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# Technological change, campaign spending and polarization ☆

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ABSTRACT

We present a model of electoral competition with endogenous platforms and campaign spending where the division of voters between impressionable and ideological is also endogenous and depends on parties' strategic platform choices. Our approach results in a tractable model that provides interesting comparative statics on the effect of recent technological advancements. For instance, we can accommodate a new justification behind the well-documented simultaneous increase in campaign spending and polarization: an increase in the effectiveness of electoral advertising, or a decrease in the electorate's political awareness, surely increases polarization and may also increase campaign spending.



## 1. Introduction

Political campaigns worldwide, shaped by institutional reforms and technological advancements, have experienced significant changes in recent decades. In the United States, the introduction of national TV gave place to the advertising industry and allowed for a gradual introduction of candidate-centered professional campaigns (Hirano and Snyder, 2019). Similarly, the spread of the internet gave place to the boom of social networks, which became voters' favorite platforms to obtain information about candidates, and vice versa.[1] Coincidentally, as these technological improvements were incorporated into campaign management strategies, there was a simultaneous increase in campaign spending and polarization in the US.[2] While it is often argued that such increases could be attributed to technological improvements, such as the fine targeting possibilities that came along with the internet and social media, spatial models of electoral competition typically do not support this argument (e.g., Herrera et al., 2008; Carrillo and Castanheira, 2008; Ashworth and Bueno de Mesquita, 2009).[3] Hence, in this paper, we

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[1] More than 60% of adults in the United States used the internet to watch news by 2016 and one year later, adults used the internet almost as often as TV to obtain news (Gottfried and Shearer, 2016; Gottfried and Shearer, 2017; Broockman and Green, 2014).

[2] Polarization (measured using DW-Nominate scores) steadily increased since the end of the Second World War (Poole and Rosenthal, 1984), notably so since the 1980's (McCarty et al., 2006). Similar trends are also reported in other OECD countries (Boxell et al., 2021). Campaign spending in the US has also increased since the 1960s (in real terms), but specially so since the turn of the century (e.g., Herrera et al., 2008; Nickerson and Rogers, 2014).

[3] Nickerson and Rogers (2014) argue that the capacity for storing data is a turning point for campaign managers to turn to fine targeting of mobilization strategies and advertisements. Additionally, Allcott et al. (2020); Allcott and Gentzkow (2017), among others, show evidence that links technology and polarization, in particular the use of social networks like Facebook.

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provide a new channel for these stylized facts, with a comprehensive spatial model that depicts a causal relationship between technological advancements and the empirical observations above.

In more detail, we propose a model of campaign spending and polarization by combining the two seminal models of Downs (1957) and Tullock (1980). Two office-motivated parties first choose their electoral platforms and then decide upon the optimal level of costly electoral advertising. Voters instead are either impressionable and vote à la Tullock, or ideological and vote à la Downs, as in the seminal papers by Baron (1994) and Grossman and Helpman (1996). Ideological voters support the party that proposes the platform closest to their bliss point. Hence, parties compete for a share of ideological voters as if they were competing in a Downsian model of electoral competition. On the contrary, impressionable voters are swayed towards one party or the other by costly electoral advertising. Given each party's electoral advertising, the effectiveness of the latter determines the fraction of impressionable voters that supports each of them.

An important novelty of our approach is that we allow for the split of voters (between impressionable and ideological) to be endogenous. The endogenous division of voters across the two types depends on the differentiation between the proposed platforms, with the fraction of ideological voters increasing in polarization. This assumption captures the idea that the more diverse platforms are, the more voters vote based on their ideological preferences since platforms become salient. On the contrary, when parties' platforms are similar, voters may have a hard time or little interest in distinguishing them, and turn to electoral advertising that determines (probabilistically) their voting behavior.[4] By endogenizing voters' types, we maintain parties' well-established incentives to strategically differentiate (polarize) and soften the advertising stage as in the endogenous valence literature with office-motivated candidates. In contrast to most of these models – that we later discuss in detail– our model has several attractive features: (i) equilibrium platforms are unique and in pure strategies when parties are symmetric, (ii) provides a tractable setting for the analysis of asymmetric parties (e.g., in their marginal cost of campaigning), and (iii) permits us to analyze three distinct channels through which technology may affect electoral competition.

The above sketched model encompasses the effect of technological changes on electoral competition through three distinct and non-mutually exclusive channels. The first two reflect the way campaigns for impressionable voters are conducted, and how technology and changes in campaign management affect (a) the *effectiveness*, and (b) the *marginal cost* of electoral advertising. One could reasonably argue that recent technological advances have increased the effectiveness of electoral advertising, since campaigns can be well targeted, and have decreased the marginal cost of advertising, given the possibility of reaching large masses. The third channel captures how technology affects electoral competition through the electorate's political awareness and the endogenous division of voters into impressionable and ideological. Political awareness is captured by the *conversion rate* at which impressionable voters become ideological as polarization increases. Although there is no consensus on the historical evolution of this conversion rate (or the citizen's awareness), we show how it affects electoral competition and discuss its implications.

[4] While intuitive, one way to formalize this behavior is that voters' preferences are described by a lexicographic semiorder (see Luce, 1956; Tversky, 1969; Rubinstein, 1988; Leland, 1994; Manzini and Mariotti, 2012). With such preferences, each individual chooses which party to support on the basis of platforms, but only if those are different "enough" (i.e., above a certain threshold). If the platforms are not sufficiently different, then the voter is influenced exclusively by parties' electoral advertising.

Consider first an increase in the *effectiveness* of electoral advertising. Since every dollar spent on campaigns leads to higher returns, parties have incentives to symmetrically increase their campaign spending (*spending effect*). To mitigate such an increase in electoral advertising, parties have incentives to polarize their platforms and reduce the number of impressionable voters, hence their spending (*polarization effect*). If the spending effect dominates, campaign spending and polarization increase simultaneously and can explain the observed trends in US politics. If instead the polarization effect dominates, an increase in electoral effectiveness is overcompensated by an increase in polarization and campaign spending decreases. Actually, the effectiveness of electoral advertising proves crucial if one wants to explain the simultaneous increase in campaign spending and polarization in terms of campaign technology. A decrease in the marginal *costs* of running a campaign does not affect either polarization or total spending. While a decrease in the marginal cost of advertising leads to more advertising, the lower marginal cost of the latter leaves campaign spending and polarization unaffected. Finally, political awareness, captured by the *conversion rate* at which impressionable voters become ideological when polarization increases, affects both campaign spending and polarization in a similar manner as a change in the effectiveness. If one wants to explain the simultaneous increase in polarization and spending exclusively through this channel, then one would require that the conversion rate has been decreasing, meaning that voters do not respond much to changes in platforms. Such a low conversion rate could be attributed for example to a "media malaise", possibly associated with mistrust of politicians and disenchantment with politics (Norris, 2000; Newton, 1999).

### 1.1. Related Literature

Downsian models of electoral competition have incorporated a vertical dimension of parties' differentiation that is commonly appreciated by all voters independently of any ideological concerns. This vertical dimension is termed as valence (Stokes, 1963), and while originally modeled as exogenous (e.g., Groseclose, 2001; Aragonès and Palfrey, 2002), subsequent work allowed candidates to spend resources to improve their performance in this dimension. As in our setup, endogenous valence models have been employed to analyze questions related to campaign spending and polarization.

In the closest works to ours where candidates are *office motivated* (e.g., Ashworth and Bueno de Mesquita, 2009; Zakharov, 2009), platform diversification softens competition in the valence accumulation stage. These well known dynamics of strategic differentiation (Tirole, 1988) are exactly the ones presented in our model through the endogenous division of voters between ideological and impressionable. Our model, however, proves more tractable than the aforementioned valence models with additive separable preferences over the horizontal and vertical dimension. In contrast to Ashworth and Bueno de Mesquita (2009), our model permits a pure strategy equilibrium in the platform stage for most parametrizations and does not require that voters' ideologies are uniformly distributed. A non-uniform distribution is also permitted in Zakharov (2009) but only when focusing on local Nash equilibria. In contrast, we are able to characterize Nash equilibria in pure strategies for a general distribution of voters' ideology (symmetric and log-concave) and perform relevant comparative statics. Also in contrast to the previous models, we are able to obtain results when parties have heterogeneous campaign costs, for example, due to an incumbency advantage (Meirowitz, 2008; Pastine and Pastine, 2012).

In the closest work to ours in terms of modeling *campaign technology*, we complement Herrera et al. (2008) and Hirsch (2019), where parties are (at least partially) *policy motivated* and incentives to polarize are very different than in our model. These two papers explicitly model campaign effectiveness and hence go beyond the usual comparative statics of a change in the marginal cost of valence accumulation (e.g., Serra, 2010; Ashworth and Bueno de Mesquita, 2009). However, for a given technology, in Herrera et al. (2008) and Hirsch (2019) higher levels of polarization induce higher spending, similar to other endogenous valence models with policy motives (e.g., Serra, 2010; Epstein and Nitzan, 2004; Cardona et al., 2018; among others). Instead, in our model polarization explicitly arises to reduce spending —as in the above-cited models with office motives.

The distinct underlying mechanisms in our paper compared to Herrera et al. (2008) and Hirsch (2019) imply that our papers differ significantly in terms of results when the technology varies. Contrary to us, in Herrera et al. (2008) an improved campaign technology reduces polarization. Same as in our model, when the effectiveness of campaign spending increases, parties anticipate greater spending that they try to mitigate in the platform selection stage. In Herrera et al. (2008), this is achieved by proposing moderate platforms and hence less polarization. Hirsch (2019) however also explains the simultaneous increase in polarization and campaign spending through an improved campaign technology, but there are several distinctive results between his paper and ours. For example, we obtain non-monotonicity in campaign spending (similar to Herrera et al., 2008), equilibrium platforms in pure strategies, and the possibility of platform convergence in equilibrium. Moreover, the way these three papers motivate the campaign technology is different, each incorporating a parallel but different version of campaign effectiveness or targeting. Finally, our work also highlights the importance of the electorate's political awareness, a channel that to the best of our knowledge has largely been unexplored.

Without aiming to review the lobbying literature in detail, among spatial models of electoral competition, lobbying models following Baron (1994) and Grossman and Helpman (1996) have also analyzed the joint determinants of polarization and campaign spending. Broadly speaking, lobbies can pull candidates to the extremes in exchange for resources that can be in turn spent to convince impressionable voters to vote for them. While this literature focuses on the interaction between the lobbyist and the party (e.g., see Rivas, 2020 and references therein), the links of these models to campaign technologies as a driver of polarization and campaign spending are less pronounced than in our setting (or in Herrera et al., 2008; Hirsch, 2019). Moreover, we contribute to this literature by endogenizing the share of impressionable and ideological voters.

Prummer (2020) also focuses on campaign spending and polarization but the analysis is performed in a non-spatial setting, focusing on changes in targeting technology and fragmentation of media networks as determinants of polarization. By characterizing candidates' optimal policy proposals and corresponding targeting strategies, Prummer (2020) shows that recent improvements in targeting technology could have led to candidates opting for extreme policy proposals that target a narrow subset of voters.

Finally, we contribute to a recent growing literature on behavioral political economy by interpreting our model as one of endogenous valence where, rather than additive separable preferences over platforms and campaigns (e.g., Carrillo and Castanheira, 2008; Zakharov, 2009; Ashworth and Bueno de Mesquita, 2009; Iaryczower and Mattozzi, 2013), individuals have semiorder lexicographic preferences (see Luce, 1956; Tversky, 1969; Rubinstein, 1988; Leland, 1994; Manzini and Mariotti, 2012). This assumption is a particular case of salience models where in general decision makers overweight attributes that exhibit greater differences in the available choice set (Bordalo et al., 2012; Bordalo et al., 2013a; Bordalo et al., 2013b; Bordalo et al., 2015; Bushong et al., 2021; Köszegi and Szeidl, 2012). Callander and Wilson (2006) and Nunnari and Zápal (2017) introduce related context-dependent preferences in political economy models. Instead, Amorós and Puy (2013, 2015, 2020) assume that parties have the ability to affect the relative salience of different issues via their strategic actions (e.g., allocation of time or effort). In our model, parties' strategic actions affect the salience of platforms versus advertising and hence whether citizens vote in an ideological or impressionable manner. Therefore, although very different in nature, our model links with recent literature where some voters may be partially informed regarding parties' policy proposals (Aragonès and Xefteris, 2017; Eguia and Nicolò, 2019).

## 2. Model

Let two political parties $i \in \{L, R\}$ first fix platforms $x_i$ in the policy space $X = \mathbb{R}$ and then choose the level of campaign advertising $e_i \geqslant 0$. Without loss of generality, we assume $x_L \leqslant x_R$. Let $S_i(x_L, x_R, e_L, e_R)$be the vote share for party $i$ and $c_i(e_i) = \mu_i e_i$ the cost of advertising, with $\mu_i > 0$ denoting the constant marginal *cost of advertising*. Without loss of generality, we assume $\mu_L \leqslant \mu_R$. Parties' are office motivated with payoffs $\Pi_i = S_i(x_L, x_R, e_L, e_R) - c_i(e_i)$.[5]

Voters have a preferred policy $x$ drawn from distribution $G(x)$ with corresponding density $g(x)$ symmetric and log-concave (i.e., $(\ln g(x))'\prime \leqslant 0$), with full support in X. Independent of their ideal policy, some voters are ideological and some are impressionable. The ideological citizens vote sincerely for the party whose proposed platform is closer to them and split their vote if indifferent (à *la Downs*). The utility of a voter with ideology $x$ that votes for party $i$ is $u_x(i) = -|x - x_i|$.[6]

The impressionable citizens' vote depends only on electoral advertising. In particular, we assume that, given parties' advertising, the probability that an impressionable citizen votes for party $i$ is determined à *la Tullock* and hence equal to $e_i^\eta / (e_L^\eta + e_R^\eta)$ for $e_L + e_R > 0$ and equal to $\frac{1}{2}$ in case of indeterminacy ($e_L = e_R = 0$). The parameter $\eta > 0$ captures the *effectiveness of electoral advertising*.[7] Parameter $\eta$ captures how relative differences in electoral spending translate into a difference in vote shares. In other words, an increase in $\eta$ makes a given difference in spending more determinant of the electoral outcome, permitting us to interpret $\eta$ as the effectiveness of electoral advertising. If $\eta \to 0$, impressionable voters split equally across the two parties for any level of electoral spending. However, as $\eta$ increases, the allocation of impressionable voters across parties becomes more responsive to electoral advertising. Impressionable voters voting on the basis of persuasive electoral advertising is a standard assumption in this literature (see, for example, seminal papers by Baron (1994) and Grossman and Helpman (1996), and a large literature thereafter). The specific proposed function is the seminal contest success function (CSF) introduced by Tullock (1980). This function is extensively used in the literature and, apart from

[5] Here each party's objective is to maximize its vote share net of campaign costs. Alternatively, $S_i(x_L, x_R, e_L, e_R)$ can also be interpreted as the probability of winning by assuming parties' uncertainty on a representative voter's preferences as in Aragonès and Xefteris (2017).

