USE OF SCATTERED AIRBORNE ACOUSTIC WAVE-FIELD TO RECOVER PROFILE AND STATISTICS OF SURFACE OF SHALLOW WATER FLOW

A. Krynkin Department of Mechanical Engineering, The University of Sheffield, Sheffield, UK
G. Dolcetti Department of Mechanical Engineering, The University of Sheffield, Sheffield, UK
K.V. Horoshenkov Department of Mechanical Engineering, The University of Sheffield, Sheffield, UK
Timothy Van Renterghem Faculty Engineering and Architecture, Ghent University, UK

1 INTRODUCTION

The surface of a shallow water flow is never flat. There has been strong evidence which suggests that the roughness pattern of the water surface of this type of flow is linked unambiguously to the underlying hydraulic processes. This area of research is new and there have been little or no systematic efforts to understand this link. The main interest of this paper is to investigate the interaction of airborne acoustic waves with dynamic roughness patterns and predict the key statistical and spectral characteristics of the water surface. It is proposed to use boundary integral equations approximated by Kirchhoff method[1] to recover profile and statistics of randomly rough surface that is described by random process in time. In particular this numerical approach allows studying acoustic scattering from inhomogeneous roughness that supports multiple scales. Stationary phase method is suggested here to approximate the numerical solution and recover the profile of the surface. This method however is limited to the particular scale of the surface roughness but can be potentially used to retrieve non-stationary process observed on the dynamic surface in time.

2 FORWARD SCATTERING FROM THE WATER FREE-SURFACE

2.1 Scattering from a rough surface

Consider semi-infinite space with the boundary composed of rough surface S which mean-plane S_0 coincides with Oxy plane in Cartesian coordinate system. In order to simplify the numerical calculations it is assumed that surface is cylindrical and acoustic source is the directional line source which directivity pattern $A(\boldsymbol{r})$ is specified later in this paper. This makes problem two dimensional. The main axis of the far-field directivity pattern is inclined at angle ψ to Ox axis. The coordinates of source and receiver are defined by (x_1, z_1) and (x_2, z_2) respectively.

In this paper the scattering from rough surface is approximated by the reflection from the plane tangential to the surface. The approximation is based on Kirchhoff method and principles of geometrical optics[1], and is valid if local curvature radius α of the rough surface is much greater than the acoustic wavelength $\lambda = 2\pi/k$. For the diffraction on a sphere, this condition can be stated in the following form

$$\sin\psi \gg \frac{1}{\sqrt[3]{k\alpha}} \quad , \qquad (1)$$

where k is wave number of the acoustic wave and α is a radius of the sphere locally inscribed in rough surface. The condition in eq. (1) can be relaxed[2] to

$$\sin\psi > \frac{1}{\sqrt[3]{k\alpha}} \quad , \qquad (2)$$

so that the Kirchhoff approach remains accurate for the incident angles far from the low grazing angles. In this paper condition (2) is used in the numerical simulation to define surface.

Assuming that the distances from the source R_1 and receiver R_2 to a fixed point on the rough surface are much greater than the acoustic wavelength and using the Kirchhoff method, the scattered acoustic pressure can be approximated by[3]

$$p(x_2, z_2) = -\frac{\mathrm{i}}{2\pi k}\int_{S_0} \frac{A(x)}{\sqrt{R_1 R_2}} \exp[\mathrm{i}k(R_1 + R_2) - \mathrm{i}q_z\zeta(x)] \left[q_z - q\frac{\partial\zeta(x)}{\partial x}\right] dx, \qquad (2)$$

where $\zeta(x)$ is surface elevation and

$$q_z = k\left(\frac{z_1}{R_1} + \frac{z_2}{R_2}\right), \qquad (3)$$
$$q = -k\left(\frac{x_1 - x}{R_1} + \frac{x_2 - x}{R_2}\right),$$
$$R_1 = \sqrt{(x - x_1)^2 + z_1^2},$$
$$R_2 = \sqrt{(x - x_2)^2 + z_2^2}.$$

Assuming that surface is smooth $\boldsymbol{\nabla}\zeta(\boldsymbol{r}) \ll 1$ equation (2) can be simplified to[3]

$$p(x_2, z_2) = -\frac{\mathrm{i}}{2\pi k}\int_{S_0} \frac{A(x)}{\sqrt{R_1 R_2}} \exp[\mathrm{i}k(R_1 + R_2) - \mathrm{i}q_z\zeta(x)]\, q_z dx, \qquad (4)$$

If the profile of surface $\zeta(x)$ is known than integral in equation (4) can be solved numerically. However the surface comes into the integral as unknown and it is acoustic pressure in the left hand side which is known. This states inversion problem where variables $\zeta(x)$ has yet to be recovered.

