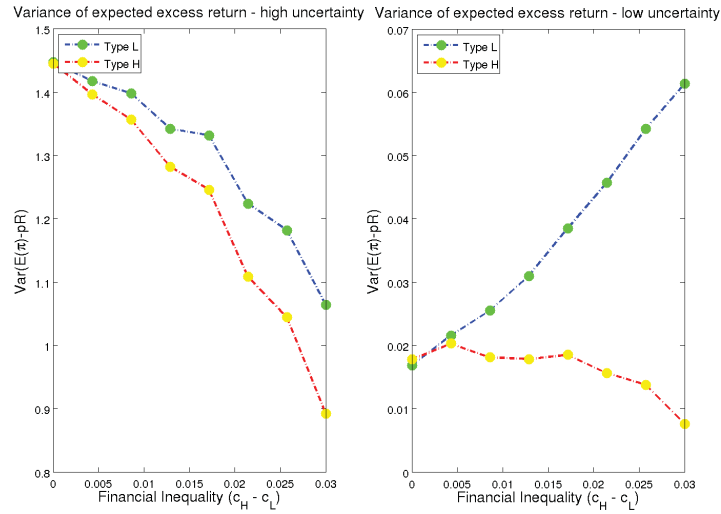
Figure 2.14: Variance of the expected excess return for both types. We make c_L to decrease, starting from $c_L = 0.045$. We distinguish between two scenarios: high uncertainty with $(\tau_\pi, \tau_\theta) = (.5, .5)$ and low uncertainty with $(\tau_\pi, \tau_\theta) = (5, 5)$. $W_1 = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$, $\lambda = 0.25$, $F = 0.1$, $c_H = 0.045$.
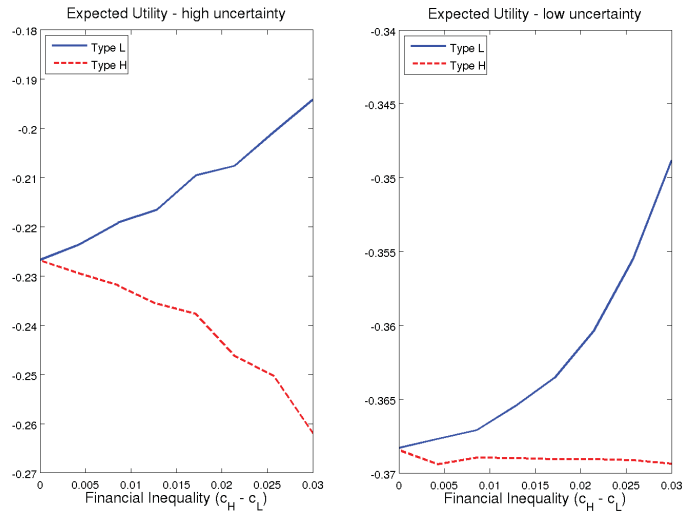

Figure 2.15: Expected Utility of both types. We make c_L to decrease, starting from $c_L = 0.045$. We distinguish between two scenarios: high uncertainty with $(\tau_\pi, \tau_\theta) = (.5, .5)$ and low uncertainty with $(\tau_\pi, \tau_\theta) = (5, 5)$. $W_1 = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$, $\lambda = 0.25$, $F = 0.1$, $c_H = 0.045$.

Chapter 3

Financial literacy, Cognitive abilities, and Asset pricing implications

3.1 Introduction

Standard asset pricing models assume rational investors who trade to maximize the utility of their wealth. Models differ by the assumptions on preferences, beliefs and the structure of market, and on how the information is shared between investors. Differences in assumptions deliver a wealth of asset pricing implications, which constitute the basis for the empirical validation of such models. An important aspect of the models is how informed the investors are about the assets return distribution.

An extensive literature documents that a large fraction of the investors population lacks of crucial information to invest in financial markets and is therefore financially illiterate. Lusardi and Mitchell (2007) show the positive association of financial literacy with planning for retirement, McArdle *et al.* (2009) with wealth, van Rooij *et al.* (2011) with stock market participation, Guiso and Jappelli (2009) with portfolio diversification. A related work by Christelis *et al.* (2010) investigates the role of cognitive ability in shaping the stock market participation decision and shows that

other things equal, including age and education, more cognitive able individuals are more likely to participate to financial market. This issue suggests a channel through which cognitive abilities can affect the households behaviour on financial market, as it is modelled by Jappelli and Padula (2011).

This chapter investigates the role of information in the formation of market prices and recognizes that investors decide how much financial literacy to acquire, trading off the benefit of financial literacy with its cost. In our model with noisy traders, a risky and a riskless asset, financial literacy lowers the cost of buying a more precise signal on the risky asset payoff, but entails a disutility cost, which varies between individuals depending on their cognitive abilities. Therefore, while the benefit is the same for all investors who acquire the same signal, the heterogeneity of cost delivers interesting general equilibrium implications that provide the basis against which to evaluate policies aimed at increasing the productivity of financial education.

The chapter emphasizes that investors decide whether to improve upon a baseline level of financial literacy by attending a financial education program. In taking this decision, investors take into account the feed-back effects on the informativeness of market prices. The incentive to acquire financial literacy increases with the productivity of financial literacy up to the point where the illiterate investors prefer no to buy the signal and consequently trade more conservatively. From the policy perspective this means that increasing the productivity of financial education programs can even decrease the share of literate investors. Related, the market price variance, which we take as a proxy of market price stability, varies in a non-monotone way with the productivity of the financial education programs, for high level of uncertainty of payoffs and noisy supply. This warns a word of caveat on policy aimed at increasing the productivity of financial education programs, which, by increasing the heterogeneity between investors, can contribute to foster the instability of financial markets.

The rest of the chapter is organized as follow. Section 3.2 sets the model up, section 3.3 characterizes the equilibrium, while in section 3.4 we discuss the main policy implications. Section 3.5 concludes and points out directions for further research. All the proofs are collected in the Appendix.

3.2 Model

Individuals are born with one unit of wealth and operate in two markets: the assets and the information markets. On the assets market, individuals trade a riskless and a risky security. On the information market, individual can acquire the precision of a private signal on the risky asset payoff at a cost that depends on financial literacy. Higher financial literacy grants a lower cost to acquire financial information, i.e. the precision of the private signal.

To increase their financial literacy, individuals can attend a financial education training. However, attending the training entails a dis-utility cost, which depends on the individuals' cognitive abilities. Better able individuals, face a lower cost of attending the financial education training. Therefore, the choice to attend the training is based on the expected utility that individuals derive from being more financially literate, when they trade the risky asset with the less literate and the noisy traders, who trade for exogenous reasons such as liquidity needs, and the like.

Hereafter, we describe the main ingredients of the model. The model has three periods and we describe the decision problem individuals face in each period.

Assets market

Agents trade a riskless and a risky security. The riskless security is the numeraire, earns the gross return $R = (1 + r)$, and is supplied inelastically. The risky security has a risky payoff π with $\pi \sim N(\mu_\pi, \tau_\pi^{-1})$ and its price, in term of the numeraire,

is p. Short selling is allowed and the per-capita supply, provided by noisy traders, is $\theta \sim N(\mu_\theta, \tau_\theta^{-1})$. Without loss of generality, we set $\mu_\theta = 0$. We assume that π and θ are mutually independent random variables and their distribution is common knowledge.

Agents

Agents are distributed over the set $J = [0, 1]$. They all maximize the same utility function of their final wealth, but differ in their cognitive abilities. Agents are ordered on the set J by increasing level of cognitive abilities. Those who attend the financial education training incur in a dis-utility cost, which is inversely related to their cognitive abilities, meaning that better able agents face lower dis-utility costs. Their dis-utility costs are indexed by the function $\gamma(j) : J \to \mathbb{R}^+$, with $\gamma(j)' < 0$. We assume that $\gamma(\cdot)$ decreases as the cognitive abilities increase and the cumulative distribution function $G(\gamma)$ represents the distribution of the agents on the set J. The agent's benefit of attending the training is a lower cost of acquiring financial information.

We define literate agents (L) those who attend the training, and illiterate (H) the others, who can acquire information at a higher cost. In this setting, we will provide, as an equilibrium result, the fraction of the agents' population $L = \lambda J$ who attends the training.

For tractability, we assume that individuals maximize a CARA utility function, with absolute risk aversion equal to ρ, and that the dis-utility cost of attending the training program is $e^{\gamma(j)}$ with $j \in J$.

Information structure

Once π is realized but not revealed, each agent $j \in J$ can purchase financial information, that we model as an unbiased signal s_j of the risky payoff, of which the agent

can privately observe realization:

$$
s_j = \begin{cases} \pi + \sqrt{\dfrac{1}{x_j}}\epsilon & \text{if} \quad x_j > 0 \\[2mm] \emptyset & \text{if} \quad x_j = 0 \end{cases}
$$

where ϵ is a white noise, independent with respect to π, θ and across agents, and x_j is the non-negative signal's precision. The signal is informative for the agent $j \in J$ if $x_j > 0$, and is otherwise uninformative. We interpret the precision of the signal as the amount of private financial information about the risky asset payoff.

