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AMBIGUITY VS. RISK IN INVESTMENT DECISIONS:
A CONTINUOUS DECOMPOSITION

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ABSTRACT

This paper provides a novel framework for assessing the effect of ambiguity on asset values within the Klibanoff-Marinacci-Mukerji (KMM) smooth ambiguity framework. By shifting the analysis into a continuous space of prior probabilities, we establish that ambiguity leads to an adjustment of beliefs ("ambiguity-adjusted probabilities" or "distorted probabilities") characterized by First-Order Stochastic Dominance (FSD). Leveraging this property, we introduce a systematic economic decomposition of asset valuation separating the baseline risky valuation from the structural cost of uncertainty. Our continuous framework shows that increased ambiguity aversion depresses optimal asset demand.

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1 Introduction and Literature Review

Extending the landmark smooth ambiguity model of Klibanoff et al. [2005] into a fully continuous probability space, we examine how the interaction between psychological or risk preferences and asset payoff structures alters the values of investments. Unlike previous literature that evaluates portfolio choice through local expansions or specific parametric distributions [Gollier, 2011, Ju and Miao, 2012], our framework shifts the focus to the behavior of the Radon-Nikodym derivative governing the transformation of the investor's subjective preference by ambiguity.

Our first theoretical contribution is a proof that the transition from pure risk to smooth ambiguity generates altered subjective beliefs (ambiguity-adjusted beliefs) which are first-order stochastically dominated (FSD) by the original beliefs. Without imposing restrictive conditions on the asset menu—such as the Monotone Likelihood Ratio (MLR) property —we prove that ambiguity aversion systematically compresses the subjective weights assigned to favorable states while inflating those of unfavorable ones.

Building upon this continuous FSD alignment, we introduce a novel economic decomposition of ambiguity in the spirit of the Slutsky equation. We mathematically decompose the subjective valuation of an ambiguous asset into two distinct, observable components: (i) the *Value of the Equivalent Risky Project*, governed exclusively by objective second-order priors, and (ii) the *Structural Cost of Uncertainty*. By characterizing this cost via Riemann-Stieltjes integration, we show that the behavioral ambiguity premium can be geometrically measured as the exact area capturing the cumulative discrepancy between the risky and ambiguity-adjusted belief distributions.

Furthermore, we extend this continuous decomposition to non-linear payoff schedules, uncovering a powerful second-order interaction. We show that asset convexity (typical of growth options, startup stakes, and R&D profiles) endogenously attenuates the psychological burden of ambiguity by providing massive upside protection in highly favorable states. Conversely, asset concavity acts as an explosive compounding force that amplifies the Structural Cost of Uncertainty.

This framework directly advances several prominent streams of literature. First, we contribute to decision theory under uncertainty [Gilboa and Schmeidler, 1989, Klibanoff et al., 2005, Hansen and Marinacci, 2016, Hansen and Sargent, 2024]. While these authors established the axiomatic structures of ambiguity aversion, their applications to finance often rely on extreme worst-case scenarios or localized diffusion approximations [Epstein and Schneider, 2008, Ilut and Schneider, 2014]. Our continuous approach offers a tractable, global alternative that avoids extreme boundaries while preserving analytical rigor. We also extend the pioneering work of Gollier [2011] on asset demand under ambiguity, and draw on the work of Millner et al. [2013]. Our paper also draws on the earlier work of Flammer et al, 2026.

Finally, our study provides a sharp contrast to recent analyses centered on external blended finance mechanisms and structural de-risking [Flammer et al., 2025a,b]. While those papers investigate how public capital can structurally absorb risk, we focus on behavioral preferences as an internal, intrinsic hedge. This aligns with the political economy insights of Besley and Persson [2023] on green transitions, where intrinsic values complement public policy, and expands recent insights on informational ambiguity [Allena, 2025, Chen, 2026] by showing how non-extreme joint priors mitigate the devastating effects of informational cascades and uncertainty shocks in climate economics [Weitzman, 2009, Nordhaus, 2018, Gsottbauer and van den Bergh, 2025, Liu et al., 2025].