[6] The assumption of a particular distance function is made without loss of generality.

[7] We assume $\eta > 0$ to focus on cases in which our model differs from a Downsian model. If $\eta = 0$, parties have no incentives to spend resources on advertising and so the vertical dimension of the model vanishes.

tractability, satisfies relevant axiomatic properties (Skaperdas, 1996). It is micro-founded in Appendix C.[8]

The endogenous division of voters across ideological and impressionable depends on the level of polarization. Let $y = x_R - x_L$ be the platforms' polarization and $F(y)$ a continuous cumulative distribution function, log-concave (i.e., $(\ln F(y))'' \leqslant 0$), with corresponding density $f(y)$ and full support in $[0, \bar{y}]$, where $\bar{y} > 0$ and $F(y) = 1$ for all $y \geqslant \bar{y}$.[9] The share of ideological voters is $F(y)$, and therefore the share of impressionable voters is $1 - F(y)$.[10] The upper bound $\bar{y}$ on the support of $F(y)$ determines the level of polarization above which all voters are ideological and vote à la Downs. The log-concavity of $F(y)$, one of the important assumptions in our model, implies that the conversion rate of impressionable voters to ideological $f(y)/F(y)$ is decreasing in $y$. Thus, incentives to polarize are higher the closer the two platforms are.

It is important to stress that the above representation of our model in aggregate terms and the division of voters according to $F(y)$ is enough to understand the dynamics in our model. For presentation purposes, and to avoid further notation, we use directly $F(y)$ as the foundation of our model. However, in the Appendix C we detail how this division can be microfounded through voters' individual behavior. In brief, consider voters with semiorder lexicographic preferences. Let the first attribute reflect policies and the second attribute electoral advertising. Each voter draws a preferred policy $x$ from $G(x)$ and a level of "sensitivity" $\phi$ from $F_\phi(y) = Pr(\phi \leqslant y)$, denoted $F(y)$ for simplicity. That is, as platforms diverge, an individual voter is ideological with ex-ante probability $F(y)$, and impressionable with ex-ante probability $1 - F(y)$. Hence, despite the stark partition of voters into ideological and impressionable presented in the main text, an individual voter's expected behavior *a priori* depends both on platforms and advertising. The microfoundations of individuals' behavior (in terms of semiorder lexicographic preferences as above and in terms of a salience model) and the effect of advertising on individual voting behavior (as well as the derivation of the Tullock CSF) are detailed in Appendix C.

The timing of the game is as follows: At $t = 1$, the political parties simultaneously choose the political platforms that maximize their payoff. At $t = 2$, having observed the platform choices and the share of impressionable voters determined by the polarization, parties choose the advertising levels. Finally, at $t = 3$, voters vote. Given the nature of our game, we focus on subgame perfect Nash equilibria (SPNE).

## 3. Results

Given the described game, let $\bar{x} = \frac{x_L + x_R}{2}$ be the indifferent ideological voter for $x_L \neq x_R$. Ideological voters with $x \leqslant \bar{x}$ vote for $L$, while the remaining vote for $R$. Thus, party $L$ obtains a share $S^L_{Idl} = G(\bar{x})$ of the ideological voters and party $R$, $S^R_{Idl} = 1 - G(\bar{x})$. If $x_L = x_R$, then $S^R_{Idl} = S^L_{Idl} = \frac{1}{2}$. Given that the individual probability that an impressionable citizen votes for party $i$ is determined à la Tullock, the expected share of impressionable votes to party $i$ is $S^i_{Imp} = \frac{e_i^\eta}{e_L^\eta + e_R^\eta}$ whenever $e_L + e_R > 0$ (or $S^i_{Imp} = \frac{1}{2}$ for the case $e_L = e_R = 0$). Hence, the expected vote share obtained by the parties can be then written as a weighted average of the previous two:

$$S_i(x_L, x_R, e_L, e_R) = F(y)S^i_{Idl}(x_L, x_R) + (1 - F(y))S^i_{Imp}(e_L, e_R). \quad (1)$$

This expression highlights the effect of platform choices in our game. First, platform choice affects how ideological voters split between the parties (via $S^i_{Idl}(x_L, x_R)$). Second, platform choice affects the ideological-impressionable composition of the electorate (via $F(y)$). As common in Downsian type models, converging towards the opponent is beneficial due to the relocation of the indifferent voter. However, in our model, platform convergence results in an increase in the share of impressionable voters, and hence a tougher competition in the (costly) advertising stage. The above trade-off is determinant for platforms' choices.

### 3.1. Symmetric parties

For illustrative purposes, and to highlight our main results in the simplest framework, we first pay attention to parties having identical marginal costs, i.e., $\mu_A = \mu_B = \mu$. Since the voters' behavior is unambiguous in this model, the last stage in our backward induction reasoning is the choice of advertising. Equilibrium advertising can be solved as effort in a Tullock contest with symmetric players, in which the prize of winning equals the share of impressionable voters. The equilibrium in this stage is described in the lemma below.

**Lemma 1.** For all $\eta \leqslant 2$ there exists a unique Nash equilibrium in the campaign stage and advertising is given by $e_i^*(x_L, x_R) = (1 - F(x_R - x_L))\frac{\eta}{4\mu}$, for all $i$.

**All proofs appear in the Appendix.**

Our first Lemma draws from previous results in the contest theory literature. It characterizes the equilibrium advertising levels while stating a condition on the campaigns' effectiveness $\eta$, such that an equilibrium in pure strategies exists. If campaigns are not effective "enough" (i.e., $\eta \leqslant 2$), an equilibrium in pure strategies exists and is unique with advertising being: a) increasing in the campaign effectiveness $\eta$, b) decreasing in the marginal cost $\mu$, and c) decreasing in the platforms' polarization $y$ (recall that $y = x_R - x_L$).[11] Note that the symmetric spending in equilibrium implies that in equilibrium impressionable voters split between the two parties (i.e., $S^L_{Imp} = S^R_{Imp} = \frac{1}{2}$).

Anticipating the advertising levels in the second stage, the political parties' maximization problem in the first stage is to choose the platform that maximizes their payoff. For instance, for party $L$, the payoff at $t = 1$ is $\Pi_L(x_L, x_R, e_L^*(x_L, x_R), e_R^*(x_L, x_R))$, which can be written as:

$$S_L(x_L, x_R) - c_L(e_L^*(x_L, x_R)) = F(x_R - x_L)S^L_{Idl}(x_L, x_R) + (1 - F(x_R - x_L))S^L_{Imp} - \mu e_L^*(x_L, x_R)$$

where $S^L_{Imp} = \frac{1}{2}$ and $S^L_{Idl} = G(\bar{x})$ if $x_L \neq x_R$ or $S^L_{Idl} = \frac{1}{2}$ if $x_L = x_R$.

[8] The campaign stage for impressionable voters is resolved via Tullock's ratio-form CSF that facilitates the comparative statics of our model. In the symmetric case, our results would have the same qualitative features if we considered the difference-form CSF proposed by Alcalde and Dahm (2007), the tractable noise CSF proposed by Amegashie (2006) or the relative-difference CSF by Beviá and Corchón (2015) under the parameter restrictions proposed by Balart et al. (2017). See Corchón (2007) and Konrad (2009) for surveys on contest theory.

[9] In a previous version of the paper, platforms could only be chosen in the continuous $[0, 1]$, i.e., $\bar{y} = 1$. Thanks to a referee's suggestion, we now allow for platforms in $\mathbb{R}$, and so for a more general $\bar{y}$.

[10] We allow the general case where $F(0) \geqslant 0$ throughout the paper. However, the distinction between $F(0) = 0$ and $F(0) > 0$ is important both in interpretation and results as, for example, only the latter is compatible with a convergent equilibrium. Assuming that $F(0) > 0$ permits that some voters remain unaffected by electoral advertising even when the two platforms are identical and vote à la Downs (hence randomizing their vote). If in contrast one assumes that $F(0) = 0$, given two identical platforms all votes are driven by electoral advertising and vote à la Tullock.

[11] For $\eta > 2$ there is no equilibrium in pure strategies in the campaign stage and the characterization of the mixed strategy is challenging (Ewerhart, 2015; Wang, 2010). However, permitting $\eta > 2$ does not render substantively new results in our model since all mixed-strategy equilibria at the spending stage are payoff equivalent with parties' expected payoffs in that stage equal to zero (Alcalde and Dahm, 2010). This equivalence result implies that platforms for $\eta > 2$ can be trivially characterized in our model by fixing $\eta = 2$.

The trade-off parties face is now evident. Consider that for a given set of platforms $(x_L, x_R)$, the leftist party chooses to propose a less extreme platform. On the one hand, the indifferent voter is more to the right, which has a positive effect on $S^L_{Idl}$ as in a standard Downsian model. On the other hand, it converts some ideological voters to impressionable ones, which by Lemma 1 increases the spending on advertising in the second stage of game. Similar to Tirole (1988) and Ashworth and Bueno de Mesquita (2009), divergence is a tool of softening competition in the vertical dimension (the advertising stage in our model).

The importance of each of the two forces present in the trade-off is determined by: i) the conversion rate at which impressionable voters become ideological as polarization increases (represented by the reversed hazard rate $\frac{f(y)}{F(y)}$) and ii) the concentration of voters around the median voter $g(0)$. A higher value of the reversed hazard rate increases the rate at which impressionable voters become ideological as polarization increases. Hence, it provides incentives to polarize and avert high advertising costs when competing for impressionable voters. Recall that $F(y)$ is log-concave. This implies that the relevant rate $\frac{f(y)}{F(y)}$ is decreasing in the levels of polarization $y$. Thus, incentives to polarize are higher the closer the two platforms are, and maximal under full convergence $y = 0$. In contrast, a higher concentration of voters around the median $g(0)$ increases incentives to converge at the median and hence reduces incentives to polarize. The relative importance of these two forces is moderated by the campaign effectiveness $\eta$. A higher $\eta$ scales-up advertising level and campaign spending, increasing the relevance of $\frac{f(y)}{F(y)}$ in platforms' selection and decreasing that of $(g(0))$.

Proposition 1 characterizes the unique equilibrium of the platform stage.[12] We refer to a convergent equilibrium when in equilibrium parties propose the same platform and hence polarization is zero. We refer to an extremism equilibrium when in equilibrium polarization is equal to $\bar{y}$, and to an interior equilibrium when in equilibrium polarization is $\hat{y} \in (0, \bar{y})$. By solving the first-order conditions $\hat{y}$ is implicitly defined as $\frac{f(\hat{y})}{F(\hat{y})} = \frac{2}{\eta} g(0)$ (see Proof of Proposition 1).

**Proposition 1.** Let $\eta \leqslant 2$. For any $\mu > 0$ there exists a unique Subgame Perfect Nash Equilibrium.

For $F(0) > 0$, the following equilibrium types arise:

- (Convergent equilibrium) $x^*_L = x^*_R = 0$ if and only if $\frac{f(0)}{F(0)} \leqslant \frac{2}{\eta} g(0)$,
- (Interior equilibrium) $x^*_L = -\frac{\hat{y}}{2}$, and $x^*_R = \frac{\hat{y}}{2}$ if and only if $\frac{f(\bar{y})}{F(\bar{y})} < \frac{2}{\eta} g(0) < \frac{f(0)}{F(0)}$.
- (Extremism equilibrium) $x^*_L = -\frac{\bar{y}}{2}$ and $x^*_R = \frac{\bar{y}}{2}$ if and only if $\frac{2}{\eta} g(0) \leqslant \frac{f(\bar{y})}{F(\bar{y})}$.

For $F(0) = 0$, the convergent equilibrium does not exist. The extremism equilibrium arises if and only if $\frac{2}{\eta} g(0) \leqslant \frac{f(\bar{y})}{F(\bar{y})}$ and the interior equilibrium arises otherwise.

Electoral advertising for each of the above SPNE is uniquely characterized in Lemma 1.

First, note that in contrast to previous literature with similar dynamics –where equilibrium platforms require mixed strategies –Proposition 1 shows that there exists a unique pure strategy SPNE.[13] In this unique equilibrium, the level of polarization $(y^*)$ can be zero (convergent equilibrium), $\hat{y}$ (interior equilibrium), or $\bar{y}$ (extremism equilibrium). Note that polarization in equilibrium can never be larger than $\bar{y}$ given that for $y^* = \bar{y}$ all voters are ideological and hence there are no incentives to polarize further. The emerging type of equilibrium depends on: a) the concentration of voters around the median $g(0)$, b) the conversion rate at which impressionable voters become ideological when polarization increases $\frac{f(y)}{F(y)}$, and c) the effectiveness of electoral campaigns $\eta$.

Starting with the *effectiveness* of the electoral campaigns, a large value of $\eta$ makes the competition for impressionable votes tougher, which exacerbates advertising costs in the second stage (Lemma 1). Therefore, a high value of $\eta$ provides incentives to polarize platforms in the first stage in order to reduce the number of impressionable voters. Indeed, if $F(0) > 0$ (i.e., there exists a share of ideological voters under convergence), as $\eta$ increases we may move across types of equilibria with convergence occurring for a smaller set of parameters. But also platforms become more polarized in the interior equilibrium (i.e., $\hat{y}$ is increasing in $\eta$). Similarly, when many voters are concentrated around the median (i.e., a high value of $g(0)$), there are strong incentives to propose moderate platforms. Thus, a strong presence of moderate voters leads to equilibria of "low" polarization (again, either across equilibria types or within the interior equilibrium). If $F(0) = 0$ (i.e., no ideological voters when parties fully converge), nothing changes except that a convergent equilibrium never arises.