Agents can acquire private financial information incurring a monetary cost that depends on the level of financial literacy. Higher illiteracy makes the cost of acquiring information higher. Acquiring the amount x of information entails a monetary cost $C(x, c_H)$ for the illiterate, and $C(x, c_L)$ for the literate, where $C(x, c_H) > C(x, c_L)$, $c_H > c_L$ and $C(x, \cdot)$ is increasing, convex, at least twice differentiable in the first argument.[1]

The monetary cost of private financial information provides the metric to evaluate the productivity of financial education programs. Becoming literate, as the result of a financial education training, reduces the cost of acquiring the amount x of information from $C(x, c_H)$ to $C(x, c_L)$.

Public financial information is freely available. We model this feature with the common knowledge about the distribution of the risky asset payoff $\pi \sim N(\mu_\pi, \tau_\pi^{-1})$ and with the assumption that all the agents know that the other agents can purchase private financial information and some of that is revealed by the market price.

---

[1]The assumptions on the cost of acquiring private information ensure the existence of an optimal information choice.

Timing

There are three periods: the training, the planning and the trading period. In the training period, the agents decide whether to attend a financial education training to improve their financial literacy from the baseline level at a cost that is inversely related to the cognitive abilities. In the planning period, the agents purchase a private signal s and choose its precision x, which amounts to acquiring private financial information. In the trading period, the agents observe the private signal realization and the market price and trade in a competitive market, choosing optimally the portfolio share α. Once uncertainty is revealed, the agents consume the proceeds from their investments.
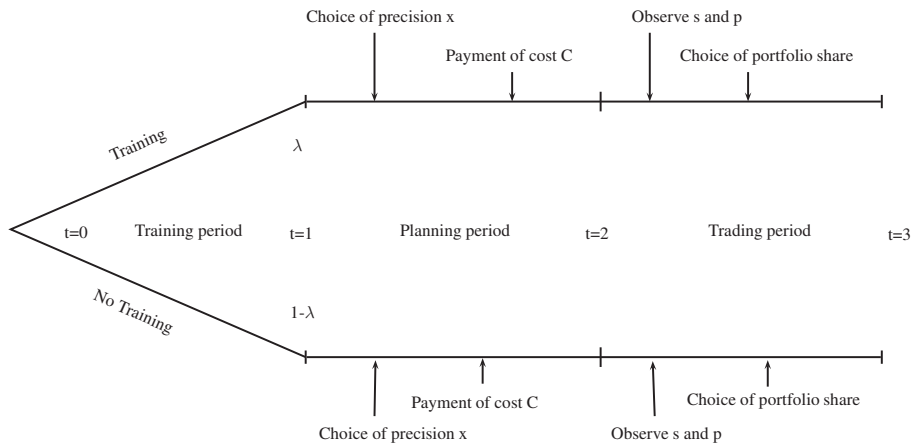
Figure 3.1 provides the timeline of the model.

Figure 3.1: Timeline of the model

To solve the model, we focus on a partially revealing noisy rational expectation equilibrium.[2] Agents conjecture a price function: $p = a + b\pi - d\theta$ where the coefficients

---

[2]All agents have rational expectations in the sense of Hellwig (1980) and use the information revealed by the price while they form their posterior beliefs. Moreover, given that we assume unbiased private signals spread among agents and this is common knowledge, agents know that the equilibrium price p contains some information about the payoff value. Therefore, they use it as an informative signal, where its precision is given by the aggregation of the private signal precisions through individual asset demands.

a, b, d are determined in the equilibrium.[3] We turn next to solve the model and describe the equilibrium.

3.3   Equilibrium

We solve the model by backward induction. At the end, each agent $j \in J$ consumes all the wealth earned by his investments. Therefore, in the trading period, the agent chooses his share of the portfolio allocated in the risky asset, α_j, in order to maximize the expected utility of his final wealth, given the precision purchased and the information cost paid. Each agent observes a private and a public signal (the equilibrium price) and computes the posterior beliefs about the final payoff value: $E[\pi|\mathcal{F}_j]$ and $Var[\pi|\mathcal{F}_j]$, where $\mathcal{F}_j = \{s_j, p\}$ is the agent $j \in J$ information set, made of the private, s_j and the public, p, signals. Recall that the private signal is informative only if agent j acquires some information precision. In the planning period, the agent chooses how much precision to acquire, x_j, and pays the monetary cost, which depends on whether he attends the financial training program in the training period. Since all agents face the same decision problem in each of the three periods, we drop the index j to ease the notation. The index will be restored to write the equilibrium conditions in the assets and in the information markets.

The trading period

In the trading period any agent maximizes:

$$\max_{\alpha} E[U(W_3)|\mathcal{F}] \tag{3.1}$$

_____

[3]The assumption of linearity is standard and is made for tractability to ensure a closed form solution for the price function.

subject to:

$$W_3 = W_2 R^p \tag{3.2}$$

$$W_2 = W_1 - C \tag{3.3}$$

$$R^p = \alpha(\frac{\pi - pR}{p}) + R \tag{3.4}$$

where the budget constraint (3.2) - (3.4) implies that the final wealth W_3 is the investment return of the agent's portfolio choice, α. The return of the portfolio R^p depends on the returns of the two assets and on the market price. The amount of wealth invested, W_2, depends on the homogeneous initial wealth W_1 and on the cost of acquiring information, C, which is sunk in the trading period and depends on the choices made in the training and in the planning period.

The optimal share of wealth invested in risky assets varies between agents, depending on the signal observed and on the precision purchased. In the trading period, all the private information choices are already done and each trading agent transfers some of his purchased information to the market price through his assets demand. Therefore, private information is partially revealed by the market price. While they are forming their posterior beliefs and formulating their assets demand, agents take into account the informativeness of p, which is transformed into an unbiased public signal, ξ.

The indirect utility of solving the trading problem is $E[U(W_3^*)|\mathcal{F}]$, which we note as $v(s, p; \Theta)$ where Θ is $\{R, W_1, \rho, \mu_\pi, \tau_\pi, \tau_\theta\}$, to emphasize its dependence on the private and public signal.

The planning period

In the planning period, each agents maximizes the indirect utility for the portfolio allocation problem with respect to the information choice, i.e.

$$\max_{x \geq 0} E[v(s, p; \Theta)]$$

subject to:

$$W_1 \geq C(x, c)$$

where the expected utility is computed over the joint distribution of s and p and $c = \{c_L, c_H\}$. Recall that the signal precision x affects, by assumption, only the distribution of the private signal s. In deciding how much precision to acquire, agents compare the costs and the benefit of private information. The former depends on financial literacy, in that higher level of financial literacy allows purchasing information at a lower cost. The latter depends on how much information is present already in the market, which in turn depends on the amount of information purchased by all agents. To the extent that p reveals a lot of information, the benefits of private information are limited.

The indirect utility of solving the planning problem is $E[v(x^*)]$, which we note as $z(c; \Theta)$.

The training period

In the training period, the agent maximizes his utility with respect to the choice to attend the financial education training. The agent problem can be summarized as:

$$\max_{D} D \, z(c_L; \Theta) e^{\gamma} + (1 - D) z(c_H; \Theta)$$

where D is binary variable which takes value 1 if the agent attends the training and 0 if he does not. The notation makes clear that the productivity of financial training has a direct effect on the decision to attend the program. Less apparently, the fraction of the agents who become literate, λ, also affects the decision to acquire information.

The equilibrium

A rational expectations equilibrium is given by the sequence of attending choices D_j, assets α_j and information x_j demands, a price function p of π and θ, and two scalar I and λ such that:

1. $D_j = D^*(\gamma(j); \Theta)$, $x_j = x^*(c_j; \Theta)$ and $\alpha_j = \alpha^*(s_j, p; \Theta)$ solve the training, the planning and the trading problems.

2. p clears the market for the risky asset:

$$\int_{j \in J} \alpha_j \frac{W_1 - C(x_j, c_j)}{p} \, dj = \theta$$

3. The informativeness of the price I, implied by aggregating individual precision choices, equals the level assumed in agents' planning maximization problem:

$$I = \int_{j \in J} x_j \, dj$$

4. The fraction of literate agents λ implied by aggregating individual training choices equals the fraction assumed in agents' training maximization problem:

$$\lambda = \int_{j \in J} D_j \, dj$$

In noisy rational expectations equilibrium models, investors make self-fulfilling conjectures about prices and the equilibrium is defined as a set of allocations such that agents maximize their utilities, their conjectures hold and markets clear.

The agents conjecture that there are λ financially literate agents in the market and they choose optimally to attend the training. Moreover, they conjecture market price informativeness I such that they choose optimally their private information. Finally, they conjecture a market price function pR such that they choose optimally their portfolio shares. The following propositions describe the equilibrium allocations.

Proposition 4 computes the price function and the optimal individual asset demand in the trading period for given price informativeness and distribution of financial literacy among the agents. Proposition 5 describes the optimal information choice, implicitly through the first order conditions. Proposition 6 computes the individuals training choice, given the cognitive abilities. Proposition 7 claims the existence of a noisy rational expectation equilibrium.

