

Johannes Kepler and the Pythagoreans

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The early modern Republic of Letters had a complicated relationship with novelty.¹ The period witnessed vast upheavals across the social and intellectual landscape, yet novelty was viewed with great suspicion, perhaps unsurprisingly, given all the turmoil. Purveyors of novelty were morally suspect, vain or seditious, or both. Sixteenth century authors rarely presented their work as new. Instead, when they put forth controversial doctrines, they cast them as ancient. If they could not plausibly trace their ideas to some snippet of Aristotle or Galen, then they appealed to figures considered even more antique: Moses, Zoroaster, Hermes Trimegistus, Hippocrates, Pythagoras. In doing so, they exercised preemptive self-defense, but it would be unfair to suggest that their motive was only a matter of camouflage or of ginning up sales in a competitive book market. Kepler's work, for example, basks in the thrill of rediscovery, of connecting with great figures in the far-away past, figures who stood closer to creation, to the divine, to universal truth. The paradigm of rediscovery encouraged a hermeneutical attention to continuity and concealment. It also implied a certain historical template, wherein truth gave way to periods of confusion and portent, then to redemption and clarity. Such a template was at work, for instance, when exegetes mined the Old Testament for echoes of Christ's birth and ministry. If most sixteenth century writers would have subscribed to a general, Christian-eschatological view of history, there remained within that framework room for cycles of collapse and restoration, such as Proclus described, concurring with Aristotle that "the sciences did not arise for the first time among us or among the men of whom we know, but at countless other cycles in the past they have appeared and vanished and will do so in the future."² The above considerations set the stage for Kepler's use of the *Timaeus*. Of course, he appealed to the work as an example of mathematical natural philosophy.³ But it meant much more. Its interest, beauty, and authority drew from an ancient tradition beginning with the first

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¹ See Daniel Garber, "Historicizing Novelty," in *What Reason Promises: Essays on Reason, Nature and History*, eds. Wendy Doniger, Peter Galison and Susan Neiman (Berlin: De Gruyter, 2016), 186-194.

² Proclus, *A Commentary on the First Book of Euclid's Elements*, trans. Glenn R. Morrow (Princeton: Princeton University Press, 1992), 51. On cyclical time in the Renaissance, see Michael J. B. Allen, "Life as a Dead Platonist," in *Marsilio Ficino: His Theology, His Philosophy, His Legacy*, eds. Michael J. B. Allen, Valery Rees and Martin Davies (Leiden: Brill, 2002), 159-178.

³ Rhonda Martens' focus is the *Timaeus* as inspiration for Kepler's mathematization of physics. Rhonda Martens "A Commentary on Genesis: Plato's *Timaeus* and Kepler's Astronomy," in *Plato's Timaeus as Cultural Icon*, ed. Gretchen J. Reydams-Schils (Notre Dame: University of Notre Dame Press, 2003), 251-266.

Pythagoreans, passing through Plato, to Euclid and Proclus, and eventually to Copernicus. Kepler's reception of the *Timaeus* as a Pythagorean text was not unique. Plato's Renaissance translator and commentator, Marsilio Ficino, had endorsed the fundamentally Pythagorean lineage of Platonic philosophy. And as Kepler would, Ficino read the Neopythagorean philosophy of late antiquity back into the earlier Pythagorean tradition.⁴ Yet perhaps no author of early modernity gave such full expression to what the Pythagorean could mean as a mathematical inquiry into nature and the divine. Kepler, as early as his first book, the *Mysterium cosmographicum* (1596), saw himself at the summit of this mathematical tradition.

Adopting the Pythagoreans

Copernicus has never ceased to be an object of fascination and an instrument of warfare. Long before the Catholic Church put *De Revolutionibus orbium coelestium* (1543) on its index of prohibited books in 1616, efforts to make sense of his impact were already underway. One reason why these sixteenth-century efforts are so interesting is lack of information from the source himself. Copernicus was not a voluminous writer, nor did he spend a career spreading his ideas far and wide in the Republic of Letters. He was certainly aware that his views could cause problems, not least with the Church, and he seems largely to have restricted their dissemination among trusted friends and professional mathematicians. He died shortly after the printing of *De revolutionibus*, so we do not know how he would have defended himself as he and his ideas became better known. The work of publicizing Copernican astronomy, and of setting it within wider epistemological and natural philosophical frameworks, was done by others: Georg Joachim Rheticus, Kepler, and Galileo are the best known names among the first two generations. Yet Copernicus provided something of an outline for how his system should be defended. We find the Pythagoreans referenced in his first known astronomical work, the *Commentariolus*, a rough summary of the heliocentric arrangement written and circulated in manuscript form decades before *De revolutionibus* appeared.⁵ There, Copernicus writes, "let

⁴ Michael J. B. Allen, "Pythagoras in the Early Renaissance," in *A History of Pythagoreanism*, ed. Carl A. Huffman (Cambridge: Cambridge University Press, 2014), 435–453, 436. Also see, Christopher S. Celenza, "Pythagoras in the Renaissance: The Case of Marsilio Ficino," *Renaissance Quarterly* 52, no. 3 (1999): 667–711. For the history of Pythagoreanism, see Walter Burkert, *Lore and Science in Ancient Pythagoreanism*, trans. Edwin L. Minar, Jr. (Cambridge, Massachusetts: Harvard University Press, 1972 [1962]); Charles H. Kahn, *Pythagoras and the Pythagoreans* (Indianapolis: Hackett, 2001); Gabriele Cornelli, Richard McKirahan and Constantinos Macris (eds.), *On Pythagoreanism* (Berlin: De Gruyter, 2013). On Kepler and the Pythagoreans, see Silvia Tangheriini, "Temi platonici et pitagorici nell'*Harmonice mundi* di Keplero," *Rinascimento* 14: 117–178.