The *conversion rate* at which impressionable voters become ideological when polarization increases (i.e, the reversed hazard rate $\frac{f(y)}{F(y)}$) also helps in explaining our result. This rate captures the incentives of increasing polarization as a way of reducing electoral advertising. By log-concavity of $F(y)$, the rate is monotonically decreasing and hence takes its maximum value at $y = 0$ and its minimum value at $y = \bar{y}$. If the maximum value of the conversion rate is small "enough" (i.e., $\frac{f(0)}{F(0)} \leqslant \frac{2}{\eta} g(0)$) the original Downsian result of platform convergence emerges. Despite full convergence, the conversion rate is so low that increasing polarization does not increase the share of ideological voters enough to diminish electoral advertising in a profitable manner. Analogously, if its minimum value is large "enough" (i.e., $\frac{2}{\eta} g(0) \leqslant \frac{f(\bar{y})}{F(\bar{y})}$), polarization is very effective in restraining electoral advertising and extremism emerges.

A distributional change in the function determining the distribution of voters across types gives interesting comparative statics.

**Notation 1.** Let $\rho$ parametrize the sensitivity of the conversion rate due to inputs other than polarization (e.g., awareness or interest in politics). If $f(y;\rho)$ satisfies the monotone likelihood ratio property (MLRP) in y for all $\rho_1 > \rho_0$, then it holds that:

$$\frac{f(y;\rho_1)}{F(y;\rho_1)} \geqslant \frac{f(y;\rho_0)}{F(y;\rho_0)}.$$

For any $f(y;\rho)$ that satisfies MLRP, any increase in $\rho$ makes the conversion rate of impressionable voters to ideological more responsive to changes in polarization. This may move platforms across types of equilibria (favoring more polarization), while also at any interior equilibrium, polarization is increasing in $\rho$ (it directly follows from applying implicit differentiation to the interior equilibrium condition $\frac{f(\hat{y})}{F(\hat{y})} = \frac{2}{\eta} g(0)$). Thus, a change in the distribution satisfying MLRP has similar effects to an increase in electoral effectiveness.

Another comparative static affecting the division between ideological and impressionable voters consists in varying $\bar{y}$, the minimum level of polarization that makes all voters ideological. Given our general assumptions on $F(y)$, a change in $\bar{y}$ must be

[12] Strictly speaking, we characterize the unique equilibrium satisfying the restriction $x_L \leqslant x_R$. A mirror equilibrium with $x_R < x_L$ also exists.

[13] The unique pure strategy equilibrium characterized in the platform substage – which is the main difference to Ashworth and Bueno de Mesquita (2009)– is also an equilibrium for $\eta > 2$ with equilibrium platforms the same as the ones characterized for $\eta = 2$ (see footNote 11).

accompanied by a specific assumption on the associated change in the distribution: let $\bar{y}_0 < \bar{y}_1$ and its associated distributions, $F_0(y)$ and $F_1(y)$. One approach is to consider the associated distributions as derived from their corresponding densities $f(y;\bar{y}_0)$ and $f(y;\bar{y}_1)$. If the MLRP is satisfied, the comparative statics and intuitions follow from the above discussion.

Finally, note that while changes in $\eta$ or the properties of $F(y)$ crucially affect platform choices, this is not the case for the costs of campaigning $\mu$. Given that this cost is symmetric for the two parties, increasing or decreasing it would only rescale the equilibrium levels of advertising $e_i$ (from Lemma 1) but will not modify the actual level of campaign spending $\mu e_i$ and hence polarization.

*3.1.1. Effects on Campaign Spending*

A technological change that increases the campaign effectiveness has an ambiguous effect on campaign spending. This is apparent when we look at the relevant expression:

$$\frac{\partial \mu e_i^*(x_L^*, x_R^*)}{\partial \eta} = \overbrace{\frac{1-F(y^*)}{4}}^{\text{Spending effect } (+)} \quad \overbrace{-f(y^*)\frac{\partial y^*}{\partial \eta}\frac{\eta}{4}}^{\text{Polarization effect } (-)} \tag{2}$$

On the one hand, ceteris paribus an increase in $\eta$ increases advertising (Lemma 1). On the other hand, it also increases equilibrium level of polarization $y^*$ (Proposition 1), which in turn, decreases the share of impressionable voters and so the levels of advertising (Lemma 1). We call the former the spending effect, while we label the latter as the polarization effect.

At the convergent and extremism equilibria there is no polarization effect, $\frac{\partial y^*}{\partial \eta} = 0$, and spending increases monotonically with $\eta$ due to the spending effect. At the interior equilibrium however, the polarization effect takes place and mitigates the spending effect. If polarization increases sufficiently with $\eta$, the polarization effect may even overturn the spending effect, and hence polarization and campaign spending move in opposite directions. In Lemma 2 below, we provide the conditions for a simultaneous increase in campaign spending and polarization, and we use an example to illustrate it.

**Lemma 2.** If the equilibrium is interior, a technological change that increases the campaign effectiveness $\eta$ – and hence polarization – also increases campaign spending when the spending effect dominates the polarization effect. This is the case if and only if effectiveness is low "enough". Formally, if the equilibrium is interior $\frac{\partial \mu e_i^*(x_L^*, x_R^*)}{\partial \eta} \geqslant 0$, this comparative statics holds if and only if $\eta \leqslant 2g(0)\frac{1-F(\hat{y})}{f(\hat{y})}\left[\frac{F(\hat{y})}{f(\hat{y})}\right]'$.

We illustrate the above intuition with the following example.

**Example 1: $F(y)$ uniformly distributed with $\bar{y} = 1$ and a mass at zero** $F(0) = \frac{1}{10}$. The conversion rate now – from impressionable to ideological voters – is proportional to polarization: $\frac{f(y)}{F(y)} = \frac{\frac{9}{10}}{\frac{1}{10}+\frac{9}{10}y}$. From Proposition 1, we are at a convergent equilibrium for $\eta \leqslant \frac{2}{9}g(0)$, at an interior equilibrium for $\frac{2}{9}g(0) < \eta < \frac{20}{9}g(0)$ and at an extremism equilibrium for $\eta \geqslant \frac{20}{9}g(0)$. These conditions highlight how the concentration of voters around the median gives rise to different equilibrium types. If, for example, $g(0)$ is large, then extremism can be excluded as an outcome for any level of campaign effectiveness. In the interior equilibrium, polarization is given by $y^* = \hat{y} = \frac{\eta}{2g(0)} - \frac{1}{9}$ and it is straightforward to see that polarization is increasing in $\eta$. Using the condition in Lemma (2), we obtain the non-monotone comparative statics on campaign spending in the interior equilibrium: campaign spending is increasing in the campaign effectiveness for $\eta$ low enough (i.e., $\eta \leqslant \frac{10}{9}g(0)$), and decreasing otherwise.

In Fig. 1 we graphically represent the comparative statics of changes of $\eta$ on polarization and campaign spending, assuming that $g(0) = \frac{1}{2}$. Consider first the equilibrium levels of polarization (solid line). For $\eta$ lower than $\frac{1}{9}$ or greater than $\frac{10}{9}$ the convergent and extremist equilibria arise, respectively. For $\frac{1}{9} < \eta < \frac{10}{9}$ the interior equilibrium arises and polarization is strictly increasing in $\eta$. Let's now turn to campaign spending (dashed line). For $\eta \leqslant \frac{1}{9}$, polarization is constant and equal to zero and campaign spending is monotonically increasing in $\eta$ (as in any standard Tullock contest). If $\eta \geqslant \frac{10}{9}$, the extremism equilibrium arises and due to all voters being ideological campaign spending is zero. Note that campaign spending is zero but would be positive if we included a mass of impressionable voters under maximal polarization. In the interior equilibrium interval (i.e., $\eta \in \left(\frac{1}{9}, \frac{10}{9}\right)$), non-monotonicity arises. Campaign spending increases until reaching $\eta = \frac{5}{9}$ due to the spending effect being larger than the polarization one. In contrast, spending decreases for values of $\eta$ larger than $\frac{5}{9}$ due to the polarization effect overcoming the spending effect, until extremism arises.

Polarization and campaign spending are also affected by the rate at which impressionable voters become ideological as polarization increases, parameterized by $\rho$. A technological change that increases $\rho$ has an ambiguous effect on campaign spending. As above, the condition comes from the derivative of campaign spending with respect to $\rho$:

$$\frac{\partial \mu e_i^*(x_L^*, x_R^*)}{\partial \rho} = \overbrace{-\frac{\partial F(y^*)}{\partial \rho}\frac{\eta}{4}}^{\text{Spending effect } (+)} \quad \overbrace{-\frac{\partial F(y^*)}{\partial y^*}\frac{\partial y^*}{\partial \rho}\frac{\eta}{4}}^{\text{Polarization effect } (-)}$$

On the one hand, an increase in $\rho$ increases the "stock" of impressionable voters $1 - F(y)$ (this follows directly from MLRP which implies first order stochastic dominance). The spending effect therefore suggests that, for a given level of polarization $y$, an increase in $\rho$ increases advertising (Lemma 1). On the other hand, an increase in $\rho$ makes voters more responsive to polarization, by affecting the conversion rate $\frac{f(y)}{F(y)}$, and therefore provides incentives to increase polarization (Proposition 1). However, this increased polarization in turn decreases the share of impressionable voters and so decreases the levels of advertising (Lemma 1). As before, we label this latter effect as the polarization effect.

At the convergent equilibrium, there is no polarization effect since $\frac{\partial y^*}{\partial \rho} = 0$ and spending increases monotonically with $\rho$. That is, an increase in $\rho$, keeping polarization constant, would increase the number of impressionable voters and their weight in the parties' maximization problem, and parties would have higher incentives to increase advertising. In the interior equilibrium, the polarization effect kicks in and the net effect on spending depends on the magnitude of these two effects. Finally, polarization is constant in $\rho$ once the extremism equilibrium is reached and spending is constant and equal to zero since there are no impressionable voters. The following lemma summarizes the above for the interior equilibrium.

**Lemma 3.** If the equilibrium is interior, a technological change that increases $\rho$ – and hence polarization – also increases campaign spending, due to the spending effect dominating the polarization effect, if and only if the effect of $\rho$ on the "stock" of impressionable voters is "large" enough. Formally, if the equilibrium is interior $\frac{\partial \mu e_i^*(x_L^*, x_R^*)}{\partial \rho} \geqslant 0$ if and only if $-\frac{\partial F(\hat{y})}{\partial \rho} \geqslant f(\hat{y})\frac{\partial \hat{y}}{\partial \rho}$.

Finally, one could consider the effects of changing $\bar{y}$ on campaign spending. As before, let $\bar{y}_0 < \bar{y}_1$ and its associated distributions, $F_0(y)$ and $F_1(y)$. If MLRP is satisfied, then the effects on spending are as in Lemma 3.

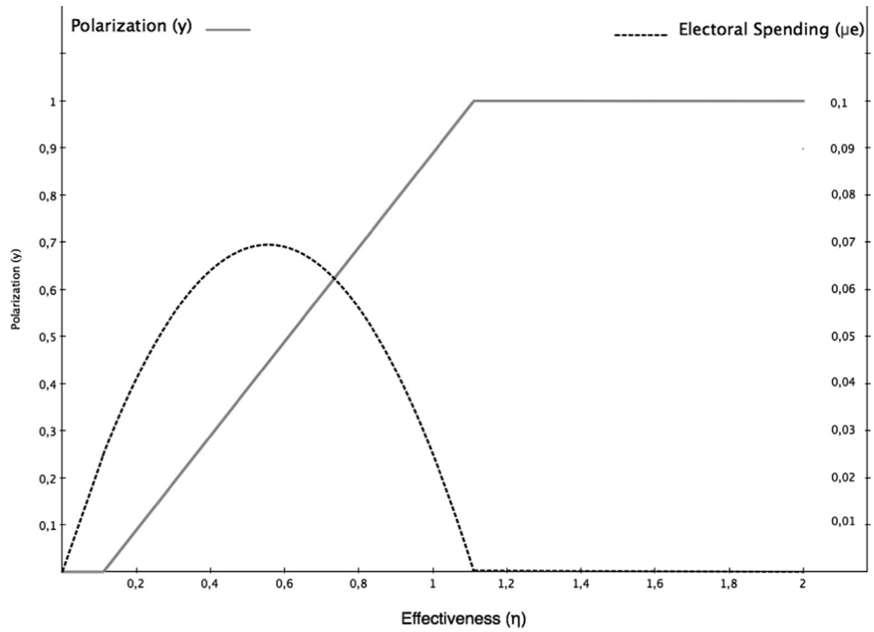


**Fig. 1.** Comparative statics on $\eta$ for campaign spending and polarization. Uniform distribution of $F(y)$ with $F(0) = \frac{1}{10}$ and $g(0) = 1/2$.

### 3.2. Asymmetric parties

In contrast to previous spatial models with endogenous valence, we can incorporate cost asymmetries in the analysis and explain our results with the same intuitive channels of the symmetric case. Such cost asymmetries could be attributed to access to diverse campaign technologies or heterogeneous fundraising skills, one example being the well-documented phenomenon of an incumbency advantage (Gelman and King, 1990; Meirowitz, 2008; Pastine and Pastine, 2012). In this section, we assume, without loss of generality, that $\mu_L < \mu_R$. We first characterize campaign spending in the second stage (Lemma 3 and then show how equilibrium platforms are affected by the asymmetry (Propositions 2 and 3). In contrast to the symmetric case, the equilibrium platforms might be in mixed strategies. For clarity of presentation, Proposition 2 focuses on all configurations where an equilibrium in pure strategies exists. Then Proposition 3 focuses on mixed strategies.