Proposition 4. *The equilibrium price is given by:*

$$pR = a + b\pi - d\theta$$

where

$$a = \frac{\mu_\pi \tau_\pi}{I + \tau_\pi + \frac{I^2}{\rho^2}\tau_\theta} \qquad b = \frac{I + \frac{I^2}{\rho^2}\tau_\theta}{I + \tau_\pi + \frac{I^2}{\rho^2}\tau_\theta} \qquad d = \frac{\rho + \frac{I}{\rho}\tau_\theta}{I + \tau_\pi + \frac{I^2}{\rho^2}\tau_\theta}$$

The optimal portfolio share for an agent $j \in J$ is given by:

$$\alpha_j^* = \frac{k_j\, p}{\rho[W_1 - C(x_j, c_j)]}(E[\pi|s_j, \xi] - pR)$$

where $c_j = \{c_L, c_H\}$

Proof. See the appendix. □

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The proof proceeds in five steps and takes λ and I as given. In the first step, we guess a price linear function and we derive the informationally equivalent public signal ξ from the price function. In the second step, we compute the mean and the variance of the posterior beliefs given the two unbiased signals, ξ and s_j. In the third step, we derive the optimal asset demand. In the fourth step, we derive market clearing conditions and, in the last step, we impose rationality and determine the coefficients of the guessed linear price function.

Here, we report the distribution of the equilibrium market price:

$$pR \sim N\left(\mu_\pi, b^2 \tfrac{1}{\tau_\pi} + d^2 \tfrac{1}{\tau_\theta}\right)$$

which variance is our proxy for the market volatility. In the next section we studied the impact of financial literacy improving policies on market stability and we focus on the changes in the market price variance.

Next, we show the existence and the uniqueness of the optimal information choice and describes an endogenous threshold that triggers the decision to acquire private information.

Proposition 5. *There exists a threshold $\overline{c}(\Theta)$ such that all agents with lower financial literacy cost purchase information. For all agents with $c < \overline{c}(\Theta)$, the optimal financial information choice x^* solves the following equation:*

$$2\rho R C'_x(x^*, c)\left(\tau_\pi + x^* + \tfrac{I^2}{\rho^2}\tau_\theta\right) = 1 \tag{3.5}$$

Proof. See the appendix. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\square$

The proof follows three steps. First we compute the indirect utility: $v(s, p; \Theta) = E[U(W_3^*)|\mathcal{F}]$ conditional to the agent's information set. This is given by:

$$v(s, p; \Theta) = -\tfrac{1}{\rho}e^{-\frac{1}{2}k(E[\pi|s,\xi]-pR)^2 - \rho R[W_1 - C(x,c)]}$$

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Then, we take the expectation with respect to the joint distribution of the private and the public signal. It depends on the first two moments of the expected excess return of purchasing a unit of risky asset. In order to simplify notation, we call f the expected excess return: $f = E[\pi|s, \xi] - pR$ where μ_f is the mean and σ_f^2 is the variance. The expected value of the indirect utility function is:

$$E[v(s, p; \Theta)] = -\tfrac{1}{\rho}\big(1 + \sigma_f^2 k\big)^{-1/2} e^{-\rho R[W_1 - C(x,c)]}$$

The last step provides conditions for the existence and the uniqueness of the optimal information choice.

The next proposition characterizes the individual optimal training choice and identifies a threshold over the cognitive abilities space according to which all the agents who are smarter attend the training.

Proposition 6. *There exists a threshold $\overline{\gamma}(c_L, c_H; \Theta)$ such that for all agents with higher cognitive ability ($\gamma < \overline{\gamma}$) it is optimal to attend the training. The agent optimally chooses to attend the training ($D = 1$) if:*

$$z(c_L; \Theta)e^\gamma > z(c_H; \Theta)$$

Proof. See the appendix. $\qquad\qquad\square$

The proof relies on the function $z(c; \Theta)$ to be non-increasing in c.

The next proposition establishes the existence of a noisy rational expectation equilibrium.

Proposition 7. *There exists a noisy rational expectation equilibrium.*

Proof. See the appendix. $\qquad\qquad\square$

To prove the existence of the equilibrium we need to check that the conjectures of the agents about the price informativeness and the distribution of financial literacy

are consistent with the aggregate amount of their optimal choices. Formally, we check that, in equilibrium, the price informativeness I solves:

$$I = \lambda \, x^*(c_L; \Theta) + (1 - \lambda) \, x^*(c_H; \Theta) \tag{3.6}$$

and the percentage of the literate agents λ is:

$$\lambda = \int_{j \in J} D^*(\gamma(j); \Theta) dG(\gamma) \tag{3.7}$$

In the appendix B, we provide a simplified example in order to understand the main intuition of the model.

In the next section, we draw the implications for policy of the above propositions. We focus on policies designed to reduce c_L vis-à-vis c_H, which amplifies the heterogeneity in financial literacy, and makes the financial training program more productive.

3.4 Policy implications

In this section we examine the effects of policies aimed at reducing c_L on the agents' choices, on price informativeness and on market stability.[4]

Policies reducing c_L but leaving c_H unchanged increase the productivity of the financial education training, but also increase the level of inequality on financial literacy. Therefore, the analysis will allow to check if increasing the productivity of

---

[4]We develop policy implications and we use some graphs to get insights. The vector of parameters Θ is set as follow: zero return for the riskless asset $R = 1$; normalized initial wealth: $W_1 = 1$; risk neutrality of the agents $\rho = 1$; the prior mean of the risky asset is equal to one: $\mu_\pi = 1$ and zero mean noisy supply $\mu_\theta = 0$. The variance of the risky asset and that of the noisy supply are our proxy to characterize two kind of scenarios, with high and low uncertainty in the fundamentals. The financial literacy cost c_H equals to 0.04, while we let c_L take values in $[0.005, 0.04]$. The cost function is linear: $C(x, c) = c \, x$. The assumed cumulative distribution function $G(\gamma)$ is a $\Gamma(1.1, 1)$.

financial education training, but also the inequality in financial literacy, fuels market instability and wealth inequalities.

Optimal information choice

We start by analyzing the effect of the training productivity on the decision to acquire private financial information. Understanding the factors leading the individual decision to acquire private information sets the stage to discuss the effect of financial education programs on the informativeness of market prices and, ultimately, on the stability of markets.

Figure 3.2 shows the effect of increasing the productivity of training on the amount of financial information acquired by those who attend and those who do not attend the training. The horizontal axis records $c_H - c_L$, the vertical one the optimal amount of financial information acquired. The Figure 3.2 focuses on two scenarios, a high ($\tau_\pi = \tau_\theta = 0.2$) and a low uncertainty ($\tau_\pi = \tau_\theta = 5$) scenario, and draws two lines, for those who attend (the continuous line) and those who do not attend (the dotted) the financial education training. We take into account the endogeneity of the attending choice and we compute, for any given level of c_L, the lowest level of cognitive abilities for which attending the training is worthwhile.[5]

If $c_H = c_L$ no one attends the training program and the two lines merge in one point. Agents still optimally acquire some financial information, which is revealed in market price. If c_L decreases, some individuals, depending on their cognitive abilities, finds it optimal to attend, others will not attend. The financial information acquired increases for the literates and decreases for the illiterates, in both the high and the low uncertainty scenario. Agents who become literate acquire more information than those who remain illiterate.

[5]We report the cognitive abilities of the marginal agent ($\overline{\gamma}$) in the third panel of the Figure 3.3.

The information is revealed in the market price and the more informative the market price, the lower the incentive to acquire private information for the illiterate. When the financial education program productivity is too high, the illiterate agent stays out of the information market and fully relies on public signal. That the illiterates do not acquire financial information makes the incentive for the literates to acquire financial information even stronger, which explains why the continuous line becomes steeper when the dotted line crosses the horizontal axis.

When the economy is in a regime of low uncertainty, we observe the same dynamics. Ceteris paribus, agents need to acquire lower amount of information in order to trade optimally. In the low uncertainty scenario, the illiterate prefer to remain uninformed for a higher value of c_L, compared to the case of high uncertainty.

Price informativeness and financial literacy distribution

Next, we turn to analyze the effect of reducing c_L on aggregate variables. The left panel of Figure 3.3 focuses on the informativeness of market prices, distinguishing between a high (continuous line) and low uncertainty (dotted line) scenario. How much informative the market price is depends on the agents' decision to acquire private information. In both scenarios, for high level of c_L, all agents decide to acquire some financial information. As c_L decreases, the productivity of the training increases and the private information acquired by the literates increases, while that of the illiterates decreases up to a point where the illiterates prefer to remain uninformed. At that point, the literates are the sole contributors to the informativeness of market prices, which keeps on increasing, since the financial information acquired by the literates explodes when the illiterates decide to remain uniformed and c_L decreases.

The effect of the training productivity on the informativeness of market prices hides the effect on the decision to attend the training program. While it is intuitive that lowering c_L cannot reduce the informativeness of market price, the effect on λ^*

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is less obvious, as shown in the central panel of Figure 3.3. The optimal fraction of literate agents is zero if there is no benefit from attending the training program, i.e. $c_H = c_L$, and is larger in the high uncertainty scenario than in the low one. As the productivity of the training increases, the fraction of agents who are attending also increases. The maximum attendance rate is given by the level of c_L such that the illiterates still acquire some information: $\overline{c}(c_L, c_H; \Theta) > c_H$. When the illiterates prefer to be uniformed, λ^* drops as the incentive of being literate in a market with uniformed and noisy traders decreases. As shown in Figure 3.4, when the productivity of the training grows too high, the illiterates have no incentives to acquire information. They realize that in the market there are well informed agents and noisy traders, aside themselves, and react by reducing their exposure to risk. Their utility decreases (the dotted line), but they limit eventual losses by trading more conservatively. This reduces the gains for the literates. Comparing the left panel of Figure 3.4 with the right panel shows that the premium of financial training (the wedge between the continuous and the dotted lines) is larger in the high that in the low uncertainty scenario. This premium is fully offset by the dis-utility cost of the marginal agent, who has the lowest amount of cognitive abilities, so that it is optimal to attend the training.