⁵ For a discussion of early Copernicans and "the invention of Pythagorean cosmology," see Pietro Daniel Omodeo, *Copernicus in the Cultural Debates of the Renaissance* (Leiden: Brill, 2014), 167–170.

no one suppose that I have gratuitously (*temere*) asserted, with the Pythagoreans, the motion of the earth; strong proof will be found in my exposition of the circles.”⁶ In other words, Copernicus will lay out solid astronomical reasons. The syntax is ambiguous, but Copernicus should not be read as claiming that the Pythagoreans themselves lacked good astronomical reasons. He only wishes to say that he does not follow them blindly. The evocation of ancient Pythagoreans is more evident in *De revolutionibus*. In the dedication to Pope Paul III, Copernicus explains that his dissatisfaction with the state of astronomy had spurred him to reread all the philosophers he could. When finally he encountered the Pythagoreans, he began to consider the possibility of the Earth’s motion. He quotes several lines from Pseudo-Plutarch, in the original Greek, to the effect that “Philolaus the Pythagorean” held the Earth to move around “the element of fire,” and that Heraclides and Ecphantus had the Earth turning from west to east on its axis.⁷ It seems unlikely, given Copernicus’s elite university education, that he had *not* heard about the Pythagoreans’ curious arrangement of the planets during his studies in hubs like Cracow, Padua, Ferrara, and Bologna, or from his network of mathematical colleagues.⁸ Be that as it may, the overriding motive of his self-presentation is to demonstrate the venerability of his ideas, and his own good intentions in adopting them, however contentious they were. Another motive is to connect himself with—and ultimately distance himself from—the code of initiation and secrecy associated with the Pythagoreans:

Therefore I debated with myself for a long time whether to publish the volume which I wrote to prove the earth’s motion or rather to follow the example of the Pythagoreans and certain others, who used to transmit philosophy’s secrets only to kinsmen and friends, not in writing but by word of mouth, as is shown by Lysis’ letter to Hipparchus.⁹

In the above, Copernicus references a spurious letter in Greek, attributed to a figure believed to have been a disciple of Pythagoras. The letter has this figure, Lysis, reminding a companion, Hipparchus, of the reasons why their master Pythagoras insisted on secrecy: without proper initiation and zeal, wisdom is corrupted by ignorance and vice. Lysis had caught

⁶ Edward Rosen, *Three Copernican Treatises* (New York: Octagon Books, 1971 [1939]), 59.

⁷ Copernicus, *De revolutionibus orbium coelestium* (Nuremberg, 1543), iii.r. Copernicus quotes from Book II, chapter 13 (“Of the motion of the Earth”) of Pseudo-Plutarch’s *Placita philosophorum*. I cite the English translation in Plutarch, *Plutarch’s Morals*, vol. III, ed. William W. Goodwin (Boston: Little, Brown, and Company, 1874), 156. This translation is available through the Perseus Digital Library at perseus.tufts.edu.

⁸ On Copernicus and his networks, see Robert S. Westman, *The Copernicus Question: Prognostication, Skepticism, and Celestial Order* (Berkeley: University of California Press, 2011).

⁹ Nicholas Copernicus, *Nicholas Copernicus on the Revolutions*, ed. Jerzy Dobrzycki, trans. Edward Rosen, *Nicholas Copernicus Complete Works*, vol. 2 (Warsaw: Polish Scientific Publishers, 1978), 3.

wind that Hipparchus was teaching their secret doctrines publicly, and he concludes by telling his erstwhile friend that if the latter does not mend his ways, “you are dead to me” (*mortuus es mihi*).¹⁰ The letter has an interesting history, appearing in antiquity and surviving among Greek scholars until it was reintroduced into Latin Europe in the fifteenth century. Cardinal Bessarion printed a Latin translation in his *In calumniatorem Platonis*. Copernicus knew this translation, but prepared his own to be published at the very end of Book I of *De revolutionibus*; for reasons unknown, the letter was crossed out in the autograph and not printed.¹¹ In any case, the point that needs to be made here is how Copernicus expresses his intellectual, indeed emotional, hesitancy to the Pope and to his readers. The Pythagoreans are not simply an example of intellectual precedence—heliocentrism as ancient doctrine—they are exemplars of a kind of *expertise*, that is, an expertise which keeps itself hidden and pure. They are a model for virtue and discretion, but a model that Copernicus does not himself follow. Instead, he projects Pythagorean authority onto professional mathematicians—the initiates—and asks that the Pope offer protection from the uninitiated, theologians included:

Perhaps there will be babblers who claim to be judges of astronomy although completely ignorant of the subject ... Astronomy is written for astronomers [*mathemata mathematicis scribuntur*]. To them my work will seem, unless I am mistaken, to make some contribution to the Church, at the head of which your Holiness now stands.¹²

We can find the same dynamic—wherein Pythagorean secrecy appreciated, then set aside—in the very first treatise of Copernican astronomy published, the *Narratio prima* of Copernicus’s first and only disciple, Georg Rheticus. The role of institutional protector there is played by the bishop Tiedemann Giese, a close friend and defender of Copernicus in Warmia. Rheticus explains that Copernicus had first wanted to use his astronomy to calculate tables that would improve on the Alfonsine Tables, while keeping the actual astronomical hypotheses or models a secret. Giese, according to Rheticus, managed to persuade Copernicus that his work would be incomplete unless he imitated Ptolemy and laid everything out, the foundations and proofs:

¹⁰ For ease, I have consulted the PDF scan of the autograph of *De revolutionibus* freely available at the Digital Library of the Jagiellonian University Library. URL:

<https://jbc.bj.uj.edu.pl/dlibra/publication/1494/edition/858/content>, p. 36 of PDF.

¹¹ On the history of this letter, see Eva Del Soldato, “The Letter of Lysis to Hipparchus in the Renaissance,” in *Platonism: Ficino to Foucault*, eds. Valery Rees et al. (Leiden: Brill, 2021), 98-122. For Copernicus and the letter, see *Ibid.*, 110-112.

¹² Copernicus, *Nicholas Copernicus on the Revolutions*, 5.