2.2 Discretization of integral equation

To be able to invert equation (4) it is proposed to use well known numerical approach to solve boundary integral equations. For this purpose the integral in (4) is discretized over the surface S_0 with the set of $\Delta x_m = x_m - x_{m-1}, m = 1..M$ elements, so that scattered acoustic pressure at point M can be approximated by

$$p(x_2, z_2) = -\frac{\mathrm{i}}{2\pi k}\sum_m \frac{A(x_m)}{\sqrt{R_{1,m} R_{2,m}}} \exp\left[\mathrm{i}k\left(R_{1,m} + R_{2,m}\right) - \mathrm{i}q_{z,m}\zeta(x_m)\right] q_{z,m}\, \Delta x_m, \qquad (5)$$

where all terms with index m are defined at points $x_m, m = 1..M$. Equation (5) can be rewritten in form of scalar product of two vectors

$$p(x_2, z_2) = \boldsymbol{V}.\mathbf{E}, \qquad (6)$$

with

$$\boldsymbol{V} = \left[-\frac{\mathrm{i}}{2\pi k}\frac{A(x_m)}{\sqrt{R_{1,m} R_{2,m}}} \exp\left[\mathrm{i}k\left(R_{1,m} + R_{2,m}\right)\right] q_{z,m}\Delta x_m\right]_{m=1..M}, \qquad (7)$$
$$\boldsymbol{E} = \left[\exp\left[-\mathrm{i}q_{z,m}\zeta(x_m)\right]\right]_{m=1..M}.$$

Equation (6) can be converted into the matrix form for the array of receivers defined by the coordinates $\left(x_{2,n}, z_{2,n}\right), n = 1..N$. In order to isolate unknown elevation of the rough surface $\zeta(x)$ at the point x_m it is assumed that variability of q_z with respect to the positon of the receiver is negligible and $q_{z,mn}\zeta(x_m) \sim 1$. This gives

$$\boldsymbol{P}^T = \boldsymbol{M}\,\mathbf{E}^{\mathrm{T}}, \qquad (8)$$

with

$$\boldsymbol{M} = \left[-\frac{\mathrm{i}}{2\pi k}\frac{A(x_{mn})}{\sqrt{R_{1,mn} R_{2,mn}}} \exp\left[\mathrm{i}k\left(R_{1,mn} + R_{2,mn}\right)\right] q_{z,mn}\Delta x_m\right]_{m=1..M, n=1..N},$$

and $\mathbf{E}$ is given by equation (7) with q_z defined by the mean value taken over all receiver positions. The number M of unknown points on the surface is assumed to be greater than the number of receivers N (M>N). This leads to undetermined linear system of equations which solution can be found using the Moore-Penrose (MP) matrix inverse method[4]. This yields

$$\boldsymbol{E}^T = \widetilde{\boldsymbol{M}}\,\mathbf{P}^{\mathrm{T}}, \qquad (9)$$

where $\widetilde{\boldsymbol{M}}$ is defined by the MP matrix inverse. The argument of each element of vector $\boldsymbol{E}$ provides information about the surface elevation. To be able to retrieve this argument from matrix equation (9) we apply natural logarithm which branch cut is assumed to be along $q_{z,m}\zeta(x_m) = \pi$. This gives

$$\boldsymbol{\zeta}^T = -\Im\left[\mathrm{Ln}(\widetilde{\boldsymbol{M}}\, \mathbf{P}^{\mathrm{T}})\right], \tag{9}$$

where vector $\boldsymbol{\zeta} = \left[q_{z,m}\zeta(x_m)\right]_{m=1..M}$ and operator Ln is applied to each element of complex valued vector $\widetilde{\boldsymbol{M}}\, \mathbf{P}^{\mathrm{T}}$.