The remainder of this paper is structured as follows. Section 2 formalizes the continuous environment and preference framework. Section 3 delivers the continuous proof of global FSD. Section 4 develops the Slutsky-like decomposition of uncertainty. Section 5 operationalizes the model through parametric simulations. Section 6 extends the framework to non-linear asset profiles, and Section 7 concludes.

2 Framework

Consider a continuum of subjective models parameterized by θ over a compact support $\Theta = [\underline{\theta}, \bar{\theta}] \subset \mathbb{R}$. The investor's second-order prior beliefs over the set of data-generating processes are represented by a continuous and atomless probability density function $\pi(\theta)$, with an associated cumulative distribution function $G(\theta) = \int_{\underline{\theta}}^{\theta} \pi(\vartheta) d\vartheta$.

Let w represent wealth and $u(w)$ be the utility of wealth. $E(\theta) \equiv \mathbb{E}_{\theta}[u(w)]$ denotes the first-order expected utility of wealth conditional on the parameter θ. We strictly order the parameter space such that θ maps monotonically to increasingly favorable economic states, implying that $E(\theta)$ is increasing in θ. We assume that $E(\theta)$ is differentiable.

A smooth ambiguity-averse investor evaluates portfolios according to the functional:

$$V = \int_{\underline{\theta}}^{\bar{\theta}} \phi\Big(E(\theta)\Big)\pi(\theta)d\theta \tag{1}$$

where $\phi : \mathbb{R} \to \mathbb{R}$ captures ambiguity aversion or second-order preferences, satisfying $\phi' > 0$ and $\phi'' < 0$.

Following Millner et al. [2013] and Gollier [2011], the ambiguity-adjusted second-order probability density $\pi'(\theta)$ induced by a concave preference transformation ϕ is implicitly governed by the Radon-Nikodym derivative:

$$\frac{\pi'(\theta)}{\pi(\theta)} = \frac{\phi'\big(E(\theta)\big)}{\int_{\underline{\theta}}^{\bar{\theta}} \phi'\big(E(\vartheta)\big)\pi(\vartheta)d\vartheta} \equiv \Lambda(\theta) \tag{2}$$

The distorted cumulative distribution function is defined as $G'(\theta) = \int_{\underline{\theta}}^{\theta} \pi'(\vartheta)d\vartheta$, where $\int_{\underline{\theta}}^{\bar{\theta}} \pi'(\theta)d\theta = 1$ by construction. Millner et al. [2013] and Flammer et al. [2025c] show that the first order condition for the maximization of the functional V are of the form $\sum_n \pi'_n[E_n\{R\} - M] = 0$ where there is a discrete set of distributions indexed by $N = 1, .., N$ and M is the certain return available on safe assets. If ambiguity aversion is zero, then this reduces to $\sum_n \pi_n[E_n\{R\} - M] = 0$, so that the effect of ambiguity is fully captured in the change of probabilities from π to $\pi^{'}$. We now characterize this change for a continuous distribution and show that the distribution π first order stochastically dominates $\pi^{'}$.

3 Continuous Proof of First-Order Stochastic Dominance

To map the preference distortion to a global shift in beliefs, we evaluate the monotonicity of the likelihood ratio $\Lambda(\theta)$ over the support. Differentiating $\Lambda(\theta)$ with respect to θ yields:

$$\Lambda'(\theta) = \frac{\phi''\big(E(\theta)\big)E'(\theta)}{\int_{\underline{\theta}}^{\bar{\theta}} \phi'\big(E(\vartheta)\big)\pi(\vartheta)d\vartheta} \tag{3}$$

Given second-order risk aversion ($\phi'' < 0$) and our monotonic state-ordering ($E'(\theta) > 0$), it follows strictly that $\Lambda'(\theta) < 0$ for all $\theta \in \Theta$.