The bishop further argued that such a procedure had produced great inconvenience and many errors in the Alfonsine Tables, since we were compelled to assume and to approve their ideas on the principle that, as the Pythagoreans used to say, “The Master said so”—a principle which has absolutely no place in mathematics.¹³

A rising disciplinary assertiveness among mathematicians of the sixteenth century has been noted by historians of science, Robert Westman chiefly.¹⁴ I would simply like to stress that Pythagorean secrecy was used as an appeal to disciplinary expertise. What Copernicus and Rheticus suggest is that if elites want to *know*, they must allow scholars to work and publish, and they must offer protection. In a world of epistolary exchange, publishing, and curious readers and rulers, scholars needed institutional protection. In a 1598 letter that Kepler wrote to his mentor Michael Maestlin, he talks about some fervent advice he had received from a friend and prominent theologian in Tübingen about refraining from publicly defending Copernican astronomy on scriptural grounds. This friend had warned Kepler that, were he to do so, he risked losing the support of his most powerful patron at the time, Duke Friedrich I of Württemberg. Kepler writes to Maestlin that, since the greater part of the learned cannot, by the force of their intellects, ascend to the heights of astronomy, it is better to imitate the Pythagoreans. Privately, astronomers can speak their minds. Publicly, “*taceamus*.”¹⁵ Kepler is not talking here about the mere teaching of Copernican astronomy; he is talking about the need to tailor that teaching, so as to compromise with the realities of the world and avoid giving offence. When astronomers engage in astronomy, “either we retain patrons of astronomy,” he writes, “or we go hungry.”¹⁶

Of course, Pythagoreanism was more than a vague moral code or mere assertion of a mobile Earth. The *Timaeus* lays out a detailed cosmological system that, in early modernity, was widely thought to represent ancient Pythagoreanism. When Copernicus and Rheticus appeal to Plato, they are evoking a core ensemble of ideas: a nature designed by mathematical proportion, a mathematically inclined creator, and a human mind that can grasp reality via clear mathematical ideas. *Symmetria* is an important concept here, bringing together divine intention, natural expression and human appreciation. Copernicus claims that his system reveals the

¹³ Rosen, *Three Copernican Treatises*, 193.

¹⁴ Robert Westman, “The Astronomer’s Role in the Sixteenth Century: A Preliminary Survey.” *History of Science* 18 (1980): 105–47.

¹⁵ Johannes Kepler, *Gesammelte Werke* [KGW], eds. Max Caspar and Walther von Dyck et al. (Munich: C. H. Beck, 1937–), vol. 13, letter 99, 231, ln. 503-507.

¹⁶ *Ibid.*, ln. 510-512.

“symmetry” (*symmetria*) of the cosmos. Plato had used *summetria* (συμμετρία) in the *Timaeus* to describe the fitness, health and beauty of the living body, the good coordination between body and soul, and the constitution of the universe. This followed its general meaning in ancient philosophy: proportionality or balance.¹⁷ But Copernicus need not have been thinking of the *Timaeus* alone. *Symmetria* was a common enough term. Vitruvius’s *De architectura* spoke at length of *symmetria* as a system of ratios with a common measure, and it has been argued that Copernicus appealed to the Vitruvian sense, since Copernican *symmetria* refers specifically to the proportions of the planetary system, with the Earth-Sun distance as the common unit.¹⁸ It is noteworthy that Rheticus describes this Copernican *symmetria* as a kind of Platonic idea:

I am persuaded that now at last I have a more accurate understanding of that delightful maxim which on account of its weightiness and truth is attributed to Plato: “God ever geometrizes”; but partly because in my teacher’s revival of astronomy I see, as the saying is, with both eyes and as though a fog had lifted and the sky were now clear, the force of that wise statement of Socrates in the *Phaedrus*: “If I think any other man is able to see things that can naturally be collected into one and divided into many, him I follow after and ‘walk in his footsteps as if he were a god.’”

Copernicus, says Rheticus, revealed how the many—the diverse motions of the heavens—are expressions of one whole and complete order. It was the nature of this order that interested Kepler, and he sought out what he calls *a priori* reasons that would reveal why God had given the universe its particular symmetry. These *a priori* reasons are essentially mathematical and metaphysical; they are appeals to the perfection of God and of certain geometrical objects that Kepler believes to be universal formal causes or, as he sometimes calls them, “archetypes.” Kepler takes from the *Timaeus* a concrete insight, that the regular solids must be among these archetypes and have a determinant role in the structure of nature.

Kepler wrote two major works of *a priori* astronomy, indeed, of *a priori* natural philosophy: the *Mysterium cosmographicum*, published in 1596 (then republished with commentary by Kepler as an older and wiser scholar in 1621), and the *Harmonice mundi* of 1619, which is best known for its announcement of what would later be called Kepler’s third

¹⁷ See Malcolm Heath, “Unity, Wholeness, and Proportion,” in *A Companion to Ancient Aesthetics*, eds. Pierre Destrée and Penelope Murray (West Sussex, UK: John Wiley & Sons, 2015), 381-392, esp. 388-390.

¹⁸ Giora Hon and Bernard R. Goldstein, *From Summetria to Symmetry: The Making of a Revolutionary Scientific Concept* (Dordrecht: Springer, 2008), 157-163.