Financial market stability

We are now ready to analyze the effect of increasing the productivity of training on market stability. We consider two proxies: the market price variance, reported in Figure 3.5, and the variance of the excess return, reported in Figure 3.6, that takes into account the covariance between the payoff and the market price.

In general, market price variance increases if c_L decreases. Therefore, increasing the inequality in financial literacy fosters market instability. The literates rely more on their private information and they trade more aggressive. Market price fluctuates

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more in order to clear excess demand. Allowing for short selling exacerbates this feature.

For low level of uncertainty (dotted line in Figure 3.5(b), $\tau_\pi = \tau_\theta = 5$), the market price variance is increasing with the price informativeness and, therefore, with productivity. Instead, for high level of uncertainty (continuous line in Figure 3.5(a), $\tau_\pi = \tau_\theta = 0.2$), the market price variance is non-monotone with respect to the effects of the policy, i.e. increased productivity of the training. For low productivity, the market price variance decreases, while for high productivity increases. The turning point corresponds to the change in the information status of the illiterates. However, with high inequality, either with low and high uncertainty, price informativeness is high enough to clear the noise due to the uncertain supply. Therefore, market price variance reflects only the prior volatility of the asset payoff.

Once we focus on the variance of the excess return, we check the incentives the agents have to trade risky asset. When the productivity of the training increases and only the literates contribute to price informativeness, the variance of the excess return of the market decreases, as shown for the high (left panel) and low (right panel) uncertainty case (see Figure 3.6). This is due to the fact that more price informativeness implies higher covariance between the market price and the asset payoff.

3.5   Conclusion

The ongoing demographic transition and the recent financial crisis have put on the agenda of policy makers around the world the issue of whether individuals are equipped to face complex financial decisions. A consensus is emerging on the need of increasing the level of financial literacy in the general population, i.e. the minimal amount of knowledge on financial matters needed to take decision involving

long-term welfare consequences, such as those regarding saving for retirement or mortgaging to purchase the primary home, Lusardi (2008) for more details.[6] To foster financial literacy, many authors have pointed out the importance of financial training programs and also emphasized the need of starting financial education early on in the school.[7] Understanding the effect of financial education on behaviour is hard, but is of paramount importance for those who advocate the need of financial education. Savy financial behaviours are associated in the data with high levels of financial literacy, which raises the issue of the endogeneity of financial literacy. Beyond the data complications, to understand the effect of financial education on individuals' welfare, one should take into account the possible general equilibrium effects of financial training programs, an exercise that is lacking in the literature.

Our work fills this gap by focussing on a particular aspect of financial training, i.e. its productivity, and endogeneizing the individuals' decision of whether or not to take financial training, as in Jappelli and Padula (2011). We assume that financial training increases financial literacy over and above some innate level at a rate that is the same for all individuals. However, individuals differ for their cognitive abilities, which in turn affect the dis-utility of taking financial education. Better able individuals face a lower dis-utility cost and, therefore, are more willing to take financial education, other things being equal. On the information market, individual can acquire the precision of the private signal on the risky asset payoff at a cost that depends on financial literacy. Attending financial education programs grants a lower cost to acquire financial information, i.e. the precision of the private signal, providing informational advantages with respect to those agents who do not attend it. Therefore, in deciding

---

[6]A concurrent line of thought, Willis (2008), suggests instead that financial products, whether dedicated to the household assets or liabilities side, should be structured in such a way to induce the optimal behaviour on the individuals' part. While not addressing the implications of changing the financial architecture, our results help at striking the balance between the financial architecture and the financial education views by describing the general equilibrium effect of increasing the productivity of the financial training.

[7]See PISA 2012, the recent large-scale international study to assess the financial literacy of young people OECD (2012).

whether to become literate through financial education programs individuals factors the decision of the other individuals' on the informativeness of market prices.

Our main results show that policies aimed at increasing the productivity of financial education programs foster the market instability and inequality among the agents. Increasing the productivity leads to higher price informativeness, that is mainly provided by the literate agents. The share of literates increases up to the point where the illiterate agents stop to acquire private financial information. Once the productivity is too high, the illiterate agents prefer to trade conservatively, without acquire any private information. The literate's expected utility drops and only the agents with lower dis-utility costs attend the training. As a result, the share of literates decreases. Our general equilibrium model enlightens non-monotone effects of the productivity of financial education programs. Policy makers should be aware of the possible feed-back effects on market stability, once they decide to provide financial education programs. Further research should focus on welfare analysis and ways to finance the programs through the general taxation.

Appendix A - Proofs

Proof of Proposition 4

The distribution of payoff, supply and signals is:[8]

$$
\begin{pmatrix} \theta \\ \pi \\ s \\ pR \end{pmatrix} \sim N \left\{ \begin{pmatrix} 0 \\ \mu_\pi \\ \mu_\pi \\ a_0 + b\mu_\pi \end{pmatrix}, \begin{pmatrix} \frac{1}{\tau_\theta} & 0 & 0 & d\frac{1}{\tau_\theta} \\ 0 & \frac{1}{\tau_\pi} & \frac{1}{\tau_\pi} & b\frac{1}{\tau_\pi} \\ 0 & \frac{1}{\tau_\pi} & \frac{1}{\tau_\pi} + \frac{1}{x} & b\frac{1}{\tau_\pi} \\ d\frac{1}{\tau_\theta} & b\frac{1}{\tau_\pi} & b\frac{1}{\tau_\pi} & b^2\frac{1}{\tau_\pi} + d^2\frac{1}{\tau_\theta} \end{pmatrix} \right\}
$$

The proof is given in five steps. In the first step, we guess a price linear function and we derive the informationally equivalent public signal ξ from the price function. In the second step, we compute the mean and the variance of the posterior beliefs given the two unbiased signals, ξ and s_j. In the third step, we derive the optimal asset demand. In the fourth step, we derive market clearing conditions and, in the last step, we impose rationality and determine the coefficients of the guessed linear price function.

Step 1 : Agents guess a price function, linear in π (future payoff) and θ (noisy supply):

$$
pR = a_0 + b \left(\lambda \int_{j \in L} s_j dj + (1-\lambda) \int_{j \in H} s_j dj \right) - d\theta
$$

Applying the law of large number within each group of traders, we have that $\int_j \epsilon_j dj = 0$ with probability one. Therefore we can rewrite the price function as:

$$pR = a_0 + b\pi - d\theta$$

Agents use the private signals to update their prior beliefs $\pi \sim N(\mu_\pi, \tau_\pi^{-1})$. The private signal is unbiased by construction $s|\pi \sim N(\pi, x^{-1})$ and conditionally independent from prior belief μ_π, $E[(\mu_\pi - \pi)(s - \pi)] = 0$. Rational agents use the price as a public signal. It is not unbiased: $E(pR|\pi) = a_0 + b\pi$. To apply Bayesian updating, agents need to transform the price in an informationally equivalent variable ξ:

$$\xi = \frac{pR - a_0}{b} = \pi - \tfrac{d}{b}\theta$$

where

$$\xi|\pi \sim N(\pi, \tfrac{d^2}{b^2}\tfrac{1}{\tau_\theta})$$

Step 2 : Agent j observes $\mathcal{F} = \{s_j, p\} \equiv \{s_j, \xi\}$ and updates his prior beliefs with the two Gaussian signals. Using the well known formula for the multivariate normal distribution (Degroot (2004), p. 55), the posterior mean is given by:

$$
\begin{aligned}
E[\pi|s_j, \xi] &= \mu_\pi + \frac{1}{k}\left\{ x_j\left(s_j - E[s_j]\right) + \tfrac{b^2}{d^2}\tau_\theta(\xi - E[\xi]) \right\} \\
&= \tfrac{1}{k_j}\left(\tau_\pi \mu_\pi + x_j s_j + \tfrac{b^2}{d^2}\tau_\theta \xi \right)
\end{aligned}
$$

where the precision of the posterior belief k_j is given by is the sum of precisions of the prior, of the private signal and of the public signal.

$$k_j = \frac{1}{Var[\pi|s_j, \xi]} = \tau_\pi + x_j + \tfrac{b^2}{d^2}\tau_\theta$$

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Step 3 : We maximize the CARA utility function with respect to the control variable α. Each agent solves the following:[9]

$$\max_{\alpha} E\left[-\tfrac{1}{\rho}e^{-\rho W(\alpha)}|s,\xi\right] = \max_{\alpha} -\tfrac{1}{\rho}e^{-\rho\{E[W(\alpha)|s,\xi]-\frac{\rho}{2}Var[W(\alpha)|s,\xi]\}}$$

where

$$E[W(\alpha)|s,\xi] = E\left[[W_1 - C(x,c)]\alpha\tfrac{\pi-pR}{p} + R[W_1 - C(x,c)]|s,\xi\right]$$

and

$$Var[W(\alpha)|s,\xi] = \tfrac{[W_1-C(x,c)]^2\alpha^2}{p^2}Var[\pi|s,\xi]$$

Substituting and deriving FOC, we get that:

$$-\rho[W_1 - C(x,c)]\tfrac{E[\pi|s,\xi]-pR}{p} + \rho^2\alpha\tfrac{[W_1-C(x,c)]^2}{p^2}Var[\pi|s,\xi] = 0$$

Therefore, the optimal risky asset demand for agent j is:

$$\alpha_j^* = \frac{1}{\rho[W_1 - C(x_j,c_j)]}\frac{\frac{E[\pi|s_j,\xi]-pR}{p}}{\frac{Var[\pi|s_j,\xi]}{p^2}} = \frac{p}{\rho[W_1 - C(x_j,c_j)]}\frac{E[\pi|s_j,\xi]-pR}{\frac{1}{k_j}}$$

The amount of wealth in risky assets depends on the precision of the posterior, the risk aversion coefficient and the expected excess return of the risky investment.