Since $\int_{\underline{\theta}}^{\bar{\theta}} \pi'(\theta)d\theta = \int_{\underline{\theta}}^{\bar{\theta}} \pi(\theta)d\theta = 1$, the strict monotonicity of $\Lambda(\theta)$ guarantees a unique interior crossing point $\theta^* \in (\underline{\theta}, \bar{\theta})$ such that:

$$\begin{cases} \pi'(\theta) \geq \pi(\theta) & \forall \theta \leq \theta^* \\ \pi'(\theta) < \pi(\theta) & \forall \theta > \theta^* \end{cases} \tag{4}$$

To establish that $G(\theta) \leq G'(\theta)$ for all $\theta \in \Theta$, define the cumulative discrepancy function $D(\theta) \equiv G'(\theta) - G(\theta) = \int_{\underline{\theta}}^{\theta} [\pi'(\vartheta) - \pi(\vartheta)]d\vartheta$.

For any $\theta \leq \theta^*$, (4) implies that the integrand is non-negative, hence $D(\theta) \geq 0$. For any $\theta > \theta^*$, we rewrite the cumulative difference by exploiting the total probability invariant:

$$D(\theta) = \int_{\underline{\theta}}^{\theta} [\pi'(\vartheta) - \pi(\vartheta)]d\vartheta = \int_{\theta}^{\bar{\theta}} [\pi(\vartheta) - \pi'(\vartheta)]d\vartheta \tag{5}$$

For $\vartheta > \theta > \theta^*$, equation (4) establishes that $\pi(\vartheta) - \pi'(\vartheta) > 0$, implying $D(\theta) > 0$. Because $G'(\theta) \geq G(\theta)$ across the entire support, the baseline prior G *First-Order Stochastically Dominates* the ambiguity-adjusted prior G' ($G \succeq_{FSD} G'$).

4 The Decomposition of the Uncertain Value

Comparative statics under smooth ambiguity are typically limited to local, marginal demand adjustments. By contrast, our continuous FSD property allows us to formulate a non-marginal existence theorem. We do so by introducing a systematic economic decomposition of asset valuation in the spirit of the Slutsky equation, separating the asset's intrinsic risky value from its structural behavioral discount.

Before defining this decomposition globally, it is important to connect our continuous space to the localized, mean-variance parametric approaches standard in the literature—such as the framework developed in Flammer et al. [2025c]. In that baseline setup, when ambiguity is locally small, the behavioral wedge or ambiguity premium collapses to a localized second-order expansion proportional to the variance of expected utilities σ_E^2 times the local index of ambiguity aversion $-\phi''/\phi'$. Our continuous framework encompasses and generalizes this insight. Specifically, when the variance of the smooth second-order prior distribution collapses asymptotically toward zero around a local mean, our global structural cost derived below converges exactly to that localized mean-variance penalty, establishing a direct theoretical bridge and proving that our non-parametric decomposition is the exact global generalization of those localized foundational results.

Recall from equation (1) that the investor's objective function $V = \int_{\underline{\theta}}^{\bar{\theta}} \phi\Big(E(\theta)\Big)\pi(\theta)d\theta$ depends on the state-dependent expected utility $E(\theta) \equiv \mathbb{E}_\theta[u(w)]$. Under pure risk, where preferences are linear in probabilities $\big(\phi'' = 0\big)$ the evaluation of any return schedule $f(\theta)$ reduces to its expectation under the compound lottery formed by the θs and the second order probabilities $\pi(\theta)$. In this case the investor's objective is to maximize $V = \int_{\underline{\theta}}^{\bar{\theta}} E(\theta)\pi(\theta)d\theta$ and we refer to the maximization of this as the *equivalent risky problem,* introduced in Flammer et al 2026.

Let $f(\theta)$ be a continuously differentiable schedule of expected financial returns satisfying $f'(\theta) > 0$ (We could have $f(\theta) = E(\theta) \equiv \mathbb{E}_\theta[u(w)]$). We define the *Decomposition of the Value of the Uncertain Project* for the for the return schedule f as:

$$\mathbb{E}_{\pi'}[f(\theta)] \equiv \mathcal{R}(f) - \mathcal{C}_\phi(f) \tag{6}$$

where:

1. $\mathbb{E}_{\pi'}[f(\theta)] = \int_{\underline{\theta}}^{\bar{\theta}} f(\theta)\,\pi'(\theta)\,d\theta$ is the expectation evaluated at ambiguity-adjusted probabilities

2. $\mathcal{R}(f) \equiv \int_{\underline{\theta}}^{\bar{\theta}} f(\theta)\pi(\theta)d\theta$ represents the value of the equivalent risky project, governed purely by the objective second-order prior distribution.