law. Kepler sets forth his “polyhedral hypothesis,” as it is sometimes called, in the *Mysterium*. There, he decided to take what seems like a very quixotic approach to defending and advancing Copernican astronomy, but it is one that ultimately drew from the Platonic promise made by both Copernicus and Rheticus: that the cosmic blueprint was based on a divine idea accessible to the human mind through mathematics. As mentioned, *if* the Earth moved, then the planetary distances, in units of Earth-Sun distance, could be determined. Copernicus touted this possibility as a distinct advantage of his system, and he calculated the distances. But this raised a question, itself suggested by the Copernican rhetoric. If the cosmos or *mundus* was the best, and made by the optimal Creator, than why did the planets have exactly *those* distances and not others? In particular, why did such a massive empty region lay between Mars and Jupiter? *Hic te Pythagoras docet omnia quinque figuris*, wrote Kepler in the dedicatory poem to his *Mysterium*: “Here, Pythagoras teaches you everything about the five solids.”¹⁹ God, it seems, had used the five regular solids—the tetrahedron, cube, octahedron, dodecahedron and icosahedron—not to make the elements but to space out the planets. Between any two planets, one could imagine interjecting a regular solid; the gap would follow from the proportion between the inscribing sphere and the circumscribed sphere.²⁰ Kepler, anxious to make a name and career for himself, wrapped his discovery in all the trappings of *prisca sapientia*. He even sealed the historical continuity with a truly awkward neologism: *Copernicopythagoraea*, as in the Copernico-Pythagorean planetary sphere.²¹ The *Mysterium* is not only meant to defend and justify Copernicus by connecting his work to ancient wisdom, although this is certainly a goal. It is also designed to attract readers, patrons and allies, with the promise of ancient secrets revealed. In Kepler’s local context, he had his eye on ingratiating himself with Duke Friedrich, an impassioned patron of alchemy and natural collection. Kepler lobbied for the Duke’s attention by proposing a sculpture of his discovery, to be funded by the Duke himself for his own collection. The sculpture was a liquor bowl, of which each planetary orb would carry a different kind of alcohol. The Copernico-Pythagorean distances would be expressed in the dimensions of the orbs; meanwhile the use of liquor and precious stones would connect the cosmological with the alchemical.²² The *Mysterium* is as much a work of style and artifice as the never-completed bowl. Appeals to Pythagoreans, *prisca sapientia*, and Christian natural

¹⁹ KGW, vol. 1, 4.

²⁰ See chapter 1 of the *Mysterium* for a description of the polyhedral scheme, KGW, vol. 1, 14-21.

²¹ KGW, vol. 1, 7, ln. 34.

²² Jonathan Regier, “Stars, Crystals and Courts: Johannes Kepler and Anselmus Boëtius de Boodt,” in *Kepler’s New Star (1604): Context and Controversy*, ed. Patrick Boner (Leiden: Brill, 2020), 107-128, esp. 113-117.

theology, are all heated up together and allowed to coalesce. At the same time, Kepler affirms the singularity of Copernicus, foregrounding him against the Pythagorean tradition.

Indeed, if somebody wanted to receive these philosophical reasons [given in support of Copernican astronomy] with only a laugh and reject them without reason, because I offer this philosophy, a new man at the tail end of so many centuries, while all the ancient luminaries of philosophy remained silent, I would introduce him to my guide, master and precursor, who comes from the most ancient age: Pythagoras.²³

Kepler reminds his readers that everybody knows from school how Pythagoras had recognized, two thousand years beforehand, the importance of the regular solids, and had understood that these mathematical objects worked as organizing principles of the physical world: Pythagoras “judged it not unworthy of the Creator’s attention that He looked to [the five solids] and that He adapted non-mathematical things to mathematical things, with regard to the various accidental qualities [of those mathematical things].”²⁴ Obviously, Kepler is thinking first and foremost of the *Timaeus* here, since he goes on to list the roles assigned to the regular solids in that work, where each shape serves as the basis of one of five elements. Of course, Kepler’s use of the solids is quite different, and he takes a moment to note that, if the *Timaeus* incorrectly made the solids the basis of the elements, Pythagoras never had a Copernicus to tell him *quid esset in mundo* (“what was in the world”). Otherwise, says Kepler, there is no doubt that Pythagoras would have discovered the real use of the solids, just as there is no doubt that “this proportion of the heavens” would be as well-received as geocentric astronomy had proved to be.²⁵

If we take the above at face value, then Kepler in the *Mysterium* endorses the *Timaeus* at the most essential level—that of a mathematician God and a mathematical template or archetype for the cosmos—but believes the Pythagoreans to have lacked astronomical sophistication. However, his opinion changed over time. His masterwork of *a priori* astronomy and natural philosophy was the *Harmonice mundi* (1619). There, he embraces another narrative, that the *Timaeus* encodes in the form of a riddle (*sub aenigmate*) the true cosmic

²³ KGW, vol. 1, 26, ln. 30-34: “Nam si quis philosophicas istas rationes, sine rationibus, et solo risu excipere atque eludere voluerit: propterea quòd nouus homo sub finem seculorum, tacentibus illis Philosophiae luminibus antiquis, philosophica ista proferam: illi ego ducem, autorem et praemonstratorem ex antiquissimo seculo proferam PYTHAGORAM.”

²⁴ Ibid., ln. 36-39: “[...] Creatoris cura non indignum censuerit ad illa respicere: atque rebus mathematicis physice, et ex sua qualibet proprietate accidentaria censitis, res non mathematicas accommodauerit.”

²⁵ Ibid., 27, ln. 2-7.

doctrine, concealing that doctrine from the uninitiated hoard whose ranks included a baffled Aristotle:

Therefore that in the secrets of the Pythagoreans on this basis the five figures were distributed not among the elements, as Aristotle believed, but among the planets themselves is very strongly confirmed by the fact that Proclus tells us that the aim of geometry is to tell how the heaven has received appropriate figures for definite parts of itself.²⁶

Hence, the cube did not actually refer to the element earth, but to the planet most at rest, Saturn. Air, which was assigned the octahedron, actually represented fleet-footed Mercury; Kepler had the octahedron separating Mercury and Venus. Venus could be “hidden” under the “disguise of water,” because Venus rules over liquids and was born from sea-foam. The “world,” or the dodecahedron, quite sensibly signified the Earth, its twelve faces representing the signs of the zodiac, which encompassed the Earth’s course through the heavens. Kepler loved playing games with geometry, with symbols and interlocking analogies.²⁷ He famously thought that God had played such games at creation, meaning that Pythagorean riddles of disguise and revelation imitated the original *ludi Dei* and so carried a certain divine quality.²⁸ We might ask why Kepler had changed his view on the *Timaeus* in the period between the *Mysterium* and the *Harmonice*. This was almost certainly the result of reading Proclus’s commentary on Book I of Euclid’s *Elements*, a point we will take up momentarily.