2.3 Stationary phase approximation

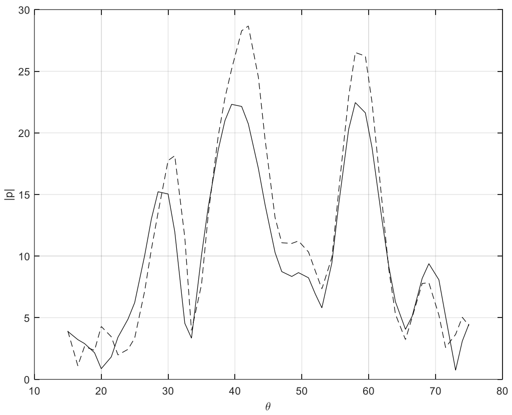
3 INVERSION OF SYHNTHETIC DATA

To illustrate the application of methods developed in the previous sections of this paper it is proposed to data synthesized with the help of Finite Difference Time Domain (FDTD) method. The FDTD results were computed for source with directivity pattern defined by

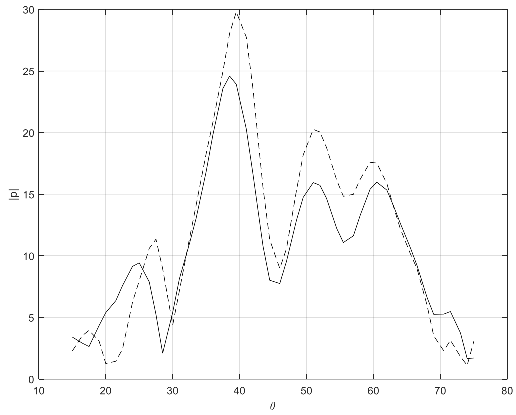
$$A(\phi) = \frac{J_1[ka\sin(\phi)]}{ka\sin(\phi)}.$$

where $a = 0.02\ m$ is the radius of the aperture. The frequency of the source is $f = 43\ kHz$.

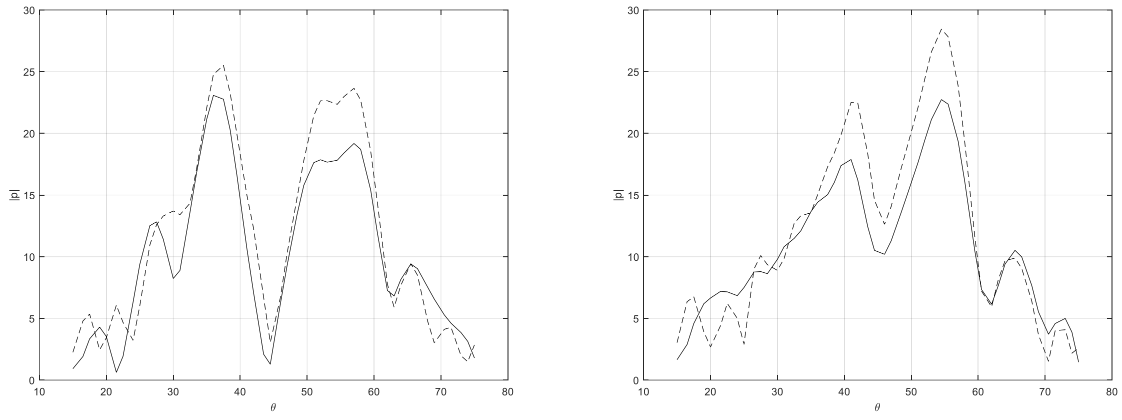
In Figure 1 the results obtained with FDTD method are compared against the scattered acoustic wave-field computed with Kirchhoff integral (4). It is observed that the approximation with the tangential plane gives adequate prediction of the acoustic waves scattered by the rough surface which satisfies the conditions (2).