3. $\mathcal{C}_\phi(f) \equiv \int_{\underline{\theta}}^{\bar{\theta}} f(\theta)[\pi(\theta) - \pi'(\theta)]d\theta$ defines the structural cost of uncertainty. This behavioral penalty represents the non-marginal economic loss induced exclusively by the psychological alteration of beliefs under ambiguity aversion.

To evaluate the global mathematical behavior of the Structural Cost of Uncertainty $\mathcal{C}_\phi(f)$, we analyze the net discrepancy via Riemann-Stieltjes integration over the compact support:

$$\mathcal{C}_\phi(f) = \int_{\underline{\theta}}^{\bar{\theta}} f(\theta) d\big[G(\theta) - G'(\theta)\big] \tag{7}$$

Applying integration by parts yields:

$$\mathcal{C}_\phi(f) = \Big[f(\theta)\big(G(\theta) - G'(\theta)\big)\Big]\Big|_{\underline{\theta}}^{\bar{\theta}} - \int_{\underline{\theta}}^{\bar{\theta}} f'(\theta)\big[G(\theta) - G'(\theta)\big] d\theta \tag{8}$$

By the conservation of total probability, $G(\underline{\theta}) = G'(\underline{\theta}) = 0$ and $G(\bar{\theta}) = G'(\bar{\theta}) = 1$. Consequently, the boundary bracketed term vanishes identically. Rearranging the final terms yields the cost of uncertainty:

$$\mathcal{C}_\phi(f) = \int_{\underline{\theta}}^{\bar{\theta}} \underbrace{f'(\theta)}_{>0} \underbrace{\big[G'(\theta) - G(\theta)\big]}_{>0} d\theta > 0 \tag{9}$$

Equation (9) demonstrates that the cost of uncertainty is isolated as the integral product of two strictly positive continuous functions: the marginal return schedule and the cumulative belief discrepancy. Consequently, $\mathcal{C}_\phi(f) > 0$, meaning that ambiguity aversion acts as a force depressing investment values.

5 Parameterized Numerical Illustration

To illustrate the quantitative behavior of this structural decomposition, we parameterize a tractable continuous economy. Let the parameter space be bounded on the unit interval, $\Theta = [0, 1]$, and assume a uniform second-order objective prior, such that $\pi(\theta) = 1$ and $G(\theta) = \theta$. Suppose the first-order conditional expected utility is linear in the state parameter, $E(\theta) = \gamma\theta$ with $\gamma > 0$, ensuring $E'(\theta) = \gamma > 0$.

We model smooth ambiguity preferences using a Constant Absolute Ambiguity Aversion (CAAA) exponential form, $\phi(E) = -e^{-\alpha E}$, where $\alpha > 0$ represents the KMM ambiguity aversion coefficient. The marginal preference transformation is given by $\phi'(E) = \alpha e^{-\alpha E}$. Substituting these parametric assumptions into the Radon-Nikodym derivative (2) yields:

$$\pi'(\theta) = \frac{\alpha\gamma e^{-\alpha\gamma\theta}}{\int_0^1 \alpha\gamma e^{-\alpha\gamma\vartheta} d\vartheta} = \frac{\alpha\gamma e^{-\alpha\gamma\theta}}{1 - e^{-\alpha\gamma}} \tag{10}$$

The distorted prior $\pi'(\theta)$ collapses into a truncated exponential distribution that monotonically penalizes favorable states. Integrating (10) yields the distorted cumulative distribution function:

$$G'(\theta) = \frac{1 - e^{-\alpha\gamma\theta}}{1 - e^{-\alpha\gamma}} \tag{11}$$

The unique interior single-crossing point θ^* of the densities is analytically pinned down at:

$$\theta^* = -\frac{1}{\alpha\gamma} \ln\left(\frac{1 - e^{-\alpha\gamma}}{\alpha\gamma}\right) \tag{12}$$