When we survey the entirety of Kepler’s work, we can conclude that the effective Pythagorean corpus at his disposal, as he saw it, was defined by four touchstones: Plato’s *Timaeus*, Euclid’s *Optics* and *Elements*, and Proclus’s commentary on the *Elements*. These works bolstered one another, spoke to one another, and they created a stable foundation for Kepler to build up a mathematical epistemology and ontology. It may be peculiar for a reader now to see Euclid’s *Optics* in this list. From Kepler’s perspective, however, the choice makes sense. The fallacy of geo-centrism, Kepler thought, arose from a misattribution of motion that, for all intents and purposes, could be considered as a kind of optical illusion. In the *Optics*, he

²⁶ KGW, vol. 6, 18, ln. 17-21. Johannes Kepler, *The Harmony of the World*, trans. E. J. Aiton, A. M. Duncan, and J. V. Field (Philadelphia: Transactions of the American Philosophical Society, 1997), 12.

²⁷ See, for instance, his musings on the pentagon, the golden ratio, and sexual generation, in KGW, vol. 6, 175-176. Kepler, *The Harmony of the World*, 240-242.

²⁸ See Nicholas Jardine, “God’s ‘Ideal Reader’: Kepler and his Serious Jokes,” in *Johannes Kepler: From Tübingen to Zagan*, eds. Richard L. Kremer and Jarosław Włodarczyk (Warsaw: Studia Copernica, 2009), 41-51.

thought, Euclid had sought to deconstruct this illusion in the propositions treating what we would now call relative motion. At least to uninitiated eyes, the propositions singled out by Kepler look like no more than an analysis of how objects seem to move, given their distance from the observer and their relative speed. So, for instance, proposition 54 reads, “When objects move at equal speed, those more remote seem to move more slowly.”²⁹ Kepler took these propositions to be astronomical. In them, he writes, “Euclid propounded pure, unadulterated Copernican astronomy.”³⁰ Of course, it was the regular solids that really had Kepler sold on Euclid’s veiled heliocentrism. Kepler believed that the solids represented the *raison d’être* of the *Elements*, that the whole work existed to culminate in Book XIII, which was devoted to them. Kepler had good reason for this belief, since it came directly from Proclus, who, in his commentary to the *Elements*, surveys the history of Greek mathematics, writing that Pythagoras, among other fundamental contributions, “discovered the doctrine of proportionals and the structure of the cosmic figures.”³¹ When Proclus’s overview reaches Euclid and the *Elements*, Proclus asserts plainly that “[...] the whole of the geometer’s discourse is obviously concerned with the cosmic figures”,³² the progression of material in the *Elements* is structured to build up to the regular solids, to a detailed treatment of the solids individually, in relation to one another, and in relation to the sphere:

Hence some have thought it proper to interpret with reference to the cosmos the purposes of individual books and have inscribed above each of them the utility it has for a knowledge of the universe.³³

The *Elements* constitutes here not just a mathematical but *natural philosophical* treatise. This *grille de lecture* would have appealed to Kepler in the most profound way, for it tied the foundational work of geometry to the structure of the cosmos, giving mathematics pride of place in natural inquiry and mathematicians pride of place over their philosopher colleagues who had never mastered the ins and outs of geometry. In turn, Kepler’s *Harmonice mundi*

²⁹ Euclid, “The Optics of Euclid,” trans. Harry Edwin Burton, *Journal of the Optical Society of America* 55, n. 5: 357-372, 371.

³⁰ Johannes Kepler, *Paralipomena to Witelo & Optical Part of Astronomy*, trans. William H. Donahue (Santa Fe, New Mexico: Green Lion Press, 2000), 342.

³¹ Proclus, *A Commentary*, 53.

³² *Ibid.*, 70.

³³ *Ibid.*

represents an extensive and detailed attempt to advance this tradition of natural philosophy built from the cosmic solids and the sphere.³⁴

***Timaeus* and Trinity**

In a well-known moment of the *Harmonice*, Kepler claims the *Timaeus* to be “beyond any doubt” a companion to the first chapter of *Genesis*, which it “transforms (*transformans*) into Pythagorean Philosophy.”³⁵ This assertion appears in Book IV of the *Harmonice*, in a marginal comment to a long translation from Proclus’s commentary of the *Elements*. Kepler had done the translation himself, obviously considering its contents of prime significance, since the passage concerned the very nature of mathematical genera and species.³⁶ Kepler added a number of marginal and parenthetical remarks to Proclus’s chapter, in order to underline the consistency of Pythagorean philosophy with Christian doctrine.³⁷ What prompted him to highlight the accord between Plato and Moses is a passage depicting the mathematical creation of the soul, such that “everything mathematical is first of all in the soul.”³⁸ In another marginal comment, he notes that the mathematical ideas structuring creation were “coeternal” with God, by which he meant that they participate in the divine essence.³⁹ This was a quite standard Christian adaptation of Platonic idealism, from Augustine onward. Where, in the *Timaeus*, the efficient cause and formal cause of the world are split between demiurge and paradigm, Christian philosophers unified the two in God.⁴⁰

³⁴ See Jonathan Regier, “An Unfolding Geometry: Appropriating Proclus in the *Harmonice mundi* (1619),” in *Unifying Heaven and Earth: Essays in the History of Modern Cosmology*, eds. Miguel A. Granada, Patrick J. Boner and Dario Tessicini (Barcelona: Universitat de Barcelona, 2016), 217-237.

³⁵ KGW, vol. 6, 221. Kepler, *The Harmony of the World*, 301.

³⁶ Proclus, *A Commentary on the First Book of Euclid’s Elements*, 10-15. Jonathan Regier, “An Unfolding Geometry,” esp. 223-226.

³⁷ Elsewhere in the *Harmonice*, Kepler gives an even stronger statement on the agreement between Proclus’s philosophy and Christianity, suggesting that Proclus had taken and adapted “what the Christians held as most divine, and most in agreement with the Platonic philosophy.” Proclus had wished to reform both pagans and Christians, says Kepler, by leading both of them away from sensible things, the gentiles from the “visible Sun,” and the Christians from the “Son of Mary.” Proclus’s flaw, adds Kepler, had been to trust too much in the natural light of his mind, an interesting criticism, since Kepler had been accused of the same by his Lutheran brethren. KGW, vol. 6, 364, ln. 31-34. Kepler, *The Harmony of the World*, 493-494.

³⁸ KGW, vol. 6., 221, ln. 12. Kepler, *The Harmony of the World*, 301.