Considering well-known results from contest theory, the advertising subgame has an equilibrium in pure strategies if and only if $\eta$ is restricted from above. This upper bound $\bar{\eta}$ is implicitly defined by $\mu_L^{\bar{\eta}} + \mu_R^{\bar{\eta}} = \bar{\eta}\mu_R^{\bar{\eta}}$ (Baik, 1994; Nti, 1999). This condition implies a one-to-one relationship between cost-asymmetry $\frac{\mu_R}{\mu_L}$ and $\bar{\eta}$ (with $\bar{\eta} \to 2$ as $\frac{\mu_R}{\mu_L} \to 1$, and $\bar{\eta} \to 1$ as $\frac{\mu_R}{\mu_L} \to \infty$). As with the symmetric case, the platform stage of our model can also be solved for any $\eta > \bar{\eta}$ because of the payoff equivalence with $\eta = \bar{\eta}$ (Alcalde and Dahm, 2010). Following Baik (1994) and Nti (1999), we can characterize equilibrium in the advertising subgame.

**Lemma 4.** For $\eta \leqslant \bar{\eta}$ there exists a unique Nash equilibrium in the campaign stage and advertising is given by $e_i^*(x_L, x_R) = (1 - F(y))\frac{\eta}{\mu_i}\frac{\mu_L^\eta \mu_R^\eta}{(\mu_L^\eta + \mu_R^\eta)^2}$, for all *i*.

Lemma 4 shows that, in equilibrium, parties choose different levels of advertising ($e_i^*(x_L, x_R)$), although they spend equal amounts ($\mu_i e_i^*(x_L, x_R)$). The share of impressionable voters is no longer equally split across parties, giving an advantage to the party with the lower marginal cost. This generates an asymmetry in parties' incentives to use polarization as a device to reduce campaign spending, and implies that in the interior or extremism equilibrium parties propose asymmetric platforms.

We present the characterization of the equilibrium in pure strategies in the following Proposition. Analogously to the symmetric case, by solving first-order conditions polarization in the interior equilibrium $\hat{y} \in (0, \bar{y})$ is implicitly defined by $\frac{f(\hat{y})}{F(\hat{y})} = \frac{g(\bar{x}^*)}{2\eta}\frac{(\mu_L^\eta + \mu_R^\eta)^2}{\mu_L^\eta \mu_R^\eta}$ where $\bar{x}^* = G^{-1}\left(\frac{\mu_R^\eta}{\mu_L^\eta + \mu_R^\eta}\right)$.

**Proposition 2.** For any $\eta \leqslant \bar{\eta}$ and $\mu_L < \mu_R$, there exists a unique Subgame Perfect Nash Equilibrium as follows:

- (Convergent equilibrium) $x_L^* = x_R^* = 0$ if and only if $F(0) > 0$ and $\frac{f(0)}{F(0)} \leqslant g(0)/\left(2\eta\frac{\mu_L^\eta \mu_R^\eta}{(\mu_L^\eta + \mu_R^\eta)^2} + \frac{\mu_R^\eta - \mu_L^\eta}{\mu_L^\eta + \mu_R^\eta}\right)$.
- (Interior equilibrium) $x_L^* = \bar{x}^* - \frac{\hat{y}}{2}$, and $x_R^* = \bar{x}^* + \frac{\hat{y}}{2}$ if and only if $\frac{f(\bar{y})}{F(\bar{y})} < \frac{g(\bar{x}^*)}{2\eta}\frac{(\mu_L^\eta + \mu_R^\eta)^2}{\mu_L^\eta \mu_R^\eta} \leqslant \frac{f(2\bar{x}^*)}{F(2\bar{x}^*)}$ and $\frac{\mu_R^\eta}{\mu_L^\eta + \mu_R^\eta} \leqslant G(\frac{\hat{y}}{2})$.
- (Extremism equilibrium) $x_L^* = \bar{x}^* - \frac{\bar{y}}{2}$ and $x_R^* = \bar{x}^* + \frac{\bar{y}}{2}$ if and only if $\frac{g(\bar{x}^*)}{2\eta}\frac{(\mu_L^\eta + \mu_R^\eta)^2}{\mu_L^\eta \mu_R^\eta} \leqslant \frac{f(\bar{y})}{F(\bar{y})}$ and $\frac{\mu_R^\eta}{\mu_L^\eta + \mu_R^\eta} \leqslant G(\frac{\bar{y}}{2})$.

Electoral advertising for each of the above SPNE is uniquely characterized in Lemma 4.

As in the symmetric case, the convergent equilibrium arises only if $F(0) > 0$, with a large concentration of voters around the median (i.e., $g(0)$ is high), and/or when the conversion rate with zero polarization is low (i.e., the conversion rate at its maximal level $\frac{f(0)}{F(0)}$ is low). Note that as the asymmetry converges to zero, the inequality characterizing a convergent equilibrium converges to that of Proposition 1. In the asymmetric case, however, the size of the asymmetry is also a determinant of platform convergence. Ceteris paribus, as the cost asymmetry increases, platform convergence becomes less likely (the denominator on the right hand side of the inequality that gives rise to convergence is increasing in the asymmetry). As the asymmetry increases, the (symmetric) convergent equilibrium becomes less attractive for party $R$ — the party in disadvantage when competing for impressionable voters. Hence, $R$ has incentives to move away from the other party, as in models of office-motivated candidates with exogenous valence (e.g., Aragonès and Palfrey, 2002).

Analogously to the symmetric case, in the unique interior equilibrium polarization is $\hat{y}$.[14] This interior equilibrium is asymmetric and the cost asymmetry plays an important role in determining the equilibrium platforms. The cost-disadvantaged party $R$ has greater incentives than the advantaged party to reduce the share of (costly) impressionable voters. Consequently, $R$ has greater incentives to separate its platform from $L$. Thus, at the interior equilibrium, platforms are shifted towards the cost-disadvantaged party $R$. More precisely, the point around which parties propose equidistant platforms is to the right of the median voter (i.e., $\bar{x}^* > 0$). Once these equidistant platforms around $\bar{x}^* > 0$ give rise to the maximum level of polarization $\bar{y}$, we obtain the extremism equilibrium where parties have no further incentives to polarize, given that all voters are ideological.

Same as in the symmetric case, for an interior or extremism equilibrium in pure strategies to arise, one needs to keep track of the presented incentives to converge or not (i.e., $\frac{f(\cdot)}{F(\cdot)}, g(\cdot), \eta$). However, note the additional equilibrium condition that reflects the cost asymmetry: $\frac{\mu_R^\eta}{\mu_L^\eta+\mu_R^\eta} \leqslant G(\frac{\bar{y}}{2})$. That is, for an interior or extremism equilibrium in pure strategies to arise, we need that the cost asymmetry is relatively "low". On the other hand, when the cost asymmetry is relatively "high" a non-convergent equilibrium in pure strategies does not exist. The typical chase-and-evade incentives arise (Aragonès and Palfrey, 2002). While the advantaged party "pushes" its platform closer to the disadvantaged one, the latter has greater incentives to play something to the left of that party, with some probability.

Even though the conditions in Proposition 2 are mutually exclusive, for some configurations of the parameters there is no pure strategy equilibrium. In this case, Dasgupta and Maskin (1986) warrants the existence of a mixed-strategy equilibrium.[15] Its characterization depends on the specific distribution function $F(y)$. We tackle this issue in Proposition 3, where we show the existence of an equilibrium in which the advantaged party $L$ proposes the platform preferred by the median, and the disadvantaged party $R$ randomizes assigning equal weights to two symmetric platforms around the median (as in Aragonès and Xefteris, 2012 with exogenous valence). A sufficient condition for the emergence of such equilibrium is that the expected payoff of party $L$ in the proposed mixed strategy equilibrium (henceforth denoted $\bar{\Pi}_L$) is quasiconcave. This holds true for a wide range of specifications; for instance, if $F(y)$ is the uniform distribution as well as for several configurations of the Beta distribution. We formally characterize this mixed strategy equilibrium in the following proposition.

**Proposition 3.** (Mixed Strategy Equilibrium) Let $\eta \leqslant \bar{\eta}$ and $\mu_L < \mu_R$ such that $\frac{F(0)}{f(0)}\frac{g(0)}{2} - \frac{1}{2}\frac{\mu_R^\eta-\mu_L^\eta}{\mu_R^\eta+\mu_L^\eta} < \eta\frac{\mu_R^\eta\mu_L^\eta}{(\mu_R^\eta+\mu_L^\eta)^2}$, and $\bar{\Pi}_L$ is quasiconcave. There exists a mixed strategy Nash equilibrium where $x_L^* = 0$ and party $R$ randomizes with equal probability over $x_R^*$ and $-x_R^*$ where:

$$x_R^* = \begin{cases} \underline{x_R} & \text{if } \frac{F(0)}{f(0)}\frac{g(0)}{2} - \frac{1}{2}\frac{\mu_R^\eta-\mu_L^\eta}{\mu_R^\eta+\mu_L^\eta} < \eta\frac{\mu_R^\eta\mu_L^\eta}{(\mu_R^\eta+\mu_L^\eta)^2} < \frac{F(\bar{y})}{f(\bar{y})}\frac{g(\bar{y}/2)}{2} + G(\bar{y}/2) - \frac{\mu_R^\eta}{\mu_R^\eta+\mu_L^\eta} \\ \bar{y} & \text{if } \frac{F(\bar{y})}{f(\bar{y})}\frac{g(\bar{y}/2)}{2} + G(\bar{y}/2) - \frac{\mu_R^\eta}{\mu_R^\eta+\mu_L^\eta} \leqslant \eta\frac{\mu_R^\eta\mu_L^\eta}{(\mu_R^\eta+\mu_L^\eta)^2} \end{cases}$$

and $\underline{x_R}$ is implicitly defined by the solution to the first-order condition for party $R$:

$$\left[1 - G\left(\frac{\underline{x_R}}{2}\right)\right] = \frac{F(\underline{x_R})}{f(\underline{x_R})}\frac{g\left(\frac{\underline{x_R}}{2}\right)}{2} + \frac{\mu_L^\eta}{\mu_R^\eta+\mu_L^\eta} - \eta\frac{\mu_R^\eta\mu_L^\eta}{(\mu_R^\eta+\mu_L^\eta)^2}$$

Campaign spending for the above SPNE is uniquely characterized in Lemma 4.

If the disadvantaged party $R$ was choosing one of the two platforms in its support deterministically, then the advantaged party $L$ would locate at some point between that platform and the median. As a consequence, a better alternative would be always available to the disadvantaged party $R$ at the other side of the median. If party $R$ randomizes between these two platforms at the opposing sides of the median, then the advantaged party has incentives to locate exactly at the median. The symmetric platforms chosen by the disadvantaged party are such that either its first-order condition is satisfied (i.e., $x_R^* = \underline{x_R}$), or the one that reaches maximal polarization (i.e., $x_R^* = \bar{y}$).

Table 1 summarizes the results of Propositions 2 and 3 by putting together the equilibrium conditions in terms of the cost asymmetry and presenting the mixed strategies only when there is no equilibrium in pure strategies. If the asymmetry is "high" (i.e., $\frac{\mu_R^\eta}{\mu_L^\eta+\mu_R^\eta} > G(\frac{\bar{y}}{2})$), party $L$ exploits its advantage by always proposing the centrist platform. In that case, both parties propose the median's preferred platform and we observe convergence when the incentives to polarize are not high enough. As the incentives to polarize increase, the disadvantaged party randomizes and we obtain a mixed strategy equilibrium (either the mixed interior or the mixed extremism). If the asymmetry instead is "low" (i.e., $\frac{\mu_R^\eta}{\mu_L^\eta+\mu_R^\eta} \leqslant G(\frac{\bar{y}}{2})$), we again observe convergence when the incentives to polarize are low. As the incentives to diverge increase, we first observe the disadvantaged party randomizing around the centrist platform proposed by the advantaged party, to be succeeded by the interior equilibrium in pure strategies where the advantaged party abandons the centrist platform. Maximal incentives to polarize give rise to the extremism equilibrium.

**Example 2: $F(y)$ uniformly distributed with $\bar{y} = 1$ and a mass at zero $F(0) = \frac{1}{4}$. $G(x)$ standard normal.** Fig. 2 illustrates the equilibria summarized in Table 1 capturing the incentives to polarize by the campaign effectiveness $\eta$, while fixing all other parameters. We are assuming that voters' ideal policies are distributed according to a standard normal distribution and $F(y)$ uniform on $[0, 1]$ with the presence of ideological voters even when parties propose identical platforms ($F(0) = 0.25$). The upper panel illustrates an example of "low" asymmetry and the lower panel an example of "high" asymmetry. The panels on the left illustrate platform choices as summarized in Table 1 when the incentives to polarize vary (i.e., $\eta$). The panels on the right instead illustrate the comparative statics of campaign effectiveness $\eta$ on polarization and campaign spending. First, note, we again encounter the situation of a non-monotone relationship between campaign spending and $\eta$ (as in Fig. 1 and the symmetric case). Second, and perhaps more importantly, our results can again sustain the simultaneous increase in polarization and campaign spending due to technological changes, even in the presence of cost asymmetry.

## 4. Empirical Implications

We suggest a tractable setting to analyze electoral competition with endogenous platforms and campaign spending. Apart from an intuitive equilibrium characterization that contributes to the corresponding endogenous valence literature, our model comple-

[14] As in the symmetric case, uniqueness is subject to the constraint $x_L \leqslant x_R$. A mirror equilibrium $x_R < x_L$ always exists.

[15] The proof follows immediately from verifying that all conditions of Theorem 5 in Dasgupta and Maskin (1986) are met.