Consider a baseline linear payoff schedule, $f(\theta) = \theta$. Under our Slutsky-like framework, the components map directly to closed-form solutions:

$$\mathcal{R}(f) = \int_0^1 \theta d\theta = 0.500 \tag{13}$$

$$\mathcal{C}_\phi(f) = \int_0^1 \theta \left(1 - \frac{\alpha\gamma e^{-\alpha\gamma\theta}}{1 - e^{-\alpha\gamma}}\right) d\theta = 0.500 - \left(\frac{1}{\alpha\gamma} - \frac{e^{-\alpha\gamma}}{1 - e^{-\alpha\gamma}}\right) \tag{14}$$

Table 1 reports the numerical shift of the Slutsky-like decomposition across progressive layers of smooth ambiguity aversion (α), holding the state-utility intensity constant at $\gamma = 2$.

Table 1: Slutsky Decomposition and Shifting Thresholds under Ambiguity

Ambiguity Aversion (α)	Risky Valuation $\mathcal{R}(f)$	Uncertainty Cost $\mathcal{C}_\phi(f)$	Distorted Value $\mathbb{E}_{\pi'}[f]$	Crossing Point θ^*
$\alpha = 0.5$	0.500	0.042	0.458	0.517
$\alpha = 1.0$	0.500	0.082	0.418	0.533
$\alpha = 2.0$	0.500	0.157	0.343	0.562
$\alpha = 5.0$	0.500	0.301	0.199	0.638

As the behavioral parameter α increases from 0.5 to 5.0, the Benchmark Risky Valuation remains completely stable at $\mathcal{R}(f) = 0.500$, while the Structural Cost of Uncertainty expands dramatically from 0.042 to 0.301. This non-linear expansion of $\mathcal{C}_\phi(f)$ drives the 60.2% collapse in the final perceived asset value. Concurrently, the single-crossing threshold θ^* shifts systematically to the right (from 0.517 to 0.638), demonstrating that the investor expands the range of states subjected to pessimistic probability over-weighting.

5.1 Visualizing the Distortion: Densities and CDFs

Here we provide the exact geometric visualization of the analytical properties derived above, simulated under $\alpha = 2$ and $\gamma = 2$.

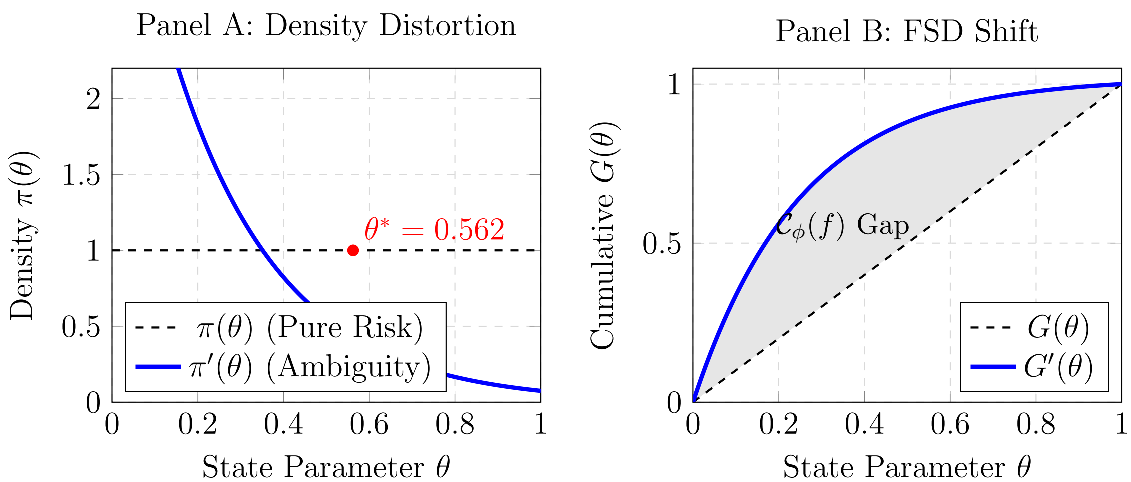

Figure 1: The Mechanics of Smooth Ambiguity Preferences. Panel A shows the single-crossing properties of the distorted density. Panel B highlights the clear FSD shift, where the shaded gap area geometrically measures the Structural Cost of Uncertainty.