³⁹ KGW, vol. 6, 220. Kepler, *The Harmony of the World*, 298-302.

⁴⁰ Augustine located the Forms in the Divine Intellect, which he also identified as the Son of God, or second person of the Trinity. Sarah Catherine Byers, “Love, Will, and the Intellectual Ascents,” in *The Cambridge Companion to Augustin’s ‘Confessions’*, ed. by Tarmo Toom (Cambridge: Cambridge University Press, 2020), 154-174, 161-162. Augustine was directly influenced by Plato and Plotinus. Later Scholastics also adopted Neoplatonic views close to Augustine’s. Aquinas, for instance, has a rich theory of divine ideas or exemplars. See, Gregory T. Doolan, *Aquinas on the Divine Ideas as Exemplar Causes* (Washington, D.C.: The Catholic University of America Press, 2008).

Between the *Timaeus* and Kepler's own view of mathematics, there is a quite important difference to discuss. This difference shows us not only *what* Kepler did not accept of the *Timaeus*, but how he considered geometry a consequence of divine attributes. The difference to which I am alluding is how the philosopher can actually *use* mathematical things. Or, to put the issue otherwise, it is the way in which mathematics connects with God and manifests in the world. Here, we need to remember the role played by the regular solids in the *Timaeus*. They account for the structure of the elements and explain how elements transform into one another through a flux of planar triangles grouping and regrouping into the various regular solids. This is quite far from Kepler's use of the solids, or of geometry in general, especially in his mature *Harmonice* and *Epitome astronomiae Copernicanae* (1618-1621).⁴¹ Kepler is always on the hunt for *proportion*, for the relations that obtain between, say, actual volumes, distances, masses. We now take for granted that physics is relational, that the ground-level activity of physics is to establish mathematical correlations. Kepler did not have the luxury of this assumption. While mixed mathematics allowed for quantitative methods of natural inquiry, only a small minority of educated people would have considered fundamental causes in nature as inherently mathematical. The power of the *Timaeus* was that it offered a template for doing just this.⁴² Actually, it offered more than a template. It made a variety of mathematical entities fundamental to nature: numbers, geometrical figures, and ratios. Some of these entities were not, in Kepler's mind, of any explanatory value. Numbers and quantities in general, he thought, were simply contingent (here, he rejected a key Pythagorean teaching). One could not explain why the cosmos held six planets rather than seven or eight on the basis of number alone; there was nothing special about six, or ten, or one hundred. Form needed to be imposed, and geometry provided that form. Thus, geometrical forms could provide mathematical causes, that is, certain forms could furnish governing proportions in nature. His position can be expressed rather bluntly: geometrical objects are wholes whose parts are related by set proportions. The proportions internal to the regular solids account for the distances between planets. Even the fact that no more than five such regular solids can exist explains why God decided on six planets (one solid "between" each pair of planets). Hence, Kepler's project required that we recognize the importance or nobility of certain geometrical forms: the regular solids (because

⁴¹ Kepler, on the other hand, has almost no matter theory. On the issue of his matter theory, and how it connects with his views on geometry, see Johann Kepler, *L'éternelle ou la neige sexangulaire*, trans. Robert Halleux (Paris: Vrin, 1977); Jonathan Regier "Stars, Crystals and Courts: Johannes Kepler and Anselmus Boëtius de Boodt."

⁴² Martens makes a similar argument. Martens "A Commentary on Genesis: Plato's *Timaeus* and Kepler's Astronomy," 262.

of their internal uniformities) and most of all the sphere. The sphere was maximally simple and regular, the primary form from which all geometry flowed: “As God is the norm and measure of created things, so is the Sphere for geometrical bodies.”⁴³

From his earliest writings, Kepler asserted in the most unambiguous terms that the sphere was the very image of the Trinity. Kepler’s entire philosophy might, at base, be called “Trinitarian.” In his early letters he speaks of how geometry and nature are both founded on the Trinity. The sphere serves as the Trinitarian image *par excellence*, specifically because it represents the uniqueness, unity and equality of Trinitarian Persons. As he puts it in a significant 1595 letter to his confidant and former professor of mathematics at Tübingen, Michael Maestlin:

For before the world there was no number beyond the Trinity, which is God Himself. [...] The Trinity, then, is in the globe: the spherical [surface], the center, the volume. Thus, in the immobile world: the fixed stars, the sun, and the *aura* or intermediate aether. And in the Trinity: Father, Son, Spirit.⁴⁴

Kepler was fond of his sphere-as-Trinity idea, and he brought it up in works throughout his career, first publishing it in the preface of the *Mysterium*. The sphere, he thought, captured both the uniqueness of the Persons and their inherent inseparability. Three entities constitute the sphere—center, surface, the volume—, and should any of these be removed, the whole and parts would cease to exist. The sphere, then, is one yet three: “none of them, even in thought, can be absent without destroying the whole.”⁴⁵ Kepler saw the center as the Father, the surface as the Son, and the volume as the Holy Spirit, that equal *σχέσις* or relation between the point and circumference.⁴⁶ Hence, in establishing a spherical universe, just so, God relied on the Trinity, his own essence. Kepler only quotes the *Timaeus* once in his first work, the *Mysterium*: “*Fas enim nec est nec unquam fuit (ut loquitur ex Timaeo Platonis Cicero in libro de universitate) quicquam nisi pulcherrimum facere eum, qui esset optimus.*”⁴⁷ The best creator can create nothing but the most beautiful cosmos. God, Kepler asserts, by necessity drew from

⁴³ “*Sicut enim norma et regula creaturarum Deus est: sic Sphaera corporum.*” KGW, vol. 1, 27, ln. 35-36.

⁴⁴ “*Nam ante mundum nullus erat numerus, praeter Trinitatem, quae est ipse deus. [...] In globo igitur est trinitas, Sphaericum, Centrum, Capacitas. Sic in mundo quieto: Fixae, Sol, Aura, sive aethra intermedia: Et in trinitate Filius, Pater, Spiritus.*” KGW, vol. 13, letter 23 [3 October, 1595], 35, ln. 55-57 and 72-74.