**Table 1**
A summary of the conditions characterized in Propositions 2 and 3 presenting the mixed strategies only when there is no equilibrium in pure strategies. With "high" asymmetry the advantaged party always proposes the centrist platform and hence the only equilibrium in pure strategies is the convergent. The interior and extremism equilibria can only arise with "low" asymmetry. The relative value of $\eta \frac{\mu_R^\eta \mu_L^\eta}{\left(\mu_R^\eta + \mu_L^\eta\right)^2}$ determines the equilibrium type.

| **Equilibrium:** | **"Low" Asymmetry:** $\frac{\mu_R^\eta}{\mu_L^\eta + \mu_R^\eta} \leqslant G\left(\frac{\bar{y}}{2}\right)$ | **"High" Asymmetry:** $\frac{\mu_R^\eta}{\mu_L^\eta + \mu_R^\eta} > G\left(\frac{\bar{y}}{2}\right)$ |
|---|---|---|
| **Convergent:** $x_L^* = 0, x_R^* = 0$ | $\eta \frac{\mu_R^\eta \mu_L^\eta}{\left(\mu_R^\eta + \mu_L^\eta\right)^2} \leqslant \frac{F(0)}{f(0)} \frac{g(0)}{2} - \frac{1}{2} \frac{\mu_R^\eta - \mu_L^\eta}{\mu_R^\eta + \mu_L^\eta}$ | $\eta \frac{\mu_R^\eta \mu_L^\eta}{\left(\mu_R^\eta + \mu_L^\eta\right)^2} \leqslant \frac{F(0)}{f(0)} \frac{g(0)}{2} - \frac{1}{2} \frac{\mu_R^\eta - \mu_L^\eta}{\mu_R^\eta + \mu_L^\eta}$ |
| **Mixed Interior:** $x_L^* = 0$$R$ with equal prob. $\underline{x_R}$ and $-\underline{x_R}$ | $\frac{F(0)}{f(0)} \frac{g(0)}{2} - \frac{1}{2} \frac{\mu_R^\eta - \mu_L^\eta}{\mu_R^\eta + \mu_L^\eta} < \eta \frac{\mu_R^\eta \mu_L^\eta}{\left(\mu_R^\eta + \mu_L^\eta\right)^2} \leqslant \frac{F(2\bar{x}^*)}{f(2\bar{x}^*)} \frac{g(\bar{x}^*)}{2}$ | $\frac{F(0)}{f(0)} \frac{g(0)}{2} - \frac{1}{2} \frac{\mu_R^\eta - \mu_L^\eta}{\mu_R^\eta + \mu_L^\eta} < \eta \frac{\mu_R^\eta \mu_L^\eta}{\left(\mu_R^\eta + \mu_L^\eta\right)^2} < \frac{F(\bar{y})}{f(\bar{y})} \frac{g(\bar{y}/2)}{2} + G(\bar{y}/2) - \frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta}$ |
| **Interior:** $x_L^* = \bar{x}^* - \frac{\hat{y}}{2}, x_R^* = \bar{x}^* + \frac{\hat{y}}{2}$ | $\frac{F(2\bar{x}^*)}{f(2\bar{x}^*)} \frac{g(\bar{x}^*)}{2} < \eta \frac{\mu_R^\eta \mu_L^\eta}{\left(\mu_R^\eta + \mu_L^\eta\right)^2} < \frac{F(\bar{y})}{f(\bar{y})} \frac{g(\bar{x}^*)}{2}$ | |
| **Mixed Extremism:** $x_L^* = 0$ $R$ with equal prob. $\bar{y}$ and $-\bar{y}$ | | $\frac{F(\bar{y})}{f(\bar{y})} \frac{g(\bar{y}/2)}{2} + G(\bar{y}/2) - \frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta} \leqslant \eta \frac{\mu_R^\eta \mu_L^\eta}{\left(\mu_R^\eta + \mu_L^\eta\right)^2}$ |
| **Extremism:** $x_L^* = \bar{x}^* - \frac{\bar{y}}{2}, x_R^* = \bar{x}^* + \frac{\bar{y}}{2}$ | $\frac{F(\bar{y})}{f(\bar{y})} \frac{g(\bar{x}^*)}{2} \leqslant \eta \frac{\mu_R^\eta \mu_L^\eta}{\left(\mu_R^\eta + \mu_L^\eta\right)^2}$ | |

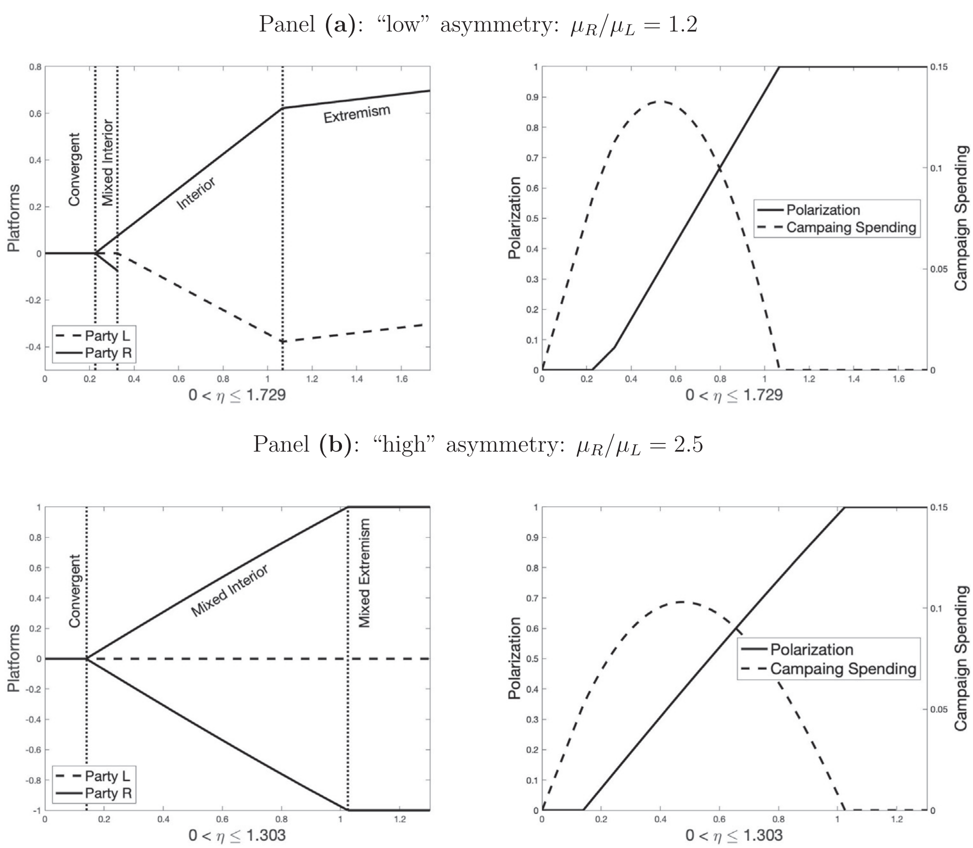


**Fig. 2.** The left panels illustrate equilibrium platforms and the right panels comparative statics on polarization and campaign spending as the campaign effectiveness $\eta$ varies. Graphs are plotted on the interval of $0 < \eta \leqslant \bar{\eta}$ that guarantees an equilibrium in pure strategies in the advertising stage (Lemma 4). For these graphs, $G(x)$ is the standard normal and $F(y)$ is the uniform distribution with $F(0) = 1/4$.

ments existing models of campaign spending and polarization by providing new comparative statics on the effects of technological advances on electoral competition. Below we offer a structured approach on how our model's predictions can be empirically tested and distinguished from other theories.

Our paper considers two main channels through which technology can affect electoral competition. Through political awareness (determined through $F(y)$), technology may allow voters to have a better grasp of politics (e.g., a better scrutiny of proposals) or a worse one (e.g., fake news). Similarly, technological changes may improve the effectiveness of electoral advertising ($\eta$). The first channel is a new contribution of our model which, conditional on the availability of empirical measurements of political awareness, provides testable predictions. Our results show that an increase in polarization and spending is only compatible with a technological change that makes voters less aware. The second channel was also considered in previous literature and below we provide ideas on how empirical work could falsify our model and differentiate it from others.

As the measurement of technological change is always challenging, we discuss the falsifiability of the model around two broad cases. First, when researchers can observe and measure campaigning technology (and its change). Second, when they do not observe it, but may control for it.

In the first place, suppose the econometrician can measure technological change, and so they can investigate its effects on electoral competition. Such setup could be used to empirically distinguish our theory from the two closest models in terms of modeling *campaign technology* (Herrera et al., 2008; Hirsch, 2019). In our model, the technology determining campaign effectiveness, as captured by parameter $\eta$, has the following testable empirical implications: First, it has an unambiguous positive effect on polarization —an implication at odds with Herrera et al. (2008) but in line with Hirsch (2019). Second, it has a non-monotone effect on campaign spending since the latter is mediated by the choice of polarization. On the one hand, $\eta$ increases spending (directly) through the spending effect; on the other hand, it decreases spending (indirectly) through the polarization effect (see Eq. 2). As a result, polarization may not only mitigate the effect of technology on spending, but it may also counter it, diminishing spending. Hence, the correlation between campaign technology and spending in our model is concave and non-monotone. A similar non-monotone effect is described in Herrera et al. (2008) but not in Hirsch (2019). Put together, the distinct empirical implications of changes in technology on polarization and campaign spending should help distinguish our model from the closest in the literature. To sum up, an econometrician interested in testing the empirical implications of our model should look for: (i) an increase in the campaign effectiveness that increases polarization, (ii) concavity: which could be tested by including the square of $\eta$ in the right-hand side of a regression, and (iii) non-monotonicity: which implies that there is level of $\eta < \infty$ that maximizes campaign spending, and so besides the term $\eta^2$, the econometrician should test for a reversed U-shape relationship (i.e., for a change of slope, as proposed in Lind and Mehlum, 2010).

In the second place, suppose the econometrician does not observe the campaigning technology but can control for it. Keeping the technology constant, there are two potential avenues of empirical research: i) the effect of platforms on campaign spending (and vice versa), and ii) the effect of those variables on vote shares.

*i) Relation between polarization and campaign spending:* In our model, the co-movement of polarization and spending is attributed to changes in the campaign technology that trigger parties' strategic differentiation to save on campaign costs. In other models, the co-movement of polarization and spending could also arise even in the absence of technology changes. For example, parties may choose more extreme platforms to attract larger campaign contributions (à la Grossman and Helpman, 1996). Similarly, policy-motivated candidates who choose (endogenous) valence may afford to be closer to their (extreme) platforms when they can spend more on political advertising (Herrera et al., 2008; Hirsch, 2019). Instead, there is a negative causal relationship between these two variables in our model, ceteris paribus; for any given level of technology, polarization induces lower spending. Thus, an empirical researcher could test our model and differentiate it from the above-mentioned ones by exploiting these differences in the relationship between polarization and spending. However, this exercise should be done carefully, as the failure to control for technology (or address reverse causality issues) will result in biased estimates, as illustrated by our model.

*ii) Effects on electoral results:* Finally, our model proposes that citizens vote based on platforms only when the proposed platforms are different ''enough", otherwise they focus on campaign spending. This feature of the model has testable empirical implications. Typically, to estimate the effect of spending on votes, researchers look for an ''instrument" or shock to spending (e.g., Da Silveira and De Mello, 2011; Larreguy et al., 2018; Spenkuch and Toniatti, 2018; Bekkouche et al., 2022). On these lines, suppose that *after* the platforms have been chosen, there is a shock to the cost of advertising that incentivizes unanticipated increases in spending. Empirically, since platforms are fixed, the marginal effect of advertising on vote shares should be greater in districts with larger shares of impressionable voters —the ones more likely to change their vote due to advertising. If platforms varied between regions (like in parliamentary elections), the marginal effect of advertising should be greater in districts with larger shares of impressionable voters, that is, districts with low polarization. Instead, the predictions would be different if there is an anticipated exogenous increase in spending that occurs *before* platforms are chosen (for instance, when spending caps are weakened, as in Fouirnaies, 2021). When platforms can react to such a shock, parties would choose greater polarization, increasing the total share of ideological voters at the expense of the impressionable share.

## 5. Conclusion

It is often argued that technology, especially the fine targeting possibilities that come along with the internet and social media, can be responsible for this simultaneous increase in polarization and spending in the United States. For example, Herrera et al. (2008) argue that "*commentators have suggested that the reason for both the increased polarization and campaign spending is that skilled political operatives using sophisticated statistical tools and purchasing advertising in local markets are better able to target particular voters*" (citation from Herrera et al. (2008, p. 502), also see NBC (2017) for a recent example). However, the results of Herrera et al. (2008) linked such technological advances with a reduction in polarization, and therefore favored alternative channels that may drive polarization such as more volatile preferences. In contrast, our results justify the simultaneous increase in polarization and campaign spending due to recent technological changes and better targeting of electoral campaigns, and are complementary to those of Hirsch (2019) and Prummer (2020). Moreover, our model can address several follow-up questions.

Given that one would naturally expect further advances in campaign technology, two natural questions arise: (a) should we expect a further increase in polarization?, and (b) what about campaign spending? Our theory suggests that further advances in targeting lead to further polarization. Ways to go against this trend would require policies that improve the awareness of the electorate and induce a shift of voters' attention from persuasive campaigns to political platforms. These implications on the electorate's awareness are particularly relevant in recent times, given the exposure of voters to a plethora of fake news and false information. Regarding campaign spending, an improvement of the targeting technology leads parties to increase their campaign spending, but at the same time to polarize (which reduces political expenditure). Hence, our non-monotone result of campaign effectiveness on spending would not be incompatible with a potential reduction in campaign spending at the cost of extreme levels of polarization. Alternatively, a reduction in campaign spending could again be achieved by improving the awareness of the electorate.

## Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Appendix A. Proofs

Before moving to the proofs, we define and write down some equations that will be useful in the following lemmata and subsections. Notice that everything is written for the asymmetric case, as the symmetric case is a subcase.

### A.1. Preliminaries

We assume without loss of generality that $x_L \leqslant x_R$. Throughout this section, let $y = x_R - x_L, \bar{x} = \frac{x_L + x_R}{2}$, and $S^L_{Idl}(x_L, x_R) = G(\bar{x}) = 1 - S^R_{Idl}(x_L, x_R)$ for $x_L \neq x_R$ and $S^L_{Idl}(x_L, x_R) = S^R_{Idl}(x_L, x_R) = \frac{1}{2}$ otherwise. By backward induction and using the equilibrium expressions of the advertising subgame, we can write the first-stage payoff for the political parties as:

$$\Pi_i(x_L, x_R) = F(y) S^i_{Idl}(x_L, x_R) + (1 - F(y)) \frac{\mu^\eta_{-i}}{\mu^\eta_L + \mu^\eta_R} - (1 - F(y))\eta \frac{\mu^\eta_R \mu^\eta_L}{(\mu^\eta_L + \mu^\eta_R)^2}, \; i = L, R. \tag{3}$$

The first derivative of the objective functions above are:

$$\Pi'_L \equiv \frac{\partial \Pi_L(x_L, x_R)}{\partial x_L} = F(y)\frac{g(\bar{x})}{2} - f(y)G(\bar{x}) + f(y)\frac{\mu^\eta_R}{\mu^\eta_L + \mu^\eta_R} - f(y)\eta \frac{\mu^\eta_R \mu^\eta_L}{(\mu^\eta_L + \mu^\eta_R)^2}, \tag{4}$$

$$\Pi'_R \equiv \frac{\partial \Pi_R(x_L, x_R)}{\partial x_R} = -F(y)\frac{g(\bar{x})}{2} + f(y)[1 - G(\bar{x})] - f(y)\frac{\mu^\eta_L}{\mu^\eta_L + \mu^\eta_R} + f(y)\eta \frac{\mu^\eta_R \mu^\eta_L}{(\mu^\eta_L + \mu^\eta_R)^2} \tag{5}$$

### A.2. Lemmata

We use lemmata A.1–A.6 to restrict pure-strategy equilibrium candidates and show the sufficiency of the first-order conditions. In these lines, they will be useful in Sections A.3 and A.4

**Lemma A.1.** If $0 < x_{-i} < x_i$ $(x_i < x_{-i} < 0)$, then $x_i$ is strictly dominated by some $x'_i < x_{-i}$ $(x'_i > x_{-i})$, where $i \neq -i, i = L, R$.