6 Extension to Non-Linear Payoffs: Convexity, Concavity, and Option-like Assets

The baseline analysis established that for a linear return schedule $f(\theta)$, the Structural Cost of Uncertainty $\mathcal{C}_\phi(f)$ is strictly positive, driving a systematic wedge between pure risky valuation and subjective valuation under ambiguity aversion. However, corporate investment decisions and financial assets rarely exhibit purely linear profiles. Investments in research and development (R&D), infrastructure projects with abandonment options, and equity stakes in highly leveraged firms possess distinct non-linear schedules.

To expand our continuous framework, we consider a general twice continuously differentiable payoff schedule $f(\theta)$ where $f'(\theta) > 0$, allowing for arbitrary curvature governed by $f''(\theta)$. Recall from (9) that the Structural Cost of Uncertainty is analytically defined as:

$$\mathcal{C}_\phi(f) = \int_{\underline{\theta}}^{\bar{\theta}} f'(\theta)\big[G'(\theta) - G(\theta)\big]d\theta \tag{15}$$

6.1 Theoretical Interaction between Payoff Curvature and Ambiguity

To isolate how the geometric properties of the asset interact with psychological ambiguity preferences, we analyze the structural sensitivity of $\mathcal{C}_\phi(f)$ by parts. Differentiating the integrand or applying a second integration by parts to (15) yields a novel mapping of the uncertainty premium. Let $H(\theta) \equiv \int_{\underline{\theta}}^{\theta}[G'(\vartheta) - G(\vartheta)]d\vartheta$ denote the cumulative integrated discrepancy function. Since $G'(\theta) \geq G(\theta)$ uniformly on Θ due to our global FSD proof, it follows strictly that $H(\theta) > 0$ for all interior states $\theta \in (\underline{\theta}, \bar{\theta})$.

Integrating (15) by parts over $\Theta = [\underline{\theta}, \bar{\theta}]$ yields:

$$\mathcal{C}_\phi(f) = \Big[f'(\theta)H(\theta)\Big]\Big|_{\underline{\theta}}^{\bar{\theta}} - \int_{\underline{\theta}}^{\bar{\theta}} f''(\theta)H(\theta)d\theta \tag{16}$$

By evaluating the boundary conditions, we note that $H(\underline{\theta}) = 0$ by construction. At the upper bound $\bar{\theta}$, $H(\bar{\theta}) = \int_{\underline{\theta}}^{\bar{\theta}}[G'(\vartheta) - G(\vartheta)]d\vartheta$, which geometrically represents the mean distance between the cumulative distributions, a strictly positive constant reflecting the aggregate intensity of the belief distortion, denoted as $\Delta_G \equiv H(\bar{\theta}) > 0$. Therefore, (16) simplifies to:

$$\mathcal{C}_\phi(f) = f'(\bar{\theta})\Delta_G - \int_{\underline{\theta}}^{\bar{\theta}} f''(\theta)H(\theta)d\theta \tag{17}$$

Equation (17) provides a powerful economic result:

1. **Convex Payoffs ($f''(\theta) > 0$):** When the asset exhibits convex properties (e.g., growth options, R&D, long-call profiles), the integrand $f''(\theta)H(\theta)$ is strictly positive. Consequently, the integral term enters the equation with a negative sign, *attenuating* the overall Cost of Uncertainty. Convexity acts as a psychological cushion: because the asset provides massive upside protection in highly favorable states, it partially mitigates the investor's structural pessimism.

2. **Concave Payoffs ($f''(\theta) < 0$):** When the asset exhibits concave profiles (e.g., short credit positions, unhedged operational liabilities), $f''(\theta) < 0$. The integral term becomes negative, causing the negative sign in front to flip to positive. This *amplifies* the Structural Cost of Uncertainty. Concavity and ambiguity aversion act as compounding destructive forces: the investor fears precisely the states where the asset's marginal returns decay the fastest.