⁴⁵ Johannes Kepler, *Paralipomena to Witelo & Optical Part of Astronomy*, 19.

⁴⁶ “*Σχέσις*” was in theology associated with the relation between the Persons of Trinity. KGW, vol. 1, 23, ln. 20-22.

⁴⁷ KGW, vol. 1, 23 (ln. 35) - 24 (ln. 1).

what was best, His Trinitarian essence. Hence the sphere, from which all geometry flows, serves as the first organizing principle of nature. What we find here, perhaps, is Kepler reconsidering the *Timaeus* in light of a long tradition of Christian mathematical theology, whose great Renaissance exponent was Nicholas of Cusa, a major early inspiration for Kepler.⁴⁸ A difference with Cusa, whose theology was ultimately apophatic, is that Kepler advocated rational-geometrical access to the Trinity, just as he believed that reason and geometry comment meaningfully on the mystery of the Eucharist.⁴⁹ In this, he stood once again on the side of the Melanchthonians, who defended the importance of reason and philosophy in theology, often against the fideist wing of Lutheranism.⁵⁰ Kepler actually goes much farther than Philip Melanchthon, who had even chastised Augustine for finding echoes of Christian revelation in Platonic philosophy, and who was suspicious of philosophical approaches to the Trinity.⁵¹ Kepler fully “Pythagoreanized” the Lutheran vein of natural theology.

Almost from the beginning, Trinitarian doctrine, indebted to middle Platonism and Plotinus, was intertwined with the *Timaeus*. Christian philosophers and theologians could read the *Timaeus* as expressing the three-ness of the Creator: the demiurge as Father, the model as Son, and the world soul as Holy Spirit. In the medieval period, the most famous or notorious Trinitarian readings of the *Timaeus* were those of twelfth-century Chartres, especially those of William of Conches and Thierry of Chartres.⁵² Twelfth century Platonist theology, judged by its monastic opponents, ran the risk of heresy, of identifying God with nature. The specific danger was to identify the Holy Spirit with the *anima mundi*, or, as William of Saint-Thierry put it, to assert that God’s presence in the world reduced to the “*concursum elementorum et*

⁴⁸ For a history of Christian Neopythagoreanism, see David Albertson, *Mathematical Theologies: Nicholas of Cusa and the Legacy of Thierry of Chartres* (Oxford: Oxford University Press, 2014).

⁴⁹ Aviva Rothman, *The Pursuit of Harmony* (Chicago: University of Chicago Press, 2017), 70-108.

⁵⁰ For an introduction to Kepler and Melanchthon on natural light, see Peter Barker and Bernard R. Goldstein, “Theological Foundations of Kepler’s Astronomy,” *Osiris* 16, 2001: 88-113. Around 1600, a revealing conflict broke out among, to be a bit reductive, Melanchthonians and fideists at the University of Helmstedt; see Markus Friedrich, *Die Grenzen der Vernunft: Theologie, Philosophie und gelehrte Konflikte am Beispiel des Helmstedter Hofmannstreits und seiner Wirkungen auf das Luthertum um 1600* (Göttingen: Vandenhoeck & Ruprecht, 2004).

⁵¹ Philip Melanchthon, *Orations on Philosophy and Education*, ed. Sachiko Kusukawa, trans. Christine F. Salazar (Cambridge, Cambridge University Press 1999), 25. Melanchthon was devoted to a biblical grounding of the Trinity and generally rejected philosophical speculation on the subject. However, he did from around 1527 show a tentative acceptance of Augustine’s mind-as-Trinity analogy. For Melanchthon and the Trinity as accessible to reason, see Samuel M. Powell, *The Trinity in German Thought* (Cambridge: Cambridge University Press, 2000), 12-30, esp. 26-28.

⁵² Two studies that have been particularly useful to me in understanding the history of mathematical and natural-philosophical views of the Trinity are the following: Tulio Gregory, *Anima mundi. La filosofia di Guglielmo di Conches e la Scuola di Chartres* (Rome: Pubblicazioni dell’Istituto di filosofia dell’Università di Roma, 1955); David Albertson, *Mathematical Theologies*.

temperaturam naturae,” the assembly of elements and constitution of nature.⁵³ The *Timaeus* also, of course, suggested a disunity of the Three, and even a relative hierarchy. Ficino, in his revival of Platonic philosophy, sought to diffuse this particular worry:

The Platonists often call the Good 'the father,' Intellect 'the son,' and the World-Soul 'spirit'-'Spirit nourishes all' [*Aeneid* 6. 726]. But if they mean their trinity to be of the same substance, they are, as it were Catholic Christians; but if they mean it to be three substances, they are Arians virtually.⁵⁴

It might be argued that there was a naturalization of the Holy Spirit in sixteenth-century natural philosophy, an identification of the Holy Spirit with the Platonic world soul, such as it is found in the *Timaeus* or in the celebrated passage from Book 6 of the *Aeneid*, which describes an animating spirit within nature and was widely thought to express Pythagorean doctrine. Two important exemplars of this current both died at the stake as heretics, Michel Servet and Giordano Bruno, one executed by the Calvinists in Geneva, the other by the Inquisition in Rome.⁵⁵ In a 1592 attestation that Bruno gave to Venetian authorities, shortly after his arrest, he explains that he was never able to understand the Holy Spirit according to the faith (*il modo che si deve credere*), that is, as the Third Person. Instead, he understood it “following the Pythagorean way” as “the soul of the universe, or helpmate to the universe.” He then quotes, presumably from memory, “the Pythagorean doctrine explained by Virgil in the sixth book of the *Aeneid*.”⁵⁶ Kepler quotes the very same passage in his 1606 *De stella nova*, referring to it as Pythagorean, in order to explain how new stars could form in the heavens.⁵⁷ Kepler does not believe in a soul literally filling and animating the entire universe, such as we find it in the *Timaeus* or *Aeneid*. In Kepler’s view, souls are situated within concrete bodies, as in the Earth or Sun; these souls fill space with generative light and heat, rather like medical spirits fill the bodies of animals. The most important soul, of course, is the Sun’s, whose *spiritus*, as light and magnetism, illuminates the world and make it turn. In this sense Kepler modifies the *Timaeus*, giving it a Galenic twist. To the extent that there is a world soul, it is lodged in the Sun, whose

⁵³ Gregory, *Anima mundi*, 151.