Step 4 : The equilibrium price clears the market for the risky asset. Aggregating over all traders yields the aggregate demand:

$$\lambda \int_{j\in L} \alpha_j^* \frac{W_1 - C(x_j,c_j)}{p}dj + (1-\lambda)\int_{j\in H}\alpha_j^*\frac{W_1 - C(x_j,c_j)}{p}dj = \theta$$

---

[9]We use the log-normal distribution properties and we drop subscript j to simplify notation. We will use it again later when we aggregate individual demands.

We apply the weak law of large numbers for independent and identically distributed random variable with the same mean, such that $\int_j s_j dj = \pi$. Therefore, imposing market clearing condition holds the following equation:

$$\left[\lambda \left(\mu_\pi \tau_\pi + x_L \pi + \tfrac{b^2}{d^2} \tau_\theta \xi - pRk_L \right) + (1 - \lambda) \left(\mu_\pi \tau_\pi + x_H \pi + \tfrac{b^2}{d^2} \tau_\theta \xi - pRk_H \right) \right] = \theta$$

$$\mu_\pi \tau_\pi + \tfrac{b^2}{d^2} \tau_\theta \xi + \pi [\lambda x_L + (1 - \lambda) x_H] - pR[\lambda k_L + (1 - \lambda) k_H] = \theta$$

Using the definition of aggregate informativeness $I = \lambda x_L + (1 - \lambda) x_H$, we can rewrite the price equation as:

$$pR = (\tau_\pi + I + \tfrac{b^2}{d^2} \tau_\theta)^{-1} \left(\mu_\pi \tau_\pi + \tfrac{b^2}{d^2} \tau_\theta \xi + \pi I - \theta \right)$$

Step 5 : We impose rationality. ξ involves undetermined coefficients b, d. We substitute the expression for $\xi = \pi - \tfrac{d\theta}{b}$ and, rearranging the terms, we have:

$$pR = (\tau_\pi + I + \tfrac{b^2}{d^2} \tau_\theta)^{-1} \left(\mu_\pi \tau_\pi + \pi (I + \tfrac{b^2}{d^2} \tau_\theta) - \theta \left(\tfrac{b}{d} \tau_\theta + \rho \right) \right)$$

We derive $\tfrac{b}{d} = \tfrac{I}{\rho}$ and we substitute it back in the price function. We find out the following determined coefficients.

Coefficient of θ:

$$d = K^{-1} \left(\rho + \tfrac{I \tau_\theta}{\rho} \right)$$

Coefficient of π:

$$b = K^{-1} \left(I + \tfrac{I^2 \tau_\theta}{\rho^2} \right)$$

Constant term, a_0:

$$a_0 = K^{-1} \mu_\pi \tau_\pi$$

where $K = \tau_\pi + I + \tfrac{I^2}{\rho^2} \tau_\theta$.

Proof of Proposition 5

In order to solve for the information choice x^*, we need to compute the indirect utility: $v(s_j, p; \Theta) = E[U(W_3(\alpha_j^*))|\mathcal{F}_j]$.

Lemma 11. *For agent $j \in J$, the indirect utility function is:*

$$v(s_j, p; \Theta) = -\frac{1}{\rho} e^{-\frac{1}{2} k_j (E[\pi|s_j, \xi] - pR)^2 - \rho R[W_1 - C(x_j; c_j)]}$$

Proof. For each agent, final wealth is $W_3(\alpha^*) = [W_1 - C(x, c)][R + \alpha^*(\frac{\pi - pR}{p})]$. In order to compute the indirect utility:

$$v(s, p; \Theta) = E[-\frac{1}{\rho} e^{-\rho W_3(\alpha^*)}|\mathcal{F}] = -\frac{1}{\rho} e^{-\rho E[W_3(\alpha^*)|s, \xi] - \frac{\rho}{2} Var[W_3(\alpha^*)|s, \xi]}$$

we need to compute the conditional mean $E[W_3(\alpha^*)|s, \xi]$ and variance $Var[W_3(\alpha^*)|s, \xi]$. At the trading period, wealth W_3 is given by normal random variables π, θ, s and p and a constant term $\rho R[W_1 - C(x, c)]$. Therefore, conditional mean is:

$$
\begin{aligned}
E[W_3(\alpha^*)|s, \xi] &= E[[W_1 - C(x, c)]\alpha^* \tfrac{\pi - pR}{p} + R[W_1 - C(x, c)]|s, \xi] \\
&= [W_1 - C(x, c)]\alpha^* E[\tfrac{\pi - pR}{p}|s, \xi] + R[W_1 - C(x, c)] \\
&= [W_1 - C(x, c)]\tfrac{pk}{\rho[W_1 - C(x, c)]}(E[\pi|s, \xi] - pR)\tfrac{E[\pi|s, \xi] - pR}{p} + R[W_1 - C(x, c)] \\
&= \tfrac{k}{\rho}(E[\pi|s, \xi] - pR)^2 + R[W_1 - C(x, c)]
\end{aligned}
$$

and conditional variance is:

$$
\begin{aligned}
Var[W_3(\alpha^*)|s, \xi] &= \tfrac{[W_1 - C(x, c)]^2 (\alpha^*)^2}{p^2} Var[\pi|s, \xi] = \tfrac{[W_1 - C(x, c)]^2 p^2 k^2 (E[\pi|s, \xi] - pR)^2}{p^2 [W_1 - C(x, c)]^2 \rho^2} \tfrac{1}{k} \\
&= \tfrac{k}{\rho^2}(E[\pi|s, \xi] - pR)^2
\end{aligned}
$$

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Thus, we can rewrite the indirect utility of agent j, as:

$$v(s_j, p; \Theta) = -\frac{1}{\rho} e^{-\rho\{\frac{k_j}{\rho}(E[\pi|s_j,\xi]-pR)^2 + R[W_1 - C(x_j, c_j)] - \frac{\rho}{2}\frac{k_j}{\rho^2}(E[\pi|s_j,\xi]-pR)^2\}}$$
$$= -\frac{1}{\rho} e^{-\frac{1}{2}k_j(E[\pi|s_j,\xi]-pR)^2 - \rho R[W_1 - C(x_j; c_j)]}$$

$\square$

Once we derive the indirect utility, we need to compute the expected value. It depends on the first two moments of the expected excess return of purchasing a unit of risky asset. For simplifying notation, we call f the expected excess return: $f = E[\pi|s_j, \xi] - pR$ where μ_f is the unconditional mean and σ_f^2 is the unconditional variance.

Lemma 12. *For agent $j \in J$, the expected value of the indirect utility function is:*

$$E[v(s_j, p; \Theta)] = -\frac{1}{\rho}\left[(\frac{1}{k_j} + \sigma_f^2)k_j\right]^{-1/2} e^{-\rho R[W_1 - C(x_j; c_j)]}$$

Proof. The expected excess return $f = (E[\pi|s,\xi] - pR)$ is a linear function of two normal distribution random variables. Therefore, it is a normal random variable. In order to compute the expected value of the indirect utility function, we need to compute the mean, $\mu_f = E[E[\pi|s,\xi] - pR]$, and the variance, $\sigma_f^2 = Var[E[\pi|s,\xi] - pR]$, of the expected excess return. We start computing the expectation of the posterior belief:

$$
\begin{aligned}
E[E[\pi|s,\xi]] &= E\left[\mu_\pi + \frac{1}{k}[x\,(s - E[s]) + \frac{l^2}{\rho^2}\tau_\theta(\xi - E[\xi])]\right] \\
&= \mu_\pi
\end{aligned}
$$

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and the expectation of the market price:

$$E[pR] = E[a_0 + b\pi - d\theta]$$

$$= a_0 + b\mu_\pi = \mu_\pi$$

Therefore, the mean of the expected excess return is zero, $\mu_f = 0$. To compute the variance we need the variance of the posterior belief, the market price variance and the covariance of two terms.

$$\sigma_f^2 = Var[E[\pi|s,\xi]] + Var[pR] - 2Cov[E[\pi|s,\xi], pR]$$

where the variance of the posterior belief is:[10]

$$
\begin{aligned}
Var[E[\pi|s,\xi]] &= \frac{x^2}{k^2}(\frac{1}{\tau_\pi} + \frac{1}{x}) + \frac{I^4 \tau_\theta^2}{\rho^4 k^2}(\frac{1}{\tau_\pi} + \frac{\rho^2}{I^2 \tau_\theta}) + 2\frac{xI^2 \tau_\theta}{\rho^2 k^2}\frac{1}{\tau_\pi} \\
&= \frac{1}{\tau_\pi k^2}(x^2 + x\tau_\pi + \frac{I^4}{\rho^4}\tau_\theta^2 + \frac{I^2}{\rho^2}\tau_\theta\tau_\pi + 2x\frac{I^2}{\rho^2}\tau_\theta) \\
&= \frac{1}{\tau_\pi k^2}(x + \frac{I^2}{\rho^2}\tau_\theta)k \\
&= \frac{1}{\tau_\pi} - \frac{1}{k}
\end{aligned}
$$