**Proof.** Consider divergent platforms $0 < x_{-i} < x_i$. Then, party $i$ is strictly better off by deviating to $x'_i = 2x_{-i} - x_i < x_{-i}$, which maintains unchanged the proportion of each type of voter and the share of impressionable votes and, by the symmetry of $g(y)$ around the median, strictly increases the share of ideological votes of party $i$. Analogously, we can show that party $i$ always has a profitable deviation to $x'_i = 2x_{-i} - x_i > x_{-i}$ when $x_i < x_{-i} < 0$

**Lemma A.2.** If $0 < x_{-i}$ $(x_{-i} < 0)$, then $x_i = x_{-i}$ is strictly dominated by $x'_i = x_{-i} - \epsilon$ $(x'_i = x_{-i} + \epsilon)$ for party $i$, where $i \neq -i, i = L, R$, and $\epsilon$ is strictly positive and arbitrarily small.

**Proof.** This claim follows immediately from noting that, according to the discontinuity in the definition of $S^i_{Idl}(x_L, x_R)$ and by the symmetry of $g(x)$ around zero: $\Pi_i(x_{-i}, x_{-i}) < \lim_{x_i \to x^-_{-i}} \Pi_i(x_i, x_{-i})$ for $0 < x_{-i}$. Similarly, $\Pi_i(x_{-i}, x_{-i}) \leqslant \lim_{x_i \to x^+_{-i}} \Pi_i(x_i, x_{-i}, x_{-i})$ for $x_{-i} < 0$.

**Lemma A.3.** The objective functions are continuous everywhere except for $x_L = x_R \neq 0$ and differentiable everywhere except for $x_L = x_R \neq 0$ and $x_R - x_L = \bar{y}$.

**Proof.** We have denoted, without loss of generality, $x_L \leqslant x_R$. All the elements in the objective functions in (3) are continuous except for $x_L = x_R \neq 0$, where $S^L_{Idl}(x_L, x_R) = G(\bar{x}) = 1 - S^R_{Idl}(x_L, x_R)$ for $x_L \neq x_R$ and $S^L_{Idl}(x_L, x_R) = S^R_{Idl}(x_L, x_R) = \frac{1}{2}$ for $x_L = x_R$. By symmetry of $G(.), G(0) = \frac{1}{2}$, thus $\lim_{x_i \to x_{-i}} S^i_{Idl}(x_i, x_{-i}) = S^i_{Idl}(x_i, x_{-i})$ if and only if $x_{-i} = 0$.

When the objective functions are continuous, the derivatives in (4) and (5) are defined everywhere except at $y = \bar{y}$, where $f(\bar{y}) > 0$, by full support of $F(y)$ over $[0, \bar{y}]$, but $f(y) = 0$ for all $y > \bar{y}$.

**Lemma A.4.** Let $F(x)$ be log-concave and $g(x)$ be symmetric and log-concave. The objective functions are strictly quasiconcave for $x_L \leqslant x_R$ with the last inequality being strict if $x_{-i} \neq 0, i \neq -i, i = L, R$.

**Proof.** We have denoted, without loss of generality, $x_L \leqslant x_R$.

We start by showing the objective function is strictly quasiconcave for party $L$. As the objective function is continuous but not differentiable at $x_R - x_L = \bar{y}$ (Lemma A.3), to prove quasiconcavity of $\Pi_L(x_L, x_R)$, we fix $x_R$ and show that: i) $\Pi_L(x_L, x_R)$ is strictly increasing in $x_L$ for $x_L < x_R - \bar{y}$, and ii) $\Pi_L(x_L, x_R)$ is quasiconcave for $x_R - \bar{y} \leqslant x_L \leqslant x_R$ (with last inequality being strict if $x_R \neq 0$). The combination of i) and ii) together with continuity imply that $\Pi_L(x_L, x_R)$ is quasiconcave for any $x_L \leqslant x_R$ (with last inequality being strict if $x_R \neq 0$).

Consider first the case $x_L < x_R - \bar{y}$. Note that in this case (4) reduces to $\Pi'_L = \frac{g(\bar{x})}{2} > 0$ showing that the objective function is strictly increasing for $x_L < x_R - \bar{y}$.

Consider now the case $x_R - \bar{y} \leqslant x_L \leqslant x_R$. Let us modify Eq. (4) by dividing it over the densities $f(y)$ and $g(\bar{x})$:

$$\tilde{\Pi}'_L \equiv \frac{\Pi'_L}{f(y)g(\bar{x})} = \frac{F(\hat{y})}{2f(\hat{y})} - \frac{1}{g(\bar{x})}\left[G(\bar{x}) - \frac{\mu^\eta_R}{\mu^\eta_L + \mu^\eta_R} + \eta \frac{\mu^\eta_R \mu^\eta_L}{(\mu^\eta_L + \mu^\eta_R)^2}\right] \tag{6}$$

Let $\tilde{\Pi}_L$ be the primitive of $\tilde{\Pi}'_L$. $\tilde{\Pi}_L$ is strictly quasiconcave if and only if $\tilde{\Pi}'_L(x)(x' - x) > 0$ whenever $\tilde{\Pi}_L(x') > \tilde{\Pi}_L(x)$. Since strict quasiconcavity is determined by the sign of $\tilde{\Pi}'_L(x)$, which is the same of the sign of $\Pi'_L(x)$ (because $f(y)g(\bar{x})$ is strictly positive for $x_R - x_L \leqslant \bar{y}$), the strict quasiconcavity of $\tilde{\Pi}_L(x)$ guarantees the strict quasiconcavity of $\Pi_L(x)$.

Therefore, by showing the strict concavity of $\tilde{\Pi}_L$ (i.e., $\tilde{\Pi}''_L = \frac{\partial \frac{\Pi'_L}{f(y)g(\bar{x})}}{\partial x_L} < 0$), we will be proving that $\Pi_L(x_L, x_R)$ is strictly quasiconcave too. Hence $\tilde{\Pi}''_L$ is;

$$\frac{\partial \frac{\Pi'_L}{f(y)g(\bar{x})}}{\partial x_L} = -\frac{1}{2}\left[\frac{F(\hat{y})}{f(\hat{y})}\right]' - \frac{1}{2g(\bar{x})^2}\left\{g(\bar{x})^2 - g'(\bar{x})\left[G(\bar{x}) - \frac{\mu^\eta_R}{\mu^\eta_L + \mu^\eta_R} + \eta \frac{\mu^\eta_R \mu^\eta_L}{(\mu^\eta_L + \mu^\eta_R)^2}\right]\right\}$$

By log-concavity of $F(y)$, the term $-\left[\frac{F(\hat{y})}{2f(\hat{y})}\right]'$ is negative, so we can focus on the negativity of $H = -\left\{g(\bar{x})^2 - g'(\bar{x})\left[G(\bar{x}) - \frac{\mu^\eta_R}{\mu^\eta_L + \mu^\eta_R} + \eta \frac{\mu^\eta_R \mu^\eta_L}{(\mu^\eta_L + \mu^\eta_R)^2}\right]\right\}$ in the expression above to guarantee strict concavity of $\tilde{\Pi}_L(x)$ (and hence strict quasiconcavity of $\Pi_L(x)$). Let us consider two cases.

- If $g'(\bar{x}) \geqslant 0$, then $-g'(\bar{x})\left[\frac{\mu_R^\eta}{\mu_L^\eta+\mu_R^\eta} - \eta\frac{\mu_R^\eta\mu_L^\eta}{(\mu_L^\eta+\mu_R^\eta)^2}\right]$ is negative (strictly negative for $g'(\bar{x}) > 0$) because the term in brackets is always positive given $\eta < \bar{\eta}$ (see Lemma 4). Log-concavity of $g(x)$ implies log-concavity of $G(x)$, thus $-\left[g(\bar{x})^2 - g'(\bar{x})G(\bar{x})\right]$is negative (strictly negative for $g'(\bar{x}) = 0$). Hence $H$ is strictly negative and $\Pi_L(x)$ strictly quasiconcave.
- If $g'(\bar{x}) < 0$, suppose there exists $\hat{x}_L : G\left(\frac{\hat{x}_L+x_R}{2}\right) = \frac{\mu_R^\eta}{\mu_L^\eta+\mu_R^\eta} - \eta\frac{\mu_R^\eta\mu_L^\eta}{(\mu_L^\eta+\mu_R^\eta)^2}$. Since $G(x)$ is increasing in $x$, for $x_L > \hat{x}_L$, $H$ would be strictly negative and $\Pi_L(x)$ strictly quasiconcave. For $x_L \leqslant \hat{x}_L, G(\bar{x}) \leqslant \frac{\mu_R^\eta}{\mu_L^\eta+\mu_R^\eta} - \eta\frac{\mu_R^\eta\mu_L^\eta}{(\mu_L^\eta+\mu_R^\eta)^2}$ implies that $\frac{\partial\Pi_L(x_L,x_R)}{\partial x_L}$ is strictly positive which directly implies that $\Pi_L(x_L, x_R)$ is strictly quasiconcave for $x_L \in [0, \hat{x}_L]$.

We can proceed similarly to show that $\Pi_R(x_L, x_R)$ is also strictly quasiconcave for $x_L \leqslant x_R$ with the last inequality being strict if $x_L \neq 0$ (in that case we use that log-concavity of $g(x)$ implies that $1 - G(x)$ is log-concave and that $\Pi'_R = -\frac{g(\bar{x})}{2} < 0$ for $x_R > x_L + \bar{y}$).

**Lemma A.5.** In any equilibrium in pure strategies $x_L^* \leqslant 0$ and $x_R^* \geqslant 0$.

**Proof.** We have denoted, without loss of generality, $x_L \leqslant x_R$. For a divergent equilibrium, i.e., $x_L^* < x_R^*$, the proof follows by contradiction from Lemma A.1. That is, if $x_L^* \geqslant 0$, then any $x_R > x_L^*$ is strictly dominated. Similarly, any $x_L \leqslant x_R^*$ is strictly dominated if $x_R^* \leqslant 0$. Lemma A.2 excludes any convergent equilibrium candidate where $x_L = x_R \neq 0$.

**Lemma A.6.** In any equilibrium in pure strategies $x_R^* - x_L^* \leqslant \bar{y}$.

**Proof.** We have denoted, without loss of generality, $x_L \leqslant x_R$. Recall that for $y > \bar{y}, F(y) = 1$ and $f(y) = 0$. Thus, fixing $x_R^*, x_L < x_R^* - \bar{y}$ cannot be an equilibrium as Eq. (4) reduces to $\frac{\partial\Pi_L(x_L,x_R^*)}{\partial x_L} = \frac{g(\bar{x})}{2} > 0$ for any $x_L < x_R^* - \bar{y}$. Similarly, we can fix $x_L^*$ to see that (5) implies $\frac{\partial\Pi_R(x_L^*,x_R)}{\partial x_R} = -\frac{g(\bar{x})}{2} < 0$ for any $x_R > x_L^* + \bar{y}$.

### A.3. Proof of Propositions 1 and 2

Combining Lemma A.5 and Lemma A.6 pure strategy equilibrium candidates, where $x_L^* \leqslant x_R^*$, are restricted to:

- $x_L^* = 0 = x_R^*$ (Convergent equilibrium)
- $x_R^* - \bar{y} < x_L^* \leqslant 0 < x_R^*$ (Interior equilibrium)
- $x_R^* - x_L^* = \bar{y}$ and $x_L^* < 0 < x_R^*$ (Extremism equilibrium)
- $x_R^* - \bar{y} < x_L^* < 0 = x_R^*$

Below, we consider each of these candidates.

#### A.3.1. Convergent equilibrium, $x_L^* = 0 = x_R^*$.

By Lemma A.5, $x_L^* = 0 = x_R^*$ is the only convergent equilibrium that can exist. We look for the conditions guaranteeing that parties have no incentives to deviate from $x_L = x_R = 0$.

By Lemma A.4, when $x_{-i} = 0$, the objective functions are continuous and quasiconcave for $x_L \leqslant x_R$, so $\frac{\partial\Pi_L(0,0)}{\partial x_L} \geqslant 0$ and $\frac{\partial\Pi_R(0,0)}{\partial x_R} \leq 0$ are necessary and sufficient conditions to disregard deviations to $x_L < x_L^* = 0$ or $x_R > x_R^* = 0$. Given that $G(0) = \frac{1}{2}$, these two inequalities can be written as:

$$\frac{F(0)}{f(0)}g(0) \geqslant 2\eta\frac{\mu_L^\eta\mu_R^\eta}{(\mu_L^\eta+\mu_R^\eta)^2} + \frac{\mu_i^\eta - \mu_{-i}^\eta}{\mu_L^\eta+\mu_R^\eta}.$$

where $i = L, R$ and $-i \neq i$. Given that $\mu_R \geqslant \mu_L$, if the equation for $R$ is satisfied, it will also be so for $L$. Note that for $F(0) = 0$, the above inequality is never satisfied and hence there is no convergent equilibrium. For $F(0) > 0$ and rearranging the above expression, the convergent equilibrium exists if and only if $\frac{f(0)}{F(0)} \leqslant g(0)/\left(2\eta\frac{\mu_L^\eta\mu_R^\eta}{(\mu_L^\eta+\mu_R^\eta)^2} + \frac{\mu_R^\eta-\mu_L^\eta}{\mu_L^\eta+\mu_R^\eta}\right)$.