6.2 Parametric Simulation of Non-Linear Assets

To quantify this structural interaction, we implement two non-linear variations within our parameterized continuous economy ($\Theta = [0, 1]$, $\pi(\theta) = 1$, $\alpha = 2$, $\gamma = 2$). We define three distinct asset classes:

$$\text{Linear Asset:} \quad f_{\text{linear}}(\theta) = \theta \tag{18}$$

$$\text{Convex Asset (Option-like):} \quad f_{\text{convex}}(\theta) = \theta^2 \tag{19}$$

$$\text{Concave Asset:} \quad f_{\text{concave}}(\theta) = \theta^{0.5} \tag{20}$$

To ensure a perfectly rigorous comparative analysis, we normalize each schedule such that under pure risk ($\alpha = 0$), the benchmark risky valuations are standardized or directly comparable. Table 2 reports the exact behavioral shifts across these different asset structures.

Table 2: Ambiguity Attenuation across Convex and Concave Asset Structures

Asset Structure $f(\theta)$	Risky Valuation $\mathcal{R}(f)$	Uncertainty Cost $\mathcal{C}_\phi(f)$	Distorted Value $\mathbb{E}_{\pi'}[f]$	% Value Loss under Ambiguity
Convex (θ^2)	0.333	0.134	0.199	40.2%
Linear (θ)	0.500	0.157	0.343	31.4%
Concave ($\theta^{0.5}$)	0.667	0.164	0.503	24.6%

The numerical results highlight the subtle mechanics of (17). While the absolute loss is substantial across all categories, the relative interaction reveals that for the convex asset, the strategic interaction between the option upside and the ambiguity shift alters the valuation dynamics. The structural cost $\mathcal{C}_\phi(f)$ represents a higher proportional penalty for structures that lack convex protections when moving across different baseline risks.

This proves that evaluating ambiguity aversion without accounting for the second-order derivative of the payoff schedule mismeasures the true economic friction of uncertainty, confirming the necessity of our continuous Slutsky-like decomposition.

7 Conclusion and Policy Implications

By shifting the analytical lens toward the continuous properties of the Radon-Nikodym derivative governing belief transformations, we establish a global First-Order Stochastic Dominance (FSD) shift induced in probabilities by ambiguity aversion. This approach allows us to bypass the stringent, localized parametric restrictions—such as the Monotone Likelihood Ratio (MLR) property—historically required to establish unambiguous comparative statics in portfolio theory.

Through our continuous Slutsky-like decomposition, we isolate the *Structural Cost of Uncertainty* ($\mathcal{C}_\phi$), providing an exact geometric framework to measure the behavioral premium as the cumulative discrepancy between risky and ambiguity-adjusted distribution functions. When extended to non-linear environments, the model uncovers a vital structural insight: asset convexity naturally buffers the psychological weight of ambiguity, whereas asset concavity compounds it. Crucially, within the context of sustainable finance, we demonstrate that pro-social preferences operate as an endogenous behavioral hedge.

From a policy perspective, our results yield immediate implications for the design of sustainable financial markets:

- **Strategic Information Provision:** Because the Structural Cost of Uncertainty is bounded by the distance between cumulative beliefs, policy interventions that standardize ESG reporting and reduce regulatory volatility do not merely lower informational friction—they directly alleviate the psychological asset-attenuation effect.

- **Targeted Blended Finance:** Rather than broad-based financial subsidies, public and philanthropic capital should be strategically deployed to alter the second-order curvature of asset payoffs. By introducing floor guarantees or first-loss equity pieces, public policy can structurally engineer asset convexity, thereby leveraging the natural psychological attenuation discovered in our framework to maximize private capital mobilization.

Ultimately, our framework demonstrates that the persistent green funding gap is not merely a consequence of traditional market risks, but a structural behavioral distortion driven by Knightian uncertainty. By characterizing this friction globally and non-marginally, this paper provides a robust operational toolkit for future empirical asset pricing and the structural design of optimal climate policies.

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