Market price variance is:
$$Var[pR] = b^2\frac{1}{\tau_\pi} + d^2\frac{1}{\tau_\theta}$$

and covariance between posterior beliefs and market price is:

$$Cov[E[\pi|s,\xi], pR] = b\frac{1}{\tau_\pi}$$

[10]We use the law of total variance: $Var[Y] = E[Var[Y|X]] + Var[E[Y|X]]$

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Thus we can rewrite σ_f^2 as:

$$
\begin{aligned}
\sigma_f^2 &= \tfrac{1}{\tau_\pi} - \tfrac{1}{k} + b^2 \tfrac{1}{\tau_\pi} + d^2 \tfrac{1}{\tau_\theta} - 2b \tfrac{1}{\tau_\pi} \\
&= (1-b)^2 \tfrac{1}{\tau_\pi} + d^2 \tfrac{1}{\tau_\theta} - \tfrac{1}{k} \\
&= \tfrac{d^2}{\tau_\theta} + \tfrac{1}{K} \left(\tfrac{\tau_\pi}{K} - \tfrac{K}{k} \right)
\end{aligned}
$$

Once we have the first two moments of the excess return, we can compute the expected value of the indirect utility:

$$
\begin{aligned}
E[v(s,p;\Theta)] \quad &= E[-\tfrac{1}{\rho} e^{-\frac{1}{2}kf^2 - \rho R[W_1 - C(x,c)]}] \\
&= -\tfrac{1}{\rho} e^{-\rho R[W_1 - C(x,c)]} E[e^{-\frac{1}{2}k\sigma_f^2 \left(\frac{f}{\sigma_f}\right)^2}]
\end{aligned}
$$

We know that $f \sim N(\mu_f, \sigma_f^2)$ and $\left(\frac{f}{\sigma_f}\right)^2 \sim \chi_1^2$. Therefore, we can use the moment generating function of a χ_1^2. This is given by the following formula:

$$
M(t,h) = E(e^{tz}) = (1-2t)^{-h/2}
$$

In our case, we have:

$$
h = 1 \qquad\qquad t = -\tfrac{1}{2}k\sigma_f^2 \qquad\qquad 1 - 2t = k(\tfrac{1}{k} + \sigma_f^2)
$$

Therefore,

$$
E[e^{-\frac{1}{2}k\sigma_f^2 \left(\frac{f}{\sigma_f}\right)^2}] = \left[k(\tfrac{1}{k} + \sigma_f^2) \right]^{-1/2}
$$

Now, rearranging all the terms, the expected value of the indirect utility for agent j is:

$$
E[-\tfrac{1}{\rho} e^{-\rho[W_3(\alpha_j^*)]}] = -\tfrac{1}{\rho} \left[k_j(\tfrac{1}{k_j} + \sigma_f^2) \right]^{-1/2} e^{-\rho R[W_1 - C(x_j; c_j)]}
$$

$\square$

Lemma 13. *Given $c > \underline{c}(\Theta)$ and $I < \infty$, there exists an endogenous thresholds $\bar{c}(\Theta)$.*
The optimal information choice is:

$$
x^*(c, I, \Theta) = \begin{cases} 0 & \text{if} \quad c > \bar{c} \\ \hat{x} \mid 2\rho R C_x(\hat{x}, c)\left(\tau_\pi + \hat{x} + \frac{I^2}{\rho^2}\tau_\theta\right) = 1 & \text{if} \quad c < \bar{c} \end{cases}
$$

In order to prove the existence and the uniqueness of the solution, we apply the concave maximum theorem. We called ϕ the positive expression, independent from the control variable x:

$$
\phi = \tfrac{1}{\rho}\left(\tfrac{1}{k} + \sigma_f^2\right)^{-1/2} e^{-\rho R W_1}
$$

and we rewrite the expected value of the indirect utility function as:

$$
E[-\tfrac{1}{\rho}e^{-\rho W_3}] = -\phi k^{-1/2} e^{\rho R C(x,c)} = -\phi(\tau_\pi + x + I^2 \tfrac{\tau_\theta}{\rho^2})^{-1/2} e^{\rho R C(x,c)} \tag{3.8}
$$

The objective function (3.8) is strictly concave and defined over a compact domain $[0, \bar{x}(c)]$ where $\bar{x}(c)$ solves $W_1 = C(\bar{x}, c)$. The concave maximum theorem guarantees the existence and the uniqueness of the solution. It could be interior or a corner solution: $x^* = 0$ or $x^* = \bar{x}(c)$. We need to specify conditions over parameter space in order to characterize the solution. First, we derive FOC and we compute it at $x = 0$.

$$
\frac{\partial E[v]}{\partial x}\Big|_{x=0} = -\gamma(\tau_\pi + \tfrac{I^2}{\rho^2}\tau_\theta)^{-1/2}\left[\rho R C_x'(0, c) - \frac{1}{2(\tau_\pi + \frac{I^2}{\rho^2}\tau_\theta)}\right]
$$

When it is positive, agent has incentive to acquire information, $x^* > 0$. We call $\bar{c}(\Theta)$ the threshold over the financial literacy parameter space such that agents with this amount of financial literacy are indifferent between being informed or remain uniformed. Formally, it is implicitly given by:

$$
[2\rho R(\tau_\pi + \tfrac{I^2}{\rho^2}\tau_\theta)]^{-1} = C_x(0; \bar{c})
$$

Therefore, given strictly convexity of cost function, $\forall c < \bar{c}$, $x^*(c, I; \Theta) > 0$.

In order to get an interior solution, it is enough to show that there exists an $x \in [0, \bar{x}(c)]$ such that FOC is negative. Formally, we check when the following condition holds:

$$\frac{1}{2\rho R(\bar{x}(c) + \tau_\pi + \frac{I^2}{\rho^2}\tau_\theta)} < C_x(\bar{x}(c), c)$$

We identify a second threshold $\underline{c}(\Theta)$ that it is implicitly given by

$$\frac{1}{2\rho R(\bar{x}(\underline{c}) + \tau_\pi)} = C_x(\bar{x}(\underline{c}); \underline{c})$$

For all $\underline{c} < c < \bar{c}$, $x^*(c, I; \Theta)$ is an interior solution belonging to the set $[0, \bar{x}]$ and it is given by:

$$2\rho R C_x(x, c)\left(\tau_\pi + x + \frac{I^2}{\rho^2}\tau_\theta\right) = 1$$

We derive implicitly the amount of information $x^*(c, I; \Theta)$ that an agent optimally acquires. It depends on the exogenous financial literacy cost c and on the endogenous price informativeness I.

The agent will be indifferent between being informed or uniformed when I goes to infinity (fully revealing market price) or if own financial literacy is equal to $\bar{c}$. For any $c > \underline{c}$, the optimal information choice is:[11]

$$x^*(c, I, \Theta) = \begin{cases} 0 & \text{if } c > \bar{c} \\ \hat{x} \mid 2\rho R C_x(\hat{x}, c)\left(\tau_\pi + \hat{x} + \frac{I^2}{\rho^2}\tau_\theta\right) = 1 & \text{if } c < \bar{c} \end{cases}$$

---

[11]We want to avoid the case where agents prefer to spend their whole initial wealth in the information market and nothing in the asset market. This case is possible given the form of the CARA utility function where agents consider both the mean and the variance of the final wealth.

Proof of Proposition 6

Recall that

$$z(c,\Theta) = E[v(x^*(c,\Theta))] = -\tfrac{1}{\rho}\left(1 + \sigma_f^2 k^*\right)^{-1/2} e^{-\rho R[W_1 - C(x^*,c)]}$$

For any given λ, we can show that $z(c,\Theta)$ is negative and non increasing in c: $\forall c_L, c_H$ with $c_L < c_H$, $z(c_L,\Theta) \geq z(c_H,\Theta)$. Formally, monotonicity is satisfied when

$$\tfrac{\partial z(c,\Theta)}{\partial c} = -\phi \left[\tfrac{\partial x^*}{\partial c} \left(1 + k^*\rho R C_x\right) + k^*\rho R C_c \right] < 0$$

where $\phi = \dfrac{e^{-\rho R[W_1 - C(x^*,c)]}}{\rho\sqrt{\tfrac{1}{k^*} + \sigma_f^2}} > 0$ and $\dfrac{\partial x^*}{\partial c} = -\dfrac{k^* C_{xc} + C_x \tfrac{\partial k}{\partial I}\tfrac{\partial I}{\partial c}}{k^* C_{xx} + C_x}$. This holds when:

$$\frac{\partial x^*}{\partial c} > -\frac{k^*\rho R C_c}{1 + k^*\rho R C_x}$$

Let the function $f : \mathbb{R}^+ \to \mathbb{R}$ define as follow:

$$f(\gamma) = z(c_L,\Theta)e^\gamma - z(c_H,\Theta)$$

It is continuous and monotone ($f'_\gamma < 0$). For any given λ and for all c_L, c_H with $c_L < c_H$, we can show that the optimal attending choice is:

$$D^*(\gamma,\Theta) = \begin{cases} 1 & \text{if } f(\gamma) > 0 \\ 0 & \text{if } f(\gamma) < 0 \end{cases}$$

Moreover, we can show there exists a $\overline{\gamma}$ such that the marginal agent $j|\gamma(j) = \overline{\gamma}$ is indifferent between attending or not attending the training. For γ equals zero, $f(0) = z(c_L,\Theta) - z(c_H,\Theta) > 0$, by monotonicity of function $z(\cdot)$. For γ goes to

infinity, $\lim_{\gamma \to +\infty} f(\gamma) < 0$. Therefore, applying Bolzano's theorem, there exists a $\overline{\gamma} \in \mathbb{R}^+$ such that $f(\overline{\gamma}) = 0$, by continuity of the function $f(\cdot)$.