To conclude the proof, the symmetry of $g(x)$ guarantees that no profitable deviations exist to $x_L > x_R^* = 0$ or $x_R < x_L^* = 0$, using the same arguments as above.

Finally, note that for the symmetric cost ($\mu_L = \mu_R$) case the above condition for a convergent equilibrium simplifies to $\frac{f(0)}{F(0)} \leqslant \frac{2}{\eta}g(0)$.

#### A.3.2. Interior equilibrium, $x_R^* - \bar{y} < x_L^* \leqslant 0 < x_R^*$

By Lemma A.1 and by transitivity, we can disregard any deviation $\tilde{x}_L > x_R^*$ ($x_L^* > \tilde{x}_R$) as it will be strictly dominated by some $x_L < x_R^*$ ($x_L^* < x_R$). By Lemma A.2, we can ignore any deviation to $\tilde{x}_L = x_R^*$ or $\tilde{x}_R = x_L^*$. By Lemma A.3 and A.4, the objective functions are continuous and quasiconcave for $x_L < x_R$, so the FOCs are necessary and sufficient to guarantee that there are no profitable deviations from an interior equilibrium.

To solve the FOCs, we begin by proving that there is a unique $(\bar{x}^*, \hat{y})$ that simultaneously equalizes (4) and (5) to zero. From $\frac{\partial\Pi_L(x_L,x_R)}{\partial x_L} + \frac{\partial\Pi L_R(x_L,x_R)}{\partial x_R} = 0$ and $G(x)$ strictly increasing in $x$, we obtain the unique $\bar{x}^*$ that solves this system of equations:

$$G(\bar{x}^*) = \frac{\mu_R^\eta}{\mu_L^\eta+\mu_R^\eta} \iff \bar{x}^* = G^{-1}\left(\frac{\mu_R^\eta}{\mu_L^\eta+\mu_R^\eta}\right).$$

Plugging this solution $\bar{x}^*$ in either of the two equations, let $\hat{y}$ be implicitly defined by the solution to $\frac{f(\hat{y})}{F(\hat{y})} = \frac{g(\bar{x}^*)}{2\eta}\frac{(\mu_L^\eta+\mu_R^\eta)^2}{\mu_L^\eta\mu_R^\eta}$. Since $\frac{f(y)}{F(y)}$ is decreasing in $y$ and $g(\bar{x}^*)$ can be treated as a constant, note there is a unique polarization level $\hat{y}$ that solves the latter condition.

Using $\bar{x}^* = (x_L^* + x_R^*)/2$ and $\hat{y} = x_R^* - x_L^*$, we obtain the unique interior solution:

$$x_L^* = \bar{x}^* - \frac{\hat{y}}{2}$$

$$x_R^* = \bar{x}^* + \frac{\hat{y}}{2}$$

Next, we have to verify when the solution satisfies the conditions for an interior equilibrium $x_R^* - \bar{y} < x_L^* \leqslant 0 < x_R^*$. First, $x_R^* > 0 \iff \hat{y} > -2G^{-1}\left(\frac{\mu_R^\eta}{\mu_L^\eta+\mu_R^\eta}\right)$, which is always true. Second, $x_L^* \leqslant 0 \iff \hat{y} \geqslant 2G^{-1}\left(\frac{\mu_R^\eta}{\mu_L^\eta+\mu_R^\eta}\right) = 2\bar{x}^*$. Given the definition of $\hat{y}$ and that $\frac{f(y)}{F(y)}$ is decreasing in $y$, we can rewrite this condition as:

$$\frac{f(\hat{y})}{F(\hat{y})} = \frac{g(\bar{x}^*)}{2\eta}\frac{(\mu_L^\eta+\mu_R^\eta)^2}{\mu_L^\eta\mu_R^\eta} \leqslant \frac{f(2\bar{x}^*)}{F(2\bar{x}^*)}. \tag{7}$$

Last, $x_R^* - x_L^* < \bar{y}$ can be written $\hat{y} < \bar{y}$. Given the definition of $\hat{y}$ and that $\frac{f(y)}{F(y)}$ is decreasing in $y$, we can rewrite the last condition as:

$$\frac{f(\hat{y})}{F(\hat{y})} = \frac{g(\bar{x}^*)}{2\eta}\frac{(\mu_L^\eta+\mu_R^\eta)^2}{\mu_L^\eta\mu_R^\eta} > \frac{f(\bar{y})}{F(\bar{y})}. \tag{8}$$

Note that from (7), (8), and $\frac{f(y)}{F(y)}$ decreasing in $y$ (log-concavity), it follows that a necessary condition for the existence of an interior equilibrium is $2\bar{x}^* \leqslant \bar{y} \iff \frac{\mu_R^\eta}{\mu_L^\eta+\mu_R^\eta} \leqslant G(\frac{\bar{y}}{2})$.

Finally, note that for the symmetric cost case ($\mu_L = \mu_R$), the above necessary and sufficient conditions for an interior equilibrium simplify to $\frac{f(\bar{y})}{F(\bar{y})} < \frac{2}{\eta}g(0) < \frac{f(0)}{F(0)}$ with $x_L^* = -\frac{\bar{y}}{2}$ and $x_R^* = \frac{\bar{y}}{2}$.

*A.3.3. Extremism equilibrium, $x_R^* - \bar{y} = x_L^* < 0 < x_R^*$.*

By Lemma A.1 and by transitivity, we can disregard any deviation $\tilde{x}_L > x_R^*$ ($x_L^* > \tilde{x}_R$) as it will be strictly dominated by some $x_L < x_R^*$ ($x_L^* < x_R$). By Lemma A.2, we can ignore any deviation to $\tilde{x}_L = x_R^*$ or $\tilde{x}_R = x_L^*$.

By Lemma A.4, the objective function of party $L$ (party $R$) is continuous and quasiconcave whenever $x_L < x_R^*$ ($x_L^* \leqslant x_R$). Lemma A.3 shows that it is not differentiable at $|x_i - x_{-i}| = \bar{y}, -i \neq i, i = L, R$. Thus, we need to show that there is an inflection point at $x_i : |x_i - x_{-i}| = \bar{y}$ for each party. Then, fixing $x_R^*$, party $L$ has no incentives to deviate from an equilibrium where $x_R^* - x_L^* = \bar{y}$ if and only if i) $\lim_{x_L \to (x_R^* - \bar{y})^-} \frac{\partial \Pi_L(x_L, x_R^*)}{\partial x_L} > 0$, and ii) $\frac{\partial \Pi_L(x_R^* - \bar{y}, x_R^*)}{\partial x_L} \leqslant 0$. Similarly, fixing $x_L^*$, party $R$ has no incentives to deviate from an equilibrium where $x_R^* - x_L^* = \bar{y}$ if and only if iii) $\lim_{x_R \to (x_L^* + \bar{y})^+} \frac{\partial \Pi_R(x_L^*, x_R)}{\partial x_R} < 0$ and iv) $\frac{\partial \Pi_R(x_L^*, x_L^* + \bar{y})}{\partial x_R} \geqslant 0$. Conditions i) and iii) follow directly from the proof of Lemma A.6. By the definition of an extremism equilibrium, we can use $\bar{x}^* = x_R^* - \frac{\bar{y}}{2} = x_L^* + \frac{\bar{y}}{2}$ in expressions (4) and (5) above, so that conditions ii) and iv) can be written:

$$\frac{F(\bar{y})}{f(\bar{y})}\frac{g(\bar{x}^*)}{2} - \eta\frac{\mu_R^\eta \mu_L^\eta}{(\mu_L^\eta + \mu_R^\eta)^2} \leqslant G(\bar{x}^*) - \frac{\mu_R^\eta}{\mu_L^\eta + \mu_R^\eta} \tag{9}$$

$$\frac{F(\bar{y})}{f(\bar{y})}\frac{g(\bar{x}^*)}{2} - \eta\frac{\mu_R^\eta \mu_L^\eta}{(\mu_L^\eta + \mu_R^\eta)^2} \leqslant -G(\bar{x}^*) + \frac{\mu_R^\eta}{\mu_L^\eta + \mu_R^\eta} \tag{10}$$

Adding (9) and (10) it follows that:

$$\frac{F(\bar{y})}{f(\bar{y})}\frac{g(\bar{x}^*)}{2} \leqslant \eta\frac{\mu_R^\eta \mu_L^\eta}{(\mu_L^\eta + \mu_R^\eta)^2}$$

Using this condition in (9) and (10) we obtain:

$G(\bar{x}^*) \geqslant \frac{\mu_R^\eta}{\mu_L^\eta+\mu_R^\eta}$ and $G(\bar{x}^*) \leqslant \frac{\mu_R^\eta}{\mu_L^\eta+\mu_R^\eta}$ Rearranging the above expressions, it follows that in an extremism equilibrium $G(\bar{x}^*) = \frac{\mu_R^\eta}{\mu_L^\eta+\mu_R^\eta}$ and $\frac{g(\bar{x}^*)}{2\eta}\frac{(\mu_L^\eta+\mu_R^\eta)^2}{\mu_L^\eta \mu_R^\eta} \leqslant \frac{f(\bar{y})}{F(\bar{y})}$. Then, the equilibrium platforms in an extremism equilibrium are uniquely characterized as:

$$x_L^* = \bar{x}^* - \frac{\bar{y}}{2}$$

$$x_R^* = \bar{x}^* + \frac{\bar{y}}{2}$$

where $\bar{x}^* = G^{-1}\left(\frac{\mu_R^\eta}{\mu_L^\eta+\mu_R^\eta}\right)$. Plugging these two expressions in the conditions of Lemma A.5, a necessary condition for extremism equilibrium is $-\frac{\bar{y}}{2} \leqslant \bar{x}^* \leqslant \frac{\bar{y}}{2}$. While the first inequality necessarily holds (as $\bar{x}^* > 0$), note that the second implies $\frac{\mu_R^\eta}{\mu_L^\eta+\mu_R^\eta} \leqslant G(\frac{\bar{y}}{2})$.

Finally, note that for the symmetric cost case ($\mu_L = \mu_R$), the above necessary and sufficient conditions for an extremism equilibrium simplify to $\frac{2}{\eta}g(0) \leqslant \frac{f(\bar{y})}{F(\bar{y})}$ with $x_L^* = -\frac{\bar{y}}{2}$ and $x_R^* = \frac{\bar{y}}{2}$.

*A.3.4. Equilibrium with $x_R^* - \bar{y} < x_L^* < 0 = x_R^*$*

To complete the proof we need to show that there is no equilibrium in pure strategies such that $x_R^* - \bar{y} < x_L^* < 0 = x_R^*$. By Lemma A.3 and Lemma A.4, the objective functions are quasiconcave, continuous and differentiable for $x_R^* - \bar{y} < x_L^* < 0 = x_R^*$. Thus, necessary conditions to assure no deviations exist are $\frac{\partial \Pi_L(x_L^*, 0)}{\partial x_L} = \frac{\partial \Pi_L(x_L^*, 0)}{\partial x_R} = 0$. Proceeding as in the proof of the interior equilibrium above, we can see that $\frac{\partial \Pi_L(x_L^*, 0)}{\partial x_L} = \frac{\partial \Pi_L(x_L^*, 0)}{\partial x_R} = 0 \iff G\left(\frac{x_L^*}{2}\right) = \frac{\mu_R^\eta}{\mu_L^\eta+\mu_R^\eta} > 0$, where the later inequality follows from $\mu_L \leqslant \mu_R$ and is only possible if $x_L^* > 0$ which disregards this equilibrium candidate.

*A.4. Proof of Proposition 3 (Mixed Strategies)*

We want to show that the following is a mixed strategy equilibrium: $x_L^* = 0$ and party $R$ randomizes with equal probability over $x_R^* > 0$ and $-x_R^*$, where:

$$x_R^* = \begin{cases} \bar{y} \text{ if } \frac{F(\bar{y})}{f(\bar{y})}\frac{g(\bar{y}/2)}{2} + G(\bar{y}/2) - \frac{\mu_R^\eta}{\mu_R^\eta+\mu_L^\eta} \leqslant \eta\frac{\mu_R^\eta\mu_L^\eta}{(\mu_R^\eta+\mu_L^\eta)^2} \\ \underline{x_R} \text{ if } \frac{F(0)}{f(0)}\frac{g(0)}{2} - \frac{1}{2}\frac{\mu_R^\eta-\mu_L^\eta}{\mu_R^\eta+\mu_L^\eta} < \eta\frac{\mu_R^\eta\mu_L^\eta}{(\mu_R^\eta+\mu_L^\eta)^2} < \frac{F(\bar{y})}{f(\bar{y})}\frac{g(\bar{y}/2)}{2} + G(\bar{y}/2) - \frac{\mu_R^\eta}{\mu_R^\eta+\mu_L^\eta} \end{cases}$$

and $\underline{x_R}$ is implicitly defined by the solution to the first-order condition for party $R$:

$$\left[1 - G\left(\frac{\underline{x_R}}{2}\right)\right] = \frac{F(\underline{x_R})}{f(\underline{x_R})}\frac{g\left(\frac{\underline{x_R}}{2}\right)}{2} + \frac{\mu_L^\eta}{\mu_R^\eta + \mu_L^\eta} - \eta\frac{\mu_R^\eta\mu_L^\eta}{(\mu_R^\eta + \mu_L^\eta)^2}$$

**Proof.** To show that the above-described strategy is an equilibrium, it suffices to check that none of the two parties have incentives to deviate:

**Party *R*.** By Lemma A.4 and fixing $x_L = 0$, it follows that $\Pi_R(0, x_R)$ is quasiconcave for $0 \leqslant x_R$. Let $\underline{x_R}$ solve the FOC of party $R$. If $\underline{x_R} > 0$ then the payoff of party $R$ is strictly greater at $\underline{x_R}$ than for any other $x_R \geqslant 0$. Given $x_L = 0$, the FOC in (5) can be written:

$$\left[1 - G\left(\frac{\underline{x_R}}{2}\right)\right] = \frac{F(\underline{x_R})}{f(\underline{x_R})}\frac{g\left(\frac{\underline{x_R}}{2}\right)}{2} + \frac{\mu_L^\eta}{\mu_R^\eta + \mu_L^\eta} - \eta\frac{\mu_R^\eta\mu_L^\eta}{(\mu_R^\eta + \mu_L^\eta)^2}$$

By Lemma A.3 and Lemma A.4, the above condition can only be satisfied if: $\frac{\partial \Pi_R(0,0)}{\partial x_R} > 0$ and $\frac{\partial \Pi_R(0,\bar{y})}{\partial x_R} < 0$. Using Eq. (5), these two inequalities can be written as follows:

$$\frac{F(0)}{f(0)}\frac{g(0)}{2} - \frac{1}{2}\frac{\mu_R^\eta - \mu_L^\eta}{\mu_R^\eta + \mu_L^\eta} < \eta\frac{\mu_R^\eta\mu_L^\eta}{(\mu_R^\eta + \mu_L^\eta)^2} < \frac{F(\bar{y})}{f(\bar{y})}\frac{g(\bar{y}/2)}{2} + G(\bar{y}/2) - \frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta},$$

which are the conditions for $x_R^* = \underline{x_R}$. Similarly, the payoff of party $R$ is strictly greater at $x_R = \bar{y}$than for any other $x_R \geqslant 0$ if and only if:

$$\frac{\partial \Pi_R(0,\bar{y})}{\partial x_R} \geqslant 0 \iff \frac{F(\bar{y})}{f(\bar{y})}\frac{g(\bar{y}/2)}{2} + G(\bar{y}/2) - \frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta} \leqslant \eta\frac{\mu_R^\eta\mu_L^\eta}{(\mu_R^\eta + \mu_L^\eta)^2},$$

which is the condition for $x_R^* = \bar{y}$.