⁵⁴ Michael J. B. Allen, “Marsilio Ficino on Plato, the Neoplatonists and the Christian Doctrine of the Trinity,” *Renaissance Quarterly* 37, n. 4, 1984: 555-584, 563-564.

⁵⁵ On Servet and the Holy Spirit, see D. P. Walker, “Medical Spirits and God and the Soul,” in *Spiritus. IVo Colloquio Internazionale Roma, 7-9 gennaio 1983*, eds. Marta Fattori and M. Bianchi (Rome: Edizioni dell'Ateneo, 1984), 223-44, 230-233. On Bruno, see below.

⁵⁶ Luigi Firpo, *Il processo di Giordano Bruno* (Rome: Salerno, 1993), 169.

⁵⁷ KGW, vol. 1, 267, ln. 21-26.

light and magnetic force reach outward and turn the planets in a long cosmic symphony composed by God before creation.⁵⁸ One reason why Kepler might have preferred this scheme is that it allows him to cast God as an artist standing outside of nature—a composer or clockmaker (albeit a clockmaker working with souls). Elsewhere in the *De stella nova*, he criticizes the ancient Pythagoreans for conflating God and *mundus*.⁵⁹ Kepler's God transcends nature, although his image is reflected throughout the natural world as in a hall of mirrors. It is in this sense—the Trinity as a structuring *imago*—that we find in Kepler's thought the risky comparison that Christian Platonists had long made between *anima mundi* and *Spiritus Sanctus*. In a letter of 1595, right at the beginning of his career, we find him transforming the old *anima mundi* of the Christian *Timaeus* into a quantifiable solar force that would serve as the foundation of his *astronomia nova*, his new astronomy:

In this way, then, the Sun, itself at rest in the middle and yet the fount of motion, carries the image of God the Father and creator. For what creation is to God, motion is to the Sun. Moreover, it moves [the planets] in a fixed place, as the Father creates in the Son. Unless the fixed stars offered a place, thanks to their motionlessness, no movement could exist. I defended this axiom while still in Tübingen. The Sun distributes motive virtue through the medium space, in which the planets are found: just as the Father creates by spirit or by the virtue of His spirit. And from the necessity of these presuppositions, it follows that motion is in proportion with distance.⁶⁰

Kepler, it should be remembered, revolutionized astronomy by making it an inquiry into the causes and forces that drive celestial motions. He was the first astronomer to consider the Sun as the seat of a force turning the planets, an insight fueling the reformation of astronomy which he put forth in his *Astronomia nova* and *Epitome*. It is fascinating to witness how, early on, he

⁵⁸ “For it seems that (if there is some such world soul) it resides in the center of the world, which for me is the Sun, and from there it is propagated over the length and breadth of [the world] by the agency of the rays of light, which are equivalent to spirits in the animate body.” KGW, vi, 265, ln. 20-24. Kepler, *Harmony of the World*, 359.

⁵⁹ KGW, vol. 1, 284, ln. 2: “[...] ipsum Mundum Deum facit cum Pythagoraeis veteribus.”

⁶⁰ KGW, vol. 13, letter 23, 35, ln. 78-86: “Sic igitur Sol in medio mobilium quietus ipse et tamen fons motus gerit imaginem Dei patris creatoris. Nam quod est deo creatio, hoc est Solis motus. Movet autem in fixis, ut pater in filio creat. Fixae enim nisi locum praebent sua quiete, nihil movere possent. Quod axioma etiam dum Tubingae tenui. Dispertitur autem Sol virtutem motus per medium, in quo sunt mobilia: sicut pater per spiritum, vel virtute spiritus sui creat. Atque jam ex necessitate praesuppositorum, sequitur, motum cum distantijs in proportione esse.”

saw the existence of a solar force as motivated or at least justified by a Trinitarian paradigm which, albeit indirectly, drew from the long tradition of the Christian reception of the *Timaeus*.

Conclusion

If the *Timaeus* was a landmark and a reference for early modern readers eager to see nature as inherently mathematical, Kepler shows us how it could function largely at the level of inspiration: it offered a cohesive picture that was both epistemological and ontological, wherein mathematics was equally innate and external, woven into the fabric of the soul and the world. The *Timaeus* is a complex text, but Kepler did not engage with its details. What interested him was straightforward enough: a mathematician God, a mathematical soul, and geometry (specifically, the regular solids) as fundamental to nature. Yet his reception of the *Timaeus* was by no means simple. In this essay, I have tried to capture how it was mediated—indeed, saturated—by his appreciation of later authors, his reconstruction of the Pythagorean tradition and of Copernicus's place within that tradition, and his own unique metaphysical and theological views. I began this chapter with a few words on the question of novelty. Kepler's best known treatise, the *Astronomia nova* (1609), openly embraced newness: it was to be new kind of astronomy, at once Copernican and engaged with the question of physical forces and material constitutions. It was also very much an empirical work, based on close examination of the highly accurate observations accrued over decades by Tycho Brahe and his team. In parallel, Kepler carried out another kind of astronomical and natural-philosophical inquiry, one that connected astronomical novelty to the ancient Pythagorean tradition. In doing so, he of course gave us his own reconstruction of that tradition, while recognizing how times had changed, how far astronomy had come in the meantime, how its methods and observations had advanced, particularly in the century or so that had preceded him. "It is my pleasure," he writes in the *Harmonice*, "to taunt mortal men with the candid acknowledgment that I am stealing the golden vessels of the Egyptians to build a tabernacle to my God from them, far, far away from the boundaries of Egypt."⁶¹ This image—of melting down idols to create a new temple—might best encapsulate how the old and the new relate in his thought. The ancient Pythagorean insights are to be reduced to their essence and recast afresh into novel patterns and shapes, the same material, but brighter and truer.

⁶¹ KGW, vol. 6, 290, ln. 3-6. Kepler, *The Harmony of the World*, 391.

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