Therefore we can rewrite the optimal attending choice as:

$$
D^*(\gamma, \Theta) = \begin{cases} 1 & \text{if } \gamma \leq \overline{\gamma} \\ 0 & \text{if } \gamma > \overline{\gamma} \end{cases}
$$

where $\overline{\gamma} = \log z(c_H, \Theta) - \log z(c_L, \Theta)$. Moreover, $\overline{\gamma}$ is continuous with respect to λ.

Proof of Proposition 7

To prove the existence of a noisy rational expectation equilibrium, we follow two steps: in the first one, for given distribution of financial literacy, λ, we check the equilibrium condition in the information market, i.e. the price informativeness that is consistent with the aggregate amount of the optimal individual information choices. In the second step, we check the equilibrium financial literacy distribution that it is consistent with the optimal attending choice of the agents.

For any $\lambda \in [0,1]$ and for any $c_H > c_L > \underline{c}$, let the compact set $\mathcal{I} = [0, \frac{1}{2\rho R C'_x(0,\underline{c})}] \subset \mathbb{R}^+$ the domain of the following mapping operator $F : \mathcal{I} \to \mathbb{R}^+$:

$$F(I) = \lambda x^*(c_L, I; \Theta) + (1-\lambda) x^*(c_H, I; \Theta)$$

where I is an element of $\mathcal{I}$ and $x^*(c, I; \Theta)$ is implicitly given by:

$$2\rho C'_x(x^*, c) \left(x^* + \tau_\pi + \frac{I^2}{\rho^2} \tau_\theta \right) = 1$$

Continuity of $F(I)$ is guaranteed by the assumption that C'_x is continuous. We need to prove that F maps into itself to apply fixed-point Brouwer's theorem ($F(I) = I$).

For all $c > \underline{c}$ and $I \in \mathcal{I}$, $x^*(c, I; \Theta) \geq 0$. Moreover, given strictly convexity of cost function, $C'_x(0, \underline{c}) \leq C'_x(x^*, c)$. This implies that:

$$x^* = \frac{1}{2\rho C'_x(x^*,c)} - \left(\tau_\pi + \frac{I^2}{\rho^2} \tau_\theta \right) \leq \frac{1}{2\rho C'_x(x^*,c)} \leq \frac{1}{2\rho C'_x(0,\underline{c})}$$

For all $c > \underline{c}$ and $I \in \mathcal{I}$,

$$0 \leq x^*(c, I; \Theta) \leq \frac{1}{2\rho C'_x(0,\underline{c})}$$

The linear combination of two terms belonging to $\mathcal{I}$, still belongs to $\mathcal{I}$:

$$0 \leq F(I) = \lambda x^*(c_L, I; \Theta) + (1 - \lambda)x^*(c_H, I; \Theta) \leq \frac{1}{2\rho C'_x(0, \underline{c})}$$

Thus, $F(I)$ maps into itself and we prove the existence of the equilibrium condition in the information market for any given λ.

The equilibrium condition for the financial literacy distribution implies that the amount of agents who optimally choose to attend the training is consistent with the financial literacy distribution conjectured. Let the compact set $\Lambda = [0, 1]$ be the domain of the following mapping operator $S : \Lambda \to \mathbb{R}^+$:

$$S(\lambda) = \int_j D^*(\gamma(j), \Theta)dG(\gamma)$$

Recalling the results of the proposition 6, according to which $D^*(\gamma(j), \Theta)$ takes value one if $\gamma(j) < \overline{\gamma}$ and recalling that $\gamma(\cdot)$ is a continuous and monotone function, we can show that $\int_j D^*(\gamma(j), \Theta)dG(\gamma) = G(\overline{\gamma})$ where $G(\cdot)$ is the cumulative distribution function, derived by the distribution of the agents over the cognitive abilities space. Given that $G(\gamma) \in [0, 1]$, by the properties of the cumulative distribution function, and $\overline{\gamma}(\lambda)$ is continuous, by proof of proposition 6, the function $S(\lambda)$ is continuous and maps into itself. Thus, the Brouwer's theorem applies and we prove the existence.

Appendix B - Example and Graphs

Example. *We specify a linear cost function:* $C(x, c) = c\,x$. *The domain of the private signal precision is* $X = [0, \overline{x}]$ *where* $\overline{x} = \frac{W_1}{c}$ *is the maximum amount of information that an agent can purchase. The threshold that trigger the choice to be informed is:*

$$\overline{c} = \left[2R\rho(\tau_\pi + \tfrac{I^2}{\rho^2}\tau_\theta) \right]^{-1}$$

and, for any positive $c \leq \overline{c}$, *the optimal information choice is:*

$$x^*(c; \Theta) = \tfrac{1}{2c\rho R} - \tau_\pi - \tfrac{I^2}{\rho^2}\tau_\theta$$

It is easy to check that it is decreasing in price informativeness I *and decreasing in* c. *The expected indirect utility is:*

$$z(c; \Theta) = \begin{cases} -\frac{1}{\rho}\sqrt{2\rho R c}\; e^{-c\rho R(\tau_\pi + \frac{I^2}{\rho^2}\tau_\theta)} \sqrt{\frac{1}{K^2}(K + \frac{\rho^2}{\tau_\theta} + I)^{-1}}\, e^{\frac{1}{2} - \rho R W_1} & \text{if} \quad c < \overline{c} \\[2ex] -\frac{1}{\rho}\sqrt{(\tau_\pi + \frac{I^2}{\rho^2}\tau_\theta)\frac{1}{K^2}(K + \frac{\rho^2}{\tau_\theta} + I)^{-1}}\, e^{-\rho R W_1} & \text{if} \quad c > \overline{c} \end{cases}$$

We need to distinguish between two cases. Case A, when both types optimally prefer to be informed: $c_L < c_H < \overline{c}$, *and case B, when the illiterates prefer to remain uniformed,* $c_L < \overline{c} < c_H$.

In case A, the agent attends the training if:

$$\frac{z(\gamma, c_L; \Theta)e^\gamma}{z(\gamma, c_H; \Theta)} > 1$$

which holds when:

$$\sqrt{\frac{c_L}{c_H}}\, e^{-\rho R(c_L - c_H)(\tau_\pi + \frac{I^2}{\rho^2}\tau_\theta) + \gamma} < 1$$

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i.e. for any $\gamma < \overline{\gamma}$ such that:

$$\overline{\gamma} = \rho R(c_L - c_H)(\tau_\pi + \tfrac{I^2}{\rho^2}\tau_\theta) - \tfrac{1}{2}\log\tfrac{c_L}{c_H}$$

In case B, the agent attends the training if:

$$\tfrac{z(\gamma,c_L;\Theta)e^\gamma}{z(\gamma,c_H;\Theta)} > 1$$

which holds when:

$$\sqrt{\tfrac{2\rho R c_L}{\tau_\pi + \tfrac{I^2}{\rho^2}\tau_\theta}}\, e^{-\rho R c_L(\tau_\pi + \tfrac{I^2}{\rho^2}\tau_\theta) + \tfrac{1}{2} + \gamma} < 1$$

i.e. for any $\gamma < \overline{\gamma}$ such that:

$$\overline{\gamma} = \rho R c_L(\tau_\pi + \tfrac{I^2}{\rho^2}\tau_\theta) - \tfrac{1}{2}[1 + \log\tfrac{2\rho R c_L}{\tau_\pi + \tfrac{I^2}{\rho^2}\tau_\theta}]$$

In equilibrium, the price informativeness is given by:

$$I = \tfrac{\rho^2}{\tau_\theta}\left[-1 + \sqrt{1 + \tfrac{\tau_\theta}{\rho^2}\left(\tfrac{1}{2\rho R}\tilde{c} - \tau_\pi\right)}\right]$$

where $\tilde{c} = \lambda\tfrac{1}{c_L} + (1 - \lambda)\tfrac{1}{c_H}$.