Moreover, fixing $x_L = 0$ and by the symmetry of $g(x)$, it follows that $\Pi_R(0, x_R)$ is symmetric around zero, which implies that, if any of the above solutions hold, i) no profitable deviation exists to $x_R \leqslant 0$, and ii) that party $R$ must be indifferent between $x_R^*$ and $-x_R^*$. Thus, if the above conditions hold, party $R$ has no incentives to deviate from the proposed mixed strategy.

**Party *L*.** Considering the mixed strategy of party *R*, note that for any $x_R^* > 0$ it must be that $x_L \in [-x_R^*, x_R^*]$. Any $x_L$ outside this interval can not be a best response for the same reasons we showed in Lemma A.5 that we can not have $x_L < x_R < 0$. Hence, the expected payoff for party *L* given *R*'s randomization is:

$$\bar{\Pi}_L = \frac{1}{2}\Pi_L(x_L, x_R^*) + \frac{1}{2}\Pi_L(x_L, -x_R^*)$$

where

$$\Pi_L(x_L, x_R^*) = F(x_R{}^* - x_L)G\left(\frac{x_R{}^* + x_L}{2}\right) + [1 - F(x_R{}^* - x_L)] \times \left[\frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta} - \eta\frac{\mu_R^\eta\mu_L^\eta}{(\mu_R^\eta + \mu_L^\eta)^2}\right]$$

and

$$\Pi_L(x_L, -x_R^*) = F(x_L + x_R^*)\left[1 - G\left(\frac{-x_R{}^* + x_L}{2}\right)\right] + [1 - F(x_L + x_R^*)] \times \left[\frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta} - \eta\frac{\mu_R^\eta\mu_L^\eta}{(\mu_R^\eta + \mu_L^\eta)^2}\right]$$

The first-order condition for party *L* is:

$$-\frac{1}{2}f(x_R^* - x_L)G\left(\frac{x_R^* + x_L}{2}\right) + \frac{1}{4}F(x_R^* - x_L)g\left(\frac{x_R^* + x_L}{2}\right) + \frac{1}{2}f(x_R^* - x_L)\left[\frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta} - \eta\frac{\mu_R^\eta\mu_L^\eta}{(\mu_R^\eta + \mu_L^\eta)^2}\right] + \frac{1}{2}f(x_L + x_R^*)\left[1 - G\left(\frac{-x_R^* + x_L}{2}\right)\right] - \frac{1}{4}F(x_L + x_R^*)g\left(\frac{-x_R^* + x_L}{2}\right) - \frac{1}{2}f(x_L + x_R^*)\left[\frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta} - \eta\frac{\mu_R^\eta\mu_L^\eta}{(\mu_R^\eta + \mu_L^\eta)^2}\right]$$

Given the symmetry of $g(\cdot)$, note that for $x_L = 0$ each term in the above equation cancels out. Thus, the first-order condition is satisfied for $x_L = 0$. Finally, if $\bar{\Pi}_L$ is quasiconcave, then $x_L = 0$ is a global maximum and party *L* has no incentive to deviate.

## Appendix B. Comparative statics

**Proof of Lemma (2).**

**Proof.** By Proposition (1), the interior equilibrium arises if $\eta \in \left[2g(0)\frac{F(0)}{f(0)}, 2g(0)\frac{F(\hat{y})}{f(\hat{y})}\right]$. Using the implicit function theorem we can write: $\frac{\partial\hat{y}}{\partial\eta} = \frac{1}{2g(0)\left[\frac{F(\hat{y})}{f(\hat{y})}\right]'}$. Then,

$$\frac{\partial\mu e_i^*(x_L^*, x_R^*)}{\partial\eta} = -f(\hat{y})\frac{\partial\hat{y}}{\partial\eta}\frac{\eta}{4} + \frac{1 - F(\hat{y})}{4} = -\frac{f(\hat{y})}{2g(0)\left[\frac{F(\hat{y})}{f(\hat{y})}\right]'}\frac{\eta}{4} + \frac{1 - F(\hat{y})}{4}. \tag{11}$$

Hence,

$$\frac{\partial\mu e_i^*(x_L^*, x_R^*)}{\partial\eta} \geqslant 0 \iff 1 - F(\hat{y}) \geqslant \frac{f(\hat{y})}{2g(0.5)\left[\frac{F(\hat{y})}{f(\hat{y})}\right]'}\eta \iff$$

$$\eta \leqslant \frac{1 - F(\hat{y})}{f(\hat{y})}2g(0)\left[\frac{F(\hat{y})}{f(\hat{y})}\right]' \tag{12}$$

where $\left[\frac{F(\hat{y})}{f(\hat{y})}\right]'$ is a positive number by log-concavity of $F(y)$.

**Proof of Lemma (3)**

**Proof.** By Proposition (1), the interior equilibrium arises if $\eta \in \left[2g(0)\frac{F(0)}{f(0)}, 2g(0)\frac{F(\hat{y})}{f(\hat{y})}\right]$. Then;

$$\frac{\partial\mu e_i^*(x_L^*, x_R^*)}{\partial\rho} = -\frac{\partial F(\hat{y})}{\partial\rho}\frac{\eta}{4} - f(\hat{y})\frac{\partial\hat{y}}{\partial\rho}\frac{\eta}{4}$$

and hence $\frac{\partial\mu e_i^*(x_L^*, x_R^*)}{\partial\rho} \geqslant 0$ if and only if $-\frac{\partial F(\hat{y})}{\partial\rho} \geqslant f(\hat{y})\frac{\partial\hat{y}}{\partial\rho}$.

## Appendix C. Voters' behavior

In this section, we formalize and derive two main assumptions of the model. Namely, a) the endogenous division of voters across ideological and impressionable (via semiorder lexicographic preferences or a model of salience), and b) the impressionable voters' behavior that results in a Tullock contest during the campaign stage.

In their "strict" formulation, lexicographic preferences require a tie in the first attribute to compare alternatives over a second attribute. In their "weak" formulation (or semiorder, Tversky, 1969), small differences between alternatives in the first attribute also lead to indifference in that attribute. In other words, small differences on the first attribute are disregarded (Fishburn, 1974). This is the exact intuition we presented in our model where, as polarization decreases and platforms look more alike, individuals turn their attention to campaigns instead of platforms. In particular, semiorder preferences with binary choices (i.e., two platforms in our setup) are unrestrictive in the sense of Manzini and Mariotti (2012), and the use of semiorder lexicographic preferences microfounds the aggregate behavior presented in the main text.

Assume a population of measure one, in which voters have semiorder lexicographic preferences and are heterogeneous in two dimensions. First, they draw an ideal policy $x$ from $G(x)$. Second, voters also draw a level of sensitivity, $\phi \in [0, 1]$, towards differences in the ideology space, from $F_\phi(y) = Pr(\phi \leqslant y)$, where $\phi$ is the minimal distance between the two platforms that a voter considers to be "relevant" or "distinguishable".[16] Voters with $\phi > x_R - x_L$ vote according to the dominated attribute (i.e., electoral advertising $e_i$). Voters with $\phi \leqslant x_R - x_L$ vote à la Downs.

Note that for the case where $x_L = x_R$, only voters with $\phi = 0$ are Downsian (and as the two platforms are identical, they vote for any of the two parties with equal probability). That is, if one assumes that $F(0) > 0$ then there is a mass of Downsian voters not paying attention to the campaigns even though the platforms are identical. If instead one assumes that $F(0) = 0$ when the platforms are identical, all voters vote according to campaign spending.[17]

Although $x$ is ex-post irrelevant for impressionable voters, all individuals are identified by the pair $(x, \phi)$. The above features can be represented in an analytical manner by adapting the semiorder lexicographic structure proposed by Luce (1978). Considering a voter $(x, \phi)$, we can write the voter's evaluation of party $i$ as:

$$\vartheta_{x,\phi}(i) = -|x - x_i|\Upsilon(\phi \leqslant x_R - x_L) + t^i\left(e_R, e_L, \theta^i\right)[1 - \Upsilon(\phi \leqslant x_R - x_L)] \tag{13}$$

where $\Upsilon(\phi \leqslant x_R - x_L)$is an indicator function taking value 1 when a voter is ideological, i.e., $\phi \leqslant x_R - x_L = y$, and 0 when a voter is

[16] In terms of the experimental literature in human perception (or psychophysics), $\phi$ can be interpreted as the just-noticeable difference.

[17] Note that assuming $F(0) = 0$ or $F(0) > 0$ affects the results; for instance, $F(0) > 0$ is required for the existence of a convergent equilibrium. In the main text, we solve the model for the general case $F(0) \geqslant 0$ and highlight this distinction.

impressionable, i.e., $\phi > x_R - x_L = y$, as in the main text. Without loss of generality $-|x - x_i|$ is the utility derived from voting for party $i \in \{L, R\}$ according to ideology; and $t^i\left(e_R, e_L, \theta^i\right)$ is the impressionable voter's utility from voting for party $i \in \{L, R\}$ according to advertisements.

Let $\theta^i$ in $t^i\left(e_R, e_L, \theta^i\right)$ be a random variable (from the candidates' points of view) that captures how much of a party's advertisement "leaks" to voters in the following way $t^i\left(e_R, e_L, \theta^i\right) = log(e_i^\eta) + \theta^i$. Similarly to multinomial applications in industrial organization (Nevo, 2000) and political economy (Casas et al., 2016), assume $\theta^i$ to be drawn *i.i.d.* from a type *I* extreme-value distribution. As in McFadden (1974), the probability $Pr(t^L > t^R)$ – which in our case is the probability that an impressionable voter votes for $L$ – becomes the contest success function $Pr(t^L > t^R) = \frac{e_L^\eta}{e_L^\eta + e_R^\eta}$. With a continuum of voters, we interpret this probability as the share of impressionable voters that vote for $L$. On the same lines, Jia (2008) and Jia et al. (2013) show that for $t^i\left(e_R, e_L, \theta^i\right) = e_i\theta^i$, if $\theta^L$ and $\theta^R$ follow independent inverse exponential distributions with parameter $\eta > 0$, the probability also becomes $Pr(t^L > t^R) = \frac{e_L^\eta}{e_L^\eta + e_R^\eta}$.[18]

For simplicity, let $F_\phi(y)$ be written as $F(y)$, which can be used to denote the proportion of ideological voters given a level of polarization $y$. Platform preferences and ideology sensitivity are assumed to be independent. Consequently, for a given pair of policy platforms $x_L$ and $x_R$, the votes of ideological and impressionable voters can be independently aggregated (integrating over $\phi$). By integrating over $\theta_L$ and $\theta_R$ under the above distributional assumptions, the impressionable vote share of party $i$ is $S^i_{Imp} = \frac{e_i^\eta}{e_L^\eta + e_R^\eta}$. For ideological voters, by integrating over $x$ one immediately obtains that for party $L, S^L_{Idl} = G(\bar{x})$ if $x_L \neq x_R$ and $S^L_{Idl} = \frac{1}{2}$ if $x_L = x_R$. By taking into account the above, the parties' vote shares can be immediately written as in Eq. (1).

**Salience and attention**

Our model can also be interpreted as an extreme case of Bordalo et al. (2012), Bordalo et al. (2013a), Bordalo et al. (2013b) and Bordalo et al. (2015), in which "salient thinkers" give more weight to attributes that exhibit greater heterogeneity in the available choice set.

In the papers on salience and attention, the salient attribute is the one in which the differences are more pronounced. The less salient attribute receives less weight. For instance, in Bordalo et al. (2015) they look at price and quality: if quality is the salient attribute, the utility from consuming good $k$ is $q_k - \omega p_k$ with $\omega$ exogenously determined and in $[0, 1]$. Instead of comparing attributes, we compare the platform differentiation with a baseline level of polarization, $\phi$, which is exogenously drawn from $F_\phi(y)$. Thus, for $\phi$ smaller than the equilibrium polarization, voters take platforms as the salient attribute. In particular, for $\phi \leqslant y, \omega = 0$. And for $\phi \geqslant y, 1 - \omega = 0$. Thus, one can consider the rank-based weighting salience function proposed by Bordalo et al. (2012),Bordalo et al. (2013b) and Bordalo et al. (2015), where:

$$\vartheta_{x,\phi}(i) = -(|x - x_i|)\omega + t^i\left(e_L, e_R, \theta^i\right)(1 - \omega)$$

The heterogeneity of "salient thinkers" is given by $F_\phi(y)$, and everything else is exactly as in the previous section.

[18] The assumption that party shocks are identical to all individuals is made without loss of generality, as long as they are independent of ideology.

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