(a)



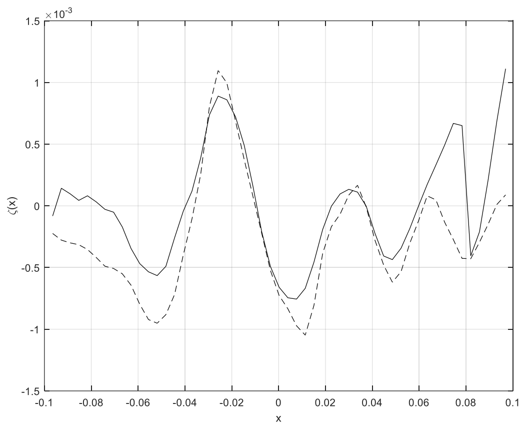
(b)



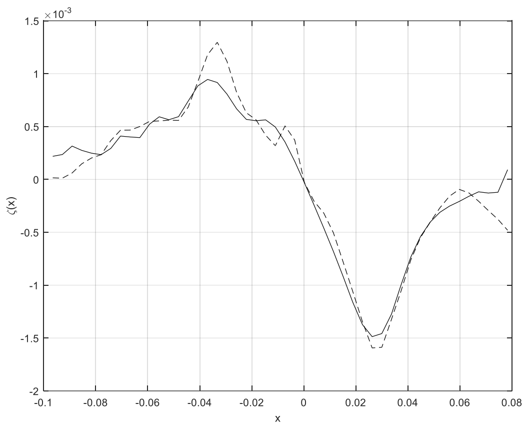
(c) (d)

Figure 1. Scattered acoustic wave-field obtained with Kirchhoff integral (3) (solid line) and FDTD method (dashed line). (a) flow conditions 1, (b) flow conditions 2, (c) flow conditions 3, (d) flow conditions 4.

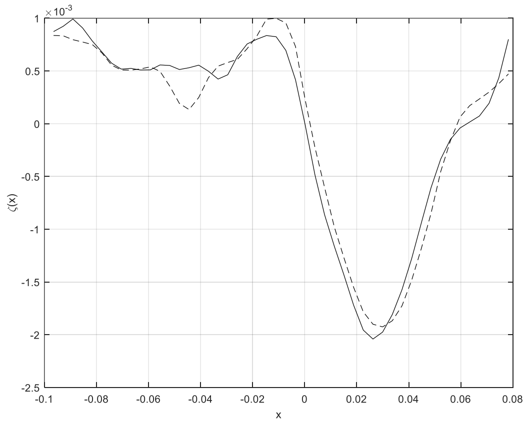
Figure 2 demonstrates the application of the PM inverse method to the equation (8) in order to retrieve the profile of rough surface.



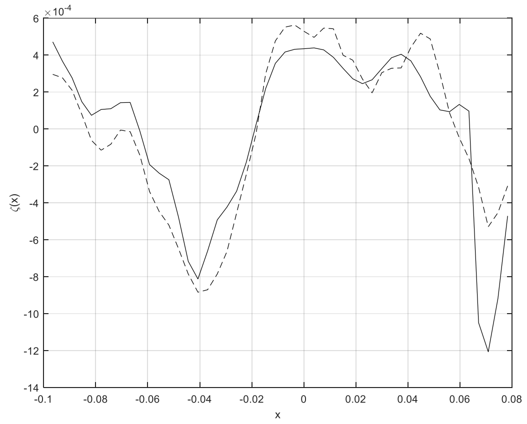
(a)



(b)



(c)



(d)

Figure 2. Exact surface (dashed line) against retrieved surface with PM inverse method. (a) flow conditions 1, (b) flow conditions 2, (c) flow conditions 3, (d) flow conditions 4.

4 CONCLUSIONS

In this paper we demonstrate two inversion methods based on approximations of the boundary integral equation.

5 REFERENCES

1. F.G. Bass and I.M. Fuks. Wave Scattering from Statistically Rough Surfaces. Oxford, Pergamon Press. (1979).
2. E. I. Thorsos, The validity of the Kirchhoff approximation for rough surface scattering using a Gaussian roughness spectrum. The Journal of the Acoustical Society of America, 83 (1), 78-92, (1988).
3. A. Krynkin, K. V. Horoshenkov, A. Nichols and S. J. Tait, A non-invasive acoustical method to measure the mean roughness height of the free surface of a turbulent shallow water flow, Review of Scientific Instruments, 85 (11), 114902, (2014).
4. S.L. Campbell and C.D. Meyer. Generalized Inverses of Linear Transformations. Dover Publications. (1991)