I is strictly positive when $\tilde{c} < \tfrac{\lambda}{2\rho R \tau_\pi}$. The fraction of the literate agents is given by:

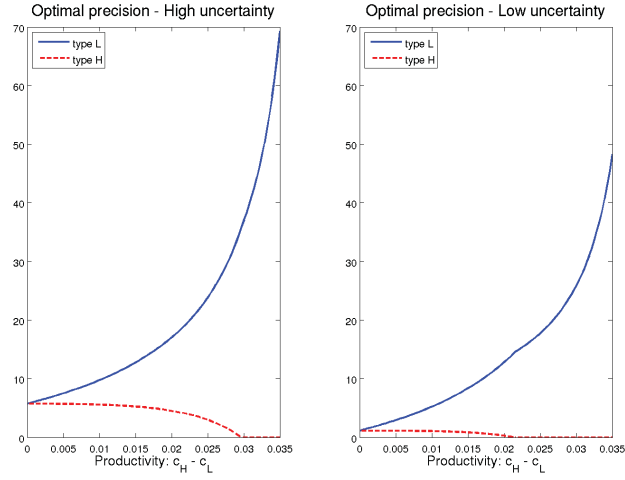$$\lambda = \int D^*(\gamma(j); \Theta)dG(\gamma) = \int_0^{\overline{\gamma}} dG(\gamma) = G(\overline{\gamma})$$

Figure 3.2: Optimal information choice for both types in high ($\tau_\pi = 0.2$, $\tau_\theta = 0.2$) or low ($\tau_\pi = 5$, $\tau_\theta = 5$) regime of uncertainty: $R = 1$, $W_1 = 1$, $\rho = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$.
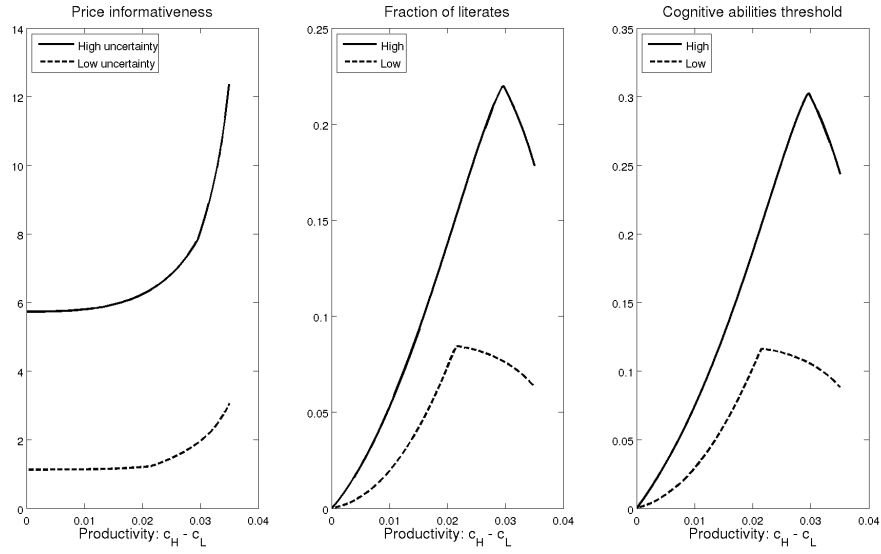

Figure 3.3: Price informativeness, optimal fraction of literate agents and threshold over cognitive abilities domain in high ($\tau_\pi = 0.2$, $\tau_\theta = 0.2$) or low ($\tau_\pi = 5$, $\tau_\theta = 5$) regime of uncertainty: $R = 1$, $W_1 = 1$, $\rho = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$.
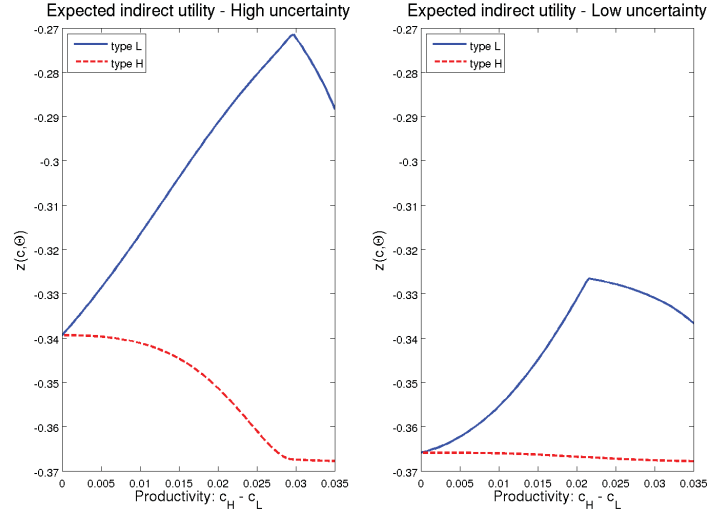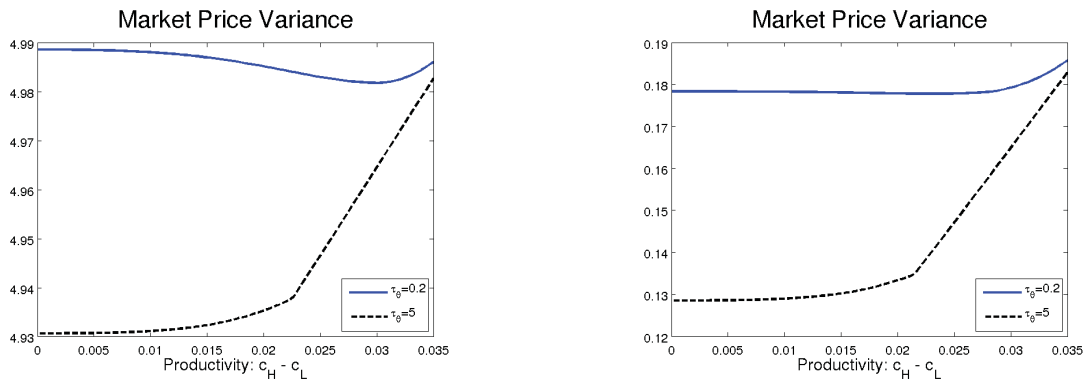
Figure 3.4: Expected utility for both types in high ($\tau_\pi = 0.2$, $\tau_\theta = 0.2$) or low ($\tau_\pi = 5$, $\tau_\theta = 5$) regime of uncertainty: $R = 1$, $W_1 = 1$, $\rho = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$.

(a) Prior precision: $\tau_\pi = .2$

(b) Prior precision: $\tau_\pi = 5$

Figure 3.5: We report market price variance, computed in equilibrium. We set c_H equal to 0.04 and we let c_L decreases up to 0.005. Moreover, $R = 1$, $W_1 = 1$, $\rho = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$.
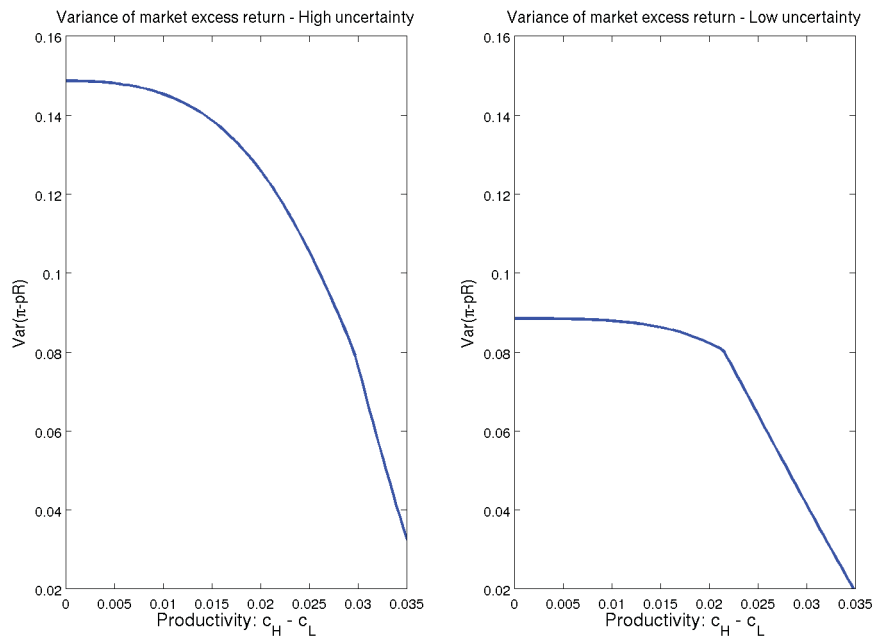
Figure 3.6: Variance of the excess return. $R = 1$, $W_1 = 1$, $\rho = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$.

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Estratto per riassunto della tesi di dottorato

Studente: Yuri Pettinicchi
Matricola: XXXX
Dottorato: Economia
Ciclo: XXIII

Titolo della tesi: Three Essays in Financial Literacy

Abstract: This dissertation focuses on the effects of financial literacy on individual financial behaviour, within a general equilibrium framework. Through a simple asset pricing model with heterogeneous beliefs about future prices, I analyse three different features: the information acquisition process, the market participation choice and the financial literacy production. I check the effects of financial literacy improving policies on individual welfare. Furthermore, I provide policy implications, especially with respect to the market stability.

Estratto: Questa tesi analizza l'effetto delle conoscenze finanziarie sul comportamento degli individui, all'interno di un modello di equilibrio economico generale. In particolare, la tesi affronta tre differenti aspetti: l'acquisizione di informazioni finanziarie, la partecipazione al mercato finanziario e la formazione di conoscenze finanziarie. Lo studio si concentra sulle conseguenze dell'attuazione di politiche economiche volte a migliorare le conoscenze finanziarie degli individui. Gli ambiti di interesse sono il benessere degli individui e la stabilità dei mercati finanziari.